

Outage Performance Analysis of Full-Duplex Relay-Assisted Device-to-Device Systems in Uplink Cellular Networks

Shuping Dang, *Student Member, IEEE*, Gaojie Chen, *Member, IEEE*, and Justin P. Coon, *Senior Member, IEEE*

Abstract—This paper proposes a full-duplex cooperative device-to-device (D2D) communication system, where the relay employed can receive and transmit signals simultaneously. We adopt such a system to assist with D2D transmission. We first derive the conditional cumulative distribution function (CDF) and the probability density function (PDF) of a series of channel parameters when the interference to the base station (BS) is taken into consideration and power control is applied at the D2D transmitter and the relay node. Then, we obtain an exact expression for the outage probability as an integral and also a closed-form expression for a special case, which can be used as a good approximation to the general case when residual self-interference (SI) is small. Additionally, we also investigate the power allocation problem between the source and the relay and formulate a sub-optimal allocation problem, which we prove to be quasi-concave. Our analysis is verified by Monte Carlo simulations and a number of important features of full-duplex cooperative D2D communications can be thereby revealed.

Index Terms—Cooperative device-to-device (D2D) communications, full-duplex system, outage performance, power allocation.

I. INTRODUCTION

Underlay device-to-device (D2D) communication coexisting with traditional cellular communication has been a frequent topic of research in both academia and industry for years, because of its high power efficiency, high spectral efficiency and low transmission delay [1]–[3]. Meanwhile, cooperative communication has also gained interest, since it can effectively enhance network reliability and performance [4]. Recently, researchers have tried to combine the merits of both communication systems and have proposed the concept of cooperative D2D communication [5]. However, most recent works only treat the combination of D2D communication with half-duplex relays, which will degrade the system throughput by a fraction due to the use of multiple orthogonal time or frequency slots for one complete transmission. On the other hand, full-duplex relaying is capable of overcoming this shortcoming, but at the cost of producing residual self-interference (SI) [6]. A simplified full-duplex D2D network model is proposed in [7]. The effects of residual SI are analyzed and a numerical optimization of the total transmit power in this full-duplex D2D network model is carried out without presenting analytical results in [8]. Power allocation problems in full-duplex D2D networks are analyzed in [9]. However, all aforementioned works have not considered cooperative relaying between the D2D transmitter and receiver, which restricts the reliability

and effectiveness of D2D communication. A cooperative D2D network with a half-duplex relay is analyzed in [10], which exhibits undesirable performance characteristics. A two-pair case in which a transmitter in one pair can assist as a full-duplex relay for the other pair when idle is analyzed in [11]. However, the model considered in that paper is oversimplified and the interference between two pairs is not considered. The most relevant network model related to full-duplex cooperative D2D communication is proposed and analyzed in [12]. However, that paper makes a number of assumptions, e.g. the authors suppose that a relay node is able to transmit the separated signals to two destinations simultaneously by different powers without considering mutual interference. These assumptions can be viewed as impractical in some circumstances.

To provide a comprehensive study of a full-duplex cooperative D2D system, we analyze the outage performance of a novel full-duplex cooperative D2D communication system in which a relay is able to assist the D2D pair only. Our analysis is verified by Monte Carlo simulations. The contributions of this paper can be summarized as follows:

- We propose a full-duplex relay-assisted D2D communication system, in which power control and the interference from the CUE to the relay and the D2D receiver are considered.
- We obtain a single integral expression for the end-to-end outage probability of the proposed system as well as a closed-form approximation to the outage probability when residual SI is small.
- We formulate a suboptimal power allocation method that is easily implemented due to its quasi-concave nature.

The rest of this paper is organized as follows. In Section II, we present the system model. Then, we analyze the outage performance and power allocation problem of the proposed system in Section III and verify the analysis by simulations in Section IV. Finally, the paper is concluded in Section V.

