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**MATCHING WITH CONTRACTS:
AN EFFICIENT MARRIAGE MARKET?**

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Abstract

This paper studies a marriage market with two-sided information asymmetry in which the gains from marriage are stochastic. Contracts specify divisions of ex-post realized marital surplus. I first study a game in which one side of the matching market offers contracts. I show that when expected marital surplus is strictly monotonic in agents' types, no separating equilibrium that achieves matching efficiency exists. I then study a social planner's problem, finding necessary and sufficient conditions for a truthful direct revelation mechanism to achieve matching efficiency. These conditions become more stringent as the number of agents in the matching market increases.

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1. Introduction

This paper studies the marriage market: Who marries whom and how couples share their resources within marriage. In a transferable utility model, I examine the possibility of achieving an efficient matching outcome in which the aggregate gains from marriage are maximized. I focus on settings where i) there is two-sided information asymmetry, and ii) the gains from marriage are stochastic.

1.1. Motivation

Marriages are not formed randomly. Marriage patterns observed in society suggest that individuals often sort themselves into marriages based on the attributes of partners. Starting with Becker, economists have long pointed out that marriages are formed because interactions in individual attributes generate mutual gains from marriage. To study who marries whom, Becker [1] and Shapley and Shubik [2] analyzed a frictionless (no information asymmetry, no search cost) marriage market with transferable utility. With transferable utility, the problem of obtaining an efficient and stable matching outcome simplifies to a problem of maximizing the aggregate matching output. Therefore complementarity of traits, or supermodularity of the match production function, must lead to positive assortative matching (PAM), where people of similar traits are matched together.

However, in real life the matching process is characterized by scarcity of information about potential matches. One strand of the literature focuses on the meeting technology and assumes search costs (Diamond [3]; Moretensen [4]; Burdett and Coles [5, 6]; Shimer and Smith [7]). The alternative approach to add realism to the matching model is to introduce information asymmetry (Damiano and Li [8]; Hoppe, Moldovanu and Sela [9]). Individuals' attributes may not be fully revealed to one another before a match. Even after a match has formed, not all attributes are observable to researchers.

This paper takes the second approach: The type of each agent is only privately known. Assuming supermodularity of the match production function, I examine whether PAM can be achieved in the presence of information asymmetry.

Unlike previous literature, I analyze an incomplete-information model where the sharing rules and the matching outcomes are jointly determined within the model. In models of frictionless marriage markets (Gale and Shapley [2]; Becker [1]; Browning, Chiappori and Weiss [10]), the division of marital output and matching outcomes are determined simultaneously. In a negotiation of sharing the match surplus, bargaining power comes from one's ability to replace his or her current spouse with another. An efficient and stable matching must be supported by some specific sharing rules on how to divide the match

surplus. However, in the literature on matching markets with information asymmetry, how to split the shares are often pre-imposed, so there is no room for negotiation. For instance, both Damiano and Li [8] and Hoppe et.al.[9] examined the setting in which a man x matches with a woman y , together they produce $2xy$ with certainty and each partner gets half of the produced output. The half-half sharing rule is fixed and known ex-ante to all types of agents. It seems inconsistent that, whereas in models of frictionless marriage markets, the sharing rule and the matching outcome are simultaneously determined, in models of markets with information asymmetry, the sharing rule is exogenously imposed.

The second novelty of this paper's analysis is that the gains from marriage are stochastic. The stochastic component of marital surplus is often ignored in the literature on matching markets with information asymmetry. An individual's uncertainty about match surplus comes only from uncertainty about the type of his or her partner. Such an assumption ignores the possibility of shocks to the marital surplus after matching, yet the shocks can have important consequences. In fact, in most of the literature on divorce and re-marriages, shocks to marital output play an important role in generating equilibria. Browning, Chiappori and Weiss [10] show that high aggregate divorce rates can be beneficial because they facilitate the recovery from negative shocks to match quality, allowing couples to replace bad marriages by better ones. This paper assumes divorce is hugely costly; therefore before any match, people use the available information as much as possible in order to achieve a good matching outcome and avoid future separation. I show that the stochastic component of the marital surplus is important in determining whether or not a mechanism achieving efficient matching exists.

A related paper which also relates noise to efficiency is by Bahkar and Hopkin [11] as they examine pre-marital investment with stochastic returns. The main difference is that their main focus is on efficiency of pre-marital investment rather than efficient matching, and their setting is a frictionless marriage market with non-transferable utility.

Another related work in progress by Dizdar and Moldovanu [12] (written independently) asks questions in the same spirit: What sharing rules are compatible with a truthful direct revelation mechanism that achieves matching efficiency? This paper differs mainly in two aspects. First, unlike their paper, where agents are allowed to compete with ex-ante monetary transfers before the match process, I exclude any ex-ante monetary transfer and investigate whether matching efficiency can be achieved by endogenous sharing rules alone. Second, while their analysis rules out stochastic shocks to matching output and examines an ex-post equilibrium, I allow marital gains to be stochastic and characterize the conditions under which a Bayesian Nash equilibrium may be found.

To examine how the sharing rules and the matching outcomes are endogenously deter-

mined in a model with two-sided information asymmetry, I adopt the standard assumption in the marriage literature¹ regarding how couples bargain over the surplus: Prospective couples can make *binding, costlessly enforceable, prenuptial agreements* to transfer resources within marriage. This assumption has been labeled Binding Agreements on the Marriage Market (BAMM)².

Since contracts proposing how to divide the match surplus are credible, binding and costlessly enforceable, this paper asks: Do there exist contracts that help achieve an efficient matching outcome? To answer this question first note that individuals in this setting face two types of uncertainty. The first type arises from asymmetry of information; ex-ante a woman (or a man) cannot observe her (his) potential partners' types, so she (he) does not know whom to match with and therefore is not sure about the amount of marital output produced after a match. If we let agents on one side of the matching market offer contracts, the sharing rules proposed need to serve two purposes: revealing one's type (signaling) and attracting a suitable partner (screening). There may exist trade-offs between these two functions. The second type of uncertainty arises because the marital output contains random elements; even without information asymmetry, after a match has formed, the total marital output produced is subject to a random shock. Therefore contracts can only specify how to divide each ex-post *realized* surplus. This worsens the trade-offs between signaling and screening of a contract. I show that because of the stochastic nature, prenuptial contracts proposing divisions of ex-post realized marital surplus may be *ineffective* in achieving matching efficiency.

1.2. Outline

This paper is organized as follows: Section 2 studies a game in which one side of the matching market offers contracts proposing sharing rules, each agent on the other side chooses a contract to accept. I show that when expected marital surplus is strictly monotonic in agents' types, matching efficiency cannot be achieved.

Section 3 proceeds to a social planner's problem where a social planner aims to maximize the aggregate matching surplus. He commits to a matching rule and designs contracts specifying sharing rules conditional on the reported types and the realization of the surplus of each matched pairs. A social planner can do strictly better in terms of achieving

¹See Gale and Shapley [13]; Becker [1]; Choo and Siow [14]; Iyigun and Walsh [15]; Chiappori, Iyigun and Weiss [16]; Browning, Chiappori and Weiss [10].

²This assumption is not as strong as it sounds. Any transfer that (i) is decided before marriage, and (ii) can be used to alter the spouse's respective bargaining positions after marriage is considered as a BAMM framework (Browning, Chiappori and Weiss [10]).

matching efficiency than the equilibrium of the game in which one side offers contracts. I derive necessary and sufficient conditions for the truthful direct revelation mechanism to achieve matching efficiency. I show that these conditions become more stringent as the number of agents in the matching market increases. When the number of agents becomes very large, matching efficiency is not achievable even in the social planner's setting.

Comparisons of alternative models and some extensions are discussed in Section 4. Section 5 concludes.

1.3. Assumptions in this paper

Main assumptions in this paper are the following:

- Transferable Utility.
- Binding Agreements Costlessly enforceable (BAMM).
- Asymmetric Information on both sides of the market.
- Gains from Marriage are Stochastic.

2. Game with One Side Offering Contracts

There are N women and N men, all are risk neutral. Each one is independently drawn from a distribution with probability $\frac{1}{2}$ to be a high type agent and probability $\frac{1}{2}$ to be a low type agent. Each agent observes only his or her own type. The payoff of an unmatched agent is assumed to be 0 regardless of types.

2.1. Benchmark Model: Matching Surplus is Deterministic

For each possible combination of types, we have the following:

$$\begin{aligned} W^H + M^H &= z_2 \\ W^H + M^L &= z'_1 \\ W^L + M^H &= z_1 \\ W^L + M^L &= z_0 \end{aligned}$$

The first equation denotes that when a high type woman, denoted by W^H , matches with a high type man, denoted by M^H , together they will produce a matching surplus of magnitude z_2 . Similarly for other three combination of types. The match surplus z_2 , z'_1 , z_1 and z_0 are different values³, and satisfy the supermodularity condition:

$$z_2 + z_0 \geq z_1 + z'_1 \tag{1}$$

³The special case where $z_1 = z'_1$ is discussed in Section 4 and Appendix 6.2.

which implies that positive assortative matching (PAM) is efficient. Without loss of generality I assume $z_2 > z_0$.

The game with one side offering contracts proceeds in the following:

First consider $N=2$. Each woman, having observed only her own type, offers a contract proposing a sharing of each possible realization of the surplus. An example of a contract she offers is illustrated below, which suggests that a man only gets ε when the total surplus turns out to be z_2 , in all other cases he gets only 0.

Surplus	Woman	Man
z_2	$z_2 - \varepsilon$	ε
z'_1	z'_1	0
z_1	z_1	0
z_0	z_0	0

Each man, having observed his own type and the two offered contracts, updates his beliefs about the types of the two women, and uses these updated beliefs to calculate his expected payoff for each contract offered. A man will first go to a contract which gives him a higher expected payoff. If not successful, he will then go to the second contract if the second contract offers him a non-negative payoff. He will not sign any contract which gives him a negative expected payoff but to remain single. If a man is indifferent between being single and signing a contract, assume he will sign the contract. If both men go for the same contract, the woman offering the contract will randomly pick one to match with.

2.1.1. No competition between W^H and W^L

Proposition 1 *When $z'_1 \leq z_2$ and $z_1 \leq z_0$, there exists a separating equilibrium that achieves matching efficiency where W^H offers contract A and W^L offers contract B:*

Contract A			Contract B		
Surplus	Woman	Man	Surplus	Woman	Man
z_2	z_2	0	z_2	0	z_2
z'_1	z'_1	0	z'_1	0	z'_1
z_1	0	z_1	z_1	z_1	0
z_0	0	z_0	z_0	z_0	0

let $\mu(W^k|\text{contract } i)$ denotes men' belief on the probability assigned to a woman is of type k when she offers contract i .

Equilibrium beliefs: $\mu(W^H|\text{contract A}) = 1$ and $\mu(W^L|\text{contract B}) = 1$

Equilibrium response rule: When being indifferent between payoffs of contract A and contract B, M^H goes for contract A and M^L goes for contract B.

Off-equilibrium beliefs: $\mu(W^H | \text{any other off-equilibrium contracts}) \in [0, 1]$

Off-Equilibrium response rule: When being indifferent between an equilibrium contract and an off-equilibrium contract, both M^H and M^L always go to an **off-equilibrium** contract first.

Proof. When the four agents turn out to be exactly one W^H , one W^L , one M^H and one M^L , and when W^H offers contract A and W^L offers contract B, M^H will go for contract A and M^L will go for contract B. Matching efficiency is achieved.

Table 1: Matching Outcomes of Equilibrium Contracts

Prob	Other three agents	Contract A	Contract B
$\frac{1}{4}$	$W^L; M^H, M^L$	M^H	$\frac{1}{2}M^H + \frac{1}{2}M^L$
$\frac{1}{4}$	$W^H; M^H, M^L$	$\frac{1}{2}M^H + \frac{1}{2}M^L$	M^L
$\frac{1}{8}$	$W^L; M^H, M^H$	M^H	M^H
$\frac{1}{8}$	$W^H; M^H, M^H$	M^H	M^H
$\frac{1}{8}$	$W^L; M^L, M^L$	M^L	M^L
$\frac{1}{8}$	$W^H; M^L, M^L$	M^L	M^L
Expected matching outcome		$\frac{5}{8}M^H + \frac{3}{8}M^L$	$\frac{3}{8}M^H + \frac{5}{8}M^L$

Table 1 calculates the expected matching outcomes of offering the two equilibrium contracts respectively. Given a woman knows her own type, she needs to first work out the possible scenarios she will face before offering a contract. The first line in Table 1 summarizes the following: The probability that of the other three agents, one turns out to be a low type woman, one a high type man and one a low type man is $\frac{1}{4}$, given that each agent is independently drawn from a population with probability $\frac{1}{2}$ to be a high type agent and probability $\frac{1}{2}$ to be a low type agent. Since the other woman is of low type, and W^L offers contract B in the equilibrium, if the woman offers contract A, she will be regarded as a high type woman and hence attract only M^H . If she offers contract B, then men believe there are two low type women and are indifferent between them. She has half probability to be matched with M^H and half probability to be matched with M^L . The reasoning is the same for other scenarios. Comparing the two expected matching

outcomes, we see that offering contract A has a higher probability of matching with M^H and offering contract B has a higher probability of matching with M^L .

