# The Holographic Swampland and other topics in String Phenomenology




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# Statement of originality

This thesis is wholly the work of the author, except for where it has been specified in the text. No part of this thesis has been submitted for any other qualification, at the University of Oxford or elsewhere. It is based on the following papers, all completed in collaboration, and referenced as [1, 2, 3, 4] in the text.

- [1] J. P. Conlon and F. Revello, "Moduli Stabilisation and the Holographic Swampland," LHEP **2020** (2020), 171 [arXiv:2006.01021].

- [2] J. P. Conlon, S. Ning and F. Revello, "Exploring the holographic Swampland," JHEP **04** (2022), 117 [arXiv:2110.06245].

- [3] F. Apers, J. P. Conlon, S. Ning and F. Revello, "Integer conformal dimensions for type IIa flux vacua," Phys. Rev. D **105** (2022) no.10, 106029 [arXiv:2202.09330].

- [4] J. P. Conlon and F. Revello, "Catch-Me-If-You-Can: The Overshoot Problem and the Weak/Inflation Hierarchy," [arXiv:2207.00567].

I gratefully acknowledge the contributions of all my coauthors. In particular, Section 3.4 and Appendix B (from [2]) were written primarily by Sirui Ning, and included here for completeness. Similarly, Sections 4.2.1 and 4.2.2 (from [3]) are based on calculations mainly performed by Fien Apers, who also wrote most of the text.

Other works completed by the author during the course of the DPhil, but not included in this thesis, are [5, 6, 7].

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# Abstract

The main topic of this thesis is to explore the consistency and structure of realistic, 4d vacua of String Theory through the AdS/CFT correspondence. As a first step, we investigate whether Swampland constraints on the low-energy dynamics of weakly coupled, moduli-stabilised string vacua in AdS can be related to inconsistencies of their putative holographic duals or, more generally, recast in terms of CFT data. We find that various swampland consistency constraints are equivalent to a negativity condition on the sign of certain mixed anomalous dimensions. This condition is similar to well-established CFT positivity bounds arising from causality and unitarity, but not known to hold in general. The studied scenarios include LVS, KKLT, the type IIA DGKT flux vacua and both perturbative and racetrack stabilisation. We also point out an intriguing connection to the Swampland Distance Conjecture, both in its original and refined versions. A second, unexpected consequence of this analysis is that the CFT data for some of these vacua takes a universal form, independent of the compactification details. For the type IIA examples, we then give a concise argument that supersymmetric AdS DGKT flux vacua on general Calabi-Yaus always have integer conformal dimensions for all low-lying scalar primaries in the dual CFT. These integers are independent of any compactification details, such as the background fluxes or triple intersection numbers of the compact manifold. We give a tentative discussion of the origin of these integers and effects that would modify these results.

In the last part of the thesis, we take a more phenomenological approach and study the overshoot problem in the context of post-inflationary string cosmology (in particular LVS). We show that in LVS, provided the existence of a large hierarchy between the inflationary and the weak scales, initial seed radiation will localize the system onto a tracker solution where the problem is solved. The consistency requirement of ending in a stable vacuum containing the weak hierarchy therefore gives a preference for high inflationary scales – an anthropic argument, if one likes, for a large inflation/weak hierarchy. We discuss various origins (both universal and model-dependent) of the initial seed radiation, with a particular emphasis on the possibility of having an LVS fundamental string network.

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# Chapter 1

# Introduction and motivations

This Chapter contains mostly introductory material, although there may be occasional overlap with [1, 2].

## 1.1 Introduction

From its foundations, String Theory has not only been employed as a theory of Quantum Gravity (QG), but has found wide and disparate applications in many branches of Physics and Mathematics. An incomplete list includes the study of Black Holes and an interpretation of their thermodynamic properties in terms of microstates [8], the Anti de Sitter/Conformal Field Theory (AdS/CFT) correspondence [9] and its applications to strongly interacting Quantum Field Theories (QFTs) [10] or Condensed Matter Physics (See [11] for a review), the discovery of Mirror symmetry [12, 13] and so on. However, one of its central and oldest tasks that has not yet been accomplished is to provide a microscopic description of reality that can be tested through experiment. This long-standing endeavour goes by the name of String Phenomenology, and its ultimate aim is to provide an experimental verification for String Theory as the fundamental theory governing the laws of our universe. It has been known since the early days of the subject that the theory contains all of the necessary building blocks to reproduce the main ingredients of Quantum Field Theories, the current paradigm in our understanding of fundamental interactions. Both gravity and gauge theories, for example, naturally emerge from the spectrum of closed and open strings respectively. However, working out the full details has proven to be a much more elusive task. A significant complication is provided by the fact that Superstring Theory is naturally formulated in 10 or 11 dimensions, in sharp contrast to the 4 that

we appear to live in. To reconcile this with observations, the extra dimensions must be *compactified* on some internal manifold, and confined to distances much smaller than the ones that can be detected experimentally. Properties of the 4-dimensional physics, such as particle spectra and interactions, are then encoded in terms of the extra-dimensional geometry and topology. An important consequence is that there are almost no free parameters, and the values of all fundamental constants emerge dynamically.

More precisely, the crucial question is whether it is possible to reproduce the Standard Model (SM) of Particle Physics, as well as other fundamental features of the universe we observe (such as a tiny and positive cosmological constant), by means of a string theoretic construction. It is further hoped that a concrete and successful realization of this program would be able to shed light on some of the mysteries and patterns of the SM, from a solution to the hierarchy problem to the existence of dark matter or supersymmetry. A significant amount of effort has, for example, been devoted to embed realistic models of cosmic inflation in a string theoretic construction [14, 15]. Although a sizeable amount of progress has been made on the matter in the past few decades, many obstacles still lie ahead on the road to what could be considered one of the holy grails of Theoretical Physics. These challenges are both technical and conceptual in nature, and it might be conceivable that a more complete understanding of the theory be needed before they can be fully addressed. From a technical point of view, full worldsheet string computations are often intractable in practical cases, and one has to resort to studying successive approximations of the original theory through an effective field theory (EFT) approach. Indeed, as we shall see shortly, it is often not clear to what extent such solutions can be trusted. On a conceptual level, the existence of an enormously vast space of solutions [16, 17] also poses the question of why we should be living in a particular vacuum with the precise features that are observed. So far, only the principle of anthropic selection [18] has been advocated as a possible resolution to this conundrum, but this is hardly a satisfactory explanation. From these premises, String Phenomenology might seem a hopeless enterprise, or even an oxymoron [19]. On the other hand, there are properties of such solutions which appear rather generically, and one might hope that they would be sufficient to capture at least some aspects of the observable physics. A typical ingredient appearing in the spectrum of higher dimensional string and supergravity theories is the existence of $p$-form gauge fields, under which D-branes are charged. Upon dimensional reduction, these can give rise to approximately massless particles such as axion-like particles (ALPs), and could potentially give rise to a plenitude

of experimental signatures. Indeed, the idea of a "String Axiverse" [20], is now considered to be a generic prediction of String Theory. This framework would also allow for an explicit realisation of the QCD axion [21, 22], potentially solving one of the longest-standing problems of the SM.

The two types of difficulties that may be encountered, technical and conceptual, are also mirrored in the different approaches one can adopt in the study of String Phenomenology. On the one hand, one can work towards manufacturing explicit constructions of realistic vacua and testing their trustworthiness. Whereas initial work in this area mainly dealt with the elaboration of new proposals, the focus has recently shifted on questioning their validity. This is the approach that we will be following in Chapters 2-4, where we will explore an alternative way of testing the consistency of moduli stabilisation scenarios using AdS/CFT. On the opposite end, one can take some general aspects of these constructions for granted, and try to explore the consequences of their dynamical aspects, for example in a cosmological setting. In the last chapter of the thesis, we will show how two very desirable requirements in model building - a high inflationary scale and a low susy breaking scale - naturally seem connected from an anthropic argument in the context of the Large Volume Scenario (LVS) [23, 24]. This is also connected to the search for new observational consequences of string theory, such as a cosmic network of fundamental strings in this case.

The traditional approach to understanding string compactifications was based on top-down analyses and constructions of string vacua (for reviews see Refs. [25, 26, 27, 28]). This aimed at obtaining such 4-dimensional vacua by starting with the full 10-dimensional theory and then proceeding through a series of controlled approximations to emerge eventually with controlled 4-dimensional solutions. Typically, the progression was from 10-dimensional string theory to 10-dimensional supergravity to 4-dimensional supergravity effective field theories – and then, finally, to a vacuum derived within this framework of 4-dimensional effective field theory. The claim to have a controlled solution was provided in three ways. The first means of control was through working with approximately supersymmetric compactifications (typically with Calabi-Yau internal manifolds) and having supersymmetry broken only at scales far below the compactification scale. This allowed higher-dimensional questions to remain within the framework of supersymmetry and benefit from the additional control provided. The second means of control was to insist that the internal manifold was in the geometric regime, so that the $\alpha'$ expansion was parametrically controlled. The final means of control was to work at small string coupling, thereby allowing

control of the string loop expansion. In terms of obtaining string vacua, these restrictions are (mostly) not ones of principle. They are imposed not because of any belief that vacua *cannot* exist at strong coupling with badly-broken supersymmetry, but rather through the (correct) sense that it is extremely hard to *establish in a convincing fashion* that such vacua exist. While there are clear limitations in restricting ourselves to vacua which are at weak coupling, there is one important physical reason to focus on such vacua – the fact that the observed universe, both in particle physics and cosmology, is characterised by small numbers and weak couplings. This statement becomes more, rather than less, true as we move to the highest energy scales. Examples are the inflationary density perturbations (characterised by $\frac{\delta\rho}{\rho} \sim 10^{-5}$), the gauge couplings of the Standard Model $\alpha_{SU(3)}$, $\alpha_{SU(2)}$ and $\alpha_{U(1)_Y}$ (which all become weakly coupled at high but sub-Planckian energies) or the Yukawa couplings of the Standard Model.

Recent years have seen an alternative approach to understanding string vacua: the Swampland program [29, 30, 31]. This aims at determining general methods to exclude many low-energy Lagrangians as possible vacua of string theory, by finding principles that must *always* be satisfied within quantum gravity. In this way it can (potentially) bypass the many steps involved in more traditional approaches using dimensional reduction and effective field theory. While such bold conjectures offer hopes of circumventing the need for laborious construction and control of individual vacua, it replaces the hard task of ensuring parametric control of vacua with the equally hard task of proving, or at least establishing significant evidence for, such general conjectures. With a somewhat analogous philosophy, but using very different methods, the Bootstrap program has been working in the direction of determining restrictions on consistent Conformal Field Theories (CFTs) based only on the assumption of certain fundamental requirements such as unitarity and crossing symmetry (an introduction to the topic can be found in [32]). This has had considerable success in carving out the parameter space of consistent CFTs through a clever combination of numerical [33, 34] and analytical techniques [35, 36], showing that certain interesting theories (such as the Ising model) live near the boundary of the allowed regions. One of the main themes of this thesis is to explore whether or not these two programs, so different in principle, can be connected through the AdS/CFT correspondence. The attractiveness of such an approach is twofold. Firstly, AdS/CFT (at least in its strongest formulation) can be thought to apply more generally than just String Theory and therefore be more general than arguments based on weakly coupled string theory. Secondly, it may provide an alternative, possibly more fundamental, perspective on

the swampland conjectures. This is also suggested by the fact that some possible swampland properties, for example scale separation, have a very clear interpretation when translated to the dual CFT description. The low-energy effective Lagrangians for string compactifications often arise after a series of several steps of dimensional reduction. By necessity, these steps leading to $\mathcal{N} = 0$ minima of the effective potential rely on perturbative expansions and weak coupling approximations. AdS/CFT may provide an alternative means to test the correctness of these ideas. In this thesis we will only focus on AdS constructions and do not discuss subsequent uplifts to de Sitter space – although there are also interesting conceptual and consistency questions associated to de Sitter constructions, the use of AdS/CFT techniques restricts our analysis to the first step involving AdS vacua, which in any case already have many phenomenologically interesting features.

This line of investigation has two purposes. The first is to determine whether manifestly 'swampland' modifications within a supergravity effective field theory can be related to defined modifications within a dual CFT. In an ideal world, this may provide a hope of understanding swampland constraints in terms of allowed behaviour within CFTs [37, 1]. This is the subject of Chapters 2 and 3. While CFT approaches to the swampland have already attracted a significant degree of attention in the literature (See Section 1.5), not much work has been carried out in the context of specific realisations. There was some discussion in older literature about properties of duals to explicit moduli stabilisation constructions [38, 39, 40], but the description is very complex and many details still remain to be worked out. In this thesis, we shall take a different approach and exploit parametric limits (such as large volume) which allow one to isolate treatable, effective CFTs with a large gap to the heavier modes. The second is to determine whether the holographic perspective reveals any interesting structure within established models of moduli stabilisation which may be opaque in more traditional treatments. As a byproduct of this analysis, we will uncover some unexpected features in the CFT dual description of certain classes of vacua. In particular, we shall see how the low-lying sectors of these vacua (*i.e.* the light moduli) seem to admit a universal description from the CFT point of view in the limit of control (large volume and small string coupling). This is characterised by a spectrum of low-dimensional, primary operators which does not depend on the underlying compactification structure, such as the geometry of the manifold and fluxes. In the case of DGKT [41], the conformal dimensions are also integer, for reasons that are not yet understood. This phenomenon, along with a tentative discussion of its origin, will be discussed in Chapter 4.

The last portion of the thesis, Chapter 5, takes a more phenomenological turn and discusses cosmological implications of the overshoot problem in the context of realistic compactifications. Assuming inflation happens at some high scale (on the order of the GUT scale), the volume modulus running down its potential can 'overshoot' and run away to decompactification, without settling in its minimum. If enough radiation is present after inflation, however, the dynamical evolution of the system can be driven to a tracker solution where this problem is avoided. In the context of LVS, such a mechanism requires the existence of a large hierarchy between the inflationary and susy-breaking scales. In the spirit of the previous discussion, this addresses some of the conceptual difficulties associated with reproducing the specific characteristic of our universe within the landscape, through an anthropic argument. The sources of radiation also represent possible observational targets for models such as LVS. In particular, we shall discuss the possibility of a fundamental cosmic string network arising from brane-antibrane inflation.

The thesis is organised as follows. Chapter 1 contains an introduction to Moduli Stabilisation, the Swampland and Holography, with a focus on the relations that exists between these topics. In Chapter 2, we propose a novel way to study Swampland constraint from an AdS/CFT perspective, using LVS as a motivating example. This is further analysed in Chapter 3, where we formulate a general conjecture and discuss a variety of other examples. In Chapter 4 we show how a similar analysis for the DGKT vacua comes with the surprising phenomenon of integer (and universal) conformal dimensions for all light operators. Finally, Chapter5 deals with the overshoot problem and its implications in known scenarios of moduli stabilization.

## 1.2 Moduli stabilisation in type II String Theory

### 1.2.1 Generalities

As mentioned in the introduction, realistic vacua require the compactification of 10 or 11-dimensional String Theory (or its Supergravity approximation) down to 4 dimensions. For reasons that will be explained shortly, the most well-studied class of compactifications involves a product spacetime

$$\mathcal{M}_4 \times \mathrm{CY}_6, \tag{1.1}$$

where $\mathcal{M}_4$ is a 4-dimensional maximally symmetric space (Minkowski, de Sitter or Anti de Sitter) and $\mathrm{CY}_6$ a Calabi-Yau manifold. The latter is a particular class of manifolds which is Ricci flat, *i.e.* $R_{ij} = 0$, automatically a solution to the Einstein equations along the 6 compact dimensions. More generally, one can consider a *warped* Calabi-Yau compactification, with a metric

$$ds^2 = (\underbrace{e^{2A(y)} g_{\mu\nu}(x) dx^\mu dx^\nu}_{\text{4D}} + \underbrace{e^{-2A(y)} g_{mn}(y) dy^m dy^n}_{\text{6D}}), \tag{1.2}$$

where the warp factor $A(y)$ can only depend on the extra-dimensional coordinates (to preserve 4d Poincaré invariance), and $g_{mn}(y)$ is a CY metric. Crucially, compactifying on a CY manifold preserves 3/4 of the original 10d supersymmetry - $\mathcal{N} = 1$ for the heterotic string and $\mathcal{N} = 2$ in the type II case. Originally, low-scale supersymmetry was motivated as a solution to the Hierarchy Problem, but from a technical point of view it also makes computations much more tractable. The holomorphic structure of the superpotential and non-renormalisation theorems enable one to keep corrections under control. Furthermore, $\mathcal{N} = 0$ models often exhibit tachyonic instabilities (See [42] for a review).

More precisely, a Calabi-Yau $n-$fold can be defined as a complex Kähler manifold with $\mathrm{SU}(n)$ holonomy. A $2n-$dimensional real manifold is said to be complex if the complex structure tensor

$$J = i dz^i \otimes \frac{\partial}{\partial z^i} - i d\bar{z}^{\bar{i}} \otimes \frac{\partial}{\partial \bar{z}^{\bar{l}}} \tag{1.3}$$

is globally well defined and the corresponding Nijenhuis tensor vanishes. Endowed with the notion of a metric, the manifold is said to be Kähler if it is hermitian

$$g_{ab} = g_{\bar{a}\bar{b}} = 0, \tag{1.4}$$

and if there exists a holomorphic function $K(z, \bar{z})$ such that the metric can locally be expressed as

$$g_{i\bar{j}} = \frac{\partial^2 K}{\partial z^i \partial z^{\bar{j}}}. \tag{1.5}$$

This allows one to define the form

$$J = i\, g_{i\bar{j}}\, dz^i \wedge d\bar{z}^{\bar{j}}, \tag{1.6}$$

which is closed for a Kähler metric. A celebrated theorem, due by Yau, states the following:

**Yau Theorem**: *Let $\mathcal{M}$ be a Kähler manifold, with a Kähler form $J$. If the first Chern class vanishes, there exists unique a Ricci flat metric with Kähler form in the same cohomology class as $J$.*

A recurring theme in string compactifications is the fact that topological data encodes information of the particle content in 4-dimensions. A particular relevant case is that of the Dolbeault cohomology groups $H^{p,q}(\mathcal{M})$, which count the number of independent, complex $(p,q)-$forms. Their dimensions $h^{p,q}$ are a topological invariant, and can be organised into a *hodge diamond*. In the specific case of a Calabi-Yau, the latter can be simplified as

$$\begin{array}{ccccccc} & & h^{3,3} & & & & \\ & h^{3,2} & & h^{2,3} & & & \\ h^{3,1} & & h^{2,2} & & h^{1,3} & & \\ h^{3,0} & & h^{2,1} & & h^{1,2} & & h^{0,3} \\ h^{2,0} & & h^{1,1} & & h^{0,2} & & \\ & h^{1,0} & & h^{0,1} & & & \\ & & h^{0,0} & & & & \end{array} \quad = \quad \begin{array}{ccccccc} & & 1 & & & & \\ & 0 & & 0 & & & \\ 0 & & h^{1,1} & & 0 & & \\ 1 & & h^{2,1} & & h^{2,1} & & 1 \\ 0 & & h^{1,1} & & 0 & & \\ & 0 & & 0 & & & \\ & & 1 & & & & \end{array} . \tag{1.7}$$

The reflection symmetry with respect to the horizontal or vertical axis derives from complex conjugation symmetry and Poincaré duality respectively, so they are generic for a complex 3−fold. Similarly, the fact that $h^{0,0} = h^{3,3} = 1$ is just the statement that the manifold is connected. However, the additional features $h^{1,0} = h^{2,0} = 0$ and $h^{3,0} = 1$ are specific to a Calabi-Yau. The last one in particular implies the existence of a unique holomorphic, harmonic $(3,0)-$form,

$$\Omega = \Omega_{ijk}(z) dz^i \wedge dz^j \wedge dz^k. \tag{1.8}$$

Since $h^{(3,3)} = 1$ ensures there is a unique $(3,3)-$form on a CY, the volume form can be expressed as

$$d\,V = \frac{1}{6} J \wedge J \wedge J = \frac{i}{||\Omega||^2} \Omega \wedge \bar{\Omega}. \tag{1.9}$$

The absence of $(3,2)$-forms also implies $\Omega \wedge J = 0$.

### 1.2.2 Moduli

Moduli are massless scalars whose vacuum expectation values parametrise the geometry of the compactification manifold. A particular class of moduli, generic in string compactifications, are the so-called *bulk* moduli,[1] arising from zero modes of the graviton or higher dimensional forms. The set of all possible deformations preserving the topology of a CY is known as the *moduli space* of the theory.

One can start by analysing possible deformations of the metric. Requiring Ricci flatness to hold for an infinitesimal variation of the metric, *i.e.*,

$$R_{mn}(g) = 0 \quad \text{and} \quad R_{mn}(g + \delta g) = 0, \tag{1.10}$$

one obtains the Lichnerowicz equation

$$\nabla^l \nabla_l \delta g_{mn} + 2R_m{}^l{}_n{}^r \delta g_{lr} = 0. \tag{1.11}$$

For a Kähler manifold, the equations (1.11) for the pure ($\delta g_{ab}$) and mixed ($\delta g_{a\bar{b}}$) components decouple. In the former case, the hermiticity assumption would naively be violated, but it can compensated by a change of the complex structure. Such complex structure deformations can be counted by the (2,1)-forms

$$\Omega_{abc} g^{c\bar{d}} \delta g_{\bar{d}\bar{e}} dz^a \wedge dz^b \wedge d\bar{z}^{\bar{e}}. \tag{1.12}$$

The mixed transformations are associated to the Kähler form variation

$$J = i\, \delta g_{a\bar{b}} dz^a \wedge d\bar{z}^{\bar{b}} \to i(\delta g_{a\bar{b}} + \delta g_{a\bar{b}}) dz^a \wedge d\bar{z}^{\bar{b}}. \tag{1.13}$$

Therefore, a generic Calabi-Yau will come with $h^{1,1}$ Kähler moduli and $h^{2,1}$ complex structure ones. In retrospect, this can also be seen as a consequence of the Yau theorem, since a Ricci-flat Calabi-Yau metric is unique once the complex structure and Kähler classes are fixed.

Let us turn our attention to higher forms $F_p = dA_{p-1}$, whose 10d equations of motion are given by

$$\Delta A_{p-1} = d \star dA_{p-1} = 0. \tag{1.14}$$

[1] We will not discuss other types of moduli which may be present, such as brane moduli.

While the precise form content differs between the different string theories, there is always a NS-NS 2-form $B_2$, with $H_3 = dB_2$. The number of massless modes arising after dimensional reduction is counted by the zero modes of the 6−dimensional laplacian, given by the Betti numbers

$$b_n = \sum_{p+q=n} h^{p,q} \tag{1.15}$$

for a generic $p-$form. On a CY 3−fold, the $B_2$ form gives rise to $h^{1,1}$ massless fields in 4 dimensions. These can be paired up with the Kähler moduli by defining the complexified form

$$\mathcal{J} = B + iJ. \tag{1.16}$$

Generally speaking, in type II string theories,the $B_2, C_2, C_3$ and $C_4$ forms can all give rise to massless particles, which can be paired with other moduli in an effective $\mathcal{N} = 1$ description. However, in realistic compactifications one has to resort to Calabi-Yau *orientifolds*, and some of these components may be projected out. Since these particles are massless and couple to U(1)'s through Chern-Simons couplings, they are sometimes referred to as the axion component of an $\mathcal{N} = 1$ multiplet.

### 1.2.3 Flux compactifications

From an observational perspective, the existence of massless scalars poses various phenomenological problems. Although their couplings to the Standard Model are Planck-scale suppressed, they would mediate long-range gravitational forces which could be detected in experiments. Sub-millimetre tests of gravity, for example, already exclude moduli masses below the meV scale[2] [43, 44]. Furthermore, in a post-inflationary cosmology setting, light (and therefore long-lived) moduli also give rise to the so-called *Cosmological moduli problem* [45, 46, 47, 48]. Reheating is expected to happen through decays of the moduli but, if they are too long-lived, this can spoil the predictions of Big-Bang Nucleosynthesis (BBN). Since the lifetime of a typical modulus can be estimated as $\tau = \Gamma^{-1} \sim 8\pi \frac{M_P^2}{m_\Phi^3}$, this puts a bound of $m_\Phi \gtrsim 30\,\text{TeV}$. From these premises, it is clear that in a realistic compactification the moduli will have to be stabilised by a potential.

[2]There is an exception for axions, since they couple to matter only at a derivative level and do not give rise to any long-range forces.

The problem of moduli stabilisation has played a central role in the field of String Phenomenology (See the reviews [25, 26, 27, 28]). In the context of type IIA/IIB, a key technique to stabilise moduli involves the use of fluxes threading the extra dimensions. To illustrate with a toy example how fluxes can stabilise a cycle, let us consider a (non-realistic) $S_1^A \times S_1^B$ compactification [49], with an $F_1$ flux on $S_B$. By flux quantization $\int_{S_B} F_1 \sim n$. The contribution to the 4d action can be readily computed as

$$S \supset -\int d^4x \int dy_A dy_B \frac{1}{4g^2} F_1 \wedge^* F_1 \sim -n^2 \int d^4x \frac{R_A}{R_B}, \tag{1.17}$$

showing how a non-zero $F_1$ prevents the $S_B$ cycle from collapsing.

In a realistic setting, it is possible to turn on a multitude of fluxes along the various cycles of a CY, which in a typical example will be of $\mathcal{O}(100)$. Along each cycle, they will have to obey a *Dirac* quantisation condition

$$\int_{\Sigma_p} F_p = (2\pi)^{p-1} \alpha'^{(p-1)/2} n, \qquad n \in \mathbb{Z}. \tag{1.18}$$

As already mentioned, the $H_3 = B_2$ 2-form gauge field is universal in any string theory, but the $R-R$ sector in type II gives rise to other options. In type IIA, one has the odd forms $F_2 = dC_1, F_4 = dC_3$ gauge fields, plus $F_0$ in *Romans Supergravity* [50] (also known as massive type IIA). Given that $p-$branes couple to $p+1$ gauge fields, the spectrum also contains $D0, D2, D4$ and $D6$ branes.[3] In type IIB, instead, there exist even form gauge fields $F_1 = dC_0, F_3 = dC_2, F_5 = dC_4$, paired to $D1, D3, D5$ and $D7$ branes. One can also consider $D(-1)$ branes coupling to $0-$forms; they are localised at one point in spacetime, and interpreted as instantons. An important consequence is that, since complex structure moduli are associated to $(2,1)$-forms, in both type IIA and type IIB they can be stabilised by fluxes at tree level. On the other hand, for the $(1,1)-$form Kähler moduli this can only happen in type IIA, thanks to the existence of the $F_2$ flux.

[3] $D8$ branes are excluded as they would require the existence of a non-dynamical 9-form gauge field, but they can be physical in certain circumstances.

### 1.2.4 Type IIB moduli stabilisation

Flux compactifications are most well developed in the case of type IIB orientifold models. In type IIA, the best known examples are the DGKT class of models [41], which will be reviewed in Sections 4.1.1 and 4.2. To compensate the positive contribution to the 6-dimensional curvature coming from fluxes, it is necessary to introduce negative tension objects, *i.e.* $O-$planes. These are localised objects corresponding to the fixed points of the transformation

$$\mathcal{O} = \Omega_p (-1)^{F_L} \sigma, \tag{1.19}$$

where $\Omega_p$ is worldsheet parity, the $(-1)^{F_L}$ left moving fermion number and $\sigma$ an involution of the complex coordinates acting as $\sigma(J) = -J, \sigma(\Omega) = \bar{\Omega}$ in type IIA and $\sigma(J) = J, \sigma(\Omega) = \pm\bar{\Omega}$ in type IIB. To cancel the RR tadpole, it will also be necessary to introduce $D-$branes.

The main class of compactifications that we shall be considering is type IIB SUGRA with $O3/O7$-planes and $D3/D7$ branes, and will be based on the GKP construction [51]. The type IIB, bosonic SUGRA action is given by

$$\begin{aligned} S_{IIB} =& \frac{1}{2\kappa_{10}^2} \int dx^{10} \sqrt{-g_s} \left\{ e^{-2\phi} \left[ R_s + 4(\nabla\phi)^2 \right] - \frac{1}{2} F_1^2 - \frac{1}{12} G_3 \cdot \bar{G}_3 - \frac{1}{4 \cdot 5!} \tilde{F}_5^2 \right\} \\ &+ \frac{1}{8i\kappa_{10}^2} \int e^{\phi} C_4 \wedge G_3 \wedge \bar{G}_3 + S_{\text{loc}} \end{aligned} \tag{1.20}$$

in the string frame. The higher form fluxes are defined as

$$\begin{aligned} G_3 &\equiv F_3 - \tau H_3 \\ \tilde{F}_5 &\equiv F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3 \end{aligned} \tag{1.21}$$

in terms of the elementary fluxes described at the end of Section 1.2.3, and $\tilde{F}_5$ must satisfy the additional constraint of self-duality. The spacetime-filling, localised objects are described by the action

$$S_{\text{loc}} = -T_p \int_{R^4 \times \Sigma} d^{p+1}\xi \sqrt{-g} + \mu_p \int_{R^4 \times \Sigma} C_{p+1}, \tag{1.22}$$

where $T_p$ and $\mu_p$ are their tension and charge, and $\Sigma$ the cycle they wrap in the extra

dimension. The fluxes $H_3$ and $F_3$ will be quantised along 3−cycles as in (1.18), and they will have to satisfy the tadpole cancellation condition

$$\frac{1}{2\kappa_{10}^2 T_3}\int H_3 \wedge F_3 + Q^{\rm loc} = 0, \tag{1.23}$$

where $Q^{\rm loc}$ is the charge of the $D-$branes and $O-$planes. In the GKP setup, a solution to the 10d equations of motion is obtained with an ansatz of the form (1.2) for the metric, and

$$\tilde{F}_5 = (1+*)d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \tag{1.24}$$

for the 5-form flux, making the self-duality condition explicit. The Bianchi identities and the Einstein equations imply that the 3-form flux has to satisfy

$$*_6\, G_3 = iG_3, \tag{1.25}$$

while the warp factor is related to the 5-form flux by

$$\alpha = {\rm e}^{4A} \sim 1 + \frac{1}{\mathcal{V}^{2/3}}. \tag{1.26}$$

Finally, the presence of the $O-$planes breaks the 4d $\mathcal{N}=2$ supersymmetry to $\mathcal{N}=1$.

From a 4-dimensional point of view, one can obtain a $\mathcal{N}=1$ description in terms of the chiral coordinates

$$T_\alpha = \tau_\alpha + ia_\alpha, \tag{1.27}$$

for the Kähler moduli, where $\tau_\alpha$ is the dimension of a 4-cycle $\Sigma_4^\alpha$ (in the Einstein frame), and

$$a_i = \int_{\Sigma_4^\alpha} C_4. \tag{1.28}$$

The Complex structure moduli can be described in terms of the periods

$$\Pi_i = \int_{\Sigma_3^i} \Omega, \tag{1.29}$$

integrated over all 3-cycles. Due to invariance under rescalings, they can be used as a set of projective coordinates, although will shall not go into the details. The

corresponding superfields will denoted by $U_a$. Finally, the axio-dilaton superfield is

$$S \equiv i\, C_0 + e^{-\phi}. \tag{1.30}$$

The resulting tree level Kähler potential is

$$K = -2\ln(\mathcal{V}) - \ln(S+\bar{S}) - \ln\left(-i\int_{CY_6}\Omega\wedge\bar{\Omega}\right), \tag{1.31}$$

while the superpotential is of the Gukov-Vafa-Witten (GVW) form [52]

$$W = \int_{\mathcal{M}} G_3\wedge\Omega, \tag{1.32}$$

and depends implicitly on the complex structure moduli. Together, they determine the scalar potential through the $F-$term supergravity potential

$$V = e^K\left(\sum_{S,T_\alpha,U_a} K^{i\bar{j}} D_i W D_{\bar{j}}\bar{W} - 3|W|^2\right). \tag{1.33}$$

The specific form of the Kähler potential (1.31) is such that the sum over the Kähler moduli cancels exactly with the $-3|W|^2$ coming from the superpotential, so that (1.33) reduces to

$$V_{\text{no-scale}} = e^K \sum_{S,U_a} K^{i\bar{j}} D_i W D_{\bar{j}}\bar{W}. \tag{1.34}$$

This expression depends only on the dilaton and complex structure moduli, satisfying what is called the *no-scale* property. One can minimise the expression (1.34) by imposing $D_a W = 0$, which resulting in a set of $h^{2,1}+1$ complex equations. For future purposes, the value of the superpotential at the minimum

$$W_0 = \langle\int_{\mathcal{M}} G_3\wedge\Omega\rangle \tag{1.35}$$

will be referred to as the tree level superpotential. Therefore, for generic values of the fluxes it is possible to stabilise all complex structure moduli and the dilaton this way. [4] However, all the Kähler moduli are still unfixed: this agrees with our

[4] While this seems to be in tension with the recent Tadpole conjecture [53], we remark that the latter applies to generic points of moduli space. To achieve moduli stabilisation, it is sufficient to find a single solution for any given value of the fluxes.

previous expectation that it would not be possible to stabilise them at tree-level and with fluxes in type IIB. To do so, one would need to introduce corrections to the Kähler potential or superpotential that go beyond leading order and break the no-scale structure. The superpotential is exact to all orders in perturbation theory, so it can only receive non-perturbative corrections. These form the basis of the so-called KKLT construction, which will be further analysed in Section 3.2.3. These AdS vacua are also the starting point of the most well-known example of de Sitter vacuum, which has recently come under criticism as part of the Swampland program[5] [55, 56] (See Section 1.3.3). In KKLT, the possibility to stabilise the AdS vacuum in its regime of validity, *i.e.* weak couplings and large volumes, requires an exponentially small value of $W_0$, whereas typical models have $W_0$ of $\mathcal{O}(1)$. For recent attempts to realise this condition in practice, see [57, 58, 59]. On the other hand, the leading corrections to the Kähler potential, together with the non-perturbative terms, also provide a mechanism to stabilise all the moduli with $W_0$ of $\mathcal{O}(1)$, in what is known as the Large Volume Scenario (LVS). In LVS, this can be achieved for exponentially large values of the volume, unlike KKLT where $\mathcal{V} \sim \ln(1/|W_0|)$. From a model-building perspective, this is instrumental in providing a mechanism to realise the hierarchies of particle physics. The LVS scenario is analysed in detail in Section 2.1.1, and will be the starting point of our holographic analysis. Other examples of moduli stabilisation in type IIB, which are essentially based on the same ingredients as LVS and KKLT, are discussed in Sections 3.2.2, 3.2.4 and 3.4.1.

## 1.3 The Swampland: a brief review

### 1.3.1 An overview

The last few years have seen a resurgence of interest in the identification of criteria that distinguish between the low-energy effective theories which admit an ultraviolet completion in String Theory, or more generally within the framework of Quantum Gravity, from those which do not. The former are commonly referred to as the *Swampland* [29, 30], as opposed to the *Landscape* [18] of acceptable theories (see [31] for a comprehensive review, and also [60, 61, 62]). The underlying idea for the emergence of such constraints is that self-consistency conditions should become more

[5] Although the uplift to de Sitter is perhaps the most controversial aspect, a holographic obstruction to the AdS KKLT vacua has recently appeared in [54].

and more restrictive as one moves towards the UV, culminating with string theory whose structure at high energies is essentially determined. This program has resulted in a large number of conjectures which vary greatly in their predictive power and their degree of rigorous support. They are often linked by a deep web of connections, as shown for some notable cases in Fig. 1.1. Another characteristic trait that is displayed in Fig. 1.1 is that the most useful conjectures are usually the hardest to prove or for which the least empirical support (in terms of string theory examples) exists, while the more established ones do not often lead to stringent implications in the IR.

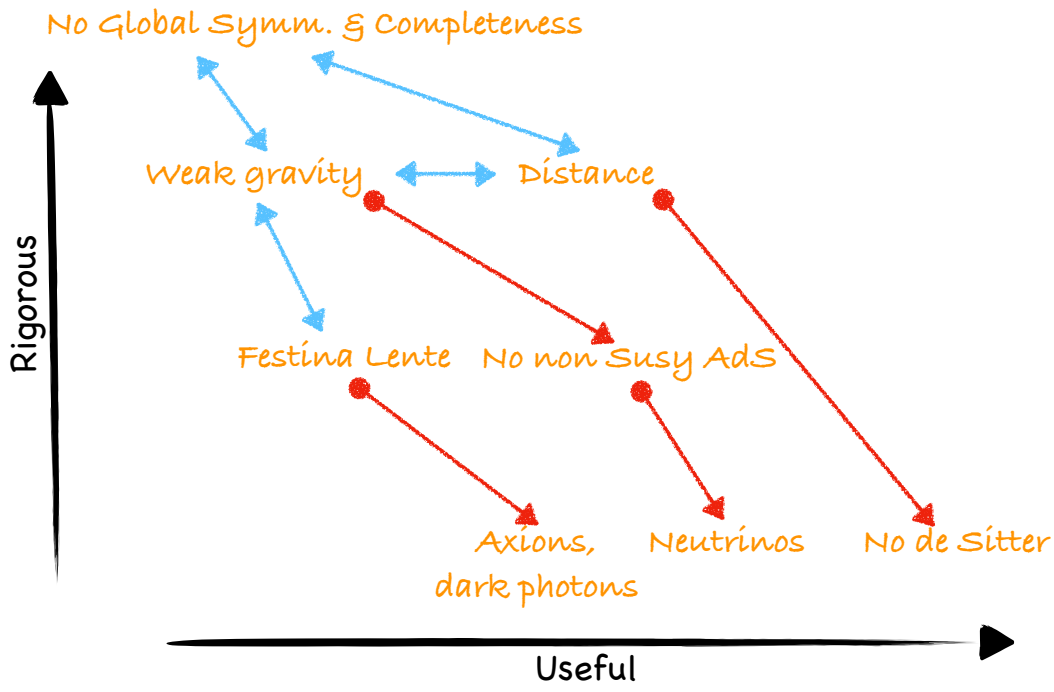



Figure 1.1: Schematic depiction of some well-known Swampland conjectures (later described in the text), organised according to the level of rigour (increasing from bottom to top) and their usefulness (increasing from left to right). Figure in large part inspired from Hiroshi Ooguri's lectures at [63].

Some of them, such as the No Global Symmetries [64, 65] and Weak Gravity conjectures (WGC) [66], have existed for quite some time and have a high level of support, relying both on a UV stringy logic and also IR arguments involving black hole physics. They have been thoroughly tested in a variety of string theory settings

(See 1.3.2 for more detail), and also analysed through the lens of AdS/CFT (as discussed in 1.5.1). However, they do not seem to imply any direct consequences for low energy physics that we can observe, either in the Standard Model (SM) or any of its extensions. One could naively think, for instance, that the absence of global symmetries should be incompatible with the SM, given the existence of the global, conserved $\mathrm{U}(1)_{\mathrm{B-L}}$ at the renormalizable level. But such a symmetry is only accidental, and could either be gauged or broken in the UV. The No Global Symmetries conjecture does not say anything about the magnitude of possible breaking terms, and is hence not useful in this setting. As another example, the WGC applied to the SM of particle physics implies the existence of at least one particle with a mass smaller than its charge, $m \leq \sqrt{2} g q M_P$. It is satisfied parametrically by the electron,

$$\frac{m_e}{\sqrt{2} M_p} \sim 10^{-22} \quad g_{EM} q_e \sim 10^{-1}, \tag{1.36}$$

by an astonishing 21 orders of magnitude. In a similar fashion, the closely related Swampland Distance Conjecture [30] (SDC) - according to which towers of massless states appear when taking infinite distance limits in moduli space - only applies to the case of massless scalars. The refined version of the conjecture [67, 68], which includes the phenomenologically interesting case of fields with a potential, is indeed much more speculative. On the other end of the spectrum, there are speculative hypotheses such as the dS Conjecture [56], which are motivated by the difficulties encountered in the construction of explicit counterexamples in String Theory and have less rigorous support, but carry broad phenomenological implications [56, 69].

Broadly speaking, Swampland conjectures (or constraints) have been proposed as the result of one of the following:

- **Examples in String Theory**

  They generally arise from explicit examples in String Theory, which can themselves have a varying degree of rigour (See the distinction between *string-derived* and *string-inspired* conjectures in [31]). Perhaps the most famous example of a conjecture which was mainly based on empirical evidence (at least in its original formulation) is the de Sitter conjecture of [56]. While this approach is perhaps the one that has led to the most progress in terms of number of new hypotheses and reach of their implications, there are certain limitations associated with it. The most obvious one is that no number of examples, however large, will be

enough to prove the validity of a statement. Another fundamental obstruction is that most known computations rely on a high degree of symmetry and/or an expansion in weak couplings $(g_s, \alpha')$, thus exploring only a limited part of the landscape. It might be the case that some properties which appear universal are an artefact of the "lamppost principle", and only apply to the limited set of vacua we can construct.

- **Semi-classical arguments involving black holes or extended objects**

  Given the absence of internal structure, formalised by the No Hair Theorem, black holes can be thought of in a certain sense as "elementary particles" of gravity at very high energies [70]. Therefore, they represent good candidates to study in the search for universal properties of Quantum Gravity. This line of reasoning is mostly relevant for the No Global Symmetries conjectures [64, 65] and some of its descendants, such as the WGC [66] and Festina Lente [71]. While the applicability is limited only to certain contexts, they can be thought to be relevant for more generic theories of Quantum Gravity, and not just String Theory. Indeed, the derivations usually assume some semi-classical properties of black holes only such as Hawking radiation, which can account for processes such as their decay. A possible pitfall is that these arguments usually proceed by showing how certain properties imply possible inconsistencies in terms of an infinite number of black hole remnants, but it is not totally clear where the contradiction lies in such cases. These aspects will be analysed more in detail in Section 1.3.2.

- **Holography**

  AdS/CFT offers some hope for a rigorous formulation of non-perturbative String Theory. Therefore, it should not come as a surprise that many have resorted to it as a tool to make certain Swampland statements more rigorous. The most well-known example is an argument for the absence of global symmetries given in [72, 73]. An intrinsic limitation of this approach is that it only applies to Anti-de Sitter space, but in practice the few known de Sitter constructions all rely on the uplift of an anti-de Sitter solution. This is also the path that will be taken in this thesis, although with a different angle with respect to most of the work that has been carried out in this area. In particular, we shall not focus on the generic properties of holographic theories *per se*, but rather concentrate on

the duals to some specific 4-dimensional vacua. For a more complete analysis of the interplay between holography and the swampland, see Section 1.5.

- **Positivity bounds from dispersion relations**

  It has been known for many years that unitarity and causality, encoded in the analytical properties of correlators, impose non-trivial restrictions in the space of possible Quantum Field Theories. More recently, these constraints have started to be exploited in a more systematic fashion [74], resulting in the formulation of various types of inequalities which must be satisfied by the Wilson coefficients of a generic EFT. The typical derivation involves the use of dispersion relations obtained by integrating scattering amplitudes in the complex plane, and thus relies on the analytic structure as well as a controlled UV behaviour of the amplitude. Under the assumption of Regge boundedness, they give very solid constraints which define a Swampland in the space of QFTs. Such bounds also exhibit a close relationship with "standard" swampland conjectures of a stringy origin, such as the WGC[75, 76, 77]. See Section 2.4 for more details.

### 1.3.2 From no global symmetries to Weak Gravity

Perhaps the oldest and most fundamental Swampland principle, which lies at the heart of many other conjectures, is the idea that global symmetries should not exist in a theory of Quantum Gravity, meaning they have to be either broken or gauged. Of course, the global part of a gauge symmetry (such as electric charge) is allowed - by global symmetry we mean one under which there exists at least one charged, gauge-invariant operator. The original formulation only concerned the existence of continuous global symmetries, but it has also been generalised to the discrete case. It can be formalised as follows:

**No Global Symmetries Conjecture** [64, 65]

*A consistent theory of Quantum Gravity cannot admit any exact, global symmetries.*

There exist a few, distinct motivations for this idea, both in String Theory and from a bottom-up perspective. In the context of perturbative String Theory, any global symmetry on the world-sheet corresponds to a gauge symmetry in the bulk

[64]. This is in accordance with explicit examples, and apparent global symmetries which appear at the classical level always turn out to be gauged or broken by quantum effects: let us now mention a few. In 10 dimensional supergravity there is a global $\mathrm{SL}(2,\mathbb{R})$ symmetry, broken to $\mathrm{SL}(2,\mathbb{Z})$ in string theory because $p$-brane charges have to be quantised. However, the surviving $\mathrm{SL}(2,\mathbb{Z})$ is actually a duality rather than a global symmetry: it maps a given physical configuration to itself, meaning it is gauged. From the F-theory picture [78] this is even clearer, as the diffeomorphisms of the torus are precisely a gauge symmetry for the metric. The story is similar for type II compactified on $T^6$, whose low-energy limit in 4 dimensions is given by $\mathcal{N}=8$ supergravity. The latter has an $E_{7(7)}(\mathbb{R})$ global symmetry broken to $E_{7(7)}(\mathbb{Z})$ by stringy effects, and it corresponds to U-duality [79]. [6] In recent years, a suitably formalised version of the statement has been shown to hold through the AdS/CFT correspondence [72, 73] (See also section 1.5.1). However, the most traditional arguments involve the physics of black holes, and they are also the ones that have provided more connections and inspirations for other conjectures. The argument goes as follows. In the presence of an exact U(1) global symmetry one could imagine throwing states charged under it into a black hole to make its charge arbitrarily high.[7] It is a well-known fact that the metric of scalar, stationary black holes can only depend on their mass (also known as the No-Hair Theorem in the literature), so that this charge would be invisible to an external observer. While this would be perfectly consistent in a classical theory, quantum black holes can decay through Hawking radiation, which cannot carry any of the global charge (the horizon is not sensitive to it in any way). Therefore, such black holes can become lighter without losing any of the global charge. It is here that we arrive at a contradiction: a black hole of fixed mass[8] can be constructed with any values of the charge, giving rise to an arbitrarily large Hilbert space. However, the entropy has to be finite, and in particular it is given by the Bekenstein-Hawking formula. A closely related justification proceeds by saying that, in the presence of a global symmetry, there would exist an infinite number of stable black holes of different charges in a finite mass range close to $M_P$, *i.e.* black holes at the end of their evaporation process. The existence of such objects, known as *remnants*, would also be in conflict with entropy considerations [81].

[6] $\mathcal{N}=8$ SUGRA also has an $\mathrm{SU}(8)_R$ R-symmetry. However, it is obtained by combining the global part of a gauged SU(8) and the global SU(8) contained in $E_{7(7)}(\mathbb{R})$, so it is also partially broken (tree level string amplitudes already break SU(8) to $\mathrm{SU}(4)\times\mathrm{SU}(4)$ [80]) and what is leftover is still part of the U-duality transformations.

[7] This can be easily extended to the case of non-abelian global symmetries. In the case of a global SU(2), one can for example construct black holes of arbitrarily high u-spin $j$.

[8] This is at least true in the classical regime where $M_{\mathrm{BH}} \gg M_P$.

The absence of global symmetries has also been extended to cover the case of $p$-form global symmetries [82], under which extended objects with $p$ spacetime dimensions are charged. This helps in making the connection to one of the cases that aren't covered by the global symmetries conjecture, namely that of gauge groups. For a compact and connected gauge group, the absence of 1-form symmetries under which Wilson lines are charged implies charged matter should exist in all possible representations [83]. If the group is disconnected, the same statement holds in the absence of non-invertible global symmetries [84], and similar conditions can also be proved for discrete groups [83, 84]. This is known as the completeness hypothesis, and had already been argued for in a similar spirit to the No Global Symmetries conjecture [65, 85].

**Completeness Hypothesis** [65, 85]

*Any theory containing a gauge field, if coupled to gravity, must contain states with all the charges compatible with the Dirac quantization condition.*

There is a sense in which a global symmetry can be considered as the limit of a gauge symmetry when the coupling $e \to 0$, where the gauge field becomes non-dynamical but the selection rules induced by its presence still apply to the physical matrix elements. It is then natural to wonder whether the arguments against the existence of global symmetries in Quantum Gravity can be used to constrain the existence of gauge symmetries. For convenience, let us consider Einstein-Maxwell theory in 4 dimensions, *i.e* an action of the form

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \dots\right) + ... \tag{1.37}$$

where the dots also allow for the presence of charged matter in the spectrum. In contrast to the global case, stationary but charged black holes in this theory are described by the Reissner-Nordström metric

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2, \tag{1.38}$$

where

$$f(r) = 1 - \frac{2M_{\rm ADM}}{r} + \frac{2e^2Q^2}{r^2}. \tag{1.39}$$

Since the function $f(r)$ is quadratic, they have an inner and outer horizon for

$$M_{\rm ADM} \geq \sqrt{2}eQM_p, \tag{1.40}$$

which collapse onto each other when the equality in (1.40) is satisfied. Such black holes are called *extremal*, and have the largest possible charge-to-mass ratio. Indeed, if (1.40) were not satisfied, there would be no horizons protecting the singularity at $r = 0$, in contrast with the Cosmic Censorship (CC) hypothesis [86]. An immediate consequence of (1.40) is that only a finite number of black holes (states) can exist for a fixed mass, and the infinite entropy considerations discussed in the case of global symmetries do not apply as straightforwardly.[9] However, one could still worry about the presence of a large number of charged, stable remnants, although it is not totally clear they would give rise to an inconsistency [81, 66]. It is true that black holes are now able to shed charge through the emission of charged particles, but they can only decay to lighter states only if at least one of the decay products has a small enough charge-to-mass ratio. Indeed, if any object of mass $M$ and charge $Q$ can decay to a set of lighter ones with masses $m_i$ and charges $q_i$, the inequality

$$\frac{M}{Q} \geq \frac{1}{Q}\sum_i m_i = \frac{1}{Q}\sum_i \left(\frac{m_i}{q_i}\right) q_i \geq \frac{1}{Q}\left(\frac{m}{q}\right)\Big|_{\rm min} \sum_i q_i = \left(\frac{m}{q}\right)\Big|_{\rm min} \tag{1.41}$$

has to be satisfied [31]. If one requires that extremal black hole should be able to decay, the theory should contain at least one particle of charge $m$ and mass $q$ such that

$$m \leq \sqrt{2}eqM_p. \tag{1.42}$$

This is known as the electric version of the Weak Gravity Conjecture (WGC) [66]. For an arbitrary number of dimensions, it can be stated as follows.