II. SYSTEM MODEL

The model of the proposed full-duplex cooperative D2D system is given in Fig. 1, where one base station (BS), one cellular user equipment (CUE)¹, one D2D user equipment (DUE) transmitter, one DUE receiver and one full-duplex relay² are considered. They are denoted as B , C , S , D and R , respectively and organized in the set $\Theta = \{B, C, S, D, R\}$. Therefore, $\forall i \neq j$ and $i, j \in \Theta^3$, the channel gain denoted as G_{ij} is assumed to be independent and non-identically exponentially distributed with average channel gain μ_{ij} ⁴. Hence, the PDF and CDF corresponding to each channel gain are

$$f_{G_{ij}}(g) = e^{-g/\mu_{ij}}/\mu_{ij} \Leftrightarrow F_{G_{ij}}(g) = 1 - e^{-g/\mu_{ij}}. \quad (1)$$

¹This one-CUE assumption is validated by the scenario in which multiple CUEs are assigned resource blocks in modern cellular systems, and thus we would only expect to receive interference from at most one user in a cell [13].

²The full-duplex relay is capable of transmitting and receiving simultaneously, while other nodes are assumed to be half-duplex in this paper.

³An exception is given by $i = j = R$, and G_{RR} is employed to denote the instantaneous loop channel gain leading to residual SI.

⁴Here all channels are assumed to be bidirectional and thus $G_{ij} = G_{ji}$ and $\mu_{ij} = \mu_{ji}$ [14]. Besides, after SI elimination processing, the loop channel gain can also be regarded as exponentially distributed [15].

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The authors are with the Department of Engineering Science, University of Oxford, Parks Road, Oxford, UK, OX1 3PJ; tel: +44 (0)1865 283 393, (e-mail: {shuping.dang, gaojie.chen, justin.coon}@eng.ox.ac.uk).

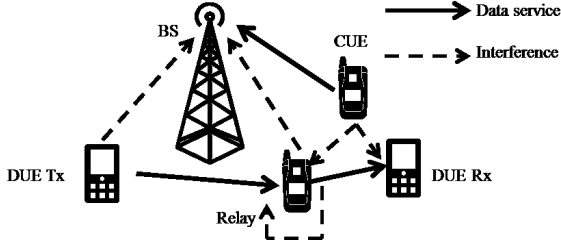


Fig. 1. Network model for the proposed full-duplex cooperative D2D system, containing one BS, one CUE, one relay, one DUE transmitter (source) and one DUE receiver (destination).

Besides, we assume that D2D transmissions occupy the uplink channel and the channel state information (CSI) G_{SB} and G_{RB} are perfectly known at the DUE transmitter and the relay respectively in order to implement power control⁵. As a result, all interference can be classified into three categories, which are the interference from the CUE to the DUE receiver and the relay, the interference from the DUE and the relay to the BS and the residual SI due to adopting full-duplex forwarding protocol. In order to mitigate the interference from the DUE and the relay to the BS in this underlay system, a power control strategy is adopted. Consequently, the transmit power at the DUE transmitter and the relay should be constrained by

$$P_S = \min\left(\frac{\alpha\eta}{G_{SB}}, \bar{P}_S\right) \quad \text{and} \quad P_R = \min\left(\frac{(1-\alpha)\eta}{G_{RB}}, \bar{P}_R\right), \quad (2)$$

where η is a predefined interference threshold at the BS; \bar{P}_S and \bar{P}_R are the maximum transmit power of the DUE transmitter and the relay due to their hardware design specifications and are fixed; $\alpha \in (0, 1)$ is the power allocation factor used to coordinate P_S and P_R , so that the relation infra can be satisfied:

$$G_{SB}P_S + G_{RB}P_R \leq \alpha\eta + (1-\alpha)\eta = \eta. \quad (3)$$

Considering an interference-limited environment (i.e., assuming receiver noise as negligible [17]), the instantaneous signal-to-interference ratios (SIR) from the source to the relay and from the relay to the destination are

$$\Gamma_{SR} = \frac{G_{SR}P_S}{G_{CR}P_C + G_{RR}P_R} \quad \text{and} \quad \Gamma_{RD} = \frac{G_{RD}P_R}{G_{CD}P_C}, \quad (4)$$

where P_C is the transmit power of the CUE. Hence, by adopting the decode-and-forward (DF) relaying protocol, the equivalent end-to-end instantaneous SIR can be expressed by⁶

$$\Gamma_{SRD} = \min(\Gamma_{SR}, \Gamma_{RD}). \quad (5)$$

From a link capacity viewpoint, we define the outage probability for such a two-hop system by

$$P_{out}(s) = \mathbb{P}\{\Gamma_{SRD} < 2^T - 1\}, \quad (6)$$

⁵This can be achieved by feeding back CSI from the BS to the DUE transmitter, e.g. by a pilot signals from the BS [16].