Remark 1 *Only when there is exactly one M^H and one M^L , offering different contracts may make a difference to the matching outcome. If the two men turn out to be of the same type, all contracts offering men non-negative payoffs achieve the same matching outcome and there is **no** efficiency loss in matching due to information asymmetry.*

Therefore in subsequent analysis, I mainly focus on scenarios with exactly one M^H and one M^L . To prove such a separating equilibrium exists, we need to show that given the other woman is offering the right equilibrium contract, the woman will not deviate from offering her equilibrium contract.

1. There is *no competition between W^H and W^L* as they desire different partners in terms of total surplus produced. $z_1 \leq z_0$ implies a low type woman prefers a low type man to a high type man in terms of total expected matching surplus produced. $z'_1 \leq z_2$ means a high type woman prefers a high type man to a low type man. When the two women turn out to be one W^H and one W^L , and there are one M^H and one M^L , each woman gets her desired partner by offering her equilibrium contract. It is clear that W^L does not have any incentive to compete against W^H for a high type man, and vice versa.

2. Now consider *competition between women of the same type*. To prevent deviations, I specify men's indifference behavior as the following: When being indifferent between an equilibrium contract and an off-equilibrium contract, both M^H and M^L always go to an **off-equilibrium** contract first.

2.1. When there are two W^H , one M^H , and one M^L , in the equilibrium both women offer contract A, so each has half probability matching with M^H and half probability matching with M^L . A high type woman may deviate to offer an off-equilibrium contract in order to achieve "attract only M^H " against contract A that is offered by the other W^H . If the off-equilibrium contract successfully achieves "attract only M^H ", she achieves a better matching outcome — matching with M^H with probability 1. However, such contract does not exist as any deviating contract will also attract a low type man. Since M^L gets 0 from contract A, given the specified men's indifference behavior, anything equal to or greater than 0 will attract him over from contract A. There does not exist a deviating contract achieves "attract only M^H ", hence no profitable deviation for W^H .

2.2. When there are two W^L , one M^H and one M^L , similar analysis as in 2.1, no deviating contract achieves "attract only M^L ", therefore no profitable deviation for W^L .

From 1 and 2, we conclude that when $z'_1 \leq z_2$ and $z_1 \leq z_0$, there exists a separating equilibrium that achieves matching efficiency. \square

Remark 2 When the four agents turn out to be exactly one W^H , one W^L , one M^H and one M^L , given the supermodularity condition $z_2 + z_0 \geq z_1 + z'_1$, a separating equilibrium that achieves matching efficiency requires that a high type woman matches with a high type man, and a low type woman matches with a low type man. In the presence of information asymmetry, the contracts offered serve two purposes: First they allow women to signal their types, offering contract A signals a woman is of high type, and offering contract B signals a woman is of low type. Second they serve as a screening device for women to attract a partner of the same type, contract A attracts M^H and contract B attracts M^L . Matching efficiency is achieved.

As the surplus value reveals information about the type of each agent with certainty, signaling (punishing a mimic) is costless. When we observe z_2 , we know it must be produced by a couple of high type agents; when we observe z'_1 , we know it must be produced by a high type woman and a low type man. As z_1 and z_0 will never be produced by a high type woman, in order to signal her type, she could offer men the full amount conditional on the output turns out to be z_1 or z_0 . By doing so a low type woman will not deviate to mimic her, as offering contract A means W^L always gets 0 surplus. Such signaling is costless for W^H as her actual payoff which is conditioned on z_2 or z'_1 , is not affected. Similarly for a low type woman, to signal her type (punishing a high type woman mimicking to be a low type woman), she offers full amount to men conditional on the output which turns out to be z_2 or z'_1 , which does not affect her actual payoff neither.

Second, when $z'_1 \leq z_2$ and $z_1 \leq z_0$, screening (attracting a partner of the same type) is costless for women. There is no competition between W^H and W^L as they desire different types of men: $z'_1 \leq z_2$ and $z_1 \leq z_0$ implies that W^H prefers M^H and W^L prefers M^L in terms of total surplus produced. In a separating equilibrium that achieves matching efficiency, both W^H and W^L achieve the best possible matching outcome by i) matching with a partner of the same type with the highest possible probability, and ii) extracting all the matching surplus since they are the side offering contracts.

2.1.2. Competition between W^H and W^L

Proposition 2 When $z_1 > z_0$, there exists a separating equilibrium that achieves matching efficiency where W^H offers contract A and W^L offers contract B:

Contract A		
Surplus	Woman	Man
z_2	$z_2 - \beta_2$	β_2
z'_1	z'_1	0
z_1	0	z_1
z_0	0	z_0

Contract B		
Surplus	Woman	Man
z_2	0	z_2
z'_1	0	z'_1
z_1	z_1	0
z_0	z_0	0

where $\beta_2 \in [\frac{z_1 - z_0}{4}, \frac{2(z_2 - z'_1)}{5}]$.

The best separating equilibrium for W^H is to offer $\beta_2 = \frac{z_1 - z_0}{4}$.

Equilibrium beliefs: $\mu(W^H | \text{contract A}) = 1$ and $\mu(W^L | \text{contract B}) = 1$

Equilibrium response rule: M^H strictly prefers contract A to contract B. When being indifferent between payoffs in contract A and contract B, M^L goes for contract B.

Off-equilibrium beliefs: $\mu(W^H | \text{contract C or F}) = 0$ and $\mu(W^H | \text{contract E}) = 1$

Off-Equilibrium response rule: When being indifferent between an equilibrium contract and an off-equilibrium contract, M^H will always go to an **equilibrium** contract first and M^L will always go to an **off-equilibrium** contract first.

We divide all the possible off-equilibrium contracts⁴ into 3 categories according to the amount offered to men conditional on the total surplus turns out to be z_1 . Contract C offers 0 to men when total surplus is z_1 , contract F offers men a strictly positive amount but no greater than β_2 , and contract E offers men an amount strictly greater than β_2 but no greater than z_1 .

Contract C		
Surplus	Woman	Man
z_2		
z'_1		
z_1	z_1	0
z_0		

Contract F		
Surplus	Woman	Man
z_2		
z'_1		
z_1	$z_1 - \gamma_1^F$	$0 < \gamma_1^F \leq \beta_2$
z_0		

Contract E		
Surplus	Woman	Man
z_2		
z'_1		
z_1	$z_1 - \gamma_1^E$	$\beta_2 < \gamma_1^E \leq z_1$
z_0		

Proof. When the four agents turn out to be exactly one W^H , one W^L , one M^H and one M^L , and when W^H offers contract A and W^L offers contract B, M^H will go for contract A and M^L will go for contract B. Matching efficiency is achieved.

To examine whether a deviation—either by offering an off-equilibrium contract or a “wrong” equilibrium contract (W^H offers contract B, or W^L offers contract A)—is profitable, I categorize all the contracts in terms of matching outcomes achieved when com-

⁴Note because they are off-equilibrium contracts, Contract C will not be the same as Contract B, and Contract E will not be the same as Contract A.

peting against contract A and against contract B (see Table 2). “Attract both” means both types of men go for that contract first, only when not successfully matched, a man will go for the remaining equilibrium contract. “Attract only M^H ” means only M^H prefers the contract to the other contract (one of the equilibrium contracts), while M^L prefers an equilibrium contract. Similar definitions for “Attract only M^L ” and “Attract both”.

Table 2: Categorization of Contracts
in terms of Matching Outcome Achieved when Competing Against Equilibrium Contracts

W^H/W^L		Competing against contract A			
		Attract neither	Attract only M^L	Attract only M^H	Attract both
Competing against contract B	Attract neither	Not Exist	Not Exist	Not Exist	Not Exist
	Attract only M^L	Not Exist	Contract C Contract E_1	Not Exist	Not Exist
	Attract only M^H	Not Exist	Not Exist	Not Exist	Contract A
	Attract both	Not Exist	Contract F Contract E_2 Contract B	Not Exist	Contract E_3

There does not exist a deviating off-equilibrium contract that achieves “attract neither” or “attract only M^H ” when competing against contract B or contract A. Since both contracts offer 0 to M^L , for any deviating contract offering M^L a non-negative payoff, he prefers the deviating contract to an equilibrium contract. This explains the “Not Exist” in row 1, row 3, column 1 and column 3 in Table 2.

Only the equilibrium contract A achieves “attract only M^H ” when competing against contract B. When the other woman also offers contract A, there is half probability the woman gets M^H and half probability she gets M^L . I put contract A into the category “attract both” when competing against contract A. Similarly I put contract B into the category “attract only M^L ” when competing against contract A and “attract both” when competing against contract B.

As contract B offers 0 to M^H , to achieve “attract only M^L ” against contract B, a deviating contract must also offer M^H 0 expected payoff. Contract C and contract E_1 below achieve “attract only M^L ” when competing against contract B, and contract A⁵:

Contract C		
Surplus	Woman	Man
z_2		
z'_1		
z_1	z_1	0
z_0		

Contract E_1		
Surplus	Woman	Man
z_2	z_2	0
z'_1		
z_1	$z_1 - \gamma_1^{E_1}$	$\gamma_1^{E_1} > \beta_2$
z_0		

Any deviating contract offering M^H a positive expected payoff will attract him over from contract B. Therefore contract E_2 , contract F and contract E_3 achieve “attract both” when competing against contract B. As the expected payoff in contract E_2 and contract F is smaller than or equal to a high type man’s payoff of matching with contract A,⁶ they achieve “attract only M^L ” when competing against contract A:

Contract E_2		
Surplus	Woman	Man
z_2	$z_2 - \gamma_2^{E_2}$	$0 < \gamma_2^{E_2} \leq \beta_2$
z'_1		
z_1	$z_1 - \gamma_1^{E_2}$	$\gamma_1^{E_2} > \beta_2$
z_0		

Contract F		
Surplus	Woman	Man
z_2		
z'_1		
z_1	$z_1 - \gamma_1^F$	$\gamma_1^F \leq \beta_2$
z_0		

Contract E_3 achieves “attract both” when competing against contract A, and against contract B⁷.

Contract E_3		
Surplus	Woman	Man
z_2	$z_2 - \gamma_2^{E_3}$	$\gamma_2^{E_3} > \beta_2$
z'_1		
z_1	$z_1 - \gamma_1^{E_3}$	$\gamma_1^{E_3} > \beta_2$
z_0		

⁵ $\mu(W^H|\text{contract C}) = 0$, and $\mu(W^H|\text{contract } E_1) = 1$. $P_C^{M^H} = 0$; $P_{E_1}^{M^H} = 0$, both are equal to $P_B^{M^H}$ and smaller than $P_A^{M^H}$, where P_K^i denotes the expected payoff of matching with contract K for agent i .

⁶ $\mu(W^H|\text{contract } E_2) = 1$ and $\mu(W^H|\text{contract } F) = 0$. $P_{E_2}^{M^H} = \gamma_2^{E_2} \leq \beta_2 = P_A^{M^H}$; $P_F^{M^H} = \gamma_1^F \leq \beta_2$.

⁷ $\mu(W^H|\text{contract } E_3) = 1$, $P_{E_3}^{M^H} = \gamma_2^{E_3} > \beta_2 = P_A^{M^H}$.