[9] A second objection is that for a gauge symmetry the charge can be measured from the flux of the electric field at infinity.

**Weak Gravity Conjecture (WGC)** [66]

*Consider a d-dimensional theory of Quantum Gravity containing a* U(1) *gauge group, whose coupling is denoted by* $e$*. Then, the following will have to be satisfied:*

**Electric version:** *At least one particle with charge* $q$ *and mass* $m$ *satisfying*

$$m \leq \sqrt{\frac{d-2}{d-1}} eq \left(M_P^d\right)^{\frac{d-2}{2}}. \tag{1.43}$$

*must exist in the spectrum.*

**Magnetic version:** *The cutoff of the theory has an upper bound*

$$\Lambda_{\text{UV}} \lesssim e\left(M_P^d\right)^{\frac{d-2}{2}}. \tag{1.44}$$

The Magnetic version derives from the application of the Electric WGC after performing electro-magnetic dualisation. In particular, an application of the WGC to the magnetic field implies the existence of a magnetic monopole with a mass (in Planck units) smaller than the dual coupling, *i.e.* $1/e$. The mass of the monopole can be estimated by integrating the electrostatic energy up to the cutoff of the theory, so that (1.44) follows from

$$m \sim \frac{\Lambda}{e^2} \lesssim \frac{1}{e}\left(M_P^d\right)^{\frac{d-2}{2}} \tag{1.45}$$

A straightforward application of (1.44) is that, when $e \to 0$ (corresponding to a global symmetry), the cutoff scale goes to zero. This inconsistency is exactly what we would expect from the No Global Symmetries conjecture.

The WGC statement can be generalised to the case of higher-dimensional objects, that is $p$-branes, by considering the case of black branes which are charged under some form. From the requirement that super-extremal black branes be able to decay, one obtains the generalisation below.

**Weak Gravity Conjecture for $p$-forms** [66, 87]

*Any consistent, d-dimensional theory coupled to gravity which includes p-form of gauge coupling $g_p$ should contain a $(p-1)$-dimensional object with tension $T_p$ and charge $Q_p$ satisfying*

$$\frac{p(d-p-2)}{d-2} T_p^2 \leq Q_p^2 g_p^2 \left(M_p^d\right)^{d-2} \tag{1.46}$$

The coupling in (1.46) is normalized in such a way that the kinetic term of the $p$-form is

$$\mathcal{L}_{\text{kin}} = \frac{1}{2g_p^2} \frac{1}{p!} F^{\mu_1 \dots \mu_p} F_{\mu_1 \dots \mu_p}. \tag{1.47}$$

Furthermore, (1.46) is only valid for $1 \leq d \leq p-3$, as the LHS degenerates for $p = 0, d-2$. The former case, however, is particularly interesting, as $p = 0$ corresponds to axions. They are coupled to (-1)-forms, corresponding to instantons. Schematically, the form of the low energy action describing an axion is

$$\mathcal{L}_a \supset \frac{1}{2} \partial_\mu a \partial^\mu a + \Lambda^4 \sum_n e^{-nS_a} \left(1 - \cos(n\, a/f)\right) \tag{1.48}$$

Therefore, there is no direct way to derive the bound (1.46) from black holes or branes, but one might still expect the generic form of the inequality (with an unspecified $\mathcal{O}(1)$ coefficient [10]) to hold. The role of the tension is played by the action $S_a$ of the instanton, since the two coincide for an object localised in a point in spacetime. The coupling is given by $1/f_a$, as can be seen from the fact that the instanton charge is given by

$$n = \frac{i}{f^2} \int_{S^{d-1}} \star da. \tag{1.49}$$

This gives the Axion Weak Gravity Conjecture, originally formulated in [66] (and further argued for in [90, 91, 92]).

[10] There have been attempts to fix the value of the coefficient in (1.50), using semi-classical solutions such as gravitational instantons [87] or wormholes [88, 89]. However, there is still no clear consensus in the literature as to what the right choice should be.

**Axion Weak Gravity Conjecture** [66]

*An axion with decay constant $f_a$ can only couple to instantons with an action $S_a$ satisfying*

$$S_a f_a \lesssim M_p \tag{1.50}$$

It can be rephrased as the statement that axion decay constants in String Theory should not be transplanckian, which had already been proposed "empirically" in [93], and also in subsequent works [94, 67, 95]. A relevant consequence for phenomenology is that the axion WGC forbids the existence of monotonous regions in the potential of size $\Delta a \gtrsim M_P$, with striking implications for large field inflation[96, 91].

A final development to consider in this section are possible extensions of the WGC in De Sitter spacetime, or any other consequences that might follow from similar lines of reasoning. For a black hole much smaller than the dS radius, one can neglect spacetime curvature and expect the standard form of the WGC to hold, using the same arguments. On the other hand, when the black hole radius starts to become comparable with the dS one, the different nature of black holes must be taken into account. Indeed, dS black holes cannot be arbitrarily large, as their horizon must always be smaller than the corresponding cosmological one. In the case where this bound is saturated, corresponding to the largest possible mass for a fixed charge, they are known as Nariai black holes. By demanding that charged, extremal Nariai black holes be able to decay one obtains an upper bound on the mass of *any* charged particle [71].

**Festina Lente (FL)** [71]

*Consider a d-dimensional theory of Quantum Gravity in (approximately) dS spacetime with a* U(1) *gauge group. Denoting the gauge coupling by $e$ and the Hubble radius as $H$, every particle in the spectrum must obey the bound*

$$m^2 \gtrsim qeM_P H. \tag{1.51}$$

Combined with (1.44), it gives a finite range for the mass of the WGC particle. For reference, the allowed window for the SM electron would be

$$10^{-3}\,\text{eV} < m_e < 3 \cdot 10^{40}\text{eV}. \tag{1.52}$$

Although the body of evidence accumulated so far is more limited with respect to the original WGC, the range of possible phenomenological applications is much wider. In the context of BSM physics, it has been successfully applied to argue for the existence of an electroweak hierarchy [97], or to constrain hidden sectors [98, 99] and axions [100].

### 1.3.3 From the distance to the de Sitter Conjectures

As shown in the previous section for the case of the WGC, limiting cases or approximations to the No Global Symmetries principle provide a rich and fruitful way to think about Swampland conjectures more in general. In particular, one can consider a gauge symmetry with coupling $e$ and ask what happens when $e \to 0$ and the corresponding global symmetry is restored. To further explore the physics of such scenarios, it is useful to approach them from the perspective of asymptotic limits in moduli space. The idea behind this is that, in String Theory compactifications, all the low energy couplings and mass scales are *dynamical* and set by the vacuum expectation value (vev) of certain scalar fields, called moduli, associated to the geometry of the internal space. If fluxes are not turned on, these fields are characterised by a flat potential thanks to the high degree of supersymmetry and one can move freely (*i.e.* at no energy cost) along this manifold, known as moduli space. In contrast, fluxes will partially break susy and induce a potential for the moduli, and in that case one speaks of an approximate moduli space.

To be concrete, the universal part of the action for such scalars coupled to gravity schematically reads

$$S \supset M_p^{D-2} \int d^d x \sqrt{-g} \left( \frac{R}{2} - \frac{1}{2} G_{ij}(\Phi_k) \partial_\mu \phi^i \partial^\mu \phi^j \right). \tag{1.53}$$

The kinetic term naturally induces the notion of a metric $g_{ij}(\Phi_k)$ in moduli space. From this point of view, all limits of vanishing gauge coupling are then 'protected' by an infinite *proper distance* in moduli space, which is the one computed using the

metric in field space. This signals the fact that effective field theory descriptions are expected to break down when one moves along large distances in moduli space, as formalised (and quantified) by the Swampland Distance Conjecture (SDC) [30].

**Swampland Distance Conjecture (SDC)** [30]

*Consider a theory of Quantum Gravity with an exact moduli space $\mathcal{M}$, as parametrised by the action 1.53. Then,*

- *$\mathcal{M}$ is non compact. Given any point $P \in \mathcal{M}$, there always exists a point $Q \in \mathcal{M}$ such that the geodesic distance $d(P,Q)$ is larger than any given distance $d_0$.*
- *If one approaches the limit where $P \to Q$, an infinite tower of exponentially light states with masses scaling as*

$$m(Q) = m(P)e^{-\lambda d(P,Q)} \tag{1.54}$$

*appears, with $\lambda$ an $\mathcal{O}(1)$ coefficient.*

A refined version of the conjecture has also been formulated to deal with the case of approximate moduli space [68] (See section 3.3.1 for a possible puzzle involving the latter)

To grasp some intuition as to why the SDC should hold, and to see in practice what these towers of states could be, it is very instructive to consider the toy example of $d+1$-dimensional string theory in flat spacetime with one direction compactified on a circle of radius $R$, that is $\mathcal{M}_d \times S^1$. Dimensional reduction of the $d+1$ dimensional gravity theory (contained in the low energy spectrum of the string theory) gives rise, amongst other things, to Einstein gravity in $d$ dimensions and a scalar. The expectation value of the canonically normalised field $\phi$ is related to the radius of the circle as

$$2\pi R = e^{-\sqrt{\frac{d-2}{2(d-1)}}\phi}, \tag{1.55}$$

so that the infinite distance limits correspond to $R \to 0$ and $R \to \infty$. Along the

compact direction, the momentum of the string is quantized

$$p^d = \frac{1}{2}\left(p_L^d + p_R^d\right) = \frac{n}{R}, \tag{1.56}$$

as happens for ordinary particles (or equivalently, scalar fields). Unlike point particles, however, a string can wind an integer $w$ number of times along the circle. Hence, the left and right-moving momenta along the circle will have to satisfy the additional relation

$$\frac{\alpha'}{2}\left(p_L^d - p_R^d\right) = wR. \tag{1.57}$$

From the standard mass relation $M^2 = -p_\mu p^\mu$, the ground state (with no oscillators) of a string with momentum $n$ (in units of $1/R$) and winding number $w$ has a mass

$$m_{n,w}^2 = \left(\frac{1}{2\pi R}\right)^{\frac{2}{d-2}} \left(\frac{n}{R}\right)^2 + (2\pi R)^{\frac{2}{d-2}} \left(\frac{wR}{\alpha'}\right)^2. \tag{1.58}$$

It is then clear that the two infinite distance limits for the field $\phi$ both give rise to infinite towers of light states, with masses that decrease exponentially in the proper field distance $d(\phi_1, \phi_2) = \phi_1 - \phi_2$. They can be identified with Kaluza-Klein (KK) modes for $R \to \infty$, and the winding modes of the string for $R \to 0$. It is important to emphasize that while the former would arise in any QFT dimensionally reduced on a compact manifold, the latter tower of states is an intrinsically stringy feature. Its existence is tied to a very deep feature of String Theory, the invariance under *T-duality*. It is a symmetry which relates two theories compactified on a circle and whose radii are one the inverse of the other, exchanging the roles of the KK and winding towers. In the open string sector, *T-duality* exchanges Neumann and Dirichlet boundary conditions. It will turn a $Dp$ brane into a $D(p-1)$ brane if performed along an $S_1$ parallel to the brane, or a $D(p+1)$ brane if the $S_1$ is transverse. More generally, T-duality maps type IIA string theory to type IIB and vice versa, with the identification

$$R' = \frac{\alpha'}{R} R \qquad g_s' = \frac{\sqrt{\alpha'}}{R} g_s. \tag{1.59}$$

It can also be generalised to compactifications on much more complicated spaces, such as Calabi-Yau's. In that context, it is deeply related to mirror symmetry [13].

Perhaps the most surprising development stemming from the SDC is the conjecture that String Theory cannot admit stable de Sitter vacua. Given the observed positive value of the cosmological constant, this carries enormous implications for any

attempt to reconcile String theory with cosmological observations (also see for other cosmological implications [69]). Difficulties in obtaining de Sitter through explicit constructions have been observed for quite some time, formalised by a series of no-go theorems and explicit examples. The most famous is the *Maldacena-Nunez* theorem of [101], which forbids the existence of classical (*i.e.*, without relying on any quantum corrections) vacua arising from 10-dimensional supergravity with fluxes and positive tension objects such as D-branes. Another well-known example, which only applies to type IIA flux compactifications on Calabi-Yau's, is the result of [102]. Allowing for the presence of both $D-$ and $O-$planes, it was proven that the scalar potential should obey the inequality

$$\frac{|\nabla V|}{V} > \sqrt{\frac{54}{13}}, \tag{1.60}$$

thus forbidding any dS vacua. In [55], it was suggested explicitly that perhaps such vacua should not exist at all, which was further argued for in the de Sitter conjecture of [56].

**de Sitter conjecture** [56]

*In any consistent theory of Quantum Gravity, the scalar field potential has to satisfy*

$$\frac{|\nabla V|}{V} > c, \tag{1.61}$$

*where the norm is calculated with the metric in moduli space and $c$ is a constant of $\mathcal{O}(1)$.*

This formulation of the conjecture is mostly based on explicit examples in String Theory, such as those coming from type II and heterotic $\mathrm{O}(16) \times \mathrm{O}(16)$ compactifications with D-branes and orientifolds. As another motivation, the authors of [56] also showed how an inequality à la *Maldacena-Nunez* applies to a subset of examples coming from M-theory, namely

$$\frac{|\nabla V|}{V} > \frac{6}{\sqrt{(d-2)(11-d)}}. \tag{1.62}$$

The inequality in (1.61) is a rather strong statement, as not only does it forbid the existence of de Sitter minima, but more in general any critical point. It was indeed pointed out in [103] that, in its original formulation, the dS conjecture would also rule out certain supersymmetric AdS vacua whose existence has been less disputed.

A refined version, introduced in [104], also allows for the existence of unstable dS critical points.

**Refined de Sitter conjecture** [104]

*In any consistent theory of Quantum Gravity, the scalar field potential has to satisfy either of the following constraints*

$$\frac{|\nabla V|}{V} > c_1, \tag{1.63}$$

or

$$\text{Min}\Big(\nabla_i \nabla_j V\Big) \leq -c_2 \frac{V}{M_P^2} \tag{1.64}$$

*where the Hessian is minimised along orthonormal directions and $c_1, c_2$ are constants of $\mathcal{O}(1)$.*

The statement seems to be well established at the boundary of moduli space, *i.e.* in the control limit of weak couplings and large volumes.

Unlike the original version, the refined dS conjecture can also be advocated for with a bottom-up approach, by combining the SDC with entropic reasonings. In particular, one can associate a temperature and then an entropy to de Sitter as

$$T_{\text{dS}} = \frac{1}{2\pi R_{\text{dS}}} \qquad S_{\text{dS}} = 8\pi^2 M_P^2 R_{\text{dS}}^2 \propto \frac{1}{\Lambda} \tag{1.65}$$

where $R_{\text{dS}}$ is the Hubble radius. If one moves a large distance in field space, an exponentially light tower of states is supposed to appear, implying an increase in entropy scaling positively with the number of states. One can then constrain the scalar potential by postulating this increase in entropy should be compensated by a decrease in the cosmological constant[11] (and hence the value of V). Given the

[11] Away from a local minimum, it is not as immediate to associate an entropy to something that

exponential nature of the tower, by equating the two entropies one saturates the condition (1.63), obtaining $|V'| \sim V$. The second condition, Eq. (1.64), is also necessary because if the Hessian has a negative eigenvalue of order $M_P^2/R_{\rm dS}^2$, the minimum is unstable and this semi-classical description is expected to break down.

### 1.3.4 Scale separation

Given that many compactifications - and especially those relevant to phenomenology[12] - involve an AdS vacuum with a potential for the moduli, it is relevant to ask how the SDC may be modified in this context. As regards scalar potentials, the SDC was extended to the case of fields with a non-zero expectation value in [67, 68]. For attempts to establish this through the AdS/CFT correspondence, see the discussion in section 1.5. In an orthogonal direction, one might also wonder whether large displacements along a "new" direction that appears for AdS vacua, that of varying radius (or cosmological constant), also imply the breakdown of effective field theory. Indeed, an application of the SDC to the metric (rather than scalar fields) yields the so-called AdS Distance Conjecture [107]

**AdS Distance Conjecture (ADC)** [107]

*Take Quantum Gravity on $AdS_d$, with a cosmological constant $\Lambda$. A tower of states with masses*

$$m \sim |\Lambda|^{\alpha}, \qquad \alpha > 0. \tag{1.66}$$

*should appear in the limit where $\Lambda \to 0$.*

There is also a strong version of the conjecture

is not quite dS. One can use the covariant entropy bound of [105, 106] to recover the same scaling $S_{\rm dS} \sim 1/\Lambda$.

[12] Strictly speaking, the compactifications compatible with phenomenology are those that realise a 4-dimensional De Sitter vacuum, given the observed value of the cosmological constant. In practice, the few known dS constructions all involve AdS vacua as an intermediate step.

**Strong AdS Distance Conjecture (SADC)** [107]

*For supersymmetric AdS, the exponent takes the value* $\alpha = \frac{1}{2}$.

In the SADC, the simplest example for such a tower of states are Kaluza-Klein modes of the internal manifold, as can be seen in the case of Freund-Rubin $AdS_5 \times S^5$ compactifications of type IIB with $F_5$ flux. In these constructions, the internal radius of the sphere is equal to that of AdS, so that when the AdS radius goes to infinity, the $S^5$ grows in volume and the Kaluza-Klein modes become light

$$m_{KK} \sim \frac{1}{R_{AdS}} \sim \Lambda^{1/2}. \tag{1.67}$$

What this means from a more physical point of view is that a consistent description of the vacuum cannot neglect the states arising from dimensional reduction on the internal manifold, and as such it cannot be described from an inherently lower dimensional perspective. This obviously poses an obstruction to any sort of realistic phenomenology, and is known as the problem of Scale Separation (SS). In particular, the strong ADC implies that there cannot be any scale-separated, supersymmetric AdS vacua. It has been known for a long time that is difficult to obtain vacua with Kaluza-Klein masses parametrically heavier than the AdS scale, and the problem was recently emphasized in [108, 109]. The analysis of [108] showed that, in the same spirit of the *Maldacena-Nunez* theorem, scale separation requires negative tension objects such as $O$-planes. However, it was later emphasized [110, 111] that it might not necessarily be the case.

In [109], a weaker version of the SADC had already been proposed, suggesting that there should not be a separation of scales between the radius of AdS and the lightest massive states in the theory. This differs from the SADC in that it does not involve towers of particles such as KK modes, but rather zero modes such as (massive) 4d moduli. It is known as the AdS Moduli Conjecture (AMC), and its precise formulation is given below.

**AdS Moduli Conjecture (AMC)** [109]

*Any stabilised vacua on $AdS_d$ where the mass of the lightest modulus satisifes*

$$m_{\text{mod}} R_{AdS} \gg 1 \tag{1.68}$$

*belongs to the Swampland.*

This statement has a very simple interpretation from a holographic point of view: there have to be moduli with a conformal dimension of order $\Delta \sim \mathcal{O}(1)$.

Nonetheless, explicit proposals for (supersymmetric) scale-separated vacua do exist; a notable example are the Type IIA DGKT vacua, which will be a central ingredient in our holographic analysis (See Sections 4.1 and 4.2 for a review of their properties). There also exists a class of $AdS_3$ vacua arising from massive type IIA on $G_2$ manifolds [112, 113, 114, 115], whose dual CFT spectrum was analysed for the first time in [114]. The construction of DGKT crucially relies on the presence of $O$-planes (to evade the no-go theorems), which are treated as if they were smeared along the extra dimensions although they are localised sources. Therefore, the 10-dimensional equations of motions are not necessarily satisfied locally, but only in an integrated way. It is still not totally clear whether the inclusion of the back-reaction from the $O$-planes would change the nature of the solutions or even make the construction impossible to realise. For this reason, its validity has been questioned [116, 117, 118], and several authors have tried to take the backreaction of the O-planes into account [117, 119, 120]. In Ref. [121], the 10d equations of motions were solved for the first time in a large-flux expansion everywhere in the compact space which only breaks down close to the compact sources, providing much stronger evidence for the existence of the original DGKT proposal (See also [122, 123]). This expansion is reliable as it corresponds to the original controlled limit of DGKT where the string coupling becomes weak and the volume large. In particular, if one considers large $F_4$ fluxes proportional to $\int F_4 \sim \bar{e}$, the string coupling and volume scale as

$$g_s \sim \bar{e}^{-3/2} \qquad \mathcal{V} \sim \bar{e}^{3/2} \tag{1.69}$$

for $\bar{e} \gg 1$. Furthermore, the AdS and KK radius behave as

$$R_{\text{AdS}} \sim \bar{e}^{9/4} \qquad R_{KK} \sim \bar{e}^{-7/4}, \tag{1.70}$$

so that scale separation is parametrically verified in this limit. We also notice the AdS moduli conjecture of [109] is automatically verified by the light moduli. Although the strong formulation of the ADC is obviously satisfied in DGKT, it is ruled in again by a refined formulation of the SADC proposed in [124].

**Refined Strong AdS Distance Conjecture (RSADC)** [124]

*If the vacuum has a discrete $\mathbb{Z}_k$ symmetry for domain walls the relation* (1.66) *regulating the appearance of a tower of states has to be modified as*

$$m \sim |k\Lambda|^{1/2}. \tag{1.71}$$

In DGKT, there exists one such symmetry with $k \sim \bar{e}$, and the conjecture is satisfied.

## 1.4 Holography and AdS/CFT

### 1.4.1 Statement of the correspondence

The fundamental statement at the basis of the AdS/CFT correspondence is that String Theory on an asymptotically locally Anti de Sitter (AdS) spacetime is dual to a Conformal Field Theory (CFT) living on its conformal boundary, following the seminal work of Maldacena [9] (See Refs. [125, 126, 127] for the many excellent reviews on the topic). This means that any physical quantity (for example correlation functions) on either side of the duality may, at least in principle, be computed in either description. Any operator $\mathcal{O}(x)$ in the CFT is associated to a field in AdS with the same quantum numbers, whose boundary value acts as a source for its correlation functions. In particular, the generating functional $W_{\text{CFT}}$ for the connected diagrams

of $\mathcal{O}(x)$ satisfies

$$e^{W_{\mathrm{CFT}[h_0]}} \equiv \left\langle \exp \int \mathrm{d}^d x h_0(x) \mathcal{O}(x) \right\rangle_{\mathrm{CFT}} = \mathcal{Z}_{\mathrm{string}} \left[ h(x,z)|_{z\to 0} = f(z) h_0(x) \right], \quad (1.72)$$

where $\mathcal{Z}_{\mathrm{string}}$ is the string partition function evaluated for fields which reduce to $h_0(x)$ on the boundary. An additional factor of $f(z)$ is needed to enforce the correct boundary conditions, because solutions to the classical EOMs typically diverge near the boundary. [13]

Since AdS/CFT is a weak-strong duality, relating the weakly coupled regime of string theory to strongly interacting CFTs, this turns out to be particularly useful in practice, allowing for an analytical description of QFT regimes otherwise inaccessible with the standard techniques of perturbation theory. The main example the correspondence is based on is the specific case of type IIB string theory compactified on $AdS_5 \times S^5$ dual to $\mathcal{N} = 4$ Super Yang-Mills (SYM) in 4 dimensions, but it can be extended to a variety of other examples, such as $AdS_4 \times S^7$ or $AdS_7 \times S^4$ compactifications in M-Theory.

### 1.4.2 Original formulation

At the heart of AdS/CFT lies an idea known as the *Holographic Principle*, *i.e.* the proposal that the bulk dynamics of a $d+1$-dimensional theory of gravity be encoded into the degrees of freedom and dynamics of a $d$-dimensional theory living on the boundary, as first proposed by 't Hooft [128] and Susskind [129]. This was partly inspired by progress in the study of Black Hole Thermodynamics by Bekenstein and Hawking, and especially the celebrated entropy formula [130, 131, 132, 133]

$$S_{\mathrm{BH}} = \frac{A_{\mathrm{BH}}}{4G_{\mathrm{N}}}. \quad (1.73)$$

At a fundamental level, Eq. (1.73) relates the entropy of a black hole to the area of its horizon, leading to the interpretation that the maximal amount of information that can be stored in a region of space is determined by its surface area rather than its volume [134]. In a string theory context, the formula (1.73) was explicitly verified by Strominger and Vafa [8] for the specific case of extremal black holes charged under multiple U(1)'s in 5-dimensional supergravity, arising from the compactification of

[13] See Section 1.4.3 for the explicit expression in the case of a scalar field.

type IIB string theory on a 5-dimensional compact manifold. This system admits a different, microscopic description in terms of a $D1-D5$ brane bound state, by which the authors of [8] managed to reproduce the correct black hole entropy at large values of the charges. One might take this as an indication that $p$-branes should be a crucial ingredient in a proper stringy realisation of holography, and indeed the original approach of [9] proceeds by examining the near-horizon limit of a stack of $N$, coincident $D3$ branes.

A p-brane can be viewed as a localised solution to the supergravity equations of motion, charged under the corresponding flux. In the string frame, the bosonic action for type IIB Supergravity is given by

$$\begin{aligned} S_{IIB} =& \frac{1}{(2\pi)^7 (\alpha')^4} \int dx^{10} \sqrt{g} e^{-2\phi} \left( R + 4(\partial\phi)^2 - \frac{1}{12} H^2_{\mu\nu\tau} \right) \\ & - \frac{\sqrt{g}}{2} \left( (F_\mu)^2 + \frac{\tilde{F}^2_{\mu\nu\tau}}{3!} + \frac{\tilde{F}^2_{\mu\nu\tau\rho\sigma}}{5!} \right) - \frac{1}{2} \epsilon_{\mu_1 \cdots \mu_{10}} C_{\mu_1 \cdots \mu_4} H_{\mu_5 \cdots \mu_7} F_{\mu_8 \cdots \mu_{10}}. \end{aligned} \tag{1.74}$$

An extremal p-brane lying along the first $p+1$ spacetime directions with charge $N$ then corresponds to the solution

$$\begin{aligned} ds^2 &= H^{-1/2}(r) dx^2_\mu + H^{1/2}(r) dy^2 \\ A_{0\cdots p} &= H(r) \\ e^\phi &= g_s H(r)^{(3-p)/4} \\ H(r) &= 1 + \frac{(4\pi)^{(5-p)/2} \Gamma\left(\frac{7-p}{2}\right) g_s N (\alpha')^{(7-p)/2}}{r^{7-p}} \equiv 1 + \frac{L^{(7-p)}}{r^{7-p}} \end{aligned} \tag{1.75}$$

The $p$-form flux along the transverse $\mathbb{R}^{8-p}$ space is quantised as

$$\int_{S^{8-p}} \star F_{p+2} = N, \tag{1.76}$$

so that (1.75) can also be viewed as a stack of $N$ coinciding, unit charge branes. The value of $p = 3$ is rather special; it is the only value for which the metric remains non-singular at the horizon (where the Ricci scalar behaves as $R \sim r^{-(p-3)^2/8}$), and the dilaton maintains a constant profile everywhere (in particular, the string coupling does not blow up close to the brane). From a field theory perspective, the massless modes of open strings stretching between $N$ parallel branes realise a $\mathrm{U}(1)^N$ gauge

theory on their world-volume, which is enhanced to U($N$) if the branes are brought to coincide. In the case of $D3$ branes, the theory is $\mathcal{N} = 4$ SYM with gauge group SU($N$). Furthermore, one can identify the complexified gauge coupling with the axio-dilaton

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{YM}^2} = \frac{C_0}{2\pi} + \frac{i}{g_s}, \tag{1.77}$$

relating the string coupling to the YM one. Written in this form, Eq. (1.77) is reminiscent of the fact that both theories have a self-duality SL$(2,\mathbb{Z})$ symmetry, sending $\tau \to (a\tau + b)/(c\tau + d)$ (with $a, b, c, d \in \mathbb{Z}$ and $ad - bc = 1$).

The key insight of Maldacena was to examine the behaviour of a stack of $N$ nearly coinciding[14] $D3$ branes in a decoupling limit where $\alpha'$ is sent to zero, keeping $g_s$ and $N$ fixed, corresponding to an effective description where only the excitations with $E \ll 1/\ell_s$ are considered. In the supergravity description, this amounts to taking a near horizon limit where $r \ll L$, since any excitation close to the branes is redshifted by an amount proportional to $H(r)^{-1/4}$ from the point of view of an observer located at infinity. In this limit, the metric (1.75) becomes that of $AdS_5 \times S^5$, where the radius of $AdS$ is to be identified with the parameter $L$ as

$$R = L \equiv 4\pi g_s (\alpha')^2 N. \tag{1.78}$$

Therefore, the physics of the small radius region can be described by IIB string theory on $AdS_5 \times S^5$. On the other hand, from the point of view of the open strings living on the branes all the heavy string modes become increasingly massive and can be integrated out, leaving just the SU($N$) gauge theory on the worldvolume of the branes. Furthermore, all the higher derivative terms in the DBI action can be neglected, as well as couplings to bulk gravity, so that in this limit the pure gauge theory is all that is left. This lead to conjecture that type IIB string theory on $AdS_5 \times S^5$ be equivalent to $\mathcal{N} = 4$ SYM in 3+1 dimensions with gauge group U($N$), for any values of $g_s$ and $N$. Notice that Eqs. (1.77) and (1.78) already imply a *dictionary* between quantities string and gauge theory sides: in terms of the 't Hooft coupling $\lambda = g_{YM}^2 N$,

$$\lambda = 4\pi g_S N \qquad \lambda = \left(\frac{R}{\ell_S}\right)^4. \tag{1.79}$$

The weakly coupled string theory regime is valid when both loop and curvature corrections can be neglected, that is $g_s \ll 1$ and $R \gg \ell_S$ respectively. Therefore,

[14] That is separated by a distance $\ell \ll \ell_s$.

a striking consequence of (1.79) is that it corresponds to a strongly coupled SYM theory, with $\lambda \gg 1$ and $N \gg 1$. It is also interesting to see how in this limit the large-N expansion of gauge theories emerges naturally, with the string coupling playing the role of $1/N$ in a genus expansion.

A first test of the correspondence concerns the symmetries of the two theories. The isometry group of $AdS_5$ is $SO(4,2)$, which is the $4-$dimensional conformal group in Lorentzian signature. This actually holds for $AdS_{d+1}$ in any dimension, as the $d-$dimensional conformal group is $SO(d,2)$. On the internal space, the $S_5$ has an isometry group of $SO(6)$ (corresponding to the rotations of the sphere), which matches with the $SU(4)_R$ R-symmetry of the gauge theory. Finally, the 32 supersymmetries of the field theory are recovered in the near horizon limit of the $D3$-brane solution on the supergravity side.

### 1.4.3 A bottom up approach

An alternative perspective on the AdS/CFT correspondence is provided by the bottom-up approach of [127], where one starts with a theory of Einstein gravity plus matter on an asymptotically AdS spacetime and essentially defines an effective CFT through the boundary correlation functions. In global coordinates, the AdS metric is given by

$$ds^2 = \frac{1}{\cos^2\left(\frac{\rho}{R}\right)}\left(dt^2 - d\rho^2 - \sin^2\left(\frac{\rho}{R}\right) d\Omega_{d-1}^2\right), \tag{1.80}$$

where $\rho \in [0, \frac{\pi}{2})$ and $t$ runs from minus to plus infinity. Alternatively, one can also express everything in the projective formalism of [135, 136], defined in terms of the coordinates $X_A$ of the embedding space

$$P_A \equiv \varepsilon X_A \quad \text{with} \quad \varepsilon \to 0, \quad \text{so} \quad P_A P^A = 0 \qquad P_A \cong \lambda P_A. \tag{1.81}$$

To define the would-be CFT operators, it is easier to start by considering the dual to a set of free massive fields on AdS. The ground state solution to the Klein-Gordon equation for a scalar field is

$$\psi_0(t,\rho) = e^{i\Delta t}\cos^{\Delta}(\rho), \tag{1.82}$$

where

$$\Delta(\Delta - d) = m^2 R_{\text{AdS}}^2. \tag{1.83}$$

Near the conformal boundary of AdS, located at $\rho = \frac{\pi}{2}$, the ground state solution (1.82) scales as $\psi_0 \sim \rho^\Delta$, so that the boundary operators can be defined in terms of the bulk fields $\{\varphi\}$ as [15]

$$\mathcal{O}(t, \Omega) \equiv \lim_{\epsilon \to 0} \frac{\varphi(t, \rho(\epsilon), \Omega)}{\varepsilon^\Delta} \qquad \rho(\epsilon) = \frac{\pi}{2} - \epsilon e^{-\tau}. \tag{1.84}$$

Their 2-point functions behave as

$$\langle \mathcal{O}(P_1)\, \mathcal{O}(P_2) \rangle = \frac{1}{(P_1 \cdot P_2)^\Delta} \sim \frac{1}{|x_1 - x_2|^{2\Delta}} \tag{1.85}$$

and therefore they can be identified as primary operators of conformal dimension $\Delta$. More in general, $\mathcal{O}$ will inherit the appropriate transformation properties under the conformal group, from the $\text{SO}(d, 2)$ isometries of $\text{AdS}_{d+1}$. The resulting theory is known as a Generalized Free Theory (GFT), which consists in the free theory of some number of scalar primaries with arbitrary conformal dimensions[16] and correlation functions which factorize exactly in terms of 2-point functions. For these reasons, such theories are also known as CFTs at infinite N. Despite being conformally invariant, they are not quite local CFTs as they do not have a conserved stress-energy tensor $T_{\mu\nu}$. [17] With a very similar argument to standard CFTs, it can however be shown that they satisfy an OPE, starting from a modified version of the state-operator correspondence where AdS states are taken at $t \to -\infty$ (instead of $r = 0$ in radial quantization). Furthermore, the correlators satisfy crossing symmetry and one can write down an equivalent version of the bootstrap equation.

At this point it is possible to start adding interactions perturbatively (including gravity), and it can be proven that under suitable conditions these do not break the conformal symmetry [135, 136]. At leading order in a $1/N$ expansion, it has been shown that the set of all possible higher derivative terms appearing in a scalar Lagrangian maps exactly to the solution of the bootstrap equations from the CFT point of view [85]. Furthermore, one can adopt an effective field theory approach as

[15] The particular form of $\rho(\varepsilon)$ is chosen to obtain a CFT in flat spacetime. In general, one can approach the boundary points at different speeds (corresponding to a Weyl rescaling), to obtain a CFT on any conformally flat manifold (for example the cylinder).

[16] In contrast to standard field theories, with dimension $\Delta = \frac{d-2+s}{2}$

[17] Unless $\Delta = \frac{d-2}{2}$, when the theory reduces to that of a free scalar.

for standard QFTs [137]. Since the role of energy is played by the conformal dimension of operators, the EFT picture is valid up to a cutoff $\Delta_i \ll \Delta_{\text{gap}}$, where the $\Delta_i$ refer to any operators being considered (and not just the primaries). This idea was made rigorous in [138], which examined the effects of neglecting operators with dimension higher than $\Delta_{\text{gap}}$ in the OPE expansion of two scalar primaries. In particular, the truncated expansion of a Euclidean, 4-point scalar correlator

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{1}{(x_{13})^{2\Delta}(x_{24})^{2\Delta}} \sum_{\Delta<\Delta_{\text{gap}},\ell} \lambda^2_{\Delta,\ell} g_{\Delta,\ell}(u,v) \tag{1.86}$$

was shown to differ from the exact result by an exponentially small correction in $\Delta_{\text{gap}}/\Delta_{\mathcal{O}}$ for fixed conformal ratios. For a generic EFT on AdS, one might expect this picture to break down eventually, unless the theory admits a well-defined UV completion. In the latter case, it is natural to assume that the "effective CFT" will be UV completed into a fully consistent CFT. It is precisely in this sense that we will take the duals to 4d scenarios of moduli stabilisation seriously, although their origin is very different from the original examples of the AdS/CFT correspondence.

## 1.5 Connections to the Swampland

### 1.5.1 General properties

In the context of the Swampland, holography has emerged as a tool to put on a more solid footing or even prove some of the proposed conjectures. The reason is that AdS/CFT may be thought to provide a non-perturbative definition of String Theory (or even Quantum Gravity) on AdS, rooted in the more rigorous CFT axioms. On the other hand, an obvious difficulty with this approach is that the weak/strong nature of duality, which has been so helpful in the study of CFTs, now becomes a hindrance. Indeed, putative swampland constraints in weakly coupled string theory on low curvature backgrounds now translate to statements on strongly coupled CFTs, which are hard to establish in general.

A number of recent works have tried to exploit this rather broad idea (see also the discussion in section 3.3.5 of the review [31]). The absence of global symmetries, which can be regarded as the most fundamental of all Swampland conjectures, was recently argued for in quantum gravity theories with a holographic dual [72, 73]. A naive argument proceeds by showing how a global symmetry in the bulk implies the

existence of a global symmetry from a boundary CFT [139]; by the holographic dictionary this would give rise to a gauge symmetry in the bulk, leading to a contradiction. The derivation in [72, 73] makes this rigorous by using concepts from information theory, and using a generalised version of Noether's theorem which also applies to discrete global symmetries. Furthermore, with similar arguments they also show the Completeness Hypothesis and prove that gauge groups must be compact.

For the case of the closely related WGC, no general proof has so far been obtained in AdS/CFT, although there have been numerous partial results. A notable exception is the case of $AdS_3$, since for 2d CFTs modular invariance has allowed to prove forms of the conjecture [140, 141, 142, 143]. Similar arguments have been applied to the physically distinct cases of $AdS_3$ vacua dual to a $CFT_2$ [142], or the to worldsheet of perturbative string theory with U(1) gauge fields[141]. Recently, the closely related Festina Lente proposal was also studied from a holographic perspective [144], within the Karch-Randall setup [145].

In the general case, an intrinsic difficulty lies in the fact that one of the main physical motivations (beyond examples in string theory) is that small, extremal black holes should be able to decay, but these are still poorly understood in AdS. To connect with this line of reasoning, it was shown in [146] that large, near extremal black holes in AdS should be unstable from an analysis of the dual CFT. However, these black holes do not necessarily have to decay through the emission of charged particles satisfying the WGC, and this alone does not imply the conjecture.

A first formulation of the WGC in AdS was proposed in [147], by simply trading the mass for $\Delta/R$ in the inequality (1.43).

**AdS Weak Gravity Conjecture** [147]

*In a holographic CFT with a* U(1) *global symmetry, there must exist at least one operator with dimension* $\Delta$ *and charge* $Q$ *such that*

$$e^2 Q^2 \geq \frac{D-3}{D-2}\frac{1}{M_P^2}\frac{\Delta^2}{R^2}. \tag{1.87}$$

A very closely related conjecture was put forward in [148, 149], taking inspiration from a particular formulation of the WGC [150, 151] which posits the existence of a charged, self repulsive particle. Given that binding energies in AdS appear as

anomalous dimensions of certain multi-trace operators, the precise statement can be phrased in terms of a convexity relation for the charged spectrum.

**Abelian Convex Charge Conjecture [?]**

*Consider a CFT with a* U(1) *global symmetry. If we define* $\Delta(q)$ *as the dimension of the lowest dimensional operator of charge* $q$*, the relation*

$$\Delta\left(n_1 q_0 + n_2 q_0\right) \geq \Delta\left(n_1 q_0\right) + \Delta\left(n_2 q_0\right) \tag{1.88}$$

*must hold for any* $n_1, n_2$ *and* $q_0$ *of order one.*

As happens for the original WGC, the inequality is satisfied by BPS states, and has been shown to hold in all known, consistent examples[18] The advantage of this formulation is that can be tested for any CFT, irrespectively of the existence of a gravitational dual, and that includes non-supersymmetric examples (from which most of the string theoretic evidence in favour of the WGC comes from).

In Refs [152, 153], studies on the distance conjecture for AdS vacua were initiated from a holographic perspective. The idea consists in relating the moduli space of such vacua to the space of relevant deformations of the corresponding CFT, *i.e* those obtained from a modification of the action of the form

$$S_{\text{CFT}} = S^0_{\text{CFT}} + \sum_i \lambda_i \int \mathrm{d}^d x \mathcal{O}_i. \tag{1.89}$$

This space is known as the conformal manifold, and is naturally equipped with a notion of distance, descending from the Zamolodchikov metric [154]

$$|x-y|^{2d}\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\rangle = g_{ij}(t^\alpha), \tag{1.90}$$

where the $\{t^\alpha\}$ are coordinates for the moduli. This allows one to rephrase questions for infinite distance limits in moduli space to statements on the infinite distance points in the conformal manifold. In particular, examples of infinite distance limits

[18] It can be violated, for example, in non-unitary CFTs.

were shown to be associated with a towers of high spin operators appearing in the CFT, and this behaviour was conjectured to be universal.

### 1.5.2 A novel approach to the Holographic Swampland

What all of the examples in the previous section have in common is that they are either based on rather generic properties of AdS/CFT, or they arise from controlled settings - such as highly supersymmetric examples or $AdS_3/CFT_2$ - and are not very useful from a phenomenological point of view. On the other hand, our approach aims at analysing the properties of the specific CFTs which would be dual to weakly coupled, effective theories arising from realistic string compactification scenarios. The focus is therefore on models with $\mathcal{N} \leq 1$ supersymmetry and potentials for moduli. Although the exploration of CFT duals to weakly-coupled low energy theories of moduli stabilisation has been the subject of occasional studies [37, 38, 39, 40], the topic remains relatively unexplored, in part due to the belief that the dual CFTs would have extremely complex properties. The main reason is that such vacua do not usually admit a 10-dimensional description due to the presence of non-perturbative ingredients (D-branes, O-planes, instantons etc), and as such the dual theory cannot be explicitly realised in terms of a near horizon limit of some brane configuration. While such considerations could cast doubts on the validity or the existence of the dual CFT, one can also take the bottom-up approach sketched in section, where the weakly coupled AdS theory is taken as a definition of a CFT, perturbatively in $\mathcal{O}(1/N)$. If the AdS vacua correspond to a genuine solution in string theory, it is very natural to expect that the operators dual to the light fields be the low-dimensional subsector of a full-fledged and consistent CFT. This is the point of view we shall take in this thesis. On the other hand, one might suspect this will not be the case for generic EFTs that belong to the Swampland. The ultimate goal would be to develop a set of criteria that can be used to discern whether a given scenario is consistent or not by analysing the properties of the low energy sector of its dual EFT.

To implement this approach, one would ideally like to find examples that are characterised by

- A simple enough (at least treatable) CFT description, so that putative Swampland constraints can be easily translated from the CFT side to AdS and vice versa.

- Relevance to phenomenology and the Swampland program. For this reason, we will focus on $AdS_4$ vacua with $\mathcal{N} \leq 1$ supersymmetry, where all (or most) of the moduli are stabilised.

In general, one technical complication is that typical Lagrangians arising in these scenarios can contain a large number of light moduli $\varphi_i$, typically of order a few hundred (from typical Calabi-Yau values for $h^{1,1}$ and $h^{2,1}$). In a holographic picture, each particle in the bulk corresponds to a single trace primary operator in the CFT$_d$ of conformal dimension $\Delta$ given by

$$\Delta(\Delta - d) = m^2 R_{AdS}^2. \tag{1.91}$$

For 'generic' $\mathcal{N} = 1$ supergravity models, $V_{AdS} \sim -m_{3/2}^2 M_P^2, R_{AdS} \sim m_{3/2}^{-1}$ and $m_{\varphi_i} \sim m_{3/2}$, translating into a large number of operators of low conformal dimension. However, it was found in [37] that the Large Volume Scenario [23, 24], viewed holographically, admits a relatively simple and unique form for the low-lying operators and their conformal dimensions. For LVS in the large volume limit, the only operators to retain a small $\mathcal{O}(1)$ conformal dimension $\Delta$ are the massless graviton, the volume modulus and its axionic partner; all other dimensions diverge and the corresponding fields can be integrated out. Above these, there is a large $\Delta_{gap}$, not just to the higher spin states but also to the other moduli. Furthermore, the limit of infinite $\mathcal{V}$ also provides a well-defined and universal limit, where all the interactions in the effective theory are fixed and do not depend on the fine details of the compactification. This makes it a relatively well-posed question whether Swampland-like constraints on the corresponding low-energy Lagrangians can be recast in the language of conformal theories, and it will be the starting point of our analysis. Chapter 2 will therefore be devoted to the exploration of the holographic dual to LVS and some closely related constructions, such as perturbative stabilisation [155, 156] or LVS with one (or more) fibred directions [157, 158]. We will also consider the well-known case of the KKLT scenario [159] (and a variation on the theme known as Racetrack stabilisation [160]), focusing on the AdS vacuum before uplifting. Although KKLT does not seem to exhibit such a clear and universal limit for the CFT spectrum at large volumes, it is still possible to isolate a low-lying subsector that can be analysed using our techniques.

On this point, it is also worth mentioning another claim, that recently appeared in the literature [54], that AdS KKLT vacua may fully be inconsistent from a CFT point of view. The argument proceeds by showing that the central charge of the dual CFT

can be bound by the Euler characteristic of the compactification space (a Calabi-Yau four-fold[19] ), as

$$R^2_{\rm AdS} \lesssim \chi(CY_4). \tag{1.92}$$

Below the string and KK scales, the degrees of freedom will be those coming from the moduli, so of the order of $h^{3,1} + h^{1,1}$ for a four-fold (or $h^{2,1} + h^{1,1}$ for the orientifold). Since

$$\chi(CY_4) = 6(8 + h^{3,1} + h^{1,1} - h^{2,1}), \tag{1.93}$$

one hence concludes that the number of light degrees of freedom will be of the order of $\chi(CY_4)$. From (1.92), the inverse AdS radius is then larger than the species scale $M_P/\sqrt{N_{\rm dof}}$, placing the AdS theory beyond the regime of effective field theory, and making it inconsistent.

Another class of vacua that will prove fruitful to examine in this context are the type IIA DGKT models [41], which are described in detail in Chapter 3. Remarkably, it is one of the few settings where full moduli stabilisation can be achieved only through fluxes, and also the most prominent example to exhibit scale separation in 4d (See also Subsection 1.3.4). We anticipate that, even in this case, it will be possible to isolate a set of low dimensional operators in the CFT that can be parametrically decoupled from the rest of the spectrum. Furthermore, we will show that their dimensions are always integers and universal, i.e. independent of the compactification space; they will be the same for any Calabi-Yau. From a holographic perspective, the feature of scale separation is particularly interesting, since it is intimately tied to the existence of operator gaps in the spectrum of the dual CFTs [161, 162]. In particular, the strong form of the ADC is equivalent to having no parametric gap between the low-lying CFT sector and the towers of states verifying the conjecture (*e.g* the KK modes.) More generally, an AdS vacuum with only positive masses would be dual to a so-called 'dead-end' CFT, *i.e.* a CFT with no marginal or relevant scalar deformations. Although a proof is still lacking, in $d > 2$ no examples are known, [20] and it is possible they might be severely constrained or even not exist at all (For example, there are no perturbative dead end CFTs in 3d [163]).[21] This is similar in spirit to the AdS moduli conjecture of [109] (also see the discussion surrounding Eq.(1.68)), which is however satisfied by DGKT. With similar motivations, it was

[19] Although KKLT was originally formulated in the context of type IIB theory compactified on Calabi-Yau three-folds, it can be generalised to F-Theory compactified on four-folds.

[20] The Monstrous Moonshine is a counter-example in $d = 2$, with the first scalar operator appearing at $\Delta = 4$.

[21] See [164] for a recent discussion of the would-be dual to pure gravity on $\text{AdS}_d$.

proposed in [165] that there be an upper bound on the scaling dimension of the first spin 2 operator (excluding the stress-energy tensor) appearing in a given CFT. The statement was shown to hold in a large class of CFTs arising from branes probing internal singularities, and also conjectured to be valid in general. If true, this would rule out scale separation, with the dual to the spin-2 operator playing the role of a graviton KK mode that becomes light for large AdS radius.

