Game-Theoretic Network Centrality

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For my parents, who always supported me.

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Abstract

Payoff division schemes from cooperative game theory, such as the Shapley value and Banzhaf index, have recently gained popularity as measures of node centrality in networks. The general idea in this line of research is to treat nodes as players in a cooperative game, to treat a group centrality measure as a characteristic function and to apply a division scheme as a centrality measure. While this research direction is promising, the computational problems that surround it are challenging and are mostly open. On the one hand, the current research lacks sufficient breadth to understand the field and its computational intricacy as a whole. Existing literature has focused on combining a single division scheme (or class of division schemes) with a single group measure and computing it. On the other hand, the current game-theoretic measures lack sufficient depth to be applicable to certain complex but often-encountered settings, such as networks with an overlapping community structure.

The aim of this thesis is to (1) provide a broad computational analysis of a wide class of game-theoretic measures, (2) develop a new class of measures for networks with overlapping community structures, (3) propose high-performance game-theoretic measures.

With respect to the first issue, we present a generic framework for defining game-theoretic measures and prove that all measures that can be expressed in our framework are computable in polynomial time. The framework applies to a broad class of division schemes called semivalues in combination with a very wide range of group centrality measures, which is the first result of its kind to date.

Our second focus is on a class of real-life networks that have an overlapping community structure. We develop a new class of solution concepts—configuration semivalues—and use them to propose a class of centrality measures based on closeness. We develop polynomial algorithms for its computation and apply them to the Warsaw public transportation network, where they produce a highly interpretable ranking that is—arguably—better than other measures.

Finally, we propose the generalised closeness interaction index for the purposes of link prediction. We develop polynomial-time algorithms for the computation of our measure and find that it achieves the best results when compared to the state-of-the-art link prediction methods in the literature.

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List of Notation

V	The set of nodes.
E	The set of edges.
ω	$\omega : E \to \mathbb{R}$ is a weight function that assigns to each edge a real number.
$G = (V, E)$ . . .	Undirected graph/network consisting of the set of nodes V and edges E.
$D = (V, E)$ . . .	Directed graph/network consisting of the set of nodes V and edges E.
$G = (V, E, \omega)$.	Undirected weighted graph/network consisting of the set of nodes V and edges E.
$D = (V, E, \omega)$.	Directed weighted graph/network consisting of the set of nodes V and edges E.
C **or** S	A coalition.
I	A set of players of a cooperative game.
n	The number of players in a cooperative game (i.e., the size of the set I).
$\nu : 2^I \to \mathbb{R}$ . . .	A characteristic function.
$\nu(C)$	A value of the coalition, where ν is characteristic function.
$\Delta_\nu(C)$	The Harsanyi Dividend of the coalition C in the game described by the characteristic function ν.
$\beta(k)$	For a given semivalue, this is the probability that a coalition of size k is chosen.
$SV_i(\nu)$	The Shapley value of the player i in game ν.
$u, v, s, t \in V$ . .	The node from the set V.
$deg(v)$	Degree of the node v.
$E(v)$	Set of neighbours of node $v \in V$.

$E_{IN}(v)$ . . . . . Set of in neighbours of node $v \in V$.

$E_{OUT}(v)$ . . . . Set of out neighbours of node $v \in V$.

$dist(v, u)$ . . . . The distance from the node v and u.

$MC(u, v)$ . . . . Marginal contribution that node u makes through node v.

ψ . . . . . A group centrality measure.

ϕ . . . . . A payoff division scheme.

Θ . . . . . A set of items.

$\Theta_l \subseteq \Theta$ . . . . . A group of items from the set Θ.

$\vartheta \in \Theta$ . . . . . An item from the set Θ.

$R(C, \vartheta)$ . . . . . The positive association relation between coalition C and item ϑ.

$\widetilde{R}(C, \vartheta)$ . . . . . The negative association relation between coalition C and item ϑ.

$N(C, \vartheta)$ . . . . . The neutral association relation between coalition C and item ϑ.

$Nod_{\sim d}(v)$ . . . . The set of nodes $u \in V$ such that $dist(v, u) \sim d$, where $\sim$ is either $<$, $\leq$, $>$, $\geq$ or $=$.

CS . . . . . A community structure (cover) of the set V.

Q . . . . . A community such that $Q \in CS$.

T . . . . . A coallition of communities, i.e. $T \subseteq CS$.

$Com_{\sim d}(v)$ . . . The set of communities $Q \in CS$ such that $dist(Q, u) \sim d$, where $\sim$ is either $<$, $\leq$, $>$, $\geq$ or $=$.

$Nod^{Q}_{\sim d}(u)$ . . . . The set of nodes $u \in Q$ such that $dist(v, u) \sim d$, where $\sim$ is either $<$, $\leq$, $>$, $\geq$ or $=$.

$S_\nu(C, i, j)$ . . . The synergy between players i and j in the context of coalition C.

1 Introduction

1.1 Motivation

Determining the significance or *centrality* of a node or edge in a network is an important research topic, the modern approach to which goes back to at least the work of Moreno and Jennings on the inmates of a prison and the residents of a reform school for girls [1–4]. The problem has been extensively studied in biology, social network analysis, and computer science, where it has led to such diverse applications as, for instance, identifying proteins that are critical for the survival of cells [5], the maximisation of influence in social networks [6], and ranking websites in order to improve web search result accuracy [7].

In the literature, a wide range of *centrality measures* have been proposed, each presenting a different perspective on how to quantify the centrality of a node or edge in a network. Among these, the four most fundamental and prominent are *degree centrality*, *closeness centrality*, *betweenness centrality*, and *eigenvector centrality* [8, 9]. In this thesis, we will be mostly interested in the first three of these—we refer to them as *classic* centrality measures. Degree centrality ranks nodes according to the number of direct connections they have to other nodes, i.e., their degree: nodes with more direct connections are ranked higher than those that have fewer. By contrast,

closeness centrality considers the distances between nodes: if, on average, a node is close to all other nodes in the network, then it will be ranked highly. Finally, betweenness centrality considers the shortest paths between any pair of nodes in the network: the larger the number of these paths a node lies on, the higher its ranking.

A common feature of classic centrality measures is that they focus on individual nodes, without taking into account the various *synergies* that may occur when nodes are considered in groups. Such synergies are crucial for a variety of applications. For example, if we consider an epidemic spreading through a city, it is desirable to isolate those who are infected from those who are not. This can be achieved both through quarantine and vaccination. Although it is often not feasible to quarantine and vaccinate everyone in a short period of time, it may be possible to vaccinate those who act as bridges between infected and healthy groups of people. In effect, this can disconnect one (infected) part of the network from another (healthy) part. Even when vaccinating a single node cannot achieve this, vaccinating a group may. We can thus say that vaccinating groups achieves a type of *positive synergy*. Yet, some groups are more effective for this purpose. Hence, it would seem natural that groups themselves also should be ranked according to their importance.

Another example of synergies between nodes can be seen in the *Influence Maximisation Problem (IMP)* or *top-k* nodes problem [6]. The aim of the top-k nodes problem is to find the group of size k that has the largest influence over the rest of the network, which has applications in viral marketing [6]. This is not as straightforward as it may seem, since the portions of the network that nodes influence may overlap. We call this *negative synergy*, since the influence of a group is less than the sum of the influences of the individual nodes that belong to the group. For this reason, simply choosing the nodes with the largest individual influence (e.g., according to some classic centrality) is suboptimal. In fact, the optimal (although computationally infeasible) solution involves ranking every group of size k according to its influence and choosing the top one.

Recognising the significance of this point, Everett and Borgatti [10] were the first to propose extensions of centrality measures to groups of nodes. In particular, they defined group degree centrality, group betweenness centrality, and group closeness centrality, each of which ranks groups of nodes based on their combined performance. This enables us to address in a principled fashion such questions as: which group is most central, or how important is a certain arm of a larger institution (e.g., a department of a university).

In view of these considerations, it has been argued that the centrality of a single node should not only depend on individual traits like degree, closeness, or betweenness, but also on the centrality of the groups of nodes it belongs to. Returning to the example of an epidemic, priority needs to be given to inoculating groups of people who bridge parts of the network. However, it is individuals—not groups—who are inoculated, and therefore some ranking giving priority to individuals must be formed. Similarly, in a top-k nodes viral marketing situation, where the budget allows for a marketing campaign to k individuals, it is much more convenient to have a top-down ranking of the individuals who it is best to target, rather than just a group of k individuals. If the budget changes, allowing for the targeting of less or more individuals, the latter information may be useless, since the optimal groups of various sizes may have little or no individuals in common. Indeed, some of the best state-of-the-art algorithms for this problem solve it by ranking individual nodes [11]. Kempe, Kleinberg, and Tardos [12] provide an overview the state of the art in maximising the spread of influence in social networks.

Pursuing this line of thought, for certain applications the centrality of individual nodes should be a function of the centralities of groups of nodes. In a broader setting, the issue of how to aggregate group values into individual ones is addressed by cooperative game theory [13]. A key element of cooperative game theory is the study of cooperative games. Such a game consists of a set of players and a valuation function, called the *characteristic function*, which evaluates (with a real number) the value or function of every group (i.e., subset) of players. In this, there is a clear parallel between characteristic functions, which evaluate every

group of players, and group centrality measures, which evaluate every group of nodes. The players in a cooperative game, however, may have various strategic considerations, such as which other players to join.

Given that all players decide to work together, a key question addressed by cooperative game theory is how to divide the value of the group among the individuals that comprise it. One such so-called *payoff division scheme* (often also referred to as a *solution concept*) is the *Shapley value* [14], which is the unique method of dividing the value of the group among its members that satisfies certain notions of fairness. Along with the Banzhaf power index [15], the Shapley value is perhaps the most prominent member of an important class of cooperative solution concepts called *semivalues*. When considering the value of all groups, it is possible to capture certain interactions and synergies between players, such as when either one of two players is actually necessary to raise the value of a group, but having the both of them does not raise the value of the group any further. Since both players perform the same function but have still decided to work together in the group, the Shapley value would determine that they should split the value of this work in half between them. In the same fashion, payoff division schemes can be applied to group centrality measures to evaluate the importance of nodes in contributing to the functioning of a network, even when nodes do not necessarily have strategic considerations and will not "leave" the network if treated unfairly.

In their 1994 paper, Brink and Borm [16] therefore proposed to use the Shapley value [14] to quantify node centrality as a function of the marginal contributions of a node to the centrality of groups of nodes [16, 17]. This approach, which also had a precursor in the work by Grofman and Owen [18], is now generally known as *game-theoretic network centrality*. In the wake of these pioneering papers, game-theoretic centrality has gained traction and a number of centrality measures involving semivalues and other solution concepts have been proposed in the literature [19–22].

A key element of the game-theoretic approach to centrality is thus to treat nodes in a graph as players in a cooperative game. Some group centrality measure usually

defines this cooperative game, and applying a cooperative solution concept then yields the game-theoretic centrality of the individual nodes. Generalising, we arrive at a generic concept of a game-theoretic centrality measure as a pair consisting of a group centrality measure and a cooperative solution concept.

In more detail, a cooperative game is a game in which every coalition (i.e., subset) of players has some payoff (or value), and the players can form coalitions and share the payoff generated by cooperation. Alternatively, coalitions may be associated with some cost, which must be shared among the players. Of particular interest to us will be payoff (alternatively: cost) division schemes. Assuming that a coalition of nodes forms, these division schemes try to quantify to what extent each individual in the coalition is responsible for the value or cost of the coalition. The methods of dividing this payoff or cost have been studied for decades in cooperative game theory, and have resulted in systematic and organized analysis of the synergies achieved between nodes in groups and how they impact the worth of individuals.

Below, we list a number of attractive properties of game-theoretic centrality measures and discuss how these have been applied in various cases.

- Various payoff or cost division schemes have certain connotations of fairness. Whenever the nodes in a network are actually agents that receive payoff or incur cost based on their topological position within the network, division schemes can be used to determine how the centrality of these nodes impacts their economic standing, and therefore how to fairly divide payoff or cost. In this case, the strategic considerations of the nodes are important, as they may take action (such as refuse to take part in the activity of the network) if they deem that they are being treated unfairly. For example, Kimms and Kozeletskyi [23] considered a setting where a salesman must travel across a network in order to deliver goods to nodes. Given a delivery route, it would be unfair to charge those who live close to the delivery depot (and therefore make little impact on the cost of the route) the same as those who live far (and therefore impact the cost significantly). The authors used the Shapley

value, which satisfies certain "fairness" axioms, as the only fair method of charging customers for delivery. Similarly, Aziz et al. [24] use the Shapley value to study transport costs in a delivery network. They prove, however, that approximating the Shapley value in this setting within a constant factor is NP-hard and propose a number of alternatives (such as sampling methods).

- Some payoff division schemes can be interpreted as a player's expected contribution to coalitions of certain sizes. Szczepański, Michalak, and Rahwan [25] use this fact to protect networks against random node failures more efficiently than some alternative approaches. By analysing how—upon failure—nodes would "contribute" to the negative effects observed within the network, the authors are able to leverage this property in order to rank nodes in terms of their importance for protection.

- Game-theoretic measures can also be interpreted as capturing the somewhat elusive idea of synergy. In this case, nodes need not have strategic considerations, nor are we necessarily concerned with their expected contribution. In settings where the synergies or interactions between nodes are very complex or not fully understood, payoff division schemes provide a systematic approach for their study and may help understand these interactions and how they impact the importance of individual nodes. As an example of such an approach, Keinan et al. [26] identified the key sections of an animal brain and organized them into an interconnected network. Next, the authors temporarily lesioned groups of brain sections (i.e., groups of nodes) in the animal, and scores were assigned to these groups based on how well the animal performed certain cognitive tasks. Finally, the Shapley value was used to assess and rank how central individual sections of the brain were for certain cognitive functions. In this case, the interaction and synergy achieved by the various sections of the brain is critical to performing even the simplest of tasks and therefore must be taken into account when ranking nodes in terms of their importance for

performing them. After all, these tasks are the very "synergy" achieved by the combined work of the various sections of the brain.

1.2 Our Contributions

In this thesis, we are concerned with the computational analysis of existing and the development of new game-theoretic centralities.

A straightforward method to compute a game-theoretic centrality measure is to successively compute the group centrality measure for each group and then the solution concept. There are two obvious computational issues associated with this successive algorithm (which is also the optimal algorithm for certain game-theoretic centralities). First, since the number of groups in a network is exponential in the size of the graph, the cooperative game that the group centrality measure defines is also exponential size (if represented extensively). Second, even given an extensive representation of a cooperative game, computing solution concepts tends to be hard. In particular, computing the Shapley value for several compact representations is #P-complete [27, 28].

It is therefore intriguing that recent research has shown that there are instances of game-theoretic centrality measures that can be computed in polynomial time [25, 29, 30]. However, existing positive computational results concern only a small number of centrality measures and each of the polynomial algorithms for these game-theoretic network centralities required an individual approach that could not be generalised to other types of centrality measures. In other words, the current research in game-theoretic network centrality lacks breadth. For example, it is not currently known exactly what class of group centrality measures facilitates polynomial computation of which classes of solution concepts. In this thesis, we take the first steps towards such a result by developing a framework for defining group centrality measures, which leads us to the discovery of a very broad class of those group centrality measures for which any semivalue can be computed in polynomial time.

On the other hand, the existing literature lacks sufficient depth to be applicable to various realistic domains—such as networks with overlapping community structures, multi-layer networks and hyper-graphs. In particular, although a class of game-theoretic measures—coalitional semivalue degree centrality, which takes into account non-overlapping communities—exists [31], this class cannot be applied in the case when communities overlap. Even in the application proposed by the authors of these measures—coauthorship networks, where a community is defined as the set of authors who have published at a given venue—this restriction is prohibitive. To address this, we develop a new family of solution concepts—configuration semivalues—that allows communities to overlap and generalises coalitional semivalues. Moreover, we create a new family of game-theoretic centrality measures by combining our solution concepts with a generalised form of group closeness centrality (which also generalises degree) and develop polynomial algorithms for their computation.

We further develop the above concepts in order to develop a game-theoretic measure of node similarity that is based on generalised group closeness centrality. As opposed to node centrality measures, such similarity measures rank pairs of nodes and are useful—for example—in predicting whether edges exist between nodes in an incomplete information setting. The first such game-theoretic measure for networks was degree-based and proposed by Szczepański et al. [32]. In comparison, our measure is more sophisticated and achieves better empirical results when predicting links in the real-life networks that we study in Chapter 6. We develop very efficient, polynomial-time algorithms for the computation of our measure.

The computational contributions of our work are summarised in Table 1.1. We present our contributions in the following chapters of this thesis:

- In Chapter 3, we present the literature on game-theoretic network centrality that is related to the topics discussed in the thesis. We note that this is the first such organised review of literature on game-theoretic network centrality to date.

Centrality Measure	Solution Concept	First Algorithm Proposed In:
Degree	Shapley value	[21]
	Banzhaf Index	**this thesis**
	Semivalues	**this thesis**
	Owen value	[31]
	Coalitional semivalues	[31]
	Configuration value	**this thesis**
	Configuration semivalues	**this thesis**
Betweenness	Shapley value	[22, 25]
	Banzhaf Index	**this thesis**
	Semivalues	**this thesis**
	Owen value	Open
	Coalitional semivalues	Open
	Configuration value	Open
	Configuration semivalues	Open
Closeness	Shapley value	[21]
	Banzhaf Index	**this thesis**
	Semivalues	**this thesis**
	Owen value	**this thesis**
	Coalitional semivalues	**this thesis**
	Configuration value	**this thesis**
	Configuration semivalues	**this thesis**

Table 1.1: The papers that proposed polynomial algorithms for various game-theoretic centrality measures. We stress that the algorithms presented in this thesis are capable of computing any one of the the centralities in this table (except those that are open) since semivalues generalise the Shapley value, although dedicated Shapley value algorithms are usually faster.

- In Chapter 4, we present our framework for building centrality measures based on semivalues. We show that with just a few restrictions and the computation of four values related to our framework, it is possible to compute semivalues in polynomial time and we provide algorithms for doing so. We hope that this analysis will aid researchers in developing new centrality measures that account for synergies and in subsequently computing and applying them. We define a broad class of semivalue-based centrality measures that can be computed in polynomial time. This is the first such general analysis of the computation of game-theoretic centrality measures in the literature.

- In Chapter 5, we develop a new solution concept that we call configuration

semivalues. This is the broadest such solution concept to date, which generalises both the configuration value and coalitional semivalues. We then pair this solution concept with a general form of closeness centrality in order to develop a new centrality measure. The key feature of this measure is that it is the first such measure to analyse the centrality of nodes based on an overlapping community structure. We apply this measure to the Warsaw public transportation network and show that this is the only measure that yields an intuitive ranking.

- In Chapter 6, we combine the game-theoretic interaction index with a general form of closeness centrality in order to predict the existence of edges in networks. We develop an efficient algorithm for the computation of our measure and show that it achieves favourable results when compared with the state-of-the-art link prediction methods in the literature.

We conclude with a discussion of the presented research and closing remarks.

1.3 The Publications Stemming from this Thesis

The contributions made in this thesis are listed in the following publications of peer-reviewed papers and papers in progress.

Chapter 3:

Mateusz Krzysztof Tarkowski, Tomasz Paweł Michalak, Talal Rahwan, and Michael Wooldridge. "Game-theoretic Network Centrality—A Critical Review". *Mimeo*, University of Oxford.

Chapter 4:

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Mateusz Krzysztof Tarkowski, Piotr Lech Szczepański, Talal Rahwan, Tomasz Paweł Michalak, and Michael Wooldridge. "Closeness Centrality for Networks with Overlapping Community Structure". In: *AAAI Conference on Artificial Intelligence*. 2016

Chapter 6:

Mateusz Krzysztof Tarkowski, Tomasz Paweł Michalak, and Michael Wooldridge. "A Game-theoretic Algorithm for Link Prediction". Submitted to *Thirty-Second AAAI Conference on Artifical Intelligence 2018*.

In addition, Chapters 1, 2 and 7 also make use of material from the publications listed above.

2

Preliminaries

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In this chapter, we will introduce the key concepts necessary for the understanding of the thesis pertaining to cooperative game theory and graph theory. Throughout the thesis we will also use key concepts from probability theory and complexity theory [13, 28].

2.1 Game-Theoretic Concepts

In this section, we formally introduce the game-theoretic concepts used in the thesis. We first discuss the basic model of cooperative games, and then its generalisation. We introduce the concepts of the unanimity basis and dividends. These concepts play a fundamental role in a game-theoretic formalization of the notion of synergy. Finally, we introduce a number of solution concepts in cooperative game theory—the cornerstone on which all the game-theoretic network centralities discussed in this thesis are built.

Cooperative games: A cooperative game consists of a set of *players*, $I = \{1, 2, \ldots, n\}$, and a *characteristic function*, $\nu : 2^I \to \mathbb{R}$, that assigns to each *coalition* $C \subseteq I$ of players a real value indicating its performance (we assume $\nu(\emptyset) = 0$). A cooperative game in characteristic function form, then, is a pair (I, ν). Following convention, we will refer to such a game by ν, omitting the set of players.

Example 1 *Take, for instance, three programmers:* p_1, p_2, *and* p_3. *Their task is to write a piece of software worth* 300 *monetary units, and each one of them can choose whether to join in on this activity or not. If only* p_1 *or* p_2 *choose to perform this task, alone they can implement* $\frac{2}{3}$*'s of the functionality, and so we will say that their individual work is worth:* $300 \times \frac{2}{3} = 200$ *(in other words* $\nu(\{1\}) = \nu(\{2\}) = 200$*). Together, if* p_1 *and* p_2 *join together, they can implement the entire piece of software without errors, and hence we will rank their joint work as* 300 *(i.e.,* $\nu(\{1, 2\}) = 300$*). The third programmer,* p_3*, does not have the required speciality and cannot contribute to the project whether he joins the others or not, thereby not affecting the worth of the finished product.*

Types of Cooperative Games: In order to introduce some of the literature related to this thesis, we will single out the following types of cooperative games:

- ν is *superadditive* if and only if $\nu(S \cup T) \geq \nu(S) + \nu(T)$ for all $S, T \subseteq I$ such that $S \cap T = \emptyset$. This means that when two disjoint coalitions join, their worth is at least the sum of their values. In other words, there is no negative synergy;

- ν is *convex* if and only if $\nu(S \cup T) + \nu(S \cap T) \geq \nu(S) + \nu(T)$ for all $S, T \subseteq I$. This property captures the "snowballing" effect. In other words, the incentive to join a coalition rises as the coalition increases in size; and

- ν is *symmetric* if and only if there exists a function f such that $\nu(S) = f(|S|)$ for all $S \subseteq I$. That is, the value of a coalition depends only on its size, and not on the individual properties of its members.

Convex games were defined by Shapley [35]. Superadditive and symmetric games are also discussed in this paper.

Unanimity Basis and Harsanyi Dividends for Cooperative Games: It is possible to express any characteristic function ν as a linear combination of the *unanimity basis* for I. For every set of players I there exists a unanimity basis consisting of $2^n - 1$ *unanimity games* (one for each subset of I, excluding the empty set $\emptyset$), where n is the number of players in I. A unanimity game, u_S (where $S \subseteq I$), has the following form:

$$u_S(C) = \begin{cases} 1 \text{ if } S \subseteq C \\ 0 \text{ otherwise} \end{cases} .$$

In other words, we say a coalition C is *winning* if it contains all of the players in S, and *losing* otherwise. It is not difficult to see that the unanimity games do indeed form a basis. Since the space of characteristic functions with n players can be interpreted as a $(2^n - 1)$-dimensional vector space, and it is easy to check that each of the $2^n - 1$ elements in the unanimity basis is linearly independent, then it follows that the unanimity basis has to span the whole space.

For any cooperative game, we call its coordinates in the unanimity basis the *Harsanyi Dividends* [36], where the coordinate of u_s is denoted by $\Delta_\nu(S)$. This results in the following:

$$\nu(C) = \sum_{S \subseteq I} \Delta_\nu(S) u_S(C). \tag{2.1}$$

It can be proven that:

$$\Delta_\nu(S) = \sum_{T \subseteq S} (-1)^{|S|-|T|} \nu(T). \tag{2.2}$$

Perhaps a more illuminating (albeit recursive) definition of Harsanyi dividends is as follows:

$$\Delta_\nu(S) = \nu(S) - \sum_{T \subset S} \Delta_\nu(T). \tag{2.3}$$

Synergy: Using the definition of Harsanyi dividends in Equation (2.3), let us consider how they relate to the informal concept of *synergy*. Synergy is an important topic for the purpose of this thesis, but also for cooperative game theory in general. For example, if two nodes achieve "negative" synergy, this means that their value as a group is less than the sum of their individual values. This may indicate that their function is redundant and, therefore, that they are *similar*. We leverage this concept in Chapter 6 in order to predict links in a network, since it is likely that similar nodes in a network are actually connected.

Colloquially, we can define synergy as the extra worth (or loss) that is generated by players when cooperating with each other. For formal purposes, we can define the synergy of a single player simply by his or her value (i.e., for player i this would be $\nu(\{i\})$). For a coalition of two players i and j, the synergy of this coalition can be simply viewed as the value of this coalition minus the individual values of the players, i.e., $\nu(\{i,j\}) - \nu(\{i\}) - \nu(\{j\})$, which is exactly the excess (or deficit) that results from the interaction of two players.

So far, this definition of synergy falls in line with the definition of Harsanyi dividends. For larger coalitions, however, the concept of synergy becomes somewhat more complicated. One could imagine the definition of synergy being $\nu(\{C\}) - \sum_{i \in C} \nu(\{i\})$, but is this correct? This value cannot be said to be the pure synergy achieved by the coalition C, since many individual synergies come into play in this value. For example, for the synergy of coalition $\{1,2,3\}$, we should not consider the synergy achieved by the joining of coalitions $\{1\}$ and $\{2\}$, since this synergy is strictly associated with the coalition $\{1,2\}$.

A more reasonable approach for defining the synergy of larger coalitions is to consider the *extra* worth (or loss) achieved by the joining of any two coalitions. From this perspective, only two coalitions may merge at any given time, and synergy should be measured based solely on this type of mergers. For instance, given $\{1,2,3\}$, we should consider the synergy achieved by $\{1,2\}$ joining $\{3\}$, $\{2,3\}$ joining $\{1\}$ and $\{3,1\}$ joining $\{2\}$. However, we cannot simply take the values of the coalitions

$\{1,2\}$, $\{2,3\}$ and $\{3,1\}$, because these values encompass the synergies within their corresponding coalitions, and $\{1,2,3\}$ should not take credit for those synergies. This gives rise to a definition of synergy in line with Equation (2.3), where we do not take the value of the coalition, but rather its Harsanyi dividend (which we equate with synergy). The synergy achieved by a coalition is its value, minus all of the synergies achieved by any two coalitions that can possibly join in order to form it. Since for any subset S_1 of C, we can find another subset S_2 of C such that $S_2 = C \setminus S_1$, then it suffices to take the value of a coalition and subtract the synergy achieved by all of its subsets.

Example 2 *Consider again our running example, which was introduced in Example 1. We will calculate the Harsanyi dividends for this game and see how they relate to the informal concept of synergy. The easiest cases are as follows:*

$$\begin{aligned}
&\Delta_\nu(\emptyset) = \nu(\emptyset) = 0,\\
&\Delta_\nu(\{1\}) = \nu(\{1\}) = 200,\\
&\Delta_\nu(\{2\}) = \nu(\{2\}) = 200, \text{ and}\\
&\Delta_\nu(\{3\}) = \nu(\{3\}) = 0.
\end{aligned}$$

For coalitions of size 2*, we have the following:*

$$\begin{aligned}
&\Delta_\nu(\{1,2\}) = \nu(\{1,2\}) - \Delta_\nu(\{1\}) - \Delta_\nu(\{2\}) = -100,\\
&\Delta_\nu(\{2,3\}) = \nu(\{2,3\}) - \Delta_\nu(\{2\}) - \Delta_\nu(\{3\}) = 0,\\
&\Delta_\nu(\{3,1\}) = \nu(\{3,1\}) - \Delta_\nu(\{3\}) - \Delta_\nu(\{1\}) = 0.
\end{aligned}$$

Finally,

$$\begin{aligned}
&\Delta_\nu(\{1,2,3\})\\
=&\nu(\{1,2,3\}) - \Delta_\nu(\{1\}) - \Delta_\nu(\{2,3\}) - \Delta_\nu(\{2\})\\
&- \Delta_\nu(\{3,1\}) - \Delta_\nu(\{3\}) - \Delta_\nu(\{1,2\})\\
=&0.
\end{aligned}$$

We see that the only Harsanyi dividend for a non-singleton coalition that is not equal to zero is for the coalition $\{1, 2\}$*. This is because something unusual happens in terms of synergy:* p_1 *and* p_2 *do not achieve the simple sum of their worth. Perhaps they have overlapping skills that can only be used by one person at a time? Since we would expect the both of them together to achieve a worth of* 400*, while they in fact only achieved a worth of* 300*, then the resulting synergy of their coalition is equal to* -100*. We will use this idea in discussing the next concept in this section—the Shapley value—a solution concept/payoff division scheme that is tied to the concept of fairness. Since both players are responsible for this synergy, then they should both be equally punished (or rewarded), i.e., each of their paychecks should be reduced by a value of* 50*.*

Following Owen [37], we can broaden the definition of synergy between two players to account for the context of them belonging to a given coalition. Given a coalition C, the synergy between two players i and j is the difference between the marginal contribution of the group $\{i, j\}$ and the *marginal contributions* of each node separately. In particular, the marginal contribution of player i to coalition C (where $i \notin C$) in a game with characteristic function ν is denoted by $MC_\nu(C, i)$, and is equal to $\nu(C \cup \{i\}) - \nu(C)$. We refer to MC_ν simply as MC when the characteristic function ν is obvious in a given context. Formally, synergy between the players i and j in the context of the coalition C is defined as follows:

$$S_\nu(C, i, j) = MC_\nu(C, \{i, j\}) - MC_\nu(C, \{i\}) - MC_\nu(C, \{j\}) \tag{2.4}$$

In particular, the Harsanyi dividend for $\{i, j\}$ is equal to the synergy between i and j in the context of the empty coalition, i.e. $\Delta_\nu(\{i, j\} = S_\nu(\emptyset, i, j)$.

Solution Concepts for Cooperative Games: Assuming that the *grand coalition*—the coalition consisting of all players in the game—is formed, and assuming that it has the highest value[1] according to ν, a key question of cooperative game

[1]Of course, there exist solution concepts for games where the grand coalition is not optimal.

theory is how to distribute the value of the coalition among its members (i.e., to give each player their payoff from joining the coalition).

The Shapley Value: An important solution concept was proposed by Shapley [14], based on the concept of marginal contribution. The Shapley value of player i, denoted by $SV_i(\nu)$, is then equal to a weighted average of the marginal contributions of i to every coalition that she or he can belong to. In more detail, let $\Pi(I)$ be the set of all permutations of the set of n players, I, i.e., every $\pi \in \Pi(I)$ is a bijection from $\{1, 2, \ldots, n\}$ to itself, where $\pi(i)$ is the position of player i. Let $C_\pi(i)$ be the coalition consisting of the players that precede i in π. That is, $C_\pi(i) = \{j \in I : \pi(j) < \pi(i)\}$. Then, the Shapley value of i is the average marginal contribution of player i to $C_\pi(i)$ over all permutations π of I. Formally,

$$SV_i(\nu) = \frac{1}{n!} \sum_{\pi \in \Pi(I)} [\nu(C_\pi(i) \cup \{i\}) - \nu(C_\pi(i))]. \tag{2.5}$$

The intuition to the Shapley value is as follows: imagine that all players arrive at a meeting point in a random order. Every player i, upon arrival, receives her or his marginal contribution to the group of players already present. Averaging these contributions over all possible orders of arrival produces the Shapley value. In other words, it is the expected marginal contribution of a player. Equivalently, we can write the Shapley value in a (computationally) simpler form:

$$SV_i(\nu) = \sum_{C \subseteq I \setminus \{i\}} \frac{|C|!(n - |C| - 1)!}{n!} [\nu(C \cup \{i\}) - \nu(C)]. \tag{2.6}$$

Example 3 *Let us return to p_1, p_2, and p_3 from Example 1. Given that the players all work together on writing the piece of software, what should their salaries (payoffs) be? Let us start by formally and fully defining the characteristic function:*

$$\nu(C) = \begin{cases} 200 & \text{if } C = \{1\} \text{ or } C = \{2\} \text{ or } C = \{1, 3\} \text{ or } C = \{2, 3\} \\ 300 & \text{if } C = \{1, 2\} \text{ or } C = \{1, 2, 3\} \\ 0 & \text{otherwise} \end{cases}$$

We see that the marginal contributions of p_1 *and* p_2 *are each equal to* 200 *in two possible cases: when the coalition they are joining is empty or consists only of* p_3*. On the other hand, if the coalition consists of* p_2 *or of* p_2 *and* p_3*, then* p_1*'s contribution is* 100*. The same is true for* p_2*, when the coalition consists of* p_1 *or of* p_1 *and* p_3*. For this reason, a "fair" payoff division scheme would give* p_1 *and* p_2 *the same fraction of the* 300 *monetary units. On the other hand,* p_3 *does not contribute at all to any coalition. If* p_3 *were to receive any payoff, then* p_1 *and* p_2 *might feel that* p_3 *is being rewarded for their work, which is "unfair".*

Using Equation (2.7)*, we find that the Shapley values of* p_1 *and* p_2 *are each equal to* 150*, whereas the Shapley value of* p_3 *is* 0*. Alternatively, one can consider the following argument:* p_1 *and* p_2 *each contributes* 200 *to two coalitions and* 100 *to two other coalitions. On average, then, each of them is expected to contribute* 150 *to the value of the software that is produced by a random coalition. On the other hand,* p_3 *is expected to contribute* 0 *to the value of the software.*

The Shapley value may also be expressed in terms of the Harsanyi Dividends and synergies that we discussed earlier in this section. Intuitively, the Harsanyi dividend of a coalition represents the unique synergy achieved by the coalition. Since this synergy is achieved jointly by the coalition members, then it is only "fair" that it is distributed evenly among them. In formal terms:

$$SV_i(\nu) = \sum_{C \in \{C : C \subseteq I \text{ and } i \in C\}} \frac{\Delta_\nu(C)}{|C|}. \tag{2.7}$$

We refer the reader to the work of Pruzhansky [38] and Macho-Stadler, Pérez-Castrillo, and Wettstein [39] for an in-depth introduction to Harsanyi dividends and their relation to solution concepts in cooperative game theory.

Among the many interesting properties of the Shapley value is the fact that it is the unique solution concept satisfying the following four desirable axioms:

- **Efficiency**—the whole value of the grand coalition is distributed. That is, $\sum_{i \in I} SV_i(\nu) = \nu(I)$;

- **Symmetry**—the payoffs of the players do not depend on their identities;
- **Null-player**—players whose marginal contributions are equal to zero for all coalitions receive zero payoff; and
- **Additivity**—for any two games with the same player set, (I, ν_1) and (I, ν_2), the sum of the payoffs for both games is equal to the payoff of the sum of the games for all players. That is, for every player i in I, $SV_i(\nu_1) + SV_i(\nu_2) = SV_i(\nu_1 + \nu_2)$, where $(\nu_1 + \nu_2)(C) = \nu_1(C) + \nu_2(C)$.

Young [40] showed that the Null-Player and Additivity axioms can be replaced by the strong monotonicity axiom:

- **Strong Monotonicity**—for any two games ν and ω with the same player set I, if for a player i we have $MC_\nu(S, i) \geq MC_\omega(S, i)$ for every coalition $S \subseteq (I \setminus \{i\})$, then $SV_i(\nu) \geq SV_i(\omega)$.

In the network theory context, this axiom can be interpreted as follows. If we consider two graphs with the same node set, then node i will be worth more in the graph in which it has a better (i.e., more *central*) position. Note that there exist in the literature many other characterisations of the Shapley value, i.e., properties or axioms that uniquely define it (the set of axioms above is just one example).

Semivalues: The Shapley value is an example of a more general class of division schemes called semivalues. The Banzhaf index of power [15], which was originally introduced for voting games, is another example. In particular, semivalues assign to each player the expected value of his or her marginal contribution to any coalition with some probability distribution over the size of the coalition. In the context of semivalues, one can interpret the Shapley value as the expected marginal contribution of a player i such that the probability of i joining a random coalition of a certain size is equal to the probability of i joining a random coalition of any other size. The Banzhaf index, on the other hand, can be interpreted as the expected

marginal contribution such that a player joins any random coalition (regardless of size) with uniform probability. In general, semivalues allow for any distribution over the size of coalitions, but the probability of drawing any two coalitions of the same size must be the same. We will denote by $\beta(k)$ the probability that a player joins a random coalition of size k. Formally, the general formula for semivalues is as follows:

$$\phi_i^{\beta}(\nu) = \sum_{0 \leq k \leq n-1} \beta(k) \mathbb{E}[MC_\nu(C^k(I \setminus \{i\}), i)], \tag{2.8}$$

where $C^k(I \setminus \{i\})$ is a random variable ranging over the set of all coalitions of size k being drawn with uniform probability from the set $I \setminus \{i\}$ and $\mathbb{E}[\cdot]$ is the expected value operator. Alternatively, we present a formulation of semivalues without the need for an expected value operator below:

$$\phi_i(\nu) = \sum_{k=0}^{n-1} \sum_{C \in C^k(I \setminus \{i\})} \beta(k) \frac{MC_\nu(i, C)}{\binom{n-1}{k}}, \tag{2.9}$$

where we slightly abuse notation for $C^k(I \setminus \{i\})$ now referring to all subsets of size k of the set $I \setminus \{i\}$. The Shapley value [14] and the *Banzhaf index* of power [15] are defined by the β-functions $\beta^{Shapley}$ and $\beta^{Banzhaf}$, respectively, where:

$$\beta^{Shapley}(k) = \frac{1}{n} \quad \text{and} \quad \beta^{Banzhaf}(k) = \frac{\binom{n-1}{k}}{2^{n-1}}.$$

It is worth noting that the Banzhaf index is not an *efficient* semivalue, i.e., it does not necessarily distribute the entire worth of the grand coalition among the players. In this sense, the Banzhaf index is not suitable as a payoff-division scheme.[2] Interestingly, the Shapley value is in fact the only efficient semivalue, i.e., the only one satisfying the aforementioned Efficiency property.

The significance of semivalues for this thesis is that they provide a method for specifying the importance of certain types of synergy in a given context. A trivial example is to give a weight of 1 to the empty coalition (i.e., $\beta(0) = 1$) and 0

[2]It is possible to rescale the Banzhaf index, however it then loses the property of *Additivity*.

to all other coalitions. In this case, synergy is not considered at all. If this semivalue is applied to group degree, group betweenness or group closeness, then the resulting measure is the standard degree, betweenness or closeness measure. In essence, this answers the question "what can a player achieve on his or her own." Going further, by varying the ratio between the weight of the empty coalition and the non-empty coalitions, it is possible to specify the relative impact of synergy on the ranking of players. On the other hand, giving coalitions of size $n-1$ a weight of 1 (i.e., $\beta(n-1) = 1$) and all other coalitions a weight of 0, the resulting semivalue answers the question "how much would all the players lose were a single player to leave." Semivalues give the possibility of fine-tuning the types of contributions and synergies that are important in determining the worth of players or—in the case of networks—nodes.

Example 4 *Continuing our running example that was started in Example 1, consider a scenario where there is a probability that the team of programmers,* $\{p_1, p_2, p_3\}$*, will be downsized. If we can determine the probability with which the team will be downsized, then we can determine the worth of every programmer in a more informed fashion using semivalues.*[3]

Suppose there is a 20% *chance of the team being downsized to a single developer, a* 30% *chance that it will be downsized to two developers and a* 50% *chance that it will not be downsized. As far as* p_3 *is concerned, her or his expected marginal contribution is still* 0 *in all three cases. As far as* p_2 *is concerned, there is a* 20% *chance that she or he produces* 200 *by working alone, there is a* 30% *chance that she or he will be working with one colleague; if that colleague is* p_1 *then the marginal contribution of* p_2 *would be 100, while if the colleague is* p_3 *then the marginal contribution of* p_2 *would be 200. Finally, there is a* 50% *chance that* p_2 *will be working with two colleagues, in which case her or his marginal contribution is 100. The situation of*

[3]Note that this does not work as a payoff-division scheme after the job is completed since we will have known by then whether the team had been downsized or not. Instead, what this semivalue approach provides is a ranking of employees before the job is started.

p_1 is symmetrical to that of p_2. This situation is reflected by the semivalue defined by:

$$\beta(k) = \begin{cases} 0.2 & \text{if } k = 0 \\ 0.3 & \text{if } k = 1 \\ 0.5 & \text{if } k = 2 \end{cases}$$

With this semivalue, the payoff of p_3 is 0, while the payoffs of p_1 and p_2 are each equal to $0.2 \times 200 + 0.3 \times \frac{100+200}{2} + 0.5 \times 100 = 135$. This is less than their Shapley values, partly because the team is unlikely (with probability 20%) to be downsized to just 1 person, where they make their highest marginal contribution of 200.

Owen Value: Importantly, semivalues assume that all types of interactions between players are possible (i.e., anyone can contribute to any coalition). To relax this assumption, Owen [41] introduced a solution concept—now known as the Owen value—that limits interactions of players to only the other players in their communities. This requires the extension of the standard cooperative setting to include a community/coalition structure $CS = \{Q_1, Q_2, \dots Q_m\}$, which is simply a non-overlapping partition of the player set I. The Owen value divides the payoff of any *a priori* coalition structure CS among the communities in CS and the value of each of these communities is distributed fairly and efficiently among its members. Assuming $i \in Q \in CS$, formally, the Owen Value is defined as follows:

$$OV_i(\nu, CS) = \sum_{T \in CS \setminus Q} \sum_{C \in Q} \frac{1}{|CS| \binom{|CS|-1}{|T|}} \frac{1}{|Q| \binom{|Q|-1}{|C|}} MC\Big(\big(\bigcup T\big) \cup C\Big) \tag{2.10}$$

Now, when $CS = \{I\}$ or $CS = \{\{i\}\}_{i \in I}$, the Owen value is equivalent to the Shapley value. As such, the Owen value is a generalization of the Shapley value; one that does not generalise the β function (as semivalues do), but rather generalises the assumed coalition structure CS. While the details are beyond the scope of this thesis, we will mention that the Owen value is uniquely characterised by the following axioms: Efficiency, Null Player, Symmetry, Linearity, and Coalitional Symmetry. We have already introduced the first three of these axioms when discussing the

Shapley value. If a payoff division scheme, $\phi_i(\nu)$, satisfies linearity, then for any $\lambda \in \mathbb{R}$ and cooperative games ν_1 and ν_2, we have $\phi_i(\lambda\nu_1 + \nu_2) = \lambda\phi_i(\nu_1) + \phi_i(\nu_2)$. Coalitional Symmetry implies that if two communities in CS contribute the same value to all coalitions of communities, then their payoffs (i.e., the sum of the payoffs of their members) are the same.

Coalitional Semivalues: Another step in this line of research was taken by Szczepański, Michalak, and Wooldridge [31], who proposed a generalisation combining both the Owen value and semivalues; they called it *coalitional semivalues*. Formally, given a coalition structure, CS, and discrete probability distributions: $\beta : \{0, \ldots, |CS| - 1\} \rightarrow [0, 1]$ and $\alpha_Q : \{0, \ldots, |Q| - 1\} \rightarrow [0, 1]$ coalitional semivalues are defined by:

$$\gamma_i(\nu, CS) = \sum_{k=0}^{|CS|-1} \beta(k) \sum_{l=0}^{|Q|-1} \alpha_Q(l) \mathbb{E}\left[MC\Big(\Big(\bigcup T^k(CS \setminus \{Q\})\Big) \cup C^l(Q \setminus \{i\}), i\Big)\right] \tag{2.11}$$

where Q is the coalition in CS that player i belongs to, $T^k(CS \setminus \{Q\})$ is a random variable over subsets of size k chosen from $CS \setminus \{Q\}$ with uniform probability, $C^l(Q \setminus \{i\})$ is a random variable over subsets of size l chosen from $Q \setminus \{i\}$ with uniform probability, and $\mathbb{E}$ is the expected value operator. The coalitional semivalue is equivalent to the Owen value when $\beta(k) = \frac{1}{|CS|}$ and $\alpha_Q(l) = \frac{1}{|Q|}$.

Solution Concepts for Games with Overlapping Communities: None of the solution concepts discussed thus far considers overlapping communities. To address this issue, Albizuri, Aurrecoechea, and Zarzuelo [42] generalised the Owen value to situations where the *a priori* coalition structure CS contains overlapping communities; they called this generalisation the *Configuration value*. Formally, it is defined as follows, where $\mathcal{T}(i) = \{Q : Q \in CS \text{ and } i \in Q\}$:

$$\chi_i(\nu, CS) = \sum_{\substack{T \subseteq CS \\ T \cap \mathcal{T}(i) = \emptyset}} \sum_{Q \in \mathcal{T}(i)} \sum_{\substack{C \subseteq Q \\ i \notin C}} \lambda MC\Big(\Big(\bigcup T\Big) \cup C, i\Big), \tag{2.12}$$

where

$$\lambda = \frac{|T|!(|CS| - |T| - 1)!}{|CS|!} \frac{|C|!(|Q| - |C| - 1)!}{|Q|!}$$

Interaction Indices: The concept of Interaction indices is intimately connected to payoff division schemes and synergies. We already mentioned that synergies can be used to evaluate the similarity of nodes, since if two players have negative synergy, then this is likely to indicate that their functions are similar and therefore redundant. Given this, Owen [37] defined the interaction between two players as their expected synergy given a random coalition. We refer to this index as the Shapley value interaction index, which is defined as follows:

$$I_{i,j}^{Shapley}(\nu) = \sum_{\pi \in \Pi(I^{i \wedge j})} \frac{S_\nu(C_\pi(\{i,j\}), i, j)}{(n-1)!}, \tag{2.13}$$

where $I^{i \wedge j}$ is the set I such that the elements $i \in I$ and $j \in I$ are replaced by a single element $\{i, j\}$. In other words, this can be interpreted as the Shapley value of the player $\{i, j\}$ in the game where i and j join to create one player minus the Shapley value of player i in the game where j is removed minus the Shapley value of player j in the game where i is removed. Grabisch and Roubens [43] continued this work and introduced the Banzhaf interaction index. Szczepański et al. [32] generalised these concepts further by introducing semivalue interaction indices, defined below:

$$I_{i,j}^{SEMI}(\nu) = \sum_{k=0}^{n-2} \sum_{C \in C^k(I \setminus \{i,j\})} \beta(k) \frac{S_\nu(C, i, j)}{\binom{n-2}{k}} \tag{2.14}$$

Szczepanski et al. [30] extended the concept further to allow for a priori community structures and defined the Coalitional semivalue interaction indices. Assuming i belongs to community P, and j to Q, we define them as follows:

$$I_{i,j}^{CoSemi}(\nu) = \sum_{k=0}^{|CS|-2} \beta(k) \sum_{l=0}^{|Q|+|P|-2} \alpha_{|P|+|Q|-2}(l) \sum_{T \in C^k(CS \setminus \{P,Q\})} \sum_{C \in C^l(N \setminus \{i,j\})} \frac{S_\nu\Big(\bigcup T \cup C, i, j\Big)}{\binom{|CS|-2}{k}\binom{|P|+|Q|-2}{l}}, \tag{2.15}$$

where $C^k(P) = \{C : C \subseteq P \text{ and } |C| = k\}$ are subsets of size k of the set P.

We finish our discussion of concepts from cooperative game theory by introducing a generalisation of characteristic functions, where the order of group of players impacts their value.

Generalised cooperative games: Of relevance to some of the work related to this thesis are generalised cooperative games. Like its standard counterpart, a generalised cooperative game also contains a set of n players, $I = \{1, 2, \ldots, n\}$. However the function that defines the game, which is called a *generalised characteristic function*, is slightly different. Generalised characteristic functions take into account the *order* of players. Consequently, the value of an *ordered coalition* not only depends on the identities of the players therein, but also their order. Formally, a generalised characteristic function is a function from the set of all ordered subsets of I to the real numbers. We will denote by $\pi_C : C \mapsto \{1, 2, \ldots, |C|\}$ a permutation of the set C and by $\Omega(I)$ the set of all ordered coalitions of I, i.e., $\Omega(I) = \bigcup_{C \subseteq I} \Pi(C)$. A generalised characteristic function, then, is denoted as follows: $\nu^* : \Omega(I) \to \mathbb{R}$.

Unanimity Basis and Dividends for generalised Cooperative Games: The unanimity basis for generalised cooperative games is defined in a similar fashion to the unanimity basis for regular cooperative games. First, we must define a partial order, $\widetilde{\subseteq}$, on the set of ordered coalitions $\Omega(I)$. For $S, C \in I$ we will define $\widetilde{\subseteq}$ as follows:

$$\pi_S \widetilde{\subseteq} \pi_C \text{ if and only if } S \subseteq C \text{ and } \forall_{s,t \in S} \pi_C(s) < \pi_C(t) \text{ implies } \pi_S(s) < \pi_S(t)$$

If $S \subset C$, i.e., the inclusion is strict, then we will write $\pi_S \widetilde{\subset} \pi_C$. The generalised unanimity basis, then, is the set of all generalised characteristic functions indexed by ordered coalitions of the form:

$$w^*_{\pi_C}(\pi_S) = \begin{cases} 1 & \text{if } \pi_C \widetilde{\subseteq} \pi_S \\ 0 & \text{otherwise} \end{cases}$$

In other words, any generalised characteristic function ν^* can be written in the form:

$$\nu^*(\pi_C) = \sum_{\pi \in \Omega(I)} \Delta^*_{\nu^*}(\pi) w^*_\pi(\pi_C),$$

where $\Delta^*_{\nu^*}(\pi), \forall \pi \in \Omega(I)$ are coefficients, which we call generalised dividends. Sanchez and Bergantiños [44] showed that:

$$\Delta^*_{\nu^*}(\pi_C) = \nu^*(\pi_C) - \sum_{\pi \widetilde{\subset} \pi_C} \Delta^*_{\nu^*}(\pi). \tag{2.16}$$

Now if, for all ordered coalitions π_1, π_2 with the same set of players S (i.e., $\pi_1, \pi_2 \in \Pi(S)$) we have a generalised cooperative game such that $\nu^*(\pi_1) = \nu^*(\pi_2)$, then this game is just a regular cooperative game. We can rewrite the basis for this game as $u^*_S = \sum_{\pi_S \in \Pi(S)} w_{\pi_S}$, which is simply equal to the unanimity basis of a regular cooperative game with Harsanyi dividends (with the difference, that it accepts ordered coalitions, rather than unordered ones).

2.2 Graph-Theoretic Concepts

A *network* is a *weighted digraph* $D = (V, E, \omega)$, where V is a set of nodes, E is a set of edges, i.e., ordered pairs (v, u) of nodes in V with $v \neq u$, and $\omega : E \to \mathbb{R}^+$ is a weight function from edges to the positive real numbers. A weighted digraph $D = (V, E, \omega)$ is said to be undirected if (i) $(v, u) \in E$ implies $(u, v) \in E$ and (ii) $\omega((v, u)) = \omega((u, v))$ for all $(u, v) \in E$. In that case, D is also said to be a weighed graph. A weighted graph or digraph $D = (V, E, \omega)$ is said to be *unweighted* if $\omega(e) = 1$ for all $e \in E$. Unweighted graphs and unweighed digraphs we also refer to as simply graphs and digraphs, respectively, and, in that case, we will generally omit the reference to ω in the signature.

A *path* π_{st} from a source node s to destination node t in a graph G is an ordered set $(v_0, v_1, \ldots, v_k)$ such that $v_0 = s$ and $v_k = t$ and $(v_i, v_{i+1}) \in E$ for all i with $1 \leq i < k$. We will define $Paths(s,t)$ as the set of paths between nodes s and t and $SP(G)$ as the set of all shortest paths in G. We define the set of *neighbours* of a node v by $E(i) = \{j : (i,j) \in E\}$. The neighbour set of a subset C of nodes is defined as $E(C) = \bigcup_{i \in C} E(i) \setminus C$. We refer to the *degree* of a node v by $deg(v) = |E(v)|$. The *distance* from a node s to a node t is denoted by $dist(s,t)$ and is defined as the length of the shortest path (i.e., sum of the edge weights between subsequent nodes in the path) between s and t. The distance between a node v and a subset of nodes $C \subseteq V$ is denoted by $dist(C,v) = \min_{u \in C} dist(u,v)$. Paths, neighbours, and distances are prominent in definitions of *network centrality measures*, i.e., functions that associate with each node a real value that represents its centrality, and we will refer to them as *(graph) items.*

By $G[S]$, where $G = (V,E)$ and $S \subseteq V$, we denote the subgraph of G induced by S. In other words, $G[S] = (S, E')$, where $E = \{(u,v) : (u,v) \in E$ and $u \in S$ and $v \in S\}$. We say that a graph is *connected* if and only if there exists a path between any two nodes in the graph. We define the *connected components* of a graph $G(V,E)$, denoted by $\mathcal{K}_G(V)$, as the set of sets of nodes that represent all maximal sub-graphs of G such that they are connected. In other words, $C \in \mathcal{K}_G(V)$ if and only if C is connected, and for any nonempty $C' \subseteq V \setminus C$, $C \cup C'$ is not connected. For $C \subseteq V$ we will denote by $\mathcal{K}_G(C)$ the maximal connected components of $G[C]$ (or just $\mathcal{K}(C)$ when G is apparent from the context).

Example 5 *Figure 2.1 presents a sample graph of* 9 *nodes, which we will refer to as* G*. Since* G *is connected, then* $\mathcal{K}(V) = \{\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}\}$*. Let us consider a disconnected group of nodes,* $C = \{v_1, v_2, v_3, v_6, v_7, v_8\}$*. The maximal subsets of* C *that are connected in* G *(i.e., the connected components of* C*), are given by:* $\mathcal{K}(C) = \{\{v_1, v_2, v_3\}, \{v_6, v_7, v_8\}\}$*.*

The adjacency matrix of a graph G, denoted by A_G, is the matrix indexed by the set of nodes V in $G(V,E)$ such that $A_G[i,j] = 1$ if $(i,j) \in E$ and $A_G[i,j] = 0$ otherwise.

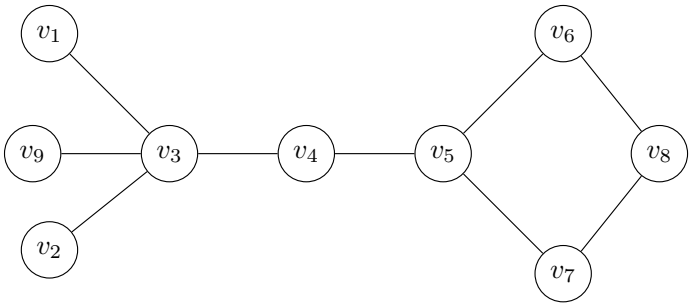

Figure 2.1: Sample network of 8 nodes.

Classic Node Centrality: We now present the three classic notions of node centrality as introduced by Freeman [8] and Eigenvector centrality due to Bonacich [9].

1. *Degree Centrality* ranks nodes according to how many neighbours they have. The larger the neighbourhood of a node, the more central it is. Formally:

$$degree(v) = |E(v)|$$

Example 6 *In Figure 2.1, the node with the highest degree centrality is* v_3*, having a degree of* 4 *(its neighbors are* v_1, v_2, v_4*, and* v_9*). Next,* v_5 *has 3 neighbors (*v_4, v_6*, and* v_7*). The third place in the ranking is a four-way tie between the nodes* v_4*,* v_6*,* v_7*, and* v_8 *each of which have degree* 2*. Finally, nodes* v_1*,* v_9 *and* v_2 *are ranked last.*

2. *Betweenness Centrality* ranks a node according to how many shortest paths in the network it lies on. Often, the additional constraint is added that paths do not contribute to the rank of their source and destination nodes. Formally, let $\sigma_{(s,t)}$ be the number of shortest paths between s and t, and $\sigma_{(s,t)}(v)$ be the number of shortest paths between s and t that visit v. Given this, betweenness centrality is defined as follows:

$$betweenness(v) = \sum_{s,t \in V \setminus \{v\}} \frac{\sigma_{(s,t)}(v)}{\sigma_{(s,t)}}$$

A simpler version of betweenness is called stress centrality. As opposed to betweenness, which normalises the paths that a node lies on by the number

of paths between any source and target node, stress centrality simply counts the number of paths that a node lies on. It is defined as follows:

$$stress(v) = \sum_{s,t \in V \setminus \{v\}} \sigma_{(s,t)}(v)$$

Example 7 *In Figure 2.1, the node with the highest betwenness centrality is* v_3*, with a centrality of* 36*. Nodes* v_4 *and* v_5 *follow with a centrality of* 33 *and* 32*, respectively. Continuing, nodes* v_6 *and* v_7 *have a centrality of* 6 *each. Finally,* v_8 *has a centrality of* 1 *and all the other nodes have a centrality of* 0*, since they do not play any intermediary role in any of the paths in the network.*

3. *Closeness Centrality* ranks nodes based on their distances to other nodes. The most classical version of the centrality ranks a node according to the sum of the distances from it to any other node in the network:

$$closeness(v) = \sum_{u \in V} dist(v, u).$$

This results in an inverse ranking (i.e., nodes that are more important have smaller value). A popular variation on closeness [21], which results in a ranking such that more important nodes have higher value, uses some non-increasing function of distance, $f : \mathbb{R} \to \mathbb{R}$, rather than distance itself. This generalised form of closeness is referred to as the influence game by Michalak et al. [21]. For clarity, however, we refer to it simply as generalised closeness throughout the thesis. Formally, generalised closeness is defined as follows:

$$generalised_closeness(v) = \sum_{u \in V} f(dist(v, u)).$$

When $f(k) = \frac{1}{k}$ the resulting measure is called *harmonic closeness* [45].

Example 8 *In Figure 2.1, the node with the best closeness centrality is* v_4*, since the sum of its distances to other nodes is* 15*. Next, for nodes* v_3 *and* v_5 *this value is* 16*, for nodes* v_6 *and* v_7 *it is* 21*, for* v_1*,* v_2 *and* v_9 *it is* 23*, and node* v_8 *comes in last with* 26*.*

4. *Eigenvector Centrality*, $C_E(v)$, assigns to each node v in G the v^{th} entry of an eigenvector with the largest eigenvalue of the adjacency matrix of G. More formally, let $\vec{x}$ be an eigenvector with the largest eigenvalue, λ. That is, if A_G is the adjacency matrix of G, then λ is the largest value such that there exists a vector $\vec{y}$ for which we have $A_G\vec{y} = \lambda\vec{y}$. Moreover, let $\vec{x}$ be an eigenvector for this value, i.e., $A_G\vec{x} = \lambda\vec{x}$. Then:

$$eigenvector(v) = \vec{x}[v]$$

Example 9 *In Figure 2.1, the node with the highest eigenvector centrality is* v_4*, with a centrality of* 0.486919*. Next is* v_5 *with* 0.401595*, then* v_6 *and* v_7 *with* 0.395147*,* v_3 *with* 0.350707*,* v_8 *with* 0.255378*, and* v_1*,* v_2 *and* v_9 *with* 0.210938*.*

Group centrality: For the purposes of this thesis, we define group centrality as follows:

Definition 1 (Group centrality) *In general, we will use the variable* ψ *to denote a group centrality measure. Formally,* ψ *maps any graph* $G = (V, E)$ *onto a characteristic function* $\nu : 2^V \to \mathbb{R}$*.*

We present below some classic group centralities due to Everett and Borgatti [10]. These group centrality measures are a natural extension of the centrality measures introduced by Freeman [8].

1. *Group Degree Centrality* of a group of nodes S is defined as the size of the neighbourhood of this group. Formally, group degree centrality in a graph G is defined as:

$$\psi^D(G)(S) = \nu_G^D(S) = |\{v : v \in E(S) \setminus S\}|;$$

2. *Group Betweenness Centrality* of a group of nodes S is defined as the number of shortest paths that visit at least one node in S. Often (but not always), the additional assumption is made that the source and destination nodes of these paths cannot belong to S. Let $\sigma_{(s,t)}$ be the number of shortest paths between the nodes s and t, and $\sigma_{(s,t)}(S)$ be the number of shortest paths between s and t that visit at least one node in S. Formally, group betweenness centrality in a graph G is defined as:

$$\psi^B(G)(S) = \nu_G^B(S) = \sum_{s,t \in V \setminus S} \frac{\sigma_{(s,t)}(S)}{\sigma_{(s,t)}}.$$

3. *Group Closeness Centrality* of a group of nodes S is defined as the sum of the distances from S, to any node outside of S. This results in a ranking, where groups with lower value are—on average—closer to the nodes outside of the group, and therefore more central. Formally, group closeness centrality in a graph G is defined as:

$$\psi^{CL}(G)(S) = \nu_G^{CL}(S) = \sum_{v \in V \setminus S} dist(S, v);$$

Similarly to closeness centrality, group closeness centrality can be generalised such that more central groups have a higher rank. This extension was called the *influence game* by Michalak et al. [21], and it is defined with the help of a non-increasing function $f : \mathbb{R} \to \mathbb{R}$ in the following manner:

$$\psi_f^{CL}(G)(S) = \nu_{G,f}^{CL}(S) = \sum_{v \in V \setminus S} f(dist(S, v)); \tag{2.17}$$

When $f(k) = \frac{1}{k}$, the resulting measure is called *harmonic group closeness* [45].

In all instances, when the graph G in question is obvious in a given context, we skip the subscript G when referring to the characteristic function. For $\psi^{CL}(f, G)(S)$ and $\nu_{G,f}^{CL}(S)$, we skip the subscript f whenever it is obvious in a given context. This allows us to reduce a clutter of notation.

We further discuss additional variations of (group) degree, betweenness and closeness centralities and their origins in Section 4.4 of Chapter 4.

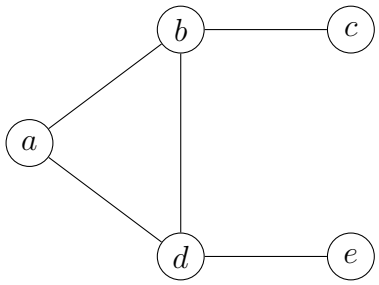

Figure 2.2: A simple example.

Game-theoretic centrality: We are now in a position to define game-theoretic centrality, which can be viewed as a method to rank nodes by aggregating the centralities of (typically all) groups of nodes in a network. In particular, in this approach nodes are treated as players in a cooperative game and their groups as coalitions. The characteristic function is typically some group centrality measure (such as those described above) and a chosen solution concept is the method of aggregation. In the context of this thesis, a pair of any given group centrality measure and solution concept can be considered a game-theoretic centrality measure, since it yields a ranking of nodes in any graph.

Let us now look at an example to illustrate the concepts discussed thus far.

Example 10 *Consider betweenness centrality and group betweenness centrality in the network G depicted in Figure 2.2. Observe that, besides the direct edges, there are also the following shortest paths between nodes :*

$$abc, ade, bde, cba, cbd, cbde, dbc, eda, edb, edbc.$$

It can then easily be established that the betweenness centrality for nodes a, c, and e is 0, whereas that of b and d totals to 6. The group centralities $\nu^B(C)$ (omitting the reference to G) of the groups of nodes (coalitions) C are summarised in Table 2.1. We can now use the Shapley value to aggregate the group centralities for the individual nodes. In this way, we find that the game-theoretical betweenness centralities for the Shapley value of the nodes a, b, c, d, and e are, respectively, $\frac{-2}{5}$, $\frac{3}{10}$, $\frac{-7}{6}$, $\frac{3}{10}$, and $\frac{-7}{6}$.

C	$\nu^B(C)$	C	$\nu^B(C)$	C	$\nu^B(C)$	C	$\nu^B(C)$
$\emptyset$	0	$\{a,b\}$	4	$\{a,b,c\}$	0	$\{a,b,c,d\}$	0
$\{a\}$	0	$\{a,c\}$	0	$\{a,b,d\}$	2	$\{a,b,c,e\}$	0
$\{b\}$	6	$\{a,d\}$	4	$\{a,b,e\}$	2	$\{a,b,d,e\}$	0
$\{c\}$	0	$\{a,e\}$	0	$\{a,c,d\}$	2	$\{a,c,d,e\}$	0
$\{d\}$	6	$\{b,c\}$	0	$\{a,c,e\}$	0	$\{b,c,d,e\}$	0
$\{e\}$	0	$\{b,d\}$	6	$\{a,d,e\}$	0	$\{a,b,c,d,e\}$	0
		$\{b,e\}$	4	$\{b,c,d\}$	2		
		$\{c,d\}$	4	$\{b,c,e\}$	0		
		$\{c,e\}$	0	$\{b,d,e\}$	2		
		$\{d,e\}$	0	$\{c,d,e\}$	0		

Table 2.1: The betweenness centralities of the coalitions of the network in Figure 2.2.

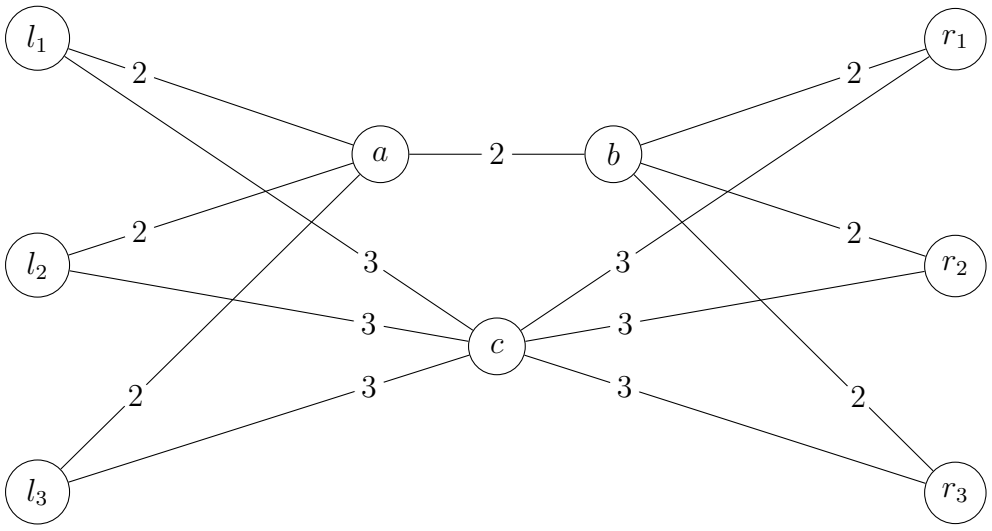

Figure 2.3: A graph representing a delivery network. Edge weights represent the physical distance between nodes.

Before we move on to the literature review, let us illustrate the differences between the three approaches to centrality that we mentioned—classic, group and game-theoretic—via an example.

Example 11 *Consider a delivery network, where parcels (for instance of data or physical mail) are transferred between nodes. Figure 2.3 presents an example of such a network. Edge weights represent the distance between nodes. Assume that we want to intercept parcels travelling through this network, while the only thing we know is that deliveries are always made via the shortest route. We can try to achieve this by placing a "mole" at any of the nodes. For this task, we can model*

the importance of a node v using a variant of betweenness centrality that we call stress centrality *after Szczepański, Michalak, and Rahwan [22].*[4]

$$stress(v) = \sum_{s \in V} \sum_{t \in V} \sigma_{(s,t)}(v)$$

If we can only place one mole in this setting, then this "classic" approach to betweenness centrality would give us the correct answer: nodes a and b are the best nodes to place the mole, since they control the flow of 44 *different shortest paths each, which is more than any other node. However, what if we can place two moles? Unfortunately, the classic approach to centrality is of little little help here, since placing moles at the two nodes with the highest stress centrality—a and b—does not guarantee us the greatest success. We could, for instance, do better by placing a mole on b and another one on c, as together they control a larger number of delivery paths. For this analysis, we must consider the group stress centrality, which is defined for any group of nodes S as follows:*

$$group_stress(S) = \sum_{s \in V} \sum_{t \in V} \sigma_{(s,t)}(S)$$

We can now see that whereas the group $\{a, b\}$ controls a combined number of 56 *shortest paths, the two optimal groups of size* 2*—$\{a, c\}$ and $\{b, c\}$—control* 80 *shortest paths each. Although the distinction between the definitions of the node centrality and group centrality in this case may seem trivial given that only the domain changes between the two definitions, two decades separate the two areas of research.*

On the other hand, what would be the best approach to defend against the random placement of moles? Assume our objective is to secure as many of the shortest paths as possible, i.e., clearing them from all moles that may be on them. Also assume that we can send a single team to a single node to investigate whether it has been compromised, and, if so, to neutralise the resident mole. Which node should we send the team to first? This depends strongly on the number of nodes that we

[4]Since moles can also affect parcels at their own location, we make the slight modification of also counting towards a node's centrality the shortest paths that originate and end at this node.

believe are compromised. The catch is that by neutralising a mole at a node, we may not succeed in securing any shortest paths through it, as each of them may also be compromised by other moles. Consider the case where we know that two moles are compromised (although any probability distribution that models how many moles are likely to exist in the network can be used).

The question of where to send the team first is a complex issue and it is not immediately obvious what we should optimise for. For instance, there is no "quickest" way to ensure that all paths are not compromised. Given that the placement of moles is random, this would require the checking of all nodes. One criterion that we can optimise for is to ensure that we clear as many shortest paths as possible with our first visit. Intuitively, this would mean that we should visit a or b first, since they control the most paths. This, however, is not the case given the following two considerations:

1. *The shortest paths that a lies on contain more nodes on average than those that c lies on. We note that it is more likely that shortest paths that contain more nodes are compromised, since a mole at any one node on the path would suffice to tamper with deliveries. Given this, if we know that there are two moles, then the likelihood of the second mole lying on a large number of the same shortest paths that a lies on is higher than for c. This means that even if we clear a of moles, the likelihood that the shortest paths it lies on are still compromised is higher than for c.*

2. *Node a lies on many of the same shortest paths as b. This means that if b is the other infected node, then clearing a of a mole leaves most of its paths still compromised. However—given that c has few common shortest paths with other nodes—clearing c of a mole will clear most of its shortest paths with absolute certainty.*

As an example, consider all shortest paths from some l_i to some r_j for $i, j \in \{1, 2, 3\}$. There are 9 paths of the form (l_i, a, b, r_j) and 9 of the form (l_i, c, r_j). We see both considerations at work here:

1. *The paths of this form that* a *lies on contain* 4 *nodes, whereas those that* c *lies on contain* 3 *nodes, and*

2. *All* 9 *of the paths of this form that* a *lies on overlap with those that* b *lies on. This means that if* b *also has a mole, then clearing* a *will leave all of these paths compromised. On the other hand, if* c *has a mole, then clearing it* guarantees *that at least* 6 *paths of this form are not compromised.*

In order to capture the complex interactions between nodes and the synergies (both positive and negative) that nodes exhibit when considered in groups, the semivalue approach needs to be considered in order to analyse the expected number of shortest paths "sterilised" after clearing a node. The semivalue for group stress centrality of a node v *with the* β *function such that* $\beta(k) = 1$ *for* $k = 1$ *and* $\beta(k) = 0$ *otherwise gives us exactly the number of shortest paths that we will cure upon removing a mole at a given location, given that all other nodes are equally likely to be compromised. If* c *is compromised, then removing a mole placed there will cure an expected number of* $24\frac{3}{4}$ *shortest paths, whereas removing a mole at* a *or* b *will cure only an expected number of* $16\frac{3}{4}$ *shortest paths.*

3

Literature Review

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3.1 Introduction

In this chapter we discuss the existing literature on game-theoretic network centrality; the recent research direction that addresses the issue of network centrality by applying techniques rooted in cooperative game theory.[1] The literature on the topic has been expanding over the past years, and is scattered among game-theoretic and computer science venues.

We identify two separate lines of research in game-theoretic network centrality. The first is primarily concerned with applying existing and developing new cooperative solution concepts that essentially depend on an underlying network structure. In other words, these solution concepts can be applied only to networks and are not well-defined otherwise. We refer to the centrality measures from this line of research as game-theoretic measures based on connectivity, since the solution concepts that it is concerned with typically attempt to account for the role played by nodes in connecting otherwise disconnected groups in the network. The Myerson value [47] is probably the most well-known such concept.

On the other hand, the second line of research is concerned with applying (both new and existing) cooperative solution concepts for regular cooperative games to specific classes of cooperative games defined on networks through group centrality measures. We refer to the measures from this line of research as game-theoretic centrality measures not based on connectivity.

In what follows, we present a comprehensive review of both types of game-theoretic centrality, with particular emphasis on computational complexity. Section 3.2 deals with game-theoretic network centrality measures based on connectivity, while Section 3.3 deals with those that are not. In Section 3.4 we present some applications of game-theoretic network centrality considered in the literature. Section 3.5 concludes the chapter.

[1] We note that there exists a literature on a game-theoretic approach to network centrality based on *non-cooperative* game theory, e.g., Calvó-Armengol [46]. Nevertheless, it is customary in the literature to use the term *game-theoretic network centrality* for techniques pertaining to *cooperative* game-theory applied to the analysis of node centrality in networks. We will follow this convention also in this thesis.

3.2 Game-Theoretic Node Centrality Measures Based on Connectivity

In this section, we review the game-theoretic centralities that are based on connectivity. Typically, this means that the centralities are based on, or inspired by, the Myerson value and Myerson's *graph-restricted game* [47]. In this approach, the model consists of a graph and a value function defined over all subsets of nodes (who are treated as players in a cooperative game).

The next step is to construct a new cooperative game based on the above value function and graph. In this context, we will say that the network topology "restricts" the game, i.e., it is "graph restricted". For now, we will leave this as an abstract concept, and discuss its variations when we get to the specific centrality measures. In particular, in Section 3.2.1, we formally introduce the Myerson value and his graph-restricted game. In Section 3.2.2, we analyze the centrality proposed by Gómez et al. [19], which is based on the Myerson value. Section 3.2.3 presents the centrality due to Pozo et al. [48] for digraphs. The centralities due to Amer, Giménez, and Magaña [49] and Belau [50] then follow in Sections 3.2.4 and 3.2.5, respectively. Section 3.2.6 discusses the Banzhaf network centrality, proposed by Grofman and Owen [18], which shares certain (but not all) traits with the connectivity approach. Finally, in Sections 3.2.7 and 3.2.8 we discuss alternative concepts of connectivity and graph-restricted games.

3.2.1 The Myerson Value and its Computational Properties

Graph-restricted games were introduced in Myerson's seminal work from 1977 [47]. In the model analysed by Myerson, a network consists of players (nodes of the graph) who can communicate either directly, via the links that directly connect them, or through intermediaries, i.e., when there exists a path in the network between them. This network is represented by a graph, G. A natural assumption, then, is that only players who communicate are able to cooperate. This idea was

embodied in a coalitional game, by introducing a value function, $\nu : 2^V \to \mathbb{R}$, where $\nu(\emptyset) = 0$, that assigns to all coalitions a real value. However, disconnected coalitions cannot communicate and therefore achieve this value. In this context, a *connected coalition* is defined as a coalition for which there exists a path between any two nodes belonging to the coalition, that visits nodes only within the coalition. In other words, a connected coalition is one that induces a connected subgraph. We will write $\mathcal{C}(G)$ to denote the set of all connected coalitions that can be made of nodes within G.

Myerson [47] proposed that the value of a disconnected coalition C, should be the sum of the maximal connected components of the subgraph induced by C. We will denote the set of all such connected components by $\mathcal{K}(C)$. Given this we can formulate the definition of the characteristic function for Myerson's graph-restricted game, which we denote by $\nu_G^{\mathcal{M}}(S)$. Formally:

$$\nu_G^{\mathcal{M}}(S) = \begin{cases} \nu(S) & \text{if } S \in \mathcal{C}(G) \\ \sum_{K_i \in \mathcal{K}(S)} \nu(K_i) & \text{otherwise.} \end{cases} \tag{3.1}$$

Myerson's celebrated result [47] is that the Shapley value of a node v in this graph-restricted game, $SV_v(G; \nu_G^{\mathcal{M}})$, is equal to his solution concept—now known as the Myerson value—of v in the original game, denoted by $MV_v(G; \nu)$. The Myerson value of node v is formally and uniquely defined by the following axioms:

1. **Efficiency in connected components:** The entire payoff of a connected component is distributed among the coalition members.

   Formally: $\forall_{K_i \in \mathcal{K}(V)} \sum_{v \in K_i} MV_v(G; \nu) = \nu(K_i)$;

2. **Fairness:** Any two players that are neighbours are rewarded equally for the edge between them. In other words, if we add an edge (s, t) to G (denoted by $G \cup (s, t)$), then the Myerson value of s and t changes by the same amount.

   Formally: $MV_s(G \cup (s, t); \nu) - MV_s(G; \nu) = MV_t(G \cup (s, t); \nu) - MV_t(G; \nu)$;

3. **Stability:** if ν is superadditive, then adding an edge (s, t) will not decrease the payoff of s or t.

In the remainder of this section, we discuss an algorithm to compute the Myerson value and then discuss how it can be used as a centrality measure.

An Algorithm for Computing the Myerson Value

A general algorithm for computing the Myerson value was proposed by Skibski et al. [51]. This algorithm runs in $O(|\mathcal{C}(G)||E|)$ time, where $|\mathcal{C}(G)|$ is the number of connected subgraphs of G. In general, this is difficult to improve upon, since the number of steps to consider the values of all subgraphs (which are given by the characteristic function ν) is $O(|\mathcal{C}(G)|)$, and for every subgraph $O(|E|)$ steps are required to identify its set of neighbors. Interestingly, given a polynomial number of connected subgraphs, this algorithm also runs in polynomial time.

The pseudocode for the algorithm due to Skibski et al. [51] is presented in Algorithm 1. We briefly explain how it works. The basic idea is to traverse all connected coalitions using depth-first search (DFS). During this process, an internal variable—S—represents the "current" connected coalition. The main function—$DFSMyersonVRec$—is called recursively: once, where the node currently being processed by the DFS algorithm is included in S, and once, where that node is forbidden from ever being included in S. By recursively calling this function in such a manner, all connected coalitions are eventually traversed. During traversal for some connected coalition, S, the algorithm stores in XN the neighbor set of S. The Myerson value of node v is computed using Equation (2.5)—the formula of the Shapley value—with the characteristic function for the graph-restricted game. Recall that Equation (2.5) iterates over all permutations in $\Pi(N)$, and for every such permutation, π, computes the marginal contribution that the node v makes to $C_\pi(v)$—the coalition of nodes preceding v in the permutation π. The important observation here, is that when we take the graph-restricted characteristic function defined in Equation (3.1), the marginal contribution of v to $C_\pi(v)$ becomes:

$$\nu_G^{\mathcal{M}}(C_\pi(v)\cup\{v\})-\nu_G^{\mathcal{M}}(C_\pi(i)) \;=\; \nu(K_1\cup\cdots\cup K_d\cup\{i\})-\nu(K_1)-\cdots-\nu(K_d), \quad (3.2)$$

Algorithm 1: The DFS-based algorithm for calculating the Myerson value due to Skibski et al. [51].

Input: Graph $G{=}(V, E)$, function $\nu : \mathcal{C}(G) \to \mathbb{R}$
Output: Myerson value for game ν_G

```
1  DFSMyersonV begin
2      sort nodes and list of neighbors by degree desc.;
3      for i ← 1 to |V| do MV_i(ν_f) ← 0;
4      for i ← 1 to |V| do
5          DFSMyersonVRec(G, (v_i), {v_i}, {v_1, ..., v_{i-1}}, ∅, 1);
6  DFSMyersonVRec(G, path, S, X, XN, startIt) begin
7      v ← path.last();
8      for it ← startIt to |E(v)| do
9          u ← N(v).get(it); // it's neighbor of v
10         if u ∉ S ∧ u ∉ X then
11             DFSMyersonVRec(G, (path, u), S ∪ {u}, X, XN, 1);
12             X ← X ∪ {u}; XN ← XN ∪ {u};
13         else if u ∈ X then XN ← XN ∪ {u};
14     path.removeLast();
15     if path.length() > 0 then
16         startIt ← N(path.last()).find(v) + 1);
17         DFSMyersonVRec(G, path, S, X, XN, startIt);
18     else
19         foreach v_i ∈ S do
20             MV_i(ν_G) ← MV_i(ν_G) + ((|S|−1)!(|XN|)!)/((|S|+|XN|)!) ν(S);
21         foreach v_i ∈ XN do
22             MV_i(ν_G) ← MV_i(ν_G) − ((|S|)!(|XN|−1)!)/((|S|+|XN|)!) ν(S);
```

where $K_1, \ldots, K_d$ are the maximal connected subsets of $C_\pi(i)$ that are connected to v. In other words, if v joins $C_\pi(v)$, then it connects these components, but not others. It is therefore possible to express the Myerson value as a weighted sum over connected subsets S of the value $\nu(S)$. In order to do so, we must compute for each such connected subset how many times it appears in the sum in Equation (2.5). Based on these observations, for every connected coalition, S, when computing the Myerson value of the node v, the algorithm distinguishes between the following two possibilities:

- If $v \in S$, then $\nu(S)$ appears in the following part of Equation (3.2): $\nu(K_1 \cup$

$\cdots \cup K_d \cup \{v\}$). This happens whenever $S = K_1 \cup \cdots \cup K_d \cup \{v\}$. More specifically, it happens in a permutation π if and only if $S \setminus \{v\} \subseteq C_\pi(v)$ and $C_\pi(v) \cap XN = \emptyset$. Let us compute how many such permutations there are by constructing them. First of all, since the players in $N \setminus (XN \cup S)$ are completely irrelevant here, they can be placed in π in any position. The number of ways in which this can be done is:

$$\binom{|N|}{|N| - |XN \cup S|}(|N| - |XN \cup S|)! = \frac{|N|!}{|XN \cup S|!}.$$

After placing those players, we still have $|S \cup NX|$ unassigned slots in π, which should be assigned as follows. The members of $S \setminus \{v\}$ should be placed in the first $(|S| - 1)$ unassigned slots; there are $(|S| - 1)!$ different ways in which this can be done. After that, the first of the remaining slots should be occupied by v, while the rest should be occupied by the members of XN; there are $|XN|!$ different ways to do this. The total number of permutations becomes:

$$\frac{|N|!(|S| - 1)!|XN|!}{|XN \cup S|!}.$$

Now, since Equation (2.5) averages each marginal contribution over all $|N|!$ possible permutations, the value of $S \setminus \{v\}$ must be divided by this coefficient when computing the Myerson value of v. In other words, for any connected coalition S that satisfies the above requirements, the value $\nu(S)$ appears the following number of times when computing the Myerson value:

$$\frac{(|S| - 1)!|XN|!}{|S \cup XN|!}.$$

- On the other hand, if $v \notin S$, then $\nu(S)$ can only appear in the following part of Equation (3.2): $-\nu(K_1) - \cdots - \nu(K_d)$. This happens whenever $S = K_j$ for some $j \in \{1, \ldots, d\}$. More specifically, it happens in a permutation π if and only if $S \subseteq C_\pi(v)$ and $C_\pi(v) \cap XN = \emptyset$ and $v \in XN$. Just as in the previous case, it is possible to compute the number of ways in which such a

permutation can be constructed. We arrive at the following coefficient, which is the number of times $\nu(S)$ appears when computing the Myerson value of v:

$$-\frac{|S|!(|XN|-1)!}{|S\cup XN|!}.$$

The above two coefficients are used in lines 20 and 22 of Algorithm 1. We provide a closed-form formula for the Myerson value based on these observations:

$$\begin{aligned} MV_v(\nu) = &\sum_{v\in S\in\mathcal{C}(V)} \frac{(|S|-1)!|E(S)|!}{|S\cup E(S)|!}\nu(S\cup\{v\}) \\ &- \sum_{S\in\mathcal{C}(V),v\in E(S)} \frac{|S|!(|E(S)|-1)!}{|S\cup E(S)|!}\nu(S), \end{aligned} \tag{3.3}$$

where $E(S)$ denotes the neighbor set of S, i.e., $|E(S)| = XN$ from Algorithm 1. We note that another expression that allows for polynomial computation of the Myerson value given a polynomial number of connected coalitions was developed by Elkind [52]. Elkind further characterised the class of graphs for which the number of connected coalitions is polynomial in $|V|$, thereby admitting polynomial computation of the Myerson value and other solution concepts.

This concludes the discussion of the computation of the Myerson value.

The Myerson Value as a Centrality Measure

Since the Myerson value assigns values to individual nodes by taking into account the topology of the network, it can be thought of as a centrality measure, as demonstrated in the following example:

Example 12 *Let us consider the network presented in Figure 3.1 and the characteristic function $\nu(C) = |C|^2$. This particular characteristic function was chosen since it is superadditive, and it gains more and more value the more neighbours join. In particular, if no network were present, each player would have an equal Shapley value, which is 5. However, as soon as we take the network topology into consideration,*

we find that not all players can participate as often in the larger (more rewarding) coalitions. Since the game is quadratic (making it strictly convex), rather than linear, this makes being in a central position in the network even more beneficial. The Myerson values for the nodes v_1, v_2, v_3, v_4 *and* v_5 *are* $3.167, 6, 8.167, 3.333$ *and* 3.333*, respectively. Clearly, node* v_3 *receive the highest payoff, which reflects the fact that it plays the most important role in connecting the other nodes. For instance, adding* v_3 *to the coalition* $\{v_2, v_4, v_5\}$ *connects them, changing the value of the coalition from* 3 *to* 16.

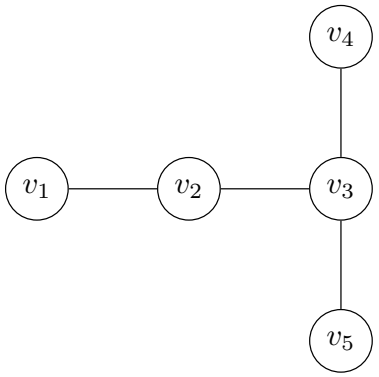

Figure 3.1: A sample network used to illustrate connectivity and how it impacts the Myerson value

Having introduced graph-restricted games due to Myerson, over the next five sections, we will now show how these games were used in the literature to construct different game-theoretic network centralities.

3.2.2 Centrality due to Gómez et al. [19]

Gómez et al. [19] propose to use the difference between the Myerson value and the Shapley value of nodes within a network as a measure of centrality. Formally, this measure is given by:

$$MV_v(G;\nu) - SV_v(\nu), \tag{3.4}$$

where $MV_v(G;\nu)$ is the Myerson value of node v and $SV_v(\nu)$ is its Shapley value. Intuitively, this measure takes the Shapley value, which accounts for the importance

of nodes without considering the network topology, and subtracts it from the Myerson value, which accounts for the importance of nodes in a network context. Hence, this measure of centrality quantifies how much power a node has gained (or lost) due to its position in the network. However, note that when ν is symmetric, then $SV_v(\nu)$ is equal for all $v \in V$. In this case, the centrality measure is equivalent to the Myerson value (shifted by some constant).

Since the Shapley value is simply used as an index of power of a player in the game, the authors note that the Banzhaf index could be used instead. Indeed, we note that any semivalue could be used depending on the application at hand. As such, an interesting research direction would be to study what these different power indices (i.e., solution concepts such as semivalues) would imply for the centrality measure; clearly, many properties of the measure rely not only on ν but also on the adopted solution concept.

Example 13 *To demonstrate the basic idea behind the centrality measure due to Gómez et al. [19], consider again the network in Figure 3.1. Here, given the characteristic function* $\nu(C) = |C|^2$*, the centrality of* v_1, v_2, v_3, v_4 *and* v_5 *is:* $-2.167, 1, 3.167, -2.333$ *and* -2.33*, respectively. Since this game is symmetric, these values are basically shifted Myerson values. It is possible, however, to define a game that is not symmetric (and in this sense includes information that is extraneous to the network), in which case the Gomez centrality can produce a ranking different to that produced by the Myerson value.*

3.2.3 Centrality in directed social networks by Pozo et al. [48]

Pozo et al. [48] build upon their previous work [19] presented in the above section and define a family of centrality measures for directed social networks. In the previous section, Gómez et al. [19] used the Shapley value as a measure of power that disregards the topology of the network, whereas the Myerson value was used as a measure of power within the network. The difference between the two values

was then used as a measure of how much power a node gains (or loses) due to its position in the network. Pozo et al. [48] extend this measure to digraphs by proposing their version of *digraph-restricted games* (as opposed to graph-restricted games). Here, the Shapley value is used just as before, as a standard measure of power disregarding the network. However, the Myerson value is replaced with a generalised solution concept (and the digraph-restricted game is a generalised cooperative game), where the order of the players matters in determining its value.

This approach makes sense for digraphs, since an ordered group of nodes may communicate in a digraph only if each consequent node is connected. To achieve this, the authors take an underlying model of a digraph, D, and cooperative game ν. They then produce a generalised cooperative game that they refer to as the digraph-restricted game, $\nu_D^{\mathcal{P}*}$, where $\mathcal{P}$ stands for Pozo, and develop a family of generalised solution concepts in order to account for the importance of nodes in the digraph-restricted game. This family of measures, referred to by χ_α, is parametrised by α, which takes any value in $[0, 1]$. On one extreme, when $\alpha = 0$, χ_α is equal to a solution concept proposed by Nowak and Radzik [53], whereby players may only join an ordered coalition at the very end. On the other extreme, when $\alpha = 1$, χ_α, it is equal to a solution concept proposed by Sanchez and Bergantiños [44], whereby players can join an ordered coalition at any position (including in between other nodes). This consequently impacts the types of marginal contributions that players can make in a generalised cooperative game. Setting α anywhere between the two extremes results in a hybrid of the two aforementioned solution concepts. When α decreases, more value is given to the players that join an ordered coalition last. Let us now formally introduce the two generalised solution concepts below.

Solution Concepts for Generalised Cooperative Games: Here, we briefly introduce two prominent solution concepts for generalised cooperative games.[2] To this end, let $\pi_C = \{c_1, c_2, \ldots, c_{|C|}\}$ be a permutation of C. We will denote

[2]The specific formulations we use here are different from the ones that were originally proposed by the authors of the solution concepts. The formulations used here can be found, for example, in the work by Pozo et al. [48].

by $\pi(i \to l)$ a permutation of $C \cup \{i\}$ such that i is inserted in π immediately after the l'th player (if $l = 0$, then i is inserted as the first player). Formally, $\pi(i \to l) = \{c_1, c_2, \ldots, c_l, i, \ldots, c_{|C|}\}$. Let us first introduce the solution concept, proposed by Sanchez and Bergantiños [44]. This solution concept, now known as the Sanchez-Bergantinos value, is computed for a player i in a game with a set of n players, I, and defined by the generalised characteristic function ν^*, as follows:

$$\begin{aligned}\Psi_i^{SB}(\nu^*) &= \sum_{\pi \in \Pi(C), C \subseteq I \setminus \{i\}} \frac{(n - |C| - 1)!}{n!(|C| + 1)} \sum_{l=0}^{|C|} (\nu^*(\pi(i \to l)) - \nu^*(\pi)) \\ &= \sum_{\pi_S \in \Omega(n), i \in S} \frac{\Delta^*_{\nu^*}(\pi_S)}{|S|!|S|}, \end{aligned} \tag{3.5}$$

where $\Delta^*_{\nu^*}(\pi_S)$ denotes the generalised dividends, discussed earlier in Chapter 2 and defined formally in Equation (2.16), and $\Omega(I)$ is the set of all ordered coalitions of I, i.e., $\Omega(I) = \bigcup_{C \subseteq I} \Pi(C)$. The other solution concept we wanted to discuss is the Nowak-Radzik value, which was proposed by Nowak and Radzik [53]. This solution concept is computed for a player i in a game with a set of n players, I, and defined by the generalised characteristic function ν^*, as follows:

$$\begin{aligned}\Psi_i^{NR}(\nu^*) &= \sum_{\pi \in \Pi(C), C \subseteq I \setminus \{i\}} \frac{(n - |C| - 1)!}{n!} (\nu^*(\pi(i \to |C|)) - \nu^*(\pi)) \\ &= \sum_{\pi_S \in \Omega(I), \pi_S(i) = |S|} \frac{\Delta^*_{\nu^*}(\pi_S)}{|S|!}. \end{aligned} \tag{3.6}$$

The difference between these two solution concepts is that the former distributes the generalised dividend among all players in π_S equally (and hence the requirement $i \in S$), whereas the latter gives the entire generalised dividend to the last contributing player in π_S (and hence the requirement $\pi(i) = |S|$, i.e., that i is the last player). Having defined the two solution concepts, we will now formally define the centrality measure due to Pozo et al. [48], which builds upon a parametrised hybrid of the two.

Centrality according to Pozo et al. [48] : The authors define a family of solution concepts for generalised cooperative games, parametrised by α, as follows:

$$\Psi_i^{\alpha}(\nu^*) = \sum_{\pi_S \in \Omega(I), i \in S} \Delta^*_{\nu^*}(\pi_S) \frac{\alpha^{|S|-\pi_S(i)}}{|S|! \sum_{j=0}^{|S|-1} \alpha^j}. \tag{3.7}$$

Note that we abuse notation and assume that $0^0 = 1$. From the above equation, it is clear that this family of solution concepts is in fact a hybrid of Equations (3.5) and (3.6). Specifically, when $\alpha = 0$ we obtain the Nowak and Radzik [53] value as in Equation (3.6), and when $\alpha = 1$ we obtain the Sanchez and Bergantiños [44] value, as in Equation (3.5). Between the two extremes, a smaller α implies that a higher proportion of a dividend is gradually given to the later players in an ordered coalition, whereas a larger α implies a more even distribution of power regardless of the order of players.