Table 3: Matching Outcomes of Offering Different Contracts

Prob	Three other agents	contract A	contract B	contract C & contract E_1	contract F & contract E_2	contract E_3
$\frac{1}{4}$	$W^L; M^H, M^L$	M^H	$\frac{1}{2}M^H + \frac{1}{2}M^L$	M^L	$\frac{1}{2}M^H + \frac{1}{2}M^L$	$\frac{1}{2}M^H + \frac{1}{2}M^L$
$\frac{1}{4}$	$W^H; M^H, M^L$	$\frac{1}{2}M^H + \frac{1}{2}M^L$	M^L	M^L	M^L	$\frac{1}{2}M^H + \frac{1}{2}M^L$
$\frac{1}{8}$	$W^L; M^H, M^H$	M^H	M^H	M^H	M^H	M^H
$\frac{1}{8}$	$W^H; M^H, M^H$	M^H	M^H	M^H	M^H	M^H
$\frac{1}{8}$	$W^L; M^L, M^L$	M^L	M^L	M^L	M^L	M^L
$\frac{1}{8}$	$W^H; M^L, M^L$	M^L	M^L	M^L	M^L	M^L
Expected matching		$\frac{5}{8}M^H + \frac{3}{8}M^L$	$\frac{3}{8}M^H + \frac{5}{8}M^L$	$\frac{1}{4}M^H + \frac{3}{4}M^L$	$\frac{3}{8}M^H + \frac{5}{8}M^L$	$\frac{1}{2}M^H + \frac{1}{2}M^L$

Following Table 2, given the other woman is offering the “right” equilibrium contract, Table 3 summarizes the different expected matching outcomes when a woman offers different contracts. To prove such a separating equilibrium exists, I proceed to show that the most profitable contract for W^H is contract A for $\beta_2 \in [\frac{z_1 - z_0}{4}, \frac{2(z_2 - z'_1)}{5}]$, and contract B dominates any other contract for W^L . See Appendix for details. \square

When $z_1 > z_0$, W^L may prefer to match with M^H since they together produce a larger surplus than if she is matched with M^L . Furthermore, $z_1 > z_0$ and $z_2 + z_0 \geq z_1 + z'_1$ imply $z_2 > z'_1$, which means W^H also prefers M^H to M^L in terms of total expected matching surplus produced. **There exists competition between W^H and W^L , attracting a high type man therefore becomes costly:** In an equilibrium that achieves matching efficiency, a high type woman needs to offer a certain amount in order to attract M^H , the amount offered prevents a low type woman from attempting to win the high type man over. Here W^H has to offer at least $\frac{z_1 - z_0}{4}$ to M^H , anything less than this amount will lead a low type woman to deviate to offer up to $\frac{z_1 - z_0}{4} - \varepsilon$ in order to attract M^H over and achieve a better matching outcome, and subsequently a higher expected payoff.

Second, for a high type woman, deterring a low type man is costless; she offers 0 to M^L conditional on z_1 and M^H 's payoff of matching with contract A is not affected.

Remark 3 *In the benchmark model, even we do not observe agents' types before a match, after observing the surplus, we can deduce the type of each agent with certainty. This enables different types of women to signal their types costlessly. When there is no competition between women of different types, women (the side offering contracts) extract all the surplus. When there is competition, men of the type desired by both women will get*

a positive amount while the not-preferred type gets 0 surplus⁸. A separating equilibrium with an efficient matching outcome can always be found in the contracts-offering game.

2.2. The Model: Matching Surplus is Stochastic

For each possible combination of types, we have the following:

$$\begin{aligned} W^H + M^H &: p_2 Y_G + (1 - p_2) Y_B = z_2 \\ W^H + M^L &: p'_1 Y_G + (1 - p'_1) Y_B = z'_1 \\ W^L + M^H &: p_1 Y_G + (1 - p_1) Y_B = z_1 \\ W^L + M^L &: p_0 Y_G + (1 - p_0) Y_B = z_0 \end{aligned}$$

Where $Y_G > Y_B \geq 0$, denote the good and the bad realization of surplus respectively. $p_i \in [0, 1]$, where $i = 0, 1, 1', 2$, denotes probability of realizing the good surplus state. The first line represents that when a high type woman matches with a high type man, they will realize the good surplus state with probability p_2 and the bad surplus state with probability $1 - p_2$. Their expected match surplus is z_2 . Similarly for the rest combination of types. As before, the expected match surplus z satisfies the supermodularity condition:

$$z_2 + z_0 \geq z_1 + z'_1$$

which implies PAM is efficient. Without loss of generality I assume $z_2 > z_0$.

The set-up captures the stochastic nature of the marital surplus. Even for a given partner's type, there is still uncertainty about the match surplus. Unlike the benchmark model, the ex-post surplus value no longer contains any information about agents' types. Although as before the different values of z represent different combinations of types, they are now magnitudes only *in expectation*. Remark 4 explains the implications.

Remark 4 *In the benchmark model, each combination of types generates a distinct value of surplus with certainty. Sharing rules conditional on different values indicate clearly which type of agent gets what amount. For example, the sharing of surplus conditional on z_1 specifies how much a woman, who must be of low type, gets out of z_1 ; and a man, who must be of high type, gets the rest of z_1 . However, when random shocks are introduced into the marital surplus in the model, although each combination of types still generates a distinct value of surplus, the values are only in expectation. The actual amount of surplus that could be conditioned on in a contract has to be the **realized** surplus state, Y_G and Y_B , which can be observed and verified ex-post. As Y_G and Y_B could be realized by any*

⁸I have shown in proposition 2 when $z_1 > z_0$, high type men are desired by both women hence M^H gets a positive payoff in contract A. I omit the analysis of the case $z_1 < z_0$ and $z_2 < z'_1$ where a low type man is desired by both types of women, the analysis is essentially the same, where M^L gets a positive payoff in contract B and M^H gets 0 in contract A.

combination of types, **any sharing rule conditional on any surplus state affects all types of agents**. In contrast with the benchmark model, the sharing of surplus no longer indicates which type of agents get what amount.

The game with one side offering contracts proceeds exactly as in the benchmark model, except that now each woman offers a contract proposing a *sharing of each possible realization of the surplus*. An example of a contract is illustrated below, a man gets ε when the total surplus turns out to be Y_G and gets 0 when it turns out to be Y_B .

Surplus	Woman	Man
Y_G	$Y_G - \varepsilon$	ε
Y_B	Y_B	0

2.2.1. No competition between W^H and W^L

Proposition 3 When $z_1 < z_0$ and $z'_1 < z_2$, there exists a separating equilibrium achieves matching efficiency where W^H offers contract A and W^L offers contract B:

Contract A			Contract B		
Surplus	Woman	Man	Surplus	Woman	Man
Y_G	$Y_G - \frac{p_1}{p_2}\varepsilon$	$\frac{p_1}{p_2}\varepsilon$	Y_G	$Y_G - \varepsilon$	ε
Y_B	Y_B	0	Y_B	Y_B	0

Equilibrium beliefs: $\mu(W^H | \text{contract A}) = 1$ and $\mu(W^L | \text{contract B}) = 1$

Equilibrium response rule: M^L prefers contract B to contract A. M^H is indifferent between payoffs of contract A and contract B, he goes for contract A.

Off-equilibrium beliefs: $\mu(W^H | \text{any off-equilibrium contracts}) = \frac{p_0 - p_1}{p_2 + p_0 - p'_1 - p_1} \in [0, 1]$

Off-equilibrium response rule: When being indifferent between an equilibrium contract and an off-equilibrium contract, both M^H and M^L will always go to an **off-equilibrium** contract first.

Proof. When the four agents turn out to be exactly one W^H , one W^L , one M^H and one M^L , and when W^H offers contract A and W^L offers contract B, a high type man's expected payoff of matching with contract A is $p_2 \frac{p_1}{p_2} \varepsilon = p_1 \varepsilon$, and is also $p_1 \varepsilon$ when being matched with contract B. He is indifferent between the payoffs and will go for contract A. A low type man's expected payoff of matching with contract A is $p'_1 \frac{p_1}{p_2} \varepsilon = \frac{p_1 p'_1}{p_2} \varepsilon$, which is smaller than $p_0 \varepsilon$, the expected payoff of being matched with contract B, since $z_1 < z_0$ and $z'_1 < z_2$ implies $p_1 < p_0$ and $p'_1 < p_2 \Rightarrow p_1 p'_1 < p_0 p_2$. He will go for contract B. Matching efficiency is achieved.

To show there is no profitable deviation for women to offer other contracts, the reasoning is essentially the same as Proposition 1 in the benchmark model. Competition exists

only between women of the same type. The off-equilibrium response rule helps to ensure that there does not exist an off-equilibrium contract that achieves a better matching outcome for women. See appendix. \square

I draw the similarities and differences of the model with the benchmark model in the case $z_0 > z_1$ and $z_2 > z'_1$.

Similar to the benchmark model:

1. Screening (attracting a partner of the same type) is costless for women. $z_0 > z_1$ and $z_2 > z'_1$ imply that there is no competition between W^H and W^L as they desire different types of men in terms of the expected total surplus produced. A woman desires a partner of the same type, and she is able to match with her desired type with the maximum probability in a separating equilibrium that achieves matching efficiency. This leads to women, the side offering contracts, to extract all the surplus.

2. When there is no competition between woman of different types, a separating equilibrium that achieves matching efficiency always exists, both in the benchmark model and in the model with stochastic marital surplus. No restrictions on the parameters of the model are required to find a separating equilibrium.

In contrast with the benchmark model: Signaling (punishing a mimic) is costless in the benchmark model. Offering men the full amount conditional on z_1 and z_0 prevents a low type woman from mimicking, and W^H 's payoff is not affected. However, in the model with stochastic marital surplus, since any sharing rule will affect all types of agents, there is no costless sharing rule to signal a woman's type. Despite that, since there is no competition between women of different types, the cost of signaling is very small: It only cost ε for low type women and $\frac{p_1}{p_2}\varepsilon$ for high type women. Offering a very small but different amount to men are enough to signal women's types.

2.2.2. Competition between W^H and W^L

Theorem 1 *When $z_1 > z_0$, a necessary condition for a separating equilibrium that achieves matching efficiency to exist is $p'_1 = 0$.*

Proof. For a separating equilibrium to achieve matching efficiency, we need women of different types offer different contracts, a high type man prefers a contract offered by a high type woman, and a low type man prefers a contract offered by a low type woman. Suppose there exists such a separating equilibrium with W^H offering contract A and W^L offering contract B:

Contract A		
Surplus	Woman	Man
Y_G	$Y_G - \beta_1$	β_1
Y_B	$Y_B - \beta_0$	β_0

Contract B		
Surplus	Woman	Man
Y_G	$Y_G - \alpha_1$	α_1
Y_B	$Y_B - \alpha_0$	α_0

where $\beta_1, \alpha_1 \in [0, Y_G]$, $\beta_0, \alpha_0 \in [0, Y_B]$

Equilibrium beliefs: $\mu(W^H|\text{contract A}) = 1$ and $\mu(W^L|\text{contract B}) = 1$

M^H prefers contract A to contract B if and only if

$$P_A^{M^H} = p_2\beta_1 + (1 - p_2)\beta_0 \geq p_1\alpha_1 + (1 - p_1)\alpha_0 = P_B^{M^H} \quad (2)$$

M^L prefers contract B to contract A if and only if

$$P_B^{M^L} = p_0\alpha_1 + (1 - p_0)\alpha_0 \geq p'_1\beta_1 + (1 - p'_1)\beta_0 = P_A^{M^L} \quad (3)$$

For contract A to be different from contract B, we need $\beta_1 \neq \alpha_1$ or $\beta_0 \neq \alpha_0$, or both.

As $z_1 > z_0$ and $z_2 > z'_1$,⁹ both W^H and W^L prefer M^H to M^L in terms of the total expected matching surplus produced. To prevent a profitable deviation to the largest extent, I specify men's indifference behavior as the following: When being indifferent between expected payoffs of contracts, M^H will always first go for an *equilibrium* contract (contract A or contract B), and M^L will always first go for an *off-equilibrium* contract.

Step 1: First I show that $p_0\alpha_1 + (1 - p_0)\alpha_0 = 0$ and $p'_1\beta_1 + (1 - p'_1)\beta_0 = 0$ are necessary conditions for such a separating equilibrium to exist.

Proof of Step 1: Prove by contradiction. Suppose $p_0\alpha_1 + (1 - p_0)\alpha_0 > 0$, then there exists a profitable deviation for W^L to offer contract C which offers 0 to men:

Contract C		
Surplus	Woman	Man
Y_G	Y_G	0
Y_B	Y_B	0

Given specified men's indifference behavior, when a high type man sees an equilibrium contract and contract C which offers him nothing, he will always go for an equilibrium contract first. Contract C never achieves the matching outcome “attracts W^H ”.

When a low type man sees contract B and contract C, he compares $p_0\alpha_1 + (1 - p_0)\alpha_0$ and 0. Since $p_0\alpha_1 + (1 - p_0)\alpha_0 > 0$, he will first go for contract B. Contract C achieves “attracts neither” when competing against contract B.

When M^L sees contract A and contract C, he compares $p'_1\beta_1 + (1 - p'_1)\beta_0$ and 0. If $p'_1\beta_1 + (1 - p'_1)\beta_0 > 0$, he will first go for contract A; then contract C achieves “attracts neither” when competing against contract A. If $p'_1\beta_1 + (1 - p'_1)\beta_0 = 0$, he will first go for contract C, then contract C achieves “attracts only M^L ” when competing against contract A. Table 4 summarizes the matching outcomes of offering contract C against equilibrium contracts. Table 5 then summarizes the different matching outcomes when W^L offers contract B and contract C, given the other woman is offering her equilibrium contract.