### 1.5.3 No SUSY AdS?

Before we proceed, let us pause to make a few remarks on a possible objection to the existence of CFT duals for some of the scenarios discussed above. According to a well-known swampland conjecture, non-supersymmetric AdS vacua will generically be unstable, raising doubts about the possibility of applying holography in such a setting. This will mostly be relevant for the LVS scenario, but also for the non-supersymmetric branches of the DGKT vacua. The conjecture proposed in [166] states that

**No Supersymmetric AdS Instability Conjecture (NSAIC)** [166]

*Any non-supersymmetric, $AdS_d$ vacuum is unstable (or metastable at best).*

and it was later speculated the same conclusion might apply to non-AdS vacua as well [167]. The original motivation comes from applying the WGC to the study of vacua stabilised by fluxes, which allows them to decay through a Coleman-de Luccia (CdL) bubble. The codimension one domain walls separating vacua with different fluxes have to satisfy the sharpened version of the WGC, which in absence of supersymmetry implies their tensions must obey the strict inequality $T < Q$ [22]. Intuitively, for a spherically symmetric solution the electromagnetic repulsion between points on the surface of the brane is greater than the gravitational attraction between them, and this represents a bubble instability. In formulas, if one takes the AdS Euclidean metric

$$ds^2 = R_{\text{AdS}}^2 \left( \cosh^2 \rho d\tau^2 + d\rho^2 + \sinh^2 \rho d\Omega_{D-2}^2 \right), \tag{1.94}$$

[22] More precisely, at least one kind of $d-2$ brane satisfying $T < Q$ should exist.

the Euclidean instanton describing a spherical bubble solution centred around the origin is described by the action [168]

$$S_E = R_{\rm AdS}^{D-1}\Omega_{D-2}\int d\tau \left[T\sinh^{D-2}\rho\sqrt{\cosh^2\rho+\left(\frac{d\rho}{d\tau}\right)^2}-Q\sinh^{D-1}\rho\right]. \qquad (1.95)$$

The equations of motion imply

$$\cosh\rho = \frac{\cosh\rho_{\rm max}}{\cosh\tau} \qquad \tanh(\rho_{\rm max}) = \frac{T}{Q}, \qquad (1.96)$$

so that if $T/Q < 1$ the maximum radius $\rho_{\rm max}$ reached for $\tau = 0$ is finite. After tunnelling has taken place, one can match with the Lorentzian solution for $\tau > 0$, corresponding to a bubble which expands outwards at the speed of light

$$\cosh\rho = \frac{\cosh\rho_{\rm max}}{\cos\tau}. \qquad (1.97)$$

It will reach the AdS boundary (located at $\rho = \infty$) in a finite time $\tau = \frac{\pi}{2}$, and that corresponds to the maximum lifetime of the vacuum.

From the point of view of holography, the consequences can be catastrophic. The vacuum decay rate per unit volume is finite, as it is given by $\Gamma \sim e^{-S_E}$, with $S_E$ the action for the Euclidean instanton. Since an observer at the boundary is surrounded by an infinite volume, they will be able to notice the instability instantaneously [169], no matter how small the decay rate. It has therefore been suggested that this might make it impossible to define a unitary CFT on the boundary, given that the gravitational theory might suffer from an unbounded spectrum or the violation of the asymptotic symmetries of AdS (corresponding to loss of conformal invariance on the CFT side) [170].

However, the application of the argument to 4-dimensional scenarios of moduli stabilisation is still unclear. An initial counter-objection is that such an explicit description of the transitions between different vacua is not available. Although one expects the landscape of string theory to be continuously connected, it would perhaps still be speculative to assume the existence of an analogous mechanism without the confirmation of a concrete computation. On the other hand, in the 4-dimensional effective theory context the stability of the LVS minimum has been established in detail. For a fixed flux configuration (and superpotential), the LVS minimum is the

only (absolute) minimum for the potential, so that decays to vacua of lower energy are not possible. Furthermore, in [171] it was shown that even for AdS vacua with different fluxes the energy gap between different vacua is too large with respect to the domain wall tension to allow nucleation, and the corresponding decays cannot take place. The non-supersymmetric branches of DGKT analysed in [172] were also found to be stable at the perturbative level, and in some cases it was shown that the conclusion is valid even non-perturbatively [173, 174, 123], for the same reason as LVS. Finally, another perplexing feature of the no non-supersymmetric holography claim is the fact that it holds for an arbitrarily small tunnelling rate. In the limit of vanishing (but non-zero) rate, one might expect the boundary theory to describe an *approximate* CFT, whose departure from a standard CFT behaviour is controlled by the magnitude of the rate itself. There are indeed examples in the literature of theories which are not, strictly speaking, unitary CFTs but can usefully be approximated by a CFT in a suitable regime. A well known case is that of QCD, which is of course non-conformal, and an example in a different context is that of (non-unitary) Complex CFTs [175]. In this thesis we will therefore take the point of view that the no-susy AdS conjecture of [166] is insufficient to completely rule out the possibility of non-supersymmetric holography.

# Chapter 2

# Towards a holographic description of realistic vacua

This Chapter is based on the paper [1].

## 2.1 The Large Volume Scenario and its Holographic Properties

We start with a brief review of the Large Volume Scenario of moduli stabilisation (LVS), focusing on properties of both the AdS vacuum and its putative holographic dual CFT.

### 2.1.1 Introduction to LVS and its low energy dynamics

The Large Volume Scenario [23, 24] is a particular model of IIB compactification with fluxes, where all moduli are stabilized in a susy-breaking AdS vacuum at an exponentially large value of the volume $\mathcal{V}$. This is achieved through a combination of perturbative corrections arising from the $\alpha'^3\mathcal{R}^4$ term and non-perturbative effects in the superpotential. A pedagogical account of the basic construction is given in [176], and some of the relevant concepts have already been introduced in Section 1.2.3. The dilaton and complex structure moduli are heavy and stabilised by fluxes. These can then be integrated out, leading to an effective theory described by a Kähler potential

$K$ and superpotential $W$ of the form

$$K = -2\log\left(\mathcal{V} + \frac{\xi}{g_s^{3/2}}\right), \tag{2.1}$$

$$W = W_0 + \sum_i A_i e^{-\alpha_i T_i}, \tag{2.2}$$

which depends on the Kähler moduli $T_i = \tau_i + ia_i$. The coefficient $\xi$ of the $\alpha'^3$ correction is determined by the Euler characteristic of the compactification manifold, $\xi = \frac{\zeta(3)\chi(M)}{2(2\pi)^3}$, while the $\alpha_i$ in the exponential can assume the values $\alpha_i = \frac{2\pi}{N}$.

In the simplest scenario (which we restrict to here), there are only two Kähler moduli, one large cycle corresponding to the size of the overall volume ($T_b$) and one small internal 'blow-up' cycle ($T_s$). The volume is then written as

$$\mathcal{V} = \frac{1}{\kappa}\left(\tau_b^{3/2} - \tau_s^{3/2}\right), \tag{2.3}$$

with the constant $\kappa$ depends on the specific Calabi-Yau. Inserting this expression into (2.1), one can use the form of the $\mathcal{N} = 1$ supergravity potential

$$V = e^K\left(G^{T\bar{T}} D_T W \bar{D}_T W - 3\left|W\right|^2\right), \tag{2.4}$$

to obtain an effective potential

$$V = \frac{A a_s^2 \sqrt{\tau_s} e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{B W_0 a_s \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{C\xi W_0^2}{g_s^{3/2}\mathcal{V}^3}, \tag{2.5}$$

where $A$, $B$ and $C$ are numerical constants, which is minimized for

$$\begin{aligned} \langle\tau_s\rangle &\sim \tfrac{\xi^{2/3}}{g_s}, \\ \langle\mathcal{V}\rangle &\sim e^{a_s\tau_s}. \end{aligned} \tag{2.6}$$

Here $\tau_s$ is the small, heavy modulus, while the large light modulus is associated to the overall breathing mode corresponding to volume rescalings.

One can usefully obtain a 1-field effective potential for the light modulus by integrating out the heavy modulus. In terms of the canonically normalized field

$\varphi = \sqrt{\frac{2}{3}} \ln \mathcal{V}$, the resulting potential is

$$V = e^{-\frac{\lambda\varphi}{M_P}} \Big( A' - \varphi^{3/2} \Big), \tag{2.7}$$

for a constant $A'$, where in LVS the coefficient $\lambda = \sqrt{\frac{27}{2}}$. This value of $\lambda$ corresponds to the overall $\mathcal{V}^{-3}$ scaling of the potential and so is fixed in LVS. There is a single minimum located at a critical value $\varphi_c = \langle \phi \rangle$ of the field satisfying

$$\frac{3}{2} \lambda \varphi_c^{\frac{3}{2}} - \varphi_c^{\frac{1}{2}} = \lambda A'. \tag{2.8}$$

This corresponds to $\mathcal{V} \gg 1$ since $\varphi$ is logarithmic in the volume and $A'$ takes typical $\mathcal{O}(1-10)$ values. It is worth emphasising that as $\ln \mathcal{V} \sim \frac{1}{g_s}$, and there is a large discretuum of flux choices to fix $g_s$ to small values, in practice $\langle \mathcal{V} \rangle$ can be made as large as one wishes and is effectively a free parameter. The corresponding vacuum realizes an AdS solution and one can verify that

$$V''(\varphi_c) = -\lambda^2 V_{min} \left( 1 - \frac{1}{2\lambda\varphi_c} \right). \tag{2.9}$$

Using $V_{min} \equiv -\frac{3M_P^2}{R_{AdS}^2}$, this can be rewritten [37]

$$V''(\varphi_c) = \frac{3\lambda^2}{R_{AdS}^2} \Big( 1 + \mathcal{O}(\ln \mathcal{V}^{-1}) \Big). \tag{2.10}$$

From a holographic perspective, this equation (combined with Eq. (1.91)) carries a striking implication: the conformal dimension of the volume modulus is fixed in the infinite volume limit, approaching in the $\mathcal{V} \to \infty$ limit a universal value (with subleading corrections in $(\ln \mathcal{V})^{-1}$

$$\Delta_\varphi = \frac{3\Big( 1 + \sqrt{1 + \frac{4}{3}\lambda^2} \Big)}{2}. \tag{2.11}$$

Moreover, a similar phenomenon holds for the interaction terms; repeated differenti-

ation of Eq. (2.7) give

$$V^{(n)}(\varphi) = (-\lambda)^n \frac{3M_P^2}{R_{AdS}^2} \frac{n-1}{n!} \Big(\frac{\varphi}{M_P}\Big)^n \Big(1 + \mathcal{O}\Big(\frac{1}{\lambda\varphi_c}\Big)\Big), \tag{2.12}$$

whose form is now independent of specific details of the UV theory (such as $W_0$, the details of the Calabi-Yau or the choice of fluxes). The volume axion has a shift symmetry and so its potential vanishes, leaving it massless, with the only interactions arising from its kinetic term

$$\mathcal{L}_{aa\varphi^{n-2}} = \left(-\sqrt{\frac{8}{3}}\right)^{n-2} \frac{1}{2(n-2)!} \Big(\frac{\varphi}{M_P}\Big)^n. \tag{2.13}$$

Furthermore, the structure of Eqs. (2.12) and (2.13) is radiatively stable, since quantum corrections give sub-leading effects in the large volume expansion [37] and so leave the basic form of the potential of Eq. (2.7) unaltered.

### 2.1.2 CFT interpretation

Given the uniqueness of the spectrum and interactions of low conformal dimension operators in the infinite volume limit, it is natural to view the low-dimension sector of LVS vacua as a small perturbation about this unique theory, which thus acquires a special status. In particular, the low dimensional sector of the putative CFT dual is characterized by a sparse spectrum, as the only single trace operators which retain a finite conformal dimension in the large volume limit are (as above) the volume modulus and its axion, plus the stress-energy tensor. Crucially, the dimensions of the other moduli diverge as $\mathcal{V}$ is taken to infinity, as shown in Table 2.1. Therefore, this sector of the theory is fully specified by the conformal dimensions of Table 2.2 and the interactions of (2.12) - (2.13). The latter can then be recast into $\mathcal{O}(\frac{1}{N^2})$ CFT data (OPE coeffiecients and anomalous dimensions) through the AdS/CFT correspondence by computing correlation functions.

From a holographic perspective, the appeal of LVS lies precisely in the fact that such computations become tractable due to the small number of states propagating inside Witten diagrams. It therefore represents both a serious scenario of moduli stabilisation and also an ideal playground to study the questions outlined in the

| Mode | Mass | $\Delta$ |
|---|---|---|
| $\tau_s$ | $\frac{M_P \ln \mathcal{V}}{\mathcal{V}}$ | $\mathcal{V}^{\frac{1}{2}} \ln \mathcal{V}$ |
| $T, U$ | $\frac{M_P}{\mathcal{V}}$ | $\mathcal{V}^{\frac{1}{2}}$ |
| $\psi_{3/2}$ | $\frac{M_P W_0}{\mathcal{V}}$ | $\mathcal{V}^{\frac{1}{2}}$ |

Table 2.1: Large $\mathcal{V}$ asymptotics of the masses and conformal dimensions for the single trace operators dual to the other moduli in the minimal realization of LVS, specifically the complex structure moduli, the small Kähler modulus and the gravitino. A similar decoupling occurs for other modes like brane moduli. The KK and string modes are even heavier with $\Delta_{KK} \sim \mathcal{V}^{5/6}$ and $\Delta_{string} \sim \mathcal{V}$, while black hole states start occurring at $\Delta_{BH} \sim \mathcal{V}^{3/2}$.

| Operator | Spin | Parity | $\Delta$ |
|---|---|---|---|
| $a$ | 0 | $-$ | 3 |
| $\varphi$ | 0 | $+$ | $\frac{3}{2}(1+\sqrt{19})$ |
| $T_{\mu\nu}$ | 2 | $+$ | 2 |

Table 2.2: Low dimensional single-trace operators in the spectrum of the CFT dual to LVS in the $\mathcal{V} \to \infty$ limit. Table taken from [37].

introduction, namely whether Swampland constraints in AdS can be rephrased or understood in terms of the corresponding CFT.

### 2.1.3 Connection to the Swampland

The first observation is that some alterations of Eqs. (2.12) - (2.13) are clearly inconsistent from a stringy point of view, even if they may not appear fatal within the effective theory. Perhaps the simplest such class of modifications comprises changes in the structure of the axion kinetic term, modifying the large volume behaviour of the axion decay constant. With the original kinetic term

$$\mathcal{L} \supset e^{-\sqrt{\frac{8}{3}}\frac{\varphi}{M_P}} \partial_\mu a \partial^\mu a, \tag{2.14}$$

the axion decay constant scales as $f_a \sim M_P \mathcal{V}^{-\frac{2}{3}}$ and vanishes in the $\mathcal{V} \to \infty$ limit. However, were $f_a$ to instead remain constant or even diverge, this would be in stark contrast with the evidence for sub-Planckian decay constants within quantum gravity as well as known behaviour in string theory. Indeed, explicit examples in String Theory show control issues systematically occurring when $f_a \gg M_P$ (e.g see [93, 94, 67, 95]).

Furthermore, according to the axionic version of the Weak Gravity Conjecture [66](See the discussion surrounding Eq. 1.50), $f_a$ should satisfy the inequality

$$f_a S \lesssim M_P, \tag{2.15}$$

where $S$ is the action of the leading instanton depending on the axion and $S$ is required to be smaller than one in order for the single instanton approximation to be valid. Here, the action of a bulk D3-instanton is $S \sim \mathcal{V}^{2/3}$ and so Eq. (2.15) would be violated by $f_a \sim M_P$.

This inconsistent behaviour can, for example, be achieved by switching the sign of the exponential in Eq. (2.14) – corresponding to $f_a \sim M_P \mathcal{V}^{\frac{2}{3}}$ – or by including only a finite number of terms in its expansion. Such statements can now be translated into statements on the structure of the EFT Lagrangian; the above sign inversion, for instance, corresponds to an additional factor of $(-1)^n$ in Eq.(2.13). More generally, when expanding about any vacuum the sign of the linear coupling $\frac{\varphi}{M_P}\partial_\mu a \partial^\mu a$ is clearly equivalent to the sign of $\frac{\partial f_a(\varphi)}{\partial \varphi}$, so any requirement that $\frac{\partial f_a}{\partial \varphi} < 0$ in the asymptotic regime is equivalent to a negative sign for the latter coupling as well.

As another example, one might wonder whether potentials resembling LVS – but with a generic $\lambda$ – can be realized in string compactifications. One reason to suppose this is *not* possible is that, if $\lambda \leq \sqrt{6}$, the dependence on the volume is such that the potential grows parametrically faster than the string scale $M_s^4$, so $V/M_s^4 \to \infty$ as $\mathcal{V} \to \infty$ – a behaviour that looks problematic in a limit of weak coupling and large volumes. A value of $\lambda = \sqrt{6}$ corresponds to

$$V \propto \frac{M_P^4}{\mathcal{V}^2} \sim M_S^4 \tag{2.16}$$

since $M_S \sim \frac{M_P}{\sqrt{\mathcal{V}}}$.

In the other direction, for the case of $\lambda \gg 1$ the leading contribution to the potential would scale with a very high power of the volume. This also appears unstable against quantum loop corrections which would be expected to re-introduce a scaling with a lower power of the volume [177, 178]. One may therefore hope that all values of $\lambda$ outside a finite interval should lie in the Swampland, or even adopt the extreme point of view that only $\lambda = \sqrt{\frac{27}{2}}$ is allowed.

The above arguments then provide a motivation for a holographic approach. Within this holographic approach, there are two basic questions to be asked:

- Can one turn swampland constraints on the AdS Lagrangian into well-defined statements about CFT properties?
- Do swampland modifications to the AdS side clearly translate into violations of fundamental properties of the putative CFT dual?

Anticipating subsequent results, we can say that, although the first question seems to admit a positive response, the second question seems harder to answer. In particular, we will show how the above Swampland conditions translate into a statement on CFT anomalous dimensions which is surprisingly similar to known causality and unitarity constraints, but is nonetheless not known to be valid in general.

## 2.2 Holographic CFTs and Consistency Conditions

We now turn to a general discussion of holographic CFTs, and the form of constraints on them that can exist, before returning to applications to LVS.

### 2.2.1 Holographic CFTs: a primer

According to the AdS/CFT correspondence, discussed in Section 1.4, correlation functions in a conformal field theory can be calculated on the gravity side by evaluating boundary correlators involving the corresponding fields in AdS. If the gravity dual is weakly coupled, such computations can be carried out explicitly in perturbation theory, and the CFT is said to be holographic. From the point of view of string theory, this is the case when $R_{AdS} \gg \ell_S$ and $g_s \ll 1$, so quantum corrections (both in spacetime and on the worldsheet) can be neglected. According to the AdS/CFT dictionary the 't Hooft coupling $\lambda = g^2 N$ is mapped to $\lambda \sim (R/\ell_S)^d$ and so must be much larger than one in this limit, implying that holographic CFTs are always characterized by $N \gg 1$, or an equivalent expansion parameter when the theory does not admit an explicit large-$N$ description. With a slight abuse of notation (which conforms to what has become standard use in the literature), we shall always denote this expansion parameter as $1/N$, and operators will be referred to as single or double trace according to the scaling of their two-point function with $N$. In the LVS minimum, $R_{AdS} \sim \mathcal{V}^{3/2}$, and so the stabilised volume plays the role of the large $N$ parameter.

A second peculiarity of holographic CFTs is that they are characterized by a large gap in the spectrum of conformal dimensions, which are related to the AdS mass through the formula

$$\Delta(\Delta - d) = m^2 R_{AdS}^2 \tag{2.17}$$

already cited in the introduction. In holographic CFTs there is a large $\Delta_{Gap} \gg 1$ between the low-lying scalar, vector and graviton modes to higher-spin operators. For example, all the heavy stringy modes have $m \sim 1/\ell_S \gg 1/R_{AdS}$ and correspond to fields with large conformal dimensions in the weak coupling regime.

It is interesting to note that the converse of this statement has been conjectured to hold, i.e. that all large-$N$ CFTs with a gap should be characterized by a weakly coupled, local bulk dual [179, 180, 181]. In [179, 180], this was proven at order $1/N^2$ through a bijective mapping of solutions to the Bootstrap crossing equations to bulk interaction vertices for scalars. Incidentally, this shows that the requirement of crossing symmetry alone is not enough to constrain the form of the low-energy Lagrangians mentioned at the beginning. More recently, a version of this conjecture was proven with conformal bootstrap techniques, by bounding Wilson coefficients as a function of $\Delta_{\text{gap}}$ [182].

From a practical point of view, the spectrum of holographic CFTs at the first non-trivial order in $1/N^2$ includes a finite number of single-trace primaries $\mathcal{O}_1, \mathcal{O}_2, \ldots \mathcal{O}_m$ plus double-trace (and higher-trace) operators, which are built from any two (or more) of the $\mathcal{O}_i$. For scalars, the double-trace operators are schematically of the form

$$\mathcal{O}_i \Box^n \partial_{\mu_1} ... \partial_{\mu_\ell} \mathcal{O}_j, \tag{2.18}$$

and there is a single double-trace primary for each value of $n$ and $\ell$, denoted as $[\mathcal{O}_i \mathcal{O}_j]_{n,\ell}$. Their conformal dimension can be written as

$$\Delta_{n,\ell} = \Delta_i + \Delta_j + 2n + \ell + \gamma(n, \ell), \tag{2.19}$$

where the classical contribution $\Delta_i + \Delta_j + 2n + \ell$ can be split from the smaller anomalous dimension $\gamma(n, \ell)$ that is $\mathcal{O}(1/N^2)$. From an AdS perspective, the double trace operator corresponds to a 2-particle state in AdS and the anomalous dimension corresponds to its binding energy. Of course, in a full-fledged CFT scalars are not the whole story, and there will also be the stress-energy tensor plus the operators built by combining the stress tensor with other primaries.

### 2.2.2 The Bootstrap

The bootstrap program, originally dating to the early 1970s [183, 184, 185], rests on the philosophy that conformal symmetry alone is powerful and restrictive enough to impose significant constraints on the space of allowed theories, allowing interesting results to be derived with little external input other than the CFT axioms. On the technical side, a fundamental property is the existence of a convergent OPE expansion

$$\mathcal{O}_1(x_1)\mathcal{O}_2(x_2) = \sum_{k \text{ primary}} f_{12}^k C(x_{12}, \partial_{12})\mathcal{O}_k(x_2), \tag{2.20}$$

where the operator $C(x, \partial)$ is fully determined by conformal symmetry, and $f_{12k}$ is the only arbitrary coefficient appearing in the $\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_k \rangle$ three-point function, namely

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_k(x_3)\rangle = \frac{f_{12k}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_k}|x_{23}|^{\Delta_k+\Delta_2-\Delta_1}|x_{13}|^{\Delta_1+\Delta_k-\Delta_2}}. \tag{2.21}$$

When inserted inside a 4-point correlator, this generates an expansion in a universal basis of functions, the conformal blocks. For identical operators,

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{1 + \sum_{\Delta,\ell}\left(f_{\mathcal{O}\mathcal{O}}^{\Delta,\ell}\right)^2 G_{\Delta,\ell}(u,v)}{|x_{12}|^{2\Delta_1}|x_{34}|^{2\Delta_3}} \equiv \frac{1 + \mathcal{A}(u,v)}{|x_{12}|^{2\Delta_1}|x_{34}|^{2\Delta_3}}, \tag{2.22}$$

where $u$ and $v$ are the conformal cross ratios, defined as

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}. \tag{2.23}$$

Although they are completely fixed by conformal invariance, the explicit form of the conformal blocks is only known in even dimension, where they satisfy a separable differential equation.

Thus, all the dynamical information about the CFT is contained in the spectrum of the theory, given by the pairings $\Delta_i, \ell_i$, and the OPE coefficients $f_{12}^k$. In particular, with a 4-point function one can perform the contraction (A.5) in two inequivalent ways, to obtain the celebrated Bootstrap equation

$$\sum_{\Delta,\ell}\left(f_{\mathcal{O}\mathcal{O}}^{\Delta,\ell}\right)^2 \left(\frac{v^{\Delta_\phi}G_{\Delta,\ell}(u,v) - u^{\Delta_\phi}G_{\Delta,\ell}(v,u)}{u^{\Delta_\phi} - v^{\Delta_\phi}}\right) = 1. \tag{2.24}$$

In principle, with a holographic CFT it is possible to calculate any correlator with Witten diagrams in AdS, and then expand into conformal blocks to recover OPE coefficients and anomalous dimensions. However, this is hardly ever achievable in practice – computations are extremely long even for the simplest of diagrams and 3d conformal blocks are not known in a closed form. Furthermore, the full correlators often contain much more information than is needed, and one is left wondering whether it would be possible to skip some of the intermediate steps.

For this reason, bootstrap techniques often yield powerful insights, both in terms of uncovering general properties and also performing specific calculations

## 2.3 Calculational tools

### 2.3.1 Mellin Amplitudes

The S-matrix does not exist in AdS as any definition of physical asymptotic states is troublesome; the only real observables are correlators or functions thereof. Heuristically, this arises because AdS behaves like a system of "particles in a box", where interactions between different constituents cannot be switched off and there are no 'states at infinity'.

However, there does exist a representation of AdS correlators which exhibits striking similarities to flat space scattering amplitudes, and can be used to derive the latter in certain limits where the radius of AdS goes to infinity. These are the Mellin amplitudes.

Starting from an $n$-point correlator, the Mellin transform is defined as

$$A\left(x_i\right) \supset \left\langle \mathcal{O}_1\left(x_1\right) \ldots \ldots \mathcal{O}_n\left(x_4\right)\right\rangle_{\mathrm{c}} = \prod_{1 \leq i<j \leq n} \int_{-i \infty}^{+i \infty} \frac{\mathrm{d} \delta_{ij}}{2 \pi i} M(\delta_{ij}) \Gamma\left(\delta_{ij}\right)\left(x_{ij}^2\right)^{-\delta_{ij}}, \quad (2.25)$$

with the quantity $M(\delta_{ij})$ known as the Mellin amplitude [135, 186, 187]. In Eq. (2.25) the integrations should be carried out in such a way that the poles arising from the Mellin amplitude or a given gamma function all be placed on one side of the contour. Defining $\delta_{ii} = -\Delta_i$, one can show that conformal invariance implies

$$\sum_i \delta_{ij} = 0. \quad (2.26)$$

After imposing these constraints, there are $n(n-3)/2$ independent variables $\delta_{ij}$ for an $n$-point function, which is the same as the number of kinematic invariants for an $n$-particle amplitude.[1] It is then possible to introduce fictitious variables $p_i$ defined by

$$p_i \cdot p_j = \delta_{ij}, \qquad \sum_{i=1}^{n} p_i = 0, \tag{2.27}$$

representing the analogue of momentum conservation for conformal correlators. For 4-point correlators, it is then natural to use Mandelstam-like variables[2]

$$\begin{gathered} s = -(p_1+p_2)^2 = \Delta_1 + \Delta_2 - 2\delta_{12}, \qquad u = -(p_1+p_4)^2 = \Delta_1 + \Delta_4 - 2\delta_{14}, \\ t = -(p_1+p_3)^2 - \Delta_1 - \Delta_4 = \Delta_3 - \Delta_4 - 2\delta_{13}, \end{gathered} \tag{2.28}$$

which obey the relation

$$s + t + u = \Delta_2 + \Delta_3. \tag{2.29}$$

For a 4-point function, the reduced amplitude $\mathcal{A}(u,v)$ is defined as

$$A\left(x_i\right) = \frac{1}{\left(x_{12}^2\right)^{\frac{\Delta_1+\Delta_2}{2}} \left(x_{34}^2\right)^{\frac{\Delta_3+\Delta_4}{2}}} \left(\frac{x_{24}^2}{x_{14}^2}\right)^{\frac{\Delta_1-\Delta_2}{2}} \left(\frac{x_{14}^2}{x_{13}^2}\right)^{\frac{\Delta_3-\Delta_4}{2}} \mathcal{A}(u,v), \tag{2.30}$$

and it can be expressed as a function of the Mellin amplitude

$$\begin{aligned} \mathcal{A}(u,v) = \int_{-i\infty}^{i\infty} \frac{dtds}{(4\pi i)^2} M(s,t) u^{s/2} v^{-(s+t)/2} \Gamma\left(\frac{\Delta_1+\Delta_2-s}{2}\right) \Gamma\left(\frac{\Delta_3+\Delta_4-s}{2}\right) \\ \Gamma\left(\frac{\Delta_{34}-t}{2}\right) \Gamma\left(\frac{-\Delta_{12}-t}{2}\right) \Gamma\left(\frac{t+s}{2}\right) \Gamma\left(\frac{t+s+\Delta_{12}-\Delta_{34}}{2}\right). \end{aligned} \tag{2.31}$$

Further evidence for the analogy comes from the explicit computation of Mellin amplitudes, at least in the simple cases where it is possible to do so. For a scalar contact interaction between $n$ different fields

$$\mathcal{L}_{int} = g\varphi_1\varphi_2...\varphi_n, \tag{2.32}$$

[1] In both cases, this counting is modified for $n > d+2$.

[2] Here the definition of $t$ has a constant shift with respect to the canonical one only to simplify some formulas involving Mack polynomials.

the Mellin amplitude is a constant,

$$M(\delta_{ij}) = \frac{g\pi^{\frac{d}{2}}}{2}\Gamma\Big(\frac{\sum \Delta_i - d}{2}\Big)\prod_{i=1}^{n}\frac{1}{\Gamma(\Delta_i)}. \tag{2.33}$$

If derivatives are added to the vertex, as in

$$\mathcal{L}_{int} = g\nabla...\nabla\varphi_1\nabla...\nabla\varphi_2...\nabla...\nabla\varphi_n, \tag{2.34}$$

then the Mellin amplitude picks up powers of the fictional momenta corresponding to the number of derivatives acting on the field [126]

$$M(\delta_{ij}) = \frac{g\pi^{\frac{d}{2}}}{2}\Gamma\Big(\frac{\sum \Delta_i - d + 2N}{2}\Big)\prod_{i=1}^{n}\frac{1}{\Gamma(\Delta_i + \beta_i)}\prod_{i<j}^{n}(-2\delta_{ij})^{\alpha_{ij}} + ..., \tag{2.35}$$

where $\alpha_{ij}$ is the number of derivatives acting on $\varphi_i$ and $\varphi_j$, $\beta_i = \sum_{j\neq i}\alpha_{ij}$ and $2N$ is the total number of derivatives. Going one step further, a diagram describing the exchange of a bulk scalar dual to a single trace operator $\mathcal{O}$ of dimension $\Delta_{\mathcal{O}}$ and spin $\ell_{\mathcal{O}}$, say in $s$-channel, has the form [188]

$$M(s,t) = f_{12\mathcal{O}}f_{\mathcal{O}34}\sum_{m}\frac{\mathcal{Q}_{\ell_{\mathcal{O}},m}(t)}{s - \Delta_{\mathcal{O}} + \ell_{\mathcal{O}} - 2m}, \tag{2.36}$$

where the $\mathcal{Q}_{\ell_{\mathcal{O}},m}(s,t)$, known as Mack Polynomials, are completely determined by conformal symmetry. Introducing the Pochhammer symbol

$$(a)_m = \frac{\Gamma(a+m)}{\Gamma(a)} = a(a+1)...(a+m-1), \tag{2.37}$$

their explicit form can be conveniently parametrized by the new polynomials $Q_{J,m}(s)$ satisfying [189]

$$\begin{aligned}\mathcal{Q}_{J,m}(s) = &-\frac{2\Gamma(\Delta+J)(\Delta-1)_J}{4^J\Gamma\left(\frac{\Delta+J+\Delta_{12}}{2}\right)\Gamma\left(\frac{\Delta+J-\Delta_{12}}{2}\right)\Gamma\left(\frac{\Delta+J+\Delta_{34}}{2}\right)\Gamma\left(\frac{\Delta+J-\Delta_{34}}{2}\right)}\\ &\frac{Q_{J,m}(s)}{m!(\Delta-h+1)_m\Gamma\left(\frac{\Delta_1+\Delta_2-\Delta+J}{2}-m\right)\Gamma\left(\frac{\Delta_3+\Delta_4-\Delta+J}{2}-m\right)}.\end{aligned} \tag{2.38}$$

Imposing the normalization $Q_{J,m}(s) = s^J + \mathcal{O}(s^{J-1})$, these are given by

$$Q_{J,0}(s) = \frac{2^J \left(\frac{\Delta_{12}+\tau}{2}\right)_J \left(\frac{\Delta_{34}+\tau}{2}\right)_J}{(\tau+J-1)_J} {}_3F_2\left(-J, J+\tau-1, \frac{\Delta_{34}-s}{2}; \frac{\tau+\Delta_{12}}{2}, \frac{\tau+\Delta_{34}}{2}; 1\right). \tag{2.39}$$

The appearance of poles in the Mandelstam-like variables corresponding to the exchange of operators of different spin is reminiscent of flat space scattering amplitudes. Quite remarkably, this is not only a feature of weakly coupled gravity duals, but instead a generic property valid for all conformal theories. Indeed, given the OPE expansion

$$\mathcal{O}_1(x_1)\,\mathcal{O}_1(x_2) = \sum_k f_{12k} \left(x_{12}^2\right)^{\frac{\Delta_k-\Delta_1-\Delta_2}{2}} \left[\mathcal{O}_k(x_2) + c x_{12}^2 \partial^2 \mathcal{O}_k(x_2) + \ldots\right], \tag{2.40}$$

in the OPE limit $x_{12}^2 \to 0$ one can perform the integral over $\delta_{12}$ by summing over the corresponding poles, denoted by a tilde.

$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_1(x_2)\ldots\rangle = \sum_{\tilde{\delta}_{12}} \left(x_{12}^2\right)^{-\tilde{\delta}_{12}} \prod_{\substack{1<i<j,\\ 2<j}} \int \frac{\mathrm{d}\delta_{ij}}{2\pi i} \mathrm{Res}_{\tilde{\delta}_{12}} M(\delta_{ij})\,\Gamma(\delta_{ij}) \left(x_{ij}^2\right)^{-\delta_{ij}}, \tag{2.41}$$

Matching terms in an expansion in inverse powers of $x_{12}^2$ it is then possible to recover poles at

$$s = \Delta_{\mathcal{O}} + 2m, \qquad m = 1, 2\ldots, \tag{2.42}$$

with a residual that is proportional to $f_{12\mathcal{O}}$. A careful derivation of factorization formulas for Mellin amplitudes can be found in [190].

### 2.3.2 Applications: a formula for $\gamma(0,\ell)$

As an application of this formalism, we now derive a tree-level formula to compute the $\gamma(0,\ell)$ anomalous dimensions of a holographic CFT in terms of the associated Mellin amplitude, used for the first time in [191] for the study of gravitational exchange diagrams.

Given two single trace primaries $\mathcal{O}_1, \mathcal{O}_3$ of dimensions $\Delta_1$ and $\Delta_3$, the anomalous

dimensions of the double trace operators $[\mathcal{O}_1\mathcal{O}_3]_{0,\ell}$ are given by

$$\begin{aligned}\gamma(0,\ell) = -\int_{-i\infty}^{+i\infty}\frac{ds}{2\pi i}M(s,0)\ {}_3F_2(-\ell,\Delta_1+\Delta_3+\ell-1,\frac{s}{2};\Delta_1,\Delta_3;1)\\ \times\Gamma\Big(\Delta_1-\frac{s}{2}\Big)\Gamma\Big(\Delta_3-\frac{s}{2}\Big)\Gamma\Big(\frac{s}{2}\Big)^2,\end{aligned} \tag{2.43}$$

where $M(s,t)$ is the Mellin amplitude corresponding to the correlator

$$\langle\mathcal{O}_1(x_1)\mathcal{O}_1(x_2)\mathcal{O}_3(x_3)\mathcal{O}_3(x_4)\rangle. \tag{2.44}$$

This expression only includes the analytic contribution in the spin arising from exchange diagrams, and is not sensitive to the presence of contact terms.[3]

To prove it, we compare the conformal block decomposition of the correlator with an appropriate expansion of the Mellin amplitude. As the double trace operators above are only contained in the OPE of $\mathcal{O}_1$ with $\mathcal{O}_3$, the conformal block decomposition contains $\gamma(0,\ell)$ at order $\frac{1}{N^2}$ only in the mixed channels. If we denote by $\mathcal{A}'(u,v)$ the stripped correlator associated to $\langle\mathcal{O}_3(x_1)\mathcal{O}_1(x_2)\mathcal{O}_1(x_3)\mathcal{O}_3(x_4)\rangle$, then

$$\mathcal{A}(u,v)v^{\frac{\Delta_1+\Delta_3}{2}} = \mathcal{A}'(v,u)u^{2\Delta_1}, \tag{2.45}$$

and we can decompose $\mathcal{A}'(v,u)$ in s-channel conformal blocks.

$$\mathcal{A}(u,v) \supset \left(\frac{u}{v}\right)^{\frac{\Delta_1+\Delta_3}{2}} u^{\frac{\Delta_1-\Delta_3}{2}}\sum_{\Delta,\ell}\left(f^{1,3}_{[13]_{0,\ell}}\right)^2 G(v,u)^{\Delta_{31},\Delta_{13}}_{\Delta_1+\Delta_3+2n+\ell+\frac{\gamma(n,\ell)}{N^2},\ell} \tag{2.46}$$

where the $f^{1,3}_{[13]_{0,\ell}}$ are OPE coefficients of $[\mathcal{O}_1\mathcal{O}_3]_{0,\ell}$ with $\mathcal{O}_1$ and $\mathcal{O}_3$. To isolate the contribution of $\gamma(0,\ell)$,

$$G(v,u)^{\Delta_{31},\Delta_{13}}_{\Delta_1+\Delta_3+2n+\ell+\frac{\gamma(n,\ell)}{N^2},\ell} = G(v,u)^{\Delta_{31},\Delta_{13}}_{\Delta_1+\Delta_3+2n+\ell,\ell} + \frac{\gamma(n,\ell)}{N^2}\partial_\Delta G^{\Delta_{31},\Delta_{13}}_{\Delta_1+\Delta_3+2n+\ell,\ell}. \tag{2.47}$$

Since conformal blocks can always be expanded as

$$G^{\{\Delta_i\}}_{\Delta,\ell}(u,v) = u^{\frac{\Delta-\ell}{2}}\sum_{m=0}^{\infty} g^{\Delta,\ell}_{\{\Delta_i\},m}(v)u^m, \tag{2.48}$$

[3] See 3.1.3.

the derivatives translate into the appearance of logarithms which, by analyticity of the conformal blocks, cannot be generated anywhere else in the sum. Therefore, the only log-singular term multiplying the lowest power of $u$ is

$$\mathcal{A}(u,v) \supset \frac{u^{\Delta_1}\log(v)}{2}\sum_{\ell}\gamma(0,\ell)\Big(f^{1,3}_{[13]_{0,\ell}}\Big)^2 g^{\Delta,\ell}_{\{\Delta_i\},0}(u). \tag{2.49}$$

At this point it is convenient to introduce an integral representation [189] for $g_0$, in terms of the modified Mack polynomials defined in (2.39) :

$$g^{\Delta,\ell}_{\{\Delta_i\},0}(u) = \alpha(\Delta_i,\ell)u^{-\Delta_1}\int_{-i\infty}^{+i\infty}\frac{ds}{8\pi i}u^{-\frac{s}{2}}Q_{J,0}(s-\Delta_1+\Delta_3)\Gamma\Big(-\frac{s}{2}\Big)^2\Gamma\Big(\Delta_1+\frac{s}{2}\Big)\Gamma\left(\Delta_3+\frac{s}{2}\right). \tag{2.50}$$

Inserting this last equation into (2.49) results in an expression that is highly reminiscent of Mellin amplitudes, to which we now turn our attention.

In fact, the same quantity can be rewritten as

$$\mathcal{A}(u,v) = \int_{-i\infty}^{i\infty}\frac{dtds}{(4\pi i)^2}M(s,t)u^{s/2}v^{t/2}\Gamma\Big(\Delta_1-\frac{s}{2}\Big)\Gamma\Big(\Delta_3-\frac{s}{2}\Big)\Gamma\Big(\frac{s+t}{2}\Big)^2\Gamma\Big(-\frac{t}{2}\Big)^2, \tag{2.51}$$

where we have used the reflection property $M(s,t) = M(s,-s-t)$ [4] to switch integration variables with respect to (2.31). Since the OPE limit in the crossed channel corresponds to $v \to 0$, one can integrate with respect to $t$ by closing the contour on the right-hand plane. On the positive axis, there are double poles located at integer values of $t = 2n$. Using

$$\mathrm{Res}\Big(\Gamma\Big(-\frac{t}{2}\Big)^2 f(t)\Big)_{t=2n} = \frac{4}{(n!)^2}f'(2n) + \frac{2(-1)^n C_n}{n!}f(2n), \tag{2.52}$$

where the form of the coefficients $C_n$ is irrelevant for our purposes. We can then isolate the logarithmic contribution

$$\mathcal{A}(u,v) = \frac{\log v}{2}\sum_{n=0}^{\infty}\frac{v^n}{(n!)^2}\int_{-i\infty}^{+i\infty}\frac{ds}{2\pi i}M(s,2n)u^{s/2}\Gamma\Big(\Delta_1-\frac{s}{2}\Big)\Gamma\Big(\Delta_3-\frac{s}{2}\Big)\Gamma\left(\frac{s+2n}{2}\right)^2. \tag{2.53}$$

Since the integrals in (2.49) and (2.53) are equal for any value of $u$, we can equate

[4] A consequence of $1 \leftrightarrow 2/3 \leftrightarrow 4$ symmetry.

the integrands[5] to obtain

$$\sum_{\ell=0}^{\infty}\gamma(0,\ell)\Big(f^{1,3}_{[13]_{0,\ell}}\Big)^2 Q_{\ell,0}(-s+\Delta_1-\Delta_3)\alpha(\Delta_i,\ell)=-4M(s,0). \tag{2.54}$$

Incidentally, in the case of a polynomial amplitude (dual to contact vertices on AdS) Eq. (2.54) becomes a finite-dimensional linear system for the $\gamma(0,\ell)$, correctly reproducing some results contained in [179] (and not captured by (2.43)). Finally, it is possible to isolate the individual $\gamma(0,\ell)$ through a projection on the corresponding Mack polynomials, by integrating (2.54) with respect to the measure in (2.53). This is because they satisfy the orthogonality relation [189]

$$\begin{aligned}&\int_{-i\infty}^{i\infty}\frac{ds}{4\pi i}Q_{J,0}(s)Q_{J',0}(s)\Gamma\left(\frac{\Delta_{34}-s}{2}\right)\Gamma\left(\frac{-\Delta_{12}-s}{2}\right)\Gamma\left(\frac{\tau+s}{2}\right)\times\\ &\times\Gamma\left(\frac{\tau+s+\Delta_{12}-\Delta_{34}}{2}\right)=\delta_{J,J'}\frac{(-4)^J\Gamma\Big(\Delta_1+J\Big)^2\Gamma\Big(\Delta_3+J\Big)^2 J!}{\Gamma(\Delta_1+\Delta_3+2J)(\Delta_1+\Delta_3+J-1)_J},\end{aligned} \tag{2.55}$$

with the choice of normalization in (2.39) The result then follows from the generalized free theory coefficients in [192]

$$\Big(f^{1,3}_{[13]_{0,J}}\Big)^2=\frac{(-1)^J(\Delta_1)_J(\Delta_3)_J}{(\Delta_1+\Delta_3+J-1)_J J!} \tag{2.56}$$

and the normalization factor

$$\alpha(\Delta_i,J)=\frac{2\Gamma(\Delta_1+\Delta_3+2J)(\Delta_1+\Delta_3+J-1)_J}{4^J\Gamma\Big(\Delta_1+J\Big)^2\Gamma\Big(\Delta_3+J\Big)^2}. \tag{2.57}$$

### 2.3.3 Anomalous Dimensions at Large Spin

The purpose of this section is to state explicitly how large spin anomalous dimensions of the form $\gamma(0,\ell)$ can be calculated from the effective action on AdS, using results from the previous section. Using (2.43), we will later see (Section 3.1.3) that only exchange diagrams contribute to the sum at leading order in the $1/\ell$ expansion, and in particular only in the t-channel. Therefore, the final result only depends on certain cubic couplings. Contact terms, for example, contribute up to a finite spin $\ell_f=N_\partial/2$,

[5] Up to terms which are not compatible with the structure of a Mellin amplitude $M(s,0)$ in a large-N theory with a weakly coupled dual.

where $N_\partial$ is equal to number of derivatives it contains. In particular, the exchange of a given primary operator $\mathcal{O}$ induces an anomalous dimension for the mixed double trace operators $[\mathcal{O}_1\mathcal{O}_2]_{0,\ell}$ of

$$\gamma_{12}^{\text{mix}}(0,\ell) \underset{\ell\gg 1}{\simeq} -\frac{2\Gamma(\Delta_{\mathcal{O}})}{\Gamma(\frac{\Delta_{\mathcal{O}}}{2})^2}\frac{\Gamma(\Delta_1)\Gamma(\Delta_2)}{\Gamma(\Delta_1-\frac{\Delta_{\mathcal{O}}}{2})\Gamma(\Delta_2-\frac{\Delta_{\mathcal{O}}}{2})}\frac{C_{11\mathcal{O}}C_{22\mathcal{O}}}{\ell^{\Delta_{\mathcal{O}}}}, \tag{2.58}$$

where $C_{11\mathcal{O}}, C_{22\mathcal{O}}$ are the coefficients of the three-point functions $\langle\mathcal{O}_{1(2)}\mathcal{O}_{1(2)}\mathcal{O}\rangle$. Although the same result could have been more easily obtained by solving the bootstrap crossing equation at leading order [193], we emphasize that formula (2.43) also provides a systematic way to expand the answer in subsequent powers of $1/\ell$, and eventually resum the series to obtain the finite spin result. It also allows one to keep the analogy with positivity bounds from flat space more manifest, given the similarity of Mellin amplitudes to scattering amplitudes in flat spacetime.

The three-point function coefficients in (2.58) can be computed by evaluating the corresponding Witten diagram in AdS, which is most easily performed in the embedding formalism [135, 126]. Given the particular structure of the Lagrangians we are going to consider, the cubic coupling will always be of the general form

$$\mathcal{L} \supset c_1\varphi_A^2\varphi_B + c_2\partial_\mu\varphi_C\partial_\mu\varphi_C\varphi_D. \tag{2.59}$$

Then, the corresponding coefficients take the form [24]

$$C_{AAB} = c_1(2+4\delta_{AB})\frac{\pi^{d/2}\Gamma\left(\frac{2\Delta_A+\Delta_B-d}{2}\right)}{\Gamma(\Delta_A)^2\Gamma(\Delta_B)}. \tag{2.60}$$

$$C_{CCD} = c_2\frac{\pi^{d/2}\Gamma\left(\frac{2\Delta_C+\Delta_D-d}{2}\right)}{\Gamma(\Delta_C)^2\Gamma(\Delta_D)}(\Delta_C^2-2\Delta_C-\Delta_D+3). \tag{2.61}$$

### 2.3.4 A Special Case: Degenerate Conformal Dimensions

When a CFT contains two primaries with integer spaced conformal dimensions, the spectra of the corresponding double trace operators overlap, and one needs to be more careful in the computation of the anomalous dimensions. To be explicit, let us consider two scalars $\mathcal{O}_1$ and $\mathcal{O}_2$ with scaling dimensions $\Delta_1$, $\Delta_2$. At zeroth order in the large-$N$ expansion, the double trace operators $[\mathcal{O}_1\mathcal{O}_1]_{n_1,\ell}$ and $[\mathcal{O}_2\mathcal{O}_2]_{n_2,\ell}$ will be

degenerate if the dimensions satisfy

$$n_1 - n_2 = \Delta_2 - \Delta_1, \qquad n_1, n_2 \in \mathbb{N} \tag{2.62}$$

and analogous conclusions hold with the mixed operators $[\mathcal{O}_1\mathcal{O}_2]_{n,\ell}$ too. While from an AdS perspective Eq. (2.62) would seemingly correspond to a fine-tuning conspiracy on the value of the masses,[6] there is one notable exception where this condition naturally occurs, namely when two fields are massless and $\Delta_1 = \Delta_2 = d$. One such example is provided by the case where the potential has flat directions corresponding to certain moduli or, alternatively, for axions protected by shift-invariance. In large volume models, the latter scenario is quite robust, since any additional non-perturbative effects lifting the axion mass are suppressed as $m \propto e^{-b\mathcal{V}^{2/3}}$, which does not appear at any order in a $1/N$ expansion.[7]

Of course, in any realistic stringy example – such as those we will consider in sections 3.4 and 4.1 – particle masses (and hence the dimensions of primaries) may receive higher order contributions that break the degeneracy. Since we are working in a perturbative framework, this is only relevant if the corrections to the conformal dimensions dominate parametrically over the anomalous dimensions, i.e. they have to be strictly larger than $\mathcal{O}(1/N^2)$. If this happens for $\Delta_1 \neq \Delta_2$, the operator mixing is effectively eliminated, and anomalous dimensions can be calculated as usual. On the other hand, if $\Delta_1 = \Delta_2$ the degeneracy is still resolved in principle (unless the symmetries of the problem conspire to leave the degeneracy still unbroken at this subleading order), but mixing effects can appear between $\mathcal{O}_1$ and $\mathcal{O}_2$. Indeed, the true primaries are the eigenstates of the degeneracy-breaking perturbation, which can be rotated from the original fields by an unknown angle.[8] Therefore, the form of the perturbation becomes necessary to calculate the anomalous dimensions even at leading order, and in such cases we will not be able to provide an expression for $\gamma(0,\ell)$. Otherwise, the operators should effectively be considered degenerate, and it will be necessary to use the techniques outlined in this section.

For concreteness, let us focus on the case where $\Delta_1 = \Delta_2 \equiv \Delta$ and no degeneracy breaking is present. For each given pairing $(n,\ell)$, there is a three-dimensional

[6] In the type IIA models of section 4.1 we will surprisingly find examples where this condition is satisfied. However, we expect it to be broken by subleading volume effects.

[7] Since the volume plays the role of large-$N$ in such models.

