

The Effects of Regime-Switching Uncertainty on Irreversible Investment Decisions

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Abstract: This paper focuses on how uncertainty about the sustainability of reform policies could affect the irreversible investment decisions of a tradable sector firm. Demand, costs and productivity are stochastic and are correlated with real exchange rate trends for the firm producing a tradable good. The firm's investment decisions are influenced by the variability of these business conditions processes, as well as by uncertainty about the timing of a large change in reform policies that dramatically alters the trend growth rates of these processes. Findings are that a firm that expects an unfavourable regime change is more hesitant to invest, reduces investment more in the face of ongoing variability of business conditions, and has a smaller investment response to favourable current trends. Ghana provides one example of a country where in the early stages of the Economic Recovery Program, entrepreneurs were uncertain about whether reform policies would be maintained.

1. Introduction

This paper analyses firm investment behaviour when there is an expectation that the government may not maintain reform policies. The focus is not on *why* reforms may not be credible, but rather on the *effect* that an expectation of reform reversal has on the irreversible investment decision of a firm. A recent literature has suggested that the impact of uncertainty on investment may be large. The reason is the combination of irreversibility and the option to wait: most investment expenditures are at least partly sunk costs that cannot be recovered if market conditions turn out to be worse than expected; and firms can delay committing resources until new information arrives. This paper extends that literature to consider a risk neutral firm facing both ongoing demand and cost uncertainty, as well as uncertainty about the *timing* of a regime change that dramatically alters the trend growth rates of the demand and cost processes.

Ghana provides one example of a country where the expectation of reform reversal may have delayed investment in the early stages of the reform. Beginning in 1983, the Economic Recovery Program's liberalizations and fiscal and exchange rate reforms contributed to achieving a large depreciation of the real exchange rate (RER). Together with other structural reforms and improvement of the infrastructure, the RER depreciation was expected to improve the profitability of investment in tradable sectors. There was, however, considerable uncertainty about the sustainability of the fiscal and exchange rate reforms in the early phase. In addition, the government's ambivalent attitude to the private sector raised questions about its true commitment to reforms beneficial to private entrepreneurs. Also contributing to the uncertainty was the memory of a previous attempt at liberalization in Ghana that had been abandoned and ended with a coup.

The impact of possible reform reversal on a tradable sector firm is modelled in a very simple way: the firm must consider that if reform collapses, then a business conditions indicator will change from a favourable to an unfavourable trend. The path followed by the business conditions indicator maps directly into the path of the marginal revenue product of capital, or returns to capital. A continuous time stochastic model is presented to analyse the irreversible investment decisions of firms facing both ongoing uncertainty, as well as uncertainty about the *timing* of a regime change that dramatically alters the trend growth rates of the business conditions indicator. The firm has to worry about *future* business conditions when making its current investment decision because investment is irreversible. This means that the firm could get stuck with "too much" capital should business conditions turn unfavourable in the future. The objective of the paper is to determine how expectations of a regime change affect investment. The method is to examine the difference between the investment decision of a firm that takes into account the fact that business conditions may switch from a positive to a negative trend at some uncertain future date (regime-switching uncertainty), and a hypothetical firm that acts myopically. The reason for comparing to a hypothetical myopic firm is to provide a benchmark for investment behaviour when there is no expectation of a regime change.

The firm's business conditions indicator depends on demand, real wage costs and productivity. For a tradable sector firm, this indicator, denoted Z_{2t} , will be correlated with RER trends. A favourable trend growth rate of Z_{2t} will be correlated with a depreciating RER, as well as productivity-enhancing liberalizations. Thus, when firms expect that the government will not persist with reform policies, they expect the RER to revert to an appreciating trend, as well as a return to price and quantity controls. In this case, firms expect the indicator Z_{2t} will switch to an unfavourable trend growth rate.

The model will be solved to determine the investment trigger points for two types of firms. The firm that behaves myopically assumes that Z_{2t} will continue indefinitely on its favourable trend growth rate. This myopic firm still cares about the future, however, since Z_{2t} follows a geometric Brownian motion, so that the variance increases with the time horizon. The other type of firm takes into account that the reform may collapse and the business conditions indicator Z_{2t} will switch from

a favourable to an unfavourable trend. Since the firm is uncertain about the timing of the switch, the arrival date of this event will be modelled as a Poisson process.

Because of the dependence on the business conditions indicator, the firm's return to capital also follows a geometric Brownian motion with drift. This stochastic process is the continuous time equivalent of a random walk. The process is nonstationary; meaning roughly that its statistical properties are not constant over long periods of time. Instead, the expected value can grow without bound, and the variance increases with the time horizon.

The approach taken in this paper contrasts with a set up in which before the reform the return to capital is a particular and deterministic low value, the reform increases the return to a new higher level, but there is uncertainty about a reform collapse that would imply the return to capital revert to the pre-reform level. In contrast, the idea here is that even without uncertainty about a reform collapse, the return to capital follow a nonstationary, stochastic process due to underlying shocks to demand, costs and productivity. Uncertainty about the timing of the regime switch implies an expectation that the trend rate around which this stochastic process fluctuates may jump to a new lower level.

After solving the model the following questions will be addressed: (1) Is the trigger point higher for the firm that expects the switch? That is, is the firm that expects the collapse of the reform more hesitant to invest? (2) Are the investment decisions of a firm expecting a regime switch more or less sensitive to ongoing uncertainty about business conditions, relative to the myopic firm? (3) Do firms care about how bad the business conditions trend is after a regime switch? (4) Do favourable trends during a reform period encourage investment even if the reform is not expected to last? (5) How is the investment decision affected by the expected time until the regime switch?

This paper contributes to the investment under uncertainty literature by solving the firm's problem when business conditions are modelled as a stochastic process that combines continuous diffusion with a discrete jump. Although the finance literature has considered asset pricing with mixed jump-diffusion processes (Trippi *et al.* 1992; Jones 1984; Mason and Bhattacharya 1981), the literature on irreversible capital investment has focused on continuous Brownian motion processes. A recent exception is Hassett and Metcalf (1994) who model an investment tax credit as following a Poisson process which switches between a low and high level.

The paper is organized as follows. Section 2 explains the reasons for choosing the particular model specification; Section 3 describes the model and solves for the investment trigger point for a myopic firm; Section 4 presents analytic results in the solution of the investment trigger problem under regime-switching uncertainty; Section 5 discusses numerical results for a central case and the effects of changes in the parameters; and Section 6 discusses possible extensions of the framework and concludes.

2. Choice of Model Specification

There are numerous models of investment under uncertainty. Many of the recent models specify investment expenditures as involving some type of sunk cost, and an economic environment with ongoing uncertainty where information arrives over time. Since it is possible to wait and invest later, the problems involve the choice of when to invest.

The model that will be described below has the following features: (1) there are first-order or linear costs of adjustment in changing capital; (2) investment is irreversible; i.e., the cumulative gross investment process is restricted to have non-negative increments; (3) the firm makes marginal investment decisions and could be viewed as solving a sequence of decisions about when to install each marginal unit of capital; and (4) the reduced form operating profits function results from constant elasticity demand and Cobb–Douglas production; thus, different cases of market structure

and returns to scale can be studied.

The choice of model specification involving these particular features was driven by two requirements. First, for the dynamic problem to be interesting, there must be some type of sunk or irreversible nature to the investment expenditure. Second, to solve the problem when there is regime-switching uncertainty, it must be possible to derive a closed form solution for a problem in which the drift rate is expected to remain constant, such as in the myopic case.¹

The reason for considering a problem where the firm faces either decreasing returns to scale and/or downward sloping demand is that these features create a link between current and future investment, so that irreversibilities matter. In models with constant returns to scale and perfectly elastic demand such as Abel (1983) and Abel and Eberly (1993), irreversibilities play no role. As discussed in Pindyck (1993), a firm with linearly homogenous technology that faces perfectly elastic demand will have a profit function that is linear in the capital stock. Thus convex costs are used to bound the size of the firm, since otherwise it would expand indefinitely if the present value the stream of marginal products exceeded the marginal cost. Since convex adjustment costs limit the size of the firm by making the marginal cost of the firm an increasing function of the level of investment, investment in each period is independent of investment in any other period. The effect of irreversibility-- being constrained by investment undertaken last period, and perhaps regretted if demand turns out lower than expected--cannot arise.