⁹ $z_1 > z_0$ and $z_2 + z_0 \geq z_1 + z'_1 \Leftrightarrow z_2 - z'_1 \geq z_1 - z_0 > 0 \Leftrightarrow z_2 - z'_1 > 0$

Table 4: Contract C's Matching Outcome
when Competing Against Equilibrium Contracts

Contract C's matching outcome	$p_0\alpha_1 + (1 - p_0)\alpha > 0$
① $p'_1\beta_1 + (1 - p'_1)\beta_0 > 0$	against contract B: attracts neither against contract A: attracts neither
② $p'_0\beta_1 + (1 - p'_1)\beta_0 = 0$	against contract B: attracts neither against contract A: attracts only M^L

Table 5: Matching outcome of W^L Offering Contract B and Contract C

Prob	Other three agents	Contract B	Contract C	
			①	②
$\frac{1}{4}$	$W^L; M^H, M^L$	$\frac{1}{2}M^H + \frac{1}{2}M^L$	$\frac{1}{2}M^H + \frac{1}{2}M^L$	$\frac{1}{2}M^H + \frac{1}{2}M^L$
$\frac{1}{4}$	$W^H; M^H, M^L$	M^L	$\frac{1}{2}M^H + \frac{1}{2}M^L$	M^L
$\frac{1}{8}$	$W^L; M^H, M^H$	M^H	M^H	M^H
$\frac{1}{8}$	$W^H; M^H, M^H$	M^H	M^H	M^H
$\frac{1}{8}$	$W^L; M^L, M^L$	M^L	M^L	M^L
$\frac{1}{8}$	$W^H; M^L, M^L$	M^L	M^L	M^L
Expected matching		$\frac{3}{8}M^H + \frac{5}{8}M^L$	$\frac{1}{2}M^H + \frac{1}{2}M^L$	$\frac{3}{8}M^H + \frac{5}{2}M^L$
Profitable deviation			Yes	Yes

Table 5 suggests that contract C is a profitable deviation when $p_0\alpha_1 + (1 - p_0)\alpha_0 > 0$. By offering contract C, the low type woman's chances of matching with M^H is either improved in case ① or remain unchanged in case ②, so her expected payoff becomes higher as contract C allows women to extract all the surplus¹⁰. Therefore for a separating equilibrium to exist, we need $p_0\alpha_1 + (1 - p_0)\alpha_0 = 0$. This means a low type man gets 0 expected payoff from contract B.

From inequality (3), we must have $p'_1\beta_1 + (1 - p'_1)\beta_0 = 0$, that M^L also gets 0 payoff from contract A. \square

Step 2: Discuss two cases: either $\alpha_1 = \alpha_0 = 0$ or $\alpha_0 = 0, \alpha_1 > 0$.

Since $z_0 < z_1 \Rightarrow 0 \leq p_0 < p_1 \leq 1 \Rightarrow 1 - p_0 > 0$, for $p_0\alpha_1 + (1 - p_0)\alpha_0 = 0$ we need

¹⁰In expectation she gets $\frac{3}{8}z_1 + \frac{5}{8}z_0$ in case ② and $\frac{1}{2}z_1 + \frac{1}{2}z_0$ in case ①, while her expected payoff from offering the equilibrium contract B is $\frac{3}{8}z_1 + \frac{5}{8}z_0 - \frac{3}{8}[p_1\alpha_1 + (1 - p_1)\alpha_0] - \frac{5}{8}[p_0\alpha_1 + (1 - p_0)\alpha_0]$ which is strictly less than $\frac{3}{8}z_1 + \frac{5}{8}z_0$ and $\frac{1}{2}z_1 + \frac{1}{2}z_0$ when $p_0\alpha_1 + (1 - p_0)\alpha > 0$.

$\alpha_0 = 0$. $z_2 - z'_1 > 0 \Rightarrow (p_2 - p'_1)(Y_G - Y_B) > 0 \Rightarrow p_2 - p'_1 > 0 \Rightarrow p'_1 < p_2 \leq 1 \Rightarrow 1 - p'_1 > 0$. For $p'_1\beta_1 + (1 - p'_1)\beta_0 = 0$ we need $\beta_0 = 0$. In order to find values of β_1 and α_1 , I discuss two cases: $\alpha_1 = \alpha_0 = 0$ or $\alpha_0 = 0, \alpha_1 > 0$.

Case 1: $\alpha_1 = \alpha_0 = 0$

Since $\beta_0 = 0$, we need $\beta_1 > 0$ for contract A to be different from contract B.

Case 2: $\alpha_0 = 0, \alpha_1 > 0$.

For $p_0\alpha_1 + (1 - p_0)\alpha_0 = 0$, we need $p_0 = 0$. Condition (2) implies $p_2\beta_1 \geq p_1\alpha_1 > 0$ since $p_1 > p_0 \geq 0$ and $\alpha_1 > 0$. Therefore we need $p_2 > 0$ and $\beta_1 > 0$.

Both cases need $\beta_1 > 0$. For $p'_1\beta_1 + (1 - p'_1)\beta_0 = 0$, it must be the case that $p'_1 = 0$. \square

The requirement for efficiency when the marital surplus is stochastic is that we need $p'_1 = 0$ or equivalently $W^H + M^L = Y_B$. Only if the combination of an agent ranked high in the side offering contracts and an agent ranked low in the side choosing contracts, leads to a *production of the lowest possible marital surplus with certainty*, an efficient matching outcome may exist. The reasoning is explained below:

In a separating equilibrium that achieves matching efficiency, when there exists competition for a high type man, a low type woman always loses the competition to a high type woman. She matches with M^H with a positive probability only when there are two M^H or the other woman is also of low type. Therefore the best for her is to offer 0 to M^H . As M^L is desired by W^L , she offers 0 to M^L . A low type woman therefore offers 0 to men in the equilibrium. Conditional on the matching outcome, offering nothing to men achieves the highest possible expected surplus for W^L .

For a high type woman, in order to attract only high type men and prevent low type men from contract B, she needs to offer 0 expected surplus to M^L , as any positive expected payoff will attract M^L since he only gets 0 from W^L . However, in order to signal her type, W^H needs to offer a contract different from W^L . This is possible only if $p'_1 = 0$ or $W^H + M^L = Y_B$. When a low type man and a high type woman produce Y_B with certainty, it allows W^H to offer M^L 0 conditional on Y_B , and offer M^H some positive amount conditional on Y_G to attract M^H , such contract will attract only M^H . Only when $p'_1 = 0$, there is no trade-off between screening (attracting a man only of the same type) and signaling (offering a contract different from a low type woman) for W^H .

Theorem 2 *When $z_1 > z_0$ and $p'_1 = 0$, there exists a separating equilibrium that achieves matching efficiency where W^H offers contract A and W^L offers contract B:*

Contract A		
Surplus	Woman	Man
Y_G	$Y_G - \beta_1$	β_1
Y_B	Y_B	0

Contract B		
Surplus	Woman	Man
Y_G	Y_G	0
Y_B	Y_B	0

where $\beta_1 \in [\frac{2(z_1 - z_0)}{5p_1 + 3p_0}, \frac{2}{5}(Y_G - Y_B)]$.

The best separating equilibrium for W^H is to offer $\beta_1 = \frac{2(z_1 - z_0)}{5p_1 + 3p_0}$.

Equilibrium beliefs: $\mu(W^H | \text{contract A}) = 1$ and $\mu(W^L | \text{contract B}) = 1$

Equilibrium response rule: M^H will go for contract A as the expected payoff is strictly higher than that in contract B. M^L is indifferent between payoffs of contract A and contract B, he goes for contract B.

Off-equilibrium beliefs: $\mu(W^H | \text{any off-equilibrium contracts}) = \frac{1}{2}$

Off-equilibrium response rule: When being indifferent between an equilibrium contract and an off-equilibrium contract, M^H will always go to an **equilibrium** contract first and M^L will always go to an **off-equilibrium** contract first.

Proof. Consider the case that the four agents turn out to be exactly one W^H , one W^L , one M^H and one M^L . When W^H offers contract A and W^L offers contract B, a high type man's expected payoff of matching with contract A is $p_2\beta_1$, and is 0 when being matched with contract B. He prefers contract A since $p_2\beta_1 > 0$. A low type man's expected payoff of matching with contract A is $p'_1\beta_1 = 0$ since $p'_1 = 0$, and is also 0 when being matched with contract B. He is indifferent between the expected payoffs and will go for contract B. Matching efficiency is achieved.

To show there is no profitable deviation for women to offer other contracts when there exists competition for M^H , the proof is technically more complicated than the proof in Proposition 2 as now there is more uncertainty about the matching outcome, but the reasoning is essentially the same. First I categorize all the contracts in terms of matching outcomes achieved when competing against contract A and against contract B. Then I proceed to show given the specified off-equilibrium beliefs and off-equilibrium response rule, the most profitable contract for W^H is contract A with $\beta_2 = \frac{2(z_1 - z_0)}{5p_1 + 3p_0}$, and contract B dominates any other contract for W^L . See appendix. \square

I draw the similarities and differences of the model with the benchmark model in the case $z_1 > z_0$.

Similar to the benchmark model:

1. When there exists competition for M^H between women of different types, a low type woman always offers nothing to men. Because she always loses the competition to a high type woman in the equilibrium, and M^L is not her desired type, she offers 0 to both M^H and M^L to maximize her expected payoff. A high type woman needs to offer a positive amount to M^H to prevent a low type woman from attempting to win him over, and to offer 0 to M^L to deter him. In both the benchmark model and in the model with stochastic marital surplus, high type men's expected payoff is strictly positive and low type men's payoff is 0.

2. In the benchmark model, the amount offered to M^H cannot be greater than $\frac{2}{5}(z_2 - z'_1)$. Similarly in the model with stochastic marital surplus, the amount offered to M^H conditional on Y_G cannot be greater than $\frac{2}{5}(Y_G - Y_B)$, this is equivalent to M^H 's expected payoff being no greater than $p_2 \frac{2}{5}(Y_G - Y_B) = \frac{2}{5}(z_2 - z'_1)$. The upper bound is to prevent a high type woman from giving up attracting high type men: As the cost of attracting M^H becomes too high, high type women may find offering nothing to men becomes a profitable deviation despite a worse matching outcome.

In contrast to the benchmark model:

1. In the benchmark model, a separating equilibrium that achieves matching efficiency always exists. However, in the model with stochastic marital surplus, a separating equilibrium exists only if $p'_1 = 0$. When random shocks are introduced into the marital surplus with *non-trivial* probabilities (neither 0 nor 1), there is no separating equilibrium in the contracts-offering game¹¹. As pointed out in Remark 4, since Y_G and Y_B could be realized by any combination of types, any sharing rule conditional on any surplus state affects all types of agents. A high type woman needs to deter a low type man by offering him 0, which means she has to offer men 0 conditional on both Y_G and Y_B . However, such contract does not enable the high type woman to signal her type since a low type woman also offers 0 to men in the equilibrium. The conflict between deterring a low type partner (offering him zero amount) and signalling her high type (offering a contract different from a low type woman) renders contracts ineffective in achieving an efficient matching.

2. In the benchmark model, the amount offered to a high type man by a high type woman needs to be at least $\frac{z_1 - z_0}{4}$. This is to prevent a low type woman from attempting to offer slightly more to win the high type man over. *It is not related to preventing a mimic as women have costlessly signaled their types.*

In the model with stochastic marital surplus, preventing a mimic may become costly for high type women (depends on whether $p_2 > p_1$). Consider a low type woman deviates to mimic M^H by offering contract A. She will be regarded as a high type woman: When there is only one high type man, not only she wins the competition against another W^L , but also she has half chance of matching with M^H when competing against W^H .

In fact, offering contract A always enables women to match with M^H with a higher probability than offering contract B. In the benchmark model, this better matching does not lead to a higher expected payoff for a deviating W^L since W^H can costlessly punish a mimic by offering full amount to men conditional on z_1 and z_0 . However, in the model

¹¹In this case I have specified women to be the side offering contracts, there could exist a separating equilibrium if we let men offer contracts instead in this case, see Figure 2. However in the men offering contracts game, we then require $p_1 = 0$ when $z'_1 > z_0$. The main message is that even if we can let either side offer contracts, in the case $z_2 > z_1 > z_0$ and $z_2 > z'_1 > z_0$, there is still no separating equilibrium that achieves matching efficiency, see Claim 1.

with stochastic marital surplus, the only way she can punish a mimic is to offer a large amount to men conditional on Y_G , since she has to offer 0 to men conditional on Y_B .

Suppose W^H offers β to M^H conditional on Y_G . M^H calculates his expected payoff as $p_2\beta$ when he sees contract A. A deviating M^L offering contract A will actually offer M^H expected payoff of only $p_1\beta$. If $p_2 > p_1$, then it means low type women gain from mimicking. As a result high type women have to bear the cost of signalling, offering a larger¹² expected amount to high type men than what she offers M^H in the benchmark model, in order to prevent W^L from mimicking.

Corollary 1 *When $z_1 > z_0$, a necessary and sufficient condition for finding a separating equilibrium that achieves matching efficiency is $p'_1 = 0$.*

Figure 1 summarizes the range of z that a separating equilibrium with matching efficiency can be found in the game with women offering contracts. The vertical red line represents Corollary 1: Only when $p'_1 = 0$ ($z'_1 = Y_B$), there exists a separating equilibrium with matching efficiency. Similarly the horizontal red line represents a symmetric case where women of different types desire a low type man¹³: Only when $p_1 = 0$ ($z_1 = Y_B$), there exists a separating equilibrium. The red region represents Proposition 3: When there is no competition between women of different types, a separating equilibrium that achieves matching efficiency can always be found.

