

A note on Dolby and Gull on radar time and the twin “paradox”

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Abstract

Recently a suggestion was made that standard textbook representations of hypersurfaces of simultaneity for the traveling twin in the twin paradox’ are incorrect. This suggestion is false: the textbooks are in agreement with a correct understanding of the relativity of simultaneity.

I. INTRODUCTION

It is common wisdom that there is nothing paradoxical about the twin “paradox,” and that appropriate attention to the unambiguous differences between the proper times of the traveling twin (whom we shall call Barbara) and the stay-at-home twin (Alex) resolves any hint of paradox. Indeed, the reader might well feel exasperated that work is still being done on this fallacious argument.

However, in a recent article,¹ Dolby and Gull argue that the standard resolutions of the twin paradox incorrectly answer the question of how the traveling twin should assign times of occurrence to distant events, that is, how Barbara should represent her hypersurfaces of simultaneity. They claim that the proper application of radar time to the accelerated twin allows us to sort out an unambiguous time of happening to each event from Barbara’s perspective.

I have no complaint with their mathematics (although see Sec. IV). My query is with the point of their exercise. For correct attention to both the concept of simultaneity and to what it means for an observer to assign a “time of happening” to an event shows that there is no problem with the standard textbook resolutions.

II. DOLBY AND GULL’S CONCERN

The standard textbook resolutions of the twin paradox (in the instantaneous turnaround case) claim that the hypersurfaces of simultaneity for Barbara are as shown in Fig. 1.^{2,3} Dolby and Gull¹ point out that when Barbara instantaneously turns around, the points G and H in Fig. 1 are regarded as simultaneous by Barbara. They acknowledge that this problem disappears if we make the situation more physically realistic, and adopt a turnaround of extended duration, as in Fig. 2. (This move still leaves untouched the conceptual problem in the original, instantaneous case.) They also note that moving to an extended turnaround “cannot resolve the more serious problem . . . that occurs to Barbara’s left. Here her hypersurfaces of simultaneity are overlapping, and she assigns three times to every event!”¹ They make no further comment, presuming that the exclamation mark is sufficient to convey their thinking about the standard treatment, that is, the fact of assigning three times to every event is an absurdity, and the textbook resolutions must be wrong because they entail the

absurd claim that there can be overlapping hypersurfaces of simultaneity.

III. DEFUSING THE CONCERN

Is the claim that more than one time of happening can be assigned to a single event such a patent absurdity as Dolby and Gull seem to regard it? I will now give a reinterpretation of Fig. 1, which will show that it is far from absurd.

Let us call the point of turnaround T . Consider an observer, O_1 , who travels inertially until point T along the same path as Barbara, and whose worldline then terminates. Consider another observer, O_2 , who springs into existence at T , and shadows Barbara thereafter. The hypersurfaces of simultaneity for O_1 and O_2 are as in Fig. 1, and these hypersurfaces overlap without problem or absurdity. It is a central fact of relativistic theories that observers who are moving differently are in different frames of reference, and hence their partitions of events into simultaneity equivalence classes can differ radically. Thus, it cannot be the mere fact that the hypersurfaces of simultaneity overlap that worries Dolby and Gull. It must be that it is possible that *one observer*, who happens to move differently at different times, can assign the same event to different equivalence classes under simultaneity, again at different times.

But why should this possibility be a problem? An absurd situation would be if at one and the same time, an observer assigned the same event to two different hypersurfaces of simultaneity, because that would (at the least) involve the absurdity that the observer was in two distinct frames of reference at one and the same time, among other problems. But there is no absurdity to the claim that, at different times, while moving differently and hence in different frames, Barbara assigns a single event to two different simultaneity classes. Hence, there was no need to tinker with the original textbook presentations of her hypersurfaces of simultaneity: they are perfectly adequate to capture Barbara's judgements of simultaneity.