Modelling firm-level investment as irreversible seems particularly relevant in countries that have recently undertaken large scale reforms, so that profitable opportunities are arising in new sectors which may require high sunk costs. Considering the Ghanaian example: (1) For firms seeking to enter new export markets following the reform, capital expenditure can be highly irreversible in an environment with thin resale markets and impediments to business liquidations. Also, "marketing" and distribution capital are sunk expenditures. (2) The fact that business people cite the low sunk costs of distributive trade relative to manufacturing investments as important for investment decision indicates these costs are considered significant (Aryeetey (1994)). (3) Investment in the mining sector, which produces the second largest share of exports, requires specialized capital expenditures largely unrecoverable upon exit. (4) Investment in cocoa, a tree crop with a long gestation period, entails a largely sunk cost.

3. Solving for the Investment Trigger for a Myopic Firm

The procedure followed to solve for the investment trigger point closely follows that used in a number of existing papers, particularly Bertola and Caballero (1994) . Even so, the derivations will be presented in detail because the solution method for the investment trigger point in the case of regime-switching uncertainty is a simple extension of the steps in this section. The specific functional forms used are chosen for tractability, and are also used by Abel and Eberly (1995), Bertola (1988) and Bentolila and Bertola (1990).

3.1 The Model

The firm maximizes the expected present discounted value of profits by choosing an optimal investment rule. The reduced form operating profits are a constant elasticity function of K , the installed capital stock, and Z , an index of business conditions. The functional form for operating

¹If numerical methods were used to solve the problem with regime-switching uncertainty, then a closed form solution from the constant drift rate case would not be necessary.

profits is a loglinear expression that results from constant elasticity demand and production that is Cobb–Douglas in K and other factors of production. The reduced form operating profits are derived as follows.

The production function is Cobb–Douglas:

$$(1) \quad Q_t = [K_t^\alpha (AL_t)^{1-\alpha}]^\phi, \quad 0 < \alpha < 1, \quad \phi > 0, \quad \text{where } \phi \text{ indexes returns to scale.}$$

The firm faces a constant elasticity demand function:

$$(2) \quad B_t = D_t Q_t^{\mu-1}$$

where B_t is the product price, Q_t is output, and μ is the inverse of the markup factor, which indexes the firm's monopoly power. Monopoly power increases as μ goes to zero; μ equals 1 for a perfectly competitive firm. D_t influences position of demand curve; if μ equals 1, then D_t equals B_t which is the market price that the firm takes as given. L is labour, a perfectly flexible production factor that can be rented at the rate w_t . A_t is a technological progress indicator.

Define the operating profits function:

$$(3) \quad A(K_t, w_t, D_t) = \max_{L_t} B_t Q_t - w_t L_t$$

s.t. (1) and (2) above. The concavity of the operating profits function requires $(1-\alpha)\phi\mu < 1$ and $(1+\beta) > 0$. Operating profits can then be written in reduced form as:

$$(4) \quad A(K_t, w_t, D_t) = \frac{1}{1+\$} K_t^{1+\$} Z_{2t}$$

where:

$$\$ = \frac{\mu N - 1}{1 - (1 - \alpha) N \mu}, \quad -1 < \$ < 0$$

β indexes the concavity of the reduced form profit function with respect to the installed capital stock. High absolute values of β indicate greater monopoly power and/or more strongly decreasing returns to scale.

The business condition parameter Z_{2t} can be written:

$$Z_{2t} = f(\alpha, \phi, \mu, \beta; D_t, w_t, A_t)$$

Z_{2t} depends positively on the strength of demand and on productivity, and negatively on the cost of factors other than capital. The firm under consideration produces a tradable good, and Z_{2t} is positively correlated with RER depreciation and productivity enhancing structural reforms. Z_{2t} follows a geometric Brownian motion process with constant mean growth rate η_2 and variance σ^2 :

$$(5) \quad dZ_2/Z_2 = \eta_2 dt + \sigma dW_t$$

where W_t is a standard Wiener process with independent, normally distributed increments. By this formulation the business condition parameter Z_{2t} is expected to grow at an exponential rate η_2 , but the actual rate is random, and the outlook further in the future is increasingly uncertain.

From (4) we can calculate the MRPK as $K_t^\beta Z_{2t}$. Since constant elasticity functions of geometric Brownian motions follow geometric Brownian motions, we know that the MRPK is a geometric Brownian motion. Since Z_{2t} is a reduced form business conditions parameter, we can consider the expected regime switch as changing the trend growth rate of this reduced form variable, due to underlying changes in demand, costs and productivity.

Investment is irreversible; i.e., the cumulative gross investment process X_t is restricted to have non-negative increments. We can then define the value function, and the related dynamic optimization problem, as follows:²

$$V^*(K_t, Z_{2t}) = \text{Max}_{\{x(J)\}} E_t \int_t^\infty e^{-r(J-t)} \left[\frac{1}{1+\$} K_J^{1+\$} Z_{2J} dJ - P_J dX_J \right]$$

subject to $dK_J = -\delta K_J dJ + dX_J$

and to $dX_t \geq 0$.

(6)

Here, r is the constant required rate of return, δ is the depreciation rate of the installed capital stock, dX_t is gross investment, and the E_t is taken over the joint distribution of the K_t and Z_{2t} processes, conditional on information available at time t . P should be viewed as a per-unit cost of adjusting the capital stock. Although its main component is the purchase price of capital, it also includes other per-unit adjustment costs.

3.2 Solution Method

3.2.1 Necessary Conditions

If the firm is maximizing expression (6), then the following necessary conditions must be true at all times:

$$E_t \int_t^\infty e^{-(r+\delta)(J-t)} K_J^\$ Z_{2J} dJ = P \quad \text{if} \quad dX_t > 0,$$

(7)

$$E_t \int_t^\infty e^{-(r+\delta)(J-t)} K_J^\$ Z_{2J} dJ < P \quad \text{if} \quad dX_t = 0.$$

(8)

The term on the left hand side of (7) and (8) is the derivative of V^* with respect to K_t , the Hamiltonian shadow value of capital. In this stochastic framework, the shadow value of capital is

²Sufficient conditions for the existence of a solution to (6) can be demonstrated. Provided that $r > \{\sigma^2/2\beta - \eta_2/\beta\}$, the value function is bounded.

the conditional expectation of all sample–path expressions for the marginal revenue product of capital (MRPK). Note that the conditional expectation in (7) and (8) is formed taking into account the probability distribution generated by Z_{2t} , as well as the fact that future capital stocks are random, given information at time t . Future capital stocks will depend on information that arrives after time t . These necessary conditions imply that when the firm is investing, the discounted expected MRPK compensates the purchase and adjustment cost paid today. When the purchase and installation cost exceeds the discounted expected MRPK then the firm does not invest.

It can be shown that the optimal policy for the firm is to allow the MRPK/P to fluctuate between zero and a constant upper control barrier, U . (Note that zero is an absorbing barrier for a geometric Brownian motion process.) If a more depreciated RER causes the MRPK/P to rise above U , then more capital is installed to stop it from going above U . The next step, therefore, is to derive the optimal U^* — the MRPK/P ratio that triggers irreversible investment for the myopic firm — as a function of the parameters of the problem. Bentalila and Bertola (1990) confront this problem by computing the conditional expectations of the discounted MRPK streams. They derive a differential equation that can be used to solve for the value of the MRPK streams, subject to a control policy with any given upper barrier, and then use the necessary conditions to choose the optimal U^* . Bertola and Caballero (1994) provide a simpler method by working directly with the Bellman equation for the problem. The steps below follow their solution method.

The Bellman equation for the dynamic optimisation problem in (5) can be written:

$$rV(K_t, Z_{2t}) = \max_{\{dX_t\}} \left[\frac{1}{1+\$} K_t^{1+\$} Z_{2t} dt - P dX_t + E_t (dV(K_t, Z_{2t})) \right],$$

subject to $dX_t \geq 0$

(9)

Optimality implies that the required return on the value of the firm equal the expected return, which consists of the profit flow less the purchase price of capital when the firm is investing plus the expected capital gain.