⁶Here, we assume that the direct transmission link between source and destination does not exist due to deep fading. Besides, AF relaying can also be considered here. Because of the limit of length, we only analyze DF relaying.

where T is a predefined outage threshold and we organize $s = 2^T - 1$ for convenience; $\mathbb{P}\{\cdot\}$ denotes the probability of the event enclosed.

Similarly, we can also obtain the outage probability for half-duplex cooperative D2D communications in (7), which is shown at the top of the next page, as a benchmark for comparison purposes. In (7), we denote $\xi = 2^{2T} - 1$ for convenience, so that the outage performances of both full-duplex and half-duplex systems can be compared fairly given the same T .

III. PERFORMANCE ANALYSIS

A. Exact outage performance analysis

As observed in (4), the random instantaneous channel gain G_{RB} is a common term in both equations and thus will result in correlation when analyzing the minimum function in (5). To temporarily eliminate this correlative effect, we condition on $P_R = p$, so that we can first analyze the outage probability at the first and second hop $\mathbb{P}\{\Gamma_{SR} < s|p\}$ and $\mathbb{P}\{\Gamma_{RD} < s|p\}$, independently, and thus analyze the distribution of the minimum function in (5). Now, let us take a close look at the denominator of Γ_{SR} as given in (4). For brevity, we set $Z = G_{CR}P_C + G_{RR}p$ when $\mu_{RRP} \neq \mu_{CR}P_C$, where the CDF of z given $P_R = p$ can be derived to be

$$F_Z(z|p) = \frac{\mu_{RRP}\left(1 - e^{-\frac{z}{\mu_{RRP}}}\right) - \mu_{CR}P_C\left(1 - e^{-\frac{z}{\mu_{CR}P_C}}\right)}{\mu_{RRP} - \mu_{CR}P_C}. \quad (8)$$

A special case is obtained when $\mu_{RRP} = \mu_{CR}P_C = \Upsilon$, in which case we can obtain the CDF:

$$F_Z(z|p) = 1 - (z + \Upsilon)e^{-\frac{z}{\Upsilon}}/\Upsilon. \quad (9)$$

Hence, the PDF of z given $P_R = p$ is given by

$$f_Z(z|p) = \begin{cases} \frac{e^{-\frac{z}{\mu_{RRP}}} - e^{-\frac{z}{\mu_{CR}P_C}}}{\mu_{RRP} - \mu_{CR}P_C} & , \mu_{RRP} \neq \mu_{CR}P_C \\ ze^{-\frac{z}{\Upsilon}}/\Upsilon^2 & , \mu_{RRP} = \mu_{CR}P_C \end{cases}. \quad (10)$$

Therefore, the conditional CDF of Γ_{SR} of the case when $G_{SB} < \alpha\eta/\bar{P}_S$, is given in (11) at the top of the next page.

Similarly, for the second case when $G_{SB} > \alpha\eta/\bar{P}_S$, we can first obtain the distribution of $W = \frac{\alpha\eta G_{SR}}{G_{SB}}$ to be

$$F_W(w) = e^{-\frac{\alpha\eta}{\mu_{SB}\bar{P}_S}} \left(1 - \frac{\alpha\eta\mu_{SR}e^{-\frac{w}{\mu_{SR}\bar{P}_S}}}{\alpha\eta\mu_{SR} + \mu_{SB}w}\right). \quad (12)$$