[8] Crucially, this angle remains finite if the magnitude of the perturbation approaches zero.

degenerate space, corresponding to the operators

$$[\mathcal{O}_1\mathcal{O}_1]_{n,\ell}\,;\;\; [\mathcal{O}_2\mathcal{O}_2]_{n,\ell}\,;\;\; [\mathcal{O}_1\mathcal{O}_2]_{n,\ell}. \tag{2.63}$$

They will be denoted by the double indices $[ij]$, taking values in $[11], [22], [12]$. In this subspace, the dilatation operator is diagonal only at order zero in the $1/N$ expansion, and the scaling dimensions are given by its eigenvalues: this is totally analogous to what happens in standard QM perturbation theory. In the approach of [137], anomalous dimensions are calculated by applying QM perturbation theory to the dilatation operator, and in a non-degenerate system this amounts to evaluating the expectation value of the interaction term $\Delta\hat{D}$ on a given state. In a degenerate system, one instead has to diagonalize

$$\bar{\gamma}_{[ij]\,[kl]}(n,\ell) = {}_{ij}\langle n,\ell|\,\Delta\hat{D}\,|n,\ell\rangle_{kl} \tag{2.64}$$

where $|n,\ell\rangle_{ij}$ is the state created by acting with $[\mathcal{O}_i\mathcal{O}_j]_{n,\ell}(0)$ on the vacuum, and the bar is used to distinguish between the matrix elements and the actual anomalous dimensions, given by the eigenvalues. The eigenvectors of the matrix determine the basis of double trace operators in which the dilatation operator is diagonal at the first non-trivial order in $1/N$, as a linear combination of the three operators in (2.63).

Although our approach to the evaluation of large spin anomalous dimensions is technically (and conceptually) different, anomalous dimensions can still be obtained as the eigenvalues of an analogous mixing matrix $\tilde{\gamma}_{[ij]\,[kl]}(0,\ell)$. In particular, we show in Appendix A that the matrix elements represent the "effective" anomalous dimension extracted from the mixed correlators

$$\langle\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k\mathcal{O}_l\rangle \supset \frac{f_0(n,\ell)^2\tilde{\gamma}_{[ij][kl]}(n,\ell)}{N^2}\partial_\Delta G_{2\Delta+2n+\ell,\ell}, \tag{2.65}$$

where the $f_0(0,\ell)$ are the generalized free theory OPE coefficients. They can be calculated with the analogue of Eq.(2.58) for the Mellin amplitude corresponding to the $\langle\mathcal{O}_i\mathcal{O}_k\mathcal{O}_j\mathcal{O}_l\rangle$ correlator.

If we assume that at least one of the two primaries is odd with respect to a $\mathbb{Z}_2$ symmetry, as is the case for operators dual to an axion on AdS, the system becomes two-dimensional. This is the only case we will encounter in practice, as the degeneracy will always be broken in other systems. In this case, only the $[\mathcal{O}_1\mathcal{O}_1]^i_{n,\ell}$

and $[\mathcal{O}_2\mathcal{O}_2]^i_{n,\ell}$ operators have the same transformation property under the discrete symmetry and can mix amongst themselves. In this particular case, the eigenvalues take the simplified form

$$\gamma_\pm = \frac{\tilde{\gamma}_{[11][11]} + \tilde{\gamma}_{[22][22]} \pm \sqrt{(\tilde{\gamma}_{[11][11]} - \tilde{\gamma}_{[22][22]})^2 + 4\tilde{\gamma}^2_{[11][22]}}}{2}, \tag{2.66}$$

where the indices $n, \ell$ have been suppressed for simplicity. An interesting consequence of the modified formula (2.66) is that one of the anomalous dimensions can apparently be positive,[9] when $\tilde{\gamma}^2_{[11][22]} > \tilde{\gamma}_{[11][11]}\tilde{\gamma}_{[22][22]}$. Even though we are dealing with identical operators, there does not seem any reason why this behaviour should be inconsistent. The reason is that in the OPE of two identical primaries, only the lowest twist operators have to obey a convexity condition (implying negativity) - therefore it is enough to have just one double trace operator with negative anomalous dimension appearing for every value of $\ell$. For instance, the OPE of $\mathcal{O}_1$ with itself will contain two classes of double trace operators, with positive and negative anomalous dimensions respectively. The lowest twist operators will be those belonging to the latter class, and these satisfy the required condition irrespective of the existence of those with a positive anomalous dimension.

## 2.4 Positivity bounds

In Section 2.1.3, we saw that sign changes in the effective field theory Lagrangians of string vacua are able to move a theory in or out of the swampland. This is reminiscent of positivity bounds in QFTs deriving from unitarity and analyticity. For our purposes, we will mostly be concerned with two specific realizations of this idea:

- S-Matrix positivity bounds, derived in [74]. These crucially rely on the assumption of analyticity[10] in the kinematic variables, an idea that is closely related to causality.

- Constraints on the sign of the anomalous dimensions in Lorentzian CFTs, such as those derived in [35, 194, 195].

[9] Since $\tilde{\gamma}_{[11][11]}$ and $\tilde{\gamma}_{[22][22]}$ will still be positive by construction.

[10] excluding branch cuts on the real axis and isolated singularities, as is usual in physics.

These results often trace back to an application of the optical theorem, which relates a generic amplitude to an integral over intermediate states. In its most general form, it can be stated as [196]

$$\mathcal{A}(i \to f) - \mathcal{A}^*(f \to i) = (2\pi)^4 \delta^{(4)}(\sum_{j \in I} p_j - P_X) \sum_X \int \mathrm{d}\Pi_X \mathcal{A}(i \to X) \mathcal{A}^*(X \to f). \tag{2.67}$$

In the case of elastic scattering (i.e when the initial and final states coincide), the LHS of (2.67) reduces to the imaginary part of the amplitude, while the RHS becomes manifestly positive: it is this sign that acts as a source for the positivity bounds. If one further specializes to two-particle states, the RHS becomes the total cross section for the process under consideration.

### 2.4.1 Causality and analyticity of scattering amplitudes

It has been known for some time that there are non-trivial constraints arising from the combination of causality and S-matrix analyticity for EFTs in flat space, but only recently this program has become more systematic. Starting from [74], it was shown that apparently healthy low energy theories cannot be UV completed into local quantum field theory or perturbative string theory if the signs of certain irrelevant operators are chosen appropriately. For simplicity, we only examine results for scalars, although there exist generalizations for non-trivial spins as well.

The simplest example is the theory of a shift symmetric, massless scalar $\pi$, which exemplifies the prototypical Nambu-Goldstone boson. The effective Lagrangian describing its self-interactions has the form

$$\mathcal{L} = (\partial\pi)^2 + a\frac{(\partial\pi)^2 \Box\pi}{\Lambda^3} + c\frac{(\partial\pi)^4}{\Lambda^4} + \dots \tag{2.68}$$

where the coefficients of each term are naively unconstrained.[11] Then, assuming the theory admits an expansion in a weak coupling $g$, it can be shown that the forward scattering amplitude $\mathcal{M}(s) = \mathcal{M}(s, t \to 0)$ admits a positive expansion, i.e. can be expanded in powers of $s$ as

$$\mathcal{A}_{\text{tree}}(s) = g \sum_{n=1}^{\infty} c_n \left(\frac{s^2}{\Lambda^4}\right)^n, \tag{2.69}$$

[11] Although in specific cases their magnitude might be estimated, for example using Naive Dimensional Analysis (NDA).

in such a way that the coefficients always satisfy $c_n > 0$. This conclusion can be reached order by order by evaluating the complex contour integrals

$$I_n = \oint_\gamma \frac{ds}{2\pi i} \frac{\mathcal{M}(s)}{s^{2n+1}} \tag{2.70}$$

along an infinite semi-circle lying on the axis, exploiting the analyticity of $\mathcal{M}(s)$. At this order in the coupling there can be no cuts in the amplitude, and all the poles give a positive contribution due to the Optical Theorem. To finish the argument, the Froissart bound $\mathcal{M}(s) < s \ln^2 s$ is used to kill the contribution at infinity for $n \geq 1$.

From an IR perspective, these bounds can also be understood to be closely connected with causality, since the wrong signs would give faster than light propagation of fluctuations in the field over non-trivial backgrounds. This last aspect is best illustrated through a simple application: according to (2.69), a Lagrangian of the form

$$\mathcal{L} = (\partial\pi)^2 + \mu \frac{(\partial\pi)^4}{\Lambda^4} \tag{2.71}$$

requires $\mu > 0$, exactly as in Eq. (2.79). If one quantizes over a non-zero background $\partial_\mu \pi = C_\mu$, with $C^2 \ll \Lambda^4$, the linearised equations of motion imply the following dispersion relation for small excitations:

$$k^2 - 4\mu \frac{(C \cdot k)^2}{\Lambda^4} = 0, \tag{2.72}$$

which clearly admits super-luminal modes for a negative $\mu$.

As a crucial remark, let us mention that there is one caveat to (2.69), which amounts to the requirement that $\Lambda \ll M_P$. The reason is that if gravity is not weak enough (compared to the irrelevant operators) to be integrated out, any scattering amplitude will receive the universal contribution coming from the tree-level t-channel exchange

$$\mathcal{M}(s,t) \propto G_N \frac{s^2}{t} \tag{2.73}$$

which diverges in the forward limit, invalidating the above discussion. From the point of view of causality, this happens because gravity bends all trajectories inside the lightcone [74].

Moving away from the forward scattering limit, more complete sets of constraints have been worked out for both scalars [197, 198, 199] and particles with spin [200, 201]. Another development has consisted in the discovery of new, non linear constraints

delineating an 'EFT-hedron' [202], enabling the formulation of two-sided bounds on Wilson coefficients [203, 204]. These ideas have also found a variety of different practical applications. Just to name a few examples, positivity has been used to derive constraints on higher order terms in the Einstein action [205], single-field inflation [206], massive gravity [207], in the SM Effective Field Theory [208, 209, 210] and pions in large-$N$ QCD [211]. An exhaustive list is beyond the scope of the thesis, so we refer to [212] for a complete review. In a Swampland context, they have been used to investigate both the original [77, 213] and axionic [214] versions of the Weak Gravity Conjecture, as well as EFTs which include gravity [215].

### 2.4.2 Positivity in CFTs

In Conformal Field Theories, the idea of scattering is ill-defined, and so it is not straightforward to make contact with the usual implications of unitarity stemming from the Optical Theorem. However, an ingenious way to circumvent the problem (first devised in [35, 36]) is to modify a CFT by a relevant perturbation that causes it to flow to a gapped phase in the infrared. In particular, the authors considered elastic scattering between the lowest-mass state in the theory, which now has a mass gap, and an arbitrary external operator $\mathcal{O}$ through a space-like virtual exchange. This procedure is in close analogy with Deep Inelastic Scattering (DIS) processes used to study the structure of hadrons, by bombarding them with energetic light leptons, and can thus be referred to as a DIS Gedanken experiment.

With the additional assumption of polynomial boundedness on off-shell amplitudes[12] and exploiting the positivity of the total cross-section, it was shown in [35] that the minimal twist operators appearing in the OPE of $\mathcal{O}$ with its adjoint have to obey the convexity property

$$\frac{\tau^*(\ell_3)-\tau^*(\ell_1)}{\ell_3-\ell_1} \leq \frac{\tau^*(\ell_2)-\tau^*(\ell_1)}{\ell_2-\ell_1} \qquad \text{for} \quad \ell_1<\ell_2<\ell_3. \tag{2.74}$$

Here the twist of an operator is defined as $\tau = \Delta - \ell$, and the relevance lies in the fact that the Lorentzian OPE is dominated in the OPE limit by low twist operators. More precisely, the $\tau^*(\ell)$ in (2.74) is the twist of the lowest dimension operator of spin $\ell$ appearing in the OPE of a given $\mathcal{O}$ with itself, and as such is called a minimal twist operator. This property holds only if $\ell$ is higher than a certain critical spin

[12] Which is not guaranteed by the Froissart bound.

$\ell \geq \ell_c$, which is determined by the leading exponent in the (polynomial) high energy limit of some amplitude, and is therefore not calculable a priori within this approach. However, in [216], the authors used the Lorentzian OPE inversion formula of [217] to extend these relations for any continuous value of the spin $\ell > 1$, without having to resort to any of the assumptions cited above.

The lower bound is in accordance with practical examples, where the theorem is found to hold for $\ell_c \geq 2$, and usually satisfied by the stress-energy tensor – which has the lowest twist possibly allowed by unitarity, $\tau = d-2$. More generally, the minimal twist operators that saturate the bounds for an arbitrary spin $\ell$ are appropriate combinations involving stress-energy tensors. A notable exception is when gravitational interactions are very suppressed with respect to other couplings in the theory – for example if the AdS dual contains interactions suppressed by a scale $\Lambda \ll M_P$ and gravity can be integrated out. In a holographic CFT of this kind, one can turn (2.74) into

$$\frac{\gamma(0,\ell_3)-\gamma(0,\ell_1)}{\ell_3-\ell_1} \leq \frac{\gamma(0,\ell_2)-\gamma(0,\ell_1)}{\ell_2-\ell_1} \qquad \text{for} \quad \ell_1 < \ell_2 < \ell_3, \tag{2.75}$$

where $\gamma(n,\ell)$ is the anomalous dimension of the double trace operators made out of the lowest dimension scalar in the theory. An immediate consequence of the convexity of Eq. (2.75) is that, at least asymptotically, the above anomalous dimensions must be negative. In the context of QCD similar negativity properties are a well-known fact, going by the name of the Nachtmann theorem [218].

Recently, a generalization of (2.74) was proven in [195], allowing one to make statements of OPEs involving different operators. If we denote by $\tau^*_{ij}(\ell)$ the minimal twist contained in the OPE of $\mathcal{O}_i$ with $\mathcal{O}_j$, the following inequalities have to be satisfied, for even spins $\ell_e \geq 2$ and odd spins $\ell_o \geq 3$ respectively:

$$\tau^*_{12}(\ell_e) \geq \frac{1}{2}\Big(\tau^*_{11}(\ell_e)+\tau^*_{22}(\ell_e)\Big) \tag{2.76}$$

$$\tau^*_{12}(\ell_o) \geq \frac{1}{2}\left(\frac{\tau^*_{11}(\ell_o+1)+\tau^*_{22}(\ell_o+1)}{2}+\frac{\tau^*_{11}(\ell_o-1)+\tau^*_{22}(\ell_o-1)}{2}\right) \tag{2.77}$$

Although this seems to point in the direction of what we aim to establish, these bounds are automatically satisfied at leading order in $1/\ell$, for all choices of OPE coefficients and anomalous dimensions.

Finally, similar results were proven in a totally different context [194] using causality bulk techniques, by examining the two-point function of an arbitrary operator $\mathcal{O}$

over a shock-wave background. There, the author argues that in the limit where the stress-energy tensor is decoupled

$$\gamma(0,2) < 0 \qquad \text{for} \qquad \mathcal{O}\partial_\mu\partial_\nu\mathcal{O}. \tag{2.78}$$

From a holographic point of view, this implies that a theory in AdS with the scalar Lagrangian

$$S = \int d^4x\sqrt{-g}\Big[(\partial_\mu\varphi)^2 + \mu(\partial_\mu\varphi)^4\Big] \tag{2.79}$$

is only consistent if $\mu > 0$. This is the AdS generalization of a well-known result in flat spacetime [74] derived using the analyticity and causality structure of scattering amplitudes. Closely related bounds for EFTs on AdS (which include the effect of dynamical gravity) have been obtained in [182, 219]. In [220, 221, 222, 223], CFT dispersion relations involving Mellin amplitudes were also developed, again strongly reminiscent of the flat spacetime story.

# Chapter 3

# The Holographic Swampland

This Chapter is based on the papers [1, 2].

## 3.1 String Compactifications and Holography

### 3.1.1 General considerations

In this section we are concerned with relating swampland modifications on LVS and other low-energy string compactification Lagrangians to constraints on QFTs and CFTs. A first crucial point to appreciate is that the established bounds do not directly lead to constraints for any of the stringy models under consideration.

From the perspective of the dual CFT, some difficulties arise when the bounds (2.74) are applied in presence of gravity. One reason is that, as has already been mentioned, the actual minimal twist operators constrained by causality involve combinations of stress-energy tensors, and not of the scalars. Secondly, the behaviour of the anomalous dimensions for large spin, i.e. where computations are most easily carried out, is dominated by graviton exchanges, saturating the $\ell$ exponent of the leading order term. If scalar couplings were all parametrically stronger than gravity, then it could be possible to neglect the gravitational exchanges. However, since moduli typically couple with gravitational-like strength it is not clear that it would be consistent to attempt to neglect dynamical gravity.

In the case of double trace operators built out of identical fields,[1] negativity of the large-$\ell$ anomalous dimension is automatically satisfied and so does not lead to constraints: the sign is always determined by the product of the couplings, which are

[1] $[\mathcal{O}_\varphi \mathcal{O}_\varphi]_{n,\ell}$ and $[\mathcal{O}_a \mathcal{O}_a]_{n,\ell}$ in the case at hand.

identical when the external legs are the same: hence $\gamma(n,\ell) \propto -g^2$. On the other hand, the constraints for different fields are always satisfied at large $\ell$.

A further limitation is that contact diagrams only give rise to tree-level anomalous dimensions up to a finite value of the spin, while exchanges contribute for any value of $\ell$. In particular, a 4-point interaction with $2j$ derivatives only contributes to anomalous dimensions $\gamma(n,\ell)$ with $n \leq j$ because of angular momentum conservation [179, 137]. For exchange diagrams, the precise statement is that a 4-point Witten diagram, say in s-channel, contains an infinite number of spins when decomposed in conformal blocks in the t and u channels [224].[2] As the contact operators in the LVS Lagrangian have $\ell_c \leq 1$ and bounds on anomalous dimensions are only valid for $\ell_c \geq 2$, positivity constraints do not apply directly to such operators.

It is interesting to notice that similar difficulties would appear if one tried to use the analyticity bounds to constrain the same coefficients for LVS in flat space-time, in the limit where $R_{AdS}$ is sent to infinity. In that case, only interactions with at least four derivatives are constrained since the bounds on the amplitude start at order $s^2$. Moreover, as observed in the previous section, the inclusion of dynamical gravity presents some obstructions to the use of these bounds.

### 3.1.2 Holographic Analysis for LVS and Other Scenarios

Having laid out these caveats, we now want to determine some basic properties of LVS and other moduli stabilisation scenarios when viewed from a holographic perspective.

One fundamental computation, which will prove useful later, is that of the three-point functions of the single trace scalars. As mentioned in section 2.2.2, these are fully constrained by conformal symmetry up to one coefficient $f_{ij}^k$. To include all of our successive examples, we consider a generic theory containing a modulus and an axion, whose interactions can be parametrized up to cubic order as

$$\mathcal{L} \supset \frac{g}{M_P R_{AdS}^2}\varphi^3 - \frac{\mu}{M_P}\varphi\,\partial_\mu a \partial^\mu a + \frac{\kappa}{M_P R_{AdS}^2}\varphi a^2. \tag{3.1}$$

The last coupling involving $\varphi a^2$ is absent in the basic LVS scenario, but is present in other scenarios (such as KKLT). Because of the discrete symmetry $a \to -a$, the only non-trivial coefficients are $f_{\varphi\varphi}^{\varphi}$ and $f_{aa}^{\varphi}$ (parity implies $f_{\varphi a}^{\varphi} = f_{aa}^{a} = 0$).

[2]Notice that here the same terminology (s,t and u) is used for two conceptually distinct concepts; the former usage refers to the form of the diagram, while the latter to the different OPE contractions used to expand the correlator.

We shall adopt the embedding formalism [135, 136]: points of $AdS_{d+1}$ are represented as null rays in $\mathbb{R}^{d+2,2}$,

$$X_0^2 - X_1^2 \ldots - X_d^2 + X_{d+1}^2 = 1, \tag{3.2}$$

on which the conformal group $SO(d,2)$ has a natural action. The boundary is identified with the projective null cone

$$P^A \simeq \lambda P^A, \qquad \lambda \in \mathbb{R}. \tag{3.3}$$

CFT operators can then be extended on the full space by imposing the homogeneity property

$$\mathcal{O}(\lambda P^A) = \lambda^{-\Delta}\mathcal{O}(P^A), \tag{3.4}$$

and conformal invariance becomes manifest. In this formalism, the bulk-to-boundary propagator of an operator with dimension $\Delta$ acquires the simple form

$$\Pi(P,X)_\Delta = \frac{1}{(2P\cdot X)^\Delta}. \tag{3.5}$$

The 3-point correlator arising from the $\varphi^3$ contact Witten diagram can then be written as

$$\langle\varphi(x_1)\varphi(x_2)\varphi(x_3)\rangle = g\int_{AdS}\mathrm{dX}\frac{1}{(2P_1\cdot X)^{\Delta_\varphi}(2P_2\cdot X)^{\Delta_\varphi}(2P_3\cdot X)^{\Delta_\varphi}}. \tag{3.6}$$

Using the integral representation formula

$$\frac{1}{(2P\cdot X)^\Delta} = \frac{1}{\Gamma(\Delta)}\int \mathrm{d}s\, s^\Delta e^{-2sP\cdot X}, \tag{3.7}$$

it follows (e.g. see [126]) that the correlator has the form of Eq. (2.21), with a coefficient

$$f^\varphi_{\varphi\varphi} = \frac{3g\pi^{d/2}\Gamma\left(\frac{3\Delta_\varphi - d}{2}\right)}{\Gamma(\Delta_\varphi)^3}. \tag{3.8}$$

Note that, for $g > 0$, this is positive for all $\Delta_\varphi > 1$.

For the correlation function involving axions, the appearance of derivatives can

be translated into

$$\begin{aligned}\langle\varphi(x_1)a(x_2)a(x_3)\rangle = -\mu\int_{AdS}\mathrm{dX}\frac{1}{(2P_1\cdot X)^{\Delta_\varphi}}\\ \times(\eta^{AB}+X^AX^B)\frac{\partial}{\partial X^A}\frac{1}{(2P_2\cdot X)^{\Delta_a}}\frac{\partial}{\partial X^B}\frac{1}{(2P_3\cdot X)^{\Delta_a}}.\end{aligned} \tag{3.9}$$

With a few manipulations, and using $-P_1\cdot P_2 = x_{12}^2$, the integral can now be turned into a sum of terms with the same structure as (3.6), resulting in

$$f_{aa}^{\varphi} = \frac{\pi^{d/2}\Gamma\left(\frac{2\Delta_a+\Delta_\varphi-d}{2}\right)}{\Gamma(\Delta_\varphi)\Gamma(\Delta_a)^2}\Big[\mu(\Delta_\varphi+2\Delta_a-\Delta_a^2-3)+\kappa\Big]. \tag{3.10}$$

For future reference, we note that when $\Delta_a = 3$ and $\kappa = 0$ (corresponding to an axion with a flat potential, as in LVS) this becomes

$$f_{aa}^{\varphi} = \frac{\mu\pi^{d/2}\Gamma\left(\frac{2\Delta_a+\Delta_\varphi-d}{2}\right)}{\Gamma(\Delta_\varphi)\Gamma(\Delta_a)^2}(\Delta_\varphi-6), \tag{3.11}$$

a quantity which changes sign for $\Delta_\varphi > 6$.

### 3.1.3 Mixed anomalous dimensions

We reviewed in section 2.4 how consistency constraints on EFTs often involve the signs of certain operators, and so we will be interested in quantities within the CFT that are sensitive to such signs. One object in particular that we will be concerned with is the anomalous dimension of mixed double-trace states, namely operators of the form $[\mathcal{O}_\varphi\mathcal{O}_a]_{n,\ell}$. Although no *a priori* constraints are known on their sign, we will see that for these operators negativity of the anomalous dimension appears to be crucial for satisfying many swampland conjectures. This work is exploratory in nature. We shall here propose and explore the consequences of the following:

**Anomalous dimension conjecture** [1, 2]

*Large spin anomalous dimensions of mixed double trace operators consisting of an axion and a saxion are negative for duals to scenarios of moduli stabilisation.*

The two primaries which constitute the double trace operators have to be a saxion and an axion because, as we will see shortly, there is a counterexample involving operators made out of two axions.

From the point of view of the Analytic Bootstrap [35, 36], crossing symmetry does require in this context one specific behaviour for the anomalous dimensions as a function of the spin. Given two primaries $\mathcal{O}_1$ and $\mathcal{O}_2$ with twists $\tau_{\mathcal{O}_1}$ and $\tau_{\mathcal{O}_2}$, the bootstrap equations in Lorentzian signature[3] imply the existence of an infinite series of operators whose twist is arbitrarily close to the sum of the two. Furthermore, the twist of these operators asymptotically behaves as

$$\tau(\ell) \simeq \tau_{\mathcal{O}_1} + \tau_{\mathcal{O}_2} - \frac{c}{\ell^{\tau^*}} \quad \text{for} \qquad \ell \gg 1, \tag{3.12}$$

where $\tau^*$ is the minimal twist amongst all operators contained in both the OPEs of $\mathcal{O}_1$ and $\mathcal{O}_2$ with themselves. While these results are easily seen to be a consequence of (2.43) at tree level (see below), we emphasize that they apply in full generality.

Assuming $\Delta_\varphi + \Delta_a \notin \mathcal{Z}$, the only tree-level Witten diagrams contributing to the mixed anomalous dimension are those associated with the correlation function

$$G(x_i) = \langle \mathcal{O}_\varphi(x_1)\mathcal{O}_\varphi(x_2)\mathcal{O}_a(x_3)\mathcal{O}_a(x_4)\rangle, \tag{3.13}$$

and they are shown in Figure 3.1. Furthermore, we make the claim that at $\mathcal{O}(\frac{1}{N^2})$, the leading contribution in the $1/\ell$ expansion only comes from the s-channel diagram. Let us first give a generic argument based on the claim in the first paragraph, before reaching the same conclusion with the formula described in the last section. One can consider the following example: three scalars on AdS, with the only cubic vertex of the form

$$\mathcal{L} \supset g\varphi_1\partial...\partial\varphi_2\partial...\partial\Phi, \tag{3.14}$$

where any possible combination of derivatives is allowed. Since vertices with more than three fields do not affect the anomalous dimension for large $\ell$ at this order in the coupling expansion, they can all be set to zero for the sake of the discussion. The Witten diagram $\varphi_1\varphi_1 \to \varphi_2\varphi_2$ only contains t and u channels with an exchange of a $\Phi$, at order $1/N^2$. Now, neither of the OPEs $\mathcal{O}_{\varphi_1}\mathcal{O}_{\varphi_1}$ or $\mathcal{O}_{\varphi_2}\mathcal{O}_{\varphi_2}$ contains $\Phi$ at any order in perturbation theory, since the correlator $\langle \mathcal{O}_{\varphi_1}\mathcal{O}_{\varphi_1}\mathcal{O}_\Phi\rangle$ vanishes as a result of

[3] $d > 2$ is also assumed.

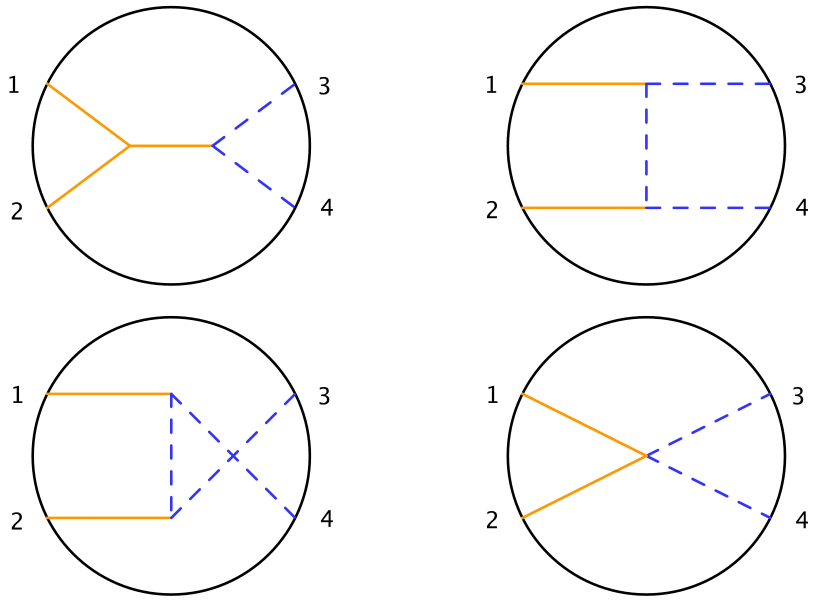



Figure 3.1: Witten diagrams contributing to the anomalous dimensions of double trace operators of the form $[\mathcal{O}_{\varphi_1}\mathcal{O}_{\varphi_2}]_{(n,\ell)}$. The volume modulus corresponds to the continuous orange lines, and the axion to the blue dashed ones. The last diagram, without internal propagators, only contributes to anomalous dimensions for small $\ell$.

the discrete symmetry

$$\mathcal{P}_1 : \quad \varphi_1 \longrightarrow -\varphi_1, \qquad\qquad \Phi \longrightarrow -\Phi, \tag{3.15}$$

and the same result holds for $\mathcal{O}_{\varphi_2}$. More generally, the two OPEs do not contain *any* common single trace operators. It is thus possible to conclude that there is no contribution to the anomalous dimensions at leading order in the coupling from scalar diagrams built out of three-field interactions in the $t$ and $u$ channels. The first correction will be due to the presence of the double trace operators $[\mathcal{O}_{\varphi_1}\mathcal{O}_{\varphi_1}]_{0,\ell}$[4] in the OPE $\mathcal{O}_{\varphi_2} \times \mathcal{O}_{\varphi_2}$, which results in a scaling behaviour of $1/\ell^{2\Delta_1}$ at $\mathcal{O}\left(\frac{1}{N^4}\right)$. The intuitive meaning of this result is that the mixed anomalous dimension for the $\mathcal{O}_{\varphi_1}\mathcal{O}_{\varphi_2}$ operator is equivalent to the binding energy of the two-particle $\varphi_1\varphi_2$ state in AdS. This binding energy would arise from particle exchange, which would correspond to the s-channel for the $\varphi_1\varphi_1 \to \varphi_2\varphi_2$ topology.

A more direct way to see this is through the formula (2.43), reported here for

[4]Assuming (without loss of generality) $\Delta_1 < \Delta_2$.

convenience:

$$\gamma(0,\ell) = -\int_{-i\infty}^{+i\infty} \frac{ds}{2\pi i} M(s,0)\ {}_3F_2(-\ell, \Delta_1+\Delta_3+\ell-1, \frac{s}{2}; \Delta_1, \Delta_3; 1) \\ \times \Gamma\Big(\Delta_1 - \frac{s}{2}\Big)\Gamma\Big(\Delta_3 - \frac{s}{2}\Big)\Gamma\Big(\frac{s}{2}\Big)^2, \tag{3.16}$$

where $M(s,t)$ is the Mellin amplitude defined with the conventions of paragraph 2.43.[5] For large $\ell$, it is possible to make the approximation

$${}_3F_2(-\ell, \Delta_1+\Delta_3+\ell-1, \frac{s}{2}; \Delta_1, \Delta_3; 1) \simeq \frac{\Gamma(\Delta_1)\Gamma(\Delta_3)}{\Gamma(\Delta_1-\frac{s}{2})\Gamma(\Delta_3-\frac{s}{2})}\frac{1}{\ell^s} \tag{3.17}$$

inside the integral (3.16) [191]. [6] Given that the contour is closed on the right half-plane, the integral is dominated by the lowest pole in $s$ of $M(s,0)$ - that is the conformal dimension of the exchanged scalar. For a generic exchange diagram, the Mellin amplitude will have the form

$$M(s,t) = f_{12X}f_{X34}\sum_m \frac{Q_{J_X,m}(t)}{s-\Delta_X+J_X-2m} + ... \tag{3.18}$$

where contributions from the crossed channels have been omitted. In our case, the t-poles are set to zero and the only relevant ones are those coming from the $s$ and $u$ channels. The latter, however, translate into a series of $s$ poles all on the left of the integration contour, because of the relation

$$s+t+u = \sum_i \Delta_i, \tag{3.19}$$

and as such do not give any contribution to the integral. Therefore, the anomalous dimensions will only be sensitive to the s-channel amplitude, and scale as $1/\ell^{\Delta_{ex}}$ in the large $\ell$ limit - we therefore recover a special case of (3.12). Moreover, equation

[5] There is a constant shift with respect to the canonical definition of the $t$ variable in order to simplify the form of the Mack polynomials.

[6] Eq. (3.17) is not valid in general. However, since the anomalous dimensions for a polynomial Mellin amplitude have to vanish for large enough $\ell$, the only relevant terms in the expansion of the ${}_3F_2$ in (3.16) are those which cancel out the poles arising from the factor $\Gamma\Big(\Delta_1-\frac{s}{2}\Big)\Gamma\Big(\Delta_3-\frac{s}{2}\Big)$. As the ${}_3F_2$ is a finite sum (thanks to the negative argument $-\ell$), one can obtain (3.17) by expanding each of the coefficients in powers of $1/\ell$ and keeping only the contribution proportional to $1/\Gamma\Big(\Delta_1-\frac{s}{2}\Big)\Gamma\Big(\Delta_3-\frac{s}{2}\Big)$ in the sum.

(3.16) easily allows to go one step further and actually compute the large-$\ell$ behaviour of $\gamma(0,\ell)$. Specialising to a Lagrangian of the form (3.1), we obtain the formula presented in Section 2.3.3

$$\gamma^{\varphi a}(0,\ell) = -2 f_{\varphi\varphi\varphi} f_{\varphi aa} \frac{\Gamma(\Delta_a)\Gamma(\Delta_\varphi)^2}{\Gamma\left(\frac{2\Delta_a-\Delta_\varphi}{2}\right)\Gamma\left(\frac{\Delta_\varphi}{2}\right)^3} \frac{1}{\ell^{\Delta_\varphi}} + \mathcal{O}\left(\frac{1}{\ell}\right), \tag{3.20}$$

which agrees with the results of [193]. Although their derivation is significantly shorter and does not require the introduction of Mellin amplitudes, we emphasize that our formalism provides a systematic procedure to calculate corrections at each order in $1/\ell$. In principle, it is also possible to resum all these contributions and derive the value of the anomalous dimensions for finite spin, up to the effect of contact terms [225, 226]. The latter can always be added as a homogeneous solution to the crossing equation, and correspond to the existence of $\phi^4$ interactions in the bulk lagrangian with up to two derivatives.

The asymptotic series is guaranteed to converge for any spin $\ell > 1$ thanks to the analyticity properties proven in [217]. Furthermore, we hope this approach might help to highlight possible connections with the standard bounds for scattering amplitudes in Minkowski space.

We flag up here the factor of

$$\frac{1}{\Gamma\left(\frac{2\Delta_a-\Delta_\varphi}{2}\right)}$$

in Eq. (3.20), as for the case of light axions with $\Delta_a = 3$ this has interesting sign dependence on $\Delta_\varphi$.

## 3.2 Explicit examples

We now study the anomalous dimensions within different scenarios of moduli stabilisation.

### 3.2.1 LVS

For the case of LVS, there are no $\varphi aa$ couplings in the potential. Then, inserting the value of the three-point function OPE coefficients from Eqs. 3.8 and 3.10 immediately leads to

$$\gamma^{\varphi a}(0,\ell) = -\frac{g\mu(\Delta_\varphi - 6)}{M_P^2 R_{AdS}^2}\frac{3\pi^3}{\Gamma(\Delta_\varphi)^2}\frac{\Gamma\left(\frac{3\Delta_\varphi-3}{2}\right)\Gamma\left(\frac{\Delta_\varphi+3}{2}\right)}{\Gamma\left(\frac{6-\Delta_\varphi}{2}\right)\Gamma^3\left(\frac{\Delta_\varphi}{2}\right)}\frac{1}{\ell^{\Delta_\varphi}} + \mathcal{O}\left(\frac{1}{\ell}\right) \tag{3.21}$$

where we have also specialized to $\Delta_a = 3$ and $d = 3$. As all other terms are automatically positive and $\Delta_\varphi \geq \frac{3}{2}$ due to the standard unitarity bound.[7], the sign of the $\phi a$ anomalous dimensions is governed, at least asymptotically, by the sign of the combination

$$\gamma^{\varphi a}(0,\ell) \sim -g\mu\frac{(\Delta_\varphi - 6)}{\Gamma\left(\frac{6-\Delta_\varphi}{2}\right)}. \tag{3.23}$$

We can now consider the meaning of equation (3.23) in more detail. In particular, an interesting correlation appears between the allowed values of the parameter space in String Theory and the sign of the anomalous dimension, which is highly reminiscent of the bounds discussed in the previous sections.

In particular, the RHS is negative if the variables take values in the original LVS model in the $\mathcal{V} \to \infty$ limit (with $\Delta_\varphi = 8.04$), but switches to positive if certain swampland-like transformations are performed on the AdS theory. The most obvious example of these is the sign exchange $\mu \to -\mu$, corresponding on the string theory side (as discussed in 2.1.3) to an axion decay constant that diverges in the large volume limit - which is in strong disagreement with the axion WGC. A similar behaviour occurs for the change $g \to -g$. A simultaneous change of both signs is allowed, as this is equivalent to the field redefinition $\varphi \to -\varphi$, but the product of the two signs is a physical quantity independent of the field redefinition.

[7]This is only true in absence of dynamical gravity, which induces correction saturating the unitarity bound on the $1/\ell$ exponent and thus scaling as $1/\ell^{d-2}$. Since the moduli interactions are suppressed by the same powers of the Planck mass as standard gravitational interactions, it is only possible to integrate out such effects if $g\mu \gg 1$, and otherwise Eq. (3.23) actually parametrizes to the leading behaviour of

$$\gamma^{\varphi a}(0,\ell) - \gamma^{\varphi a}(0,\ell)_{grav} \tag{3.22}$$

for sufficiently high values of $\ell$. In principle, one could also suspect that subleading terms in the $1/N$ expansion might decrease with a lower power of $\ell$ and dominate asymptotically if the coupling is fixed. In our example, however, $R_{AdS}$ diverges in the large volume limit and it is legitimate to consider the $\frac{1}{N^2}$ correction only.

There is also an interesting dependence on $\Delta_\varphi$, as any value $\Delta_\varphi < 8$ (and consistent with the unitarity bound) also gives rise to a positive anomalous dimension, as can be seen in Fig.3.2. Recall that the value of $\Delta_\varphi$ arises from the volume scaling of the

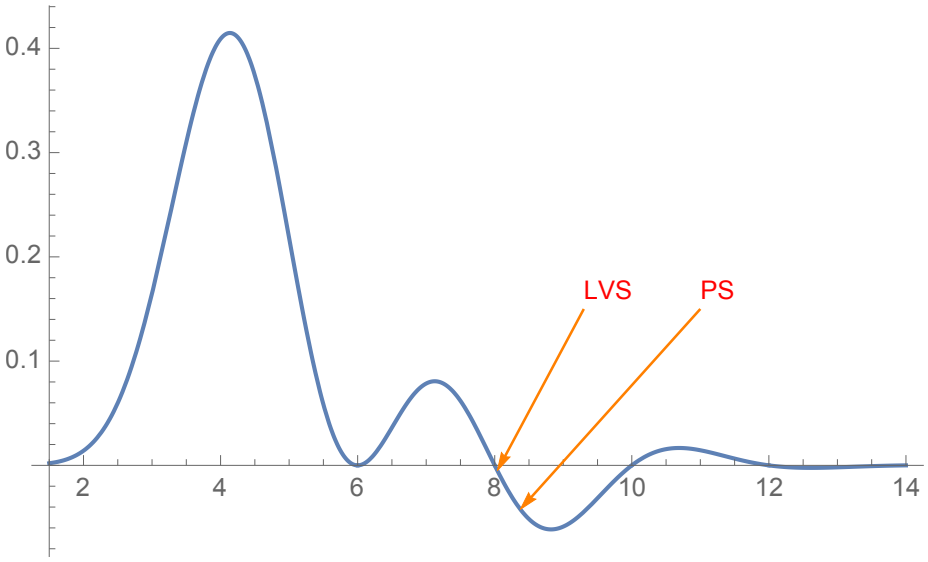



Figure 3.2: Prefactor $C(\Delta_\varphi) = \gamma(0,\ell)\ell^{\Delta_\varphi}$ as a function of $\Delta$. For graphical purposes, it has been rescaled by a factor of $\left(\frac{\Delta}{10}\right)^6$. The lowest possible value admitting a negative anomalous dimension is $\Delta = 8$. From then onwards, the sign oscillates between positive and negative values with decreasing amplitude and a period of 2. The two arrows show the location of the LVS and Perturbative Stabilization models respectively, in the $\mathcal{V} \to \infty$ limit .

potential. Quite remarkably, LVS is found to reside in a rather special place, right at the edge of the lower boundary of the 'allowed' region with negative anomalous dimension.

While the region where $\Delta_\varphi < 6$ can be understood as corresponding to a potential growing parametrically faster than the string scale $M_s^4$, we have no valid interpretation as to why $\Delta_\varphi = 8$ should be regarded as a critical value. We also note that $\gamma(0,\ell)$ goes to zero exactly at the point where $\Delta_\varphi = 6$, which by the above argument is expected to exhibit some kind of transition.

It is intriguing that in the $\mathcal{V} \to \infty$ limit of LVS, the value of $\Delta_\varphi$ is so close to 8. At finite volume, $\Delta_\varphi$ approaches the asymptotic value of $\frac{3}{2}\left(1+\sqrt{19}\right) \simeq 8.038$ from below as the volume is brought into infinity, with corrections that scale as $\delta\Delta_\varphi \sim \frac{1}{\ln \mathcal{V}}$. Including these corrections in the 1-modulus potential explicitly, one finds that the

finite volume necessary to achieve $\Delta_\varphi = 8$ is given by

$$\langle\varphi\rangle = \frac{1}{2\left(\lambda - \frac{40}{3\lambda}\right)} \tag{3.24}$$

Numerically, this amounts to $\mathcal{V} \sim 10^5/10^6$.

However, one cannot conclude that at such volumes it is definitively true that $\Delta_\varphi < 8$, as once volumes become smaller corrections that are higher-order in $\alpha'$ (and so can normally be neglected) become relevant for the question of whether $\Delta_\varphi > 8$ or $\Delta_\varphi < 8$. Specifically, the fractional difference between 7.99 and 8.00 is $\mathcal{O}(10^{-3})$ and so effects that are far too small to modify the existence of the minimum are capable of changing the sign of $(\Delta_\varphi - 8)$. This can be verified by a numerical study of the full 2-modulus LVS stabilisation. Here one sees that the sign of $\Delta_\varphi - 8$ at finite $\mathcal{V}$ (e.g. $\mathcal{V} \sim 10^6$) can easily be modified by corrections of the form

$$K = -2\ln\left(\mathcal{V} + \xi\right) \to K = -2\ln\left(\mathcal{V} + \xi + \frac{\xi^2}{\mathcal{V}}\right).$$

This is illustrated in figure 3.3. As the determination of the coefficients of such

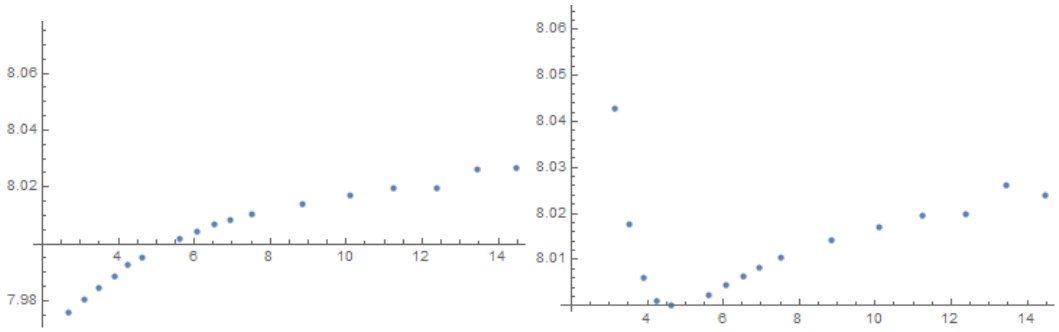



Figure 3.3: This plot illustrates the effect of a higher-order correction to the Kähler potential. The x-axis shows the logarithm (in base 10) of the stabilised volume while the y-axis shows the conformal dimension of the dual volume operator, $\Delta_\varphi$. The case on the left is LVS based on the original $\mathbb{P}^4_{1,1,1,6,9}$, with a volume $\mathcal{V} = \frac{1}{9\sqrt{2}}\left(\tau_b^{3/2} - \tau_s^{3/2}\right)$, a Kähler potential $-2\ln\left(\mathcal{V} + \xi\right)$, and a superpotential $W = W_0 + A_s e^{-2\pi T_S}$ with $W_0 = 1$ and $A_s = 1$. Different values of the stabilised volume have been obtained by taking different values of $\xi$. In this case, the logarithmic corrections are seen to bring $\Delta_\varphi$ below 8 at smaller volumes. The case on the right is identical, except for a Kähler potential $K = -2\ln\left(\mathcal{V} + \xi + \frac{\xi^2}{\mathcal{V}}\right)$. In this case, $\Delta_\varphi > 8$ throughout.

corrections is beyond any current control, this demonstrates why there is no controlled

region of LVS in which one can be sure that $\Delta_\varphi < 8$.

It is suggestive to find a connection between the Swampland and something so closely resembling CFT consistency conditions. Furthermore, the $\gamma^{\varphi a}$'s are essentially the only OPE data (at least at tree-level) sensitive to the sign of the product $g\mu$. However we stress again that negativity of the anomalous dimensions is not established in general for double trace operators constructed of two different primaries.

### 3.2.2 Perturbative Stabilisation

We can perform a similar analysis for other models of IIB moduli stabilisation, even if they are not as fully worked out as LVS. In particular, they do not offer the cleanliness of the $\mathcal{V} \to \infty$ limit of LVS (the holographic study of IIA moduli stabilisation scenarios [41] offers a different large-volume limit driven by large flux quanta, and will be discussed in Chapter 3). One example is that of the so-called perturbative stabilization [155, 156], in which the volume is stabilised by the competing effects of two separate perturbative corrections to the scalar potential. Here the dynamics of the volume modulus is governed by a potential of the form

$$V(\varphi) = Ae^{-\frac{\lambda_1\varphi}{M_P}} - Be^{-\frac{\lambda_2\varphi}{M_P}}, \tag{3.25}$$

which reflects competition between two effects scaling with different powers of the volume. Such a potential could, for instance, be generated by a combination of $\alpha'$ corrections scaling as $\mathcal{V}^{-3}$, as in LVS, and string loop effects of order $\mathcal{V}^{-\frac{10}{3}}$. The minimum is located at

$$\varphi_c = \frac{\log\left(\frac{A\lambda_1}{B\lambda_2}\right)}{\lambda_1 - \lambda_2}, \qquad \text{with} \qquad V_{min} = Be^{-\lambda_2\varphi_c}\left(\frac{\lambda_2}{\lambda_1} - 1\right), \tag{3.26}$$

and results in an AdS vacuum for $\lambda_1 > \lambda_2$. In a top-down scenario, the coefficients $\lambda_1$ and $\lambda_2$ would be fixed. However, for purpose of exploration we may consider them arbitrary, allowing for the case that $|\lambda_1 - \lambda_2| \ll |\lambda_1 + \lambda_2|$ which leads to large volumes (alternatively, one could demand that the coefficients are fine-tuned with $A \gg B$). Expanding about the minimum, one recovers n-point interactions for the volume modulus

$$\mathcal{L}_{\varphi^n} = (-1)^{n-1}\frac{3M_P^2\lambda_1\lambda_2}{R_{AdS}^2}\frac{\lambda_1^{n-1} - \lambda_2^{n-1}}{\lambda_1 - \lambda_2}(\varphi)^n. \tag{3.27}$$

Consequently the conformal dimension of the corresponding CFT operator is

$$\Delta_\varphi = \frac{3(1 \pm \sqrt{1 + \frac{4}{3}\lambda_1\lambda_2})}{2}. \tag{3.28}$$

The resemblance with the result for LVS should not be too surprising, as the potential actually reduces to that of standard LVS in the limit $\lambda_1 \to \lambda_2$. The generalization of Eq. (3.23) is

$$\gamma^{\varphi a}(0,\ell) \sim -\mu\lambda_1\lambda_2(\lambda_1+\lambda_2)\frac{(\Delta_\varphi - 6)}{\Gamma\left(\frac{6-\Delta_\varphi}{2}\right)}, \tag{3.29}$$

which again lends itself to an interpretation in terms of Swampland constraints. For the values considered above, namely

$$\lambda_1 = \frac{10}{3}\sqrt{\frac{3}{2}}, \qquad \lambda_2 = 3\sqrt{\frac{3}{2}}, \tag{3.30}$$

the anomalous dimensions are again negative. However, the sign changes under any of the following:

- $\mu \to -\mu$, corresponding to $f_a \sim M_P \mathcal{V}^{\frac{2}{3}}$.
- $\lambda_2 \to -\lambda_2$, which leads to an unbounded potential for large values of the volume. Notice that the same does not happen if one inverts the sign of $\lambda_1$, since $\lambda_1 > \lambda_2$ and the anomalous dimension is proportional to the product $(\lambda_1 + \lambda_2)$
- $0 \leq \lambda_1\lambda_2 \leq \frac{40}{3}$: This is exactly analogous to LVS. If $0 \leq \lambda_1\lambda_2 \leq 6$, there will be at least one term in the potential growing faster than $M_s^4$, but otherwise there is no clear interpretation. As a further comment, this is slightly reminiscent (but qualitatively different) from similar constraints arising from the De Sitter Swampland conjecture, of the schematic form $\lambda_i < c$ with $c$ an order one constant [56].