Ito's formula can be used to expand the expected capital gain (omitting time subscripts and the arguments of $V(\cdot)$).

$$E_t [dV(\cdot)] = E_t \left[\partial_K V(\cdot) dK + \partial_Z V(\cdot) dZ_2 + \frac{1}{2} \partial_{ZZ} V(\cdot) (dZ_2)^2 \right]$$

$$= \partial_K V(\cdot) [-*K dt + dX] + \partial_Z V(\cdot) Z_2 0_2 dt + \frac{1}{2} \partial_{ZZ} V(\cdot) Z_2^2 F^2 d$$

(10)

Now using (10) in (9), the necessary conditions can be expressed:

$$\partial_K V(K_t, Z_{2t}) \leq P \quad \forall t$$

$$\partial_K V(K_t, Z_{2t}) = P \quad \forall t ; \quad dX_t > 0$$

(11)

Using the necessary conditions in the Bellman equation yields:

$$r V(\cdot) = \frac{1}{1+\$} K^{1+\$} Z_2 + \partial_K V(\cdot) (-*K) + \partial_Z V(\cdot) dZ_2 + \frac{1}{2} \partial_{ZZ} V(\cdot) (dZ_2)^2 \quad (12)$$

Since this condition holds over the optimal path, it can be differentiated with respect to K . Defining $v(K, Z_2) \equiv \partial_K V(K, Z_2)$, the Bellman equation can now be written:

$$(r + *) v(\cdot) = K^{\$} Z_2 + \partial_K v(\cdot) (-*K) + \partial_Z v(\cdot) dZ_2 + \frac{1}{2} \partial_{ZZ} v(\cdot) (dZ_2)^2 \quad (13)$$

3.2.2 Explicit Solution

The Bellman equation has now been expressed as a differential equation that must be satisfied by $v(K, Z_2)$. Solutions to (13) can be written as linear combinations of the sum of a particular solution and the solution to the homogeneous equation. A particular solution is:

$$v_p(K, Z_2) = \frac{K^{\$} Z_2}{r + * (1 + \$) - 0_2} \quad (14)$$

The complementary function has the form:

$$v(K, Z_2) = B_2 (K^{\$} Z_2)^{\alpha} + B_3 (K^{\$} Z_2)^{\beta}$$

However, in the special one-sided barrier case the requirement that $v(K, Z_2)$ be bounded means that $B_3 = 0$, since in the absence of negative control ($K^{\beta} Z_2$) could become arbitrarily close to zero. The sufficient conditions that guarantee that the value function defined in (6) is bounded also imply that $\alpha > 0$. Thus the general solution is of the form:

$$v(K, Z_2) = v_c(K, Z_2) + v_p(K, Z_2) = B_2 (K^{\$} Z_2)^{\alpha} + \frac{K^{\$} Z_2}{r + * (1 + \$) - 0_2} \quad (16)$$

Solutions of the form $v(K, Z_2) = B_2 (K^{\$} Z_2)^{\alpha}$ satisfy the homogenous form of the

differential equation (12) provided that α is a root of the characteristic equation:

$$\frac{1}{2} F^2 (\alpha - 1)'' + (0_2 - * \$)'' - (r + *) = 0 \quad (17)$$

The positive root α is:

$$\alpha = \frac{-\left[(0_2 - * \$) - \frac{F^2}{2}\right] + \sqrt{\left[(0_2 - * \$) - \frac{F^2}{2}\right]^2 + 2(r + *)F^2}}{F^2} > 1 \quad (18)$$

Comparative statics using these expressions demonstrate that the change in α with respect to an increase in σ is positive. This result will be used below to analyse the impact of increases in uncertainty on the investment trigger points.

The remaining unknowns are the integration constant B_2 and the trigger U^* , which is the level of the MRPK at which the firm invests. These can be determined using the boundary conditions of the problem. First, recall the necessary conditions in equation (10). The optimal investment policy prevents the MRPK from ever exceeding the purchase price of capital. Define a desired capital process $K^d(Z_{2t})$ such that $K_t \geq K^d(Z_{2t})$ for all t , and $K_t = K^d(Z_{2t})$ if $dX_t > 0$. Since $K_t = K^d(Z_{2t})$ if the firm is investing, the boundary condition can be written as:

$$v(K^d(Z_2), Z_2) = P \quad (19)$$

This is called a “value matching” condition as can be seen by imposing it on the general solution (16):³

$$v(K^d(Z_2), Z_2) = B_2(K^d Z_2) + \frac{K^d Z_2}{r + * (1 + \$) - 0_2} = P \quad (20)$$

The term $(K^d Z_2) / (r + * (1 + \$) - 0_2)$ is the expected contribution of the marginal investment dK to the discounted operating profit flow. The term $B_2(K^d Z_2)$ is the present value of the expansion option of currently marginal unit of capital. Equation (20) says that at the threshold that

³ The explicit designation of the K terms on the left hand side of the equation as desired capital values is omitted for convenience.

triggers the incremental investment, its expected contribution to the discounted capital flow equals the purchase price and the opportunity cost of the option to wait.

The other optimality condition is the “smooth pasting” condition, which requires that the slope of the marginal value function in (19) be smoothly differentiable at the optimum. This yields:

$$\begin{aligned}\partial_K v(K^d(Z_2), Z_2) &= 0 \\ \partial_{Z_2} v(K^d(Z_2), Z_2) &= 0\end{aligned}\tag{21}$$

By imposing the value matching condition (19) and the smooth pasting conditions in (21) on the form of the solution to the Bellman equation (16), we can determine the remaining integration constant B_2 ⁴ and the MRPK that triggers investment. This yields:

$$\frac{|(K^d(Z_2))^\sigma Z_2|}{P} = \frac{\sigma}{\sigma - 1} (r + \sigma(1 + \beta) - \delta_2)\tag{22}$$

Consider rearranging the equation above as follows:

$$\frac{|(K^d(Z_2))^\sigma Z_2|}{(r + \sigma(1 + \beta) - \delta_2)} dK = \frac{\sigma}{\sigma - 1} P dK\tag{22'}$$

The numerator of the left hand side is the contribution of a marginal unit of capital to the profit flow. Since the business conditions process Z_2 is expected to grow at the rate η_2 , future profits are discounted at the rate r , and the marginal unit depreciates at the rate δ , the left hand side is the expected present value of the marginal unit's contribution to the profit flow. The per unit purchase and adjustment cost is $P dK$. Thus the marginal irreversible investment takes place when the expected present value exceeds the cost of the investment by the multiple $\alpha / (\alpha - 1)$ ⁵. Since the multiple is increasing in σ , higher uncertainty in the business conditions indicator increases the investment trigger point.

Defining U^* as the level of the MRPK/ P that triggers investment, we can write:

$$U^* = \frac{\sigma}{\sigma - 1} (r + \sigma(1 + \beta) - \delta_2)\tag{23}$$

⁴ B_2 can be expressed as a function of a given U : $B_2 = - (UP)^{(1-\sigma)} / (r + \sigma - (\delta_2 - \sigma\beta))^\sigma$

⁵ This is one of the fundamental insights in Dixit and Pindyck (1994).

This optimality condition can be compared to the Jorgensonian investment condition. First note that the characteristic equation (17) can be rewritten in the form:

$$\frac{\eta}{\eta - 1} = \frac{r + \eta + (\sigma^2/2)\eta}{r + \eta + (\sigma^2/2)\eta - \alpha} \quad (17')$$

Substituting this expression into the optimal trigger expression in the form (22) yields:

$$\left[K^d(Z_2) \right]^\alpha Z_2 = (r + \eta + (\sigma^2/2)\eta) P \quad (24)$$

The term on the right hand side is larger than Jorgensonian user cost of capital because of the addition of the $(\sigma^2/2)\alpha$ term. Thus for the firm to undertake irreversible investment, the MRPK that triggers investment exceeds the neoclassical cost of capital.

The optimal U^* derived is the trigger for a firm facing ongoing uncertainty with respect to demand and cost processes, summarized by the business conditions indicator Z_{2t} . Note that it is the optimal investment trigger when the growth rate of the business conditions indicator, η_2 is expected to be constant.