We are almost ready to introduce the measure proposed by Pozo et al. [48]. The last definition we need to introduce is that of *digraph-restricted games*. To this end, let $\nu_D^{\mathcal{P}*}$ denote the game ν restricted by the digraph D, and let Paths(D) denote the set of all paths without loops in the digraph D. We will first show how games belonging to the unanimity basis can be restricted by digraphs (see Chapter 2 for more on unanimity bases). Recall that when u_S^* is a generalised characteristic function for a (regular, not generalised) unanimity game for the coalition S, then we have $u_S^* = \sum_{\pi_S \in \Pi(S)} w^*_{\pi_S}$, where $w^*_{\pi_S}$ is the generalised unanimity game for π_S. The digraph restriction of the unanimity game u_S^* according to Pozo et al., denoted by $u_{D,S}^{\mathcal{P}*}$, is defined as follows:

$$u_{D,S}^{\mathcal{P}*} = \sum_{\pi_S \in \Pi(S) \cap \text{Paths}(D)} w_{\pi_S}.$$

In other words, only consecutively connected ordered coalitions (i.e., those that are paths) that contain every member of S have value 1. The general formula for digraph restricted games not belonging to the unanimity basis, then, is as follows:

$$\nu_D^{\mathcal{P}*} = \sum_{S \in V} \Delta_\nu(S) u_{D,S}^{\mathcal{P}*},$$

where $\Delta_\nu(S)$ are the regular (not generalised) Harsanyi dividends for the cooperative game ν. Note that—unlike ν—$\nu_D^{\mathcal{P}*}$ is a generalised characteristic function. For this reason, solution concepts for generalised cooperative games must be used as a measure of power. Now, we are ready to present the centrality measure due to Pozo et al. [48]. Given a digraph D and a (regular, not generalised) cooperative game ν, the centrality of a node v is:

$$\Psi_v^\alpha(\nu_D^{\mathcal{P}*}) - SV_v(\nu). \tag{3.8}$$

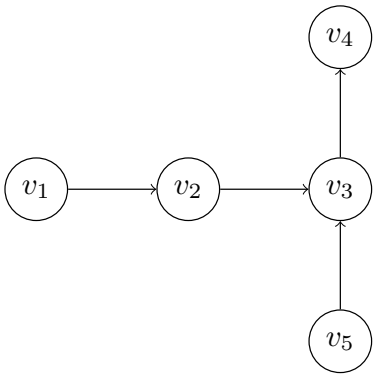

Figure 3.2: A sample digraph used to compare a number of centrality measures based on connectivity.

Example 14 *Consider the directed network in Figure 3.2 along with the characteristic function* $\nu(C) = |C|^2$. *Let us start by calculating the Harsanyi dividends for this game. Since* ν *is symmetric, we will write* $\Delta(|C|)$ *to be equivalent to* $\Delta_\nu(C)$ *(which will allow us to define the dividends for* 5 *possible coalition sizes, rather than for all* 2^5 *coalitions). First, we have* $\Delta(1) = 1^2 = 1$. *Next,* $\Delta(2) = 2^2 - 2\Delta(1) = 2$. *Moving on,* $\Delta(3) = 3^2 - 3\Delta(1) - 3\Delta(2) = 0$, $\Delta(4) = 4^2 - 4\Delta(1) - 6\Delta(2) - 4\Delta(3) = 0$, *and finally* $\Delta(5) = 5^2 - 5\Delta(1) - 10\Delta(2) - 10\Delta(3) - 5\Delta(4) = 0$. *The Shapley value of any player is simply* $\Delta(1) + \frac{4\Delta(2)}{2} = 1 + \frac{4}{2} = 5$, *since every node belongs to only one coalition of size* 1 *and* 4 *coalitions of size* 2. *Alternatively, the value of the grand coalition is* 25, *and since the game is symmetric, then this value is split equally between all players, i.e.,* $\frac{25}{5} = 5$ *each.*

We will now express $\nu_D^{\mathcal{P}*}$ *given the digraph* D *from Figure 3.2. We have* $\nu_D^{\mathcal{P}*}(\pi_C) = \sum_{S\subseteq V} \Delta_\nu(S) u_{D,S}^{\mathcal{P}*}(\pi_C)$*. Recall that* $u_{D,S}^{\mathcal{P}*}(\pi_C) = 1$ *if and only if* π_C *is a path in* D *and* C *contains all the players from* S*. Since the Harsanyi dividends are equal to* 0 *for all coalitions of size greater than* 2*, our job is not too difficult.* $\nu_D^{\mathcal{P}*}(\pi_C) = |C| + 2 \times [$*number of paths in* D *of length* 2 *in* $\pi_C]$*. In terms of the unanimity basis for generalised cooperative games, we have:* $\nu_D^{\mathcal{P}*} = w^*_{\{1\}} + w^*_{\{2\}} + w^*_{\{3\}} + w^*_{\{4\}} + w^*_{\{5\}} + 2 \times [w^*_{\{1,2\}} + w^*_{\{2,3\}} + w^*_{\{3,4\}} + w^*_{\{5,3\}}]$*. This form of* $\nu_D^{\mathcal{P}*}$ *is very convenient, as we now have the coefficients in the unanimity basis, and hence the dividends required to calculate the new centrality.*

The Nowak-Radzik value vector for the nodes v_1, v_2, v_3, v_4, v_5 *is therefore* $1, 2, 3, 2, 1$*, whereas Sanchez-Bergantios value vector is* $1.5, 2, 2.5, 1.5, 1.5$*, and the centrality due to Pozo et al. [48] is simply a shift of these values by* -5*. Note that the sum of the centralities is equal for both solution concepts; only the distribution has changed. We can now see how important the edges and their direction is for the Nowak-Radzik value. For example, the benefit of the edge* $v_3 \to v_4$ *is split equally between nodes* v_3 *and* v_4 *in the Sanchez-Bergantios value, however in the Nowak-Radzik value* v_3 *gains nothing from it, and the entire worth of the edge is given to the last node—*v_4*.*

The complexity of this measure has not been extensively studied. The characterisation of the centrality measure given by Pozo et al. [48] requires the calculation of all Harsanyi dividends for all paths in the graph. A naive algorithm would have to traverse all paths without loops in a graph, possibly resulting in $O(\sum_{k=1}^{n} \binom{n}{k} k!)$ calculations. Furthermore, calculating all Harsanyi dividends requires $O(\sum_{k=1}^{n} \binom{n}{k} 2^k)$ calculations. It is an open problem whether these complexities can be improved upon in general or for certain classes of graphs.

One criticism of the above family of centrality measures is as follows: why does it not allow—through the parameter α or otherwise—for the option of the first nodes in a path to benefit most? It is purely by convention that we equate paths with ordered coalitions such that the source of the path is the first node in the coalition, and the destination is the last. This can, of course, be remedied by reversing the

edges in the graph depending on the application, but it is not an inherent parameter of the centrality. We will return to this question in Section 3.3.5; the centrality measure presented therein does the opposite of the Nowak-Radzik value, giving more value to nodes at the beginning of paths.

In the next section, we move on to a measure of centrality developed by Amer, Giménez, and Magaña [49]. Although the centrality presented in this section stems from the work by Gómez et al. [19], which makes prominent use of the Myerson value, it is not in fact based on the idea of graph-restricted games according to Myerson. The idea of digraph-restricted games presented here is somewhat convoluted, and the result is not intuitive. The centrality in the next section, however, is based on almost a direct translation of Myerson's graph-restricted games to digraphs.

3.2.4 Accessibility in Digraphs and Centrality due to Amer, Giménez, and Magaña [49]

Amer, Giménez, and Magaña [49] define the notion of accessibility in digraphs based on an idea similar to Myerson's graph-restricted games. Although they do not use the term "centrality", they do talk about the ranking of nodes in a network. In essence, their notion of accessibility is a centrality measure suitable for applications in which it is important for central nodes to be reachable (i.e., accessible). In a sense, this is the opposite of closeness centrality, where it is desirable to be able to reach other nodes quickly *from* central nodes. Amer, Giménez, and Magaña [49] define a digraph-restricted, generalised cooperative game on a directed network, in much the same way that Myerson defined a graph-restricted cooperative game on networks. The approach taken is to partition ordered coalitions into a maximal set of smaller ordered coalitions that form paths in the digraph, and to equate their value with the sum of the values of these maximal coalitions. Formally, let $\Pi(V)$ be the set of all permutations on a set of nodes, V. Furthermore, let $\pi \in \Pi(V)$ be a permutation of V and denote by $\pi(j)$ the position of node j in π, and by π_j the j'th element in π. Next, we must introduce connected consecutive subcoalitions of π according to a digraph D. We say $\pi = \{\pi_p, \pi_{p+1}, \ldots, \pi_{p+u}\}$ is such a coalition

whenever $(\pi_j, \pi_{j+1}) \in E(D)$. Additionally, if $p = 1$ or $(\pi_{p-1}, \pi(p)) \notin E(D)$, then π cannot be expanded to the left. If $p + u = |V|$ or $(\pi(p + u), \pi(p + u + 1)) \notin E(D)$, then π cannot be expanded to the right. If π cannot be expanded to the left nor to the right, then it is a maximal connected consecutive subcoalition. We denote by π/D the set of maximal consecutive subcoalitions of π according to D.

Let us now define the *digraph-restricted generalised characteristic functio*n of a (regular, non-generalised) cooperative game that makes use of the above concepts. Recall that Equation (2.5)—which defines the Shapley value—considers different permutations of players in each coalition. Now, according to Amer, Giménez, and Magaña [49], in the case of digraphs, the characteristic function needs to be redefined, because some of the aforementioned permutations may not be feasible (i.e., not all of them form paths and may therefore not be able to communicate). We will denote by $\nu_D^{\mathcal{AG}*}$ the generalised characteristic function obtained from restricting ν by the digraph D. For an ordered coalition, $\pi_S \in \Pi(S)$, we define:

$$\nu_D^{\mathcal{AG}*}(\pi_S) = \sum_{\pi_C \in \pi_S/D} \nu(C). \tag{3.9}$$

Note that $\nu_D^{\mathcal{AG}*}$ is defined over *ordered* coalitions, as opposed to ν, which is defined over unordered coalitions.

The authors used these digraph-restricted games to characterise the *accessibility* of a node in much the same way as Myerson can be equated to the Shapley value of the graph-restricted game. The accessibility of a node v in the digraph D with the cooperative game ν, $AC_v(D; \nu)$ is defined as the Nowak-Radzik value (as defined in the previous section) of the digraph-restricted game:

$$AC_v(D; \nu) = \frac{1}{|V|!} \sum_{\pi \in \Pi(V)} [\nu_D^{\mathcal{AG}*}(\pi_{|v} \cup v) - \nu_D^{\mathcal{AG}*}(\pi_{|v})], \tag{3.10}$$

where $\pi_{|v}$ is the ordered coalition π cut off just before the node v (i.e., if $\pi = \{v_1, v_2, \ldots, v_{k-1}, v_k, v, \ldots, v_n\}$, then $\pi_{|v} = \{v_1, v_2, \ldots, v_{k-1}, v_k\}$), and where $\pi \cup v$ is the ordered coalition with v added at the end of π.

Example 15 *Let us consider the centrality due to Amer, Giménez, and Magaña [49] with the characteristic function* $\nu_2(C) = |C|^2$ *and the digraph presented in Figure 3.2—the same directed graph for which we calculated the centrality due to Pozo et al. [48] in the previous section. The accessibility of the nodes* v_1, v_2, v_3, v_4, v_5 *are* $1, 1.4, 1.9, 1.63, 1$*, respectively. One can see that the paths ending at* v_4 *are—on average—longer than those ending at* v_3*. However, more short paths exist that end at* v_3 *than those that end at* v_4*. To give more value to longer paths, one might consider the game* $\nu_{10}(C) = |C|^{10}$*. Now, the resulting ranks are* 1*,* 205.4*,* 3259.9*,* 21430.6 *and* 1*, respectively. The exponent—in effect—specifies the extent of the synergy that happens when large groups collaborate. In this case, the longer paths that end with* v_4 *dominate the larger number of short paths ending with* v_3*. We saw the opposite for* ν_2*, where synergy between nodes resulted in less value.*

Accessibility satisfies the following properties:

- Linearity: for any node, v, any pair of characteristic functions, μ and ν, and any pair of real numbers, x and y, we have: $AC_v(D; x\nu + y\mu) = x \cdot AC_v(D; \nu) + y \cdot AC_v(D; \mu)$;

- Dummy-player: Amer, Giménez, and Magaña [49] use a somewhat broader definition of the classic dummy player axiom. It is defined such that for any node v that contributes exactly its singleton value to any coalition we have: $AC_v(D; \nu) = \nu_D^{\mathcal{AG}*}(\{v\})$;

- Average efficiency: $\sum_v AC_v(D; \nu) = \frac{1}{|V|!} \sum_{\pi \in \Pi(V)} \nu_D^{\mathcal{AG}*}(\pi)$;

- Given a complete digraph D (i.e., one in which all possible edges are present), for every node $v \in V$, $AC_v(D; \mu)$ coincides with the Shapley value of v in the game (I, ν);

- If a node v is inaccessible in D (i.e., if the indegree of v equals 0), then $AC_v(D; \nu) = \nu_D^{\mathcal{AG}*}(\{v\})$;

- For any node v in any digraph D, $AC_v(D;\nu)$ does not change by adding to D an edge from v to any other player;

- If ν is superadditive, then adding an edge from some node to the node v does not decrease the value $AC_v(D;\mu)$.

These properties, however, do not uniquely characterise the accessibility measure. In contrast to the Shapley value, no set of axioms have yet been identified that would fully characterise accessibility.

The accessibility centrality measure was extended by Amer, Giménez, and Magaña [20] to something akin to that of semivalue as follows:

$$AC_v(D;\nu,\phi^\beta) = \phi_v^\beta(\nu_D) = \sum_{C\in V\setminus\{v\}} \beta(|C|)\frac{(|V|-1-|C|)!(|C|)!}{|V|!}\sum_{\pi\in\Pi(C)}[\nu^{\mathcal{AG}*}(\pi\cup\{v\})-\nu^{\mathcal{AG}*}(\pi)], \quad (3.11)$$

where ϕ^β is the semivalue with which the probability of a coalition of size k being chosen is equal to $\beta(k)$. Now, let us say that a game (I,ν) is *zero-normalized* whenever $\nu(\{i\})=0$ for all $i\in I$. Then, the accessibility centrality measure defined in Equation (3.11) satisfies the following properties:

- If ν is zero-normalized, then for every *inaccessible* node v (i.e., a node whose indegree is 0), we have: $AC_v(D;\nu,\phi^\beta)=0$;

- For every digraph $D=(V,E)$ and every node $v\in V$, if an edge (v,u) is added to E, then the accessibility of v stays the same, i.e., $AC_v((V,E);\nu,\phi^\beta) = AC_v((V,E\cup(v,u));\nu,\phi^\beta)$;

- If ν is monotonic and zero-normalized, then for every digraph $D=(V,E)$ and every node $v\in V$, adding an edge (u,v) will not decrease the accessibility of v. That is, $AC_v((V,E);\nu,\phi^\beta)\leq AC_v((V,E\cup(u,v));\nu,\phi^\beta)$;

- Given a complete digraph D (i.e., one in which all possible edges are present), for every node $v\in V$, $AC_v(D;\nu,\phi^\beta)$ coincides with $\phi_v^\beta(\nu)$—the semivalue of v in the game (V,ν);

To date, the complexity of this measure has not been extensively studied, however a naive algorithm would have to traverse all ordered coalitions, of which there are $O(\sum_{k=1}^{n} \binom{n}{k} k!)$. In the next section, we discuss a recent measure of centrality proposed by Belau [50]. As opposed to the previous measures, this centrality is tailored for weighted networks. It makes use of the graph-restricted game by Myerson, but in a much different manner from the centrality measures presented up until now. This will be the last measure that we discuss that features Myerson's notion of connectivity.

3.2.5 Cohesion centrality due to Belau [50]

In this section we introduce cohesion centrality [50]. The defining feature of this node centrality measure is that it goes through the intermediary step of ranking all edges in terms of their importance for the *cohesion* of the network. The underlying model for this measure consists of a weighted (or unweighted) digraph and a cooperative game on the set of nodes. From this, a new cooperative game *on the set of edges* is defined, which is meant to represent how important each set of edges is for the cohesion, or interconnectedness, of the network. Next, either the Shapley value or the Banzhaf index is used to assess the importance for cohesion of every edge. The rank of each edge and its weight are then combined and—finally—one of Freeman's [8] measures (degree, closeness or betweenness) is used to determine the weight of each node.

The first step of the cohesion centrality is to assign new weights to the edges in the network $D = (V, E)$. This is done according to the *link-game* [54, 55]—$\nu_V^{\mathcal{B}} : 2^E \to \mathbb{R}$—of a cooperative game $\nu : 2^V \to \mathbb{R}$, which is defined as follows:

$$\nu_V^{\mathcal{B}}(S_E) := \nu_{(V,S_E)}^{\mathcal{M}}(V),$$

where $S_E \in E$ and $\nu_{(V,S_E)}^{\mathcal{M}}$ is Myerson's graph-restricted game for ν and the graph (V, S_E). In short, the link game defines a process through which we take a cooperative game (ν) on the set of nodes (V), and from it define a cooperative game ($\nu_V^{\mathcal{B}}$) on the set of edges (E). Myerson's graph-restricted game is used to asses the value

of the set of all nodes (i.e., the whole network) under different subsets of edges. The next step in defining the cohesion centrality of nodes is to use either the Shapley value or Banzhaf index to measure the value of each edge. The weight of the edges and their payoffs are then normalised to sum up to 1 (this is done by dividing the payoff of each edge by $|E|$ and its weight by the sum of all weights) and combined according to the formula:

$$\alpha\omega(s,t) + (1-\alpha)\phi_{(s,t)}(\nu_V^{\mathcal{B}}),$$

for some parameter α and where $\phi_{(s,t)}$ is either the Shapley value or Banzhaf index of the edge (s,t) in the link game, and $\omega(s,t)$ is the weight of the edges (s,t). Finally, one of the classical centralities defined by Freeman [8] is used to define the cohesion centrality.

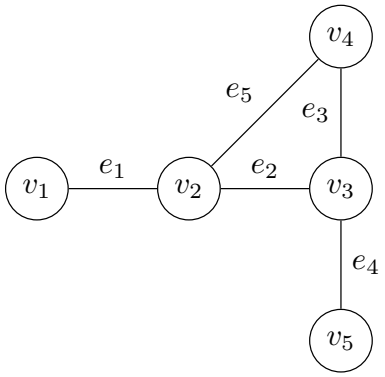

Figure 3.3: A sample weighted graph used to illustrate cohesion centrality due to Belau [50].

Example 16 *Let us consider the cohesion centrality of the nodes in Figure 3.3. Recall, that in order to define the underlying model we need both a graph and cooperative game on the set of nodes. For the latter, we will consider the cohesion game defined by Belau [50], which is defined as follows:* $c(S,G) = \sum_{v\in S} c_v(G)$*, where* $c_v(G)$ *is the number of nodes reachable from the node* v *in the graph* G*. The link game (on the set of nodes) for this cohesion game then follows:* $c_V^{\mathcal{B}}(S_E) = \sum_{v\in V} c_v((V,S_E))$*, where* $S_E \subseteq E$*. The Shapley value of the edges* e_1*,* e_2*,* e_3*,* e_4*, and* e_5 *in the cohesion*

link game is 3.7*,* 3.6*,* 2.7*,* 3.7*, and* 2.7*, respectively. By using Freeman's Degree Centrality with these weights, we obtain a ranking of individual nodes. The rank of the nodes* v_1*,* v_2*,* v_3*,* v_4*, and* v_5 *is* 3.7*,* 10*,* 10*,* 5.4*, and* 3.7*, respectively. We note that—if removed—*v_2 *and* v_3 *both disconnect one node from the rest of the network, making them the most important nodes for the cohesion of the network. Nodex* v_1 *and* v_5 *both only have one edge, which connects them to the rest of the network, making them the least important. Node* v_4*, on the other hand, has two edges, but they are not particularly important towards cohesion, making* v_4 *not much more important than* v_1 *or* v_5*.*

In the next section, we discuss the centrality measure due to Grofman and Owen [18]. This centrality considers all the paths within a directed network (just like the centralities due to Pozo et al. [48] and Amer, Giménez, and Magaña [49]), but it considers connectivity in a slightly different manner than the centrality measures that we have discussed up until now.

3.2.6 Banzhaf Centrality due to Grofman and Owen [18]

In this section we discuss the Banzhaf path-based centrality [18]. This approach to connectivity is somewhat different than what we have discussed thus far, as it does not involve graph-restricted games of any manner. The centrality considers all connected ordered coalitions (i.e., paths without loops), much like the centrality due to Pozo et al. [48], but without an explicit evaluation of groups of nodes. Rather, the authors of the measure go back to the roots of the Banzhaf index [15], which was defined as the relative number of possible swing votes that players have in weighted voting games.

Banzhaf Index of Power for weighted voting games: The Banzhaf power index is a very prominent solution concept in cooperative game theory. Originally proposed for voting games, it is defined as the probability that a player can change the outcome of an election. Here, we will give a slightly more general definition

for weighted voting games (the original definition is equivalent to the one below, where all weights are equal to 1).

In a weighted voting game, players attempt to pass a motion. A coalition is considered *winning*, if the motion is carried. Consider a set I of n players, $\{1, \ldots, n\}$, where player i has weight w_i, $0 < w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$. We define the quota, q, with $0 < q \leq 1$, as the sum of the weights of votes that is necessary for the motion to be carried. A coalition is *winning* if the sum of the weights of its members is equal to or greater than q.

A *swing* vote for player i is a winning coalition that contains i (i.e., i was one of the players that voted for the motion to be carried) and his or her defection would change the outcome (the motion would not be carried if i left it). Let the number of such coalitions for i be denoted by S_i. The relative Banzhaf power index for player i, B_i, is then defined as

$$B_i = \frac{S_i}{\sum_{i=1}^{n} S_i}. \tag{3.12}$$

Since such an index is game-specific (i.e., it heavily depends on the number of players), a total/normalised Banzhaf index, B'_i, is defined for comparison of nodes between games.

$$B'_i = \frac{S_i}{2^n - 1}. \tag{3.13}$$

Example 17 *Continuing the running example we started in Example 1, consider a situation in which the software must necessarily be error free, otherwise the customer will not purchase it. In this case, the winning coalitions are* $\{1, 2\}$ *and* $\{1, 2, 3\}$. *The Banzhaf power index of both* p_1 *and* p_2 *is clearly* $\frac{2}{4}$, *since there are* 2 *swing votes for each of them.*

Recall that we defined the Banzhaf index in Chapter 2 as the expected contribution of a player in a cooperative game. Consider the cooperative game ν for a weighted voting game such that $\nu(C) = 1$ if C is winning and $\nu(C) = 0$ otherwise. Given

this definition, the Banzhaf index of ν coincides with the Banzhaf index for the weighted voting game.

Banzhaf centrality due to Grofman and Owen [18]: The idea behind this centrality measure is to count the number of times when a path is irreparably broken by the removal of a node. If the removal of v from a path, π, means that no subset of the remaining nodes can form another path between the source and destination nodes, then v clearly plays a significant role in π. The number of times a node can "break" a path in such a way determines its rank.

More formally, let us defined the winning and losing coalitions. First and foremost, it is important to note that Grofman and Owen [18] consider coalitions as ordered sets. The set of winning coalitions, then, is defined as the set of simple paths (paths without cycles) of length 2 or greater. Thus $(1, 2, 3)$ represents a path from node 1 to node 3, whereas $(1, 3, 2)$ a path from node 1 to 2. A node v has a swing vote in a winning coalition (i.e., path) π that visits v whenever the set of nodes that are visited by π without v can in no way form another path from the source to the destination. With these definitions of swing and winning coalition, the traditional definition of the Banzhaf power index (Equations 3.12 and 3.13) can be applied directly.

As a critique of this approach, its connection to cooperative game theory and the Banzhaf index is slightly forced. Why do the authors go through modifying the original Banzhaf setting of weighted voting to networks? From a purely network science point of view, it seems perfectly reasonable to consider the number of paths that a node is essential to as its centrality measure and no new insight is brought from an analogy to the Banzhaf index.

Example 18 *Consider again the nodes in Figure 3.3 and their Banzhaf path-based centrality [18]. In particular, we will consider a variant of the centrality that was proposed by the authors, where swing votes of the first and last nodes in a path are not counted. After all, it is obvious that if the source or destination is removed, then the other nodes will not be able to reach these nodes; a source or destination node* always *has a swing vote.*

Given the graph in Figure 3.3, the centrality of each of the nodes v_1*,* v_4 *and* v_5 *according to Grofman and Owen [18] is* 0*. Indeed, nodes* v_1 *and* v_5 *are simply connected by one edge to the rest of the graph and lead nowhere else, making it clear that they do not have a single swing vote. As for* v_4*, it brings no indispensable edges to the graph. For all paths that it belongs to, it can be bypassed. In contrast, nodes* v_2 *and* v_3 *have a centrality of* $\frac{12}{24}$ *each. This is because each of them is indispensable for* 12 *paths, and there are a total of* 24 *swing votes. More specifically, node* v_2 *is indispensable to the following paths:*

$$v_1 \to v_2 \to v_3, \qquad v_1 \to v_2 \to v_4 \to v_3,$$
$$v_1 \to v_2 \to v_3 \to v_4, \qquad v_1 \to v_2 \to v_4,$$
$$v_1 \to v_2 \to v_3 \to v_5, \qquad v_1 \to v_2 \to v_4 \to v_3 \to v_5,$$
$$v_3 \to v_2 \to v_1, \qquad v_3 \to v_4 \to v_2 \to v_1,$$
$$v_4 \to v_3 \to v_2 \to v_1, \qquad v_4 \to v_2 \to v_1,$$
$$v_5 \to v_3 \to v_2 \to v_1, \qquad v_5 \to v_3 \to v_4 \to v_2 \to v_1.$$

On the other hand, node v_3 *is indispensable to the following paths:*

$$v_4 \to v_3 \to v_5, \qquad v_4 \to v_2 \to v_3 \to v_5,$$
$$v_1 \to v_2 \to v_3 \to v_5, \qquad v_1 \to v_2 \to v_4 \to v_3 \to v_5,$$
$$v_2 \to v_3 \to v_5, \qquad v_2 \to v_4 \to v_3 \to v_5 \to,$$
$$v_5 \to v_3 \to v_4, \qquad v_5 \to v_3 \to v_2 \to v_4,$$
$$v_5 \to v_3 \to v_2 \to v_1, \qquad v_5 \to v_3 \to v_4 \to v_2 \to v_1,$$
$$v_5 \to v_3 \to v_2, \qquad v_5 \to v_3 \to v_4 \to v_2.$$

The complexity of computing this centrality has not been studied in the literature. A naive algorithm considers all paths within the network (of which there are potentially $O(\sum_{k=1}^{n} \binom{n}{k} k!)$), and for each node in the path checks whether the rest of the nodes can rearrange themselves in order to form a different path between the same pair of nodes.

In the following section, we will discuss two other variatoions of connectivity that have also been applied to game-theoretic network centrality.

3.2.7 Connectivity Games due to Amer and Giménez [56]

In approaching connectivity, Amer and Giménez [56] formalised an approach that differs from Myerson's graph restricted games in its evaluation of disconnected coalitions. Whereas Myerson [47] assumed that disconnected coalitions cannot fully cooperate and therefore their value should be the sum of their maximally connected parts, Amer and Giménez [56] postulate that the value of such coalitions should be 0. In line with this, Amer and Giménez [56] defined the following characteristic function of a simple game:[3]

$$\nu_G^{\mathcal{AG}}(S) = \begin{cases} 1 & \text{if } S \in \mathcal{C}(G) \\ 0 & \text{otherwise,} \end{cases} \tag{3.14}$$

where $\mathcal{AG}$ stands for Amer and Gimenez. The game is referred to as the 0-1-*connectivity game*. This characteristic function was later generalised by Lindelauf, Hamers, and Husslage [57] to:

$$\nu_G^{f}(S) = \begin{cases} f(S,G) & \text{if } S \in \mathcal{C}(G) \\ 0 & \text{otherwise,} \end{cases} \tag{3.15}$$

where f is an arbitrary function to the real numbers, allowing for any valuation of connected coalitions.

Amer and Giménez [56] combined the game $\nu_G^{\mathcal{AG}}(S)$ with semivalues in order to rank individual nodes in terms of their importance to connectivity. Lindelauf, Hamers, and Husslage [57] considered the Shapley value of ν_G^{f}, although—in principle—any other semivalue could also be used.

As a rather interesting application, Lindelauf, Hamers, and Husslage [57] used their centrality measure to analyse terrorist networks. However, Michalak et al. [21] later argued that, for terrorist networks, this centrality measure overstates the importance

[3]Simple coalitional games are a popular class of games, where every coalition has a value of either 1 or 0.

of nodes for the connectivity of the network and thus is not suitable for this application. In terms of complexity, Michalak et al. [21] also showed that, even for the simplest variant of connectivity games, where $\forall_{S \in \mathcal{C}(G)} f(S, G) = 1$, computing the Shapley value of ν_G^f is #P-Complete and therefore not tractable for larger networks.

In the next section based on connectivity, we discuss an alternative (and in some respects more sophisticated) notion of connectivity for game-theoretic payoff allocation in directed networks, which was introduced by Kim and Tackseung [58].

3.2.8 Weak Connectivity due to Kim and Tackseung [58]

Kim and Tackseung [58] introduce the notion of *weak connectivity*, and use it to develop a payoff division scheme (that can be interpreted as a centrality measure) that is similar to the Myerson value. In fact, this division scheme is equal to the Myerson value if the network is undirected. However, rather than requiring a path between any pair of nodes (i.e., strong connectivity), weak connectivity attempts to account for the fact that a single node in a digraph can "gather" the information from within a coalition. Such a node (which need not be unique) is referred to by the authors as the *informational coordinator*. Let E^* be the relation that represents the transitive closure of the relation defined by the edges of a digraph, $D(V, E)$. In other words, we have $E^*(s, t)$ if and only if there exists a path from s to t. For $C \subseteq V$, if there exists a node $v \in C$ such that for all nodes $u \in C$ we have $E^*(u, v)$ (i.e., v is an informational coordinator), then we say that C is *weakly connected*. By C/E we denote the set of weakly connected components of $C \in V$ and define it as the coarsest partition of C into weakly connected sets (i.e., such that no two weakly connected sets can be joined to form an even larger weakly connected set). Note that this partition need not be unique. If the partition is unique, then the authors define digraph restricted games in the following manner:

$$\nu_D^{\mathcal{KT}}(S) = \sum_{C \in S/E} \nu(C).$$

Not weakly connected	Weakly connected components
$\{1,3\}$	$\{1\},\{3\}$
$\{1,4\}$	$\{1\},\{4\}$
$\{2,4\}$	$\{2\},\{4\}$
$\{1,2,4\}$	$\{1,2\},\{4\}$
$\{1,3,4\}$	$\{1\},\{3,4\}$
$\{1,5\}$	$\{1\},\{5\}$
$\{2,5\}$	$\{2\},\{5\}$
$\{1,2,5\}$	$\{1,2\},\{5\}$
$\{1,3,5\}$	$\{1\},\{3,5\}$
$\{4,5\}$	$\{4\},\{5\}$
$\{1,4,5\}$	$\{1\},\{4\},\{5\}$
$\{2,4,5\}$	$\{2\},\{4\},\{5\}$
$\{1,2,4,5\}$	$\{1,2\},\{4\},\{5\}$
$\{1,3,4,5\}$	$\{1\},\{3,4,5\}$

Table 3.1: The breakdown of the not weakly connected subgraphs of $\{1,2,3,4,5\}$ in Figure 3.2 into weakly connected components

When the partition is not unique, the value of the digraph restricted game is defined as the maximum value over all partitions:

$$\nu_D^{\mathcal{KT}}(S) = \max_{S/E \in \mathcal{KT}(S)} [\sum_{C \in S/E} \nu(C)],$$

where $\mathcal{KT}(S)$ denotes the set of all possible partitions of the set S into maximally weakly connected components. Next, the authors used the following allocation rule:

$$\phi_i^{\mathcal{KT}}(\nu) = SV_i(\nu_D^{\mathcal{KT}}).$$

When the network in question is undirected, then weak connectivity and strong connectivity are equivalent. Therefore, for undirected networks, this allocation is equal to the Myerson value. Its complexity for undirected networks is therefore also the same as that of the Myerson value. For directed networks, however, it is potentially worse (although this has not been studied in the literature to date).

Example 19 *Consider again Figure 3.2 and the characteristic function* $\nu(C) = |C|^2$. *There are* 31 *non-empty subsets of the set* $\{1,2,3,4,5\}$. *Of these,* 17 *are*

weakly connected. As for the other 14 *subsets, they are listed in Table 3.1 along with their weakly connected components. In this case, all of the partitions are unique. None of these subgraphs (except for singletons) would be considered connected in the classical sense, since we never have a situation where every node can communicate with every other node.*

The centralities of the nodes v_1*,* v_2*,* v_3*,* v_4*, and* v_5 *according to Kim and Tackseung [58] are* $7.36667, 8.5, 9.33333, 7.46667$*, and* 7.46667*, respectively.*

In the next section, we discuss an axiomatic approach to connectivity due to Skibski et al. [59]. This will also be the last centrality based on connectivity that we discuss.

3.2.9 Attachment Centrality due to Skibski et al. [59]

Skibski et al. [59] developed an axiomatic approach to study connectivity in networks. In particular, they postulated the following axioms for a connectivity-based centrality measure:

- **Locality:** The centrality of a node depends only on its connected component. In other words, let C be the connected component of v in some graph G. Then the centrality of v in G should be the same as its centrality in the subgraph of G induced by C, $G[C]$
- **Normalisation:** The centrality of any node is inbetween the values 0 and $|V| - 1$, inclusive.
- **Fairness:** If we add an edge (u, v) to G, then the magnitude of the resulting change to the centrality of u is equal to the magnitude of the change to the centrality of v. This is the same axiom that is used to characterise the Myerson value.
- **Gain-loss:** Adding any edge to any connected graph, G, will not change the sum of the centralities of the nodes.

The authors show that these four axioms uniquely characterise the following centrality, which they call attachment centrality:

$$A_v(G) = \sum_{S \in V \setminus \{v\}} 2\beta(S,V)(|\mathcal{K}(C)| - |\mathcal{K}(C \cup \{v\})| + 1),$$

where $\beta(S,V) = \frac{|S|!(|V|-|S|-1)!}{|V|!}$ and $\mathcal{K}(C)$ is the set of components of $G[C]$, the subgraph induced by C. The authors also note that $A_v(G)$ is equal to the Shapley value of the following game:

$$\nu(C) = 2(|C| - |\mathcal{K}(C)|)$$

In this, the approach of the authors to centrality based on connectivity is similar to Myerson, whereby it is defined by a set of axioms. This is highly desirable, since the axioms intuitively identify the centrality and therefore help identify those contexts where it is best to apply it.

With this, we conclude our review of centrality measures based on connectivity. In the next section, we discuss those that are not.

3.3 Game-Theoretic Node Centrality Measures Not Based on Connectivity

The second general approach to game-theoretic network centrality places emphasis on choosing a characteristic function that evaluates the role of nodes according to their topological position and combining this with some solution concept for normal cooperative games (without networks). In this approach, the connectivity of nodes does not need to have any direct bearing on their centrality, although this can still be expressed in the choice of the characteristic function. The general idea behind most of the centrality measures discussed in this section is to take some group centrality measure and treat it as a cooperative game on the set of nodes. Next, solution concepts such as the Shapley value, semivalues or others are used in conjunction with group centrality measures in order to yield a ranking of nodes.

This section is structured as follows. In Section 3.3.1, we discuss the work by Brink and Borm [16] and the follow-up work by Brink and Gilles [17], who defined a group centrality that reflects the *dominance* of groups of nodes in directed graphs, and then proposed a Shapley value-based centrality that captures synergies within the network. This idea was rediscovered by Ramasuri and Narahari [60] for the application of information diffusion and influence propagation, and was extended by Michalak et al. [29] to a broader range of group centrality measures. Both publications are discussed in Section 3.3.2, where Shapley value degree and Shapley value closeness centrality measures are discussed. Michalak et al. [29] developed polynomial algorithms for computing these—and other—game-theoretic centrality measures. Continuing, in Section 3.3.3 we discuss the work by Szczepański, Michalak, and Rahwan [22], who proposed the Shapley value betweenness centrality and also developed polynomial-time algorithms for calculating it. In Section 3.3.4, we discuss a generalisation of the Shapley value approach to centrality that was developed by Szczepański, Michalak, and Wooldridge [31]. This approach, which is based on the Owen value, applies to networks that have an extraneously defined community structure. Finally, in Section 3.3.5 we discuss the VL control measure, which shares some, but not all aspects of the other centrality measures discussed in this section.

3.3.1 Centrality based on Dominance due to Brink and Borm [16]

Brink and Borm [16] and Brink and Gilles [17] propose a measure of *dominance* in directed networks called the *β-measure*.[4] Although this measure was apparently not intended to be game-theoretic in nature, the authors show that it coincides with the Shapley value of the *score game* in networks (which is a generalisation of the so-called *score measure*). Moreover, the authors axiomatically characterize both the β-measure and score measure. Although it is not stated explicitly as such, the β-measure can be interpreted as a measure of centrality in networks.

[4] A preliminary version of this article appeared as a working paper [16] and was later on extended by Brink and Borm [61].

The remainder of this section formally introduces the score measure, the score game, and—finally—the β-measure.

In the context of this section, a directed network is interpreted as a representation of the dominance between nodes, whereby a directed edge (v, u) represents the fact that v dominates u. Let $E(v) = \{u : (v, u) \in D\}$ denote the set of nodes dominated by s, and let $E^{-1}(v) = \{u : (u, v) \in D\}$ denote the set of nodes that dominate v. The *score measure*, then, is a function, $\sigma_D : V \to \mathbb{N}$ that assigns to every node, $v \in V$, the number of nodes that are dominated by v (i.e., the outdegree of v). Formally:

$$\sigma_D(v) = |E_D(v)| .$$

This measure is well known in the literature and mentioned, for example, by Behzad, Chartrand, and Lesniak [62]. It is typically used to rank participants in a tournament, where a tournament is defined as an irreflexive digraph where for all $u, v \in V$ such that $u \neq v$ it holds that either $(u, v) \in D$ or $(v, u) \in D$. Brink and Gilles [17] axiomatically characterized this measure, showing that it uniquely satisfies the following properties or axioms:

- *Score normalisation*: For every digraph, D, the sum of the scores of all nodes add up to $|E|$.
- *Dummy-node*: If v does not dominate any node, its score is 0;
- *Symmetry*: For any $u, v \in V$ such that $E_D(u) = E_D(v)$ and $E_D^{-1}(u) = E_D^{-1}(v)$, the score of u is equal to the score of v;
- *Additivity over independent partitions*: The sum of the scores of nodes in D is equal to the sum of the scores of nodes in the subgraphs generated by all the independent partitions of D (where independent partition are such that for any node u, if the set of nodes that dominates u is non-empty, then all of the nodes dominating u belong to the same part in the partition), the score of any node v is the sum of the values that v receives in each part in the partition.

Inspired by the score measure, Brink and Borm [16] introduced the *score game* of a digraph D, which is a cooperative game with the set V of players and the characteristic function—$\nu_D^{\mathcal{BB}}$—that generalises the score measure as follows:

$$\nu_D^{\mathcal{BB}}(C) = \left|E_D^{\mathcal{BB}}(C)\right|,$$

where $E_D^{\mathcal{BB}}(C) = \bigcup_{v \in C} E_D(v)$ (note that $E_D^{\mathcal{BB}}(C)$ may or may not contain members of C). In a sense, this is a version of group degree centrality for digraphs, where the out-degree of a coalition determines its score. Note, however, that for a coalition of nodes C, whereas group out-degree centrality does not usually count the out-neighbours of the members of C that also belong to C towards its value, the score game does.

Finally, the β-measure can be defined as the Shapley value of the score game. Equivalently, the authors show that for every node v in D, we have:

$$\beta_v(D) = SV_v(\nu_D^{\mathcal{BB}}) = \sum_{u \in E_D(v)} \frac{1}{|E_D^{-1}(u)|}.$$

Intuitively, the measure indicates that a reward of 1 is given for dominating any node. If a node, v, is dominated by multiple nodes, then this reward is equally distributed among all of the nodes that dominate it. Brink and Gilles [17] axiomatically characterize their β-measure by the properties: dummy-node, symmetry, additivity over independent partitions, and dominance normalisation. Whereas we are familiar with the first four axioms, dominance normalisation is defined as follows:

- *Dominance normalisation*: For every digraph D, the sum of the scores of all players is equal to the number of players that are each dominated by at least one node (i.e., have an in-degree that is not equal to 0).

As a criticism of this axiomatisation, we note that it axiomatises a dominance relation through the use of axioms that inherently account for dominance. In this, the axiomatisation adds little new insight into understanding the measure. To compute the β-measure of a node v, first the sets $E_D^{\mathcal{BB}}(v)$ and $|E_D^{-1}(u)|$ must

be computed for any node u dominated by v. This can be done in $O(|V| + |E|)$ time. Brink and Gilles [17] later generalise the score measure and β-measure for weighted digraphs, however they do not show any link between these generalised measures and cooperative game-theory.

3.3.2 The Top-k Nodes Problem and the Sphere of Influence

In this section, we review various versions of Shapley value degree and closeness centrality. The one thing that these centrality measures have in common is that they were proposed in order to model the sphere of influence of nodes. Following Brink and Borm [16], the second game-theoretic centrality based on degree was proposed by Ramasuri and Narahari [60]. The authors consider a cooperative game where the characteristic function is defined by the group degree centrality of each coalition. In other words, the value of a coalition of nodes is equal to the number of neighbours of this coalition. Next, the authors propose to use the Shapley value of this game as a centrality measure. Michalak et al. [29] follow by introducing a slightly different variant of a cooperative game based on degree centrality. In particular, the value of a coalition is equal to the size of its "fringe" set, which is defined as the set of its members and neighbours. Formally, the "fringe" of a subset $C \subseteq V$ is defined as follows:

$$\mathit{fringe}(C) = \left\{ v \in V : \Big(v \in C \Big) \text{ or } \Big(\exists u \in C : (u, v) \in E \Big) \right\}.$$

Building upon this, the authors define the following group centrality measure

$$g_1(C) = |\text{fringe}(C)|. \tag{3.16}$$

We see that this approach is very similar to the one introduced in the previous section. Just like [16] and Ramasuri and Narahari [60], Michalak et al. [29] use the Shapley value of this game as a centrality measure. In terms of computation, whereas Ramasuri and Narahari [60] approximated their centrality measure using

Monte Carlo techniques, Michalak et al. [29] developed polynomial algorithms for the computation of the Shapley value of any node given the game g_1. Michalak et al. [29] proposed four additional centrality measures, which correspond to the Shapley values of nodes in the following games:

- $\mathbf{g_2^{(k)}}$: This game is inspired by the general-threshold model introduced by Kempe, Kleinberg, and Tardos [63]. It is parameterized by a threshold value—k—and the general idea is that the value of a coalition is equal to the number of players that are either in C or have at least k neighbors in C. In a sense, whereas g_1 assumes that controlling any neighbour of a node, v, is sufficient to control v, $g_2^{(k)}$ assumes that at least k neighbours of v must be controlled in order to influence it. Formally, $g_2^{(k)}(G)(C) = |\{v \in V : (v \in C) \text{ or } (|E(v) \cap C| \geq k)\}|$.

- $\mathbf{g_3}$: Unlike the previous games, g_3 is defined over weighted graphs. Under this game, the value of a coalition, C, is equal to the number of nodes that are within a certain "cutoff distance" from C. Formally, $g_3(G)(C) = |\{v \in V : dist(v, C) \leq d_{cutoff})\}|$. Intuitively, the cutoff distance can be interpreted as the "radius" within which any node can influence other nodes. We note that g_1 is a special case of g_3, where $d_{cutoff} = 1$ and the weight of each edge is 1.

- $\mathbf{g_4}$: According to this game, the value of a coalition C is the sum over all nodes not belonging to C of a positive-valued, non-increasing function, $f : \mathbb{R} \to \mathbb{R}$ of the distances from C to those nodes. In other words, this is exactly the generalised form of group closeness, $\psi_f^{CL}(G)(S)$, discussed in Chapter 2. We note that g_3 is a special case of g_4 where $f(dist(v, C)) = 1$ if $dist(v, C) \geq d_{cutoff}$, and $f(dist(v, C)) = 0$ otherwise.

- $\mathbf{g_5}$: This game is also defined over weighted graphs, but unlike g_3 and g_4, the weight of an edge is not interpreted as its length, but rather its *power of influence*. The influence of a coalition, C, on a node, v, is defined as the sum of all influences induced upon v by the members of C. Now, for every

Game	Complexity	Accuracy
g_1	$O(\|V\| + \|E\|)$	exact
$g_2^{(k)}$	$O(\|V\| + \|E\|)$	exact
g_3	$O(\|V\|\|E\| + \|V\|^2 \log\|V\|)$	exact
g_4	$O(\|V\|\|E\| + \|V\|^2 \log\|V\|)$	exact
g_5	$O(\|V\|\|E\|)$	approx.

Table 3.2: Games considered by Michalak et al. [29] and their computational results.

node v, let us introduce a threshold, $W_{cutoff}(v)$, that is necessary to influence it. The value of a coalition, C, is then equal to the number of nodes that are either in C or are influenced by C. Formally, $\nu_5(C) = |\{v \in V : (v \in C) \text{ or } (\sum_{u \in C} \omega(u, v) \geq W_{cutoff}(v))\}|$, where $\omega(u, v)$ is the weight of the edge between u and v. Note that $g_2^{(k)}$ is a special case of g_5 where $W_{cutoff}(v) = k$ for all $v \in V$ and $\omega(u, v) = 1$ for all $(u, v) \in E$.

Michalak et al. [29] proposed exact and efficient algorithms for computing the Shapley value-based centrality measures based on g_1 to g_4. As for the Shapley value of g_5, the authors propose an approximation algorithm. A summary of the computational complexity of their algorithms for computing the Shapley value of the five games is presented in Table 3.2. Additionally, the authors evaluate the effectiveness of their algorithms on two real-life networks: an infrastructure network that represents the topology of the Western States Power Grid and a collaboration network from the field of astrophysics.

Ramasuri and Narahari [64] considered another characteristic function for the application of information diffusion. In order to express the fact that information has reached a node, we will say that a node has been *activated*. Once a node has been activated, it can share this information with its neighbors. These types of situations can appear in ad campaigns, social networks or politics. Within this context, Ramasuri and Narahari [64] are concerned with determining which nodes are the most important in disseminating information.

To model dissemination of information, Ramasuri and Narahari [64] base their characteristic function on the number of activated nodes in the Linear Threshold model. Next, the Shapley value is used as a measure of centrality. The Linear Threshold model consists of a weighted graph with active and/or inactive nodes, each of which has a threshold value. Nodes are activated in iterative fashion by their neighbors provided that the sum of the weights of an inactive node's neighbors surpasses its threshold value. The value of a coalition according to the characteristic function is then defined to be the number of active nodes in a graph after information has been disseminated in so many rounds, such that another iteration would yield no more active nodes. Ramasuri and Narahari [64] use Monte Carlo approximations in order to compute this centrality.

Matejczyk, Adamczewski, and Michalak [65] study the appropriateness of the Shapley value to the influence maximisation problem. They propose two new game-theoretic algorithms for this problem: the discount Shapley value algorithm and a refinement of the Local Directed Acyclic Graph (LDAG) algorithm. They conclude that the greedy LDAG algorithm is currently the best solution, however their Shapley value LDAG algorithm works almost as well and can be implemented in a parallel map-reduce fashion. Moreover, the Shapley value LDAG algorithm produces a larger cover (i.e., the set of nodes that is influenced) than any other previously studied game-theoretic solution.

We have now surveyed various models of domination and information diffusion and game-theoretic centrality measures that account for it. In doing so, we introduced three variations of Shapley value degree centrality [16, 29, 60] and a generalised Shapley value closeness centrality (based on g_4). The natural next step is to apply the Shapley value to group betweenness centrality, which presents a useful measure of network flow. We address this in the next section.

3.3.3 The Shapley Value Betweenness Centrality due to Szczepański, Michalak, and Rahwan [22]

In the previous section we introduced game-theoretic network centrality measures that were developed by applying the Shapley value to variations of both group degree centrality and group closeness centrality. In this section, we introduce the Shapley value betweenness centrality, which was proposed by Szczepański, Michalak, and Rahwan [22, 25].

Formally, the centrality is defined as the Shapley value for the cooperative game defined by group betweenness centrality, ν^B, which we defined in the previous chapter. Szczepański, Michalak, and Rahwan [22] show the centrality can be reformulated as follows:

$$SV_v(\nu_B) = \sum_{s \neq v} \sum_{t \neq v} \left(\frac{\sigma_{s,t}(v)}{\sigma_{s,t} dist(s,t)} + \frac{2 - dist(s,v)}{2 dist(s,v)} \right), \tag{3.17}$$

where $\sigma_{s,t}$ is the number of shortest paths in the graph G between the nodes s and t, and $\sigma_{s,t}(v)$ is the number of those paths that contain v. The authors show that this formulation naturally leads to a polynomial algorithm for the computation of their centrality measures.

Example 20 *Consider Figure 3.4 (a) and (b). Some of the centrality measures we have seen thus far would place most importance on either the outer nodes (v_3, v_5, v_7 and v_9) or the inner node (v_1). Both the standard betweenness centrality and its Shapley value extension, however, place most emphasis on the intermediary nodes—v_2, v_4, v_6, v_8. In fact, the central and outer nodes in the networks all have centrality* 0 *according to standard and Shapley value betweenness centrality, since none of them are intermediary nodes in any paths. Consider also the Banzhaf path-based centrality due Grofman and Owen [18], which considers a coalition "winning" when it forms a simple path, and focuses on players that have "swing" votes (i.e., whose removal makes it impossible for the two ends of the path to communicate). Since all*

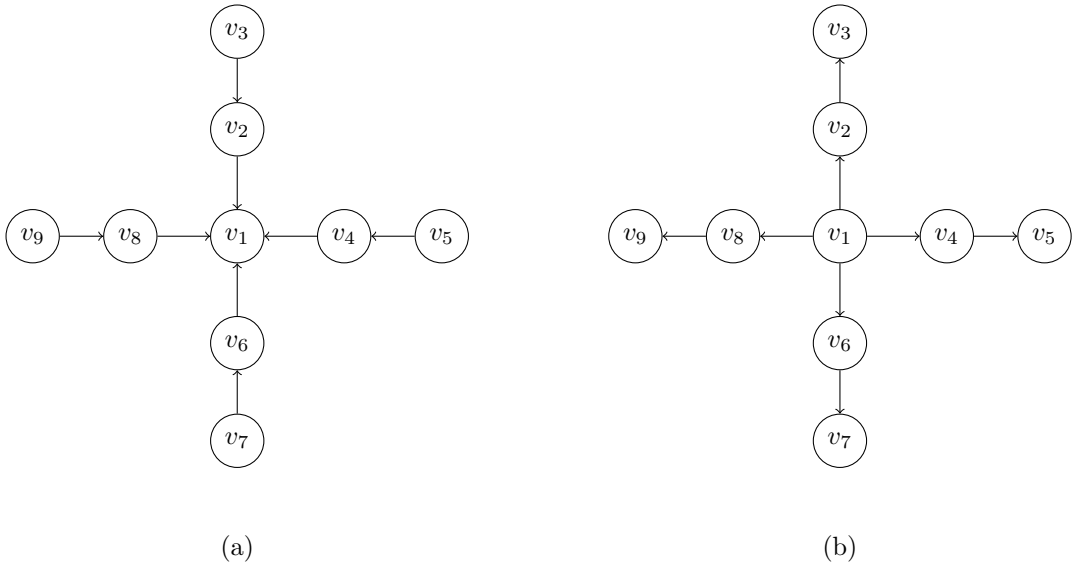

Figure 3.4: Two sample networks showing one of the difficulties with node centrality in digraphs. Depending on the application, any nodes could be considered as "central."

the possible paths in Figure 3.4 (a) and (b) are shortest paths, the Banzhaf path-based centrality [18] will result in the exact same ranking as betweenness centrality (assuming that the beginning and end nodes of a path are not allowed to have swing votes).

In the next section, we discuss a generalisation of the centrality measures from this section that considers a more complex setting, where nodes belong to communities.

3.3.4 The Owen value-based Centrality due to Szczepański, Michalak, and Wooldridge [31]

In this section we discuss a recent paper by Szczepański, Michalak, and Wooldridge [31], who developed the first measure of centrality that takes into account the *community structure* of the underlying network. The measure is based on a generalisation of the Shapley value known as *the Owen value*—a well-known solution concept from cooperative game theory that focuses on games with *a priori*-given unions (i.e., cooperative arrangements) of players. Moreover, the authors develop a class of solution concepts that they call coalitional semivalues, which are a

generalisation of the Owen value and semivalues. We refer the reader to Chapter 2, where we introduced these concepts.

Szczepański, Michalak, and Wooldridge [31] propose to use coalitional semivalues in combination with group degree centrality as a measure of centrality in graphs with a community structure. Much like in the general Shapley-value approach used by Ramasuri and Narahari [60] and Michalak et al. [29], coalitional semivalues can be used to modify classical centrality measures to account for synergies. The authors provide a polynomial algorithm to compute their measure.

The main advantage of this approach is that it takes into account the additional information afforded by the community structure in determining the centrality of nodes. The authors argue that the community to which a node belongs should also impact its centrality. That is, if a node's community is weak, then this should impact the centrality of the node negatively, and if it is strong, then it should impact it positively. In particular, the authors use their centrality measure to analyse a coauthorship network, where communities are identified by the various venues (journals, conferences, etc.) where the nodes (i.e., authors) have published. The main advantage of the Owen value approach is that it accounts for the fact that being a top contributor to weak venues does not necessarily make one a good researcher, whereas having fewer publications in top venues may be more advantageous. We discuss this further in Section 3.4.

The next section introduces the last measure from the literature—the VL Control Measure, as introduced by Hendrickx et al. [66]. Although this measure also focuses on synergies, it does so in a much different manner than the other measures in this section.

3.3.5 VL Control Measure for symmetric networks due to Hendrickx et al. [66]

In this section, we discuss the VL control measure (where VL stands for Vorobev Liapounov) due to Hendrickx et al. [66]. This measure—much like those surveyed

in Section 3.3.2—also has applications in influence propagation and information diffusion. To illustrate the measure, let us analyse a fictitious context considered by the authors. Imagine that some amount of a noxious substance is hidden in the nodes of a network. The object is to eliminate all of the substance. To do so, a *searcher* assigns x_v resources to each node v, with the constraint that $\sum_{v \in V} x_v = 1$. That is, the searcher has 1 unit of resource that can be freely divided up among the nodes. The probability of finding the substance at any given node v is $y_v = x_v + \sum_{(v,u) \in E} x_u$. That is, resources can search for the noxious substance in the node they are assigned to and in its neighbors. The probability of removing all of the substance, then, is $\prod_{v \in V} y_v$. Alternatively, we can think of allocating resources in order to spread information in a network, or any similar setting. Finally, the VL control measure of a given node, v, is the amount of resources that needs to be placed at v according to the strategy that maximises the probability of removing all of the substance. In other words, the centrality of nodes is the solution to the maximisation problem $\max(\prod_{v \in v} y_v)$ with the constraints $y_v = x_v + \sum_{(v,u) \in E} x_u$ for all v, $\sum_{v \in V} x_v = 1$ and $\forall_{v \in v} x_v \geq 0$. By placing resources to appropriate nodes we want to maximise the probability that all of the substance will be removed, while staying within the restrictions of the problem (e.g., not using more resources than we have at our disposal). Note that this solution need not be unique.

Example 21 *Consider the nodes in Figure 3.3. First, let us think how we can raise the probability of removing all of the substance up from* 0*. Putting any amount of resources at any one node will not suffice to achieve this. Clearly, then, resources must be placed in at least* 2 *nodes. Moreover, placing any resources at* v_1 *or* v_5 *makes no sense, since a better effect can be achieved by placing the resources in nodes* v_2 *or* v_3*, respectively (since they additionally cover* v_4*). The answer we need is the solution to the maximisation problem* $r_2 * (r_2 + r_3 + r_4) * (r_2 + r_3 + r_4) * (r_2 + r_3 + r_4) * r_3$ *subject to* $\sum_{i=1}^{5} r_i = 1$*, where* r_i *is the amount of resources placed in node* v_i *(we have already informally established that* $r_1 = 0$ *and* $r_5 = 0$*). Again, we can correctly surmise that no resources should be placed in* v_4*, since placing them in* v_2 *or* v_3

would achieve the same effect while also protecting v_1 and v_5. The answer to the problem, then, is placing half of the resource in v_2, and the other half in v_3, yielding a centrality ranking vector of $0, 0.5, 0.5, 0, 0$.

At first glance, it does not look like this measure has anything to do with cooperative game theory. However, the centrality of each node according to this measure turns out to be a *proper Shapley value* of the game $\nu(S) = |\{v \in S : E(v) \subseteq S\}|$, i.e., the number of nodes in a coalition that do not have connections to nodes outside of it. The Harsanyi dividends for this game are easily expressed as follows: $\Delta_\nu(S) = |\{v \in S : E(v) = S\}|$. To continue, we will introduce the *weighted Shapley value*, which will lead us to the definition of the proper Shapley value.

The Weighted and Proper Shapley Values: Harsanyi Dividends can be used to develop a payoff division scheme, where the value is not shared equally among players. One such division scheme is called the weighted Shapley value. In this payoff division scheme, it is possible to give more importance to certain players (i.e., to disregard the Symmetry axiom). The idea is that a weight is assigned to every player through a weight vector $\omega = \{\omega_1, \omega_2, \ldots, \omega_i, \ldots, \omega_n\}$, where ω_i is the weight of player i.[5] The weighted Shapley value with weight vector ω is then:

$$SV_i^\omega(\nu) = \sum_{C \in \{C : C \subseteq I \text{ and } i \in C\}} \frac{\omega_i}{\sum_{j \in C} \omega_j} \Delta_\nu(C). \tag{3.18}$$

Whenever for all players $i \in I$ the case is that $SV_i^\omega(\nu) = \omega_i$, then $SV_i^\omega(\nu)$ is referred to as a *Proper Shapley value* for ν.

Example 22 *Continuing our running example that was started in Example 1, let us consider the situation that p_1 is a manager and has more responsibility than p_2 or p_3 within the company. Let us quantify this by giving p_2 and p_3 weight* 0.3*, and p_1 weight*

[5] Note that the weights do not need to sum up to 1, since the weighted Shapley value considers the relative power of players in every coalition in accordance with the weights. For example, for the coalition $C = \{2, 3, 5\}$, player 2 would be assigned $\frac{\omega_2}{\omega_2+\omega_3+\omega_5}$ power, player 3 would be assigned $\frac{\omega_3}{\omega_2+\omega_3+\omega_5}$ power and player 5 would be assigned $\frac{\omega_5}{\omega_2+\omega_3+\omega_5}$ power.

0.4. *Formally,* $\omega_2 = \omega_3 = 0.3$ *and* $\omega_1 = 0.4$. *First, let us compute the Harsanyi Dividends. We have* $\Delta(\{1\}) = \Delta(\{2\}) = 200 = 200$, $\Delta(\{1,3\}) = -200 + 200 = 0$, $\Delta(\{1,2\}) = -200 - 200 + 300 = -100$, $\Delta(\{1,2,3\}) = +200 + 200 - 200 - 200 - 300 + 300 = 0$. *If we consider the weighted Shapley values, then* p_2 *receives* $200 - \frac{0.3}{0.7} \times 50$ *payoff, whereas* p_1 *receives* $200 - \frac{0.4}{0.7} \times 50$ *payoff. In this situation—counter-intuitively—*p_1 *receives less payoff despite having a higher weight. This is due to the fact that weights are only used to calculate relative power within coalitions (this is also why these weights do not need to sum up to* 1*).*

When p_2 *is working alone, she/he can make* 200 *monetary units. Working with* p_1*, however, results in negative synergy, i.e., if we consider the unique fair division scheme that is the Shapley value then she/he now only makes* 150 *monetary units (since together,* p_1 *and* p_2 *are less productive than the sum of their individual achievements). By raising* p_1*'s weight with respect to* p_2*, we are making* p_1 *more responsible for their joint work. In this case, we are making* p_1 *more responsible for the negative synergy that is achieved. As a leader, it is* p_1*'s responsibility to coordinate the team and the synergies that are achieved, which is why she/he is punished more than* p_2 *for this negative synergy.*

But what happens in the case of positive synergy? Let us consider modifying the characteristic function such that $\nu(\{1,2\} = \nu(\{1,2,3\}) = 500$. *We now have positive synergy between* p_1 *and* p_2*, since they both achieve* 100 *more monetary units than the sum of their individual work. Their Shapley values are now* 250 *each, but* p_1*'s weighted Shapley value (using the same weights as before) is now* $200 + \frac{0.4}{0.7} \times 50$*, whereas* p_2*'s is* $200 + \frac{0.3}{0.7} \times 50$. *Now,* p_1 *is more responsible for the positive synergy that was achieved with* p_2.

Equivalence of the concepts: The equivalence of the maximisation problem to the proper Shapley value was showed by Vorob'ev and Liapounov [67]. In particular, the authors showed that the solution to the following maximisation problem is a proper Shapley value of ν:

$$\max_{x \in \mathcal{S}^I} \prod_{S \subseteq I} (\sum_{j \in S} x_j)^{\Delta_\nu(S)},$$

where $\mathcal{S}^I$ is the unit simplex and x_j is the j'th element of the vector x. This can be rewritten as:

$$\begin{aligned}&\max \prod_{S\subseteq I}(\sum_{j\in S} x_j)^{\Delta_\nu(S)},\\ &\sum_{i=1}^{n} x_i \leq 1,\\ &\forall_{i\in I} x_i \geq 0.\end{aligned}$$

For the game ν this can be applied as follows:

$$\max_{x\in\mathcal{S}^N} \prod_{S\subseteq N}(\sum_{j\in S} x_j)^{|\{i\in S:E(i)\subseteq S\}|},$$

which is equivalent to the maximisation problem:

$$\max_{x\in\mathcal{S}^I} \prod_{i\in I} \sum_{j\in E(i)} x_j,$$

which is the VL control measure that we introduced at the beginning of this section.

Unfortunately, Bellare and Rogaway [68] showed that there is no polynomial algorithm for non-linear programming even for a quadratic polynomial unless P = NP. Moreover, there is no polynomial time approximation for this problem (even with a very poor guarantee) unless P = NP. However, even though the VL control measure can be reduced to an instance of a nonlinear programming problem, the actual computational complexity of the VL control measure has yet to be extensively studied.

Example 23 *Consider now Figure 3.5. We reach a fundamental question for centrality in directed networks: in which of these networks (all of which are "stars") is the center node—v_1—central? The answer is usually straightforward in the undirected graph (a). Both Pozo et al. [48] and Hendrickx et al. [66] (with a strictly convex game), for example, agree that v_1 should be the most central node. However, if we assume that v_1 is the most central in digraph (b) (as it is according to Hendrickx et al. [66]), then we require central nodes to be able to travel, or send information to other nodes in the network. For this same reason, v_1 cannot be the most central*

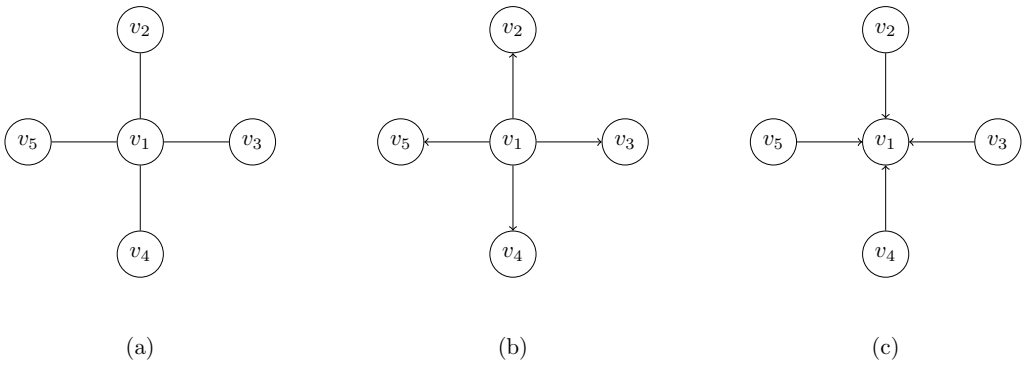

Figure 3.5: Three sample networks showing one of the difficulties with node centrality in digraphs. Most centrality measures agree that v_1 is the most central node in network (a). However, if a centrality measure admits that node v_1 is central in network (b), then it should not be central in (c) (and vice versa).

node in digraph (c). On the other hand, if we assume v_1 to be the most central node in digraph (c) (as it would be according to Pozo et al. [48]), then our application requires central nodes to be reachable from other nodes in the network. Clearly, then, v_1 cannot be the most central node in digraph (b), since no other node can reach it.

3.4 Applications

One weakness of the literature on game-theoretic network centrality is that it has not been applied very extensively. This is perhaps due to a lack of algorithms proposed and the computational difficulties inherent in the field. We list below a few of the applications for which game-theoretic network centrality has been used.

Ramasuri and Narahari [60] apply their Shapley value-based degree centrality to a co-authorship network of 8361 high-energy Physics researchers. They show that their centrality for the top-k nodes problem achieved better results than a well-known algorithm from the literature—the maximum degree heuristic.

Skibski et al. [51] and Michalak et al. [69] use the Myerson value in order to identify key nodes in terrorist networks. The authors argue that this helps understand the hierarchy of such organisations and can facilitate efficient deployment of investigation resources.

Szczepański, Michalak, and Wooldridge [31] applied their Owen value degree centrality to a citation network. The unique feature of this approach is that it is able to take into account the importance of the community to which a node belongs in evaluating its centrality. Unfortunately, however, the method cannot be applied when nodes belong to more than one community (i.e., communities overlap). The citation network that was studied consists of 2 084 055 publications and 2 244 018 citation relationships. A total of 22 954 unique communities representing journals, conference proceedings or single book titles were identified by the authors by using basic text mining techniques. The authors found that the Owen value-based degree centrality has a significant advantage over other centralities in ranking the authors. This is shown through a comparison to two other centrality measures—weighted degree centrality and Shapley value-based degree centrality. The authors show that some nodes are significantly less powerful according to the Owen value centrality, since the communities to which they belong are weak. They argued that the Owen value-based centrality is able to account for the fact that being a strong author in only weak journals does not make one a strong author in general.

Szczepański, Michalak, and Rahwan [25] proposed to use semivalue betweenness centrality in order to protect networks. The idea is that random node failures may close lines of communication in a network, and semivalues can help rank nodes in terms of how important it is to protect them. They find that their methods perform favourably when compared to a number of other centrality measures.

Matejczyk, Adamczewski, and Michalak [65] use the Shapley value and Banzhaf index on games with networks to solve the top-k nodes problem. They find that their method performs comparatively (although a little worse) to the state-of-the-art.

Keinan et al. [26] use the Shapley value in order to rank how important sections of the brain are for certain cognitive functions. Given the complex interactions between sections of the brain, they argue that the Shapley value can account for the synergies achieved by them.

Skibski et al. [70] proposed the first game-theoretic centrality measure that is based on the extension of the Shapley value to games with externalities [71] and advocated its usefulness to the analysis of the well-known board game of Diplomacy.

Michalak et al. [72] argue that game-theoretic centralities can be considered as novel measures of social capital that address two key deficiencies of standard measures. Firstly, while the standard measures focus separately on various types of social capital (e.g., on either individual or group social capital), game-theoretic centralities (e.g., those based on the Owen value) can be used to measure interactions between such different types of social capital. Secondly, network-based standard social capital measures focus solely on the network topology; hence, they do not take into account various additional information about the nodes, groups and connections between them. In contrast, such additional information can be embodied in the characteristic function upon which game-theoretic centralities are built upon.

Game-theoretic centralities become power indices when applied to weighted voting games restricted by a graph. For instance, the Myerson value is the extension of the Shapley-Shubik index to graph-restricted weighted voting games. It is argued that restricted weighted voting games better model reality as we cannot expect that cooperation of any subset of political parties is always feasible [73]. For instance, it is hard to expect that a new entrant to the German Bundestag—the AfD party—will be treated by CDU/CSU in the same way as SPD. Unfortunately, power indices are computationally challenging [27, 74]. Hence, various authors have recently proposed dedicated algorithms for computing power indices in special cases of graph-restricted weighted games [73, 75–77].

Narayanam et al. [78] introduce a game-theoretic centrality measure based on the Shapley value in which the characteristic function is especially designed to promote "gatekeepers"—nodes that play an important role in connecting their communities with the remainder of the network. They analyse two application of their centrality measure: community detection and limiting the spread of misinformation over the network.

Szczepański et al. [32] use a class of solution concepts for pairs of players in cooperative games that is based on semivalues—semivalue interaction indices (introduced in Chapter 2). They pair this class of solution concepts with group k-degree centrality (where the value of a coalition, C, is the number of nodes at least k nodes away from it) in order to analyse the similarity of nodes in networks. This has two main applications: link prediction and community detection. In a missing information scenario, where random edges are removed from a network, the authors show that their approach is competitive when compared with the state-of-the-art methods in the literature. Szczepanski et al. [30] continued this research and develop a measure of node similarity for players that belong to different communities. They base their measure on coalitional semivalues and refer to it as coalitional semivalue interaction indices, which we also introduced in Chapter 2. The authors also pair this solution concept with group k-degree centrality and find that it is well-suited to predicting whether edges exist between nodes that belong to different communities (i.e., the inter-links prediction problem). This is more complicated than the standard link-prediction problem, since there are usually very few edges from nodes in one community to those in any other community, meaning there is less information to base the similarity measure on. We expand on this research in Chapter 6.

3.5 Conclusions

We have organised the literature on game-theoretic network centrality measures into two categories and presented the relevant centrality measures and included a short discussion of the computational complexity of each measure. We note that whereas the measures based on connectivity are generally intractable, those that are present in the literature that are not based on connectivity, are often computable in polynomial time. This is a surprising result, given the complexity of the solution concepts that these measures have adopted from cooperative game theory. We also mention the work due to Skibski, Michalak, and Rahwan [79], who organised game-theoretic centrality measures according to their axiomatic properties.

In the following three chapters we present our contribution to the literature. We note that our work falls into the second category of game-theoretic centrality measures—those that are not based on connectivity. In Chapter 4, we present a general analysis of semivalues for game-theoretic network centrality. In Chapter 5 we introduce a new class of solution concepts for cooperative games with overlapping community structures and combine them with general group closeness centrality to develop a robust and general centrality measure for graphs where the communities of nodes can overlap. In Chapter 6 we continue this research and combine generalised closeness with the interaction index in order to develop a measure of node similarity for link prediction. We develop polynomial algorithms for the computation of our measure and find that it generally performs better than the state-of-the-art methods in the literature.

4

Computation of Semivalues for Game-Theoretic Network Centrality

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4.1 Introduction

The aim of this chapter is to provide a broad and systematic study of which group centrality measures allow for polynomial computation of semivalues. In doing so, we unify the approach to computing many of the game-theoretic network centrality measures developed to date, and many others. We hope that this will provide researchers with a convenient method for building and computing new, readily applicable centrality measures. This is especially significant given that there is no wide consensus as to which game-theoretic centrality measures are appropriate in which circumstances.

To this end, we develop a framework for defining such measures. We show that, given a formulation of some group centrality measure within our framework, all that is required to compute its semivalue is the computation of four specific values introduced within our framework. Given this, we present an algorithm that computes any semivalue of the group centrality measure in polynomial time. In more detail, the contributions of this chapter can be summarised as follows:

- Whereas the focus of most previous work has been the computation of a single solution concept for a single group centrality, we present a generic framework for defining group centralities. Here we rely on the definition of a game-theoretic network centrality as a pair of a group centrality measure and a cooperative solution concept and exploit the structural relationships that exist between the two. If the group centrality is defined within our framework, using the conventions that we introduce, the general semivalue algorithm developed in this chapter can be readily used in a plug-and-play fashion;

- By using our framework, it is possible to study whether any semivalue of a group centrality measure can be computed in polynomial time. We show that this also facilitates the complexity analysis and the development of algorithms for classes of group centrality measures (e.g., parametrised measures) at once, rather than analysing every measure individually;

- We leverage our results to analyse the computational complexity of classes of game-theoretic measures based on the semivalue of various notions of group degree, closeness, and betweenness. In addition, we present the first game-theoretic centrality measures, along with their polynomial-time algorithms, for weighted and normalised degree centrality, impact factor centrality, distance-scaled and normalised betweenness centrality, and normalised closeness centrality. Although similar game-theoretic centrality measures have been proposed, the particular variation of degree, betweenness, or closeness appears to have been chosen arbitrarily. Our framework facilitates the systematic study and principled comparison of many game-theoretic measures based on the many variations of these types of measures;

- We show that our framework can also be used to study game-theoretic centrality measures based on specific semivalues. Moreover, the resulting algorithms may have an even lower complexity than our general semivalue algorithm. We demonstrate the merits of this technique by developing the first polynomial-time algorithms that compute the game-theoretic centrality measures based on the Banzhaf index of power of various group degree, betweenness and closeness centrality measures.

In what follows, we present our framework for developing game-theoretic network centrality measures based on semivalues, and in Section 4.3 we show when these centrality measures can be computed in polynomial time and present an algorithm for doing so. The subsequent section presents a more focused analysis and algorithms for computing semivalues in particular of group parameterised degree, betweenness and closeness centralities. Section 4.5 presents a specific analysis for the Banzhaf index (as opposed to a general semivalue one) with faster (yet less general) algorithms when compared to those from the previous section. Finally, we present our conclusions and avenues for future work.

4.2 A Generic Framework for Game-Theoretic Network Centrality

In this section, we will formally introduce our generic framework for modeling group centrality measures (i.e., characteristic functions on graphs). Before we do so however, let us define a game-theoretic centrality measure for the purposes of this chapter:

Definition 2 (Game-theoretic centrality measure) *For the purposes of this thesis, we define a game-theoretic centrality measure as a pair (ψ, ϕ) consisting of a group centrality ψ and a cooperative solution concept ϕ.*

Example 24 *Let us consider the game-theoretic centrality measure $(\psi^B, \phi^{Shapley})$. For a specific graph $G = (V, E)$, the importance of each node $u \in V$ according to this measure is evaluated by the Shapley value of the game $\psi^B(G)$, i.e., $\phi_u^{Shapley}(\psi^B(G))$. Since we started off with a group centrality measure called group betweenness centrality and the Shapley value to it, we call the resulting centrality measure Shapley betweenness centrality.*

We will refer to the class of group centrality measures (which we will later combine with semivalues in order to form game-theoretic centrality measures) that can be expressed by using our framework as $\mathscr{M}$, which is an intermediary step in our ultimate goal of finding within $\mathscr{M}$ those group centralities that allow for polynomial computation of semivalues. $\mathscr{M}$ will allow us to abstract away the various types of considerations that make nodes central (i.e., shortest paths, neighbours, or closeness to other nodes).[1] The remainder of this section will be dedicated to introducing our framework, which consists of two main components: the first we call *items*, and the second *relations*. In the two following subsections we will introduce both components, and in the third we will show how they are combined to model characteristic functions (i.e., group centrality measures) on graphs.

[1] Next, in Section 4.3 we are able to put just a few restrictions on our abstract framework such that polynomial computation is guaranteed, resulting in a subclass of $\mathscr{M}$ that can be computed in polynomial time.

4.2.1 Items

Items refer to the graph-theoretic entities that influence the centrality of groups of nodes given a particular group centrality measure. For instance, for group betweenness centrality, the items that influence centrality are the shortest paths in the network. In the case of group degree centrality, items are simply nodes, since centrality is derived from having neighbours. If a node neighours a group, then it contributes to the group's centrality. In the case of group closeness centrality, nodes impact centrality like in degree centrality, but the magnitude of their impact on the ranking depends on their distance from the group.

Formally, we assume that the characteristic function ν_G (as given by some group centrality measure, i.e. $\nu_G = \psi(G)$) can be computed by counting these *items*. We will typically denote a single item by ϑ. Each item ϑ has its own real value, denoted by $f(\vartheta) \in \mathbb{R}$, and it is this value that impacts the value of nodes and coalitions. Given a group centrality measure, the set of all items in a graph G is denoted by Θ_G. We will also partition Θ_G into pairwise disjoint *groups* $\Theta_1, \Theta_2, \ldots, \Theta_l, \ldots, \Theta_{h(G)}$ of items according to their value. That is, items from the same group should have the same value, whereas the value of any two items from different groups should be different. We will denote the size of the partition, which depends on the graph in question and the group centrality measure under consideration, by $h(G)$. This value will be central to the complexity analysis of our algorithms. Later we will see how such partitions facilitate the computation of semivalues. In particular, even in the presence of an exponential number of items (such as in the case of betweenness), the size, $h(G)$, of such a partition may be polynomial.

4.2.2 Relations

Relations are the second component of our framework and control how items affect the value of nodes and coalitions (in effect, defining a characteristic function). In particular, we define three types of relations: positive, negative and neutral. In simplest terms, the value of a coalition will be the sum of the values of items that it

is positively related to. Deciding which items are positively related to the coalition, however, depends on the individual relations of the nodes to these items. For example, in the context of group degree centrality, nodes are positively related to their neighbours, negatively related to themselves and neutrally related to all other nodes in the network. These three relations are pairwise mutually exclusive and otherwise independent of one another. We will later see how we can use them to determine the marginal contributions of nodes, and—in effect—compute semivalues. For individual nodes, the relations are as follows:

N: is the *neutral* relation. $N(u, \vartheta)$ simply indicates that the node u and item ϑ are unrelated, and u does not impact the value of any coalition through ϑ.

R: is the *positive* relation. $R(u, \vartheta)$ indicates that u is positively related to ϑ, meaning that u *contributes* the value $f(\vartheta)$ when joining a coalition C as long as all nodes in C are neutrally related to ϑ.

$\widetilde{R}$: is the *negative* relation. $\widetilde{R}(u, \vartheta)$ indicates that u is negatively related to ϑ, meaning that u *subtracts* the value $f(\vartheta)$ when joining a coalition C *if* it has already been contributed by some node in C but not yet taken away by another node in C that is negatively related to ϑ.

Now that we have introduced relations between nodes and items, we can formally extend them to relations between groups of nodes/coalitions and items.

$$\begin{aligned}
\widetilde{R}(C, \vartheta) &\text{ if and only if } \exists_{v \in C} \widetilde{R}(v, \vartheta), \\
R(C, \vartheta) &\text{ if and only if } \exists_{v \in C} R(v, \vartheta) \text{ and } \neg \exists_{u \in C} \widetilde{R}(u, \vartheta), \\
N(C, \vartheta) &\text{ if and only if } \forall_{v \in C} N(v, \vartheta).
\end{aligned}$$

Observe that the relations as defined for coalitions are also pairwise mutually exclusive. Intuitively, this means that if $\widetilde{R}(C, \vartheta)$ holds, then some node belongs to C that precludes C from ever benefiting from the item ϑ, even when other nodes join the coalition. $R(C, \vartheta)$, on the other hand, means that $f(\vartheta)$ (the value

of ϑ) is counted towards the value of C. As mentioned previously, the value of a coalition is the sum of the values of the items to which it is positively related. In this case, if a node u such that $\widetilde{R}(u, \vartheta)$ ever joins C, then it will subtract $f(\vartheta)$ from the value of the coalition. Finally, $N(C, \vartheta)$ means that $f(\vartheta)$ can still be contributed by any node u for which $R(u, \vartheta)$ holds.

Example 25 *Consider again Figure 2.2, which was discussed in Example 10 from Chapter 2. In particular, we will be concerned with the characteristic function ν^B that represents the betweenness group centrality. This characteristic function/group centrality we can now model by choosing appropriate items, relations, and the function f. For each coalition C, we can then express $\nu^B(C)$ as a function of these. As the set of items we choose the set of shortest paths π_{st} between any two distinct nodes s and t. For each item, i.e., shortest path π_{st}, we set $f(\pi_{st}) = 1$. We say that node x is positively related to the shortest path π_{st} (or, in symbols, $\pi_{st}\ R\ x$) if π_{st} visits x and $x \neq s, t$. Moreover, $\pi_{st}\ \widetilde{R}\ x$ if $x = s$ or $x = t$, and $\pi_{st}\ N\ x$ if neither $\pi_{st}\ R\ x$ nor $\pi_{st}\ \widetilde{R}\ x$.*

In the example, we see that, for instance, node b is positively related to the paths

$$abc,\ cba,\ cbd,\ cbde,\ dbc,\ \text{and}\ edbc.$$

This reflects b's betweenness centrality of 6. Similarly, node d is positively related to

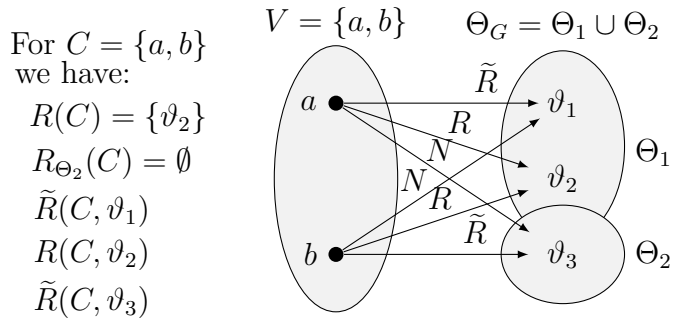
$$ade,\ bde,\ cbde,\ eda,\ edb,\ \text{and}\ edbc,$$

signifying that the betweenness centrality of d is likewise 6. As b and d are negatively related to a path if they occur as one of the endpoints, we find that the coalition $\{b, d\}$ is positively related to the paths

$$abc,\ ade,\ cba,\ cbde,\ eda,\ \text{and}\ edbc.$$

We thus find that $\nu^B(\{b, d\})$ equals

$$\sum_{\vartheta \in R(\{b,d\})} f(\vartheta) = f(abc) + f(ade) + f(cba) + f(cbde) + f(eda) + f(edbc) = 6.$$



$$R^{-1}(\vartheta_2) = \{\{a\}, \{b\}, \{a, b\}\}\, R^{-1}_{\#1}(\vartheta_2) = \{\{a\}, \{b\}\}$$
$$\widetilde{R}^{-1}(\vartheta_3) = \{\{b\}, \{a, b\}\}$$

Figure 4.1: An example of notation used to describe relations between nodes and items.

In the next section we will generalise this idea. Observe that node a is negatively related to abc, ade, cba, and eda. Hence, if a were to join $\{b, d\}$, we find that $\sum_{\vartheta \in R(\{a,b,d\})} f(\vartheta) = f(cbde) + f(edbc) = 2$, which neatly coincides with $\nu^B(\{a, b, d\})$.

For clarity, we will use the following notation for the relation R (and similarly for $\widetilde{R}$ and N). This notation will allow us to express the class $\mathscr{M}$ formally and concisely.

$$R(C) = \{\vartheta \in \Theta_G : R(C, \vartheta)\} \qquad R_{\Theta_l}(C) = \{\vartheta \in \Theta_l : R(C, \vartheta)\},$$
$$R^{-1}(\vartheta) = \{C \subseteq V : R(C, \vartheta)\} \qquad R^{-1}_{\#k}(\vartheta) = \{C \subseteq V : R(C, \vartheta) \text{ and } |C| = k\}.$$

Thus, $R(C)$ denotes the set of items positively related to the coalition C, $R_{\Theta_l}(C)$ the set of items in group Θ_l that is positively related to C, $R^{-1}(\vartheta)$ the set of coalitions to which item ϑ is positively related, and $R^{-1}_{\#k}(\vartheta)$ the set of coalitions of size k to which ϑ is positively related. Note that in the case of a singleton coalition we write $R(u)$ instead of $R(\{u\})$ to avoid cluttered notation.

4.2.3 Model of a Characteristic Function

We are now in a position to combine all of the elements of our framework and show how they can be used to model characteristic functions on graphs.

Definition 3 (Model of a characteristic function) *Given a graph $G = (V, E)$ and a characteristic function $\nu : 2^V \to \mathbb{R}$, a* model *of ν is a tuple*

$$M_G = \Big(\Theta_G, \{\Theta_1, \ldots, \Theta_{h(G)}\}, R, \widetilde{R}, N, f, g\Big),$$

where $\{\Theta_1, \ldots, \Theta_{h(G)}\}$ is a partition of a set Θ_G of items, R, $\widetilde{R}$ and N are the positive, negative and neutral association relations, respectively, and $f : \Theta_G \to \mathbb{R}$ and $g : \mathbb{N} \to \mathbb{R}$ are real-valued functions, such that for all coalitions $C \subseteq V$ the following property holds:

$(M1)$ $\nu(C) = g(|C|) \sum_{\vartheta \in R(C)} f(\vartheta)$.

Observe that the relation R depends on $\widetilde{R}$, which plays a role in determining whether a coalition can be positively related to an item. Moreover, any pair of relations from R, $\widetilde{R}$, and N can be used to determine the third, however we include all three for the sake of completeness and convenience. For the purposes of this chapter, we want to group similar items together using the partition of items, and so we focus on models for which a number of structural regularity constraints hold:

$(M2)$ $\forall_{1 \leq l \leq h(G)} \forall_{\vartheta_i, \vartheta_j \in \Theta_l}\ f(\vartheta_i) = f(\vartheta_j)$;

$(M3)$ $\forall_{1 \leq l \leq h(G)} \forall_{\vartheta_i, \vartheta_j \in \Theta_l}\ |N^{-1}_{\#k}(\vartheta_i)| = |N^{-1}_{\#k}(\vartheta_j)|$;

$(M4)$ $\forall_{1 \leq l \leq h(G)} \forall_{\vartheta_i, \vartheta_j \in \Theta_l}\ |R^{-1}_{\#k}(\vartheta_i)| = |R^{-1}_{\#k}(\vartheta_j)|$.

We note that in cases where centrality is based on degree, closeness and betweenness these properties are relatively straightforward to verify, which we show in Section 4.4. On the other hand, if these properties are not easy to verify, then this may indicate that a given centrality is difficult to compute. Given that these properties hold and $\vartheta \in \Theta_l$, we also let, with a slight abuse of notation:

$$f(\Theta_l) = f(\vartheta), \quad N^{-1}_{\#k}(\Theta_l) = N^{-1}_{\#k}(\vartheta), \quad R^{-1}_{\#k}(\Theta_l) = R^{-1}_{\#k}(\vartheta), \text{ and } \widetilde{R}^{-1}_{\#k}(\Theta_l) = \widetilde{R}^{-1}_{\#k}(\vartheta).$$

Property $(M1)$ states that the value of a coalition is the product of a function of its size and the sum of the contributions of its members over all positively related

items. Property $(M2)$ states that the value of every item in a part must be the same. Moreover, Properties $(M3)$, and $(M4)$ state that the set of coalitions of size k to which any item is related (by relation R, N, and consequently by $\widetilde{R}$) is the same for items in the same part of the partition.

Note that the properties $(M2)$, $(M3)$ and $(M4)$ do not limit the expressiveness of the model. Rather, they are introduced as a matter of convention in order to group similar items together in the partition of items. A partition that contains only singletons will always satisfy these properties. However, such a partition will often be prohibitively large. We will later see that graph structure from which the centrality of groups is derived imposes structure on the group centrality itself (thereby not being fully expressive) that—in some cases—reduces the computational complexity of game-theoretic solution concepts. In the following sections, our framework will allow us to study in a principled fashion how group centrality, solution concepts and classes of cooperative games that can be described by graphs interact to impact complexity.

First, however, we have to show how models of characteristic functions relate to group centrality measures, i.e., functions mapping graphs to characteristic functions. We do this by introducing the class $\mathscr{M}$ of group centrality measures:

Definition 4 (The class $\mathscr{M}$) *$\mathscr{M}$ is the class of all group centrality measures such that for all $\psi \in \mathscr{M}$, and for every graph G, there exists a model M_G of $\psi(G)$ such that M_G has the properties $(M1)$, $(M2)$, $(M3)$, and $(M4)$.*

Let us illustrate using an example how the various elements of a model interact in order to define a characteristic function.

Example 26 *Let us consider Figure 4.1. We want to find a model of the characteristic function ν_G, i.e., $M_G = (\Theta_1, \Theta_2, R, \widetilde{R}, f, g)$, such that it models $\nu_G(\{a\}) = 1, \nu_G(\{b\}) = 1, \nu_G(\{a,b\}) = 2$. We define $f(\vartheta_1) = f(\vartheta_2) = 1$, $f(\vartheta_3) = 2$ (satisfying $(M2)$) and $g(k) = k$. Property $(M1)$ holds, since $\nu_G(\{a\}) = \nu_G(\{b\}) =$*

$g(1)f(\vartheta_2)$ and $\nu_G(\{a,b\}) = g(2)f(\vartheta_2)$. However, property $(M3)$ is not satisfied since $|N_{\#1}^{-1}(\vartheta_1)| \neq |N_{\#1}^{-1}(\vartheta_2)|$. This can be fixed by defining three groups of singletons instead. In fact, a partition of singletons will always satisfy properties $(M2)$, $(M3)$ and $(M4)$. The motivation for introducing groups will become apparent in the next section, where we will show how restricting the number of groups (without restricting the number of items) will yield polynomial computation.

4.3 Computational Analysis

For any group centrality measure ψ and semivalue ϕ, we define the problem SEMIVALUE(ψ, ϕ) to compute the centrality of a node u according to the game-theoretic centrality measure (ψ, ϕ):

SEMIVALUE(ψ, ϕ)
Given: Graph $G = (V, E)$, node $u \in V$
Problem: Compute $\phi_u(\psi(G))$

In general, this problem is intractable [27, 28] and it is not immediately obvious what restrictions on the game-theoretic centrality measures would yield polynomial computation. Even if we restrict ourselves to the class of group centrality measures $\mathscr{M}$, there are still functions in this class for which SEMIVALUE(ψ, ϕ) is intractable (for example, this could happen if $h(G)$ is exponential in the size of V). However, we will define a subclass $\mathscr{M}^*$ of $\mathscr{M}$ such that SEMIVALUE(ψ, ϕ) can be computed in polynomial time for all $\psi \in \mathscr{M}^*$.

Definition 5 (The subclass $\mathscr{M}^*$ of $\mathscr{M}$) *$\mathscr{M}^*$ is the subclass of group centrality measures in $\mathscr{M}$ for which additionally the following properties hold:*

$(M1^*)$ $\exists_{n\in\mathbb{N}} \forall_G \exists_{M_G}$ *M_G models $\psi(G)$ and $h(G) \leq O(|V|^n)$;*

$(M2^*)$ $\exists_{n\in\mathbb{N}}$ $\forall_G$ *there exists a model $(\Theta_G, \{\Theta_1, \ldots, \Theta_{h(G)}\}, R, \widetilde{R}, f, g)$ of $\psi(G)$ such that the function that for each $u \in V$, k, and l returns the values $f(\Theta_l)$, $g(k)$, $|R_{\Theta_l}(u)|$, $|\widetilde{R}_{\Theta_l}(u)|$, $|N_{\#k}^{-1}(\Theta_l)|$, and $|R_{\#k}^{-1}(\Theta_l)|$ is computable in time $O(|V|^n)$.*

The first of these additional constraints—Property ($M1^*$)—puts a polynomial size restriction on the partition of items in the model of group centrality measure for every possible graph. In practice, this means that items cannot have more than a polynomial number of values, as this would prohibit polynomial computation. The second constraint—Property ($M2^*$)—reduces the overall computational problem to the computation of a number of values. It is necessary to compute the values $|R_{\Theta_l}(u)|$, $|\widetilde{R}_{\Theta_l}(u)|$, $|N^{-1}_{\#k}(\Theta_l)|$, and $|R^{-1}_{\#k}(\Theta_l)|$ in polynomial time, since this allows us to reduce exponential sums in the definition of semivalues into a polynomial number of multiplications. We show in Section 4.4 that these restrictions are sufficiently broad to capture all fundamental network centrality measures and their variants known in the literature. In particular, in Proposition 1 we prove that generalised group degree centrality belongs to $\mathscr{M}^*$, in Proposition 2 we prove that generalised group betwenness centrality belongs to $\mathscr{M}^*$, and in in Proposition 3 we prove that generalised group closeness centrality belongs to $\mathscr{M}^*$. Our key result is as follows:

Theorem 1 *For all $\psi \in \mathscr{M}^*$ and semivalues ϕ,* SEMIVALUE(ψ, ϕ) *is solvable in polynomial time.*

Proof: In order to solve SEMIVALUE(ψ, ϕ) in polynomial time, we will use equation (2.8) and show how to compute $\mathbb{E}_{C_k}[\mathrm{MC}(C_k, u)]$ in polynomial time for all k. To this end, we need to consider the contributions of the node u to C_k through the various items that u is in relation with (as defined by $R(u)$, $\widetilde{R}(u)$ and $N(u)$).

We will denote by $\mathrm{MC}(C, u, \vartheta)$ the marginal contribution of node u to C through the item ϑ (which we will define below). In effect, we have

$$\mathrm{MC}(C, u) = \sum_{\vartheta \in \Theta_G} \mathrm{MC}(C, u, \vartheta).$$

Two computational issues arise. First, we cannot immediately use this result, as it would require iteration over all coalitions. Second, iterating over a possibly exponential number of items within this sum is quite troubling. To address the first issue, we will denote by $\mathrm{MC}(k, u, \vartheta)$ the sum of contributions of u through ϑ

to all coalitions of size k. To address the second, for all groups Θ_l we will consider instead $\mathrm{MC}(k, u, \Theta_l) = \sum_{\vartheta \in \Theta_l} \mathrm{MC}(k, u, \vartheta)$. We get:

$$\mathbb{E}_{C_k}[\mathrm{MC}(C_k, u)] = \sum_{1 \leq l \leq h(G)} \mathbb{E}_{C_k}[\mathrm{MC}(C_k, u, \Theta_l)] = \sum_{1 \leq l \leq h(G)} \frac{\mathrm{MC}(k, u, \Theta_l)}{\binom{|V|-1}{k}}. \quad (4.1)$$

The remainder of the proof will focus on defining $\mathrm{MC}(C, u, \vartheta)$ and computing $\mathrm{MC}(k, u, \vartheta)$. Polynomial computation of $\mathrm{MC}(k, u, \Theta_l)$ will follow. A node u can contribute (possibly negative) value to a coalition C through $\vartheta \in \Theta_l$ in one of three mutually exclusive ways. These are:

[1] $R(u, \vartheta)$ and $N(C, \vartheta)$,

[2] $\widetilde{R}(u, \vartheta)$ and $R(C, \vartheta)$, and

[3] $R(u, \vartheta)$ or $N(u, \vartheta)$, and $R(C, \vartheta)$.

For all other cases $\mathrm{MC}(C, u, \vartheta) = 0$. Since the remainder of the proof hinges on the understanding of these cases, let us illustrate them with our running example from Figure 4.1.

Example 27 *Let $S = \{a\}$. We will compute the contribution of node b to coalition S. We have $MC(S, b, \vartheta_1) = 0$, since $\widetilde{R}(S, \vartheta_1)$ (meaning there is nothing to take away). Evidently, none of the cases [1], [2], [3] are satisfied. We also have $\widetilde{R}(b, \vartheta_3)$ and $N(S, \vartheta_3)$, which implies $MC(S, b, \vartheta_3) = 0$. A contribution can only be made through the item ϑ_2. We have: $R(b, \vartheta_2)$ and $R(S, \vartheta_2)$, which satisfies condition [3]. It is not immediately obvious why a contribution is made, since ϑ_2 is already positively related with S. However, it is necessary to keep in mind that the size of the coalition will change. Due to the function g, this means that the value added by item ϑ_2 will now be $g(k+1)f(\vartheta_2)$ instead of $g(k)f(\vartheta_2)$. We say, then, that b contributes the value $g(k+1)f(\vartheta_2) - g(k)f(\vartheta_2)$ through the item ϑ_2 to coalition S.*

We compute $\mathrm{MC}(k, u, \vartheta)$ in these three cases. In equations (4.2), (4.3) and (4.4) we will use properties $(M2)$ and $(M3)$ to give a closed formula that depends just

on Θ_l and the size of the coalition, but not on ϑ itself. These formulas can be computed in polynomial time due to $(M2^*)$.