Figure 2 is the mirror image of Figure 1 when men become the side offering contracts.

Suppose we have the freedom to choose which side to offer contracts, then combining Figure 1 and Figure 2, Figure 3 represents the largest range of z where a separating equilibrium with matching efficiency can be found. As Figure 3 suggests, when the probability of realizing the good surplus state is non-trivial (neither 0 nor 1), and when $z_2 > z_1 > z_0$ and $z_2 > z'_1 > z_0$, that is, the expected marital surplus is strictly monotonic in agents' types, there is no separating equilibrium that achieves matching efficiency in the contracts-offering game.

To summarize main findings of the model:

Claim 1 *When there is two-sided information asymmetry and the marital surplus is stochastic, and PAM is the efficient matching, let one side of the market, say women, offers contracts that propose some sharing rule of the realized surplus to men:*

1. *When there exists competition between women of different types, a necessary and sufficient condition to find a separating equilibrium that achieves matching efficiency*

¹² $p_2 \frac{2(z_1 - z_0)}{5p_1 + 3p_0} > \frac{z_1 - z_0}{4}$ if $p_2 > p_1$. This is analogous to the classical Spence's signalling model that high ability workers have to **over-invest** in education in order to signal their types.

¹³ See appendix Lemma 1 and Lemma 2 in the appendix for the analysis of the case $z_1 < z_0$ and $z_2 \leq z'_1$ where there exists competition for M^L between women of different types. The proofs and reasoning are essentially the same as Theorem 1 and Theorem 2 where there exists competition for M^H .

Figure 1: Game with Women Offering Contracts

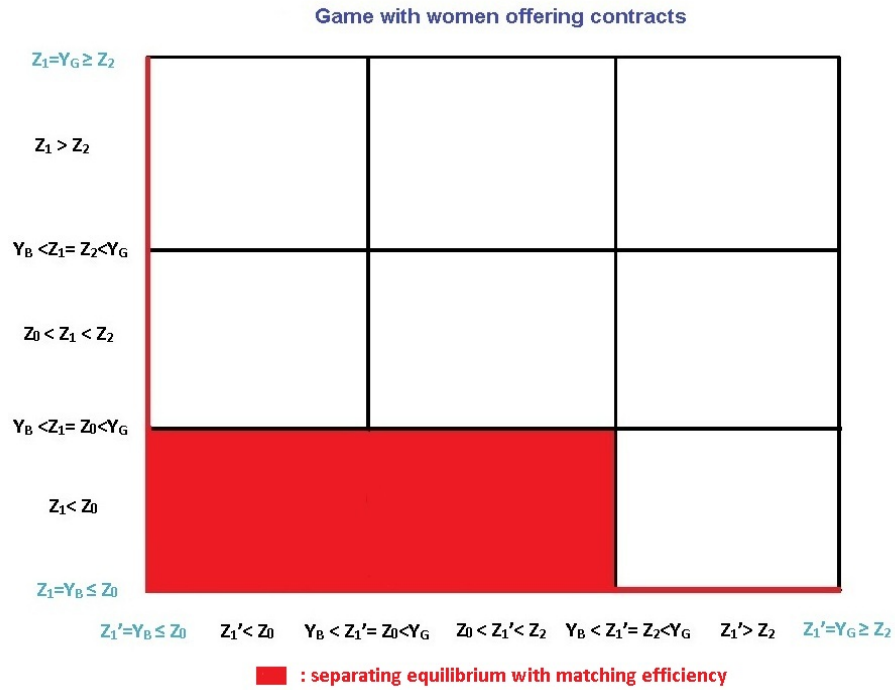


Figure 2: Game with Men Offering Contracts

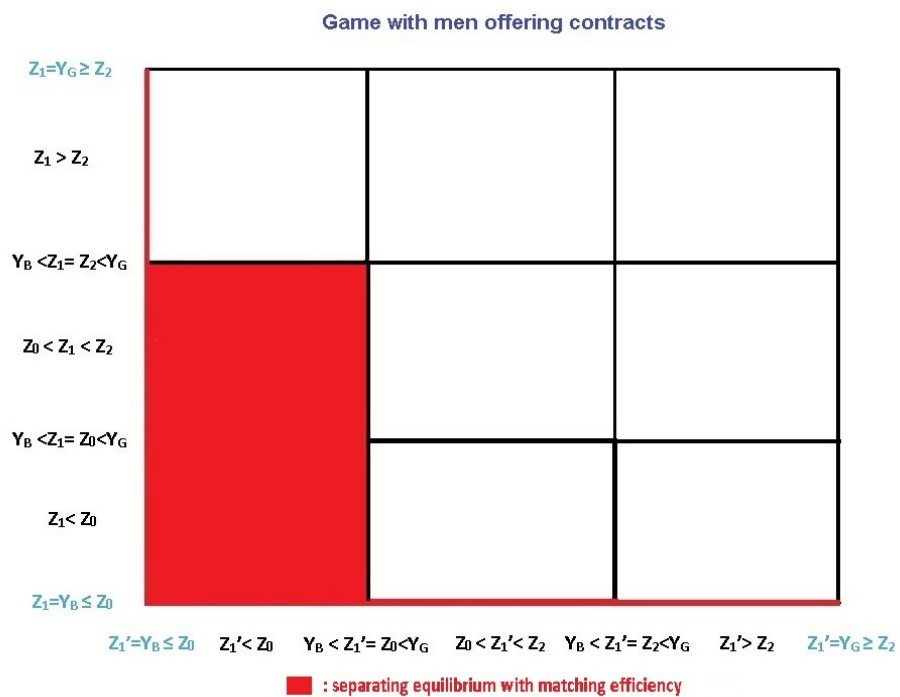
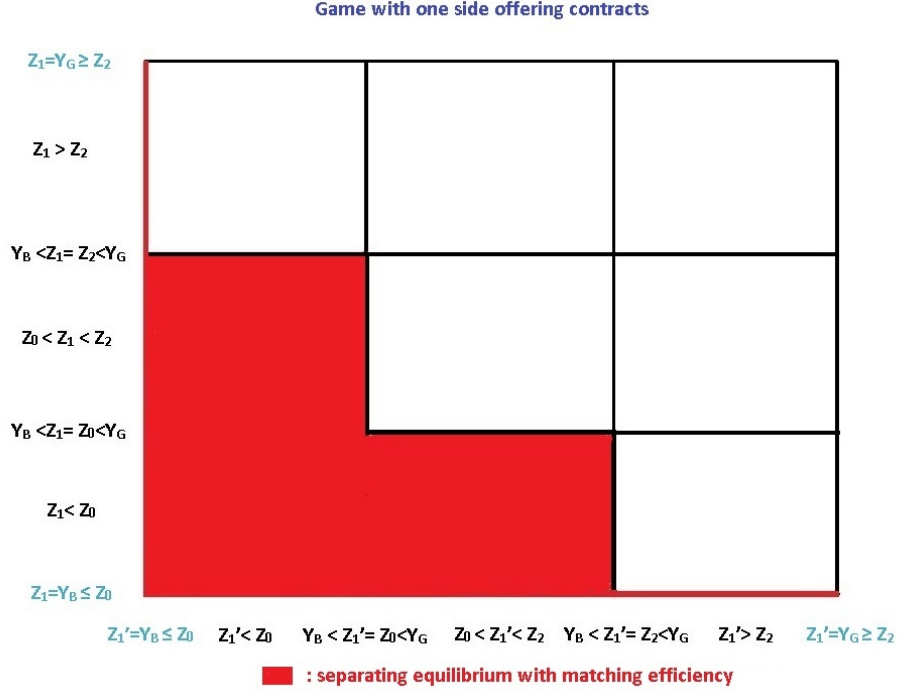


Figure 3: Game with Either Side Offering Contracts



is that the combination of a man of the non-desired type and a woman of a different type, leads to a production of the **lowest possible surplus with certainty**.

2. When expected surplus is strictly monotonic in agents' types, or agents of different types have the same rank of potential partners in terms of generating the total expected match surplus, there is **no** separating equilibrium that achieves an efficient matching outcome.

3. Social Planner's Mechanism

As pointed by Claim 1.2, when the probabilities of realizing the good surplus state are non-trivial (neither 0 nor 1), and when the side offering contracts have the same preference about partners' types in terms of total expected match surplus produced, the contracts-offering game in Section 2 cannot achieve matching efficiency. Section 3 asks is there any other mechanism able to achieve matching efficiency?

The set-up of a social planner's problem is the following:

Each agent, having observed only his or her own type, could either simultaneously and costlessly report his or her type to a social planner or choose not to report. A social planner commits to a mechanism specifying who matches with whom and how each

surplus is shared as a function of the reported types. The social planner designs contracts $C1-4$, choosing β , μ , τ , and α to help achieve matching efficiency:

Contract $C_1: W^H + M^H$		
Surplus	Woman	Man
Y_G	$Y_G - \beta_1$	β_1
Y_B	$Y_B - \beta_0$	β_0

Contract $C_2: W^H + M^L$		
Surplus	Woman	Man
Y_G	$Y_G - \mu_1$	μ_1
Y_B	$Y_B - \mu_0$	μ_0

Contract $C_4: W^L + M^L$		
Surplus	Woman	Man
Y_G	$Y_G - \alpha_1$	α_1
Y_B	$Y_B - \alpha_0$	α_0

Contract $C_3: W^L + M^H$		
Surplus	Woman	Man
Y_G	$Y_G - \tau_1$	τ_1
Y_B	$Y_B - \tau_0$	τ_0

For $N=2$, if there is exactly one reported “high” type agent on each side of the matching market, the social planner matches “ W^H ” with “ M^H ” with the sharing rule specified in contract C_1 and matches “ W^L ” with “ M^L ” with the sharing rule specified in contract C_4 . Matching efficiency is achieved. In other cases the social planner randomly matches the agents, as any re-arranging of the matching does not affect matching efficiency.

For general N , to achieve matching efficiency, the social planner makes sure the reported “high” type agents on the side which has fewer “high” type agents will all be matched with “high” type agents on the other side of the matching market.

Each agent, having observed his or her own type and knowing the contracts and matching rules, chooses whether to report his or her type truthfully.

By the *Revelation Principle*, Section 3 restricts attention to a direct revelation mechanism where each agent chooses to report truthfully to the social planner.

3.1. The two men-two women case

Proposition 4 *In a symmetric setting ($z_1 = z'_1$), if matching efficiency can be achieved, it can be achieved by a symmetric mechanism in which $\beta_1 = \alpha_1 = \frac{Y_G}{2}$, $\beta_0 = \alpha_0 = \frac{Y_B}{2}$ and $\mu_1 = Y_G - \tau_1$, $\mu_0 = Y_B - \tau_0$.*

Contract $C_1: W^H + M^H$		
Surplus	Woman	Man
Y_G	$\frac{Y_G}{2}$	$\frac{Y_G}{2}$
Y_B	$\frac{Y_B}{2}$	$\frac{Y_B}{2}$

Contract $C_2: W^H + M^L$		
Surplus	Woman	Man
Y_G	$Y_G - \mu_1$	μ_1
Y_B	$Y_B - \mu_0$	μ_0

Contract $C_4: W^L + M^L$		
Surplus	Woman	Man
Y_G	$\frac{Y_G}{2}$	$\frac{Y_G}{2}$
Y_B	$\frac{Y_B}{2}$	$\frac{Y_B}{2}$

Contract $C_3: W^L + M^H$		
Surplus	Woman	Man
Y_G	μ_1	$Y_G - \mu_1$
Y_B	μ_0	$Y_B - \mu_0$

Proof. See appendix. \square

Intuitively, when the genders are symmetric in terms of generating the surplus¹⁴, a symmetric mechanism which treats genders equally supports a separating equilibrium to the largest extent. If a symmetric mechanism cannot achieve matching efficiency in this setting, no asymmetric mechanism can. Therefore I focus the analysis on a symmetric mechanism: When the agents of the same types are matched, they always share the surplus equally. When agents of different types are matched, the high type agent gets the same amount regardless of one's gender. Hence, when $z_1 = z'_1$, the problem simplifies to can we find some $\mu_1 \in [0, Y_G]$ and some $\mu_0 \in [0, Y_B]$ for a truthful direct revelation mechanism (TDRM) to achieve an efficient matching outcome?

Proposition 5 *In a symmetric setting ($z_1 = z'_1$) with $N=2$, there exists a truthful direct revelation mechanism where a social planner designs contracts to achieve matching efficiency if and only if*

$$\frac{2(p_1^2 - p_2 p_0)}{3(p_2 - p_0)} \leq \frac{Y_B}{Y_G - Y_B}.$$

Proof. See appendix. \square

The condition $\frac{2(p_1^2 - p_2 p_0)}{3(p_2 - p_0)} \leq \frac{Y_B}{Y_G - Y_B}$ is to ensure that the smallest μ_0^* , which satisfies both high type and low type agents' incentive compatibility constraints, is no greater than Y_B .