We will use this optimal investment trigger in two different ways. First, the next section will determine the optimal U^* when there is regime-switching uncertainty by solving recursively. The solution that would obtain *after* a regime-switch is used in a problem to solve for the optimal investment rule during the "first phase" when there is also uncertainty about the arrival date of the switch. Supposing that the drift rate switches from η_1 to η_2 and then stays constant, we know that after the switch has taken place the firm would solve the same problem as in this section. Thus the solution to the value function after a regime switch has just been derived above.⁶

Second, the optimal U^* can also be used to define *myopic* behaviour in the face of uncertainty about the timing of a regime change which causes the business conditions to switch from a favourable to an unfavourable trend. Define the drift rate η_1 as the expected rate of change in the business conditions process *before* the regime change. If a firm solved the same problem as in this section, replacing η_2 with η_1 , the solution U^* would represent the optimal trigger point for a myopic firm before the regime change (U_m^* , for myopic $\equiv U^*$). Myopic behaviour in the face of regime-switching uncertainty can be defined in the following way. If the firm does not take into consideration that the drift rate may switch to a lower level at some uncertain date in the future, then its trigger point will be constant at U_m^* , and would then shift up to a new level at the time of the switch. The new trigger point would be another myopic level corresponding to the new drift rate.

It could be expected, however, that if the firm *does* take into account the probability of future lower growth in its summary business conditions indicator, then it would be more hesitant to invest, and that the pre-switch trigger would be higher. In order to make this comparison, the next section will derive an equation that implicitly defines the optimal U^* when there is

⁶ Note that in order to clarify the solution, only a one-time switch in the drift rate is considered. This is the most relevant case for the expectation of a reform reversal. However, the same recursive method could be used to analyse multiple switches.

regime-switching uncertainty.

4. Solving for the Investment Trigger under Regime-Switching Uncertainty

4.1 Validity of Solution Method

Uncertainty about reform sustainability is modelled here as uncertainty about the date when the drift rate of the business conditions indicator switches to a new rate that is known with certainty. The rate that the drift switches to will be called η_2 . Events such as this can be modelled by a Poisson process where in any small time interval $(t, t+\Delta t)$ the probability of a switch occurring is $\lambda\Delta t + o(\Delta t)$. Since the probability of this single Poisson event is distributed through time by $\lambda e^{-\lambda t}$, at any point in time the expected time until the occurrence of the event is $1/\lambda$.

Let T represent the date of the expected regime change. After time T the firm would solve the same problem as described in Section 3 above as relevant for a myopic firm. Thus we can use that value to solve recursively for the investment trigger point in the "first phase" of a problem where there is uncertainty about the arrival date of the switch.

The structure of the firm's problem in the first phase is similar to a optimal consumption/portfolio selection problem faced by an agent whose utility depends on lifetime consumption and a bequest at death, where the age of death is uncertain. This problem is considered by Merton (1971). He shows that using the Ito formula for a Poisson process is valid whenever an event occurs that causes a state variable to be incremented, provided that the probability of the event occurring during a given time interval has the probability structure of a Poisson process. Consider an individual whose age of death is a random variable, where the event of death at each instant of time is an independent Poisson process. The age of death, τ , is the first time that the event of death occurs. The value function can be defined:⁷

$$J(W, t) = \max_{C, W} E_0 \left\{ \int_0^J U(C, t) dt + B(W(J), J) \right\} .$$

(25)

Merton constructs an "artificial" state variable $x(t)$, with $x(t) = 0$ while the individual is alive and $x(t) = 1$ in the event of death. The stochastic process generating x is defined by $dx = dq$, where $q(t)$ is a Poisson process, and τ is defined by $\tau = \min\{t | t > 0 \text{ and } x(t) = 1\}$. The derived utility function J is now a function of the state variables W , x , and t , and subject to the boundary condition $J(W, x, t) = B(W, t)$ when $x = 1$. The optimality equation associated with the problem is $0 = U(C^*, t) + \lambda[B(W, t) - J(W, t)]$. The last term arises by using the Ito formula applicable to Poisson processes, and applying the boundary condition.

The investment decision problem under regime-switching uncertainty can be written in the form of the consumption/bequest problem. First note that in the firm's investment problem, the maximized present discounted value of the firm after the drift rate has switched is the equivalent of the bequest function $B(W, t)$ in Merton's set up. Define:

⁷ $C(t)$ is consumption per unit time during period t , $W(t)$ is wealth at time t , and $B(\cdot)$ is the bequest function, assumed to be concave in W .

$$V^*(K(T), Z_2(T)) = \max_{x(J)} E_T \int_0^\infty e^{-r(J-T)} \left[\frac{1}{1+\$} K_J^{1+\$} Z_{2J} dJ - P_J dX_J \right]$$

$$\text{subject to } dK_J = -*K_J dJ + dX_J$$

$$\text{and to } dX_t \geq 0$$

(26)

T is generated by a Poisson process with density function $\Gamma(T) = \lambda e^{-\lambda T}$. Z_{2t} follows a geometric Brownian motion process with drift rate η_2 and variance σ^2 . Thus $V^*(K(T), Z_2(T))$ is the maximized value of the firm after the drift rate has switched to η_2 .

The investment decision problem *before* the drift rate switches can be cast in the form of Merton's problem:

$$K_t, Z_{1t}) = \max_{\{x(J)\}} E_0 \int_0^T e^{-rt} \left[\frac{1}{1+\$} K_t^{1+\$} Z_{1t} dt - P_t dX_t \right] + V^*(K(T), Z_2(T))$$

(27)

Z_{1t} follows a geometric Brownian motion process with drift rate η_1 and variance σ^2 . Now, the artificial state variable $x(t)$ can be constructed where $x(t) = 0$ before the switch takes place, and $x(t) = 1$ in the event of a regime change that leads to a change in the drift rate. As above, the stochastic process generating x is defined by $dx = dq$, where $q(t)$ is an independent Poisson process, and T is defined by $T = \{\min t | t > 0 \text{ and } x(t) = 1\}$. The value function V can be considered a function of the state variables K , Z , and x subject to the boundary condition :

$$(28) \quad V_1(K_t, Z_{1t}, x) = V^*(K(T), Z_2(T)) \text{ when } x(t) = 1$$

The Bellman equation for the firm's problem takes the same form as before (equation (9)). The Poisson probability of a regime change implies that when Ito's formula is used to expand the expected capital gain, we obtain:

$$\begin{aligned} E_t[dV(\cdot)] &= E_t \left[\partial_K V(\cdot) dK + \partial_Z V(\cdot) dZ_1 + \frac{1}{2} \partial_{ZZ} V(\cdot) (dZ_1)^2 \right. \\ &\quad \left. + 8(V^*(K(T), Z_2(T)) - V_1(K_t, Z_{1t})) \right] \\ (29) \quad &= \partial_K V(\cdot) [-*K dt + dX] + \partial_Z V(\cdot) Z_1 0_1 dt + \frac{1}{2} \partial_{ZZ} V(\cdot) Z_1^2 F^2 d \\ &\quad + 8(V^*(K(T), Z_2(T)) - V_1(K_t, Z_{1t})) \end{aligned}$$

The solution proceeds along the same lines as in the previous section. The necessary conditions (11) are the same and can be substituted into the expression for the Bellman equation above. Also, since the resulting expression holds along the optimal path, it can be differentiated term by term with respect to K . Defining $v_1(K, Z_1) \equiv \partial_K V_1(K, Z_1)$, this yields:

$$\begin{aligned}
(r + *) v_1(\cdot) &= K^{\$} Z_1 + \partial_K v_1(\cdot) (-*K) + \partial_Z v_1(\cdot) dZ_1 + \\
&\quad \frac{1}{2} \partial_{ZZ} v_1(\cdot) (dZ_1)^2 + 8 \left[v^*(K^d(Z_2), Z_2) - v_1(K, Z_1) \right]
\end{aligned}
\tag{30}$$

