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AMBIGUITY AVERSION AND THE ABSENCE OF INDEXED DEBT

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Ambiguity aversion and the absence of indexed debt[1]

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Abstract

If an agent's (subjective) beliefs are *ambiguous* then the beliefs may not be represented by a unique probability distribution in the standard Bayesian fashion but instead by a set of probabilities. Roughly put, an *ambiguity averse* decision maker evaluates an act by the minimum expected value that may be associated with it.

Inspite of wide and long-standing support among economists for indexation of loan contracts there has been relatively little use of indexation, except in situations of extremely high inflation. The object of this paper is to provide a (theoretical) explanation for this puzzling phenomenon based on the hypothesis that economic agents are *ambiguity averse.* The present paper considers a competitive general equilibrium model of goods, money and bond markets populated by agents with Choquet expected utility preferences, where both nominal and indexed bonds are available for trade and prices of all goods and bonds are determined endogenously. We obtain conditions which prompt an *endogenous* cessation of trade in indexed bonds: i.e., conditions under which there is no trade in indexed bonds in *any* equilibrium; only nominal bonds are traded.

1 Introduction

Inspite of wide and long-standing support among economists for indexation of loan contracts there has been relatively little use of indexation, except in situations of extremely high inflation. Indeed, except in cases where inflationary circumstances forced them to do so, few governments, and fewer private borrowers, have issued indexed bonds. People seem to have a preference for specifying their obligations and opportunities in nominal units. As Shiller (1997) remarks:

> That the public should generally want to denominate contracts in currency units–despite all the evidence that it is not wise to do so and despite the obvious examples from nominal contracts of redistributions caused by unexpected inflation–should be regarded as one of the great economic puzzles of all time.

The object of this paper is to provide a (theoretical) explanation for this puzzling phenomenon based on the hypothesis that economic agents are *ambiguity averse*. The analysis throws up testable hypotheses and insights on policy that are distinctive, compared to what a more standard analysis based on the assumption that decision makers are (subjective) expected utility maximizers would suggest.

Suppose an agent's subjective knowledge about the likelihood of contingent events is consistent with more than one probability distribution. And further that, what the agent knows does not inform him of a (second order) probability distribution over the set of 'possible' (first order) probabilities. Roughly put, we say then that the agent's beliefs about contingent events are characterized by *ambiguity.* If ambiguous, the agent's beliefs are captured not by a unique probability distribution in the standard Bayesian fashion but instead by a set of probabilities. Thus not only is the outcome of an act uncertain but *also* the expected payoff of the action, since the payoff may be measured with respect to more than one probability. An *ambiguity averse* decision maker evaluates an act by the minimum expected value that may be associated with it: the decision rule is to compute all possible expected values for each action and then choose the act which has the best minimum expected outcome. This notion of ambiguity aversion, an intuition about behavior under subjective uncertainly famously noted in Ellsberg (1961) and earlier by Knight (1921), inspires the formal model of Choquet expected utility (CEU) preferences introduced in Schmeidler (1989). The present paper considers a competitive general equilibrium model of goods, bonds and money markets populated by agents with CEU preferences[1], where both nominal and indexed bonds are available for trade and prices of all goods and bonds are determined endogenously. We obtain conditions which prompt an *endogenous* cessation of trade in indexed bonds among private agents: i.e., conditions under which there is no trade in indexed bonds in *any* equilibrium and only nominal bonds are traded. It is worth clarifying, at this point, that while we "explain" the veritable absence of indexed debt by showing that no trade in indexed bonds is the unique equilibrium outcome under certain conditions, the analysis does not imply that this is an efficient outcome. Indeed, as will be evident from the analysis, making appropriate changes to the way price indices are constructed would lead, if the theory presented in this paper is empirically valid, to more widespread indexation of debt *and* an accompanying Pareto improvement.

An important point of inspiration for the analysis was to note that indexing does not eliminate all (price) risk—rather it substitutes one risk for another— an observation, we believe,

[1] Recent literature has debated the merits of the CEU framework as a model of ambiguity aversion. For instance, Epstein (1999) contends that CEU preferences associated with *convex capacities* (see section 2, below) do not always conform with a "natural" notion of ambiguity averse behavior. On the other hand, Ghirardato and Marinacci (1997) argue that ambiguity aversion is demonstrated in the CEU model by a broad class of capacities which includes convex capacities. As it happens, all beliefs identified as ambiguous in the model constructed in this paper satisfy criteria set by both the cited references.

originally due to Magill and Quinzii (1997). An indexed bond, whose payoffs by definition are denoted in units of a reference bundle of goods and services, will be secure against the aggregate price level risk arising from changes in the money supply—*the monetary risk*, but unavoidably picks up the *real risks* arising from fluctuations in the relative prices of the goods in the reference bundle. A nominal contract on the other hand implies susceptibility to monetary risk but less so to real risk. The basic intuition is straightforward. Being paid in terms of an index essentially amounts to being paid units of the reference bundle of goods. Typically, the reference bundle contains items that are not part of a given individual's consumption basket. Hence, effectively the individual is left to exchange goods in the reference bundle not in his consumption basket with goods he actually consumes. Thus, a change in the price of goods not in his basket will affect the "worth" of his remuneration in terms of the goods he does consume. Since the presence of both types of risk is typical, standard portfolio analysis will advise that the optimal portfolio should contain *both* nominal and indexed contracts (or to put it somewhat differently, partial indexation). Given this one would expect, and a result in this paper confirms, trade in indexed bonds will always be observed in a market consisting of SEU (subjective expected utility) agents so long as there were *some* inflation, however small. Under ambiguity aversion the market outcome, though, may be dramatically different.

More specifically suppose, with respect to *any* two agents wishing to trade in indexed bonds, the following is true:

1. the indexation bundle contains at least one good which is not consumed by either of the agents;
2. the agents' beliefs about the change in the price of good(s) not consumed by either of them, relative to the average price level, is ambiguous;
3. agents are ambiguity averse.

The main result of the paper shows that, if agents believe general inflation will not exceed a given bound and if ambiguity of beliefs about the relative price movements is sufficiently high, then agents will have zero holdings of the indexed bond in *any* equilibrium.

The result appears to fit very well with what is commonly observed, both, in terms of the plausibility of the hypotheses it rests on and in terms of its consistency with regularities widely associated with trade in indexed debt. First, consider the plausibility of the assumptions. Even at that best of times and even in the most developed nations, information (say, formal forecasts) about *relative* price movements are very hard to come by. Pick any two agents in the economy; it is inevitable that the consumer price index will include goods and services that are not part of the consumption basket of either agent. For instance, it will inevitably include housing in regions that the agents have no interest in. It is a plausible assumption that agents will have, at best, very sparse informal knowledge about possible (relative) price movements of goods and services that they never consume. Thus, if agents are typically ambiguity averse when confronted with vague information, as much experimental evidence suggests (see Camerer (1995)), then it would seem compelling (a priori) to argue that they would behave in an ambiguity averse manner when acting on beliefs about relative price movements of goods that never figure in their consumption plans[2].

Next, consider some "stylized facts" about trade in indexed debt. As has been already mentioned, barring certain exceptions trade in indexed bonds, especially private bonds, is negligible.

[2]It is interesting to note in this context, as reported in Shiller (1997), when asked for reasons for not opting for indexation many agents say that they are inhibited by their doubts that the government inflation numbers were valid for their individual circumstances (pp 183, 188-190, 208). (Apparently, the concern was that the official price index referred to a basket of goods that was possibly different from the individual's.)

The exceptions are, one, situations of extremely high and variable inflation, and two, situations (like in U.K., Israel) where even though inflation is currently moderate, many wage payments are statutorily index linked. A second exception occurs in economies which have tamed inflation in the recent past but have experienced bouts of high inflation in the more distant past (many S. American countries and also, Israel). It will be explained that the logic of the main result does not only imply that trade in indexed debt will be observed during episodes of relatively high inflation; such an episode actually serves to ensure that trade in indexed debt would endure for many periods beyond the original trigger even if agents do not expect further bouts of high inflation. It is also observed that individuals in countries with high and variable inflation commonly denominate their debt (or even transactions like rental contracts for housing, as in Israel) in U.S. Dollars even though they may rarely choose to tie payments to an index like the CPI. This practice, at the least, demonstrates that agents understand the vulnerability of nominal contracts denominated in terms of the (relatively inflationary) domestic currency and *do act* on their understanding. They are not as "naive" or "confused" as may be supposed in the first instance. If anything, this observation deepens the puzzle: if they do understand the point of indexation why do they refuse to use CPI and use the Dollar instead? The results presented in the paper provide an explanation of this practice. Finally, even though ours is an analysis of the private debt market in that we do not model government behavior per se, we will argue the phenomenon actually modeled here also provides an explanation for the lack of trade in indexed public debt.

The rest of the paper is organized as follows. The following section provides an introduction to the formal model of Choquet expected utility. Section 3 works through a leading example with the aim of conveying the essential intuition of the argument in a partial equilibrium setting. Section 4 contains the general equilibrium model and the main result. Section 5 concludes the paper with a discussion of the related literature as well as the interpretation and empirical significance of the findings.

2 Ambiguity aversion

2.1 The Ellsberg urn

One classic experiment illustrating how ambiguity aversion may affect *behavior*, due to Daniel Ellsberg (1961), runs as follows:

> There are two urns each containing one hundred balls. Each ball is either red or black. The subjects are told of the fact that there are fifty balls of each color in urn I. But no information is provided about the proportion of red and black balls in urn II. One ball is chosen at random from each urn. There are four events, denoted IR, IB, IIR, IIB, where IR denotes the event that the ball chosen from urn I is red, etc. On each of the events a bet is offered: \$100 if the events occurs and \$0 if it does not.

The modal response is for a subject to prefer every bet from urn I (IR or IB) to every bet from urn II (IIR or IIB). That is, the typical revealed preference is $IB \succ IIB$ *and* $IR \succ IIR$. (The preferences are strict.) Clearly, DM's beliefs about the likelihood of the events, as revealed in the preferences, is not consistent with a unique probabilistic prior. The story goes: People dislike the ambiguity that comes with choice under uncertainty; they dislike the possibility that they may have the odds wrong and so make a wrong choice (*ex ante*). Hence they go with the gamble where they know the odds — betting from urn I. A slight restatement of the intuition conveyed by the observed (modal) choice provides a useful perspective on what is to follow.

Notice, *betting on IIR* is the same as *betting against IIB*, and vice versa, since the events are complementary. But to decide whether to bet on or against IIR requires information about relative likelihood of the event IIR and its complement. That is, of course, what ambiguity about IIR precludes. On the other hand, betting on, say, IB allows the DM to choose a prospect whose evaluation is unaffected by ambiguity.

2.2 Choquet expected utility

Let $\Omega = \{\omega_i\}_{i=1}^N$ be a finite state space, and assume that the decision maker (DM) chooses among acts with state contingent payoffs, $z : \Omega \to \mathbb{R}$. In the CEU model (Schmeidler (1989)) an ambiguity averse DM's subjective belief is represented by a *convex non-additive probability* function (or a *convex capacity*), ν such that, (i) $\nu(\emptyset) = 0$, (ii) $\nu(\Omega) = 1$ and, (iii) $\nu(X \cup Y) \geq \nu(X) + \nu(Y) - \nu(X \cap Y)$, for all $X, Y \subset \Omega$. Define the *core* of ν, (notation: $\Delta(\Omega)$ is the set of all additive probability measures on Ω):

$$\mathcal{C}(\nu) = \{\pi \in \Delta(\Omega) \mid \pi(X) \geq \nu(X), \text{ for all } X \subseteq \Omega.\}$$

Hence, $\nu(X) = \min_{\pi \in \mathcal{C}(\nu)} \pi(X)$. Hence, convex capacity may be interpreted as representing a convex set of (additive) probabilities. The *ambiguity*[3] of the belief about an event X is measured by the expression $\mathcal{A}(X; \nu) \equiv 1 - \nu(X) - \nu(X^c) = \max_{\pi \in \mathcal{C}(\nu)} \pi(X) - \min_{\pi \in \mathcal{C}(\nu)} \pi(X)$.

Like in SEU, a *utility function* $u : \mathbb{R}_+ \to \mathbb{R}$, $u'(\cdot) \geq 0$, describes DM's attitude to risk and wealth. Given a convex non-additive probability ν, *the Choquet expected utility*[4] of an act is simply the minimum of all possible 'standard' expected utility values obtained by measuring the contingent utilities possible from the act with respect to each of the additive probabilities in the core of ν:

$$\mathbb{CE}_\nu u(z) = \min_{\pi \in \mathcal{C}(\nu)} \left\{ \sum_{\omega \in \Omega} u(z(\omega)) \pi(\omega) \right\} \equiv \int_\Omega u(z(\omega))\, d\nu$$

The fact that the same additive probability in $\mathcal{C}(\nu)$ will not in general 'minimize' the expectation for two different acts, explains why the Choquet expectations operator is not additive, i.e., given any acts z, w : $\mathbb{CE}_\nu(z) + \mathbb{CE}_\nu(w) \leq \mathbb{CE}_\nu(z + w)$. The operator is additive, however, if the two acts z and w are *comonotonic*, i.e., if $(z(\omega_i) - z(\omega_j))(w(\omega_i) - w(\omega_j)) \geq 0$.