For the first case, $\mathrm{MC}(C, u, \vartheta) = g(k+1)f(\vartheta)$. There are $|N^{-1}_{\#k}(\vartheta)|$ coalitions of size k such that $N(C, \vartheta)$. Thus, for $\vartheta \in \Theta_l$ we have:

$$\mathrm{MC}^{[1]}(k, u, \vartheta) \;=\; g(k+1)f(\vartheta)|N^{-1}_{\#k}(\vartheta)| \;=\; g(k+1)f(\Theta_l)|N^{-1}_{\#k}(\Theta_l)|. \tag{4.2}$$

For the second case, $\mathrm{MC}(C, u, \vartheta) = -g(k)f(\vartheta)$. There are $|R^{-1}_{\#k}(\vartheta)|$ coalitions of size k such that $R(C, \vartheta)$. We have:

$$\mathrm{MC}^{[2]}(k, u, \vartheta) \;=\; -g(k)f(\vartheta)|R^{-1}_{\#k}(\vartheta)| \;=\; -g(k)f(\Theta_l)|R^{-1}_{\#k}(\Theta_l)|. \tag{4.3}$$

For the third case, $\mathrm{MC}^{[3]}(C, u, \vartheta) = \Delta^k f(\vartheta)$, where $\Delta^k = g(k+1) - g(k)$. There are exactly $|R^{-1}_{\#k}(\vartheta)|$ coalitions of size k such that $R(C, \vartheta)$. We therefore have:

$$\mathrm{MC}^{[3]}(k, u, \vartheta) \;=\; \Delta^k f(\vartheta)|R^{-1}_{\#k}(\vartheta)| \;=\; \Delta^k f(\Theta_l)|R^{-1}_{\#k}(\Theta_l)|. \tag{4.4}$$

Since $\mathrm{MC}^{[i]}(k, u, \vartheta)$ depends on the group Θ_l such that $\vartheta \in \Theta_l$, equivalently we write $\mathrm{MC}^{[i]}(k, u, \Theta_l)$. Using equations (4.2), (4.3) and (4.4), we find that $\mathrm{MC}(k, u, \Theta_l)$ equals

$$\begin{aligned}\mathrm{MC}(k, u, \Theta_l) = &\sum_{\vartheta \in R_{\Theta_l}(u)} \mathrm{MC}^{[1]}(k, u, \vartheta) + \sum_{\vartheta \in \widetilde{R}_{\Theta_l}(u)} \mathrm{MC}^{[2]}(k, u, \vartheta) \\ &+ \sum_{\vartheta \in R_{\Theta_l}(u) \cup N_{\Theta_l}(u)} \mathrm{MC}^{[3]}(k, u, \vartheta).\end{aligned}$$

Using equations (4.2), (4.3) and (4.4) this term can be rewritten as

$$\begin{aligned}\mathrm{MC}(k, u, \Theta_l) = \; &|R_{\Theta_l}(u)|\mathrm{MC}^{[1]}(k, u, \Theta_l) + |\widetilde{R}_{\Theta_l}(u)|\mathrm{MC}^{[2]}(k, u, \Theta_l) \\ &+ \Big(|R_{\Theta_l}(u)| + |N_{\Theta_l}(u)|\Big)\mathrm{MC}^{[3]}(k, u, \Theta_l).\end{aligned} \tag{4.5}$$

Observe that the summations over the potentially exponentially growing sets $R_{\Theta_l}(u)$, $\widetilde{R}_{\Theta_l}(u)$, and $N_{\Theta_l}(u)$ fall out of the equation. Moreover, in virtue of property $(M2^*)$, this term can be computed in polynomial time.

Finally, equations (4.5), (4.1) and (2.8) yield the following closed formula for the semivalue of node v, which also furnishes us with a polynomial algorithm and concludes the proof:

$$\phi_v(\nu) = \sum_{0 \leq k < |V|} \beta(k) \sum_{1 \leq l \leq h(G)} \frac{\mathrm{MC}(k, u, \Theta_l)}{\binom{|V|-1}{k}}. \tag{4.6}$$

This concludes the proof. □

Of course, since the Shapley value and Banzhaf index are instances of semivalues, we can also obtain the following:

Corollary 1 *For all $\psi \in \mathscr{M}^*$ we can compute the Shapley value-based and Banzhaf centralities in polynomial time.*

Algorithm 2: (**SEMI**) The semivalue-based centrality

Input: Graph $G = (V, E, \omega)$, node $v \in V$, function β, functions c_G^N and c_G^R, precomputed vectors $\{|R_{\Theta_l}(\{v\})| : 1 \leq l \leq h(G) \wedge v \in V\}$, and $\{|\widetilde{R}_{\Theta_l}(\{v\})| : 1 \leq l \leq h(G) \wedge v \in V\}$

Output: semivalue, $\phi_v(\Psi(G))$, for the node v

$\phi_v \leftarrow 0$;
for $k \leftarrow 0$ **to** $|V| - 1$ **do**
  $\mathrm{MC}_k \leftarrow 0$;
  for $l \leftarrow 1$ **to** $h(G)$ **do**
    // **********[$R(v, \vartheta)$ and $N(C, \vartheta)$]**********
    $|N_{\#k}^{-1}(\Theta_l)| \leftarrow c_G^N(\vartheta \in \Theta_l, k)$;
    $\mathrm{MC}^{[1]} \leftarrow g(k+1) f(\Theta_l) |N_{\#k}^{-1}(\Theta_l)|$;
    $\mathrm{MC}_k \leftarrow \mathrm{MC}_k + |R_{\Theta_l}(\{v\})| \mathrm{MC}^{[1]}$;
    // **********[$\widetilde{R}(v, \vartheta)$ and $R(C, \vartheta)$]**********
    $|R_{\#k}^{-1}(\Theta_l)| \leftarrow c_G^R(\vartheta \in \Theta_l, k)$;
    $\mathrm{MC}^{[2]} \leftarrow g(k) f(\Theta_l) |R_{\#k}^{-1}(\Theta_l)|$;
    $\mathrm{MC}_k \leftarrow \mathrm{MC}_k - |\widetilde{R}_{\Theta_l}(\{v\})| \mathrm{MC}^{[2]}$;
    // ******[$R(v, \vartheta)$ or $N(v, \vartheta)$, and $R(C, \vartheta)$]******
    $\mathrm{MC}^{[3]} \leftarrow \Big(g(k+1) - g(k)\Big) f(\Theta_l) |R_{\#k}^{-1}(\Theta_l)|$;
    $\mathrm{MC}_k \leftarrow \mathrm{MC}_k + |R_{\Theta_l}(\{v\}) \cup N_{\Theta_l}(\{v\})| \mathrm{MC}^{[3]}$;
  $\mathrm{MC}_k \leftarrow \frac{\beta(k)}{\binom{|V|-1}{k}} \mathrm{MC}_k$;
  $\phi_v \leftarrow \phi_v + \mathrm{MC}_k$;

Equation (4.6) in the proof of Theorem 1 yields an algorithm to compute the problem Semivalue(ψ, ϕ). Algorithm 2 (SEMI) is a direct implementation. In

lines 5–7, 8–10 and 11–12, it computes the contribution of v to C_k associated with the expressions $\mathrm{MC}^{[1]}(k, v, \Theta_l)$, $\mathrm{MC}^{[2]}(k, v, \Theta_l)$ and $\mathrm{MC}^{[3]}(k, v, \Theta_l)$, respectively. Recall that polynomial computation of $|N^{-1}_{\#k}(\Theta_l)|$ and $|R^{-1}_{\#k}(\Theta_l)|$ was one of our assumptions in $\mathscr{M}^*$. The functions for computing these values are represented in the pseudocode by c^N_G and c^R_G and must be provided in the input. The values $|R_{\Theta_l}(\{v\})|$ and $|\widetilde{R}_{\Theta_l}(\{v\})|$ must be precomputed for all $1 \leq l \leq h(G)$ and $v \in V$. Given those precomputations, $|N_{\Theta_l}(\{v\})|$ can also be computed in constant time. Given this, we can draw the following additional conclusion as to the asymptotic complexity of Algorithm 2.

Corollary 2 *If $\psi \in \mathscr{M}^*$, then there exist natural numbers m_1, m_2, and m_3 such that*

1. *f can be computed in time $O(|V|^{m_1})$;*
2. *g can be computed in time $O(|V|^{m_2})$; and*
3. *$|N^{-1}_{\#k}(\Theta_l)|$, $|R^{-1}_{\#k}(\Theta_l)|$, $|R_{\Theta_l}(u)|$ and $|\widetilde{R}_{\Theta_l}(u)|$ can be computed in time $O(|V|^{m_3})$.*

For a given network $G = (V, E)$ and node u, Algorithm 2 runs in time

$$O\Big(h(G)|V|^{m_1} + |V|^{m_2+1} + h(G)|V|^{m_3+1}\Big).$$

4.4 Semivalues of Parametrised Group Degree, Betweenness and Closeness

In this section, we show that our framework can be applied to a wide variety of group centrality measures. We identify efficiently computable game-theoretic centrality measures that correspond to the semivalue of parametrised group degree, parametrised group betweenness, and parametrised group closeness. These parametrised measures are more general than those presented earlier and define classes of group centrality measures based on degree, betweenness and closeness. Such an approach allows us to generalise the computational results in the literature

to whole classes rather than individual measures. The other obvious generalisation of our work in comparison to the literature is that previous work has usually focused on just a single semivalue such as the Shapley value or the Banzhaf index. For example Michalak et al. [21] and Szczepański, Michalak, and Rahwan [22] developed polynomial-time algorithms to compute the Shapley value of group degree, generalised closeness and betweenness centralities, respectively. The remainder of this section is devoted to analysing the parametrised measures. We show how to build the model for each group centrality in our framework and then derive the individual components and functions required to invoke the method SEMI in Algorithm 2 in order to compute the semivalue of each measure.

4.4.1 Parametrised degree centrality

In this section, we define in our framework the class of cooperative games, where a node's value is based on its degree. We consider the general parametrised group degree centrality of a coalition C in graph G, which is defined by:

$$\psi^D(G)(C) = \nu_G^D(C) = g(|C|) \sum_{v \in E(C)} f(v),$$

where $E(C)$ is the set of neighbours of C, and f and g are polynomially computable parameters such that $deg(u) = deg(v)$ implies $f(u) = f(v)$ for all $u, v \in V$. Using this characteristic function, we build semivalue degree, semivalue weighted degree, semivalue impact factor and semivalue normalised degree centralities (Table 4.1).

Proposition 1 *The parametrised group centrality measure ψ^D (for polynomially computable parameters f and g) belongs to $\mathscr{M}^*$. Consequently,* SEMIVALUE(ψ^D, ϕ) *can be solved in polynomial time and the corresponding semivalue-based centrality can be computed in time $O(|V|^2 + |V|^{m_1+1} + |V|^{m_2+1})$.*

Centrality measure	Introduced in	$f(v)$	$g(\|C\|)$	Complexity
degree$^{(*)}$	[10]	1	1	$O(\|V\|^2)$
weighted degree	[80]	$\frac{1}{deg(v)}$	1	$O(\|V\|^2)$
impact factor	[81]	1	$\frac{1}{\|C\|}$	$O(\|V\|^2)$
normalised degree	[10]	1	$\frac{1}{\|V\|-\|C\|}$	$O(\|V\|^2)$

$^{(*)}$ Michalak et al. [21] presented the first algorithm for Shapley value degree centrality, whereas our algorithm applies to all semivalues.

Table 4.1: The functions f and g for various group degree centralities and where they were first introduced. The semivalues of these group centralities can be computed for the first time in polynomial time using our framework.

Proof: We will prove that given ν_G^D for any graph G, we can find a model $M_G = (\left\{\Theta_l\right\}_{l=1}^{h(G)}, R, \widetilde{R}, N, f, g)$ that models it, proving $\psi^D(G)$ is in the class $\mathscr{M}$. Next, we will show that all properties necessary for $\mathscr{M}^*$ are also satisfied. To this end, for all graphs G we define the set of items $\Theta_G = V$. The two relations, $R, \widetilde{R} \subseteq V \times V$, will be as follows:

$$\begin{aligned} R(v,u) &\quad \text{if and only if} \quad \exists_{(v,u)\in E}\ v \neq u \\ \widetilde{R}(v,u) &\quad \text{if and only if} \quad v = u. \end{aligned}$$

We define the partitions of Θ_G by $\Theta_l = \{v \in V : deg(v) = l\}$. Hence, we can rewrite $\nu_G^D(C) = g(|C|)\sum_{\vartheta\in R(C)} f(\vartheta)$, satisfying property $(M1)$. $(M2)$ is satisfied due to our assumption on f and all the examples in Table 4.1 satisfy it as well. Next, we consider the relations with respect to coalition sizes k and for $v \in \Theta_G$ we have:

$$|N_{\#k}^{-1}(v)| = \begin{cases} 0 & \text{if } |V| - 1 - deg(v) < k \\ \binom{|V|-1-deg(v)}{k} & \text{otherwise.} \end{cases}$$

$$|R_{\#k}^{-1}(v)| = \begin{cases} 0 & \text{if } |V| = k \\ \binom{|V|}{k} - |N_{\#k}^{-1}(v)| - \binom{|V|-1}{k-1} & \text{otherwise.} \end{cases}$$

Note that for all Θ_l and all $\vartheta_i, \vartheta_j \in \Theta_l$ we have that $|N_{\#k}^{-1}(\vartheta_i)| = |N_{\#k}^{-1}(\vartheta_j)|$ and $|R_{\#k}^{-1}(\vartheta_i)| = |R_{\#k}^{-1}(\vartheta_j)|$, satisfying $(M3)$ and $(M4)$. Hence, $\psi \in \mathscr{M}$.

Regarding $\mathscr{M}^*$, property $(M1^*)$ holds trivially, since clearly $h(G) \leq |V|$. For each $v \in V$ and $1 \leq l \leq h(G)$ it is easy to compute $|R_{\Theta_l}(\{v\})|$ and $|\widetilde{R}_{\Theta_l}(\{v\}|$. To do so, we must first compute the degree of each node, which takes $O(|V| + |E|)$ time.

Next, the number of neighbours of v with degree l must be counted to compute $|R_{\Theta_l}(\{v\})|$, and $|\widetilde{R}_{\Theta_l}(\{v\})| = 1$ if and only if $l = deg(v)$ and $|\widetilde{R}_{\Theta_l}(\{v\})| = 0$ otherwise. Both f and g can be computed in polynomial time. Finally, $|N^{-1}_{\#k}(\vartheta)|$ and $|R^{-1}_{\#k}(\vartheta)|$ can be computed in polynomial time simply from their definitions shown above, satisfying property $(M2^*)$.

Due to Theorem 1, we can compute semivalue-based centralities based on v^D_G in polynomial time. Due to Corollary 2, we can compute $\phi_v(\nu^D_G)$ in time $O(|V|^2 + |V|^{m_1+1} + |V|^{m_2+1})$. □

Algorithm 10 is presented in Appendix A.1 and invokes the method SEMI (Algorithm 2) in order to compute parametrised degree centrality. Our algorithm takes as an argument the graph G, defines the two functions, c^N_G and c^R_G. They compute $|N^{-1}_{\#k}(\Theta_l)|$ and $|R^{-1}_{\#k}(\Theta_l)|$, respectively, in polynomial time. Note that Algorithm 10 is also parametrised by the probability distribution β, which allows for the computation of any semivalue-based centrality. Pre-computing $|R_{\Theta_l}(\{v\})|$ for each node v comes down to computing the number of neighbours of v with degree l. This operation takes no more than $O(|V|^2)$ time. Precomputing $|\widetilde{R}_{\Theta_l}(\{v\})|$ is trivial, since it is always either equal to 0 or 1. Thus, these pre-computations take $O(|V|^2)$ time. Due to Corollary 2, this algorithm runs in $O(|V|^2+|V|^{m_1+1}+|V|^{m_2+1})$ time (recall that m_1 and m_2 are such that f can be computed in $O(|V|^{m_1})$ time and g can be computed in $O(|V|^{m_2})$ time).

4.4.2 Parametrised betweenness centrality

In this subsection, we show how to express in our framework the class of cooperative games, where a node's value is based on its betweenness centrality. Recall that we denote the set of paths between s and t by $Paths(s,t)$ and the set of all shortest paths in G by $\mathrm{SP}(G)$. We denote by σ_{st} the number of shortest paths between s and t (if $s = t$ then $\sigma_{st} = 1$) and by $\sigma_{st}(C)$ the number of shortest paths between s and t that pass through some node $v \in C$.

The general parametrised group betweenness centrality of a coalition C in graph G is defined by:

$$\psi^B(G)(C) = \nu_G^B(C) = g(|C|) \sum_{s,t \notin C} \sigma_{st}(C) f(s,t),$$

where f and g polynomially computable parameters.[2] Using this characteristic function, we build semivalue stress, semivalue betweenness, semivalue distance-scaled betweenness and semivalue normalised betweenness centralities (Table 4.2).

Centrality measure	Introduced in	$f(s,t)$	$g(\|C\|)$	Complexity
stress$^{(*)}$	[22]	1	1	$O(\|V\|^3)$
betweenness$^{(*)}$	[10]	$\frac{1}{\sigma_{st}}$	1	$O(\|V\|^3)$
distance-scaled betw.	[82]	$\frac{1}{\sigma_{st}\, dist(s,t)}$	1	$O(\|V\|^3)$
normalised betw.	[10]	$\frac{1}{\sigma_{st}}$	$\frac{2}{(\|V\|-\|C\|)(\|V\|-\|C\|-1)}$	$O(\|V\|^3)$

$^{(*)}$ Szczepański, Michalak, and Rahwan [22] presented the first algorithm for Shapley value stress and betweenness centralities, whereas our algorithm applies to all semivalues.

Table 4.2: The functions f and g for various group betweenness centralities and where they were first introduced. The semivalues of these group centralities can be computed for the first time in polynomial time using our framework.

Proposition 2 *The parametrised group centrality measure ψ^B (for polynomially computable parameters f and g) belongs to $\mathscr{M}^*$. Consequently,* SEMIVALUE(ψ^B, ϕ) *can be solved in polynomial time and the corresponding semivalue-based centrality can be computed in time $O(|V|^3 + |V|^{m_1+2} + |V|^{m_2+1})$.*

Proof: We will prove that given ν_G^B for any graph G, we can find a model $M_G = (\left\{\Theta_l\right\}_{l=1}^{h(G)}, R, \widetilde{R}, f, g)$ that models it and satisfies the conditions $(M1)$ through $(M4)$ for class $\mathscr{M}$. Next, we will show that all properties of class $\mathscr{M}^*$ are also satisfied. To this end, we define $\Theta_G = SP(G)$ (set of all shortest paths in graph G). Also, we define the two relations, $R, \widetilde{R} \subseteq V \times SP(G)$, as follows:

$$R(v, \pi_{st}) \quad \text{if and only if} \quad v \in \pi_{st} \text{ and } v \neq s \text{ and } v \neq t$$
$$\widetilde{R}(v, \pi_{st}) \quad \text{if and only if} \quad v = s \text{ or } v = t.$$

[2] If $\sigma_{xy}(C') = 0$, we postulate $\frac{0}{0} = 0$.

and we define the set of partitions of Θ_G as:

$$\Theta_{st} = \{\pi_{st} \in SP(G) : \pi_{st} \in Paths(s,t)\}.$$

We slightly abuse notation here by indexing groups of items by s,t for $s,t \in V$. We refer to each group of items therefore by Θ_{st} and define it as the set of all the shortest paths between s and t. In this way $\nu_G^B(C) = g(|C|)\sum_{\vartheta \in R(C)} f(\vartheta)$, satisfying properties $(M1)$ and $(M2)$. Next, we look at the three relations with respect to coalitions of different sizes. The set of nodes C is in relation N with shortest path $\pi_{st} \in SP(G)$ if and only if $\pi_{st} \cap C = \emptyset$, so we get:

$$|N_{\#k}^{-1}(\pi_{st})| = \begin{cases} 0 & \text{if } |V| - |\pi_{st}| < k \\ \binom{|V|-|\pi_{st}|}{k} & \text{otherwise.} \end{cases}$$

Let $e(\pi_{st}) = \min(2, |\pi_{st}|)$. Then,

$$|\widetilde{R}_{\#k}^{-1}(\pi_{st})| = \begin{cases} 0 & \text{if } k = 0 \\ e(\pi_{st}) & \text{if } k = 1 \\ 1 & \text{if } k = |V| \\ \sum_{i=1}^{e(\pi_{st})} \binom{e(\pi_{st})}{i}\binom{|V|-e(\pi_{st})}{k-i} & \text{otherwise.} \end{cases}$$

Since all coalitions are exclusively in relation R, $\widetilde{R}$ or N with every item, we have:

$$|R_{\#k}^{-1}(\pi_{st})| = \binom{|V|}{k} - |N_{\#k}^{-1}(\pi_{st})| - |\widetilde{R}_{\#k}^{-1}(\pi_{st})|.$$

Properties $(M3)$ and $(M4)$ may seem troubling at first, but note that all shortest paths between two nodes s and t must necessarily be of the same length. Therefore, $\psi^B \in \mathscr{M}$.

We note that $h(G) = |V|^2$, satisfying property $(M1^*)$. Property $(M2^*)$ is satisfied by our assumption on f and g and for all functions in Table 4.2. Moreover, for each $v \in V$ and $s,t \in V$ it is easy to compute $|R_{\Theta_{s,t}}(\{v\})|$ and $|\widetilde{R}_{\Theta_{s,t}}(\{v\})|$ for some $v \in V$ and all $s,t \in V$ in time $O(|V|^2 + |V||E|)$ using the BFS algorithm [83] for unweighted graphs, and in time $O(|V|^2 \log(|V|) + |V||E|)$ using the Dijkstra algorithm [84] for weighted graphs, again satisfying $(M2^*)$. An example of how to perform such computations for all nodes in a graph is presented in Appendix A.2 in Algorithm 11. For each $\pi_{st} \in \Theta_G$ and $k \in \{0, \ldots, |V|-1\}$ we can compute

Centrality measure	Introduced in	$f(d)$	$g(\|C\|)$	Complexity
closeness	[10]	1	1	$O(\|V\|^3)$
harmonic(*)	[45]	$\frac{1}{d}$	1	$O(\|V\|^3)$
influence game(*)	[21]	positive value decreasing	1	$O(\|V\|^3)$
normalised closeness	[10]	$\frac{1}{d}$	$(\|V\| - \|C\|)$	$O(\|V\|^3)$

(*) Michalak et al. [21] presented the first algorithm for Shapley value harmonic and influence game centralities, whereas our algorithm applies to all semivalues.

Table 4.3: The functions f and g for various group closeness centralities and where they were first introduced. The semivalues of these group centralities can be computed for the first time in polynomial time using our framework.

$|N_{\#k}^{-1}(\pi_{st})|$ and $|R_{\#k}^{-1}(\pi_{st})|$ in polynomial time simply from the characterisations of these sets that we have shown above (a modified BFS or Dijkstra algorithm can also return the value of $e(\pi_{st}) = \min(2, |\pi_{st}|)$), which satisfies $(M2^*)$.

It follows that $\psi^B \in \mathscr{M}^*$ and due to Theorem 1 we can compute any semivalue-based centrality based on the game v_G^B in polynomial time. Due to Corollary 2, we can compute $\phi_v(\nu_G^B)$ for the node $v \in V$ in time $O(|V|^3 + |V|^{m_1+2} + |V|^{m_2+1})$. □

Algorithm 12 is presented Appendix A.2 and computes the semivalue betweeness centrality.

4.4.3 Parametrised closeness centrality

The last class of characteristic functions that we will define in our framework assigns a value to a node based on its distance to other nodes. The parametrised closeness centrality is defined as follows:

$$\psi^{CL}(G)(C) = \nu_G^{CL}(C) = g(|C|) \sum_{v \in V} f(dist(C, v)),$$

where f and g are polynomially computable parameters. Using this characteristic function, we build semivalue closeness, semivalue harmonic, semivalue influence game and semivalue normalised closeness centralities (Table 4.3).

Proposition 3 *The parametrised group centrality measure ψ^{CL} (for polynomially computable parameters f and g) belongs to $\mathscr{M}^*$. Consequently,* SEMIVALUE(ψ^{CL}, ϕ) *can be solved in polynomial time and the corresponding semivalue-based centrality can be computed in time $O(|V|^3 + |V|^{m_1+2} + |V|^{m_2+1})$.*

Proof: We will prove that given ν_G^{CL} for any graph G, we can find a model $M_G = (\left\{\Theta_l\right\}_{l=1}^{h(G)}, R, \widetilde{R}, f, g)$ that models it, satisfying the necessary conditions for class $\mathscr{M}$. Next, we will prove that all properties of class $\mathscr{M}^*$ are also satisfied. To this end, we will define items as pairs of the form $\langle u, d\rangle \subseteq V \times \mathit{dists}(G)$, where $\mathit{dists}(G)$ is the set of distances between any pair of nodes in G. They will represent nodes and all possible distances between them. We define $\Theta_G = \{\langle u, d\rangle \in V \times \mathbb{N} : \exists_{v \in V}\, \mathit{dist}(u, v) = d\}$ and the partition $\{\{\vartheta\} | \vartheta \in \Theta_G\}$ of singletons. We slightly abuse notation here by indexing groups of items (i.e., sets in the partition) by u, d. We refer to a group of items therefore by $\Theta_{u,d}$. Next, we define the two relations, $R, \widetilde{R} \subseteq V \times V \times \{0, \ldots, |V| - 1\}$, as follows:

$$R(v, \langle u, d\rangle) \text{ if and only if } \mathit{dist}(v, u) = d$$
$$\widetilde{R}(v, \langle u, d\rangle) \text{ if and only if } \mathit{dist}(v, u) < d.$$

In this way $\nu_G^{CL}(C) = g(|C|) \sum_{\vartheta \in R(C)} f(\vartheta)$, satisfying $(M1)$.[3] Also $(M2)$ holds for all examples in Table 4.3, and $(M3)$ and $(M4)$ hold trivially since groups of items are singletons. Thus, $\psi^{CL} \in \mathscr{M}$.

Property $(M1^*)$ holds, since $h(G)$ is bounded by $|V|^2$ (number of nodes and all possible distances between them). Functions f and g can be computed in polynomial time and, therefore, $(M2^*)$ also holds. We note that $h(G) \leq |V|^2$ and that for each $v \in V$ and $1 \leq d \leq h(G)$ it is easy to compute $|R_{\Theta_{u,d}}(\{v\})|$ and $|\widetilde{R}_{\Theta_{u,d}}(\{v\})|$ with the help of the BFS algorithm for unweighted graphs, or the Dijkstra algorithm for weighted graphs, thus also satisfying $(M2^*)$. Once the distances between all nodes are computed either using BFS or the Dijkstra algorithm, $|R_{\Theta_{u,d}}(\{v\})|$ is

[3] In our examples f ignores the first parameter of ϑ and is simply a function of distance.

equal to 1 if $dist(v,u) = d$ and 0 otherwise, whereas $|\widetilde{R}_{\Theta_{u,d}}(\{v\})|$ is computed identically with the condition $dist(v,u) < d$.

Next, we will consider the three relations with respect to coalition sizes. To this end, we will define

$$Nod_{\sim d}(u) = |\{v : dist(u,v) \sim d\}|, \text{ and}$$
$$NodSet_{\sim d}(u) = \{v : dist(u,v) \sim d\}.$$

where $\sim$ can be either $<$ or $=$. These values are simple to compute for all u, d in $O(|V|^2 + |V||E|)$ time using the BFS algorithm for unweighted graphs or $O(|V|^2 \log(|V|) + |V||E|)$ time for weighted graphs using the Dijkstra algorithm. Note that:

$$R(C, \langle u, d\rangle) \text{ iff } NodSet_{<d}(u) \cap C = \emptyset \text{ and } NodSet_{=d}(u) \cap C \neq \emptyset$$
$$\widetilde{R}(C, \langle u, d\rangle) \text{ iff } NodSet_{<d}(u) \cap C \neq \emptyset.$$

Using the above relations we can now calculate:

$$|R^{-1}_{\#k}(\langle u, d\rangle)| = \sum_{i=1}^{Nod_{=d}(u)} \binom{Nod_{=d}(u)}{i} \binom{|V| - Nod_{=d}(u) - Nod_{<d}(u)}{k-i}$$
$$|\widetilde{R}^{-1}_{\#k}(\langle u, d\rangle)| = \sum_{i=1}^{Nod_{<d}(u)} \binom{Nod_{<d}(u)}{i} \binom{|V| - Nod_{<d}(u)}{k-i}.$$

Since all coalitions are exclusively in relation R, $\widetilde{R}$ or N with every item, we have:

$$|N^{-1}_{\#k}(\langle u, d\rangle)| = \binom{|V|}{k} - |R^{-1}_{\#k}(\langle u, d\rangle)| - |\widetilde{R}^{-1}_{\#k}(\langle u, d\rangle)|.$$

Due to the above characterisations, for each $\vartheta \in \Theta_G$ and $k \in \{0, .., |V| - 1\}$, we can compute $|N^{-1}_{\#k}(\vartheta)|$ and $|R^{-1}_{\#k}(\vartheta|$ in polynomial time $O(V)$, thus satisfying $(M2^*)$. We show in Appendix B.4 how this computation can be improved to $O(1)$ time.

Hence, $\psi^{CL} \in \mathscr{M}^*$. Consequently, SEMIVALUE(ψ^{CL}, ϕ) can be solved in polynomial time. By virtue of Corollary 2, we can compute every semivalue-based centrality for node $v \in V$ in time $O(|V|^3 + |V|^{m_1+2} + |V|^{m_2+1})$ □

Algorithm 13 is presented in Appendix A.3 and uses the method SEMI to compute the parametrised closeness centrality of a node. Evaluating the functions c_G^N and c_G^R takes $O(1)$ time. Precomputing $|N_{u,d}(\{v\})|$ and $|R_{u,d}(\{v\})|$ for each node v can be done with the help of the BFS algorithm for unweighted networks to compute distances between all nodes in the graph, or the Dijkstra algorithm for weighted networks. This takes $O(|V|^2+|V||E|)$ or $O(|V|^2\log(|V|)+|V||E|))$ time, respectively. In particular, for all nodes v and possible distances l, we need to store the number of nodes that are closer to v than l and the number of nodes that are at a distance equal to l. Due to Corollary 2, this algorithm computes the parametrised closeness centrality for the node v in $O(|V|^3 + |V|^{m_1+2} + |V|^{m_2+1})$ time.

In the following section, we present algorithms for computing the Banzhaf index (the most prominent semivalue next to the Shapley value) of the parametrised group centrality measures presented in this section, with the additional constraint that $g(|C|) = 1$. Focusing on these specific cases allows us to achieve even faster polynomial computation, often by a factor of $O(|V|)$.

4.5 Computational Analysis of the Banzhaf Index

In this section, we show how to compute the Banzhaf index of power of the parametrised measures from Section 4.4. Although the algorithms presented in Section 4.4 can also perform these computations, if we focus on just the Banzhaf index and restrict the model of the characteristic function such that $g(|C|) = 1$ (the proofs work for any constant function), then quantification over k (the size of the coalition) can be removed from the algorithm and computation generally improves by a factor of $O(|V|)$. Let us define

$$\mathcal{N}^l = \sum_{0\leq k<|V|} \beta(k)\frac{|N^{-1}_{\#k}(\Theta_l)|}{\binom{|V|-1}{k}}, \qquad \text{and} \qquad \mathcal{R}^l = \sum_{0\leq k<|V|} \beta(k)\frac{|R^{-1}_{\#k}(\Theta_l)|}{\binom{|V|-1}{k}}.$$

We can then reorder Equation (4.6) from Theorem 1—see Appendix B.1—and obtain the following equation:

$$\phi_v(\nu) = \sum_{1 \leq l \leq h(G)} |R_{\Theta_l}(\{u\})| f(\Theta_l)\mathcal{N}^l - \sum_{1 \leq l \leq h(G)} |\widetilde{R}_{\Theta_l}(\{u\})| f(\Theta_l)\mathcal{R}^l. \tag{4.7}$$

It should be clear from this that, if computing $\mathcal{N}^l$ and $\mathcal{R}^l$ can be achieved in constant time for a given group centrality measure, then this will rid us of the quantification over k. This leads to a simplification of the general semivalue algorithm (Algorithm 2), which we present in Algorithm 3. This algorithm computes the Banzhaf centrality of a given node, v. As in our previous algorithm, $R_{\Theta_l}(\{v\})$ and $\widetilde{R}_{\Theta_l}(\{v\})$ must be precomputed for every item group index l. The specifics of this for every centrality can be found in the previous section. In addition, $\mathcal{N}^l$ and $\mathcal{R}^l$ must be computed. The complexity of this algorithm is $O(h(G)(|V|^{m_1} + |V|^{m_4}))$, where $|V|^{m_1}$ is the time required to compute $f(\Theta_l)$ and $|V|^{m_4}$ is the time required to compute $\mathcal{N}^l$, $\mathcal{R}^l$, $R_{\Theta_l}(\{v\})$ and $\widetilde{R}_{\Theta_l}(\{v\})$. We discuss the specifics of computing $\mathcal{N}^l$ and $\mathcal{R}^l$ for each of the parametrised group degree, betweenness and closeness centralities in what follows.

Algorithm 3: General Banzhaf Centrality

Input: Graph $G = (V, E)$, node $v \in V$, function f
Output: Banzhaf value for the node v

```
1 φ_v ← 0;
2 for l ← 1 to h(G) do
3     if R(v, Θ_l) then
4         φ_v ← φ_v + R_{Θ_l}({v}) f(Θ_l) N^l;
5     if R̃(v, Θ_l) then
6         φ_v ← φ_v − R̃_{Θ_l}({v}) f(Θ_l) R^l;
```

4.5.1 A faster algorithm for Banzhaf Degree Centrality

In this section we will show how to compute $\mathcal{N}^l$ and $\mathcal{R}^l$ for group degree centrality and develop algorithms to compute the Banzhaf index. If $g(|C|) = 1$, then this centrality takes the following form:

$$\psi^{D'}(G)(C) = \nu_G^D(C) = \sum_{v \in E(C)} f(v),$$

Proposition 4 *Given a constant function g,* BANZHAF(ψ^D, ϕ) *can be solved in* $O(|V|^{1+m_1})$ *time if* $deg(v)$ *can be solved in constant time, and* $O(|V|^{1+m_1} + |E|)$ *otherwise.*

The proof of Proposition 4 can be found in Appendix B.2

and its main finding is as follows:

$$\mathcal{N}^l = \mathcal{N}^{deg(v)} = \frac{1}{2^{deg(v)}} \quad \text{and} \quad \mathcal{R}^l = \mathcal{R}^{deg(v)} = 1 - \frac{1}{2^{deg(v)}}$$

In order to compute the Banzhaf degree centrality these arguments must be passed to Algorithm 3.

4.5.2 A faster algorithm for Banzhaf Betweenness Centrality

We will now show how to compute $\mathcal{N}^l$ and $\mathcal{R}^l$ for the Banzhaf betweenness centrality. First, recall that if $g(|C|) = 1$:

$$\psi^B(G)(C) = \nu_G^B(C) = \sum_{s,t \notin C} \sigma_{st}(C) f(s,t).$$

Proposition 5 *Given a constant function g,* BANZHAF(ψ^B, ϕ) *can be solved in* $O(|V|^{2+m_1}+|V||E|)$ *time for undirected graphs, and* $O(|V|^{2+m_1}+|V||E|+|V|^2 \log(|V|))$ *time for directed graphs.*

The proof of Proposition 5 can be found in Appendix B.3 and its main finding is as follows:

$$\mathcal{N}^{s,t} = \frac{2^{|V|-|\pi_{st}|}}{2^{|V|-1}}$$

Assuming $e(\pi_{st}) = 2$,

$$\widetilde{\mathcal{R}}^{st} = \frac{1}{2^{|V|-1}} \left(3(2^{|V|-2}) - |V| \right)$$

and assuming $e(\pi_{st}) = 1$,

$$\widetilde{\mathcal{R}}^{s,t} = \frac{1}{2},$$

which leads to:

$$\mathcal{R}^{s,t} = \frac{2^{|V|}-1}{2^{|V|-1}} - \widetilde{\mathcal{R}}^{s,t} - \mathcal{N}^{s,t}.$$

In order to compute the Banzhaf betweenness centrality these arguments must be passed to Algorithm 3.

4.5.3 A faster algorithm for Banzhaf Closeness Centrality

We will now show how to compute $\mathcal{N}^l$ and $\mathcal{R}^l$ for the Banzhaf closeness centrality. First, recall that if $g(|C|) = 1$:

$$\nu_G^{CL}(C) = \sum_{v \in V} f(dist(C, v)),$$

Proposition 6 *Given a constant function g,* $\textsc{Banzhaf}(\psi^{CL}, \phi)$ *can be solved in $O(|V|^{2+m_1} + |V||E|)$ time for undirected graphs, and $O(|V|^{2+m_1} + |V||E| + |V|^2 \log(|V|))$ time for directed graphs.*

The proof of Proposition 6 can be found in Appendix B.4 and its main finding is as follows: Assuming $Nod_{<d}(u) > 0$,

$$\mathcal{R}^{u,d} = \frac{1}{2^{|V|-1}}\left(2^{|V|-Nod_{<d}(u)} - 2^{|V|-Nod_{=d}(u)-Nod_{<d}(u)}\right)$$

and assuming $Nod_{<d}(u) = 0$,

$$\mathcal{R}^{u,d} = \frac{1}{2^{|V|-1}}\left(2^{|V|-Nod_{<d}(u)} - 2^{|V|-Nod_{=d}(u)-Nod_{<d}(u)} - 1\right)$$

.

Continuing, if $Nod_{<d}(u) > 0$

$$\widetilde{\mathcal{R}}^{u,d} = \frac{1}{2^{|V|-1}}\left(2^{|V|} - 1 - 2^{|V|-Nod_{<d}(u)}\right).$$

and if $Nod_{<d}(u) = 0$,

$$\widetilde{\mathcal{R}}^{u,d} = \frac{1}{2^{|V|-1}}\left(2^{|V|} - 2^{|V|-Nod_{<d}(u)}\right),$$

which allows us to compute

$$\mathcal{N}^{u,d} = \frac{2^{|V|}-1}{2^{|V|-1}} - \mathcal{R}^{u,d} - \widetilde{\mathcal{R}}^{u,d}$$

In order to compute the Banzhaf closeness centrality these arguments must be passed to Algorithm 3.

4.6 Conclusions and Future Work

In this chapter, we have proposed a general framework to facilitate computation of semivalues for a wide range of game-theoretic network centrality measures. Whereas previous work has focused on computing a single solution concept for a given characteristic function, we have shown that it is possible to consolidate and generalise certain computational properties. Given that the model satisfies just a few properties, we show that our algorithm runs in polynomial time. In fact, our algorithm covers all previously proposed centrality measures based on some semivalue (see Table 1.1) and many more (see Tables 4.1, 4.2 and 4.3). We trust that this will facilitate the study of computational properties and implementation of such measures in the future.

A number of future research directions stem from this work. First, we would like to investigate whether it would be possible to broaden the scope of our framework. For example, it may be possible to expand the class $\mathscr{M}^*$ to include additional group centrality measures. Another direction is to study the computation complexity of computing solution concepts other than semivalues and the classes of group centrality measures for which these solution concepts can be computed in polynomial time. There are many solution concepts in the literature on game theory, such as probabilistic values, quasivalues, weighted values, indices of power [85], the core [86] and nucleoulus [87] some of which have not yet been applied to networks or centrality.

Second, pertaining specifically to our framework rather than the computational complexity of game-theoretic network centrality as a whole, it may be possible to learn certain features of the model of a cooperative function using machine learning methods. For example, the value of items may be discovered given ground-truth rankings, or a number of intuitive rankings given by humans that the measure should replicate. In this case, it would be possible to include many different types of items in the model, and those that do not matter for the ranking would simply be given value zero.

Third, it may be possible to use game-theoretic approaches in order to analyse more general types of networks. For example, it may be possible to incorporate additional, extraneous information about nodes and their relationships in order to rank nodes. For example, *Coalitional semivalues* [31] have already been used as game-theoretic centrality measures that incorporates an extraneously defined, non-overlapping community structure in the ranking. We expand on this work to incorporate overlapping communities in the following Chapter. Other interesting directions for future research include whether the game-theoretic approach can also be used to analyse multi-layer networks (where multiple types of relationships may exist between nodes), hypergraphs (where relationships can exist between more than two nodes), or other types of networks.

Fourth, the axiomatic study of game-theoretic network centrality measures is still in its infancy. Such an analysis is very important in order to characterise and study the properties of such measures. Brink and Gilles [17] axiomatised their degree-based centrality measure and—interestingly—developed a closed-form formula for it that easily translates to a polynomial algorithm. Skibski, Michalak, and Rahwan [79] have also studied axiomatisation of game-theoretic network centrality, but did not analyse computational properties. It would be interesting to investigate whether there is any relation between the axioms and the computational properties of a given measure (e.g., a given axiom might be very desirable but too computationally complex to implement).

Finally, the study of the applications of game-theoretic network centrality and the evaluation of the quality and comparative studies of network centrality measures in general require more attention.

5

Closeness Centrality for Networks with Overlapping Community Structure

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5.1 Introduction

One aspect of networks that has been largely ignored in the literature on centrality is the fact that *certain real-life networks have a predefined community structure*. In public transportation networks, for example, bus stops are typically grouped by the bus lines (or routes) that visit them. In coauthorship networks, the various venues where authors publish can be interpreted as communities [31]. In social networks, individuals grouped by similar interests can be thought of as members of a community. Clearly for such networks, it is desirable to have a centrality

measure that accounts for the predefined community structure. Yet, to the best of our knowledge, only one such measure has been developed to date [31], which extends Degree centrality to networks with community structure. Despite this recent development, one important aspect of real-life networks remains missing from existing centrality measures: the ability to consider *overlapping communities*. Take social networks, for example, where such overlaps are widespread due to the various affiliations and interests of the individuals involved [88] (e.g., a person may belong to a certain society of lawyers and at the same time belong to a certain golf club). Similarly, in our example of transportation networks, a bus stop may be on the route of multiple (i.e., overlapping) bus lines. If such a stop becomes inaccessible, then all the bus lines that visit it could no longer operate. As such, the importance of a bus stop is connected to the importance of the bus lines that visit it.

With this in mind, we focus in this chapter on a generalised form of closeness centrality. This measure considers the important nodes to be those that are relatively close to all other nodes in the network: the closer a node is to the others, the higher its centrality. Closeness centrality has many applications, from coauthorship networks [89], through tourism [90], to social networks [91, 92]. It is also more general than the degree centrality used by Szczepański, Michalak, and Wooldridge [31] in developing their Owen value degree-based centrality that accounts for communities. In particular, we propose the first extension of closeness centrality to networks with either overlapping or non-overlapping community structure. To this end, we propose four game-theoretic variants of Closeness centrality, three of which are based on existing solution concepts, namely: the *Owen value* [41], the *configuration value* [42], and the *coalitional semivalue* [31]. The fourth and most general variant is based on a new solution concept proposed in this chapter, which generalises the aforementioned three concepts, and offers greater flexibility in modeling the underlying community structure. We call it the *Configuration semivalue*.

Crucially, all of the aforementioned solution concepts (including our own) are hard to compute given an arbitrary group centrality. Nevertheless, when paired with group closeness centrality, we prove that they can be computed in polynomial time and

Harmonic	Shapley Value
1. v_7, v_9	v_7, v_9
2. v_1, v_2	v_1, v_2
3. v_5, v_6, v_8, v_{10}	v_3, v_4
4. v_3, v_4	v_5, v_6, v_8, v_{10}

Table 5.1: *Harmonic closeness and Shapley value rankings for Figure 5.1 (d).*

propose polynomial-time algorithms for computing the corresponding game-theoretic solution concept. This result fills several gaps in the computational-complexity analysis of game-theoretic centrality measures (see Table 1.1).

Finally, to demonstrate the applicability of our approach, we apply it to the Warsaw public transportation network, identifying the most central stops and routes therein, from the perspective of Closeness centrality.

5.2 Our Centrality Measures

The first game-theoretic extension of Closeness centrality was introduced by Michalak et al. [29]. In particular, the authors defined a game in which the players are the nodes of the network, and the characteristic function is the generalised group closeness centrality introduced in Chapter 2: ν_f^{CL}. The centrality of each node was then determined using the Shapley value. The resulting game-theoretic centrality measure is called the *Shapley value-based Closeness centrality*. Roughly speaking, the harmonic Closeness centrality (an instance of the generalised closeness measure such that $f(d) = \frac{1}{d}$) evaluates how close a node is to others, whereas the Shapley value-based variant evaluates the role that a node plays in *bringing other nodes closer together*. To illustrate this difference, consider networks **(a)** and **(b)** from Figure 5.1. Here, according to harmonic Closeness, v_1 is relatively more important in **(b)** than in **(a)** since it is close to more nodes in **(b)** than in **(a)**. In contrast, according to the Shapley-value based Closeness, v_1 is actually more important in **(a)**, since the removal of v_1 from **(a)** has a greater impact on the distances between the other nodes, compared to the removal of v_1 from **(b)**.

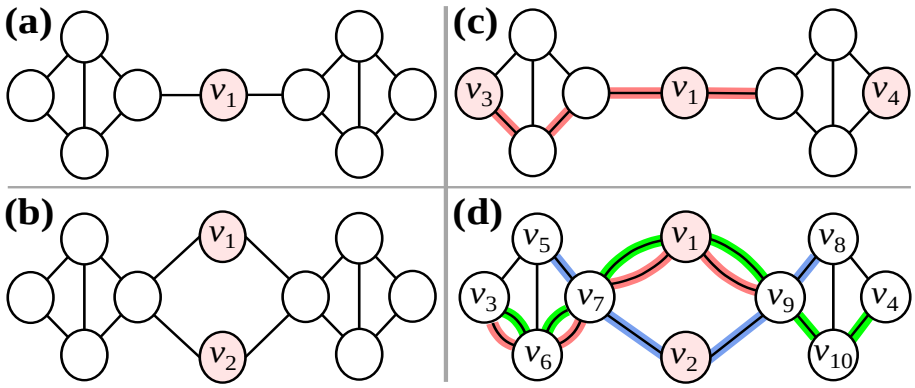

Figure 5.1: Sample networks. In **(c)** and **(d)**, communities are highlighted by same-coloured edges.

Configuration Semivalues: We now propose a family of solution concepts, which we believe to be the most general of its kind to date, as it not only allows for an arbitrary β, α and CS, but also allows for overlapping coalitions. Recall that CS is the set of communities, $\beta(k)$ represents the probability that a node v contributes to a set of k communities and $\alpha(l)$ represents the probability that v contributes to l members of its own community. We call it the *Configuration semivalue*, and define it as follows, where T^k is a random variable over subsets of size k of $CS \setminus \tau_i$, C^l is a random variable over subsets of size l from Q and $\mathbb{E}$ is the expected value operator:

Definition 6 (Configuration Semivalue)

$$\phi_i(\nu, CS) = \sum_{k=0}^{|CS|-1} \beta(k) \sum_{Q \in \mathcal{T}(i)} \sum_{l=0}^{|Q|-1} \alpha_Q(l)\ \mathbb{E}\left[MC\left(\left(\bigcup T^k\right) \cup C^l, i\right)\right] \tag{5.1}$$

In particular, given a community structure CS in which communities do not overlap, this family of solution concepts is equivalent to *coalitional semivalues*. Given $CS = \{N\}$, it is equivalent to *semivalues*. Further restrictions on β and α_Q (as discussed in the preliminaries) lead to the *Shapley value*, the *Banzhaf index*, or the *Owen value*. Compared to the configuration *value*, our configuration *semivalues* offer greater control over the contributions of players, due to a probability distribution over the number of communities to which a player contributes and a distribution over the number of nodes from her or his own community that the player can contribute to. In applications such as counter-terrorism [21, 57], this can represent the expected size of an attack on a network or the number of targeted communities.

Configuration Semivalue Community Index: Whereas to measure the importance of a community using the Owen value it suffices to sum up the power of the nodes comprising it, this is not the case for Configuration semivalues. In particular, the power of a node may be the result of its membership to many communities. For this reason, the distinction must be made as to which of the player's marginal contributions are made through of which community. To this end, we propose the following measure of community strength:

Definition 7 (Configuration Semivalue Community Power Index)

$$CP_Q(\nu, CS) = \sum_{i \in Q} \sum_{k=0}^{|CS|-1} \beta(k) \sum_{l=0}^{|Q|-1} \alpha_Q(l)\ \mathbb{E}\left[MC\Big(\Big(\bigcup T^k\Big) \cup C^l, i\Big)\right],$$

where $C^l \subseteq Q \setminus \{i\}$ and $T^k \subseteq CS \setminus Q$ are random variables. Although an axiomatic characterisation of this community index is outside the scope of this thesis, we mention that in the case of the Configuration value, the index is efficient, and in the case of the Owen value, it is equal to the sum of the Owen values of the members of a community.

Configuration Semivalue Closeness Centrality: Our extension of closeness centrality (which accounts for overlapping and non-overlapping community structures) involves using our Configuration semivalue (see Equation 5.1) with the characteristic function for group Closeness centrality (see Equation (2.17)). More formally, it is: $\phi_v(\nu_f^{CL}, CS)$.

Example 28 *As stated earlier, no centrality measures to date can readily be applied to networks with overlapping community structures. An example of such networks is depicted in Figure 5.1* ***(c)*** *and* ***(d)****. Specifically, in network* ***(c)****, nodes* v_3 *and* v_4 *are symmetric except that* v_3 *belongs to a seemingly-important community; one that connects the two parts of the network. Arguably, when taking this additional information into consideration,* v_3 *should be considered more important than* v_4*. A more complicated example is presented in network* ***(d)****, where node* v_1 *belongs to*

more communities than v_2. Moreover, each of the communities to which v_1 belongs seem to be of similar or more importance than the single community to which v_2 belongs. In a real-life network, this could mean, for example, that more bus, tram or train routes visit (and rely on) the bus stop v_1 than the bus stop v_2, implying that v_1 should be more central once the underlying community structure is taken into consideration.

We present in Table 5.1 the harmonic and Shapley value closeness rankings for Figure 5.1 (d).[1] *The configuration value harmonic closeness ranking is as follows: v_9, v_7, v_1, v_6, v_3, v_8, v_5, v_2, v_{10}, v_4. The configuration value makes use of community information, promoting nodes v_9, v_1, v_6 and v_3. This ranking is also more fine-grained (i.e., there are no ties) than either the harmonic or Shapley-value harmonic rankings, because it draws upon additional community information, which is different for most nodes in the example.*

5.3 Algorithms for Configuration Semivalue Closeness Centrality

In this section, we show that any *configuration semivalue* of generalised group Closeness centrality is computable in polynomial time (Theorem 2), and obtain better time complexity for the *configuration value* (Theorem 3). We note that since our choice of generalised closeness centrality generalises degree, then our results and algorithms can be readily applied to the configuration semivalues of group degree centrality.

Theorem 2 *Any configuration* semivalue *of group Closeness centrality for all nodes in a weighted digraph $D = (V, E, \omega)$ can be computed in $O(|V|^3|CS|^2)$ time.*

[1] We note that the ranking alone does not capture the subtlety of valuations. For example, harmonic closeness ranks v_7 as slightly more important than v_1, however Shapley value closeness ranks it as much more important.

Proof: Starting from Equation (5.1), which defines the configuration semivalue, let us first replace the arbitrary characteristic function therein, i.e., ν, with that of group Closeness centrality (defined in Equation (2.17)). We get:

$$\phi_v(\nu, CS) = \sum_{Q \in \mathcal{T}(v)} \sum_{k=0}^{|CS|-1} \sum_{l=0}^{|Q|-1} \sum_{\substack{T^k \subseteq CS \setminus \mathcal{T}(v) \\ |T^k|=k}} \sum_{\substack{C^l \subseteq Q \setminus \{v\} \\ |C^l|=l}} \alpha_Q(l)\beta(k) \frac{\sum_{u \in V} f(dist(\bigcup T^k \cup C^l \cup \{v\}, u)) - f(dist(\bigcup T^k \cup C^l, u))}{\binom{|CS|-1}{k}\binom{|Q|-1}{l}}.$$

Although this may seem inconsequential, our next step is to rearrange the summation over u and bring it to the forefront:

$$\phi_v(\nu, CS) = \sum_{u \in V} \sum_{! \in \mathcal{T}(v)} \sum_{k=0}^{|CS|-1} \sum_{l=0}^{|Q|-1} \sum_{\substack{T^k \subseteq CS \setminus \mathcal{T}(v) \\ |T^k|=k}} \sum_{\substack{C^l \subseteq Q \setminus \{v\} \\ |C^l|=l}} \alpha_Q(l)\beta(k) \frac{f(dist(\bigcup T^k \cup C^l \cup \{v\}, u)) - f(dist(\bigcup T^k \cup C^l, u))}{\binom{|CS|-1}{k}\binom{|Q|-1}{l}}.$$

Next, for $v \in Q$ we will split the equation into more manageable parts:

$$MC^+_{k,l}(v, u, Q) = \sum_{\substack{T^k \subseteq CS \setminus \mathcal{T}(v) \\ |T^k|=k}} \sum_{\substack{C^l \subseteq Q \setminus \{v\} \\ |C^l|=l}} f(dist(v, u)), \tag{5.2}$$

and

$$MC^-_{k,l}(v, u, Q) = \sum_{\substack{T^k \subseteq CS \setminus \mathcal{T}(v) \\ |T^k|=k}} \sum_{\substack{C^l \subseteq Q \setminus \{v\} \\ |C^l|=l}} f(dist(\bigcup T^k \cup C^l, u)), \tag{5.3}$$

with the additional constraint on T^k and C^l such that:

$$dist(\bigcup T^k \cup C^l, u)) \neq dist(\bigcup T^k \cup C^l \cup \{v\}, u)). \tag{5.4}$$

We can now state the following:

$$\phi_v(\nu, CS) = \sum_{u \in V} \sum_{Q \in \mathcal{T}(v)} \sum_{k=0}^{|CS|-1} \sum_{l=0}^{|Q|-1} \alpha_Q(l)\beta(k) \frac{MC^+_{k,l}(v, u, Q) - MC^-_{k,l}(v, u, Q)}{\binom{|CS|-1}{k}\binom{|Q|-1}{l}}. \tag{5.5}$$

The constraint in Equation (5.4) simply allows us to avoid redundant computations (the marginal contribution of v in the opposite case is trivially zero, since by entering such a coalition, v does not change the distance to u). The remainder of the proof will focus on computing Equations (5.2) and (5.3). We will first focus on Equation (5.2).

Note that due to the constraint in Equation (5.4), we can be sure that

$$f(dist(\bigcup T^k \cup C^l \cup \{v\}, u)) = f(dist(u, v)),$$

since by entering the coalition, v must have brought it closer to u. Thus, it suffices to count the number of coalitions from Q (of size l)—C^l—and coalitions of communities (of size k)—T^k—such that $dist(\bigcup T^k \cup C^l, u)) > dist(v, u)$, since only then Equation (5.4) will hold. To do this, let us introduce the following notation:

- $Com_{\sim d}(u) = \{Q : Q \in M \text{ and } dist(Q, u) \sim d)\}$;
- $Nod^Q_{\sim d}(u) = \{s : s \in Q \text{ and } dist(s, u) \sim d)\}$,

where $\sim$ will be one of $<$, $>$, $\leq$, $\geq$ or $=$. These sets are fairly simple to precompute, and will allow us to count the number of required coalitions. In this particular case, let $d = dist(u, v)$. We will use $Com_{>d}(u)$ to count the number of communities farther from u than v. Similarly, $Nod^Q_{>d}(u)$ counts the number of nodes in the community Q that are farther from u than the distance from u to v. Altogether, there are $\binom{Com_{>d}(u)}{k}$ coalitions of communities of size k and $\binom{Nod^Q_{>d}(u)}{l}$ coalitions from Q of size l satisfying the requirements. Finally:

$$MC^+_{k,l}(v, u, Q) = \binom{Nod^Q_{>d}(u)}{l}\binom{Com_{>d}(u)}{k} f(d).$$

As for Equation (5.3), we divide computations as follows:

$$MC^-_{k,l}(v, u, Q) = \sum_{\substack{d \in dists(u) \\ d > dist(v,u)}} MC^-_{k,l}(v, u, Q, d),$$

where $dists(u)$ is the set of distances of any node to the node u, and

$$MC^-_{k,l}(v, u, Q, d) = \sum_{\substack{T^k \subseteq CS \setminus \mathcal{T}(v) \\ |T^k| = k}} \sum_{\substack{C^l \subseteq Q \setminus \{v\} \\ |C^l| = l}} \alpha_Q(l)\beta(k) \frac{f(dist(\bigcup T^k \cup C^l, u))}{\binom{|CS|-1}{k}\binom{|Q|-1}{l}},$$

keeping in mind the constraint from Equation (5.4) and adding the constraint on C^l and T^k such that

$$dist(\bigcup T^k \cup C^l, u) = d. \tag{5.6}$$

Now, the computation of $MC^-_{k,l}(v, u, Q, d)$ reduces to computing the number of coalitions C^l and coalitions of communities T^k that satisfy both constraints. By using the inclusion-exclusion principle, and assuming that $dist(v, u) < d$, we have the following:

$$MC^-_{k,l}(v, u, Q, d) = \binom{Com_{\geq d}(u)}{k}\binom{Nod^Q_{\geq d}(u)}{l} - \binom{Com_{>d}(u)}{k}\binom{Nod^Q_{>d}(u)}{l}.$$

To explain this, we first compute the number of coalitions of communities of size k that are at distance d or farther from u. We do the same for coalitions of nodes from Q. However, we must take away the number of occurrences when no nodes from C^l and communities from T^k are at exactly distance d from u, in order to satisfy the constraint from Equation (5.6).

Computing all $MC^-_{k,l}(v, u, Q)$ variables is the most time-consuming, as it takes $O(|V|^4|CS|)$ time. This may be counter-intuitive, since computing all of the $MC^-_{k,l}(v, u, Q, d)$ variables would imply a time complexity of $O(|V|^5|CS|)$, but dynamic programming eliminates this need. We introduce algorithms for computing the configuration semivalue generalised closeness centrality. Some required precomputations are presented in Algorithm 4 and the main computations of the centrality is presented in Algorithm 5. □

Theorem 3 *The configuration* value *of group Closeness centrality for all nodes in a weighted graph $G = (V, E, \omega)$ can be computed in $O(|V|^2[\log(|V|) + |CS|] + |V||E|)$ time.*

Proof: We will work in a similar manner to the proof of Theorem 2, but consider an alternate formula for the configuration value. This formula considers permutations, instead of coalitions. The idea is, that a node makes a contribution to the coalition of nodes that precede it in a permutation and the average contribution among all permutations is the configuration value. Moreover, nodes from the same communities must be grouped together as follows:

$$\phi_v(\nu, CS) = \sum_{u \in V} \sum_{Q \in \mathcal{T}(v)} \sum_{\pi' \in \Pi(CS)} \sum_{\pi \in \Pi(Q)} \frac{f(dist(C_{\pi'}(Q) \cup C_\pi(v) \cup \{v\}, u)) - f(dist(C_{\pi'}(Q) \cup C_\pi(v), u))}{|CS|!|Q|!}, \tag{5.7}$$

where $\Pi(X)$ refers to the set of permutations of the set X, and $C_\pi(v)$ is the set of elements preceding v in the permutation π, and $C_{\pi'}(Q)$ is the set of nodes belonging to the communities that precede Q in the permutation π'.

We will introduce the following notation for $Q \in \mathcal{T}(v)$:

$$MC^+(v, u, Q) = \sum_{\pi' \in \Pi(CS)} \sum_{\pi \in \Pi(Q)} f(dist(v, u)) \tag{5.8}$$

and

$$MC^-(v, u, Q) = \sum_{\pi' \in \Pi(CS)} \sum_{\pi \in \Pi(Q)} f(dist(C_{\pi'}(Q) \cup C_\pi(v), u)), \tag{5.9}$$

with the following constraint: $dist(v, u) < dist(C_{\pi'}(Q) \cup C_\pi(v), u)$. If we manage to compute these two values in polynomial time for all $u, v \in V$ and $Q \in \mathcal{T}(V)$, then we will achieve polynomial computation, since we have:

$$\phi_v(\nu, CS) = \sum_{u \in V} \sum_{Q \in \mathcal{T}(v)} \frac{MC^+(v, u, Q) - MC^-(v, u, Q)}{|CS|!|Q|!},$$

The rest of the proof will focus on computing these two values.

$\mathbf{MC^+(v, u, Q)}$: This value corresponds to the number of permutations $\pi' \in \Pi(CS)$ multiplied by the number of permutations $\pi \in \Pi(Q)$ such that $dist(v, u) < dist(C_{\pi'}(Q) \cup C_\pi(v), u)$, multiplied by the value $f(dist(v, u))$. Let $d = dist(v, u)$. First, we must find the number of permutations of communities, such that the

communities before the community Q are further away from u than d. We first choose $Com_{\leq d}(u)$ spots for the communities that are at distance d or closer to u: $\binom{|CS|}{Com_{\leq d}(u)}$. The first community of these must be Q, otherwise our constraint will not hold. All the other communities from this set can be arranged in $(Com_{\leq d}(u) - 1)!$ orders. The rest of the communities can be placed in the remaining slots and arranged in any of $(|CS| - Com_{\leq d}(u))!$ orders. Continuing, we have $\binom{|CS|}{Com_{\leq d}(u)}(Com_{\leq d}(u) - 1)!(|CS| - Com_{\leq d}(u))!$, or simply $\frac{(|CS|)!}{Com_{\leq d}(u)}$ such permutations. Similarly, the number of permutations of nodes within Q is $\frac{(|Q|)!}{Nod^{Q}_{\leq d}(u)}$. In effect, we have:

$$MC^{+}(v, u, Q) = \frac{f(d)(|CS|)!(|Q|)!}{Nod^{Q}_{\leq d}(u)Com_{\leq d}(u)}. \tag{5.10}$$

$\mathbf{MC}^{-}(\mathbf{v}, \mathbf{u}, \mathbf{Q})$: Similarly to the previous proof in this chapter, we introduce the following notation:

$$MC^{-}(v, u, Q, d) = \sum_{\pi' \in \Pi(CS)} \sum_{\pi \in \Pi(Q)} f(dist(C_{\pi'}(Q) \cup C_{\pi}(v), u))$$

with the additional constraint that $dist(C_{\pi'}(Q) \cup C_{\pi}(v), u) = d$ for some d. In effect, we have:

$$MC^{-}(v, u, Q) = \sum_{d \in dists(u) d > dist(v,u)} MC^{-}(v, u, Q, d),$$

where $dists(u)$ is the set of distances from any node to u. We can compute this value using the inclusion/exclusion principle, which results in the following:

$$MC^{-}(v, u, Q, d) = f(d)\left[\frac{(|CS|)!(|Q|)!}{Nod^{Q}_{<d}(u)Com_{<d}(u)} - \frac{(|CS|)!(|Q|)!}{Nod^{Q}_{\leq d}(u)Com_{\leq d}(u)}\right].$$

The precomputations required to compute the configuration value closeness centrality are shared with the configuration semivalue and presented in Algorithm 4. The main computations are presented in Algorithm 6. □

In Algorithm 4, lines 1 to 20 compute node distances using Johnson's algorithm [93] and sort them in a descending order, allowing us to use dynamic programming to

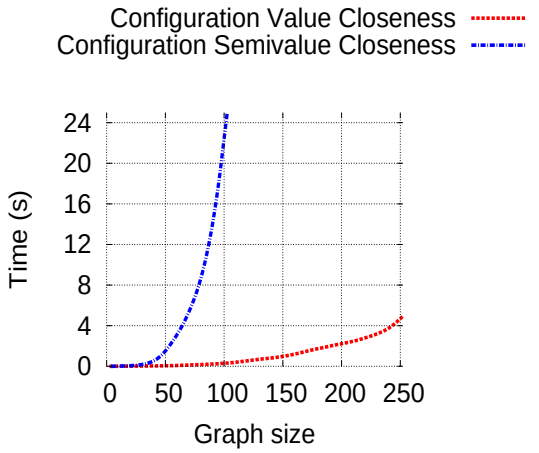

Figure 5.2: Empirical running times for Algorithms 5 and 6.

avoid redundant computation. As an example, if d' directly follows d in such a list, then $COM_{\geq d'}(u) = COM_{=d}(u) + COM_{\geq d}(u)$. The last step removes duplicate distances from the community distances list, which also helps avoid redundant computation.

Algorithm 5 computes marginal contributions by moving backwards (largest to smallest) through the possible distances, and also uses dynamic programming. The data computed thus-far is held in the variable $prev_val$ and accumulated in ϕ_v. $s_merge(a, b)$ is a function that takes two sorted sets (descending order) and returns a sorted set, such that any items in b that are smaller than the smallest item in a are not included. Finally, the array CP must be mentioned, which collects the power of communities as a whole. If a marginal contribution is made by a node through community Q, then CP_Q is incremented by the value of the contribution.

We conclude by noting that the configuration value is a generalisation of the Shapley value. Algorithm 6 can compute the Shapley value-based Closeness centrality with the same time-complexity as the best currently known algorithm due to Michalak et al. [29]. In fact, computing distances between all nodes is the most time-consuming aspect of the algorithm. Empirical running times for random graphs are presented in Figure 5.2.

Algorithm 4: Precomputations for Configuration Semivalue Closeness.

input : Graph $G = (V, E, \omega)$, Closeness function $f : \mathbb{R} \to \mathbb{R}$, Overlapping Community Structure CS, Probability distribution functions $\beta : 0, 1, \ldots |V| - 1 \to \mathbb{R}$, $\forall_{Q \in CS}\ \alpha_Q : 0, 1, \ldots |Q| - 1 \to \mathbb{R}$

output : Configuration Semivalue

$dist[V, V] \leftarrow Johnson(G, \omega)$;
for $v \in V$ **do**
  $COM[v] \leftarrow \emptyset$;
for $v \in V$ **do**
  $\phi_v \leftarrow 0$;
  $distances[v] \leftarrow$ empty ordered set;
  $c_dists[v] \leftarrow$ empty ordered set;
  for $Q \in CS$ **do**
    $CP[Q] \leftarrow 0$; $com_dist[Q][v] \leftarrow \infty$;
    $g_dists[Q][v] \leftarrow$ empty ordered set;
    for $u \in Q$ **do**
      $COM[u] \leftarrow COM[u] \cup Q$;
      $com_dist[Q][v] \leftarrow \min(dist(v, u), com_dist[Q][v])$;
      $distances[v] \leftarrow distances[v] \cup \langle u, dist[u, v] \rangle$;
    $c_dists[v] \leftarrow c_dists[v] \cup \langle Q, com_dist[Q][v] \rangle$;
  $sort_desc(c_dists[v]); sort_desc(distances[v])$;
for $u \in V$ **do**
  for $\langle v, d \rangle \in distances[u]$ **do**
    for $Q \in COM[v]$ **do**
      $g_dists[Q][u] \leftarrow g_dists[Q][u] \cup \langle v, d \rangle$;
for $u \in V$ **do**
  $m1 \leftarrow$ largest distance in $c_dists[u]$; $prev \leftarrow m1$;
  $Com_{>prev}(u) \leftarrow 0$; $Com_{\geq prev}(u) \leftarrow 0$;
  for $\langle Q, d \rangle \in c_dists[u]$ **do**
    $m2 \leftarrow$ largest distance in g_dists[Q][u];
    if $m2 > m1$ **then**
      $Com_{>m2}(u) \leftarrow 0$; $Com_{\geq m2}(u) \leftarrow 0$;
    if $d \neq prev$ **then**
      $Com_{>d}(u) \leftarrow Com_{\geq prev}(u)$;
      $Com_{\geq d} \leftarrow Com_{\geq prev}(u)$; $prev = d$;
    $Com_{\geq d}(u) \leftarrow Com_{\geq d}(u) + 1$;
    $prevnod \leftarrow$ largest distance in $distances[u]$;
    $Nod^Q_{>prevnod}(u) \leftarrow 0$; $Nod^Q_{\geq prevnod}(u) \leftarrow 0$;
    for $\langle v, dnod \rangle \in distances[u]$ **do**
      if $dnod \neq prevnod$ **then**
        $Nod^Q_{>dnod}(u) \leftarrow Nod^Q_{\geq prevnod}(u)$;
        $Nod^Q_{\geq dnod}(u) \leftarrow Nod^Q_{\geq prevnod}(u)$;
        $prevnod = dnod$;
      if $Q \in COM(v)$ **then**
        $Nod^Q_{\geq dnod}(u) \leftarrow Nod^Q_{\geq dnod}(u) + 1$;
for $u \in V$ **do**
  $remove_repeated_distances(c_dists[u])$;

Algorithm 5: Efficient Algorithm for Configuration Semivalue Closeness (continued from Algorithm 4).

for $u \in V$ **do**
  for $Q \in CS, k \in [0, |CS|), l \in [0, |Q|)$ **do**
    $prev_d \leftarrow -1$; $prev_val \leftarrow 0$; $MC^- \leftarrow 0$;
    for $\langle x, d\rangle \in s_merge(g_dists[Q][u], c_dists[u])$ **do**
      if $x \in V$ *and* $prev_d \neq -1$ **then**
        $Com_{\geq d}(u) \leftarrow Com_{\geq prev_d}(u)$;
        $Com_{>d}(u) \leftarrow Com_{>prev_d}(u)$;
      if $prev_d \neq d$ **then**
        $MC^- \leftarrow prev_val$; $prev_d \leftarrow d$;
        $prev_val \leftarrow$
        $prev_val + \left[\binom{Com_{\geq d}(u)}{k}\binom{Nod^Q_{\geq d}(u)}{l} - \binom{Com_{>d}(u)}{k}\binom{Nod^Q_{>d}(u)}{l}\right] f(d)$;
      if $x \in V$ **then**
        $MC^+ \leftarrow f(d)\binom{Com_{>d}(u)}{k}\binom{Nod^Q_{>d}(u)}{l}$;
        $\phi_x \leftarrow \phi_x + \beta(k)\alpha_Q(l)\frac{MC^+ - MC^-}{\binom{|CS|-1}{k}\binom{|Q|-1}{l}}$;
        $CP_Q \leftarrow CP_Q + \beta(k)\alpha_Q(l)\frac{MC^+ - MC^-}{\binom{|CS|-1}{k}\binom{|Q|-1}{l}}$;

rank	Harmonic	Shapley Value	Configuration Value
1.	Świętokrzyska	Politechnika	Centrum
2.	Ratusz	PKP Falenica	Dw.Wileński
3.	Ron.Starzyńskiego	Erazma z Zakr.	PKP Falenica
4.	Dw.Wileński	PKP Radość	Świętokrzyska

Table 5.2: *Top four hubs in the Warsaw transportation network, according to different Closeness measures.*

node	Harmonic	Shapley value	Configuration value	lines
Centrum	234	0.23	4.96	68
Świętokrzyska	235	0.50	4.22	29
Politechnika	224	1.04	3.07	17
PKP Falenica	108	1.00	4.91	8
PKP Radość	122	0.80	3.45	8

Table 5.3: *Properties of hubs in Warsaw.*

5.4 Warsaw Public Transportation Network

Algorithms 4, 5 and 6 were implemented in Java by Piotr Szczepański with the aid of Mateusz Tarkowski. The configuration value-based harmonic closeness centrality

Algorithm 6: Efficient Algorithm for Configuration Value Closeness (continued from Algorithm 4).

```
1  for u ∈ V do
2      for Q ∈ CS do
3          prev_d ← −1; prev_val ← 0; MC⁻ ← 0;
4          for ⟨x, d⟩ ∈ s_merge(g_dists[Q][u], c_dists[u]) do
5              if x ∈ V and prev_d ≠ −1 then
6                  Com_{≤d}(u) ← Com_{≤prev_d}(u);
7                  Com_{<d}(u) ← Com_{<prev_d}(u);
8              if prev_d ≠ d then
9                  MC⁻ ← prev_val; prev_d ← d;
10                 prev_val ← prev_val + [ f(d)/((Nod^Q_{<d}(u))(Com_{<d}(u))) − f(d)/((Nod^Q_{≤d}(u))(Com_{≤d}(u))) ];
11             if x ∈ V then
12                 MC⁺ ← f(d)/((Nod^Q_{≤d}(u))(Com_{≤d}(u)));
13                 φ_x ← φ_x + MC⁺ − MC⁻;
14                 CP_Q ← CP_Q + MC⁺ − MC⁻;
```

(i.e., CV-based harmonic closeness centrality) was used to analyse the Warsaw public transportation network.[2] Edge-weights were defined as the average travel times between nodes. In total, the weighted network consists of 1425 nodes, 2135 edges and 380 communities formed by bus lines, trams, and underground and suburbia trains. We note that the ranking of nodes according to our Configuration value harmonic closeness differs significantly from the Harmonic Closeness and Shapley value Harmonic Closeness rankings. We present the four most prominent nodes according to these centralities in Table 5.2. Each of the three rankings according to each centrality and the ranking of each community (i.e., bus, tram metro or train line) can be found in Appendix C.

Configuration-based Closeness ranks *Centrum* (or "city center") as the most important. This result is intuitive and makes sense, since this stop is a large hub in downtown Warsaw, where passengers can choose from 68 bus, tram and train connections, and one metro line (see Table 5.3). The top ranked nodes according to the other centrality measures are also near the city center, but provide less

[2]The data, experiment results and programs can be downloaded from https://github.com/szczep/gtna.

Line	Community Index
401	4.35
ZS1	3.68
409	3.61
M1	3.51

Table 5.4: *Important lines.*

connections that are not as important as those in the city center. Were this stop to shut down, this would dramatically disrupt the rest of the network.

On the other hand, the rankings for *PKP Falenica* and *PKP Radość*, which are railway stations far from downtown Warsaw, are rather surprising. On further analysis, however, it becomes clear why they are ranked so high. Both stops are important because—for certain source-destination pairs—it is almost impossible to find routes that omit these stations. Additionally, trains play an important role in shortening the travel time between distant nodes, since they are the fastest means of transportation. In this case, both train stations connect parts of Warsaw to which it would take much longer to travel to from other districts (such as the city center) without a connection by train.

Interestingly, the fifth node according to configuration value harmonic closeness, *Płowiecka*, is not in the top ten according to the other measures. Harmonic Closeness misses the fact that this stop is not easily replaced. Shapley value-based Closeness misses that the bus lines that stop at *Płowiecka* are very important. Even if the stop is omitted, a traveler must often still use bus lines that visit *Płowiecka* for further travel. As for communities—unsurprisingly—long routes that bring in commuters from all of Warsaw (including the metro line M1) are ranked as most important.

Harmonic Closeness centrality prioritises the topologically most *central* nodes. The Shapley value harmonic centrality considers a new dimension, promoting irreplaceable nodes. We go one step further and provide a new configuration value-based measure that promotes hubs with powerful connections, while taking into account the previous two considerations. Importantly, only our measure is able to detect the most central and popular hub in Warsaw.

5.5 Conclusions

We have developed a general class of solution concepts—configuration semivalues—that encompasses both coalitional semivalues [31] and the configuration value [42]. We used this class in order to develop the first network centrality measure that accounts for an overlapping community structure. We based our centrality on the notion of closeness, developed polynomial-time algorithms for its computation and used it to analyse the Warsaw public transportation network. In the next chapter, we develop the semivalue closeness interaction indices for link prediction.

6

Link Prediction in Networks

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6.1 Introduction

In this chapter we are concerned with link prediction—an important research problem in social network analysis [94–96]. Its aim is to reason about between which pairs of nodes unknown links exist or should form based on the known structural characteristics of the network. Applications include biological networks, where some of the network is known, but checking whether other links actually exist is very costly (e.g., food web, protein-protein and metabolic networks [96]); social networks, where certain information about links is impossible to attain or verify

(e.g., terrorist networks [95]); and identifying potentially fruitful collaboration in organisations [95]. Predicting whether links exist or will form between any pair of nodes is strongly associated with the concept of how similar they are [96]. If two nodes are similar, then there is a greater chance that they are connected. The three most common approaches to computing node similarity take into account either local, quasi-local or global topological information about nodes. In general, global methods produce better results than local ones, but are more computationally involved. Furthermore, quasi-local methods, which only consider the network within a certain radius k around the pair of nodes, can sometimes do better than global ones [96]. This is because it is unlikely that features of the network that are far away from a pair of nodes that are taken into account by global methods will actually impact the chance of those two nodes being connected.

Recently, Szczepański et al. [32] proposed a new method to tackle the link prediction problem based on group r-degree centrality and the concept of the game-theoretic interaction index. In particular, group r-degree centrality is a valuation of groups of nodes according to the number of nodes that are at a distance r or closer to the group (not including the members of the group themselves). In turn, the interaction index can be interpreted as a measure of the synergy or similarity of players in a cooperative game. When applied to group r-degree centrality, the interaction index becomes a sophisticated measure of the similarity of the topological placement of nodes. In particular, two nodes produce negative synergy according to group r-degree centrality whenever they have common neighbours. The interaction index is used to measure such negative synergies between any pair of nodes in the context of all the possible groups of nodes that these two nodes belong to. Considering all these groups allows the measure to prioritise the impact of unique common neighbours within the context of various groups of nodes. The intuition behind considering all groups of nodes is as follows: if most of the nodes in a network neighbour a certain node, then this does not necessarily mean that they are similar; on the other hand, if only two nodes neighbour a certain node, then this is a unique characteristic shared only by these two nodes and may suggest that they are similar.

Szczepański et al. [32] show that their approach produces a very competitive link prediction method when compared to a selection of quasi-local measures. The authors develop an algorithm for computing semivalue interaction indices of r-degree centrality that runs in $O(|V|^4)$ and an algorithm specifically for the Shapley value interaction index, which runs in $O(|V|^3)$ time.

In what follows, we propose an alternative game-theoretic method to tackle the link prediction problem. Our method builds upon the following intuition: while the r-degree centrality used by Szczepański et al. [32] postulates that all nodes within a distance r of a pair of nodes impact their similarity equally, we postulate that those that are farther away should be considered less important than those that are closer to the pair. In order to develop a method based on this intuition, we use the concept of generalised group closeness centrality, which has exactly this property: *the magnitude of the impact of a node on the value of a group reduces with an increased distance from this group.* As compared to r-degree centrality, this allows us to reduce the impact of far-away nodes on synergy. As we are interested in developing a quasi-local measure, similarly to r-degree centrality, we introduce a restriction on the generalised group closeness centrality such that only nodes inside of a radius r around a given group impact the value of this group.

Interestingly, despite the fact that our method is more general that the one developed by Szczepański et al. [32], our algorithm for computing it is less computationally complex. In particular, we compute our measure given any generalised closeness semivalue interaction index with a complexity of $O(V_r^2|V|^2)$, where V_r is the average number of nodes within a distance r of any node. We also develop an algorithm to compute the generalised closeness Shapley value interaction index that runs in $O(V_r^2|V| + |V|^2)$ time. We show that often V_r is much smaller than $|V|$. The generality of our algorithm stems from the fact that we allow for any function of distance, f, to be used with our measure. For $f(d) = 1$ whenever $d < r$, our measure is equivalent to the r-degree interaction index.

We evaluate our approach by considering 11 real-life networks on which we run a series of experiments. We find that—in most cases—our approach produces equal or better results than the state-of-the-art quasi-local measures in the literature. Among others, it outperforms the game-theoretic method proposed by Szczepański et al. [32].

Furthermore, we consider an aspect of quasi-local measures that has thus-far been overlooked in the literature. In particular, most authors present the result of their measure given an optimal parameter r [30, 32, 96]. However, the optimal value of this parameter is unknown and, to the best of our knowledge, there exist no straightforward methods to find it for an arbitrary network. Therefore, we study how the measures perform given a selection of values for the parameter r, and not only the optimal value of this parameter.

Two cases can be distinguished. If a value for r is chosen that is smaller than the optimal value of this parameter, then this results in a computational/accuracy trade-off. The measure can usually be computed faster, but it could perform better by having more information about the network (i.e., a larger value for r). For large networks, it may simply take too long to compute a quasi-local measure with an optimal r, and such a trade-off is necessary. In the worst case, $r = 1$ may be chosen, and the quasi-local measure becomes local. On the other extreme, if r is equal to the diameter of the network, then it can be said that the quasi-local measure becomes global.

What happens, then, if the value of r is larger than the optimal value? In our study, we show that such a k can result in a significant reduction of the quality of the similarity ranking for quasi-local measures. We find, however, that our proposed measure does not suffer from this phenomenon to such an extent. For example, in one network that we study—the Football network [97, 98]—the precision of the ranking produced by the measure due to Szczepański et al. [32] decreases by approximately 37% when the value of r is 3 instead of the optimal value 1, whereas the precision of our measure decreases by less than 1%. Interestingly, we also note that there are cases where the quality of other measures is reduced given a larger r,

while the quality of our measure improves. This means that whereas other measures pick up more noise given more information about the network, our measure is still able to gather more insight in order to improve the ranking of potential edges.

6.2 Performance Metrics for Link Prediction

In this section, we present the main metrics used to measure the performance of link prediction methods. The first of these is the *area under curve* (AUC), and the second is *precision* [96].

The goal of a link prediction algorithm is to successfully identify those pairs of nodes in a graph that are not connected by an edge but which actually do exist in the real world. For example, people in an online social network are typically connected by their friendships. However, this is not to say that people who have not indicated their friendship via the social platform are not actually friends. Ideally, a link prediction algorithm would identify those pairs of individuals who are friends in real-life but not on the social network.

In order to test link prediction methods, it is typical to take a real life network and remove a certain percent of links from it [30, 32, 96]. Next, this altered network (with removed links) is given as input to a link-prediction algorithm, which ranks all disconnected pairs of nodes in terms of the likelihood that they belong to the "removed," or "missing" set.

6.2.1 Area Under Curve

The area under curve is a good overall indicator of the quality of the ranking of edges that is produced by a link prediction algorithm. Formally, let m be the number of edges in the network that are "missing." In other words, these edges do not exist in the graph that the link-prediction algorithm received as input, but do exist in the real world. These are the edges that we would like a link-prediction algorithm to discover and therefore give them a high rank. Let l be the number

of edges that do not exist in the graph that are not missing. In other words, these are the edges that we would like a link-prediction algorithm to recognise as non-existent and therefore give them a low rank.

There are a total of $n = m \times l$ comparisons between missing and non-existent edges. Let n' be the number of such comparisons where a missing edge is ranked over a non-existent edge, and n'' be the number of comparisons where a missing edge is given the same rank as a non-existent edge. AUC, then, is defined as follows:

$$AUC = \frac{n' + \frac{n''}{2}}{n}$$

If all missing edges are ranked higher than non-existent edges, the resulting AUC is equal to 1. If the opposite is true (i.e., all non-existent edges are ranked higher than all missing edges), then this results in an AUC of 0. A random link-prediction algorithm results in an average AUC of 0.5. In short, AUC is the percentage of comparisons between ranked edges that are "correct." For this reason, we present AUC in this thesis as a percentage value.

6.2.2 Precision

For a given $i \in \mathbb{N}$, let $Top(i)$ be the set of top i edges according to the ranking generated by a link-prediction algorithm. Let $Correct(i)$ be the set of edges in $Top(i)$ that are "missing" in the network (i.e., they are not present in the graph, but do exist in the real-world). In other words, these are the edges that the algorithm guessed correctly. The precision of the algorithm, then, is defined as follows:

$$Precision(i) = \frac{Correct(i)}{i}$$

If all of the top i edges ranked by an algorithm are missing edges, then the precision is equal to 1. If none of them are, then the precision is equal to 0. Note that this metric is dependent on the variable i, which indicates the depth to which the ranking is studied. In particular, this depth should never be higher than the

actual number of missing edges. Whereas previous comparisons have focused on a static depth, such as 100 [96], we take a different approach. Since the sizes of our networks vary greatly, having a static depth makes no sense. In particular, some networks that we study have fewer than 100 edges. On the other hand, for other large networks taking just the top 100 ranked edges may not be a good indicator of the quality of the ranking. This is why we take i to be equal to the number of missing edges in the network. In short, precision is the percent of the top i ranked edges that are ranked correctly. For this reason, we also present it as a percentage value. In the edge case where there is a tie in the ranking between x missing edges and y non-existent edges and there are z edges ranked above them such that $z + x + y > i$ and $z < i$, then precision is defined as $\frac{Correct(z)}{i} + \frac{x}{(x+y)i}$. In other words, this is the expected result of ranking the z ties in a random order.

6.3 Semivalue Closeness Interaction Index & Its Computation

In this section, we introduce our family of quasi-local measures for link prediction. Generally, quasi-local measures are characterised by requiring a parameter, r. When a quasi-local algorithm evaluates the likelihood of an edge existing between the nodes u and v, it only ever considers the nodes that are at distance of at most r away from u or v. Conversely, local algorithms are equivalent to quasi-local algorithms with $r = 1$ and global algorithms consider the whole graph in evaluating the likelihood of an edge existing.

Following Szczepański et al. [32], we apply the family of solution concepts known as semivalue interaction indices (see Equation (2.14) in Chapter 2 for more details) to a group centrality measure as a means to measure the similarity of disconnected nodes. Whereas Szczepański et al. used group r-degree centrality, we use a broader class of group centrality measures—general group closeness centrality:

$$\nu_f^{CL}(C) = \sum_{v \in V} f(dist(C, v)).$$

In particular, if for any natural number r we define f as

$$f(d) = \begin{cases} 1 & \text{if } d \leq r \\ 0 & \text{otherwise,} \end{cases}$$

then ν_f^{CL} is equivalent to r-degree centrality. To develop our measure, however, we use the following distance function:

$$f(d) = \begin{cases} \frac{1}{d^2} & \text{if } d \leq r \\ 0 & \text{otherwise.} \end{cases}$$

This choice has two benefits:

1. It retains the computational advantage of r-degree group centrality, whereby faraway nodes do not impact the ranking. This not only improves computational performance, but also the accuracy of the resulting similarity measure, since Szczepański et al. [32] and Szczepanski et al. [30] showed that faraway nodes are less likely to impact the similarity of nodes and therefore reduce the accuracy of the measure. In fact, this is why some quasi-local measures outperform global ones in certain networks [96].

2. Out of the nodes that are meaningful for link-prediction, it is likely that common first-order relations are more meaningful than those of a higher order. For this reason, it makes sense to use a function such as $\frac{1}{d^2}$ for those nodes where $d < r$. We also considered functions such as $\frac{1}{d}$ or $\frac{1}{2^d}$, but found that $\frac{1}{d^2}$ produced the best results.

Let us now introduce our main computational results. We start off by proving that the generalised closeness semivalue interaction index can be computed in polynomial time of $O(V_r^2|V|^2)$. Next, we consider a particular case of this general result, i.e., the generalised closeness Shapley value interaction index. We prove that it can be computed even faster, in $O(V_r^2|V| + |V|^2)$ time. Importantly, for both algorithms, we leave the choice of f open, meaning that the analysis and algorithms can be used with any decreasing function.

Theorem 4 *$I_{s,t}^{Semi}(\nu_f^{CL})$ can be computed in $O(V_r^2|V|^2)$ time.*

Proof: Our goal is to compute Equation (2.14)—

$$I_{s,t}^{SEMI}(\nu_f^{CL}) = \sum_{k=0}^{|V|-2} \sum_{C \in C^k(V \setminus \{s,t\})} \beta(k) \frac{S(C,s,t)}{\binom{|V|-2}{k}} \tag{6.1}$$

—in polynomial time. We will be counting the number of coalitions for which certain common expressions appear in this sum. By multiplying these expressions by their number of appearances, we will achieve polynomial computation. Let us first look closer at the definition of ν_f^{CL} in Equation (2.17). In particular, the equation itself consists of a sum over nodes u. We will only focus on one of these elements at a time, keeping in mind that $\sum_{u \in V} I_{s,t}^{SEMI}(f(dist(C,u)) = I_{s,t}^{SEMI}(\nu_f^{CL})$. Let us define $\nu_u = f(dist(C,u))$ for the remainder of our proof and focus on computing $I_{s,t}^{SEMI}(\nu_u)$ for some s and t.

Moreover, we will focus on computing the inner sum of Equation (2.14)—

$$\sum_{C \in C^k(V \setminus \{s,t\})} \beta(k) \frac{S(C,s,t)}{\binom{|V|-2}{k}}$$

—for an arbitrary k, and our resulting algorithm will then sum the value of this inner part for all k such that $0 \leq k \leq |V| - 2$.Recall from Equation (2.4), that $S(C,s,t)$ consists of three parts: $MC(C,\{s\})$, $MC(C,\{t\})$ and $MC(C,\{s,t\})$. This is where our first important observation takes place. Without loss of generality, assume that $dist(s,u) \leq dist(t,u)$. This means that $dist(s,u) = dist(\{s,t\},u)$ and, consequently, $MC(C,\{s\}) = MC(C,\{s,t\})$. We arrive at the following simplification: $S(C,s,t) = -MC(C,\{t\}) = -\Big(\nu(C \cup \{t\}) - \nu(C)\Big)$. Moreover, only coalitions C such that $dist(C,u) > dist(t,u)$ matter (since otherwise $S(C,s,t) = 0$). In effect, we arrive at the following simplification:

$$\sum_{C \in C^k(V \setminus \{s,t\})} -\beta(k)\Big(\frac{\nu_u(\{s\})}{\binom{|V|-2}{k}} - \frac{\nu_u(C)}{\binom{|V|-2}{k}}\Big)$$

.

We will use the following notation:

$$MC^+(s,t,u,k) = \sum_{C \in \{C : C \in C^k(V \setminus \{s,t\}) \text{ and } dist(C,u) > dist(s,u)\}} \beta(k) \frac{\nu_u(\{s\})}{\binom{|V|-2}{k}}$$

and

$$MC^{-}(s,t,u,k) = \sum_{C \in \{C : C \in C^k(V \setminus \{s,t\}) \text{ and } dist(C,u) > dist(s,u)\}} \beta(k) \frac{\nu_u(C)}{\binom{|V|-2}{k}}$$

The rest of the proof will focus on computing these two values. In order do this, let us first introduce the following notation:

$$Nod_{\sim d}(u) = \{v : v \in V \text{ and } c \sim u\},$$

where $\sim$ is one of $<$, $>$, $\leq$, or $\geq$.

$\boldsymbol{MC^{+}(s,t,u,k)}$: Let $d = dist(t,u)$. Computing $MC^{+}(s,t,u,k)$ is equivalent to computing the following $|\{C : C \in C^k(V \setminus \{s,t\}) \text{ and } dist(C,u) > d\}|$ and multiplying this value by $\beta(k)\frac{f(d)}{\binom{|V|-2}{k}}$. We need to count the number of coalitions C of size k such that $dist(C,u) > d$. Counting the number of such coalitions is as simple as the number of ways to choose k elements from $Nod_{>d}$. In other words:

$$\binom{Nod_{>d}}{k}$$

This gives us the desired result:

$$MC^{+}(s,t,u,k) = \beta(k) \frac{f(d)}{\binom{|V|-2}{k}} \binom{Nod_{>d}}{k}$$

$\boldsymbol{MC^{-}(s,t,u,k)}$: Let us define

$$MC^{-}(s,t,u,k,d) = \sum_{C \in \{C : C \in C^k(V \setminus \{s,t\}) \text{ and } dist(C,u) = d\}} \beta(k) \frac{\nu_u(C)}{\binom{|V|-2}{k}}.$$

We now have $MC^{-}(s,t,u,k) = \sum_{d \in \{d : d \in dists(u) \text{ and } d < dist(s,u)\}} MC^{-}(s,t,u,k,d)$. We therefore have to find the number of coalitions of size k such that $dist(C,u) = d$. In other words, they need to have at least some node at distance d from u and no nodes that are closer. The answer is as follows:

$$\binom{Nod_{\geq d}}{k} - \binom{Nod_{>d}}{k}$$

This gives us the desired result and completes our proof:

$$MC^{-}(s,t,u,k,d) = \frac{f(d)}{\binom{|V|-2}{k}}\left(\binom{Nod_{\geq d}}{k} - \binom{Nod_{>d}}{k}\right)$$

Algorithm 7 implements certain precomputations that are required to compute the index, and Algorithm 8 computes the generalised closeness semivalue interaction index in the required time. □

As for the generalised closeness Shapley value interaction index, the following result holds.

Theorem 5 *$I_{s,t}^{Shapley}(\nu_f^{CL})$ can be computed in $O(V_r^2|V| + |V|^2)$ time.*

Proof: Our proof will be based on dissecting Equation (2.13) from Chapter 2. First, note that this equation is a sum of multiple expressions over various permutations. To achieve polynomial computation, we will group expressions that are equal to one another and count how many permutations these expressions appear in. Finally, by multiplying the expressions by their respective number of appearances we will achieve polynomial computation.

As in the previous proof, we will focus on computing the interaction index of ν_u rather than ν_f^{CL}. First, let us dissect the formula for $I_{s,t}^{Shapley}(\nu_u)$ into smaller parts. Also, as previously, We will assume without loss of generality that $dist(s,u) \leq dist(t,u)$, meaning that $S(C,s,t) = MC(C,t)$. Moreover, only coalitions C such that $dist(C,u) > dist(t,u)$ matter (since otherwise $S(C,s,t) = 0$).

Continuing, we note that $MC(C,\{t\}) \neq 0$ if and only if $dist(t,u) < dist(C,u)$. In this case $\nu(C \cup \{t\}) = \nu(\{t\})$ is independent of C. We refer to this as the left, or *positive*, part of the sum that constitutes the marginal contribution. We refer to $\nu(C)$ as the negative part. We will use the following notation:

$$MC^{+}(t,u) = \sum_{\pi \in \{\pi : \pi \in \Pi(V^{s \wedge t}) \text{ and } dist(t,u) < dist(\pi_t,u)\}} \nu_u(\{t\})$$

and

$$MC^{-}(t,u) = \sum_{\pi \in \{\pi : \pi \in \Pi(V^{s \wedge t}) \text{ and } dist(t,u) < dist(\pi_t,u)\}} \nu_u(C_\pi(\{s,t\})),$$

where $C_\pi(t)$ is the set of elements in the permutation π that precedes t and arrive at the following simplification:

$$I_{s,t}^{Shapley}(\nu_u) = -\frac{(MC^{+}(t,u) - MC^{-}(t,u))}{(|V|-1)!}.$$

The remainder of the proof will focus on computing $MC^{+}(t,u)$ and $MC^{-}(t,u)$.