### 3.2.3 KKLT

Another well-studied case where these ideas can be tested is the KKLT construction [159]. Although this ultimately aims at de Sitter space, KKLT starts with an AdS vacuum that is subsequently uplifted to dS: it is the former that we will discuss here.

The Kähler potential is given by the standard tree-level expression

$$K = -3\ln[-i(\rho - \bar{\rho})], \tag{3.31}$$

while the superpotential includes a tree-level contribution from fluxes and non-perturbative effects.

$$W = W_0 + Ae^{i\alpha\rho}. \tag{3.32}$$

The supergravity potential

$$V = e^K\left(G^{\rho\bar{\rho}}D_\rho W\bar{D}_\rho W - 3\left|W\right|^2\right) \tag{3.33}$$

can be minimised while simultaneously preserving supersymmetry, for a critical value of the modulus field $\sigma_{cr}$ satisfying

$$\frac{W_0}{A} = -e^{-\alpha\sigma_{cr}}(1 + \frac{2\alpha\sigma_{cr}}{3}). \tag{3.34}$$

Since the supergravity approximation and the single instanton approximation require $\sigma_{cr} \gg 1$ and $\alpha\sigma_{cr} > 1$ respectively, $W_0$ has to be tuned to be very small.

The situation when considering low energy excitations around this vacuum is qualitatively different from LVS. As the instanton breaks the continuous shift symmetry, the axion now has both a mass and also new potential-type interactions with the volume modulus. From a holographic perspective, the first consequence is that the conformal dimension of the axion is no longer $\Delta = 3$, but instead

$$\Delta_a = \frac{3}{2}\left(1 \pm \sqrt{1 + \frac{8}{9}\alpha\sigma_{cr}(2\alpha\sigma_{cr} + 3)}\right), \tag{3.35}$$

while for the volume modulus, the result is

$$\Delta_\varphi = \frac{3}{2}\left(1 \pm \sqrt{1 + \frac{8}{9}(2 + \alpha\sigma_{cr})(1 + 2\alpha\sigma_{cr})}\right). \tag{3.36}$$

A noteworthy point is that the last two equations imply $\Delta\varphi < 2\Delta a$, so that the oscillating sign behaviour due to the negative gamma function is not reproduced in this case.

The cubic interaction terms in the potential now read

$$\begin{aligned} V^{(3)}(\varphi,a) \supset & -\frac{1}{M_P R_{AdS}^2}\sqrt{\frac{2}{3}}\Big[2+\alpha\sigma_{cr}(5+\alpha\sigma_{cr}(5+2\alpha\sigma_{cr}))\Big]\varphi^3 \\ & -\frac{1}{M_P R_{AdS}^2}\sqrt{\frac{2}{3}}\alpha\sigma_{cr}(2+\alpha\sigma_{cr})(3+2\alpha\sigma_{cr})\varphi a^2. \end{aligned} \tag{3.37}$$

The interactions of the form $\varphi^n \partial_\mu a \partial^\mu a$ are left unchanged as they arise from the axion kinetic term. For completeness, we also report the quartic interactions in the potential:

$$\begin{aligned} V^{(4)}(\varphi,a) \supset & \frac{1}{18 M_P^2 R_{AdS}^2}\Big[(1+\alpha\sigma_{cr})(14+\alpha\sigma_{cr}(21+\alpha\sigma_{cr}(11+14\alpha\sigma_{cr})))\varphi^4 \\ & +6\alpha\sigma_{cr}(3+2\alpha\sigma_{cr})(4+\alpha\sigma_{cr}(3+\alpha\sigma_{cr}))a^2\varphi^2 - 6\alpha\sigma_{cr}^3(3+2\alpha\sigma_{cr})a^4\Big]. \end{aligned} \tag{3.38}$$

In particular, the anomalous dimensions can be computed as in section 3.1.3 (note that the fact that $\Delta\varphi < 2\Delta a$ also implies that the dominating contribution to anomalous dimensions at large $\ell$ will always be the one coming from the exchange of a $\varphi$, and one does not have to worry about potential loop effects proportional to $\ell^{-2\Delta_a}$). The main difference is that there are now two contributions to $f_{aa}^{\varphi}$, arising from both the $\varphi a^2$ and $\varphi \partial_\mu a \partial^\mu a$ vertices, and with opposing signs. As the sign of the $\varphi^3$ coefficient in (3.37) is fixed it is the sum of these two compensating effects that must be considered, according to

$$\gamma(0,\ell) \propto -g\Big[\alpha\sigma_{cr}(2+\alpha\sigma_{cr})(3+2\alpha\sigma_{cr}) + 2\Big(\Delta_\varphi(\alpha\sigma_{cr}) - \Delta_a^2(\alpha\sigma_{cr}) + 2\Delta_a(\alpha\sigma_{cr}) - 3\Big)\Big]. \tag{3.39}$$

The anomalous dimensions are then negative provided

$$\alpha\sigma_{cr} \gtrsim 2.24, \tag{3.40}$$

which is always true in KKLT's regime of validity of $W_0 \sim e^{-\alpha\sigma} \ll 1$ where any multi-instanton effects can be neglected.

We note that negativity was not automatic and required the presence of the additional $\varphi a^2$ term in the potential, as the values of the conformal dimensions were such that the contribution from the derivative vertex switched sign with respect to LVS and the potential contribution was necessary to produce negative anomalous

dimensions.

### 3.2.4 Racetrack

Another popular scenario has been that of racetrack stabilisation, where the tree-level flux superpotential is set to zero and the dominant effect instead results from an interplay between two different non-perturbative effects (for example see [160]),

$$W = Ae^{i\alpha\rho} - Be^{i\beta\rho}. \tag{3.41}$$

Assuming without loss of generality $\alpha > \beta > 0$ a supersymmetric minimum of the potential is found at

$$e^{-(\alpha-\beta)\sigma_{cr}} = \frac{B(3+2\beta\sigma_{cr})}{A(3+2\alpha\sigma_{cr})}. \tag{3.42}$$

Using the same techniques, it is possible to compute the supergravity potential in Eq. (3.33) and expand around its minimum to obtain the conformal dimensions and low-point interactions of the axion and volume modulus.

$$\Delta_a = \frac{3}{2}\left(1 \pm \sqrt{1 + \frac{8}{243}(3+2\alpha\sigma_{cr})(3+2\beta\sigma_{cr})(2\alpha\beta\sigma_{cr}^2 + 3\alpha\sigma_{cr} + 3\beta\sigma_{cr})}\right), \tag{3.43}$$

$$\Delta_\varphi = \frac{3}{2}\left(1 \pm \sqrt{1 + \frac{8}{243}(6 + 3\alpha\sigma_{cr} + 3\beta\sigma_{cr} + 2\alpha\beta\sigma_{cr}^2)(3 + 4\alpha\beta\sigma_{cr}^2 + 6\alpha\sigma_{cr} + 6\alpha\sigma_{cr}))}\right). \tag{3.44}$$

They can be shown to satisfy $\Delta_a < \Delta_\varphi < 2\Delta_a$, so that again there is no oscillating behaviour and one should not worry about higher-order effects becoming relevant for large $\ell$.

Here, the cubic terms in the potential read

$$V^{(3)}(\varphi, a) \supset -\frac{\sqrt{\frac{2}{3}}}{9M_P R_{AdS}^2}\Big[(\alpha\sigma(2\beta\sigma+3) + 3\beta\sigma + 3)(\sigma(\alpha+\beta)(2\alpha\sigma+3)(2\beta\sigma+3) + 6)\varphi^3$$
$$- \sigma(2\alpha\sigma+3)(2\beta\sigma+3)\left(\alpha^2\sigma(2\beta\sigma+3) + 2\alpha(\beta\sigma+1)(\beta\sigma+3) + 3\beta(\beta\sigma+2)\right)\varphi a^2\Big], \tag{3.45}$$

while the quartic ones are

$$
\begin{aligned}
V^{(4)}(\varphi,a) \supset \frac{4}{162 M_P^2 R_{AdS}^2}\Big[&\varphi^4\sigma^3(\alpha-\beta)^2(2\alpha\sigma+3)(2\beta\sigma+3)(2\alpha\beta\sigma+3(\alpha+\beta)) \\
&- 6\sigma(2\alpha\sigma+3)(2\beta\sigma+3)(\alpha^3\sigma^2(2\beta\sigma+3)+\alpha^2\sigma(\beta\sigma(4\beta\sigma+11)+9) \\
&+ \alpha(\beta\sigma+2)^2(2\beta\sigma+3)+3\beta(\beta\sigma(\beta\sigma+3)+4))a^2\varphi^2 \\
&- \Big(14\alpha^4\sigma^4(2\beta\sigma+3)^2+5\alpha^3\sigma^3(2\beta\sigma+3)(8\beta\sigma(\beta\sigma+3)+15) \\
&+ \alpha^2\sigma^2(2\beta\sigma+3)(\beta\sigma(2\beta\sigma(14\beta\sigma+69)+169)+96) \\
&+ 3\alpha\sigma(2\beta\sigma+3)(\beta\sigma(\beta\sigma(28\beta\sigma+43)+52)+35) \\
&+ 9(\beta\sigma+1)(\beta\sigma(\beta\sigma(14\beta\sigma+11)+21)+14)\Big)a^4\Big].
\end{aligned}
\tag{3.46}
$$

This time, the sign of the anomalous dimensions is determined by the combination

$$
\begin{aligned}
\gamma(0,\ell) \propto -\mu\sqrt{\frac{2}{3}}\Big[&\frac{1}{9}(3+2\beta\sigma_{cr})(3+2\alpha\sigma_{cr})(2\alpha^2\beta\sigma_{cr}^3+2\alpha\beta^2\sigma_{cr}^3 \\
&+ 3\alpha^2\sigma_{cr}^2+3\beta^2\sigma_{cr}^2+8\alpha\beta\sigma_{cr}^2+2\alpha\beta^2\sigma_{cr}^2+6\alpha\sigma_{cr}+6\beta\sigma_{cr}) \\
&+ 2\Big(\Delta_\sigma(\alpha\sigma_{cr},\beta\sigma_{cr})-\Delta_a^2(\alpha\sigma_{cr},\beta\sigma_{cr})+2\Delta_a(\alpha\sigma_{cr},\beta\sigma_{cr})-3\Big)\Big].
\end{aligned}
\tag{3.47}
$$

Numerically, one can verify that this is negative in the controlled region $(\alpha+\beta)\sigma_{cr} \gtrsim 0.75$, which again contains the region of parameter space where the single-instanton approximation is valid for both terms in the superpotential.

The summary of this is that in a variety of examples negativity of $\gamma_{\varphi a}$ is satisfied within the controlled region, with a variety of interlocking parts leading to this conclusion.

## 3.3 Heavy States and the Swampland Distance Conjecture

So far our swampland analysis has focused on the question of transPlanckian axion decay constants. However we can also make connections to the swampland distance conjectures [30, 68] (See Section 1.3.3). These conjectures are statements about the

appearance of light towers of states as one moves through large (in general trans-Planckian) distances in moduli space. They come in both original [30] and refined versions [68].

The original version states that as one moves an asymptotic geodesic distance $d(P,Q)$ (from $P$ to $Q$ in moduli space), a tower of states becomes light, with the masses of the tower particles descending as

$$M(P) = M(Q)e^{-\lambda d(P,Q)/M_P}$$

in the limit that $d(P,Q) \to \infty$. Examples of this behaviour are the towers of Kaluza-Klein or string modes that become light on moving to asymptotically large geometric volumes, or alternatively the towers of winding/wrapped brane modes in the limit that cycle sizes collapse to zero volume.

The refined version states that the constant $\lambda$ is $\mathcal{O}(1)$ and that the behaviour is realised not merely asymptotically but already once $d(P,Q) \gtrsim M_P$. Equivalently, the tower of light modes occurs not simply at the asymptotic boundaries of moduli space but already once transPlanckian field displacements occur.

For the purpose of constraining low-energy effective field theory Lagrangians, it is the refined swampland distance conjecture that is much more powerful. The original swampland distance conjecture constrains behaviour at asymptotic distances in field space. However, the low-energy physicist is only sensitive to a particular EFT and perturbations around it, while such asymptotic displacements are not accessible as a small perturbation about any low-energy Lagrangian – and so cannot be used to say that a particular effective field theory is or is not in the swampland.

This is not so for the refined distance conjecture. The couplings and interactions of gravitationally coupled moduli $\varphi$ can be ordered by an expansion in $\left(\frac{\varphi}{M_P}\right)$, $\left(\frac{\varphi}{M_P}\right)^2$. Constraints on the behaviour of masses (for example) for transPlanckian moduli displacement constrain the form and signs of such couplings, as it is essential to the refined distance conjecture that the behaviour kick in once $\Delta\varphi \sim M_P$. This therefore provides constraints both on models of large-field inflation and also on the leading perturbative interactions of gravitationally coupled scalars (such as moduli).

### 3.3.1 A Puzzle

Although a lot of evidence exists for some form of the refined distance conjecture [227, 228, 229], at this point we want to note a puzzle concerning the refined dis-

tance conjecture. The conjecture states that towers of states should become light for geodesic displacements $\Delta\varphi \gtrsim M_P$ in moduli space. We consider an ordinary type IIB Calabi-Yau orientifold compactification in a limit of large internal geometric internal volume $\mathcal{V} \gg l_s^6$ (such as LVS). As the canonically normalised volume field is $\varphi = M_P\sqrt{\frac{3}{2}}\ln\tau_b = M_P\sqrt{\frac{2}{3}}\ln(\mathcal{V}/l_s^6)$, large field displacements ($|\Delta\varphi| \gg M_P$) can be achieved by exponential rescalings $\mathcal{V} \to e^{\pm\sqrt{\frac{3}{2}}|\Delta\varphi|/M_P}\mathcal{V}$.

For displacements with positive $\Delta\varphi > 0$, it is clear how the conjecture is satisfied. In this case there are multiple towers of modes becoming light, for example the towers of string or KK modes. Taking these as canonical examples, their masses behave as

$$M_{string} \propto \frac{M_P}{\sqrt{\mathcal{V}}}, \qquad M_{KK} \propto \frac{M_P}{\mathcal{V}^{2/3}},$$

using the standard relationship between the string scale and the 4-d Planck scale. As $M < M_P$, these indeed correspond to particle states in 4d QFT. For positive displacements $\Delta\varphi > 0$ the masses of the states in these towers then behave as

$$M_{string} = e^{-\frac{1}{2}\sqrt{\frac{3}{2}}\Delta\varphi/M_P}, \qquad M_{KK} = e^{-\frac{2}{3}\sqrt{\frac{3}{2}}\Delta\varphi/M_P}.$$

However, the refined distance conjecture refers simply to transPlanckian displacements $d(P,Q) > M_P$ and so should therefore apply equally to displacements with $\Delta\varphi < 0$ – corresponding to rescalings $\mathcal{V} \to e^{-\sqrt{\frac{3}{2}}|\Delta\varphi|}\mathcal{V}$. Equivalently, when writing an effective Lagrangian that is valid about a large-volume locus in moduli space, we are free to make a field redefinition $\varphi \to -\varphi$, while any general statement about low-energy effective Lagrangians must remain valid.

This is where the puzzle lies – for if we start at large volumes $\mathcal{V} \ggg 1$ and move inwards in moduli space towards smaller volumes, with $|\Delta\varphi| > M_P$ but $\Delta\varphi < 0$, there is no apparent tower of particle states that becomes light. It is clear that as $\mathcal{V}$ decreases the string and KK tower become heavier. What about the tower of winding states? As these have masses $M_{winding} \sim RM_{string}$, it is true that $\frac{M_{winding}}{M_{string}}$ decreases as the volume decreases. However, as $M_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}$, we have overall $M_{winding} \sim \frac{M_P}{\mathcal{V}^{1/3}}$, and so the tower of winding states also *increases* in mass under a displacement $\Delta\varphi < 0, |\Delta\varphi| > M_P$.

We can also consider states arising from wrapped branes. A D$(p+1)$-brane wrapped on an internal $p$-cycle $\Sigma_p$ corresponds to a particle state in spacetime with a mass $M \sim \left(\mathrm{Vol}(\Sigma_p)/\sqrt{\mathcal{V}}\right) M_P$. For branes wrapping internal 1-cycles or 2-cycles,

the mass of the resulting state increases under reductions in the bulk volume (as $\text{Vol}(\Sigma_p) \propto \mathcal{V}^{1/6}$ or $\mathcal{V}^{1/3}$ for bulk 1- or 2-cycles). For a brane wrapping an internal 3-cycle, the volume factors cancel and so the tower remains unaltered in mass.

These volume scalings imply that a tower of particle states decreasing in mass under reductions in volume, if coming from wrapped branes, would require the branes to wrap bulk 4-cycles (or 5-cycle or 6-cycles) in the internal space. However, in the limit of large radii $R \gg l_s$ this is immediately problematic. For a brane wrapped on a 4-cycle, the mass of such 'states' behaves as $M \sim RM_P$, and so in a large radius limit such wrapped branes have masses above the 4-d Planck scale. As such, they cannot be interpreted as particles within the 4d effective field theory and instead correspond to black holes. In the large radius limit of $R \gg l_s$ they do not provide examples of particle states that become lighter on reduction of the internal volume.

It is still true that in the strict limit of $d(P,Q) \to \infty$ (as in the original distance conjecture), this is not an issue – in the limit of infinite $d(P,Q)$ the compactification reaches down to (formally) zero radius, entering a regime where winding modes become light and wrapped brane states do correspond to particles. However, this puzzle can be formulated for any arbitrarily large but finite value of $d(P,Q)$: by working with exponentially large internal volumes, and starting arbitrarily far away from the centre of moduli space, the geodesic distance to the self-dual radius can be made arbitrarily large.

The large volume limit therefore appears to create a problem for the ordinary formulation of the refined swampland distance conjecture: under displacements $|\Delta\varphi| \gg M_P$, $\Delta\varphi < 0$, corresponding to exponential reductions in the internal volume, all the towers of heavy particle states appear to be *increasing* in mass, with $M_{tower} \to e^{+\lambda|\Delta_\varphi/M_P|}M_{tower}$, rather than decreasing.

### 3.3.2 Heavy Modes and Holographic Anomalous Dimensions

We now extend our earlier results involving the $[\varphi a]$ mixed correlator to the case of mixed correlators involving heavy modes. In particular, we examine the implications of requiring a negative anomalous dimension for double trace CFT states in which one operator corresponds to a heavy field $\psi$. 'Heavy' here is defined by a condition that the conformal dimension $\Delta_\psi$ of the field diverges in the limit that $\mathcal{V} \to \infty$. For the case of LVS, such modes can correspond either to certain moduli (such as the complex structure moduli or the small Kähler moduli) or to the KK, string or winding modes.

We restrict our analysis in this section to LVS. There are then three types of mixed states involving a heavy mode that we can consider. These are mixed states containing two heavy modes, $[\psi\psi']$, mixed states of a heavy mode and the volume modulus, $\psi\varphi$, and mixed states of a heavy mode with the light axion, $\psi a$. In each case we want to determine the signs of the anomalous dimensions $\gamma_{\psi\psi'}$, $\gamma_{\psi\varphi}$ and $\gamma_{\psi a}$. Following section 4, we restrict the analysis to scalar modes only.

As for the treatment of the light modes, the underlying parity properties of the $\varphi$ and $a$ fields imply that the relevant Mellin diagram is one involving the exchange of the $\varphi$ field – the $a$ field has odd parity and so a single $a$ field cannot be exchanged. This leaves the $\varphi$ field as the only scalar with a low conformal dimension, requiring us to determine the $C_{\psi\psi\varphi}$ structure coefficient as the only additional feature compared to the earlier analysis involving only the light modes.

In a similar fashion to the axion in KKLT, there are two contributions to this. One arises from the kinetic term coupling $f(\varphi)\partial_\mu\psi\partial^\mu\psi$, and the other from the mass term $m^2(\varphi)\frac{\psi^2}{2}$. The linear coupling can be obtained by deriving the general form of these couplings and then expanding to linear order.

For a general heavy mode $\psi$, we can determine the appearance of the volume modulus in the kinetic and mass terms as follows, by temporarily using the formalism of $\mathcal{N}=1$ supergravity. As the volume field originates as a Kähler modulus, it cannot appear in the superpotential. In an $\mathcal{N}=1$ supergravity Lagrangian, the kinetic terms and mass fields for the heavy field originate from

$$K_{\psi\bar{\psi}}\partial_\mu\psi\partial^\mu\bar{\psi} + e^K\left(K^{\psi\bar{\psi}}D_\psi W D_{\bar{\psi}}\bar{W} + \ldots\right) \in \mathcal{L}. \tag{3.48}$$

The point is that as the Kähler moduli $T$ cannot appear in the superpotential, the dependence on the volume has to go via the Kähler potential (in particular through the kinetic terms $K_{\psi\bar{\psi}}$). As the overall Kähler potential for a IIB compactification is $K = -2\ln\mathcal{V} + \ldots$, the effective Lagrangian for terms quadratic in $\psi$ is

$$\mathcal{L}_{\psi\psi} = K_{\psi\bar{\psi}}\partial_\mu\psi\partial^\mu\bar{\psi} + \frac{1}{\mathcal{V}^2}K^{\psi\bar{\psi}}\psi\bar{\psi}.$$

This allows the volume dependence of $K_{\psi\bar{\psi}}$ to be determined, as it is fixed by the requirement that the physical mass

$$m^2 = \frac{(K^{\psi\bar{\psi}})^2}{\mathcal{V}^2}$$

scale correctly as a function of the volume.

The scaling of the physical mass of a heavy mode with the volume (in the large volume limit) is determined on general principles. As examples, a string mode behaves as $M_s \propto \frac{M_P}{\sqrt{\mathcal{V}}}$, a bulk KK mode behaves as $M_{KK} \propto \frac{M_P}{\mathcal{V}^{2/3}}$ and a bulk winding mode behaves as $M_{winding} \propto \frac{M_P}{\mathcal{V}^{1/3}}$.

For example, for the case of a bulk KK mode, it follows that $K_{\psi\bar{\psi}} \sim \mathcal{V}^{-1/3}$, and so using $\varphi = \sqrt{\frac{2}{3}} \ln \mathcal{V}$, we obtain a kinetic term coupling that scales as $e^{-\sqrt{\frac{1}{6}}\varphi/M_P}$. There will be additional overall prefactors depending on complex structure (and other) moduli, but as our interest is in the coupling to the volume mode we can neglect these.

The Lagrangian describing the coupling of the light volume modulus to the heavy bulk KK modes is then

$$\frac{1}{2} e^{-\sqrt{\frac{1}{6}}(\varphi-\varphi_0)/M_P} \partial_\mu \psi \partial^\mu \psi - m_\psi^2 e^{-\frac{5}{\sqrt{6}}(\varphi-\varphi_0)/M_P} \frac{\psi^2}{2}. \tag{3.49}$$

We shift the origin of $\varphi$ by an amount $\varphi_0 = \langle\varphi\rangle$, so that the heavy field $\psi$ is canonically normalised in the vacuum. Expanding the exponential to first order, the 3-pt couplings are

$$\mathcal{L}_{\varphi\psi\psi} = -\sqrt{\frac{1}{6}} (\delta\varphi) \frac{\partial_\mu \psi \partial^\mu \psi}{2} + m_\psi^2 \frac{5}{\sqrt{6}} (\delta\varphi) \frac{\psi^2}{2}. \tag{3.50}$$

The resulting structure function is

$$f_{\varphi\psi\psi} = \frac{\Gamma\left(\frac{2\Delta_\psi + \Delta_\varphi - 3}{2}\right)}{2\Gamma(\Delta_\varphi)\Gamma(\Delta_\psi)^2} \left( \sqrt{\frac{1}{6}} \left( \Delta_\varphi + 2\Delta_\psi - \Delta_\psi^2 - 3 \right) + \frac{5}{\sqrt{6}} \Delta_\psi \left( \Delta_\psi - 3 \right) \right). \tag{3.51}$$

As once $\Delta_\psi \gg 1$ the quadratic terms in $\Delta_\psi$ dominate, this clearly satisfies $f_{\varphi\psi\psi} > 0$ as – analogously to what occurs in KKLT – the contribution from the mass term is larger than the contribution from the kinetic term. Naively, this looks reminiscent of the bound discussed in [230], since the sign remains negative as long as the coefficient in the mass term exponential is larger than the $1/\sqrt{6}$ coming from the kinetic term. However, the two numbers are not really independent in this case, and it does not appear possible to draw similar conclusions.

We can ask what condition would lead to opposite sign behaviour, $f_{\varphi\psi\psi} < 0$. This would require the kinetic coupling in Eq. (3.51) to be greater than the mass coupling. It is easy to see that the critical volume behaviour here is $K_{\psi\bar{\psi}} = \frac{1}{\mathcal{V}}$, for which the

two terms quadratic in $\Delta_\psi$ cancel (the subleading terms are $\mathcal{O}(1/N)$ suppressed and so are not trustworthy at this order). Physically this would correspond to a state that remains unaltered in mass as $\mathcal{V} \to \infty$, for example particles arising from D3-branes wrapped on holomorphic 3-cycles. Positive sign behaviour, $f_{\varphi\psi\psi} < 0$ is then equivalent to having a state that grows in mass in the $\mathcal{V} \to \infty$ limit, although as previously discussed such 'states' would have masses $m_\psi > M_P$ and so cannot be regarded as particle states in the effective field theory.

We therefore see that the overall behaviour for the heavy modes can be summarised as

$$f_{\varphi\psi\psi} > 0 \qquad \equiv \frac{\partial m^2(\psi)}{\partial \varphi} < 0.$$

$$f_{\varphi\psi\psi} < 0 \qquad \equiv \frac{\partial m^2(\psi)}{\partial \varphi} > 0.$$

That is, the sign of the structure function is determined by the behaviour of the massive states: a positive structure function is equivalent to states decreasing in mass with increased volume, and a negative structure function to the opposite.

The anomalous dimension is determined by the (negative) product of the two structure functions,

$$\gamma \propto -f_{\varphi\psi\psi} f_{\varphi\varphi\varphi}.$$

Using the earlier results for LVS, it then follows that a negative anomalous dimension for mixed states involving the heavy modes is equivalent to $\frac{\partial m^2(\psi)}{\partial \varphi} < 0$. Put another way, a requirement of a negative anomalous dimension is equivalent to requiring heavy states to decrease in mass as the field $\varphi$ moves towards the asymptotic large volume regime of moduli space. This is interesting as it shows that, within this context, the physics of the distance conjecture can be derived from a simple statement about the sign of anomalous dimensions of mixed double trace operators.

Interestingly, this condition also addresses the puzzle raised above with the refined distance conjecture. Under the redefinition $\varphi \to -\varphi$, the sign of *both* structure coefficients $C_{\varphi\psi\psi}$ and $C_{\varphi\varphi\varphi}$ (or alternatively $C_{\varphi aa}$) change signs. Although in this Lagrangian, the heavy tower of states all now *increase* in mass for $\Delta\varphi > 0$, as we are headed towards the centre of moduli space, the anomalous dimensions remain negative as they are sensitive to the product of the signs of the structure coefficients. So the formulation in terms of signs of anomalous dimensions appears more fundamental as it captures the correct behaviour in this regime as well.

### 3.3.3 The Presence of the Tower of States

We therefore see that our hypothetical constraint on the sign of anomalous dimensions for mixed double trace operators captures some of the correct behaviour associated to the swampland distance conjecture. However, the swampland distance conjecture also has a first element - the *existence* of towers of heavy states that become light for transPlanckian field displacements. The above constraints on anomalous dimensions constrain the behaviour of the heavy states – assuming they exist. However, can CFT arguments be used to understand the *existence* of this tower of states?

What does it mean for a tower of heavy states to exist? This can be formulated in terms of the density of operators by conformal dimension, $\rho(\Delta)$. LVS has a small number of single-trace primary states with $\Delta \sim \mathcal{O}(1)$, as well as other states are built up from these as double-trace (or higher) operators. For conformal dimensions $\Delta \lesssim \mathcal{V}^{5/6}$ the spectrum of states (on the AdS side) consists of quantum field theory on AdS space, and is built up as the Fock space of states. The analogous behaviour on the CFT side tells us that for $\Delta \lesssim \mathcal{V}^{5/6}$, $\rho(\Delta)$ grows as for the Fock space of the Generalised Free Field Theory.

However, once $\Delta \gtrsim \mathcal{V}^{5/6}$, the operators dual to the tower of KK modes enter the spectrum, and for $\Delta \gtrsim \mathcal{V}$ the operators dual to perturbative string states enter the spectrum. These states are still far below the Planck scale (as $m_s \ll M_P$ in the asymptotic large volume regime), and indeed it is not until $\Delta \gtrsim \mathcal{V}^{3/2}$ that the CFT will enter the regime in which operators dual to black hole states appear as part of the spectrum. As the number of KK modes grows power-law with energy and the number of string states grows exponentially, this leads to a qualitative change in the functional form of $\rho(\Delta)$ for $\Delta \gtrsim \mathcal{V}^{5/6}$.

So – in the context of LVS – the statement that a tower of heavy states exists is equivalent to the statement that the growth in operator density increases dramatically beyond a certain $\Delta_{crit} \ll \Delta_{BH}$, where $\Delta_{BH}$ is the conformal dimension corresponding to operators dual to black hole states, in a way that no longer can be described by the QFT of single particles on $AdS_4$ space.

Such statements about the spectrum of operator densities have been proven in the context of 2d CFTs in the context of the idea of string universality [231] and analogous statements may also hold for higher-dimensional CFTs (see [162] for some work in this direction).

## 3.4 Counter-examples? Fibred Calabi-Yau Moduli Stabilization

In the previous sections we have studied holographic aspects of certain moduli stabilisation scenarios, in particular LVS, KKLT and racetrack scenarios. These displayed interesting properties. LVS displayed a universality of conformal dimensions in the large-volume limit, and all scenarios had an interesting negative behaviour for the mixed anomalous dimensions. We are therefore interested in extending these studies to a wider range of scenarios, and in this section we consider fibred versions of the original LVS.

### 3.4.1 A Brief Review of Fibred Calabi-Yau Scenarios

One interesting variation on the original LVS is to consider models with fibred Calabi-Yau (CY) structures. In such models, there are multiple large moduli associated both to base and fibre moduli [157, 158]. We shall take the form of the volume of a fibred Calabi-Yau to be given as the following (based on [157]):

$$\mathcal{V} = f_{\frac{3}{2}}(\tau_j) - \sum_{i=1}^{N_{small}} \lambda_i \tau_i^{3/2}, \tag{3.52}$$

where $f$ is a homogeneous function of degree 3/2, $j$ runs from 1 to $N_{large}$ and $h_{1,1} = N_{small} + N_{large}$, where $N_{large}$ is the number of large moduli and $N_{small}$ is the number of blow up moduli.

The fibred CY shares a similar feature with the "Swiss Cheese" form used in the original LVS, in terms of the blow-up moduli included in the volume. The difference lies in the function $f$, as this now involves multiple moduli: both the overall volume modulus and the fibration moduli. Different moduli play different roles. The volume modulus controls the LVS potential. The fibre modulus is a flat direction of the LVS potential, which means it will be absent in the LVS potential at leading order. However, it will appear in the kinetic terms, and also at subleading order in the string loop expansion. The blow-up modulus controls the heavy mode which will subsequently be integrated out of the LVS potential within the low-energy effective field theory.

In the next two subsections, we will use two specific examples to illustrate these ideas which will involve respectively one or two fibre moduli.

### 3.4.2 Fibred Calabi-Yau: Example with One Fibre Modulus

We will first look at moduli stabilisation in order to establish the AdS effective action (essentially a review of [26]), before considering the holographic implications of this effective action.

#### Moduli Stabilisation

We assume as in [157] that the volume takes the form:

$$\mathcal{V} = \alpha(\sqrt{\tau_1}\tau_2) - \gamma\tau_3^{3/2}, \tag{3.53}$$

where $\alpha$ is a constant determined by the topology of the manifold. Here $\tau_1$ controls the volume of the fibre, $\tau_2$ controls the volume of the base and $\tau_3$ describes the blow-up modulus.

The Large Volume Scenario is a specific type IIB flux compactification scenario, which stabilises all moduli in a non-SUSY AdS vacuum at an exponentially large volume $\mathcal{V}$ (See Section 2.1 and [23, 24, 27]). Following the usual approach in LVS, after stabilising and integrating out the complex structure moduli and dilaton, the Kähler potential and superpotential read:

$$K = -2\ln(\mathcal{V} + \hat{\xi}), \tag{3.54}$$

$$W = W_0 + A_3 e^{-a_3 T_3}, \tag{3.55}$$

$$\hat{\xi} = \frac{\xi}{g_s^{\frac{3}{2}}}, \tag{3.56}$$

$$\xi = -\frac{\chi(X)\zeta(3)}{2(2\pi)^3}, \tag{3.57}$$

Here $\chi(X)$ is the Euler number of the Calabi-Yau manifold X, $\zeta$ is the Riemann zeta function with $\zeta(3) \approx 1.2$. $T_3$ is the chiral multiplet for the blow-up Kähler modulus, $T_3 = \tau_3 + iC_3$.

The $\mathcal{N} = 1$ F-term supergravity scalar potential takes the following form:

$$V = e^K(K^{i\bar{j}} D_i W D_{\bar{j}} \overline{W} - 3|W|^2), \tag{3.58}$$

where we have set $M_P = 1$. After substituting the Kähler potential and superpotential into the scalar potential, the effective potential becomes:

$$V_{scalar} = \frac{Aa_3^2\sqrt{\tau_3}}{\mathcal{V}}e^{-2a_3\tau_3} - \frac{BW_0a_3A_3\tau_3}{\mathcal{V}^2}e^{-a_3\tau_3} + \frac{C\xi W_0^2}{g_s^{\frac{3}{2}}\mathcal{V}^3}, \tag{3.59}$$

where $A, B, C$ are numerical constants. The important point is that the fibre modulus does not appear in the effective potential at tree level, and so at this order is a flat direction of the scalar potential. Therefore there is an additional light scalar degree of freedom, which is an interesting feature of fibred Calabi-Yau models compared to the original LVS.

As with standard LVS, this scalar potential has a minimum at exponentially large volumes,

$$\langle\tau_3\rangle \sim \frac{\xi^{\frac{2}{3}}}{g_s}, \tag{3.60}$$

$$\langle\mathcal{V}\rangle \sim e^{a_3\langle\tau_3\rangle}. \tag{3.61}$$

After integrating out the blow-up modulus $\tau_3$, the effective potential for the LVS volume modulus reads [37]:

$$V_{scalar} = [-A^{'}\left(\ln(f\mathcal{V})\right)^{\frac{3}{2}} + B^{'}]\frac{W_0^2}{\mathcal{V}^3}. \tag{3.62}$$

where $A^{'}$, $B^{'}$ are numerical constants.

**Kinetic Terms and Canonical Fields**

The effective field theory of these fibred models contains four light moduli: one overall volume modulus which is massive, plus one fibre modulus and two axions, which are approximately massless. In order to see how they interact with each other, we write down the kinetic terms for the theory in the large volume limit using standard EFT techniques. Neglecting higher order terms, the kinetic terms read:

$$L_{kin} = \frac{1}{4\tau_1^2}\partial_u\tau_1\partial^u\tau_1 + \frac{1}{2\tau_2^2}\partial_u\tau_2\partial^u\tau_2 + \frac{1}{4\tau_1^2}\partial_u a_1\partial^u a_1 + \frac{1}{2\tau_2^2}\partial_u a_2\partial^u a_2. \tag{3.63}$$

As a first step in diagonalising the action, we re-express the mode $\tau_2$ in terms of the

overall LVS volume:

$$\tau_2 = \frac{\mathcal{V}}{\alpha\sqrt{\tau_1}}. \tag{3.64}$$

The kinetic terms then become

$$\mathcal{L} \supset \frac{3}{8\tau_1^2}\partial_u\tau_1\partial^u\tau_1 - \frac{1}{2\tau_1\mathcal{V}}\partial_u\tau_1\partial^u\mathcal{V} + \frac{1}{2\mathcal{V}^2}\partial_u\mathcal{V}\partial^u\mathcal{V} + \frac{1}{4\tau_1^2}\partial_u a_1\partial^u a_1 + \frac{\alpha^2\tau_1}{2\mathcal{V}^2}\partial_u a_2\partial^u a_2. \tag{3.65}$$

We next diagonalise both kinetic terms and mass terms in order to get a canonically normalised theory,

$$\ln\tau_1 = a\Phi_1 + b\Phi_2, \tag{3.66}$$

$$\ln\mathcal{V} = c\Phi_1 + d\Phi_2. \tag{3.67}$$

Here $\Phi_1$ and $\Phi_2$ will be the canonically normalised fields. Before we go into the details of the calculation, we make an observation. As the mass term only depends on the overall LVS volume, we must choose either $c = 0$ or $d = 0$ when diagonalising. Here we choose $d = 0$, so our ansatz becomes:

$$\begin{aligned} \ln\tau_1 &= a\Phi_1 + b\Phi_2, \\ \ln\mathcal{V} &= c\Phi_1. \end{aligned} \tag{3.68}$$

We refer interested readers to appendix B.1 for the details. Here we just state the final results:

$$\begin{aligned} \ln\tau_1 &= \sqrt{\frac{2}{3}}(\Phi_1 + \sqrt{2}\Phi_2), \\ \ln\mathcal{V} &= \sqrt{\frac{3}{2}}\Phi_1. \end{aligned} \tag{3.69}$$

The kinetic terms then read:

$$\mathcal{L} \supset \frac{1}{2}(\partial_u\Phi_1)^2 + \frac{1}{2}(\partial_u\Phi_2)^2 + \frac{1}{2}e^{-2\sqrt{\frac{2}{3}}(\Phi_1+2\Phi_2)}\partial_u a_1\partial^u a_1 + \frac{1}{2}e^{\left(\sqrt{\frac{2}{3}}-2\sqrt{\frac{3}{2}}\right)\Phi_1 + 2\sqrt{\frac{2}{3}}\Phi_2}\partial_u a_2\partial^u a_2, \tag{3.70}$$

where the axions have also been rescaled by a constant factor to achieve canonical normalization. The potential only depends on $\Phi_1$ and so is diagonal in this basis. Having expressed the Lagrangian in canonical form, we can now move to a discussion of the interactions and their holographic interpretation.

### 3.4.3 Holographic Interpretation

From the couplings in (3.70) and in the scalar potential, we can easily read off the OPE coefficients following Eqs (2.60)-(2.61). The results are the following:

$$
\begin{aligned}
f^{\Phi_1}_{a_1a_1} &= \sqrt{\frac{2}{3}}\frac{\pi^{\frac{d}{2}}\Gamma\left(\frac{2\Delta_{a_1}+\Delta_{\Phi_1}-d}{2}\right)}{\Gamma\left(\Delta_{\Phi_1}\right)\Gamma\left(\Delta_{a_1}\right)^2}\left(\Delta_{\Phi_1}+2\Delta_{a_1}-\Delta_{a_1}^2-3\right), \\
f^{\Phi_2}_{a_1a_1} &= 2\sqrt{\frac{2}{3}}\frac{\pi^{\frac{d}{2}}\Gamma\left(\frac{2\Delta_{a_1}+\Delta_{\Phi_2}-d}{2}\right)}{\Gamma\left(\Delta_{\Phi_2}\right)\Gamma\left(\Delta_{a_1}\right)^2}\left(\Delta_{\Phi_2}+2\Delta_{a_1}-\Delta_{a_1}^2-3\right), \\
f^{\Phi_1}_{a_2a_2} &= \left(\sqrt{\frac{3}{2}}-\frac{1}{2}\sqrt{\frac{2}{3}}\right)\frac{\pi^{\frac{d}{2}}\Gamma\left(\frac{2\Delta_{a_2}+\Delta_{\Phi_1}-d}{2}\right)}{\Gamma\left(\Delta_{\Phi_1}\right)\Gamma\left(\Delta_{a_2}\right)^2}\left(\Delta_{\Phi_1}+2\Delta_{a_2}-\Delta_{a_2}^2-3\right), \\
f^{\Phi_2}_{a_2a_2} &= -\sqrt{\frac{2}{3}}\frac{\pi^{\frac{d}{2}}\Gamma\left(\frac{2\Delta_{a_2}+\Delta_{\Phi_2}-d}{2}\right)}{\Gamma\left(\Delta_{\Phi_2}\right)\Gamma\left(\Delta_{a_2}\right)^2}\left(\Delta_{\Phi_2}+2\Delta_{a_2}-\Delta_{a_2}^2-3\right), \\
f^{\Phi_1}_{\Phi_1\Phi_1} &= \frac{81\sqrt{6}}{2}\frac{3\pi^{\frac{d}{2}}\Gamma\left(\frac{3\Delta_{\Phi_1}-3}{2}\right)}{\Gamma\left(\Delta_{\Phi_1}\right)^3}.
\end{aligned}
\tag{3.71}
$$

The key difference between the fibred Calabi-Yau LVS model and the original LVS model is that there are a total of 3 massless moduli in the effective field theory rather than just one. This leads to operator mixing in the operator product expansion. On the other hand, the volume modulus behaves exactly as in the pure LVS scenario, obtaining a conformal dimension of [1, 37]

$$
\Delta_{\Phi_1} = \frac{3(1+\sqrt{19})}{2}, \tag{3.72}
$$

and with the same self-couplings.

In principle, the following double trace operators are degenerate

$$
[a_1a_1]_{n,\ell}, \quad [a_2a_2]_{n,\ell}, \quad [a_1a_2]_{n,\ell}, \quad [\Phi_2\Phi_2]_{n,\ell}, \quad [a_1\Phi_2]_{n,\ell}, \quad [a_2\Phi_2]_{n,\ell}, \tag{3.73}
$$

while

$$
[a_1\Phi_1]_{n,\ell}, \quad [a_2\Phi_1]_{n,\ell} \tag{3.74}
$$

also mix within themselves. Each axion is invariant under an independent discrete $\mathbb{Z}_2$ symmetry acting as $a_i \to -a_i$, which results in selection rules for the double trace operators that can appear in a given OPE. This implies that states in different

symmetry classes cannot mix with one another even though their classical dimension is the same. This is shown in Table 3.1.

| | $\mathbb{Z}_2(a_1)$ | $\mathbb{Z}_2(a_2)$ |
|---|---|---|
| $[a_1\Phi_2]_{n,\ell}$ | -1 | 1 |
| $[a_2\Phi_2]_{n,\ell}$ | 1 | -1 |
| $[a_1a_2]_{n,\ell}$ | -1 | -1 |
| $[a_1a_1]_{n,\ell}, [a_2a_2]_{n,\ell}, [\Phi_2\Phi_2]_{n,\ell}$ | 1 | 1 |
| $[a_1\Phi_1]_{n,\ell}$ | -1 | 1 |
| $[a_2\Phi_1]_{n,\ell}$ | 1 | -1 |

Table 3.1: Irreducible representation of the double trace operators under the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry. The two different blocks correspond to distinct subspaces, where the degeneracies are partly broken by the discrete symmetries.

For the non-degenerate operators involving different fields $[a_1\Phi_1]_{0,\ell}$, $[a_2\Phi_1]_{0,\ell}[a_1\Phi_2]_{0,\ell}$, $[a_2\Phi_2]_{0,\ell}$, $[a_1a_2]_{0,\ell}$, we can directly refer to (2.58) to work out the anomalous dimension. In particular, since the kinematic factors are equal for the two axions

$$\begin{aligned}
&\gamma(0,\ell)_{a_1\Phi_1} \propto -f_{a_1a_1\Phi_1}f_{\Phi_1\Phi_1\Phi_1} < 0,\\
&\gamma(0,\ell)_{a_2\Phi_1} = 0,\\
&\gamma(0,\ell)_{a_1\Phi_2} = 0,\\
&\gamma(0,\ell)_{a_2\Phi_2} = 0.\\
&\gamma(0,\ell)_{a_1a_2} \propto -f_{a_1a_1\Phi_2}f_{a_2a_2\Phi_2} > 0,
\end{aligned} \tag{3.75}$$

The first two anomalous dimensions are exactly those of LVS without fibred directions, and their agreement with our original conjecture should not come as a surprise. However, $\gamma(0,\ell)_{a_1a_2}$ is positive, violating the expected behaviour. The interpretation for this is that maintaining a fixed volume requires one cycle to increase in size while the other decreases, and so the coefficients reflecting the coupling of the associated axions to the modulus have opposite signs.

Let us now analyse the properly degenerate subspace. Taking perturbative corrections into account [232, 156, 155, 177], D-brane loops lift the flat potential for $\Phi_2$ with a scaling

$$V_{\text{loop}} \sim \frac{1}{\mathcal{V}^{10/3}}, \tag{3.76}$$

which is parametrically faster than $\mathcal{O}(1/N)$ and thus dominates over anomalous dimensions. Therefore the degeneracy between $a_1, a_2$ and $\Phi_2$ breaks to $a_1$ and $a_2$ only. Since there are no interactions mixing $a_1$ and $a_2$ in the Lagrangian, the resulting matrix is diagonal, with

$$\begin{aligned}\gamma(0,\ell)_{a_1a_1} &\propto -f^2_{a_1a_1\Phi_2} < 0,\\ \gamma(0,\ell)_{a_2a_2} &\propto -f^2_{a_2a_2\Phi_2} < 0.\end{aligned} \tag{3.77}$$

Similarly, the anomalous dimension is negative (by construction) for $[\Phi_2\Phi_1]_{0,\ell}$.

### 3.4.4 Fibred Calabi-Yau: Example with Two Fibred Moduli

A similar analysis can be carried out for other fibred Calabi-Yau models (here Appendix B.2 gives the full details). One direct extension is to consider a model with two fibred moduli, in which the volume of the fibred Calabi-Yau manifold reads:

$$\mathcal{V} = \sqrt{\tau_1\tau_2\tau_3} - \tau_s^{3/2}. \tag{3.78}$$

Here $\tau_1, \tau_2, \tau_3$ control the volumes of individual 4-cycles, with the overall volume direction corresponding in the rescaling $\tau_i \to \lambda\tau_i$. The $\sqrt{\tau_1\tau_2\tau_3}$ form is reminiscent of toroidal orbifolds based on $T_2 \times T_2 \times T_2$, while $\tau_s$ is the blow up mode necessary to realize LVS.