Hence, the CDF of Γ_{SR} when $G_{SB} > \alpha\eta/\bar{P}_S$ is given by

$$F_{C2}(t|p) = \begin{cases} e^{-\frac{\alpha\eta}{\mu_{SB}\bar{P}_S}} + \frac{\alpha\eta\mu_{SR}\Phi(t)}{\mu_{SB}(\mu_{RRP} - \mu_{CR}P_C)t}, & \mu_{RRP} \neq \mu_{CR}P_C \\ e^{-\frac{\alpha\eta}{\mu_{SB}\bar{P}_S}} - \frac{\alpha\eta\mu_{SR}\bar{P}_S}{\mu_{SB}(\mu_{SR}\bar{P}_S + \Upsilon t)\Upsilon t} + \Omega(t), & \mu_{RRP} = \mu_{CR}P_C \end{cases} \quad (13)$$

where

$$\Phi(t) := e^{\frac{\alpha\eta\mu_{SR}}{\mu_{RRP}\mu_{SB}P_C t}} \text{Ei}\left(-\frac{\alpha\eta(\mu_{SR}\bar{P}_S + \mu_{RRP}t)}{\mu_{RRP}\mu_{SB}\bar{P}_S P_C t}\right) - e^{\frac{\alpha\eta\mu_{SR}}{\mu_{CR}\mu_{SB}P_C t}} \text{Ei}\left(-\frac{\alpha\eta(\mu_{SR}\bar{P}_S + \mu_{CR}P_C t)}{\mu_{CR}\mu_{SB}\bar{P}_S P_C t}\right), \quad (14)$$

$$\Omega(t) := \left(\frac{\alpha\eta\mu_{SR}}{\mu_{SB}\Upsilon t}\right)^2 e^{\frac{\alpha\eta\mu_{SR}}{\mu_{SB}\Upsilon t}} \text{Ei}\left(\frac{\alpha\eta(\mu_{SR}\bar{P}_S + \Upsilon t)}{\mu_{SB}\bar{P}_S \Upsilon t}\right) \quad (15)$$

$$P_{half-out}(\xi) = \mathbb{P} \left\{ \min (G_{SR} \min (\alpha\eta/G_{SB}, \bar{P}_S)/(G_{CR}P_C), G_{RD} \min ((1-\alpha)\eta/G_{RB}, \bar{P}_R)/(G_{CD}P_C)) < \xi \right\} \quad (7)$$

$$F_{C1}(t|p) = \begin{cases} \frac{\left(1 - e^{-\frac{\alpha\eta}{\mu_{SB}\bar{P}_S}}\right)t[\mu_{CR}\mu_{SR}P_C\bar{P}_S + \mu_{RR}p(\mu_{SR}\bar{P}_S + \mu_{CR}P_Ct)]}{\left(1 - e^{-\frac{\mu_{SR}\bar{P}_S + \mu_{RR}pt}{\mu_{SB}\bar{P}_S}}\right)\Upsilon t(2\mu_{SR}\bar{P}_S + \Upsilon t)/(\mu_{SR}\bar{P}_S + \Upsilon t)^2} & , \mu_{RR}p \neq \mu_{CR}P_C \\ \left(1 - e^{-\frac{\mu_{SR}\bar{P}_S + \mu_{RR}pt}{\mu_{SB}\bar{P}_S}}\right)\Upsilon t(2\mu_{SR}\bar{P}_S + \Upsilon t)/(\mu_{SR}\bar{P}_S + \Upsilon t)^2 & , \mu_{RR}p = \mu_{CR}P_C \end{cases} \quad (11)$$

and $\text{Ei}(\cdot)$ is the exponential integral function defined as $\text{Ei}(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt$. Finally, $P_{out1}(s|p) = \mathbb{P} \{\Gamma_{SR} < s|p\}$ is determined by

$$P_{out1}(s|p) = \mathbb{P} \{\Gamma_{SR} < s|p\} = F_{C1}(s|p) + F_{C2}(s|p). \quad (16)$$

For the second hop, we similarly derive $P_{out2}(s|p) = \mathbb{P} \{\Gamma_{RD} < s|p\}$ to be

$$P_{out2}(s|p) = \mathbb{P} \{\Gamma_{RD} < s|p\} = \frac{\mu_{CD}P_C s}{\mu_{RD}p + \mu_{CD}P_C s}. \quad (17)$$