$\boldsymbol{MC^{+}(t,u)}$ **:** Our goal here is to simply find the number of permutations $\pi \in \Pi(V^{s \wedge t})$ such that $dist(t,u) < C_\pi(\{s,t\})$ and multiply this number by $\nu(\{t\})$. Let $d = dist(t,u)$. We can construct all such permutations in the following manner:

1. First, choose $Nod_{\leq d}(u) - 1$ positions (we have to subtract 1, since s and t are treated as one node) for all of the nodes in $V^{s \wedge t}$ that are as close to u as t or closer. Out of all of these positions, t has to be first, otherwise $dist(t,u) < C_\pi(t)$ will not be satisfied. There are $\binom{|V|-1}{Nod_{\leq d}(u)-1}$ ways that we can choose the positions and all of the nodes except the first can be permuted in $(Nod_{\leq d}(u) - 2)!$ ways.

2. Next, the rest of the nodes are placed in the rest of the positions, which can be permuted in $(|V| - 1 - (Nod_{\leq d}(u))! - 1)$ ways.

When we combine both steps, there are $\binom{|V|-1}{Nod_{\leq d}(u)-1}(Nod_{\leq d}(u)-2)!(|V|-Nod_{\leq d}(u))!$ such permutations, which simplifies to:

$$MC^{+}(t,u) = \frac{(|V|-1)!}{Nod_{\leq d}(u) - 1}$$

***MC*$^-$*(t, u)* :** For this case, let us introduce the following notation

$$MC^-(t,u,d) = \sum_{\pi \in \{\pi : \pi \in \Pi(V^{s \wedge t}) \text{ and } dist(C_\pi(\{s,t\}),u) = d\}} \nu_u(C_\pi(\{s,t\}))$$

and let $dists(u)$ be the set of distances from any node to u. In effect, we have

$$MC^-(t,u) = \sum_{d \in \{d : d \in dists(u) \text{ and } dist(t,u) < dist(d,u)\}} MC^-(t,u,d).$$

For a given d, then, let us focus on computing $MC^-(t,u,d)$. This is slightly more difficult than the previous case, since we need to capture all the permutations π for which the coalition of nodes preceding $\{s,t\}$ (i.e., $C_\pi(\{s,t\})$ is exactly at distance d from u. The requirement can be summarised as follows: there needs to be at least one node x preceding $\{s,t\}$ such that $dist(x,u) = d$ and no nodes that are closer than x to u preceding t. This can be counted using the inclusion/exclusion principle. If we take all permutations such that $dist(C_\pi(\{s,t\}),u) \geq d$ and subtract from the those such that $dist(C_\pi(\{s,t\}),u) > d$, then we have what we are looking for. We can use the techniques for counting $MC^+(t,u)$ to arrive at the following:

$$MC^-(t,u,d) = \frac{(|V|-1)!}{Nod_{<d}(u) - 1} - \frac{(|V|-1)!}{Nod_{\leq d}(u) - 1}$$

Algorithm 7 implements certain precomputations that are required to compute the index, and Algorithm 9 computes the generalised closeness semivalue interaction index in the required time. □

Since our Algorithms require information about the distances between certain nodes to be presented in a sorted fashion and then this information is used multiple times, it is advisable to compute these sorted vectors in a precomputation phase. Furthermore, since these precomputations are common between both the generalised closeness Shapley value interaction index and generalised closeness semivalue interaction index, we present these in a common pre-computation algorithm the pseudo-code of which can be found in Algorithm 7. Algorithms 8 and 9 continue where the precomputations leave off to compute their respective indices.

In particular, pre-computation Algorithm 7 uses a modified Dijkstra's algorithm [84] in order to compute the distance between the r nearest nodes to each node. This allows for a running time of $O(|V|(V_r \log(V_r) + E_r))$, where V_r is the average number of nodes at distance r from any node, and E_r is the average number of edges within a distance of r around any node. The algorithm also sets up the data structures required for the actual computation of the interaction index. Most importantly, for every node v, an ordered set of the nodes that are at a distance of at most r is created, and the interaction index of every pair of nodes is set to 0. In fact, setting every index initially to 0 is one of the most time consuming aspects of the algorithm, and takes $O(|V|^2)$ time. In general, if the algorithm is to rank every disconnected pair of nodes, then this is unavoidable. Algorithms 8 and 9 use dynamic programming in order to compute the negative part of the marginal contributions (MC^- from our proof). Algorithm 8 runs in $O(|V|^2 V_r^2)$ time, and Algorithm 9 runs in $O(|V|V_r^2 + |V|^2)$ time.

We note that, given the parameter

$$f(d) = \begin{cases} 1 \text{ when } d \leq r \\ 0 \text{ otherwise,} \end{cases}$$

our similarity measure is equivalent to the r-Degree Shapley interaction index. Our algorithm for computing the Shapley value closeness interaction index, however, is more general and sophisticated than the one presented by Szczepański et al. [32]. Most importantly, the algorithm presented by Szczepański et al. [32] requires the evaluation of the ranking between all disconnected pairs of nodes. If, however, there is node s within a distance, r, of both u and v, then the interaction index between the two nodes is 0. The authors quote the complexity of their algorithm as $O(|V|^3)$.

Remark 1 *We note that the authors did not take into account the time complexity of finding the intersection of two sets. It is actually possible to compute the intersection of sets in linear, i.e., $O(V_k)$, time, which results in an algorithm with an overall time complexity of $O(|V|^2 V_k)$. This, however, first requires the neighbour sets to be sorted according to their index, which the authors do not mention.*

As opposed to considering every pair of nodes and then every node s within the radius r of the pair, our algorithm reverses this, and first considers any node s and then all pairs of nodes within its vicinity. In doing this, we avoid altogether querying pairs of nodes that are far away (except to first initialise the distance between each node to infinity). Combined with our restricted Dijkstra algorithm, this results in a total time complexity of $O(|V|V_r^2 + |V|^2)$. Although this may seem to be a marginal difference when compared with the algorithm due to Szczepański et al. [32], we note that for many networks V_r tends to be much smaller than $|V|$, resulting in a faster running time of our algorithm.

We note also that when V_r approaches $|V|$ (equivalently when r approaches the diameter of the network), r-degree stops being useful as a measure, since its aim is to capture those nodes that are close to each other, and for large enough r it will decide that all nodes are close to each other (for a connected network). Consequently, the interaction index according to r-degree between all nodes is of no use as a measure for large r, since all nodes interact equally. In short, when V_r is small, then our algorithm has a computational advantage, and when V_r is large then the ranking produced by our algorithm is likely to be superior since we account for the distance between nodes (i.e., when far-away, insignificant nodes are considered given a large r, then they have little impact on the ranking).

We study the running time of both algorithms using randomly generated graphs according to the preferential attachment (PA) model due to Barabási and Albert [99]. In particular, we study two cases: a relatively sparse network, where we start with a clique of 3 nodes and in each iteration add a node with 2 edges, and a denser network that starts with a clique of 5 nodes and with each node adds 3 edges. The running times of both algorithms are presented in Tables 6.1 and 6.2, where m_0 is the size of the initial clique and m is the number of edges added during each iteration of the preferential attachment algorithm. We note that the comparison is heavily dependent on our implementation of the algorithms, and that we implemented the algorithm due to Szczepański et al. [32] with the benefit of our restricted Dijkstra algorithm. We note that although our algorithm is significantly

$\|V\|$	r	V_r	Algorithm 9	Szczepański et al. [32]
500	1	3.788	3.153	21.789
	2	18.2011	17.417	32.648
	3	65.1542	106.436	107.116
400	1	3.785	2.331	16.412
	2	17.9544	12.295	24.916
	3	57.734	70.226	72.582
300	1	3.78	1.669	10.096
	2	17.1246	9.199	15.626
	3	60.0592	50.777	49.086
200	1	3.77	1.053	6.169
	2	15.5977	5.517	8.668
	3	46.0083	26.046	25.923
100	1	3.74	0.476	2.874
	2	12.0278	2.324	3.416
	3	30.4716	8.147	8.757

Table 6.1: Average running time (in milliseconds) of Algorithm 9 compared to Szczepański et al. [32] for 1000 random PA graphs using the parameters $m_0 = 3$ and $m = 2$.

$\|V\|$	r	V_r	Algorithm 9	Szczepański et al. [32]
500	1	5.184	4.14	39.241
	2	38.5843	39.503	66.001
	3	158.143	360.747	318.472
400	1	5.18	3.437	25.969
	2	34.162	27.707	45.686
	3	139.181	251.65	206.952
300	1	5.1733	2.329	18.799
	2	33.2759	21.42	31.384
	3	133.982	160.85	140.103
200	1	5.16	1.397	10.756
	2	26.6135	11.172	17.757
	3	88.1525	77.701	75.709
100	1	5.12	0.691	5.189
	2	22.4189	4.253	6.685
	3	65.1413	16.191	16.797

Table 6.2: Average running time (in milliseconds) of Algorithm 9 compared to Szczepański et al. [32] for 1000 random PA graphs using the parameters $m_0 = 5$ and $m = 3$.

faster given a sparse network or low radius r, due to the more complicated nature of our algorithm it actually becomes somewhat slower given a high enough V_r. This is because our algorithm performs more complicated operations, which cannot be expressed by its asymptotic complexity alone.

Algorithm 7: Precomputations.

input : Graph $G = (V, E, \omega)$, Closeness function $f : \mathbb{R} \to \mathbb{R}$, Probability distribution function $\beta : 0, 1, \ldots |V| - 1 \to \mathbb{R}$, radius r

output : Configuration Semivalue

```
1  dist[V][V];
2  for v ∈ V do
3  |  for u ∈ V do
4  |  |  dist[v][u] = ∞
5  |  distance[v] ← empty set;
6  |  visited ← empty set;
7  |  φ_v ← 0;
8  |  Q ← Priority Queue;
9  |  Q.enqueue(⟨v, 0⟩);
10 |  dist[v][v] = 0;
11 |  while Q not Empty do
12 |  |  ⟨u, d⟩ ← Q.pop();
13 |  |  II[v, u] = 0;
14 |  |  visited.insert(u);
15 |  |  for s ∈ E(u) do
16 |  |  |  if (s ∉ visited or dist[v][s] > dist[v][u] + ω(u, s)) and
   |  |  |    (dist[v][u] + ω(u, s) ≤ r) then
17 |  |  |  |  dist[v][s] = dist[v][u] + ω(u, s);
18 |  |  |  |  Q.enqueue(⟨s, dist[v][s]⟩);
19 |  for u ∈ visited do
20 |  |  distances[v] ← distances[v] ∪ ⟨u, dist[u, v]⟩;
21 |  sort_in_descending_order(distances[v]);
```

6.4 Empirical Evaluation

In this section, we compare our generalised closeness Shapley interaction index to four other state-of-the-art link prediction methods from the literature on 11 real-life networks.

6.4.1 Setting & Datasets

We compare our algorithm to the Shapley r-degree interaction index [32], to r-Common Neighbours [32], and to the SRW and LRW algorithms [100]. We briefly introduce these algorithms below:

Algorithm 8: Semivalue Closeness Interaction Index.

```
for u ∈ V do
    prev_d ← largest distance in distances[u];
    Nod_>[prev_d] ← |V| − distances[u].size();
    Nod_≥[prev_d] ← |V| − distances[u].size();
    for (v, d) ∈ distances[u] do
        if d ≠ prev_d then
            Nod_>[d] ← Nod_≥[prev_d];
            Nod_≥[d] ← Nod_≥[prev_d];
            prev_d ← d;
        Nod_≥[d] ← Nod_≥[d] + 1;
    prev_d ← largest distance in distances[u];
    for k ∈ [0, |V|] do
        MC⁻ ← 0;
        for (s, d) ∈ distances[u] do
            if d ≠ prev_d then
                MC⁻ ← MC⁻ + f(prev_d)( C(Nod_≥[d], k) C(Nod_>[d], k) );
                prev_d ← d;
            for (t, dt) ∈ distances[u] do
                d ← max(d, dt);
                MC⁺ ← f(d) C(Nod_≥[d], k);
                II[s, t] = MC⁻ − MC⁺;
```

- **Shapley r-Degree Interaction Index:** This similarity measure is equivalent to our measure with the parameter $f(d) = 1$ for $d \leq r$ and $f(d) = 0$ otherwise.

- **r-Common Neighbours (CN):** According to this measure, which is used to rank undirected, unweighted graphs, the rank of every non-existing edge between a pair of nodes is the number of common r-neighbours between the two nodes. Let $E^r(v) = \{u : v \neq u \text{ and } dist(v, u) < r\}$. r-Common Neighbours, then, is defined as follows:

$$CN^r(u, v) = |E^r(u) \cap E^r(v)|.$$

- **Local Random Walk (LRW):** This measure ranks the similarity of nodes based on the concept of a random walk. Assume that at time step $t = 0$ a

Algorithm 9: Shapley Value Closeness Interaction Index.

for $u \in V$ **do**
  $prev_d \leftarrow$ largest distance in $distances[u]$;
  $Nod_{>}[prev_d] \leftarrow |V| - distances[u].size()$;
  $Nod_{\geq}[prev_d] \leftarrow |V| - distances[u].size()$;
  for $(v, d) \in distances[u]$ **do**
    if $d \neq prev_d$ **then**
      $Nod_{>}[d] \leftarrow Nod_{\geq}[prev_d]$;
      $Nod_{\geq}[d] \leftarrow Nod_{\geq}[prev_d]$;
      $prev_d \leftarrow d$;
    $Nod_{\geq}[d] \leftarrow Nod_{\geq}[d] + 1$;
  $prev_d \leftarrow$ largest distance in $distances[u]$;
  $MC^{-} \leftarrow 0$;
  for $(s, d) \in distances[u]$ **do**
    if $d \neq prev_d$ **then**
      $MC^{-} \leftarrow MC^{-} + \frac{f(prev_d)}{Nod_{<}[prev_d]-1} - \frac{f(prev_d)}{Nod_{\leq}[prev_d]-1}$;
      $prev_d \leftarrow d$;
    for $(t, dt) \in distances[u]$ **do**
      $d \leftarrow \max(d, dt)$;
      $MC^{+} \leftarrow \frac{f(d)}{Nod_{\leq}[d]-1}$;
      $II[s, t] = MC^{-} - MC^{+}$;

walker starts at node u. In other words, there is 100% probability that the current node is u at time step $t = 0$. At any other time step, the walker can visit any of the neighbours of the current node with equal probability. Denote by $P_{uv}(t)$ the probability that a walker that started at u is at node v at time step t. LRW, then, is defined as follows:

$$LRW^{r}(u, v) = \frac{|E(u)|}{2|E|} P_{uv}(r) + \frac{|E(v)|}{2|E|} P_{vu}(r)$$

- **Superimposed Random Walk (SRW):** This measure is considered by the authors to be a slightly more version of LRW. It is defined as the sum of all LRW measures from time step 0 to time step r. Formally, we have:

$$SRW^{r}(u, v) = \sum_{t=0}^{r} LRW^{t}(u, v)$$

An important facet of our analysis is that whereas, as far as we are aware, the analysis of quasi-local similarity measures in the literature has focused on the

performance of algorithms given an optimal choice of r, there is no analysis on the impact of a suboptimal r on the algorithms. To combat this, we chose to compare all algorithms using r values of 1, 2 and 3. For the datasets we evaluated, we found that none of the methods significantly benefited from a higher r value (in fact, in most cases a higher value was detrimental), but comparing these 3 values was sufficient to highlight the differences between the methods.

In particular, experiments were performed for up to $r = 5$, but these values did not impact the visible trends for $r = 3$. We see in Table 6.3 that increasing the r value up to 3 means that for most networks already more than half of all nodes are—on average—already in the vicinity of all other nodes. Even though the Amazon network is the exception to this, we see that increasing r further increases the average neighbourhood size by extremely little, meaning that the algorithms change very little given a larger r. The Youtube network is another exception, however we see a consistent drop in quality of all algorithms given an r higher than 3.

In order to compare the methods, we take 11 networks and randomly remove 30% of the edges from each of them. Next, we rank non-existing edges within the networks (including those that were removed) according to each algorithm. We use the area under the curve (AUC) and precision to compare the results. We repeat this process 1000 times for all networks, methods and parameters r and present the average AUC and precision in Tables 6.4, and 6.5, respectively. The networks that we evaluate the algorithms on[1] are as follows:

- **Youtube 20:** A network of users belonging to the 20 top groups in the SNAP dataset from the popular video-sharing website Youtube [101]. Connections between users indicate friendship between their user accounts. Link prediction can predict friendships between users whose user accounts are not formally connected as friends on the website.

[1] All datasets but the last are available at `http://vlado.fmf.uni-lj.si/pub/networks/data/` [98], `https://snap.stanford.edu/data/` [101], or `http://konect.uni-koblenz.de/networks/` [102]

- **Amazon 100:** A network of products from the 100 top product categories in the SNAP dataset from the Amazon online store website. The authors of the dataset mined connections between products using the "Customers Who Bought This Item Also Bought" feature [101]. Link prediction methods can be used to discover new product associations and therefore improve the impact of the recommendation service.
- **US AIR:** A network of airports in the USA and their connections [98].
- **Football:** A network of college football teams [97, 98]. Edges represent matches between the teams.
- **Taro:** A network of gift-giving (taro) between households in a Papuan village [102–104].
- **Jazz:** A collaboration network of jazz musicians from 2003 [102, 105]. Edges indicate that two musicians performed together in a band.
- **Zachary:** A friendship network of the Zachary karate club [102, 106].
- **Surfers:** A network of the interpersonal contacts of windsurfers in southern California in the fall of 1986 [102, 107].
- **Dolphins:** A network representing a community bottlenose dolphins off Doubtful Sound and their associations observed between 1994 and 2001 [102, 108].
- **Terrorists:** The network of suspected terrorists who orchestrated the 2004 Madrid train bombing [102, 109]. A connection between two terrorists indicates that they communicated with each other.
- **Polbooks:** This network represents books about politics sold through Amazon. Two books are connected if they were frequently co-purchased [110]. This dataset was kindly provided by Szczepanski et al. [30].

We present some of the characteristics of the networks in Table 6.3.

Dataset	$\|V\|$	$\|E\|$	r	Average V_r with 30% edges randomly removed
Youtube 20	436	1384	1	3.22018
			2	16.8829
			3	40.7019
Amazon 100	433	2014	1	4.25173
			2	6.39215
			3	6.81899
US Air	332	4252	1	9.96386
			2	94.7047
			3	215.447
Football	115	1226	1	8.46087
			2	34.3531
			3	87.2797
Taro	22	78	1	3.45455
			2	7.18136
			3	11.602
Jazz	198	5484	1	20.3838
			2	113.99
			3	177.578
Zachary	32	156	1	4.27647
			2	14.1438
			3	22.6532
Surfers	43	672	1	11.9302
			2	38.1082
			3	42.939
Dolphins	62	318	1	4.58065
			2	13.7555
			3	26.2656
Terrorists	64	486	1	6.3125
			2	21.6096
			3	38.7756
Polbooks	105	822	1	6.86667
			2	28.5871
			3	54.9784

Table 6.3: Networks' characteristics.

6.4.2 Results

We note that in all but the Zachary network (where LRW^3 achieved the best result), our closeness Shapley interaction index produced the best ranking according to the AUC measure. For precision our measure also achieved the best results, except for the Football network, where—interestingly—the simplest common neighbour algorithm with a radius of 1 achieved the best result. Even in these cases, however, the advantage of other methods was marginal. Interestingly, in the Football and Dolphins networks the quality of our ranking according to AUC increased when r was raised from 1 to 2, however the quality of the Shapley degree interaction

Dataset	r	Shapley Closeness	Shapley Degree	CN	SRW	LRW
Youtube 20	1	61.3505	61.3505	61.1019	58.7498	58.7498
	2	70.0164	69.7227	69.1751	63.5614	63.6256
	3	66.8958	66.5265	65.8977	62.3907	62.6129
Amazon 100	1	92.5538	92.5538	92.4659	74.4255	74.4255
	2	96.9606	96.9144	96.8751	91.641	91.5938
	3	96.9922	96.9897	96.8335	92.0309	91.9374
US Air	1	92.9399	92.9399	91.6535	86.4357	86.4357
	2	91.5911	88.6927	85.5982	91.6951	91.8008
	3	91.2845	84.295	83.8852	91.4204	90.8273
Football	1	81.3609	81.3609	81.3816	67.1795	67.1795
	2	82.8605	80.3923	77.99	77.559	78.4229
	3	81.2913	54.9976	53.0419	76.8319	74.9382
Taro	1	59.2821	59.2821	59.087	48.4095	48.4095
	2	51.2598	49.354	42.4937	48.6618	49.1697
	3	48.9925	44.5161	40.7339	45.9852	44.04
Jazz	1	95.8357	95.8357	94.4122	84.5605	84.5605
	2	94.4967	84.42589	80.3607	90.4417	90.7296
	3	94.9948	74.3762	72.735	89.4337	85.3518
Zachary	1	65.8832	65.8832	63.4124	54.1989	54.1989
	2	67.7601	63.6074	59.9963	66.0652	67.4467
	3	67.8417	62.803	60.9969	67.0779	68.3511
Surfers	1	82.091	82.091	80.6681	58.7149	58.7149
	2	81.3624	69.8356	68.6895	66.9169	69.0184
	3	81.825	52.1366	52.131	65.8986	63.6896
Dolphins	1	71.3971	71.3971	71.4927	62.996	62.996
	2	77.1422	76.716	74.6696	72.7849	73.1603
	3	76.8158	74.8411	71.3779	72.618	72.2354
Terrorists	1	89.5731	89.5731	88.1727	68.4043	68.4043
	2	88.992	85.6847	79.7279	82.8721	84.334
	3	88.4692	78.2818	75.6315	83.1278	82.8595
Polbooks	1	83.7946	83.7946	83.0612	73.5109	73.5109
	2	87.8976	85.8275	83.3794	83.6285	84.6427
	3	87.5154	81.1343	79.5508	83.7414	83.3007

Table 6.4: The average AUC (in percent) of our Shapley value Interaction Index and other quasi-local link prediction methods on 11 datasets. The best AUC for each dataset is highlighted in gray.

Data Set	r	Shapley Closeness	Shapley Degree	CN	SRW	LRW
Youtube 20	1	9.23527	9.23527	6.11643	4.17198	4.17198
	2	8.73816	3.22899	1.35217	5.58599	7.39179
	3	9.37343	5.70435	4.96667	6.82899	7.91884
Amazon 100	1	60.5944	60.5944	58.0997	49.1292	49.1292
	2	60.9884	60.5868	49.0636	57.1358	58.5904
	3	60.8652	64.7351	36.1613	57.443	58.2556
US Air	1	53.8612	53.8612	45.1463	41.0411	41.0411
	2	49.186	37.0044	34.402	47.1418	48.3915
	3	50.9319	27.8438	28.8824	47.0859	44.7281
Football	1	40.8557	40.8557	41.1049	21.9579	21.9579
	2	40.5464	20.4885	17.5956	28.2721	32.8016
	3	40.1038	3.3776	3.18852	28.5202	26.277
Taro	1	15.9455	15.9455	12.9455	14.1636	14.1636
	2	15.2364	11.4091	3.8664	13.7182	10.7909
	3	12.9273	4.00909	3.12727	12.4273	7.73636
Jazz	1	63.6966	63.6966	57.6658	36.83	36.83
	2	60.8122	28.051	24.8321	42.3302	42.2436
	3	62.2605	19.9005	18.588	40.8583	36.6822
Zachary	1	24.487	24.487	15.5435	10.5652	10.5652
	2	19.387	9.05217	7.56957	15.1783	20.0957
	3	20.587	13.9739	13.7783	17.7043	19.913
Surfers	1	49.378	49.378	47.155	26.213	26.213
	2	49.287	33.981	33.552	30.671	33.475
	3	49.391	25.035	25.052	30.424	28.723
Dolphins	1	18.4468	18.4468	20.7532	13.8787	13.8787
	2	20.2319	18.0021	12.1723	15.9745	16.5149
	3	19.7277	11.3553	9.70851	14.9766	15.4957
Terrorists	1	65.9319	65.9319	55.8806	26.1722	26.1722
	2	64.4972	41.8264	35.7958	37.4875	45.5889
	3	64.8903	28.5861	28.5806	39.3431	39.0875
Polbooks	1	29.3667	29.3667	26.6636	16.8841	16.8841
	2	29.7386	22.3629	21.1856	20.0485	22.4038
	3	29.8985	15.1152	13.0773	20.7667	20.4273

Table 6.5: The average precision (in percent) of our Shapley value Interaction Index and other quasi-local link prediction methods on 11 datasets. The best precision for each dataset is highlighted in gray.

index fell. We attribute this to the fact that the latter algorithm over-stresses the importance of second- and third-order relationships. For precision, we see this phenomenon in the Amazon 100, Dolphins, and most prominently Surfers and Polbooks networks (where our ranking was slightly improved, but the Shapley degree ranking decreased by approximately 24% and 14%, respectively).

In general, we note that the quality of all algorithms falls when the r value is too high. With the exception of the Taro network (where, given $r = 3$ the quality of our ranking fell by approximately 10%), it seems that the Shapley value closeness measure is very resilient to this phenomenon. We note, however, that even though SRW and LRW seem to be more resilient in this regard, they tend to produce worse results at all r values. In fact, the results of both algorithms given any r were worse than random in the Taro network. This network seems especially difficult, with many of the results being worse than random. We also note that our algorithm is—generally—more resilient than LRW and SRW when choosing a r value that is too low.

We see that the Shapley degree interaction index and common neighbours are the most likely to under-perform given a high r value. In fact, this can decrease the AUC of both measures by nearly 30% (as seen in the Surfers network), and precision by approximately 40% (in the case of the Jazz and Football networks). Given this, it is difficult to recommend these algorithms as quasi-local link prediction methods, given that providing them with too much information (i.e., a r parameter that is too high) can result in a ranking that is little better than random. Although it is possible to estimate this parameter when using these methods, there is no way to know whether the parameter is too high, potentially dramatically decreasing the effectiveness of the measure.

Finally, we note that whereas SRW is generally viewed as being superior to LRW, it is LRW that usually achieves the better result in our experiments when each algorithm is given its own, respective optimal paramter r. We note, however, that given the same r for a high value of r, it is usually SRW that is superior.

When we consider the network characteristics in Table 6.3, we can explain some of the algorithm behaviours that we see. In particular, for the Amazon network increasing r past 2 adds very little nodes to the average neighbourhood size of a node. It is also for this reason that the Shaple r-degree interaction index does relatively well in this network. On the other hand, in the Jazz network for $r = 3$ we already have 177 nodes on average of a total 198 nodes in the vicinity of any node. The algorithm therefore bases its ranking on which of a maximum possible 21 nodes on average are not in the vicinity of a pair of nodes, which explains why the algorithm fails with a high k for this network.

6.5 Conclusions and Future Work

We developed a new game-theoretic quasi-local algorithm for link prediction that is based on generalized closeness centrality. Our approach achieves competitive results when compared to the state of the art. It is especially resilient when provided with a suboptimal radius, r, within which to query for similarities between nodes.

As a potential future research direction, it may be worthwhile to study a variable radius for different pairs of nodes. It goes to reason that if a different radius is required to achieve the optimal result in various networks, perhaps various sections of the network (such as connected components) should also be studied with different radii. Moreover, other group centrality measures may prove to be even more effective when paired with the interaction index in predicting links between nodes. Group betweenness centrality, for example, has not been studied for this purpose.

7

Concluding Remarks

Our goals in this thesis were threefold: (1) to provide a broad computational analysis of a wide class of game-theoretic measures, (2) to develop a new class of measures for networks with overlapping community structures, which broadens the types of networks that can be analysed using game-theoretic methods; and (3) to improve the effectiveness of game-theoretic measures.

To address the first issue, we developed a framework for defining group centrality measures. This allowed us to identify a subclass of group centrality measures, such that when they are paired with any semivalue, the resulting game-theoretic centrality measure can be computed in polynomial time. We provided algorithms for computing any such measure. These algorithms require the user to define the group centrality measure within our framework and to provide four functions that compute values associated with the framework. In essence, we were able to distill computing the centrality measure in polynomial time to computing just these four values.

A limitation of this approach is that, although it is broader than what has been presented in the literature thus far (encompassing all previous results involving semivalues and much more), it is still not fully general. The question of exactly which game-theoretic measures based on semivalues can be computed in polynomial time

is still left open. Moreover, other classes of solution concepts such as probabilistic values, quasivalues, weighted values, indices of power [85], the core [86] and nucleoulus [87] are still yet to be studied. However, our work can be seen as a first step towards a characterisation of the polynomially computable game-theoretic measures. Moreover, one can argue that our framework—although still with certain limitations—provides very plausible measures. By using our framework, it is possible to create new, tractable game-theoretic measures simply by identifying those factors that are meaningful for the application (i.e., graph items such as shortest paths), their numerical impact on the network and whether these factors are affected positively or negatively by certain nodes.

The second issue addressed by this thesis is centrality measures for networks with overlapping community structure. To the best of our knowledge, we have presented the first centrality measures in the literature that consider the impact of potentially overlapping communities on the centrality of nodes. We have developed new class of solution concepts for this purpose that we call configuration semivalues. We have paired configuration semivalues with a general form of group closeness centrality and developed polynomial algorithms for the computation of the resulting class of game-theoretic centrality measures. We have applied our approach to the Warsaw public transportation network, showing that it results in an interpretable ranking that is—arguably—more intuitive than that of other measures.

Finally, to address the third issue, we have tackled the problems of link prediction in networks. Our approach was to pair semivalue interaction indices with a general form of group closeness centrality. This lead to an algorithm that achieved some of the best results when compared to the state-of-the-art in the literature, and addressed the main issue with the first—degree-based—game theoretic measure in the literature: a vulnerability such that if the measure was given a radius parameter that was too high, then this would drastically reduce its performance.

Naturally, our approach to game-theoretic centrality in addressing networks with overlapping communities is itself not very broad in its scope. Nevertheless, our

approach is broader than the current literature (both in our choice of solution concepts, which encompass those in the literature, and in the choice of our group centrality measure, since general closeness centrality encompasses degree centrality). However, a general study of which group centrality measures allow for polynomial computation of configuration semivalues and semivalues indice would be useful. In addition, we have not yet demonstrated the wider applicability of our approach. Outside of networks with a community structure, it is not yet clear whether the game-theoretic approach (or perhaps even configuration semivalues) can also be used to analyse multi-layer networks (where multiple types of relationships may exist between nodes), hypergraphs (where relationships can exist between more than two nodes), or other types of networks.

In general, another limitation of the thesis is that we have not studied the axiomatisation for the game-theoretic network centralities that we have proposed. Indeed, research on the axiomatisation of such centralities is in its infancy. Brink and Gilles [17] axiomatised their degree-based centrality measure and—interestingly—developed a closed-form formula for it that easily translates to a polynomial algorithm. Skibski, Michalak, and Rahwan [79] and Skibski et al. [59] also studied axiomatisation of game-theoretic network centrality measures, and the latter work developed an algorithm of a new, axiomatised game-theoretic centrality measure—attachment centrality. It would be interesting to investigate whether there is any relation between axioms and the computational properties of a given measure (e.g., a given axiom might be very desirable but too computationally complex to implement). In general, an axiomatic analysis and characterisation of game-theoretic centrality measures would help better understand their properties and the types of situations where it is best to apply them.

In general, the study of the applications of game-theoretic network centrality is only beginning. To complement the axiomatic approach, an empirical one could show the real-world implications of such measures. Although we have provided some such results in this thesis, many more such studies (including comparative

ones of other measures) need to be done to assess the performance and viability of game-theoretic measures.

In conclusion, game-theoretic centrality is a promising research direction, although it is still largely unexplored. We have contributed both to the analysis of the computational complexity of the field and also expanded it by developing novel methods to address networks with overlapping community structure. Moreover, we were able to improve upon the state-of-the-art for the link prediction problem.

Appendices

A

Algorithms for Section 4.4

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In this appendix, we present the algorithms from Section 4.4. Each of the algorithms makes use of Algorithm 2 and passes to it the required arguements in order to compute the respective centrality measure.

A.1 Semivalue Degree Algorithm

In this section we present Algorithm 10 for computing the semivalue degree centrality.

Algorithm 10: The parametrised degree centrality

Input: Graph $G = (V, E)$, node $v \in V$, functions f and g, function β
Output: $\phi_v(\nu_G^D)$, for the node v

Function $c_G^N(v, k)$
  if $|V| - 1 - deg(v) < k$ **then**
    0;
  else
    $\binom{|V|-1-deg(v)}{k}$;
Function $c_G^R(v, k)$
  if $|V| = k$ **then**
    0;
  else
    $\binom{|V|}{k} - c_G^N(v, k) - \binom{|V|-1}{k-1}$;
for $v \in V$ **do**
  for $m \in [0, max_deg]$ **do**
    $|R_{\Theta_l}(\{v\})| \leftarrow 0$;
    $|\widetilde{R}_{\Theta_l}(\{v\})| \leftarrow 0$;
for $v \in V$ **do**
  for $u \in E(v)$ **do**
    $|R_{\Theta_{deg(u)}}(\{v\})| \leftarrow |R_{\Theta_{deg(u)}}(\{v\})| + 1$;
  $|\widetilde{R}_{\Theta_{deg(v)}}(\{v\})| \leftarrow 1$;
$\phi_v \leftarrow$
  $\mathbf{SEMI}\Big(G, v, \beta, c_G^N, c_G^R, \{|R_{\Theta_l}(\{v\})|\}_{v\in V, l\in[0,max_deg]}, \{|\widetilde{R}_{\Theta_l}(\{v\})|\}_{v\in V, l\in[0,max_deg]}\Big)$;

A.2 Semivalue Betweenness Algorithm

In this section, we compute semivalue betweenness centrality. Algorithm 11 shows how to count the number of shortest paths between each pair of nodes and the number of these paths that visit any node. We present the main computations in Algorithm 12.

Algorithm 11: Dijkstra algorithm for computing distance and counting shortest paths

input : Graph $G = (V, E, \omega)$
output : Configuration Semivalue

```
1  dist[V][V];
2  shortest_paths[V][V];
3  for v ∈ V do
4      for u ∈ V do
5          dist[v][u] = ∞;
6          shortest_paths[v][u] = 0;
7      Q ← Priority Queue;
8      Q.enqueue(⟨v, 0⟩);
9      dist[v][v] = 0;
10     while Q not Empty do
11         ⟨u, d⟩ ← Q.pop();
12         for s ∈ E(u) do
13             if (dist[v][s] > dist[v][u] + ω(u, s)) then
14                 dist[v][s] = dist[v][u] + ω(u, s);
15                 Q.enqueue(⟨s, dist[v][s]⟩);
16                 shortest_paths[v][s] ← shortest_paths[v][u];
17             if (dist[v][s] = dist[v][u] + ω(u, s)) then
18                 shortest_paths[v][s] ← shortest_paths[v][s] + shortest_paths[v][u];
19 for s ∈ V do
20     for t ∈ V do
21         |R̃_{Θ_st}({s})| ← |R̃_{Θ_st}({s})| + shortest_paths[s][t];
22         |R̃_{Θ_st}({t})| ← |R̃_{Θ_st}({t})| + shortest_paths[s][t];
23         for v ∈ V \ {s, t} do
24             if dist[s][t] = dist[s][v] + dist[v][t] then
25                 |R_{Θ_st}({v})| ← shortest_paths[s][v] × shortest_paths[v][t];
```

Algorithm 12: The parametrised betweenness centrality

Input: Graph $G = (V, E, \omega)$, node $v \in V$, functions f and g, function β
Output: $\phi_v(\nu_G^B)$, for the node v

Function $c_G^N(\pi_{st}, k)$
  if $|V| - |\pi_{st}| < k$ **then**
    0;
  else
    $\binom{|V|-|\pi_{st}|}{k}$;
Function $c_G^{\widetilde{R}}(\pi_{st}, k)$
  if $k = 0$ **then**
    0;
  else if $k = 1$ **then**
    $e(\pi_{st})$;
  else if $k = |V|$ **then**
    1;
  else
    $\sum_{i=1}^{e(\pi_{st})} \binom{e(\pi_{st})}{i}\binom{|V|-e(\pi_{st})}{k-i}$;
Function $c_G^R(\pi_{st}, k)$
  $\binom{|V|}{k} - c_G^N(\pi_{st}, k) - c_G^{\widetilde{R}}(\pi_{st}, k)$;
// See Algorithm 11 for computation of $|R_{\Theta_{s,t}(\{v\})}|$ and $|\widetilde{R}_{\Theta_{s,t}(\{v\})}|$
$\phi_v \leftarrow$ **SEMI**$\Big(G, v, \beta, c_G^N, c_G^R, \{|R_{\Theta_{s,t}}(\{v\})|\}_{s,t,v\in V}, \{|\widetilde{R}_{\Theta_{s,t}}(\{v\})|\}_{s,t,v\in V}\Big)$;

A.3 Semivalue Closeness Algorithm

In this section we present Algorithm 13 for computing the semivalue closeness centrality. First, however, let us improve computation of $|R_{\#k}^{-1}(\langle u, d\rangle)|$ and $|N_{\#k}^{-1}(\langle u, d\rangle)|$ from $O(|V|)$ to $O(1)$. Assuming $Nod_{=d}(u) > 0$ (otherwise $|R_{\#k}^{-1}(\langle u, d\rangle)|$ is equal to 0), we can use the well-known combinatoric property $\sum_{0\le i\le m}\binom{m}{i}\binom{n-m}{k-i} = \binom{n}{k}$ to simplify:

$$|R_{\#k}^{-1}(\langle u, d\rangle)| = \binom{|V| - Nod_{<d}(u)}{k} - \binom{|V| - Nod_{=d}(u) - Nod_{<d}(u)}{k}$$

Next, using similar techniques we can simplify $|\widetilde{R}_{\#k}^{-1}(\langle u, d\rangle)|$ as follows:

$$|\widetilde{R}_{\#k}^{-1}(\langle u, d\rangle)| = \binom{|V|}{k} - \binom{|V| - Nod_{<d}(u)}{k}$$

Algorithm 13: The parametrised closeness centrality

Input: Graph $G = (V, E, \omega)$, node $v \in V$, functions f and g, function β
Output: $\phi_v(\nu_G^{CL})$, for the node v

Function $c_G^R(\langle u, d\rangle, k)$
  $\left\lfloor \binom{|V|-Nod_{<d}(u)}{k} - \binom{|V|-Nod_{=d}(u)-Nod_{<d}(u)}{k} \right\rfloor$;
Function $c_G^{\widetilde{R}}(\langle u, d\rangle, k)$
  $\left\lfloor \binom{|V|}{k} - \binom{|V|-Nod_{<d}(u)}{k} \right\rfloor$;
Function $c_G^N(\langle u, d\rangle, k)$
  $\left\lfloor \binom{|V|}{k} - c_G^R(\langle u, d\rangle, k) - c_G^{\widetilde{R}}(\langle u, d\rangle, k) \right\rfloor$;
for $u \in V$ **do**
  for $v \in V$ **do**
    for $d \in dists(G)$ **do**
      if $dist(u, v) = d$ **then**
        $|R_{\Theta_{u,d}(\{v\})}| \leftarrow 1$;
      else
        $|R_{\Theta_{u,d}(\{v\})}| \leftarrow 0$;
      if $dist(u, v) < d$ **then**
        $|\widetilde{R}_{\Theta_{u,d}(\{v\})}| \leftarrow 1$;
      else
        $|\widetilde{R}_{\Theta_{u,d}(\{v\})}| \leftarrow 0$;
$\phi_v \leftarrow$
  $\mathbf{SEMI}\Big(G, v, \beta, c_G^N, c_G^R, \{|R_{\Theta_{u,d}}(\{v\})|\}_{u,v\in V, d\in dists(G)}, \{|\widetilde{R}_{\Theta_{u,d}}(\{v\})|\}_{u,v\in V, d\in dists(G)}\Big)$;

B

Proofs for Section 4.5

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In this appendix, we provide the proofs for computing the Banzhaf index from Section 4.5.

B.1 General Derivations for the Banzhaf Index

In this section, we will reorder Equation 4.6 in order to group all terms with the variable k in one spot. This will allow us to get rid of the variable altogether in the next sections, yielding faster computation.

Proof: From Theorem 1 we have:[1]

$$
\begin{aligned}
&\phi_v(\nu)\\
&= \sum_{0\le k<|V|} \beta(k) \sum_{1\le l\le h(G)} \frac{\mathrm{MC}(k,u,\Theta_l)}{\binom{|V|-1}{k}}.\\
&= \sum_{0\le k<|V|} \beta(k) \sum_{1\le l\le h(G)} \frac{|R_{\Theta_l}(\{u\})|\mathrm{MC}^{[1]}(k,u,\Theta_l)}{\binom{|V|-1}{k}}\\
&\quad - \sum_{0\le k<|V|} \beta(k) \sum_{1\le l\le h(G)} \frac{|\widetilde{R}_{\Theta_l}(\{u\})|\mathrm{MC}^{[2]}(k,u,\Theta_l)}{\binom{|V|-1}{k}}\\
&= \sum_{0\le k<|V|} \beta(k) \sum_{1\le l\le h(G)} \frac{|R_{\Theta_l}(\{u\})|f(\Theta_l)|N^{-1}_{\#k}(\Theta_l)|}{\binom{|V|-1}{k}}\\
&\quad - \sum_{0\le k<|V|} \beta(k) \sum_{1\le l\le h(G)} \frac{|\widetilde{R}_{\Theta_l}(\{u\})|f(\Theta_l)|R^{-1}_{\#k}(\Theta_l)|}{\binom{|V|-1}{k}}\\
&= \sum_{1\le l\le h(G)} |R_{\Theta_l}(\{u\})|f(\Theta_l) \sum_{0\le k<|V|} \beta(k)\frac{|N^{-1}_{\#k}(\Theta_l)|}{\binom{|V|-1}{k}}\\
&\quad - \sum_{1\le l\le h(G)} |\widetilde{R}_{\Theta_l}(\{u\})|f(\Theta_l) \sum_{0\le k<|V|} \beta(k)\frac{|R^{-1}_{\#k}(\Theta_l)|}{\binom{|V|-1}{k}}\\
&= \sum_{1\le l\le h(G)} |R_{\Theta_l}(\{u\})|f(\Theta_l)\mathcal{N}^l - \sum_{1\le l\le h(G)} |\widetilde{R}_{\Theta_l}(\{u\})|f(\Theta_l)\mathcal{R}^l
\end{aligned}
$$

□

B.2 Derivations for Banzhaf Degree Centrality

In this section, we prove Proposition 4.

Proof: Recall that:

$$
|N^{-1}_{\#k}(v)| = \begin{cases} 0 & \text{if } |V|-1-deg(v) < k \\ \binom{|V|-1-deg(v)}{k} & \text{otherwise.} \end{cases}
$$

and

$$
|R^{-1}_{\#k}(v)| = \begin{cases} 0 & \text{if } |V| = k \\ \binom{|V|}{k} - |N^{-1}_{\#k}(v)| - \binom{|V|-1}{k-1} & \text{otherwise.} \end{cases}
$$

[1]Note that $\mathrm{MC}^{[3]}(k,u,\Theta_l) = 0$ since $\Delta^k = g(k+1) - g(k) = 0$ and it is therefore omitted in the below formula

Using the property $\sum_{0\leq k\leq n}\binom{n}{k} = 2^n$ we have:

$$\begin{aligned}\mathcal{N}^l &= \sum_{0\leq k<|V|}\beta(k)\frac{|N^{-1}_{\#k}(\Theta_l)|}{\binom{|V|-1}{k}}\\ \mathcal{N}^{deg(v)} &= \frac{1}{2^{|V|-1}}\sum_{0\leq k<|V|}\binom{|V|-1-deg(v)}{k} = \frac{2^{|V|-1-deg(v)}}{2^{|V|-1}} = \frac{1}{2^{deg(v)}},\end{aligned}$$

and

$$\begin{aligned}\mathcal{R}^l &= \sum_{0\leq k<|V|}\beta(k)\frac{|R^{-1}_{\#k}(\Theta_l)|}{\binom{|V|-1}{k}}\\ \mathcal{R}^{deg(v)} &= \frac{1}{2^{|V|-1}}\sum_{0\leq k<|V|}\binom{|V|-1}{k} - \binom{|V|-1-deg(v)}{k} = 1-\frac{1}{2^{deg(v)}}\end{aligned}$$

Since $h(G) = |V|$, $\mathcal{N}^l$ and $\mathcal{R}^l$ can be computed in constant time, and $R_{\Theta_l}(\{v\})$ and $\widetilde{R}_{\Theta_l}(\{v\})$ can be computed in $O(|V|+|E|)$ time, then the Banzhaf generalised degree centrality (with $g(|C|) = 1$) can be computed by Algorithm 3 in $O(|V|^{1+m_1}+|E|)$ time.

□

B.3 Derivations for Banzhaf Betweenness Centrality

In this section, we prove Proposition 5.

Proof: Recall that:

$$|N^{-1}_{\#k}(\pi_{st})| = \begin{cases}0 & \text{if } |V|-|\pi_{st}|<k\\ \binom{|V|-|\pi_{st}|}{k} & \text{otherwise.}\end{cases}$$

and

$$|\widetilde{R}^{-1}_{\#k}(\pi_{st})| = \begin{cases}0 & \text{if } k=0\\ e(\pi_{st}) & \text{if } k=1\\ 1 & \text{if } k=|V|\\ \sum_{i=1}^{e(\pi_{st})}\binom{e(\pi_{st})}{i}\binom{|V|-e(\pi_{st})}{k-i} & \text{otherwise,}\end{cases}$$

where $e(\pi_{st}) = \min(2,|\pi_{st}|)$, and

$$|R^{-1}_{\#k}(\pi_{st})| = \binom{|V|}{k} - |\widetilde{R}^{-1}_{\#k}(\pi_{st})| - |N^{-1}_{\#k}(\pi_{st})|.$$

We can now calculate values:

$$\begin{aligned}\mathcal{N}^l &= \sum_{0\le k<|V|} \beta(k)\frac{|N^{-1}_{\#k}(\Theta_l)|}{\binom{|V|-1}{k}}\\ \mathcal{N}^{s,t} &= \frac{1}{2^{|V|-1}}\sum_{0\le k<|V|}\binom{|V|-|\pi_{st}|}{k}\\ &= \frac{2^{|V|-|\pi_{st}|}}{2^{|V|-1}}\end{aligned}$$

and define

$$\begin{aligned}\widetilde{\mathcal{R}}^l &= \sum_{0\le k<|V|} \beta(k)\frac{|R^{-1}_{\#k}(\Theta_l)|}{\binom{|V|-1}{k}}\\ \widetilde{\mathcal{R}}^{s,t} &= \frac{1}{2^{|V|-1}}\left(e(\pi_{st})+1+\sum_{2\le k<|V|-1}\sum_{i=1}^{e(\pi_{st})}\binom{e(\pi_{st})}{i}\binom{|V|-e(\pi_{st})}{k-i}\right)\end{aligned}$$

Assuming $e(\pi_{st}) = 2$ we have:

$$\begin{aligned}&= \frac{1}{2^{|V|-1}}\left(3+2\sum_{2\le k<|V|-1}\binom{|V|-2}{k-1}+\sum_{2\le k<|V|-1}\binom{|V|-2}{k-2}\right)\\ &= \frac{1}{2^{|V|-1}}\left(3(2^{|V|-2})-|V|\right)\end{aligned}$$

Assuming $e(\pi_{st}) = 1$, we get the following:

$$\begin{aligned}&= \frac{1}{2^{|V|-1}}\left(2+\sum_{2\le k<|V|-1}\binom{|V|-2}{k-1}\right)\\ &= \frac{2^{|V|-2}}{2^{|V|-1}}\\ &= \frac{1}{2}\end{aligned}$$

Finally, we have

$$\mathcal{R}^{s,t} = \frac{2^{|V|}-1}{2^{|V|-1}} - \widetilde{\mathcal{R}}^{s,t} - \mathcal{N}^{s,t}$$

Since $h(G) = |V|^2$, $\mathcal{N}^l$ and $\mathcal{R}^l$ can be computed in constant time, and $R_{\Theta_{s,t}}(\{v\})$ and $\widetilde{R}_{\Theta_{s,t}}(\{v\})$ can be precomputed in $O(|V|^2 + |V||E|)$ time for undirected graphs and $O(|V||E| + |V|^2\log(|V|))$ time for directed graphs, then the Banzhaf generalised betweenness centrality (with $g(|C|) = 1$) can be computed by Algorithm 3 in $O(|V|^{2+m_1} + |V||E|)$ time for undirected graphs, and $O(|V|^{2+m_1} + |V||E| + |V|^2\log(|V|))$ time for directed graphs. □

B.4 Derivations for Banzhaf Closeness Centrality

In this section, we prove Proposition 6.

Proof: Recall that:

$$|R^{-1}_{\#k}(\langle u,d\rangle)| = \sum_{i=1}^{Nod_{=d}(u)} \binom{Nod_{=d}(u)}{i}\binom{|V| - Nod_{=d}(u) - Nod_{<d}(u)}{k-i},$$
$$|\widetilde{R}^{-1}_{\#k}(\langle u,d\rangle)| = \sum_{i=1}^{Nod_{<d}(u)} \binom{Nod_{<d}(u)}{i}\binom{|V| - Nod_{<d}(u)}{k-i}$$

and

$$|N^{-1}_{\#k}(\langle u,d\rangle)| = \binom{|V|}{k} - |R^{-1}_{\#k}(\langle u,d\rangle)| - |\widetilde{R}^{-1}_{\#k}(\langle u,d\rangle)|.$$

$$\mathcal{R}^l = \sum_{0\le k<|V|} \beta(k)\frac{|R^{-1}_{\#k}(\Theta_l)|}{\binom{|V|-1}{k}}$$
$$\mathcal{R}^{u,d} = \frac{1}{2^{|V|-1}} \sum_{0\le k<|V|} \sum_{i=1}^{Nod_{=d}(u)} \binom{Nod_{=d}(u)}{i}\binom{|V| - Nod_{=d}(u) - Nod_{<d}(u)}{k-i}$$

Assuming $Nod_{=d}(u) > 0$ (otherwise the value is equal to 0), we can use the well-known combinatoric property $\sum_{0\le i\le m}\binom{m}{i}\binom{n-m}{k-i} = \binom{n}{k}$ to simplify:

$$= \frac{1}{2^{|V|-1}}\left(\sum_{0\le k<|V|}\binom{|V| - Nod_{<d}(u)}{k} - \sum_{0\le k<|V|}\binom{|V| - Nod_{=d}(u) - Nod_{<d}(u)}{k}\right)$$

Assuming $Nod_{<d}(u) > 0$ we have:

$$= \frac{1}{2^{|V|-1}}\left(2^{|V|-Nod_{<d}(u)} - 2^{|V|-Nod_{=d}(u)-Nod_{<d}(u)}\right)$$

and assuming $Nod_{<d}(u) = 0$ we have:

$$= \frac{1}{2^{|V|-1}}\left(2^{|V|-Nod_{<d}(u)} - 2^{|V|-Nod_{=d}(u)-Nod_{<d}(u)} - 1\right)$$

Next, let us define:

$$\widetilde{\mathcal{R}}^{u,d} = \sum_{0\leq k<|V|} \beta(k)\frac{|\widetilde{R}^{-1}_{\#k}(\Theta_{u,d})|}{\binom{|V|-1}{k}}$$
$$= \frac{1}{2^{|V|-1}} \sum_{0\leq k<|V|} \sum_{i=1}^{Nod_{<d}(u)} \binom{Nod_{<d}(u)}{i}\binom{|V|-Nod_{<d}(u)}{k-i}$$

We can simplify this value as above, which leads to the following result:

$$= \frac{1}{2^{|V|-1}} \sum_{0\leq k<|V|} \binom{|V|}{k} - \binom{|V|-Nod_{<d}(u)}{k}$$
$$= \frac{1}{2^{|V|-1}}\left(2^{|V|} - 1 - \sum_{0\leq k<|V|} \binom{|V|-Nod_{<d}(u)}{k}\right).$$

Again, assuming $Nod_{<d}(u) = 0$ we have:

$$= \frac{1}{2^{|V|-1}}\left(2^{|V|} - 1 - 2^{|V|-Nod_{<d}(u)}\right).$$

and assuming $Nod_{<d}(u) > 0$ we have:

$$= \frac{1}{2^{|V|-1}}\left(2^{|V|} - 2^{|V|-Nod_{<d}(u)}\right).$$

With this, we have all the tools to complete our proof:

$$\mathcal{N}^l = \sum_{0\leq k<|V|} \beta(k)\frac{|N^{-1}_{\#k}(\Theta_l)|}{\binom{|V|-1}{k}}$$
$$\mathcal{N}^{u,d} = \frac{2^{|V|}-1}{2^{|V|-1}} - \mathcal{R}^{u,d} - \widetilde{\mathcal{R}}^{u,d}$$

Since $h(G)\widehat{=}O(|V|^2)$, $\mathcal{N}^l$ and $\mathcal{R}^l$ can be computed in constant time, and $R_{\Theta_{u,d}}(\{v\})$ and $\widetilde{R}_{\Theta_{u,d}}(\{v\})$ can be precomputed in $O(|V|^2 + |V||E|)$ time for undirected graphs and $O(|V||E| + |V|^2\log(|V|))$ time for directed graphs, then the Banzhaf generalised closeness centrality (with $g(|C|) = 1$) can be computed by Algorithm 3 in $O(|V|^{2+m_1} + |V||E|)$ time for undirected graphs, and $O(|V|^{2+m_1} + |V||E| + |V|^2\log(|V|))$ time for directed graphs. □

C

Warsaw Public Transportation Network Data

In this section, we present the full rankings of the Warsaw public transportation network. Table C.1 presents the ranking according to Configuration value harmonic closeness, Table C.2 presents the ranking according to Shapley value harmonic closeness and Table C.3 presents the ranking according to harmonic closeness centrality. In Table C.4 we present the community ranking of the bus, tram, metro and train stops that comprise the network.

Station	Ranking
CENTRUM (WARSZAWA)	4.96229
DW.WILEŃSKI (WARSZAWA)	4.9267
PKP FALENICA (WARSZAWA)	4.91329
METRO ŚWIĘTOKRZYSKA (WARSZAWA)	4.22121
PŁOWIECKA (WARSZAWA)	3.66108
SASKA (WARSZAWA)	3.47287
PKP RADOŚĆ (WARSZAWA)	3.45012
KRUCZA (WARSZAWA)	3.38518
PL.SZEMBEKA (WARSZAWA)	3.37
OKULAROWA (WARSZAWA)	3.16705
RZYMOWSKIEGO (WARSZAWA)	3.15756
DICKENSA (WARSZAWA)	3.07248
METRO POLITECHNIKA (WARSZAWA)	3.06621
AL.LOTNIKÓW (WARSZAWA)	3.03703
MEHOFFERA (WARSZAWA)	3.03335
PKP ŻERAŃ (WARSZAWA)	2.95572
DOLINA SŁUŻEWIECKA (WARSZAWA)	2.91898
KAJKI (WARSZAWA)	2.89078
POLNA (KONSTANCIN-JEZIORNA)	2.8328
METRO URSYNÓW (WARSZAWA)	2.77918
BIAŁOŁĘKA-RATUSZ (WARSZAWA)	2.74449
RONDO WASZYNGTONA (WARSZAWA)	2.68935
SZYSZKOWA (WARSZAWA)	2.68049
BOGUSŁAWSKIEGO (WARSZAWA)	2.65612
PKP WŁOCHY (WARSZAWA)	2.64855
PARK MIEJSKI (OTWOCK)	2.64442
MŁODZIEŃCZA (WARSZAWA)	2.56406
PL.ZAWISZY (WARSZAWA)	2.44779
HENRYKÓW (WARSZAWA)	2.36987
PIOTRA SKARGI (WARSZAWA)	2.3535
ŻERAŃ FSO (WARSZAWA)	2.34391
PKP URSUS (WARSZAWA)	2.34359
METRO RATUSZ ARSENAŁ (WARSZAWA)	2.33095
SOKRATESA (WARSZAWA)	2.3205
ŚW.BONIFACEGO (WARSZAWA)	2.31902
MIĘDZYBORSKA (WARSZAWA)	2.28924
TARCHOMIN (WARSZAWA)	2.20346
ŚMIGIELSKA (WARSZAWA)	2.19144
URZĄD GMINY (WIĄZOWNA)	2.18609
REMBELSZCZYZNA (REMBELSZCZYZNA)	2.14451
PKP MIĘDZYLESIE (WARSZAWA)	2.12111
RYNEK (STARE BABICE)	2.11637
AL.WITOSA (WARSZAWA)	2.10924
PKP REMBERTÓW (WARSZAWA)	2.09509
SANKTUARIUM (OŻARÓW MAZOWIECKI)	2.07801
RONDO ONZ (WARSZAWA)	2.06501
BEMOWO-RATUSZ (WARSZAWA)	2.03819
RONDO STARZYŃSKIEGO (WARSZAWA)	2.00668
WSPÓLNA DROGA (WARSZAWA)	1.98929
1. PRASKIEGO PUŁKU (WARSZAWA)	1.97593
POLECZKI (WARSZAWA)	1.97294
SZWEDZKA (WARSZAWA)	1.96775
KIJOWSKA (WARSZAWA)	1.95741
NOWOURSYNOWSKA (WARSZAWA)	1.9525
REDUTA WOLSKA (WARSZAWA)	1.92055
CIEPŁOWNIA WOLA (WARSZAWA)	1.8945
URZĄD MIASTA (LEGIONOWO)	1.86028
ZEGRZYŃSKA (LEGIONOWO)	1.84013
WOJSKA POLSKIEGO (PIASECZNO)	1.83496
RONDO DASZYŃSKIEGO (WARSZAWA)	1.82423
METRO POLE MOKOTOWSKIE (WARSZAWA)	1.82257
DW.CENTRALNY (WARSZAWA)	1.80021
RONDO ŻABA (WARSZAWA)	1.79243
ZAMIENIECKA (WARSZAWA)	1.7714
KĄTY GRODZISKIE (WARSZAWA)	1.75762
MURANOWSKA (WARSZAWA)	1.74801
GOŁKÓW-LETNISKO (PIASECZNO)	1.74679
PL.WILSONA (WARSZAWA)	1.74478
SZPITAL BRÓDNOWSKI (WARSZAWA)	1.73746
ROZBRAT (WARSZAWA)	1.72605
KONWALIOWA (WARSZAWA)	1.70575
STRAŻACKA (WARSZAWA)	1.7029
TROCKA (WARSZAWA)	1.68976
CZARNIECKIEGO (WARSZAWA)	1.68238
WYŚCIGI (WARSZAWA)	1.67099
MOST PONIATOWSKIEGO (WARSZAWA)	1.66304
KONDRATOWICZA (WARSZAWA)	1.65841
DW.GDAŃSKI (WARSZAWA)	1.655
URZĄD MIASTA (PIASECZNO)	1.65061
STRZYGŁOWSKA (WARSZAWA)	1.64969
METRO WIERZBNO (WARSZAWA)	1.64715
METRO MARYMONT (WARSZAWA)	1.64412
MYSIADŁO (MYSIADŁO)	1.63805
KARCZUNKOWSKA (WARSZAWA)	1.62874
RYŻOWA (WARSZAWA)	1.62272
MYŚLIBORSKA (WARSZAWA)	1.62176
HYNKA (WARSZAWA)	1.59477
WROCŁAWSKA (WARSZAWA)	1.58913
OS.PIASKI (LEGIONOWO)	1.58071
CHEŁMSKA (WARSZAWA)	1.57331
WIATRACZNA (WARSZAWA)	1.56801
MOST SIEKIERKOWSKI (WARSZAWA)	1.55638
AL.ZIELENIECKA (WARSZAWA)	1.55046
PARK KASKADA (WARSZAWA)	1.53351
PŁOCHOCIŃSKA (WARSZAWA)	1.52634
PKP ANIN (WARSZAWA)	1.51999
PIASTA (KONSTANCIN-JEZIORNA)	1.48966
PŁOCKA-SZPITAL (WARSZAWA)	1.48451
ZĄBKOWSKA (WARSZAWA)	1.457

PROZY (WARSZAWA)	1.45673
BITWY WARSZAWSKIEJ 1920 (WARSZAWA)	1.43415
METRO KABATY (WARSZAWA)	1.43355
DOLNA (WARSZAWA)	1.42032
PRZYCZÓŁEK GROCHOWSKI (WARSZAWA)	1.41336
METRO SŁUŻEW (WARSZAWA)	1.41179
METRO WILANOWSKA (WARSZAWA)	1.40501
BAZYLIAŃSKA (WARSZAWA)	1.40364
KINO FEMINA (WARSZAWA)	1.40273
OS.GÓRCZEWSKA (WARSZAWA)	1.397
RONDO ZESŁAŃCÓW SYBERYJSKICH (WARSZAWA)	1.38863
SOBIESKIEGO (WARSZAWA)	1.38678
MUZEUM NARODOWE (WARSZAWA)	1.38573
FSO (WARSZAWA)	1.35491
BYSŁAWSKA (WARSZAWA)	1.34532
PL.GRUNWALDZKI (WARSZAWA)	1.34523
PL.STARYNKIEWICZA (WARSZAWA)	1.34402
BORA-KOMOROWSKIEGO (WARSZAWA)	1.3437
PKP KOŁO (WARSZAWA)	1.3435
ŚWIDER (OTWOCK)	1.34287
UMIŃSKIEGO (WARSZAWA)	1.34276
PL.NA ROZDROŻU (WARSZAWA)	1.33879
CENTRUM ONKOLOGII (WARSZAWA)	1.33755
GUS (WARSZAWA)	1.3304
POSTĘPU (LESZNOWOLA)	1.32917
PRATULIŃSKA (WARSZAWA)	1.31333
JÓRSKIEGO (WARSZAWA)	1.30613
NA SKRAJU (WARSZAWA)	1.30578
JABŁONIOWA (SŁOMCZYN)	1.30412
METRO STARE BIELANY (WARSZAWA)	1.30084
KRÓLEWSKA (WARSZAWA)	1.29354
GORASZEWSKA (WARSZAWA)	1.27671
SASANKI (WARSZAWA)	1.27637
TRAKT LUBELSKI (WARSZAWA)	1.27631
BRATA ALBERTA (WARSZAWA)	1.26664
OS.POTOK (WARSZAWA)	1.25619
PIASKOWA (ŁAJSKI)	1.2559
PKP NIEPORĘT (NIEPORĘT)	1.24827
MOZAIKOWA (WARSZAWA)	1.2423
CHŁOPICKIEGO (WARSZAWA)	1.23811
PL.KONSTYTUCJI (WARSZAWA)	1.23781
CYBERNETYKI (WARSZAWA)	1.21989
ESTRADY (WARSZAWA)	1.21878
WAFLOWA (WARSZAWA)	1.21774
STOICKA (WARSZAWA)	1.21692
CONRADA (WARSZAWA)	1.21243
KOBIAŁKA (WARSZAWA)	1.20767
SŁOMCZYN-SZKOŁA (SŁOMCZYN)	1.20694
DĄBROWSZCZAKÓW (WARSZAWA)	1.20438
BEMOWO (WARSZAWA)	1.19326
PKP MIEDZESZYN (WARSZAWA)	1.19212
NAPOLEONA BONAPARTE (WARSZAWA)	1.18716
RAKOWIEC (WARSZAWA)	1.18545
POPIELA (WARSZAWA)	1.18455
POLINEZYJSKA (WARSZAWA)	1.17587
OŁÓWKOWA (WARSZAWA)	1.17341
KOLEJKA PIASECZYŃSKA (PIASECZNO)	1.17306
RUSKOWY BRÓD (WARSZAWA)	1.16202
SPACEROWA (WARSZAWA)	1.16153
PKP LEGIONOWO (LEGIONOWO)	1.15968
VOGLA (WARSZAWA)	1.15673
BOROWA (KONSTANCIN-JEZIORNA)	1.15554
ŁOPUSZAŃSKA (WARSZAWA)	1.14736
CHYLICZKOWSKA-TECHNIKUM (PIASECZNO)	1.14456
WÓLKA WĘGLOWA (WARSZAWA)	1.14284
METRO STADION NARODOWY (WARSZAWA)	1.14141
ESKULAPÓW (WARSZAWA)	1.13459
PKP KASPRZAKA (WARSZAWA)	1.13283
NOWODWORY (WARSZAWA)	1.13083
DZIECI WARSZAWY (WARSZAWA)	1.11194
LUCERNY (WARSZAWA)	1.10658
ADAMIECKIEGO (WARSZAWA)	1.10414
SZKOLNA (PIASECZNO)	1.10207
TORUŃSKA (WARSZAWA)	1.09213
PRZEDPEŁSKIEGO (WIELISZEW)	1.08997
TELIGI (WARSZAWA)	1.08819
OS.WOLSKA (WARSZAWA)	1.07891
SIENKIEWICZA (IZABELIN)	1.07058
RONDO RADOSŁAWA (WARSZAWA)	1.06357
WIERTNICZA (WARSZAWA)	1.0588
RUMIANKOWA (WARSZAWA)	1.05644
AL.REYMONTA (WARSZAWA)	1.04524
STRYJEŃSKICH (WARSZAWA)	1.04482
ANIELEWICZA (WARSZAWA)	1.04046
ALUZYJNA (WARSZAWA)	1.03363
JURANDA ZE SPYCHOWA (WARSZAWA)	1.0299
CM.PÓŁNOCNY-BRAMA GŁ. (WARSZAWA)	1.02476
ŻÓŁKIEWSKIEGO (WARSZAWA)	1.02338
SIERAKOWSKA (IZABELIN)	1.01827
REMBIELIŃSKA (WARSZAWA)	1.00966
CMENTARZ (KONSTANCIN-JEZIORNA)	1.0086
SZKOLNA (MARKI)	1.00665
SPORTOWA (RASZYN)	1.00422
MŁYNÓW (WARSZAWA)	1.00179
ŚW.WINCENTEGO (WARSZAWA)	1.0015
OKOPOWA (WARSZAWA)	1.00065
KUSOCIŃSKIEGO (PIASECZNO)	0.999349
PŁOCKA (WARSZAWA)	0.992169
PKP RAKOWIEC (WARSZAWA)	0.985324
POSTĘPU (NOWA WOLA)	0.985008

WAWELSKA (WARSZAWA)	0.978434
SIEKIERKI-SANKTUARIUM (WARSZAWA)	0.967633
DZIERŻONIOWSKA (WARSZAWA)	0.959712
KULESZÓWKA (KULESZÓWKA)	0.95543
METRO STOKŁOSY (WARSZAWA)	0.953755
HALA WOLA (WARSZAWA)	0.952849
TRAUGUTTA (PIASTÓW)	0.946266
DYWIZJONU 303 (WARSZAWA)	0.933329
MŁOCINY (WARSZAWA)	0.930968
PL.BANKOWY (WARSZAWA)	0.930324
GORZYKOWSKA (WARSZAWA)	0.923191
DW.ZACHODNI (WARSZAWA)	0.919026
ZALESIE DOLNE (PIASECZNO)	0.918716
WYGODA (WÓLKA KOSOWSKA)	0.901887
IX POPRZECZNA (WARSZAWA)	0.900653
RADIOWA (WARSZAWA)	0.892674
KLAUDYNY (WARSZAWA)	0.886893
WILANÓW (WARSZAWA)	0.884048
TARGÓWEK-RATUSZ (WARSZAWA)	0.877699
WESOŁA-RATUSZ (WARSZAWA)	0.873158
KWITNĄCA (WARSZAWA)	0.872894
KOŚCIUSZKI (PRUSZKÓW)	0.872007
NOVOTEL (WARSZAWA)	0.869874
PKP TORUŃSKA (WARSZAWA)	0.868775
SIELCE (WARSZAWA)	0.863568
SZOSA LUBELSKA (WARSZAWA)	0.858847
POŁCZYŃSKA-PARKING P+R (WARSZAWA)	0.854514
KOMBAJNISTÓW (WARSZAWA)	0.851851
RZECZNA (WARSZAWA)	0.849873
GRODZISK (WARSZAWA)	0.847801
STRUGA (MARKI)	0.845214
PL.KASZTELAŃSKI (WARSZAWA)	0.828556
STATKOWSKIEGO (WARSZAWA)	0.827729
GRUSZY (WARSZAWA)	0.827025
GRZYMAŁY-SOKOŁOWSKIEGO (WARSZAWA)	0.82072
POLNA (WIELISZEW)	0.816561
LIPKÓW (LIPKÓW)	0.806627
MOST GDAŃSKI (WARSZAWA)	0.805376
ŻUROMIŃSKA (WARSZAWA)	0.80066
ZŁOTA (WĄGRODNO)	0.792722
PRASOWA (WARSZAWA)	0.792467
STRZELECKIEGO (WARSZAWA)	0.790465
KRYNOLINY (WARSZAWA)	0.789132
BERESTECKA (WARSZAWA)	0.78504
ZGORZAŁA (ZGORZAŁA)	0.784777
KOŚCIELISKA (WARSZAWA)	0.781816
PKP POWIŚLE (WARSZAWA)	0.780731
STAWKI (WARSZAWA)	0.776425
WOLA-RATUSZ (WARSZAWA)	0.776301
CENTRUM ZDROWIA DZIECKA (WARSZAWA)	0.770487
17 STYCZNIA (WARSZAWA)	0.757951
TAWUŁKOWA (WARSZAWA)	0.756155
ŁABISZYŃSKA (WARSZAWA)	0.752234
OŚRODEK SZKOLENIOWY (WARSZAWA)	0.752187
WOŁOSKA (WARSZAWA)	0.74909
CIECISZEW-OSP (CIECISZEW)	0.747711
ZIELONA (GOŁKÓW)	0.746306
SZCZAKI (SZCZAKI)	0.742963
PKP WKD AL.JEROZOLIMSKIE (WARSZAWA)	0.737798
SĘKOCIN LAS (SĘKOCIN LAS)	0.737024
KLARYSEW (KONSTANCIN-JEZIORNA)	0.734001
ZALESINEK (PIASECZNO)	0.731186
WYGLĘDY (WYGLĘDY)	0.729216
TYTUŁOWA (WARSZAWA)	0.726183
RAKOWIECKA (WARSZAWA)	0.723781
SZELIGOWSKA (WARSZAWA)	0.712616
DOSTAWCZA (WARSZAWA)	0.712069
KOŚCIÓŁ (RASZYN)	0.709014
PARKOWA (LEGIONOWO)	0.708775
GOCŁAWEK (WARSZAWA)	0.705778
MIRKOWSKA (KONSTANCIN-JEZIORNA)	0.699135
PRZY AGORZE (WARSZAWA)	0.699127
WIEJSKA (MARKI)	0.698564
SZKOLNA (JABŁONNA)	0.694939
MAGDALENKA (MAGDALENKA)	0.694137
RAWAR (WARSZAWA)	0.693773
SZCZĘŚLIWICKA (WARSZAWA)	0.692949
LAMINA (PIASECZNO)	0.689504
PIĘKNA (WARSZAWA)	0.684073
POTOCKICH (WARSZAWA)	0.679568
POWSTAŃCÓW (ZĄBKI)	0.677281
NOWOWIEJSKA (WARSZAWA)	0.674982
NADARZYŃSKA (PIASECZNO)	0.673375
MIKLASZEWSKIEGO (WARSZAWA)	0.672014
DALIBORA (WARSZAWA)	0.669469
KOROTYŃSKIEGO (WARSZAWA)	0.669368
HENRYKA BRODATEGO (WARSZAWA)	0.663064
BORKÓW (WARSZAWA)	0.661773
INSTYTUT LOTNICTWA (WARSZAWA)	0.661308
BIAŁOŁĘKA (WARSZAWA)	0.660415
WARSZAWA AL.JEROZOLIMSKIE (WARSZAWA)	0.659966
REMBERTÓW-RATUSZ (WARSZAWA)	0.656233
CZŁUCHOWSKA (WARSZAWA)	0.65513
AL.KRAKOWSKA (WARSZAWA)	0.64597
CIOŁKA (WARSZAWA)	0.645715
FALENTY NOWE (FALENTY)	0.644887
METRO IMIELIN (WARSZAWA)	0.641931
WIEKOWEJ SOSNY (ZALESIE GÓRNE)	0.634742
URSUS-NIEDŹWIADEK (WARSZAWA)	0.634343

PARK TRAUGUTTA (WARSZAWA)	0.632593
KWITNĄCEJ AKACJI (WARSZAWA)	0.631425
ŁOJEWSKA (WARSZAWA)	0.629513
BIELANY-RATUSZ (WARSZAWA)	0.629023
PKP PIASECZNO (PIASECZNO)	0.627295
KADETÓW (WARSZAWA)	0.625615
INSTYTUT ENERGETYKI (WARSZAWA)	0.616032
KOŚCIÓŁ (JAZGARZEW)	0.61369
WÓLKA KOSOWSKA (WÓLKA KOSOWSKA)	0.60168
PODBIPIĘTY (WARSZAWA)	0.600648
ABRAHAMA (WARSZAWA)	0.588904
KRÓLOWEJ MARYSIEŃKI (WARSZAWA)	0.585951
PL.ZBAWICIELA (WARSZAWA)	0.582928
STEFANIKA (WARSZAWA)	0.582069
PAPIERNIA (KONSTANCIN-JEZIORNA)	0.581892
LEGIONOWO (LEGIONOWO)	0.579074
SMOLUCHOWSKIEGO (WARSZAWA)	0.576205
DWORKOWA (WARSZAWA)	0.575589
NIZINNA (WARSZAWA)	0.574579
PARK DRESZERA (WARSZAWA)	0.57298
OS.PRZYLESIE (JABŁONNA)	0.570556
PL.SPORTOWY (KONSTANCIN-JEZIORNA)	0.562391
EDISONA (WARSZAWA)	0.55797
OS.LAS (NIEPORĘT)	0.556054
GOCŁAW (WARSZAWA)	0.553626
PIASTÓW ŚLĄSKICH (WARSZAWA)	0.552755
PRUSZKOWSKA (WARSZAWA)	0.552699
ŁANOWA (SZCZAKI)	0.549031
ZDZIARSKA-KANAŁ (WARSZAWA)	0.548173
WARSZAWA WSCHODNIA (WARSZAWA)	0.543036
METRO MŁOCINY (WARSZAWA)	0.537689
KRAKOWIAKÓW (WARSZAWA)	0.532653
SPISAKA (WARSZAWA)	0.531923
SUWALSKA (WARSZAWA)	0.526945
CODZIENNA (WARSZAWA)	0.522964
STARODĘBY (WARSZAWA)	0.521323
BACZYŃSKIEGO (PARCELA)	0.518766
OS.DERBY VI (WARSZAWA)	0.518413
WOLNOŚCI (ZĄBKI)	0.51174
PADEREWSKIEGO (WARSZAWA)	0.507782
SIWIŃSKIEGO (LEGIONOWO)	0.507487
BLIZNE ŁASZCZYŃSKIEGO (BLIZNE ŁASZCZYŃSKIEGO)	0.501894
ARMII KRAJOWEJ (WARSZAWA)	0.501726
KOCZARGI STARE (KOCZARGI STARE)	0.500797
PARK OLSZYNA (WARSZAWA)	0.500621
ALEKSANDRÓW (STANISŁAWÓW PIERWSZY)	0.495438
LESZNOWOLA-POLE (LESZNOWOLA)	0.494674
STADION (RASZYN)	0.491896
SZARYCH SZEREGÓW (JABŁONNA)	0.489185
SPORTOWA (ZIELONKI)	0.488638
OŁÓWKOWA (PRUSZKÓW)	0.487849
PRZYBYSZEWSKIEGO (WARSZAWA)	0.485792
BOHOMOLCA (WARSZAWA)	0.48151
WARSZAWSKA (ZĄBKI)	0.479033
SZWOLEŻERÓW (WARSZAWA)	0.478532
SOSNKOWSKIEGO (WARSZAWA)	0.477531
ODYŃCA (WARSZAWA)	0.474749
KOLONIA LESZNOWOLA (KOLONIA LESZNOWOLA)	0.474276
PL.KRASIŃSKICH (WARSZAWA)	0.470498
GŁOSKÓW-ZIELONE (GŁOSKÓW)	0.462851
LUBELSKA (WARSZAWA)	0.460003
METRO CENTRUM NAUKI KOPERNIK (WARSZAWA)	0.459027
INŻYNIERSKA (WARSZAWA)	0.457615
LANGIEWICZA (IZABELIN)	0.454516
DZIERZGOŃSKA (WARSZAWA)	0.452556
ARKADYJSKA (WARSZAWA)	0.45235
PASZKÓW (PASZKÓW)	0.450516
CENTRUM MODY (NADARZYN)	0.450516
RADIOWO-BOCZNICA (WARSZAWA)	0.44662
MOŚCISKA (WARSZAWA)	0.44662
KOŚCIELNA (KĄTY WĘGIERSKIE)	0.443204
SADY ŻOLIBORSKIE (WARSZAWA)	0.442929
ORLICH GNIAZD (WARSZAWA)	0.4382
CM.PÓŁNOCNY-BRAMA PŁN. (WARSZAWA)	0.43749
TOPIEL (WARSZAWA)	0.435176
ZAJEZDNIA WORONICZA (WARSZAWA)	0.434783
17 STYCZNIA (PIASECZNO)	0.434056
POWSINEK (WARSZAWA)	0.433561
PAŁACOWA (WARSZAWA)	0.433561
HOŻA (OTWOCK)	0.431315
PAŁAC (JABŁONNA)	0.430933
BRACI WAGÓW (WARSZAWA)	0.429758
SZANCERA (WARSZAWA)	0.426989
KOCHANOWSKIEGO (WARSZAWA)	0.426289
POMNIK LOTNIKA (WARSZAWA)	0.426032
ORZECHOWA (LEGIONOWO)	0.425021
3 MAJA (LEGIONOWO)	0.422179
DEOTYMY (WARSZAWA)	0.422139
BORZĘCIN DUŻY (BORZĘCIN DUŻY)	0.418362
OŚRODEK ZDROWIA (GLINIANKA)	0.417797
PARK MOCZYDŁO (WARSZAWA)	0.417243
PL.TRZECH KRZYŻY (WARSZAWA)	0.416452
PIASECZNO (PIASECZNO)	0.415811
ŚNIADECKICH (KONSTANCIN-JEZIORNA)	0.414847
HOŁUBCOWA (WARSZAWA)	0.413952
WIERUCHOWSKA (STARE BABICE)	0.405192
FORT BLIZNE (WARSZAWA)	0.405062
MODLIŃSKA (WIELISZEW)	0.402342
MŁYN (RASZYN)	0.396953

POETÓW (WARSZAWA)	0.396747
ORLA (OTWOCK)	0.39632
KINOWA (WARSZAWA)	0.395072
WORONICZA (WARSZAWA)	0.393926
PARK PRASKI (WARSZAWA)	0.393881
OCH-TEATR (WARSZAWA)	0.387556
ŚLIWICE (WARSZAWA)	0.384751
REMBERTÓW-AON (WARSZAWA)	0.381193
ELBLĄSKA (WARSZAWA)	0.374997
CHOPINA (KONSTANCIN-JEZIORNA)	0.367831
OS.GRAPA (KONSTANCIN-JEZIORNA)	0.366871
NOWOLIPKI (WARSZAWA)	0.366634
OKNICKA (WARSZAWA)	0.365495
AFRYKAŃSKA (WARSZAWA)	0.363532
ZIELONKI-WIEŚ (ZIELONKI-WIEŚ)	0.361619
SZPITAL WOLSKI (WARSZAWA)	0.358034
FABRYCZNA (WARSZAWA)	0.356434
BORZĘCIN DUŻY-SZKOŁA (BORZĘCIN DUŻY)	0.35481
BARTYCKA (WARSZAWA)	0.352921
KOŁOBRZESKA (KONSTANCIN-JEZIORNA)	0.351543
BERENSONA (WARSZAWA)	0.351536
POŻAROWA (WARSZAWA)	0.349723
WARTKA (WARSZAWA)	0.348632
EMILII PLATER (WARSZAWA)	0.343963
ERAZMA Z ZAKROCZYMIA (WARSZAWA)	0.338726
AL.WOJSKA POLSKIEGO (KONSTANCIN-JEZIORNA)	0.337082
BOHUNA (WARSZAWA)	0.336917
LANCIEGO (WARSZAWA)	0.334223
KAPELI (WARSZAWA)	0.331586
JAGIELSKA (WARSZAWA)	0.331586
OS.DERBY III (WARSZAWA)	0.328925
SKOLIMÓW (KONSTANCIN-JEZIORNA)	0.327739
PL.UNII LUBELSKIEJ (WARSZAWA)	0.3277
WARSZAWA SŁUŻEWIEC (WARSZAWA)	0.325804
ŁYCZYN (ŁYCZYN)	0.321099
RUDZKA (WARSZAWA)	0.319522
BEŁDAN (WARSZAWA)	0.318796
LWOWSKA (LEGIONOWO)	0.317637
WARSZAWA OCHOTA (WARSZAWA)	0.316492
REDUTOWA (WARSZAWA)	0.313361
POWĄZKOWSKA (WARSZAWA)	0.31236
JANKI (JANKI)	0.310058
PORAJÓW (WARSZAWA)	0.30877
MUZEUM POWSTANIA WARSZAWSKIEGO (WARSZAWA)	0.308756
MIRKÓW-SZKOŁA (KONSTANCIN-JEZIORNA)	0.307752
OS.KONSTANCJA (BIELAWA)	0.30708
KOŁOWA (WARSZAWA)	0.304441
KUŹNICY KOŁŁĄTAJOWSKIEJ (WARSZAWA)	0.29973
DUNIKOWSKIEGO (WARSZAWA)	0.298646
STARE ŚWIDRY (WARSZAWA)	0.29864
ŁAZY (ŁAZY)	0.290787
WSPÓLNA (WARSZAWA)	0.288676
MŁYNARSKA (WARSZAWA)	0.287889
PUCHAŁY (FALENTY)	0.28491
STERNICZA (WARSZAWA)	0.282431
MERA-PNEFAL (WARSZAWA)	0.282406
MAKLAKIEWICZA (WARSZAWA)	0.281656
POMORSKA (PIASECZNO)	0.281314
WIERNA (WARSZAWA)	0.279469
KOSZYKOWA (WARSZAWA)	0.277526
GLINIANKA (GLINIANKA)	0.276875
SZAMOCIN (WARSZAWA)	0.274134
MANGALIA (WARSZAWA)	0.270687
HALA KOPIŃSKA (WARSZAWA)	0.26344
PELIKANÓW (WARSZAWA)	0.261924
WIERZBIN (WIERZBIN)	0.259106
SULEJÓWEK (SULEJÓWEK)	0.258066
BENNETTA (WARSZAWA)	0.257698
RYCERSKA (LEGIONOWO)	0.255622
NIEDZIAŁKOWSKIEGO (WARSZAWA)	0.255339
STACJA KRWIODAWSTWA (WARSZAWA)	0.255093
ZWOLEŃSKA (WARSZAWA)	0.255036
EUROPEJSKA (WARSZAWA)	0.254867
TRAWIASTA (WARSZAWA)	0.254089
SIEKIERKOWSKA (WARSZAWA)	0.253144
ZACHODNIA (WARSZAWA)	0.2509
ZAJĘCZA (WARSZAWA)	0.248541
DUCHNICKA (WARSZAWA)	0.248322
AL.WOJSKA POLSKIEGO (PRUSZKÓW)	0.248116
TUŻYCKA (WARSZAWA)	0.243391
DŁUGA (WOLICA)	0.236621
WARSZAWA WŁOCHY (WARSZAWA)	0.234071
SKOROSZEWSKA (WARSZAWA)	0.230836
PERZYŃSKIEGO (WARSZAWA)	0.230237
DANUSI (WARSZAWA)	0.229203
PODGÓRNA (WIELISZEW)	0.228486
WOŁOMIŃSKA (WARSZAWA)	0.227993
TABITA (KONSTANCIN-JEZIORNA)	0.227678
PL.BERNARDYŃSKI (WARSZAWA)	0.22656
SUCHY LAS (WARSZAWA)	0.226388
CYNKOWA (LEGIONOWO)	0.225722
RADIOSTACJA ŁAZY (ŁAZY)	0.224951
WARSZAWA ZACHODNIA (WARSZAWA)	0.224869
GÓRALSKA (WARSZAWA)	0.222736
KOCJANA (WARSZAWA)	0.212732
WAŁOWICKA (WARSZAWA)	0.212704
KOSAĆCOWA (WARSZAWA)	0.210967
KOŚCIELNA (WIELISZEW)	0.210694
MIĘDZYNARODOWA (WARSZAWA)	0.209728
KANAŁ ŻERAŃSKI (WARSZAWA)	0.206937

GAZOWNIA (REMBELSZCZYZNA)	0.20499
SÓJKI (WARSZAWA)	0.202426
HRABSKA DROGA (FALENTY)	0.202232
WSPÓLNA (MARKI)	0.201489
LEKTYKARSKA (WARSZAWA)	0.197933
SOBIESKIEGO (KONSTANCIN-JEZIORNA)	0.19202
WYBRZEŻE HELSKIE (WARSZAWA)	0.189388
OS.BATOREGO (LEGIONOWO)	0.189364
POLONEZA (WARSZAWA)	0.189308
WŁOCHY-RATUSZ (WARSZAWA)	0.188791
BRONOWSKA (WARSZAWA)	0.18872
CYLICHOWSKA (WARSZAWA)	0.188579
MARYSIN (WARSZAWA)	0.18686
CMENTARZ (LEGIONOWO)	0.183971
WIERUCHÓW (WIERUCHÓW)	0.181497
PKP PŁUDY (WARSZAWA)	0.180506
OGRODY DZIAŁKOWE BEMOWO (WARSZAWA)	0.180153
ŻWIROWA (CIECISZEW)	0.179771
PRZEKORNA (WARSZAWA)	0.179697
DOMANIEWSKA (WARSZAWA)	0.179517
BRZEZIŃSKA (WARSZAWA)	0.178392
ROSOCHATA (WARSZAWA)	0.178278
LIMANOWSKIEGO (WARSZAWA)	0.178001
METRO NATOLIN (WARSZAWA)	0.17725
BIBLIOTEKA UNIWERSYTECKA (WARSZAWA)	0.176821
PL.HALLERA (WARSZAWA)	0.176627
POWOJOWA (WARSZAWA)	0.17351
KLASZTORNA (WARSZAWA)	0.171082
ŻYWIECKA (WARSZAWA)	0.170024
ZAŚCIANKOWA (WARSZAWA)	0.169965
POLSKIE RADIO (WARSZAWA)	0.169687
SIENKIEWICZA (KONSTANCIN-JEZIORNA)	0.16925
POLITECHNIKA (WARSZAWA)	0.165802
KOŁO (WARSZAWA)	0.160932
RADIOWO (WARSZAWA)	0.160483
MADALIŃSKIEGO (WARSZAWA)	0.160244
OBRAZKOWA (WARSZAWA)	0.15955
NOSKOWSKIEGO (WARSZAWA)	0.159501
SZAFARZY (WARSZAWA)	0.15861
PRZYDROŻNA (WARSZAWA)	0.157507
ŁUKOWSKA (WARSZAWA)	0.156291
KARTEZJUSZA (WARSZAWA)	0.15279
MICKIEWICZA (LEGIONOWO)	0.148518
WODOCIĄG PŁN. (WIELISZEW)	0.148284
GRÓDEK (EMÓW)	0.146782
OSMAŃCZYKA (WARSZAWA)	0.14611
KADETÓW-SZKOŁA (WARSZAWA)	0.143533
STARE MIASTO (WARSZAWA)	0.143344
BYDGOSKA (KONSTANCIN-JEZIORNA)	0.142817
TOMASZEW (WARSZAWA)	0.142479
MROKÓW (MROKÓW)	0.142022
WARSZAWA TORUŃSKA (WARSZAWA)	0.139956
MALCZEWSKIEGO (WARSZAWA)	0.139937
BORYSZEWSKA (WIĄZOWNA)	0.139069
CEDROWA (KONSTANCIN-JEZIORNA)	0.138863
WARSZAWA ŻWIRKI I WIGURY (WARSZAWA)	0.138349
LALKI (WARSZAWA)	0.137779
INSTYTUT FIZYKI (WARSZAWA)	0.136188
SAMOCHODOWA (WARSZAWA)	0.136019
PELCOWIZNA (WARSZAWA)	0.131162
ORZECHOWA (STRZYKUŁY)	0.130002
SADKOWSKA (WARSZAWA)	0.129703
WARSZAWA PŁUDY (WARSZAWA)	0.129054
SZPITAL ONKOLOGICZNY (WIELISZEW)	0.129025
ŻOŁNY (WARSZAWA)	0.127431
POPŁAWSKA (RZAKTA)	0.126875
OSIEDLOWA (STARE BABICE)	0.123984
POPULARNA (WARSZAWA)	0.122951
SIKORSKIEGO (MOŚCISKA)	0.121567
ZIELONKI (ZIELONKI)	0.119608
GEN.KUTRZEBY (STARE BABICE)	0.119608
PL.NARUTOWICZA (WARSZAWA)	0.118723
ROZJAZD OBORSKI (KONSTANCIN-JEZIORNA)	0.117273
CZARNOMORSKA (WARSZAWA)	0.116248
KS.POPIEŁUSZKI (WARSZAWA)	0.114396
NOWY ŚWIAT (WARSZAWA)	0.114354
WARSZAWA STADION (WARSZAWA)	0.112825
CMENTARZ (ŁAZY)	0.112696
GEN.MACZKA (WARSZAWA)	0.110797
HALLERA (MARKI)	0.109928
OBJAZDOWA (WARSZAWA)	0.109343
WAŁ GOCŁAWSKI (WARSZAWA)	0.108695
GRANICZNA (MARKI)	0.105147
CH TARGÓWEK (WARSZAWA)	0.104647
ORZESZKOWEJ (PIASTÓW)	0.104201
PRZEŁĘCZY (WARSZAWA)	0.103294
KOBIAŁKA-SZKOŁA (WARSZAWA)	0.102415
EC SIEKIERKI (WARSZAWA)	0.101224
ZAPUSTNA (WARSZAWA)	0.0992596
WARSZAWA ŚRÓDMIEŚCIE (WARSZAWA)	0.0991496
ŻUPNICZA (WARSZAWA)	0.0985544
TERMINAL CARGO (WARSZAWA)	0.0983045
WARSZAWA POWIŚLE (WARSZAWA)	0.0948904
ZERZEŃ (WARSZAWA)	0.0942094
WILKOWIECKA (WARSZAWA)	0.092729
SAMOTNEJ MATKI (WARSZAWA)	0.0881525
HANDLOWA (WARSZAWA)	0.0878622
WARSZAWSKA (BIELAWA)	0.087128
KU SŁOŃCU (MOCZYDŁÓW)	0.0870566
SYTA (WARSZAWA)	0.0858294

BYSTRA (WARSZAWA)	0.0854347
WÓLKA ZAŁĘSKA (WÓLKA DWORSKA)	0.0845766
POLNA (KRUBIN)	0.0839732
GOŁKÓW-WIEŚ (GOŁKÓW)	0.0837543
BARTNICZA (WARSZAWA)	0.0828876
CEMENTOWNIA (WARSZAWA)	0.0818922
GONTARSKA (WARSZAWA)	0.0813957
OBOZOWA (WARSZAWA)	0.0770913
PIMOT (WARSZAWA)	0.076983
CM.ŻYDOWSKI (WARSZAWA)	0.0753218
ŁUBINOWA (WARSZAWA)	0.0731356
CH PROMENADA (WARSZAWA)	0.073113
DWERNICKIEGO (WARSZAWA)	0.0729352
UKSW (WARSZAWA)	0.0722554
NORBLIN (WARSZAWA)	0.0708345
BIELSZOWICKA (WARSZAWA)	0.070704
DŁUGA (ŁOMIANKI)	0.069256
SKARBKA Z GÓR (WARSZAWA)	0.0681485
WARSZAWA RAKOWIEC (WARSZAWA)	0.0643157
NACZELNIKOWSKA (WARSZAWA)	0.0638166
CHĘCIŃSKA (WARSZAWA)	0.0629875
ŻEROMSKIEGO (WARSZAWA)	0.061383
TURMONCKA (WARSZAWA)	0.059523
OBORSKA (KONSTANCIN-JEZIORNA)	0.0593103
NARODOWA (WARSZAWA)	0.0590716
FABRYCZNA (MARKI)	0.0569164
KOŚCIUSZKI-SZKOŁA (IZABELIN)	0.0565737
KŁOBUCKA (WARSZAWA)	0.0560714
SZPITALNA (MARKI)	0.0550897
PARK SKARYSZEWSKI (WARSZAWA)	0.0547684
PLATER (WARSZAWA)	0.0531103
KROKWI (WARSZAWA)	0.0512109
KLEMENSIEWICZA (WARSZAWA)	0.0508019
1 SIERPNIA (WARSZAWA)	0.0504925
SZUBERTA (WARSZAWA)	0.0503415
MYSZCZYN (MYSZCZYN)	0.0499199
PODBIAŁOWA (WARSZAWA)	0.049666
WENECKA (MARKI)	0.0494913
MARYMONT-POTOK (WARSZAWA)	0.0493156
HALA SPORTOWA BEMOWO (WARSZAWA)	0.0477582
SYMPATYCZNA (WARSZAWA)	0.0454834
BELGRADZKA (WARSZAWA)	0.0449291
OS.LEWANDÓW I (WARSZAWA)	0.0440489
REJTANA (MARKI)	0.0436871
JÓZEFÓW (JÓZEFÓW)	0.0431337
MODRA (KAJETANY)	0.0424963
KONIK NOWY (KONIK NOWY)	0.0422392
MAJOWA (WÓLKA RADZYMIŃSKA)	0.0407197
JEZIORKI (WARSZAWA)	0.040506
ZGRUPOWANIA AK KAMPINOS (WARSZAWA)	0.0377392
WIERZBOWA (HIPOLITÓW)	0.0375313
PIECHURÓW (WARSZAWA)	0.0373262
LITEWSKA (WARSZAWA)	0.0370244
DĄBRÓWKA (WARSZAWA)	0.03625
ŚMIGŁOWCA (WARSZAWA)	0.0360127
LOKAJSKIEGO (WARSZAWA)	0.0355557
ŻOŁNIERSKA (WARSZAWA)	0.0349478
KS.SŁOJEWSKIEGO (ŁAZY)	0.034151
FALENTY-OSIEDLE (FALENTY)	0.03071
JASNA (KONSTANCIN-JEZIORNA)	0.0300032
UNIWERSYTET (WARSZAWA)	0.0294205
MAŃKI-WOJDY (WARSZAWA)	0.0290544
PKP GOCŁAWEK (WARSZAWA)	0.027919
TOŁSTOJA (WARSZAWA)	0.0277903
BATUMI (WARSZAWA)	0.0272446
CMENTARZ (ZĄBKI)	0.0270299
STARY SĘKOCIN (STARY SĘKOCIN)	0.0268872
MOCHTYŃSKA (WARSZAWA)	0.0268273
FRACHTOWA (WARSZAWA)	0.0268273
MALCANÓW-BOOS (MALCANÓW)	0.0261484
KLAMROWA (WARSZAWA)	0.0258239
PKP URSUS PŁN. (WARSZAWA)	0.0257864
BUKÓW (WARSZAWA)	0.0249702
ZBYSZKA Z BOGDAŃCA (WARSZAWA)	0.0249457
MAŁCUŻYŃSKIEGO (WARSZAWA)	0.0245207
POLNA (WŁADYSŁAWÓW)	0.0243593
TROMBITY (WARSZAWA)	0.0242104
BENIAMINÓWKA (WÓLKA RADZYMIŃSKA)	0.0228586
POMOLOGICZNA (WARSZAWA)	0.0223695
KASZTANOWA (JEZIÓRKO)	0.0213202
UWIELINY-SZKOŁA (UWIELINY)	0.0198483
SMOCZA (WARSZAWA)	0.0196703
INSTYTUT NEUROLOGII (WARSZAWA)	0.0190241
REYMONTA (OTWOCK)	0.0187187
SOWIŃSKIEGO (LEGIONOWO)	0.0182599
OBORY (OBORY)	0.0180774
ŁYSAKOWSKA (WARSZAWA)	0.0173655
GÓRASZKA (GÓRASZKA)	0.0172252
TOR STEGNY (WARSZAWA)	0.0164492
ZAJEZDNIA WOLA (WARSZAWA)	0.0142813
BRÓDNOWSKA (WARSZAWA)	0.0138383
DOM ARTYSTÓW (KONSTANCIN-JEZIORNA)	0.0137933
WYSZYŃSKIEGO (GÓRA KALWARIA)	0.0134476
POSTĘPU (WARSZAWA)	0.012202
MAJDAN (MAJDAN)	0.012124
WCZASOWA (BIAŁOBRZEGI)	0.0116465
UMIASTOWSKA (KAPUTY)	0.00994437
MARYSIN (MARYSIN)	0.00984998
PODSKARBIŃSKA (WARSZAWA)	0.00772964