Next, we state the notion of independence of convex non-additive probabilities, proposed by Gilboa and Schmeidler (1989), used in this paper. Essentially, the idea is as follows. Start with the set of probabilities in the core of each capacity, select a probability from each such set and multiply to obtain the corresponding product probability in the usual way and repeat for all possible selections, thereby obtaining a set of product probabilities. The lower envelope of the set of product probabilities, obtained in this way, is the product capacity.

Definition 1 *Let ν and μ be two convex non-additive probabilities, defined on contingency spaces Ω_ν and Ω_μ respectively. The independent product of ν and μ , denoted $\nu \bigotimes \mu$, is defined as follows*

$$\left(\nu \bigotimes \mu\right)(A) \equiv \min\left\{(\pi_\nu \times \pi_\mu)(A) : \pi_\nu \in \mathcal{C}(\nu), \pi_\mu \in \mathcal{C}(\mu)\right\}$$

for every $A \subseteq \Omega_\nu \times \Omega_\mu$.

[3] Fishburn (1993) provides an axiomatic framework for this definition of ambiguity and Mukerji (1997) demonstrates its equivalence to a more primitive and epistemic notion of ambiguity (expressed in term's of the DM's knowledge of the state space).

[4] The Choquet expectation operator may be directly defined with respect to a non-additive probability, see Schmeidler (1989). Also, for an intuitive introduction to the CEU model see Section 2 in Mukerji (1998).

2.3 A bid-ask spread

Dow and Werlang (1992), identified an important implication of Schmeidler's model. They showed, in a model with one risky and one riskless asset, that if a CEU maximizer has a riskless endowment then there exists a *set* of asset prices that support the optimal choice of a riskless portfolio. The intuition behind this finding may be grasped in the following example. Consider an asset that pays off 1 in state L and 3 in state H and assume that $\nu(L) = 0.3$ and $\nu(H) = 0.4$. Assuming that the DM has a linear utility function, the expected payoff of buying an unit of z, the act z_b, is given by $\mathbb{CE}_\nu(z_b) = 0.6 \times 1 + 0.4 \times 3 = 1.8$. On the other hand, the payoff from going short on an unit of z (the act z_s) is higher at L than at H. Hence, the relevant minimizing probability when evaluating $\mathbb{CE}_\nu(z_b)$ is that probability in $\mathcal{C}(\nu)$ that puts most weight on H. Thus, $\mathbb{CE}_\nu(z_s) = 0.3 \times (-1) + 0.7 \times (-3) = -2.4$. Hence, if the price of the asset z were to lie in the open interval $(1.8, 2.4)$, then the investor would strictly prefer a zero position to either going short or buying. Unlike in the case of unambiguous beliefs there is no single price at which to switch from buying to selling. Taking a zero position on the risky asset has the unique advantage that its evaluation is not affected by ambiguity.

3 The single decision maker's problem: the intuition in a simplified set up

In this section we consider the problem of a decision maker (DM) who wants to transfer an amount S, $S > 0$, from today (Period 0) to tomorrow (Period 1). Goods prices that will obtain tomorrow are uncertain at the moment and, for the purposes of this section, taken to be exogenously determined. We will examine, in particular, the DM's choice of portfolio given that the DM has access to only two kinds of assets, nominal bonds and indexed bonds, whose prices are known and exogenously given. While the model in this section is simpler in many details than the one in the next section, it is instructive in that it will reveal to us how the trade-offs involved, given ambiguity aversion, are such that the DM will strictly prefer to maintain a zero holding of the indexed bond over a *non-degenerate* interval of indexed bond prices. This, as we will see in the next section, is a key intuition to understanding why no trade in indexed bonds might emerge as an equilibrium outcome.

3.1 A simple portfolio problem

We assume that there are just two goods in the economy: x and y. The agent consumes only good x and is endowed in Period 1 with a (non-random) endowment of that good, $\bar{x}$. The agent does not consume good y nor is he endowed with that good. However, the indexed bond pays off a unit of good x and a unit of good y. The nominal bond pays in units of money.

The money supply in the economy in Period 1 can be either high (M) or low (m). When the money supply is low, suppose that prices can be equal to either (p_x, p_y^L) or (p_x, p_y^H), with $p_y^H > p_y^L$, *i.e.*, we assume that the price of good y can be affected by factors that do not affect the price of good x. When the money supply is high, we assume that prices can be either equal to $\lambda(p_x, p_y^L)$ or $\lambda(p_x, p_y^H)$ where $\lambda = M/m > 1$. This is reminiscent of the quantity theory of money.

The following four states exhaustively describe the price uncertainty faced by the individual:

State	Prices	Return from an indexed bond
1	$\left(p_x, p_y^H\right)$	$p_x + p_y^H$
2	$\left(\lambda p_x, \lambda p_y^H\right)$	$\lambda\left(p_x + p_y^H\right)$
3	$\left(p_x, p_y^L\right)$	$p_x + p_y^L$
4	$\left(\lambda p_x, \lambda p_y^L\right)$	$\lambda\left(p_x + p_y^L\right)$

In this section we leave the decision problem concerning the Period 0 consumption unspecified and simply assume that the agent wants to save a given amount S. Let x^s denote the agent's consumption in state s, b^i the agent's indexed bond holding, q^i its price, b^n the agent's nominal bond holding, and q^n its price. The DM's budget constraints may then be rewritten to obtain:

$$\begin{aligned}
x^1 &= \bar{x} + \left(1 + \frac{p_y^H}{p_x}\right) b^i + \frac{b^n}{p_x} &&= \bar{x} + \left(1 + \frac{p_y^H}{p_x} - \frac{q^i}{q^n p_x}\right) b^i + \frac{S}{q^n p_x} \\
x^2 &= \bar{x} + \left(1 + \frac{p_y^H}{p_x}\right) b^i + \frac{b^n}{\lambda p_x} &&= \bar{x} + \left(1 + \frac{p_y^H}{p_x} - \frac{q^i}{q^n \lambda p_x}\right) b^i + \frac{S}{q^n \lambda p_x} \\
x^3 &= \bar{x} + \left(1 + \frac{p_y^L}{p_x}\right) b^i + \frac{b^n}{p_x} &&= \bar{x} + \left(1 + \frac{p_y^L}{p_x} - \frac{q^i}{q^n p_x}\right) b^i + \frac{S}{q^n p_x} \\
x^4 &= \bar{x} + \left(1 + \frac{p_y^L}{p_x}\right) b^i + \frac{b^n}{\lambda p_x} &&= \bar{x} + \left(1 + \frac{p_y^L}{p_x} - \frac{q^i}{q^n \lambda p_x}\right) b^i + \frac{S}{q^n \lambda p_x}
\end{aligned}$$

The budget constraints reveal how each of the two kinds of bonds provide a hedge against a particular type of risk while simultaneously making the agent vulnerable to another type of risk. The agent does not consume y, hence given that the indexed bond pays a unit each of x and y, on maturity (of the indexed bond) the agent is effectively left to exchange units of y obtained for units of x. Therefore, even though payoff from an indexed bond is immune to monetary shocks (it is independent of λ) it changes with changes in the price of y, relative to the price of x. On the other hand, while the payoff (to the agent) of a nominal bond is not affected by shocks to the relative price of y, it is affected by monetary shocks (i.e., the value of λ). Hence, if $b^i = 0$, $x^1 = x^3$ and $x^2 = x^4$, while, if $b^n = 0$, then $x^1 = x^2$ and $x^3 = x^4$. Notice also that, given our assumptions, if $b^i > 0$, then $x^1 > x^3$ and $x^2 > x^4$, while, if $b^i < 0$, then $x^1 < x^3$ and $x^2 < x^4$; i.e., the agent's ranking of the states (1,3 and 2,4) according to consumption *reverses* when switching from a long to a short position on the indexed bond.

We next explore the consequences of ambiguity of beliefs about relative price movements on the agent's decision whether or not hold indexed bonds. We assume that the agent is risk averse, with a utility index $u : \mathbb{R}_+ \to \mathbb{R}$, which is increasing, strictly concave and differentiable. Suppose the agent has precise probabilistic beliefs concerning the money supply[5] (and consequently whether the "price level" is high or low) but has ambiguous beliefs concerning the realization of the idiosyncratic shock that affects only the price of good y (a good he is not endowed with, and which he does not consume). More precisely, we assume that the agent can assess the probability of the event $\{1, 3\}$ to be, say, μ and that of event $\{2, 4\}$ to be $1 - \mu$. On the other hand, conditional on a monetary state, the agent has only vague beliefs on whether the price of good y is high or low, which is represented by the fact that subjective beliefs are described by capacities $\nu^H \equiv \nu(\{1, 2\})$ and $\nu^L \equiv \nu(\{3, 4\})$, with $\nu^L + \nu^H < 1$. We then assume that the overall beliefs of the agent are simply the independent product of μ and ν. The preferences of the agent are then represented by a utility functional, denoted $V(x^1, x^2, x^3, x^4)$, obtained by taking the Choquet integral of $u(x^s)$ with respect to the independent product belief $\mu \otimes \nu$.

If $b^i > 0$, then $V(x^1, x^2, x^3, x^4)$ is given by:

$$\mu\left(\nu^H u(x^1) + (1 - \nu^H) u(x^3)\right) + (1 - \mu)\left(\nu^H u(x^2) + (1 - \nu^H) u(x^4)\right)$$

[5] The actual equilibrium model in the next section allows beliefs about the money supply to be ambiguous too.

If $b^i < 0$, then $V(x^1, x^2, x^3, x^4)$ is given by:

$$\mu\left((1-\nu^L)u(x^1) + \nu^L u(x^3)\right) + (1-\mu)\left((1-\nu^L)u(x^2) + \nu^L u(x^4)\right)$$

Note that if $\nu^H + \nu^L = 1$ then the two expressions above coincide.

3.2 A price interval supporting zero holding of the indexed bond

We now establish, following Dow and Werlang (1992) that there is a non-degenerate interval of relative bond prices, $\frac{q^i}{q^n}$, at which the agent optimally wants to hold a zero position in the indexed bond. Below, we present an informal, intuitive argument. A more formal argument appears in the Appendix.

Suppose the agent presently holds only nominal bonds and is considering buying/selling an arbitrarily small unit of indexed bonds. The agent's present utility level in each of the four states may then be represented generically by $U(\lambda) \equiv u\left(\bar{x} + \frac{S}{q^n \lambda p_x}\right)$, where $\lambda = 1$ in states 1 and 3 and $\lambda = 1 + \varepsilon$, $\varepsilon > 0$, in states 2 and 4. Since $u''(x) < 0$, the marginal utility in each state, $U'(\lambda) \equiv u'\left(\bar{x} + \frac{S}{q^n \lambda p_x}\right)$, is increasing in λ; the intuition being that a higher inflation will affect the saver adversely. Now consider the gross increase (decrease) in welfare at each state if the agent were to buy (sell) an infinitesimal unit of an indexed bond:

$$\begin{aligned} \text{State 1}: &\quad \left(1 + \frac{p_y^H}{p_x}\right) U'(1) \\ \text{State 2}: &\quad \left(1 + \frac{p_y^H}{p_x}\right) U'(1+\varepsilon) \\ \text{State 3}: &\quad \left(1 + \frac{p_y^L}{p_x}\right) U'(1) \\ \text{State 4}: &\quad \left(1 + \frac{p_y^L}{p_x}\right) U'(1+\varepsilon) \end{aligned}$$

Figure 1, depicts these "payoffs" for an ε "small enough". Notice, as arranged in the figure the payoffs are increasing from left to right. Since $U'(\lambda)$ is increasing in λ and $\varepsilon > 0$, $\left(1 + \frac{p_y^L}{p_x}\right) U'(1) < \left(1 + \frac{p_y^L}{p_x}\right) U'(1+\varepsilon)$ and $\left(1 + \frac{p_y^H}{p_x}\right) U'(1) < \left(1 + \frac{p_y^H}{p_x}\right) U'(1+\varepsilon)$. Since u is continuous and $p_y^H > p_y^L$, for ε small enough $\left(1 + \frac{p_y^L}{p_x}\right) U'(1+\varepsilon) < \left(1 + \frac{p_y^H}{p_x}\right) U'(1)$. Now, to simplify matters dramatically, suppose $\nu^H = \nu^L = 0$. Hence, if the agent were to go long in the indexed bond the payoffs in events where the monetary shock is low and high are $\left(1 + \frac{p_y^L}{p_x}\right) U'(1)$ and $\left(1 + \frac{p_y^L}{p_x}\right) U'(1+\varepsilon)$, respectively. Similarly, if the agent were to go short in the indexed bond the payoffs in events where the monetary shock is low and high are $\left(1 + \frac{p_y^H}{p_x}\right) U'(1)$ and $\left(1 + \frac{p_y^H}{p_x}\right) U'(1+\varepsilon)$, respectively. Hence, the most (in terms of the relative price $\frac{q^i}{q^n}$) the agent would want to bid for an unit of the indexed bond is $\left(1 + \frac{p_y^L}{p_x}\right) U'(1+\varepsilon)$. On the other hand the minimum the agent would ask for going short on an indexed bond is $\left(1 + \frac{p_y^H}{p_x}\right) U'(1)$. Given the "bid-ask" spread, there must be a non-degenerate interval of prices at which the agent strictly prefers to maintain a zero holding of the indexed bond. Notice, as depicted in the figure, the effect of increasing ε would be to increase the distance l, since both $\left(1 + \frac{p_y^L}{p_x}\right) U'(1+\varepsilon)$ and $\left(1 + \frac{p_y^H}{p_x}\right) U'(1+\varepsilon)$ will rise relative to the other payoffs. This implies that, for ε large enough, the bid-ask spread will collapse. Notice, it is not necessary that $\mathcal{A} = 1 - \nu^H - \nu^L = 1$ for the bid-ask spread to emerge. A bid-ask spread will exist for $\mathcal{A} < 1$, as long $\mathcal{A}$ is high enough.