Perhaps Dolby and Gull are worried that a single, persisting observer cannot (without considerable mental problems) assign one and the same event to different simultaneity classes, even if the observer never does that at a single time. This concern seems unwarranted, for these differing assignments are what we should expect in a relativistic scenario. Maybe Barbara is a determinedly pre-relativistic individual, in which case she may have some conceptual problems with the different assignments, perhaps because she still operates

with a hidden Newtonian assumption that there is one universal time. But that cannot be at the root of Dolby and Gull's concern, as they make no such assumption, see, however, Sec. V.

Dolby and Gull go on to say that:¹ “[I]f Barbara's hypersurfaces of simultaneity at a certain time depend so sensitively on her instantaneous velocity as these diagrams suggest, then she would be forced to conclude that the distant planets swept backwards and forwards in time whenever she went dancing!” Perhaps if we can understand this remark, we will get at the root of their concerns.

As far as I can tell, their concern here is that, as Barbara's instantaneous velocity changes from moment to moment, she will be forced to conclude that some events that are in her current subjective future, that is, lie within the future light cone of some event on her current hypersurface of simultaneity, were, at some point on her past worldline, judged to be in the past (lying within the past light cone of some event on her past (then current) hypersurface of simultaneity). Of course, this situation is no absurdity: it has long been clear that the pre-scientific concepts of past and future do not mesh perfectly with their relativized versions. I cannot see anything more to Dolby and Gull's concern, other than that it is motivated by a pre-theoretical intuition about distant assignments of past and future to events, an intuition that should by now be seen as very doubtful in a relativistic universe.

One residual concern remains: What happens at point T ? In Fig. 1, it does seem that at T , Barbara has two conflicting simultaneity assignments, which is related to Dolby and Gull's first concern. The resolution is simple: we need to assign to her at most one instantaneous frame of reference. The obvious one to choose is that at the instant T , she is at rest; her frame of reference then yields a hypersurface of simultaneity that is orthogonal to Alex's worldline, running horizontally across the page. This assignment has the virtue that it shows Fig. 1 to be the limit of Fig. 2 as the duration of acceleration decreases, although we might think that at this instantaneous point, there is no sense to be made of the observer's frame of reference, and hence we cannot assign a hypersurface of simultaneity. This latter option has problems of its own, but in principle, either choice serves to resolve the residual concern.

IV. THE CONVENTIONALITY OF SIMULTANEITY

On the basis of these considerations, I see no force to the motivating remarks that Dolby and Gull provide, and hence I wonder that their mathematical analysis was needed.

Setting that issue aside, some interesting details emerge when we consider their positive proposal. They begin by defining the *radar time* of an event e as follows. Let t_1 be the (proper) time at which the observer sends a signal to e , and let t_2 be the (proper) time at which the observer receives a return signal from e . The radar time $\tau(e)$ of e is defined as

$$\tau(e) = t_1 + \frac{1}{2}(t_2 - t_1). \quad (1)$$

A hypersurface of simultaneity for an observer at e σ_e is a set of events with the same radar time for that observer as e ($\sigma_e = \{x : \tau(x) = \tau(e)\}$); it is obviously an equivalence class. This same relation of simultaneity is, as Dolby and Gull are well aware, Einstein's standard convention for simultaneity.⁴

There exists a large body of work on the status of this definition of simultaneity.⁵⁻⁷ Although they make passing reference to some of this work,⁶ Dolby and Gull do not engage it thoroughly. If they had, they would have noticed that although their radar time definition of simultaneity is an acceptable definition, it is not the only available option.

Debs and Redhead⁶ maintain that any definition of radar time is acceptable if it is compatible with the following:

$$\tau_\epsilon(e) = t_1 + \epsilon(t_2 - t_1). \quad (0 < \epsilon < 1) \quad (2)$$

They assume that any particular choice of value of ϵ in Eq. (2) is *conventional*, that is, not fixed by the physical facts, but rather by our conventional decision to use the term “simultaneous” to pick out an equivalence class under τ_ϵ . No contradiction with any physical fact is possible for any of these relations defined by different ϵ values, because only proper time has an “objective status in special relativity.”⁸ If Debs and Redhead were correct, then no special significance would attach to assignments of distant simultaneity; they are arbitrary and hence without physical importance. If that is true, then the purported conflict over assignments which interested Dolby and Gull is of even smaller significance than I discussed previously.