KACZEŃCÓW (ŁAZY)	0.00692686
HORBACZEWSKIEGO (WARSZAWA)	0.0066724
KLARYSEWSKA (WARSZAWA)	0.00665889
CZERWONA DROGA (WARSZAWA)	0.00541259
DŁUGA (DAWIDY)	0.005105
WARSZAWA PRAGA (WARSZAWA)	0.00359532
WARSZAWSKA (OLSZEWNICA STARA)	0.00355607
MŁOCIŃSKA (ŁOMIANKI)	0.00169223
OŚRODEK WOJSKOWY (WARSZAWA)	0.000717972
WILEŃSKA (WARSZAWA)	2.29022e-009
ŻOLIBORZ (WARSZAWA)	0
ŻERAŃ WSCHODNI (WARSZAWA)	0
ŚRÓDBORÓW (OTWOCK)	0
ZIELONA (WARSZAWA)	0
ZEGRZE PŁD. (ZEGRZE PŁD.)	0
ZBÓJNA GÓRA (WARSZAWA)	0
ZALESIE GÓRNE (ZALESIE GÓRNE)	0
ZAJEZDNIA UTRATA (WARSZAWA)	0
ZACISZE (WARSZAWA)	0
WOLICA K.PŁOCHOCINA (WOLICA k. PŁOCHOCINA)	0
WIŚNIOWA GÓRA (WARSZAWA)	0
WITOLIN (WARSZAWA)	0
WIELISZEW (WIELISZEW)	0
WARSZAWA ZACISZE-WILNO (WARSZAWA)	0
WARSZAWA ZACHODNIA WKD (WARSZAWA)	0
WARSZAWA LOTNISKO CHOPINA (WARSZAWA)	0
URZĄD MIASTA (NOWY DWÓR MAZ.)	0
URZĄD GMINY (PRAŻMÓW)	0
TOR SŁUŻEWIEC (WARSZAWA)	0
SZCZĘŚLIWICE (WARSZAWA)	0
SULEJÓWEK MIŁOSNA (SULEJÓWEK)	0
STRUSIA (WARSZAWA)	0
STARA MIŁOSNA (WARSZAWA)	0
STADIONOWA (RASZYN)	0
SPARTAŃSKA (WARSZAWA)	0
SOBIKÓW (SOBIKÓW)	0
SKIBIŃSKIEGO (TRUSKAW)	0
RZAKTA (RZAKTA)	0
RYNEK (GÓRA KALWARIA)	0
REMBERTÓW-KOLONIA (WARSZAWA)	0
REGULSKA (WARSZAWA)	0
RECHNIEWSKIEGO (WARSZAWA)	0
PŁOCHOCIN (PŁOCHOCIN)	0
PL.SZWEDZKI (JANKI)	0
PL.KOŚCIUSZKI (LEGIONOWO)	0
PKP ZALESIE GÓRNE (ZALESIE GÓRNE)	0
PKP PRAGA (WARSZAWA)	0
PKP OLSZYNKA GROCHOWSKA (WARSZAWA)	0
PKP GOŁĄBKI (WARSZAWA)	0
PALUCH (WARSZAWA)	0
PALMIRY-MUZEUM (KAMPINOSKI PN)	0
OS.VICTORIA (RADZYMIN)	0
OS.STASZICA (PRUSZKÓW)	0
OS.NIEPODLEGŁOŚCI (WOŁOMIN)	0
OS.KABATY (WARSZAWA)	0
OLESIN (WARSZAWA)	0
OGIŃSKIEGO (PIASTÓW)	0
NOWE ZAŁUBICE (NOWE ZAŁUBICE)	0
MŁOCINY-UKSW (WARSZAWA)	0
MIĘDZYLESIE (WARSZAWA)	0
MARIEW (MARIEW)	0
LESZNO (LESZNO)	0
LEGIONOWO PRZYSTANEK (LEGIONOWO)	0
KUPIECKA (WARSZAWA)	0
KUPIECKA (OTWOCK)	0
KOŚCIELNY (WARSZAWA)	0
INSTALATORÓW (WARSZAWA)	0
GASSY-PĘTLA (GASSY)	0
DZIKIEJ RÓŻY (BORZĘCIN MAŁY)	0
CZYNSZOWA (WARSZAWA)	0
CZARNA STRUGA (MARKI)	0
CM.PÓŁNOCNY-BRAMA ZACH. (WARSZAWA)	0
CH MARKI (MARKI)	0
AGRICOOP (KOPRKI)	0
KIELECKA (STARA IWICZNA)	-2.79853e-005
LESZCZYNKA (MROKÓW)	-0.00268506
MACIERZANKI (NADARZYN)	-0.00337389
TOR KOLARSKI (WARSZAWA)	-0.00502589
KORALOWYCH DĘBÓW (ZALESIE GÓRNE)	-0.00503497
PRAGA-PŁD.-RATUSZ (WARSZAWA)	-0.00645839
3 MAJA (TRUSKAW)	-0.00697209
KOMOROWSKA (NADARZYN)	-0.00743789
WARSZAWA WOLA GRZYBOWSKA (WARSZAWA)	-0.00748047
WAWERSKA (WARSZAWA)	-0.00829271
TKACZY (WARSZAWA)	-0.00986573
POZNAŃSKA (OŁTARZEW)	-0.0108551
MICHALIN (JÓZEFÓW)	-0.0110945
DKS TARGÓWEK (WARSZAWA)	-0.0114036
GOŁASZEWSKA (OŁTARZEW)	-0.0118157
OSTRÓDZKA (WARSZAWA)	-0.0124369
URZĄD GMINY (IZABELIN)	-0.0135454
WOLICA (WOLICA)	-0.0142072
PL.TEATRALNY (WARSZAWA)	-0.0160157
CZARNÓWKA (CZARNÓWKA)	-0.0160539
NAD ŚWIDREM (WOLA KARCZEWSKA)	-0.0197484
MOCZYDŁÓW (MOCZYDŁÓW)	-0.0203131
GŁĘBOCKA (WARSZAWA)	-0.0203683
URSUS-RATUSZ (WARSZAWA)	-0.0213754

OS.WEDEL (WOLA DUCKA)	-0.0216116
WARSZAWA GOCŁAWEK (WARSZAWA)	-0.0216462
WŁADYSŁAWA HERMANA (WARSZAWA)	-0.0224135
BATALIONÓW CHŁOPSKICH (OTWOCK)	-0.0227408
MAGELLANA (WARSZAWA)	-0.022954
OGRODOWA (KIEŁPIN STARY)	-0.0231364
OS.JANTAR (WARSZAWA)	-0.0245234
EKSPRESOWA (WARSZAWA)	-0.0246025
WARSZAWA MIĘDZYLESIE (WARSZAWA)	-0.0251269
OGRODOWA (SIERZCHÓW)	-0.0262139
PIASKI (WARSZAWA)	-0.0266414
BRUSZEWSKA (WARSZAWA)	-0.0269857
OSMAŃSKA (WARSZAWA)	-0.027036
ŁADY (ŁADY)	-0.0277031
PIASTÓW (PIASTÓW)	-0.0289011
SZKOLNA (GLINIANKA)	-0.0297663
LEŚNEJ POLANKI (WARSZAWA)	-0.0301003
WICHROWA (JESÓWKA)	-0.0309531
CM.POŁUDNIOWY-BRAMA WSCH. (ANTONINÓW)	-0.0310989
PALMIRY (KAMPINOSKI PN)	-0.0312796
3 MAJA (ZŁOTOKŁOS)	-0.0314936
KOSTOWIEC (KOSTOWIEC)	-0.0329787
BORZĘCIN DUŻY-WIEŚ (BORZĘCIN DUŻY)	-0.0333939
MENNICA (WARSZAWA)	-0.0337781
PRYSTORA (WARSZAWA)	-0.0354768
INSTRUKTORSKA (WARSZAWA)	-0.0360109
SZKOLNA (LESZNOWOLA)	-0.036809
WARSZAWA WAWER (WARSZAWA)	-0.0371512
MAJDAN-STACJA PALIW (MAJDAN)	-0.0375878
PAŁACOWA (ROZALIN)	-0.0378427
ROZALIN (ROZALIN)	-0.0378753
WARSZAWA ANIN (WARSZAWA)	-0.0382489
WIELOWIEJSKA (WARSZAWA)	-0.0383492
WINOGRONOWA (ROZALIN)	-0.0398433
ASPEKT (WARSZAWA)	-0.0406073
MYSZKOWSKA (WARSZAWA)	-0.0426885
BUDOWLANA (WARSZAWA)	-0.0436631
BURSZTYNOWA-SZPITAL (WARSZAWA)	-0.0437758
PARKOWA (TRUSKAW)	-0.0438599
THOMME (STARE BABICE)	-0.044286
KWIATOWA (ZĄBKI)	-0.0444426
SOBIESKIEGO (LEGIONOWO)	-0.0461808
WĘDKARSKA (WARSZAWA)	-0.0470334
BARTOSZYCKA (WARSZAWA)	-0.047088
STARE BEMOWO (WARSZAWA)	-0.0481654
ŁAGIEWNICKA (WARSZAWA)	-0.0482314
URSUS-SANKTUARIUM (WARSZAWA)	-0.0492134
KOŚCIÓŁ (ŁOMIANKI)	-0.0492871
BOROWINA III (BOROWINA)	-0.0497702
PL.MAŁACHOWSKIEGO (WARSZAWA)	-0.0508051
WARSZAWA RADOŚĆ (WARSZAWA)	-0.052041
BRZÓZEK (ORZESZYN)	-0.0520946
KAZUBÓW (WARSZAWA)	-0.0523386
GŁOSKÓW-LETNISKO (GŁOSKÓW-LETNISKO)	-0.0527525
NOWY PODOLSZYN (NOWY PODOLSZYN)	-0.0535827
PRZEBUDZENIA WIOSNY (ZALESIE GÓRNE)	-0.0540961
METRO RACŁAWICKA (WARSZAWA)	-0.0543266
GRABALÓWKI (WARSZAWA)	-0.0554657
SOLEC (WARSZAWA)	-0.0561755
WRZOSOWA (MICHAŁÓW-REGINÓW)	-0.0566305
TRENÓW (LASKI)	-0.0566979
KLASYKÓW (WARSZAWA)	-0.0579262
WARSZAWA MIEDZESZYN (WARSZAWA)	-0.0579852
BIBLIOTEKA NARODOWA (WARSZAWA)	-0.0598255
ZAWODOWA (WARSZAWA)	-0.0598377
WERSALSKA (WARSZAWA)	-0.0611043
WAT (WARSZAWA)	-0.0612292
ARMII POZNAŃ (ŁOMIANKI)	-0.0614734
GRENADY (WARSZAWA)	-0.0616497
RUNÓW OSADA (RUNÓW)	-0.0618283
DROGOWCÓW (JÓZEFÓW)	-0.0626394
MICHAŁÓW-REGINÓW (MICHAŁÓW-REGINÓW)	-0.0628717
WARSZAWA URSUS-NIEDŹWIADEK (WARSZAWA)	-0.0635734
WARSZAWA WESOŁA (WARSZAWA)	-0.0637663
LEWINOWSKA (WARSZAWA)	-0.0649355
KRYNICZNA (WARSZAWA)	-0.0655877
WARSZAWA FALENICA (WARSZAWA)	-0.065991
ZIELENIECKA (ZĄBKI)	-0.0665002
MOST ŚWIĘTOKRZYSKI (WARSZAWA)	-0.0675885
ŚWIERK-SZKOŁA (OTWOCK)	-0.0677153
KSIĘŻYCOWA (WARSZAWA)	-0.0678613
SZACHOWA (WARSZAWA)	-0.0683287
NIZINNA (OŁTARZEW)	-0.0683751
SŁONECZNA (WIELISZEW)	-0.0685732
GROMADZKA (HENRYKÓW-UROCZE)	-0.0690463
AWF (WARSZAWA)	-0.0692739
DWORCOWA (GÓRA)	-0.0697998
WSPÓLNA (DUCHNÓW)	-0.0701851
POŻARYSKIEGO (WARSZAWA)	-0.0702546
ICDS (ŁOMIANKI)	-0.0703328
IZABELIŃSKA (IZABELIN)	-0.0714158
WIŚLANA (PODŁĘCZE)	-0.0719334
OKUNIN (OKUNIN)	-0.0725042
WŁOŚCIAŃSKA (WARSZAWA)	-0.0725093
WARSZAWA CENTRALNA (WARSZAWA)	-0.0735348
WARSZAWA CHOSZCZÓWKA (WARSZAWA)	-0.0735808
ORĘŻNA (PIASECZNO)	-0.0740824
OSP ZŁOTOKŁOS (ZŁOTOKŁOS)	-0.0747741

LESZNOWOLA (LESZNOWOLA)	-0.0751562
STAWOWA (OTWOCK)	-0.0758143
VIVALDIEGO (KLAUDYN)	-0.0759761
SZYMANOWSKIEGO (KLAUDYN)	-0.0759761
OGRODNICZA (SULEJÓWEK)	-0.0761101
ANUSINEK (KĄTY WĘGIERSKIE)	-0.0769487
PRZYJACIÓŁ (NIEPORĘT)	-0.0771478
STAROWIEJSKA (NADARZYN)	-0.0773821
CMENTARZ (WÓLKA KOZODAWSKA)	-0.079049
MORSKIE OKO (WARSZAWA)	-0.0791541
RYNEK (LEGIONOWO)	-0.0799724
WARSZAWA REMBERTÓW (WARSZAWA)	-0.0806566
ZEGRZYŃSKA (STARE ZAŁUBICE)	-0.0811376
KĘPA OKRZEWSKA (KĘPA OKRZEWSKA)	-0.0815484
DĘBÓWKA II (DĘBÓWKA)	-0.0820881
BOGATKI (WARSZAWA)	-0.0839242
SIARCZANA (WARSZAWA)	-0.0841293
KWIATOWA (JANÓWEK PIERWSZY)	-0.0842695
OŻARÓW MAZ. (OŻARÓW MAZOWIECKI)	-0.0843617
KAŁUSZYN (KAŁUSZYN)	-0.0845769
PRUSA (PRUSZKÓW)	-0.0851279
KOLONIA WARSZAWSKA (KOLONIA WARSZAWSKA)	-0.0853703
KIERBEDZIA (OŻARÓW MAZOWIECKI)	-0.0855375
URZUT (URZUT)	-0.0863973
IZABELIŃSKA (STARE BABICE)	-0.086574
GÓRECKA (OTWOCK)	-0.0870054
WARSZAWA JEZIORKI (WARSZAWA)	-0.0881079
KLAUDYN (KLAUDYN)	-0.0882776
NOWA IWICZNA (NOWA IWICZNA)	-0.0883988
LUBICZÓW (LUBICZÓW)	-0.0896319
ZAJEZDNIA URSUS PŁN. (WARSZAWA)	-0.0897826
NOWY SĘKOCIN (NOWY SĘKOCIN)	-0.089852
PERKUNA (WARSZAWA)	-0.0911904
SZKOLNA (SKRZESZEW)	-0.091591
NOWA (OTWOCK)	-0.0918727
LEGIONÓW (WOŁOMIN)	-0.0923757
ROSY (WARSZAWA)	-0.0931642
PUSTELNIK (MARKI)	-0.093262
TEKLIN (OTWOCK)	-0.0936808
TATRZAŃSKA (OTWOCK)	-0.0937595
OGRODOWA (WÓLKA PRACKA)	-0.0939895
LEŚNA (JAROSZOWA WOLA)	-0.0943215
KOSMOWSKA II (BORZĘCIN DUŻY)	-0.0945001
WARSZAWA OLSZYNKA GROCHOWSKA (WARSZAWA)	-0.0949779
BORYSZEW (BORYSZEW)	-0.0950612
PRZYJAZNA (WARSZAWA)	-0.0951927
SŁONECZNA (LEGIONOWO)	-0.0952935
WARSZAWA URSUS (WARSZAWA)	-0.0960489
CENDROWICE (CENDROWICE)	-0.096331
KAMPINOSKA (ŁOMIANKI)	-0.0967458
DZIECHCINIEC-KOLONIA (DZIECHCINIEC)	-0.0976326
REZEDOWA (WARSZAWA)	-0.0984215
STANIEWICKA (WARSZAWA)	-0.0987613
SPOKOJNA (ŻANĘCIN)	-0.0988952
STRZYKUŁY (STRZYKUŁY)	-0.0990215
PÓŁNOCNA (WOLA PRAŻMOWSKA)	-0.0994666
INSTYTUT EKOLOGII PAN (DZIEKANÓW LEŚNY)	-0.0997183
ŁAŹNIEWSKA (ŚWIĘCICE)	-0.100136
GRZYBOWA (ŁOMIANKI)	-0.100948
HUTNICZA (WARSZAWA)	-0.100978
DOBKA Z OLEŚNICY (WARSZAWA)	-0.101441
BIAŁOŁĘKA-KANAŁ (WARSZAWA)	-0.101441
GWARNA (SULEJÓWEK)	-0.10207
ZABOROWSKA (ŚWIĘCICE)	-0.102567
EKOLOGICZNA (KLAUDYN)	-0.102826
URZĄD GMINY (NADARZYN)	-0.102833
PIOTRKÓWEK MAŁY (PIOTRKÓWEK MAŁY)	-0.103
ANDRUTOWA (WARSZAWA)	-0.103083
WARSZAWA GDAŃSKA (WARSZAWA)	-0.103683
ZAPLECZE (WARSZAWA)	-0.105652
ZAKŁADY ZBOŻOWE (WARSZAWA)	-0.105652
DZIECHCINIEC-SKLEP (DZIECHCINIEC)	-0.105744
ZDOBYWCÓW KOSMOSU (NOWY DWÓR MAZ.)	-0.105831
NOWA WIEŚ (JÓZEFÓW)	-0.10634
CHABROWA (WARSZAWA)	-0.106823
ZAKRĘTOWA (WARSZAWA)	-0.106858
ŻANĘCIN (ŻANĘCIN)	-0.106999
WARSZAWA URSUS PŁN. (WARSZAWA)	-0.107018
MAZURSKA (OTWOCK)	-0.107157
KUBUSIA PUCHATKA (OTWOCK)	-0.107166
PĘCZKOWSKIEGO (NIEPORĘT)	-0.108409
DZIAŁKOWA (WOLA PRAŻMOWSKA)	-0.109231
ROSY (HENRYKÓW-UROCZE)	-0.110014
MAGISTRACKA (WARSZAWA)	-0.110381
OSP SKRZESZEW (SKRZESZEW)	-0.110575
GRABINA (REMBELSZCZYZNA)	-0.111175
DŁUGORZECZNA (WARSZAWA)	-0.111175
PL.INWALIDÓW (WARSZAWA)	-0.111666
STAROSTWO (OŻARÓW MAZOWIECKI)	-0.111796
MIASTA (PIASECZNO)	-0.112427
STAWY (WARSZAWA)	-0.112593
PILAWA (PILAWA)	-0.112657
DURACZA (WARSZAWA)	-0.112821
MARIENSZTAT (WARSZAWA)	-0.11305
WOLSKA I (WOLA ALEKSANDRA)	-0.11313
POGROSZEW (POGROSZEW)	-0.113247
IMBIROWA (HENRYKÓW-UROCZE)	-0.113667
PRZEMYSŁOWA (OKUNIN)	-0.114539

WARSZAWA DAWIDY (WARSZAWA)	-0.114847
PIJARSKA (GÓRA KALWARIA)	-0.115127
ROZWOJOWA (SKRZESZEW)	-0.115999
DOBIESZ (DOBIESZ)	-0.116141
OKRZESZYN (OKRZESZYN)	-0.116247
AKACJOWA (SIERZCHÓW)	-0.116393
ŚWIĘCICKA (PŁOCHOCIN)	-0.116836
BALETOWA (DAWIDY)	-0.116954
WARSZAWA GOŁĄBKI (WARSZAWA)	-0.117616
WYSZYŃSKIEGO (JÓZEFÓW)	-0.118071
PONIATÓW (PONIATÓW)	-0.118442
TURÓW (KOBYŁKA)	-0.118528
ROLNA (KAJETANY)	-0.118703
PKP ŚRÓDBORÓW (OTWOCK)	-0.118779
GOK (STANISŁAWÓW DRUGI)	-0.118912
BATALIONÓW CHŁOPSKICH (WARSZAWA)	-0.118969
PARKOWA (PĘCICE)	-0.119098
BANACHA-SZPITAL (WARSZAWA)	-0.119205
ROZCHODNIKOWA (WARSZAWA)	-0.119309
NOWOLIPIE (NIEPORĘT)	-0.119792
KASZTANOWA (ORZESZYN)	-0.120319
HERBSTA (WARSZAWA)	-0.120654
BASENY INFLANCKA (WARSZAWA)	-0.120694
ŚWIDRY MAŁE (JÓZEFÓW)	-0.1207
SPACEROWA (OLSZEWNICA NOWA)	-0.120832
HASSA (WARSZAWA)	-0.121025
SZKOŁA (NOWA IWICZNA)	-0.121347
ARENA LEGIONOWO (LEGIONOWO)	-0.122022
KS.CZARTORYSKIEJ (RADZYMIN)	-0.122218
JAŁOWCOWA (OTWOCK)	-0.122781
ŹRÓDLANA (ŚWIĘCICE)	-0.123558
WILANÓWKA (OKRZESZYN)	-0.124506
CZUMY (WARSZAWA)	-0.124782
RUSIEC (RUSIEC)	-0.124788
ENERGETYCZNA (WARSZAWA)	-0.124936
NISKA (WARSZAWA)	-0.12567
ŁAWKI (ŁAWKI)	-0.127235
WÓLKA MŁĄDZKA (OTWOCK)	-0.128483
WIERZBOWA (RADZYMIN)	-0.128501
KRUBIN (KRUBIN)	-0.129269
CZOŁCHAŃSKIEGO (PRAŻMÓW)	-0.129811
WIEJSKA (WOŁOMIN)	-0.130036
FIELDORFA (WOŁOMIN)	-0.130036
OKRĄGŁA (WARSZAWA)	-0.130093
ZABORÓWEK-WIEŚ (ZABORÓWEK)	-0.130829
CHABROWA (WIKTORÓW)	-0.131354
SYPNIEWSKA (WARSZAWA)	-0.131621
PKP OKĘCIE (WARSZAWA)	-0.131634
KOLEJOWA (ZIELONKA)	-0.132322
ŚW.HIERONIMA (WARSZAWA)	-0.132487
PL.PONIATOWSKIEGO (NADARZYN)	-0.132645
PODŁĘCZE (PODŁĘCZE)	-0.133117
KOPRKI (KOPRKI)	-0.133754
CZAJKOWSKIEGO (KLAUDYN)	-0.13396
JUHASÓW (WARSZAWA)	-0.134513
PRUSZKÓW (PRUSZKÓW)	-0.134515
CMENTARZ 1920 R. (RADZYMIN)	-0.134567
PISKÓRKA (PISKÓRKA)	-0.135479
JANIN (ZABORÓW)	-0.135586
PL.TYSIĄCLECIA (WARSZAWA)	-0.135798
JAWCZYCE (JAWCZYCE)	-0.135848
CZOŁGISTÓW (WARSZAWA)	-0.136353
ZACHĘTA (WARSZAWA)	-0.136674
SPIRALNA (WARSZAWA)	-0.136764
PARLAMENTARNA (DAWIDY BANKOWE)	-0.137079
POCZTOWA (LASKI)	-0.137193
AKCENT (WARSZAWA)	-0.137217
ŁĄCZNICZEK AK (PRUSZKÓW)	-0.137345
JANÓW (JANÓW)	-0.138574
MAGNOLII (JÓZEFOSŁAW)	-0.138587
KAWĘCZYNEK I (KAWĘCZYNEK)	-0.139
OLSZEWNICA STARA (OLSZEWNICA STARA)	-0.139049
ZABORÓWEK-OSIEDLE (ZABORÓWEK)	-0.139621
POLNA (JÓZEFÓW)	-0.139819
KANAŁ NOWE UJŚCIE (WARSZAWA)	-0.139884
WARSZAWA ŻERAŃ (WARSZAWA)	-0.140442
OGÓRKOWA (WARSZAWA)	-0.140638
OS.ARBUZOWA (WARSZAWA)	-0.141016
PĘCICE-MAJĄTEK (PĘCICE)	-0.141149
PILASZKÓW (PILASZKÓW)	-0.141313
NORWIDA (RADZYMIN)	-0.14143
RĘCZAJSKA (KOBYŁKA)	-0.141555
GÓRKI (SŁUPNO)	-0.142931
KANARKOWA (MŁOCHÓW)	-0.143135
DZIAŁKOWA (JANKI)	-0.143279
PĘCICE-KOŚCIÓŁ (PĘCICE)	-0.144071
KOMOROWSKA-HOTEL (NADARZYN)	-0.144251
LESZNO-GS (LESZNO)	-0.144381
JANA OLBRACHTA (WARSZAWA)	-0.14484
SZKOLNA (IZABELIN)	-0.145462
BIAŁY ŁUG (BIAŁY ŁUG)	-0.145585
WARSZAWSKA (KONIK STARY)	-0.145638
GMINNEJ RADY NARODOWEJ (LESZNOWOLA)	-0.145903
ARCHIMEDESA (WARSZAWA)	-0.146512
PONIATOWSKIEGO (OTWOCK)	-0.146905
WARSZAWSKA (LESZNO)	-0.146952
STEFANÓWKA (BORYSZEW)	-0.147848
ŁĄKOWA (LASKI)	-0.147979

LEŚNY (DZIEKANÓW LEŚNY)	-0.148632
MARCINKI (STARE ZAŁUBICE)	-0.148746
KOŚCIUSZKI (NADARZYN)	-0.148934
SOCHACZEWSKA (MACIERZYSZ)	-0.150153
ZŁOTOKŁOS (ZŁOTOKŁOS)	-0.15198
BRONISZE (BRONISZE)	-0.152044
GOŁĘBIA (KAJETANY)	-0.152977
ZABRANIECKA (WARSZAWA)	-0.153459
LIPOWA (NADARZYN)	-0.154316
MARCELIN (WARSZAWA)	-0.15447
WARSZAWSKA (NADARZYN)	-0.154483
TERMINAL AUTOKAROWY (WARSZAWA)	-0.154741
PEONII (WARSZAWA)	-0.154976
PAROWCOWA (WARSZAWA)	-0.155251
NOWE WĄGRODNO (WĄGRODNO)	-0.15555
DŁUGA (USTANÓW)	-0.155611
HIPOLITOWSKA (KONIK NOWY)	-0.155881
WARSZAWA KOŁO (WARSZAWA)	-0.156386
DĄBRÓWKA SZLACHECKA (WARSZAWA)	-0.156478
OS.WYŻYNY (WARSZAWA)	-0.156544
BŁOTA (WARSZAWA)	-0.158084
NADZIEI (PIASECZNO)	-0.158398
ŚWIDRY WIELKIE (OTWOCK)	-0.158493
KĄPIELOWA (BIAŁOBRZEGI)	-0.158603
HABDZIN IV (HABDZIN)	-0.159216
POGROSZEW KOLONIA (POGROSZEW KOLONIA)	-0.159407
MORY (MORY)	-0.159467
KRÓLOWEJ MARYSIEŃKI (WOJCIESZYN)	-0.159697
SIELANKOWA (LEGIONOWO)	-0.159955
KORCZAKA (RADZYMIN)	-0.160109
MALINOWA (WARSZAWA)	-0.1603
DZIECHCINIEC (DZIECHCINIEC)	-0.160375
WARSZAWA ZOO (WARSZAWA)	-0.160379
JESIENNYCH LIŚCI (WARSZAWA)	-0.16075
KRÓLIKARNIA (WARSZAWA)	-0.16081
UMIASTOWSKA (OŻARÓW MAZOWIECKI)	-0.161228
WARYŃSKIEGO (ZIELONKA)	-0.161272
WŁADYSŁAWÓW (WŁADYSŁAWÓW)	-0.161948
RUCZAJ (WARSZAWA)	-0.162334
POD WILANÓWKA (WARSZAWA)	-0.162334
WIERZBOWA (LIPOWO)	-0.162829
WAWRZYSZEWSKA (WARSZAWA)	-0.163279
KARCZOCHA (WARSZAWA)	-0.163347
SIWEK (BORKI)	-0.164252
KOMPUTEROWA (WARSZAWA)	-0.164264
BIAŁOBRZEGI (BIAŁOBRZEGI)	-0.164296
GOCŁAWSKA (WARSZAWA)	-0.165638
URBANISTÓW (JULIANÓW)	-0.165764
CZEKOLADOWA (WARSZAWA)	-0.166117
KALINOWA DROGA (JESIONOWA)	-0.166223
JANCZEWICE (JANCZEWICE)	-0.166324
SZLACHECKA (OTWOCK)	-0.166358
ZAWADZKA (ZŁOTOKŁOS)	-0.166933
GMINY (LESZNOWOLA)	-0.167199
WÓLCZYŃSKA (MARIEW)	-0.167635
POW (PRUSZKÓW)	-0.168177
ZORZY (KLAUDYN)	-0.169074
3 MAJA (OTWOCK)	-0.169183
PRĄDZYŃSKIEGO (WARSZAWA)	-0.169919
URSYNÓW PŁN. (WARSZAWA)	-0.170638
WARSZAWA RAKÓW (WARSZAWA)	-0.171765
SOKOŁOWSKIEGO (LASKI)	-0.171993
WILCZA GÓRA (WILCZA GÓRA)	-0.172535
POD KAJETANY (SZAMOTY)	-0.17283
PARKOWA (USTANÓW)	-0.172949
ŻWIROWA (BOBROWIEC)	-0.173771
OTWOCK (OTWOCK)	-0.174208
BOBROWIECKA (BOBROWIEC)	-0.175284
MAJERANKOWA (WARSZAWA)	-0.175356
WRÓBLA (WARSZAWA)	-0.175388
ZAJEZDNIA KLESZCZOWA (WARSZAWA)	-0.17602
WASILKOWSKA (WARSZAWA)	-0.176237
WOJCIESZYN (WOJCIESZYN)	-0.176585
KĘDZIERÓWKA (KĘDZIERÓWKA)	-0.176692
JODŁOWA (STRZENIÓWKA)	-0.177567
3 MAJA (OŻARÓW MAZOWIECKI)	-0.177691
PĘCHERY (PĘCHERY)	-0.177912
MŁODZIEŻOWA (LIPOWO)	-0.17813
PROJEKTOWANA (WIĄZOWNA KOŚCIELNA)	-0.178295
KWIATÓW POLNYCH (DAWIDY BANKOWE)	-0.178398
PKP ZACISZE-WILNO (WARSZAWA)	-0.178441
GRANICZNA (JEZIÓRKO)	-0.178825
SPORTOWA (WIĄZOWNA)	-0.179042
BATOREGO (OTWOCK)	-0.179066
KOTLINY (MALCANÓW)	-0.179288
TERESPOLSKA (KONIK NOWY)	-0.179317
RACJONALIZACJI (WARSZAWA)	-0.180188
GRABÓW (WARSZAWA)	-0.181229
KORSAKA (WARSZAWA)	-0.18125
OS.LEŚNE (WARSZAWA)	-0.181854
LIPOWO (LIPOWO)	-0.181868
ZABORÓW-SZKOŁA (ZABORÓW)	-0.182016
WOJEWÓDZKA (ZŁOTOKŁOS)	-0.182041
TOLAK (JANCZEWICE)	-0.18258
SŁONECZNA (KRUPIA WÓLKA)	-0.182838
ŁADY-SZKOŁA (DAWIDY BANKOWE)	-0.182992
WOLSKA II (STANISŁAWÓW DRUGI)	-0.18315
STAROSTWO POWIATOWE (LEGIONOWO)	-0.18429

UMIASTÓW (UMIASTÓW)	-0.184963
MOKOTÓW (WARSZAWA)	-0.185066
WIŚNIOWA (WIELISZEW)	-0.185472
SIEMIATYCKA (WARSZAWA)	-0.186243
BASZKÓWKA (BASZKÓWKA)	-0.186333
NOWOLIPIE (WARSZAWA)	-0.186736
WIOSENNA (ZABORÓW)	-0.187172
KLONOWA (STANISŁAWÓW PIERWSZY)	-0.187984
WOŁODYJOWSKIEGO (STARE BABICE)	-0.188243
KISIELEWSKIEGO (CHOTOMÓW)	-0.188274
ODRĘBNA (WARSZAWA)	-0.1889
MURANÓW (WARSZAWA)	-0.189033
REJTANA (WARSZAWA)	-0.189231
SŁUPNO (SŁUPNO)	-0.189316
JESIONOWA (MAGDALENKA)	-0.189763
LEGIONOWO PIASKI (LEGIONOWO)	-0.189786
ENERGETYCZNA (PIASECZNO)	-0.190294
IWICZNA (PIASECZNO)	-0.190351
PKP MICHAŁÓW-REGINÓW (MICHAŁÓW-REGINÓW)	-0.190533
WOLICA (STARE ZAŁUBICE)	-0.191316
ORLA (ZĄBKI)	-0.191857
BRYLOWSKA (WARSZAWA)	-0.192193
MYCIELSKIEGO (WARSZAWA)	-0.192203
NAMYSŁOWSKA (WARSZAWA)	-0.192349
SPACEROWA (DUCHNÓW)	-0.192533
SOKOŁÓW (SOKOŁÓW)	-0.192603
ROBOTNICZA (OŻARÓW MAZOWIECKI)	-0.192865
KOSSAKA (OŻARÓW MAZOWIECKI)	-0.192865
SOSENKOWA (ALEKSANDRÓW)	-0.192982
BYSTRZYCKA (WARSZAWA)	-0.193145
NA BATERYJCE (WARSZAWA)	-0.194654
AL.BOHATERÓW WRZEŚNIA (WARSZAWA)	-0.194654
WARSZAWA SALOMEA (WARSZAWA)	-0.194809
RUDKA (RUDKA)	-0.195687
KRÓTKA (STARA IWICZNA)	-0.195747
WIELKIEGO DĘBU (WARSZAWA)	-0.19608
AL.PIŁSUDSKIEGO (SULEJÓWEK)	-0.196374
KOLEJOWA (WARSZAWA)	-0.196547
BARTYKI (WARSZAWA)	-0.196792
TOPOLOWA (KAWĘCZYNEK)	-0.196981
KS.SKORUPKI (ZĄBKI)	-0.197434
DOMINIKAŃSKA (WARSZAWA)	-0.197936
BOROWA (OTWOCK)	-0.19831
RADIÓWEK (RADIÓWEK)	-0.19847
LOTNISKO CHOPINA-PRZYLOTY (WARSZAWA)	-0.198539
MOŚCICKIEGO (LIPKÓW)	-0.199092
WIRAŻOWA (WÓLKA RADZYMIŃSKA)	-0.199364
KSIĘŻNEJ ANNY (WARSZAWA)	-0.19977
EC ŻERAŃ (WARSZAWA)	-0.199903
MOŁDAWSKA (WARSZAWA)	-0.200261
BLUE CITY (WARSZAWA)	-0.200324
ZNANA (WARSZAWA)	-0.200421
IMUZ (FALENTY)	-0.200521
POLNA (GABRYELIN)	-0.20063
PIASKOWA (LEGIONOWO)	-0.201327
WITEBSKA (WARSZAWA)	-0.201383
ZIELONA (WŁADYSŁAWÓW)	-0.201601
MOKRY ŁUG (WARSZAWA)	-0.201959
INSUREKCJI (WARSZAWA)	-0.20227
GRANICZKA (KIEŁPIN STARY)	-0.204135
SZPITAL POWIATOWY (PRUSZKÓW)	-0.204712
SOSNOWA (WARSZAWA)	-0.206178
TEOFILÓW (WOLA DUCKA)	-0.206661
DĄBKOWIZNA (DĄBKOWIZNA)	-0.207209
WOLA KARCZEWSKA (WOLA KARCZEWSKA)	-0.207978
WÓLKA DWORSKA-WISŁA (WÓLKA DWORSKA)	-0.208754
ZAŁUBICE-SZKOŁA (STARE ZAŁUBICE)	-0.208809
PRAŻMOWSKA (WARSZAWA)	-0.209116
JULIANÓW (JULIANÓW)	-0.209544
PIASTA KOŁODZIEJA (WARSZAWA)	-0.210006
KULSKIEGO (WARSZAWA)	-0.210553
HOSPICJUM ONKOLOGICZNE (WARSZAWA)	-0.21139
PŁYTOWA (WARSZAWA)	-0.211873
KOSYNIERÓW (ZĄBKI)	-0.212037
GÓRKI (JÓZEFÓW)	-0.212574
WOJSŁAWICKA (WARSZAWA)	-0.212961
PKP PRUSZKÓW (PRUSZKÓW)	-0.212976
MIKLASZEWSKIEGO (DAWIDY BANKOWE)	-0.213126
BORYNY (DAWIDY BANKOWE)	-0.213126
SGGW-KLINIKA (WARSZAWA)	-0.214188
CISZEWSKIEGO (WARSZAWA)	-0.214188
PODGÓRNA (KONIK NOWY)	-0.214511
MARYNIN (WARSZAWA)	-0.214716
STEFANÓW (ŻABIENIEC)	-0.214863
ŚWIERK (OTWOCK)	-0.215314
WOLA MROKOWSKA (WOLA MROKOWSKA)	-0.215505
WIERZBOWA (BASZKÓWKA)	-0.216308
KSIĘCIA ZIEMOWITA (WARSZAWA)	-0.216355
URZĄD MIASTA (ZĄBKI)	-0.216364
CM.WAWRZYSZEWSKI (WARSZAWA)	-0.216807
KAJETANY (KAJETANY)	-0.217674
POWĄZKI-CM.WOJSKOWY (WARSZAWA)	-0.217933
OBORY-PAŁAC (OBORY)	-0.218381
METRYCZNA (WARSZAWA)	-0.218572
PKP OŻARÓW MAZOWIECKI (OŻARÓW MAZOWIECKI)	-0.218623
RYBNICKA (WARSZAWA)	-0.218677
PAŃSKA (KONSTANCIN-JEZIORNA)	-0.218749
JESÓWKA (JESÓWKA)	-0.219463

OLSZYNOWA (NOWY PODOLSZYN)	-0.221483
WDW RYNIA (RYNIA)	-0.221693
ŻYZNA (KAPUTY)	-0.221767
PASCHALISA-JAKUBOWICZA (LIPKÓW)	-0.222775
FALENCKA (JANKI)	-0.223682
AL.JEDNOŚCI (LESZNOWOLA)	-0.22381
POLNA (KOMORNICA)	-0.223946
KOSODRZEWINY (WARSZAWA)	-0.224038
KUKUŁCZA (JASTRZĘBIE)	-0.22426
STEGNY (WARSZAWA)	-0.224552
KOMETY (JÓZEFOSŁAW)	-0.225129
KSIĄŻĘCA (WARSZAWA)	-0.225464
KWIRYNÓW (KWIRYNÓW)	-0.22589
MAJAKOWSKIEGO (WARSZAWA)	-0.226137
SZWOLEŻERÓW (ZĄBKI)	-0.226149
WARSZAWSKA (MICHAŁÓW-REGINÓW)	-0.226554
CUKROWNICZA (WARSZAWA)	-0.227439
PODŁĘCZE II (PODŁĘCZE)	-0.227548
JASNA (CHOTOMÓW)	-0.227874
MONTAŻOWA (WARSZAWA)	-0.228149
ZAŁUBICE-POCZTA (STARE ZAŁUBICE)	-0.228368
WSPÓŁCZESNA (WARSZAWA)	-0.22901
SZOBERA (WARSZAWA)	-0.229521
PRZEBIŚNIEGÓW (NOWA IWICZNA)	-0.230141
OS.KWIATOWA (NOWA IWICZNA)	-0.230141
CM.WŁOSKI (WARSZAWA)	-0.230464
BUKOWIŃSKA (WARSZAWA)	-0.2305
WINNICA (WARSZAWA)	-0.230868
HABDZIN III (HABDZIN)	-0.231341
HABDZIN II (HABDZIN)	-0.231341
ANUSIN (HABDZIN)	-0.231341
MOSTOWA (ŻABIENIEC)	-0.231344
WARSZAWA REDUTA ORDONA (WARSZAWA)	-0.23183
RODZINNA (STANISŁAWÓW PIERWSZY)	-0.231856
AL.SOLIDARNOŚCI (WARSZAWA)	-0.231932
DĘBOWA (UWIELINY)	-0.232391
MEISSNERA (WARSZAWA)	-0.232431
SADOWA (BASZKÓWKA)	-0.232692
BLISKA (WARSZAWA)	-0.233125
HENRYKÓW-UROCZE (HENRYKÓW-UROCZE)	-0.233545
SZYSZAKÓW (WARSZAWA)	-0.234163
LISTONOSZY (WARSZAWA)	-0.234163
TRUSKAWIECKA (WARSZAWA)	-0.234243
OKRZEI (OTWOCK)	-0.234571
PROTAZEGO (NIEPORĘT)	-0.234988
WARSZAWA KASPRZAKA (WARSZAWA)	-0.235026
RZESZOTARSKIEJ (WARSZAWA)	-0.235321
MALBORSKA (WARSZAWA)	-0.235507
ŚWIDERSKA (WARSZAWA)	-0.235531
MIKOŁAJSKA (WARSZAWA)	-0.235717
MATUSZEWSKA (WARSZAWA)	-0.236761
JACHOWICZA (WARSZAWA)	-0.237144
JADŹWINGÓW (WARSZAWA)	-0.237276
GASSY (GASSY)	-0.237599
GARWOLIŃSKA (WARSZAWA)	-0.23778
SZKOLNA (JÓZEFÓW k. LEGIONOWA)	-0.238012
BRZEŚCE (BRZEŚCE)	-0.23811
POLNA (LEGIONOWO)	-0.239318
ZALEŚNA (KAMIONKA)	-0.239636
LEŻAJSKA (WARSZAWA)	-0.239799
HALINÓW (HIPOLITÓW)	-0.240908
TRENÓW-LAS (WARSZAWA)	-0.241249
NOWE SŁUPNO (SŁUPNO)	-0.241987
OBJAZDOWA (JAWOROWA)	-0.242596
SYNÓW PUŁKU (WARSZAWA)	-0.243503
MARSZAŁKOWSKA (WARSZAWA)	-0.244049
NOWA (STARA IWICZNA)	-0.244091
OS.BUKOWY DWOREK (JABŁONNA)	-0.244339
GAJOWA (ZĄBKI)	-0.244658
PROMYKA (JAWOROWA)	-0.2448
WIŚLANA (WARSZAWA)	-0.245288
MAKOLĄGWY (WARSZAWA)	-0.245751
GROTY (WARSZAWA)	-0.245832
LUTOSŁAWSKIEGO (KLAUDYN)	-0.245915
CIEĆWIERZA (KLAUDYN)	-0.245915
PIASKOWA (JABŁONNA)	-0.24632
WÓLKA DWORSKA (WÓLKA DWORSKA)	-0.246813
OS.MAJDAŃSKA (WARSZAWA)	-0.246863
KOCJANA-BOCZNICA (WARSZAWA)	-0.247117
FORT WOLA (WARSZAWA)	-0.247303
NAŁĘCZOWSKA (WARSZAWA)	-0.247929
FALENICA (WARSZAWA)	-0.248588
ORZECHOWA (WOLA GOŁKOWSKA)	-0.248624
DZIKIEJ RÓŻY (NOWA WOLA)	-0.248711
MAŁE SIEKIERKI (WARSZAWA)	-0.24888
OS.BERNARDYŃSKA (WARSZAWA)	-0.248905
MARZYCIELI (KĄTY WĘGIERSKIE)	-0.249125
OGRODOWA (GŁOSKÓW-LETNISKO)	-0.249168
PATKOWSKIEGO (WARSZAWA)	-0.250394
URZĄD GMINY (NIEPORĘT)	-0.251157
DWORKOWA (JABŁONNA)	-0.252012
FIGOWA (WARSZAWA)	-0.252031
CMENTARZ (JABŁONNA)	-0.252461
WIELISZEW-PLAŻA (WIELISZEW)	-0.252715
KOZAKÓW (SZAMOTY)	-0.252995
JARZĘBINOWA (WARSZAWA)	-0.253314
BERENTA (PRUSZKÓW)	-0.254325
STRAŻACKA-MAGAZYNY (WARSZAWA)	-0.254771

SPÓŁDZIELNIA (DAWIDY)	-0.256993
OBJAZDOWA (JÓZEFÓW k. LEGIONOWA)	-0.257295
PKP JÓZEFÓW (JÓZEFÓW)	-0.257769
OS.BUKOWIEC (LEGIONOWO)	-0.258268
GEN.ROI (LEGIONOWO)	-0.258268
LIPY (NIEPORĘT)	-0.258463
LOTNISKO CHOPINA-ODLOTY (WARSZAWA)	-0.258928
RUNÓW OSADA II (RUNÓW)	-0.258974
WILANOWSKA (JÓZEFOSŁAW)	-0.259009
LEGIA-STADION (WARSZAWA)	-0.259017
CHOPINA (WARSZAWA)	-0.259194
ŁBISKA (ŁBISKA)	-0.259426
TARNOWIECKA (WARSZAWA)	-0.259495
NORWIDA (PIASTÓW)	-0.259765
OS.LEWANDÓW II (WARSZAWA)	-0.259932
INTERNETOWA (WARSZAWA)	-0.259932
OKRĄG (NOWA IWICZNA)	-0.259978
BOROWINA II (BOROWINA)	-0.260038
BOROWINA I (BOROWINA)	-0.260038
TANGO (WARSZAWA)	-0.260211
URSYNÓW ZACH. (WARSZAWA)	-0.260374
SZPITAL BIELAŃSKI (WARSZAWA)	-0.26083
LEGIONOWA (MARKI)	-0.26098
ZEGARYNKI (WARSZAWA)	-0.261766
ORACZY (WARSZAWA)	-0.261804
ZAJEZDNIA ŻOLIBORZ (WARSZAWA)	-0.262291
ŁĘG (ŁĘG)	-0.262469
ZAWADOWSKA (WARSZAWA)	-0.263114
HOSERÓW (WARSZAWA)	-0.263114
ATEŃSKA (WARSZAWA)	-0.264001
DROBIARSKA (WARSZAWA)	-0.265154
BOHATERÓW WARSZAWY (PRUSZKÓW)	-0.26613
RUNOWSKA (ZŁOTOKŁOS)	-0.266393
IWICKA (WARSZAWA)	-0.267368
DĄBROWSKIEGO (PIASTÓW)	-0.267855
KAPUTY (KAPUTY)	-0.268132
UNIWERSYTET MEDYCZNY (WARSZAWA)	-0.26878
OCHOTA-RATUSZ (WARSZAWA)	-0.269236
KAMELIOWA (NOWA WOLA)	-0.26969
BADYLARSKA (MICHAŁOWICE)	-0.269926
CICHA (JÓZEFÓW)	-0.27087
LIPOWA (MAGDALENKA)	-0.271127
MAGIERA (WARSZAWA)	-0.271284
POTULICKA (WARSZAWA)	-0.271621
GAJOWA (WOLA ALEKSANDRA)	-0.271727
NOWA WOLA (NOWA WOLA)	-0.272006
NOWY ZJAZD (WARSZAWA)	-0.27243
MILENIJNA (WARSZAWA)	-0.273294
ŁOMIANKI-CENTRUM (ŁOMIANKI)	-0.27389
PODOLSZYN (PODOLSZYN)	-0.273932
WOLSKA (MICHAŁÓW-REGINÓW)	-0.274229
POD CZERNIAKÓW (WARSZAWA)	-0.275145
DĘBÓWKA (DĘBÓWKA)	-0.275593
LEŚNA (WIKTORÓW)	-0.275699
ZAGÓRZE-SANATORIUM (ZAGÓRZE)	-0.276349
ELEKCYJNA (WARSZAWA)	-0.276449
STARZYŃSKIEGO (DAWIDY BANKOWE)	-0.27646
POSTĘPU (JASTRZĘBIEC)	-0.276724
PUŁASKIEGO (ZĄBKI)	-0.277854
TELEWIZJA POLSKA (WARSZAWA)	-0.278086
GRANICZNA (NOWA IWICZNA)	-0.278252
SREBRNYCH ŚWIERKÓW (PIASECZNO)	-0.278317
RYBNA (WOLA GOŁKOWSKA)	-0.278706
KAMERALNA (JÓZEFOSŁAW)	-0.278723
DWERNICKIEGO (OTWOCK)	-0.279203
KAWĘCZYN (KAWĘCZYN)	-0.279408
PKP SŁUŻEWIEC (WARSZAWA)	-0.27945
PL.POWSTAŃCÓW WARSZAWY (WARSZAWA)	-0.279976
NOWINEK (ZALESIE GÓRNE)	-0.280227
GIERDZIEJEWSKIEGO (WARSZAWA)	-0.280369
BORZĘCIN MAŁY (BORZĘCIN MAŁY)	-0.280461
REKRUCKA (WARSZAWA)	-0.280707
CM.PÓŁNOCNY-BRAMA PŁD. (WARSZAWA)	-0.280905
PKP WAWER (WARSZAWA)	-0.282289
POWĄZKI-IV BRAMA (WARSZAWA)	-0.282486
BRZOZOWA (WOLA MROKOWSKA)	-0.282506
CHOTOMÓW (CHOTOMÓW)	-0.282647
ZAGŁOBY (IZABELIN)	-0.282733
KS.SZULCZYKA (WARSZAWA)	-0.283207
PL.PIŁSUDSKIEGO (WARSZAWA)	-0.283237
SOSNOWA (PODDĘBIE)	-0.283446
PL.PRZYMIERZA (WARSZAWA)	-0.283602
IDZIKOWSKIEGO (SULEJÓWEK)	-0.284558
KOŚCIUSZKI (LEGIONOWO)	-0.284929
WÓLKA (WÓLKA)	-0.285123
CZERNIDŁA (CZERNIDŁA)	-0.285538
SKOROCHÓD-MAJEWSKIEGO (WARSZAWA)	-0.28656
GRZYBOWA (WARSZAWA)	-0.286903
AUTOKAROWY (WARSZAWA)	-0.286954
MUZEALNA (WARSZAWA)	-0.287832
ZAWODZIE (WARSZAWA)	-0.288762
SANTOCKA (WARSZAWA)	-0.288762
SOLIDNA (WARSZAWA)	-0.28916
POD SZCZAKI (SZCZAKI)	-0.290204
WIŚNIOWA (STANISŁAWÓW DRUGI)	-0.290276
KAWĘCZYN II (KAWĘCZYN)	-0.29063
OLSZOWA (OTWOCK)	-0.290782
WANILIOWA (PIASECZNO)	-0.293202
UTRATA (WARSZAWA)	-0.293219

LIPOWA (GŁOSKÓW-LETNISKO)	-0.295233
MARKOWSKA (WARSZAWA)	-0.295502
LATOSZKI (WARSZAWA)	-0.295582
GRYFITÓW (WARSZAWA)	-0.296624
KRÓLOWEJ BONY (WARSZAWA)	-0.296806
SPADOWA (WARSZAWA)	-0.29706
WYSPIAŃSKIEGO (ZĄBKI)	-0.297204
POD GŁOSKÓW (GŁOSKÓW)	-0.297437
PARTYZANTÓW (PRUSZKÓW)	-0.298202
WOJCIESZYN-GÓRKI (WOJCIESZYN)	-0.298403
GRZYMALITÓW (WARSZAWA)	-0.298419
OKRZEWSKA-CMENTARZ (KĘPA OKRZEWSKA)	-0.298658
LATCHORZEW (LATCHORZEW)	-0.299196
PUSZKINA (SULEJÓWEK)	-0.299627
ZALESIE (WIERZBIN)	-0.300384
LOKALNA (ŁAZY)	-0.301054
OBROŃCÓW TOBRUKU (WARSZAWA)	-0.301778
RATAJA (SULEJÓWEK)	-0.303312
WIERZCHOWSKIEGO (WARSZAWA)	-0.30424
ŚNIEŻNA (WARSZAWA)	-0.304857
SKOLIMÓW PÓŁNOCNY (KONSTANCIN-JEZIORNA)	-0.305148
SAPIEHY (IZABELIN)	-0.305405
SŁOWICZA (USTANÓW)	-0.306657
KONOPACKA (WARSZAWA)	-0.306669
OBROŃCÓW WARSZAWY (OŻARÓW MAZOWIECKI)	-0.307026
CHEMICZNA (WARSZAWA)	-0.307451
CH BŁĘKITNE (LEGIONOWO)	-0.307567
ĆWIKLIŃSKIEJ (WARSZAWA)	-0.308201
GROSZÓWKA (WARSZAWA)	-0.308338
ŁOZISKA (PIASECZNO)	-0.308417
CIOŁKOSZA (WARSZAWA)	-0.308768
DZIEWANOWSKA (WARSZAWA)	-0.308787
MIŁOSNA (SULEJÓWEK)	-0.309879
TOPOLOWA (WYGLĘDY)	-0.310515
PAROWOZOWA (WARSZAWA)	-0.311046
PIUSA XI (CHOTOMÓW)	-0.312231
ĆMIELOWSKA (WARSZAWA)	-0.312813
PODLEŚNA (JÓZEFÓW)	-0.313352
POLNYCH KWIATÓW (WARSZAWA)	-0.313793
CHODKIEWICZA (WARSZAWA)	-0.314245
DZIKA (ZĄBKI)	-0.314315
RASZYŃSKA (WARSZAWA)	-0.314372
POLNA (ŻABIENIEC)	-0.314757
BRONISŁAWA CZECHA-LAS (WARSZAWA)	-0.315954
DYREKCJA FSO (WARSZAWA)	-0.317133
KIEPURY (WARSZAWA)	-0.317627
KONSTRUKTORSKA (WARSZAWA)	-0.317729
NOWE BABICE (NOWE BABICE)	-0.318641
CZATÓW (WARSZAWA)	-0.319463
LEBIODOWA (WARSZAWA)	-0.319543
PŁD. (ANTONINÓW)	-0.320241
ZAWADY (WARSZAWA)	-0.321591
ARMATNIA (WARSZAWA)	-0.322984
WYNALAZEK (WARSZAWA)	-0.323341
INSTYTUT KARDIOLOGII (WARSZAWA)	-0.323366
PIELĘGNIAREK (WARSZAWA)	-0.323947
KAWĘCZYNEK II (KAWĘCZYNEK)	-0.32532
MARMUROWA (WARSZAWA)	-0.325734
CM.BRÓDNOWSKI (WARSZAWA)	-0.326
BLIZNE JASIŃSKIEGO (BLIZNE JASIŃSKIEGO)	-0.326271
OS.PATIO (WARSZAWA)	-0.326284
NOWOKABACKA (WARSZAWA)	-0.326284
KOSMOWSKA I (BORZĘCIN DUŻY)	-0.32644
BIAŁOBRZESKA (WARSZAWA)	-0.326721
SIEDLISKOWA (WARSZAWA)	-0.326746
ZIMORODKA (JASTRZĘBIE)	-0.327002
TABOROWA (WARSZAWA)	-0.327494
BOKSERSKA (WARSZAWA)	-0.327494
SZYMAŃSKIEGO (WARSZAWA)	-0.328453
OD LASU (KONSTANCIN-JEZIORNA)	-0.329081
CZEKANOWSKA (WARSZAWA)	-0.329406
SOBOCKA (WARSZAWA)	-0.330546
SZKOLNA (MACIERZYSZ)	-0.330996
OCHOCZA (WARSZAWA)	-0.331393
MIGDAŁOWA (WARSZAWA)	-0.332141
ZAJEZDNIA PRAGA (WARSZAWA)	-0.335591
GŁOGI (NIEPORĘT)	-0.336104
TORUŃSKA (PIASTÓW)	-0.336642
STRĄKOWA (WARSZAWA)	-0.337268
FARADAYA (WARSZAWA)	-0.337861
WAGONOWNIA (WARSZAWA)	-0.338117
OS.ISKRA (WARSZAWA)	-0.338941
SŁUŻEWIEC (WARSZAWA)	-0.339531
PODZAMCZE (WARSZAWA)	-0.339571
KLIMONTOWSKA (WARSZAWA)	-0.33971
BAREI (WARSZAWA)	-0.340823
PRZYSTANEK (LEGIONOWO)	-0.343943
ESPERANTO (WARSZAWA)	-0.344391
OCZAPOWSKIEGO (WARSZAWA)	-0.345071
KLUB LIRA (WARSZAWA)	-0.345106
GALLA ANONIMA (WARSZAWA)	-0.345118
TUZINOWA (WARSZAWA)	-0.345256
HABDZIN I (HABDZIN)	-0.345332
WOJNICKA (WARSZAWA)	-0.346377
KOCZARGI NOWE (KOCZARGI NOWE)	-0.347759
GRANICZNA (MAGDALENKA)	-0.348205
CHODAKOWSKA (WARSZAWA)	-0.348223
SZPITAL MSW (WARSZAWA)	-0.3483
FABRYCZNA (ŁAJSKI)	-0.348665

PARTYZANTÓW (OTWOCK)	-0.348717
REBUSOWA (WARSZAWA)	-0.34919
POGODNA (WARSZAWA)	-0.349209
ZAJEZDNIA OSTROBRAMSKA (WARSZAWA)	-0.349227
SZAMOTY (SZAMOTY)	-0.349646
DRZEWIECKIEGO (WARSZAWA)	-0.349654
PIASTOWSKA (WARSZAWA)	-0.34971
OLCHY (WARSZAWA)	-0.349985
JONTKA (WARSZAWA)	-0.349985
OKRĘŻNA (WARSZAWA)	-0.350996
CZERNIAKOWSKA (WARSZAWA)	-0.351277
ODRODZENIA (WARSZAWA)	-0.351702
TURKAWKI (JASTRZĘBIE)	-0.35353
MEGASAM (WARSZAWA)	-0.353685
PKP LEGIONOWO PIASKI (LEGIONOWO)	-0.353758
WALECZNYCH (WARSZAWA)	-0.35441
AL.SYBIRAKÓW (LEGIONOWO)	-0.354693
HANKI ORDONÓWNY (WARSZAWA)	-0.354812
URZĄD GMINY (LESZNOWOLA)	-0.355053
JESIONOWA (JESIONOWA)	-0.355219
STRUMYKOWA (PIASECZNO)	-0.35555
MILENIUM (GŁOSKÓW)	-0.35659
BANDURSKIEGO (LEGIONOWO)	-0.356634
BAJECZNA (BOBROWIEC)	-0.357053
PKP NOWA IWICZNA (NOWA IWICZNA)	-0.357936
SKALNICOWA (WARSZAWA)	-0.360194
SZARYCH SZEREGÓW (LEGIONOWO)	-0.360579
INSTYTUT HEMATOLOGII (WARSZAWA)	-0.361522
RADAROWA (WARSZAWA)	-0.361569
KOLONIA HELENÓW (WARSZAWA)	-0.361917
ROMANTYCZNA (WARSZAWA)	-0.362248
TRAKT LUBELSKI-LAS (WARSZAWA)	-0.362409
OKĘCIE BUSINESS PARK (WARSZAWA)	-0.362588
ZARUSKIEGO (WARSZAWA)	-0.362606
NARWIK (WARSZAWA)	-0.362868
PLANTOWA (PRUSZKÓW)	-0.362961
SZPITAL POWIATOWY (OTWOCK)	-0.363766
WIESIOŁKA (WARSZAWA)	-0.363815
PRZELOTOWA (WARSZAWA)	-0.364125
METRO WAWRZYSZEW (WARSZAWA)	-0.364488
KAROLIN (WARSZAWA)	-0.364676
RZEMIEŚLNICZA (OTWOCK)	-0.36541
BUKOWIECKA (WARSZAWA)	-0.36569
CENTRUM OLIMPIJSKIE (WARSZAWA)	-0.366428
RKS ORZEŁ (WARSZAWA)	-0.366521
SOSNKOWSKIEGO (ZĄBKI)	-0.36692
HALLERA (ZĄBKI)	-0.36692
BARTOSZKA (WARSZAWA)	-0.367712
KRAKUSA (LEGIONOWO)	-0.367931
ŁUCZNICZA (WARSZAWA)	-0.368093
POSAG 7 PANIEN (WARSZAWA)	-0.368861
BANACHA (WARSZAWA)	-0.368989
BUCHALTERYJNA (WARSZAWA)	-0.369061
WĄWOZOWA (WARSZAWA)	-0.369225
MOCHTYŃSKA-KANAŁ (WARSZAWA)	-0.370106
CYGAŃSKA (WARSZAWA)	-0.370125
(TUNELOWA) (WARSZAWA)	-0.37014
ŁOWICKA (WARSZAWA)	-0.370882
SMOLNA (OTWOCK)	-0.37114
ARONIOWA (WARSZAWA)	-0.371301
GŁOWACKIEGO (KONSTANCIN-JEZIORNA)	-0.371743
DOLNA (KONSTANCIN-JEZIORNA)	-0.371743
ŁOSOSIOWA (WARSZAWA)	-0.371795
STARA (NADMA)	-0.372715
SIERAKÓW (SIERAKÓW)	-0.373129
LERCHA (WARSZAWA)	-0.373402
KAMPINOSKA (WARSZAWA)	-0.3735
SCHRONISKO DLA ZWIERZĄT (WARSZAWA)	-0.373809
OGRODY DZIAŁKOWE NA SKRAJU (WARSZAWA)	-0.373809
WARSZAWSKA (ZIELONKI-WIEŚ)	-0.374212
SŁOWACKIEGO (LEGIONOWO)	-0.376039
LIMBY (WARSZAWA)	-0.376454
NOWE BEMOWO (WARSZAWA)	-0.377452
SADY SGGW (WARSZAWA)	-0.379337
WARSZAWA OKĘCIE (WARSZAWA)	-0.380219
TORFOWA (WARSZAWA)	-0.380336
WKD RAKÓW (WARSZAWA)	-0.380526
TOPOGRAFICZNA (WARSZAWA)	-0.381091
OS.WILGA (WARSZAWA)	-0.381369
CHMIELNA (WARSZAWA)	-0.381977
ŁUKOWA (WARSZAWA)	-0.382606
BIAŁOWIEJSKA (WARSZAWA)	-0.383183
GLEBOWA (WARSZAWA)	-0.383389
RATUSZOWA-ZOO (WARSZAWA)	-0.384149
DROGOWA (WARSZAWA)	-0.385362
BRACI ZAWADZKICH (WARSZAWA)	-0.385362
ADAMPOLSKA (WARSZAWA)	-0.386125
TUŁOWICKA (WARSZAWA)	-0.387107
DANKOWICKA (WARSZAWA)	-0.387107
ZAMIENIE (ZAMIENIE)	-0.387352
SIELSKA (STANISŁAWÓW PIERWSZY)	-0.387375
HOŻA (WARSZAWA)	-0.388241
MACZKA (ZĄBKI)	-0.388391
WODNIAKÓW (WARSZAWA)	-0.389067
KONARSKIEGO (WARSZAWA)	-0.389609
CYKLAMENÓW (WARSZAWA)	-0.390983
POMIECHOWSKA (WARSZAWA)	-0.391005
TOMASZOWSKA (WARSZAWA)	-0.3911
PARK MŁOCIŃSKI (WARSZAWA)	-0.39188
SKOROSZE (WARSZAWA)	-0.392975
ŻONKILOWA (WARSZAWA)	-0.393139

ZDZIARSKA (WARSZAWA)	-0.393294
PUSTOLA (WARSZAWA)	-0.393351
ZARUBY (WARSZAWA)	-0.393421
PODLEŚNA (ŁAZY)	-0.393487
NIK (WARSZAWA)	-0.393977
KSIĘCIA BOLESŁAWA (WARSZAWA)	-0.396067
JEZIOROWA (WARSZAWA)	-0.396241
ANTKA ROZPYLACZA (WARSZAWA)	-0.398128
TUROWICE (TUROWICE)	-0.398334
KŁOBUCKA-HCB (WARSZAWA)	-0.398468
OGRÓD BOTANICZNY (WARSZAWA)	-0.400012
STĘPIŃSKA (WARSZAWA)	-0.40064
KAWĘCZYN POD (WARSZAWA)	-0.401137
KRZYŻANOWSKIEGO (WARSZAWA)	-0.403172
RYBAŁTÓW (WARSZAWA)	-0.404486
BUCHNIK (JABŁONNA)	-0.404944
ORCHOWIECKA (WARSZAWA)	-0.406917
PLANETOWA (WARSZAWA)	-0.407131
PRZYLEŚNA (STANISŁAWÓW PIERWSZY)	-0.407405
AMBARAS (WARSZAWA)	-0.408105
WIĄZANA (WARSZAWA)	-0.408738
MIELCZARSKIEGO (WARSZAWA)	-0.40911
OSIEDLE MŁODYCH (WARSZAWA)	-0.409128
SAGALLI (WARSZAWA)	-0.410257
PÓŁNOCNA (KAMIONKA)	-0.410352
DŁUGOSZA (WARSZAWA)	-0.410696
ORDYNACKA (WARSZAWA)	-0.411076
TECHNIKÓW (WARSZAWA)	-0.411333
SIERAKOWSKIEGO (WARSZAWA)	-0.412234
GŁOSKÓW-SZKOŁA (GŁOSKÓW)	-0.412246
MAŁEJ ŁĄKI (WARSZAWA)	-0.412756
JULIANOWSKA (WARSZAWA)	-0.413291
HALA MIROWSKA (WARSZAWA)	-0.414163
DWORSKA (WARSZAWA)	-0.414448
MEKSYKAŃSKA (WARSZAWA)	-0.415036
GRANICZNA (SZCZAKI)	-0.415194
ŚWIDER-KOŚCIÓŁ (OTWOCK)	-0.415536
ALEJA RÓŻ (LEGIONOWO)	-0.415904
FOKSAL (WARSZAWA)	-0.417548
MAŁYCH DĘBÓW (WARSZAWA)	-0.418266
ANDERSENA (WARSZAWA)	-0.419398
INFLANCKA (WARSZAWA)	-0.420956
KONCERTOWA (WARSZAWA)	-0.421088
STĘŻYCKA (WARSZAWA)	-0.421685
(KIJOWSKA) (WARSZAWA)	-0.423037
GRENADIERÓW (WARSZAWA)	-0.424109
KROKUSÓW (WARSZAWA)	-0.425032
ZAGŁOBY (WARSZAWA)	-0.425042
HUBALA-DOBRZAŃSKIEGO (WARSZAWA)	-0.425042
PARCELACYJNA (WARSZAWA)	-0.425806
PODLEŚNA (KOCZARGI STARE)	-0.426168
BOHATEREWICZA (WARSZAWA)	-0.426267
AGATOWA (JÓZEFOSŁAW)	-0.426343
POWSTAŃCÓW WARSZAWY (OTWOCK)	-0.427684
KOPIJNIKÓW (WARSZAWA)	-0.428331
KICKIEGO (WARSZAWA)	-0.428766
MASZEWSKA (WARSZAWA)	-0.431439
STANISŁAWÓW (STANISŁAWÓW PIERWSZY)	-0.431691
WKD OPACZ (MICHAŁOWICE)	-0.432006
SIKORECZKI (JASTRZĘBIE)	-0.433118
BOCIANA CZARNEGO (JASTRZĘBIE)	-0.433118
PIĘCIOLINII (WARSZAWA)	-0.433392
KRASICKIEGO (NOWA WOLA)	-0.433565
OPOLSKA (STARE ZAŁUBICE)	-0.434591
SZASERÓW (WARSZAWA)	-0.435946
PODOLSKA (WARSZAWA)	-0.435946
FRANCUSKA (WARSZAWA)	-0.436703
ŻUBROWA-UKSW (WARSZAWA)	-0.437599
MAŃKOWSKA (WARSZAWA)	-0.438246
BALEYA (WARSZAWA)	-0.438896
KOLIBRA (ZGORZAŁA)	-0.440209
MAŁEGO RYCERZA (WARSZAWA)	-0.440332
KOLARSKA (WARSZAWA)	-0.440777
BUDEXPO (WARSZAWA)	-0.440961
MARKOWSKIEGO (PIASECZNO)	-0.4415
DZIELNICOWA (WARSZAWA)	-0.443377
ANTONIEWSKA (WARSZAWA)	-0.444396
ANANASOWA (WARSZAWA)	-0.444396
REYMONTA (LATCHORZEW)	-0.445322
JANOWSKIEGO (LATCHORZEW)	-0.445322
WARSZAWSKIEJ 1920 (WARSZAWA)	-0.446192
PORT NIEPORĘT (NIEPORĘT)	-0.44705
SZKOLNA (WIĄZOWNA)	-0.447125
SZKOLNA (ŁAJSKI)	-0.447306
GRUSZCZYŃSKIEGO (WARSZAWA)	-0.447413
JARA (WARSZAWA)	-0.447503
KOCJANA-SĄDY (WARSZAWA)	-0.447745
KWATERY GŁÓWNEJ (WARSZAWA)	-0.448326
ŚNIEGOCKIEJ (WARSZAWA)	-0.449357
WILANOWSKA (WARSZAWA)	-0.449847
KRZYŻY (WARSZAWA)	-0.451448
POMNIK SAPERA (WARSZAWA)	-0.45172
SOLIDARNOŚCI (WIELISZEW)	-0.451941
BOTANICZNY (WARSZAWA)	-0.4522
SZKOLNA (RASZYN)	-0.457096
LELEWELA (PIASTÓW)	-0.45734
PKP LEGIONOWO-PRZYSTANEK (LEGIONOWO)	-0.457371
JAROSŁAWSKA (JÓZEFÓW)	-0.458067
URBANISTÓW (WARSZAWA)	-0.458284

WYCZÓŁKI (WARSZAWA)	-0.458629
ODKRYTA (WARSZAWA)	-0.459082
BARSZCZEWSKA (WARSZAWA)	-0.459122
PRZEPIÓRKI (NOWA WOLA)	-0.46065
ZAJEZDNIA REDUTOWA (WARSZAWA)	-0.462157
STRUŻAŃSKA (LEGIONOWO)	-0.464005
KIELECKA (WARSZAWA)	-0.464349
FELIŃSKIEGO (WARSZAWA)	-0.464628
LEŚNYCH BOGINEK (ZALESIE GÓRNE)	-0.465668
POD BUDOWLANI II (WARSZAWA)	-0.466197
KALORYCZNA (WARSZAWA)	-0.466197
WAWRZYSZEW (WARSZAWA)	-0.466515
STRUMYKOWA (WARSZAWA)	-0.467379
WITWICKIEGO (KONSTANCIN-JEZIORNA)	-0.46792
GOŁKOWSKA (WARSZAWA)	-0.468672
PILECKIEGO (WARSZAWA)	-0.470087
NADARZYŃSKA (WARSZAWA)	-0.471463
KSIĘŻYCOWA (STANISŁAWÓW PIERWSZY)	-0.472576
GDECKA (WARSZAWA)	-0.47279
BOLEŚĆ (WARSZAWA)	-0.473456
ŁAZY-SZKOŁA (ŁAZY)	-0.473484
ROZŁOGI (WARSZAWA)	-0.474161
WYTWÓRNIA MAS BITUMICZNYCH (WARSZAWA)	-0.475073
DANISZEWSKA (WARSZAWA)	-0.475073
ORLA (SULEJÓWEK)	-0.4754
CZERWONYCH BERETÓW (WARSZAWA)	-0.476086
WIDAWSKA (WARSZAWA)	-0.477733
MIŁOBĘDZKA (WARSZAWA)	-0.480403
CYNAMONOWA (WARSZAWA)	-0.480941
GRZYBOWSKA (PARCELA)	-0.481027
PONIATOWSKIEGO (OŻARÓW MAZOWIECKI)	-0.481323
OŚ KRÓLEWSKA (WARSZAWA)	-0.481977
HIPODROM (WARSZAWA)	-0.482688
SŁOWACKIEGO (WARSZAWA)	-0.482689
WSCHODNI (WARSZAWA)	-0.484106
WRZOSOWA (KOCZARGI STARE)	-0.484776
FABRYKA POMP (WARSZAWA)	-0.485327
CHODECKA (WARSZAWA)	-0.485673
KOSTRZEWSKIEGO (WARSZAWA)	-0.485731
MIASTA (OTWOCK)	-0.486442
BUDZIŃSKIEJ-TYLICKIEJ (WARSZAWA)	-0.487312
FORT BEMA (WARSZAWA)	-0.487359
DO FORTU (WARSZAWA)	-0.487377
KAFLOWA (WARSZAWA)	-0.487544
CERAMICZNA (WARSZAWA)	-0.487544
PRZY PARKU (WARSZAWA)	-0.488044
RYBNA (WARSZAWA)	-0.488177
SGGW-REKTORAT (WARSZAWA)	-0.490149
SGGW-BIBLIOTEKA (WARSZAWA)	-0.490149
CH WÓLKA KOSOWSKA (WÓLKA KOSOWSKA)	-0.490838
(RYDYGIERA) (WARSZAWA)	-0.490937
OS.DOMANIEWSKA (WARSZAWA)	-0.491432
ŁOWCZA (WARSZAWA)	-0.491601
KOSIARZY (WARSZAWA)	-0.49178
RYCHNOWSKA (WARSZAWA)	-0.492685
LAWINOWA (WARSZAWA)	-0.493207
WĄSKA (JÓZEFÓW)	-0.494073
WIŚNIOWA (WARSZAWA)	-0.494234
REMBERTÓW (WARSZAWA)	-0.4944
SZEROKA (WARSZAWA)	-0.495369
JEZIORKO CZERNIAKOWSKIE (WARSZAWA)	-0.495394
KADETÓW-KOŚCIÓŁ (WARSZAWA)	-0.496346
MOTYLKOWA (WARSZAWA)	-0.496967
MANDARYNKI (WARSZAWA)	-0.497238
OLSZYNA ŁYCZYŃSKA (PARCELA)	-0.497446
ANNY JAGIELLONKI (WARSZAWA)	-0.497462
POCZTY GDAŃSKIEJ (WARSZAWA)	-0.497591
POŁUDNIOWA (KONSTANCIN-JEZIORNA)	-0.498007
AKADEMIJNA (JABŁONNA)	-0.498504
POCISK (WARSZAWA)	-0.500067
KINO BAŚŃ (PIASTÓW)	-0.500418
KAWĘCZYN DOLNY (KAWĘCZYN)	-0.503467
WILLOWA (NOWY PODOLSZYN)	-0.50494
CHŁODNIA (WARSZAWA)	-0.505845
PRZEBIEG (WARSZAWA)	-0.506496
BUŁAWY (WARSZAWA)	-0.506496
BOLESŁAWICKA (WARSZAWA)	-0.508359
OPŁOTEK (WARSZAWA)	-0.508924
PARCELA I (PARCELA)	-0.509083
MASZYNOWA (WARSZAWA)	-0.510793
SZARADY (WARSZAWA)	-0.511737
SUZINA (WARSZAWA)	-0.512737
OŚRODEK AZS (WARSZAWA)	-0.513081
GEN.ZAJĄCZKA (WARSZAWA)	-0.513223
PĘCLIŃSKA (WIĄZOWNA KOŚCIELNA)	-0.513909
SPB KAWĘCZYN (WARSZAWA)	-0.513941
EC KAWĘCZYN (WARSZAWA)	-0.513941
MIŃSKA (WARSZAWA)	-0.514891
LIPKÓW-MUZEUM (LIPKÓW)	-0.515742
LEŚNA (JABŁONNA)	-0.516051
RADIOWA-WAT (WARSZAWA)	-0.516571
GWARKÓW (WARSZAWA)	-0.516758
STASZICA (WARSZAWA)	-0.517044
TYMOTKI (WARSZAWA)	-0.517841
CM.PRAWOSŁAWNY (WARSZAWA)	-0.518548
ORDONA (WARSZAWA)	-0.519348
ZAKRZEWSKA (WARSZAWA)	-0.521612
MOKOTOWSKA (WARSZAWA)	-0.522086
BATALIONU WŁOCHY (WARSZAWA)	-0.522347
CHUDOBY (WARSZAWA)	-0.522957

ŁASAKA (WARSZAWA)	-0.524592
GOTARDA (WARSZAWA)	-0.525047
HORTENSJI (WARSZAWA)	-0.525475
DRACENY (WARSZAWA)	-0.525475
PKP MICHALIN (JÓZEFÓW)	-0.525951
FALENTY DUŻE (FALENTY DUŻE)	-0.526686
ARBUZOWA (WARSZAWA)	-0.527109
KSAWERÓW (WARSZAWA)	-0.528239
ZBÓJNOGÓRSKA (WARSZAWA)	-0.528306
AKACJOWA (KOCZARGI STARE)	-0.528551
RAGINISA (WARSZAWA)	-0.52888
PL.GRZYBOWSKI (WARSZAWA)	-0.52905
OGRODY DZIAŁKOWE PALUCH (WARSZAWA)	-0.530937
ZAUŁEK (WARSZAWA)	-0.533378
ZGK (KONSTANCIN-JEZIORNA)	-0.537062
RONDO ŻABCZYŃSKIEGO (WARSZAWA)	-0.537582
KSIĄŻĄT MAZOWIECKICH (PIASECZNO)	-0.537588
SPÓŁDZIELNIA NOMA (WARSZAWA)	-0.538666
JAZGARZEW-SZKOŁA (JAZGARZEW)	-0.539114
GŁÓWNA (JAZGARZEW)	-0.539114
OPACZEWSKA (WARSZAWA)	-0.540056
ZASADOWA (WARSZAWA)	-0.540331
SZASERÓW-SZPITAL (WARSZAWA)	-0.540612
BUKOWSKIEGO (WARSZAWA)	-0.541396
POBORZAŃSKA (WARSZAWA)	-0.541886
WARCHAŁOWSKIEGO (WARSZAWA)	-0.543231
OS.MŁODYCH (LEGIONOWO)	-0.543243
SOKOŁOWSKA (WARSZAWA)	-0.543289
BIAŁOSTOCKA (KONSTANCIN-JEZIORNA)	-0.543681
SŁOMCZYN-KOŚCIÓŁ (SŁOMCZYN)	-0.543802
PARCELA II (PARCELA)	-0.543989
DZIAŁYŃCZYKÓW (WARSZAWA)	-0.544237
DW.WSCHODNI (KIJOWSKA) (WARSZAWA)	-0.544294
ODLEWNICZA (WARSZAWA)	-0.544531
AKACJOWA (KĄTY WĘGIERSKIE)	-0.545321
PŁASKOWICKIEJ (WARSZAWA)	-0.545354
NOWODWORSKA (WARSZAWA)	-0.545411
SANGUSZKI (WARSZAWA)	-0.545873
PALISADOWA (WARSZAWA)	-0.5474
RADOŚĆ-WODOCIĄG (WARSZAWA)	-0.550048
NATOLIN PŁN. (WARSZAWA)	-0.550412
PARK BRÓDNOWSKI (WARSZAWA)	-0.550573
FRANCISZKAŃSKA (WARSZAWA)	-0.550613
STAREGO DĘBU (CZARNÓW)	-0.550759
ŻYRARDOWSKA (WARSZAWA)	-0.551009
WAŁUSZEWSKA (WARSZAWA)	-0.551009
OSTRÓDZKA-SZKOŁA (WARSZAWA)	-0.551925
BOROWIECKA (WARSZAWA)	-0.552171
CMENTARZ (STARE BABICE)	-0.55242
TEATR KOMEDIA (WARSZAWA)	-0.552971
WÓLCZYŃSKA-BOCZNICA (WARSZAWA)	-0.553935
URZĄD GMINY (WIELISZEW)	-0.555583
AUGUSTÓWKA (WARSZAWA)	-0.555724
PORT PILAWA (BIAŁOBRZEGI)	-0.556306
PŁUŻAŃSKA (WARSZAWA)	-0.55929
UROCZA (WARSZAWA)	-0.559839
11 LISTOPADA (WARSZAWA)	-0.560784
GRUNTOWA (WARSZAWA)	-0.561124
RUBINOWA (JÓZEFOSŁAW)	-0.563957
STOCER (KONSTANCIN-JEZIORNA)	-0.563965
GWIAŹDZISTA (WARSZAWA)	-0.5642
DĄBRÓWKA WIŚLANA (WARSZAWA)	-0.564414
ROGALIŃSKA (WARSZAWA)	-0.564535
DOJAZDOWA (WARSZAWA)	-0.564607
MONIUSZKI (WARSZAWA)	-0.564799
GŁÓWNA (GOŁKÓW)	-0.565458
GERBERA (PIASECZNO)	-0.566608
WYSZOGRODZKA (WARSZAWA)	-0.566876
ROGOWSKA (WARSZAWA)	-0.566881
ŚWIĄTYNIA OPATRZNOŚCI BOŻEJ (WARSZAWA)	-0.56717
KOZIELSKA (WARSZAWA)	-0.567177
HETMAŃSKA (WARSZAWA)	-0.567265
OŚRODEK CORA (WARSZAWA)	-0.568348
KAKOWSKIEGO (WARSZAWA)	-0.570823
KAPITULNA (WARSZAWA)	-0.570989
KOMPASOWA (WARSZAWA)	-0.572644
BROŻKA (WARSZAWA)	-0.573299
PRZASNYSKA (WARSZAWA)	-0.573774
LIPOWA (BIELAWA)	-0.574168
BIELAWA-OSP (BIELAWA)	-0.574168
DOBOSZA (WARSZAWA)	-0.574426
SOWIŃSKIEGO (WARSZAWA)	-0.576505
URZĄD SKARBOWY (PRUSZKÓW)	-0.577699
IZBICKA-CMENTARZ (WARSZAWA)	-0.577962
DROHOBYCKA (WARSZAWA)	-0.577962
CZERWIŃSKA (WARSZAWA)	-0.578609
WRZECIONO (WARSZAWA)	-0.579748
STOCZNIOWCÓW (WARSZAWA)	-0.579858
NIEPODLEGŁOŚCI (WIELISZEW)	-0.580413
ŁYCZYŃSKA (SŁOMCZYN)	-0.580771
WCZASOWA (KONSTANCIN-JEZIORNA)	-0.580813
SEZAM (WARSZAWA)	-0.58134
ARESZT ŚLEDCZY SŁUŻEWIEC (WARSZAWA)	-0.582424
WARNEŃSKA (WARSZAWA)	-0.582509
NISKA (BLIZNE ŁASZCZYŃSKIEGO)	-0.582581
DWORCOWA (PIASECZNO)	-0.583796
OSTROROGA (WARSZAWA)	-0.584135
RYBIE (RYBIE)	-0.584543

MRĄGOWSKA (WARSZAWA)	-0.587095
BOBROWIECKA (WARSZAWA)	-0.587179
ZASOBNA (WARSZAWA)	-0.587195
BEREZYŃSKA (WARSZAWA)	-0.58739
ROLNA (ŁAZY)	-0.587864
CHYLICZKI (CHYLICZKI)	-0.589671
MALINOWSKIEGO (WARSZAWA)	-0.592677
KS.SKORUPKI (LEGIONOWO)	-0.593833
OGRODY DZIAŁKOWE PIASKI (WARSZAWA)	-0.595511
BEŁZY (WARSZAWA)	-0.596006
CZEREMCHOWA (WARSZAWA)	-0.596645
AL. 3 MAJA (WARSZAWA)	-0.597871
TRZYKROTKI (WARSZAWA)	-0.597918
CYTADELA (WARSZAWA)	-0.598214
GRZEGORZEWSKIEJ (WARSZAWA)	-0.598916
TŁUCHOWSKA (WARSZAWA)	-0.5994
KORDIANA (WARSZAWA)	-0.599559
ORZYCKA (WARSZAWA)	-0.599655
RUTY (WARSZAWA)	-0.600104
KAROLKOWA (WARSZAWA)	-0.601642
WILLOWA (WIELISZEW)	-0.601778
LESZCZYŃSKIEGO (WARSZAWA)	-0.601895
JACHTOWA (WARSZAWA)	-0.603447
ARESZT ŚLEDCZY GROCHÓW (WARSZAWA)	-0.606168
MOKRY ŁUG-LEŚNICZÓWKA (WARSZAWA)	-0.606651
PKP JEZIORKI (WARSZAWA)	-0.606999
FORTOWA (BLIZNE ŁASZCZYŃSKIEGO)	-0.607304
JAGIELLOŃSKA (WARSZAWA)	-0.608568
WIŚLICKA (WARSZAWA)	-0.60914
GĄSOCIŃSKA (WARSZAWA)	-0.610251
WÓJTYSZKI (WARSZAWA)	-0.610514
SZOLC-ROGOZIŃSKIEGO (WARSZAWA)	-0.61063
LIPOWCZANA (WARSZAWA)	-0.612852
KAMBUD (WARSZAWA)	-0.616016
WIOLINOWA (WARSZAWA)	-0.617028
OS.JAGIELLOŃSKA (LEGIONOWO)	-0.617156
RAABEGO (WARSZAWA)	-0.617263
PL.POLITECHNIKI (WARSZAWA)	-0.617285
OLSZOWA (WARSZAWA)	-0.618138
PICASSA (WARSZAWA)	-0.619695
OSIEDLE (WARSZAWA)	-0.621835
ŻYTNIA (WARSZAWA)	-0.623069
ZIELONA (PIASECZNO)	-0.624033
MARSA-LAS (WARSZAWA)	-0.624958
ROLANDA (WARSZAWA)	-0.625518
DOM KULTURY (PIASECZNO)	-0.626035
ZYNDRAMA Z MASZKOWIC (WARSZAWA)	-0.626277
SMUGOWA (WARSZAWA)	-0.626277
PAŃSKA (JABŁONNA)	-0.627325
OŚRODEK LOK (WARSZAWA)	-0.627351
ORLĄT LWOWSKICH (WARSZAWA)	-0.629466
WESOŁA (WARSZAWA)	-0.629752
TORWAR (WARSZAWA)	-0.629991
PIASECZYŃSKA (KONSTANCIN-JEZIORNA)	-0.63089
KOWIEŃSKA (WARSZAWA)	-0.632552
JERZEGO (WARSZAWA)	-0.635634
GMINY (WIĄZOWNA)	-0.636515
KAZIMIERZOWSKA (KONSTANCIN-JEZIORNA)	-0.640508
CEGLANA (NADMA)	-0.64098
CHYLICE (CHYLICE)	-0.64187
AL.WIELKOPOLSKI (WARSZAWA)	-0.642626
TALAROWA (WARSZAWA)	-0.643787
PRZEPIÓRKI (WARSZAWA)	-0.643795
BIELAWA I (BIELAWA)	-0.644453
STOKROTKI (NIEPORĘT)	-0.644571
SIEKIERKI (WARSZAWA)	-0.647099
STEFANA BATOREGO (WARSZAWA)	-0.64811
WOLA GRZYBOWSKA (WARSZAWA)	-0.648407
RAKOWIECKA-SANKTUARIUM (WARSZAWA)	-0.649848
SIENNICKA (WARSZAWA)	-0.650006
GŁOWACKIEGO (WARSZAWA)	-0.652678
SPICHRZOWA (WARSZAWA)	-0.653574
KĄTY GRODZISKIE-OSIEDLE (WARSZAWA)	-0.653574
HUSARSKA (LEGIONOWO)	-0.655389
ZAKAMAREK (WARSZAWA)	-0.658314
HOTELOWA (SULEJÓWEK)	-0.65867
KRZYWICKIEGO (WARSZAWA)	-0.659005
JURANDA ZE SPYCHOWA-SZKOŁA (WARSZAWA)	-0.65958
ECHA LEŚNE (WARSZAWA)	-0.65958
KANAŁ GOCŁAWSKI (WARSZAWA)	-0.66082
JARACZA (WARSZAWA)	-0.661639
INSTALATORÓW-KIS (WARSZAWA)	-0.66231
KS.MALINOWSKIEGO (JÓZEFÓW)	-0.663933
ŁOKIETKA (WARSZAWA)	-0.664789
DZIĘCIOŁA (WARSZAWA)	-0.666848
CM.OFIAR WOJNY (WARSZAWA)	-0.666966
NIEDŹWIEDZIA (WARSZAWA)	-0.667562
CM.WOLSKI (WARSZAWA)	-0.668661
SYRENY (WARSZAWA)	-0.66992
KRAŃCOWA (WARSZAWA)	-0.670741
NOWODWORSKA (ŁAJSKI)	-0.671899
KRZESIWA (WARSZAWA)	-0.671958
TAGORE'A (WARSZAWA)	-0.673182
CHOMICZÓWKA (WARSZAWA)	-0.674108
SAMARYTANKA (WARSZAWA)	-0.674554
GŁ. (WARSZAWA)	-0.674606
URZĄD STATYSTYCZNY (WARSZAWA)	-0.675436
LINDLEYA (WARSZAWA)	-0.675514
MPT GROCHÓW (WARSZAWA)	-0.678437
INOWŁODZKA (WARSZAWA)	-0.678625

WARZELNICZA (WARSZAWA)	-0.679514
SCHROEGERA (WARSZAWA)	-0.679572
PLANTY (IZABELIN)	-0.679574
CHODKIEWICZA (IZABELIN)	-0.679574
GNIEWKOWSKA (WARSZAWA)	-0.679661
JADWISIN (WARSZAWA)	-0.679801
SADYBA (WARSZAWA)	-0.679818
P+R AL.KRAKOWSKA (WARSZAWA)	-0.68514
PL.ZAMKOWY (WARSZAWA)	-0.685281
KOLUMBA (WARSZAWA)	-0.687138
HENRYKA PROBUSA (WARSZAWA)	-0.688165
KS.BOGUCKIEGO (WARSZAWA)	-0.688324
PANCERA (WARSZAWA)	-0.688756
ŚW.A.BOBOLI (WARSZAWA)	-0.691159
DIAMENTOWA (WARSZAWA)	-0.691374
BOHUSZEWICZÓWNY (WARSZAWA)	-0.692642
PL.OSTREJ BRAMY (WARSZAWA)	-0.694743
WYSZKOWSKA (WARSZAWA)	-0.694968
TATARSKA (WARSZAWA)	-0.695505
WALEREGO SŁAWKA (WARSZAWA)	-0.695546
ZAGÓRZAŃSKA (WARSZAWA)	-0.696515
SAMORZĄDOWA (WARSZAWA)	-0.696515
CENTRUM TARGOWE (WARSZAWA)	-0.696683
SALOMEA (WARSZAWA)	-0.699675
KOŁŁĄTAJA (ZĄBKI)	-0.700022
FIELDORFA (WARSZAWA)	-0.700104
POWSTAŃCÓW WARSZAWY (PIASECZNO)	-0.701751
NALEWKI-MUZEUM (WARSZAWA)	-0.70184
HALE BANACHA (WARSZAWA)	-0.702356
KLIMATYCZNA (WARSZAWA)	-0.703278
PŁUŻNICKA (WARSZAWA)	-0.704708
ROKOKOWA (WARSZAWA)	-0.704736
HARFOWA (WARSZAWA)	-0.705209
WÓLCZYŃSKA (WARSZAWA)	-0.706816
LESZNO (WARSZAWA)	-0.707627
NIEŚWIESKA (WARSZAWA)	-0.708865
PREFABET (WARSZAWA)	-0.713983
KAMYKOWA (WARSZAWA)	-0.715049
OS.OLSZYNY (WARSZAWA)	-0.715544
NOWAKA-JEZIORAŃSKIEGO (WARSZAWA)	-0.715701
SOŁTYKA (WARSZAWA)	-0.716519
ZAOLZIAŃSKA (WARSZAWA)	-0.717228
MELCERA (WARSZAWA)	-0.717777
LNIANA (WARSZAWA)	-0.720107
ŚWIĘTOJERSKA (WARSZAWA)	-0.72016
PIRENEJSKA (WARSZAWA)	-0.720493
LASEK NA KOLE (WARSZAWA)	-0.722062
KASZTANOWA (WARSZAWA)	-0.722197
JESIONOWA (OPACZ-KOLONIA)	-0.722672
RATUSZOWA (WARSZAWA)	-0.724744
ROGATKOWA (WARSZAWA)	-0.724884
KONDUKTORSKA (WARSZAWA)	-0.7271
WAŁBRZYSKA-CMENTARZ (WARSZAWA)	-0.727963
KAMIŃSKIEGO (WARSZAWA)	-0.729832
NOWE WŁOCHY (WARSZAWA)	-0.731831
DS RIVIERA (WARSZAWA)	-0.733204
LAUROWA (WARSZAWA)	-0.734022
KOŃSKI JAR (WARSZAWA)	-0.737797
OS.MARYWILSKA (WARSZAWA)	-0.738143
ZBARASKA (WARSZAWA)	-0.742722
STADION NARODOWY (WARSZAWA)	-0.749519
TADŻYCKA (WARSZAWA)	-0.750613
HORNÓWEK (HORNÓWEK)	-0.755549
SZPITAL GROCHOWSKI (WARSZAWA)	-0.755557
ŁĄCZYNY (WARSZAWA)	-0.757014
KRASICKIEGO (WARSZAWA)	-0.757534
DZIELNA (WARSZAWA)	-0.761024
ŁAZIENKI KRÓLEWSKIE (WARSZAWA)	-0.762037
BŁĘKITNE (LEGIONOWO)	-0.764317
CZORSZTYŃSKA (WARSZAWA)	-0.765574
ZABRANIECKA-MAGAZYNY (WARSZAWA)	-0.766897
ZABRANIECKA-BOCZNICA (WARSZAWA)	-0.766897
NAUCZYCIELSKA (MARKI)	-0.768552
ORZECHOWA (WARSZAWA)	-0.769737
TARGÓWEK (WARSZAWA)	-0.771754
BARTŁOMIEJA (WARSZAWA)	-0.774477
ZAMENHOFA (WARSZAWA)	-0.774922
MIŁOSNA (WARSZAWA)	-0.77552
NOWOTYSKA (WARSZAWA)	-0.783876
AL.JANA PAWŁA II (WARSZAWA)	-0.784017
FLETNIOWA (WARSZAWA)	-0.784284
NOCZNICKIEGO (WARSZAWA)	-0.785887
KRÓLEWSKA (JABŁONNA)	-0.786339
ZABŁUDOWSKA (WARSZAWA)	-0.78675
BACHA (WARSZAWA)	-0.786946
JOLIOT-CURIE (WARSZAWA)	-0.787893
KOŚCIUSZKI (MARKI)	-0.789086
WIRAŻOWA (WARSZAWA)	-0.789766
CZAPELSKA (WARSZAWA)	-0.79171
LAS BIELAŃSKI (WARSZAWA)	-0.792432
MODZELEWSKIEGO (WARSZAWA)	-0.793867
SZYMANOWSKIEGO (WARSZAWA)	-0.79732
SIEMIEŃSKIEGO (WARSZAWA)	-0.7983
KS POLONIA (WARSZAWA)	-0.800019
OKRZEI (WARSZAWA)	-0.800384
GEODEZYJNA (WARSZAWA)	-0.800836
BATALIONU MIOTŁA (WARSZAWA)	-0.801179
WOJCIECHOWSKIEGO (WARSZAWA)	-0.803075
RUDNICKIEGO (WARSZAWA)	-0.803987
OS.DERBY (WARSZAWA)	-0.805012

WIELICKA (WARSZAWA)	-0.805708
AGRYKOLA (WARSZAWA)	-0.806387
PRZEDPOLE (WARSZAWA)	-0.807083
WIEJSKA (WARSZAWA)	-0.808893
(LUBELSKA) (WARSZAWA)	-0.809133
GOLĘDZINÓW (WARSZAWA)	-0.813034
BATALIONU PLATERÓWEK (WARSZAWA)	-0.814541
URSYNÓW PŁD. (WARSZAWA)	-0.815489
KULTURY (PIASECZNO)	-0.818151
BELLOTTIEGO (WARSZAWA)	-0.819121
POMORSKA (WARSZAWA)	-0.819944
OS.ORLIK (WARSZAWA)	-0.821963
PŁN. (WARSZAWA)	-0.822927
RAFII (WARSZAWA)	-0.826484
RODŁA (WARSZAWA)	-0.82741
JASIELSKA (WARSZAWA)	-0.835422
SPISKA (WARSZAWA)	-0.836038
KRAŚNICKA (WARSZAWA)	-0.839618
NUGAT (WARSZAWA)	-0.846872
KOPCIŃSKIEGO (WARSZAWA)	-0.847467
INSTYTUT REUMATOLOGII (WARSZAWA)	-0.847565
WARSZAWSKA (WARSZAWA)	-0.849007
KS POLONEZ (WARSZAWA)	-0.849118
DW.WSCHODNI (LUBELSKA) (WARSZAWA)	-0.849365
KS.SITNIKA (WARSZAWA)	-0.850248
SMOLEŃSKIEGO (WARSZAWA)	-0.85058
BRAZYLIJSKA (WARSZAWA)	-0.850908
GOŚCIERADOWSKA (WARSZAWA)	-0.852948
HŁASKI (WARSZAWA)	-0.853681
METRO SŁODOWIEC (WARSZAWA)	-0.853945
OS.ZIELONY STAW (WARSZAWA)	-0.856994
MONTE CASSINO (WARSZAWA)	-0.85973
CHŁODNA (WARSZAWA)	-0.863161
SOCHACZEWSKA (WARSZAWA)	-0.863399
WALEWSKA (WARSZAWA)	-0.865351
NIEMCEWICZA (WARSZAWA)	-0.865623
NARCYZOWA (WARSZAWA)	-0.865914
ZWYCIĘZCÓW (WARSZAWA)	-0.867592
METRO NOWY ŚWIAT-UNIWERSYTET (WARSZAWA)	-0.871129
WAPOWSKIEGO (WARSZAWA)	-0.872336
MARYMONT (WARSZAWA)	-0.873371
PODCZASZYŃSKIEGO (WARSZAWA)	-0.877116
ROKOSOWSKA (WARSZAWA)	-0.880373
ROZŁUCKA (WARSZAWA)	-0.882551
RADZYMIŃSKA (ZĄBKI)	-0.88354
NOAKOWSKIEGO (WARSZAWA)	-0.884927
KIERBEDZIA (WARSZAWA)	-0.888171
REDUTA (WARSZAWA)	-0.88898
SIODLARSKA (WARSZAWA)	-0.890029
SIEDMIOGRODZKA (WARSZAWA)	-0.890921
ROLNICZA (WARSZAWA)	-0.891296
DARWINA (WARSZAWA)	-0.892275
DROHICKA (WARSZAWA)	-0.892517
ŚWIECKA (WARSZAWA)	-0.897453
GILARSKA (WARSZAWA)	-0.912912
MADALIŃSKIEGO-SZPITAL (WARSZAWA)	-0.916303
TRASA ŁAZIENKOWSKA (WARSZAWA)	-0.916862
WYBRZEŻE SZCZECIŃSKIE (WARSZAWA)	-0.924521
MIŁA (WARSZAWA)	-0.92897
SIENNA (WARSZAWA)	-0.937003
WIDOKOWA (WARSZAWA)	-0.947112
DOBRA (WARSZAWA)	-0.978283
TARCHOMIN KOŚCIELNY (WARSZAWA)	-1.00076
PODKOWY (WARSZAWA)	-1.10715
DZIKA (WARSZAWA)	-1.26228

Table C.1: Warsaw public transportation network stop rankings according to Configuration value harmonic closeness.