Remark 5 *With $N = 2$, a social planner can do **strictly better** in terms of achieving matching efficiency than the equilibrium of the game in which one side offers contracts.*

Whenever the contracts-offering game achieves matching efficiency with W^H offering contract A and W^L offering contract B, by replicating the contracts, matching efficiency can always be achieved in the social planner's setting. In addition, the social planner restricts the possible deviations of sharing the surplus to a much smaller set. In the contracts-offering game, for a separating equilibrium to exist, not only we need to show a woman will not deviate to mimic the other type, but also need to show that she will not deviate to offer *any other* contracts. In the social planner's setting, signaling becomes a *binary* choice as an agent can only report either high or low. This is equivalent to restrict a woman to offer either contract A or contract B in the contracts-offering game.

When expected surplus is strictly monotonic in agents' types, the social planner achieves an efficient matching under the condition $\frac{2(p_1^2 - p_2 p_0)}{3(p_2 - p_0)} \leq \frac{Y_B}{Y_G - Y_B}$, even the one side offering contracts game does not.

Combining the two cases, we conclude that when $N=2$, a social planner does strictly better in terms of achieving matching efficiency than the contracts-offering game.

¹⁴A high type woman matches with a low type man, or a low type woman matches with a high type man, both pairs generate the same amount of surplus: $z_1 = z'_1$.

3.2. The N men- N women Case

Proposition 6 Let P_N denotes the probability of matching with a high type man, M^H , when a woman reports “high” type to the social planner.

$$P_N = 1 - \frac{1}{2^{2N-1}} \binom{2N-1}{N}$$

It has the following properties:

1. $P_N > \frac{1}{2}$;
2. P_N increases with N and $\frac{2P_N-1}{1-P_N}$ increases with N ;
3. $1 - P_N \approx \frac{1}{\sqrt{\pi N}}$ when N is large.

Table 6: Probability of Matching with M^H after Reporting as W^H

		No. of high type men, M^H							
		0	1	2	3	...	$N-2$	$N-1$	N
Total No. of reported W^H	1	0	1	1	1	...	1	1	1
	2	0	$\frac{1}{2}$	1	1	...	1	1	1
	3	0	$\frac{1}{3}$	0	1	...	1	1	1

	$N-2$	0	$\frac{1}{N-2}$	$\frac{2}{N-2}$	$\frac{3}{N-2}$...	1	1	1
	$N-1$	0	$\frac{1}{N-1}$	$\frac{2}{N-1}$	$\frac{3}{N-1}$...	$\frac{N-2}{N-1}$	1	1
	N	0	$\frac{1}{N}$	$\frac{2}{N}$	$\frac{3}{N}$...	$\frac{N-2}{N}$	$\frac{N-1}{N}$	1

Proof. Table 6 explains calculation of P_N , the probability of matching with M^H when a woman reports “high” type to the social planner. Whether she can be matched with M^H depends on i) how many high type men are there, and ii) how many women are also reporting “high” type. For instance, when all the other women are reporting “low” type, the woman becomes the only one reporting “high” type, she will be matched with a high type man with probability 1 when the number of M^H equal to or greater than one (see line 1). If all the other $N-1$ women are also reporting “high” type, and there is only one M^H , then her probability of matching with M^H becomes $\frac{1}{N}$ (see the last line, column 2). To calculate P_N , I sum up all conditional probabilities in Table 6. See appendix. \square

The properties of P_N are quite intuitive. 1. With random matching, a woman will be matched with M^H with probability half. In a mechanism that aims to achieve matching efficiency, a woman reporting “high” type will be matched with M^H with a probability

greater than half as the social planner attempts to match W^H with M^H whenever it is possible. 2. As the number of agents increases on both sides of the market, a woman is more likely to face the scenario that there are $\frac{N-1}{2}$ high type women out there and $\frac{N}{2}$ high type men. The probability of the woman reporting “high” type and then matching with M^H becomes $\frac{N/2}{(N-1)/2+1} = \frac{1}{1+\frac{1}{N}}$ which increases with N . 3. As N becomes very large, the probability approaches 1.

Corollary 2 *The probability of matching with a low type man, M^L , when a woman reports “low” type to the social planner, is also P_N .*

Theorem 3 *In the symmetric setting with N men and N women, there exists a truthful direct revelation mechanism which achieves an efficient matching if and only if*

$$\frac{(2P_N - 1)(p_1^2 - p_2 p_0)}{(1 - P_N)(p_2 - p_0)} \leq \frac{Y_B}{Y_G - Y_B}$$

and

$$(1 - P_N) z_1 \geq P_N \left(\frac{z_1 - z_0}{2} \right)$$

$$\text{where } P_N = 1 - \frac{1}{2^{2N-1}} \binom{2N-1}{N}.$$

Proof. See appendix. □

The condition $\frac{(2P_N-1)(p_1^2-p_2p_0)}{(1-P_N)(p_2-p_0)} \leq \frac{Y_B}{Y_G-Y_B}$ is to ensure that the smallest μ_0^* , which satisfies both high type and low type agents’ incentive compatibility constraints, is no greater than Y_B . As the number of agents increases, $\frac{2P_N-1}{1-P_N}$ increases, the inequality is harder to sustain. Similarly for $\frac{P_N}{1-P_N} \leq \frac{2z_1}{z_1-z_0}$. The two conditions to ensure matching efficiency become more stringent as N increases.

Condition (4) represents low type agents’ incentive compatibility constraint:

$$\underbrace{(1 - P_N) z_1}_{\text{Largest expected loss by misreporting as “H”}} \geq \underbrace{P_N \left(\frac{z_1 - z_0}{2} \right)}_{\text{Expected gain by misreporting as “H”}} \quad (4)$$

The right-hand side of inequality (4) represents the expected gain from mis-reporting as a “high” type agent. For a low type women (same analysis for a low type men), if she truthfully reports as W^L , she will be matched with M^L with probability P_N and sharing the surplus z_0 equally. If she mis-reports as W^H , she will be matched with M^H with probability P_N and share the surplus z_1 equally. Therefore her expected gain from mis-reporting is $P_N(\frac{z_1-z_0}{2})$. The left-hand side represents the largest expected loss from mis-reporting as a “high” type agent. In a symmetric mechanism, the most severe

punishment to a deviating low type agent is to set $\mu_1 = Y_G$ and $\mu_0 = Y_B$.¹⁵ After reporting truthfully as “low” type, she matches with M^H with probability $1 - P_N$ and gets z_1 ; after mis-reporting as “high” type, she matches with M^L with probability $1 - P_N$ and gets 0. The largest expected loss from mis-reporting is therefore $(1 - P_N)z_1$. Condition (4) is necessary to prevent low type agents from mis-reporting.

Note that when N is large, P_N approaches 1, condition (4) becomes $0 \geq \frac{z_1 - z_0}{2}$ which does not hold when $z_1 > z_0$. Matching efficiency therefore cannot be achieved in the social planner’s setting. Because in a symmetric mechanism, agents of the same reported types share the surplus equally, when a woman reports as “low” type, she will match with a low type man with probability almost equal to 1 and gets an expected surplus of $\frac{z_0}{2}$. When she reports as “high” type, she will match with a high type man with probability almost equal to 1 and gets an expected surplus of $\frac{z_1}{2}$. It is always profitable for a low type agent to mis-report when N is large.

To summarize main findings of the social planner’s problem:

Claim 2 *When there is two-sided information asymmetry and the marital surplus is stochastic, consider a truthful direct revelation mechanism where a social planner designs contracts proposing some sharing rules of the realized surplus:*

1. *With finite number of agents, the social planner **can do strictly better** than the contracts-offering game.*
2. *His ability to achieve matching efficiency **decreases** with the number of agents in the matching market.*
3. *When N is very large, an efficient matching cannot be achieved.*

4. Discussion

1. First Mover Advantage in the contracts-offering game. When there is no competition between women of different types, and if women are the side offering contracts, they are able to extract almost all the surplus. The signalling costs are very small. When there is competition for M^H , in the equilibrium W^L extracts all the surplus, and W^H offers a positive amount to M^H conditional on Y_G to prevent W^L from mimicking. Nevertheless, a high type man’s expected payoff is always strictly less than that of a high type woman.¹⁶ Similar results hold when there is competition for M^L . Hence, a separating equilibrium that achieves matching efficiency in the women offering contracts game is *strictly preferred* to the men offering contracts game by all women. There is

¹⁵It specifies that when a high type agent matches with a low type agent, the high type agent gets 0 surplus and the low type agent gets the full surplus, this provides the low type agents the largest incentive to report truthfully.

¹⁶ $P_A^{M^H} = p_2\beta_1 \leq \frac{2p_2(Y_G - Y_B)}{5} = \frac{2(z_2 - z_1')}{5} < \frac{5}{8}z_2 + \frac{3}{8}z_1' = P_A^{W^H}$

first mover advantage in the contracts-offering game. This is analogous to the first mover advantage in the Gale-Shapley Algorithm, that in a non-transferrable utility model, *for all women*, a stable matching outcome obtained when women propose to men (weakly) *Pareto dominates* the stable matching when men propose to women.

2. In the contracts-offering game, the harder to verify agents' types from the surplus value, the more difficult to achieve matching efficiency. Section 2 shows that in the benchmark model where each realized surplus state fully reveals the combination of agents' types, a separating equilibrium that achieves matching efficiency can always be found. However, in the model with stochastic marital surplus, the realized surplus states do not reveal any information about agents' types. When the expected marital surplus is strictly monotonic in agents' types, no separating equilibrium that achieves matching efficiency exists. The set-up below provides an intermediate case with partial verifiability of agents' types. Comparing the different results suggests that the harder to verify agent's types, the more difficult to achieve matching efficiency.

$$\begin{aligned} W^H + M^H &= z_2 \\ W^H + M^L = W^L + M^H &= z_1 \\ W^L + M^L &= z_0 \end{aligned}$$

The partial non-verifiability comes from z_1 : z_1 reveals that is must be produced by a combination of a high type agent and a low type agent, but it does not tell exactly who is of low type. As a result any sharing rule conditional on z_1 affects agents of all types. It can be shown that in the intermediate case, the range of z supporting matching efficiency is larger than that in the model with stochastic marital surplus, and smaller than the range of z in the benchmark model.¹⁷

3. Alternative “global” sharing rule. This paper focuses analysis on sharing rules only *within*, but not across couples. There is no transfer of resources between couples. Section 3 shows that an agent's incentive compatibility constraint is not always satisfied in a truthful DRM, hence matching efficiency is not always achievable. An alternative approach to deal with agents' incentives is to propose a “global” sharing rule. For example, the social planner can specify that any agent will get $\frac{1}{4}$ of the *aggregate* matching surplus in the two men-two women case. Such a global sharing rule aligns agents' incentives with the social planner's. However, such analysis has to assume that the social planner can

¹⁷The analysis is essentially the same as in Section 2. In the equilibrium W^L offers 0 to men. In order to deter W^L , W^H offers 0 conditional on z_1 . However, this will induce W^L to mimic as now she is able to get the full surplus when she matches with M^H . To punish a mimic, W^H offers the full amount to men conditional on z_0 . Therefore, to sustain a separating equilibrium with matching efficiency, the benefit of mimicking, z_1 , cannot be too large relative to z_0 , the cost of mimicking. With $N = 2$, we need $z_1 \leq \frac{5}{2}z_0$. When N is very large, the probability of matching with W^H approaches 1, W^L will always mimic.

observe and verify all couples' marital output. If it is not the case, then a couple may hide their assets and always claim the bad surplus state even if they have reached the good surplus state, since by doing so they avoid subsidizing the other couple and may even get transfers from them. The analysis in this paper does not require such an assumption. For instance, even if a high type woman wants to hide the assets and claim the bad surplus state, her partner will stop her from doing so as he has an incentive to reveal the true surplus state to get a larger amount. Since there is no transfer across couples, no couple has an incentive to hide their assets.

4. Type-independent v.s. type-dependent outside option. This paper assumes that the payoff of an unmatched agent is 0 regardless of types. It automatically puts lower and upper bounds on the amount can be shared between a couple. A woman can not offer a negative amount to men, neither can she offer an amount greater than the total surplus produced. Second, it implies that any amount of surplus is verifiable, hence contracts proposing sharing rules can condition on the realized surplus states.

Such an assumption rules out type-dependent outside options. A possible extension is to assume a high type agent can have a better outside option than a low type agent. Then when agents' types are only privately known, an agent's outside option also becomes private information. Although the absolute amount of a couple's marital output can be verified, the marital surplus—total output subtracting the sum of the two agents' outside options—is no longer verifiable. As a result, sharing rules have to condition on the total marital output instead. *The bounds on what can be shared enlarges.* As now in some states an agent may get less than what he or she produces when being single, the new analysis needs to take into account each agent's participation constraint: Ex-ante an agent expects non-negative gains from marriage.