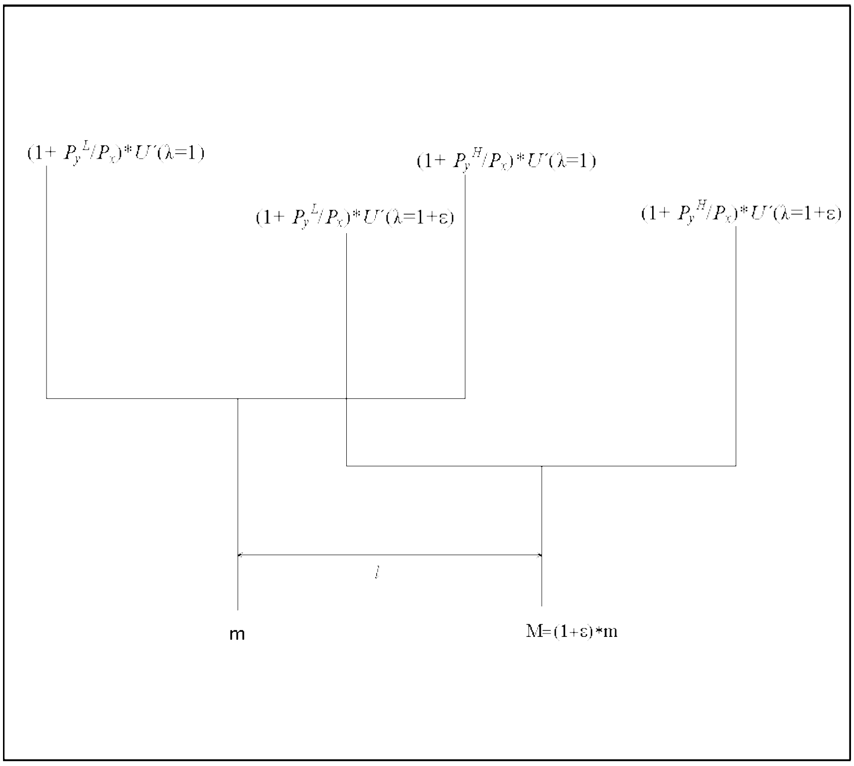

Figure 1: Contingent payoffs from an indexed bond

Since the ranking of states (according to consumption) reverses when switching between a long position and a short position, the "relevant" probability also switches between evaluating a long and a short position. It were as if when evaluating a long position the priors are "concentrated" on states 1 and 2, while they are concentrated on 3 and 4 when evaluating a short position. It is this feature which causes a "kink" in the utility functional at the zero holding position and leads to the bid-ask spread.

Remark 1 An analogous argument, mutatis mutandis, shows that even for a lender (i.e., $S < 0$) one would obtain a bid-ask spread. While reconstructing the argument, essentially, the only adjustment one needs to make is to bear in mind is that for a lender $U'(\lambda)$ is decreasing in λ and hence the "ordering" of the states is reversed (compared to the ordering in the case of a saver).

Remark 2 One could also wonder whether there is a range of prices at which there is a zero holding of the nominal bond. Indeed, in the set up we have described so far, if we were to have, in addition, ambiguous beliefs about the inflation (i.e., $\mu(\{1,3\}) + \mu(\{2,4\}) < 1$), then for a high enough level of this ambiguity, by an argument analogous to the one given above, there will be an interval of relative bond prices, $\frac{q^i}{q^n}$, at which the agent will only have a zero holding of the nominal bond. However, this is not very compelling as the result is not robust in an important way. It does not hold any more if the agent were to have some second period income, preset in nominal terms, that is not derived from bond holdings.

Imagine for instance that the agent receives a state contingent income stream, $\{\overline{m}^s\}_{s=1}^4$, preset in nominal terms. This could represent any previously contracted arrangement like, wage income, pension, social security benefits, etc., that are no more than partially indexed (i.e., they have a nominal component). In this “richer” model, the second period budget constraints are as follows:

$$\begin{aligned}
x^1 &= \bar{x} + \tfrac{\left(p_x+p_y^H\right)}{q^i p_x} S + \left(1 - \left(p_x + p_y^H\right) \tfrac{q^n}{q^i}\right) \tfrac{b^n}{p_x} + \tfrac{\overline{m}^1}{p_x} \\
x^2 &= \bar{x} + \tfrac{\left(p_x+p_y^H\right)}{q^i p_x} S + \left(\tfrac{1}{\lambda} - \left(p_x + p_y^H\right) \tfrac{q^n}{q^i}\right) \tfrac{b^n}{p_x} + \tfrac{\overline{m}^2}{\lambda p_x} \\
x^3 &= \bar{x} + \tfrac{\left(p_x+p_y^L\right)}{q^i p_x} S + \left(1 - \left(p_x + p_y^L\right) \tfrac{q^n}{q^i}\right) \tfrac{b^n}{p_x} + \tfrac{\overline{m}^3}{p_x} \\
x^4 &= \bar{x} + \tfrac{\left(p_x+p_y^L\right)}{q^i p_x} S + \left(\tfrac{1}{\lambda} - \left(p_x + p_y^L\right) \tfrac{q^n}{q^i}\right) \tfrac{b^n}{p_x} + \tfrac{\overline{m}^4}{\lambda p_x}
\end{aligned}$$

Now, it follows immediately, except in the very specific case wherein $\left(\overline{m}^2, \overline{m}^4\right) = \lambda\left(\overline{m}^1, \overline{m}^3\right)$, having a zero holding of nominal bonds does not allow the agent to be rid of the inflation risk. Indeed, $b^n = 0$ implies $x^1 = x^2$ and $x^3 = x^4$ if and only if $\left(\overline{m}^2, \overline{m}^4\right) = \lambda\left(\overline{m}^1, \overline{m}^3\right)$. This, of course, is the case only if <u>*all*</u> income is <u>*fully*</u> indexed or there is no preset nominal income $\left(\overline{m}^s = 0, \forall s\right)$. Hence, whenever, $\left(\overline{m}^2, \overline{m}^4\right) \neq \lambda\left(\overline{m}^1, \overline{m}^3\right)$ and $b^n = 0$ consumption is <u>*stricly*</u> ordered across states 1 and 2 and across states 3 and 4. (For instance, if $\overline{m}^s = \overline{m} > 0$ $\forall s$, one has $x^1 > x^2$ and $x^3 > x^4$.) This strict ordering is preserved in the ϵ-neighborhood $b^n \in (-\epsilon, \epsilon)$. Hence, there is no switch in the probability the DM “applies” when evaluating going short and going long on the nominal bond. Thus the usual expected utility logic applies and the derivative (of the utility functional) is continuous at the point of zero holding, implying that there is no non-degenerate interval of (bond) prices at which the agent holds zero nominal bond.

4 No-trade in indexed bonds: a general equilibrium framework

4.1 The institutional setup

The previous section shows that for both types of agents, those who save and those who borrow, there exists a range of relative bond prices, corresponding to each agent, at which the agent maintains a zero holding in indexed bonds. However, this does not immediately translate into a conclusion about conditions under which no-trade in indexed bonds is the *unique equilibrium* outcome. To be able to get to that conclusion several questions remain to be answered. What would ensure that the bid-ask price intervals of the various agents “overlap”? Why should equilibrium bond price fall within the zone of “overlap”? Further, since we know that for a bid-ask spread to emerge the goods prices have to vary across states in particular ways, a related question is are such state-contingent price variations consistent with competitive equilibrium in money, goods and bond markets? To deal with such issues we turn next to a two-period monetary general equilibrium model, general in the sense that *all* prices are obtained endogenously by (simultaneous) market clearing in bond, goods and money markets. Since the overall aim is to lay out the logic of no trade as transparently as possible, we have chosen the simplest model we could. For instance, given the crucial role of the movement of goods prices in obtaining no-trade, the relationship between such prices and the parameters of the model has been kept as tractable as possible. Arguably, the more realistic source of sectoral price movements are shifts in preferences and/or technological shocks. However, the analysis here is exposited within a framework of the simplest general equilibrium model known to economists, an exchange economy without any production, wherein relative-price movements are derived by perturbing endowments. The point, we emphasize, is transparency, not realism *per se*.

There are two groups of agents in the model. The first group (whose agents are indexed by $h = 1, \ldots, H$) are those who trade on financial markets, while the second group (whose agents are indexed by $k = 1, \ldots, K$) has no access to any financial markets and therefore all the agents in this group consume all the revenue from their endowment spot by spot. There are three goods in this economy, x, y, and z. Agents h consume only goods x and z while agents k consume only goods y and z. We also assume that agents h have real endowments only in goods x and z, while agents k have real endowments only in goods y and z. In addition, agents h may have nominal endowments. Nominal endowments are any precontracted transfers, positive or negative, between agents that are (at least partly) set in nominal terms. Examples include, wages and salaries, house rents or even something like alimony or child support payments. (In the U.S. alimony and child support payments are almost never indexed, see Shiller (1997).)

To see the rationale of "type-casting" agents as above recall, from what was noted in the introductory section, we want to ensure in the model that with respect to *any* two agents wishing to trade in indexed bonds, it is true that the indexation bundle contains at least one good which is not consumed by either of the agents. This condition, of course, would not be satisfied if an h-type agent were to trade bonds with a k-type agent. We have each type of agent consuming two goods rather than one, unlike in the model in the previous section, so that there may be market exchange among agents, thereby obtaining well-defined prices at equilibrium (reflecting the common utility gradients). Informally put, the focus of the "show" will be the intertemporal exchange between the h-type agents, with the role of k-type agents being essentially that of a necessary "prop", enabling the determination of the relative price of the good not figuring in the consumption baskets of agents trading bonds.

There are two periods in the model; uncertainty essentially comes into play in the final period. The endowment of h-type agents is uncontingent, given by $((\bar{x}_h^0, \bar{z}_h^0), (\bar{x}_h, \bar{z}_h, \overline{m}_h))$, where $(\bar{x}_h^0, \bar{z}_h^0)$ is the endowment in the initial period, Period 0, and $(\bar{x}_h, \bar{z}_h, \overline{m}_h)$ is the endowment in the final period, Period 1. $\overline{m}_h$ denotes the nominal endowment, so that $\overline{m}_h \lesseqqgtr 0$ and since transfers should balance across households, we have

$$\sum_{h=1}^{H} \overline{m}_h = 0.$$

Note though, the endowments vary across households; this heterogeneity is the reason why h-type agents trade intertemporal transfers. The endowment of k-type agents are given by $(\bar{y}_k^0, \bar{z}_k^0)$ in the initial period[6]. Their final period endowment in good z is uncontingent and equal to $\bar{z}_k$. We assume, though, their endowments in good y *is* contingent: in state t, $\bar{y}_k^t$, are such that $\sum_{k=1}^{K} \bar{y}_k^t = y^L$ for, say, $t = 1, \ldots, \tau$ and $\sum_{k=1}^{K} \bar{y}_k^t = y^H$ for $t = \tau + 1, \ldots, T$. Thus, in terms of real endowments, there are two aggregate states: one where the total endowment of good y is low (y^L) and another, where the total endowment of y is high (y^H). As will be seen, it is this variation in aggregate endowment which completely determines the variation in the relative price of y.

There is also (outside) money in the model, whose supply in the Period 0 is fixed at M^0 but may take on two values in the Period 1, m or M, where $M \equiv \lambda m$, $\lambda > 1$. The role of money is simply to facilitate exchange. Hence, at each spot, we assume the standard fiction that agents sell to a central authority *all* their endowments against currency issued by the central authority and then buy back from that authority the goods they want to consume (see Magill and Quinzii (1992)). The money obtained from the central authority by agent h (respectively, k) from the sale of endowments in state s is denoted m_h^s (respectively, m_k^s).