The recursive nature of the solution method will become clear by considering the term $v^*(K^d, Z_2) = \partial_K V^*(K(T), Z_2(T))$. $v^*(K^d, Z_2)$ represents the maximized, expected discounted stream of benefits--MRPKs of the currently marginal unit of capital--that will accrue to the firm *after* the regime change. In Section 3 we were able to obtain a closed form solution for this marginal payoff stream: $v^*(K^d, Z_2) = B_2(K^{\$}Z_2)'' + (K^{\$}Z_2)/(r + *(1 + \$) - 0_2)$. Using this solution in the Bellman equation yields:

$$\begin{aligned}
(r + * + 8) v_1(\cdot) &= K^{\$} Z_1 + \partial_K v(\cdot) (-*K) + \partial_Z v_1(\cdot) dZ_1 + \frac{1}{2} \partial_{ZZ} v_1(\cdot) (dZ_1)^2 \\
&\quad + 8 \left[B_2(K^{\$}Z_2)'' + \frac{(K^{\$}Z_2)}{(r + *(1 + \$) - 0_2)} \right]
\end{aligned}
\tag{31}$$

Again the general form of the solution to (31) is the sum of a particular solution and the solution to the homogeneous equation. A particular solution has the form:

$$v_{1p}(K, Z_1) = C(K^{\$}Z_1) + D(K^{\$}Z_1)''
\tag{32}$$

The values of the constants C and D can be determined in terms of the known parameters and integration constant B_2 :

$$C = \frac{1 + 8(1/(r + *(1 + \$) - 0_2))(Z_2/Z_1)}{(r + * + 8) - (0_1 - *\$)} ,
\tag{33}$$

$$D = \frac{-8B_2(Z_2/Z_1)''}{\frac{F^2}{2}''(' - 1) + (0_1 - *\$)'' - (r + * + 8)} .
\tag{34}$$

The solution to the homogeneous part of the equation has the form:

$$v_{1c}(K^{\$}Z_1) = A(K^{\$}Z_1)^{\gamma}.$$

(35)

It is straightforward to show that for the differential equation (30) to be satisfied, γ must be the solution to a characteristic equation similar to the one derived in the myopic case, with the value of the positive root γ given by:

$$\gamma = \frac{-\left[(0_1 - \pi^{\$}) - \frac{F^2}{2}\right] + \sqrt{\left[(0_1 - \pi^{\$}) - \frac{F^2}{2}\right]^2 + 2(r + \pi^{\$} + 8)F^2}}{F^2}.$$

(36)

In order to solve for the integration constant A and the optimal trigger U^* for the regime switching problem, we can use the same value matching and smooth pasting conditions. This allows us to solve for the constant A . The general solution can be written:

$$v_1(K, Z_1) = v_{1c} + v_{1p} = A(K^{\$}Z_1)^{\gamma} + C(K^{\$}Z_1) + D(K^{\$}Z_1)^{\eta}$$

(37)

Imposing the value matching and smooth pasting conditions on the form of the solution yields an expression:

$$v_1(K^d(Z_1), Z_1) = A(K^{\$}Z_1)^{\gamma} + C(K^{\$}Z_1) + D(K^{\$}Z_1)^{\eta} = P$$

(38)

Note that the values of the constants A , C , and D have been determined and can be written in terms of the exogenous parameters of the problem. Thus, although it is not possible to derive a closed-form solution to (38), the equation implicitly defines U_s^* , the optimal trigger point for irreversible investment when there is uncertainty about the arrival date of a regime change.

5. Numerical Results

5.1 Questions to be Addressed

It is now possible to compare the optimal investment trigger points for a myopic firm (U_m^*) and a firm taking into account that the drift rate may switch (U_s^*). Recall the example that is considered: Ghana's Economic Recovery Program has resulted in achieving a real depreciation and improving the growth trend of productivity. There is an expectation that the RER will continue depreciating since there is a considerable way to go before any "equilibrium" level will be reached, barring further shocks. However, there is also some probability that the program will collapse and the RER will

return to a sharply appreciating trend. The reinstated control regime will also lower productivity for tradable firms. These expected changes will be correlated with a switch in the drift rate of the business conditions indicator Z_t . Firms use an investment decision rule: invest whenever the $MRPK/P$ reaches a particular U^* . Given the model specified above, is it true that a firm which takes into account the probability that the drift rate may switch unfavourably will invest less readily than a firm that ignores this possibility?

U_m^* and U_s^* are increasing functions of σ . In addition to comparing U_m^* and U_s^* for a given σ , it is also important to examine the differential effects of increases in σ on U_m^* and U_s^* . In terms of its investment decisions, does a firm which considers that the business conditions indicator may switch to an unfavourable trend at some uncertain future date react differently to a given increase in the volatility of the indicator compared to the reaction of a myopic firm?

Numerical results will be used to explore how the investment trigger points for both types of firms depend on the parameters of the problem. Numerical results can also shed light on the factors that influence the relationship between U_m^* and U_s^* , and between the slopes of U_m^* and U_s^* as functions of σ .

5.2 Results

5.2.1 Base Case

The first step is to compare U_m^* and U_s^* for two central cases of parameter values. From the properties of the Poisson distribution, the arrival date of the switch (the first time that the event occurs) is an exponentially distributed random variable with parameter λ . The realistic central case sets $\lambda = 0.25$, (see Figure II.1) so that the expected waiting time until the drift switch is four years; however, some of the results will be compared to the same central case with $\lambda = 10$ (see Figure II.2), implying that firms expect a switch extremely soon.⁸

U_m^* is a function of the discount rate r , the depreciation rate δ , the drift rate before the switch η_1 , the standard deviation of the RER process σ , and β , an index of the concavity of the reduced form profit function with respect to capital. Higher absolute values of β ($|\beta|$) indicate greater monopoly power and/or more strongly decreasing returns to scale. The central case takes $r = 0.10$, $\delta = 0.07$, $\eta_1 = 0.10$, and calculates U_m^* as σ ranges from 0.02 to 0.25. As shown in Figures II.1 and II.2, this is done for $\beta = -0.8$, $\beta = -0.6$, and $\beta = -0.4$.

In addition to the parameters defined above, U_s^* also depends on η_2 , the drift rate after the switch, P , the purchase price of capital, and λ , the arrival rate of the Poisson process. The central case sets $\eta_2 = -0.06$, $P = 1$, and $\lambda = 0.25$ or 10 as discussed above.

Before discussing the relationship between U_m^* and U_s^* , two general points will be noted. First, Dixit and Pindyck (1994) provide intuition for why the investment trigger points are increasing in σ by illustrating how optimal investment rules can be determined using methods developed for pricing options in financial markets. For marginal or incremental investment problems, Bertola (1988) and Pindyck (1988) have demonstrated that the marginal irreversible investment decision problem can also be solved as a marginal option valuation problem. Pindyck solves by first determining the value of an incremental unit of capital, given the level of the existing capital stock and the demand parameter. He then finds the value of the option to invest in the unit and the optimal exercise rule. This method provides insight as to why higher demand variability raises the investment trigger. Given a profit function that is convex in prices, although an increase

⁸ The results follow a similar pattern where λ is not as large, implying that expected time until the switch is not as extremely short as considered here.

in the variance of demand increases the value of an incremental unit of capital, which would increase investment, it increases the value of the option to invest even more. Since the lost option value is an opportunity cost of investment, the increase in uncertainty increases the trigger required for irreversible investment.

The second general point is that for a given σ , the investment trigger point is lower for higher $|\beta|$. The basic intuition is that since higher absolute values of β imply that the operating profit function is more concave in K , firms with higher $|\beta|$ have more incentive to always stay close to the optimum point.

5.2.2 Investment Triggers for the Myopic Firm and the Firm Expecting Drift Rate Switch

Turning to the comparison of U_m^* and U_s^* , it is clear from both Figures II.1 and II.2 that for any given σ , U_s^* is higher than the trigger for the myopic firm as hypothesized. Thus the firm that takes into account the probability of a regime change will invest less readily than a firm that ignores this possibility.

5.2.3 Effects of Increased Volatility on the Two Types of Firms

In comparing the slopes of U_m^* and U_s^* with respect to σ in order to discuss the differential effect of increased variability on the two types of firms, the graphs show that the patterns are different for the alternate cases of large and small λ . Figure II.2 is the case of a large λ value, which indicates that the switch is expected in the near future. The trigger points are more sharply increasing in σ when the expected drift rate switch is taken into account, as compared to the myopic case.

Thus while an increase in σ would increase the value of a marginal unit of capital by the same amount for both types of firms, the option value of waiting would increase more for the firm that considers the possibility of a switch.