Because the outage event in a two-hop system is the union of the outage event in each hop, $P_{out}(s|p)$ is given by [18]

$$P_{out}(s|p) = 1 - (1 - P_{out1}(s|p))(1 - P_{out2}(s|p)). \quad (18)$$

Subsequently, we should remove the condition on $P_R = p$ in (18). The PDF of p can be obtained as

$$f_P(s) = \frac{(1-\alpha)\eta e^{-\frac{(1-\alpha)\eta}{\mu_{RB}s}} H(\bar{P}_R - s)}{\mu_{RB}s^2} + \delta(s - \bar{P}_R) \left(1 - e^{-\frac{(1-\alpha)\eta}{\mu_{RB}s}}\right), \quad (19)$$

where $H(\cdot)$ and $\delta(\cdot)$ denote the Heaviside step function and the Dirac delta function, respectively. Hence, the unconditional outage probability independent of p is given by

$$P_{out}(s) = \int_0^{\infty} P_{out}(s|p) f_P(p) dp. \quad (20)$$

To the best of the authors' knowledge, there is no closed-form expression for (20). To understand the property of the proposed system better, we can approximate the outage probability under some special conditions and obtain a closed-form approximate expression for the approximated outage probability. This is detailed in the following subsection.

B. Outage performance approximation

Considering the state-of-the-art progress in SI elimination, we can focus on a cooperative D2D communication system in which SI can be mitigated to a noise level and thus becomes negligible [16], [19]–[21]. Therefore, by simplifying Γ_{SR} in (4), the outage probabilities in the first and second hop can be obtained from (16), (17) and (19) in closed-form expressions as given in (21) and (22) at the top of the next page. Then, we can obtain the approximated outage probability along the entire transmission link by

$$\tilde{P}_{out}(s) = \tilde{P}_{out1}(s) + \tilde{P}_{out2}(s) - \tilde{P}_{out1}(s)\tilde{P}_{out2}(s). \quad (23)$$

This is an accurate approximation of (20) when μ_{RR} and/or \bar{P}_R is small⁷. Moreover, it can also be proved that the asymptotic relation between $P_{out}(s)$ and μ_{RR} is given by $P_{out}(s) = \tilde{P}_{out}(s) + O(\mu_{RR})$ for $\mu_{RR} \simeq 0$, which indicates $P_{out}(s)$ decreases linearly (to first order) with μ_{RR} .

C. Optimization of the power allocation factor α

There exists a trade-off of α in the end-to-end outage performance, which is related to the instantaneous channel gains and power allocation between source and relay. Therefore, there should be an optimal α which is capable of minimizing the outage probability. The optimization for minimizing the outage probability is equivalent to maximizing the average end-to-end SIR, i.e. the average of (5), $\mathbb{E} \{\Gamma_{SRD}\}$. This equivalence has been proved in [23] and applied in [24]. However, due to the mathematical intractability of the integral in (20), it is also impossible to obtain a close-form expression for $\mathbb{E} \{\Gamma_{SRD}\}$. Hence, we can loosen the optimization condition and obtain a suboptimal solution to α . We formulate the problem as

$$\max_{\alpha} \left\{ \min \left(\frac{\mu_{SR} \min (\alpha\eta/\mu_{SB}, \bar{P}_S)}{\mu_{CR}P_C + \mu_{RR} \min ((1-\alpha)\eta/\mu_{RB}, \bar{P}_R)}, \frac{\mu_{RD} \min ((1-\alpha)\eta/\mu_{RB}, \bar{P}_R)}{\mu_{CD}P_C} \right) \right\} \quad (24)$$

$$\text{s.t. } 0 < \alpha < 1.$$

It turns out that this problem is quasi-concave (see the appendix for a proof). Since the formulated problem is quasi-concave, it can be solved efficiently using standard techniques to find out the sub-optimal α^* (e.g. CVX in MATLAB).