Uncertainty in the model is exhaustively represented by the state space

$$\mathcal{S} \equiv \{0\} \cup \{\{1, ..., T\} \times \{m, M\}\},$$

[6] Note, h-type agents do not have nominal endowments in the initial period. k-type agents do not have nominal endowments at all. This is just to save on notation; introducing such endowments would not make the slightest difference to any of our results.

where, $\{0\}$ refers to the only state of the world in Period 0, $\{1,...,T\}$ indexes contingencies in Period 1 obtaining due to variation in real endowments of agents, $\{m, M\}$ indexes the variation in money supply. Let $s \in \mathcal{S}$ be an index for states, $s = 0, 1, \ldots, S$. We denote the (absolute) prices of goods x, y, and z as p_x^s, p_y^s, and p_z^s, respectively, in state s.

There are two financial assets in the model, traded in Period 0. The first is a nominal bond, b^n, that pays off one unit of money in all states and with its price denoted q^n. The second is an indexed bond, b^i, that pays off a bundle of goods at each state in Period 1. We take this bundle to be state-independent and comprising of a unit of each good traded in the economy. Hence, the monetary return to holding a unit of this indexed bond is $p_x^s + p_y^s + p_z^s$ in state $s = 1, \ldots, S$. Its price is denoted q^i.

For the moment, denote an agent h's preferences by a functional $V_h\left((x_h^0, z_h^0), \ldots, (x_h^S, z_h^S)\right)$, on which we'll impose assumptions detailed later on. His maximization problem is hence:

$$
\begin{array}{ll}
Max_{x_h, z_h, b_h^i, b_h^n} & V_h\left((x_h^0, z_h^0), \ldots, (x_h^S, z_h^S)\right) \\
s.t. & \left\{ \begin{array}{rcl}
p_x^0 \bar{x}_h^0 + p_z^0 \bar{z}_h^0 & = & m_h^0 \\
p_x^0 x_h^0 + p_z^0 z_h^0 & = & m_h^0 - q^i b_h^i - q^n b_h^n \\
p_x^s \bar{x}_h^s + p_z^s \bar{z}_h^s + \bar{m}_h & = & m_h^s \\
p_x^s x_h^s + p_z^s z_h^s & = & m_h^s + b_h^n + (p_x^s + p_y^s + p_z^s) b_h^i \;\; s = 1, \ldots, S
\end{array} \right.
\end{array}
$$

Agents k, who have no access to financial markets, have to solve $S+1$ separate maximization programs. We assume that their preferences at each spot take the simple form of a Cobb-Douglas function: $(y_k^s)^\alpha (z_k^s)^{1-\alpha}$ for $\alpha \in (0,1)$. Hence, their maximization problem for $s \in \mathcal{S}$, is:

$$
\begin{array}{ll}
Max_{y_k^s, z_k^s} & (y_k^s)^\alpha (z_k^s)^{1-\alpha} \\
s.t. & \left\{ \begin{array}{rcl}
p_y^s \bar{y}_k^s + p_z^s \bar{z}_k^s & = & m_k^s \\
p_y^s y_k^s + p_z^s z_k^s & = & m_k^s
\end{array} \right.
\end{array}
$$

An equilibrium of this model is therefore an allocation $(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{m}, \mathbf{b}^i, \mathbf{b}^n)$ and prices $(p_x, p_y, p_z, q^i, q^n)$ such that, given these prices agents solve their maximization problems and markets clear.

Observe that money as we introduced it is simply a veil and we can rewrite the budget constraints as follows, for agent h:

$$
\left\{ \begin{array}{rcll}
p_x^0 x_h^0 + p_z^0 z_h^0 & = & p_x^0 \bar{x}_h^0 + p_z^0 \bar{z}_h^0 - q^i b_h^i - q^n b_h^n & \\
p_x^s x_h^s + p_z^s z_h^s & = & \overline{m}_h + p_x^s \bar{x}_h^s + p_z^s \bar{z}_h^s + b_h^n + (p_x^s + p_y^s + p_z^s) b_h^i & s = 1, \ldots, S
\end{array} \right.
$$

and, for agent k in state s ($s = 0, 1, \ldots, S$):

$$
p_y^s y_k^s + p_z^s z_k^s \;\; = \;\; p_y^s \bar{y}_k^s + p_z^s \bar{z}_k^s
$$

One can also use the particular structure of the model to simplify the market clearing condition for good z. Indeed, adding the budget constraints in state s of agents h, one gets, at equilibrium, $\sum_{h=1}^H z_h^s = \sum_{h=1}^H \bar{z}_h$ (under the assumption that $p_x^s > 0$, which is met since preferences are assumed strictly monotonic). Similarly, for agents k, one obtains, from adding their budget constraints in state s and using equilibrium condition on the market for good y (plus the fact that, at an equilibrium, $p_z^s > 0$), that $\sum_{k=1}^K z_k^s = \sum_{k=1}^K \bar{z}_k$. Thus, the market clearing conditions on the market for good z can be split in two equalities as follows:

$$
\sum_{h=1}^H z_h^s = \sum_{h=1}^H \bar{z}_h \;\text{ and }\; \sum_{k=1}^K z_k^s = \sum_{k=1}^K \bar{z}_k \quad s = 0, \ldots, S
$$

Hence, the market for good z can be "divided in two", agents h exchanging among themselves, and similarly for agents k. The intuition for this is fairly obvious once one lifts the "veil of money" and considers the nature of "real" exchange in the model. The point is, given that the two types of agents share only one good between their respective consumption baskets, there cannot be any "real" exchange between these groups on spot markets.

Finally, notice that the market clearing condition on the money market can be written as

$$p_x^s \sum_{h=1}^{H} \bar{x}_h + p_y^s \sum_{k=1}^{K} \bar{y}_k^s + p_z^s (\sum_{h=1}^{H} \bar{z}_h + \sum_{k=1}^{K} \bar{z}_k) = M^s \quad s = 0, \ldots, S$$

while the market clearing condition on the bond markets are $\sum_{h=1}^{H} b_h^i = \sum_{h=1}^{H} b_h^n = 0$.

4.2 Equilibrium prices in goods markets

We can further reduce the model by noticing that only aggregate states "matter". Indeed, note that there are two sources of (aggregate) uncertainty in this model: one is linked to the money supply, the second stems from the randomness in the (aggregate) endowment in good y of agents k. As we will be only interested in the equilibrium allocation of the h agents (and in particular whether they hold indexed bonds or not), the only way this last source of uncertainty is relevant to h agents is through the effect it has on prices. Now, observe that we can solve for the equilibrium relative price of y with respect to z, spot by spot. Indeed, agents k demand functions are easily computed and are equal to:

$$y_k^s(p_y^s, p_z^s) = \alpha \frac{p_y^s \bar{y}_k^s + p_z^s \bar{z}_k^s}{p_y^s} \text{ and } z_k^s(p_y^s, p_z^s) = (1-\alpha) \frac{p_y^s \bar{y}_k^s + p_z^s \bar{z}_k}{p_z^s}$$

Hence, at equilibrium,

$$\frac{p_y^s}{p_z^s} = \frac{\alpha}{1-\alpha} \frac{\sum_{k=1}^{K} \bar{z}_k}{\sum_{k=1}^{K} \bar{y}_k^s}$$

and therefore, the ratio of the prices p_y^s and p_z^s depends only on the *aggregate* (among k agents) endowments of good y and z, and thus, can take on only two values, whether aggregate endowment in y is high (y^H) or low (y^L). Note that the price levels do depend on the money supply. To sum up, we need, for our purposes, concentrate only on four states $\omega \in \Omega \equiv \{1, 2, 3, 4\}$ defined as follows:

State $\omega = 1$: low money supply (m) and low aggregate y-endowment (y^L)
State $\omega = 2$: high money supply (M) and low aggregate y-endowment (y^L)
State $\omega = 3$: low money supply (m) and high aggregate y-endowment (y^H)
State $\omega = 4$: high money supply (M) and high aggregate y-endowment (y^H)

We now describe agent's h preferences, following partly a specification due to Magill and Quinzii (1997). At state 0, h-type agents' utility function is written as $u(x_h^0, z_h^0)$, where $u : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, concave, differentiable and homogeneous of degree 1. In state $\omega = 1, 2, 3, 4$, the spot utility function of agent h is given by $f_h(u(x_h^\omega, z_h^\omega))$, where $f_h : \mathbb{R} \to \mathbb{R}$ is strictly increasing, strictly concave and differentiable, and u is as defined above. The linear homogeneity of u will facilitate the tractability of the equilibrium (contingent) price function (essentially, the assumption ensures that goods prices, in each spot market, are independent of the distribution of wealth among agents) while the concavity of f_h is a simple way of endowing agents with risk aversion as well as a desire to smooth consumption across periods.

Type-h agents are endowed with (common) beliefs about the money supply process and the process generating the (aggregate) y-endowments. $\mu = (\mu^m, \mu^M)$ denotes the capacity describing their (marginal) belief about the money supply: μ^m is the (possibly non-additive) probability the money supply is m and μ^M is the probability that it is M; $\mu^m + \mu^M \leq 1$. Their (marginal) beliefs on the process generating the y-endowments (a good they do not consume and are not endowed with) are represented[7] by $\nu = (\nu^L, \nu^H)$, with $\nu^L + \nu^H \leq 1$. The overall beliefs on Ω is given by the independent product $\mu \otimes \nu$. Preferences of agent h are thus represented by the functional V_h, where,

$$V_h\left((x_h^0, z_h^0), \ldots, (x_h^4, z_h^4)\right) \equiv u(x_h^0, z_h^0) + \mathbb{CE}_{\mu\otimes\nu} f_h\left(u(x_h^\omega, z_h^\omega)\right).$$

Under our assumptions (essentially, the linear homogeneity of u) at an equilibrium, the maximization problem of each h agent can be decomposed to separate the financial decision about the allocation of resources across states from the decision about the consumption mix at each state. Turning first to the latter, given a stream $(R_h^0, R_h^1, \ldots, R_h^4)$ of income with $R_h^0 = p_x^0 \bar{x}_h^0 + p_z^0 \bar{z}_h^0 - q^i b_h^i - q^n b_h^n$ and $R_h^\omega = p_x^\omega \bar{x}_h^\omega + p_z^\omega \bar{z}_h^\omega + (p_x^\omega + p_y^\omega + p_z^\omega) b_h^i + b_h^n + \bar{m}_h$, $\omega = 1, \ldots, 4$, the agent solves the problem:

$$\begin{array}{cl} Max_{x_h^\omega, z_h^\omega} & u(x_h^\omega, z_h^\omega) \\ \text{s.t.} & p_x^\omega x_h^\omega + p_z^\omega z_h^\omega = R_h^\omega \end{array}$$

At the optimal choice, one gets for all ω:

$$\frac{\nabla_1 u(x_h^\omega, z_h^\omega)}{\nabla_2 u(x_h^\omega, z_h^\omega)} = \frac{p_x^\omega}{p_z^\omega}$$

where $\nabla_i u(x_h^\omega, z_h^\omega)$ is the derivative of u with respect to its i^{th} component. By homogeneity of degree one, the gradients are collinear among agents only if their consumption vectors are collinear as well. Now, remember that, at an equilibrium agents h only trade among themselves and do not trade with the k agents. Hence, each agent h's consumption in state ω is a fraction α_h^ω of total endowment of h-agents with

$$\alpha_h^\omega = \frac{p_x^\omega \bar{x}_h + p_z^\omega \bar{z}_h + (p_x^\omega + p_y^\omega + p_z^\omega) b_h^i + b_h^n + \bar{m}_h}{p_x^\omega \sum_{h=1}^H \bar{x}_h + p_z^\omega \sum_{h=1}^H \bar{z}_h}$$

Hence, at an equilibrium,

$$(x_h^\omega, z_h^\omega) = \alpha_h^\omega \left(\sum_{h=1}^H \bar{x}_h, \sum_{h=1}^H \bar{z}_h \right)$$

Therefore, agent h's utility, at an equilibrium, can be rewritten $u\left(x_h^\omega, z_h^\omega\right) = \alpha_h^\omega u$, where u is simply the utility at the endowment point, i.e., $u \equiv u\left(\sum_{h=1}^H \bar{x}_h, \sum_{h=1}^H \bar{z}_h\right)$. Finally, since relative prices of good x and z in state ω are equal to the gradient of an agent h's utility function, it is easy to see that

$$\frac{p_x^1}{p_z^1} = \frac{p_x^2}{p_z^2} = \frac{p_x^3}{p_z^3} = \frac{p_x^4}{p_z^4} \equiv \zeta$$

given that endowments of goods x and z are constant across states. In fact, it turns out that, the absolute price of goods x and z do not depend on the amount of good y available in the

[7] Note that, ν^H refers now to the state with high y-endowment, and therefore low p_y, while it referred to the high p_y in the previous section.

economy. In other words, and since there is no uncertainty on the total endowments of goods x and z, their price depends only on the money supply. This is the content of Proposition 1, below, which is proved in the appendix. A direct corollary is that the price of y, conditional on the monetary state, is completely determined by the aggregate endowment in y. Proposition 2 (proved in the appendix) which essentially shows that monetary equilibrium requires the price vector in state 2 (respectively, 4) is simply a λ-multiple of prices in state 1 (respectively, 3), completes the required characterization of equilibrium prices.