Note, however, that when the expected drift rate change is long time off (λ small), the slope of U_m^* and U_s^* as functions of σ are basically the same. Therefore, if a regime change is expected in the near future, investment decisions of the firm expecting the change are significantly influenced by increasing volatility of the business conditions process. Given an increase in volatility, the firm expecting the switch is more reluctant to invest than a myopic firm. If, however, the expected regime change is quite far off, then the two types of firms react similarly to increases in volatility.

5.2.4 Effect of Monopoly Power for the Two Types of Firms

Recall that for a given σ , firms with higher degrees of monopoly power have lower investment trigger points. Figures II.1 and II.2 show that the inverse relationship between $|\beta|$ and the investment triggers is more pronounced for the expected switch case. The extent of monopoly power matters more for the investment decision for the firm that is expecting a regime change.⁹

Recall that firms with higher degrees of monopoly power have greater aversion to deviating from the desired capital stock. The costs of deviating are greater in the case of the expected switch. An unfavourable drift rate makes it more likely that the firm could get stuck with too much capital,

⁹ Since the reform's trade liberalization may increase competition in the domestic market, the extent of a firm's monopoly power could be affected by the reform.

greatly lowering its operating profits.

5.2.5 Effect of Post Regime Change Drift Rate

Figures II.3 and II.4 look at the effects of different post-switch drift rates on the investment trigger points. The results are the same for both large and small λ , and make intuitive sense. For any given σ , the lower the η_2 that the firm expects the business condition indicator may switch to, the higher the investment trigger point. This implies that if a firm expects the business conditions trend to be quite bad after a possible regime change, it will be more hesitant to invest now.

5.2.6 Effect of Pre-switch Drift Rate

Figure II.6 demonstrates that when the drift rate switch is expected to occur extremely soon, the level of the current drift rate does not matter much for determination of the investment trigger. Figure II.5 illustrates that the current drift rate *does* matter when the switch is a fairly long time away, and that a firm will invest more readily the "better" current conditions are. Thus, if the reform policies are expected to persist for some time before a change, the firm will respond to the favourable trend in Z_{1t} . If, however, reform policies are expected to be abandoned soon, then favourable current trends do not lead the firm to invest.

5.2.7 Effects of Changes in λ

Figure II.7 looks at how investment triggers vary with differences in the expected time until a regime change. For any given σ , the trigger point is higher for higher λ . This indicates that the firm will invest less readily when the expected time until the switch is shorter. We can also see that the trigger points are more sharply increasing in σ for higher values of λ . The firm that is expecting a regime change to occur extremely soon is also more concerned about ongoing volatility.

5.2.8 Desired Capital Stock for the Two Types of Firms

The final figure (II.8) shows the firm's desired capital stocks, calculated as follows. Recall the investment rule for the myopic case: whenever possible the firm should install more capital so as to satisfy:

$$\frac{K_t^\$ Z_{1t}}{P} = U_m^* = \frac{''}{'' - 1} (r + *(1 + \$) - 0_1) .$$

(39)

This condition can be inverted to find an expression for the desired capital stock as a function of Z_1 and P . A numerical result is available for the regime-switching case.

Figure II.8 plots the desired capital stock for given levels of Z_1 and P , as a function of σ , for different values of $|\beta|$. Since higher uncertainty increases the optimal investment trigger, it also implies a lower desired capital stock. We can also see that for given σ , P , and Z_1 , the desired capital is lower for firms with higher degrees of monopoly power. Finally, for given σ , P , and Z , the desired

capital is lower for firms taking into account the possibility of a drift rate switch compared to the myopic firms.

6. Conclusion

This paper analysed a model of a firm making irreversible investment decisions under uncertainty. It was found that a tradable sector firm that is uncertain about the timing of a regime change, identified here as a reversal of reform policies, is more hesitant to invest.¹⁰ It is worthwhile to stress that these are firm level results, and further work would be required to determine if the results carry over to the industry or aggregate level.

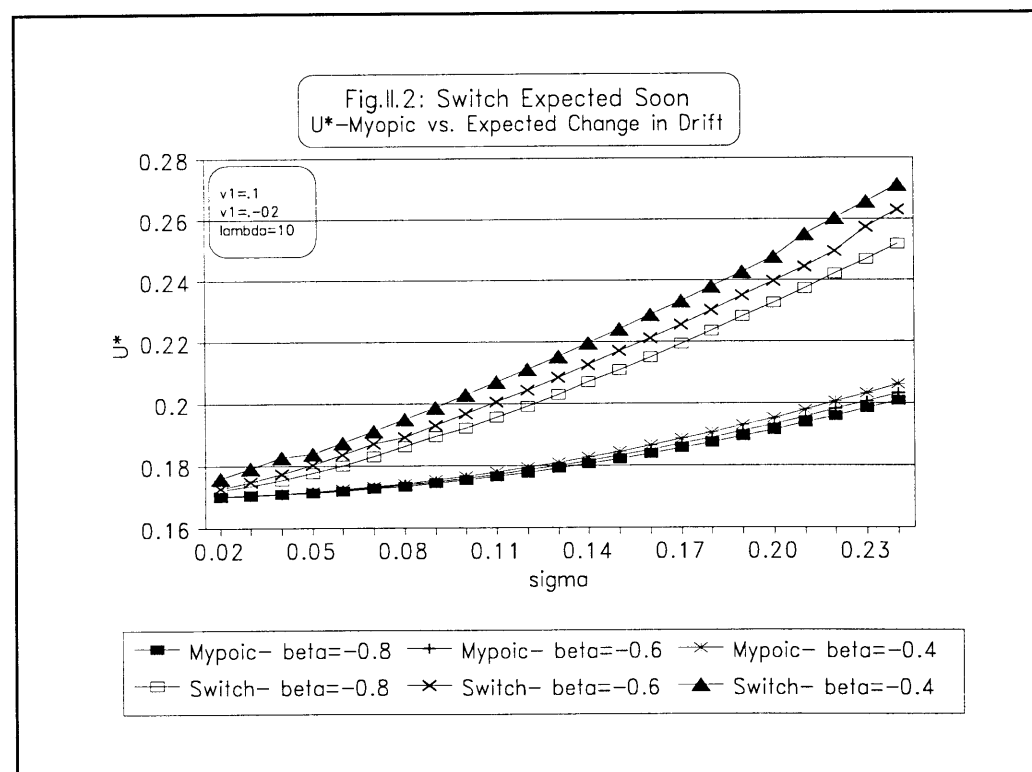
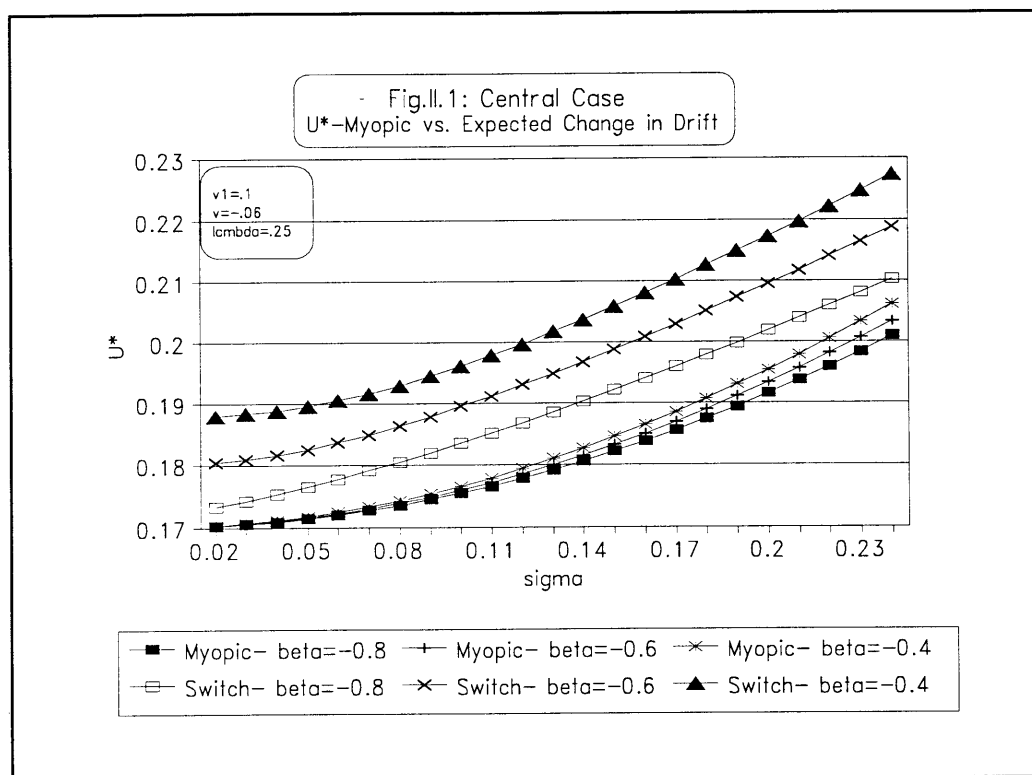
The main results from the numerical analysis are summarized as follows. (1) The firm that takes into account the probability of a regime change will invest less readily than a firm that ignores this possibility. (2) As business conditions become more variable, the firm that is expecting a regime change in the near future reduces investment more, relative to the myopic firm. (3) If a firm expects that after a regime change the trend in its business conditions indicator will be quite unfavourable, it will be more hesitant to invest now. (4) If reform policies are expected to be abandoned soon, then more favourable current trends do not make the firm invest more readily. If, however, the reform policies are expected to persist for some time before a change, the firm will invest more readily the more favourable current trends are. (5) The firm will invest less readily when it is expected that the regime change will occur soon.