IV. NUMERICAL RESULTS

To verify our analysis presented in the last section, we carried out four Monte Carlo simulations. In all simulations, we let $\bar{P}_S = \bar{P}_R = P_C = 1$ (normalized), $\mu_{SB} = \mu_{RB} = 10$ dB and $\mu_{CR} = \mu_{CD} = 2$ dB. In the first simulation, we also assumed $\mu_{RR} = 5$ dB, $s = 1$ and $\alpha = 0.5$. For different $\eta \in \{1, 2, 8\}$, we let $\mu_{RD} = \mu_{SR}$ and varied μ_{SR} to verify (20) and (23). As shown in Fig. 2, the theoretical results match the simulation results and this validates the correctness of (20). Also, the approximation gets accurate when decreasing η .

Meanwhile, the simulation results regarding the effects of μ_{RR} on the relation between α and outage probability are presented in Fig. 3. Here we assumed $\mu_{RD} = \mu_{SR} = 30$ dB,

⁷It can also be noticed that if we replace s in this expression by ξ , this expression becomes the exact outage probability of half-duplex cooperative D2D communication systems [22].

$$\tilde{P}_{out1}(s) = \frac{\left(1 - e^{-\frac{\alpha\eta}{\mu_{SB}\bar{P}_S}}\right) \mu_{CR}P_Cs}{(\mu_{SR}\bar{P}_S + \mu_{CR}P_Cs)} + e^{-\frac{\alpha\eta}{\mu_{SB}\bar{P}_S}} + \frac{\alpha\eta\mu_{SR}}{\mu_{SB}\mu_{CR}P_Cs} e^{\frac{\alpha\eta\mu_{SR}}{\mu_{CR}\mu_{SB}\bar{P}_C}} \text{Ei}\left(-\frac{\alpha\eta(\mu_{SR}\bar{P}_S + \mu_{CR}P_Cs)}{\mu_{CR}\mu_{SB}\bar{P}_C}\right) \quad (21)$$

$$\tilde{P}_{out2}(s) = \frac{\left(1 - e^{-\frac{(1-\alpha)\eta}{\mu_{RB}\bar{P}_R}}\right) \mu_{CD}P_Cs}{(\mu_{RD}\bar{P}_R + \mu_{CD}P_Cs)} + e^{-\frac{(1-\alpha)\eta}{\mu_{RB}\bar{P}_R}} + \frac{(1-\alpha)\eta\mu_{RD}}{\mu_{RB}\mu_{CD}P_Cs} e^{\frac{(1-\alpha)\eta\mu_{RD}}{\mu_{CD}\mu_{RB}\bar{P}_C}} \text{Ei}\left(-\frac{(1-\alpha)\eta(\mu_{RD}\bar{P}_R + \mu_{CD}P_Cs)}{\mu_{CD}\mu_{RB}\bar{P}_C}\right) \quad (22)$$

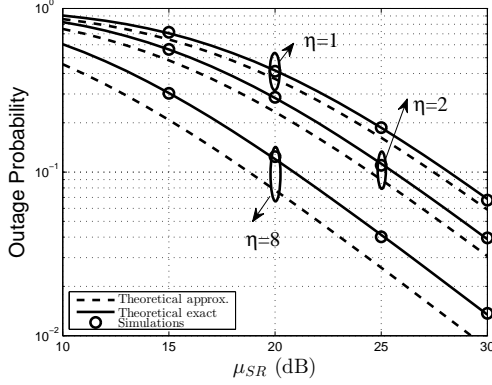


Fig. 2. Outage performance vs. the channel average gain $\mu_{SR} = \mu_{RD}$, when $\eta \in \{1, 2, 8\}$, $\mu_{RR} = 5$ dB, $s = 1$ and $\alpha = 0.5$.