Proposition 1 *At an equilibrium, $p_x^1 = p_x^3$, $p_x^2 = p_x^4$, $p_z^1 = p_z^3$, and $p_z^2 = p_z^4$.*

From this proposition, it is easy to show that, at an equilibrium, $\frac{p_y^1}{p_y^3} = \frac{y^H}{y^L} = \frac{p_y^2}{p_y^4}$ (this follows from the fact that $\frac{p_y^1}{p_z^1} = \frac{p_y^3}{p_z^3}\frac{y^H}{y^L}$ and $p_z^1 = p_z^3$).

Proposition 2 *At an equilibrium, $(p_x^1, p_y^1, p_z^1) = \frac{1}{\lambda}(p_x^2, p_y^2, p_z^2)$ and $(p_x^3, p_y^3, p_z^3) = \frac{1}{\lambda}(p_x^4, p_y^4, p_z^4)$.*

The following table, then, summarizes the equilibrium prices, and the corresponding return from an unit of an indexed bond at each state ω, $\omega \in \Omega$. The reader will recall the table is identical to the one presented in Section 3.

State ω	Prices	Return from an indexed bond
1	$\left(p_x, p_y^H\right)$	$p_x + p_y^H$
2	$\left(\lambda p_x, \lambda p_y^H\right)$	$\lambda\left(p_x + p_y^H\right)$
3	$\left(p_x, p_y^L\right)$	$p_x + p_y^L$
4	$\left(\lambda p_x, \lambda p_y^L\right)$	$\lambda\left(p_x + p_y^L\right)$

4.3 The nature of equilibrium in bond markets

We now turn to the intertemporal maximization problem of agent h and derive the principal formal conclusions of our analysis. The first result, which is in the nature of a benchmark, shows that, for generic endowments, if beliefs are not ambiguous, there will always be trade in indexed bonds. The second and main result shows that, at equilibrium, if ambiguity of belief about the $y-$prices ($\mathcal{A}(\nu) \equiv 1 - \nu^L - \nu^H$) is large enough and inflation (λ) is not too high, the indexed bond is not traded and only the nominal bond is traded. As we show, this result holds irrespective of the degree of ambiguity about the money supply. In what follows, we first explain an intuition of the equilibrium reasoning underlying the results and then state the theorems, with the formal proofs appearing in the appendix.

We begin by considering the nature of the equilibrium in the indexed bond market at two values of λ, $\lambda = 1$ and $\lambda = 1 + \varepsilon$, where ε is a positive number arbitrarily close to 0. Consider, first, the case wherein $\lambda = 1$. Without any inflation at all, clearly, all borrowing and lending will be done through nominal bonds, at equilibrium. Take two h-type agents, h' and h who save and borrow, respectively, in the initial period. Their utility in the final period, with slight abuse of the notation, may be written as $f_{h'}\left(u(\bar{x}_{h'}, \bar{z}_{h'}; \frac{S_{h'}}{\lambda q^n})\right)$ and $f_h\left(u(\bar{x}_h, \bar{z}_h; \frac{S_h}{\lambda q^n})\right)$, where $S_{h'} > 0$ and $S_h < 0$ denote the amount saved and the amount borrowed by h' and h respectively. Define the "marginal utilities", $U_h'(\lambda) \equiv f_h'\left(u(\bar{x}_{h'}, \bar{z}_{h'}; \frac{S_h}{\lambda q^n})\right)$ and $U_{h'}'(\lambda) \equiv f_{h'}'\left(u(\bar{x}_{h'}, \bar{z}_{h'}; \frac{S_{h'}}{\lambda q^n})\right)$. Notice, $U_{h'}'(\lambda) \uparrow$ in λ while $U_h'(\lambda) \downarrow$ in λ, since $S_{h'} > 0$ and $S_h < 0$; intuitively, inflation affects the welfare of savers and borrowers differently. Furthermore, it must be necessarily true at an equilibrium that $U_h'(\lambda = 1) = U_{h'}'(\lambda = 1)$. This is so since if there is no inflation, h-agents are effectively trading in a complete market when trading only in nominal bond, and thus the equilibrium is Pareto optimal.

First suppose, ceteris paribus, there were no ambiguity, i.e., $1-\nu^L-\nu^H=0$ and $1-\mu^m-\mu^M=0$, so that the DM's behavior were that of an SEU agent. At $\lambda=1$, the "utility return" from an (infinitesimal) unit of an indexed bond at equilibrium must be $U'_h(\lambda=1)\times\mathbb{E}\left(p_x+p_y^\omega\right)=U'_{h'}(\lambda=1)\times\mathbb{E}\left(p_x+p_y^\omega\right)\equiv q^i(\lambda=1;h,h')$. Putting it differently, $q^i(\lambda=1;h,h')$ is the price at which the agents (h and h') are indifferent between not trading and trading an infinitesimal amount of indexed bonds. Similarly, for an arbitrary λ, define $q^i(\lambda;h)$ (respectively, $q^i(\lambda;h')$) as the minimum (maximum) price h (h') is willing to accept (pay) to trade in the indexed bond. Next, consider a perturbation of λ to $\lambda=1+\varepsilon$. Recalling the effect of a change in λ on $U'_h(\lambda)$ and $U'_{h'}(\lambda)$, it is straightforward to see that

$$\begin{aligned} q^i\left(\lambda=1+\varepsilon;h'\right) &\equiv U'_{h'}(\lambda=1+\varepsilon)\times\mathbb{E}\left(p_x+p_y^\omega\right)\\ &> q^i\left(\lambda=1;h,h'\right)\\ &> U'_h(\lambda=1)\times\mathbb{E}\left(p_x+p_y^\omega\right)\equiv q^i(\lambda=1+\varepsilon;h)\,. \end{aligned}$$

Intuitively, since the saver is affected adversely by inflation, relative to the debtor, the indexed bond is more valuable to the saver in the presence of inflation, and also, more valuable than it is to the debtor. Hence, inevitably, with inflation creeping up there will be gains from trading in indexed bonds and indexed bonds will be traded at equilibrium under SEU. This is depicted in Figure 2, below, in the left-hand-side panel and formally stated in our first theorem[8].

Theorem 1 *Suppose,* $\mu^m+\mu^M=1$ *and* $\nu^L+\nu^H=1$. *Then, for generic first period aggregate endowments, there is trade in the indexed bond whenever* $\lambda\neq1$.

Next suppose, agents have CEU preferences and beliefs about the y-prices are ambiguous, i.e., $1-\nu^L-\nu^H>0$. As before, we first consider the equilibrium at $\lambda=1$. As would be evident from our discussion in Section 3, there would exist a bid-ask spread: there will be a bid price corresponding to (perceived) marginal gain from moving (infinitesimally) into a indexed bond, and an ask price, that is strictly lower, corresponding to the (perceived) marginal gain from going short on the indexed bond. The bid-ask interval will correspond to an interval of expected marginal utilities, where the lower end of the interval is evaluated by applying the probability measure that minimizes the expectation for an agent going long and the upper end is evaluated by applying the probability measure that minimizes the expectation for an agent going short:

$$\begin{aligned} &\left[U'_h(\lambda=1)\times\underline{\mathbb{E}}\left(p_x+p_y^\omega\right),U'_h(\lambda=1)\times\bar{\mathbb{E}}\left(p_x+p_y^\omega\right)\right]\\ =&\left[U'_{h'}(\lambda=1)\times\underline{\mathbb{E}}\left(p_x+p_y^\omega\right),U'_{h'}(\lambda=1)\times\bar{\mathbb{E}}\left(p_x+p_y^\omega\right)\right]\\ \equiv&\left[\underline{q}^i\left(1;h,h'\right),\hat{q}^i\left(1;h,h'\right)\right] \end{aligned}$$

Next, consider the equilibrium given the perturbation $\lambda=1+\varepsilon$. Noting again the effect of the perturbation on $U'_h(\lambda)$ and $U'_{h'}(\lambda)$, it is straightforward to see that for h', the saver, the *entire* interval moves *up*, whereas for h, the debtor, the *entire* interval moves *down*. The extent of movement is greater, the greater increase in λ. Hence, for $\lambda>1$, but small enough, the intervals overlap and the bid price of the saver remains (strictly) lower than the ask price required by the lender:

$$\underline{q}^i\left(1+\varepsilon;h'\right)\equiv U'_{h'}(1+\varepsilon)\times\underline{\mathbb{E}}\left(p_x+p_y^\omega\right)<U'_h(1+\varepsilon)\times\bar{\mathbb{E}}\left(p_x+p_y^\omega\right)\equiv\hat{q}^i(1+\varepsilon;h)$$

[8]Both theorems refer to properties that hold "generically". The term is applied in a way that is now standard in economic theory. Notice, endowments are points in $\mathbb{R}^6$ and the μ-beliefs are simply points on the 2-dimensional simplex. We say that a property is satisfied for generic endowments (respectively, μ's) if, for every endowment (respectively, μ) vector there is an open dense neighborhood of endowment (respectively, μ) vectors that generate economies that satisfy this property. Thus, if a property is satisfied for all generic endowments (respectively, μ's), small perturbations of the endowment (respectively, μ) in any economy can generate a new economy that satisfies this property robustly, even if the original economy does not.

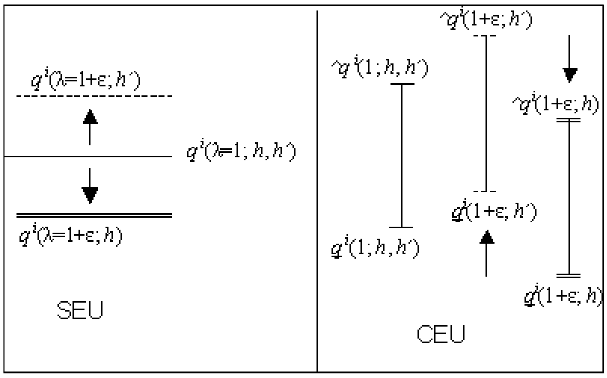

Figure 2: Equilibrium in the indexed bond market

This is represented in Figure 2 in the right-hand side panel. It is also evident in the figure that if the increase in λ were large enough then the intervals move apart enough not to overlap. Hence, we have, for λ small enough, there is no trade in indexed bonds at equilibrium. This is formally stated in the theorem below.

Finally, recalling the Remark 2, if there is at least one agent with a positive nominal endowment (and hence, at least one other agent with a negative nominal endowment) it follows that there is at least a pair of agents who do not optimally choose a zero-holding of the nominal bond over a non-degenerate interval of relative bond prices. Hence, for these agents the situation is no different from the SEU case described earlier (and depicted on the left-hand-side panel of Figure 2). "Typically" such agents would want to trade in nominal bonds. The discussion is summarized by the two parts of Theorem 2, below.

Theorem 2 *Suppose,* $\mu^m + \mu^M \leq 1$ *and* $\nu^L + \nu^H < 1$. *Then, there exists a bound* δ, $\delta > 1$, *such that, if* $\lambda < \delta$, *there exists* γ, $0 < \gamma < 1$, *such that if* $\mathcal{A}(\nu) > \gamma$, *then at an equilibrium,*

(a) *the indexed bond is not traded, i.e.,* $b_h^i = 0$ *for all* h,

(b) *for generic* μ*-beliefs, there is trade in the nominal bond as long as there exists an* h *such that* $\bar{m}_h > 0$.

Remark 3 Notice, both parts of the theorem hold regardless of the level of ambiguity w.r.t. the μ-beliefs. Indeed, the formal proof is a lot shorter (and simpler) if one were to assume that μ-beliefs were unambiguous. We, however, do not impose this restriction in the analysis so that one may obtain a more informed idea of the nature of robustness of the result. Of course, nominal endowments would no longer play a role in the argument if μ-beliefs were assumed to be unambiguous.

Remark 4 The proof of the theorem, in the Appendix, provides an explicit characterization of the bound δ in terms of the fundamentals of the model. As would have been intuitively evident from the discussion preceding the theorem, the degree to which inflation would be "tolerated" before indexed bonds are traded depends (positively) on the "variability" of the relative prices. For instance, the δ is equal to $\frac{2+\sum_{h=1}^{H} \bar{x}_h / y^L}{2+\sum_{h=1}^{H} \bar{x}_h / y^H}$ when all agents (h and k) have Cobb-Douglas $(.5, .5)$ as their respective $u(.,.)$ functions.