The model of firm behaviour could be extended in a number of ways. One straightforward generalization would be to model the size of the change in the drift rate as a random variable. This would be more realistic since firms do not know what the exact trend growth rate of the business conditions indicator would be after a regime change. Another extension would relax the assumption of risk neutrality, thereby recognizing that the capital markets the firms operate in are highly imperfect.

A more ambitious improvement would involve specifying the source of uncertainties about reform sustainability. Currently, the expected regime change is modelled as exogenous. It is important to endogenize the expectation of reform reversal, and specify the source of the credibility problem, as a step toward developing testable hypotheses from the model.

Assuming that these results also held at the sectoral level, what are the implications for policy? (1) When reforms are not credible, so that there is some expectation they will not be sustained, investment will not respond to reforms that greatly improve returns. (2) Since investment will be low even if current trends in the fundamentals are quite favourable, then policies that provide cost or production subsidies and further improve returns will not induce investment in this environment. (3) Uncertainty about reform sustainability makes firms more concerned about ongoing volatility in returns. Therefore, high volatility in exchange rates or interest rates, for example, will have an even greater negative impact on investment when reforms are not credible.

¹⁰Uncertainty about the timing of the shock can be seen as creating an implicit tax on current investment. This is also the finding of Rodrick (1989) from a different type of model.



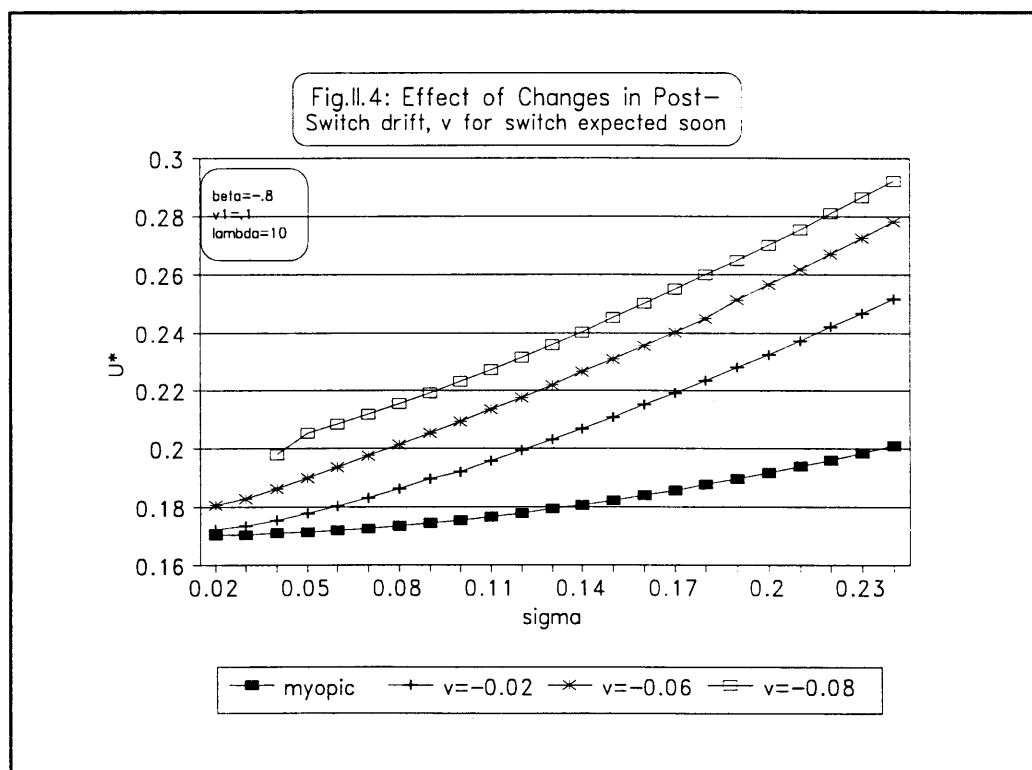
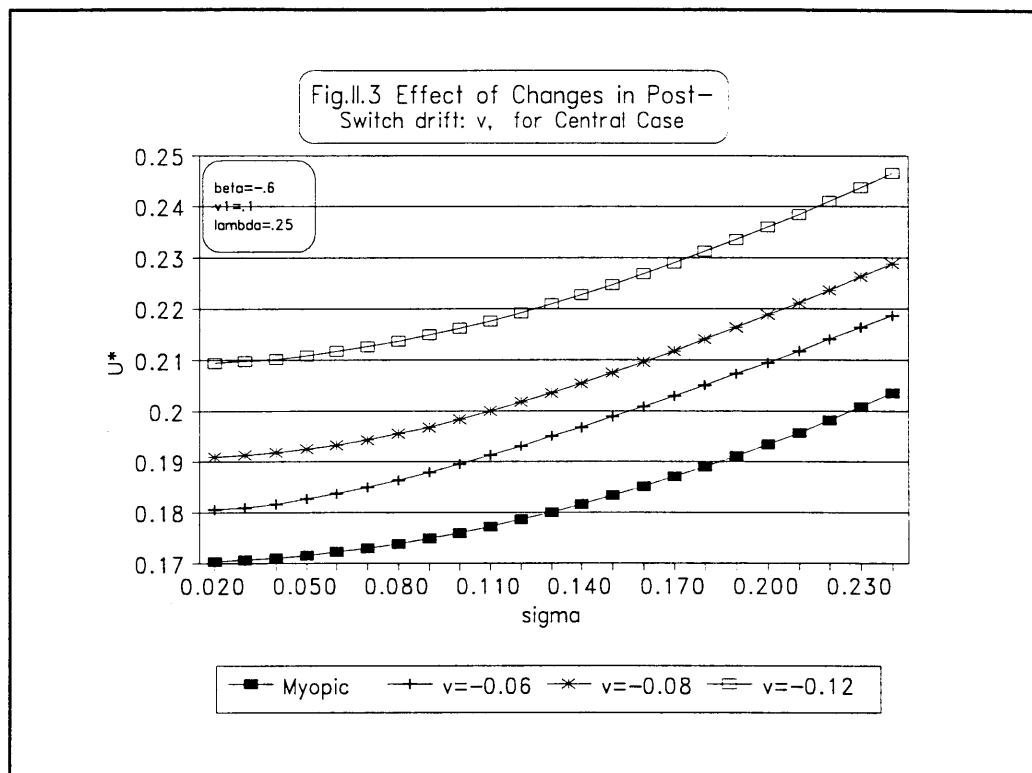