Of course, others have rushed to defend the Einsteinian convention, proving its unique adequacy if we set certain conditions on a plausible candidate for simultaneity.^{5,10} But these

further conditions have been disputed too. Although I have no wish to defend it here, it seems that conventionalism about simultaneity remains an open possibility.^{6,7,9} If that possibility turns out to be true, then Dolby and Gull have done some excellent work to define a potential candidate simultaneity relation, but one that loses its importance once we see that any candidate relation, within wide bounds, will do. In particular, the textbook presentations of the hypersurfaces of simultaneity are perfectly good candidates for us to adopt as our convention.

V. FURTHER EXTERNAL CONSTRAINTS

It seems that Dolby and Gull's motivation for proposing a unique foliation of spacetime (a partition of events into hypersurfaces of simultaneity) for any given observer does not rest on considerations intrinsic to special relativity.^{12,13} Rather, they have a particular application in mind, for which they regard unique foliation as crucial. The particular application is to fermionic particle creation in relativistic quantum field theory. Obviously if this application requires unique foliation, then my remarks about conventionality of simultaneity will be misapplied, because we are now in effect imposing additional physical constraints that suffice for uniqueness. Moreover, their particular proposal requires consistency of foliation for a given observer. Hence, the textbook overlapping hypersurfaces at different times also will be incorrect.

One small concern about this approach is that we have strictly gone beyond the conceptual content of special relativity, and therefore it is inappropriate to represent the Einsteinian convention as the only one possible for special relativity. Rather, the Einstein proposal is the only one that is pragmatically appropriate to use when applying special relativity, given the further constraints imposed by our beliefs concerning what the actual world is like. But strictly speaking, simultaneity may yet be conventional in standard special relativity without external constraints.

A bigger concern is with the particular application that Dolby and Gull have in mind. The crucial point for them is that if we cannot establish a unique foliation for a given observer, certain features of the article distribution for that observer will change. They argue that it is odd that a mere change in velocity (or a merely conventional choice of simultaneity relation) could result in the appearance or disappearance of particles.

It is now clear that there is no relativistic quantum theory of localizable particles.^{14,15} The most plausible and standard response to this result is to argue that because different observers observe different particle distributions, we should be *anti-realists* (or conventionalists) about particle distribution. If this standard plausible response were accepted, it would undercut Dolby and Gull’s motivation for wanting a unique foliation of the spacetime to ensure reliable and constant particle distributions.

In summary, if we are irrealists about particle distributions, we have no motivation to demand unique foliations for given observers, and we have every reason to be irrealist about particle distributions, given the impossibility of giving an adequate characterization of local particles within relativistic quantum field theory. Moreover the latter “*itself* shows how the ‘illusion’ of localizable particles can arise, and how talk about localizable particles can be a useful fiction.”¹⁴ Insofar as we are prepared to deploy a particle number observable, we do so from the perspective of an instantaneous rest frame of an observer, and the foliation consequent upon that choice of rest frame, without demanding that this foliation be constant for a given observer no matter her motion.

VI. CONCLUSION

Dolby and Gull have given an elegant and precise example of how to apply the standard Einsteinian convention to assignments of distant simultaneity in the twin paradox case. It very usefully illustrates that the standard convention can be applied alike to accelerating and inertial observers, contrary to some of the thinking in the field.¹¹

However, their example was not necessary because there is no intuitive paradox or problem with the standard textbook presentations, as shown in Sec. III. Also, many people regard the definition of simultaneity as a conventional matter in any case, as discussed in Sec. IV. Hence no acceptable choice for this convention can be criticized as mistaken on physical grounds, but only on grounds of usefulness for our purposes. In addition, even if we undermine the conventionality response by adducing some further actual physical fact, that would not show the conceptual commitments of special relativity to be different. Moreover, I suspect that any such additional physical fact will not conflict with the standard representations of hypersurfaces of simultaneity (see Sec. V). Given these observations, I believe that the standard textbook suggestion concerning what should be regarded as the hypersurfaces

of simultaneity is at least as successful as Dolby and Gull’s more complicated alternative proposal.¹

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¹ C. E. Dolby and S. F. Gull, “On radar time and the twin ‘paradox’,” *Am. J. Phys.* **69** (12), 1257–1261 (2001).