From the large volume form of the Kähler metric (See Appendix B.2), the kinetic terms for the moduli $\tau_1, \tau_2, \tau_3$ and their axion partners can be expressed as the following:

$$\begin{aligned}\mathcal{L} \supset \frac{1}{4\tau_1^2}\partial^u\tau_1\partial_u\tau_1 + \frac{1}{4\tau_1^2}\partial^u a_1\partial_u a_1 + \frac{1}{4\tau_2^2}\partial^u\tau_2\partial_u\tau_2 + \\ \frac{1}{4\tau_2^2}\partial^u a_2\partial_u a_2 + \frac{1}{4\tau_3^2}\partial^u\tau_3\partial_u\tau_3 + \frac{1}{4\tau_3^2}\partial^u a_3\partial_u a_3.\end{aligned} \tag{3.79}$$

Following similar techniques to the last section, we can substitute one of the flat moduli with the volume modulus, diagonalise the kinetic term and write down the scalar potential. After putting all the constituents together, the resulting effective field theory reads:

$$\begin{aligned}\mathcal{L} =&\frac{1}{2}(\partial_u\Phi_1)^2+\frac{1}{2}(\partial_u\Phi_2)^2+\frac{1}{2}(\partial_u\Phi_3)^2+\frac{1}{2}e^{-2\sqrt{\frac{2}{3}}\Phi_1-2\sqrt{\frac{1}{3}}\Phi_2-2\Phi_3}(\partial_u a_1)^2\\&+\frac{1}{2}e^{-2\sqrt{\frac{2}{3}}\Phi_1-2\sqrt{\frac{1}{3}}\Phi_2+2\Phi_3}(\partial_u a_2)^2+\frac{1}{2}e^{\left(2\sqrt{\frac{2}{3}}-2\sqrt{\frac{3}{2}}\right)\Phi_1+4\sqrt{\frac{1}{3}}\Phi_2}(\partial_u a_3)^2-V_{LVS},\end{aligned} \tag{3.80}$$

where the axions have also been rescaled to give canonical kinetic terms. As before, the OPE coefficients can be determined with Eqs (2.60)-(2.61). In particular, they are

$$\begin{aligned}
f^{\Phi_1}_{a_1a_1} &= \sqrt{\frac{2}{3}}\frac{\pi^{\frac{d}{2}}\Gamma\left(\frac{2\Delta_{a_1}+\Delta_{\Phi_1}-d}{2}\right)}{\Gamma\left(\Delta_{\Phi_1}\right)\Gamma\left(\Delta_{a_1}\right)^2}\left(\Delta_{\Phi_1}+2\Delta_{a_1}-\Delta_{a_1}^2-3\right),\\
f^{\Phi_2}_{a_1a_1} &= \sqrt{\frac{1}{3}}\frac{\pi^{\frac{d}{2}}\Gamma\left(\frac{2\Delta_{a_1}+\Delta_{\Phi_2}-d}{2}\right)}{\Gamma\left(\Delta_{\Phi_2}\right)\Gamma\left(\Delta_{a_1}\right)^2}\left(\Delta_{\Phi_2}+2\Delta_{a_1}-\Delta_{a_1}^2-3\right),\\
f^{\Phi_3}_{a_1a_1} &= \frac{\pi^{\frac{d}{2}}\Gamma\left(\frac{2\Delta_{a_1}+\Delta_{\Phi_3}-d}{2}\right)}{\Gamma\left(\Delta_{\Phi_3}\right)\Gamma\left(\Delta_{a_1}\right)^2}\left(\Delta_{\Phi_3}+2\Delta_{a_1}-\Delta_{a_1}^2-3\right),\\
f^{\Phi_1}_{a_2a_2} &= \sqrt{\frac{2}{3}}\frac{\pi^{\frac{d}{2}}\Gamma\left(\frac{2\Delta_{a_2}+\Delta_{\Phi_1}-d}{2}\right)}{\Gamma\left(\Delta_{\Phi_1}\right)\Gamma\left(\Delta_{a_2}\right)^2}\left(\Delta_{\Phi_1}+2\Delta_{a_2}-\Delta_{a_2}^2-3\right),\\
f^{\Phi_2}_{a_2a_2} &= \sqrt{\frac{1}{3}}\frac{\pi^{\frac{d}{2}}\Gamma\left(\frac{2\Delta_{a_2}+\Delta_{\Phi_2}-d}{2}\right)}{\Gamma\left(\Delta_{\Phi_2}\right)\Gamma\left(\Delta_{a_2}\right)^2}\left(\Delta_{\Phi_2}+2\Delta_{a_2}-\Delta_{a_2}^2-3\right),\\
f^{\Phi_3}_{a_2a_2} &= -\frac{\pi^{\frac{d}{2}}\Gamma\left(\frac{2\Delta_{a_2}+\Delta_{\Phi_3}-d}{2}\right)}{\Gamma\left(\Delta_{\Phi_3}\right)\Gamma\left(\Delta_{a_2}\right)^2}\left(\Delta_{\Phi_3}+2\Delta_{a_2}-\Delta_{a_2}^2-3\right),\\
f^{\Phi_1}_{a_3a_3} &= \left(\sqrt{\frac{3}{2}}-\sqrt{\frac{2}{3}}\right)\frac{\pi^{\frac{d}{2}}\Gamma\left(\frac{2\Delta_{a_3}+\Delta_{\Phi_1}-d}{2}\right)}{\Gamma\left(\Delta_{\Phi_1}\right)\Gamma\left(\Delta_{a_3}\right)^2}\left(\Delta_{\Phi_1}+2\Delta_{a_3}-\Delta_{a_3}^2-3\right),\\
f^{\Phi_2}_{a_3a_3} &= -2\sqrt{\frac{1}{3}}\frac{\pi^{\frac{d}{2}}\Gamma\left(\frac{2\Delta_{a_3}+\Delta_{\Phi_2}-d}{2}\right)}{\Gamma\left(\Delta_{\Phi_2}\right)\Gamma\left(\Delta_{a_3}\right)^2}\left(\Delta_{\Phi_2}+2\Delta_{a_3}-\Delta_{a_3}^2-3\right),\\
f^{\Phi_3}_{a_3a_3} &= 0,\\
f^{\Phi_1}_{\Phi_1\Phi_1} &= \frac{81\sqrt{6}}{2}\frac{3\pi^{\frac{d}{2}}\Gamma\left(\frac{3\Delta_{\Phi_1}-3}{2}\right)}{\Gamma\left(\Delta_{\Phi_1}\right)^3}.
\end{aligned} \tag{3.81}$$

Analogously to what happens in the previous examples, all the axions and the two fibre moduli are massless at zeroth order, leading to a large degeneracy between the double trace operators. A first simplification is again provided by the existence of a $\mathbb{Z}_2\times\mathbb{Z}_2\times\mathbb{Z}_2$, symmetry, which partially breaks it down to a subset of operators in

| | $\mathbb{Z}_2(a_1)$ | $\mathbb{Z}_2(a_2)$ | $\mathbb{Z}_2(a_3)$ |
|---|---|---|---|
| $[a_1 a_2]_{n,\ell}$ | -1 | -1 | 1 |
| $[a_1 a_3]_{n,\ell}$ | -1 | 1 | -1 |
| $[a_2 a_3]_{n,\ell}$ | 1 | -1 | -1 |
| $[a_1\Phi_2]_{n,\ell}, [a_1\Phi_3]_{n,\ell}$ | -1 | 1 | 1 |
| $[a_2\Phi_2]_{n,\ell}, [a_2\Phi_3]_{n,\ell}$ | 1 | -1 | 1 |
| $[a_3\Phi_2]_{n,\ell}, [a_3\Phi_3]_{n,\ell}$ | 1 | 1 | -1 |
| $[a_1 a_1]_{n,\ell}, [a_2 a_2]_{n,\ell}, [a_3 a_3]_{n,\ell}, [\Phi_2\Phi_2]_{n,\ell}, [\Phi_3\Phi_3]_{n,\ell}, , [\Phi_2\Phi_3]_{n,\ell}$ | 1 | 1 | 1 |
| $[a_1\Phi_2]_{n,\ell}, [a_1\Phi_1]_{n,\ell}$ | -1 | 1 | 1 |
| $[a_2\Phi_2]_{n,\ell}, [a_2\Phi_1]_{n,\ell}$ | 1 | -1 | 1 |
| $[a_3\Phi_2]_{n,\ell}, [a_3\Phi_1]_{n,\ell}$ | 1 | 1 | -1 |

Table 3.2: Irreducible representation of the double trace operators under the $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry. The two different blocks correspond to distinct subspaces, where the degeneracies are partly broken by the discrete symmetries.

the same representations. A classification is provided in Table 3.2.

We are now ready to discuss the anomalous dimensions. For operators involving axions only,

$$\begin{aligned}
\gamma_{a_1 a_2} &\propto -(f^{\Phi_2}_{a_1 a_1} f^{\Phi_2}_{a_2 a_2} + f^{\Phi_3}_{a_1 a_1} f^{\Phi_3}_{a_2 a_2}) > 0, \\
\gamma_{a_1 a_3} &\propto -(f^{\Phi_2}_{a_1 a_1} f^{\Phi_2}_{a_3 a_3}) > 0, \\
\gamma_{a_2 a_3} &\propto -(f^{\Phi_2}_{a_2 a_2} f^{\Phi_2}_{a_3 a_3}) > 0.
\end{aligned} \tag{3.82}$$

We have obtained the same, puzzling result encountered in the previous section - positive anomalous dimensions for operators involving two axions. Moreover, all three anomalous dimensions are identical in this case. On the other hand, for the ones involving the volume modulus

$$\begin{aligned}
\gamma_{a_1\Phi_1} &\propto -f^{\Phi_1}_{a_1 a_1} f^{\Phi_1}_{\Phi_1\Phi_1} < 0, \\
\gamma_{a_2\Phi_1} &\propto -f^{\Phi_1}_{a_2 a_2} f^{\Phi_1}_{\Phi_1\Phi_1} < 0, \\
\gamma_{a_3\Phi_1} &\propto -f^{\Phi_1}_{a_3 a_3} f^{\Phi_1}_{\Phi_1\Phi_1} < 0
\end{aligned} \tag{3.83}$$

are all negative as in previous studies. Again, all of the operators of the form $[a_i\Phi_2]_{0,\ell}, [a_j\Phi_3]_{0,\ell}$ decouple through a combination of the discrete symmetries and sub-leading volume correction, which lift the degeneracy between $\Phi_2$ and $\Phi_3$ (and with the axions too). In particular they are all zero at this order in perturbation theory. The same happens for $[\Phi_2\Phi_2]_{0,\ell}, [\Phi_2\Phi_3]_{0,\ell}, [\Phi_3\Phi_3]_{0,\ell}$, which do not mix with

the axion double trace operators for the same reason. Finally,

$$[a_1 a_1]_{n,\ell}, \quad [a_2 a_2]_{n,\ell}, \quad [a_3 a_3]_{n,\ell}$$

are still degenerate, but the corresponding anomalous dimension matrix is diagonal and all entries are necessarily negative, since they are proportional to a sum of squared OPE coefficients (with a minus sign in front).

## 3.5 Bounds from anomalous dimensions: an outlook

Moduli stabilisation is central for reconciling String Theory with experimental observations. Many of the known constructions, however, rely on a series of approximations which may not fully be justified and often involve complicated ingredients that have to be added in 'by hand'. In this chapter, we have argued that the study of CFT duals to the low energy sector of weakly coupled AdS vacua may help to shed some light on the validity of such constructions, in a way that is complementary to standard arguments. Furthermore, this fits in the recent line of developments concerning the Swampland program, a tentative search of general criteria which low energy theories must satisfy in order to be compatible with Quantum Gravity.

Interestingly, one quantity - the sign of the mixed anomalous dimension $\gamma^{\varphi a}(0,\ell)$, corresponding to double trace operators built out of two non-identical primaries - appears to correlate well with swampland constraints on the effective low-energy Lagrangians for moduli-stabilised string vacua. In particular, the requirement that this takes a negative sign appears to reproduce various swampland constraints, both for LVS and the related methods of perturbative stabilisation, as well as for the qualitatively different scenarios of KKLT and racetrack stabilisation. This analysis has also revealed that LVS is rather close to a 'critical' point where the sign of the anomalous dimension would change. Furthermore, the same requirement seems to generically relate to the distance conjecture in the large volume limit. In the context of LVS, this is also connected to a possible puzzle within the refined version of the conjecture pointed out in 3.3.1. However, in the IIB fibred CY construction, this is only true for operators involving an axion and a modulus. Indeed, all the double trace operators made out of two separate axions now have positive anomalous dimensions.

CFT methods offer promise for rigorous formulations for swampland constraints on semi-realistic string vacua, at least for the first step of AdS vacua. This thesis has

made some exploratory steps in this direction. However, there remain some unclear points which require further investigation:

- Our focus has been on contributions and anomalous dimensions arising from scalar exchange. Thus gravity appears to be left out, in the sense that graviton exchange contributions to the Mellin amplitudes will always dominate at large enough $\ell$ as $\Delta_{g_{\mu\nu}u} = 3$. One potential approach would be to compute $\gamma(0,\ell)$ at finite values of $\ell$ and verify that the sign is negative for some intermediate finite value $\ell_f \geq 2$, but computations away from the asymptotic limit $\ell \gg 1$ are technically more complicated.

- For these studied cases of moduli stabilisation, the negativity of the mixed anomalous dimensions arises from the exchange of a scalar modulus that corresponds to a volume modulus. Morally, this mode corresponds to a dimensional reduced 10d graviton, and so 'ought' to lead to negative anomalous dimensions as gravity is universally attractive. It would be interesting to know whether this extends to more general scenarios, and also under what conditions negativity of the mixed anomalous dimensions occur in arbitrary CFTs.

- Anomalous dimensions of double trace operators can be interpreted as the binding energies of two-particle states on AdS [35, 193]. One might wonder whether this could provide a more transparent interpretation for the negativity of $\gamma^{\varphi a}(0,\ell)$.

- It would be interesting if the low-spin behaviour of anomalous dimensions - namely $\gamma(0,0)$ and $\gamma(0,1)$ - could be used to infer something about the quartic vertices, perhaps with some additional assumptions. From a Minkowski perspective, this would be equivalent to asking whether anything can be said on the first two coefficients in the $s$ expansion of a forward scattering amplitude.

Aside from these issues, our work can ideally be expanded in two different kinds of directions, corresponding to the trade-off between applicability and rigour that is typical of swampland conjectures. The first possibility consists in testing our speculative criterium for other classes of phenomenologically interesting compactifications. In the case of the type IIA DGKT vacua, this will be done in Chapter 3. At the other end of the spectrum, it would be useful to investigate precisely to what extent the well-established bounds discussed in 2.4.2 carry implications for low energy theories of Quantum Gravity on AdS.

# Chapter 4

# Integer conformal dimensions

This Chapter is based on the papers [2, 3].

## 4.1 DGKT: a simple example

Up to this point, we have exclusively focused on scenarios arising in type IIB compactifications: LVS, KKLT and variations thereof. However, it is clearly right that we also explore qualitatively different moduli stabilisation scenarios, in particular those arising from IIA flux compactifications. One significant difference with these is that, in a type IIA setting, moduli stabilisation with fluxes can be achieved at tree-level due to the simultaneous presence of the NS-NS $B_2$ form and the R-R odd $p$-forms. In particular, this is obtained by compactifying massive IIA string theory on Calabi-Yau orientifolds with O6-planes, which allows for stable $\mathrm{AdS}_4$ vacua [41, 233]. These are known as the DGKT vacua [41], and are one of the few known examples to exhibit the remarkable property of scale separation. For this reason, DGKT vacua have received considerable attention lately [111, 112, 114, 121, 122, 234, 235], as the scale separation property has been conjectured to lie in the Swampland [107, 109, 124, 236].[1] From a holographic point of view, scale-separated vacua also admit a clear interpretation in terms of CFTs with a gapped spectrum, which could also point towards inconsistencies [165]. Moreover, another puzzling behaviour was pointed out in the early investigations of [39, 237]: the DGKT dual has a central charge growing with $N$ as $c \sim N^{9/2}$, which is larger than any known system. [2] Therefore, DGKT provides a

[1] However, both the AdS moduli conjecture of [109] and a refined version of the Strong AdS distance conjecture [107] presented in [124] are consistent with DKGT.

[2] The fastest known scaling is $c \sim N^3$, from 6d SCFTs [238] dual to M-theory on $AdS_7 \times S^4$ [239, 240].

particularly interesting example to attack with our techniques.

In particular, the work of [41] found that type IIA Calabi-Yau orientifolds with fluxes admit stable $AdS_4$ vacua in the controlled regime of large volume and weak coupling (for earlier work on the effective action of IIA orientifolds see [241, 242]), and presented an explicit example where all moduli are stabilized, which will form the basis of our analysis. This construction is based on the toroidal orientifold $T^6/\mathbb{Z}_3^2$, where the discrete symmetries fix the shape of the tori and thus eliminate all complex structure moduli ($h^{2,1} = 0$). Aside from the blow-up modes associated to the singularities, the only moduli are the Kähler moduli determining the size of the three tori, with their associated $B$-field axions, and the dilaton, with an axionic partner coming from the $C_3$ three-form. They are all stabilised by fluxes. The construction can also be generalised for an arbitrary orientifold, as will be shown in Section 4.2.

In 4D, the low energy theory is then characterized by a small number of fields which are light in the holographic sense; i.e. whose conformal dimensions remain finite in the limit of an infinite volume. While previous investigations into the nature of the dual to this scenario [39] have concentrated on more generic properties of the whole spectrum, we again focus on the detailed properties of this low-lying CFT subsector. From our perspective, the low number of moduli is thus crucial to ensure a manageable effective CFT.

### 4.1.1 Effective Action for the toroidal orbifold

Let us start by analysing the EFT of the particular orientifold model mentioned above. In 4d, the dynamics of the type IIA vacua can be conveniently analysed with the formalism of $\mathcal{N} = 1$ supergravity. The superfield coordinates corresponding to the Kähler moduli and the dilaton can be written as

$$t_i = b_i + iv_i \qquad S = e^{-D} + i\frac{\xi}{\sqrt{2}}. \tag{4.1}$$

They are related to the overall volume of the manifold $\mathcal{V}$ and the 10-dimensional dilaton $\phi$ by

$$\mathcal{V} = \kappa\, v_1 v_2 v_3, \qquad e^D = \frac{e^\phi}{\sqrt{\mathcal{V}}}, \tag{4.2}$$

where $\kappa$ is the triple intersection number of the three 2-forms. The scalars $b_i$ and $\xi$ arise from the dimensional reduction of the $B_2$ and $C_3$ forms respectively. In terms

of these variables, the Kähler potential is [41, 241]

$$\mathcal{K} = -\log(\mathcal{V}) - 4\log(S+\bar{S}). \tag{4.3}$$

This gives rise to a kinetic term

$$\mathcal{L} \supset \partial_\mu D \partial^\mu D + \frac{1}{2}e^{2D}\partial_\mu \xi \partial^\mu \xi + \sum_i \frac{1}{2}\partial_\mu \phi_i \partial^\mu \phi_i + \frac{1}{4}e^{-2\sqrt{2}\phi_i}\partial_\mu b_i \partial^\mu b_i, \tag{4.4}$$

where the canonically normalized fields $\phi_i = \frac{1}{\sqrt{2}}\log(v_i)$ have been used to make contact with the conventions of [41].[3] The moduli are then stabilized by turning on background fluxes for the NS-NS 3-form $H_3$, a constant $F_0$ and the four-form $F_4$, quantized according to

$$\int F_p = (2\pi)^{p-1}\alpha'^{(p-1)/2} f_p. \tag{4.5}$$

These three fluxes can also be rewritten in terms of integers $f_p$ as

$$m_0 = \frac{f_0}{2\sqrt{2}\pi\sqrt{\alpha'}}, \quad p = (2\pi)^2\alpha' h_3, \quad e_i = \frac{\kappa^{1/3}}{\sqrt{2}}\left(2\pi\sqrt{\alpha'}\right)^3 f_4^i, \tag{4.6}$$

where the index $i$ in the last equation refers to the three non-trivial 4-cycles of the manifold. To ensure tadpole cancellation, the first two fluxes cannot be independent and must satisfy

$$m_0 p = -2\left(\sqrt{2}\pi\sqrt{\alpha'}\right). \tag{4.7}$$

In terms of these quantities, one can finally write a scalar potential for the moduli [41], which reads

$$V = \frac{p^2}{4}\frac{e^{2D}}{k}e^{-\sqrt{2}\sum_i \phi_i} + \left(\sum_i e_i^2 e^{2\sqrt{2}\phi_i}\right)\frac{e^{4D-\sqrt{2}\sum_i \phi_i}}{2k} + \frac{m_0^2}{2}e^{4D}k e^{\sqrt{2}\sum_i \phi_i} - \sqrt{2}|m_0 p|e^D. \tag{4.8}$$

In particular, the minimum occurs for

$$\bar{\phi_i} = \frac{1}{\sqrt{2}}\log\left(\frac{1}{|e_i|}\sqrt{\frac{5}{3}\left|\frac{e_1 e_2 e_3}{k m_0}\right|}\right) \qquad e^D = |p|\sqrt{\frac{27}{160}\left|\frac{k m_0}{e_1 e_2 e_3}\right|}, \tag{4.9}$$

[3] For the same reason the 4-dimensional dilaton has not yet been normalized.

where the potential takes the value

$$V|_{\min} \equiv -\frac{3}{R_{AdS}^2} = -\frac{243\sqrt{\frac{3}{5}}k^{3/2}p^4\,|m_0|^{5/2}}{12800\,|e_1e_2e_3|^{3/2}}. \tag{4.10}$$

In the presence of background fluxes, dimensional reduction of the $B_2$ and $C_3$ forms gives rise to a potential for the axions. The effective Lagrangian is of the form [41]

$$\begin{aligned}\mathcal{L} \supset\ & \frac{1}{4}\sum_{i=1}^{3} e^{-2\sqrt{2}\phi_i}\partial_\mu b_i\partial^\mu b_i + \frac{1}{2}e^{2D}\partial_\mu\xi\partial^\mu\xi - \frac{e^{4D}}{\mathcal{V}}\left(b_1e_1+b_2e_2+b_3-p\xi\right)^2 \\ & - \frac{e^{4D}}{2}\sum_{i=1}^{3}\left(m_0^2e^{-2\sqrt{2}\phi_i}\mathcal{V}\,b_i^2 - 2m_0e^{2\sqrt{2}\phi_i-\sqrt{2}(\phi_1+\phi_2+\phi_3)}b_1b_2b_3\frac{e_i}{b_i}\right),\end{aligned} \tag{4.11}$$

### 4.1.2 Integer Conformal Dimensions

Expanding around the vacuum solution discussed in the previous section, we can obtain the masses and couplings for the moduli. In particular, the mass matrix is non-diagonal; full details of the mixing are provided in Appendix C. Here, we stress a surprising fact: after diagonalization, the dilatation operator eigenstates obtain conformal dimensions which do not depend on any of the compactification parameters (e.g. the values of the fluxes) and furthermore are all integers:

$$\Delta_\varphi = (10, 6, 6, 6). \tag{4.12}$$

Similarly, the couplings appearing in the interaction terms are fixed only by the AdS radius, and all come with the same factor of $R_{AdS}^{-2}$ in front. As in LVS, this again hints at the fact that the dual CFT at large volumes can be thought of as a small perturbation above some universal generalized free theory. There is also a high degree of degeneracy between the various modes. Naively this would appear to translate into mixing between the corresponding double trace operators at order $\mathcal{O}(1/N^2)$.[4] However, in practice we expect this degeneracy to be lifted by the effects of higher order operators. The degeneracy partially comes as a result of the exchange symmetries between pairs of tori and their fluxes $b_i \to b_j$ and $e_i \to e_j$[5], but the fact that there is also a third identical eigenvalue seems to be coincidental.

[4]Notice that is also true for the first scalar, since the spectra of different double trace operators overlap when their conformal dimensions are integer spaced (and not only when they are equal).

[5]And the independence from the fluxes, which effectively turns the symmetry into $b_i \to b_j$ only.

Surprisingly, the exact same phenomenon occurs for the Kähler axions, with the only difference that there is a residual imprint of the flux signs, and one can distinguish between a few discrete cases. Expanding (4.11) around the solution (4.9), the mass matrix is almost independent of the values taken by the fluxes, which only enter through the signs

$$s_i \equiv \text{sgn}(m_0 e_i). \tag{4.13}$$

While different choices of signs do not affect the vacuum, they do affect its supersymmetry - the only choice that leads to a supersymmetric vacuum is complete negativity, $s = (-1, -1, -1)$ (e.g. see [173]). However, for our purposes the presence of supersymmetry is not crucial either way.

The diagonalisation procedure is reported in Appendix C, where we also give the full set of cubic interactions relevant for the computation of anomalous dimensions. Except for a discrete sign choice associated to the $s_i$'s, couplings are entirely determined up to a global $R_{AdS}^{-2}$ factor. For the conformal dimensions, the signs $s = (1, 1, 1)$ and $s = (1, -1, -1)$ yield

$$\Delta_a = (8, 8, 8, 2) \qquad \text{or} \qquad \Delta_a = (8, 8, 8, 1). \tag{4.14}$$

Two choices are available for the last eigenvalue, since both solutions for the conformal dimension in terms of the mass satisfy the unitarity bound $\Delta \geq d/2 - 1$. [6] For the other two cases $s = (-1, 1, 1)$ and $s = (-1, -1, -1)$, there is only one possibility:

$$\Delta_a = (11, 5, 5, 5). \tag{4.15}$$

In both instances, the considerations about degeneracy apply as before, and again we only see integer conformal dimensions.

While we cannot offer an explanation of this fact within our framework, it is certainly surprising to see the emergence of such a peculiar pattern. Given the quadratic nature of the $\Delta(\Delta - d) = m^2 R_{AdS}^2$ relation, the origin of these integers cannot simply be traced back to integers on the AdS, giving more credibility to the idea that the CFT duals might offer a complementary perspective on the description of such vacua. Moreover, the (almost) complete independence on the details of the compactification is already a striking feature, also shared with LVS (but not KKLT). It is very much in

[6] Except for the supersymmetric case, where the structure of the superconformal multiplet fixes $\Delta = 2$.

line with the philosophy of the Swampland program, according to which consistency conditions should become more and more stringent in certain limits, and eventually culminate with a very restricted set of "special" theories.

Interestingly, something similar happens if we consider what happens for more general Calabi-Yau's with $h^{2,1}$ complex structure moduli $U_i$. The above example concerned a $\mathbb{T}^6/\mathbb{Z}_3 \times \mathbb{Z}_3$ geometry with no complex structure moduli at all. For more general IIA flux vacua, an orientifold projection reduces the original hypermultiplet down to two scalars. The real part corresponds to the calibrated volume of a 3-cycle $\Sigma_i$ weighted by $g_s^{-1}$, and the imaginary part is a 'true' axion arising from the reduction of the RR 3-form on the cycle, $\int_{\Sigma_i} C_3$. For models of particle physics arising from intersecting D6-branes, these $U_i$ moduli represent the gauge kinetic functions for the gauge groups living on the D6 branes, $f_a = \sum \lambda_i U_i$. The use of the expression 'true' axion refers to the origin from an RR 3-form. Within weakly coupled string theory, the only physics sensitive to the value of such axions are non-perturbative effects - either D-brane instantons or gaugino condensation - associated to the cycles wrapped by the D6 branes. All such effects are suppressed by non-perturbative exponentials, $e^{-2\pi \mathrm{Vol}(\Sigma_i)/g_s}$, which are non-perturbative in both $\alpha'$ and $g_s$ expansions. This implies that, within perturbation theory, the shift symmetry $\mathrm{Im}(U_i) \to \mathrm{Im}(U_i) + \epsilon$ is exact: if it is there at tree-level, it is also there at all orders in perturbation theory.

In IIA flux compactifications, the superpotential for the complex structure moduli is linear in the complex structure moduli. Equivalently, the presence of the fluxes *explicitly* breaks the shift symmetry for one – and only one – linear combination of the axions. By taking linear combinations of the moduli, one can regard the superpotential as depending on only one of these moduli, while the remaining moduli do not appear in the superpotential and also retain the shift symmetry, $K = K(U_i + \bar{U}_i)$.

For such moduli, their axionic components are of course massless (as nothing in the potential depends on their vev). However, at the minimum the properties of the saxions are also fixed. In fact, one can show that for a supersymmetric minimum, the above properties imply ([21] for a general argument, [172, 235] for explicit spectra and some non-supersymmetric examples)

$$M^2_{saxion} = -\frac{2}{3} V_{min} \equiv -\frac{2}{R^2_{AdS}}.$$

On a holographic interpretation, this immediately implies

$$\Delta_{saxion}\left(\Delta_{saxion}-3\right)=-2$$

and so

$$\Delta_{saxion}=1 \qquad \text{or} \qquad \Delta_{saxion}=2.$$

### 4.1.3 Degeneracy Lifting and Anomalous Dimensions

As explained in 2.3.4, the presence of degeneracies can significantly complicate the calculation of anomalous dimensions, because of the mixing between the double trace operators occurring at $\mathcal{O}(1/N^2)$. However, the effect is not present if the conformal dimensions of the single trace operators receive corrections of order $\mathcal{O}(1/N)$ or higher, as this effectively lifts the degeneracy. This is what we would expect to happen in this case, as can be inferred from the scaling of higher derivative supergravity corrections. From Eq. (4.9), the controlled limit of large volume is obtained for large four-form fluxes, when $|e_i| >> |m_0|$. Assuming all fluxes to be of the same order $e_i \sim \bar{e}$, we then have the following scalings:

$$v_i \sim \bar{e}^{\frac{1}{2}} \qquad e^D \sim \bar{e}^{-\frac{3}{2}} \qquad R_{AdS} \sim \bar{e}^{\frac{9}{4}} \qquad \frac{1}{N} \sim \bar{e}^{-\frac{9}{2}}, \tag{4.16}$$

which can be used to estimate the impact of higher dimensional operators. As noted in [241], the scalar potential will receive relative corrections of order $\bar{e}^{-\frac{3}{2}}$ from $|F_4|^4$ terms in the Lagrangian, taking all the metric contractions and $g_s$ factors into account. This will result in a mass shift for the moduli

$$\delta\Delta_\varphi = \frac{2\Delta_\varphi(\Delta_\varphi-3)}{2\Delta_\varphi-3}\left(\frac{\delta m^2}{m^2}\right) \sim \bar{e}^{-\frac{3}{2}} \gg \frac{1}{N}. \tag{4.17}$$

Similarly, $|F_4|^2\,|B_2|^2$ terms will be multiplied by additional factors of $g_s$ and $R^{-4}$, and give

$$\delta\Delta_a = \frac{2\Delta_a(\Delta_a-3)}{2\Delta_a-3}\left(\frac{\delta m^2}{m^2}\right) \sim \bar{e}^{-\frac{5}{2}} \gg \frac{1}{N}. \tag{4.18}$$

for the axions. This would suggest that the degeneracy is lifted in both cases, and anomalous dimensions can be calculated using the standard procedure.[7]

Unfortunately, it also means that the single trace operators, out of which the double trace ones are constructed, will be rotated with respect to the basis we are using. Within an exactly degenerate subspace (with two or more identical conformal dimensions), the eigenstates will depend on the degeneracy-splitting hamiltonian at leading order, and it is impossible to deduce what they are without knowing the explicit form of the perturbation. Thus, the only meaningful anomalous dimensions correspond to the double trace operators constructed with singlets of the original Hamiltonian: $\varphi_1$ and $a_1$ or $a_4$ (corresponding to cases 1 and 2, which depend on the flux signs). As shown in appendix C, cases 3 and 4 can be related to and 2 respectively by some discrete transformations which leave the anomalous dimensions invariant, so we can focus only on the first two. In the first case, formula (2.58) dictates that the anomalous dimension will be proportional to a factor

$$\gamma_{\varphi_1 a_1} \propto \frac{1}{\Gamma(\Delta_{a_1} - \frac{\Delta_{\mathcal{O}}}{2})}, \tag{4.19}$$

where $\mathcal{O}$ is the lowest dimensional operator exchanged (one of $\varphi_2, \varphi_3, \varphi_4$ in this case, with $\Delta_{\varphi_i} = 6$). For the values of $\Delta$ found above this term diverges, and it is only regulated by sub-leading corrections to the conformal dimensions. However, given the behaviour of the gamma function near its poles, the sign of the anomalous dimension will depend on the signs of such corrections, and cannot be determined at this point. The only thing that can be said is that for the two cases $\Delta_a = 1$ and $\Delta_a = 2$ the overall sign will be the same, since any correction will shift their magnitude in opposite directions. On the other hand, the relevant anomalous dimension in case 2 can be calculated meaningfully. Using the couplings derived in C,

$$C_{\varphi_1\varphi_1\varphi_2} = -\frac{315\sqrt{2}}{169} \qquad C_{\varphi_1\varphi_1\varphi_3} = \frac{105\sqrt{2}}{169} \qquad C_{\varphi_1\varphi_1\varphi_4} = \frac{105\sqrt{2}}{169} \tag{4.20}$$

[7] We should give one note of caution here. As we do not understand the origin of integer conformal dimensions, we cannot exclude the possibility that whatever structure gave rise to them also holds for the higher order terms - in which case the higher order terms would fail to lift the degeneracy.

and

$$C_{a_1a_1\varphi_2} = -\frac{60(3+100\sqrt{2})}{2197} \qquad C_{a_1a_1\varphi_3} = \frac{20(3+100\sqrt{2})}{2197} \qquad C_{a_1a_1\varphi_4} = \frac{20(3+100\sqrt{2})}{2197} \quad . \tag{4.21}$$

Since the kinematic factors appearing in (2.58) are always positive in this example,

$$\gamma_{\varphi_1 a_1}(0,\ell) \propto -\sum_{i=2}^{4} C_{\varphi_1\varphi_1\varphi_i} C_{a_1a_1\varphi_i} < 0 \tag{4.22}$$

Thus the only anomalous dimensions we can calculate, which involve an axion and a modulus (as in LVS), are negative.

## 4.2 General DGKT vacua

Similar results to the ones outlined at the beginning of the chapter apply for a generic $\mathcal{N}=1$ orientifold, where it is again possible to fix all the moduli (except for the flat directions corresponding to the complex structure axions). Following the orientifold projection, one is left with $h^{1,1}_{-}$ Kähler moduli and $h^{2,1}$ CS moduli, which can be described with the formalism of $\mathcal{N}=1$ supergravity. The Kähler moduli can be expressed in terms of the complex scalar fields

$$t_a = b_a + iv_a \qquad a = 1, \dots h^{1,1}_{-} \tag{4.23}$$

where the $v_a$ are volumes of the 2-cycles ($\int J$) and the $b_a$ axions arise from dimensional reduction of the $B_2$ form on this cycle. Their Kähler potential is given by

$$K^K = -\log\left(\frac{4}{3}\mathcal{V}\right), \tag{4.24}$$

where the volume is[8] $\mathcal{V} = \kappa_{abc}v_a v_b v_c$. This Kähler potential satisfies the well known no-scale relations

$$K^{ab}K_aK_b = 3 \qquad K^{ab}K_b = -v_a. \tag{4.25}$$

[8]Our definition differs from the proper volume by Vol $= \mathcal{V}/6$.

The complex structure moduli, together with the axio-dilaton, can be packaged into $h^{2,1}+1$ complex fields

$$\begin{aligned} N_k &= \frac{\xi_k}{2} + i\mathrm{Re}(CZ_k) \qquad k = 0 \dots \tilde{h} \\ T_\lambda &= i\tilde{\xi_\lambda} - 2\mathrm{Re}(Cg_\lambda) \qquad \lambda = \tilde{h}+1 \dots h^{2,1} \end{aligned} \tag{4.26}$$

where the $Z_k, g_\lambda$ and the $\xi_k, \tilde{\xi}_\lambda$ are the coefficients of the holomorphic 3-form $\Omega$ and of $C_3$

$$\begin{aligned} \Omega &= Z_{\hat{K}}\alpha_{\hat{K}} - g_{\hat{L}}\beta_{\hat{L}} \\ C_3 &= \xi_{\hat{K}}\alpha_{\hat{K}} - \tilde{\xi}_{\hat{L}}\beta_{\hat{L}}, \end{aligned} \tag{4.27}$$

expanded in a symplectic basis $\{\alpha_{\hat{K}}, \beta_{\hat{L}}\}$ of $H^3$. The basis can be split into an odd part $\{\alpha_\lambda, \beta_k\}$ and even part $\{\alpha_k, \beta_\lambda\}$; it is only the components with respect to the latter which survive the orientifold projection and appear in (4.26). We have also introduced the compensator $C \equiv e^{-D+K_{\mathrm{cs}}/2}$, where the 4d dilaton $D$ is related to the 10d one by $e^D = e^\phi/\sqrt{\mathrm{Vol}}$ and

$$K_{\mathrm{cs}} = -\log(i\int \Omega \wedge \bar{\Omega}). \tag{4.28}$$

Their Kähler potential is given by

$$K^Q = -2\log\left(2\int \mathrm{Re}(C\Omega) \wedge *\mathrm{Re}(C\Omega)\right) = 4D \tag{4.29}$$

and also satisfies a no-scale relation (see appendix C of [241]), namely

$$K^{N_k\bar{N}_k}K_{N_k}K_{\bar{N}_k} + K^{T_\lambda\bar{T}_\lambda}K_{T_\lambda}K_{\bar{T}_\lambda} = 4. \tag{4.30}$$

In a generic setting, both the NS-NS 3-form field strength $H_3$ and the R-R field strengths $F_0, F_2, F_4, F_6$ can thread fluxes through the internal manifold. Following the notation of [241], the background fluxes can be expressed in a basis of the appropriate

cohomologies as

$$H_3 = q_\lambda \alpha_\lambda - p_k \beta_k, \quad F_2 = -m_a w_a, \quad F_4 = e_a \tilde{w}^a, \\ F_0 = m_0, \quad F_6 = e_0. \tag{4.31}$$

The even 2-forms $\{w_a\}$ span a basis of $H^{1,1}_+$, while their duals $\{\tilde{w}^a\}$ are a basis of $H^{2,2}_+$. We briefly remark that the presence of the $F_2$ and $F_6$ fluxes is not needed to achieve moduli stabilisation, but we nevertheless include them for full generality.[9]

The resulting superpotential is given by

$$W = e_0 + e_a t^a + \frac{1}{2}\kappa_{abc} m_a t_b t_c - \frac{m_0}{6}\kappa_{abc} t_a t_b t_c \\ - 2p_k N_k - i q_\lambda T_\lambda. \tag{4.32}$$

A crucial simplification is that the superpotential (4.32) depends only on a linear combination of the complex structure moduli which, after a (holomorphic) rotation in field space, can be effectively taken to be a single modulus. When $h^{2,1} = 0$ this direction reduces entirely to the axio-dilaton,[10] and hence it will be denoted as

$$S \equiv \xi + is, \qquad \text{with} \qquad W \supset -2pS \tag{4.33}$$

as the form for the superpotential. Combined with the fact that the Kähler potential factorises as a sum of two independent terms, this ensures a decoupling between the two sectors. Other than $S$, there will now be $h^{2,1}$ complex structure moduli

$$U_\alpha \equiv a_\alpha + i u_\alpha, \qquad \alpha = 1, \dots h^{2,1} \tag{4.34}$$

which do not appear in the superpotential $W(t_a, S)$.

Assuming the tadpole conditions are satisfied (as they must be), the scalar poten-

[9] In fact, it usually leads to solutions which are either equivalent or qualitatively similar to the ones without $F_2$ or $F_6$ flux.

[10] In that case, the imaginary part of $S$ will be related to the 4d dilaton by $s = e^{-D}/\sqrt{2}$.

tial then takes the standard $\mathcal{N}=1$ supergravity form [241, 41]

$$V = e^K \left( \sum_{t_i, U_\alpha, S} K^{i\bar{j}} D_i W D_{\bar{j}} \overline{W} - 3 |W|^2 \right), \tag{4.35}$$

Supersymmetric vacua occur with vanishing F-terms:

$$D_{t_i} W = 0 \qquad D_S W = 0 \qquad D_{U_\alpha} W = 0, \tag{4.36}$$

which also ensures extrema of the potential (4.35). For the superpotential (4.32) and Kähler potential (4.24), these imply the following relationships for the Kähler moduli at the minimum [41]:

$$\begin{aligned} 3m_0^2 \kappa_{abc} v^b v^c + 10 m_0 e_a + 5 \kappa_{abc} m^b m^c = 0, \\ s = -\frac{2\, m_0}{15\, p} \mathcal{V}, \quad b_a = \frac{m_a}{m_0}, \quad \xi = -\frac{e_0}{2p}. \end{aligned} \tag{4.37}$$

This implies that at the minimum [41]

$$W = \frac{2i}{15} m_0 \mathcal{V}. \tag{4.38}$$

Eqs (4.37) and (4.38) imply the second derivatives of the superpotential take the simpler form

$$\begin{aligned} \partial_{v_a} \partial_{v_b} W &= -\partial_{t_a} \partial_{t_b} W \\ &= -\kappa_{cab} m_c + m_0 \kappa_{abc} t_c = i m_0 \kappa_{abc} v_c \\ &= i \frac{m_0 \mathcal{V}}{6} (K_a K_b - K_{ab}) = \frac{5}{4} (K_a K_b - K_{ab}) W. \end{aligned} \tag{4.39}$$

when Eqs (4.37) are satisfied. On the other hand, the F-term condition for the complex structure moduli simply implies

$$K_{U_\alpha} = \frac{1}{2i} K_{u_\alpha} = 0, \tag{4.40}$$

ensuring the absence of mixing with the Kähler moduli and $S$ for fluctuations around

the minimum. In addition, (4.40) turns the no-scale relation (4.30) into

$$K^{ss}K_sK_s = 4. \tag{4.41}$$

### 4.2.1 Mass matrices for the Kähler moduli and dilaton

To obtain the mass matrix, we compute the second derivatives of the potential (4.35),

$$\begin{aligned}\partial_m\partial_n V = &-3e^K(K_{mn}\left|W\right|^2 + K_m\partial_n\left|W\right|^2 + \partial_m\partial_n\left|W\right|^2)\\ &+ 2e^K K^{i\bar{j}}(\partial_n D_i W)(\partial_m D_{\bar{j}}\overline{W}),\end{aligned} \tag{4.42}$$

in which terms proportional to a first derivative of the potential or $D_iW$ are dropped, and $m, n$ denote saxions. To evaluate the derivatives in the first line of (4.42), we note that the conditions (4.36) imply that

$$\partial_m\left|W\right|^2 = -K_m\left|W\right|^2, \tag{4.43}$$

and the second derivatives of $\left|W\right|^2$ equal

$$\begin{aligned}\partial_s^2\left|W\right|^2 &= \frac{1}{2}K_s^2\left|W\right|^2,\\ \partial_s\partial_b\left|W\right|^2 &= \frac{1}{2}K_sK_b\left|W\right|^2,\\ \partial_b\partial_a\left|W\right|^2 &= 2(\partial_{v_a}\partial_{v_b}W)\overline{W} + 2(\partial_{v_a}W)(\partial_{v_b}\overline{W})\\ &= \left(3K_aK_b - \frac{5}{2}K_{ab}\right)\left|W\right|^2.\end{aligned} \tag{4.44}$$

For the second line in (4.42), we have [11]

$$
\begin{aligned}
&K^{i\bar{j}}(\partial_n D_i W)(\partial_m D_{\bar{j}}\overline{W}) \\
&= K^{i\bar{j}}(W_{in} + K_{in}W - \frac{K_i K_n}{2}W) \\
&\times (\overline{W}_{\bar{j}m} + K_{\bar{j}m}\overline{W} - \frac{K_{\bar{j}}K_m}{2}\overline{W}) \\
&= K^{i\bar{j}}W_{in}\overline{W}_{\bar{j}m} + 2K^{i\bar{j}}W_{in}K_{\bar{j}m}\overline{W} - K^{i\bar{j}}W_{in}K_{\bar{j}}K_m\overline{W} \\
&+ \Big[K^{i\bar{j}}K_{in}K_{\bar{j}m} - K^{i\bar{j}}K_{in}K_{\bar{j}}K_m \\
&+ \frac{1}{4}(K^{i\bar{j}}K_i K_{\bar{j}})K_n K_m\Big]|W|^2 \\
&= K^{i\bar{j}}W_{in}\overline{W}_{\bar{j}m} + 4W_{nm}\overline{W} - K^{i\bar{j}}W_{in}K_{\bar{j}}K_m\overline{W} \\
&+ \Big[K_{nm} + \frac{3}{4}K_n K_m\Big]|W|^2 .
\end{aligned}
\tag{4.45}
$$

If $n, m = a, b$ are both size moduli,

$$
K^{i\bar{j}}W_{in}\overline{W}_{\bar{j}m} = \frac{25}{16}(K_a K_b + K_{ab})|W|^2 , \tag{4.46}
$$

and if $n = a$ is a size modulus

$$
K^{i\bar{j}}W_{in}K_{\bar{j}}K_m\overline{W} = 5K_a K_b |W|^2 \tag{4.47}
$$

by substituting (4.39), and otherwise these terms vanish. Combining these leads to

$$
\partial_a\partial_b V = e^K(9K_{ab} + 8K_a K_b)|W|^2 , \tag{4.48}
$$

$$
\partial_a\partial_s V = e^K(-2K_a K_s)|W|^2 , \tag{4.49}
$$

$$
\partial_s^2 V = e^K(-K_{ss} + 3K_s^2)|W|^2 . \tag{4.50}
$$

[11] Note that because $K = K(t_a + \bar{t}_a, S + \overline{S})$, the derivatives w.r.t. the complex moduli are $\partial_{t_a}K = \partial_{v_a}K/2i$, $\partial_S K = \partial K_s/2i$. The superpotential is holomorphic and so $\partial W_{t_a} = -i\partial_{v_a}W$, $\partial_S W = -i\partial_s W$.

Hence, the mass matrix for the moduli in AdS units is given by

$$R^2_{\rm AdS}\partial_m\partial_n V = \begin{pmatrix} 9K_{ab} + 8K_aK_b & -2K_aK_s \\ -2K_aK_s & -K_{ss} + 3K_s^2 \end{pmatrix}, \tag{4.51}$$

where $R^2_{\rm AdS} = -3/V_{min}$. We can proceed similarly with expression (4.42) for the axions $b_a, \xi$, where now all derivatives of the Kähler potential will vanish, and for the superpotential derivatives we have

$$\partial_{b_a} W = -i\partial_{v_a} W, \quad \partial_\xi W = -i\partial_s W. \tag{4.52}$$

This results in

$$\partial_{b_a}\partial_{b_b} V = e^K(5K_{ab} + 12K_aK_b)\left|W\right|^2, \tag{4.53}$$

$$\partial_{b_a}\partial_\xi V = -3e^K K_aK_s\left|W\right|^2, \tag{4.54}$$

$$\partial^2_\xi V = 2e^K K_s^2\left|W\right|^2, \tag{4.55}$$

and so the mass matrix is

$$R^2_{\rm AdS}\partial_m\partial_n V = \begin{pmatrix} 5K_{ab} + 12K_aK_b & -3K_aK_s \\ -3K_aK_s & 2K_s^2 \end{pmatrix}. \tag{4.56}$$

These expressions match with the results obtained in [172] (see in particular Appendix B). However, this computation provides a more compact way to arrive at these results, using the ordinary $\mathcal{N} = 1$ structure of the potential and without having to rely on the formalism developed in [243, 172, 235]. Moreover, the derivation provided above makes it clear that the only properties feeding into the final result are the no-scale relations for the Kähler potential and the specific form of the superpotential leading to (4.38) and (4.39).

### 4.2.2 Complex structure moduli

Let us briefly review the arguments that lead both to integer conformal dimensions for the complex structure moduli (which have already been discussed (implicitly or

explicitly) in [21] and Section 4.1.2) and the absence of any mixing with the Kähler sector. To establish the latter, one can follow the same steps as the ones leading to (4.49), finding that

$$\partial_{u_\alpha}\partial_{v_b}V = -2K_{u_\alpha}K_{v_b}e^K\left|W\right|^2 = 0 \tag{4.57}$$

because of (4.40). Similarly,

$$\begin{aligned}\partial_{u_\alpha}\partial_s V = -3e^K(-K_{u_\alpha}K_s + \frac{1}{2}K_{u_\alpha}K_s)\left|W\right|^2 \\ + \frac{3}{2}K_{u_\alpha}K_s e^K\left|W\right|^2 = 0.\end{aligned} \tag{4.58}$$

Within the CS sector, non-diagonal terms are also absent

$$\partial_{u_\alpha}\partial_{u_\beta}V = 3e^K K_{u_\alpha}K_{u_\beta}\left|W\right|^2 = 0. \tag{4.59}$$

It was shown in [21] that the only non-vanishing contribution then comes from

$$\partial_{u_\alpha}\partial_{u_\alpha}V = -K_{u_\alpha u_\alpha}e^K\left|W\right|^2, \tag{4.60}$$

fixing all masses to the universal value of

$$m_u^2 = -2/3V_{\text{min}} = -2/R_{\text{AdS}}^2. \tag{4.61}$$

As the axions appear in neither the Kähler potential nor the superpotential, they are all massless.

### 4.2.3 Spectrum

Let us define the two block matrices

$$K_{mn} = \begin{pmatrix} K_{ab} & 0 \\ 0 & K_{ss}\end{pmatrix} \quad L_{mn} = \begin{pmatrix} 4K_aK_b & -K_aK_s \\ -K_aK_s & \frac{K_s^2}{4}\end{pmatrix}, \tag{4.62}$$

where $K$ is simply the Kähler metric for the moduli (in real components) evaluated at the minimum of the potential. Then, from the results of the previous section the Hessians of the moduli and axion potential can be expressed as

$$H^M_{mn} \equiv R^2_{\rm AdS}\partial_m\partial_n V = 9K_{mn} + 2L_{mn} \tag{4.63}$$

and

$$H^A_{mn} \equiv R^2_{\rm AdS}\partial_m\partial_n V = 5K_{mn} + 3L_{mn}. \tag{4.64}$$

respectively. The physical masses can be obtained as the eigenvalues of $M = 2K^{-1}H$, which is related to the mass matrix

$$\tilde{M} = 2\sqrt{K^{-1}}^T H \sqrt{K^{-1}} \tag{4.65}$$

by a similarity transformation.

In principle, the stabilized values of the $v_i$'s and the dilaton can be extracted by the system of $h^{1,1}_- + 1$ equations in the first line of (4.37). Knowing the value of the axions at the minimum, it would then be possible to write the mass matrix as a function of the flux data only. Although the equations cannot be solved explicitly for arbitrary triple intersection numbers $\kappa_{abc}$ and fluxes $e_a, m_a$, the "implicit" form of the matrix derived above is not only sufficient for our purposes, but it is more helpful to elucidate the surprising simplifications in the final expressions.

To compute the eigenvalues of $K^{-1}L$, one can begin by noticing that any vector of the form

$$x = (x_i, x_0) \quad \text{with} \quad x_0\, K_s = 4K_a x^a \tag{4.66}$$

satisfies $K^{-1}\, L\, x = 0$. This essentially descends from the fact that $M$ can be expressed as a tensor product $L_{mn} = A_m A_n$, with $A_m = (2K_a, -K_s/2)$. One therefore has a basis of $h^{1,1}_-$ eigenvectors with an eigenvalue of $\lambda^i_x = 0$. Furthermore, the no-scale relations (4.25)-(4.30) imply that the vector

$$y = (v_i, K_s^{-1}), \tag{4.67}$$

is also an eigenvector of $L$, with eigenvalue $\lambda_y = 13$.

Then, there is a basis in which the mass matrices in AdS units read

$$M^M_{mn} = 18\delta_{mn} + 52\,\delta_{1,m} \tag{4.68}$$

$$M^A_{mn} = 10\delta_{mn} + 78\,\delta_{1,m}, \tag{4.69}$$

also in agreement with the results of [172]. Through the relation $\Delta(\Delta - d) = m^2 R^2_{\rm AdS}$, they correspond to conformal dimensions of

$$\Delta_{v_1} = 10, \qquad \Delta_{v_a} = 6 \tag{4.70}$$

for the saxions, and

$$\Delta_{b_1} = 11, \qquad \Delta_{b_a} = 5. \tag{4.71}$$

for the corresponding axions, with $a = 2...h^{1,1}_- + 1$. Similar conclusions can be drawn for all the complex structure moduli as well. From Eq. (4.61), there are in principle two distinct possibilities for the saxions as both solutions to $\Delta(\Delta - d) = m^2 R^2_{\rm AdS}$ satisfy the unitarity bound. However, $\mathcal{N} = 1$ superconformal symmetry excludes the option $\Delta_{u_\alpha} = 1$,[12] as the axion and saxion are in the same $3d$ $\mathcal{N} = 1$ supermultiplet [244]. Combined with the fact the complex structure axions are all massless, one arrives at the conclusion that

$$\Delta_{u_\alpha} = 2, \quad \Delta_{a_\alpha} = 3. \tag{4.72}$$

for all complex structure moduli $U_\alpha$, where $\alpha = 1...h^{2,1}$.