Station	Ranking
POLITECHNIKA (WARSZAWA)	1.03683
PKP FALENICA (WARSZAWA)	1.00225
ERAZMA Z ZAKROCZYMIA (WARSZAWA)	0.817167
PKP RADOŚĆ (WARSZAWA)	0.802163
DICKENSA (WARSZAWA)	0.765829
REDUTA WOLSKA (WARSZAWA)	0.664405
PKP ŻERAŃ (WARSZAWA)	0.647429
ZAJEZDNIA URSUS PŁN. (WARSZAWA)	0.643024
SUCHY LAS (WARSZAWA)	0.640742
URZĄD GMINY (WIĄZOWNA)	0.63064
CHEMICZNA (WARSZAWA)	0.625797
STATKOWSKIEGO (WARSZAWA)	0.61392
METRO URSYNÓW (WARSZAWA)	0.613555
KAJKI (WARSZAWA)	0.605197
REMBERTÓW-RATUSZ (WARSZAWA)	0.604758
CZEKANOWSKA (WARSZAWA)	0.601995
SKOLIMÓW (KONSTANCIN-JEZIORNA)	0.598091
PODSKARBIŃSKA (WARSZAWA)	0.59759
KROKWI (WARSZAWA)	0.591612
URZĄD MIASTA (PIASECZNO)	0.568503
NACZELNIKOWSKA (WARSZAWA)	0.565998
SOKRATESA (WARSZAWA)	0.559739
PIASTÓW ŚLĄSKICH (WARSZAWA)	0.551631
SYTA (WARSZAWA)	0.550583
PRZEKORNA (WARSZAWA)	0.545609
HRABSKA DROGA (FALENTY)	0.540738
SASKA (WARSZAWA)	0.539542
PIĘKNA (WARSZAWA)	0.538759
MURANOWSKA (WARSZAWA)	0.533817
CMENTARZ (ŁAZY)	0.531623
DANUSI (WARSZAWA)	0.53138
DĄBROWSZCZAKÓW (WARSZAWA)	0.527322
KOŚCIÓŁ (RASZYN)	0.525928
PKP MIĘDZYLESIE (WARSZAWA)	0.52417
METRO WIERZBNO (WARSZAWA)	0.519696
MAKLAKIEWICZA (WARSZAWA)	0.518529
PŁOWIECKA (WARSZAWA)	0.515073
WSPÓLNA (WARSZAWA)	0.51316
MYSIADŁO (MYSIADŁO)	0.512704
RONDO ŻABCZYŃSKIEGO (WARSZAWA)	0.512
MIĘDZYBORSKA (WARSZAWA)	0.508299
DOLINA SŁUŻEWIECKA (WARSZAWA)	0.506587
BERENSONA (WARSZAWA)	0.498495
METRO ŚWIĘTOKRZYSKA (WARSZAWA)	0.496392
PKP WŁOCHY (WARSZAWA)	0.4948
CYNKOWA (LEGIONOWO)	0.491582
1. PRASKIEGO PUŁKU (WARSZAWA)	0.49001
METRO STARE BIELANY (WARSZAWA)	0.48806
WOJSKA POLSKIEGO (PIASECZNO)	0.487139
EDISONA (WARSZAWA)	0.483951
EUROPEJSKA (WARSZAWA)	0.480862
CYBERNETYKI (WARSZAWA)	0.477281
CYLICHOWSKA (WARSZAWA)	0.472948
KOCJANA (WARSZAWA)	0.472349
WIATRACZNA (WARSZAWA)	0.47093
URSYNÓW PŁD. (WARSZAWA)	0.469064
CISZEWSKIEGO (WARSZAWA)	0.468669
BRATA ALBERTA (WARSZAWA)	0.468412
METRO STADION NARODOWY (WARSZAWA)	0.464169
PERKUNA (WARSZAWA)	0.457579
RZYMOWSKIEGO (WARSZAWA)	0.454959
KĄTY GRODZISKIE (WARSZAWA)	0.4529
PARK MOCZYDŁO (WARSZAWA)	0.452428
UMIŃSKIEGO (WARSZAWA)	0.451953
PKP POWIŚLE (WARSZAWA)	0.451811
OS.BERNARDYŃSKA (WARSZAWA)	0.448852
WROCŁAWSKA (WARSZAWA)	0.442866
DW.WILEŃSKI (WARSZAWA)	0.441347
GIERDZIEJEWSKIEGO (WARSZAWA)	0.436747
PIASTA (KONSTANCIN-JEZIORNA)	0.436565
OS.JANTAR (WARSZAWA)	0.43309
MAŁCUŻYŃSKIEGO (WARSZAWA)	0.43244
PKP URSUS (WARSZAWA)	0.431453
HENRYKÓW (WARSZAWA)	0.430436
PKP KOŁO (WARSZAWA)	0.43013
CERAMICZNA (WARSZAWA)	0.429093
RADIOWO-BOCZNICA (WARSZAWA)	0.425914
METRO POLE MOKOTOWSKIE (WARSZAWA)	0.422949
TORUŃSKA (WARSZAWA)	0.420555
PROZY (WARSZAWA)	0.420047
PARK MIEJSKI (OTWOCK)	0.412622
PLATER (WARSZAWA)	0.401021
NOWOLIPIE (WARSZAWA)	0.399668
SZWEDZKA (WARSZAWA)	0.398369
PL.SZEMBEKA (WARSZAWA)	0.39616
PIMOT (WARSZAWA)	0.392675
CHŁOPICKIEGO (WARSZAWA)	0.389662
PL.GRUNWALDZKI (WARSZAWA)	0.389233
MOZAIKOWA (WARSZAWA)	0.388294
BRACI ZAWADZKICH (WARSZAWA)	0.386429
METRO SŁUŻEW (WARSZAWA)	0.38479
KRUCZA (WARSZAWA)	0.38468
GRZYMAŁY-SOKOŁOWSKIEGO (WARSZAWA)	0.383888
SZYSZKOWA (WARSZAWA)	0.379939
ŚW.BONIFACEGO (WARSZAWA)	0.374773
JANOWSKIEGO (LATCHORZEW)	0.373928
STRAŻACKA (WARSZAWA)	0.372191
ZAGŁOBY (WARSZAWA)	0.371186
KOŁOWA (WARSZAWA)	0.370196

PRZYBYSZEWSKIEGO (WARSZAWA)	0.368752
STRUGA (MARKI)	0.368248
TARCHOMIN (WARSZAWA)	0.366953
BAZYLIAŃSKA (WARSZAWA)	0.364673
DŁUGORZECZNA (WARSZAWA)	0.364589
MENNICA (WARSZAWA)	0.364011
SCHRONISKO DLA ZWIERZĄT (WARSZAWA)	0.362886
RUCZAJ (WARSZAWA)	0.361372
SANTOCKA (WARSZAWA)	0.361077
LITEWSKA (WARSZAWA)	0.35984
RYŻOWA (WARSZAWA)	0.357331
WYTWÓRNIA MAS BITUMICZNYCH (WARSZAWA)	0.356824
MŁODZIEŃCZA (WARSZAWA)	0.354222
ANUSIN (HABDZIN)	0.353258
DZIKA (WARSZAWA)	0.352008
BIAŁOŁĘKA-KANAŁ (WARSZAWA)	0.350952
PKP REMBERTÓW (WARSZAWA)	0.348016
ECHA LEŚNE (WARSZAWA)	0.347719
SZPITAL BRÓDNOWSKI (WARSZAWA)	0.347127
ANTONIEWSKA (WARSZAWA)	0.345702
MŁYNÓW (WARSZAWA)	0.34567
RONDO DASZYŃSKIEGO (WARSZAWA)	0.344611
METRO WILANOWSKA (WARSZAWA)	0.340895
KOSSAKA (OŻARÓW MAZOWIECKI)	0.34089
SIERAKOWSKA (IZABELIN)	0.339762
KONDRATOWICZA (WARSZAWA)	0.339714
MAGDALENKA (MAGDALENKA)	0.339474
MOST GDAŃSKI (WARSZAWA)	0.337846
OBJAZDOWA (WARSZAWA)	0.337724
METRO MARYMONT (WARSZAWA)	0.334695
DWERNICKIEGO (WARSZAWA)	0.333779
ZIELONKI (ZIELONKI)	0.333207
DZIAŁYŃCZYKÓW (WARSZAWA)	0.333157
KROKUSÓW (WARSZAWA)	0.332575
JAGIELSKA (WARSZAWA)	0.331706
ZAKŁADY ZBOŻOWE (WARSZAWA)	0.328504
POD BUDOWLANI II (WARSZAWA)	0.326362
HOŁUBCOWA (WARSZAWA)	0.325983
ZABRANIECKA-MAGAZYNY (WARSZAWA)	0.323395
ZGRUPOWANIA AK KAMPINOS (WARSZAWA)	0.323276
POLINEZYJSKA (WARSZAWA)	0.32247
STRYJEŃSKICH (WARSZAWA)	0.32084
GUS (WARSZAWA)	0.320165
PL.ZBAWICIELA (WARSZAWA)	0.319358
ŻYWIECKA (WARSZAWA)	0.318186
BIELAWA-OSP (BIELAWA)	0.317099
PIASKOWA (ŁAJSKI)	0.314433
MIRKOWSKA (KONSTANCIN-JEZIORNA)	0.314068
FALENTY NOWE (FALENTY)	0.313998
SGGW-REKTORAT (WARSZAWA)	0.312629
BOROWA (KONSTANCIN-JEZIORNA)	0.310985
TUŁOWICKA (WARSZAWA)	0.308353
RZECZNA (WARSZAWA)	0.308216
REMBELSZCZYZNA (REMBELSZCZYZNA)	0.307338
KOSZYKOWA (WARSZAWA)	0.306934
SPB KAWĘCZYN (WARSZAWA)	0.306851
SPICHRZOWA (WARSZAWA)	0.304246
MIKLASZEWSKIEGO (DAWIDY BANKOWE)	0.303933
BEMOWO (WARSZAWA)	0.303884
ŻYRARDOWSKA (WARSZAWA)	0.303707
HOSPICJUM ONKOLOGICZNE (WARSZAWA)	0.303512
SANKTUARIUM (OŻARÓW MAZOWIECKI)	0.302166
ROZBRAT (WARSZAWA)	0.302121
SIKORECZKI (JASTRZĘBIE)	0.30125
MOCHTYŃSKA (WARSZAWA)	0.301193
KOCHANOWSKIEGO (WARSZAWA)	0.30079
PRZEBIEG (WARSZAWA)	0.297309
INTERNETOWA (WARSZAWA)	0.297019
BEMOWO-RATUSZ (WARSZAWA)	0.296792
PKP NIEPORĘT (NIEPORĘT)	0.296292
PŁOCHOCIŃSKA (WARSZAWA)	0.295337
OS.KWIATOWA (NOWA IWICZNA)	0.294995
METRO RACŁAWICKA (WARSZAWA)	0.294013
RONDO ŻABA (WARSZAWA)	0.29169
SZKOLNA (JABŁONNA)	0.291508
TABOROWA (WARSZAWA)	0.290835
PKP ANIN (WARSZAWA)	0.290481
OLCHY (WARSZAWA)	0.289164
ODYŃCA (WARSZAWA)	0.288612
OCH-TEATR (WARSZAWA)	0.287168
POPIELA (WARSZAWA)	0.286869
CENTRUM ONKOLOGII (WARSZAWA)	0.286748
CODZIENNA (WARSZAWA)	0.286347
MOKOTOWSKA (WARSZAWA)	0.283456
CZARNIECKIEGO (WARSZAWA)	0.281241
BIAŁOŁĘKA-RATUSZ (WARSZAWA)	0.280017
WIERNA (WARSZAWA)	0.279927
KARCZUNKOWSKA (WARSZAWA)	0.277646
CMENTARZ (KONSTANCIN-JEZIORNA)	0.276948
PRATULIŃSKA (WARSZAWA)	0.276918
DRACENY (WARSZAWA)	0.273667
POWSINEK (WARSZAWA)	0.272901
DOSTAWCZA (WARSZAWA)	0.272003
OŁÓWKOWA (PRUSZKÓW)	0.270016
SZYSZAKÓW (WARSZAWA)	0.267186
GŁÓWNA (JAZGARZEW)	0.264603
DYWIZJONU 303 (WARSZAWA)	0.264048
PŁOCKA-SZPITAL (WARSZAWA)	0.263753
DROHOBYCKA (WARSZAWA)	0.263684
PODOLSKA (WARSZAWA)	0.263074

SZWOLEŻERÓW (WARSZAWA)	0.262739
ALUZYJNA (WARSZAWA)	0.261464
PIOTRA SKARGI (WARSZAWA)	0.260817
ŚW.WINCENTEGO (WARSZAWA)	0.259953
SAMORZĄDOWA (WARSZAWA)	0.259551
SUWALSKA (WARSZAWA)	0.258477
RUMIANKOWA (WARSZAWA)	0.257776
SZARYCH SZEREGÓW (JABŁONNA)	0.257174
SIENKIEWICZA (IZABELIN)	0.255853
ŻOŁNIERSKA (WARSZAWA)	0.255676
SZELIGOWSKA (WARSZAWA)	0.254966
SŁOMCZYN-SZKOŁA (SŁOMCZYN)	0.254832
SIEKIERKOWSKA (WARSZAWA)	0.249854
STARODĘBY (WARSZAWA)	0.249478
17 STYCZNIA (WARSZAWA)	0.24909
NOVOTEL (WARSZAWA)	0.24778
WENECKA (MARKI)	0.247208
FORT BLIZNE (WARSZAWA)	0.246332
LUBELSKA (WARSZAWA)	0.2462
DWORKOWA (WARSZAWA)	0.245544
WIERTNICZA (WARSZAWA)	0.244705
AL.WITOSA (WARSZAWA)	0.244522
KWITNĄCA (WARSZAWA)	0.244349
STRZYGŁOWSKA (WARSZAWA)	0.244272
CENTRUM MODY (NADARZYN)	0.243322
OŁÓWKOWA (WARSZAWA)	0.242692
BIELANY-RATUSZ (WARSZAWA)	0.240706
METRO STOKŁOSY (WARSZAWA)	0.23998
BORKÓW (WARSZAWA)	0.236915
KADETÓW-SZKOŁA (WARSZAWA)	0.236888
WIEKOWEJ SOSNY (ZALESIE GÓRNE)	0.236537
BOGUSŁAWSKIEGO (WARSZAWA)	0.236185
CIECISZEW-OSP (CIECISZEW)	0.236134
DW.GDAŃSKI (WARSZAWA)	0.235881
MARYSIN (WARSZAWA)	0.235137
OKULAROWA (WARSZAWA)	0.234623
SZKOLNA (MARKI)	0.233368
PL.ZAWISZY (WARSZAWA)	0.233162
LUCERNY (WARSZAWA)	0.232765
BENNETTA (WARSZAWA)	0.232217
PKP JEZIORKI (WARSZAWA)	0.231306
POLSKIE RADIO (WARSZAWA)	0.231055
CENTRUM (WARSZAWA)	0.230881
REMBERTÓW-AON (WARSZAWA)	0.230817
OS.WOLSKA (WARSZAWA)	0.226321
KIJOWSKA (WARSZAWA)	0.225565
PL.KASZTELAŃSKI (WARSZAWA)	0.22397
OS.PIASKI (LEGIONOWO)	0.223843
MOKRY ŁUG (WARSZAWA)	0.223659
POSTĘPU (NOWA WOLA)	0.223179
METRO CENTRUM NAUKI KOPERNIK (WARSZAWA)	0.222887
STAWKI (WARSZAWA)	0.221572
JUHASÓW (WARSZAWA)	0.221451
SOBIESKIEGO (WARSZAWA)	0.22124
BYSTRZYCKA (WARSZAWA)	0.22053
TROCKA (WARSZAWA)	0.220099
HALLERA (ZĄBKI)	0.219744
LEŚNEJ POLANKI (WARSZAWA)	0.219073
HALA WOLA (WARSZAWA)	0.218819
WSPÓLNA DROGA (WARSZAWA)	0.218056
POTOCKICH (WARSZAWA)	0.217268
MIKLASZEWSKIEGO (WARSZAWA)	0.216676
SOSNKOWSKIEGO (WARSZAWA)	0.215252
ESTRADY (WARSZAWA)	0.214718
17 STYCZNIA (PIASECZNO)	0.214235
DZIECI WARSZAWY (WARSZAWA)	0.213184
PARK OLSZYNA (WARSZAWA)	0.212021
KS.POPIEŁUSZKI (WARSZAWA)	0.211658
SZOSA LUBELSKA (WARSZAWA)	0.211425
GOŁKÓW-LETNISKO (PIASECZNO)	0.211112
SZCZĘŚLIWICE (WARSZAWA)	0.2097
TAWUŁKOWA (WARSZAWA)	0.209465
IX POPRZECZNA (WARSZAWA)	0.208947
DOLNA (WARSZAWA)	0.208332
POSTĘPU (LESZNOWOLA)	0.207776
METRO KABATY (WARSZAWA)	0.207652
HALA KOPIŃSKA (WARSZAWA)	0.207596
PARK KASKADA (WARSZAWA)	0.20692
ELBLĄSKA (WARSZAWA)	0.206684
RAKOWIEC (WARSZAWA)	0.206257
ZYNDRAMA Z MASZKOWIC (WARSZAWA)	0.205688
RYNEK (LEGIONOWO)	0.205416
ARMII KRAJOWEJ (WARSZAWA)	0.205311
PRUSZKOWSKA (WARSZAWA)	0.205093
POLNA (WIELISZEW)	0.202713
CHODKIEWICZA (IZABELIN)	0.202359
RUDZKA (WARSZAWA)	0.202154
CENTRUM ZDROWIA DZIECKA (WARSZAWA)	0.201664
PRZEDPEŁSKIEGO (WIELISZEW)	0.201017
AL.LOTNIKÓW (WARSZAWA)	0.200056
GROTY (WARSZAWA)	0.199804
KADETÓW (WARSZAWA)	0.199676
MARIENSZTAT (WARSZAWA)	0.199461
WESOŁA-RATUSZ (WARSZAWA)	0.199457
TYTUŁOWA (WARSZAWA)	0.199127
BORA-KOMOROWSKIEGO (WARSZAWA)	0.198999
PRZYCZÓŁEK GROCHOWSKI (WARSZAWA)	0.198433
KOŚCIELNA (KĄTY WĘGIERSKIE)	0.197621
MAGIERA (WARSZAWA)	0.195931
RYCERSKA (LEGIONOWO)	0.195619
STADION (RASZYN)	0.194233

PL.KRASIŃSKICH (WARSZAWA)	0.193386
CHYLICZKOWSKA-TECHNIKUM (PIASECZNO)	0.19241
METRO POLITECHNIKA (WARSZAWA)	0.192232
JURANDA ZE SPYCHOWA (WARSZAWA)	0.192186
MOST SIEKIERKOWSKI (WARSZAWA)	0.191202
STARE ŚWIDRY (WARSZAWA)	0.189609
KWITNĄCEJ AKACJI (WARSZAWA)	0.18481
BIAŁOBRZESKA (WARSZAWA)	0.184018
NAPOLEONA BONAPARTE (WARSZAWA)	0.183563
TELIGI (WARSZAWA)	0.182553
ZIELONA (GOŁKÓW)	0.179523
KOŚCIELISKA (WARSZAWA)	0.178296
ZEGRZYŃSKA (LEGIONOWO)	0.178246
JÓRSKIEGO (WARSZAWA)	0.178245
KLAUDYNY (WARSZAWA)	0.177882
NOWOURSYNOWSKA (WARSZAWA)	0.177659
KRÓLEWSKA (WARSZAWA)	0.177206
REMBIELIŃSKA (WARSZAWA)	0.176334
MUZEUM POWSTANIA WARSZAWSKIEGO (WARSZAWA)	0.176166
RUSKOWY BRÓD (WARSZAWA)	0.175074
STRZELECKIEGO (WARSZAWA)	0.173322
KOŁOBRZESKA (KONSTANCIN-JEZIORNA)	0.173116
PKP MIEDZESZYN (WARSZAWA)	0.172806
WÓLKA WĘGLOWA (WARSZAWA)	0.172276
BORZĘCIN DUŻY-SZKOŁA (BORZĘCIN DUŻY)	0.172145
CZŁUCHOWSKA (WARSZAWA)	0.171717
PKP PIASECZNO (PIASECZNO)	0.171408
POWSTAŃCÓW (ZĄBKI)	0.171348
OSTRÓDZKA (WARSZAWA)	0.171126
WYBRZEŻE HELSKIE (WARSZAWA)	0.170639
CM.PÓŁNOCNY-BRAMA GŁ. (WARSZAWA)	0.170374
OS.WYŻYNY (WARSZAWA)	0.169791
FSO (WARSZAWA)	0.169728
EC ŻERAŃ (WARSZAWA)	0.169574
PARKOWA (LEGIONOWO)	0.16907
URSUS-SANKTUARIUM (WARSZAWA)	0.167737
OKNICKA (WARSZAWA)	0.16734
OS.PATIO (WARSZAWA)	0.167087
MOST PONIATOWSKIEGO (WARSZAWA)	0.1639
MICKIEWICZA (LEGIONOWO)	0.16328
WYGODA (WÓLKA KOSOWSKA)	0.163093
PKP WKD AL.JEROZOLIMSKIE (WARSZAWA)	0.162698
PL.NA ROZDROŻU (WARSZAWA)	0.161809
ŻUPNICZA (WARSZAWA)	0.161596
SIKORSKIEGO (MOŚCISKA)	0.16154
OS.POTOK (WARSZAWA)	0.161443
AL.REYMONTA (WARSZAWA)	0.161212
BYSTRA (WARSZAWA)	0.159297
RYNEK (STARE BABICE)	0.158489
PKP KASPRZAKA (WARSZAWA)	0.15845
WŁOCHY-RATUSZ (WARSZAWA)	0.158093
ŚWIDER-KOŚCIÓŁ (OTWOCK)	0.156583
NOWY ZJAZD (WARSZAWA)	0.156583
OS.PRZYLESIE (JABŁONNA)	0.156428
RONDO WASZYNGTONA (WARSZAWA)	0.156092
BACZYŃSKIEGO (PARCELA)	0.156031
SZCZĘŚLIWICKA (WARSZAWA)	0.155967
LAMIŃA (PIASECZNO)	0.155858
PL.WILSONA (WARSZAWA)	0.155366
AL.BOHATERÓW WRZEŚNIA (WARSZAWA)	0.154763
AL.WOJSKA POLSKIEGO (PRUSZKÓW)	0.154714
HERBSTA (WARSZAWA)	0.154672
GLINIANKA (GLINIANKA)	0.153928
HYNKA (WARSZAWA)	0.153794
DOLNA (KONSTANCIN-JEZIORNA)	0.152953
GORZYKOWSKA (WARSZAWA)	0.152885
ZAJĘCZA (WARSZAWA)	0.152799
KŁOBUCKA (WARSZAWA)	0.152184
PKP RAKOWIEC (WARSZAWA)	0.152065
LANCIEGO (WARSZAWA)	0.151576
GEN.ROI (LEGIONOWO)	0.15139
ZGORZAŁA (ZGORZAŁA)	0.151266
WERSALSKA (WARSZAWA)	0.151213
ŁYCZYN (ŁYCZYN)	0.150703
KOBIAŁKA (WARSZAWA)	0.149864
BERESTECKA (WARSZAWA)	0.149637
PŁOCKA (WARSZAWA)	0.149573
LEGIONOWO (LEGIONOWO)	0.149455
DUCHNICKA (WARSZAWA)	0.149102
OGRODY DZIAŁKOWE PALUCH (WARSZAWA)	0.148883
KRYNOLINY (WARSZAWA)	0.148394
BOROWINA I (BOROWINA)	0.147328
WAWELSKA (WARSZAWA)	0.145762
ORZECHOWA (STRZYKUŁY)	0.145004
URZĄD MIASTA (LEGIONOWO)	0.143794
RADIOSTACJA ŁAZY (ŁAZY)	0.141667
POLONEZA (WARSZAWA)	0.140539
ŚLIWICE (WARSZAWA)	0.140096
HANDLOWA (WARSZAWA)	0.140059
ZDZIARSKA-KANAŁ (WARSZAWA)	0.139912
ZALESINEK (PIASECZNO)	0.13983
ŚMIGIELSKA (WARSZAWA)	0.139728
FIELDORFA (WOŁOMIN)	0.139349
ZŁOTOKŁOS (ZŁOTOKŁOS)	0.138461
CM.POŁUDNIOWY-BRAMA WSCH. (ANTONINÓW)	0.137325
BEŁDAN (WARSZAWA)	0.136571
NORBLIN (WARSZAWA)	0.136103
CZERWONA DROGA (WARSZAWA)	0.136039
POLNA (KONSTANCIN-JEZIORNA)	0.1356

SKARBKA Z GÓR (WARSZAWA)	0.135488
WIDOKOWA (WARSZAWA)	0.134898
POLNA (KRUBIN)	0.134411
RONDO STARZYŃSKIEGO (WARSZAWA)	0.134147
KOLONIA LESZNOWOLA (KOLONIA LESZNOWOLA)	0.133999
ĆMIELOWSKA (WARSZAWA)	0.131544
MURANÓW (WARSZAWA)	0.131437
DZIERZGOŃSKA (WARSZAWA)	0.130141
KOŚCIELNY (WARSZAWA)	0.1293
BOHATEREWICZA (WARSZAWA)	0.129277
OŚRODEK SZKOLENIOWY (WARSZAWA)	0.128504
POMNIK LOTNIKA (WARSZAWA)	0.128003
DŁUGA (DAWIDY)	0.12768
KOMBAJNISTÓW (WARSZAWA)	0.127274
TURMONCKA (WARSZAWA)	0.126664
JABŁONIOWA (SŁOMCZYN)	0.126399
MODRA (KAJETANY)	0.123544
PRZY AGORZE (WARSZAWA)	0.123523
PODBIPIĘTY (WARSZAWA)	0.123272
UWIELINY-SZKOŁA (UWIELINY)	0.122841
CEMENTOWNIA (WARSZAWA)	0.122786
KSIĘŻNEJ ANNY (WARSZAWA)	0.12256
REYMONTA (OTWOCK)	0.121184
WCZASOWA (BIAŁOBRZEGI)	0.119695
MALCANÓW-BOOS (MALCANÓW)	0.119627
HALA SPORTOWA BEMOWO (WARSZAWA)	0.118898
ZIELONKI-WIEŚ (ZIELONKI-WIEŚ)	0.118713
WIEJSKA (MARKI)	0.118488
SASANKI (WARSZAWA)	0.117739
MYSZCZYN (MYSZCZYN)	0.117468
WARSZAWSKA (ZĄBKI)	0.117218
ZACHĘTA (WARSZAWA)	0.116621
KASZTANOWA (JEZIÓRKO)	0.116027
ZAMIENIECKA (WARSZAWA)	0.115916
KULESZÓWKA (KULESZÓWKA)	0.115828
KONIK NOWY (KONIK NOWY)	0.115768
ŁABISZYŃSKA (WARSZAWA)	0.115384
RONDO ZESŁAŃCÓW SYBERYJSKICH (WARSZAWA)	0.114888
URSUS-NIEDŹWIADEK (WARSZAWA)	0.1148
BYDGOSKA (KONSTANCIN-JEZIORNA)	0.114606
SULEJÓWEK (SULEJÓWEK)	0.11441
SPACEROWA (WARSZAWA)	0.114206
NOWE ZAŁUBICE (NOWE ZAŁUBICE)	0.113898
INSTYTUT LOTNICTWA (WARSZAWA)	0.112958
UMIASTOWSKA (KAPUTY)	0.112775
KOZAKÓW (SZAMOTY)	0.112537
BYSŁAWSKA (WARSZAWA)	0.112316
KOSMOWSKA II (BORZĘCIN DUŻY)	0.112277
ZŁOTA (WĄGRODNO)	0.112057
SPORTOWA (ZIELONKI)	0.11181
PL.STARYNKIEWICZA (WARSZAWA)	0.111804
KOCZARGI STARE (KOCZARGI STARE)	0.110615
PL.BERNARDYŃSKI (WARSZAWA)	0.110279
TECHNIKÓW (WARSZAWA)	0.110022
KIELECKA (STARA IWICZNA)	0.109857
LOKAJSKIEGO (WARSZAWA)	0.10983
ORĘŻNA (PIASECZNO)	0.1088
CM.OFIAR WOJNY (WARSZAWA)	0.108333
WOLA-RATUSZ (WARSZAWA)	0.108029
CHEŁMSKA (WARSZAWA)	0.106955
MIRKÓW-SZKOŁA (KONSTANCIN-JEZIORNA)	0.106558
UKSW (WARSZAWA)	0.106339
ŚNIADECKICH (KONSTANCIN-JEZIORNA)	0.106163
POLNA (WŁADYSŁAWÓW)	0.106121
RUNÓW OSADA (RUNÓW)	0.10466
DUNIKOWSKIEGO (WARSZAWA)	0.104439
HOŻA (OTWOCK)	0.104435
GŁOSKÓW-ZIELONE (GŁOSKÓW)	0.103889
MARCELIN (WARSZAWA)	0.103537
METRO RATUSZ ARSENAŁ (WARSZAWA)	0.103456
PARK PRASKI (WARSZAWA)	0.103004
PL.BANKOWY (WARSZAWA)	0.102862
STACJA KRWIODAWSTWA (WARSZAWA)	0.102274
PKP GOŁĄBKI (WARSZAWA)	0.100909
MARYMONT-POTOK (WARSZAWA)	0.0996594
POD SZCZAKI (SZCZAKI)	0.0995547
STAWY (WARSZAWA)	0.098353
KAMELIOWA (NOWA WOLA)	0.0983373
SOBIESKIEGO (KONSTANCIN-JEZIORNA)	0.0976653
SZYMAŃSKIEGO (WARSZAWA)	0.0972932
STRAŻACKA-MAGAZYNY (WARSZAWA)	0.097141
LIPKÓW (LIPKÓW)	0.0970483
BRACI WAGÓW (WARSZAWA)	0.0962526
ANDRUTOWA (WARSZAWA)	0.0960215
TOPOLOWA (WYGLĘDY)	0.0959872
ASPEKT (WARSZAWA)	0.095956
1 SIERPNIA (WARSZAWA)	0.0958665
NADARZYŃSKA (PIASECZNO)	0.0952875
TROMBITY (WARSZAWA)	0.0952361
DZIKIEJ RÓŻY (NOWA WOLA)	0.0949758
TRAUGUTTA (PIASTÓW)	0.0946105
WIERUCHOWSKA (STARE BABICE)	0.0942541
LANGIEWICZA (IZABELIN)	0.0937148
INSTALATORÓW-KIS (WARSZAWA)	0.0936145
ZAŁUBICE-SZKOŁA (STARE ZAŁUBICE)	0.0935951
SPADOWA (WARSZAWA)	0.0932639
WAFLOWA (WARSZAWA)	0.0928094
POLECZKI (WARSZAWA)	0.0919993

WAŁOWICKA (WARSZAWA)	0.0912644
KOBIAŁKA-SZKOŁA (WARSZAWA)	0.0906606
BOROWINA III (BOROWINA)	0.0903546
PORAJÓW (WARSZAWA)	0.0897651
BROŻKA (WARSZAWA)	0.0897139
SŁONECZNA (LEGIONOWO)	0.0892668
SZPITAL WOLSKI (WARSZAWA)	0.0890146
MŁYNARSKA (WARSZAWA)	0.0885573
ANIELEWICZA (WARSZAWA)	0.0877811
WRZOSOWA (MICHAŁÓW-REGINÓW)	0.0872595
INSTYTUT HEMATOLOGII (WARSZAWA)	0.0871474
AL.KRAKOWSKA (WARSZAWA)	0.0863897
ORLICH GNIAZD (WARSZAWA)	0.0860455
NOWOLIPKI (WARSZAWA)	0.0856409
BARTNICZA (WARSZAWA)	0.0856181
LESZCZYNKA (MROKÓW)	0.0844263
WIERZBOWA (HIPOLITÓW)	0.0841672
SYMPATYCZNA (WARSZAWA)	0.0840899
MARKOWSKIEGO (PIASECZNO)	0.0840356
PELCOWIZNA (WARSZAWA)	0.0836405
GAZOWNIA (REMBELSZCZYZNA)	0.082798
LUBICZÓW (LUBICZÓW)	0.0825958
DOMANIEWSKA (WARSZAWA)	0.081637
NA SKRAJU (WARSZAWA)	0.0812184
PODBIAŁOWA (WARSZAWA)	0.0811753
PAŁAC (JABŁONNA)	0.0803508
HENRYKA BRODATEGO (WARSZAWA)	0.079825
ZAWADOWSKA (WARSZAWA)	0.0796506
EKOLOGICZNA (KLAUDYN)	0.0792801
PKP TORUŃSKA (WARSZAWA)	0.079229
AFRYKAŃSKA (WARSZAWA)	0.0791343
SOWIŃSKIEGO (LEGIONOWO)	0.0787061
INSTRUKTORSKA (WARSZAWA)	0.0780859
KOMOROWSKA (NADARZYN)	0.0758479
POŁUDNIOWA (KONSTANCIN-JEZIORNA)	0.0755905
ŚWIECKA (WARSZAWA)	0.0752855
GRODZISK (WARSZAWA)	0.0745722
PL.OSTREJ BRAMY (WARSZAWA)	0.0743703
PELIKANÓW (WARSZAWA)	0.0742811
WARSZAWA OCHOTA (WARSZAWA)	0.0741027
MILENIUM (GŁOSKÓW)	0.0740633
KULSKIEGO (WARSZAWA)	0.0738205
MAJOWA (WÓLKA RADZYMIŃSKA)	0.0735968
TOPIEL (WARSZAWA)	0.0734845
MŁOCINY (WARSZAWA)	0.0734568
WARSZAWA SŁUŻEWIEC (WARSZAWA)	0.0731591
ŻERAŃ FSO (WARSZAWA)	0.0731578
KOŚCIÓŁ (JAZGARZEW)	0.0730813
OKOPOWA (WARSZAWA)	0.0730042
STARE BEMOWO (WARSZAWA)	0.0724224
OBOZOWA (WARSZAWA)	0.0720906
OSIEDLOWA (STARE BABICE)	0.0717637
KOLEJKA PIASECZYŃSKA (PIASECZNO)	0.0716888
PARK DRESZERA (WARSZAWA)	0.0716549
NAD ŚWIDREM (WOLA KARCZEWSKA)	0.0711962
PL.HALLERA (WARSZAWA)	0.0696569
NARODOWA (WARSZAWA)	0.0696122
GŁÓWNA (GOŁKÓW)	0.0695921
DZIERŻONIOWSKA (WARSZAWA)	0.0693575
ATEŃSKA (WARSZAWA)	0.0693105
PRUSA (PRUSZKÓW)	0.0685142
SZKOŁA (NOWA IWICZNA)	0.0683058
OS.DERBY VI (WARSZAWA)	0.0680632
WARSZAWA AL.JEROZOLIMSKIE (WARSZAWA)	0.0677972
OS.LEWANDÓW I (WARSZAWA)	0.0677319
KAMBUD (WARSZAWA)	0.0676764
KU SŁOŃCU (MOCZYDŁÓW)	0.0675678
OBROŃCÓW WARSZAWY (OŻARÓW MAZOWIECKI)	0.0674552
RATUSZOWA-ZOO (WARSZAWA)	0.0674519
GŁOSKÓW-LETNISKO (GŁOSKÓW-LETNISKO)	0.0671348
SAMOCHODOWA (WARSZAWA)	0.0670557
MYŚLIBORSKA (WARSZAWA)	0.0662755
3 MAJA (LEGIONOWO)	0.0661895
SZKOLNA (WIĄZOWNA)	0.0657029
GORASZEWSKA (WARSZAWA)	0.065268
MACIERZANKI (NADARZYN)	0.0648194
RAKOWIECKA-SANKTUARIUM (WARSZAWA)	0.0647433
MOŁDAWSKA (WARSZAWA)	0.0639061
DŁUGA (WOLICA)	0.0633374
GÓRALSKA (WARSZAWA)	0.062894
KOROTYŃSKIEGO (WARSZAWA)	0.0628597
POŻAROWA (WARSZAWA)	0.0627259
RONDO ONZ (WARSZAWA)	0.0625484
KONOPACKA (WARSZAWA)	0.0624534
OKRZEI (WARSZAWA)	0.062308
BIAŁOŁĘKA (WARSZAWA)	0.0617878
ZAMENHOFA (WARSZAWA)	0.0615351
CM.ŻYDOWSKI (WARSZAWA)	0.061321
HABDZIN IV (HABDZIN)	0.0608543
SKOROCHÓD-MAJEWSKIEGO (WARSZAWA)	0.0606919
PEONII (WARSZAWA)	0.0604892
DW.ZACHODNI (WARSZAWA)	0.060342
SOCHACZEWSKA (MACIERZYSZ)	0.060214
OŚRODEK WOJSKOWY (WARSZAWA)	0.0601874
THOMME (STARE BABICE)	0.0593762
FABRYCZNA (WARSZAWA)	0.0593685
OS.WEDEL (WOLA DUCKA)	0.0591852
BARTŁOMIEJA (WARSZAWA)	0.0588777
CHABROWA (WIKTORÓW)	0.0584291

WKD RAKÓW (WARSZAWA)	0.0583808
NOSKOWSKIEGO (WARSZAWA)	0.0581477
ŻUROMIŃSKA (WARSZAWA)	0.0581441
OGRODOWA (KIEŁPIN STARY)	0.0578291
OCHOCZA (WARSZAWA)	0.0574542
ŻONKILOWA (WARSZAWA)	0.0571812
WARSZAWSKA (OLSZEWNICA STARA)	0.0571202
BRUSZEWSKA (WARSZAWA)	0.0569436
SZKOLNA (RASZYN)	0.0568194
SADY SGGW (WARSZAWA)	0.0552581
POZNAŃSKA (OŁTARZEW)	0.0550945
SPIRALNA (WARSZAWA)	0.0548159
ELEKCYJNA (WARSZAWA)	0.0538198
WARSZAWA REDUTA ORDONA (WARSZAWA)	0.0536989
BENIAMINÓWKA (WÓLKA RADZYMIŃSKA)	0.0535846
ANNY JAGIELLONKI (WARSZAWA)	0.0532883
OS.KONSTANCJA (BIELAWA)	0.0532086
GOK (STANISŁAWÓW DRUGI)	0.0531014
OGRODNICZA (SULEJÓWEK)	0.0524538
ZEGRZYŃSKA (STARE ZAŁUBICE)	0.0521169
SOBIESKIEGO (LEGIONOWO)	0.0520271
KSIĄŻĘCA (WARSZAWA)	0.0518717
GWARNA (SULEJÓWEK)	0.0516363
POŁCZYŃSKA-PARKING P+R (WARSZAWA)	0.0510532
KLEMENSIEWICZA (WARSZAWA)	0.0506403
GOŁASZEWSKA (OŁTARZEW)	0.0503948
RADIOWO (WARSZAWA)	0.0503071
CMENTARZ (STARE BABICE)	0.0500184
GRABALÓWKI (WARSZAWA)	0.0499742
PRZYJAZNA (WARSZAWA)	0.0491578
11 LISTOPADA (WARSZAWA)	0.0490685
BELGRADZKA (WARSZAWA)	0.0489995
TRENÓW (LASKI)	0.0489809
FALENTY-OSIEDLE (FALENTY)	0.0487564
KLAUDYN (KLAUDYN)	0.0487339
ESPERANTO (WARSZAWA)	0.0484513
WOLSKA (MICHAŁÓW-REGINÓW)	0.0482349
NIEPODLEGŁOŚCI (WIELISZEW)	0.0481459
PREFABET (WARSZAWA)	0.0481451
CHOPINA (KONSTANCIN-JEZIORNA)	0.0481269
DEOTYMY (WARSZAWA)	0.0477281
SZKOLNA (PIASECZNO)	0.0476908
KOŚCIÓŁ (ŁOMIANKI)	0.0468842
CHĘCIŃSKA (WARSZAWA)	0.0468434
BOHOMOLCA (WARSZAWA)	0.0466799
URBANISTÓW (JULIANÓW)	0.0466349
PRĄDZYŃSKIEGO (WARSZAWA)	0.0463557
PKP PŁUDY (WARSZAWA)	0.0463511
KINOWA (WARSZAWA)	0.0462587
ABRAHAMA (WARSZAWA)	0.0460534
WARSZAWA STADION (WARSZAWA)	0.0458823
TUŻYCKA (WARSZAWA)	0.0455485
GRANICZNA (NOWA IWICZNA)	0.0454713
BATALIONÓW CHŁOPSKICH (OTWOCK)	0.0452241
KINO FEMINA (WARSZAWA)	0.044871
NOWINEK (ZALESIE GÓRNE)	0.0446672
ŚWIDER (OTWOCK)	0.0444504
ORZECHOWA (LEGIONOWO)	0.0443544
SKOLIMÓW PÓŁNOCNY (KONSTANCIN-JEZIORNA)	0.0440433
MŁOCIŃSKA (ŁOMIANKI)	0.0439884
LAUROWA (WARSZAWA)	0.0439047
PONIATOWSKIEGO (OŻARÓW MAZOWIECKI)	0.0437619
WYSZYŃSKIEGO (GÓRA KALWARIA)	0.0437263
ŁADY (ŁADY)	0.0436991
ZORZY (KLAUDYN)	0.0436398
CMENTARZ (WÓLKA KOZODAWSKA)	0.0435708
KWIATOWA (ZĄBKI)	0.0435592
MEGASAM (WARSZAWA)	0.0434335
ZAPUSTNA (WARSZAWA)	0.0434264
CONRADA (WARSZAWA)	0.0433157
ROLANDA (WARSZAWA)	0.0432857
KALINOWA DROGA (JESIONOWA)	0.0423017
LEWINOWSKA (WARSZAWA)	0.0420118
WICHROWA (JESÓWKA)	0.0416862
ŁOWICKA (WARSZAWA)	0.0410056
PL.KONSTYTUCJI (WARSZAWA)	0.0408883
WRÓBLA (WARSZAWA)	0.0407495
BIBLIOTEKA UNIWERSYTECKA (WARSZAWA)	0.0402662
IZABELIŃSKA (IZABELIN)	0.0397905
PODZAMCZE (WARSZAWA)	0.0394682
JACHTOWA (WARSZAWA)	0.0390682
KRAKOWIAKÓW (WARSZAWA)	0.0389414
PKP LEGIONOWO (LEGIONOWO)	0.0387497
MIGDAŁOWA (WARSZAWA)	0.037854
3 MAJA (ZŁOTOKŁOS)	0.0378415
ĆWIKLIŃSKIEJ (WARSZAWA)	0.0371598
STRUŻAŃSKA (LEGIONOWO)	0.0370693
INSTYTUT ENERGETYKI (WARSZAWA)	0.0367724
WARSZAWA WSCHODNIA (WARSZAWA)	0.0366885
FRANCISZKAŃSKA (WARSZAWA)	0.0365569
BOBROWIECKA (WARSZAWA)	0.0365445
WÓLCZYŃSKA (WARSZAWA)	0.0365096
KAWĘCZYNEK I (KAWĘCZYNEK)	0.0362816
ROSY (WARSZAWA)	0.0361443
POETÓW (WARSZAWA)	0.035387
PRYSTORA (WARSZAWA)	0.0351478
WOLSKA I (WOLA ALEKSANDRA)	0.0351284
OŚRODEK ZDROWIA (GLINIANKA)	0.0348936

OGRODOWA (SIERZCHÓW)	0.0347597
SZPITAL POWIATOWY (OTWOCK)	0.0346861
MAZURSKA (OTWOCK)	0.0346846
WARSZAWA ŻWIRKI I WIGURY (WARSZAWA)	0.0343457
PARTYZANTÓW (OTWOCK)	0.0342247
MARSZAŁKOWSKA (WARSZAWA)	0.0337743
KOSIARZY (WARSZAWA)	0.0335512
SMOLNA (OTWOCK)	0.0333547
NADZIEI (PIASECZNO)	0.0330972
SĘKOCIN LAS (SĘKOCIN LAS)	0.0330893
WYŚCIGI (WARSZAWA)	0.0330702
GROMADZKA (HENRYKÓW-UROCZE)	0.0328591
WIŚLANA (PODŁĘCZE)	0.0328084
DĘBÓWKA II (DĘBÓWKA)	0.0325343
DROGOWCÓW (JÓZEFÓW)	0.0325217
DĄBROWSKIEGO (PIASTÓW)	0.0323218
MUZEUM NARODOWE (WARSZAWA)	0.032137
KOLONIA WARSZAWSKA (KOLONIA WARSZAWSKA)	0.0320901
BARTYCKA (WARSZAWA)	0.0317393
NOWA WIEŚ (JÓZEFÓW)	0.0315679
OS.MAJDAŃSKA (WARSZAWA)	0.0310054
KOWIEŃSKA (WARSZAWA)	0.0309536
STARZYŃSKIEGO (DAWIDY BANKOWE)	0.0309377
ŚWIĘTOJERSKA (WARSZAWA)	0.0308301
NOWE WĄGRODNO (WĄGRODNO)	0.0304416
PIASKOWA (LEGIONOWO)	0.0302956
OGRODOWA (WÓLKA PRACKA)	0.0300958
WIERUCHÓW (WIERUCHÓW)	0.0299634
PARLAMENTARNA (DAWIDY BANKOWE)	0.029636
PL.SPORTOWY (KONSTANCIN-JEZIORNA)	0.0294568
MIŁOBĘDZKA (WARSZAWA)	0.0294009
KAWĘCZYN (KAWĘCZYN)	0.0293959
POWĄZKI-CM.WOJSKOWY (WARSZAWA)	0.0292648
WARSZAWA TORUŃSKA (WARSZAWA)	0.0285273
MAJDAN (MAJDAN)	0.0282654
DW.CENTRALNY (WARSZAWA)	0.0281383
JANA OLBRACHTA (WARSZAWA)	0.0273192
JASNA (KONSTANCIN-JEZIORNA)	0.027291
BRONOWSKA (WARSZAWA)	0.02723
PRZYLEŚNA (STANISŁAWÓW PIERWSZY)	0.0265948
DOM KULTURY (PIASECZNO)	0.0264605
CMENTARZ (ZĄBKI)	0.026408
BRYLOWSKA (WARSZAWA)	0.02638
KUSOCIŃSKIEGO (PIASECZNO)	0.0263085
LELEWELA (PIASTÓW)	0.0259111
ORZECHOWA (WARSZAWA)	0.0257603
GWARKÓW (WARSZAWA)	0.0253935
PIASKOWA (JABŁONNA)	0.0253164
PKP URSUS PŁN. (WARSZAWA)	0.025279
OGRODY DZIAŁKOWE BEMOWO (WARSZAWA)	0.0252074
KSIĘŻYCOWA (STANISŁAWÓW PIERWSZY)	0.0244942
CIEPŁOWNIA WOLA (WARSZAWA)	0.0243891
PORT NIEPORĘT (NIEPORĘT)	0.0243026
WYGLĘDY (WYGLĘDY)	0.0242762
PKP PRUSZKÓW (PRUSZKÓW)	0.0240482
BADYLARSKA (MICHAŁOWICE)	0.0239681
ŁYSAKOWSKA (WARSZAWA)	0.0235862
KAZIMIERZOWSKA (KONSTANCIN-JEZIORNA)	0.0235272
WARSZAWA PRAGA (WARSZAWA)	0.0232211
MANGALIA (WARSZAWA)	0.0231122
FALENICA (WARSZAWA)	0.0229511
ARMII POZNAŃ (ŁOMIANKI)	0.0228423
STERNICZA (WARSZAWA)	0.0227819
ŻOŁNY (WARSZAWA)	0.0227287
BASENY INFLANCKA (WARSZAWA)	0.0224483
KOŚCIUSZKI (PRUSZKÓW)	0.0221378
ZIELENIECKA (ZĄBKI)	0.0220861
DZIKA (ZĄBKI)	0.0217635
WITWICKIEGO (KONSTANCIN-JEZIORNA)	0.0217152
KAWĘCZYN POD (WARSZAWA)	0.021325
MALCZEWSKIEGO (WARSZAWA)	0.0209888
NISKA (BLIZNE ŁASZCZYŃSKIEGO)	0.0209268
PUSTOLA (WARSZAWA)	0.0207807
CH BŁĘKITNE (LEGIONOWO)	0.0206274
TUROWICE (TUROWICE)	0.0201323
UROCZA (WARSZAWA)	0.0200828
PL.INWALIDÓW (WARSZAWA)	0.0200592
WĄSKA (JÓZEFÓW)	0.0195162
DS RIVIERA (WARSZAWA)	0.0194934
POSTĘPU (WARSZAWA)	0.0194193
ADAMIECKIEGO (WARSZAWA)	0.0193623
ICDS (ŁOMIANKI)	0.0186438
WARSZAWA PŁUDY (WARSZAWA)	0.0181519
ŻUBROWA-UKSW (WARSZAWA)	0.017903
OS.GRAPA (KONSTANCIN-JEZIORNA)	0.0177293
MADALIŃSKIEGO (WARSZAWA)	0.0173535
BRONISŁAWA CZECHA-LAS (WARSZAWA)	0.016849
LEGIONÓW (WOŁOMIN)	0.0167646
BORZĘCIN DUŻY (BORZĘCIN DUŻY)	0.0162093
KONWALIOWA (WARSZAWA)	0.0159689
WARSZAWA POWIŚLE (WARSZAWA)	0.0159448
KS.SŁOJEWSKIEGO (ŁAZY)	0.0154621
BALETOWA (DAWIDY)	0.0149414
OLSZOWA (WARSZAWA)	0.014669
STAROWIEJSKA (NADARZYN)	0.0144663
SÓJKI (WARSZAWA)	0.013247
LIMBY (WARSZAWA)	0.0129472
OS.LAS (NIEPORĘT)	0.0129101
KLAMROWA (WARSZAWA)	0.0126136

ŁUKOWSKA (WARSZAWA)	0.0122516
PERZYŃSKIEGO (WARSZAWA)	0.0117432
ARCHIMEDESA (WARSZAWA)	0.0116259
WARSZAWA GOCŁAWEK (WARSZAWA)	0.0114045
AL.PIŁSUDSKIEGO (SULEJÓWEK)	0.0112001
WARSZAWA URSUS PŁN. (WARSZAWA)	0.0110017
TOMASZOWSKA (WARSZAWA)	0.0104672
RUNOWSKA (ZŁOTOKŁOS)	0.0101541
ZĄBKOWSKA (WARSZAWA)	0.0100955
STAROSTWO POWIATOWE (LEGIONOWO)	0.00973957
BANDURSKIEGO (LEGIONOWO)	0.00958456
STANIEWICKA (WARSZAWA)	0.0093708
ZIMORODKA (JASTRZĘBIE)	0.00935921
FABRYKA POMP (WARSZAWA)	0.00917955
NALEWKI-MUZEUM (WARSZAWA)	0.00889052
URZĄD GMINY (NADARZYN)	0.00883374
WYSZKOWSKA (WARSZAWA)	0.00882995
POMOLOGICZNA (WARSZAWA)	0.00857644
SZAFARZY (WARSZAWA)	0.00815069
LEŚNA (WIKTORÓW)	0.00809077
NOWY PODOLSZYN (NOWY PODOLSZYN)	0.00806863
MAGNOLII (JÓZEFOSŁAW)	0.00759786
WOLNOŚCI (ZĄBKI)	0.00746263
PÓŁNOCNA (KAMIONKA)	0.00724888
KOŁO (WARSZAWA)	0.00723678
SZANCERA (WARSZAWA)	0.00716624
FIELDORFA (WARSZAWA)	0.00709658
BRZÓZEK (ORZESZYN)	0.00690436
WSPÓLNA (DUCHNÓW)	0.00688415
ENERGETYCZNA (PIASECZNO)	0.00683241
PL.GRZYBOWSKI (WARSZAWA)	0.00679257
KOŚCIUSZKI (LEGIONOWO)	0.0067112
GRUSZY (WARSZAWA)	0.00668695
MARCINKI (STARE ZAŁUBICE)	0.00661216
VIVALDIEGO (KLAUDYN)	0.00632416
SZYMANOWSKIEGO (KLAUDYN)	0.00632416
PL.TRZECH KRZYŻY (WARSZAWA)	0.00601879
OGRÓD BOTANICZNY (WARSZAWA)	0.0058867
SIELCE (WARSZAWA)	0.00583368
TARGÓWEK-RATUSZ (WARSZAWA)	0.00564179
PARK TRAUGUTTA (WARSZAWA)	0.00564034
WIERZBIN (WIERZBIN)	0.00558823
ŚMIGŁOWCA (WARSZAWA)	0.0055015
PLANTOWA (PRUSZKÓW)	0.00545554
DZIEWANOWSKA (WARSZAWA)	0.00534227
WARSZAWA ŚRÓDMIEŚCIE (WARSZAWA)	0.00515235
ŁAZY (ŁAZY)	0.00487479
KOSTOWIEC (KOSTOWIEC)	0.00481715
ROZJAZD OBORSKI (KONSTANCIN-JEZIORNA)	0.00468349
CHODKIEWICZA (WARSZAWA)	0.0044698
PALISADOWA (WARSZAWA)	0.00437528
ŁOPUSZAŃSKA (WARSZAWA)	0.00432993
MŁYN (RASZYN)	0.00422699
FIGOWA (WARSZAWA)	0.00395547
REKRUCKA (WARSZAWA)	0.00379836
IZABELIŃSKA (STARE BABICE)	0.00352337
METRO MŁOCINY (WARSZAWA)	0.00312661
OBORY (OBORY)	0.00284728
ŻEROMSKIEGO (WARSZAWA)	0.00283331
SOLIDARNOŚCI (WIELISZEW)	0.00282032
BOBROWIECKA (BOBROWIEC)	0.00279074
IWICKA (WARSZAWA)	0.00221241
KRÓLIKARNIA (WARSZAWA)	0.00212832
OLSZOWA (OTWOCK)	0.00212314
TERMINAL AUTOKAROWY (WARSZAWA)	0.00205986
WARSZAWA KASPRZAKA (WARSZAWA)	0.0019967
OS.DOMANIEWSKA (WARSZAWA)	0.0019952
ŚWIERK-SZKOŁA (OTWOCK)	0.00188924
LESZNOWOLA-POLE (LESZNOWOLA)	0.00140484
BATALIONÓW CHŁOPSKICH (WARSZAWA)	0.0012991
OS.GÓRCZEWSKA (WARSZAWA)	0.000553885
CZERWIŃSKA (WARSZAWA)	0.000471826
KŁOBUCKA-HCB (WARSZAWA)	0.000246333
POWĄZKI-IV BRAMA (WARSZAWA)	0.000148767
STANISŁAWÓW (STANISŁAWÓW PIERWSZY)	5.20845e-005
WILEŃSKA (WARSZAWA)	1.15011e-017
WARSZAWA ZACISZE-WILNO (WARSZAWA)	1.15011e-017
BITWY WARSZAWSKIEJ 1920 (WARSZAWA)	-7.5901e-005
WILLOWA (WIELISZEW)	-0.000157188
KRAKUSA (LEGIONOWO)	-0.000174544
DWORCOWA (PIASECZNO)	-0.000480676
AMBARAS (WARSZAWA)	-0.000490962
WARSZAWA WOLA GRZYBOWSKA (WARSZAWA)	-0.000719218
ROMANTYCZNA (WARSZAWA)	-0.000848054
PLANETOWA (WARSZAWA)	-0.000875351
GRABÓW (WARSZAWA)	-0.000898842
PONIATÓW (PONIATÓW)	-0.00170294
ŻWIROWA (CIECISZEW)	-0.00170924
PRZY PARKU (WARSZAWA)	-0.00173372
BARTOSZKA (WARSZAWA)	-0.0018458
KĘPA OKRZEWSKA (KĘPA OKRZEWSKA)	-0.00191602
NOWY ŚWIAT (WARSZAWA)	-0.00204213
WARSZAWA MIĘDZYLESIE (WARSZAWA)	-0.00206123
PRZELOTOWA (WARSZAWA)	-0.00207011
FLETNIOWA (WARSZAWA)	-0.00216943
WARSZAWA URSUS-NIEDŹWIADEK (WARSZAWA)	-0.00238312
ORLA (OTWOCK)	-0.00263333
KONCERTOWA (WARSZAWA)	-0.00289323
ŚNIEGOCKIEJ (WARSZAWA)	-0.00289548

KOLEJOWA (WARSZAWA)	-0.00306103
KUŹNICY KOŁŁĄTAJOWSKIEJ (WARSZAWA)	-0.00314774
RUBINOWA (JÓZEFOSŁAW)	-0.00316831
NIZINNA (OŁTARZEW)	-0.00329792
GRZYBOWA (WARSZAWA)	-0.00344389
ZACHODNIA (WARSZAWA)	-0.00353483
NOWOTYSKA (WARSZAWA)	-0.00358551
SIELSKA (STANISŁAWÓW PIERWSZY)	-0.00369141
PRZEBUDZENIA WIOSNY (ZALESIE GÓRNE)	-0.00380728
DWERNICKIEGO (OTWOCK)	-0.00385392
HETMAŃSKA (WARSZAWA)	-0.00395549
DĄBRÓWKA (WARSZAWA)	-0.0039763
PL.POWSTAŃCÓW WARSZAWY (WARSZAWA)	-0.00423075
MODZELEWSKIEGO (WARSZAWA)	-0.00425547
NOWODWORSKA (WARSZAWA)	-0.00435315
CUKROWNICZA (WARSZAWA)	-0.00447065
JARZĘBINOWA (WARSZAWA)	-0.00454963
KLARYSEW (KONSTANCIN-JEZIORNA)	-0.00467582
MOCZYDŁÓW (MOCZYDŁÓW)	-0.00472302
MONIUSZKI (WARSZAWA)	-0.00487464
BORYSZEW (BORYSZEW)	-0.00487962
STAWOWA (OTWOCK)	-0.00491913
MICHAŁÓW-REGINÓW (MICHAŁÓW-REGINÓW)	-0.00514366
WINOGRONOWA (ROZALIN)	-0.00518073
TOMASZEW (WARSZAWA)	-0.00551116
SADY ŻOLIBORSKIE (WARSZAWA)	-0.00561774
MERA-PNEFAL (WARSZAWA)	-0.00573128
OKUNIN (OKUNIN)	-0.00580853
ALEKSANDRÓW (STANISŁAWÓW PIERWSZY)	-0.00588132
MROKÓW (MROKÓW)	-0.00601183
MYSZKOWSKA (WARSZAWA)	-0.00621066
INSTYTUT KARDIOLOGII (WARSZAWA)	-0.00654431
AKCENT (WARSZAWA)	-0.00661672
ORCHOWIECKA (WARSZAWA)	-0.00673547
AL.WOJSKA POLSKIEGO (KONSTANCIN-JEZIORNA)	-0.00685826
WARSZAWA ZACHODNIA (WARSZAWA)	-0.00698859
ZAŁUBICE-POCZTA (STARE ZAŁUBICE)	-0.00700923
WARSZAWA WAWER (WARSZAWA)	-0.00713849
SIEKIERKI-SANKTUARIUM (WARSZAWA)	-0.00717995
BIAŁY ŁUG (BIAŁY ŁUG)	-0.00726837
ROZALIN (ROZALIN)	-0.00740935
WOLA GRZYBOWSKA (WARSZAWA)	-0.00796716
NIZINNA (WARSZAWA)	-0.00830656
OS.OLSZYNY (WARSZAWA)	-0.00881988
PL.PRZYMIERZA (WARSZAWA)	-0.00905168
WCZASOWA (KONSTANCIN-JEZIORNA)	-0.00916939
WSPÓŁCZESNA (WARSZAWA)	-0.00917401
URZĄD GMINY (NIEPORĘT)	-0.00918494
WOJEWÓDZKA (ZŁOTOKŁOS)	-0.00950069
WARSZAWSKA (MICHAŁÓW-REGINÓW)	-0.00969467
CENTRUM OLIMPIJSKIE (WARSZAWA)	-0.00974484
INSTYTUT NEUROLOGII (WARSZAWA)	-0.00985163
KORALOWYCH DĘBÓW (ZALESIE GÓRNE)	-0.00987462
PAŁACOWA (ROZALIN)	-0.00991216
WARSZAWA ANIN (WARSZAWA)	-0.0099706
RYBNA (WARSZAWA)	-0.00998044
WARSZAWA WŁOCHY (WARSZAWA)	-0.00998319
WARSZAWA RAKÓW (WARSZAWA)	-0.0102155
GÓRKI (SŁUPNO)	-0.0102207
GÓRECKA (OTWOCK)	-0.010668
HORBACZEWSKIEGO (WARSZAWA)	-0.0107158
SPOKOJNA (ŻANĘCIN)	-0.0109677
AUGUSTÓWKA (WARSZAWA)	-0.0111892
ZALEŚNA (KAMIONKA)	-0.011303
ZALESIE DOLNE (PIASECZNO)	-0.0114923
DWORCOWA (GÓRA)	-0.0115263
TELEWIZJA POLSKA (WARSZAWA)	-0.0115516
PIECHURÓW (WARSZAWA)	-0.0121126
ŚWIDRY MAŁE (JÓZEFÓW)	-0.0121357
WOLSKA II (STANISŁAWÓW DRUGI)	-0.0122663
KLONOWA (STANISŁAWÓW PIERWSZY)	-0.0124928
KAMPINOSKA (ŁOMIANKI)	-0.012565
SZEROKA (WARSZAWA)	-0.0125989
UNIWERSYTET (WARSZAWA)	-0.0126727
BOROWIECKA (WARSZAWA)	-0.0127589
TATRZAŃSKA (OTWOCK)	-0.012775
RAKOWIECKA (WARSZAWA)	-0.0130383
TARNOWIECKA (WARSZAWA)	-0.0130402
WARSZAWSKA (ZIELONKI-WIEŚ)	-0.013107
AGATOWA (JÓZEFOSŁAW)	-0.0131102
NORWIDA (RADZYMIN)	-0.0131745
PL.PONIATOWSKIEGO (NADARZYN)	-0.013241
URZĄD GMINY (IZABELIN)	-0.013313
SOLEC (WARSZAWA)	-0.0133278
MAJDAN-STACJA PALIW (MAJDAN)	-0.0133931
GDECKA (WARSZAWA)	-0.0134223
ZNANA (WARSZAWA)	-0.0134568
ENERGETYCZNA (WARSZAWA)	-0.0135566
KOLARSKA (WARSZAWA)	-0.013628
SZPITAL ONKOLOGICZNY (WIELISZEW)	-0.0139253
ROZWOJOWA (SKRZESZEW)	-0.0141923
KUBUSIA PUCHATKA (OTWOCK)	-0.0142498
PRAŻMOWSKA (WARSZAWA)	-0.0143176
HARFOWA (WARSZAWA)	-0.0143886
SOSNOWA (WARSZAWA)	-0.0144678
BATOREGO (OTWOCK)	-0.0144991
WARSZAWA ŻERAŃ (WARSZAWA)	-0.0145068
PARKOWA (PĘCICE)	-0.0146869
GEN.MACZKA (WARSZAWA)	-0.0147337

GEN.ZAJĄCZKA (WARSZAWA)	-0.0147673
ROSY (HENRYKÓW-UROCZE)	-0.0147934
RODZINNA (STANISŁAWÓW PIERWSZY)	-0.0150603
3 MAJA (OTWOCK)	-0.0150735
OS.BATOREGO (LEGIONOWO)	-0.0150856
PKP ŚRÓDBORÓW (OTWOCK)	-0.0151205
ZAGŁOBY (IZABELIN)	-0.0151515
PONIATOWSKIEGO (OTWOCK)	-0.0151893
SZKOLNA (IZABELIN)	-0.0152444
SZPITALNA (MARKI)	-0.0155292
PRZYJACIÓŁ (NIEPORĘT)	-0.0158387
LEŚNYCH BOGINEK (ZALESIE GÓRNE)	-0.016101
DZIECHCINIEC-KOLONIA (DZIECHCINIEC)	-0.0162109
KS.CZARTORYSKIEJ (RADZYMIN)	-0.0163213
WALECZNYCH (WARSZAWA)	-0.0164256
PODLEŚNA (KOCZARGI STARE)	-0.0164885
WILANÓWKA (OKRZESZYN)	-0.0165637
KRÓLOWEJ BONY (WARSZAWA)	-0.0166057
RADIÓWEK (RADIÓWEK)	-0.0166595
MOST ŚWIĘTOKRZYSKI (WARSZAWA)	-0.0167272
DZIAŁKOWA (WOLA PRAŻMOWSKA)	-0.0168018
ODRODZENIA (WARSZAWA)	-0.0168022
JANIN (ZABORÓW)	-0.0169039
ZABRANIECKA (WARSZAWA)	-0.0170273
FABRYCZNA (ŁAJSKI)	-0.0170437
SIEMIATYCKA (WARSZAWA)	-0.0172434
CH TARGÓWEK (WARSZAWA)	-0.0174253
ZAMIENIE (ZAMIENIE)	-0.0174993
IMBIROWA (HENRYKÓW-UROCZE)	-0.0175032
KOŚCIUSZKI (NADARZYN)	-0.0175466
MEHOFFERA (WARSZAWA)	-0.0175801
WARSZAWA CHOSZCZÓWKA (WARSZAWA)	-0.0176641
CZARNÓWKA (CZARNÓWKA)	-0.0177895
CZORSZTYŃSKA (WARSZAWA)	-0.0183956
SIENKIEWICZA (KONSTANCIN-JEZIORNA)	-0.0184289
PODGÓRNA (WIELISZEW)	-0.0184875
CZATÓW (WARSZAWA)	-0.018648
LWOWSKA (LEGIONOWO)	-0.0186507
URSUS-RATUSZ (WARSZAWA)	-0.0186634
WARSZAWA GOŁĄBKI (WARSZAWA)	-0.0187214
SAPIEHY (IZABELIN)	-0.0189132
FORT WOLA (WARSZAWA)	-0.0189314
CEDROWA (KONSTANCIN-JEZIORNA)	-0.018957
DKS TARGÓWEK (WARSZAWA)	-0.01908
WYSZYŃSKIEGO (JÓZEFÓW)	-0.0191409
NIEDZIAŁKOWSKIEGO (WARSZAWA)	-0.0192137
GASSY (GASSY)	-0.0195324
SKOROSZE (WARSZAWA)	-0.0197387
SZAMOCIN (WARSZAWA)	-0.0197916
ŚWIĘCICKA (PŁOCHOCIN)	-0.0199333
ROGALIŃSKA (WARSZAWA)	-0.0201575
WINNICA (WARSZAWA)	-0.0203336
OKRZESZYN (OKRZESZYN)	-0.0203669
TOR STEGNY (WARSZAWA)	-0.0205076
ŁASAKA (WARSZAWA)	-0.0206377
LIPOWA (MAGDALENKA)	-0.0206911
SPISKA (WARSZAWA)	-0.0209746
OŻARÓW MAZ. (OŻARÓW MAZOWIECKI)	-0.0209878
POMIECHOWSKA (WARSZAWA)	-0.0211317
DĄBRÓWKA SZLACHECKA (WARSZAWA)	-0.0211788
LEGIONOWO PIASKI (LEGIONOWO)	-0.0212012
ZAWADZKA (ZŁOTOKŁOS)	-0.0213704
POLNA (JÓZEFÓW)	-0.0214384
WIERZBOWA (LIPOWO)	-0.0214884
LIPOWO (LIPOWO)	-0.0216284
WARSZAWSKA (LESZNO)	-0.0217654
ZABORÓWEK-WIEŚ (ZABORÓWEK)	-0.0217737
FORT BEMA (WARSZAWA)	-0.0217813
SZARADY (WARSZAWA)	-0.0218298
OGRODOWA (GŁOSKÓW-LETNISKO)	-0.0220442
JESIONOWA (MAGDALENKA)	-0.0222938
RYBNICKA (WARSZAWA)	-0.0223538
OKRĄG (NOWA IWICZNA)	-0.0224424
POLNA (KOMORNICA)	-0.0226122
KUKUŁCZA (JASTRZĘBIE)	-0.0226284
MOSTOWA (ŻABIENIEC)	-0.022634
POLNYCH KWIATÓW (WARSZAWA)	-0.0227213
PIRENEJSKA (WARSZAWA)	-0.0228411
PROTAZEGO (NIEPORĘT)	-0.0228606
OS.LEŚNE (WARSZAWA)	-0.0229889
PODŁĘCZE (PODŁĘCZE)	-0.0229945
PALMIRY (KAMPINOSKI PN)	-0.0231101
STASZICA (WARSZAWA)	-0.0231854
MASZEWSKA (WARSZAWA)	-0.0232629
ŻANĘCIN (ŻANĘCIN)	-0.0232837
MYCIELSKIEGO (WARSZAWA)	-0.023365
AL.ZIELENIECKA (WARSZAWA)	-0.0237455
ŹRÓDLANA (ŚWIĘCICE)	-0.0237694
SIWIŃSKIEGO (LEGIONOWO)	-0.0237697
NOWA (OTWOCK)	-0.0238049
WIERZCHOWSKIEGO (WARSZAWA)	-0.0239352
TERMINAL CARGO (WARSZAWA)	-0.02396
LIPY (NIEPORĘT)	-0.0240069
TRZYKROTKI (WARSZAWA)	-0.024177
KOMPASOWA (WARSZAWA)	-0.0244153
WARSZAWA REMBERTÓW (WARSZAWA)	-0.024462
LESZNO-GS (LESZNO)	-0.0246955
NOWOWIEJSKA (WARSZAWA)	-0.0247149

OŚ KRÓLEWSKA (WARSZAWA)	-0.0248022
POCZTY GDAŃSKIEJ (WARSZAWA)	-0.0249021
PÓŁNOCNA (WOLA PRAŻMOWSKA)	-0.0249926
GROSZÓWKA (WARSZAWA)	-0.0250522
PRZEDPOLE (WARSZAWA)	-0.0250611
MICHALIN (JÓZEFÓW)	-0.0250938
WARSZAWA RAKOWIEC (WARSZAWA)	-0.0254241
SZKOLNA (SKRZESZEW)	-0.0255472
PIASKI (WARSZAWA)	-0.0256759
GŁOGI (NIEPORĘT)	-0.0257559
POLNA (LEGIONOWO)	-0.0257807
KOŚCIUSZKI-SZKOŁA (IZABELIN)	-0.0263168
OSTRÓDZKA-SZKOŁA (WARSZAWA)	-0.026351
PODLEŚNA (ŁAZY)	-0.0263696
OSP SKRZESZEW (SKRZESZEW)	-0.0265407
KRZYWICKIEGO (WARSZAWA)	-0.0265684
CHABROWA (WARSZAWA)	-0.0267119
ORLA (SULEJÓWEK)	-0.0267155
ZABOROWSKA (ŚWIĘCICE)	-0.0267415
KAŁUSZYN (KAŁUSZYN)	-0.0268844
CENDROWICE (CENDROWICE)	-0.0268884
CMENTARZ 1920 R. (RADZYMIN)	-0.027162
PIELĘGNIAREK (WARSZAWA)	-0.0272815
WIERZBOWA (RADZYMIN)	-0.0273868
INOWŁODZKA (WARSZAWA)	-0.027398
METRO WAWRZYSZEW (WARSZAWA)	-0.0275968
SZKOLNA (JÓZEFÓW k. LEGIONOWA)	-0.0276091
SZKOLNA (ŁAJSKI)	-0.027617
ŁAŹNIEWSKA (ŚWIĘCICE)	-0.0276715
PALMIRY-MUZEUM (KAMPINOSKI PN)	-0.0276951
KARCZOCHA (WARSZAWA)	-0.0276969
POPULARNA (WARSZAWA)	-0.0277287
DZIECHCINIEC-SKLEP (DZIECHCINIEC)	-0.0278359
KOŚCIELNA (WIELISZEW)	-0.0279317
ZABORÓW-SZKOŁA (ZABORÓW)	-0.028034
NOWODWORY (WARSZAWA)	-0.028129
KOCJANA-SĄDY (WARSZAWA)	-0.0283612
FELIŃSKIEGO (WARSZAWA)	-0.0284361
ZERZEŃ (WARSZAWA)	-0.0286232
KWIATOWA (JANÓWEK PIERWSZY)	-0.0286424
OBRAZKOWA (WARSZAWA)	-0.0287474
WIOSENNA (ZABORÓW)	-0.0287924
FARADAYA (WARSZAWA)	-0.0288849
MŁODZIEŻOWA (LIPOWO)	-0.0290161
HIPODROM (WARSZAWA)	-0.0292093
TEKLIN (OTWOCK)	-0.0294091
GOCŁAW (WARSZAWA)	-0.0295047
TURÓW (KOBYŁKA)	-0.0295283
WARSZAWA URSUS (WARSZAWA)	-0.0295388
SŁUPNO (SŁUPNO)	-0.0296469
SŁOWACKIEGO (WARSZAWA)	-0.0297629
GŁĘBOCKA (WARSZAWA)	-0.0298011
JOLIOT-CURIE (WARSZAWA)	-0.0298587
WILANOWSKA (JÓZEFOSŁAW)	-0.030005
LIPKÓW-MUZEUM (LIPKÓW)	-0.030384
URBANISTÓW (WARSZAWA)	-0.0304421
TOPOLOWA (KAWĘCZYNEK)	-0.0304434
WARSZAWA JEZIORKI (WARSZAWA)	-0.0305168
PAŃSKA (KONSTANCIN-JEZIORNA)	-0.0305819
HALLERA (MARKI)	-0.0306224
MARSA-LAS (WARSZAWA)	-0.0306818
ŁAWKI (ŁAWKI)	-0.0309006
GĄSOCIŃSKA (WARSZAWA)	-0.0311795
KOSAĆCOWA (WARSZAWA)	-0.0312382
DOM ARTYSTÓW (KONSTANCIN-JEZIORNA)	-0.0312787
POŻARYSKIEGO (WARSZAWA)	-0.0314102
SIEDLISKOWA (WARSZAWA)	-0.0314426
WILKOWIECKA (WARSZAWA)	-0.0317285
LEŚNA (JAROSZOWA WOLA)	-0.0317671
GARWOLIŃSKA (WARSZAWA)	-0.0320471
ORZECHOWA (WOLA GOŁKOWSKA)	-0.0322055
GOCŁAWEK (WARSZAWA)	-0.0322268
ZABORÓWEK-OSIEDLE (ZABORÓWEK)	-0.0323294
LERCHA (WARSZAWA)	-0.0325451
KOLUMBA (WARSZAWA)	-0.0325542
ZDOBYWCÓW KOSMOSU (NOWY DWÓR MAZ.)	-0.0326152
WARSZAWA CENTRALNA (WARSZAWA)	-0.0326163
WARSZAWA RADOŚĆ (WARSZAWA)	-0.0327826
WARSZAWA ZOO (WARSZAWA)	-0.033022
SADKOWSKA (WARSZAWA)	-0.033033
PAPIERNIA (KONSTANCIN-JEZIORNA)	-0.0331013
WŁADYSŁAWA HERMANA (WARSZAWA)	-0.0333285
REJTANA (WARSZAWA)	-0.0333497
PĘCICE-MAJĄTEK (PĘCICE)	-0.0334778
GÓRASZKA (GÓRASZKA)	-0.0338916
KLIMONTOWSKA (WARSZAWA)	-0.034003
BORZĘCIN DUŻY-WIEŚ (BORZĘCIN DUŻY)	-0.0340702
BURSZTYNOWA-SZPITAL (WARSZAWA)	-0.0340715
KANAŁ NOWE UJŚCIE (WARSZAWA)	-0.0341464
STEGNY (WARSZAWA)	-0.0342023
CYTADELA (WARSZAWA)	-0.0342183
JAWCZYCE (JAWCZYCE)	-0.0342518
ROKOKOWA (WARSZAWA)	-0.0343413
ROLNA (KAJETANY)	-0.0343834
PL.MAŁACHOWSKIEGO (WARSZAWA)	-0.0345001
WARSZAWSKA (NADARZYN)	-0.034605
KSAWERÓW (WARSZAWA)	-0.0353086
STEFANA BATOREGO (WARSZAWA)	-0.0353097

DYREKCJA FSO (WARSZAWA)	-0.035506
POWSTAŃCÓW WARSZAWY (PIASECZNO)	-0.0355879
CZERNIDŁA (CZERNIDŁA)	-0.0357459
CIOŁKA (WARSZAWA)	-0.0358164
SAGALLI (WARSZAWA)	-0.0358646
WYBRZEŻE SZCZECIŃSKIE (WARSZAWA)	-0.0358896
GONTARSKA (WARSZAWA)	-0.0359063
PADEREWSKIEGO (WARSZAWA)	-0.0359831
WOLA MROKOWSKA (WOLA MROKOWSKA)	-0.0361064
KAZUBÓW (WARSZAWA)	-0.0361892
CZEREMCHOWA (WARSZAWA)	-0.0362151
REBUSOWA (WARSZAWA)	-0.0362991
KWATERY GŁÓWNEJ (WARSZAWA)	-0.0364021
BOHATERÓW WARSZAWY (PRUSZKÓW)	-0.0365509
WARSZAWA GDAŃSKA (WARSZAWA)	-0.0365758
MODLIŃSKA (WIELISZEW)	-0.0365867
PKP MICHAŁÓW-REGINÓW (MICHAŁÓW-REGINÓW)	-0.0366043
OKRĘŻNA (WARSZAWA)	-0.036725
RYCHNOWSKA (WARSZAWA)	-0.0367434
ŻÓŁKIEWSKIEGO (WARSZAWA)	-0.0367532
WYSPIAŃSKIEGO (ZĄBKI)	-0.036876
GOTARDA (WARSZAWA)	-0.0368961
KOMETY (JÓZEFOSŁAW)	-0.036933
PĘCICE-KOŚCIÓŁ (PĘCICE)	-0.036953
KS.SITNIKA (WARSZAWA)	-0.0369786
KLARYSEWSKA (WARSZAWA)	-0.0371175
WÓLCZYŃSKA (MARIEW)	-0.0371642
JANCZEWICE (JANCZEWICE)	-0.037353
PKP OKĘCIE (WARSZAWA)	-0.0374534
WIELOWIEJSKA (WARSZAWA)	-0.0374603
SZKOLNA (LESZNOWOLA)	-0.0375195
WOJSŁAWICKA (WARSZAWA)	-0.03831
CZOŁGISTÓW (WARSZAWA)	-0.0385762
KORCZAKA (RADZYMIN)	-0.0385916
WIĄZANA (WARSZAWA)	-0.0387353
KWIATÓW POLNYCH (DAWIDY BANKOWE)	-0.0387679
MALINOWA (WARSZAWA)	-0.0387865
BOLEŚĆ (WARSZAWA)	-0.038857
TORWAR (WARSZAWA)	-0.0389165
SKOROSZEWSKA (WARSZAWA)	-0.0389335
PRZEMYSŁOWA (OKUNIN)	-0.0390178
POGROSZEW (POGROSZEW)	-0.0390538
NOWE SŁUPNO (SŁUPNO)	-0.0391404
PL.PIŁSUDSKIEGO (WARSZAWA)	-0.0391462
WAT (WARSZAWA)	-0.0393644
POWOJOWA (WARSZAWA)	-0.0396258
GRÓDEK (EMÓW)	-0.0397282
JÓZEFÓW (JÓZEFÓW)	-0.0397347
PĘCZKOWSKIEGO (NIEPORĘT)	-0.0399161
RONDO RADOSŁAWA (WARSZAWA)	-0.0399191
KRZESIWA (WARSZAWA)	-0.0400342
WARSZAWA WESOŁA (WARSZAWA)	-0.0401461
KIERBEDZIA (OŻARÓW MAZOWIECKI)	-0.0402342
KOLEJOWA (ZIELONKA)	-0.0402385
BERENTA (PRUSZKÓW)	-0.0402437
DO FORTU (WARSZAWA)	-0.0407196
TOLAK (JANCZEWICE)	-0.0410442
AKACJOWA (KOCZARGI STARE)	-0.0411742
URZUT (URZUT)	-0.0413107
ODLEWNICZA (WARSZAWA)	-0.0413315
ŁOSOSIOWA (WARSZAWA)	-0.0416317
LEGIONOWA (MARKI)	-0.0416381
DZIĘCIOŁA (WARSZAWA)	-0.041674
PILAWA (PILAWA)	-0.0417471
JARA (WARSZAWA)	-0.0417904
ŁUKOWA (WARSZAWA)	-0.0418804
KS.BOGUCKIEGO (WARSZAWA)	-0.0419435
BALEYA (WARSZAWA)	-0.0419675
TORFOWA (WARSZAWA)	-0.0419881
METRO IMIELIN (WARSZAWA)	-0.0421121
AKACJOWA (SIERZCHÓW)	-0.0424654
KISIELEWSKIEGO (CHOTOMÓW)	-0.0425398
MAKOLĄGWY (WARSZAWA)	-0.0425891
WODNIAKÓW (WARSZAWA)	-0.0426107
JULIANOWSKA (WARSZAWA)	-0.0428508
WILANÓW (WARSZAWA)	-0.0428548
DŁUGA (USTANÓW)	-0.0428654
WĘDKARSKA (WARSZAWA)	-0.0429217
POD KAJETANY (SZAMOTY)	-0.0430048
GŁOWACKIEGO (WARSZAWA)	-0.0430232
PŁUŻNICKA (WARSZAWA)	-0.0431909
PUSZKINA (SULEJÓWEK)	-0.0433617
KORSAKA (WARSZAWA)	-0.0434178
SMOCZA (WARSZAWA)	-0.0435316
INSTYTUT EKOLOGII PAN (DZIEKANÓW LEŚNY)	-0.0435391
ARBUZOWA (WARSZAWA)	-0.0435753
WÓLCZYŃSKA-BOCZNICA (WARSZAWA)	-0.043844
CM.WAWRZYSZEWSKI (WARSZAWA)	-0.0439351
PIASTÓW (PIASTÓW)	-0.0441885
BIELSZOWICKA (WARSZAWA)	-0.0442497
WOJCIECHOWSKIEGO (WARSZAWA)	-0.0445396
ŁĄKOWA (LASKI)	-0.0446113
GRENADIERÓW (WARSZAWA)	-0.0446199
PARKOWA (USTANÓW)	-0.0447125
WARSZAWA MIEDZESZYN (WARSZAWA)	-0.0448476
OPŁOTEK (WARSZAWA)	-0.0449222
PATKOWSKIEGO (WARSZAWA)	-0.0449527
SIENNICKA (WARSZAWA)	-0.0450108
PRASOWA (WARSZAWA)	-0.0450405

JANÓW (JANÓW)	-0.0451782
METRYCZNA (WARSZAWA)	-0.0454222
TRAKT LUBELSKI-LAS (WARSZAWA)	-0.045594
MIĘDZYNARODOWA (WARSZAWA)	-0.0456538
CM.WŁOSKI (WARSZAWA)	-0.0456786
CZOŁCHAŃSKIEGO (PRAŻMÓW)	-0.0457261
ARENA LEGIONOWO (LEGIONOWO)	-0.0461218
ZAKRĘTOWA (WARSZAWA)	-0.0462682
PRZEŁĘCZY (WARSZAWA)	-0.0464295
3 MAJA (OŻARÓW MAZOWIECKI)	-0.0465005
LEGIA-STADION (WARSZAWA)	-0.0467039
STEFANÓWKA (BORYSZEW)	-0.0467246
BATALIONU MIOTŁA (WARSZAWA)	-0.0467401
PAROWOZOWA (WARSZAWA)	-0.0469338
SZPITAL POWIATOWY (PRUSZKÓW)	-0.0469393
RĘCZAJSKA (KOBYŁKA)	-0.046988
SOKOŁOWSKA (WARSZAWA)	-0.047061
ŁOKIETKA (WARSZAWA)	-0.0472484
JAŁOWCOWA (OTWOCK)	-0.0474075
TOPOGRAFICZNA (WARSZAWA)	-0.0474416
PKP LEGIONOWO-PRZYSTANEK (LEGIONOWO)	-0.0477188
PROMYKA (JAWOROWA)	-0.0477258
PIASECZNO (PIASECZNO)	-0.0477258
SIENNA (WARSZAWA)	-0.0478764
WARSZAWA DAWIDY (WARSZAWA)	-0.0483446
POPŁAWSKA (RZAKTA)	-0.0484118
CH PROMENADA (WARSZAWA)	-0.0487148
BIAŁOBRZEGI (BIAŁOBRZEGI)	-0.0487905
NAMYSŁOWSKA (WARSZAWA)	-0.0490285
3 MAJA (TRUSKAW)	-0.049203
PKP LEGIONOWO PIASKI (LEGIONOWO)	-0.0492667
KĄPIELOWA (BIAŁOBRZEGI)	-0.0492814
WIŚNIOWA (WIELISZEW)	-0.0493545
RASZYŃSKA (WARSZAWA)	-0.0493624
NOWA IWICZNA (NOWA IWICZNA)	-0.0493863
KSIĘŻYCOWA (WARSZAWA)	-0.0494741
ŻWIROWA (BOBROWIEC)	-0.049636
ROZŁOGI (WARSZAWA)	-0.0498691
WARSZAWA KOŁO (WARSZAWA)	-0.0499112
KASZTANOWA (WARSZAWA)	-0.0503026
BRZOZOWA (WOLA MROKOWSKA)	-0.0504708
MORY (MORY)	-0.0506765
DĄBRÓWKA WIŚLANA (WARSZAWA)	-0.0507021
MARMUROWA (WARSZAWA)	-0.0507247
BACHA (WARSZAWA)	-0.0508186
WIŚNIOWA (WARSZAWA)	-0.0513756
KOLONIA HELENÓW (WARSZAWA)	-0.0521889
OLSZYNA ŁYCZYŃSKA (PARCELA)	-0.0522823
OS.DERBY III (WARSZAWA)	-0.0522912
SANGUSZKI (WARSZAWA)	-0.0522988
RAGINISA (WARSZAWA)	-0.0524014
TEATR KOMEDIA (WARSZAWA)	-0.0524441
GOŁKÓW-WIEŚ (GOŁKÓW)	-0.0525435
MAŁYCH DĘBÓW (WARSZAWA)	-0.052637
ZBYSZKA Z BOGDAŃCA (WARSZAWA)	-0.0528026
OS.ARBUZOWA (WARSZAWA)	-0.05282
FALENCKA (JANKI)	-0.0530691
SADYBA (WARSZAWA)	-0.0531276
NAŁĘCZOWSKA (WARSZAWA)	-0.0531728
ŁBISKA (ŁBISKA)	-0.0532135
TABITA (KONSTANCIN-JEZIORNA)	-0.0533116
WOŁOSKA (WARSZAWA)	-0.0533177
POCISK (WARSZAWA)	-0.053414
GRZYBOWSKA (PARCELA)	-0.0534529
ŁĘG (ŁĘG)	-0.053577
STARY SĘKOCIN (STARY SĘKOCIN)	-0.0536071
KOMOROWSKA-HOTEL (NADARZYN)	-0.053775
URZĄD STATYSTYCZNY (WARSZAWA)	-0.054193
KRAŚNICKA (WARSZAWA)	-0.0542983
INSUREKCJI (WARSZAWA)	-0.0543217
KAMYKOWA (WARSZAWA)	-0.0543899
SŁOWACKIEGO (LEGIONOWO)	-0.0544763
WALEREGO SŁAWKA (WARSZAWA)	-0.0545808
OLSZYNOWA (NOWY PODOLSZYN)	-0.0546559
CM.PRAWOSŁAWNY (WARSZAWA)	-0.0547182
OŚRODEK CORA (WARSZAWA)	-0.0547796
POCZTOWA (LASKI)	-0.0548128
SZACHOWA (WARSZAWA)	-0.0548241
ŁĄCZNICZEK AK (PRUSZKÓW)	-0.054899
POGODNA (WARSZAWA)	-0.0551909
TYMOTKI (WARSZAWA)	-0.0555451
WARSZAWA OLSZYNKA GROCHOWSKA (WARSZAWA)	-0.0556588
BATUMI (WARSZAWA)	-0.055743
KINO BAŚŃ (PIASTÓW)	-0.0557455
CMENTARZ (JABŁONNA)	-0.055807
WÓLKA MŁĄDZKA (OTWOCK)	-0.0558091
MAŃKOWSKA (WARSZAWA)	-0.0562289
ŚW.HIERONIMA (WARSZAWA)	-0.0562493
KANAŁ GOCŁAWSKI (WARSZAWA)	-0.056314
PIASTA KOŁODZIEJA (WARSZAWA)	-0.0564433
PRZEPIÓRKI (WARSZAWA)	-0.0565177
BIBLIOTEKA NARODOWA (WARSZAWA)	-0.0565517
ORZESZKOWEJ (PIASTÓW)	-0.056638
KRAŃCOWA (WARSZAWA)	-0.0567536
OS.JAGIELLOŃSKA (LEGIONOWO)	-0.0569141
ZAJEZDNIA WOLA (WARSZAWA)	-0.056956
STRĄKOWA (WARSZAWA)	-0.0572429
DZIELNICOWA (WARSZAWA)	-0.0572538
ORACZY (WARSZAWA)	-0.0573861
WÓLKA DWORSKA (WÓLKA DWORSKA)	-0.0575753