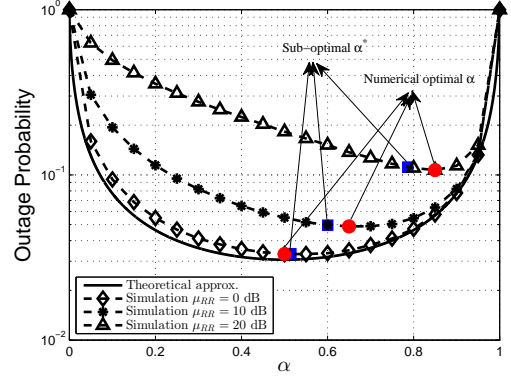


Fig. 3. Outage performance vs. the power allocation factor α , when $\mu_{RR} \in \{0, 10, 20\}$ dB, $\mu_{SR} = \mu_{RD} = 30$ dB, $s = 1$ and $\eta = 2$.

$s = 1$ and $\eta = 2$. From this figure, it is clear that the theoretical approximation derived at $\mu_{RR} = 0$ can lower bound the outage probability when $\mu_{RR} > 0$. As highlighted by the blue square and red circle, the α^* generated by our proposed sub-optimal method is close to the optimal values, and this sub-optimal value can be used as a useful reference for power allocation between source and relay. From Fig. 3, it is also clear that the optimal α will be larger than 0.5 as long as $\mu_{RR} > 0$, which aligns with our expectation. This indicates the link between the source and the relay is affected by the residual SI and more transmit power should be allocated to the source than the relay. Furthermore, it is worth noting that the optimal α is also affected by the average channel gains in the first and second hops, i.e. μ_{SR} and μ_{RD} . We simulated the relation among α , μ_{SR} and μ_{RD} and plot the results in Fig. 4. Again, these results conform to our analysis that the transmit terminal regarding the weaker hop should be allocated more power.

Also, due to the trade-off between half-duplex and full-duplex communication, it is meaningful to simulate the relation between the outage performance of our proposed system and μ_{RR} , given the performance of half-duplex systems as a benchmark. Again, we assumed $\mu_{RD} = \mu_{SR} = 30$ dB, $\eta = 2$ and $\alpha = 0.5$. For $s \in \{0.2, 1\}$, the outage performance for each forwarding protocol is illustrated in Fig. 5. It is obvious that the priority of the full-duplex protocol in cooperative D2D systems is dependent on μ_{RR} . The full-duplex mode will be preferable only when μ_{RR} is less than approximately 14 and 11 dB for both cases, respectively. Hence, without a satisfactory SI elimination technology, it is not technically feasible to implement full-duplex cooperative D2D systems.

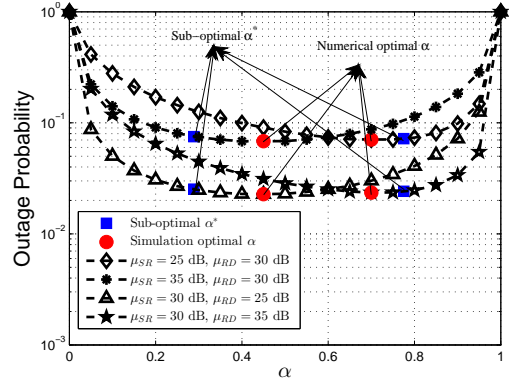


Fig. 4. Outage performance vs. the power allocation factor α , when $\mu_{SR} \in \{25, 30, 35\}$ dB, $\mu_{RD} \in \{25, 30, 35\}$ dB, $\mu_{RR} = 5$ dB, $s = 1$ and $\eta = 2$.

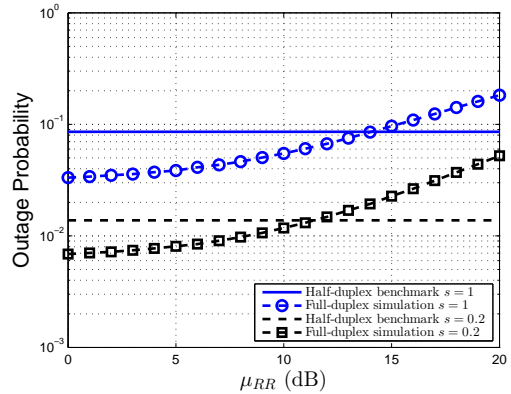


Fig. 5. Outage performance vs. the residual SI feedback channel gain μ_{RR} , when $s \in \{0.2, 1\}$, $\mu_{SR} = \mu_{RD} = 30$ dB, $\eta = 2$ and $\alpha = 0.5$.