## 4.3 Further developments

Unlike the rest of this chapter, which is based on [2, 3], this section contains some additional material on the topic, including the original results of [114, 245, 246].

[12] It would be interesting to understand how supersymmetry implements this from an AdS perspective, where the choices correspond to different boundary conditions for the scalar field.

### 4.3.1 Non-SUSY case

Although the argument that we presented in this chapter is more concise, a derivation of the mass matrix for general DGKT vacua (although without a link to conformal dimensions) also appears spread across the two papers [172, 243]. For completeness, let us sketch the approach followed in those papers, which has recently been used to derive new results on the conformal dimensions for some classes of non-supersymmetric, DGKT vacua [245].

In [243], it was argued that the full scalar potential for the general DGKT vacua of Section 4.2 can be written in the form

$$V = \frac{1}{\kappa_4^2} \vec{\rho}^{\,t}\, \mathbf{Z}\, \vec{\rho}, \tag{4.73}$$

where the axionic and saxionic components completely factorize. In particular, the vector $\vec{\rho}$ only depends on the value of the fluxes $\{m_0, e_a\, q_\lambda, p_k\}$ and the axion fields $\{b_a, a_\alpha\}$, while the matrix $\mathbf{Z}$ only on the saxions $\{v_a, u_\alpha\}$. The precise expressions can be found in [243], and will not be reported here.

In [172] (See also [247, 248]), this formalism was used to perform a more complete scan of the DGKT landscape, resulting in the identification of new non-supersymmetric minima. In [245], the spectrum of some of these branches was analysed from a holographic perspective. The ones that were studied include:

- **SUSY branch:**
  These are the 'standard' DGKT vacua we have analysed in Section 4.2. With this approach, the mass matrices and conformal dimensions agree with the ones presented in Sections 4.2.1 and 4.2.2.

- **Non-SUSY $F_4$ branch:**
  This branch is characterised by vanishing $F_0$ and $F_2$ fluxes, and also related to the susy one by $F_4^{\text{non-susy}} = -F_4^{\text{non-susy}}$. The conformal dimensions corresponding to the saxions are the same as the susy branch for both Kähler and Complex Structure moduli, i.e. Eqs (4.70)-(4.71). For the axions, the result is instead

$$\Delta_{b_1} = 1 \text{ or } 2 \qquad \Delta_{b_a} = 8 \qquad \Delta_{a_\alpha} = 3, \tag{4.74}$$

  with $a = 2...h^{1,1}_- + 1$ and $\alpha = 1...h^{2,1}$. These expressions agree with our previous results for the original $T^6/\mathbb{Z}_3^2$ example [246], and a variety of toroidal orientifolds

examined in [114]. Notice that, since supersymmetry is broken, the conformal dimension gap between axions and saxions is no longer 1.

- **Non-susy $F_2$:**
  This branch has non-vanishing $F_0$ and $F_2$ flux. In this particular case there is mixing between the axion and saxion sectors, and the conformal dimensions are

$$\begin{aligned} &\Delta_1 = \frac{1}{2}(3+\sqrt{393}) && \Delta_a = \frac{1}{2}(3+\sqrt{201}) && \Delta_\alpha = 3 \\ &\Delta_1 = \frac{1}{2}(3+\sqrt{33}) && \Delta_a = 6 && \Delta_\alpha = 3. \end{aligned} \tag{4.75}$$

  As usual, the indices take the values $a = 2...h^{1,1}_- + 1$ and $\alpha = 1...h^{2,1}$. The $\Delta$'s are still universal and independent of the flux choice, although some dimensions are not integers (nor rationals).

At this point, two comments are in order. The first one is that, in our previous analysis of the $T^6/\mathbb{Z}_3^2$ orbifold, we found integer conformal dimensions for all the non-susy branches - see Eqs. (4.14) and (4.15). In particular, there is a non-supersymmetric solution where they take the same values as the in the susy case. The second comment regards the stability of the $\mathcal{N} = 0$ branches. Although they are all perturbatively stable [172, 173], it was in found in [123] that the non-susy $F_2$ branch - which is the only example of non-integer conformal dimensions - admits a non-perturbative instability in terms of super-extremal membranes. This is not necessarily the case for the non-SUSY $F_4$ branch, where the decay was found to be marginal [174, 123] at this order of approximation.

### 4.3.2 Other examples

Following the initial observation of integer conformal dimensions in DGKT [2], other works have investigated whether this property holds more generally. In this regard, scale-separated vacua are particularly relevant candidates, as one might hope that the two properties be somehow connected. As reviewed in Section 1.3.4, however, such examples are extremely difficult to come by.

In [114], the holographic duals to a class of scale separated $AdS_3$ vacua [112, 113] were analysed for the first time. They are obtained by compactifying type IIA massive string theory on a $G_2$ holonomy manifold. In the low energy spectrum, there are no

axions, but only 7 scalars coming from the 10d metric and the dilaton. The conformal dimensions of the corresponding operators were shown to be

$$\Delta_a = (11.44, 4.48, 3.84, 3.09, 3.09, 3.09, 3.09, 3.09). \tag{4.76}$$

The fact that these numbers are not integers is in contrast with possible expectations for scale-separated vacua. However, the authors of [114] also noticed that, unlike DGKT, these models do not have the necessary discrete symmetries to satisfy the Refined Strong AdS Distance Conjecture of [124].

Finally, it was noticed in the same paper that some 4d, type IIA AdS vacua with geometric fluxes (but without Romans mass) [234] admit a low-lying spectrum of rational (but not integer) conformal dimensions,

$$\Delta = \{6, 11/3, 1 \text{ or } 2, 5, 8/3, 3\}. \tag{4.77}$$

In an orthogonal direction, the study of holographic duals to M-theory compactifications was initiated in [246]. In the case of flux stabilised vacua [249, 250], one can obtain scale separated solutions if the imaginary part of a certain complex, Chern-Simons invariant is much larger than one. In that case, the conformal dimensions for the 3−cycle size moduli and the corresponding axions are

$$\Delta_{s_1} = 8 \qquad \Delta_{s_a} = 4 \qquad \Delta_{t_1} = 7 \qquad \Delta_{t_a} = 3, \tag{4.78}$$

where $a = 2...b_3$. This is an intriguing result, but perhaps not too surprising given the similarities to DGKT in the form of the superpotential. A second class of vacua consist of M-theory compactifications with zero flux, where the moduli are stabilised by non-perturbative effects [251, 252]. In this case, complicated (and generically non-integer/rational) expressions were found for the conformal dimensions of the moduli, with some similarities to the KKLT/Racetrack cases.

## 4.4 Concluding remarks

We have shown how - if we assume they actually exist - the spectrum of the $CFT_3$ duals to supersymmetric DGKT type IIA flux vacua is characterized by a spectrum of integer conformal dimensions for the low-lying primaries. This results generalises

an observation first made in [2] for the simplest case of a $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3$ orientifold, and later extended in [114] to a wide range of examples. As well as the conformal dimensions, it is also the case that the higher-point interactions take on a universal form in the large-volume limit and 'forget' about the precise details of flux quantum numbers. The result is surprising, as we can discern no obvious reason why this should be the case. In information, the conformal dimensions are equivalent to the moduli masses, but it is only when expressed in this particular fashion that an integer structure appears - the masses are not, for example, integer multiples of $m_{BF}$, and integer masses do not always lead to integer (or even rational) conformal dimensions. This hints at some deeper structure, but we can offer no explanation as to what it is.

There are two particular aspects of this that are striking. First, the universality of the large-volume limit is reminiscent of similar behaviour for LVS. There, although the dual moduli dimensions were non-integer, the CFT also took a universal form in the large-volume limit. It is worth stressing that this runs counter to the landscape picture of string vacua being able to scan over enormously large parameter spaces in the low-energy effective field theory. Instead, these two examples of moduli stabilisation scenarios producing vacua that are in the extreme limit of Kähler moduli space instead lead to almost unique forms for the corresponding dual CFT. We note this is *not* true of KKLT or racetrack stabilisation, which are harder to stabilise in the asymptotic regions of moduli space. Even with extraordinarily small values of $W_0$ (such as the $10^{-95}$ reported in [58]) the volume in KKLT is only logarithmic in $W_0$, and can never be made as large as in LVS. It is intriguing to speculate that any stabilisation method producing vacua in the asymptotic large-volume limit (or, more generally, in the extreme limits of moduli space) corresponds to a very limited set of choices for the CFT properties. This sounds vaguely similar to ideas for classifying the extremal limits of moduli space (e.g. see [253]) and a more precise correspondence would be interesting.

Secondly, the conformal dimension for a generic modulus is given by one of the very few integer values in Eqs (4.70)-(4.72). It is surprising to see how these numbers do not bear any trace of the microscopic details of the compactification, *i.e.* the values of the fluxes and the geometry of the orientifold. Such a simple and universal behaviour is suggestive of the fact that the holographic perspective offers both a novel and insightful viewpoint on these constructions. Despite the simplicity of the result, the calculation itself does not seem to yield any clear insight as to why such a striking property should hold. Its outcome relies on the no-scale relations for the

Kähler potential, as well as some more specific conditions involving the superpotential and its derivatives, Eqs (4.38) and (4.39).

One might wonder whether supersymmetry plays a role - already at the $\mathcal{N}=1$ level, the structure of the superconformal multiplets implies that the conformal dimensions of axions and saxions should differ by one. While one of the non-supersymmetric branches of DGKT admits integer dimensions [245], it is very closely related to the susy one ($F_4^{\text{non-susy}} = -F_4^{\text{non-susy}}$). A further observation is that no-scale relations partially depend on the factorisation of the Kähler potential into a sum of two terms (depending only on the Kähler and complex structure moduli respectively), which could be seen as a remnant of a more constraining $\mathcal{N}=2$ supersymmetry of the original supergravity theory, where the Kähler potential factorises into hypermultiplet and vector multiplet terms. One would expect that $\mathcal{N}=1$ corrections would break this factorisation and introduce mixing between the Kähler and complex structure moduli. Examples of such corrections would be $\alpha'$ and $g_s$ corrections which, generically, would not preserve the $\mathcal{N}=2$ structure. The DGKT vacua have a scale-separated limit of large volume and weak coupling - one would expect this limit to suppress such corrections, with an asymptotic limit in which corrections to the integer conformal dimensions vanish asymptotically. More generally, it would be extremely fascinating to come up with a deeper explanation of either the integer dimensions or the large degeneracies amongst the operators. From an AdS field theory point of view, it was noticed in [254] that the generalised shift symmetry

$$\varphi \longmapsto \varphi + c^{(N)}_{\mu_1\cdots\mu_N} x^{\mu_1}\cdots x^{\mu_N}, \tag{4.79}$$

with $c^{(N)}_{\mu_1\cdots\mu_N}$ a totally symmetric tensor, is preserved by scalars with conformal dimensions of the form $\Delta = d+n$. However, it is not clear what this would entail from a CFT point of view.

In a long-term perspective, an interesting direction would involve trying to understand if one could either prove or disprove the consistency of these (or similar) 4d vacua using ideas from AdS/CFT.

# Chapter 5

# The Overshoot Problem and the Weak/Inflation Hierarchy

This Chapter is based on the paper [4].

## 5.1 Introduction and the Overshoot Problem

Inflation is (justly) widely believed to be the physics underlying the origin of structure in the universe, through quantum fluctuations in a primordial inflaton field during a quasi-de Sitter phase in the early universe. While the precise energy scales involved in inflation are not known, in most models these are high. $V_{inf} \sim (10^{15}\text{GeV})^4$ is typical, while even larger values arise in high-scale inflation models associated to observable levels of primordial gravitational waves. For a review of the physics of inflation, see [255]. We shall assume without further comment that the universe did indeed go through an inflationary phase in its earliest moment.

Whatever the universe was, today its vacuum state involves an electroweak symmetry broken at a characteristic energy scale $E_{weak} \sim 100\,\text{GeV}$. Despite the discovery of the Higgs boson in 2012, the origin of this scale is not understood. It appears unstable to quantum corrections, requiring either fine tuning or the appearance of additional sectors or symmetries (for example, supersymmetry) to control radiative corrections and maintain a Higgs mass and vev sixteen orders of magnitude below the Planck scale.

String theory is our leading candidate for a fundamental theory of nature that includes quantum treatments of both gauge and gravitational interactions. If string theory is the theory of *this* world, it must account for a cosmological evolution that

started with an inflationary phase and ends in a vacuum state today characterised by $E_{weak} \ll E_{inflation}$ which also includes the many other hierarchies of particle physics (for example, the Yukawa couplings).

To even talk about string vacua, we are required to confront the physics of moduli stabilisation and construct models which both stabilise moduli and exhibit a stable (ideally de Sitter) minimum. Although many aspects of our discussion extend more generally, here we will mostly consider type IIB compactifications for which the most developed models are KKLT and LVS [256, 257, 258]. However, all string theories have a vanishing energy in the decompactification limit ($\mathcal{V} \to \infty$ or $g_s \to 0$). String models therefore have a natural tendency to exhibit runaway behaviour from the interior of moduli space towards the boundary, and so all vacua with (effectively) zero vacuum energy will contain a barrier height to decompactification.

Although precise details are model-dependent, the barrier height is normally much smaller than the inflationary energy scales (as the barrier is associated to the vacuum and *not* the physics during inflation). In a typical model with $m_{3/2} \sim 100\,\mathrm{TeV}$, the barrier height may be $V_{barrier} \sim m_{3/2}^2 M_P^2 \sim 10^{-26} M_P^4$ [259, 260, 261] (or $V_{barrier} \sim m_{3/2}^3 M_P$ in the case of LVS [262]). For an inflationary scale $V_{inf} \sim (10^{15}\mathrm{GeV})^4$, this gives

$$\frac{V_{barrier}}{V_{inf}} \sim 10^{-13}. \tag{5.1}$$

This leads to the *overshoot* problem [263] - how can the minimum 'trap' the post-inflationary theory in the vacuum? To make the problem more graphic: suppose a ball rolls from the top of Mount Everest towards a hole with vertical sides of one nanometer. How do we trap the ball in the hole?

Neglecting other sources of energy density, the equations of motion for a scalar field $\phi$ in an expanding universe are[1]

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi}, \tag{5.2}$$

$$H^2 = \frac{1}{3}\left(V(\phi) + \frac{\dot{\phi}^2}{2}\right). \tag{5.3}$$

[1]When the field $\phi$ corresponds to a variation of the total volume (as in the case of the volume modulus), one might worry that the Planck mass is changing throughout its evolution, affecting the equations of motion. However, in the Einstein frame the 4d Planck mass is taken to be fixed, and the varying quantity is the string scale $m_s$. Furthermore, the transformation from the string to the Einstein frame is performed for an arbitrary configuration of the field (which could be time dependent), so a varying volume is allowed.

After inflation occurs at a scale $\Lambda_{\rm inf}$, the field starts rolling down its potential. String theory potentials are naturally steep. Potentials that are power-law in a volume are exponential in the canonical field (e.g. LVS, with $V(\Phi) \sim \exp\left(-\sqrt{\frac{27}{2}}\Phi\right)$) while non-perturbative potentials (as in KKLT) are double exponentials, $V(\Phi) \sim \exp\left(-\alpha \exp\left(\beta\Phi\right)\right)$. The initial potential energy is converted into kinetic energy of the scalar field and the universe enters a kination phase, where $\dot{\Phi}^2 \gg V(\Phi)$ and energy density scales as $\rho_\Phi \sim a^{-6}$. If no other sources of energy density are present, the field will approach the minimum of the potential still in the kination phase. In this case, the field kinetic energy will be much greater than the barrier height and the field will overshoot the barrier and run away to the decompactification limit at infinity. This is the *overshoot* problem.

Although it appears obvious, we note explicitly here that in this conventional formulation of the problem, the overshoot problem is made worse either by (a) making the inflationary scale larger or (b) reducing the barrier height. We emphasise this explicitly because one of the key points of this paper is that this conventional wisdom should be inverted in LVS.

Solutions to the overshoot problem can be put in two classes. In the first, the problem is solved by removing any hierarchy between the inflationary scale and the barrier height. This can be done either by keeping the barrier height low and associated to the physics of the weak scale but working with models of very low-scale inflation (for example, see [264]), or alternatively by continuing with high-scale inflation and bringing the barrier up to a similar scale (for example, see [259]). This latter approach is associated with models that regard the weak scale as an accidental feature generated by fine-tuning, when there is no *dynamical* reason for the hierarchical smallness of the weak scale.

The second, far more interesting, class of solutions uses dynamical effects to avoid overshooting. These exploit the fact that, as $\rho_{kination} \sim a^{-6}$, almost any other early source of energy $\rho_{extra}$ will eventually catch up with the kinetic energy of the scalar (as e.g. $\rho_m \sim a^{-3}$ and $\rho_\gamma \sim a^{-4}$).

$$H^2 = \frac{1}{3M_P^2}\left(\rho_{extra} + \frac{1}{2}\dot{\Phi}^2 + V(\Phi)\right), \tag{5.4}$$

The $\rho_{extra}$ term increases the Hubble friction to a level where it can halt the runaway behaviour. Provided $\rho_{extra}$ catches up with the kinetic energy before the field reaches the barrier, the modulus will settle into the minimum and avoid both overshooting and decompactification.

In the next section we review the dynamics of the kination epoch (while the field is fast-rolling down the exponential slope) in a language adapted to string moduli.

## 5.2 Kination

Soon after inflation ends, the field starts rolling down from the high inflationary scale and enters the (LVS) exponential slope. It rapidly converts potential into kinetic energy and the universe enters a period of kination domination. In this epoch, the energy density of the Universe is dominated by the kinetic energy of the rolling scalar and so the potential energy is, to first approximation, irrelevant.

### 5.2.1 Pure Kination Epoch

In a pure kination epoch, where we neglect all other sources of energy, the scalar field equations of motion

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi}, \tag{5.5}$$

$$H^2 = \frac{1}{3M_P^2}\left(V(\phi) + \frac{\dot{\phi}^2}{2}\right), \tag{5.6}$$

reduce to

$$\ddot{\phi} + 3H\dot{\phi} = 0, \tag{5.7}$$

$$H^2 = \frac{\dot{\phi}^2}{6M_P^2}. \tag{5.8}$$

The resulting equation for $\phi$,

$$M_P\,\ddot{\phi} + \sqrt{\frac{3}{2}}\dot{\phi}^2 = 0, \tag{5.9}$$

is solved by (denoting the initial condition $\phi(t_0)) = \phi_0$.

$$\phi = \phi_0 + \sqrt{\frac{2}{3}} M_P \ln\left(\frac{t}{t_0}\right). \tag{5.10}$$

One other integration constant has been removed by requiring that, as conventional, a time coordinate of $t = 0$ corresponds to a (formal) initial singularity where energy densities diverge. The scale factor behaves as

$$a(t) \propto t^{1/3}, \tag{5.11}$$

which follows immediately from $H^2 \equiv \frac{\dot{a}(t)^2}{a(t)^2} = \frac{\dot{\phi}^2}{6M_P^2}$. The energy density during a kination epoch, therefore, drops off as

$$\rho_{kination}(t) \propto \frac{1}{a(t)^6}. \tag{5.12}$$

Note that in a kination epoch, the field evolves through approximately one Planckian distance each Hubble time. From a stringy perspective, this is automatically interesting: transPlanckian field excursions require a theory of quantum gravity in order to ensure control of the effective field theory over such large displacements. While this result is long-standing, it has received more recent attention following the Swampland Distance Conjecture [265, 104]. It automatically follows that during an extended kination epoch lasting many Hubble times, the field must traverse a substantially transPlanckian distance. Assuming the initial inflationary epoch occurred towards the centre of moduli space, any long kination epoch propels the system towards the boundary.

In general, there are control issues for field excursions $\Delta\Phi \gg M_P$. Fortunately, in the LVS context we are primarily considering, the interpretation of the field excursion is straightforward - the field $\Phi$ controls the overall volume, and the transPlanckian excursion corresponds to a growth in the size of the extra dimensions as the field rolls towards the decompactification limit. While, as per the distance conjecture, there is a tower of states that becomes 'light' (here, the KK modes) this does not affect control – which actually improves as the volume increases, as the scale of the potential drops even faster and we move deeper into the supergravity limit.

Let us relate this kination epoch to the compactification parameters. The basic kinetic term in IIB compactifications originates from

$$K = -3\ln\left(T + \bar{T}\right), \tag{5.13}$$

where $T = \tau + ic$ is a Kähler modulus whose real part corresponds to the volume of

a 4-cycle, $\tau \sim \mathcal{V}^{2/3}$, where $\mathcal{V}$ is the overall Calabi-Yau volume, and all volumes are measured in appropriate powers of $l_s = 2\pi\sqrt{\alpha'}$. The imaginary part $c$ is an axion from an RR 4-form. The canonical field $\Phi$ is

$$\Phi = M_P\sqrt{\frac{3}{2}}\ln\tau \equiv M_P\sqrt{\frac{2}{3}}\ln\mathcal{V}, \tag{5.14}$$

It follows that for a field excursion $\Delta\Phi = \langle\Phi\rangle - \langle\Phi_0\rangle$, we can write

$$\sqrt{\frac{2}{3}}M_P\ln\left(\frac{t}{t_0}\right) = \Delta\Phi = \langle\Phi\rangle - \langle\Phi_0\rangle = \sqrt{\frac{2}{3}}M_P\ln\left(\frac{\mathcal{V}}{\mathcal{V}_0}\right). \tag{5.15}$$

In this stringy kination epoch, it follows that the evolution of the internal compactification volume is linear in cosmic time,

$$\mathcal{V} = \mathcal{V}_0\left(\frac{t}{t_0}\right). \tag{5.16}$$

In the context of LVS, as the terms contributing to the potential energy in LVS all behave as $V \sim \mathcal{V}^{-3}$, this shows that during the kination-dominated roll down the exponential slope, the magnitude of the LVS potential energy behaves as

$$V_{LVS,kination}(t) \sim t^{-3}. \tag{5.17}$$

We note that (as must be the case) this also follows from using the exponential potential $V \propto \exp\left(-\sqrt{\frac{27}{2}}\Phi\right)$ and the time-evolution of the canonical field $\Phi$,

$$V_{LVS}(\Phi) \sim \exp\left(-3\ln t\right) \sim t^{-3}. \tag{5.18}$$

As the overall energy density $V_{total} \sim t^{-2}$, the potential energy in this epoch is therefore suppressed compared to the overall energy by a factor $\frac{t_0}{t}$. This self-consistently justifies our decision to drop the potential in the analysis of the kination epoch. As a last important point, let us notice that as $m_s \sim M_P/\sqrt{\mathcal{V}}$

$$\rho_{\rm kin} = \frac{M_P^2}{3t^2} \sim m_s^4 \tag{5.19}$$

for the whole of the kination epoch. It is tempting to speculate whether the presence

of such a high energy density might be able to excite stringy degrees of freedom, making kination a useful laboratory for string theory.

### 5.2.2 The Tracker Solution

We here review the well-known existence of an attractor tracker solution for exponential potentials [266, 267, 268]. This implies that, with sufficient initial radiation, the overshoot problem can be solved. [2]

The tracker solution relies on additional contributions that redshift slower than kinetic energy. For a generic cosmic fluid with equation of state $P = (\gamma - 1)\rho$, $\rho \sim a^{-3\gamma}$, and so this condition is equivalent to $\gamma < 2$. Both matter and radiation satisfy this condition. Given the high inflationary scales, the presence of stable matter at the end of inflation appears unlikely (although we consider primordial black holes later). We mostly consider just the case of initial radiation, where $\rho_{extra} = \rho_\gamma$ (note we use $\rho_\gamma$ to denote any form of radiation, not just photons).

For this system, the Friedmann equations are

$$\dot{H} = -\frac{1}{2M_P^2}\Big(\rho_\gamma + P_\gamma + \dot{\Phi}^2\Big) = -\frac{1}{2M_P^2}\Big(\gamma\rho_\gamma + \dot{\Phi}^2\Big), \tag{5.20}$$

$$H^2 = \frac{1}{3M_P^2}\Big(\rho_\gamma + \frac{1}{2}\dot{\Phi}^2 + V(\Phi)\Big), \tag{5.21}$$

while the energy conservation equation is

$$\dot{\rho}_\gamma = -3H\Big(\rho_\gamma + P_\gamma\Big) = -3H\gamma\rho_\gamma. \tag{5.22}$$

Alternatively, one can switch to the variables

$$x = \frac{\dot{\Phi}}{M_P}\frac{1}{\sqrt{6}H}, \qquad y = \sqrt{\frac{V(\Phi)}{3}}\frac{1}{M_P H}, \tag{5.23}$$

which encode the fractional energy densities in kinetic and potential energy respectively, $\Omega_k = x^2, \Omega_p = y^2$, with a radiation energy density $\Omega_\gamma = 1 - x^2 - y^2$. The same

[2] An analogous problem concerning the stabilisation of the dilaton was also solved with the same approach in [269].

dynamical evolution can be recast as the system

$$\begin{cases} x'(N) = -3x - \dfrac{V'(\Phi)}{V(\Phi)}\sqrt{\dfrac{3}{2}}y^2 + \dfrac{3}{2}x\Big[2x^2 + \gamma(1-x^2-y^2)\Big] \\ y'(N) = \dfrac{V'(\Phi)}{V(\Phi)}\sqrt{\dfrac{3}{2}}xy + \dfrac{3}{2}y\Big[2x^2 + \gamma(1-x^2-y^2)\Big] \\ H'(N) = -\dfrac{3}{2}H(2x^2 + \gamma(1-x^2-y^2)) \\ \Phi'(N) = \sqrt{6}x \end{cases} \tag{5.24}$$

where the time variable is $N = \log a$.

A simple case (which holds in LVS) is where the potential can be approximated by a single (steep) exponential,[3]

$$V = V_0 \exp\left(-\lambda \frac{\Phi}{M_P}\right), \tag{5.26}$$

so that $V'(\Phi)/V(\Phi) = -\frac{\lambda}{M_P}$. For LVS, $\lambda = \sqrt{\frac{27}{2}}$. The precise value of $V_0$ will depend on the details of the compactification, but for reasonable values of $W_0$ we expect it to be of order $M_P^4$. In this regime, the system (5.24) is known to have a stable attractor solution where the scalar field and radiation have a fixed ratio of energy densities. The fixed point is characterised by

$$\Omega_k = x^2 = \frac{3}{2}\frac{\gamma^2}{\lambda^2} \qquad \Omega_p = y^2 = \frac{3(2-\gamma)\gamma}{2\lambda^2} \qquad \Omega_\gamma = 1 - x^2 - y^2 = 1 - \frac{3\gamma}{\lambda^2}. \tag{5.27}$$

If the attractor solution is obtained before the rolling field reaches the barrier, it will not overshoot.

### 5.2.3 Picking up the Tracker Solution

The tracker solution relies on other sources of radiation or matter 'catching up' with the kinetic energy through their slower redshift. However, this takes (a lot of) time and

[3]This expression is valid far away from the minimum, where the uplifting term can be neglected. Including the uplift, the full potential takes the form

$$V(\Phi) = V_0\big((1 - \varepsilon\Phi^{3/2})e^{-\lambda\Phi} + \delta e^{-\sqrt{6}\Phi}\big). \tag{5.25}$$

The parameter $\delta$ needs to be fine-tuned to achieve a dS vacuum at $\Phi \sim \varepsilon^{-2/3}$.

so it may appear impractical for small seed amounts of radiation to grow sufficiently. A distinctive, and almost unique, feature of LVS (which makes it highly appealing for this purpose) is that the minimum of the potential is located a long way (in principle, many Planckian distances in field space) from the centre of moduli space where we assume inflation originally happened.

As $a(t) \propto t^{1/3}$ and $\rho \propto a(t)^{-6}$ during the kination epoch, the relative proportion of initial radiation $\rho_\gamma$ grows as

$$\left(\frac{\rho_\gamma(t)}{\rho_{KE}(t)}\right) = \left(\frac{t}{t_0}\right)^{2/3} \left(\frac{\rho_\gamma(t_0)}{\rho_{KE}(t_0)}\right), \tag{5.28}$$

or equivalently,

$$\ln\left(\frac{\rho_\gamma(t)}{\rho_{KE}(t)}\right) = \frac{2}{3}\ln\left(\frac{t}{t_0}\right) + \ln\left(\frac{\rho_\gamma(t_0)}{\rho_{KE}(t_0)}\right). \tag{5.29}$$

If kination-radiation equality is obtained, the system will find its way to the tracker solution as subsequent expansion sees radiation dominate over the kinetic energy; Hubble friction then effectively halts the field on its potential until the system evolves into the tracker solution. Kination-radiation equality occurs at

$$\ln\left(\frac{t}{t_0}\right) = -\frac{3}{2}\ln\left(\frac{\rho_\gamma(t_0)}{\rho_{KE}(t_0)}\right). \tag{5.30}$$

As we would expect, the smaller the initial fraction of radiation, the longer it takes for this to be obtained. Using the field evolution Eq. (5.10) during kination, the field displacement prior to reaching the tracker solution is

$$\Delta\Phi = \langle\Phi\rangle - \langle\Phi_0\rangle = \sqrt{\frac{3}{2}}\ln\left(\frac{\rho_{KE}(t_0)}{\rho_\gamma(t_0)}\right). \tag{5.31}$$

Expressed in terms of the physical compactification volume, this gives

$$\ln\left(\frac{\mathcal{V}_f}{\mathcal{V}_0}\right) = \frac{3}{2}\ln\left(\frac{\rho_{KE}(t_0)}{\rho_\gamma(t_0)}\right). \tag{5.32}$$

The physical implications of Eqs. (5.31) and (5.32) are striking. If we assume a small fraction of initial radiation $\ln\left(\frac{\rho_\gamma(t_0)}{\rho_{KE}(t_0)}\right)$, the field evolution is able to avoid overshooting and settle in the stable vacuum *only provided the vacuum is sufficiently far away in*

*field space*. The *smaller* the initial fraction of radiation, the *further* away the vacuum has to be in order for it to be accessible. As exponential potentials lead to a rapid decrease in vacuum energy along the roll, we also see that, somewhat paradoxically, vacua associated to much lower energy scales (and so with smaller barriers) become *more* accessible not less.

This is one part of the inversion of conventional wisdom on overshoot within LVS. We now discuss the possible seeds for $\rho_\gamma(t_0)$ before returning to a discussion of physical and model-building implications.

## 5.3 Seeds of radiation

Where can the original seed radiation come from? While everything is model-dependent to some extent, there are generic reasons why radiation should be present.

### 5.3.1 The Thermal de Sitter bath

The presumed dynamics is a primordial epoch of inflation exiting into a kination phase (although we wish to be agnostic about the actual model of inflation). If we suppose, consistent with the breakdown of slow-roll conditions at the end of inflation, that this transition is more or less instantaneous, any radiation present during inflation will still be present at the start of the kination phase.

While radiation is constantly diluted during inflation, this is balanced by the continual production of particles coming from the thermal de Sitter bath at temperature

$$T_{dS} = \frac{H_{\rm inf}}{2\pi}. \tag{5.33}$$

This provides a natural source of a radiation energy density

$$\rho_{\gamma,dS} = \frac{\pi^2}{30} g_* \left(\frac{H_{\rm inf}}{2\pi}\right)^4, \tag{5.34}$$

where $H$ is the Hubble scale during inflation and $g^*$ is the effective number of massless degrees of freedom. There is no reason that $g^*$ should be small; both chiral fermions and gauge vector bosons naturally remain light during inflation. Even with scalars, although the $\eta$ problem implies that it can be difficult to hold them lighter than the inflationary Hubble scale during inflation, this is only true if their mass is unprotected

against quantum corrections. Axion fields – which are generic in string theory – are protected by their shift symmetry and remain naturally light during inflation. In a string theory context, $g_*$ could easily be rather large and $\mathcal{O}(100-1000)$, as the multiplicity of axions or gauge bosons may be set by the topology of the internal space and by Hodge numbers $h^{1,1}$ or $h^{2,1}$.

Setting $\rho_{KE}(t_0) = 3H^2M_P^2$, the thermal de Sitter bath would then lead to

$$\frac{\rho_\gamma(t_0)}{\rho_{KE}(t_0)} = \frac{\pi^2}{90} g^* \left(\frac{H_{\rm inf}^2}{(2\pi)M_P^2}\right). \tag{5.35}$$

### 5.3.2 Perturbative Modulus 'Decays'

The thermal de Sitter bath has the advantage of universality: it treats all fields democratically. However, we expect there also to be radiation whose origin is specific to the fact that it is the volume modulus rolling down the exponential slope (analogous to the more conventional conversion of energy in the volume field to radiation at reheating). Although this may be small in *overall* magnitude, it is competing with the similarly small thermal de Sitter bath (and the subsequent enhancement of radiation during the kination epoch makes even small initial quantities of radiation important).

We consider a perturbative origin for radiation from 'decays' of a field rolling on a potential $V = V_0 \exp\left(-\lambda\frac{\Phi}{M_P}\right)$. Our approach is to model the potential as an (instantaneous) quadratic potential, and argue that for infinitesimal time periods it is reasonable to treat the field as equivalent to a conventional massive scalar particle within a quadratic potential, coupled to external degrees of freedom to which it can decay.

Without loss of generality, we can consider the field starting at a value $\Phi = 0$ on the exponential potential $V = V_0 \exp\left(-\lambda\frac{\Phi}{M_P}\right)$.

At this time, we have

$$V = V_0, \tag{5.36}$$

$$\frac{dV}{d\Phi} = -\frac{\lambda}{M_P}V_0, \tag{5.37}$$

$$\frac{d^2V}{d\Phi^2} = \left(\frac{\lambda}{M_P}\right)^2 V_0. \tag{5.38}$$

We wish to consider 'instantaneous' dynamics, i.e. those over a time period $\Delta t \ll \frac{1}{H}$ for a field released at $\Phi = 0$ (from Eq. (5.8), it follows that over such a period the

field displacement is $\Delta\Phi \ll M_P$). To do so, we approximate the form of the potential in the immediate vicinity of $\Phi = 0$ as

$$U(\Phi) = U_0 + \frac{1}{2}m^2 \left(\Phi - \Phi_0\right)^2 . \tag{5.39}$$

This is satisfied with $\Phi_0 = \frac{M_P}{\lambda}$, $U_0 = \frac{V_0}{2}$ and $m^2 = \left(\frac{\lambda}{M_P}\right)^2 V_0$.

We now imagine, at the transition from the inflationary epoch, an approximately stationary field coming onto this potential at $\Phi = 0$. For a short period of time, we expect the field to 'decay' to radiation in a similar fashion as if it were on the quadratic potential (instantaneously, it cannot 'know the difference'). To model this source of initial radiation, we use the behaviour of a field starting at $\Phi = 0$ in the quadratic potential above. How long does 'instantaneous' last for? To give an estimate, we take approximately one Hubble time $\tau = \left(\frac{1}{H_0}\right) = \frac{\sqrt{3}M_P}{V_0^{1/2}}$.

We then compute the seed radiation content by regarding this field as a massive particle with

$$m_\Phi = \left(\frac{\lambda}{M_P}\right)\sqrt{V_0},$$

which decays via couplings to axions, photons or other light massless degrees of freedom. The precise couplings of LVS moduli to both matter and other dark particles, in the context of reheating, have been discussed in [270, 271, 272, 273, 274, 275, 276, 277, 278, 279]. We expect a broadly similar analysis to hold here (although notice that these papers are working around the ultimate minimum of the scalar potential, while we are interested in couplings *away* from the minimum).

While the numerical coefficients of the exact decay rate will be model-dependent, on general grounds we can write the decay rate $\Gamma_{\Phi\to X_i X_i}$ of any individual channel as

$$\Gamma_i \equiv \Gamma_{\Phi\to X_i X_i} = \frac{\alpha_i}{16\pi}\frac{m_\Phi^3}{M_P^2} = \frac{\alpha_i}{16\pi}\left(\frac{\lambda}{M_P}\right)^3 \frac{V_0^{3/2}}{M_P^2}.$$

In the absence of any additional suppression, $\alpha \sim 1$. As the energy in the quadratic part of the potential (which we re-interpret as particles) is $V_0/2$ , it follows that the *overall* fractional conversion rate of energy to radiation is $\Gamma_i/2$. As a naive estimate

for the overall fraction of energy converted to radiation, we then have

$$\sum_i \frac{\Gamma}{2}\tau = \frac{\sqrt{3}}{2}\sum_i \alpha_i \left(\frac{\lambda^3}{16\pi}\right)\frac{V_0}{M_P^4} \quad (5.40)$$

$$= \frac{3\sqrt{3}}{2}\sum_i \alpha_i \left(\frac{\lambda^3}{16\pi}\right)\frac{H^2}{M_P^2}, \quad (5.41)$$

where the index $i$ runs over all decay channels. It is easily seen that this may be dominant over the thermal bath contribution Eq. (5.35).

This formalism and in particular Eq. (5.41) also makes it clear that the dominant contribution to radiation comes from the immediate post-inflation period (due to the $\frac{H^2}{M_P^2}$ factor). As the field rolls down the exponential slope and picks up speed, the Hubble scale decreases and the rate at which seed radiation is generated falls off rapidly.

### 5.3.3 Gravitational Waves from Cosmic String Networks

We now consider possibly the most interesting scenario for the generation of seed radiation.

This involves the radiation seed coming from gravitational waves emitted by cosmic string networks that were formed at the end of (brane) inflation. Such cosmic string networks can arise naturally in scenarios involving brane-antibrane inflation [280, 281, 282, 283, 14, 284] (see [15] for a comprehensive review). In these models, the end of inflation is coincident with brane-antibrane annihilation producing a large number of both fundamental and D-strings (D1-branes). In terms of their cosmic evolution, the most important quantity is the tension of such a string network, characterised by $G\mu$ (where $G$ is Newton's constant, $8\pi G = \frac{1}{M_P^2}$). Reviews of cosmic strings can be found in [285, 286, 287].

Such string networks exhibit a scaling behaviour in which their energy density remains at a constant fraction of the overall energy density of the universe,

$$\rho_{string} \sim \mu H^2. \quad (5.42)$$

This implies that as the universe expands, this energy in the string network decreases. This is not simply associated to a reduction in number density; the energy in the string network is lost via gravitational radiation emitted through string reconnections

and from cusps. For this reason, cosmic string networks are regarded as one of the plausible sources of gravitational waves from the early universe (for example, see [288] for a search and [289] for a review).

Such cosmic string networks can survive to today, where their tension is bounded via CMB constraints to be $G\mu \lesssim 10^{-7}$. In the context of either fundamental or D-strings, the string tension $\mu \sim m_s^2$ and so observational consistency implies any fundamental string network would require either warping or a relatively low string scale.

We suppose a brane/antibrane inflationary model ending with the formation of a cosmic string network, which for simplicity we assume consists of vanilla fundamental strings (i.e. no warping). We also assume that the cosmic string scaling regime is attained rapidly and so soon after inflation the energy in the network is $\rho_{string} \sim \mu H_{\rm inf}^2$. As the Hubble scale decreases the string network rapidly loses its energy via radiation from string cusps or reconnection. We assume an $\mathcal{O}(1)$ fraction of the string emission is to gravitational waves, implying an early presence of radiation

$$\rho_{GW} \sim \mu H_{\rm inf}^2.$$

As $\mu \sim m_s^2$, we see that for a string scale $m_s \sim 10^{16}$GeV during inflation, giving $\mu \sim 10^{-4} M_P^2$, the resulting early fraction of injected radiation energy can be significant, e.g.

$$\frac{\rho_{GW}}{\rho_{total}} \sim 10^{-4}.$$

This can easily dominate over either of the two previously identified sources of seed radiation Eqs. (5.35) and (5.41).

Normally, such values of $G\mu$ would be immediately ruled out by the CMB constraint $G\mu \lesssim 10^{-7}$. However, here one appealing feature of LVS cosmology comes into play. During the long kination roll, the volume increases and so – as $m_s^2 \sim \frac{M_P^2}{\mathcal{V}}$ – the cosmic string tension also decreases by a similar factor of $\frac{\mathcal{V}}{\mathcal{V}_0}$. The CMB constraints apply to the cosmic string tension today, not at the time of inflation – and this increase in the volume reduces the string tension to a level entirely compatible with current observations.

In this scenario, the magnitude of the *initial injection* of radiation as a proportion of the total energy density is set by $G\mu_{early}$, while the *current* energy density of the cosmic string network is set by $G\mu_{today}$ – and these two can be very different.

This scenario also offers the possibility of a very intriguing aspect of the cosmological history of the universe. Suppose the initial injection of radiation from the cosmic string network was indeed dominantly in the form of gravitational waves. Then, as the universe approaches the tracker solution during the kination role, the radiation component consists of gravitational waves. While the universe is on the tracker solution, the radiation component makes up the largest individual part – and so in fact, the dominant energy density of the universe is in the form of gravitational waves (for LVS parameters, $\Omega_{GW} = 19/27$)!

This would hold during the tracker epoch. Once the modulus reaches the minimum and oscillates about it, the universe becomes matter-dominated and so this energy would then be redshifted away. Nonetheless, it would still be extremely striking if the universe did actually go through such a 'graviation' epoch, where its energy was primarily in the form of gravitational waves. The possibility of such an epoch does not appear to have been considered very much in the literature, and it would be interesting to study its properties further.

### 5.3.4 Primordial Black Holes

Another interesting scenario for the initial seed radiation can occur if the dynamics at the end of inflation results in the production of primordial black holes (PBHs). Such black holes redshift as matter for as long as they remain black holes (i.e. before they evaporate). As $\rho_m \sim a(t)^{-3}$, if such PBHs were present the universe would evolves towards a matter tracker solution.

As with cosmic strings, the production and initial density of black holes is highly model-dependent, as it crucially involves the spectrum of density perturbations in the universe and the details of the particular inflationary model (reviews of PBH can be found in e.g. [290, 291, 292]). However, if PBHs are produced, their mass when produced is given by the horizon mass,

$$M_{BH} \sim \frac{M_P^2}{H}. \tag{5.43}$$

From the point of production, the relative energy fraction in PBHs would increase throughout any kination epoch. The subsequent evolution would depend on the mass of the primordial black hole. If the black holes all evaporated while still in the kination epoch, prior to the modulus reaching the LVS minimum, their mass-energy

is converted to thermal radiation. In such a scenario, the role of the PBH has been solely to act as a radiation seed.

A more interesting scenario is when the PBHs live long enough that the universe reaches the 'matter tracker'. At this point $\Omega_m = \frac{21}{27}$, and so the majority of the energy density of the universe would be in the form of primordial black holes. This will remain the case while the field settles into the minimum of the potential – as the modulus itself also redshifts as matter, it cannot displace the black holes. Such an epoch, with energy density dominated by the mass of primordial black holes, would also be striking in itself as a feature of the (string) cosmological history of the universe.

If the PBHs were able to outlive the volume modulus, then reheating would end up proceeding via evaporation of the PBHs. Such an epoch would offer a potential solution to the problem of excessive dark radiation production in modulus decay. When the volume modulus decays, the branching ratio to axions appears to be $\mathcal{O}(1)$ as many of the Standard Model decay modes are suppressed [270, 271]. However, for evaporation of a primordial black hole, modes should be more democratically accessible, potentially reducing the dark radiation fraction (depending on the number of hidden sectors).

However – we do not expect this scenario to be viable, and would instead expect any PBHs to evaporate prior to the decay of the volume modulus. To see this, note that a black hole's lifetime behaves as $\tau_{BH} \propto \frac{M_{BH}^3}{M_P^4}$ whereas the lifetime of a modulus is $\tau_\Phi \sim \frac{M_P^2}{m_{\Phi^3}}$. We can therefore write the lifetime of a primordial black hole formed during kination as

$$\tau_{BH} \sim \frac{M_P^2}{H_{form}^3}, \tag{5.44}$$

where $H_{form}$ is the Hubble scale at the time of formation. However, as during the kination epoch $H > m_\Phi$ (as equality occurs when the field reaches the minimum of the potential), we find that

$$\tau_{BH} < \tau_\Phi \tag{5.45}$$

and so the black hole evaporates prior to modulus decay. It therefore appears unlikely that any black holes formed during the kination epoch could outlive the volume modulus.

## 5.4 Model-Building Implications

Assuming that the kination phase starts directly at the end of inflation, one can use an assumption about the source of the seed radiation to relate the inflationary scale $\Lambda_{\rm inf}$ to the minimal field displacement required to settle into the minimum without overshooting.

From (5.32), kination-radiation equality is reached for

$$\mathcal{V}_{\rm eq} = \left(\frac{V_0}{\Lambda_{\rm inf}^4}\right)^{\sqrt{\frac{3}{2\lambda^2}}} \Omega_{\gamma,0}^{-3/2}, \tag{5.46}$$

where the initial volume has been rewritten in terms of $\Lambda_{\rm inf}$ and the parameters appearing in the potential. To avoid overshooting, we require the volume at the minimum to satisfy $\mathcal{V}_{\rm min} \gtrsim \mathcal{V}_{\rm eq}$.

In the most minimal realisation (see section 5.3.1), the initial radiation will come from the thermal de Sitter radiation, viewed as a bath with $T_{\rm dS} = \frac{H}{2\pi}$. All relativistic species with $m \ll \Lambda_{\rm inf}$ will contribute to a normalised energy density of

$$\Omega_{\rm th}^0 = g^* \frac{\pi^2}{30} \left(\frac{H_{\rm inf}}{2\pi}\right)^4 \frac{1}{3H_{\rm inf}^2 M_P^2} = \frac{g^*}{270 \times 16\pi^2} \left(\frac{\Lambda_{\rm inf}}{M_P}\right)^4. \tag{5.47}$$

Under the conservative assumption that no new physics appears below the inflationary scale, the number of degrees of freedom would be $g^* = g^*_{\rm SM} \sim \mathcal{O}(10^2)$. Eq. (5.46) implies

$$\mathcal{V}_{\rm min} \gtrsim 8.0 \times 10^{17} \left(\frac{g^*}{100}\right)^{-3/2} \left(\frac{V_0}{M_P^4}\right)^{1/3} \left(\frac{\Lambda_{\rm inf}}{3 \cdot 10^{16}\,{\rm GeV}}\right)^{-22/3}, \tag{5.48}$$

where we have used the LVS value of $\lambda = \sqrt{\frac{27}{2}}$. It would be entirely reasonable, however, to have $g^* = \mathcal{O}(10^3 - 10^4)$, potentially reducing the estimate (5.48) by up to 3 orders of magnitude. As mentioned earlier, such a large number could easily arise from the high multiplicity of axions and gauge bosons in a typical compactification or from hidden sectors decoupled from the Standard Model.