RUSIEC (RUSIEC)	-0.0576728
ORZYCKA (WARSZAWA)	-0.0578442
CM.WOLSKI (WARSZAWA)	-0.0578989
OSIEDLE (WARSZAWA)	-0.0581692
KOŚCIUSZKI (MARKI)	-0.0581902
STARE MIASTO (WARSZAWA)	-0.0582028
LOKALNA (ŁAZY)	-0.0583329
PKP JÓZEFÓW (JÓZEFÓW)	-0.0583349
DOBIESZ (DOBIESZ)	-0.058375
LOTNISKO CHOPINA-ODLOTY (WARSZAWA)	-0.0585112
PISKÓRKA (PISKÓRKA)	-0.0585571
PARCELA II (PARCELA)	-0.0586556
KRÓLOWEJ MARYSIEŃKI (WOJCIESZYN)	-0.0593823
WSPÓLNA (MARKI)	-0.0594363
SŁOMCZYN-KOŚCIÓŁ (SŁOMCZYN)	-0.059532
WIERZBOWA (BASZKÓWKA)	-0.0597557
PARCELA I (PARCELA)	-0.0599463
WARYŃSKIEGO (ZIELONKA)	-0.0599967
JADWISIN (WARSZAWA)	-0.0600369
RADIOWA (WARSZAWA)	-0.0603084
ODRĘBNA (WARSZAWA)	-0.0605049
BARTYKI (WARSZAWA)	-0.0605155
BUDZIŃSKIEJ-TYLICKIEJ (WARSZAWA)	-0.0605804
GRENADY (WARSZAWA)	-0.0606926
BASZKÓWKA (BASZKÓWKA)	-0.0608623
WOJCIESZYN (WOJCIESZYN)	-0.0609575
WODOCIĄG PŁN. (WIELISZEW)	-0.0611042
KS.SKORUPKI (LEGIONOWO)	-0.0612691
OŚRODEK AZS (WARSZAWA)	-0.0612923
ZEGARYNKI (WARSZAWA)	-0.0613161
POWSTAŃCÓW WARSZAWY (OTWOCK)	-0.0614866
LESZNOWOLA (LESZNOWOLA)	-0.0616169
ZAJEZDNIA KLESZCZOWA (WARSZAWA)	-0.0616713
DOMINIKAŃSKA (WARSZAWA)	-0.0622161
CZAJKOWSKIEGO (KLAUDYN)	-0.0623936
LEKTYKARSKA (WARSZAWA)	-0.0625043
WARSZAWA FALENICA (WARSZAWA)	-0.0628542
ŚWIDRY WIELKIE (OTWOCK)	-0.0628927
TRUSKAWIECKA (WARSZAWA)	-0.0629178
DZIAŁKOWA (JANKI)	-0.0629346
PARKOWA (TRUSKAW)	-0.0629475
PŁOCHOCIN (PŁOCHOCIN)	-0.0629982
MEKSYKAŃSKA (WARSZAWA)	-0.0631012
PIASTOWSKA (WARSZAWA)	-0.0632217
PROJEKTOWANA (WIĄZOWNA KOŚCIELNA)	-0.0633226
SAMOTNEJ MATKI (WARSZAWA)	-0.0637916
MIŃSKA (WARSZAWA)	-0.0638004
CZARNOMORSKA (WARSZAWA)	-0.0638965
JEZIORKO CZERNIAKOWSKIE (WARSZAWA)	-0.0639654
SIWEK (BORKI)	-0.0639989
NOWY SĘKOCIN (NOWY SĘKOCIN)	-0.0640313
WÓLKA ZAŁĘSKA (WÓLKA DWORSKA)	-0.0642168
KASZTANOWA (ORZESZYN)	-0.064281
WILCZA GÓRA (WILCZA GÓRA)	-0.0642878
PKP WAWER (WARSZAWA)	-0.0644718
URZĄD MIASTA (ZĄBKI)	-0.0645607
CICHA (JÓZEFÓW)	-0.0645835
PUSTELNIK (MARKI)	-0.0646593
ZAJEZDNIA REDUTOWA (WARSZAWA)	-0.0655692
ORLA (ZĄBKI)	-0.0656407
MARZYCIELI (KĄTY WĘGIERSKIE)	-0.065709
POLNA (ŻABIENIEC)	-0.0659847
WARSZAWA OKĘCIE (WARSZAWA)	-0.0661004
IMUZ (FALENTY)	-0.0663638
SZUBERTA (WARSZAWA)	-0.0664405
PRUSZKÓW (PRUSZKÓW)	-0.0664941
INŻYNIERSKA (WARSZAWA)	-0.0666069
HOTELOWA (SULEJÓWEK)	-0.0666402
FORTOWA (BLIZNE ŁASZCZYŃSKIEGO)	-0.0666983
CYGAŃSKA (WARSZAWA)	-0.0667774
GMINNEJ RADY NARODOWEJ (LESZNOWOLA)	-0.0668309
ŚNIEŻNA (WARSZAWA)	-0.0668875
ANUSINEK (KĄTY WĘGIERSKIE)	-0.0669414
RUDKA (RUDKA)	-0.0669929
WIESIOŁKA (WARSZAWA)	-0.0671718
BANACHA-SZPITAL (WARSZAWA)	-0.0673012
LEBIODOWA (WARSZAWA)	-0.0674941
ŁOWCZA (WARSZAWA)	-0.0678121
OŚRODEK LOK (WARSZAWA)	-0.0678815
KLIMATYCZNA (WARSZAWA)	-0.0679157
SMOLEŃSKIEGO (WARSZAWA)	-0.06815
MILENIJNA (WARSZAWA)	-0.0683695
NOWA WOLA (NOWA WOLA)	-0.0683766
ZIELONA (PIASECZNO)	-0.0686766
ARONIOWA (WARSZAWA)	-0.0687215
MAGISTRACKA (WARSZAWA)	-0.0688898
JARACZA (WARSZAWA)	-0.0688912
KRÓLOWEJ MARYSIEŃKI (WARSZAWA)	-0.0690567
RYBIE (RYBIE)	-0.0691134
WÓLKA DWORSKA-WISŁA (WÓLKA DWORSKA)	-0.069355
TATARSKA (WARSZAWA)	-0.0695141
PANCERA (WARSZAWA)	-0.0696271
ŁOZISKA (PIASECZNO)	-0.0696454
DRZEWIECKIEGO (WARSZAWA)	-0.0696665
CEGLANA (NADMA)	-0.0697324
RYBNA (WOLA GOŁKOWSKA)	-0.0697613
SOBOCKA (WARSZAWA)	-0.0700975
STAROSTWO (OŻARÓW MAZOWIECKI)	-0.0701989
SPACEROWA (OLSZEWNICA NOWA)	-0.070334

BUDOWLANA (WARSZAWA)	-0.0705781
KRÓTKA (STARA IWICZNA)	-0.0705867
STRZYKUŁY (STRZYKUŁY)	-0.0706665
ZASADOWA (WARSZAWA)	-0.0707073
KOPIJNIKÓW (WARSZAWA)	-0.0707431
KICKIEGO (WARSZAWA)	-0.0708109
NOWOLIPIE (NIEPORĘT)	-0.0709141
LIPOWA (GŁOSKÓW-LETNISKO)	-0.0709879
METRO SŁODOWIEC (WARSZAWA)	-0.0711253
NADARZYŃSKA (WARSZAWA)	-0.0712556
WOJTYSZKI (WARSZAWA)	-0.0713059
ARESZT ŚLEDCZY SŁUŻEWIEC (WARSZAWA)	-0.0714035
DW.WSCHODNI (KIJOWSKA) (WARSZAWA)	-0.0714751
BLIZNE ŁASZCZYŃSKIEGO (BLIZNE ŁASZCZYŃSKIEGO)	-0.0716728
AL. 3 MAJA (WARSZAWA)	-0.0718864
GOCŁAWSKA (WARSZAWA)	-0.0725558
SOKOŁÓW (SOKOŁÓW)	-0.0725793
PIASECZYŃSKA (KONSTANCIN-JEZIORNA)	-0.0725909
CH WÓLKA KOSOWSKA (WÓLKA KOSOWSKA)	-0.0727557
ZAWADY (WARSZAWA)	-0.0728228
JADŹWINGÓW (WARSZAWA)	-0.0729497
PILASZKÓW (PILASZKÓW)	-0.0730332
OKĘCIE BUSINESS PARK (WARSZAWA)	-0.0730731
ZALESIE (WIERZBIN)	-0.0731973
BUKÓW (WARSZAWA)	-0.0733972
URSYNÓW PŁN. (WARSZAWA)	-0.0734773
ARKADYJSKA (WARSZAWA)	-0.0735651
GOŁĘBIA (KAJETANY)	-0.0736714
WIOLINOWA (WARSZAWA)	-0.0736944
POBORZAŃSKA (WARSZAWA)	-0.0737453
WARSZAWA LOTNISKO CHOPINA (WARSZAWA)	-0.0737761
NOWE BABICE (NOWE BABICE)	-0.0738286
KĘDZIERÓWKA (KĘDZIERÓWKA)	-0.0741126
ŚRÓDBORÓW (OTWOCK)	-0.0741412
SŁONECZNA (WIELISZEW)	-0.0750364
SYPNIEWSKA (WARSZAWA)	-0.0754455
JERZEGO (WARSZAWA)	-0.0755344
LEGIONOWO PRZYSTANEK (LEGIONOWO)	-0.0756199
BIAŁOSTOCKA (KONSTANCIN-JEZIORNA)	-0.0765381
WOLICA (STARE ZAŁUBICE)	-0.0765687
BEREZYŃSKA (WARSZAWA)	-0.0767831
MRĄGOWSKA (WARSZAWA)	-0.0768677
WALEWSKA (WARSZAWA)	-0.0769392
WARSZAWA SALOMEA (WARSZAWA)	-0.077013
KRUBIN (KRUBIN)	-0.0771632
KOTLINY (MALCANÓW)	-0.0773561
PASCHALISA-JAKUBOWICZA (LIPKÓW)	-0.0775467
DZIECHCINIEC (DZIECHCINIEC)	-0.0777973
REJTANA (MARKI)	-0.077885
REDUTOWA (WARSZAWA)	-0.0781248
JAROSŁAWSKA (JÓZEFÓW)	-0.0781623
HŁASKI (WARSZAWA)	-0.0784308
BLISKA (WARSZAWA)	-0.0787356
AL.JANA PAWŁA II (WARSZAWA)	-0.0789789
GRYFITÓW (WARSZAWA)	-0.0789969
PĘCHERY (PĘCHERY)	-0.07908
SPORTOWA (WIĄZOWNA)	-0.0792235
SZKOLNA (GLINIANKA)	-0.0797392
INSTYTUT FIZYKI (WARSZAWA)	-0.0797773
RUTY (WARSZAWA)	-0.0799021
SZLACHECKA (OTWOCK)	-0.0801434
WYSZOGRODZKA (WARSZAWA)	-0.08031
OSTROROGA (WARSZAWA)	-0.0803731
TŁUCHOWSKA (WARSZAWA)	-0.0805356
DŁUGOSZA (WARSZAWA)	-0.0805624
WOJCIESZYN-GÓRKI (WOJCIESZYN)	-0.0805978
SOŁTYKA (WARSZAWA)	-0.0808853
GRZYMALITÓW (WARSZAWA)	-0.0810202
SULEJÓWEK MIŁOSNA (SULEJÓWEK)	-0.0810495
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POMORSKA (PIASECZNO)	-0.0813755
ZAUŁEK (WARSZAWA)	-0.0814231
PAROWCOWA (WARSZAWA)	-0.0818695
STEFANÓW (ŻABIENIEC)	-0.081961
MARYSIN (MARYSIN)	-0.0821235
SOLIDNA (WARSZAWA)	-0.0822743
NAUCZYCIELSKA (MARKI)	-0.0824907
HUTNICZA (WARSZAWA)	-0.0825832
OS.ZIELONY STAW (WARSZAWA)	-0.0826063
SAMARYTANKA (WARSZAWA)	-0.0826458
JULIANÓW (JULIANÓW)	-0.0826844
POD CZERNIAKÓW (WARSZAWA)	-0.082698
MALBORSKA (WARSZAWA)	-0.0828288
DW.WSCHODNI (LUBELSKA) (WARSZAWA)	-0.0830416
ŻYTNIA (WARSZAWA)	-0.083268
ŁAZIENKI KRÓLEWSKIE (WARSZAWA)	-0.0833905
PASZKÓW (PASZKÓW)	-0.0836294
NOWE WŁOCHY (WARSZAWA)	-0.0836341
KOZIELSKA (WARSZAWA)	-0.08405
WIELISZEW (WIELISZEW)	-0.0841447
PL.NARUTOWICZA (WARSZAWA)	-0.0842104
MAŃKI-WOJDY (WARSZAWA)	-0.0844898
KOSODRZEWINY (WARSZAWA)	-0.0845108
WOLA KARCZEWSKA (WOLA KARCZEWSKA)	-0.0847169
KAPUTY (KAPUTY)	-0.0848725
BORYSZEWSKA (WIĄZOWNA)	-0.0848829
WŁOŚCIAŃSKA (WARSZAWA)	-0.0851941
GRANICZNA (JEZIÓRKO)	-0.0852101

BRONISZE (BRONISZE)	-0.0853414
SIEDMIOGRODZKA (WARSZAWA)	-0.0853683
MAŁE SIEKIERKI (WARSZAWA)	-0.0857501
PRZYSTANEK (LEGIONOWO)	-0.0861813
SŁONECZNA (KRUPIA WÓLKA)	-0.0864032
LIMANOWSKIEGO (WARSZAWA)	-0.0866623
AKACJOWA (KĄTY WĘGIERSKIE)	-0.0868207
AL.JEDNOŚCI (LESZNOWOLA)	-0.0869255
IWICZNA (PIASECZNO)	-0.0870937
BUKOWIECKA (WARSZAWA)	-0.0871009
OTWOCK (OTWOCK)	-0.0871739
SZCZAKI (SZCZAKI)	-0.0873546
OBJAZDOWA (JAWOROWA)	-0.0876359
RAWAR (WARSZAWA)	-0.087713
PL.TEATRALNY (WARSZAWA)	-0.0877882
ZAJEZDNIA PRAGA (WARSZAWA)	-0.0880152
HABDZIN I (HABDZIN)	-0.0880486
SOBIKÓW (SOBIKÓW)	-0.088131
STRUMYKOWA (PIASECZNO)	-0.088302
KAMPINOSKA (WARSZAWA)	-0.0883729
DURACZA (WARSZAWA)	-0.0886956
CHŁODNIA (WARSZAWA)	-0.0887362
ZAKRZEWSKA (WARSZAWA)	-0.0887469
OBJAZDOWA (JÓZEFÓW k. LEGIONOWA)	-0.0895063
TALAROWA (WARSZAWA)	-0.0895171
HIPOLITOWSKA (KONIK NOWY)	-0.0899259
POGROSZEW KOLONIA (POGROSZEW KOLONIA)	-0.0899617
MAJERANKOWA (WARSZAWA)	-0.0899619
OD LASU (KONSTANCIN-JEZIORNA)	-0.0899698
OLSZEWNICA STARA (OLSZEWNICA STARA)	-0.0900917
BAREI (WARSZAWA)	-0.0903992
MIKOŁAJSKA (WARSZAWA)	-0.0906903
WAWRZYSZEWSKA (WARSZAWA)	-0.0910548
MAJAKOWSKIEGO (WARSZAWA)	-0.0911277
PIOTRKÓWEK MAŁY (PIOTRKÓWEK MAŁY)	-0.0913324
WYNALAZEK (WARSZAWA)	-0.0914677
BORZĘCIN MAŁY (BORZĘCIN MAŁY)	-0.0914714
KOSYNIERÓW (ZĄBKI)	-0.0916795
KOCJANA-BOCZNICA (WARSZAWA)	-0.0918186
BLUE CITY (WARSZAWA)	-0.0920259
KOSMOWSKA I (BORZĘCIN DUŻY)	-0.0921878
ŻYZNA (KAPUTY)	-0.0922655
ZEGRZE PŁD. (ZEGRZE PŁD.)	-0.0923683
ZASOBNA (WARSZAWA)	-0.0924947
BOROWA (OTWOCK)	-0.0925304
URZĄD GMINY (PRAŻMÓW)	-0.092898
WIŚLICKA (WARSZAWA)	-0.0929286
MATUSZEWSKA (WARSZAWA)	-0.0930533
WARSZAWSKA (KONIK STARY)	-0.0932448
TERESPOLSKA (KONIK NOWY)	-0.0932574
ZALESIE GÓRNE (ZALESIE GÓRNE)	-0.0933771
GRZYBOWA (ŁOMIANKI)	-0.0935375
URZĄD GMINY (LESZNOWOLA)	-0.0940074
RADAROWA (WARSZAWA)	-0.0941491
BUCHALTERYJNA (WARSZAWA)	-0.0943271
DWORKOWA (JABŁONNA)	-0.0946061
UMIASTÓW (UMIASTÓW)	-0.0946815
WÓLKA KOSOWSKA (WÓLKA KOSOWSKA)	-0.0949071
URZĄD MIASTA (NOWY DWÓR MAZ.)	-0.0951833
WAŁ GOCŁAWSKI (WARSZAWA)	-0.0952018
BRZEŚCE (BRZEŚCE)	-0.0953428
HANKI ORDONÓWNY (WARSZAWA)	-0.09555
SZAMOTY (SZAMOTY)	-0.0961314
SYRENY (WARSZAWA)	-0.0962485
OCZAPOWSKIEGO (WARSZAWA)	-0.0962725
CHODECKA (WARSZAWA)	-0.096405
STRUMYKOWA (WARSZAWA)	-0.0965111
POW (PRUSZKÓW)	-0.0966932
POD GŁOSKÓW (GŁOSKÓW)	-0.0967344
RATAJA (SULEJÓWEK)	-0.0968409
ORDYNACKA (WARSZAWA)	-0.0970189
WIŚNIOWA (STANISŁAWÓW DRUGI)	-0.0972199
OS.BUKOWY DWOREK (JABŁONNA)	-0.0974996
ROZŁUCKA (WARSZAWA)	-0.0975331
PARTYZANTÓW (PRUSZKÓW)	-0.0975912
PL.ZAMKOWY (WARSZAWA)	-0.0978054
PARK SKARYSZEWSKI (WARSZAWA)	-0.0980408
WARSZAWA ZACHODNIA WKD (WARSZAWA)	-0.0980752
PARK BRÓDNOWSKI (WARSZAWA)	-0.0983488
PRAGA-PŁD.-RATUSZ (WARSZAWA)	-0.0984634
CIOŁKOSZA (WARSZAWA)	-0.0984978
PKP SŁUŻEWIEC (WARSZAWA)	-0.0986627
WŁADYSŁAWÓW (WŁADYSŁAWÓW)	-0.0986692
MOKOTÓW (WARSZAWA)	-0.0987263
PODGÓRNA (KONIK NOWY)	-0.0993444
KOCZARGI NOWE (KOCZARGI NOWE)	-0.0997185
PŁYTOWA (WARSZAWA)	-0.0999502
SZKOLNA (MACIERZYSZ)	-0.0999539
ŁADY-SZKOŁA (DAWIDY BANKOWE)	-0.10023
KOPRKI (KOPRKI)	-0.10048
JANKI (JANKI)	-0.100585
BEŁZY (WARSZAWA)	-0.100608
ŁOJEWSKA (WARSZAWA)	-0.100725
FOKSAL (WARSZAWA)	-0.1013
UMIASTOWSKA (OŻARÓW MAZOWIECKI)	-0.101351
LEŚNY (DZIEKANÓW LEŚNY)	-0.10149
SOSENKOWA (ALEKSANDRÓW)	-0.101821
NATOLIN PŁN. (WARSZAWA)	-0.102005
KOLIBRA (ZGORZAŁA)	-0.102209

GŁOSKÓW-SZKOŁA (GŁOSKÓW)	-0.102237
RKS ORZEŁ (WARSZAWA)	-0.102426
KIERBEDZIA (WARSZAWA)	-0.102677
MOCHTYŃSKA-KANAŁ (WARSZAWA)	-0.103243
AWF (WARSZAWA)	-0.103507
ŁUBINOWA (WARSZAWA)	-0.103763
TRAWIASTA (WARSZAWA)	-0.104242
LIPOWCZANA (WARSZAWA)	-0.104271
KAWĘCZYN II (KAWĘCZYN)	-0.10445
DŁUGA (ŁOMIANKI)	-0.104548
SŁOWICZA (USTANÓW)	-0.104624
NORWIDA (PIASTÓW)	-0.105054
GRANICZNA (SZCZAKI)	-0.105194
WIELKIEGO DĘBU (WARSZAWA)	-0.105496
ŁANOWA (SZCZAKI)	-0.105691
PAŃSKA (JABŁONNA)	-0.105959
KS.SKORUPKI (ZĄBKI)	-0.106017
WILLOWA (NOWY PODOLSZYN)	-0.106125
WARSZAWSKA (BIELAWA)	-0.106551
SIELANKOWA (LEGIONOWO)	-0.106851
TRENÓW-LAS (WARSZAWA)	-0.106936
JAGIELLOŃSKA (WARSZAWA)	-0.107077
WÓLKA (WÓLKA)	-0.107318
PIJARSKA (GÓRA KALWARIA)	-0.107321
OSP ZŁOTOKŁOS (ZŁOTOKŁOS)	-0.107365
ZIELONA (WŁADYSŁAWÓW)	-0.107765
POWĄZKOWSKA (WARSZAWA)	-0.107841
ZARUBY (WARSZAWA)	-0.108071
LIPOWA (NADARZYN)	-0.108153
MARYNIN (WARSZAWA)	-0.108271
BRÓDNOWSKA (WARSZAWA)	-0.108478
KORDIANA (WARSZAWA)	-0.109375
POTULICKA (WARSZAWA)	-0.109578
CHUDOBY (WARSZAWA)	-0.109583
RUNÓW OSADA II (RUNÓW)	-0.110389
KSIĄŻĄT MAZOWIECKICH (PIASECZNO)	-0.110494
JESÓWKA (JESÓWKA)	-0.110588
GERBERA (PIASECZNO)	-0.110879
POMNIK SAPERA (WARSZAWA)	-0.111197
CM.PÓŁNOCNY-BRAMA PŁN. (WARSZAWA)	-0.111568
OS.VICTORIA (RADZYMIN)	-0.111906
RADZYMIŃSKA (ZĄBKI)	-0.112014
PL.TYSIĄCLECIA (WARSZAWA)	-0.112296
SOKOŁOWSKIEGO (LASKI)	-0.112686
OSIEDLE MŁODYCH (WARSZAWA)	-0.112926
KS.MALINOWSKIEGO (JÓZEFÓW)	-0.112931
GRANICZNA (MAGDALENKA)	-0.113086
ZAJEZDNIA WORONICZA (WARSZAWA)	-0.113523
WORONICZA (WARSZAWA)	-0.113523
PRZEPIÓRKI (NOWA WOLA)	-0.113724
GMINY (LESZNOWOLA)	-0.113912
PKP GOCŁAWEK (WARSZAWA)	-0.114083
MOTYLKOWA (WARSZAWA)	-0.114919
AL.SYBIRAKÓW (LEGIONOWO)	-0.115102
WOŁODYJOWSKIEGO (STARE BABICE)	-0.11528
WRZOSOWA (KOCZARGI STARE)	-0.115492
SADOWA (BASZKÓWKA)	-0.11559
STOICKA (WARSZAWA)	-0.115767
KRYNICZNA (WARSZAWA)	-0.116349
ŁAZY-SZKOŁA (ŁAZY)	-0.117254
ZAKAMAREK (WARSZAWA)	-0.117698
METRO NOWY ŚWIAT-UNIWERSYTET (WARSZAWA)	-0.1177
DALIBORA (WARSZAWA)	-0.11791
LATCHORZEW (LATCHORZEW)	-0.118027
JODŁOWA (STRZENIÓWKA)	-0.118051
ROLNA (ŁAZY)	-0.118057
MIELCZARSKIEGO (WARSZAWA)	-0.118102
SIARCZANA (WARSZAWA)	-0.118104
KSIĘCIA ZIEMOWITA (WARSZAWA)	-0.118104
PŁASKOWICKIEJ (WARSZAWA)	-0.118473
FALENTY DUŻE (FALENTY DUŻE)	-0.118564
KANARKOWA (MŁOCHÓW)	-0.11871
ROLNICZA (WARSZAWA)	-0.118817
POLNA (GABRYELIN)	-0.118822
JACHOWICZA (WARSZAWA)	-0.118939
MOŚCICKIEGO (LIPKÓW)	-0.119567
PKP MICHALIN (JÓZEFÓW)	-0.119734
ZAGÓRZE-SANATORIUM (ZAGÓRZE)	-0.120056
OKRZEI (OTWOCK)	-0.120101
CHYLICZKI (CHYLICZKI)	-0.120514
WASILKOWSKA (WARSZAWA)	-0.120526
BARTOSZYCKA (WARSZAWA)	-0.120526
RAFII (WARSZAWA)	-0.120866
SIERAKOWSKIEGO (WARSZAWA)	-0.120954
GÓRKI (JÓZEFÓW)	-0.120959
STEFANIKA (WARSZAWA)	-0.12142
WOLICA (WOLICA)	-0.121425
MANDARYNKI (WARSZAWA)	-0.121553
HENRYKÓW-UROCZE (HENRYKÓW-UROCZE)	-0.122056
PĘCLIŃSKA (WIĄZOWNA KOŚCIELNA)	-0.122306
WIELISZEW-PLAŻA (WIELISZEW)	-0.122486
KAKOWSKIEGO (WARSZAWA)	-0.122702
SMOLUCHOWSKIEGO (WARSZAWA)	-0.122793
RZAKTA (RZAKTA)	-0.122831
GEODEZYJNA (WARSZAWA)	-0.122916
KONDUKTORSKA (WARSZAWA)	-0.122948
LATOSZKI (WARSZAWA)	-0.123092
ROGOWSKA (WARSZAWA)	-0.123173
BRAZYLIJSKA (WARSZAWA)	-0.123672

DĘBOWA (UWIELINY)	-0.123736
SZWOLEŻERÓW (ZĄBKI)	-0.123778
ROZCHODNIKOWA (WARSZAWA)	-0.123818
OBROŃCÓW TOBRUKU (WARSZAWA)	-0.123854
DZIELNA (WARSZAWA)	-0.123879
KAWĘCZYNEK II (KAWĘCZYNEK)	-0.123926
IDZIKOWSKIEGO (SULEJÓWEK)	-0.124316
KAJETANY (KAJETANY)	-0.12442
WARNEŃSKA (WARSZAWA)	-0.1245
WĄWOZOWA (WARSZAWA)	-0.124654
MORSKIE OKO (WARSZAWA)	-0.124705
NISKA (WARSZAWA)	-0.124744
RYBAŁTÓW (WARSZAWA)	-0.125131
GAJOWA (WOLA ALEKSANDRA)	-0.125132
CM.PÓŁNOCNY-BRAMA PŁD. (WARSZAWA)	-0.125242
ZAOLZIAŃSKA (WARSZAWA)	-0.125413
KOŃSKI JAR (WARSZAWA)	-0.125413
GRZEGORZEWSKIEJ (WARSZAWA)	-0.125445
SREBRNYCH ŚWIERKÓW (PIASECZNO)	-0.126261
DOJAZDOWA (WARSZAWA)	-0.126608
WOLICA K.PŁOCHOCINA (WOLICA k. PŁOCHOCINA)	-0.126713
WDW RYNIA (RYNIA)	-0.126964
NOWAKA-JEZIORAŃSKIEGO (WARSZAWA)	-0.127201
KRASICKIEGO (NOWA WOLA)	-0.127227
KAROLIN (WARSZAWA)	-0.127233
GNIEWKOWSKA (WARSZAWA)	-0.127447
PIĘCIOLINII (WARSZAWA)	-0.127547
MACZKA (ZĄBKI)	-0.127678
PODŁĘCZE II (PODŁĘCZE)	-0.127709
KRZYŻANOWSKIEGO (WARSZAWA)	-0.127782
CZARNA STRUGA (MARKI)	-0.127803
WKD OPACZ (MICHAŁOWICE)	-0.127856
CHYLICE (CHYLICE)	-0.128317
PODLEŚNA (JÓZEFÓW)	-0.12832
REZEDOWA (WARSZAWA)	-0.128491
ZBARASKA (WARSZAWA)	-0.128656
TKACZY (WARSZAWA)	-0.128983
CHODAKOWSKA (WARSZAWA)	-0.129399
TEOFILÓW (WOLA DUCKA)	-0.129464
AUTOKAROWY (WARSZAWA)	-0.130082
PODCZASZYŃSKIEGO (WARSZAWA)	-0.130245
BATALIONU PLATERÓWEK (WARSZAWA)	-0.130428
STARA (NADMA)	-0.130527
GRUSZCZYŃSKIEGO (WARSZAWA)	-0.130614
PKP OŻARÓW MAZOWIECKI (OŻARÓW MAZOWIECKI)	-0.130741
LESZNO (LESZNO)	-0.131146
GRANICZKA (KIEŁPIN STARY)	-0.131389
ODKRYTA (WARSZAWA)	-0.131434
OGÓRKOWA (WARSZAWA)	-0.131466
P+R AL.KRAKOWSKA (WARSZAWA)	-0.13195
CYNAMONOWA (WARSZAWA)	-0.132173
SPACEROWA (DUCHNÓW)	-0.132251
OS.NIEPODLEGŁOŚCI (WOŁOMIN)	-0.13242
BUCHNIK (JABŁONNA)	-0.132687
HALA MIROWSKA (WARSZAWA)	-0.132796
CHOPINA (WARSZAWA)	-0.132806
ANDERSENA (WARSZAWA)	-0.132809
HOŻA (WARSZAWA)	-0.133396
ZAJEZDNIA UTRATA (WARSZAWA)	-0.133428
NIK (WARSZAWA)	-0.133458
KOPCIŃSKIEGO (WARSZAWA)	-0.133551
RZEMIEŚLNICZA (OTWOCK)	-0.13363
KLASZTORNA (WARSZAWA)	-0.133892
POSAG 7 PANIEN (WARSZAWA)	-0.134079
TOR SŁUŻEWIEC (WARSZAWA)	-0.134273
GALLA ANONIMA (WARSZAWA)	-0.134282
SPÓŁDZIELNIA (DAWIDY)	-0.134628
URZĄD SKARBOWY (PRUSZKÓW)	-0.134633
BLIZNE JASIŃSKIEGO (BLIZNE JASIŃSKIEGO)	-0.134889
KOSTRZEWSKIEGO (WARSZAWA)	-0.135025
SPISAKA (WARSZAWA)	-0.135372
DROBIARSKA (WARSZAWA)	-0.135415
MIASTA (PIASECZNO)	-0.135681
SPÓŁDZIELNIA NOMA (WARSZAWA)	-0.136125
SZOLC-ROGOZIŃSKIEGO (WARSZAWA)	-0.136214
DĘBÓWKA (DĘBÓWKA)	-0.136318
URSYNÓW ZACH. (WARSZAWA)	-0.136383
MAŁEJ ŁĄKI (WARSZAWA)	-0.13704
TORUŃSKA (PIASTÓW)	-0.137294
SZPITAL GROCHOWSKI (WARSZAWA)	-0.13732
NIEŚWIESKA (WARSZAWA)	-0.137438
BARSZCZEWSKA (WARSZAWA)	-0.137977
WILANOWSKA (WARSZAWA)	-0.138165
LEŚNA (JABŁONNA)	-0.139038
KS POLONEZ (WARSZAWA)	-0.139135
ŁUCZNICZA (WARSZAWA)	-0.139574
PIUSA XI (CHOTOMÓW)	-0.139668
NOWODWORSKA (ŁAJSKI)	-0.139944
WIŚLANA (WARSZAWA)	-0.140278
OKRĄGŁA (WARSZAWA)	-0.140664
KADETÓW-KOŚCIÓŁ (WARSZAWA)	-0.140759
CZUMY (WARSZAWA)	-0.14083
BOHUSZEWICZÓWNY (WARSZAWA)	-0.141056
FABRYCZNA (MARKI)	-0.141635
STRUSIA (WARSZAWA)	-0.142618
NOWA (STARA IWICZNA)	-0.142788
ŁYCZYŃSKA (SŁOMCZYN)	-0.143148
CH MARKI (MARKI)	-0.143643

SOSNOWA (PODDĘBIE)	-0.143837
KAWĘCZYN DOLNY (KAWĘCZYN)	-0.144029
KIEPURY (WARSZAWA)	-0.144169
ZAJEZDNIA OSTROBRAMSKA (WARSZAWA)	-0.144461
TURKAWKI (JASTRZĘBIE)	-0.144858
OBORSKA (KONSTANCIN-JEZIORNA)	-0.144861
ARMATNIA (WARSZAWA)	-0.144909
STARA MIŁOSNA (WARSZAWA)	-0.144921
ESKULAPÓW (WARSZAWA)	-0.145344
PICASSA (WARSZAWA)	-0.145383
ŚWIERK (OTWOCK)	-0.145445
JEZIOROWA (WARSZAWA)	-0.145572
KAMERALNA (JÓZEFOSŁAW)	-0.145861
MARIEW (MARIEW)	-0.146477
POMORSKA (WARSZAWA)	-0.146517
OBORY-PAŁAC (OBORY)	-0.146787
WIRAŻOWA (WARSZAWA)	-0.146845
EKSPRESOWA (WARSZAWA)	-0.147417
JESIENNYCH LIŚCI (WARSZAWA)	-0.147504
TAGORE'A (WARSZAWA)	-0.147602
OS.ISKRA (WARSZAWA)	-0.147673
WYCZÓŁKI (WARSZAWA)	-0.148251
KONARSKIEGO (WARSZAWA)	-0.148324
MUZEALNA (WARSZAWA)	-0.148693
RACJONALIZACJI (WARSZAWA)	-0.149439
POSTĘPU (JASTRZĘBIEC)	-0.149533
RZESZOTARSKIEJ (WARSZAWA)	-0.149835
WANILIOWA (PIASECZNO)	-0.150161
JASNA (CHOTOMÓW)	-0.150865
KONSTRUKTORSKA (WARSZAWA)	-0.151014
ZAWODOWA (WARSZAWA)	-0.151382
SKALNICOWA (WARSZAWA)	-0.15189
SYNÓW PUŁKU (WARSZAWA)	-0.152092
SCHROEGERA (WARSZAWA)	-0.152496
JESIONOWA (JESIONOWA)	-0.152506
LUTOSŁAWSKIEGO (KLAUDYN)	-0.152519
CIEĆWIERZA (KLAUDYN)	-0.152519
ŁĄCZYNY (WARSZAWA)	-0.152675
OCHOTA-RATUSZ (WARSZAWA)	-0.15323
AGRICOOP (KOPRKI)	-0.153779
LESZCZYŃSKIEGO (WARSZAWA)	-0.153869
DZIKIEJ RÓŻY (BORZĘCIN MAŁY)	-0.153909
KWIRYNÓW (KWIRYNÓW)	-0.154323
BOGATKI (WARSZAWA)	-0.154804
FRANCUSKA (WARSZAWA)	-0.155079
MASZYNOWA (WARSZAWA)	-0.155313
CHMIELNA (WARSZAWA)	-0.155839
SZOBERA (WARSZAWA)	-0.155906
CHOMICZÓWKA (WARSZAWA)	-0.155965
LOTNISKO CHOPINA-PRZYLOTY (WARSZAWA)	-0.156503
GRUNTOWA (WARSZAWA)	-0.156861
ZDZIARSKA (WARSZAWA)	-0.157199
MARKOWSKA (WARSZAWA)	-0.157317
ZWYCIĘZCÓW (WARSZAWA)	-0.15792
PL.SZWEDZKI (JANKI)	-0.158017
AL.WIELKOPOLSKI (WARSZAWA)	-0.158018
KOŁŁĄTAJA (ZĄBKI)	-0.158062
ADAMPOLSKA (WARSZAWA)	-0.158098
DROHICKA (WARSZAWA)	-0.158172
KLUB LIRA (WARSZAWA)	-0.158671
SKIBIŃSKIEGO (TRUSKAW)	-0.159088
KUPIECKA (WARSZAWA)	-0.159161
RECHNIEWSKIEGO (WARSZAWA)	-0.159258
RYNEK (GÓRA KALWARIA)	-0.1601
ROGATKOWA (WARSZAWA)	-0.160353
TUZINOWA (WARSZAWA)	-0.160516
SŁUŻEWIEC (WARSZAWA)	-0.16101
WOJNICKA (WARSZAWA)	-0.161447
GAJOWA (ZĄBKI)	-0.161554
WIEJSKA (WARSZAWA)	-0.161811
MALINOWSKIEGO (WARSZAWA)	-0.16193
TOŁSTOJA (WARSZAWA)	-0.162336
WARCHAŁOWSKIEGO (WARSZAWA)	-0.162625
CYKLAMENÓW (WARSZAWA)	-0.163312
KAMIŃSKIEGO (WARSZAWA)	-0.163567
TANGO (WARSZAWA)	-0.163621
ŁOMIANKI-CENTRUM (ŁOMIANKI)	-0.164643
ZGK (KONSTANCIN-JEZIORNA)	-0.164831
STĘPIŃSKA (WARSZAWA)	-0.164842
PKP NOWA IWICZNA (NOWA IWICZNA)	-0.165095
LESZNO (WARSZAWA)	-0.165107
ROSOCHATA (WARSZAWA)	-0.165155
KRASICKIEGO (WARSZAWA)	-0.16595
BUDEXPO (WARSZAWA)	-0.166513
WRZECIONO (WARSZAWA)	-0.166574
LAS BIELAŃSKI (WARSZAWA)	-0.16677
KUPIECKA (OTWOCK)	-0.16693
BANACHA (WARSZAWA)	-0.167148
EC SIEKIERKI (WARSZAWA)	-0.167616
CZEKOLADOWA (WARSZAWA)	-0.167767
DOBOSZA (WARSZAWA)	-0.167891
WIRAŻOWA (WÓLKA RADZYMIŃSKA)	-0.168638
NOCZNICKIEGO (WARSZAWA)	-0.169188
WITEBSKA (WARSZAWA)	-0.169388
PARK MŁOCIŃSKI (WARSZAWA)	-0.170189
ORDONA (WARSZAWA)	-0.17022
BIAŁOWIEJSKA (WARSZAWA)	-0.170371
CZERNIAKOWSKA (WARSZAWA)	-0.170432
KANAŁ ŻERAŃSKI (WARSZAWA)	-0.17049
GLEBOWA (WARSZAWA)	-0.170659

ANTKA ROZPYLACZA (WARSZAWA)	-0.17129
MONTE CASSINO (WARSZAWA)	-0.171858
AGRYKOLA (WARSZAWA)	-0.172494
ORLĄT LWOWSKICH (WARSZAWA)	-0.17255
WIEJSKA (WOŁOMIN)	-0.172783
ŁAGIEWNICKA (WARSZAWA)	-0.17294
KACZEŃCÓW (ŁAZY)	-0.173086
PL.KOŚCIUSZKI (LEGIONOWO)	-0.173111
MONTAŻOWA (WARSZAWA)	-0.173153
STOKROTKI (NIEPORĘT)	-0.173316
PL.POLITECHNIKI (WARSZAWA)	-0.173509
GILARSKA (WARSZAWA)	-0.173959
MARYMONT (WARSZAWA)	-0.174091
SIEMIEŃSKIEGO (WARSZAWA)	-0.17432
BATALIONU WŁOCHY (WARSZAWA)	-0.174691
LAWINOWA (WARSZAWA)	-0.175398
MŁOCINY-UKSW (WARSZAWA)	-0.17545
BUKOWIŃSKA (WARSZAWA)	-0.175523
TARGÓWEK (WARSZAWA)	-0.175571
RADOŚĆ-WODOCIĄG (WARSZAWA)	-0.175882
SALOMEA (WARSZAWA)	-0.176016
WARSZAWSKIEJ 1920 (WARSZAWA)	-0.176749
PRZASNYSKA (WARSZAWA)	-0.176763
AKADEMIJNA (JABŁONNA)	-0.176799
ZBÓJNOGÓRSKA (WARSZAWA)	-0.176964
SIERAKÓW (SIERAKÓW)	-0.177458
DĄBKOWIZNA (DĄBKOWIZNA)	-0.178166
ALEJA RÓŻ (LEGIONOWO)	-0.179804
LINDLEYA (WARSZAWA)	-0.179935
WIDAWSKA (WARSZAWA)	-0.180098
SIODLARSKA (WARSZAWA)	-0.180818
PUCHAŁY (FALENTY)	-0.180957
HUSARSKA (LEGIONOWO)	-0.180969
STADION NARODOWY (WARSZAWA)	-0.181143
SUZINA (WARSZAWA)	-0.181978
CHOTOMÓW (CHOTOMÓW)	-0.183283
ROKOSOWSKA (WARSZAWA)	-0.18347
SZPITAL BIELAŃSKI (WARSZAWA)	-0.18431
HALE BANACHA (WARSZAWA)	-0.184503
GMINY (WIĄZOWNA)	-0.184699
LALKI (WARSZAWA)	-0.184837
SZARYCH SZEREGÓW (LEGIONOWO)	-0.185145
ŻOLIBORZ (WARSZAWA)	-0.185418
KIELECKA (WARSZAWA)	-0.185448
WAŁBRZYSKA-CMENTARZ (WARSZAWA)	-0.185503
OGIŃSKIEGO (PIASTÓW)	-0.185671
NUGAT (WARSZAWA)	-0.186297
RATUSZOWA (WARSZAWA)	-0.186437
STADIONOWA (RASZYN)	-0.187433
WAGONOWNIA (WARSZAWA)	-0.18765
PUŁASKIEGO (ZĄBKI)	-0.187921
KRZYŻY (WARSZAWA)	-0.188086
(KIJOWSKA) (WARSZAWA)	-0.188142
LEŻAJSKA (WARSZAWA)	-0.18863
NOAKOWSKIEGO (WARSZAWA)	-0.189245
ZARUSKIEGO (WARSZAWA)	-0.189331
KSIĘCIA BOLESŁAWA (WARSZAWA)	-0.189626
RUDNICKIEGO (WARSZAWA)	-0.189792
GŁ. (WARSZAWA)	-0.190879
TOR KOLARSKI (WARSZAWA)	-0.191361
LASEK NA KOLE (WARSZAWA)	-0.191435
UNIWERSYTET MEDYCZNY (WARSZAWA)	-0.191521
OGRODY DZIAŁKOWE PIASKI (WARSZAWA)	-0.191843
NARCYZOWA (WARSZAWA)	-0.192546
NIEDŹWIEDZIA (WARSZAWA)	-0.19278
AL.SOLIDARNOŚCI (WARSZAWA)	-0.19284
ŚWIDERSKA (WARSZAWA)	-0.193354
CM.BRÓDNOWSKI (WARSZAWA)	-0.194508
MAŁEGO RYCERZA (WARSZAWA)	-0.194898
KLASYKÓW (WARSZAWA)	-0.19518
HABDZIN III (HABDZIN)	-0.197579
HABDZIN II (HABDZIN)	-0.197579
UTRATA (WARSZAWA)	-0.197682
HALINÓW (HIPOLITÓW)	-0.198208
NARWIK (WARSZAWA)	-0.198727
JESIONOWA (OPACZ-KOLONIA)	-0.198799
JASIELSKA (WARSZAWA)	-0.199506
TADŻYCKA (WARSZAWA)	-0.200568
KS POLONIA (WARSZAWA)	-0.20094
WITOLIN (WARSZAWA)	-0.201384
TRAKT LUBELSKI (WARSZAWA)	-0.2028
HASSA (WARSZAWA)	-0.202855
SPORTOWA (RASZYN)	-0.20287
PŁUŻAŃSKA (WARSZAWA)	-0.203037
GOŚCIERADOWSKA (WARSZAWA)	-0.20318
OSMAŃCZYKA (WARSZAWA)	-0.203225
HORNÓWEK (HORNÓWEK)	-0.203485
PŁD. (ANTONINÓW)	-0.204612
OPACZEWSKA (WARSZAWA)	-0.205572
CHŁODNA (WARSZAWA)	-0.206455
KULTURY (PIASECZNO)	-0.20658
BOLESŁAWICKA (WARSZAWA)	-0.206586
REDUTA (WARSZAWA)	-0.207056
ZABŁUDOWSKA (WARSZAWA)	-0.209299
SZPITAL MSW (WARSZAWA)	-0.210462
SOWIŃSKIEGO (WARSZAWA)	-0.210881
OS.MARYWILSKA (WARSZAWA)	-0.211165
OS.MŁODYCH (LEGIONOWO)	-0.212816
PORT PILAWA (BIAŁOBRZEGI)	-0.213112

NIEMCEWICZA (WARSZAWA)	-0.214765
DARWINA (WARSZAWA)	-0.21484
WIELICKA (WARSZAWA)	-0.215332
BRZEZIŃSKA (WARSZAWA)	-0.216618
ZAŚCIANKOWA (WARSZAWA)	-0.216819
RODŁA (WARSZAWA)	-0.216938
NOWE BEMOWO (WARSZAWA)	-0.216938
KAROLKOWA (WARSZAWA)	-0.217012
PARCELACYJNA (WARSZAWA)	-0.217155
SZYMANOWSKIEGO (WARSZAWA)	-0.217264
CMENTARZ (LEGIONOWO)	-0.21733
PODOLSZYN (PODOLSZYN)	-0.21882
MADALIŃSKIEGO-SZPITAL (WARSZAWA)	-0.219021
OSMAŃSKA (WARSZAWA)	-0.219518
OS.WILGA (WARSZAWA)	-0.220133
OS.STASZICA (PRUSZKÓW)	-0.220565
WARZELNICZA (WARSZAWA)	-0.221088
DIAMENTOWA (WARSZAWA)	-0.221622
KOMPUTEROWA (WARSZAWA)	-0.223213
PKP ZALESIE GÓRNE (ZALESIE GÓRNE)	-0.223434
BIELAWA I (BIELAWA)	-0.223592
MIASTA (OTWOCK)	-0.224639
KRÓLEWSKA (JABŁONNA)	-0.225328
MELCERA (WARSZAWA)	-0.225731
BAJECZNA (BOBROWIEC)	-0.227956
(TUNELOWA) (WARSZAWA)	-0.228278
OS.ORLIK (WARSZAWA)	-0.228635
OKRZEWSKA-CMENTARZ (KĘPA OKRZEWSKA)	-0.228639
URZĄD GMINY (WIELISZEW)	-0.230092
PKP PRAGA (WARSZAWA)	-0.23145
RADIOWA-WAT (WARSZAWA)	-0.23194
KAPITULNA (WARSZAWA)	-0.233022
INSTYTUT REUMATOLOGII (WARSZAWA)	-0.233383
SPARTAŃSKA (WARSZAWA)	-0.233383
STOCER (KONSTANCIN-JEZIORNA)	-0.233519
CZYNSZOWA (WARSZAWA)	-0.235028
SEZAM (WARSZAWA)	-0.236412
SOCHACZEWSKA (WARSZAWA)	-0.236741
ZIELONA (WARSZAWA)	-0.236896
BOTANICZNY (WARSZAWA)	-0.237123
GOLĘDZINÓW (WARSZAWA)	-0.238497
RAABEGO (WARSZAWA)	-0.239054
WAPOWSKIEGO (WARSZAWA)	-0.239653
REMBERTÓW-KOLONIA (WARSZAWA)	-0.242247
WIŚNIOWA GÓRA (WARSZAWA)	-0.242628
GASSY-PĘTLA (GASSY)	-0.242985
PRZYDROŻNA (WARSZAWA)	-0.243481
WESOŁA (WARSZAWA)	-0.244733
MIĘDZYLESIE (WARSZAWA)	-0.246399
MOŚCISKA (WARSZAWA)	-0.246569
ZACISZE (WARSZAWA)	-0.246839
MIŁOSNA (WARSZAWA)	-0.247954
CZAPELSKA (WARSZAWA)	-0.24797
WOŁOMIŃSKA (WARSZAWA)	-0.248763
VOGLA (WARSZAWA)	-0.249644
BOHUNA (WARSZAWA)	-0.2498
MIŁA (WARSZAWA)	-0.252234
BŁĘKITNE (LEGIONOWO)	-0.253347
ŻERAŃ WSCHODNI (WARSZAWA)	-0.25417
STOCZNIOWCÓW (WARSZAWA)	-0.254335
ARESZT ŚLEDCZY GROCHÓW (WARSZAWA)	-0.254572
CENTRUM TARGOWE (WARSZAWA)	-0.2547
MPT GROCHÓW (WARSZAWA)	-0.255701
OLESIN (WARSZAWA)	-0.256393
PKP OLSZYNKA GROCHOWSKA (WARSZAWA)	-0.256733
ŚWIĄTYNIA OPATRZNOŚCI BOŻEJ (WARSZAWA)	-0.25704
WAWRZYSZEW (WARSZAWA)	-0.257142
MEISSNERA (WARSZAWA)	-0.261305
STAREGO DĘBU (CZARNÓW)	-0.263081
DWORSKA (WARSZAWA)	-0.263867
JAZGARZEW-SZKOŁA (JAZGARZEW)	-0.264677
GWIAŹDZISTA (WARSZAWA)	-0.265719
CZERWONYCH BERETÓW (WARSZAWA)	-0.265972
(RYDYGIERA) (WARSZAWA)	-0.266912
KARTEZJUSZA (WARSZAWA)	-0.267545
ZWOLEŃSKA (WARSZAWA)	-0.267602
ŚW.A.BOBOLI (WARSZAWA)	-0.2726
OS.DERBY (WARSZAWA)	-0.272815
ZAJEZDNIA ŻOLIBORZ (WARSZAWA)	-0.273538
ZBÓJNA GÓRA (WARSZAWA)	-0.274231
REGULSKA (WARSZAWA)	-0.275359
METRO NATOLIN (WARSZAWA)	-0.275631
OS.KABATY (WARSZAWA)	-0.278554
BELLOTTIEGO (WARSZAWA)	-0.279152
TRASA ŁAZIENKOWSKA (WARSZAWA)	-0.279271
FRACHTOWA (WARSZAWA)	-0.280304
SIEKIERKI (WARSZAWA)	-0.280781
LNIANA (WARSZAWA)	-0.284108
(LUBELSKA) (WARSZAWA)	-0.285202
GOŁKOWSKA (WARSZAWA)	-0.288205
BUKOWSKIEGO (WARSZAWA)	-0.289226
ROBOTNICZA (OŻARÓW MAZOWIECKI)	-0.289846
MOKRY ŁUG-LEŚNICZÓWKA (WARSZAWA)	-0.291341
EMILII PLATER (WARSZAWA)	-0.291829
MAGELLANA (WARSZAWA)	-0.293122
WARTKA (WARSZAWA)	-0.294402
KS.SZULCZYKA (WARSZAWA)	-0.296833
GRANICZNA (MARKI)	-0.298183
NA BATERYJCE (WARSZAWA)	-0.299645

GRABINA (REMBELSZCZYZNA)	-0.300601
WSCHODNI (WARSZAWA)	-0.304654
WARSZAWSKA (WARSZAWA)	-0.307765
PŁN. (WARSZAWA)	-0.30907
BŁOTA (WARSZAWA)	-0.310297
SGGW-KLINIKA (WARSZAWA)	-0.315364
ZAPLECZE (WARSZAWA)	-0.317236
STĘŻYCKA (WARSZAWA)	-0.320111
SOŚNKOWSKIEGO (ZĄBKI)	-0.322057
KAFLOWA (WARSZAWA)	-0.322632
DOBKA Z OLEŚNICY (WARSZAWA)	-0.323719
OGRODY DZIAŁKOWE NA SKRAJU (WARSZAWA)	-0.324323
GEN.KUTRZEBY (STARE BABICE)	-0.327131
DANKOWICKA (WARSZAWA)	-0.327715
DANISZEWSKA (WARSZAWA)	-0.329429
ZAWODZIE (WARSZAWA)	-0.329718
JEZIORKI (WARSZAWA)	-0.33528
KAPELI (WARSZAWA)	-0.337743
LIPOWA (BIELAWA)	-0.339168
PL.UNII LUBELSKIEJ (WARSZAWA)	-0.341778
WAWERSKA (WARSZAWA)	-0.342967
BORYNY (DAWIDY BANKOWE)	-0.345862
REYMONTA (LATCHORZEW)	-0.347483
INFLANCKA (WARSZAWA)	-0.348271
BOKSERSKA (WARSZAWA)	-0.353463
SZASERÓW-SZPITAL (WARSZAWA)	-0.35479
PRZEBIŚNIEGÓW (NOWA IWICZNA)	-0.355035
HUBALA-DOBRZAŃSKIEGO (WARSZAWA)	-0.356652
EC KAWĘCZYN (WARSZAWA)	-0.357223
PLANTY (IZABELIN)	-0.361754
KĄTY GRODZISKIE-OSIEDLE (WARSZAWA)	-0.363156
DROGOWA (WARSZAWA)	-0.363182
BOCIANA CZARNEGO (JASTRZĘBIE)	-0.363398
WAŁUSZEWSKA (WARSZAWA)	-0.363733
SMUGOWA (WARSZAWA)	-0.368163
OS.BUKOWIEC (LEGIONOWO)	-0.370742
REMBERTÓW (WARSZAWA)	-0.371585
POD WILANÓWKA (WARSZAWA)	-0.373207
BUŁAWY (WARSZAWA)	-0.374484
NOWOKABACKA (WARSZAWA)	-0.376763
ANANASOWA (WARSZAWA)	-0.377503
BOROWINA II (BOROWINA)	-0.379159
OS.LEWANDÓW II (WARSZAWA)	-0.380565
PAŁACOWA (WARSZAWA)	-0.381455
OPOLSKA (STARE ZAŁUBICE)	-0.381713
JONTKA (WARSZAWA)	-0.382137
DOBRA (WARSZAWA)	-0.382673
JURANDA ZE SPYCHOWA-SZKOŁA (WARSZAWA)	-0.386876
PILECKIEGO (WARSZAWA)	-0.396237
LISTONOSZY (WARSZAWA)	-0.396532
SZASERÓW (WARSZAWA)	-0.396717
HORTENSJI (WARSZAWA)	-0.398172
CM.PÓŁNOCNY-BRAMA ZACH. (WARSZAWA)	-0.400618
GŁOWACKIEGO (KONSTANCIN-JEZIORNA)	-0.401723
KALORYCZNA (WARSZAWA)	-0.40456
IZBICKA-CMENTARZ (WARSZAWA)	-0.415437
ZAGÓRZAŃSKA (WARSZAWA)	-0.421305
SGGW-BIBLIOTEKA (WARSZAWA)	-0.425385
HOSERÓW (WARSZAWA)	-0.428709
INSTALATORÓW (WARSZAWA)	-0.440465
HENRYKA PROBUSA (WARSZAWA)	-0.441554
ZABRANIECKA-BOCZNICA (WARSZAWA)	-0.457282
PALUCH (WARSZAWA)	-0.468058
PKP ZACISZE-WILNO (WARSZAWA)	-0.475919
PODKOWY (WARSZAWA)	-0.480152
TARCHOMIN KOŚCIELNY (WARSZAWA)	-0.556165

Table C.2: Warsaw public transportation network stop rankings according to Shapley value harmonic closeness.

Station	Ranking
METRO ŚWIĘTOKRZYSKA (WARSZAWA)	235.527
METRO RATUSZ ARSENAŁ (WARSZAWA)	234.738
RONDO STARZYŃSKIEGO (WARSZAWA)	234.25
DW.WILEŃSKI (WARSZAWA)	234.203
CENTRUM (WARSZAWA)	234.174
DW.GDAŃSKI (WARSZAWA)	231.751
PL.BANKOWY (WARSZAWA)	227.934
METRO STADION NARODOWY (WARSZAWA)	226.945
MUZEUM NARODOWE (WARSZAWA)	224.877
KINO FEMINA (WARSZAWA)	224.776
METRO POLITECHNIKA (WARSZAWA)	223.949
SASKA (WARSZAWA)	223.009
RONDO WASZYNGTONA (WARSZAWA)	221.486
RONDO ONZ (WARSZAWA)	221.174
ROZBRAT (WARSZAWA)	220.503
DW.CENTRALNY (WARSZAWA)	220.408
KRUCZA (WARSZAWA)	220.226
MURANOWSKA (WARSZAWA)	218.667
KRYNICZNA (WARSZAWA)	218.161
PL.NA ROZDROŻU (WARSZAWA)	218.123
METRO CENTRUM NAUKI KOPERNIK (WARSZAWA)	217.647
CIOŁKA (WARSZAWA)	217.172
ŻERAŃ FSO (WARSZAWA)	215.402
ŚLIWICE (WARSZAWA)	215.222
AFRYKAŃSKA (WARSZAWA)	214.665
RONDO DASZYŃSKIEGO (WARSZAWA)	214.408
MOST PONIATOWSKIEGO (WARSZAWA)	214.176
PARK PRASKI (WARSZAWA)	214.159
PL.WILSONA (WARSZAWA)	213.992
PL.KONSTYTUCJI (WARSZAWA)	213.356
STARE MIASTO (WARSZAWA)	213.297
PL.HALLERA (WARSZAWA)	213.025
METRO NOWY ŚWIAT-UNIWERSYTET (WARSZAWA)	212.963
PL.UNII LUBELSKIEJ (WARSZAWA)	212.454
PL.ZAWISZY (WARSZAWA)	212.438
EMILII PLATER (WARSZAWA)	212.381
OS.WOLSKA (WARSZAWA)	212.148
AL.ZIELENIECKA (WARSZAWA)	211.71
MURANÓW (WARSZAWA)	211.545
KIJOWSKA (WARSZAWA)	210.605
PIOTRA SKARGI (WARSZAWA)	210.269
KRÓLEWSKA (WARSZAWA)	209.911
PARK KASKADA (WARSZAWA)	209.624
NOWY ŚWIAT (WARSZAWA)	209.082
GUS (WARSZAWA)	208.262
MARSZAŁKOWSKA (WARSZAWA)	208.221
PKP KASPRZAKA (WARSZAWA)	208.129
DOLNA (WARSZAWA)	207.599
HALA MIROWSKA (WARSZAWA)	206.923
RONDO ŻABA (WARSZAWA)	206.796
SPACEROWA (WARSZAWA)	206.733
METRO MARYMONT (WARSZAWA)	206.38
PL.TRZECH KRZYŻY (WARSZAWA)	206.218
PL.MAŁACHOWSKIEGO (WARSZAWA)	205.942
PIMOT (WARSZAWA)	205.692
SZWEDZKA (WARSZAWA)	205.393
MOST ŚWIĘTOKRZYSKI (WARSZAWA)	205.386
METRO RACŁAWICKA (WARSZAWA)	205.27
SZPITAL WOLSKI (WARSZAWA)	205.092
JESIENNYCH LIŚCI (WARSZAWA)	204.379
ZACHĘTA (WARSZAWA)	204.242
TOPIEL (WARSZAWA)	203.985
TORWAR (WARSZAWA)	203.929
WOLA-RATUSZ (WARSZAWA)	203.823
ŚWIĘTOJERSKA (WARSZAWA)	203.809
DW.ZACHODNI (WARSZAWA)	203.803
ZĄBKOWSKA (WARSZAWA)	203.772
UNIWERSYTET (WARSZAWA)	203.728
PRZYCZÓŁEK GROCHOWSKI (WARSZAWA)	203.643
PL.TEATRALNY (WARSZAWA)	203.548
SIERAKOWSKIEGO (WARSZAWA)	203.478
PL.GRZYBOWSKI (WARSZAWA)	203.407
ZAMENHOFA (WARSZAWA)	202.909
PARK TRAUGUTTA (WARSZAWA)	202.836
PKP KOŁO (WARSZAWA)	202.816
POMNIK LOTNIKA (WARSZAWA)	202.792
STADION NARODOWY (WARSZAWA)	202.711
PARK OLSZYNA (WARSZAWA)	202.561
PKP POWIŚLE (WARSZAWA)	202.264
NAMYSŁOWSKA (WARSZAWA)	202.241
PL.KRASIŃSKICH (WARSZAWA)	202.224
PL.ZBAWICIELA (WARSZAWA)	202.074
ATEŃSKA (WARSZAWA)	202.001
BASENY INFLANCKA (WARSZAWA)	201.946
NORBLIN (WARSZAWA)	201.88
HOŻA (WARSZAWA)	201.626
NOWY ZJAZD (WARSZAWA)	201.579
INŻYNIERSKA (WARSZAWA)	201.487
WYBRZEŻE HELSKIE (WARSZAWA)	201.443
KS POLONIA (WARSZAWA)	201.253
WARNEŃSKA (WARSZAWA)	200.595
NOWOWIEJSKA (WARSZAWA)	200.348
OBOZOWA (WARSZAWA)	200.339
GEN.ZAJĄCZKA (WARSZAWA)	200.055
PL.STARYNKIEWICZA (WARSZAWA)	200.018
GOLĘDZINÓW (WARSZAWA)	199.76
OS.PÓTOK (WARSZAWA)	199.384
DS RIVIERA (WARSZAWA)	199.334
MARKOWSKA (WARSZAWA)	199.314

MOKOTOWSKA (WARSZAWA)	199.219
RATUSZOWA-ZOO (WARSZAWA)	199.064
ANIELEWICZA (WARSZAWA)	199.026
RZESZOTARSKIEJ (WARSZAWA)	198.898
WSPÓLNA (WARSZAWA)	198.754
WYBRZEŻE SZCZECIŃSKIE (WARSZAWA)	198.504
FRANCISZKAŃSKA (WARSZAWA)	198.432
PIĘKNA (WARSZAWA)	198.265
RATUSZOWA (WARSZAWA)	198.231
STACJA KRWIODAWSTWA (WARSZAWA)	198.014
BEMOWO-RATUSZ (WARSZAWA)	197.682
DZIKA (WARSZAWA)	197.661
PŁOCKA-SZPITAL (WARSZAWA)	197.575
KSIĄŻĘCA (WARSZAWA)	197.039
KONWALIOWA (WARSZAWA)	197.022
MIĘDZYNARODOWA (WARSZAWA)	196.981
ŚNIEGOCKIEJ (WARSZAWA)	196.74
PL.NARUTOWICZA (WARSZAWA)	196.706
NOWOLIPKI (WARSZAWA)	196.67
POMNIK SAPERA (WARSZAWA)	196.534
CIEPŁOWNIA WOLA (WARSZAWA)	196.45
PL.POLITECHNIKI (WARSZAWA)	196.289
OKRZEI (WARSZAWA)	196.139
HALA KOPIŃSKA (WARSZAWA)	195.615
METRO WIERZBNO (WARSZAWA)	195.518
PARK SKARYSZEWSKI (WARSZAWA)	195.09
PŁOCKA (WARSZAWA)	194.815
WAŁ GOCŁAWSKI (WARSZAWA)	194.158
RONDO ZESŁAŃCÓW SYBERYJSKICH (WARSZAWA)	194.115
KOSZYKOWA (WARSZAWA)	193.871
RASZYŃSKA (WARSZAWA)	193.781
TEATR KOMEDIA (WARSZAWA)	193.65
DOLINA SŁUŻEWIECKA (WARSZAWA)	193.375
NALEWKI-MUZEUM (WARSZAWA)	193.095
RAKOWIECKA (WARSZAWA)	192.958
WIEJSKA (WARSZAWA)	192.95
STAWKI (WARSZAWA)	192.876
PKP TORUŃSKA (WARSZAWA)	192.803
CHMIELNA (WARSZAWA)	192.786
PKP WKD AL.JEROZOLIMSKIE (WARSZAWA)	192.776
MUZEUM POWSTANIA WARSZAWSKIEGO (WARSZAWA)	192.649
BRAZYLIJSKA (WARSZAWA)	192.61
KAROLKOWA (WARSZAWA)	192.609
POLITECHNIKA (WARSZAWA)	192.579
SOLEC (WARSZAWA)	192.516
BAZYLIAŃSKA (WARSZAWA)	192.27
METRO WILANOWSKA (WARSZAWA)	192.081
BEREZYŃSKA (WARSZAWA)	191.973
RONDO RADOSŁAWA (WARSZAWA)	191.874
KOWIEŃSKA (WARSZAWA)	191.871
LEGIA-STADION (WARSZAWA)	191.689
PODZAMCZE (WARSZAWA)	191.675
FSO (WARSZAWA)	191.438
AL.WITOSA (WARSZAWA)	191.429
FRANCUSKA (WARSZAWA)	191.323
CZARNIECKIEGO (WARSZAWA)	191.084
AGRYKOLA (WARSZAWA)	191.041
ŻEROMSKIEGO (WARSZAWA)	190.93
REDUTOWA (WARSZAWA)	190.911
WAWELSKA (WARSZAWA)	190.772
KANAŁ ŻERAŃSKI (WARSZAWA)	190.772
ADAMPOLSKA (WARSZAWA)	190.726
PL.ZAMKOWY (WARSZAWA)	190.635
INFLANCKA (WARSZAWA)	190.621
BOHOMOLCA (WARSZAWA)	190.585
MŁODZIEŃCZA (WARSZAWA)	190.557
NOAKOWSKIEGO (WARSZAWA)	190.547
OKOPOWA (WARSZAWA)	190.389
MOST GDAŃSKI (WARSZAWA)	190.36
BRÓDNOWSKA (WARSZAWA)	190.244
PL.POWSTAŃCÓW WARSZAWY (WARSZAWA)	189.999
PL.INWALIDÓW (WARSZAWA)	189.824
GÓRALSKA (WARSZAWA)	189.787
BROŻKA (WARSZAWA)	189.766
ŁOPUSZAŃSKA (WARSZAWA)	189.743
SUZINA (WARSZAWA)	189.657
AL.WIELKOPOLSKI (WARSZAWA)	189.656
ŁAZIENKI KRÓLEWSKIE (WARSZAWA)	189.654
RAWAR (WARSZAWA)	189.552
ORDYNACKA (WARSZAWA)	189.371
SOKOŁOWSKA (WARSZAWA)	189.168
PL.PIŁSUDSKIEGO (WARSZAWA)	189.101
METRO SŁODOWIEC (WARSZAWA)	188.788
FOKSAL (WARSZAWA)	188.758
DĄBROWSZCZAKÓW (WARSZAWA)	188.214
OS.MAJDAŃSKA (WARSZAWA)	188.17
GRZYMAŁY-SOKOŁOWSKIEGO (WARSZAWA)	188.146
METRO SŁUŻEW (WARSZAWA)	188.021
CZERNIAKOWSKA (WARSZAWA)	187.97
BUDZIŃSKIEJ-TYLICKIEJ (WARSZAWA)	187.881
OCHOTA-RATUSZ (WARSZAWA)	187.742
BATALIONU PLATERÓWEK (WARSZAWA)	187.714
LUBELSKA (WARSZAWA)	187.526
KAPITULNA (WARSZAWA)	187.452
PŁOCHOCIŃSKA (WARSZAWA)	187.451
TROCKA (WARSZAWA)	187.365
BANACHA-SZPITAL (WARSZAWA)	187.273
SPISKA (WARSZAWA)	187.109
NIEMCEWICZA (WARSZAWA)	187.109

DYREKCJA FSO (WARSZAWA)	187.08
ODYŃCA (WARSZAWA)	186.999
BERESTECKA (WARSZAWA)	186.982
CH TARGÓWEK (WARSZAWA)	186.935
EC ŻERAŃ (WARSZAWA)	186.914
ELEKCYJNA (WARSZAWA)	186.854
CZORSZTYŃSKA (WARSZAWA)	186.791
SIENNA (WARSZAWA)	186.717
ZAKRZEWSKA (WARSZAWA)	186.439
TRASA ŁAZIENKOWSKA (WARSZAWA)	186.43
LITEWSKA (WARSZAWA)	186.43
OCH-TEATR (WARSZAWA)	186.37
MENNICA (WARSZAWA)	186.305
KS.POPIEŁUSZKI (WARSZAWA)	185.891
ABRAHAMA (WARSZAWA)	185.64
KONDUKTORSKA (WARSZAWA)	185.407
PLATER (WARSZAWA)	185.311
PL.GRUNWALDZKI (WARSZAWA)	185.283
PARK MOCZYDŁO (WARSZAWA)	185.169
SIEKIERKI-SANKTUARIUM (WARSZAWA)	185.077
BIBLIOTEKA NARODOWA (WARSZAWA)	184.904
AL.LOTNIKÓW (WARSZAWA)	184.736
KOŁOWA (WARSZAWA)	184.601
SZCZĘŚLIWICKA (WARSZAWA)	184.497
MANGALIA (WARSZAWA)	184.458
GORZYKOWSKA (WARSZAWA)	184.377
DW.WSCHODNI (KIJOWSKA) (WARSZAWA)	184.329
AL.JANA PAWŁA II (WARSZAWA)	184.3
NIK (WARSZAWA)	184.285
MARYMONT-POTOK (WARSZAWA)	184.243
MYSZKOWSKA (WARSZAWA)	184.206
MADALIŃSKIEGO (WARSZAWA)	184.069
WOŁOMIŃSKA (WARSZAWA)	183.996
BITWY WARSZAWSKIEJ 1920 (WARSZAWA)	183.879
BORA-KOMOROWSKIEGO (WARSZAWA)	183.795
DURACZA (WARSZAWA)	183.753
METRO STARE BIELANY (WARSZAWA)	183.664
GEN.MACZKA (WARSZAWA)	183.641
ROGALIŃSKA (WARSZAWA)	183.458
(KIJOWSKA) (WARSZAWA)	183.412
ORDONA (WARSZAWA)	183.41
GOCŁAW (WARSZAWA)	183.21
11 LISTOPADA (WARSZAWA)	183.16
NOSKOWSKIEGO (WARSZAWA)	183.111
AL.SOLIDARNOŚCI (WARSZAWA)	183.019
WŁOŚCIAŃSKA (WARSZAWA)	182.787
URSYNÓW PŁN. (WARSZAWA)	182.259
METRO URSYNÓW (WARSZAWA)	182.129
WERSALSKA (WARSZAWA)	182.073
STANIEWICKA (WARSZAWA)	182.038
CHŁODNA (WARSZAWA)	182.009
STĘPIŃSKA (WARSZAWA)	181.964
PRĄDZYŃSKIEGO (WARSZAWA)	181.893
KOSTRZEWSKIEGO (WARSZAWA)	181.857
OS.LEWANDÓW I (WARSZAWA)	181.756
MEISSNERA (WARSZAWA)	181.659
MOST SIEKIERKOWSKI (WARSZAWA)	181.641
ROGOWSKA (WARSZAWA)	181.629
ZAJEZDNIA WOLA (WARSZAWA)	181.614
KINOWA (WARSZAWA)	181.456
NOWOLIPIE (WARSZAWA)	181.44
ZAJĘCZA (WARSZAWA)	181.379
BIAŁOŁĘKA-RATUSZ (WARSZAWA)	181.226
RADIOWA (WARSZAWA)	181.093
SYRENY (WARSZAWA)	181.059
KONOPACKA (WARSZAWA)	180.904
ZAMIENIECKA (WARSZAWA)	180.878
BIAŁOBRZESKA (WARSZAWA)	180.719
POPULARNA (WARSZAWA)	180.673
BRATA ALBERTA (WARSZAWA)	180.641
CHEŁMSKA (WARSZAWA)	180.49
JAGIELLOŃSKA (WARSZAWA)	180.401
MARYMONT (WARSZAWA)	180.364
METRO POLE MOKOTOWSKIE (WARSZAWA)	180.345
WIATRACZNA (WARSZAWA)	180.339
PKP URSUS (WARSZAWA)	180.313
NOWOURSYNOWSKA (WARSZAWA)	179.967
RUDNICKIEGO (WARSZAWA)	179.829
LASEK NA KOLE (WARSZAWA)	179.753
SMOCZA (WARSZAWA)	179.699
ROZŁUCKA (WARSZAWA)	179.692
PODBIPIĘTY (WARSZAWA)	179.557
DARWINA (WARSZAWA)	179.47
DOBRA (WARSZAWA)	179.284
LESZNO (WARSZAWA)	179.194
TOR STEGNY (WARSZAWA)	179.075
POLSKIE RADIO (WARSZAWA)	179.006
PŁOWIECKA (WARSZAWA)	178.946
KANAŁ GOCŁAWSKI (WARSZAWA)	178.926
NA BATERYJCE (WARSZAWA)	178.825
PAROWCOWA (WARSZAWA)	178.792
TRAKT LUBELSKI (WARSZAWA)	178.577
SADY ŻOLIBORSKIE (WARSZAWA)	178.349
SZELIGOWSKA (WARSZAWA)	178.083
DOMINIKAŃSKA (WARSZAWA)	178.078
WIŚNIOWA (WARSZAWA)	177.761
KLEMENSIEWICZA (WARSZAWA)	177.664
TORUŃSKA (WARSZAWA)	177.259
OS.JANTAR (WARSZAWA)	177.226
WKD RAKÓW (WARSZAWA)	177.214

TELEWIZJA POLSKA (WARSZAWA)	177.148
LINDLEYA (WARSZAWA)	177.137
AWF (WARSZAWA)	177.116
RUDZKA (WARSZAWA)	177.075
KORSAKA (WARSZAWA)	176.869
POWĄZKOWSKA (WARSZAWA)	176.784
PARK DRESZERA (WARSZAWA)	176.377
METRO STOKŁOSY (WARSZAWA)	176.298
CZUMY (WARSZAWA)	176.288
TKACZY (WARSZAWA)	176.282
BOLEŚĆ (WARSZAWA)	176.261
SYNÓW PUŁKU (WARSZAWA)	176.016
METRO WAWRZYSZEW (WARSZAWA)	175.958
PODCZASZYŃSKIEGO (WARSZAWA)	175.882
MŁYNARSKA (WARSZAWA)	175.497
SIEDMIOGRODZKA (WARSZAWA)	175.493
METRO MŁOCINY (WARSZAWA)	175.482
SMOLUCHOWSKIEGO (WARSZAWA)	175.37
PERZYŃSKIEGO (WARSZAWA)	175.317
LAUROWA (WARSZAWA)	175.225
DWORKOWA (WARSZAWA)	175.08
ZWYCIĘZCÓW (WARSZAWA)	175.071
WALECZNYCH (WARSZAWA)	175.071
KONARSKIEGO (WARSZAWA)	175.038
SZWOLEŻERÓW (WARSZAWA)	174.985
SANGUSZKI (WARSZAWA)	174.969
EKSPRESOWA (WARSZAWA)	174.934
KAZUBÓW (WARSZAWA)	174.878
JANA OLBRACHTA (WARSZAWA)	174.807
POŁCZYŃSKA-PARKING P+R (WARSZAWA)	174.753
UNIWERSYTET MEDYCZNY (WARSZAWA)	174.701
HORBACZEWSKIEGO (WARSZAWA)	174.567
BIELANY-RATUSZ (WARSZAWA)	174.551
BARTNICZA (WARSZAWA)	174.494
HYNKA (WARSZAWA)	174.422
GRENADY (WARSZAWA)	174.381
KRZYŻY (WARSZAWA)	174.284
MORSKIE OKO (WARSZAWA)	173.905
KRASICKIEGO (WARSZAWA)	173.867
CM.ŻYDOWSKI (WARSZAWA)	173.859
ROKOSOWSKA (WARSZAWA)	173.828
PIASKI (WARSZAWA)	173.753
CHODKIEWICZA (WARSZAWA)	173.678
GŁĘBOCKA (WARSZAWA)	173.676
TARGÓWEK-RATUSZ (WARSZAWA)	173.611
BEŁDAN (WARSZAWA)	173.401
CYTADELA (WARSZAWA)	173.39
ŚW.HIERONIMA (WARSZAWA)	173.377
MIŁA (WARSZAWA)	173.275
DALIBORA (WARSZAWA)	173.158
UMIŃSKIEGO (WARSZAWA)	172.921
KSAWERÓW (WARSZAWA)	172.864
CODZIENNA (WARSZAWA)	172.805
MALBORSKA (WARSZAWA)	172.739
GOŁKOWSKA (WARSZAWA)	172.697
KRZYWICKIEGO (WARSZAWA)	172.636
WRÓBLA (WARSZAWA)	172.598
WILANOWSKA (WARSZAWA)	172.584
NIEDŹWIEDZIA (WARSZAWA)	172.584
PL.PRZYMIERZA (WARSZAWA)	172.557
ZARUSKIEGO (WARSZAWA)	172.495
SASANKI (WARSZAWA)	172.304
PELCOWIZNA (WARSZAWA)	172.281
WYŚCIGI (WARSZAWA)	172.275
KRZYŻANOWSKIEGO (WARSZAWA)	172.267
CHŁODNIA (WARSZAWA)	172.206
AL. 3 MAJA (WARSZAWA)	172.202
PKP ŻERAŃ (WARSZAWA)	171.981
OS.ISKRA (WARSZAWA)	171.964
WIELICKA (WARSZAWA)	171.947
KARCZOCHA (WARSZAWA)	171.909
PL.BERNARDYŃSKI (WARSZAWA)	171.756
CH PROMENADA (WARSZAWA)	171.74
ARMATNIA (WARSZAWA)	171.605
RZYMOWSKIEGO (WARSZAWA)	171.595
MASZYNOWA (WARSZAWA)	171.595
ŚMIGŁOWCA (WARSZAWA)	171.582
PUSTOLA (WARSZAWA)	171.538
OS.ORLIK (WARSZAWA)	171.53
TAGORE'A (WARSZAWA)	171.514
BOBROWIECKA (WARSZAWA)	171.44
ZAJEZDNIA OSTROBRAMSKA (WARSZAWA)	171.39
GRENADIERÓW (WARSZAWA)	171.371
WOLA GRZYBOWSKA (WARSZAWA)	171.366
STEGNY (WARSZAWA)	171.32
MARIENSZTAT (WARSZAWA)	171.188
CENTRUM OLIMPIJSKIE (WARSZAWA)	171.134
ŁUKOWA (WARSZAWA)	171.121
KRYNOLINY (WARSZAWA)	171.12
MONTE CASSINO (WARSZAWA)	171.078
BIAŁOWIEJSKA (WARSZAWA)	171.061
SIELCE (WARSZAWA)	170.946
PRATULIŃSKA (WARSZAWA)	170.916
DICKENSA (WARSZAWA)	170.904
CM.BRÓDNOWSKI (WARSZAWA)	170.728
BARTOSZKA (WARSZAWA)	170.712
BLISKA (WARSZAWA)	170.689
ZAJEZDNIA KLESZCZOWA (WARSZAWA)	170.556
KOŚCIELISKA (WARSZAWA)	170.543
SZPITAL GROCHOWSKI (WARSZAWA)	170.519

SAMOCHODOWA (WARSZAWA)	170.461
ŻYTNIA (WARSZAWA)	170.388
MALCZEWSKIEGO (WARSZAWA)	170.305
ANTKA ROZPYLACZA (WARSZAWA)	170.257
REDUTA WOLSKA (WARSZAWA)	170.244
MADALIŃSKIEGO-SZPITAL (WARSZAWA)	170.203
RAKOWIEC (WARSZAWA)	170.148
PL.SZEMBEKA (WARSZAWA)	170.118
DROHICKA (WARSZAWA)	170.111
KOCHANOWSKIEGO (WARSZAWA)	170.107
NARODOWA (WARSZAWA)	169.996
SZYMANOWSKIEGO (WARSZAWA)	169.822
OBRAZKOWA (WARSZAWA)	169.807
BUKOWIŃSKA (WARSZAWA)	169.804
WAŁOWICKA (WARSZAWA)	169.797
CM.PRAWOSŁAWNY (WARSZAWA)	169.698
OS.DOMANIEWSKA (WARSZAWA)	169.697
IWICKA (WARSZAWA)	169.595
SALOMEA (WARSZAWA)	169.491
NISKA (WARSZAWA)	169.469
MAGIERA (WARSZAWA)	169.349
STRZELECKIEGO (WARSZAWA)	169.319
JÓRSKIEGO (WARSZAWA)	169.29
ZBARASKA (WARSZAWA)	169.184
OSTRÓDZKA (WARSZAWA)	169.15
POTULICKA (WARSZAWA)	169.138
POD CZERNIAKÓW (WARSZAWA)	169.045
SPÓŁDZIELNIA NOMA (WARSZAWA)	169.002
TUŻYCKA (WARSZAWA)	168.981
KOMPASOWA (WARSZAWA)	168.952
DKS TARGÓWEK (WARSZAWA)	168.933
OS.WILGA (WARSZAWA)	168.93
POWĄZKI-CM.WOJSKOWY (WARSZAWA)	168.879
POŻAROWA (WARSZAWA)	168.873
DW.WSCHODNI (LUBELSKA) (WARSZAWA)	168.794
POLECZKI (WARSZAWA)	168.775
POWĄZKI-IV BRAMA (WARSZAWA)	168.578
OŚRODEK LOK (WARSZAWA)	168.524
OCZAPOWSKIEGO (WARSZAWA)	168.5
HANDLOWA (WARSZAWA)	168.468
BARTYCKA (WARSZAWA)	168.423
CZARNOMORSKA (WARSZAWA)	168.344
JARACZA (WARSZAWA)	168.238
ESPERANTO (WARSZAWA)	168.171
BUKOWSKIEGO (WARSZAWA)	168.138
BIBLIOTEKA UNIWERSYTECKA (WARSZAWA)	168.011
KRAŚNICKA (WARSZAWA)	167.921
INSTYTUT FIZYKI (WARSZAWA)	167.917
BACHA (WARSZAWA)	167.601
FORT WOLA (WARSZAWA)	167.526
WROCŁAWSKA (WARSZAWA)	167.376
ŁOWICKA (WARSZAWA)	167.273
KONCERTOWA (WARSZAWA)	167.08
OBROŃCÓW TOBRUKU (WARSZAWA)	167.076
KS POLONEZ (WARSZAWA)	166.918
PKP WŁOCHY (WARSZAWA)	166.822
OKULAROWA (WARSZAWA)	166.611
URSYNÓW PŁD. (WARSZAWA)	166.468
OKRĄGŁA (WARSZAWA)	166.462
OS.GÓRCZEWSKA (WARSZAWA)	166.443
KS.SITNIKA (WARSZAWA)	166.381
NOVOTEL (WARSZAWA)	166.38
SIEKIERKOWSKA (WARSZAWA)	166.335
DOMANIEWSKA (WARSZAWA)	166.325
NACZELNIKOWSKA (WARSZAWA)	166.299
JOLIOT-CURIE (WARSZAWA)	166.264
MYŚLIBORSKA (WARSZAWA)	166.2
WARTKA (WARSZAWA)	166.175
SOSNOWA (WARSZAWA)	166.093
SOKRATESA (WARSZAWA)	166.08
DOBOSZA (WARSZAWA)	166.022
WOŁOSKA (WARSZAWA)	165.983
WALEWSKA (WARSZAWA)	165.817
REDUTA (WARSZAWA)	165.739
KLASYKÓW (WARSZAWA)	165.734
WITEBSKA (WARSZAWA)	165.63
ZAJEZDNIA PRAGA (WARSZAWA)	165.581
HALE BANACHA (WARSZAWA)	165.558
URSUS-NIEDŹWIADEK (WARSZAWA)	165.44
MEKSYKAŃSKA (WARSZAWA)	165.428
BUDOWLANA (WARSZAWA)	165.326
CENTRUM ONKOLOGII (WARSZAWA)	165.153
BARSZCZEWSKA (WARSZAWA)	165.152
REMBIELIŃSKA (WARSZAWA)	165.097
HALA WOLA (WARSZAWA)	165.079
KS.BOGUCKIEGO (WARSZAWA)	165.068
DUNIKOWSKIEGO (WARSZAWA)	165.018
BANACHA (WARSZAWA)	164.997
KONDRATOWICZA (WARSZAWA)	164.931
GRZYBOWA (WARSZAWA)	164.779
MIĘDZYBORSKA (WARSZAWA)	164.692
ŚNIEŻNA (WARSZAWA)	164.624
MAŁE SIEKIERKI (WARSZAWA)	164.571
PL.KASZTELAŃSKI (WARSZAWA)	164.324
TRAWIASTA (WARSZAWA)	164.303
SZPITAL BRÓDNOWSKI (WARSZAWA)	164.281
RYŻOWA (WARSZAWA)	164.269
RAKOWIECKA-SANKTUARIUM (WARSZAWA)	164.269
LEKTYKARSKA (WARSZAWA)	164.253
ŁUKOWSKA (WARSZAWA)	164.206

MIKOŁAJSKA (WARSZAWA)	164.191
ZAJEZDNIA REDUTOWA (WARSZAWA)	163.854
WSPÓLNA DROGA (WARSZAWA)	163.837
DYWIZJONU 303 (WARSZAWA)	163.828
GOCŁAWEK (WARSZAWA)	163.775
POSTĘPU (WARSZAWA)	163.727
FIELDORFA (WARSZAWA)	163.711
PRUSZKOWSKA (WARSZAWA)	163.676
PRZYBYSZEWSKIEGO (WARSZAWA)	163.669
PERKUNA (WARSZAWA)	163.662
WARSZAWSKIEJ 1920 (WARSZAWA)	163.656
ŻÓŁKIEWSKIEGO (WARSZAWA)	163.38
METRO IMIELIN (WARSZAWA)	163.254
PKP WAWER (WARSZAWA)	163.253
AL.REYMONTA (WARSZAWA)	163.245
ORLICH GNIAZD (WARSZAWA)	163.153
WAŁBRZYSKA-CMENTARZ (WARSZAWA)	163.071
WIOLINOWA (WARSZAWA)	163.048
STERNICZA (WARSZAWA)	163.039
ARMII KRAJOWEJ (WARSZAWA)	162.987
RADIOWA-WAT (WARSZAWA)	162.957
PIRENEJSKA (WARSZAWA)	162.934
MARYSIN (WARSZAWA)	162.914
URSUS-RATUSZ (WARSZAWA)	162.828
DOSTAWCZA (WARSZAWA)	162.817
PKP SŁUŻEWIEC (WARSZAWA)	162.622
SOBOCKA (WARSZAWA)	162.583
ŚW.WINCENTEGO (WARSZAWA)	162.458
ŚW.BONIFACEGO (WARSZAWA)	162.44
UROCZA (WARSZAWA)	162.44
MILENIJNA (WARSZAWA)	162.345
KAJKI (WARSZAWA)	162.342
HŁASKI (WARSZAWA)	162.333
NOWAKA-JEZIORAŃSKIEGO (WARSZAWA)	162.28
KASZTANOWA (WARSZAWA)	161.96
KIELECKA (WARSZAWA)	161.912
RADAROWA (WARSZAWA)	161.806
KIERBEDZIA (WARSZAWA)	161.804
OŚRODEK AZS (WARSZAWA)	161.717
BELLOTTIEGO (WARSZAWA)	161.649
EDISONA (WARSZAWA)	161.626
KOROTYŃSKIEGO (WARSZAWA)	161.573
CZYNSZOWA (WARSZAWA)	161.533
GOŚCIERADOWSKA (WARSZAWA)	161.489
ŻUROMIŃSKA (WARSZAWA)	161.419
STEFANA BATOREGO (WARSZAWA)	161.396
SCHROEGERA (WARSZAWA)	161.379
PROZY (WARSZAWA)	161.377
HERBSTA (WARSZAWA)	161.365
PRZASNYSKA (WARSZAWA)	161.36
ZAOLZIAŃSKA (WARSZAWA)	161.314
KOŃSKI JAR (WARSZAWA)	161.314
BOGUSŁAWSKIEGO (WARSZAWA)	161.231
ZAJEZDNIA ŻOLIBORZ (WARSZAWA)	161.181
AL.BOHATERÓW WRZEŚNIA (WARSZAWA)	161.179
CERAMICZNA (WARSZAWA)	161.136
DEOTYMY (WARSZAWA)	161.112
MYCIELSKIEGO (WARSZAWA)	161.112
GALLA ANONIMA (WARSZAWA)	161.035
MALINOWA (WARSZAWA)	160.987
DWERNICKIEGO (WARSZAWA)	160.981
GOTARDA (WARSZAWA)	160.893
KRÓLIKARNIA (WARSZAWA)	160.789
ARBUZOWA (WARSZAWA)	160.764
SZPITAL BIELAŃSKI (WARSZAWA)	160.747
LALKI (WARSZAWA)	160.719
SZCZĘŚLIWICE (WARSZAWA)	160.642
PKP GOCŁAWEK (WARSZAWA)	160.641
KICKIEGO (WARSZAWA)	160.612
GOCŁAWSKA (WARSZAWA)	160.561
KOŁO (WARSZAWA)	160.537
PL.TYSIĄCLECIA (WARSZAWA)	160.18
GIERDZIEJEWSKIEGO (WARSZAWA)	160.171
WŁOCHY-RATUSZ (WARSZAWA)	160.092
BRACI ZAWADZKICH (WARSZAWA)	160.086
MŁYNÓW (WARSZAWA)	160.018
OŚRODEK CORA (WARSZAWA)	160.005
BARTŁOMIEJA (WARSZAWA)	159.838
SYMPATYCZNA (WARSZAWA)	159.824
PRAŻMOWSKA (WARSZAWA)	159.764
ŻYWIECKA (WARSZAWA)	159.732
KRAKOWIAKÓW (WARSZAWA)	159.667
MŁOCINY (WARSZAWA)	159.547
CZAPELSKA (WARSZAWA)	159.481
NOWOTYSKA (WARSZAWA)	159.43
SPADOWA (WARSZAWA)	159.407
CHODECKA (WARSZAWA)	159.385
OS.BERNARDYŃSKA (WARSZAWA)	159.383
BOHUSZEWICZÓWNY (WARSZAWA)	159.295
INOWŁODZKA (WARSZAWA)	159.243
KULSKIEGO (WARSZAWA)	159.139
KSIĘCIA BOLESŁAWA (WARSZAWA)	158.978
FABRYKA POMP (WARSZAWA)	158.975
OPACZEWSKA (WARSZAWA)	158.885
DŁUGOSZA (WARSZAWA)	158.805
PRZEDPOLE (WARSZAWA)	158.738
GEODEZYJNA (WARSZAWA)	158.717
WOJCIECHOWSKIEGO (WARSZAWA)	158.497
LEŻAJSKA (WARSZAWA)	158.484
LIMANOWSKIEGO (WARSZAWA)	158.467

TARGÓWEK (WARSZAWA)	158.393
CM.OFIAR WOJNY (WARSZAWA)	158.358
POPIELA (WARSZAWA)	158.35
ŁABISZYŃSKA (WARSZAWA)	158.095
BALEYA (WARSZAWA)	158.091
SOWIŃSKIEGO (WARSZAWA)	158.085
CM.WOLSKI (WARSZAWA)	158.085
WESOŁA-RATUSZ (WARSZAWA)	157.994
WORONICZA (WARSZAWA)	157.98
CYBERNETYKI (WARSZAWA)	157.912
17 STYCZNIA (WARSZAWA)	157.867
BATUMI (WARSZAWA)	157.867
PKP REMBERTÓW (WARSZAWA)	157.866
MODZELEWSKIEGO (WARSZAWA)	157.816
ŚWIECKA (WARSZAWA)	157.742
MIŁOBĘDZKA (WARSZAWA)	157.721
PRZEPIÓRKI (WARSZAWA)	157.693
PATKOWSKIEGO (WARSZAWA)	157.679
STAWY (WARSZAWA)	157.614
BEMOWO (WARSZAWA)	157.562
PKP RAKOWIEC (WARSZAWA)	157.556
PRZY PARKU (WARSZAWA)	157.315
TURMONCKA (WARSZAWA)	157.172
BYSTRA (WARSZAWA)	157.114
INSTRUKTORSKA (WARSZAWA)	157.082
ASPEKT (WARSZAWA)	156.989
ORLA (SULEJÓWEK)	156.915
KLAUDYNY (WARSZAWA)	156.893
ŻUPNICZA (WARSZAWA)	156.875
DO FORTU (WARSZAWA)	156.533
WIŚLICKA (WARSZAWA)	156.423
TELIGI (WARSZAWA)	156.421
SMOLEŃSKIEGO (WARSZAWA)	156.226
ELBLĄSKA (WARSZAWA)	156.202
GRODZISK (WARSZAWA)	156.181
BRYLOWSKA (WARSZAWA)	156.132
KAKOWSKIEGO (WARSZAWA)	156.039
DOJAZDOWA (WARSZAWA)	155.971
DZIELNA (WARSZAWA)	155.962
GROSZÓWKA (WARSZAWA)	155.93
KADETÓW (WARSZAWA)	155.921
ZNANA (WARSZAWA)	155.895
ORZYCKA (WARSZAWA)	155.75
CONRADA (WARSZAWA)	155.735
ORZECHOWA (WARSZAWA)	155.666
PKP ANIN (WARSZAWA)	155.607
PŁUŻNICKA (WARSZAWA)	155.528
TARCHOMIN (WARSZAWA)	155.461
PŁYTOWA (WARSZAWA)	155.437
BUDEXPO (WARSZAWA)	155.433
RODŁA (WARSZAWA)	155.431
NOWE BEMOWO (WARSZAWA)	155.431
RZECZNA (WARSZAWA)	155.378
WITOLIN (WARSZAWA)	155.33
MARCELIN (WARSZAWA)	155.294
STARE ŚWIDRY (WARSZAWA)	155.269
HENRYKÓW (WARSZAWA)	155.115
SOSNKOWSKIEGO (ZĄBKI)	155.105
DUCHNICKA (WARSZAWA)	155.041
PORAJÓW (WARSZAWA)	155.019
CZERWIŃSKA (WARSZAWA)	155.001
WYTWÓRNIA MAS BITUMICZNYCH (WARSZAWA)	154.968
PRAGA-PŁD.-RATUSZ (WARSZAWA)	154.94
ZGRUPOWANIA AK KAMPINOS (WARSZAWA)	154.906
NOCZNICKIEGO (WARSZAWA)	154.723
SKOROCHÓD-MAJEWSKIEGO (WARSZAWA)	154.682
CHŁOPICKIEGO (WARSZAWA)	154.671
WSPÓŁCZESNA (WARSZAWA)	154.651
JASIELSKA (WARSZAWA)	154.415
PARK BRÓDNOWSKI (WARSZAWA)	154.39
GDECKA (WARSZAWA)	154.345
LEWINOWSKA (WARSZAWA)	154.331
MOŁDAWSKA (WARSZAWA)	154.28
OS.MARYWILSKA (WARSZAWA)	154.279
KWATERY GŁÓWNEJ (WARSZAWA)	154.148
REJTANA (WARSZAWA)	154.075
HARFOWA (WARSZAWA)	154.056
CEMENTOWNIA (WARSZAWA)	153.953
MEHOFFERA (WARSZAWA)	153.876
MIKLASZEWSKIEGO (WARSZAWA)	153.804
HENRYKA BRODATEGO (WARSZAWA)	153.752
ŁOKIETKA (WARSZAWA)	153.612
TOR SŁUŻEWIEC (WARSZAWA)	153.597
GRUSZCZYŃSKIEGO (WARSZAWA)	153.587
STARE BEMOWO (WARSZAWA)	153.524
KRZESIWA (WARSZAWA)	153.48
CISZEWSKIEGO (WARSZAWA)	153.413
GRABÓW (WARSZAWA)	153.38
REZEDOWA (WARSZAWA)	153.287
KUPIECKA (WARSZAWA)	153.286
POTOCKICH (WARSZAWA)	153.184
ANDERSENA (WARSZAWA)	153.118
BOHATEREWICZA (WARSZAWA)	153.064
METRO NATOLIN (WARSZAWA)	152.993
KLARYSEWSKA (WARSZAWA)	152.969
WYSZOGRODZKA (WARSZAWA)	152.905
OBJAZDOWA (WARSZAWA)	152.895
BLIZNE ŁASZCZYŃSKIEGO (BLIZNE ŁASZCZYŃSKIEGO)	152.889
ŚWIDERSKA (WARSZAWA)	152.83
DZIECI WARSZAWY (WARSZAWA)	152.748