Remark 5 The logic underlying the result in Theorem 2(b) is actually instructive, indirectly, as to why ambiguity about the price movements of goods *not* in the(h-agents') consumption basket was the crucial factor in obtaining no-trade in indexed bonds. Putting it differently, if we allowed the absolute prices of say, x, to vary in response to supply shocks and assumed agents had ambiguous beliefs about such price movements, that would not obtain the no-trade in indexed bonds (without ambiguity about price of y). The reasoning here is analogous to the one showing that the presence of nominal endowments precludes no-trade in nominal bonds. Since x is present in the endowment and/or affects utility directly (of h-agents), maintaining a zero position on the indexed bond would not get rid of the risk due to the variability of the price of x. Consequently, the ordering of states would not switch at the zero holding position.

Remark 6 In the real world inflation and relative price movements are correlated. Recall, though, the model assumes that the processes generating the money supply and the real shocks (to the aggregate endowment of y) are believed to be independent. However, what is really crucial is that the process generating p_y must have at least one component that is believed to be orthogonal to the money supply: i.e., there has to be at least two states of the world across which the price of y changes even though the money supply stays constant. In other words, for the reasoning to hold, prices cannot be perfectly correlated with the money supply. Similarly, the logic behind our result does not require that p_y and p_x (or p_z) have to be independent, but that they are not perfectly correlated.

5 Concluding discussion

The discussion in this section is in two parts. In the first part we relate the results obtained in this paper to those in the relevant (theoretical) literature. The second part considers to what extent the analysis here may be thought to explain the empirics of trade in indexed debt any more than the explanations already advanced. Implications of the results in terms of policy required to encourage trade in indexed bonds is also discussed.

5.1 Related (theoretical) literature

We begin by linking the result here to the findings in the literature applying ambiguity aversion to financial markets. Then we discuss the theoretical literature in the standard Savage paradigm that seeks to explain observed facts relating to trade in indexed debt.

As has already been noted, Dow and Werlang (1992) showed that a zero position may be held on a price interval if the agent's endowments were riskless. Obviously, an economy where all agents' endowments were unvarying across *all* states the question of asset trading and risk sharing is an uninteresting question. Epstein and Wang (1994) significantly generalized the Dow and Werlang (1992) result to find that price intervals supporting the zero position occurred (in equilibrium) if there were *some* states across which asset payoffs differ while endowments remain identical; in other words, asset payoffs have component of idiosyncratic risk. However, the focus of Epstein and Wang (1994) was the issue of asset pricing. In their model endowments are Pareto optimal, and consequently, the issue of whether ambiguity aversion cause assets not to be traded is not examined. Mukerji and Tallon (1999), building on the results in the two papers cited, finds conditions for an economy wherein the agents' price intervals overlap in such a manner such that *every* equilibrium of the economy involves no trade in an asset, and more importantly, conditions under which ambiguity aversion *demonstrably* "worsens" risk sharing and incompleteness of markets. One of the conditions, the presence of idiosyncratic risk, identified in Mukerji and Tallon (1999) as necessary for no-trade, is essentially the same as in the result of Epstein and Wang (1994) explained above. Looking back at the model in this paper, it is possible to see that, for h-type agents, payoffs of indexed bonds contain an element of idiosyncratic risk derived from the risk inherent in the relative price of y. This is, essentially, how the finding in this paper links up with the results in the papers cited above.

The paper closest to ours, within the Savage paradigm, which seeks to explain the lack of indexed debt is Magill and Quinzii (1997). That paper compares the welfare improvements obtained from introducing within an incomplete markets setting, *in turn*, a nominal bond and an indexed bond. The welfare improvements derive from, essentially, the increase in the span of available assets (or, in other words, the "lessening" of incompleteness) that comes about due to the introduction of each type of bond. The more relevant result is that the welfare gain from introducing the indexed bond may be less (respectively, more) than that from introducing a nominal bond if the inflation risk was "small" (respectively, large) compared to the relative price risk. In contrast to the analysis in this paper, Magill and Quinzii (1997) does not actually obtain a equilibrium with no-trade in indexed bonds; indeed, as we confirm in Theorem 1, Savage rational agents will necessarily trade in indexed bonds as long as there is some inflation. Also, Magill and Quinzii (1997) do not allow *both* indexed and nominal bonds to be available for trade simultaneously; one or the other is available.

5.2 Explaining the empirics of trade in indexed debt

Recall the intuition underlying the main result. Taking a long or a short position on the indexed bond implies betting on or against the (ambiguous) event wherein the (relative) price of good y will be high. To decide whether to bet on or against a particular event one has to reach a fine judgement about the relative likelihood of the event compared to its complement. Hence, the attraction of the zero holding position to the ambiguity averse agent. Moving from the zero position, in either direction, requires a compensating "ambiguity premium". Hence, the bid-ask spreads for the indexed bond. At low levels of inflation the bid-ask intervals of the borrower and the saver overlap and agents only trade in the nominal bond. As inflation rises, the saver is affected adversely while the borrower is made better off. As a consequence, the most the saver is willing to pay for the indexed bond goes up and the minimum the borrower would ask decreases. Hence if inflation were high enough, agents do trade indexed bonds. We also argued that, so long as agents held (non-zero) nominal endowments, this reasoning does not apply quite symmetrically to trade in nominal bonds. For instance, with a positive holding of the nominal bond, one would be betting on the event that (average) price level will be low. But, if one had, say, positive nominal endowments, moving to a zero holding would still imply one is betting in favor of the event that the price level will be low. Indeed, this would continue to be true even if one were to move marginally into a negative holding of the nominal bond. Hence, the bid-ask spread no longer occurs at the zero holding and trade occurs. Of course, by the same token, if some *endowments* were indexed, the argument for no trade in indexed bonds will be affected similarly.

Thus, according to the theory presented in this paper, it is the comparative lack of information about relative, as opposed to average, price movements, the comparative preponderance of nominal, as opposed to indexed, endowments that explains why trade in indexed bonds is observed only in exceptional circumstances but trade in nominal bonds is so widespread. While these are testable hypotheses that could provide the basis for a specific empirical investigation, that is a matter for future research. However, a lot that we do know about trade in indexed bonds is broadly consistent with the theory. Arguably, the theory is very consistent with the fact that typically indexed bonds are traded almost exclusively under extreme inflationary circumstances. Also, while trade in indexed bonds is negligible in most non-inflationary economies, it is more than negligible (though still quite small) in the few such economies where, in addition, there are some instances of indexed endowments, statutory wage indexation, as is the case, for example, in the U.K. and in Israel (statutory wage indexation in a limited number of sectors of the economy). Indeed, the reasoning predicts that one would observe a kind of hysteresis in the market for indexed bonds. In economies with inflationary past, where indexed bonds were traded when high inflation reigned, indexed bonds would continue to be traded even after inflation has been brought down to moderate levels because of the presence of indexed bonds as endowments. Perhaps, this explains the continued trade in indexed financial instruments observed in some South American economies (and even Israel) where inflation has lately been tamed. One may also argue that an analogous reasoning explains why in countries, like Turkey, where use of dollar is widespread in spot market transactions, so is the use of dollar-indexed debt.

The theory presented, strictly interpreted, demonstrates a reason why indexed bonds are not exchanged by private individuals. But in the case of trade in government bonds, at least at the point of issue, one of the parties to the trade is not a utility maximizing private agent. However, note our theory would apply just as well to any secondary trade of indexed government bonds between private individuals. Thus our theory predicts, this secondary market typically be a rather thin market. In turn, this carries the implication of indexed government debt not being a particularly liquid asset. Clearly, given rational, forward looking individuals, this would, in itself, ensure that demand for such debt would be weak even at the point of issue.

Finally, we turn briefly to some policy implications. One obviously welfare increasing move would be to publish (trustworthy) indexes that are more particular and focused on fewer goods and services than the CPI. Looking back to the formal model, if there were an index composed purely of the prices of x and z the resulting allocation would indeed be a Pareto improvement on the allocation obtained with index composed of prices of x, z and y (the market would become as good as complete for the h-type agents). Government action in introducing statutory indexation of some payments, say of wages in some sectors, would increase the trade in indexed debt. However, it is far from clear, given the abstractions in our model, that we may conclude that issuing such a fiat would at all be welfare enhancing.

6 Appendix

6.1 Slice-comonotonicity

The computation of the Choquet expectation operator using product capacities is particularly simple for *slice comonotonic* functions (Ghirardato (1997)), defined below. Let $X_1, ..., X_n$ be n (finite) sets and let $\Omega = X_1 \times ... \times X_n$. Correspondingly, let ν_i be convex non-additive probabilities defined on algebras of subsets of X_i, $i = 1, ..., n$.

Definition 2 *Let $\varphi : \Omega \to \mathbb{R}$. We say that φ has comonotonic x_i-sections if for every $(x_1, ..., x_{i-1}, x_{i+1}, ..., x_n)$, $\left(x'_1, ..., x'_{i-1}, x'_{i+1}, ..., x'_n\right) \in X_1 \times ... \times X_{i-1} \times X_{i+1} \times ... \times X_n$, $\varphi(x_1, ..., x_{i-1}, \cdot, x_{i+1}, ..., x_n) : X_i \to \mathbb{R}$, and $\varphi\left(x'_1, ..., x'_{i-1}, \cdot, x'_{i+1}, ..., x'_n\right) : X_i \to \mathbb{R}$ are comonotonic functions. φ is called slice-comonotonic if it has comonotonic x_i-sections for every $i \in \{1, ..., n\}$.*

The utility function of agent h, $f_h(u(x_h^\omega, z_h^\omega))$, is actually slice comonotonic, since u is strictly monotone and hence Fact 1, below, which follows from Proposition 7 and Theorem 1 in Ghirardato (1997), applies to the calculation of Choquet expected utility for agent h.

Fact 1 *Suppose that $\varphi : \Omega \to \mathbb{R}$ is slice comonotonic. Then*

$$\int_\Omega \varphi(x_1, ..., x_n)\, d(\otimes \nu_i) = \int_{X_1} ... \int_{X_n} \varphi(x_1, ..., x_n)\, d\nu_n ... d\nu_1$$

6.2 A formal confirmation of the argument in Section 3:

To see the argument put more formally, compute first the right-hand-side derivative of $V(x^1, x^2, x^3, x^4)$, in which we replace x^s by its expression as a function of b^i, to obtain a function $W(b^i)$:

$$\frac{dW(b^i)}{db^i} \mid_{b^i=0^+} = \mu\left[\nu^H\left(1+\frac{p_y^H}{p_x}-\frac{q^i}{q^n p_x}\right)+(1-\nu^H)\left(1+\frac{p_y^L}{p_x}-\frac{q^i}{q^n p_x}\right)\right]u'\left(\bar{x}+\frac{S}{q^n p_x}\right)$$
$$+(1-\mu)\left[\nu^H\left(1+\frac{p_y^H}{p_x}-\frac{q^i}{q^n\lambda p_x}\right)+(1-\nu^H)\left(1+\frac{p_y^L}{p_x}-\frac{q^i}{q^n\lambda p_x}\right)\right]u'\left(\bar{x}+\frac{S}{q^n\lambda p_x}\right)$$

The same computation, but for the left-hand-side derivative yields:

$$\frac{dW(b^i)}{db^i} \mid_{b^i=0^-} = \mu\left[(1-\nu^L)\left(1+\frac{p_y^H}{p_x}-\frac{q^i}{q^n p_x}\right)+\nu^L\left(1+\frac{p_y^L}{p_x}-\frac{q^i}{q^n p_x}\right)\right]u'\left(\bar{x}+\frac{S}{q^n p_x}\right)$$
$$+(1-\mu)\left[(1-\nu^L)\left(1+\frac{p_y^H}{p_x}-\frac{q^i}{q^n\lambda p_x}\right)+\nu^L\left(1+\frac{p_y^L}{p_x}-\frac{q^i}{q^n\lambda p_x}\right)\right]u'\left(\bar{x}+\frac{S}{q^n\lambda p_x}\right)$$

Hence, if the following conditions hold, then $b^i = 0$ is optimal for the agent:

$$\mu\left[1-\tfrac{q^i}{q^n p_x}+\left(\nu^H\tfrac{p_y^H}{p_x}+(1-\nu^H)\tfrac{p_y^L}{p_x}\right)\right]u'\left(\bar{x}+\tfrac{S}{q^n p_x}\right)+$$
$$(1-\mu)\left[1-\tfrac{q^i}{q^n\lambda p_x}+\left(\nu^H\tfrac{p_y^H}{p_x}+(1-\nu^H)\tfrac{p_y^L}{p_x}\right)\right]u'\left(\bar{x}+\tfrac{S}{q^n\lambda p_x}\right)$$

$$\leq 0 \leq$$

$$\mu\left[1-\tfrac{q^i}{q^n p_x}+\left((1-\nu^L)\tfrac{p_y^H}{p_x}+\nu^L\tfrac{p_y^L}{p_x}\right)\right]u'\left(\bar{x}+\tfrac{S}{q^n p_x}\right)+$$
$$(1-\mu)\left[1-\tfrac{q^i}{q^n\lambda p_x}+\left((1-\nu^L)\tfrac{p_y^H}{p_x}+\nu^L\tfrac{p_y^L}{p_x}\right)\right]u'\left(\bar{x}+\tfrac{S}{q^n\lambda p_x}\right)$$