Pushing this reasoning to its limits, one could imagine using the largest conceivable number of degrees of freedom in the UV, *i.e.* saturating the species bound of [293, 294]. If one assumes the inflationary scale to lie below the UV cutoff, this gives an

absolute upper bound on the number of the EFT light degrees of freedom

$$g^* \lesssim \frac{M_P^2}{\Lambda_{\rm inf}^2}. \tag{5.49}$$

Substituting in (5.49), this turns into the laxer constraint

$$\mathcal{V}_{\rm min} \gtrsim 1.6 \times 10^{15} \left(\frac{V_0}{M_P^4}\right)^{1/3} \left(\frac{\Lambda_{\rm inf}}{3 \cdot 10^{16}\,{\rm GeV}}\right)^{-13/3}. \tag{5.50}$$

While values of the volume appearing in (5.50) for realistic inflationary scales are certainly closer to what one would require for phenomenology, it is not clear in practice how and if such a high number of light states could arise. Therefore, we only interpret (5.50) as an absolute lower bound on the volume of vacua which can avoid overshooting through thermal dS radiation. In both cases, typical values for the volumes as a function of the inflationary scale are reported in Table 5.1.

| | $\Lambda_{\rm inf}$(GeV) | $3 \cdot 10^{16}$ | $3 \cdot 10^{15}$ | $3 \cdot 10^{14}$ |
|---|---|---|---|---|
| $g^* = 10^2$ | $\mathcal{V}_{\rm min}$ | $8.0 \cdot 10^{17}$ | $1.7 \cdot 10^{25}$ | $3.7 \cdot 10^{32}$ |
| $g^* = 10^4$ | $\mathcal{V}_{\rm min}$ | $8.0 \cdot 10^{14}$ | $1.7 \cdot 10^{22}$ | $3.7 \cdot 10^{29}$ |
| $g^* = M_P^2/\Lambda_{\rm inf}^2$ | $\mathcal{V}_{\rm min}$ | $1.6 \cdot 10^{15}$ | $3.3 \cdot 10^{19}$ | $7.2 \cdot 10^{23}$ |

Table 5.1: *Lower bound for a stable volume (measured in units of* $(2\pi\sqrt{\alpha'})^6$*) in LVS as a function of the inflation scale, assuming only a thermal de Sitter radiation bath. The first line conservatively assumes the number of relativistic degrees of freedom after inflation to be* $g^* = g_{\rm SM} \simeq 100$*, while the second one uses the more aggressive estimate* $g^* = 10^4$*. The third line gives an absolute lower bound assuming the maximum number of degrees of freedom compatible with the species conjecture.*

In the case of perturbative decays, treated in section 5.3.2, the parametric scaling of the initial radiation density as a function of the Hubble scale is the same as for thermal de Sitter radiation ($\propto H^4$). However, the numerical coefficient in front of (5.41) gives a larger enhancement:

$$\Omega_{\rm dec}^0 = \frac{3\sqrt{3}}{2} \sum_i \alpha_i \left(\frac{\lambda^3}{16\pi}\right) \frac{H^2}{M_P^2} = \frac{9\sqrt{3}}{2} g_{\rm dec} \left(\frac{\lambda^3}{16\pi}\right) \left(\frac{\Lambda_{\rm inf}}{M_P}\right)^4. \tag{5.51}$$

The factor $g_{\rm dec}$ is the effective number of species to which the volume modulus can

decay, taking a value $\alpha_i = 1$. Then, taking again $\lambda = \sqrt{\frac{27}{2}}$, there is a relative factor

$$\frac{\Omega^0_{\rm dec}}{\Omega^0_{\rm th}} \simeq 3.3 \cdot 10^5 \frac{g_{\rm dec}}{g^*}. \tag{5.52}$$

Then,

$$\mathcal{V} \gtrsim 4.2 \times 10^9 \left(\frac{g_{\rm dec}}{100}\right)^{-3/2} \left(\frac{V_0}{M_P^4}\right)^{1/3} \left(\frac{\Lambda_{\rm inf}}{3 \cdot 10^{16}\,{\rm GeV}}\right)^{-22/3}. \tag{5.53}$$

Typical values are summarised in Table 5.2.

| | $\Lambda_{\rm inf}$(GeV) | $3 \cdot 10^{16}$ | $3 \cdot 10^{15}$ | $3 \cdot 10^{14}$ | $3 \cdot 10^{13}$ |
|---|---|---|---|---|---|
| $g_{\rm dec} = 1$ | $\mathcal{V}_{\rm min}$ | $4.1 \cdot 10^{12}$ | $8.9 \cdot 10^{19}$ | $1.9 \cdot 10^{27}$ | $4.1 \cdot 10^{34}$ |
| $g_{\rm dec} = 10^2$ | $\mathcal{V}_{\rm min}$ | $4.1 \cdot 10^{9}$ | $8.9 \cdot 10^{16}$ | $1.9 \cdot 10^{24}$ | $4.1 \cdot 10^{31}$ |
| $g_{\rm dec} = 10^4$ | $\mathcal{V}_{\rm min}$ | $4.1 \cdot 10^{6}$ | $8.9 \cdot 10^{13}$ | $1.9 \cdot 10^{21}$ | $4.1 \cdot 10^{28}$ |

Table 5.2: *Lower bound for the volume (measured in units of $(2\pi\sqrt{\alpha'})^6$) in LVS as a function of the inflation scale, assuming initial radiation from perturbative decays. The first line conservatively assumes the number of light degrees of freedom the volume can decay to after inflation to be $g_{\rm dec} = 1$, while the second and third assume the more aggressive estimates $g_{\rm dec} = g_{\rm SM} \simeq 100$ and $g_{\rm dec} = 10^4$.*

Perhaps the most interesting scenario of seed radiation involved the production of gravitational waves from cosmic string scaling networks. Our results therefore motivate the detailed study of inflationary models within LVS which end in brane/antibrane annihilation and the production of cosmic strings. So far, the most detailed analyses of brane inflation models have been carried out within KKLT constructions. LVS offers a very different phenomenology, in particular the prospect of a large reduction in the value of the string scale between the end of inflation and the present day. The low string scales in LVS would also make a fundamental cosmological superstring network observationally viable, with the tension naturally satisfying $G\mu \lesssim 10^{-7}$.

In terms of avoiding overshoot, there are two important general points to make. The first is the fact that in all the possible radiation seeds we have considered, the larger the initial ratio of $\frac{H_{\rm inf}}{M_P}$, the larger the initial radiation fraction, and so the easier it is subsequently to obtain kination-radiation equality and avoid an overshoot by locating, and switching onto, the tracker solution. This is also true for cosmic strings and PBHs, although estimates such as (5.48) or (5.53) would be more model dependent in those cases.

The second point is more LVS-specific: this is that the location of the minimum at a large distance in field space away from the 'center' of moduli space is crucial for the presence of hierarchical scale separation and, when using these hierarchies for particle physics, crucial for explaining the large ratio between the Planck and electroweak scales. In this context, the vacuum can only be located (avoiding overshooting) if the inflationary scale is sufficiently high.

This inverts the usual logic of the overshoot problem – in this framing, the problem becomes *less* severe the higher up on the potential we start. For those who find anthropic arguments appealing, in this context we can also turn this into an anthropic argument for the existence of a large hierarchy between the scale of inflation (i.e. where we start on the roll) and the electroweak scale (characterised by the location of the minimum at a large distance in field space from the inflationary locus). Indeed, the gravitino mass - which could be thought to provide the scale of SUSY breaking - scales as

$$m_{3/2} \simeq \frac{M_P |W_0|}{\mathcal{V}} \tag{5.54}$$

in LVS. The relationship of the gravitino mass and MSSM soft terms is subtle in LVS; while original analyses [258] assumed these would be comparable, more careful analyses of soft terms and sequestering [295, 296] show that we would actually expect soft terms at a scale

$$M_{\rm soft} \sim \frac{m_{3/2}^{3/2}}{M_P^{1/2}}. \tag{5.55}$$

In any case, under the hypothesis that the solution to the hierarchy problem is connected in some way to the presence of low energy supersymmetry, this gives a direct link between large volumes and a low electroweak scale. Depending on the precise model details, the appropriate LVS volume to match onto particle physics may be $10^6 \lesssim \mathcal{V} \lesssim 10^{13}$. From the estimates in Tables 5.1 and 5.2, these volumes may reasonably be large enough to avoid overshooting but with a clear requirement for relatively high-scale inflation (not necessarily transPlanckian field excursions).

## 5.5 Conclusions

String theorists should feel their ears burning and exhibit a professional interest whenever fields undergo trans-Planckian displacements $\Delta\Phi \gtrsim M_P$. This fact is widely acknowledged for models of large-field inflation. Less attention, however, has been paid to the case of a possible kination epoch in the early universe.

We have shown here that a long kination epoch is simultaneously a natural feature of cosmology in LVS and an attractive method of solving the overshoot problem: initial seed radiation from the immediate end of the inflationary epoch grows in relative importance during kination, allowing the rolling field to locate the tracker solution and avoid overshooting the minimum.

This approach requires the initial seed radiation to grow from a (presumed) small value to reach an order unity fraction of the overall energy density. There are both universal and model-dependent options for the origin of the original seed radiation, but one aspect they have in common is that the initial radiation fraction grows with positive powers of $\left(\frac{H_{\rm inf}}{M_P}\right)$. As it takes considerable time for the small initial radiation fraction to 'catch up', the avoidance of overshoot is therefore made *easier* by large inflationary scales and/or a large (significantly transPlanckian) field displacement between the inflationary epoch and the final vacuum. This takes some time; LVS is rather distinctive here as it naturally contains a large trans-Planckian displacement in the field space between the locus of inflation and the final minimum. In the context of the different scales in LVS, this offers an anthropic argument for a large hierarchy between the weak and inflationary scales.

One particularly interesting scenario is when inflation ends with brane/anti-brane annihilation and the creation of a cosmic superstring network. As the string scale decreases during the kination roll, the network can be formed with a relatively high tension (far above bounds based on the universe today). If the network dissipates energy primarily through gravitational waves, this can lead to the universe passing through an epoch where its energy density is principally in the form of gravitational waves. This can also lead to a surviving network of fundamental cosmic superstrings; as phenomenological scenarios of LVS are expected to operate with low string scales, $10^{12}\text{GeV} \lesssim m_s \lesssim 10^{16}\text{GeV}$, the string tension $G\mu$ can naturally lie in interesting observational windows $10^{-7} \lesssim G\mu \lesssim 10^{-11}$.

It will be interesting to study the physics and cosmology of these scenarios further.

# Appendix A

# Degenerate Case: the anomalous dimension matrix

In this appendix we show how the anomalous dimensions of a degenerate system can be obtained by diagonalizing the matrix of the "effective" anomalous dimensions $\tilde{\gamma}_{[ij]\,[kl]}(n,\ell)$, as stated in section **??**. The latter are obtained from the mixed correlators $\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \mathcal{O}_l \rangle$, and will be defined precisely in the following.

From a bootstrap perspective, the crucial point is that at zeroth order in the large-$N$ expansion the contribution of the various double traces in any of the OPEs cannot be distinguished from one another. Schematically, the $\mathcal{O}_1\mathcal{O}_1$, $\mathcal{O}_2\mathcal{O}_2$ and $\mathcal{O}_1\mathcal{O}_2$ OPEs' can be decomposed as

$$\mathcal{O}_i(x)\mathcal{O}_j(0) \supset c_{ijA}(n,\ell)\,[\mathcal{O}\mathcal{O}]^A_{n,\ell}(x);\;\; c_{ijB}(n,\ell)\,[\mathcal{O}\mathcal{O}]^B_{n,\ell}(x);\;\; c_{ijC}(n,\ell)\,[\mathcal{O}\mathcal{O}]^C_{n,\ell}(x) \tag{A.1}$$

where the $[\mathcal{O}\mathcal{O}]^{A,B,C}_{n,\ell}$ operators are orthogonal linear combinations of the double trace operators in absence of mixing, which diagonalize the Hamiltonian at order $1/N^2$.

In terms of these variables, the conformal block expansion of a correlator involving pairs of identical operators is

$$\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_i \mathcal{O}_j \rangle = \sum_{\Delta,\ell} c^2_{ijA}(n,\ell) G^A_{\Delta,\ell} + c^2_{ijB}(n,\ell) G^B_{\Delta,\ell} + c^2_{ijC}(n,\ell) G^C_{\Delta,\ell}, \tag{A.2}$$

where the $G^X_{\Delta,\ell}$ are the conformal blocks corresponding to the $[\mathcal{O}\mathcal{O}]^X_{n,\ell}(x)$ double trace operators. The functional form of the conformal blocks is identical in all cases, and

the subscript is just to remind they are a function of different conformal dimensions. Up to order $\mathcal{O}\left(\frac{1}{N^4}\right)$, each term can be expanded as:

$$\begin{aligned}
&\left[c_{ijA}^{(0)\,2}(n,\ell)+c_{ijB}^{(0)\,2}(n,\ell)+c_{ijC}^{(0)\,2}(n,\ell)\right]G_{2\Delta+2n+\ell,\ell}\\
&+\left[2c_{ijA}^{(0)}c_{ijA}^{(1)}+2c_{ijB}^{(0)}c_{ijB}^{(1)}+2c_{ijC}^{(0)}c_{ijC}^{(1)}\right]\frac{G_{2\Delta+2n+\ell,\ell}}{N^2}\\
&+\left[c_{ijA}^{(0)\,2}\gamma_A(n,\ell)+c_{ijB}^{(0)\,2}(n,\ell)\gamma_B(n,\ell)+c_{ijC}^{(0)\,2}\gamma_C(n,\ell)\right]\frac{\partial_\Delta G_{2\Delta+2n+\ell,\ell}}{N^2}
\end{aligned}\tag{A.3}$$

Comparing this with the standard expansion for the correlator

$$f^{(0)\,2}(n,\ell)G^{11}_{2\Delta+2n+\ell,\ell}+2f^{(0)}(n,\ell)f^{(1)}(n,\ell)\frac{G_{2\Delta+2n+\ell,\ell}}{N^2}+f^{(0)\,2}(n,\ell)\tilde{\gamma}_{[ij][ij]}(n,\ell)\frac{\partial_\Delta G_{2\Delta+2n+\ell,\ell}}{N^2},\tag{A.4}$$

we obtain the equations

$$c_{ijA}^{(0)\,2}(n,\ell)+c_{ijB}^{(0)\,2}(n,\ell)+c_{ijC}^{(0)\,2}(n,\ell)=f^{(0)\,2}(n,\ell)\tag{A.5}$$

$$c_{ijA}^{(0)\,2}\gamma_A(n,\ell)+c_{ijB}^{(0)\,2}(n,\ell)\gamma_B(n,\ell)+c_{ijC}^{(0)\,2}\gamma_C(n,\ell)=f^{(0)\,2}(n,\ell)\tilde{\gamma}_{[ij][ij]}(n,\ell)\tag{A.6}$$

The same can be done for a mixed correlator $\langle\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k\mathcal{O}_l\rangle$, with different pairs $(i,j)\neq(k,l)$. Then, the conformal block decomposition reads

$$\langle\mathcal{O}_i\mathcal{O}_j\mathcal{O}_i\mathcal{O}_j\rangle=\sum_{\Delta,\ell}c_{ijA}(n,\ell)c_{klA}(n,\ell)G^A_{\Delta,\ell}+c_{ijB}(n,\ell)c_{klB}(n,\ell)G^B_{\Delta,\ell}+c_{ijC}c_{klC}(n,\ell)G^C_{\Delta,\ell}.\tag{A.7}$$

and can be expanded as

$$\begin{aligned}
&\left[c_{ijA}^{(0)}(n,\ell)c_{klA}^{(0)}(n,\ell)+c_{ijB}^{(0)}(n,\ell)c_{klB}^{(0)}(n,\ell)+c_{ijC}^{(0)}(n,\ell)c_{klC}^{(0)}(n,\ell)\right]G_{2\Delta+2n+\ell,\ell}\\
&+\left[c_{ijA}^{(0)}c_{klA}^{(1)}+c_{ijA}^{(1)}c_{klA}^{(0)}+c_{ijB}^{(0)}c_{klB}^{(1)}+c_{ijB}^{(1)}c_{klB}^{(0)}+c_{ijC}^{(0)}c_{klC}^{(1)}+c_{ijC}^{(1)}c_{klC}^{(0)}\right]\frac{G_{2\Delta+2n+\ell,\ell}}{N^2}\\
&+\left[c_{ijA}^{(0)}c_{klA}^{(0)}\gamma_A(n,\ell)+c_{ijB}^{(0)}c_{klB}^{(0)}\gamma_B(n,\ell)+c_{ijC}^{(0)}c_{klC}^{(0)}\gamma_C(n,\ell)\right]\frac{\partial_\Delta G_{2\Delta+2n+\ell,\ell}}{N^2}.
\end{aligned}\tag{A.8}$$

Since the disconnected four-point function only receives a contribution from the identity, the same correlator can be written as

$$2f^{(0)}(n,\ell)f^{(1)}(n,\ell)\frac{G_{2\Delta+2n+\ell,\ell}}{N^2}+f^{(0)\,2}(n,\ell)\tilde{\gamma}_{[ij][kl]}(n,\ell)\frac{\partial_\Delta G_{2\Delta+2n+\ell,\ell}}{N^2},\tag{A.9}$$

resulting in the equations

$$c^{(0)}_{ijA}(n,\ell)c^{(0)}_{klA}(n,\ell) + c^{(0)}_{ijB}(n,\ell)c^{(0)}_{klB}(n,\ell) + c^{(0)}_{ijC}(n,\ell)c^{(0)}_{klC}(n,\ell) = 0 \tag{A.10}$$

$$c^{(0)}_{ijA}(n,\ell)c^{(0)}_{klA}(n,\ell)\gamma_A(n,\ell) + c^{(0)\,2}_{ijB}(n,\ell)\gamma_B(n,\ell) + c^{(0)\,2}_{ijC}\gamma_C(n,\ell) = f^{(0)\,2}(n,\ell)\tilde{\gamma}_{[ij][ij]}(n,\ell). \tag{A.11}$$

This system admits a simple geometric interpretation. Equations (A.5) and (A.10) imply that the vectors

$$\vec{c}^{(0)}_{ij}(n,\ell) = \frac{1}{f^{(0)}(n,\ell)}\Big(c^{(0)}_{ijA}(n,\ell), c^{(0)}_{ijB}(n,\ell), c^{(0)}_{ijC}(n,\ell)\Big) \tag{A.12}$$

are an orthonormal basis in $\mathbb{R}^3$. In this basis, the original anomalous dimension matrix $\mathbf{M}(n,\ell) = \mathrm{Diag}(\gamma_{\mathrm{A}}(\mathrm{n},\ell), \gamma_{\mathrm{B}}(\mathrm{n},\ell), \gamma_{\mathrm{C}}(\mathrm{n},\ell))$ has matrix elements

$$\tilde{\mathbf{M}}(n,\ell)_{[ij][kl]} = \langle c^{(0)}_{ij}(n,\ell)|\,\mathbf{M}(n,\ell)\,|c^{(0)}_{kl}(n,\ell)\rangle = \tilde{\gamma}_{[ij][kl]}, \tag{A.13}$$

where the second equality derives from (A.6) and (A.11). Therefore, the $\tilde{\gamma}_{[ij][kl]}$ are the coefficients of the anomalous dimension matrix $\mathbf{M}(n,\ell)$ expressed in a different orthonormal basis, implying the two share the same eigenvalues.

# Appendix B

# Fibred Calabi-Yau Effective Lagrangians

## B.1 One Fibre Modulus

From equation(3.70), the Lagrangian for the fibred one-modulus Calabi Yau scenario is :

$$\mathcal{L}_{kin} = \frac{3}{8\tau_1^2}\partial_u\tau_1\partial^u\tau_1 - \frac{1}{2\tau_1\mathcal{V}}\partial_u\tau_1\partial^u\mathcal{V} + \frac{1}{2\mathcal{V}^2}\partial_u\mathcal{V}\partial^u\mathcal{V} + \frac{1}{4\tau_1^2}\partial_u a_1\partial^u a_1 + \frac{\alpha^2\tau_1}{2\mathcal{V}^2}\partial_u a_2\partial^u a_2. \tag{B.1}$$

To put this in a canonically normalised form, we use the ansatz:

$$\begin{aligned} \ln\tau_1 &= a\Phi_1 + b\Phi_2, \\ \ln\mathcal{V} &= c\Phi_1. \end{aligned} \tag{B.2}$$

We substitute this ansatz into the Lagrangian:

$$\begin{aligned} \frac{3}{8\tau_1^2}\partial_u\tau_1\partial^u\tau_1 = \frac{3}{8}(\partial_u\ln\tau_1)^2 &= \frac{3}{8}(a\partial_u\Phi_1 + b\partial_u\Phi_2)^2 \\ &= \frac{3}{8}a^2(\partial_u\Phi_1)^2 + \frac{3}{8}b^2(\partial_u\Phi_2)^2 + \frac{3}{4}ab\partial_u\Phi_1\partial^u\Phi_2, \end{aligned} \tag{B.3}$$

$$-\frac{1}{2\tau_1\mathcal{V}}\partial_u\tau_1\partial^u\mathcal{V} = -\frac{1}{2}\partial_u\ln\tau_1\partial^u\ln\mathcal{V} = -\frac{1}{2}(a\partial_u\Phi_1 + b\partial_u\Phi_2)c\partial^u\Phi_1 \\ = -\frac{1}{2}ac(\partial_u\Phi_1)^2 - \frac{1}{2}bc\partial_u\Phi_2\partial^u\Phi_1, \tag{B.4}$$

$$\frac{1}{2\mathcal{V}^2}\partial_u\mathcal{V}\partial^u\mathcal{V} = \frac{1}{2}(\partial_u\ln\mathcal{V})^2 = \frac{1}{2}c^2(\partial_u\Phi_1)^2. \tag{B.5}$$

Putting them together, the kinetic term reads:

$$\mathcal{L}_{kin} = \left(\frac{3}{8}a^2 - \frac{1}{2}ac + \frac{1}{2}c^2\right)(\partial_u\Phi_1)^2 + \left(\frac{3}{4}ab - \frac{1}{2}bc\right)\partial_u\Phi_1\partial^u\Phi_2 + \frac{3}{8}b^2(\partial_u\Phi_2)^2 \tag{B.6}$$

Canonical normalisation then requires satisfying the following conditions:

$$\frac{3}{8}a^2 - \frac{1}{2}ac + \frac{1}{2}c^2 = \frac{1}{2}, \tag{B.7}$$

$$\frac{3}{4}ab - \frac{1}{2}bc = 0, \tag{B.8}$$

$$\frac{3}{8}b^2 = \frac{1}{2}. \tag{B.9}$$

A solution is:

$$\begin{aligned} a &= \sqrt{\frac{2}{3}}, \\ b &= \frac{2}{\sqrt{3}}, \\ c &= \sqrt{\frac{3}{2}}. \end{aligned} \tag{B.10}$$

After substituting this into the kinetic term, we have:

$$\frac{3}{8\tau_1^2}\partial_u\tau_1\partial^u\tau_1 = \frac{1}{4}(\partial_u\Phi_1)^2 + \frac{1}{2}(\partial_u\Phi_2)^2 + \frac{\sqrt{2}}{2}\partial_u\Phi_1\partial^u\Phi_2, \tag{B.11}$$

$$-\frac{1}{2\tau_1\mathcal{V}}\partial_u\tau_1\partial^u\mathcal{V} = -\frac{1}{2}(\partial_u\Phi_1)^2 - \frac{\sqrt{2}}{2}\partial_u\Phi_1\partial^u\Phi_2, \tag{B.12}$$

$$\frac{1}{2\mathcal{V}^2}\partial_u\mathcal{V}\partial^u\mathcal{V} = \frac{1}{2}(\partial_u\ln\mathcal{V})^2 = \frac{3}{4}(\partial_u\Phi_1)^2. \tag{B.13}$$

Adding these together, we have both diagonal kinetic terms and diagonal mass terms

$$\frac{1}{2}(\partial_u \Phi_1)^2 + \frac{1}{2}(\partial_u \Phi_2)^2 + V(\Phi). \tag{B.14}$$

Where the canonical fields $\Phi_1, \Phi_2$ relate to the original fields as:

$$\mathcal{V} = e^{\sqrt{\frac{3}{2}}\Phi_1}, \tag{B.15}$$

$$\tau_1 = e^{\sqrt{\frac{2}{3}}(\Phi_1 + 2\Phi_2)}. \tag{B.16}$$

## B.2 Two Fibre Moduli

In the large volume limit, the Kähler metric reads:

$$K_{i\bar{j}} = \begin{pmatrix} \frac{1}{4\tau_1^2} & \frac{\tau_s^{\frac{3}{2}}}{8\tau_1^{\frac{3}{2}}\tau_2^{\frac{3}{2}}\tau_3^{\frac{1}{2}}} & \frac{\tau_s^{\frac{3}{2}}}{8\tau_1^{\frac{3}{2}}\tau_2^{\frac{1}{2}}\tau_3^{\frac{3}{2}}} & -\frac{3\tau_s^{\frac{1}{2}}}{8\tau_1\mathcal{V}} \\ \frac{\tau_s^{\frac{3}{2}}}{8\tau_1^{\frac{3}{2}}\tau_2^{\frac{3}{2}}\tau_3^{\frac{1}{2}}} & \frac{1}{4\tau_2^2} & \frac{\tau_s^{\frac{3}{2}}}{8\tau_1^{\frac{1}{2}}\tau_2^{\frac{3}{2}}\tau_3^{\frac{3}{2}}} & -\frac{3\tau_s^{\frac{1}{2}}}{8\tau_2\mathcal{V}} \\ \frac{\tau_s^{\frac{3}{2}}}{8\tau_1^{\frac{3}{2}}\tau_2^{\frac{1}{2}}\tau_3^{\frac{3}{2}}} & \frac{\tau_s^{\frac{3}{2}}}{8\tau_1^{\frac{1}{2}}\tau_2^{\frac{3}{2}}\tau_3^{\frac{3}{2}}} & \frac{1}{4\tau_3^2} & -\frac{3\tau_s^{\frac{1}{2}}}{8\tau_3\mathcal{V}} \\ -\frac{3\tau_s^{\frac{1}{2}}}{8\tau_1\mathcal{V}} & -\frac{3\tau_s^{\frac{1}{2}}}{8\tau_2\mathcal{V}} & -\frac{3\tau_s^{\frac{1}{2}}}{8\tau_3\mathcal{V}} & \frac{3}{8}\frac{1}{\tau_s^{\frac{1}{2}}\mathcal{V}} \end{pmatrix},$$

while the inverse matrix is:

$$K^{-1} = \begin{pmatrix} 4\tau_1^2 & -2\frac{\tau_s^{\frac{3}{2}}\tau_1^{\frac{1}{2}}\tau_2^{\frac{1}{2}}}{\tau_3^{\frac{1}{2}}} & -2\frac{\tau_s^{\frac{3}{2}}\tau_1^{\frac{1}{2}}\tau_3^{\frac{1}{2}}}{\tau_2^{\frac{1}{2}}} & 4\tau_1\tau_s \\ -2\frac{\tau_s^{\frac{3}{2}}\tau_1^{\frac{1}{2}}\tau_2^{\frac{1}{2}}}{\tau_3^{\frac{1}{2}}} & 4\tau_2^2 & -2\frac{\tau_s^{\frac{3}{2}}\tau_2^{\frac{1}{2}}\tau_3^{\frac{1}{2}}}{\tau_1^{\frac{1}{2}}} & 4\tau_2\tau_s \\ -2\frac{\tau_s^{\frac{3}{2}}\tau_1^{\frac{1}{2}}\tau_3^{\frac{1}{2}}}{\tau_2^{\frac{3}{2}}} & -2\frac{\tau_s^{\frac{3}{2}}\tau_3^{\frac{1}{2}}\tau_2^{\frac{1}{2}}}{\tau_1^{\frac{1}{2}}} & 4\tau_3^2 & 4\tau_3\tau_s \\ 4\tau_1\tau_s & 4\tau_2\tau_s & 4\tau_3\tau_s & \frac{8}{3}V\tau_s^{\frac{1}{2}} \end{pmatrix}.$$

Therefore, the kinetic terms in the Lagrangian become

$$\begin{aligned} \mathcal{L}_{kin} =& \frac{1}{4\tau_1^2}\partial^u\tau_1\partial_u\tau_1 + \frac{1}{4\tau_1^2}\partial^u a_1\partial_u a_1 + \frac{1}{4\tau_2^2}\partial^u\tau_2\partial_u\tau_2 + \\ & \frac{1}{4\tau_2^2}\partial^u a_2\partial_u a_2 + \frac{1}{4\tau_3^2}\partial^u\tau_3\partial_u\tau_3 + \frac{1}{4\tau_3^2}\partial^u a_3\partial_u a_3 \end{aligned} \tag{B.17}$$

To keep the mass term diagonal, we must have:

$$\ln\mathcal{V} = a\Phi_1, \tag{B.18}$$

and we also anticipate:

$$\begin{aligned} \ln\tau_1 &= b\Phi_1 + c\Phi_2 + d\Phi_3, \\ \ln\tau_2 &= b\Phi_1 + e\Phi_2 + f\Phi_3. \end{aligned} \tag{B.19}$$

Similarly to the previous subsection, canonical normalisation leads to the following conditions:

$$
\begin{aligned}
&a^2+\frac{3}{2}b^2-2ab=\frac{1}{2},\\
&c^2+e^2+ce=1,\\
&d^2+f^2+df=1,\\
&a(c+e)=\frac{3}{2}b(c+e),\\
&a(d+f)=\frac{3}{2}b(d+f),\\
&c(d+\frac{1}{2}f)+e(\frac{1}{2}d+f)=0.
\end{aligned}
\tag{B.20}
$$

A solution is:

$$
\begin{aligned}
a&=\sqrt{\frac{3}{2}},\\
b&=\sqrt{\frac{2}{3}},\\
c&=e=\sqrt{\frac{1}{3}},\\
d&=-f=1.
\end{aligned}
\tag{B.21}
$$

The Lagrangian then reads:

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} =&\frac{1}{2}(\partial_u\Phi_1)^2+\frac{1}{2}(\partial_u\Phi_2)^2+\frac{1}{2}(\partial_u\Phi_3)^2+\frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\Phi_1-2\Phi_3-2\sqrt{\frac{1}{3}}\Phi_2}(\partial_u a_1)^2\\
&+\frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\Phi_1+2\Phi_3-2\sqrt{\frac{1}{3}}\Phi_2}(\partial_u a_2)^2+\frac{1}{4}e^{\left(2\sqrt{\frac{2}{3}}-2\sqrt{\frac{3}{2}}\right)\Phi_1+4\sqrt{\frac{1}{3}}\Phi_2}(\partial_u a_3)^2-V_{lvs}.
\end{aligned}
\tag{B.22}
$$

Other solutions are also possible but are physically equivalent under field redefinitions.

# Appendix C

# Type IIA Effective Lagrangian

## C.1 Scalar Potential

At the minimum, the potential takes the value in (5.46) and through the relation $V_0 = -\frac{3M_P^2}{R_{AdS}}$ one can express the various couplings as a function of the AdS radius. Surprisingly, all of the dependence on the values of the fluxes drops out, and the mass matrix is simply given by[1]

$$M_{ab}^2 = \frac{1}{R_{AdS}^2} \begin{pmatrix} 34 & 16 & 16 & 8 \\ 16 & 34 & 16 & 8 \\ 16 & 16 & 34 & 8 \\ 8 & 8 & 8 & 22 \end{pmatrix}. \tag{C.1}$$

It can be diagonalised in terms of new fields $\varphi = U\phi$, where

$$U = \begin{pmatrix} 2 & 2 & 2 & 1 \\ -1 & 0 & 0 & 2 \\ -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}, \tag{C.2}$$

[1]Since the kinetic term (4.4) is not canonically normalized, we have been careful to use $\tilde{D} = \frac{D}{\sqrt{2}}$.

yielding the mass matrix

$$M_D^2 = \frac{1}{R_{AdS}^2}\text{Diag}(70, 18, 18, 18). \tag{C.3}$$

Using the standard relation $\Delta(\Delta - d) = m^2 R_{AdS}^2$ one obtains[2]

$$\Delta_1 = 10 \qquad \Delta_2 = \Delta_3 = \Delta_4 = 6. \tag{C.4}$$

It is interesting to notice how the exchange symmetry $\phi_i \leftrightarrow \phi_j$ for the first three fields is only responsible for part of the degeneracy between the conformal dimensions. Indeed, for a generic matrix

$$M = \begin{pmatrix} a & b & b & c \\ b & a & b & c \\ b & b & a & c \\ c & c & c & d \end{pmatrix} \tag{C.5}$$

with the same symmetry properties only two of the eigenvalues are identical:

$$\lambda_1 = \lambda_2 = a - b \quad \lambda_{3,4} = \frac{1}{2}\left(a + 2b + d \pm \sqrt{(a + 2b - d)^2 + 12c^2}\right). \tag{C.6}$$

In order for three (or more) to be equal, the matrix elements must satisfy the additional constraint

$$(a - b - d)b + c^2 = 0, \tag{C.7}$$

which in the case at hand appears to be coincidental.

Expanding at higher orders, one can obtain the n-point self-interactions for the moduli. Given the exponential structure of the potential, derivatives with respect to any of the moduli evaluated at the minimum will be proportional to $V|_{\text{min}}$; hence all vertices will all come with the same factor of $R_{AdS}^{-2}$ in front. This means that also the couplings are independent of the flux choice, and universally fixed apart from an overall constant (corresponding to the $1/N$ expansion parameter) which vanishes in the infinite volume limit. We report here the cubic terms in the potential, the only

[2]This equation can have two real solutions, but we only consider the one satisfying the unitarity bound.

ones necessary to compute large-spin anomalous dimensions.

$$
\begin{aligned}
&V^{(3)}R^2_{AdS} = \\
&12\sqrt{2}\tilde{D}^2\phi_1 + 12\sqrt{2}\tilde{D}^2\phi_2 + 12\sqrt{2}\tilde{D}^2\phi_3 + \frac{27\tilde{D}^3}{\sqrt{2}} + 30\sqrt{2}\tilde{D}\phi_1^2 + 30\sqrt{2}\tilde{D}\phi_2^2 \; . \\
&+ 30\sqrt{2}\tilde{D}\phi_3^2 + 24\sqrt{2}\tilde{D}\phi_1\phi_2 + 24\sqrt{2}\tilde{D}\phi_1\phi_3 + 24\sqrt{2}\tilde{D}\phi_2\phi_3 + 18\sqrt{2}\phi_1\phi_2\phi_3
\end{aligned}
\tag{C.8}
$$

In terms of the diagonal fields $\varphi_i$, it becomes

$$
\begin{aligned}
&V^{(3)}R^2_{AdS} = \\
&\frac{\sqrt{2}}{169}\Big(1080\,\varphi_2\varphi_4^2 - 360\,\varphi_3\varphi_4^2 - 120\,\varphi_4^3 - 105\,\varphi_1^2\varphi_4 - 198\,\varphi_2^2\varphi_4 - 360\,\varphi_3^2\varphi_4 \\
&- 882\,\varphi_2\varphi_3\varphi_4 + 135\,\varphi_1^3 + 198\,\varphi_2^3 - 120\,\varphi_3^3 + 1080\,\varphi_2\varphi_3^2 + 315\,\varphi_1^2\varphi_2 - 105\,\varphi_1^2\varphi_3 - 198\,\varphi_2^2\varphi_3\Big)
\end{aligned}
\tag{C.9}
$$

We notice that, at least up to cubic order, the potential is invariant under the exchange of $\varphi_3$ and $\varphi_4$.

## C.2 Axion Kinetic Term

The same procedure can be carried out for axions, expanding the lagrangian (4.11) about the usual vacuum solution in terms of the canonically normalized fields

$$
b_i' = \frac{b_i}{\sqrt{2}\bar{v}_i} \qquad \xi' = \xi e^{\bar{D}} \tag{C.10}
$$

and the moduli fluctuations

$$
\phi_i' = \phi_i - \bar{\phi}_i \qquad D' = D - \bar{D}, \tag{C.11}
$$

where it is understood that all primes will be omitted in the following. The resulting expression can be divided into a kinetic term

$$
\mathcal{L}_{kin} = \frac{1}{2}e^{-2\sqrt{2}\phi_i}\partial^\mu b_i \partial_\mu b^i + \frac{1}{2}e^{\sqrt{2}\tilde{D}}\partial_\mu\xi\partial^\mu\xi \tag{C.12}
$$

and a potential

$$
\begin{aligned}
&\mathcal{L}_{\phi b} = \\
&\frac{15}{R_{AdS}^2} e^{2\sqrt{2}\tilde{D}} \Big[ s_1 b_2 b_3 e^{\sqrt{2}(\phi_1-\phi_2-\phi_3)} + s_2 b_3 b_1 e^{\sqrt{2}(\phi_2-\phi_3-\phi_1)} + s_3 b_1 b_2 e^{\sqrt{2}(\phi_3-\phi_1-\phi_2)} \Big] \\
&- \frac{9}{2R_{AdS}^2} e^{\sqrt{2}(2\tilde{D}-\phi_1-\phi_2-\phi_3)} \Big[ b_1^2 + b_2^2 + b_3^2 + 2 s_1 s_2 b_1 b_2 + 2 s_1 s_3 b_1 b_3 + 2 s_2 s_3 b_2 b_3 \Big] \\
&- \frac{12}{R_{AdS}^2} e^{\sqrt{2}(2\tilde{D}-\phi_1-\phi_2-\phi_3)} \Big[ s_1 b_1 + s_2 b_2 + s_3 b_3 \Big] \xi - \frac{8}{R_{AdS}^2} e^{\sqrt{2}(2\tilde{D}-\phi_1-\phi_2-\phi_3)} \xi^2 \\
&- \frac{25}{2R_{AdS}^2} e^{2\sqrt{2}\tilde{D}} \Big[ b_1^2 e^{\sqrt{2}(\phi_2+\phi_3-\phi_1)} + b_2^2 e^{\sqrt{2}(\phi_3+\phi_1-\phi_2)} + b_3^2 e^{\sqrt{2}(\phi_1+\phi_2-\phi_3)} \Big].
\end{aligned}
\tag{C.13}
$$

### C.2.1 Spectrum

Setting all of the moduli fluctuations to zero, this gives the mass matrix

$$
M_{ab}^2 = \frac{1}{R_{AdS}^2} \begin{pmatrix}
34 & 9\,s_1 s_2 - 15\,s_3 & 9\,s_1 s_3 - 15\,s_2 & 12\,s_1 \\
9\,s_1 s_2 - 15\,s_3 & 34 & 9\,s_2 s_3 - 15\,s_1 & 12 s_2 \\
9\,s_1 s_3 - 15\,s_2 & 9\,s_2 s_3 - 15\,s_1 & 34 & 12\,s_3 \\
12\,s_1 & 12\,s_2 & 12\,s_3 & 16
\end{pmatrix}, \tag{C.14}
$$

as reported in DeWolfe:2005uu. Since the matrix depends on the signs $s_i \equiv \mathrm{sgn}(m_0 e_i)$, we can distinguish four different combinations (permutations are indistinguishable):

**Case 1:** s = (1,1,1)
The matrix is diagonalised in terms of the new fields $a = U_1 b$, where

$$
U_1 = \begin{pmatrix}
2 & 0 & 0 & 1 \\
-1 & 0 & 1 & 0 \\
-1 & 1 & 0 & 0 \\
-1 & -1 & -1 & 2
\end{pmatrix}. \tag{C.15}
$$

The mass matrix is given by

$$M_D^2 = \frac{1}{R_{AdS}^2}\text{Diag}(40, 40, 40, -2), \tag{C.16}$$

resulting in

$$\Delta_a = (8, 8, 8, 2) \qquad \text{or} \qquad \Delta_a = (8, 8, 8, 1). \tag{C.17}$$

**Case 2:** s = (1,1,-1)
The matrix is diagonalised in terms of the new fields $a = U_2 b$, where

$$U_2 = \begin{pmatrix} 2 & 2 & -2 & 1 \\ -1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}. \tag{C.18}$$

The mass matrix is given by

$$M_D^2 = \frac{1}{R_{AdS}^2}\text{Diag}(88, 10, 10, 10), \tag{C.19}$$

resulting in

$$\Delta_a = (11, 5, 5, 5). \tag{C.20}$$

**Case 3:** s = (1,-1,-1)
The matrix is diagonalised in terms of the new fields $a = U_3 b$, where

$$U_3 = \begin{pmatrix} 2 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 2 \end{pmatrix}. \tag{C.21}$$

The mass matrix is given by

$$M_D^2 = \frac{1}{R_{AdS}^2}\text{Diag}(40, 40, 40, -2), \tag{C.22}$$

resulting in

$$\Delta_a = (8, 8, 8, 2) \qquad \text{or} \qquad \Delta_a = (8, 8, 8, 1). \tag{C.23}$$

**Case 4:** s = (-1,-1,-1)
The matrix is diagonalised in terms of the new fields $a = U_4 b$, where

$$U_4 = \begin{pmatrix} -2 & -2 & -2 & 1 \\ 1 & 0 & 0 & 2 \\ -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}. \tag{C.24}$$

The mass matrix is given by

$$M_D^2 = \frac{1}{R_{AdS}^2}\text{Diag}(88, 10, 10, 10), \tag{C.25}$$

resulting in

$$\Delta_a = (11, 5, 5, 5). \tag{C.26}$$

### C.2.2 Interactions

For the interactions, let's start from the ones originating from the kinetic term (C.12). Again, we can distinguish differences depending on the flux signs:

**Case 1:** s = (1,1,1)

$$
\begin{aligned}
\mathcal{L}^{1}_{kin} = -\frac{1}{637\sqrt{2}}\Big[ & \\
& (\partial b_1)^2\Big(39\varphi_1 - 78\varphi_2 + 26\varphi_3 + 26\varphi_4\Big) + \partial b_1 \partial b_2\Big(52\varphi_1 - 104\varphi_2 \\
& + 520\varphi_3 - 208\varphi_4\Big) + \partial b_1 \partial b_3\Big(52\varphi_1 - 104\varphi_2 - 208\varphi_3 + 520\varphi_4\Big) \\
& + \partial b_1 \partial b_4\Big( - 60\varphi_1 - 48\varphi_2 + 16\varphi_3 + 16\varphi_4\Big) + (\partial b_2)^2\Big(148\varphi_1 - 100\varphi_2 \\
& + 640\varphi_3 - 270\varphi_4\Big) + \partial b_2 \partial b_3\Big( - 96\varphi_1 - 4\varphi_2 - 120\varphi_3 - 120\varphi_4\Big) \\
& + \partial b_2 \partial b_4\Big( - 40\varphi_1 - 32\varphi_2 - 232\varphi_3 + 132\varphi_4\Big) + (\partial b_3)^2\Big(148\varphi_1 - 100\varphi_2 \\
& - 270\varphi_3 + 640\varphi_4\Big) + \partial b_3 \partial b_4\Big( - 40\varphi_1 - 32\varphi_2 + 132\varphi_3 - 232\varphi_4\Big) \\
& + (\partial b_4)^2\Big(8\varphi_1 - 30\varphi_2 + 10\varphi_3 + 10\varphi_4\Big)\Big]
\end{aligned}
\tag{C.27}
$$

$$
\begin{aligned}
\mathcal{L}^1_{\varphi b} = \frac{1}{637}\Big[ & \\
& b_1^2\Big((1560\sqrt{2}-360)\varphi_1 + (180-6240\sqrt{2})\varphi_2 + (2080\sqrt{2}-60)\varphi_3 \\
& + (2080\sqrt{2}-60)\varphi_4\Big) + b_1 b_2\Big((2080\sqrt{2}-480)\varphi_1 + (240-8320\sqrt{2})\varphi_2 \\
& + (8840\sqrt{2}-3720)\varphi_3 + (1740-260\sqrt{2})\varphi_4\Big) + b_1 b_3\Big((2080\sqrt{2}-480)\varphi_1 \\
& + (240-8320\sqrt{2})\varphi_2 + (1740-260\sqrt{2})\varphi_3 + (8840\sqrt{2}-3720)\varphi_4\Big) \\
& + b_1 b_4\Big((360+120\sqrt{2})\varphi_1 - (180+60\sqrt{2})\varphi_2 + (60+20\sqrt{2})\varphi_3 \\
& + (60+20\sqrt{2})\varphi_4\Big) + b_2^2\Big((330-1430\sqrt{2})\varphi_1 - (165+10205\sqrt{2})\varphi_2 \\
& + (10985\sqrt{2}+1875)\varphi_3 - (855+390\sqrt{2})\varphi_4\Big) + b_2 b_3\Big((3510\sqrt{2}-810)\varphi_1 \\
& + (4050+1885\sqrt{2})\varphi_2 - 2145\sqrt{2}\varphi_3 - 2145\sqrt{2}\varphi_4\Big) \\
& + b_2 b_4\Big((240+80\sqrt{2})\varphi_1 - (120+40\sqrt{2})\varphi_2\Big) + (1860-3020\sqrt{2})\varphi_3 \\
& + (1530\sqrt{2}-870)\varphi_4\Big) + b_3^2\Big((330-1430\sqrt{2})\varphi_1 - (165+10205\sqrt{2})\varphi_2 \\
& - (855+390\sqrt{2})\varphi_3 + (10985\sqrt{2}+1875)\varphi_4\Big) + b_3 b_4\Big((240+80\sqrt{2})\varphi_1 \\
& - (120+40\sqrt{2})\varphi_2\Big) + (1530\sqrt{2}-870)\varphi_3 + (1860-3020\sqrt{2})\varphi_4\Big) \\
& + b_4^2\Big( - (90+58\sqrt{2})\varphi_1 + (45+120\sqrt{2})\varphi_2 - (15+40\sqrt{2})\varphi_3 \\
& - (15+40\sqrt{2})\varphi_4\Big)\Big].
\end{aligned}
\tag{C.28}
$$

Both are symmetric under the simultaneous exchanges

$$
\varphi_3 \longleftrightarrow \varphi_4 \qquad b_2 \longleftrightarrow b_3. \tag{C.29}
$$

**Case 2:** s = (1,-1,1)

$$
\begin{aligned}
\mathcal{L}^2_{kin} = -\frac{1}{2197\sqrt{2}}\Big[ \\
&(\partial b_1)^2\Big(47\varphi_1 - 30\varphi_2 + 10\varphi_3 + 10\varphi_4\Big) + \partial b_1 \partial b_2\Big( - 60\varphi_1 - 48\varphi_2 \\
&+ 16\varphi_3 + 16\varphi_4\Big) + \partial b_1 \partial b_3\Big( - 20\varphi_1 - 16\varphi_2 - 896\varphi_3 + 456\varphi_4\Big) \\
&+ \partial b_1 \partial b_4\Big(20\varphi_1 + 16\varphi_2 + 456\varphi_3 - 896\varphi_4\Big) + (\partial b_2)^2\Big( - 24\varphi_1 - 222\varphi_2 \\
&+ 74\varphi_3 + 74\varphi_4\Big) + \partial b_2 \partial b_3\Big( - 16\varphi_1 - 148\varphi_2 + 500\varphi_3 - 176\varphi_4\Big) \\
&+ \partial b_2 \partial b_4\Big(16\varphi_1 + 148\varphi_2 + 176\varphi_3 - 500\varphi_4\Big) + (\partial b_3)^2\Big(448\varphi_1 - 250\varphi_2 \\
&+ 1210\varphi_3 - 480\varphi_4\Big) + \partial b_3 \partial b_4\Big(456\varphi_1 - 176\varphi_2 + 960\varphi_3 + 960\varphi_4\Big) \\
&+ (\partial b_4)^2\Big(448\varphi_1 - 250\varphi_2 - 480\varphi_3 + 1210\varphi_4\Big)\Big]
\end{aligned}
\tag{C.30}
$$

$$
\begin{aligned}
&\mathcal{L}^2_{\varphi b} = \frac{1}{2197}\Big[ \\
&b_1^2\Big((8\sqrt{2}+360)\varphi_1 - (180+7440\sqrt{2})\varphi_2 + (60+2480\sqrt{2})\varphi_3 \\
&+ (60+2480\sqrt{2})\varphi_4\Big) + b_1 b_2\Big((2080\sqrt{2}-480)\varphi_1 + (240-8320\sqrt{2})\varphi_2 \\
&+ (380\sqrt{2}-60)\varphi_3 + (380\sqrt{2}-60)\varphi_4\Big) + b_1 b_3\Big((760\sqrt{2}-120)\varphi_1 \\
&+ (60-380\sqrt{2})\varphi_2 - (6780+11140\sqrt{2})\varphi_3 + (3360+5760\sqrt{2})\varphi_4\Big) \\
&+ b_1 b_4\Big((-760\sqrt{2}+120)\varphi_1 + (-60+380\sqrt{2})\varphi_2 - (3360+5760\sqrt{2})\varphi_3 \\
&+ (6780+11140\sqrt{2})\varphi_4\Big) + b_2^2\Big((90+210\sqrt{2})\varphi_1 - (45+2640\sqrt{2})\varphi_2 \\
&+ (15+880\sqrt{2})\varphi_3 + (15+880\sqrt{2})\varphi_4\Big) + b_2 b_3\Big((60+140\sqrt{2})\varphi_1 \\
&- (30+1760\sqrt{2})\varphi_2 + (3390+6220\sqrt{2})\varphi_3 - (1680+2230\sqrt{2})\varphi_4\Big) \\
&+ b_2 b_4\Big( - (60+140\sqrt{2})\varphi_1 + (30+1760\sqrt{2})\varphi_2 + (1680+2230\sqrt{2})\varphi_3 \\
&- (3390+6220\sqrt{2})\varphi_4\Big) + b_3^2\Big( - (1680+3920\sqrt{2})\varphi_1 + (840-5645\sqrt{2})\varphi_2 \\
&+ (15965\sqrt{2}-13800)\varphi_3 + (6480-5160\sqrt{2})\varphi_4\Big) + b_3 b_4\Big( - (1710+3990\sqrt{2})\varphi_1 \\
&+ (855-4765\sqrt{2})\varphi_2\Big) + (12855\sqrt{2}-15945)\varphi_3 + (12855\sqrt{2}-15945)\varphi_4\Big) \\
&+ b_4^2\Big( - (1680+3920\sqrt{2})\varphi_1 + (840-5645\sqrt{2})\varphi_2 + (15965\sqrt{2}-13800)\varphi_3 \\
&+ (6480-5160\sqrt{2})\varphi_4\Big)\Big].
\end{aligned}
\tag{C.31}
$$

Both are symmetric under the simultaneous exchanges

$$
\varphi_3 \longleftrightarrow \varphi_4 \qquad b_3 \longleftrightarrow -b_4. \tag{C.32}
$$

**Case 3:** s = (1,-1,-1)

$$
\mathcal{L}^3_{kin}(b_1,b_2,b_3,b_4) = \mathcal{L}^1_{kin}(b_1,-b_2,-b_3,b_4) \tag{C.33}
$$

$$
\mathcal{L}^3_{\varphi_b}(b_1,b_2,b_3,b_4) = \mathcal{L}^1_{\varphi b}(b_1,-b_2,-b_3,b_4) \tag{C.34}
$$

**Case 4:** s = (-1,-1,-1)

$$\mathcal{L}^4_{kin}(b_1, b_2, b_3, b_4) = \mathcal{L}^2_{kin}(b_1, b_2, b_3, -b_4) \tag{C.35}$$

$$\mathcal{L}^4_{\varphi b}(b_1, b_2, b_3, b_4) = \mathcal{L}^2_{\varphi b}(b_1, b_2, b_3, -b_4) \tag{C.36}$$

The relations between cases 1 & 3 and 2 & 4 descend from the relations between the matrices $U_1$ & $U_2$ and $U_3$ & $U_4$ introduced above.

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