The qualitative conclusions remain. In the contracts-offering game, whether matching efficiency can be achieved depends on to what extent the marital output is informative about agents' types. In the social planner's mechanism, there exists a larger range of z supporting matching efficiency in the truthful DRM.¹⁸ However when N becomes large, matching efficiency is still not achievable.

5. Conclusion

This paper examines a marriage market with two-sided information asymmetry and in which the match surplus is stochastic. My main focus is on how endogenous determination

¹⁸Previously, the smallest μ_0^* , the amount a low type agent gets when being matched with a high type agent conditional on Y_B , which satisfies all agents' incentive compatibility constraints, needs to be no greater than Y_B . Now this condition can be relaxed to some extent, that μ_0^* can be greater than Y_B to compensate a low type agent more.

of the sharing rule can help to achieve an efficient matching. I find that when marital surplus is deterministic, matching efficiency can be achieved by a simple contracts-offering game, where contracts proposing some sharing of the surplus serve as a credible signal to reveal one's type, and as a screening device to attract a suitable partner. However, when random shocks are introduced into marital surplus, matching efficiency is not always achievable, especially when the number of agents is large. A natural next step would be to examine some costly pre-marital signals which affects agents of different types differently and helps to achieve an efficient matching. Matching efficiency with costly pre-marital signals when marital surplus is stochastic, can then be related to the literature on efficiency of pre-marital investment.

6. Appendix

6.1. Proof of Propositions, Lemmas and Theorems

Proof of Proposition 2: To check whether there is any profitable deviation for W^H and W^L , Proposition 1 has established that signaling (punishing a mimic) is costless in the benchmark model, W^H will not mimic W^L by offering contract B, and W^L will not mimic W^H by offering contract A.

For W^L , conditional on a matching outcome, contract B achieves the highest possible payoff for W^L as it offers 0 to men. Hence contract B dominates contract C, contract E_1 , contract F_1 and contract E_2 , as they achieve no better matching. W^L will prefer contract B to contract E_3 iff $P_{E_3}^{W^L} \leq P_B^{W^L} \Rightarrow \frac{1}{2}(z_1 - \beta_2) + \frac{1}{2}z_0 \leq \frac{3}{8}z_1 + \frac{5}{8}z_0 \Rightarrow \beta_2 \geq \frac{z_1 - z_0}{4}$.

For W^H , the best contract E_2 (contract in the form of E_2 that achieves the highest possible payoff for W^H) is strictly dominated by the best contract F , as both contracts achieve the same matching, but W^H could offer 0 to men conditional z_2 and z'_1 in contract F . The best contract C and contract E_1 are strictly dominated by the best contract F as they achieve no better matching outcome. W^H will not deviate to offer contract F iff $\frac{3}{8}(z_2) + \frac{5}{8}(z'_1) \leq \frac{5}{8}(z_2 - \beta_2) + \frac{3}{8}z'_1 \Rightarrow \beta_2 \leq \frac{2}{5}(z_2 - z'_1)$; and will not deviate to offer contract E_3 iff $\frac{1}{2}(z_2 - \beta_2) + \frac{1}{2}z'_1 \leq \frac{5}{8}(z_2 - \beta_2) + \frac{3}{8}z'_1 \Rightarrow \beta_2 \leq z_2 - z'_1$.

We conclude that $\beta_2 \in [\frac{z_1 - z_0}{4}; \frac{2}{5}(z_2 - z'_1)]$ will support a separating equilibrium that achieves matching efficiency. \square

Proof of Theorem 2: $\beta_1 \in [\frac{2(z_1 - z_0)}{5p_1 + 3p_0}, \frac{2}{5}(Y_G - Y_B)]$ satisfy the six conditions below, therefore a separating equilibrium that achieves matching efficiency always exists when $z_1 > z_0$ and $p'_1 = 0$.

1. $(p_2 + p_1)Y_G \geq 2(p_2\beta_1 + \varepsilon)$, i.e $p_2\beta_1 \leq \frac{p_2 + p_1}{2}Y_G$, then there exists a contract F_2

Contract F_2		
Surplus	Woman	Man
Y_G	$Y_G - \gamma_1^{F_2}$	$\gamma_1^{F_2}$
Y_B	Y_B	0

where $\gamma_1^{F_2} = \frac{2(p_2\beta_1 + \varepsilon)}{p_2 + p_1} \in [0, Y_G]$ maximizes women's expected payoff subject to the constraint that it "attract both" when competing against contract A and contract B.

2. $p_2\beta_1 \geq \frac{p_2 + p_1}{8(p_1 + p_0)}(z_1 - z_0)$, for W^L not deviate to offer contract F_2 .
3. $\left(5 - \frac{8p_2}{p_2 + p_1}\right)p_2\beta_1 \leq z_2 - z'_1$, for W^H not deviate to offer contract F_2 .
4. $\beta_1 \geq \frac{2(z_1 - z_0)}{5p_1 + 3p_0}$, for W^L not deviate to offer contract A mimicking W^H .
5. $\beta_1 \leq \frac{2}{5}(Y_G - Y_B)$, for W^H not deviate to offer contract B mimicking W^L .
6. $\beta_1 \in [0, Y_G]$ □

Proof of Proposition 3: Similar reasoning as in Proposition 1, we only need to consider competition between women of the same type. a). **When there are two W^H , one M^H and one M^L :** There exists competition between the two high type women to try to win the only high type man over to achieve "attract only M^H ", i.e. W^H may deviate to offer contract H:

Contract H		
Surplus	Woman	Man
Y_G	$Y_G - \gamma_1^H$	γ_1^H
Y_B	$Y_B - \gamma_0^H$	γ_0^H

where $\gamma_1^H \in [0, Y_G]$ and $\gamma_0^H \in [0, Y_B]$ achieves "attract both" when competing against contract A. Men's expected payoffs of being matched with contract H are the following: $P_H^{M^H} = \mu_H[p_2\gamma_1^H + (1 - p_2)\gamma_0^H] + (1 - \mu_H)[p_1\gamma_1^H + (1 - p_1)\gamma_0^H]$ and $P_H^{M^L} = \mu_H[p'_1\gamma_1^H + (1 - p'_1)\gamma_0^H] + (1 - \mu_H)[p_0\gamma_1^H + (1 - p_0)\gamma_0^H]$. I specify the off-equilibrium belief μ_H such that both high type and low type men's expected payoff when being matched with contract H is the same: $\mu_H = \frac{p_0 - p_1}{p_2 + p_0 - p'_1 - p_1} \in [0, 1] \Leftrightarrow P_H^{M^H} = P_H^{M^L}$. Hence it is not possible for a high type woman to achieve "attract only W^H " by deviating when competing against contract A.

b). **When there are two W^L , one M^H and one M^L :** Similarly there exists competition between the two low type women to attempt to win the only low type man over. However for whatever W^L offers in the off-equilibrium contract H such that $P_H^{M^L} \geq p_0\varepsilon$, we also have $P_H^{M^H} \geq p_0\varepsilon > p_1\varepsilon$ since $z_0 > z_1$ implies $p_0 > p_1$. Therefore both M^H and M^L will be attracted over. It is not possible for W^L to achieve "attract only W^L " by deviating when competing against contract B. □

Lemma 1 *When $z_1 < z_0$ and $z_2 \leq z'_1$, a necessary condition to find a separating equilibrium that achieves matching efficiency is $p_1 = 0$.*

Proof. $z_1 < z_0$ implies W^L prefers M^L in terms of total surplus produced. $z_2 \leq z'_1$ implies W^H also prefers M^L in terms of total surplus produced. There exists competition between W^H and W^L for a low type man. The same reasoning as Theorem 1, we need $p'_1 = 0$. W^H will offer 0 to men in a separating equilibrium. In order to attract **only** M^L , W^L needs to offer 0 expected surplus to M^H . However, in order to signal her type, she needs to offer a contract different from W^H . Only when $p_1 = 0$, that a low type woman and a high type man will produce Y_B with certainty, it allows W^L to offer M^H 0 conditional on Y_B , and offer M^L a positive amount conditional on Y_G to attract only a low type man. □

Lemma 2 When $z_1 < z_0$ and $z_2 \leq z'_1$, and $p_1 = 0$, there exists a separating equilibrium achieves matching efficiency where W^H offers contract A and W^L offers contract B:

Contract A			Contract B		
Surplus	Woman	Man	Surplus	Woman	Man
Y_G	Y_G	0	Y_G	$Y_G - \alpha_1$	α_1
Y_B	Y_B	0	Y_B	Y_B	0

$$\text{where } \alpha_1 \in \left[\max \left[\frac{z'_1 - z_2}{8p_0}, \frac{2(z'_1 - z_2)}{5p'_1 + 3p_2} \right], \min \left[\frac{2(z_0 - z_1)}{5p_0}, \frac{z_0 - z_1}{\left(5 - \frac{8p_0}{p_2 + p'_1}\right)p_0} \right] \right].$$

The best separating equilibrium for W^L is to offer $\alpha_1 = \max \left[\frac{z'_1 - z_2}{8p_0}, \frac{2(z'_1 - z_2)}{5p'_1 + 3p_2} \right]$

Equilibrium beliefs: $\mu(W^H | \text{contract A}) = 1$ and $\mu(W^L | \text{contract B}) = 1$

Equilibrium response rule: When being indifferent between equilibrium contracts, M^H will go for contract A. M^L prefers contract B due to the higher expected payoff.

Off-equilibrium beliefs: $\mu(W^H | \text{any other off-equilibrium contracts}) = \frac{p_2 + p'_1 - 2p_0}{2(p'_1 - p_0)}$

Off-equilibrium response rule: When being indifferent an equilibrium contract and an off-equilibrium contract, M^H will always go to an **off-equilibrium** contract first. M^L will always go to an **equilibrium** contract first.

Proof. Similar to proof in Theorem 2. $\alpha_1 \in \left[\max \left[\frac{z'_1 - z_2}{8p_0}, \frac{2(z'_1 - z_2)}{5p'_1 + 3p_2} \right], \min \left[\frac{2(z_0 - z_1)}{5p_0}, \frac{z_0 - z_1}{\left(5 - \frac{8p_0}{p_2 + p'_1}\right)p_0} \right] \right]$ satisfy the six conditions below and supports a separating equilibrium with matching efficiency.

1. $(p_2 + p'_1)Y_G \geq 2(p_0\alpha_1 + \varepsilon)$ i.e $p_0\alpha_1 \leq \frac{p_2 + p'_1}{2}Y_G$, then there exists a contract F_2

Contract F_2		
Surplus	Woman	Man
Y_G	$Y_G - \gamma_1^{F_2}$	$\gamma_1^{F_2}$
Y_B	Y_B	0

where $\gamma_1^{F_2} = \frac{2(p_0\alpha_1 + \varepsilon)}{p_2 + p'_1} \in [0, Y_G]$ maximizes women's expected payoffs while achieves "attract both" when competing against contract A and contract B.

2. $\left(5 - \frac{8p_0}{p_2 + p'_1}\right)p_0\alpha_1 \leq z_0 - z_1$, then W^L will not deviate to offer contract F_2 .
3. $\alpha_1 \geq \frac{z'_1 - z_2}{8p_0}$, then W^H will not deviate to offer contract F_2 .
4. $\alpha_1 \leq \frac{2(z_0 - z_1)}{5p_0}$, then W^L will not deviate to offer contract A mimicking W^H .
5. $\alpha_1 \geq \frac{2(z'_1 - z_2)}{3p_2 + 5p'_1}$, then W^H will not deviate to offer contract B mimicking W^L .
6. $\alpha_1 \in [0, Y_G]$

The lower bound imposed on the amount offered to men conditional on Y_G is to prevent a high type women from deviating in an attempt to increase her chance of matching with M^L . The upper bound imposed on α_1 is to prevent a low type woman from giving up attracting M^L . \square

Proof of Proposition 4: When $N = 2$, same as the contract-offering game, if a woman reports "high" (which is equivalent to offering contract A in contracts-offering game), her probability of being matched with M^H is $\frac{5}{8}$ and with M^L is $\frac{3}{8}$.