This amounts to:

$$\nu^H\tfrac{p_y^H}{p_x}+(1-\nu^H)\tfrac{p_y^L}{p_x}\leq$$
$$\frac{\mu\frac{q^i}{q^n p_x}u'\left(\bar{x}+\frac{S}{q^n p_x}\right)+(1-\mu)\frac{q^i}{q^n\lambda p_x}u'\left(\bar{x}+\frac{S}{q^n\lambda p_x}\right)}{\mu u'\left(\bar{x}+\frac{S}{q^n p_x}\right)+(1-\mu)u'\left(\bar{x}+\frac{S}{q^n\lambda p_x}\right)}-1$$
$$\leq(1-\nu^L)\tfrac{p_y^H}{p_x}+\nu^L\tfrac{p_y^L}{p_x}$$

If one looks at the limiting case in which $\lambda = 1$, this further reduces to:

$$\nu^H p_y^H+(1-\nu^H)p_y^L+p_x\leq\frac{q^i}{q^n}\leq(1-\nu^L)p_y^H+\nu^L p_y^L+p_x$$

where it is easily seen that there is a range of relative prices for the indexed bond with respect to the nominal bond such that it is not held in the optimal portfolio, as long as $\nu^L + \nu^H < 1$. If, on the other hand, the capacity ν is actually a probability measure $\nu^L + \nu^H = 1$, there is only one relative price q^i/q^n such that the agent does not want to hold any indexed bond. By continuity, the same is true when λ is strictly greater than 1. Observe that the length of the interval at which the agent does not want to hold any position in the indexed bond is increasing in the ambiguity of the beliefs, measured by $1 - \nu^L - \nu^H$.

6.3 Proofs of results in Section 4

Proof of Proposition 1.

Proof. Since $\frac{p_x^1}{p_z^1} = \frac{p_x^3}{p_z^3}$, there exists β such that $(p_x^1, p_z^1) = \frac{1}{\beta}(p_x^3, p_z^3)$. We want to show that $\beta = 1$. From the equilibrium condition on the money market and the definition of the states (which entails that $M^1 = M^3$), we have that:

$$p_x^1 \sum_{h=1}^{H} \bar{x}_h + p_y^1 y^L + p_z^1 \left(\sum_{h=1}^{H} \bar{z}_h + \sum_{k=1}^{K} \bar{z}_k \right) = p_x^3 \sum_{h=1}^{H} \bar{x}_h + p_y^3 y^H + p_z^3 \left(\sum_{h=1}^{H} \bar{z}_h + \sum_{k=1}^{K} \bar{z}_k \right)$$

Replacing p_x^3 and p_z^3 by βp_x^1 and βp_z^1 respectively, in the equation above, and recalling that, at equilibrium,

$$\frac{p_y^1}{p_z^1} = \frac{p_y^3}{p_z^3} \frac{y^H}{y^L}$$

and hence,

$$\frac{p_y^3}{p_z^1} = \beta \frac{p_y^1}{p_z^1} \frac{y^L}{y^H}$$

one gets:

$$\frac{p_x^1}{p_z^1}(1-\beta) \sum_{h=1}^{H} \bar{x}_h + \frac{p_y^1}{p_z^1}(1-\beta) y^L + (1-\beta) \left(\sum_{h=1}^{H} \bar{z}_h + \sum_{k=1}^{K} \bar{z}_k \right) = 0$$

Hence, $\beta = 1$. The same reasoning shows that $p_x^2 = p_x^4$ and $p_z^2 = p_z^4$. ■

Proof of Proposition 2

Proof. Recall that $\frac{p_x^1}{p_z^1} = \zeta = \frac{p_x^2}{p_z^2}$ and $\frac{p_y^1}{p_z^1} = \frac{p_y^2}{p_z^2}$. Hence, (p_x^1, p_y^1, p_z^1) and (p_x^2, p_y^2, p_z^2) are proportional. Furthermore,

$$p_x^1 \sum_{h=1}^{H} \bar{x}_h + p_y^1 y^L + p_z^1 \left(\sum_{h=1}^{H} \bar{z}_h + \sum_{k=1}^{K} \bar{z}_k \right) = m$$

and

$$p_x^2 \sum_{h=1}^{H} \bar{x}_h + p_y^2 y^L + p_z^2 \left(\sum_{h=1}^{H} \bar{z}_h + \sum_{k=1}^{K} \bar{z}_k \right) = M$$

and hence

$$(p_x^1, p_y^1, p_z^1) = \frac{1}{\lambda}(p_x^2, p_y^2, p_z^2)$$

An analogous argument holds for states 3 and 4. ■

Notation 1 *To simplify notation, the remaining proofs apply the terms $\alpha_h^\omega(b_h^i, b_h^n)$ and $\xi_h^\omega(b_h^i, b_h^n)$ (that we will write as α_h^ω and ξ_h^ω, respectively, for the sake of simplicity) defined as follows:*

$$\alpha_h^\omega(b_h^i, b_h^n) \equiv \frac{p_x^\omega \bar{x}_h + p_z^\omega \bar{z}_h + (p_x^\omega + p_y^\omega + p_z^\omega) b_h^i + b_h^n + \bar{m}_h}{p_x^\omega \sum_{h=1}^{H} \bar{x}_h + p_z^\omega \sum_{h=1}^{H} \bar{z}_h},$$

$$\xi_h^\omega(b_h^i, b_h^n) = \frac{f_h' \left(\alpha_h^\omega(b_h^i, b_h^n) u \right)}{p_x^\omega \sum_{h=1}^{H} \bar{x}_h + p_z^\omega \sum_{h=1}^{H} \bar{z}_h} u.$$

Lemma 1 *At an equilibrium, if* $b_h^i > 0$, *then* $\alpha_h^1 > \alpha_h^3$ *and* $\alpha_h^2 > \alpha_h^4$. *If* $b_h^i < 0$, *then* $\alpha_h^1 < \alpha_h^3$ *and* $\alpha_h^2 < \alpha_h^4$.

Proof. By definition of α_h^ω and since $p_x^1 = p_x^3$ and $p_z^1 = p_z^3$, and the endowment in x and z are non random, one has:

$$\alpha_h^1 - \alpha_h^3 = \frac{(p_y^1 - p_y^3)b_h^i}{p_x^1 \sum_{h=1}^H \bar{x}_h + p_z^1 \sum_{h=1}^H \bar{z}_h}$$

Hence, the sign of $\alpha_h^1 - \alpha_h^3$ is the same as the sign of b_h^i since $p_y^1 > p_y^3$ (recall that $p_y^1/p_y^3 = y^H/y^L > 1$). ■

Proof of Theorem 1

Proof. (Sketch.) Since the hypothesis of this theorem is that $\mu^m + \mu^M = 1$ and $\nu^H + \nu^L = 1$, we may write the proof by setting $\mu^m \equiv \mu$, $\mu^M \equiv 1 - \mu$ and $\nu^H = \nu$, $\nu^L = 1 - \nu$.

Write down the f.o.c. to agent h's program:

$$\begin{array}{l} -q^i u^0 + \alpha_h^0(b_h^i, b_h^n)\mu(1-\nu)\xi_h^1(b_h^i, b_h^n)(p_x^1 + p_y^1 + p_z^1) + (1-\mu)(1-\nu)\xi_h^2(b_h^i, b_h^n)(p_x^2 + p_y^2 + p_z^2) \\ +\mu\nu\xi_h^3(b_h^i, b_h^n)(p_x^3 + p_y^3 + p_z^3) + (1-\mu)\nu\xi_h^4(b_h^i, b_h^n)(p_x^4 + p_y^4 + p_z^4) = 0 \\ -q^n u^0 + \mu(1-\nu)\xi_h^1(b_h^i, b_h^n) + (1-\mu)(1-\nu)\xi_h^2(b_h^i, b_h^n) \\ +\mu\nu\xi_h^3(b_h^i, b_h^n) + (1-\mu)\nu\xi_h^4(b_h^i, b_h^n) = 0 \end{array}$$

Recall that $(p_x^2, p_y^2, p_z^2) = \lambda(p_x^1, p_y^1, p_z^1)$, $(p_x^3, p_y^3, p_z^3) = \lambda(p_x^4, p_y^4, p_z^4)$, $p_x^1 = p_x^3$, $p_z^1 = p_z^3$, $p_x^2 = p_x^4$, $p_z^2 = p_z^4$, and $p_y^1/p_y^3 = y^H/y^L = p_y^2/p_y^4$.

Observe that $\xi_h^1(0, b_h^n) = \xi_h^3(0, b_h^n)$ and $\xi_h^2(0, b_h^n) = \xi_h^4(0, b_h^n)$. Hence, at an equilibrium, if $b_h^i = 0$ for all h, it must be the case that:

$$\begin{array}{l} -q^i u^0 + \mu(1-\nu)\xi_h^1(0, b_h^n)(p_x^1 + p_y^1 + p_z^1) + (1-\mu)(1-\nu)\xi_h^2(0, b_h^n)\lambda(p_x^1 + p_y^1 + p_z^1) \\ +\mu\nu\xi_h^1(0, b_h^n)(p_x^1 + p_z^1 + p_y^1 y^L/y^H) + (1-\mu)\nu\xi_h^2(0, b_h^n)\lambda(p_x^2 + p_z^2 + p_y^2 y^L/y^H) = 0 \\ -q^n u^0 + \mu\xi_h^1(0, b_h^n) + (1-\mu)\xi_h^2(0, b_h^n) = 0 \end{array}$$

that is

$$\begin{array}{l} -q^i u^0 + [p_x^1 + p_z^1 + p_y^1(1 - \nu + \nu y^L/y^H)][\mu\xi_h^1(0, b_h^n) + (1-\mu)\lambda\xi_h^2(0, b_h^n)] = 0 \\ -q^n u^0 + \mu\xi_h^1(0, b_h^n) + (1-\mu)\xi_h^2(0, b_h^n) = 0 \end{array}$$

One can see straight away on these two equations that, unless $\lambda = 1$, there is "little chance" that there exists (q^i, q^n) and $(b_h^n)_{h=1,\ldots,H}$ such that they hold for all h. The following argument sketches how we might put this formally. We define a mapping ϕ from $\mathbb{R}^{H+3}$ to $\mathbb{R}^{2H+1}$ as follows:

$$\phi(b_1^n, \ldots, b_H^n, q^i, q^n, u^0) \quad = \quad \left[\begin{array}{c} \vdots \\ \mu\xi_h^1(0, b_h^n) + (1-\mu)\lambda\xi_h^2(0, b_h^n) - \frac{q^i u^0}{p_x^1 + p_z^1 + p_y^1(1-\nu+\nu y^L/y^H)} \\ \mu\xi_h^1(0, b_h^n) + (1-\mu)\lambda\xi_h^2(0, b_h^n) - q^n u^0 \\ \vdots \\ \sum_{h=1}^H b_h^n \end{array} \right]$$

Observe, the equilibria with no trade in the indexed bond are the zeros of this function. The argument then runs as follows. Note that the Jacobian of ϕ, a $(2H+1) \times (H+3)$ matrix, has full rank. Hence, by a transversality argument, we may conclude that for generic u^0, the rank of the matrix is full. Now, this implies that the system does not have a solution, since "there are more equations than unknowns". Hence, $b_h^i = 0$ for all h cannot be an equilibrium for generic first period aggregate endowments (the latter is sufficient to change u^0). ■

Proof of Theorem 2(a)

Proof. Now, to write down the Choquet integral w.r.t. an agent's portfolio, one has to consider all possible cases (drop subscript h), whether $b^i > 0$ or $b^i < 0$ and whether $b^n + \bar{m} > 0$ or $b^n + \bar{m} < 0$. Observe that:

$$\begin{array}{lll} b^i > 0 & \Rightarrow & \alpha^1 > \alpha^3 \quad \alpha^2 > \alpha^4 \\ b^i < 0 & \Rightarrow & \alpha^1 < \alpha^3 \quad \alpha^2 < \alpha^4 \\ b^n + \bar{m} > 0 & \Rightarrow & \alpha^1 > \alpha^2 \quad \alpha^3 > \alpha^4 \\ b^n + \bar{m} < 0 & \Rightarrow & \alpha^1 < \alpha^2 \quad \alpha^3 < \alpha^4 \end{array}$$