² R. D’Inverno, *Introducing Einstein’s Relativity* (Oxford University Press, Oxford, 1992).

³ L. Sartori, *Understanding Relativity: a simplified approach to Einstein’s theories* (University of California Press, Berkeley, 1996).

⁴ A. Einstein, “On the electrodynamics of moving bodies,” *Ann. Phys. (Leipzig)* **17**, 891–921 (1905).

⁵ D. Malament, “Causal theories of time and the conventionality of simultaneity,” *Noûs* **11** (3), 293–300 (1977).

⁶ T. A. Debs and M. L. G. Redhead, “The twin ‘paradox’ and the conventionality of simultaneity,” *Am. J. Phys.* **64** (4), 384–392 (1996).

⁷ S. Sarkar and J. Stachel, “Did Malament prove the non-conventionality of simultaneity in the special theory of relativity?,” *Philos. Sci.* **66** (2), 208–220 (1999).

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⁹ D. Giulini, “Uniqueness of Simultaneity,” *British J. Philos. Sci.* **52**, 651–670 (2001).

¹⁰ A. A. Robb, *A Theory of Time and Space* (Cambridge University Press, Cambridge, 1914).

¹¹ H. Bondi, *Assumption and Myth in Physical Theory* (Cambridge University Press, Cambridge, 1967).

¹² S. F. Gull, personal communication, 10 January 2005.

- ¹³ C. E. Dolby and S. F. Gull, “State-space-based approach to quantum field theory for arbitrary observers in electromagnetic backgrounds,” *Ann. Phys.* **293** (2), 189–214 (2001), especially Sec. 4.
- ¹⁴ H. Halvorson and R. Clifton, “No place for particles in relativistic quantum theories?,” *Philos. Sci.* **69** (1), 1–28 (2002).
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Figure Captions

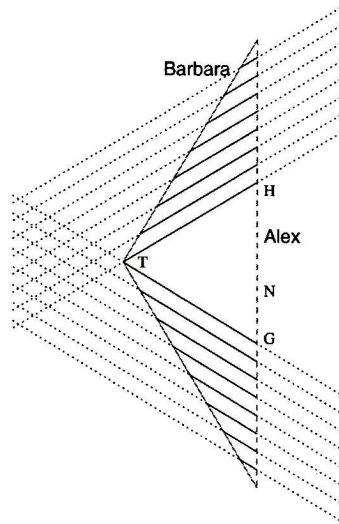


FIG. 1: A typical textbook illustration of the hypersurfaces of simultaneity of the traveling twin Barbara in the twin paradox. (Adapted from Fig. 1 in Ref. 1.)

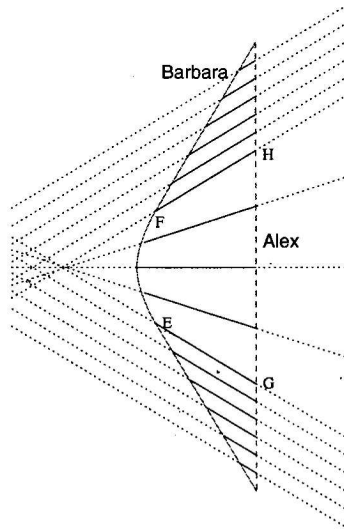


FIG. 2: Another typical textbook illustration in which Barbara's hypersurfaces of simultaneity sweep from EG to FH during the duration of turnaround (acceleration). (Adapted from Fig. 2 in Ref. 1.)