In a symmetric setting where $z_1 = z'_1(p_1 = p'_1)$, when a W^H reports truthfully as a high type woman, her expected payoff is $\frac{5}{8}z_2 + \frac{3}{8}z'_1 - \frac{5}{8}[p_2\beta_1 + (1-p_2)\beta_0] - \frac{3}{8}[p'_1\mu_1 + (1-p'_1)\mu_0]$. When she mis-reports as a low type woman, her expected payoff is $\frac{3}{8}z_2 + \frac{5}{8}z'_1 - \frac{3}{8}[p_2\tau_1 + (1-p_2)\tau_0] - \frac{5}{8}[p'_1\alpha_1 + (1-p'_1)\alpha_0]$. W^H will report truthfully if and only if

$$\begin{aligned} & 5[(p_2\beta_1 + (1-p_2)\beta_0) - (p_1\alpha_1 + (1-p_1)\alpha_0)] \\ & + 3[(p_1\mu_1 + (1-p_1)\mu_0) - (p_2\tau_1 + (1-p_2)\tau_0)] \leq 2(z_2 - z_1) \end{aligned} \quad (5)$$

Similarly W^L will report truthfully if and only if

$$\begin{aligned} & 5[(p_1\beta_1 + (1-p_1)\beta_0) - (p_0\alpha_1 + (1-p_0)\alpha_0)] \\ & + 3[(p_0\mu_1 + (1-p_0)\mu_0) - (p_1\tau_1 + (1-p_1)\tau_0)] \geq 2(z_1 - z_0) \end{aligned} \quad (6)$$

M^H will report truthfully if and only if

$$5[(p_2\beta_1 + (1-p_2)\beta_0) - (p_1\alpha_1 + (1-p_1)\alpha_0)] \geq 3[(p_2\mu_1 + (1-p_2)\mu_0) - (p_1\tau_1 + (1-p_1)\tau_0)] \quad (7)$$

M^L will report truthfully if and only if

$$5[(p_0\alpha_1 + (1-p_0)\alpha_0) - (p_1\beta_1 + (1-p_1)\beta_0)] \geq 3[(p_0\tau_1 + (1-p_0)\tau_0) - (p_1\mu_1 + (1-p_1)\mu_0)] \quad (8)$$

Consider a **symmetric** mechanism: $\beta_1 = \alpha_1 = \frac{Y_G}{2}, \beta_0 = \alpha_0 = \frac{Y_B}{2}; \mu_1 = Y_G - \tau_1, \mu_0 = Y_B - \tau_0$

Condition (5) and (7) becomes: $(p_2 + p_1)\mu_1 + (2 - p_2 - p_1)\mu_0 \leq \frac{5}{6}z_2 + \frac{1}{6}z_1$

Condition (6) and (8) becomes: $(p_2 + p_1)\mu_1 + (2 - p_2 - p_1)\mu_0 \geq \frac{5}{6}z_1 + \frac{1}{6}z_0$

Re-arranging we obtain $12(p_2 - p_0)\mu_0 \geq (6p_2 - 4p_1 - 2p_0)Y_B + 4(p_1z_1 - p_0z_2)$, with the requirement $\mu_0 \leq Y_B$. We need $\frac{Y_B}{Y_G - Y_B} \geq \frac{2(p_1^2 - p_0p_2)}{3p_2 - 3p_0}$. Furthermore, when this condition holds, there always exists some $\mu_1^* \in [0, Y_G]$ that satisfy the two inequalities above. Therefore $\frac{Y_B}{Y_G - Y_B} \geq \frac{2(p_1^2 - p_0p_2)}{3p_2 - 3p_0}$ is the necessary and sufficient condition for a social planner design the contract mechanism that achieves matching efficiency.

Now we consider an **asymmetric** mechanism. By Condition (5) and (7), we obtain:

$$(p_1 + p_2)\mu_1 + (2 - p_1 - p_2)\mu_0 \leq \frac{2}{3}(z_2 - z_1) + (p_1 + p_2)\tau_1 + (2 - p_1 - p_2)\tau_0 \quad (9)$$

By condition (6) and (8), we obtain:

$$(p_1 + p_0)\mu_1 + (2 - p_1 - p_0)\mu_0 \geq \frac{2}{3}(z_1 - z_0) + (p_1 + p_0)\tau_1 + (2 - p_1 - p_0)\tau_0 \quad (10)$$

Multiply (10) by $(p_1 + p_2)$ then subtract (9) multiplied by $(p_1 + p_0)$, we get: $2(p_2 - p_0)(\mu_0 - \tau_0) \geq \frac{2}{3}(2p_1^2 - 2p_2p_0)(Y_G - Y_B) \Rightarrow \mu_0 - \tau_0 \geq \frac{2}{3}\left(\frac{p_1^2 - p_2p_0}{p_2 - p_0}\right)(Y_G - Y_B)$. Since $\max(\mu_0 - \tau_0) = Y_B$, a necessary condition for a general **asymmetric** mechanism to achieve matching efficiency is $Y_B \geq \frac{2}{3}\left(\frac{p_1^2 - p_2p_0}{p_2 - p_0}\right)(Y_G - Y_B) \Rightarrow \frac{Y_B}{Y_G - Y_B} \geq \frac{2}{3}\left(\frac{p_1^2 - p_2p_0}{p_2 - p_0}\right)$. Recall the necessary and sufficient condition for a **symmetric** mechanism that achieves matching efficiency is $\frac{Y_B}{Y_G - Y_B} \geq \frac{2}{3}\left(\frac{p_1^2 - p_2p_0}{p_2 - p_0}\right)$. We conclude that in a symmetric set-up ($z_1 = z'_1$), a symmetric mechanism is the best mechanism. If the symmetric mechanism cannot achieve matching efficiency, then no other mechanism can. \square

Proof of Proposition 6:

$$P_N = 1 - \frac{1}{2^{2N-1}} \binom{2N-1}{N}$$

The entries Table 6, a_{ij} , is the probability of marrying a high type man M^H , when there are i contract A and j high type men. $a_{ij} = \begin{cases} \frac{j}{i} & \text{if } j \leq i \\ 1 & \text{if } j > i \end{cases}$.

$$\begin{aligned} P_N &= \sum_{1 \leq i \leq N} \sum_{0 \leq j \leq N} \frac{\binom{N-1}{i-1} \binom{N}{j}}{2^{2N-1}} a_{ij} \\ &= \frac{1}{2^{2N-1}} \left[\sum_{1 \leq j \leq i \leq N} \binom{N-1}{i-1} \binom{N}{j} a_{ij} + \sum_{1 \leq i < j \leq N} \binom{N-1}{i-1} \binom{N}{j} \right] \\ &= \frac{1}{2^{2N-1}} \left[\sum_{1 \leq j \leq i \leq N} \binom{N}{i} \binom{N-1}{j-1} + \sum_{1 \leq i < j \leq N} \binom{N-1}{i-1} \binom{N}{j} \right] \end{aligned}$$

Let $X = \sum_{1 \leq j \leq i \leq N} \binom{N}{i} \binom{N-1}{j-1}$ and $Y = \sum_{1 \leq i < j \leq N} \binom{N-1}{i-1} \binom{N}{j}$.

$$X = \sum_{1 \leq j \leq i \leq N} \binom{N}{i} \binom{N-1}{j-1} = \sum_{0 \leq \tilde{j} < i \leq N} \binom{N}{i} \binom{N-1}{\tilde{j}} \quad \text{denote } \tilde{j} = j-1$$

so we could rewrite X as $X = \sum_{0 \leq \tilde{j} < i \leq N} \binom{N}{i} \binom{N-1}{\tilde{j}}$.

Imagine there are $N-1$ white balls, W_1, W_2, \dots, W_{N-1} and N black balls, B_1, B_2, \dots, B_N , each ball is distinct. Now we consider the number of ways to select these balls, for each ball we could either select it or not. X represents the number of ways to select the balls such that there are more black balls than white balls. To compute X , we need to count the number of ways such that $|B| > |W|$. There are two situations: 1) B_N is selected, then we should have $|B| \geq |W|$ in $(N-1)$ black balls *v.s* $(N-1)$ white balls case, and 2) B_N is not selected, then we should have $|B| > |W|$ in $(N-1)$ black balls *v.s* $(N-1)$ white balls case.

$$\begin{aligned} X &= \# \{ |B| \geq |W| \text{ in } (N-1) \text{ v.s } (N-1) \} + \# \{ |B| > |W| \text{ in } (N-1) \text{ v.s } (N-1) \} \\ &= \# \{ |B| \geq |W| \text{ in } (N-1) \text{ v.s } (N-1) \} + \# \{ |B| < |W| \text{ in } (N-1) \text{ v.s } (N-1) \}, \text{ by symmetry} \\ &= \# \{ \text{any way to choose } 2N-2 \text{ balls} \} = 2^{2N-2} \end{aligned}$$

$$\begin{aligned} Y &= \sum_{1 \leq i < j \leq N} \binom{N-1}{i-1} \binom{N}{j} = \sum_{1 \leq i \leq j \leq N} \binom{N-1}{i-1} \binom{N}{j} - \sum_{1 \leq i=j \leq N} \binom{N-1}{i-1} \binom{N}{j} \\ &= X - \sum_{k=1}^N \binom{N-1}{k-1} \binom{N}{k} \end{aligned}$$

Let $Z = \sum_{k=1}^N \binom{N-1}{k-1} \binom{N}{k} = \sum_{k=1}^N \binom{N-1}{N-k} \binom{N}{k} = \binom{2N-1}{N}$ since Z represents the number of ways to choose N balls out of $2N-1$ balls. Therefore $P_N = \frac{1}{2^{2N-1}} (X + Y) = \frac{1}{2^{2N-1}} (X + X - Z) = 1 - \frac{1}{2^{2N-1}} \binom{2N-1}{N}$.

1. P_N increases with N : $P_{N+1} - P_N = \frac{1}{2^{2N}} \cdot \frac{(2N-1)!}{(N-1)!(N+1)!} > 0$, $N = 2, 3, 4, \dots$
2. $P_N > \frac{1}{2}$, $\frac{P_N}{2^{P_N-1}} > 1$: $P_2 = 1 - \frac{1}{2^3} \binom{3}{2} = \frac{5}{8} > \frac{1}{2}$ with $N=2$ and P_N increases with N .
3. $1 - P_N \approx \frac{1}{\sqrt{\pi N}}$ when N is large: apply Stirling's formula. $\frac{1}{2^{2N-1}} \cdot \binom{2N-1}{N} = \frac{1}{2^{2N-1}} \cdot \frac{(2N-1)!}{(N-1)!N!} = \sqrt{\frac{1}{\pi N}} \cdot \left(1 + \frac{(-\frac{1}{2})}{N}\right)^N \left(1 + \frac{\frac{1}{2}}{(N-1)}\right)^{N-1} \approx \sqrt{\frac{1}{\pi N}} \cdot e^{-\frac{1}{2}} \cdot e^{\frac{1}{2}}$ using the fact $\lim_{n \rightarrow \infty} (1 + \frac{x}{n})^n = e^x$, therefore when N is large, P_N is approximately $1 - \sqrt{\frac{1}{\pi N}}$. \square

6.2. Intermediate Case with partial non-verifiability about agents' types ($z_1 = z'_1$)

Proposition 7 1. With $N=2$, when $z_1 \leq 2.5z_0$, there exists a separating equilibrium where W^H offers contract A and W^L offers contract B

Contract A			Contract B		
Surplus	woman	man	Surplus	woman	man
z_2	$z_2 - \frac{z_1 - z_0}{4}$	$\frac{z_1 - z_0}{4}$	z_2	0	z_2
z_1	z_1	0	z_1	z_1	0
z_0	0	z_0	z_0	z_0	0

2. With general N , When $z_1 \leq \frac{P_N}{2^{P_N-1}} z_0$, there exists a separating equilibrium where W^H offers contract A and W^L offers contract B

Contract A			Contract B		
Surplus	woman	man	Surplus	woman	man
z_2	$z_2 - \gamma_2^A$	γ_2^A	z_2	0	z_2
z_1	z_1	0	z_1	z_1	0
z_0	0	z_0	z_0	z_0	0

where $\gamma_2^A \in \left[(2P_N - 1)(z_1 - z_0), \frac{(2P_N - 1)}{P_N} (z_2 - z_1) \right]$, $P_N = 1 - \frac{1}{2^{2N-1}} \binom{2N-1}{N}$

Equilibrium beliefs: $\mu(W^H | \text{contract } A) = 1$ and $\mu(W^L | \text{contract } B) = 1$

Equilibrium response rule: Both types of men are indifferent between payoffs of equilibrium contracts (contract A and contract B). M^H goes for contract A , M^L goes for contract B .

Off-equilibrium beliefs: $\mu(W^H | \text{any other off-equilibrium contracts}) = 0$

Off-equilibrium response rule: A man will go for the contract which gives him the maximum payoff first, if he did not get it, he will then go for the next contract which gives him the second largest payoff, and so on, until he is matched with a contract. When being indifferent between an equilibrium contract and an off-equilibrium contract: M^L will go for an **off-equilibrium** contract first; M^H will go for an **equilibrium** contract first.

Theorem 4 $z_1 \leq \frac{P_N}{2^{P_N-1}} z_0$ is a necessary condition for the existence of a separating equilibrium with matching efficiency.

The condition $z_1 \leq \frac{P_N}{2^{P_N-1}} z_0$ is to ensure that W^L will not mimic M^H by offering contract A . When $N=2$, we require $z_1 \leq 2.5z_0$. When N is very large, the condition becomes $z_1 \leq z_0$.

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