V. CONCLUSION

In this paper, we proposed a full-duplex cooperative D2D communication system, in which a relay assists the transmission of the D2D pair. To analyze the system, we obtained a single integral expression for the outage probability and also a closed-form expression for a special case, which can be used as a good approximation to the general case when residual SI is small. Also, we formulated a sub-optimal problem regarding power allocation and proved it to be quasi-concave. Our analysis has also been verified by simulation results. By these results, a number of important features of full-duplex cooperative D2D communications are thereby revealed.

APPENDIX: PROOF OF QUASI-CONCAVITY

To prove the quasi-concavity of the cost function in (24), we first need to prove the lemma given below:

Lemma 1: For bounded $x \in (x_{\min}, x_{\max})$, if $g(x)$ is a bounded, continuous and monotonically increasing function and $h(x)$ is a bounded, continuous and monotonically decreasing function, $f(x) = \min(g(x), h(x))$ will be quasi-concave.

Proof: Because of the monotonicity of $g(x)$ and $h(x)$, we can have three cases:

- Case 1: $g(x) > h(x)$, $\forall x \in (x_{\min}, x_{\max})$
- Case 2: $g(x) < h(x)$, $\forall x \in (x_{\min}, x_{\max})$
- Case 3: $g(x) < h(x)$, $\forall x \in (x_{\min}, c)$ and $g(x) > h(x)$, $\forall x \in (c, x_{\max})$, where c is the cross point of $g(x)$ and $h(x)$, where $g(c) = h(c)$.

In the first and second case, it is obvious that $f(x) = g(x)$ and $f(x) = h(x)$, and both cases satisfy

$$\begin{aligned} \forall x_1, x_2 \in (x_{\min}, x_{\max}) \text{ and } \lambda \in (0, 1), \\ \exists f(\lambda x_1 + (1 - \lambda)x_2) \geq \min(f(x_1), f(x_2)). \end{aligned} \quad (25)$$

Therefore, $f(x)$ is quasi-concave for Case 1 and Case 2 [25]. For Case 3, without losing generality, we assume $x_{\min} < x_1 < x_2 < x_{\max}$. When $x_1, x_2 \in (x_{\min}, c)$ and $x_1, x_2 \in (c, x_{\max})$, we can perform the similar method as for Cases 1 and 2 to prove its quasi-concavity. Now, let us discuss the range of λ . For $0 < \lambda < \frac{x_2 - c}{x_2 - x_1}$, we have

$$\begin{aligned} f(\lambda x_1 + (1 - \lambda)x_2) &= g(\lambda x_1 + (1 - \lambda)x_2) \\ &\geq g(x_1) = f(x_1) \geq \min(f(x_1), f(x_2)); \end{aligned} \quad (26)$$

for $\frac{x_2 - c}{x_2 - x_1} < \lambda < 1$, we can similarly have

$$\begin{aligned} f(\lambda x_1 + (1 - \lambda)x_2) &= h(\lambda x_1 + (1 - \lambda)x_2) \\ &\geq h(x_2) = f(x_2) \geq \min(f(x_1), f(x_2)). \end{aligned} \quad (27)$$

Consequently, the quasi-concavity of $f(x)$ can be proved for Case 3. Summarizing the proofs for all three cases shown above, we prove the proposed lemma. ■

Therefore, let

$$g(\alpha) = \frac{\mu_{SR} \min(\alpha\eta/\mu_{SB}, \bar{P}_S)}{\mu_{CR}P_C + \mu_{RR} \min((1 - \alpha)\eta/\mu_{RB}, \bar{P}_R)} \quad (28)$$

and

$$h(\alpha) = \mu_{RD} \min((1 - \alpha)\eta/\mu_{RB}, \bar{P}_R)/(\mu_{CD}P_C). \quad (29)$$

We can observe that $g(\alpha)$ and $h(\alpha)$ are monotonically increasing and decreasing with α . Hence, $f(\alpha) = \min(g(\alpha), h(\alpha))$, i.e. the cost function in (24) is quasi-concave.

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