For each case, there are two possible orders, but these two orders give the same (probabilistic) "decision weights". For instance, if $b^i > 0$ and $b^n + \bar{m} > 0$, the two following orders are possible: $\alpha^1 > \alpha^2 > \alpha^3 > \alpha^4$ or $\alpha^1 > \alpha^3 > \alpha^2 > \alpha^4$. If one computes the Choquet integral in these two cases, one sees that they take the same form, *i.e.* the switch in the "middle position" has no effect. Decision weights associated to the different cases:

	$\omega = 1$	$\omega = 2$	$\omega = 3$	$\omega = 4$
$b^i > 0, b^n + \bar{m} > 0$	$\nu^L \mu^m$	$\nu^L(1-\mu^m)$	$(1-\nu^L)\mu^m$	$(1-\nu^L)(1-\mu^m)$
$b^i > 0, b^n + \bar{m} < 0$	$\nu^L(1-\mu^M)$	$\nu^L \mu^M$	$(1-\nu^L)(1-\mu^M)$	$(1-\nu^L)\mu^M$
$b^i < 0, b^n + \bar{m} > 0$	$(1-\nu^H)\mu^m$	$(1-\nu^H)(1-\mu^m)$	$\nu^H \mu^m$	$\nu^H(1-\mu^m)$
$b^i < 0, b^n + \bar{m} < 0$	$(1-\nu^H)(1-\mu^M)$	$(1-\nu^H)\mu^M$	$\nu^H(1-\mu^M)$	$\nu^H \mu^M$

Suppose first that $b_h^i > 0$. One gets, from the first order conditions

$$\frac{q^i}{q^n} = \frac{\pi\nu^L\xi_h^1(p_x^1+p_y^1+p_z^1)+(1-\pi)\nu^L\xi_h^2(p_x^2+p_y^2+p_z^2)}{\pi\nu^L\xi_h^1+(1-\pi)\nu^L\xi_h^2+\pi(1-\nu^L)\xi_h^3+(1-\pi)(1-\nu^L)\xi_h^4} + \frac{\pi(1-\nu^L)\xi_h^3(p_x^3+p_y^3+p_z^3)+(1-\pi)(1-\nu^L)\xi_h^4(p_x^4+p_y^4+p_z^4)}{\pi\nu^L\xi_h^1+(1-\pi)\nu^L\xi_h^2+\pi(1-\nu^L)\xi_h^3+(1-\pi)(1-\nu^L)\xi_h^4}$$

where π depends on the agent's position on the nominal bond market: if $b_h^n + \bar{m}_h > 0$, $\pi = \mu^m$, if $b_h^n + \bar{m}_h < 0$, $\pi = (1-\mu^M)$ and, lastly, if $b_h^n + \bar{m}_h = 0$, π is some number lying in the interval $\left[\mu^m, 1-\mu^M\right]$.

Observe that $(\alpha_h^1(b_h^i, b_h^n), \alpha_h^3(b_h^i, b_h^n))$ and (p_y^1, p_y^3) are positively dependent. Hence, since f_h is concave,

$$cov\left(\left(\alpha_h^1\left(b_h^i, b_h^n\right), \alpha_h^3\left(b_h^i, b_h^n\right)\right), \left(p_y^1, p_y^3\right)\right) < 0$$

Similarly,

$$cov\left(\left(\alpha_h^2\left(b_h^i, b_h^n\right), \alpha_h^4\left(b_h^i, b_h^n\right)\right), \left(p_y^2, p_y^4\right)\right) < 0$$

Hence, a necessary condition for having $b_h^i > 0$ at an equilibrium is that:

$$\frac{q^i}{q^n} < \frac{\pi\left(\nu^L\xi_h^1+(1-\nu^L)\xi_h^3\right)\left(\nu^L(p_x^1+p_y^1+p_z^1)+(1-\nu^L)\pi(p_x^3+p_y^3+p_z^3)\right)}{\pi\left(\nu^L\xi_h^1+(1-\nu^L)\xi_h^3\right)+(1-\pi)\left(\nu^L\xi_h^2+(1-\nu^L)\xi_h^4\right)} + \frac{(1-\pi)\left(\nu^L\xi_h^2+(1-\nu^L)\xi_h^4\right)\left(\nu^L(p_x^2+p_y^2+p_z^2)+(1-\nu^L)(p_x^4+p_y^4+p_z^4)\right)}{\pi\left(\nu^L\xi_h^1+(1-\nu^L)\xi_h^3\right)+(1-\pi)\left(\nu^L\xi_h^2+(1-\nu^L)\xi_h^4\right)}$$

and therefore

$$\frac{q^i}{q^n} < \max\left(\nu^L(p_x^1+p_y^1+p_z^1)+(1-\nu^L)(p_x^3+p_y^3+p_z^3), \nu^L(p_x^2+p_y^2+p_z^2)+(1-\nu^L)(p_x^4+p_y^4+p_z^4)\right)$$

Recalling that

$$p_x^3 = p_x^1, \quad p_y^3 = \tfrac{y^L}{y^H} p_y^1, \quad \text{and } p_z^3 = p_z^1$$
$$p_x^4 = p_x^2, \quad p_y^4 = \tfrac{y^L}{y^H} p_y^2, \quad \text{and } p_z^4 = p_z^2$$

one gets that a necessary condition for $b_h^i > 0$ at equilibrium is that:

$$\frac{q^i}{q^n} < \max\left(p_x^1 + \left(\frac{y^L}{y^H} + \nu^L\left(1 - \frac{y^L}{y^H}\right)\right)p_y^1 + p_z^1, p_x^2 + \left(\frac{y^L}{y^H} + \nu^L\left(1 - \frac{y^L}{y^H}\right)\right)p_y^2 + p_z^2\right)$$

Furthermore, since $(p_x^2, p_y^2, p_z^2) = \lambda(p_x^1, p_y^1, p_z^1)$, a sufficient condition for agent h to hold a positive position in the indexed bond at equilibrium is:

$$\frac{q^i}{q^n} < \lambda\left(p_x^1 + \left(\frac{y^L}{y^H} + \nu^L\left(1 - \frac{y^L}{y^H}\right)\right)p_y^1 + p_z^1\right)$$

Observe that this condition is independent of the position of the agent on the nominal bond market, as the weights π, which depend on the sign of $b_h^n + \bar{m}_h$, do not show up in this condition.

Consider now an agent who, at an equilibrium, holds a negative amount of the indexed bond, that is $b_h^i < 0$. One gets, from the first order conditions

$$\frac{q^i}{q^n} = \frac{\pi(1-\nu^H)\xi_h^1(p_x^1+p_y^1+p_z^1)+(1-\pi)(1-\nu^H)\xi_h^2(p_x^2+p_y^2+p_z^2)}{\pi(1-\nu^H)\xi_h^1+(1-\pi)(1-\nu^H)\xi_h^2+\pi\nu^H\xi_h^3+(1-\pi)\nu^H\xi_h^4} + \frac{\pi\nu^H\xi_h^3(p_x^3+p_y^3+p_z^3)+(1-\pi)\nu^H\xi_h^4(p_x^4+p_y^4+p_z^4)}{\pi(1-\nu^H)\xi_h^1+(1-\pi)(1-\nu^H)\xi_h^2+\pi\nu^H\xi_h^3+(1-\pi)\nu^H\xi_h^4}$$

where, as above, the value of π depends on the sign of $b_h^n + \bar{m}_h$ for agent h. Note that given that we look at one particular agent, the π that appears here is the same as the one that appeared in the f.o.c. for agent h if he had a positive position in the indexed bond ($b_h^i > 0$).

Noticing that $\left(\alpha_h^1(b_h^i, b_h^n), \alpha_h^3(b_h^i, b_h^n)\right)$ and $\left(p_y^1, p_y^3\right)$ are now negatively dependent, and hence

$$cov\left(\left(\alpha_h^1\left(b_h^i, b_h^n\right), \alpha_h^3\left(b_h^i, b_h^n\right)\right), \left(p_y^1, p_y^3\right)\right) > 0$$

as well as

$$cov\left(\left(\alpha_h^2\left(b_h^i, b_h^n\right), \alpha_h^4(b_h^i, b_h^n)\right), \left(p_y^2, p_y^4\right)\right) > 0$$

one gets the following necessary condition for an agent to go short on the indexed bond:

$$\frac{q^i}{q^n} > p_x^1 + \left(\frac{y^L}{y^H} + (1-\nu^H)\left(1 - \frac{y^L}{y^H}\right)\right)p_y^1 + p_z^1$$

Here also, the exact value of π does not matter given that it does not appear in the expression.

Therefore, if

$$\lambda\left(p_x^1 + \left(\frac{y^L}{y^H} + \nu^L\left(1 - \frac{y^L}{y^H}\right)\right)p_y^1 + p_z^1\right) < p_x^1 + \left(\frac{y^L}{y^H} + (1-\nu^H)\left(1 - \frac{y^L}{y^H}\right)\right)p_y^1 + p_z^1 \quad (1)$$

then, at an equilibrium, no agent wishes to trade the indexed bond, and therefore, $b_h^i = 0$ for all h. This condition can be expressed as follows, writing $1 - \nu^H = \mathcal{A}(\nu) + \nu^L$:

$$\lambda < \frac{p_x^1 + \left(\frac{y^L}{y^H} + \left(\mathcal{A}(\nu) + \nu^L\right)\left(1 - \frac{y^L}{y^H}\right)\right)p_y^1 + p_z^1}{p_x^1 + \left(\frac{y^L}{y^H} + \nu^L\left(1 - \frac{y^L}{y^H}\right)\right)p_y^1 + p_z^1}$$

Let

$$\delta = \frac{p_x^1 + p_y^1 + p_z^1}{p_x^1 + \frac{y^L}{y^H} p_y^1 + p_z^1}$$

corresponding to $\mathcal{A}(\nu) = 1$ and $\nu^L = \nu^H = 0$. Notice that $\delta > 1$. Furthermore, if $\lambda < \delta$, there exists $\gamma < 1$ such that if $\mathcal{A}(\nu) > \gamma$, then the condition (1) is met, and, therefore, at an equilibrium, $b_h^i = 0$ for all h. Notice that δ can be expressed as a function of the fundamentals of the model, since, say p_x^1/p_z^1 and p_y^1/p_z^1 are known as functions of the aggregate endowments and the utility function u. ■

Proof of Theorem 2(b)

Proof. We show that if $(b_h^i, b_h^n) = (0,0)$ for all h is an equilibrium of the model when beliefs about the money supply are represented by the capacity μ, then, perturbing slightly μ, one gets a new equilibrium at which $b_h^n \neq 0$ and $b_{h'}^n \neq 0$, for some h, h'.Note first that, if $\bar{m}_h > 0$, $b_h^n = 0$ implies that $\alpha^1 > \alpha^2$ and $\alpha^3 > \alpha^4$, while if $\bar{m}_{h'} < 0$, $b_{h'}^n = 0$ implies that $\alpha^1 < \alpha^2$ and $\alpha^3 < \alpha^4$.

Now, the first order condition with respect to b_h^n, taken at $b_h^n = 0$ is:

$$q^n u^0 = \pi\mu^m \xi_h^1(b_h^i, 0) + \pi(1-\mu^m)\xi_h^2(b_h^i, 0) + (1-\pi)\mu^m \xi_h^3(b_h^i, 0) + (1-\pi)(1-\mu^m)\xi_h^4(b_h^i, 0)$$

where π is some decision weight related to the capacity ν that we need not specify at this stage. Indeed, observe that $\xi_h^1(0,0) = \xi_h^3(0,0)$ and $\xi_h^2(0,0) = \xi_h^4(0,0)$. Hence, given that $\bar{m}_h > 0$, a necessary condition for $(b_h^i, b_h^n) = (0,0)$ to be a solution to h's maximization program is simply that:

$$q^n u^0 = \mu^m \xi_h^1(0,0) + (1-\mu^m)\xi_h^2(0,0)$$

Similarly, a necessary condition for $(b_{h'}^i, b_{h'}^n) = (0,0)$ to be a solution to h''s maximization program is that:

$$q^n u^0 = (1-\mu^M)\xi_{h'}^1(0,0) + \mu^M \xi_{h'}^2(0,0)$$

Therefore, a necessary condition for $(b_h^i, b_h^n) = (0,0)$, h, h', at an equilibrium is that

$$\mu^m \xi_h^1(0,0) + (1-\mu^m)\xi_h^2(0,0) = (1-\mu^M)\xi_{h'}^1(0,0) + \mu^M \xi_{h'}^2(0,0)$$

Now, either this condition does not hold, and then $(0,0)$ is not an equilibrium of the economy and therefore, since we know there is no trade in the indexed bond, this means that there is some trade in the nominal bond. Or else, this condition does hold. However, this essentially means that

$$\mu^m = \frac{(1-\mu^M)\xi_{h'}^1(0,0) + \mu^M \xi_{h'}^2(0,0) - \xi_h^2(0,0)}{\xi_h^1(0,0) - \xi_h^2(0,0)}$$

Hence, if this were the case, that property would not hold for any other economy in which we would have changed μ^m ever so slightly. ■

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