Fig.II.5: Effect of Changes in Pre-Switch drift, v_1 , for Central Case

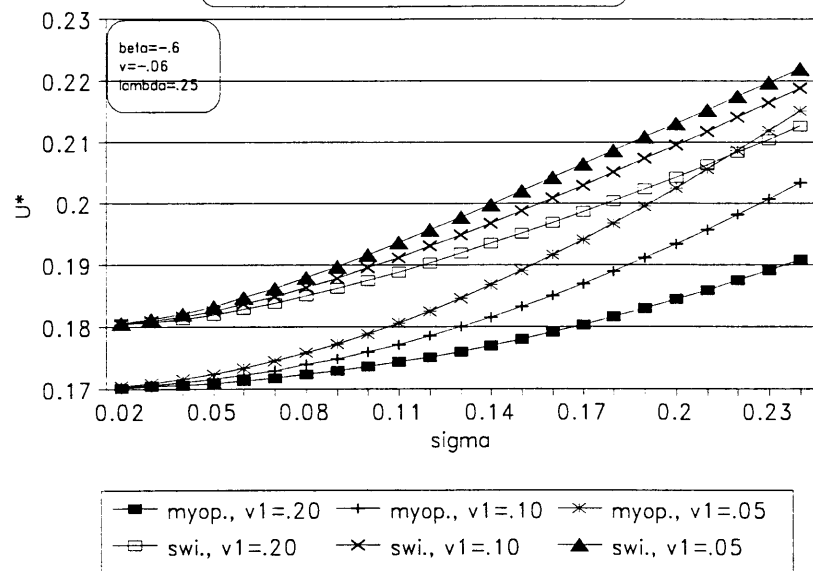
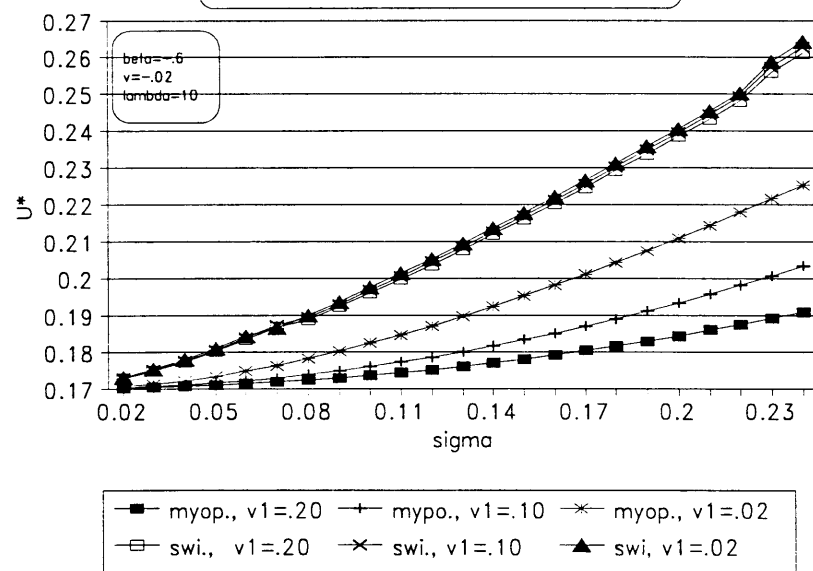
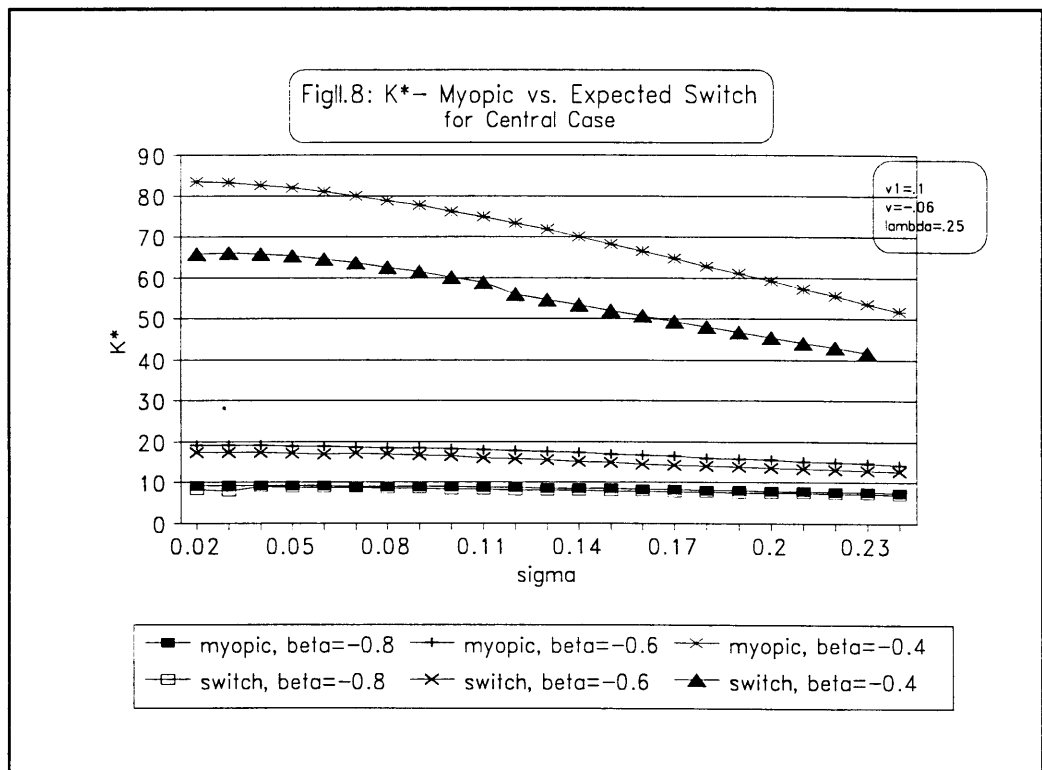
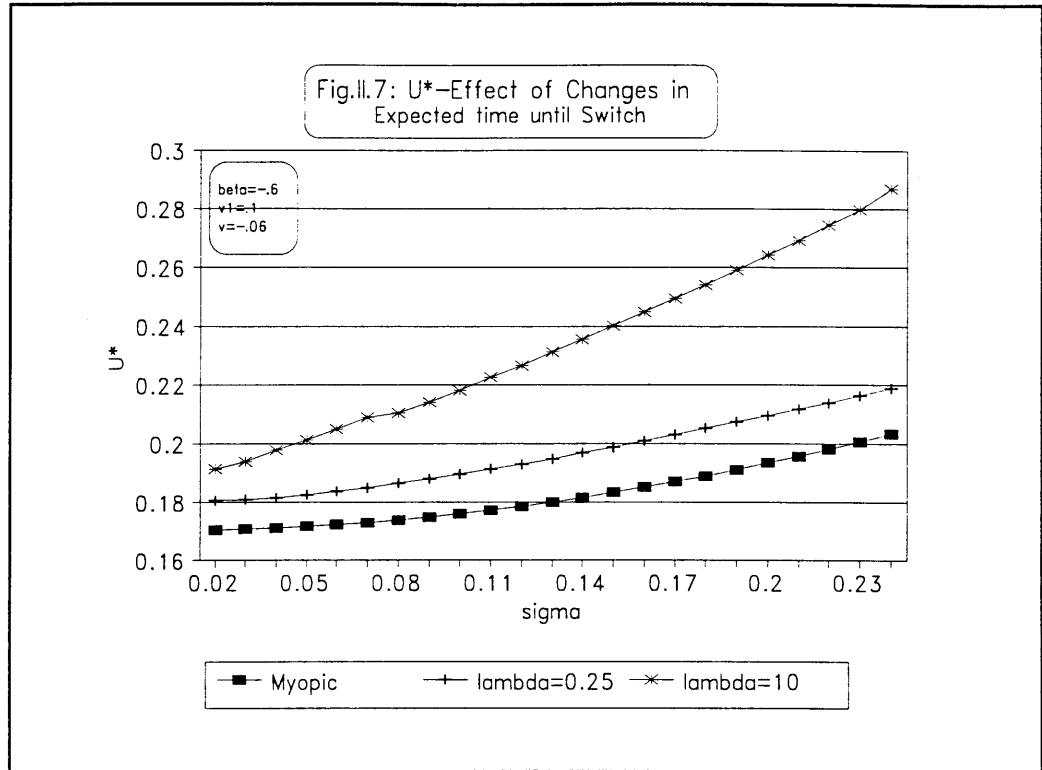


Fig.II.6: U^* —Effects of Changes in Pre-Switch drift, v_1 : switch expected soon





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