CHEMICZNA (WARSZAWA)	152.729
OGRODY DZIAŁKOWE PIASKI (WARSZAWA)	152.726
DIAMENTOWA (WARSZAWA)	152.653
PKP PRAGA (WARSZAWA)	152.453
SZPITAL MSW (WARSZAWA)	152.443
KSIĘCIA ZIEMOWITA (WARSZAWA)	152.412
TANGO (WARSZAWA)	152.332
ŚMIGIELSKA (WARSZAWA)	152.277
PIASTÓW ŚLĄSKICH (WARSZAWA)	152.262
UKSW (WARSZAWA)	152.078
ŁOSOSIOWA (WARSZAWA)	152.074
NIEDZIAŁKOWSKIEGO (WARSZAWA)	151.971
TECHNIKÓW (WARSZAWA)	151.933
ARKADYJSKA (WARSZAWA)	151.929
POLINEZYJSKA (WARSZAWA)	151.817
TARNOWIECKA (WARSZAWA)	151.806
SZOBERA (WARSZAWA)	151.774
NARWIK (WARSZAWA)	151.712
BOLESŁAWICKA (WARSZAWA)	151.62
WIDAWSKA (WARSZAWA)	151.592
MAGELLANA (WARSZAWA)	151.544
MARYNIN (WARSZAWA)	151.452
KLASZTORNA (WARSZAWA)	151.449
TOŁSTOJA (WARSZAWA)	151.447
DANISZEWSKA (WARSZAWA)	151.43
GĄSOCIŃSKA (WARSZAWA)	151.423
KAROLIN (WARSZAWA)	151.362
ŁOJEWSKA (WARSZAWA)	151.26
RAGINISA (WARSZAWA)	151.2
URSUS-SANKTUARIUM (WARSZAWA)	150.973
OS.ZIELONY STAW (WARSZAWA)	150.954
CM.WAWRZYSZEWSKI (WARSZAWA)	150.939
KONSTRUKTORSKA (WARSZAWA)	150.814
INSTYTUT HEMATOLOGII (WARSZAWA)	150.802
REKRUCKA (WARSZAWA)	150.772
URZĄD STATYSTYCZNY (WARSZAWA)	150.738
SIEMIATYCKA (WARSZAWA)	150.689
ROLNICZA (WARSZAWA)	150.636
SUWALSKA (WARSZAWA)	150.562
WOJNICKA (WARSZAWA)	150.518
POBORZAŃSKA (WARSZAWA)	150.498
PODSKARBIŃSKA (WARSZAWA)	150.477
FELIŃSKIEGO (WARSZAWA)	150.247
MAKOLĄGWY (WARSZAWA)	150.226
RYBNICKA (WARSZAWA)	150.198
ZAPUSTNA (WARSZAWA)	150.186
WARSZAWSKA (WARSZAWA)	150.162
KWITNĄCA (WARSZAWA)	150.146
FIGOWA (WARSZAWA)	150.12
URBANISTÓW (WARSZAWA)	150.115
JESIONOWA (JESIONOWA)	150.02
SOLIDNA (WARSZAWA)	149.949
KROKWI (WARSZAWA)	149.913
MALINOWSKIEGO (WARSZAWA)	149.885
SIODLARSKA (WARSZAWA)	149.881
1 SIERPNIA (WARSZAWA)	149.87
SZYMAŃSKIEGO (WARSZAWA)	149.823
BATALIONU MIOTŁA (WARSZAWA)	149.743
ŁYSAKOWSKA (WARSZAWA)	149.709
WRZECIONO (WARSZAWA)	149.678
GORASZEWSKA (WARSZAWA)	149.614
TRUSKAWIECKA (WARSZAWA)	149.593
STARODĘBY (WARSZAWA)	149.47
MOKOTÓW (WARSZAWA)	149.466
CM.WŁOSKI (WARSZAWA)	149.412
PŁUŻAŃSKA (WARSZAWA)	149.382
JULIANOWSKA (WARSZAWA)	149.332
WIEJSKA (MARKI)	149.311
JADŹWINGÓW (WARSZAWA)	149.294
PIĘCIOLINII (WARSZAWA)	149.287
SŁOWACKIEGO (WARSZAWA)	149.242
PKP URSUS PŁN. (WARSZAWA)	149.165
ODRODZENIA (WARSZAWA)	149.158
MEGASAM (WARSZAWA)	149.151
OS.LEŚNE (WARSZAWA)	149.092
PRYSTORA (WARSZAWA)	149.033
DROGOWA (WARSZAWA)	148.955
MONIUSZKI (WARSZAWA)	148.823
SPISAKA (WARSZAWA)	148.719
RAFII (WARSZAWA)	148.69
PRZY AGORZE (WARSZAWA)	148.664
OGRODY DZIAŁKOWE BEMOWO (WARSZAWA)	148.606
INSTYTUT LOTNICTWA (WARSZAWA)	148.593
CHODAKOWSKA (WARSZAWA)	148.532
KĄTY GRODZISKIE (WARSZAWA)	148.454
WYNALAZEK (WARSZAWA)	148.212
ORLĄT LWOWSKICH (WARSZAWA)	148.127
ADAMIECKIEGO (WARSZAWA)	148.104
BEŁZY (WARSZAWA)	148.044
ZIELONA (WARSZAWA)	148.042
SZANCERA (WARSZAWA)	147.907
WOLNOŚCI (ZĄBKI)	147.836
SZASERÓW-SZPITAL (WARSZAWA)	147.754
DZIAŁYŃCZYKÓW (WARSZAWA)	147.709
CHOMICZÓWKA (WARSZAWA)	147.673
RKS ORZEŁ (WARSZAWA)	147.67
RONDO ŻABCZYŃSKIEGO (WARSZAWA)	147.63
CM.PÓŁNOCNY-BRAMA PŁN. (WARSZAWA)	147.581
ROLANDA (WARSZAWA)	147.505
KAFLOWA (WARSZAWA)	147.471
1. PRASKIEGO PUŁKU (WARSZAWA)	147.413

OS.ARBUZOWA (WARSZAWA)	147.401
WIRAŻOWA (WARSZAWA)	147.399
SOCHACZEWSKA (WARSZAWA)	147.31
SGGW-BIBLIOTEKA (WARSZAWA)	147.075
RECHNIEWSKIEGO (WARSZAWA)	147.022
MAGISTRACKA (WARSZAWA)	147.001
BLUE CITY (WARSZAWA)	146.93
ŚW.A.BOBOLI (WARSZAWA)	146.918
PKP MIĘDZYLESIE (WARSZAWA)	146.798
MPT GROCHÓW (WARSZAWA)	146.78
KUŹNICY KOŁŁĄTAJOWSKIEJ (WARSZAWA)	146.694
STRĄKOWA (WARSZAWA)	146.563
WAWRZYSZEWSKA (WARSZAWA)	146.546
PUSZKINA (SULEJÓWEK)	146.515
WKD OPACZ (MICHAŁOWICE)	146.513
WIERZCHOWSKIEGO (WARSZAWA)	146.479
SOŁTYKA (WARSZAWA)	146.397
JEZIORKO CZERNIAKOWSKIE (WARSZAWA)	146.386
GILARSKA (WARSZAWA)	146.365
OKĘCIE BUSINESS PARK (WARSZAWA)	146.253
LIPÓWCZANA (WARSZAWA)	146.072
IX POPRZECZNA (WARSZAWA)	146.061
OŚRODEK WOJSKOWY (WARSZAWA)	146.053
REMBERTÓW-RATUSZ (WARSZAWA)	145.845
DZIELNICOWA (WARSZAWA)	145.804
ODLEWNICZA (WARSZAWA)	145.74
WAPOWSKIEGO (WARSZAWA)	145.563
ZAJEZDNIA WORONICZA (WARSZAWA)	145.404
PANCERA (WARSZAWA)	145.328
ARCHIMEDESA (WARSZAWA)	145.154
GNIEWKOWSKA (WARSZAWA)	145.09
MUZEALNA (WARSZAWA)	145.075
KLUB LIRA (WARSZAWA)	144.976
TRAUGUTTA (PIASTÓW)	144.959
TYTUŁOWA (WARSZAWA)	144.95
DRZEWIECKIEGO (WARSZAWA)	144.92
SIEMIEŃSKIEGO (WARSZAWA)	144.763
KOMPUTEROWA (WARSZAWA)	144.744
INSTYTUT ENERGETYKI (WARSZAWA)	144.695
KADETÓW-SZKOŁA (WARSZAWA)	144.679
HALA SPORTOWA BEMOWO (WARSZAWA)	144.673
PIASTA KOŁODZIEJA (WARSZAWA)	144.665
RADZYMIŃSKA (ZĄBKI)	144.665
SÓJKI (WARSZAWA)	144.431
STASZICA (WARSZAWA)	144.356
REMBERTÓW (WARSZAWA)	144.355
POETÓW (WARSZAWA)	144.204
MIŃSKA (WARSZAWA)	144.027
FORT BEMA (WARSZAWA)	144.016
ZAPLECZE (WARSZAWA)	143.904
STRUSIA (WARSZAWA)	143.839
CZŁUCHOWSKA (WARSZAWA)	143.795
JADWISIN (WARSZAWA)	143.741
KOPIJNIKÓW (WARSZAWA)	143.624
INTERNETOWA (WARSZAWA)	143.602
KAMBUD (WARSZAWA)	143.463
WOJSŁAWICKA (WARSZAWA)	143.45
FORT BLIZNE (WARSZAWA)	143.449
SAMARYTANKA (WARSZAWA)	143.332
KOLEJOWA (WARSZAWA)	143.295
KLAMROWA (WARSZAWA)	143.267
HALLERA (ZĄBKI)	143.147
SAGALLI (WARSZAWA)	143.063
(LUBELSKA) (WARSZAWA)	143.06
ŻOLIBORZ (WARSZAWA)	143.008
TATARSKA (WARSZAWA)	142.991
CHĘCIŃSKA (WARSZAWA)	142.854
HIPODROM (WARSZAWA)	142.819
PICASSA (WARSZAWA)	142.818
SMUGOWA (WARSZAWA)	142.795
BATALIONÓW CHŁOPSKICH (WARSZAWA)	142.671
GWIAŹDZISTA (WARSZAWA)	142.654
LAS BIELAŃSKI (WARSZAWA)	142.652
GRZEGORZEWSKIEJ (WARSZAWA)	142.561
POCISK (WARSZAWA)	142.478
NIEŚWIESKA (WARSZAWA)	142.443
HUTNICZA (WARSZAWA)	142.401
PRZEBIEG (WARSZAWA)	142.395
NUGAT (WARSZAWA)	142.297
WARCHAŁOWSKIEGO (WARSZAWA)	142.275
PIECHURÓW (WARSZAWA)	142.261
BUKOWIECKA (WARSZAWA)	142.194
BLIZNE JASIŃSKIEGO (BLIZNE JASIŃSKIEGO)	142.158
WŁADYSŁAWA HERMANA (WARSZAWA)	142.035
URSYNÓW ZACH. (WARSZAWA)	141.911
POLONEZA (WARSZAWA)	141.844
SKOROSZEWSKA (WARSZAWA)	141.816
POSAG 7 PANIEN (WARSZAWA)	141.771
ROZŁOGI (WARSZAWA)	141.706
WINNICA (WARSZAWA)	141.414
POMIECHOWSKA (WARSZAWA)	141.407
NATOLIN PŁN. (WARSZAWA)	141.373
OSIEDLE MŁODYCH (WARSZAWA)	141.293
KRÓLOWEJ BONY (WARSZAWA)	141.291
ZACISZE (WARSZAWA)	141.216
HETMAŃSKA (WARSZAWA)	141.21
KOPCIŃSKIEGO (WARSZAWA)	141.208
BOKSERSKA (WARSZAWA)	141.159
KAMIŃSKIEGO (WARSZAWA)	141.064

INSTYTUT KARDIOLOGII (WARSZAWA)	140.93
GROTY (WARSZAWA)	140.894
ŻOŁNIERSKA (WARSZAWA)	140.811
TOR KOLARSKI (WARSZAWA)	140.804
CZEREMCHOWA (WARSZAWA)	140.769
SOBIESKIEGO (WARSZAWA)	140.71
SGGW-KLINIKA (WARSZAWA)	140.589
NISKA (BLIZNE ŁASZCZYŃSKIEGO)	140.375
SIARCZANA (WARSZAWA)	140.346
NAŁĘCZOWSKA (WARSZAWA)	140.342
KALINOWA DROGA (JESIONOWA)	140.219
PKP OLSZYNKA GROCHOWSKA (WARSZAWA)	140.076
ANANASOWA (WARSZAWA)	140.066
JESIONOWA (OPACZ-KOLONIA)	139.976
P+R AL.KRAKOWSKA (WARSZAWA)	139.975
AL.KRAKOWSKA (WARSZAWA)	139.975
METRO KABATY (WARSZAWA)	139.905
SZUBERTA (WARSZAWA)	139.902
LELEWELA (PIASTÓW)	139.897
NOWE WŁOCHY (WARSZAWA)	139.888
HOSPICJUM ONKOLOGICZNE (WARSZAWA)	139.879
KOCJANA-BOCZNICA (WARSZAWA)	139.875
MAŁCUŻYŃSKIEGO (WARSZAWA)	139.873
STEFANIKA (WARSZAWA)	139.846
SEZAM (WARSZAWA)	139.845
ERAZMA Z ZAKROCZYMIA (WARSZAWA)	139.711
RACJONALIZACJI (WARSZAWA)	139.658
MARSA-LAS (WARSZAWA)	139.545
OSMAŃCZYKA (WARSZAWA)	139.523
LANCIEGO (WARSZAWA)	139.505
ĆMIELOWSKA (WARSZAWA)	139.457
WAWERSKA (WARSZAWA)	139.419
SADYBA (WARSZAWA)	139.292
SKARBKA Z GÓR (WARSZAWA)	139.051
NOWODWORSKA (WARSZAWA)	138.861
SZOLC-ROGOZIŃSKIEGO (WARSZAWA)	138.838
KRAŃCOWA (WARSZAWA)	138.795
ZAJEZDNIA URSUS PŁN. (WARSZAWA)	138.754
ORZESZKOWEJ (PIASTÓW)	138.695
AL.PIŁSUDSKIEGO (SULEJÓWEK)	138.629
FLETNIOWA (WARSZAWA)	138.616
SZPITALNA (MARKI)	138.548
RAABEGO (WARSZAWA)	138.539
MELCERA (WARSZAWA)	138.533
GARWOLIŃSKA (WARSZAWA)	138.481
SUCHY LAS (WARSZAWA)	138.279
CM.PÓŁNOCNY-BRAMA GŁ. (WARSZAWA)	138.262
OSTROROGA (WARSZAWA)	138.2
JURANDA ZE SPYCHOWA (WARSZAWA)	138.189
BATALIONU WŁOCHY (WARSZAWA)	138.038
ORACZY (WARSZAWA)	137.966
(RYDYGIERA) (WARSZAWA)	137.913
TARCHOMIN KOŚCIELNY (WARSZAWA)	137.895
PKP OKĘCIE (WARSZAWA)	137.845
SGGW-REKTORAT (WARSZAWA)	137.719
MAJAKOWSKIEGO (WARSZAWA)	137.674
SOSNKOWSKIEGO (WARSZAWA)	137.527
SKOROSZE (WARSZAWA)	137.464
OS.OLSZYNY (WARSZAWA)	137.463
HASSA (WARSZAWA)	137.454
OS.DERBY VI (WARSZAWA)	137.271
GRUSZY (WARSZAWA)	137.271
FABRYCZNA (WARSZAWA)	137.263
LESZCZYŃSKIEGO (WARSZAWA)	137.214
KLIMONTOWSKA (WARSZAWA)	137.095
BADYLARSKA (MICHAŁOWICE)	137.042
GŁOWACKIEGO (WARSZAWA)	136.984
WESOŁA (WARSZAWA)	136.976
WYSZKOWSKA (WARSZAWA)	136.972
INSTYTUT NEUROLOGII (WARSZAWA)	136.704
SYPNIEWSKA (WARSZAWA)	136.678
KOZIELSKA (WARSZAWA)	136.66
SIENNICKA (WARSZAWA)	136.589
ZAUŁEK (WARSZAWA)	136.482
STRYJEŃSKICH (WARSZAWA)	136.385
MAKLAKIEWICZA (WARSZAWA)	136.347
BENNETTA (WARSZAWA)	136.321
KADETÓW-KOŚCIÓŁ (WARSZAWA)	136.278
KOCJANA (WARSZAWA)	135.994
WAT (WARSZAWA)	135.99
HOŁUBCOWA (WARSZAWA)	135.968
WALEREGO SŁAWKA (WARSZAWA)	135.912
BERENSONA (WARSZAWA)	135.908
SŁUŻEWIEC (WARSZAWA)	135.818
LEŚNEJ POLANKI (WARSZAWA)	135.534
ZAWODOWA (WARSZAWA)	135.394
ZAKŁADY ZBOŻOWE (WARSZAWA)	135.144
ŻOŁNY (WARSZAWA)	135.11
WARSZAWSKA (ZĄBKI)	135.007
MORY (MORY)	134.993
KOLONIA HELENÓW (WARSZAWA)	134.953
ESKULAPÓW (WARSZAWA)	134.927
SZYSZKOWA (WARSZAWA)	134.858
LUCERNY (WARSZAWA)	134.771
CH MARKI (MARKI)	134.745
FARADAYA (WARSZAWA)	134.696
CENTRUM TARGOWE (WARSZAWA)	134.677
DZIERŻONIOWSKA (WARSZAWA)	134.63
KORDIANA (WARSZAWA)	134.603
NORWIDA (PIASTÓW)	134.598

DANKOWICKA (WARSZAWA)	134.527
LOTNISKO CHOPINA-ODLOTY (WARSZAWA)	134.493
ZYNDRAMA Z MASZKOWIC (WARSZAWA)	134.453
DĄBRÓWKA SZLACHECKA (WARSZAWA)	134.434
ŻONKILOWA (WARSZAWA)	134.38
PIASTOWSKA (WARSZAWA)	134.37
LUBICZÓW (LUBICZÓW)	134.334
POŻARYSKIEGO (WARSZAWA)	134.309
KANAŁ NOWE UJŚCIE (WARSZAWA)	134.227
CHABROWA (WARSZAWA)	134.119
OS.LEWANDÓW II (WARSZAWA)	134.08
KOLARSKA (WARSZAWA)	134.069
WYSPIAŃSKIEGO (ZĄBKI)	133.965
PILECKIEGO (WARSZAWA)	133.826
GRYFITÓW (WARSZAWA)	133.797
KRÓLOWEJ MARYSIEŃKI (WARSZAWA)	133.741
BRACI WAGÓW (WARSZAWA)	133.675
CZERWONA DROGA (WARSZAWA)	133.587
KŁOBUCKA (WARSZAWA)	133.492
ROKOKOWA (WARSZAWA)	133.448
CZATÓW (WARSZAWA)	133.425
OS.WYŻYNY (WARSZAWA)	133.375
WAGONOWNIA (WARSZAWA)	133.282
ROGATKOWA (WARSZAWA)	133.209
ĆWIKLIŃSKIEJ (WARSZAWA)	133.18
KSIĘŻNEJ ANNY (WARSZAWA)	133.049
ŁASAKA (WARSZAWA)	132.948
SPARTAŃSKA (WARSZAWA)	132.894
INSTYTUT REUMATOLOGII (WARSZAWA)	132.894
TERMINAL CARGO (WARSZAWA)	132.866
IDZIKOWSKIEGO (SULEJÓWEK)	132.864
WYCZÓŁKI (WARSZAWA)	132.855
STĘŻYCKA (WARSZAWA)	132.849
WIERTNICZA (WARSZAWA)	132.767
PŁASKOWICKIEJ (WARSZAWA)	132.684
OSMAŃSKA (WARSZAWA)	132.659
KOŚCIUSZKI (PRUSZKÓW)	132.655
SOCHACZEWSKA (MACIERZYSZ)	132.608
KINO BAŚŃ (PIASTÓW)	132.58
DĄBROWSKIEGO (PIASTÓW)	132.58
WIERNA (WARSZAWA)	132.561
INSTALATORÓW-KIS (WARSZAWA)	132.491
TORFOWA (WARSZAWA)	132.36
(TUNELOWA) (WARSZAWA)	132.221
CYNAMONOWA (WARSZAWA)	132.16
PIELĘGNIAREK (WARSZAWA)	131.92
NOWODWORY (WARSZAWA)	131.702
ZAGŁOBY (WARSZAWA)	131.695
TABOROWA (WARSZAWA)	131.645
KOCJANA-SĄDY (WARSZAWA)	131.594
POMORSKA (WARSZAWA)	131.453
TALAROWA (WARSZAWA)	131.262
PKP PŁUDY (WARSZAWA)	131.163
ANTONIEWSKA (WARSZAWA)	131.055
FORTOWA (BLIZNE ŁASZCZYŃSKIEGO)	131.052
SZKOLNA (MARKI)	131.04
BUŁAWY (WARSZAWA)	130.985
ZABRANIECKA-MAGAZYNY (WARSZAWA)	130.951
MIGDAŁOWA (WARSZAWA)	130.944
WIŚLANA (WARSZAWA)	130.943
SAMOTNEJ MATKI (WARSZAWA)	130.943
PODOLSKA (WARSZAWA)	130.868
NIZINNA (WARSZAWA)	130.519
STATKOWSKIEGO (WARSZAWA)	130.478
POCZTY GDAŃSKIEJ (WARSZAWA)	130.465
ORCHOWIECKA (WARSZAWA)	130.362
ŁĄCZYNY (WARSZAWA)	130.101
RĘBUSOWA (WARSZAWA)	129.877
CM.PÓŁNOCNY-BRAMA PŁD. (WARSZAWA)	129.846
BELGRADZKA (WARSZAWA)	129.845
GONTARSKA (WARSZAWA)	129.722
OŚ KRÓLEWSKA (WARSZAWA)	129.653
DROBIARSKA (WARSZAWA)	129.362
LOKAJSKIEGO (WARSZAWA)	129.349
STOICKA (WARSZAWA)	129.313
BRONISŁAWA CZECHA-LAS (WARSZAWA)	129.236
MIELCZARSKIEGO (WARSZAWA)	128.99
LATCHORZEW (LATCHORZEW)	128.874
KĄTY GRODZISKIE-OSIEDLE (WARSZAWA)	128.715
JUHASÓW (WARSZAWA)	128.614
BOHUNA (WARSZAWA)	128.591
MONTAŻOWA (WARSZAWA)	128.508
ZARUBY (WARSZAWA)	128.483
RUMIANKOWA (WARSZAWA)	128.469
ODKRYTA (WARSZAWA)	128.453
BRUSZEWSKA (WARSZAWA)	128.447
ALUZYJNA (WARSZAWA)	128.415
WILANÓW (WARSZAWA)	128.4
BOGATKI (WARSZAWA)	128.198
KAMYKOWA (WARSZAWA)	128.181
OKNICKA (WARSZAWA)	127.996
BRONOWSKA (WARSZAWA)	127.96
PADEREWSKIEGO (WARSZAWA)	127.883
CHUDOBY (WARSZAWA)	127.878
TERMINAL AUTOKAROWY (WARSZAWA)	127.755
STRUMYKOWA (WARSZAWA)	127.634
STARA MIŁOSNA (WARSZAWA)	127.543
SZAFARZY (WARSZAWA)	127.525
OS.KABATY (WARSZAWA)	127.483

JAWCZYCE (JAWCZYCE)	127.465
SZWOLEŻERÓW (ZĄBKI)	127.365
URZĄD MIASTA (ZĄBKI)	127.16
PALISADOWA (WARSZAWA)	127.118
PARTYZANTÓW (PRUSZKÓW)	127.098
SZKOLNA (MACIERZYSZ)	127
RYNEK (STARE BABICE)	126.793
OS.DERBY III (WARSZAWA)	126.764
OSTRÓDZKA-SZKOŁA (WARSZAWA)	126.722
SZARADY (WARSZAWA)	126.711
MASZEWSKA (WARSZAWA)	126.65
MARMUROWA (WARSZAWA)	126.551
KARTEZJUSZA (WARSZAWA)	126.419
WIELKIEGO DĘBU (WARSZAWA)	126.299
KSIĘŻYCOWA (WARSZAWA)	126.197
KOŚCIELNY (WARSZAWA)	126.179
RADOŚĆ-WODOCIĄG (WARSZAWA)	126.106
KOLUMBA (WARSZAWA)	125.996
GŁ. (WARSZAWA)	125.868
REMBERTÓW-AON (WARSZAWA)	125.843
POGODNA (WARSZAWA)	125.786
KOŁŁĄTAJA (ZĄBKI)	125.716
ŁOWCZA (WARSZAWA)	125.696
MANDARYNKI (WARSZAWA)	125.626
PKP PRUSZKÓW (PRUSZKÓW)	125.536
TRAKT LUBELSKI-LAS (WARSZAWA)	125.444
MOTYLKOWA (WARSZAWA)	125.293
TUŁOWICKA (WARSZAWA)	124.957
ŻERAŃ WSCHODNI (WARSZAWA)	124.932
AL.WOJSKA POLSKIEGO (PRUSZKÓW)	124.868
STRAŻACKA (WARSZAWA)	124.851
WÓLCZYŃSKA-BOCZNICA (WARSZAWA)	124.843
BURSZTYNOWA-SZPITAL (WARSZAWA)	124.687
AUTOKAROWY (WARSZAWA)	124.683
HUBALA-DOBRZAŃSKIEGO (WARSZAWA)	124.662
SANTOCKA (WARSZAWA)	124.585
ARESZT ŚLEDCZY SŁUŻEWIEC (WARSZAWA)	124.531
ANNY JAGIELLONKI (WARSZAWA)	124.195
NA SKRAJU (WARSZAWA)	124.193
ESTRADY (WARSZAWA)	124.188
REGULSKA (WARSZAWA)	124.161
KŁOBUCKA-HCB (WARSZAWA)	124.095
PARK MŁOCIŃSKI (WARSZAWA)	124.062
STRAŻACKA-MAGAZYNY (WARSZAWA)	124.031
ZABRANIECKA-BOCZNICA (WARSZAWA)	123.999
BIAŁOŁĘKA (WARSZAWA)	123.928
KALORYCZNA (WARSZAWA)	123.845
MATUSZEWSKA (WARSZAWA)	123.818
WIERUCHÓW (WIERUCHÓW)	123.717
MAŁEJ ŁĄKI (WARSZAWA)	123.656
SZOSA LUBELSKA (WARSZAWA)	123.581
GRZYMALITÓW (WARSZAWA)	123.351
POMOLOGICZNA (WARSZAWA)	123.342
HENRYKA PROBUSA (WARSZAWA)	123.326
KOŚCIUSZKI (MARKI)	123.266
PREFABET (WARSZAWA)	123.171
BUKÓW (WARSZAWA)	123.169
MŁOCINY-UKSW (WARSZAWA)	123.145
STOCZNIOWCÓW (WARSZAWA)	123.06
TORUŃSKA (PIASTÓW)	123.03
KOSIARZY (WARSZAWA)	122.958
INSTALATORÓW (WARSZAWA)	122.918
ŁAGIEWNICKA (WARSZAWA)	122.812
VOGLA (WARSZAWA)	122.806
PL.OSTREJ BRAMY (WARSZAWA)	122.716
PKP ZACISZE-WILNO (WARSZAWA)	122.716
SADKOWSKA (WARSZAWA)	122.691
PŁN. (WARSZAWA)	122.689
FABRYCZNA (MARKI)	122.436
SPICHRZOWA (WARSZAWA)	122.385
SZASERÓW (WARSZAWA)	122.254
LNIANA (WARSZAWA)	122.216
BOHATERÓW WARSZAWY (PRUSZKÓW)	122.04
ORLA (ZĄBKI)	122.038
PARCELACYJNA (WARSZAWA)	121.988
PKP RADOŚĆ (WARSZAWA)	121.96
KLIMATYCZNA (WARSZAWA)	121.928
CMENTARZ (ZĄBKI)	121.892
NOWE BABICE (NOWE BABICE)	121.827
RYBAŁTÓW (WARSZAWA)	121.755
NAUCZYCIELSKA (MARKI)	121.592
EC SIEKIERKI (WARSZAWA)	121.539
BRONISZE (BRONISZE)	121.506
PUŁASKIEGO (ZĄBKI)	121.497
WODNIAKÓW (WARSZAWA)	121.386
WÓLCZYŃSKA (WARSZAWA)	121.272
REMBERTÓW-KOLONIA (WARSZAWA)	121.268
WOLICA (WOLICA)	121.088
ECHA LEŚNE (WARSZAWA)	120.865
ZBYSZKA Z BOGDAŃCA (WARSZAWA)	120.841
CIOŁKOSZA (WARSZAWA)	120.631
ZBÓJNOGÓRSKA (WARSZAWA)	120.582
ŚWIĄTYNIA OPATRZNOŚCI BOŻEJ (WARSZAWA)	120.4
KIEPURY (WARSZAWA)	120.371
TADŻYCKA (WARSZAWA)	120.339
SIEKIERKI (WARSZAWA)	120.264
JEZIOROWA (WARSZAWA)	120.23
WĄWOZOWA (WARSZAWA)	120.154

RADIOWO (WARSZAWA)	120.08
MAŁYCH DĘBÓW (WARSZAWA)	120.004
PRUSA (PRUSZKÓW)	119.986
MŁYN (RASZYN)	119.672
DĄBRÓWKA WIŚLANA (WARSZAWA)	119.671
OPŁOTEK (WARSZAWA)	119.309
JANOWSKIEGO (LATCHORZEW)	119.097
LOTNISKO CHOPINA-PRZYLOTY (WARSZAWA)	118.97
KS.SKORUPKI (ZĄBKI)	118.903
MOZAIKOWA (WARSZAWA)	118.774
WIĄZANA (WARSZAWA)	118.764
LEBIODOWA (WARSZAWA)	118.736
ZAWODZIE (WARSZAWA)	118.731
MIŁOSNA (WARSZAWA)	118.645
PELIKANÓW (WARSZAWA)	118.613
DZIEWANOWSKA (WARSZAWA)	118.519
POWSTAŃCÓW (ZĄBKI)	118.482
DANUSI (WARSZAWA)	118.462
WOŁODYJOWSKIEGO (STARE BABICE)	118.452
ZIELENIECKA (ZĄBKI)	118.3
ZDZIARSKA-KANAŁ (WARSZAWA)	118.256
GWARKÓW (WARSZAWA)	118.178
WARYŃSKIEGO (ZIELONKA)	118.109
POD BUDOWLANI II (WARSZAWA)	117.959
WIERUCHOWSKA (STARE BABICE)	117.953
OS.PRZYLESIE (JABŁONNA)	117.93
BIAŁOŁĘKA-KANAŁ (WARSZAWA)	117.905
OŁÓWKOWA (PRUSZKÓW)	117.878
KOMBAJNISTÓW (WARSZAWA)	117.877
OS.DERBY (WARSZAWA)	117.805
BERENTA (PRUSZKÓW)	117.784
CHOPINA (WARSZAWA)	117.737
KWIATOWA (ZĄBKI)	117.624
WAWRZYSZEW (WARSZAWA)	117.604
DZIERZGOŃSKA (WARSZAWA)	117.587
URZĄD SKARBOWY (PRUSZKÓW)	117.538
JERZEGO (WARSZAWA)	117.478
TOMASZEW (WARSZAWA)	117.472
CMENTARZ (STARE BABICE)	117.435
ŻUBROWA-UKSW (WARSZAWA)	117.3
PLANETOWA (WARSZAWA)	117.163
AKCENT (WARSZAWA)	117.144
STAROSTWO (OŻARÓW MAZOWIECKI)	117.067
POWSINEK (WARSZAWA)	117.026
BIELSZOWICKA (WARSZAWA)	116.998
SPORTOWA (RASZYN)	116.931
TOPOGRAFICZNA (WARSZAWA)	116.844
ORZECHOWA (STRZYKUŁY)	116.797
WÓLKA WĘGLOWA (WARSZAWA)	116.741
ZAKRĘTOWA (WARSZAWA)	116.504
OŁÓWKOWA (WARSZAWA)	116.352
ZABRANIECKA (WARSZAWA)	116.235
JURANDA ZE SPYCHOWA-SZKOŁA (WARSZAWA)	116.165
OLSZOWA (WARSZAWA)	115.926
AUGUSTÓWKA (WARSZAWA)	115.858
PALMIRY (KAMPINOSKI PN)	115.673
HOTELOWA (SULEJÓWEK)	115.665
KAWĘCZYN POD (WARSZAWA)	115.563
RADIOWO-BOCZNICA (WARSZAWA)	115.562
SZEROKA (WARSZAWA)	115.545
REJTANA (MARKI)	115.467
KAPELI (WARSZAWA)	115.465
EC KAWĘCZYN (WARSZAWA)	115.228
EUROPEJSKA (WARSZAWA)	115.195
REYMONTA (LATCHORZEW)	115.103
BRZEZIŃSKA (WARSZAWA)	115.041
MAŃKI-WOJDY (WARSZAWA)	114.94
SANKTUARIUM (OŻARÓW MAZOWIECKI)	114.933
MAŁEGO RYCERZA (WARSZAWA)	114.91
DŁUGA (WOLICA)	114.818
CUKROWNICZA (WARSZAWA)	114.731
PKP GOŁĄBKI (WARSZAWA)	114.663
HANKI ORDONÓWNY (WARSZAWA)	114.643
OGIŃSKIEGO (PIASTÓW)	114.615
ZWOLEŃSKA (WARSZAWA)	114.484
SPB KAWĘCZYN (WARSZAWA)	114.427
BUCHNIK (JABŁONNA)	114.423
ZABŁUDOWSKA (WARSZAWA)	114.417
PRUSZKÓW (PRUSZKÓW)	114.187
PEONII (WARSZAWA)	114.181
PRZYJAŹNA (WARSZAWA)	114.162
DOBKA Z OLEŚNICY (WARSZAWA)	113.987
DWORKOWA (JABŁONNA)	113.975
TOMASZOWSKA (WARSZAWA)	113.937
OSIEDLE (WARSZAWA)	113.886
POWOJOWA (WARSZAWA)	113.676
SZKOLNA (RASZYN)	113.445
JAGIELSKA (WARSZAWA)	113.369
WIELOWIEJSKA (WARSZAWA)	113.308
MOŚCISKA (WARSZAWA)	113.289
PAŁACOWA (WARSZAWA)	113.247
THOMME (STARE BABICE)	113.031
ZERZEŃ (WARSZAWA)	112.851
SZPITAL POWIATOWY (PRUSZKÓW)	112.829
ENERGETYCZNA (WARSZAWA)	112.805
KARCZUNKOWSKA (WARSZAWA)	112.722
WILKOWIECKA (WARSZAWA)	112.718
KOŚCIÓŁ (RASZYN)	112.592
LIMBY (WARSZAWA)	112.4

AKADEMIJNA (JABŁONNA)	112.242
STRZYKUŁY (STRZYKUŁY)	112.065
PRZYDROŻNA (WARSZAWA)	111.779
TŁUCHOWSKA (WARSZAWA)	111.776
ŁUCZNICZA (WARSZAWA)	111.666
GEN.KUTRZEBY (STARE BABICE)	111.611
KOLEJOWA (ZIELONKA)	111.575
ŁĄCZNICZEK AK (PRUSZKÓW)	111.451
POW (PRUSZKÓW)	111.451
SZYSZAKÓW (WARSZAWA)	111.429
DZIKA (ZĄBKI)	111.329
MAJDAN (MAJDAN)	111.279
BUCHALTERYJNA (WARSZAWA)	111.213
PUCHAŁY (FALENTY)	111.073
ZASADOWA (WARSZAWA)	111.048
ZDZIARSKA (WARSZAWA)	111.012
LAMINA (PIASECZNO)	110.971
CENTRUM ZDROWIA DZIECKA (WARSZAWA)	110.741
BORKÓW (WARSZAWA)	110.682
CHYLICZKOWSKA-TECHNIKUM (PIASECZNO)	110.646
ZASOBNA (WARSZAWA)	110.618
MOCHTYŃSKA-KANAŁ (WARSZAWA)	110.488
STADION (RASZYN)	110.266
KUSOCIŃSKIEGO (PIASECZNO)	110.264
PIASTÓW (PIASTÓW)	110.241
BAREI (WARSZAWA)	110.17
PAŃSKA (JABŁONNA)	110.11
PLANTOWA (PRUSZKÓW)	110.084
MYSIADŁO (MYSIADŁO)	109.95
DĄBRÓWKA (WARSZAWA)	109.934
PASZKÓW (PASZKÓW)	109.918
OSIEDLOWA (STARE BABICE)	109.771
SZARYCH SZEREGÓW (JABŁONNA)	109.711
UTRATA (WARSZAWA)	109.644
WARZELNICZA (WARSZAWA)	109.567
WSPÓLNA (MARKI)	109.552
PIASKOWA (JABŁONNA)	109.527
LEŚNA (JABŁONNA)	109.42
INSUREKCJI (WARSZAWA)	109.392
SKALNICOWA (WARSZAWA)	109.342
RYBNA (WARSZAWA)	109.319
SZAMOCIN (WARSZAWA)	109.314
LAWINOWA (WARSZAWA)	109.297
WAŁUSZEWSKA (WARSZAWA)	109.294
CYLICHOWSKA (WARSZAWA)	109.272
RATAJA (SULEJÓWEK)	109.262
TRENÓW-LAS (WARSZAWA)	109.178
PIOTRKÓWEK MAŁY (PIOTRKÓWEK MAŁY)	109.173
POLNYCH KWIATÓW (WARSZAWA)	109.16
SADY SGGW (WARSZAWA)	109.13
CZERWONYCH BERETÓW (WARSZAWA)	109.048
CZEKOLADOWA (WARSZAWA)	109.044
PĘCICE-MAJĄTEK (PĘCICE)	108.994
WOJSKA POLSKIEGO (PIASECZNO)	108.908
SZACHOWA (WARSZAWA)	108.698
KOBIAŁKA (WARSZAWA)	108.627
SZKOLNA (PIASECZNO)	108.623
SIKORSKIEGO (MOŚCISKA)	108.516
NOWOKABACKA (WARSZAWA)	108.422
WIDOKOWA (WARSZAWA)	108.316
CM.PÓŁNOCNY-BRAMA ZACH. (WARSZAWA)	108.316
PKP FALENICA (WARSZAWA)	108.199
PALMIRY-MUZEUM (KAMPINOSKI PN)	108.109
OS.BUKOWY DWOREK (JABŁONNA)	108.007
JANKI (JANKI)	107.796
MOCHTYŃSKA (WARSZAWA)	107.742
ARESZT ŚLEDCZY GROCHÓW (WARSZAWA)	107.742
ARONIOWA (WARSZAWA)	107.737
ENERGETYCZNA (PIASECZNO)	107.735
TROMBITY (WARSZAWA)	107.713
WSCHODNI (WARSZAWA)	107.652
IWICZNA (PIASECZNO)	107.59
PONIATOWSKIEGO (OŻARÓW MAZOWIECKI)	107.443
WARSZAWA URSUS-NIEDŹWIADEK (WARSZAWA)	107.438
ZIELONKI (ZIELONKI)	107.327
3 MAJA (OŻARÓW MAZOWIECKI)	107.321
NADARZYŃSKA (PIASECZNO)	107.163
GRUNTOWA (WARSZAWA)	106.977
MOKRY ŁUG (WARSZAWA)	106.887
OŚRODEK SZKOLENIOWY (WARSZAWA)	106.875
CENTRUM MODY (NADARZYN)	106.866
SZKOLNA (JABŁONNA)	106.792
ROSOCHATA (WARSZAWA)	106.639
IZABELIŃSKA (STARE BABICE)	106.376
PKP MIEDZESZYN (WARSZAWA)	106.323
KAMPINOSKA (WARSZAWA)	106.198
PĘCICE-KOŚCIÓŁ (PĘCICE)	106.071
PKP OŻARÓW MAZOWIECKI (OŻARÓW MAZOWIECKI)	106.057
WARSZAWA ZACHODNIA (WARSZAWA)	106.034
MAJDAN-STACJA PALIW (MAJDAN)	106.025
MOŚCICKIEGO (LIPKÓW)	105.992
HRABSKA DROGA (FALENTY)	105.955
SPIRALNA (WARSZAWA)	105.933
TURÓW (KOBYŁKA)	105.836
WARSZAWA URSUS (WARSZAWA)	105.745
OLCHY (WARSZAWA)	105.673
WARSZAWA WŁOCHY (WARSZAWA)	105.659
LISTONOSZY (WARSZAWA)	105.657

WOJTYSZKI (WARSZAWA)	105.631
MACZKA (ZĄBKI)	105.572
SOKOŁÓW (SOKOŁÓW)	105.443
CYKLAMENÓW (WARSZAWA)	105.434
SOBIESKIEGO (LEGIONOWO)	105.405
JARZĘBINOWA (WARSZAWA)	105.15
PARKOWA (PĘCICE)	105.045
OS.PATIO (WARSZAWA)	104.982
BOROWIECKA (WARSZAWA)	104.773
PAROWOZOWA (WARSZAWA)	104.605
SPORTOWA (ZIELONKI)	104.596
PKP JEZIORKI (WARSZAWA)	104.507
PUSTELNIK (MARKI)	104.465
RYBIE (RYBIE)	104.287
URZĄD MIASTA (PIASECZNO)	104.212
TRZYKROTKI (WARSZAWA)	104.181
PAŁAC (JABŁONNA)	104.097
RUCZAJ (WARSZAWA)	104.03
DŁUGORZECZNA (WARSZAWA)	103.991
SIWIŃSKIEGO (LEGIONOWO)	103.966
ZAGÓRZE-SANATORIUM (ZAGÓRZE)	103.819
GWARNA (SULEJÓWEK)	103.758
ŁUBINOWA (WARSZAWA)	103.687
OKRĘŻNA (WARSZAWA)	103.668
PRASOWA (WARSZAWA)	103.564
MIĘDZYLESIE (WARSZAWA)	103.514
PRZEKORNA (WARSZAWA)	103.446
ŻYRARDOWSKA (WARSZAWA)	103.431
ŁĄKOWA (LASKI)	103.393
TRENÓW (LASKI)	103.303
DZIAŁKOWA (JANKI)	103.26
OS.STASZICA (PRUSZKÓW)	103.22
NOWA (STARA IWICZNA)	103.182
OKRĄG (NOWA IWICZNA)	102.912
ANDRUTOWA (WARSZAWA)	102.824
RUBINOWA (JÓZEFOSŁAW)	102.823
MARKOWSKIEGO (PIASECZNO)	102.797
STADIONOWA (RASZYN)	102.626
ZAGŁOBY (IZABELIN)	102.562
ROMANTYCZNA (WARSZAWA)	102.454
KRÓLEWSKA (JABŁONNA)	102.43
POLNA (KONSTANCIN-JEZIORNA)	102.239
POWSTAŃCÓW WARSZAWY (PIASECZNO)	102.21
STRUMYKOWA (PIASECZNO)	102.182
KOLEJKA PIASECZYŃSKA (PIASECZNO)	102.159
WARSZAWA AL.JEROZOLIMSKIE (WARSZAWA)	102.113
ZIELONA (PIASECZNO)	101.992
ZGORZAŁA (ZGORZAŁA)	101.98
WAFLOWA (WARSZAWA)	101.931
GÓRASZKA (GÓRASZKA)	101.923
IMUZ (FALENTY)	101.919
ODRĘBNA (WARSZAWA)	101.88
POSTĘPU (LESZNOWOLA)	101.847
OBROŃCÓW WARSZAWY (OŻARÓW MAZOWIECKI)	101.833
RUTY (WARSZAWA)	101.752
LIPKÓW (LIPKÓW)	101.682
URZĄD MIASTA (LEGIONOWO)	101.593
KIERBEDZIA (OŻARÓW MAZOWIECKI)	101.584
FRACHTOWA (WARSZAWA)	101.566
WARSZAWA CENTRALNA (WARSZAWA)	101.516
MAŃKOWSKA (WARSZAWA)	101.446
PKP PIASECZNO (PIASECZNO)	101.325
CZEKANOWSKA (WARSZAWA)	101.295
DWORCOWA (PIASECZNO)	101.084
WARSZAWA WSCHODNIA (WARSZAWA)	101.015
TUZINOWA (WARSZAWA)	101.008
KLARYSEW (KONSTANCIN-JEZIORNA)	100.981
FALENICA (WARSZAWA)	100.913
PL.PONIATOWSKIEGO (NADARZYN)	100.907
ZORZY (KLAUDYN)	100.821
GRABINA (REMBELSZCZYZNA)	100.783
ZAKAMAREK (WARSZAWA)	100.76
RĘCZAJSKA (KOBYŁKA)	100.711
PÓSTĘPU (NOWA WOLA)	100.573
KOŚCIUSZKI (NADARZYN)	100.529
RYCHNOWSKA (WARSZAWA)	100.485
HORTENSJI (WARSZAWA)	100.451
NOWY SĘKOCIN (NOWY SĘKOCIN)	100.426
WARSZAWSKA (NADARZYN)	100.41
PASCHALISA-JAKUBOWICZA (LIPKÓW)	100.393
STRZYGŁOWSKA (WARSZAWA)	100.392
JONTKA (WARSZAWA)	100.287
SAPIEHY (IZABELIN)	100.187
NADARZYŃSKA (WARSZAWA)	100.179
HALLERA (MARKI)	100.137
WĄSKA (JÓZEFÓW)	100.091
MOKRY ŁUG-LEŚNICZÓWKA (WARSZAWA)	99.9534
POCZTOWA (LASKI)	99.8801
GERBERA (PIASECZNO)	99.8138
POD WILANÓWKA (WARSZAWA)	99.7877
PROMYKA (JAWOROWA)	99.7331
WARSZAWSKA (BIELAWA)	99.7244
WARSZAWSKA (ZIELONKI-WIEŚ)	99.713
PODKOWY (WARSZAWA)	99.6693
BOROWA (KONSTANCIN-JEZIORNA)	99.6435
3 MAJA (LEGIONOWO)	99.6123
KOSYNIERÓW (ZĄBKI)	99.5717
GAJOWA (ZĄBKI)	99.5717
WARSZAWA OCHOTA (WARSZAWA)	99.5297
SIENKIEWICZA (IZABELIN)	99.5075

KIELECKA (STARA IWICZNA)	99.2831
KOLIBRA (ZGORZAŁA)	99.024
WARSZAWA URSUS PŁN. (WARSZAWA)	99.0017
REMBELSZCZYZNA (REMBELSZCZYZNA)	98.9985
OGRODNICZA (SULEJÓWEK)	98.9052
OS.KONSTANCJA (BIELAWA)	98.8811
BYSTRZYCKA (WARSZAWA)	98.836
TYMOTKI (WARSZAWA)	98.8242
GAZOWNIA (REMBELSZCZYZNA)	98.8029
BORYSZEWSKA (WIĄZOWNA)	98.7991
PKP LEGIONOWO (LEGIONOWO)	98.7576
WĘDKARSKA (WARSZAWA)	98.7263
PRZELOTOWA (WARSZAWA)	98.6833
STARY SĘKOCIN (STARY SĘKOCIN)	98.6753
MIASTA (PIASECZNO)	98.4752
SOKOŁOWSKIEGO (LASKI)	98.3396
KAMPINOSKA (ŁOMIANKI)	98.3065
SĘKOCIN LAS (SĘKOCIN LAS)	98.25
AMBARAS (WARSZAWA)	98.2404
KAMELIOWA (NOWA WOLA)	98.2319
WIŚNIOWA GÓRA (WARSZAWA)	98.1223
GRANICZNA (NOWA IWICZNA)	98.1146
KSIĄŻĄT MAZOWIECKICH (PIASECZNO)	98.0071
PRZEPIÓRKI (NOWA WOLA)	97.9495
ZAJEZDNIA UTRATA (WARSZAWA)	97.8602
ROZJAZD OBORSKI (KONSTANCIN-JEZIORNA)	97.8041
CH BŁĘKITNE (LEGIONOWO)	97.7576
DOM KULTURY (PIASECZNO)	97.6932
HUSARSKA (LEGIONOWO)	97.6882
KRÓTKA (STARA IWICZNA)	97.566
JEZIORKI (WARSZAWA)	97.5264
WARSZAWA RAKOWIEC (WARSZAWA)	97.4834
DRACENY (WARSZAWA)	97.4722
URZĄD GMINY (WIĄZOWNA)	97.4212
SZKOLNA (LESZNOWOLA)	97.3991
AGATOWA (JÓZEFOSŁAW)	97.397
FALENTY-OSIEDLE (FALENTY)	97.3846
CMENTARZ (JABŁONNA)	97.329
RUSKOWY BRÓD (WARSZAWA)	97.3211
KOLONIA LESZNOWOLA (KOLONIA LESZNOWOLA)	97.1907
ROBOTNICZA (OŻARÓW MAZOWIECKI)	97.091
GLEBOWA (WARSZAWA)	97.0287
CHYLICE (CHYLICE)	96.9736
ZAMIENIE (ZAMIENIE)	96.8787
PIASECZNO (PIASECZNO)	96.8787
GRANICZNA (MARKI)	96.8234
KAJETANY (KAJETANY)	96.7834
UMIASTOWSKA (OŻARÓW MAZOWIECKI)	96.7006
KOBIAŁKA-SZKOŁA (WARSZAWA)	96.6926
LESZNOWOLA-POLE (LESZNOWOLA)	96.6328
SYTA (WARSZAWA)	96.6078
ZIELONKI-WIEŚ (ZIELONKI-WIEŚ)	96.5254
EKOLOGICZNA (KLAUDYN)	96.481
LIPKÓW-MUZEUM (LIPKÓW)	96.4218
OBJAZDOWA (JAWOROWA)	96.2949
17 STYCZNIA (PIASECZNO)	96.169
BYDGOSKA (KONSTANCIN-JEZIORNA)	96.1198
FIELDORFA (WOŁOMIN)	96.0634
URBANISTÓW (JULIANÓW)	96.0427
POMORSKA (PIASECZNO)	96.0103
SOWIŃSKIEGO (LEGIONOWO)	96.0076
GRANICZNA (MAGDALENKA)	95.9482
LATOSZKI (WARSZAWA)	95.9278
MAGDALENKA (MAGDALENKA)	95.9146
TAWUŁKOWA (WARSZAWA)	95.8996
OS.GRAPA (KONSTANCIN-JEZIORNA)	95.8975
AL.WOJSKA POLSKIEGO (KONSTANCIN-JEZIORNA)	95.8432
CZOŁGISTÓW (WARSZAWA)	95.8254
WANILIOWA (PIASECZNO)	95.7257
LESZNOWOLA (LESZNOWOLA)	95.7172
SOBIESKIEGO (KONSTANCIN-JEZIORNA)	95.7079
WIESIOŁKA (WARSZAWA)	95.6425
URZĄD GMINY (NADARZYN)	95.6272
OGÓRKOWA (WARSZAWA)	95.4962
KULTURY (PIASECZNO)	95.4242
WARSZAWA TORUŃSKA (WARSZAWA)	95.413
MERA-PNEFAL (WARSZAWA)	95.3894
KOMOROWSKA-HOTEL (NADARZYN)	95.3781
OS.BATOREGO (LEGIONOWO)	95.1783
DZIĘCIOŁA (WARSZAWA)	95.1637
MIRKOWSKA (KONSTANCIN-JEZIORNA)	95.147
WASILKOWSKA (WARSZAWA)	95.1461
PKP MICHALIN (JÓZEFÓW)	95.0658
MIRKÓW-SZKOŁA (KONSTANCIN-JEZIORNA)	95.0561
MICKIEWICZA (LEGIONOWO)	95.0252
FALENTY NOWE (FALENTY)	94.9853
WARSZAWA REDUTA ORDONA (WARSZAWA)	94.9664
WARSZAWA RAKÓW (WARSZAWA)	94.9664
CHYLICZKI (CHYLICZKI)	94.815
WARSZAWA ŻWIRKI I WIGURY (WARSZAWA)	94.7637
ALEKSANDRÓW (STANISŁAWÓW PIERWSZY)	94.7481
PRZEŁĘCZY (WARSZAWA)	94.7302
PODGÓRNA (KONIK NOWY)	94.5549
WARSZAWA PRAGA (WARSZAWA)	94.5464
URZĄD GMINY (IZABELIN)	94.5163
OCHOCZA (WARSZAWA)	94.3461
OS.KWIATOWA (NOWA IWICZNA)	94.3421
WARSZAWA ŚRÓDMIEŚCIE (WARSZAWA)	94.2848
PODLEŚNA (ŁAZY)	94.2529
RYCERSKA (LEGIONOWO)	94.2104

WARSZAWA OLSZYNKA GROCHOWSKA (WARSZAWA)	94.1943
OGRODY DZIAŁKOWE NA SKRAJU (WARSZAWA)	94.1899
STRUGA (MARKI)	94.1585
BARTOSZYCKA (WARSZAWA)	93.9795
SIEDLISKOWA (WARSZAWA)	93.9638
POŁUDNIOWA (KONSTANCIN-JEZIORNA)	93.9619
DZIKIEJ RÓŻY (NOWA WOLA)	93.9426
MŁOCIŃSKA (ŁOMIANKI)	93.9204
KOŁOBRZESKA (KONSTANCIN-JEZIORNA)	93.9121
LIPOWA (MAGDALENKA)	93.8878
KOSODRZEWINY (WARSZAWA)	93.8724
WARSZAWA GOŁĄBKI (WARSZAWA)	93.7178
ŁOZISKA (PIASECZNO)	93.6685
NOWA IWICZNA (NOWA IWICZNA)	93.5893
ZACHODNIA (WARSZAWA)	93.5664
SPÓŁDZIELNIA (DAWIDY)	93.5594
ANUSINEK (KĄTY WĘGIERSKIE)	93.5523
KLAUDYN (KLAUDYN)	93.5024
NOWA WOLA (NOWA WOLA)	93.5021
ZEGRZYŃSKA (LEGIONOWO)	93.4807
ZEGARYNKI (WARSZAWA)	93.4443
ŚNIADECKICH (KONSTANCIN-JEZIORNA)	93.4437
WARSZAWA REMBERTÓW (WARSZAWA)	93.4226
MODRA (KAJETANY)	93.408
JASNA (KONSTANCIN-JEZIORNA)	93.3968
WARSZAWA SŁUŻEWIEC (WARSZAWA)	93.3674
ZALESIE DOLNE (PIASECZNO)	93.3458
WARSZAWA STADION (WARSZAWA)	93.2892
SIELANKOWA (LEGIONOWO)	93.2116
WCZASOWA (KONSTANCIN-JEZIORNA)	93.1992
KOCZARGI STARE (KOCZARGI STARE)	93.047
MRĄGOWSKA (WARSZAWA)	93.0138
PAPIERNIA (KONSTANCIN-JEZIORNA)	92.991
STARZYŃSKIEGO (DAWIDY BANKOWE)	92.9371
KOSSAKA (OŻARÓW MAZOWIECKI)	92.9248
WILANOWSKA (JÓZEFOSŁAW)	92.8308
ZAŚCIANKOWA (WARSZAWA)	92.7364
ORĘŻNA (PIASECZNO)	92.6513
AKACJOWA (KOCZARGI STARE)	92.4704
KWITNĄCEJ AKACJI (WARSZAWA)	92.4351
MAJERANKOWA (WARSZAWA)	92.4069
DOM ARTYSTÓW (KONSTANCIN-JEZIORNA)	92.3821
POZNAŃSKA (OŁTARZEW)	92.3702
KSIĘŻYCOWA (STANISŁAWÓW PIERWSZY)	92.3429
PODBIAŁOWA (WARSZAWA)	92.3303
SŁOWACKIEGO (LEGIONOWO)	92.33
WENECKA (MARKI)	92.0828
ŁAZY (ŁAZY)	92.0539
KISIELEWSKIEGO (CHOTOMÓW)	91.983
JARA (WARSZAWA)	91.9324
SIENKIEWICZA (KONSTANCIN-JEZIORNA)	91.9288
PIASECZYŃSKA (KONSTANCIN-JEZIORNA)	91.8913
LERCHA (WARSZAWA)	91.875
WARSZAWA JEZIORKI (WARSZAWA)	91.8648
SPORTOWA (WIĄZOWNA)	91.8631
SKOLIMÓW (KONSTANCIN-JEZIORNA)	91.8113
DŁUGA (DAWIDY)	91.8053
WARSZAWA OKĘCIE (WARSZAWA)	91.8039
KOCZARGI NOWE (KOCZARGI NOWE)	91.8019
WILLOWA (NOWY PODOLSZYN)	91.7907
WIEJSKA (WOŁOMIN)	91.772
SZKOLNA (WIĄZOWNA)	91.7498
KOSAĆCOWA (WARSZAWA)	91.7075
ROSY (WARSZAWA)	91.673
PRZYLEŚNA (STANISŁAWÓW PIERWSZY)	91.6439
RYNEK (LEGIONOWO)	91.6335
OBORSKA (KONSTANCIN-JEZIORNA)	91.5013
WARSZAWA POWIŚLE (WARSZAWA)	91.4899
ŻWIROWA (BOBROWIEC)	91.4545
STAROWIEJSKA (NADARZYN)	91.4143
OLESIN (WARSZAWA)	91.3945
STEFANÓWKA (BORYSZEW)	91.3505
CICHA (JÓZEFÓW)	91.3223
WARSZAWA DAWIDY (WARSZAWA)	91.3086
PRZEBIŚNIEGÓW (NOWA IWICZNA)	91.2846
NADZIEI (PIASECZNO)	91.1169
KOMOROWSKA (NADARZYN)	91.006
OS.PIASKI (LEGIONOWO)	90.9811
PAŃSKA (KONSTANCIN-JEZIORNA)	90.8591
WARSZAWA PŁUDY (WARSZAWA)	90.8392
PIASTA (KONSTANCIN-JEZIORNA)	90.8286
URZĄD GMINY (LESZNOWOLA)	90.7325
TABITA (KONSTANCIN-JEZIORNA)	90.699
KOŚCIUSZKI-SZKOŁA (IZABELIN)	90.6192
TERESPOLSKA (KONIK NOWY)	90.6052
OD LASU (KONSTANCIN-JEZIORNA)	90.5684
GOŁĘBIA (KAJETANY)	90.564
KOŚCIELNA (KĄTY WĘGIERSKIE)	90.5483
OS.LAS (NIEPORĘT)	90.5129
POLNA (ŻABIENIEC)	90.4912
PL.KOŚCIUSZKI (LEGIONOWO)	90.4668
NAPOLEONA BONAPARTE (WARSZAWA)	90.4184
GMINY (WIĄZOWNA)	90.3005
ROZCHODNIKOWA (WARSZAWA)	90.2484
CMENTARZ (WÓLKA KOZODAWSKA)	90.1822
ARENA LEGIONOWO (LEGIONOWO)	90.1684
BYSŁAWSKA (WARSZAWA)	90.1321
MIKLASZEWSKIEGO (DAWIDY BANKOWE)	90.091
ZALESIE GÓRNE (ZALESIE GÓRNE)	90.0534

SCHRONISKO DLA ZWIERZĄT (WARSZAWA)	90.05
MOSTOWA (ŻABIENIEC)	90.0211
GRZYBOWA (ŁOMIANKI)	90.0001
WARSZAWA GOCŁAWEK (WARSZAWA)	89.9726
FALENTY DUŻE (FALENTY DUŻE)	89.9109
OBORY-PAŁAC (OBORY)	89.9085
OS.JAGIELLOŃSKA (LEGIONOWO)	89.8692
CHOPINA (KONSTANCIN-JEZIORNA)	89.6806
OLSZYNOWA (NOWY PODOLSZYN)	89.6769
JESIONOWA (MAGDALENKA)	89.5634
ZALESINEK (PIASECZNO)	89.4801
PODLEŚNA (KOCZARGI STARE)	89.4346
SZKOŁA (NOWA IWICZNA)	89.3012
WARSZAWA ŻERAŃ (WARSZAWA)	89.2058
KAPUTY (KAPUTY)	89.1937
WARSZAWA KASPRZAKA (WARSZAWA)	89.1539
KRASICKIEGO (NOWA WOLA)	89.1499
WARSZAWA ZACHODNIA WKD (WARSZAWA)	89.1143
WARSZAWA SALOMEA (WARSZAWA)	89.1143
NARCYZOWA (WARSZAWA)	89.0935
OŻARÓW MAZ. (OŻARÓW MAZOWIECKI)	89.048
PL.SPORTOWY (KONSTANCIN-JEZIORNA)	89.0229
BALETOWA (DAWIDY)	89.0106
CEGLANA (NADMA)	88.9949
LIPOWA (BIELAWA)	88.8551
KOMETY (JÓZEFOSŁAW)	88.8467
WARSZAWA ZOO (WARSZAWA)	88.7987
IZBICKA-CMENTARZ (WARSZAWA)	88.7728
GOŁKÓW-LETNISKO (PIASECZNO)	88.7495
WRZOSOWA (KOCZARGI STARE)	88.6879
DWORSKA (WARSZAWA)	88.5962
PKP NOWA IWICZNA (NOWA IWICZNA)	88.5934
NIZINNA (OŁTARZEW)	88.4345
PKP JÓZEFÓW (JÓZEFÓW)	88.434
GŁOGI (NIEPORĘT)	88.3844
LEGIONOWA (MARKI)	88.3525
BORYNY (DAWIDY BANKOWE)	88.3388
WARSZAWA WESOŁA (WARSZAWA)	88.3308
TOLAK (JANCZEWICE)	88.3093
WOJCIESZYN (WOJCIESZYN)	88.2235
LUTOSŁAWSKIEGO (KLAUDYN)	88.2163
CIEĆWIERZA (KLAUDYN)	88.2163
ROLNA (KAJETANY)	88.1408
STANISŁAWÓW (STANISŁAWÓW PIERWSZY)	88.1344
LWOWSKA (LEGIONOWO)	88.1108
ROLNA (ŁAZY)	88.1077
GMINNEJ RADY NARODOWEJ (LESZNOWOLA)	88.0927
BIAŁOSTOCKA (KONSTANCIN-JEZIORNA)	88.0689
OGRÓD BOTANICZNY (WARSZAWA)	88.0473
RADIOSTACJA ŁAZY (ŁAZY)	88.0385
JACHOWICZA (WARSZAWA)	88.0029
DOLNA (KONSTANCIN-JEZIORNA)	87.9766
JANÓW (JANÓW)	87.9663
ZAWADY (WARSZAWA)	87.9513
STRUŻAŃSKA (LEGIONOWO)	87.9282
GRÓDEK (EMÓW)	87.9207
AL.JEDNOŚCI (LESZNOWOLA)	87.888
KOŚCIÓŁ (JAZGARZEW)	87.8804
WILANÓWKA (OKRZESZYN)	87.8131
SIERAKOWSKA (IZABELIN)	87.7532
KWIATÓW POLNYCH (DAWIDY BANKOWE)	87.702
SIELSKA (STANISŁAWÓW PIERWSZY)	87.6883
PĘCLIŃSKA (WIĄZOWNA KOŚCIELNA)	87.6863
STARA (NADMA)	87.6859
LEGIONÓW (WOŁOMIN)	87.685
RUSIEC (RUSIEC)	87.6622
JANCZEWICE (JANCZEWICE)	87.6317
STOCER (KONSTANCIN-JEZIORNA)	87.5513
WARSZAWA LOTNISKO CHOPINA (WARSZAWA)	87.5303
PARLAMENTARNA (DAWIDY BANKOWE)	87.4973
CYGAŃSKA (WARSZAWA)	87.4292
HABDZIN I (HABDZIN)	87.4158
WILCZA GÓRA (WILCZA GÓRA)	87.3684
ŁADY-SZKOŁA (DAWIDY BANKOWE)	87.297
ŁYCZYŃSKA (SŁOMCZYN)	87.235
PKP NIEPORĘT (NIEPORĘT)	87.23
LEGIONOWO (LEGIONOWO)	87.1491
KACZEŃCÓW (ŁAZY)	87.0365
MACIERZANKI (NADARZYN)	87.0348
KONIK NOWY (KONIK NOWY)	86.9765
SREBRNYCH ŚWIERKÓW (PIASECZNO)	86.947
JASNA (CHOTOMÓW)	86.9026
KS.SŁOJEWSKIEGO (ŁAZY)	86.8983
CMENTARZ (ŁAZY)	86.8627
WARSZAWA WAWER (WARSZAWA)	86.8533
BORYSZEW (BORYSZEW)	86.8328
TURKAWKI (JASTRZĘBIE)	86.7424
AKACJOWA (KĄTY WĘGIERSKIE)	86.6109
KS.SKORUPKI (LEGIONOWO)	86.5945
CEDROWA (KONSTANCIN-JEZIORNA)	86.5016
WARSZAWA KOŁO (WARSZAWA)	86.4803
DŁUGA (ŁOMIANKI)	86.452
MAGNOLII (JÓZEFOSŁAW)	86.393
KAZIMIERZOWSKA (KONSTANCIN-JEZIORNA)	86.3008
SZKOLNA (JÓZEFÓW k. LEGIONOWA)	86.2111
JAROSŁAWSKA (JÓZEFÓW)	86.1939
POD KAJETANY (SZAMOTY)	86.1273
OGRODY DZIAŁKOWE PALUCH (WARSZAWA)	86.1055
DROHOBYCKA (WARSZAWA)	86.0754

SAMORZĄDOWA (WARSZAWA)	85.9875
WIŚNIOWA (STANISŁAWÓW DRUGI)	85.9044
OBORY (OBORY)	85.8395
ŻYZNA (KAPUTY)	85.811
KROKUSÓW (WARSZAWA)	85.7868
WOLSKA (MICHAŁÓW-REGINÓW)	85.6943
SŁOMCZYN-SZKOŁA (SŁOMCZYN)	85.673
FALENCKA (JANKI)	85.5832
BŁĘKITNE (LEGIONOWO)	85.4914
KAMERALNA (JÓZEFOSŁAW)	85.4912
STEFANÓW (ŻABIENIEC)	85.4795
WITWICKIEGO (KONSTANCIN-JEZIORNA)	85.4259
KRÓLOWEJ MARYSIEŃKI (WOJCIESZYN)	85.404
WOJCIESZYN-GÓRKI (WOJCIESZYN)	85.4001
WARSZAWA CHOSZCZÓWKA (WARSZAWA)	85.3991
AL.SYBIRAKÓW (LEGIONOWO)	85.3582
NOWE SŁUPNO (SŁUPNO)	85.1982
GŁOWACKIEGO (KONSTANCIN-JEZIORNA)	85.1922
MARYSIN (MARYSIN)	85.1252
SKOLIMÓW PÓŁNOCNY (KONSTANCIN-JEZIORNA)	85.1012
JULIANÓW (JULIANÓW)	85.0908
URZĄD GMINY (NIEPORĘT)	84.9616
JESÓWKA (JESÓWKA)	84.9523
ALEJA RÓŻ (LEGIONOWO)	84.8683
GOŁASZEWSKA (OŁTARZEW)	84.7739
GOŁKÓW-WIEŚ (GOŁKÓW)	84.7269
PARKOWA (LEGIONOWO)	84.7082
RADIÓWEK (RADIÓWEK)	84.6463
KS.SZULCZYKA (WARSZAWA)	84.6252
ŚWIDER-KOŚCIÓŁ (OTWOCK)	84.6222
SZAMOTY (SZAMOTY)	84.5683
WARSZAWA GDAŃSKA (WARSZAWA)	84.5312
METRYCZNA (WARSZAWA)	84.4438
WARSZAWA ANIN (WARSZAWA)	84.4254
PŁOCHOCIN (PŁOCHOCIN)	84.4081
KULESZÓWKA (KULESZÓWKA)	84.3061
PROJEKTOWANA (WIĄZOWNA KOŚCIELNA)	84.2799
ZIELONA (WŁADYSŁAWÓW)	84.2738
URZUT (URZUT)	84.2533
WARSZAWA WOLA GRZYBOWSKA (WARSZAWA)	84.2459
KRAKUSA (LEGIONOWO)	84.2286
OKRZESZYN (OKRZESZYN)	84.2225
VIVALDIEGO (KLAUDYN)	84.2038
SZYMANOWSKIEGO (KLAUDYN)	84.2038
ŚWIDER (OTWOCK)	84.1963
LANGIEWICZA (IZABELIN)	84.0563
BIELAWA-OSP (BIELAWA)	84.0079
JACHTOWA (WARSZAWA)	84.0024
RODZINNA (STANISŁAWÓW PIERWSZY)	83.8764
KUKUŁCZA (JASTRZĘBIE)	83.8644
JABŁONIOWA (SŁOMCZYN)	83.8547
LIPY (NIEPORĘT)	83.8241
ŁAZY-SZKOŁA (ŁAZY)	83.8106
MARZYCIELI (KĄTY WĘGIERSKIE)	83.7588
WIERZBIN (WIERZBIN)	83.6246
HIPOLITOWSKA (KONIK NOWY)	83.5991
CZARNA STRUGA (MARKI)	83.5002
BACZYŃSKIEGO (PARCELA)	83.5
ŁADY (ŁADY)	83.4983
KOZAKÓW (SZAMOTY)	83.4163
OS.NIEPODLEGŁOŚCI (WOŁOMIN)	83.4153
OBJAZDOWA (JÓZEFÓW k. LEGIONOWA)	83.2908
JODŁOWA (STRZENIÓWKA)	83.2684
PLANTY (IZABELIN)	83.2563
WYGODA (WÓLKA KOSOWSKA)	83.2336
ŁOMIANKI-CENTRUM (ŁOMIANKI)	83.2085
HABDZIN II (HABDZIN)	83.1992
BOTANICZNY (WARSZAWA)	83.165
CYNKOWA (LEGIONOWO)	83.1567
KWIRYNÓW (KWIRYNÓW)	83.1538
ZIELONA (GOŁKÓW)	83.0951
GMINY (LESZNOWOLA)	83.057
GŁÓWNA (GOŁKÓW)	82.9624
GŁÓWNA (JAZGARZEW)	82.9192
LOKALNA (ŁAZY)	82.8297
WSPÓLNA (DUCHNÓW)	82.8236
ZALESIE (WIERZBIN)	82.8225
CMENTARZ (KONSTANCIN-JEZIORNA)	82.8032
ZBÓJNA GÓRA (WARSZAWA)	82.7755
WÓLKA KOSOWSKA (WÓLKA KOSOWSKA)	82.7197
UMIASTOWSKA (KAPUTY)	82.7162
WŁADYSŁAWÓW (WŁADYSŁAWÓW)	82.5759
GOK (STANISŁAWÓW DRUGI)	82.554
POLNA (WŁADYSŁAWÓW)	82.5231
WARSZAWA MIĘDZYLESIE (WARSZAWA)	82.5202
CH WÓLKA KOSOWSKA (WÓLKA KOSOWSKA)	82.503
ZAGÓRZAŃSKA (WARSZAWA)	82.4507
BŁOTA (WARSZAWA)	82.3757
BIAŁOBRZEGI (BIAŁOBRZEGI)	82.3303
WARSZAWSKA (MICHAŁÓW-REGINÓW)	82.1293
PALUCH (WARSZAWA)	81.967
SŁUPNO (SŁUPNO)	81.926
LEGIONOWO PIASKI (LEGIONOWO)	81.8395
ŁYCZYN (ŁYCZYN)	81.8388
RUDKA (RUDKA)	81.8013
JÓZEFÓW (JÓZEFÓW)	81.7886
PILAWA (PILAWA)	81.6931

ZGK (KONSTANCIN-JEZIORNA)	81.6613
STOKROTKI (NIEPORĘT)	81.6572
PORT NIEPORĘT (NIEPORĘT)	81.6572
ZIMORODKA (JASTRZĘBIE)	81.6179
GAJOWA (WOLA ALEKSANDRA)	81.5502
PIASKOWA (LEGIONOWO)	81.468
POLNA (LEGIONOWO)	81.4068
SPOKOJNA (ŻANĘCIN)	81.2858
PL.SZWEDZKI (JANKI)	81.2487
KOPRKI (KOPRKI)	81.2449
BARTYKI (WARSZAWA)	81.2135
WARSZAWA RADOŚĆ (WARSZAWA)	81.2088
KOSTOWIEC (KOSTOWIEC)	81.1092
WICHROWA (JESÓWKA)	80.9951
GEN.ROI (LEGIONOWO)	80.9924
PRZYSTANEK (LEGIONOWO)	80.9228
LEGIONOWO PRZYSTANEK (LEGIONOWO)	80.9228
SULEJÓWEK (SULEJÓWEK)	80.9157
PROTAZEGO (NIEPORĘT)	80.8286
ORZECHOWA (WOLA GOŁKOWSKA)	80.7844
CZAJKOWSKIEGO (KLAUDYN)	80.7712
KĘPA OKRZEWSKA (KĘPA OKRZEWSKA)	80.7567
MICHALIN (JÓZEFÓW)	80.6894
BORZĘCIN DUŻY-SZKOŁA (BORZĘCIN DUŻY)	80.622
KLONOWA (STANISŁAWÓW PIERWSZY)	80.5763
WARSZAWA MIEDZESZYN (WARSZAWA)	80.4695
WARSZAWSKA (KONIK STARY)	80.3976
OGRODOWA (GŁOSKÓW-LETNISKO)	80.347
PKP LEGIONOWO-PRZYSTANEK (LEGIONOWO)	80.3093
KOŚCIÓŁ (ŁOMIANKI)	80.2171
SIKORECZKI (JASTRZĘBIE)	80.2058
ŻWIROWA (CIECISZEW)	80.2057
WARSZAWA FALENICA (WARSZAWA)	80.2049
OLSZYNA ŁYCZYŃSKA (PARCELA)	80.0844
WOLSKA II (STANISŁAWÓW DRUGI)	80.0627
WOLSKA I (WOLA ALEKSANDRA)	80.0373
PARKOWA (TRUSKAW)	80.0319
CMENTARZ (LEGIONOWO)	80.0042
BOCIANA CZARNEGO (JASTRZĘBIE)	80.0001
NOWY PODOLSZYN (NOWY PODOLSZYN)	79.9496
PARCELA I (PARCELA)	79.9086
UMIASTÓW (UMIASTÓW)	79.8635
KOLONIA WARSZAWSKA (KOLONIA WARSZAWSKA)	79.7365
HABDZIN III (HABDZIN)	79.6579
OTWOCK (OTWOCK)	79.5294
ORZECHOWA (LEGIONOWO)	79.4957
PKP MICHAŁÓW-REGINÓW (MICHAŁÓW-REGINÓW)	79.4694
KS.MALINOWSKIEGO (JÓZEFÓW)	79.4554
CHODKIEWICZA (IZABELIN)	79.4486
BIELAWA I (BIELAWA)	79.4165
GŁOSKÓW-LETNISKO (GŁOSKÓW-LETNISKO)	79.3893
OS.BUKOWIEC (LEGIONOWO)	79.3559
RYBNA (WOLA GOŁKOWSKA)	79.3305
WÓLKA MLĄDZKA (OTWOCK)	79.3168
LIPOWA (NADARZYN)	79.3034
JAZGARZEW-SZKOŁA (JAZGARZEW)	79.2521
SŁONECZNA (LEGIONOWO)	79.2014
LIPOWA (GŁOSKÓW-LETNISKO)	79.1181
HORNÓWEK (HORNÓWEK)	79.1153
CM.POŁUDNIOWY-BRAMA WSCH. (ANTONINÓW)	79.0149
GÓRKI (SŁUPNO)	78.9753
TUROWICE (TUROWICE)	78.8909
RZEMIEŚLNICZA (OTWOCK)	78.8627
BORZĘCIN DUŻY (BORZĘCIN DUŻY)	78.8311
KĄPIELOWA (BIAŁOBRZEGI)	78.7758
KAWĘCZYNEK II (KAWĘCZYNEK)	78.7402
SPACEROWA (DUCHNÓW)	78.7378
PÓŁNOCNA (KAMIONKA)	78.7038
BOROWINA I (BOROWINA)	78.6491
ŻANĘCIN (ŻANĘCIN)	78.5979
SŁOMCZYN-KOŚCIÓŁ (SŁOMCZYN)	78.4822
PKP LEGIONOWO PIASKI (LEGIONOWO)	78.4318
BRZÓZEK (ORZESZYN)	78.3868
WINOGRONOWA (ROZALIN)	78.1649
GRABALÓWKI (WARSZAWA)	78.1625
GRZYBOWSKA (PARCELA)	77.9508
WRZOSOWA (MICHAŁÓW-REGINÓW)	77.8804
BANDURSKIEGO (LEGIONOWO)	77.8692
MICHAŁÓW-REGINÓW (MICHAŁÓW-REGINÓW)	77.8586
PRZEBUDZENIA WIOSNY (ZALESIE GÓRNE)	77.7255
PARCELA II (PARCELA)	77.6453
SZKOLNA (IZABELIN)	77.6371
STAREGO DĘBU (CZARNÓW)	77.5011
AGRICOOP (KOPRKI)	77.4698
ICDS (ŁOMIANKI)	77.4339
WIERZBOWA (HIPOLITÓW)	77.2551
POGROSZEW (POGROSZEW)	77.2169
NOWA (OTWOCK)	77.1614
LESZCZYNKA (MROKÓW)	77.1275
GŁOSKÓW-ZIELONE (GŁOSKÓW)	77.1113
CIECISZEW-OSP (CIECISZEW)	77.0804
OKRZEWSKA-CMENTARZ (KĘPA OKRZEWSKA)	77.038
ZEGRZE PŁD. (ZEGRZE PŁD.)	77.0151
PORT PILAWA (BIAŁOBRZEGI)	77.0151
PODLEŚNA (JÓZEFÓW)	76.8803
KOŚCIUSZKI (LEGIONOWO)	76.7531
OS.MŁODYCH (LEGIONOWO)	76.6964
PIASKOWA (ŁAJSKI)	76.6886

3 MAJA (TRUSKAW)	76.5608
ANUSIN (HABDZIN)	76.5449
SULEJÓWEK MIŁOSNA (SULEJÓWEK)	76.4896
MIŁOSNA (SULEJÓWEK)	76.4896
ŁBISKA (ŁBISKA)	76.2656
CMENTARZ 1920 R. (RADZYMIN)	76.2565
PODOLSZYN (PODOLSZYN)	76.2002
DZIECHCINIEC-KOLONIA (DZIECHCINIEC)	76.1572
BOROWINA II (BOROWINA)	76.1472
BORZĘCIN DUŻY-WIEŚ (BORZĘCIN DUŻY)	75.9095
WCZAŚOWA (BIAŁOBRZEGI)	75.7896
SIERAKÓW (SIERAKÓW)	75.5703
KAWĘCZYN (KAWĘCZYN)	75.4819
ŚRÓDBORÓW (OTWOCK)	75.4223
KASZTANOWA (ORZESZYN)	75.4205
KAWĘCZYNEK I (KAWĘCZYNEK)	75.3995
ROZALIN (ROZALIN)	75.3512
MROKÓW (MROKÓW)	75.3259
TEKLIN (OTWOCK)	75.2722
HOSERÓW (WARSZAWA)	75.1649
ZALEŚNA (KAMIONKA)	75.0861
KOSMOWSKA I (BORZĘCIN DUŻY)	75.0532
KORALOWYCH DĘBÓW (ZALESIE GÓRNE)	74.9772
IZABELIŃSKA (IZABELIN)	74.9734
ARMII POZNAŃ (ŁOMIANKI)	74.8193
POGROSZEW KOLONIA (POGROSZEW KOLONIA)	74.746
NOWA WIEŚ (JÓZEFÓW)	74.5755
NOWODWORSKA (ŁAJSKI)	74.5642
PŁD. (ANTONINÓW)	74.5301
SZARYCH SZEREGÓW (LEGIONOWO)	74.3619
WYGLĘDY (WYGLĘDY)	74.1502
BOROWINA III (BOROWINA)	74.069
WIELISZEW (WIELISZEW)	73.9646
DZIECHCINIEC-SKLEP (DZIECHCINIEC)	73.9267
HALINÓW (HIPOLITÓW)	73.8106
PĘCHERY (PĘCHERY)	73.7685
HABDZIN IV (HABDZIN)	73.7285
WIERZBOWA (RADZYMIN)	73.7125
TOPOLOWA (KAWĘCZYNEK)	73.6916
JAŁOWCOWA (OTWOCK)	73.6517
POD GŁOSKÓW (GŁOSKÓW)	73.5101
MILENIUM (GŁOSKÓW)	73.5101
PIUSA XI (CHOTOMÓW)	73.4904
KOŚCIELNA (WIELISZEW)	73.3722
KAWĘCZYN DOLNY (KAWĘCZYN)	73.2165
WDW RYNIA (RYNIA)	73.1497
SZKOLNA (ŁAJSKI)	72.9791
SKIBIŃSKIEGO (TRUSKAW)	72.9474
STAROSTWO POWIATOWE (LEGIONOWO)	72.9257
WIEKOWEJ SOSNY (ZALESIE GÓRNE)	72.9234
KAWĘCZYN II (KAWĘCZYN)	72.7939
DOBIESZ (DOBIESZ)	72.7046
PAŁACOWA (ROZALIN)	72.5566
BRZOZOWA (WOLA MROKOWSKA)	72.5377
SOSENKOWA (ALEKSANDRÓW)	72.5304
POLNA (JÓZEFÓW)	72.4937
MYSZCZYN (MYSZCZYN)	72.4246
OGRODOWA (KIEŁPIN STARY)	72.3318
BOROWA (OTWOCK)	72.3295
PARK MIEJSKI (OTWOCK)	72.1414
KOSMOWSKA II (BORZĘCIN DUŻY)	71.9812
FABRYCZNA (ŁAJSKI)	71.9267
DZIECHCINIEC (DZIECHCINIEC)	71.8825
ZAWADOWSKA (WARSZAWA)	71.8565
BOBROWIECKA (BOBROWIEC)	71.7603
PISKÓRKA (PISKÓRKA)	71.7083
WOLA MROKOWSKA (WOLA MROKOWSKA)	71.5677
REYMONTA (OTWOCK)	71.3768
KS.CZARTORYSKIEJ (RADZYMIN)	71.295
SZPITAL ONKOLOGICZNY (WIELISZEW)	71.2556
ŁĘG (ŁĘG)	71.1191
SZLACHECKA (OTWOCK)	70.9962
WIOSENNA (ZABORÓW)	70.9817
TOPOLOWA (WYGLĘDY)	70.9083
RUNÓW OSADA (RUNÓW)	70.78
GŁOSKÓW-SZKOŁA (GŁOSKÓW)	70.78
SIWEK (BORKI)	70.7542
BRZEŚCE (BRZEŚCE)	70.6505
ŚWIDRY MAŁE (JÓZEFÓW)	70.6112
GRANICZNA (SZCZAKI)	70.4663
DĘBÓWKA (DĘBÓWKA)	70.3958
PĘCZKOWSKIEGO (NIEPORĘT)	70.2712
PILASZKÓW (PILASZKÓW)	70.228
LEŚNA (JAROSZOWA WOLA)	70.1728
OGRODOWA (SIERZCHÓW)	70.1687
LEŚNYCH BOGINEK (ZALESIE GÓRNE)	70.1622
PODGÓRNA (WIELISZEW)	70.1024
CHOTOMÓW (CHOTOMÓW)	70.0373
MALCANÓW-BOOS (MALCANÓW)	70.0105
GRANICZKA (KIEŁPIN STARY)	69.9178
ZŁOTA (WĄGRODNO)	69.7703
KANARKOWA (MŁOCHÓW)	69.4253
WÓLKA ZAŁĘSKA (WÓLKA DWORSKA)	69.2619
BORZĘCIN MAŁY (BORZĘCIN MAŁY)	69.1198
POLNA (WIELISZEW)	68.9947
NORWIDA (RADZYMIN)	68.9475
WYSZYŃSKIEGO (JÓZEFÓW)	68.9137

PKP ZALESIE GÓRNE (ZALESIE GÓRNE)	68.7382
POD SZCZAKI (SZCZAKI)	68.7318
CZERNIDŁA (CZERNIDŁA)	68.6348
WOLICA (STARE ZAŁUBICE)	68.5428
HOŻA (OTWOCK)	68.5181
WIERZBOWA (BASZKÓWKA)	68.5126
RUNÓW OSADA II (RUNÓW)	68.5126
SZPITAL POWIATOWY (OTWOCK)	68.5103
ZABORÓW-SZKOŁA (ZABORÓW)	68.4899
KOTLINY (MALCANÓW)	68.338
LEŚNA (WIKTORÓW)	68.3343
BAJECZNA (BOBROWIEC)	68.2649
PRZYJACIÓŁ (NIEPORĘT)	68.175
ŁAŹNIEWSKA (ŚWIĘCICE)	68.1308
PARTYZANTÓW (OTWOCK)	68.1169
DĘBÓWKA II (DĘBÓWKA)	68.0365
PÓSTĘPU (JASTRZĘBIEC)	68.0193
NOWINEK (ZALESIE GÓRNE)	67.9457
AKACJOWA (SIERZCHÓW)	67.7414
SOLIDARNOŚCI (WIELISZEW)	67.7038
POWSTAŃCÓW WARSZAWY (OTWOCK)	67.6014
INSTYTUT EKOLOGII PAN (DZIEKANÓW LEŚNY)	67.4767
DROGOWCÓW (JÓZEFÓW)	67.3782
ŁANOWA (SZCZAKI)	67.3563
MODLIŃSKA (WIELISZEW)	67.3297
WIERZBOWA (LIPOWO)	66.8836
SZCZAKI (SZCZAKI)	66.6351
ZŁOTOKŁOS (ZŁOTOKŁOS)	66.626
KORCZAKA (RADZYMIN)	66.5683
HENRYKÓW-UROCZE (HENRYKÓW-UROCZE)	66.5444
BASZKÓWKA (BASZKÓWKA)	66.5444
ZEGRZYŃSKA (STARE ZAŁUBICE)	66.4734
WÓLKA DWORSKA (WÓLKA DWORSKA)	66.4194
OLSZOWA (OTWOCK)	66.4077
DĘBOWA (UWIELINY)	66.3355
JANIN (ZABORÓW)	66.2894
KU SŁOŃCU (MOCZYDŁÓW)	66.2888
NOWOLIPIE (NIEPORĘT)	66.2535
GASSY (GASSY)	66.158
ZABOROWSKA (ŚWIĘCICE)	66.1031
CHABROWA (WIKTORÓW)	66.0303
PÓŁNOCNA (WOLA PRAŻMOWSKA)	66.0274
NOWE WĄGRODNO (WĄGRODNO)	66.0274
DZIKIEJ RÓŻY (BORZĘCIN MAŁY)	66.0089
GÓRKI (JÓZEFÓW)	65.9997
SŁOWICZA (USTANÓW)	65.9923
PODŁĘCZE (PODŁĘCZE)	65.9828
LIPOWO (LIPOWO)	65.6858
NIEPODLEGŁOŚCI (WIELISZEW)	65.6066
DWERNICKIEGO (OTWOCK)	65.5829
ORLA (OTWOCK)	65.4811
CENDROWICE (CENDROWICE)	65.3108
SADOWA (BASZKÓWKA)	65.0804
GROMADZKA (HENRYKÓW-UROCZE)	65.0804
OŚRODEK ZDROWIA (GLINIANKA)	65.0626
RUNOWSKA (ZŁOTOKŁOS)	65.0365
OSP ZŁOTOKŁOS (ZŁOTOKŁOS)	65.0365
PRZEDPEŁSKIEGO (WIELISZEW)	64.9759
WILLOWA (WIELISZEW)	64.9535
SŁONECZNA (WIELISZEW)	64.9535
ŚWIDRY WIELKIE (OTWOCK)	64.9202
BATOREGO (OTWOCK)	64.8686
SMOLNA (OTWOCK)	64.8144
MAJOWA (WÓLKA RADZYMIŃSKA)	64.7059
LEŚNY (DZIEKANÓW LEŚNY)	64.6607
MARCINKI (STARE ZAŁUBICE)	64.5108
OGRODOWA (WÓLKA PRACKA)	64.3712
IMBIROWA (HENRYKÓW-UROCZE)	64.3712
WÓLKA DWORSKA-WISŁA (WÓLKA DWORSKA)	64.2996
ZABORÓWEK-OSIEDLE (ZABORÓWEK)	64.2608
WOJEWÓDZKA (ZŁOTOKŁOS)	64.2575
3 MAJA (ZŁOTOKŁOS)	64.2575
PARKOWA (USTANÓW)	64.2294
OKRZEI (OTWOCK)	64.2277
MAZURSKA (OTWOCK)	64.2154
WIŚLANA (PODŁĘCZE)	64.1646
ŹRÓDLANA (ŚWIĘCICE)	64.0998
ZAWADZKA (ZŁOTOKŁOS)	64.0768
ROSY (HENRYKÓW-UROCZE)	64.0768
UWIELINY-SZKOŁA (UWIELINY)	63.9728
MOCZYDŁÓW (MOCZYDŁÓW)	63.9114
WÓLKA (WÓLKA)	63.8517
OS.VICTORIA (RADZYMIN)	63.8093
URZĄD GMINY (WIELISZEW)	63.6593
3 MAJA (OTWOCK)	63.5651
GASSY-PĘTLA (GASSY)	63.3295
DZIAŁKOWA (WOLA PRAŻMOWSKA)	63.2953
BIAŁY ŁUG (BIAŁY ŁUG)	63.2953
MŁODZIEŻOWA (LIPOWO)	62.8662
DŁUGA (USTANÓW)	62.6244
ZAŁUBICE-POCZTA (STARE ZAŁUBICE)	62.6195
PODŁĘCZE II (PODŁĘCZE)	62.551
SOBIKÓW (SOBIKÓW)	62.5212
GLINIANKA (GLINIANKA)	62.4957
ZABORÓWEK-WIEŚ (ZABORÓWEK)	62.3364

BENIAMINÓWKA (WÓLKA RADZYMIŃSKA)	62.1017
ŚWIĘCICKA (PŁOCHOCIN)	62.0277
KĘDZIERÓWKA (KĘDZIERÓWKA)	62.0164
PONIATOWSKIEGO (OTWOCK)	61.8255
KUPIECKA (OTWOCK)	61.7725
WYSZYŃSKIEGO (GÓRA KALWARIA)	61.739
WÓLCZYŃSKA (MARIEW)	61.6633
WIŚNIOWA (WIELISZEW)	61.5639
MIASTA (OTWOCK)	61.5225
WIRAŻOWA (WÓLKA RADZYMIŃSKA)	61.435
WOLA KARCZEWSKA (WOLA KARCZEWSKA)	61.273
GRANICZNA (JEZIÓRKO)	61.1601
WODOCIĄG PŁN. (WIELISZEW)	60.8201
POLNA (KOMORNICA)	60.8201
ŁAWKI (ŁAWKI)	60.8161
CZOŁCHAŃSKIEGO (PRAŻMÓW)	60.8161
ZAŁUBICE-SZKOŁA (STARE ZAŁUBICE)	60.7511
SZKOLNA (GLINIANKA)	60.4546
WARSZAWSKA (LESZNO)	60.4532
SŁONECZNA (KRUPIA WÓLKA)	60.2974
PKP ŚRÓDBORÓW (OTWOCK)	60.2702
KASZTANOWA (JEZIÓRKO)	59.9652
NAD ŚWIDREM (WOLA KARCZEWSKA)	59.9213
PIJARSKA (GÓRA KALWARIA)	59.5873
WOLICA K.PŁOCHOCINA (WOLICA k. PŁOCHOCINA)	59.5447
PONIATÓW (PONIATÓW)	59.4658
DĄBKOWIZNA (DĄBKOWIZNA)	59.4291
MARIEW (MARIEW)	59.1015
KUBUSIA PUCHATKA (OTWOCK)	58.852
OPOLSKA (STARE ZAŁUBICE)	58.8104
OS.WEDEL (WOLA DUCKA)	58.725
CZARNÓWKA (CZARNÓWKA)	58.5661
LESZNO-GS (LESZNO)	58.5081
URZĄD GMINY (PRAŻMÓW)	58.0823
POLNA (GABRYELIN)	58.0823
ROZWOJOWA (SKRZESZEW)	57.7973
TEOFILÓW (WOLA DUCKA)	57.6496
WIELISZEW-PLAŻA (WIELISZEW)	57.5594
SOSNOWA (PODDĘBIE)	57.5594
TATRZAŃSKA (OTWOCK)	57.5437
RYNEK (GÓRA KALWARIA)	57.076
ŚWIERK (OTWOCK)	56.6835
POPŁAWSKA (RZAKTA)	56.6577
NOWE ZAŁUBICE (NOWE ZAŁUBICE)	56.4542
OSP SKRZESZEW (SKRZESZEW)	56.3606
GÓRECKA (OTWOCK)	56.3279
LESZNO (LESZNO)	56.1538
BATALIONÓW CHŁOPSKICH (OTWOCK)	55.8084
STAWOWA (OTWOCK)	55.1927
SZKOLNA (SKRZESZEW)	55.0937
ŚWIERK-SZKOŁA (OTWOCK)	55.0179
RZAKTA (RZAKTA)	54.3578
KAŁUSZYN (KAŁUSZYN)	54.0073
WARSZAWSKA (OLSZEWNICA STARA)	53.328
SPACEROWA (OLSZEWNICA NOWA)	51.7062
POLNA (KRUBIN)	50.4626
OLSZEWNICA STARA (OLSZEWNICA STARA)	50.4173
KRUBIN (KRUBIN)	49.336
KWIATOWA (JANÓWEK PIERWSZY)	48.2648
DWORCOWA (GÓRA)	47.2181
OKUNIN (OKUNIN)	46.1686
PRZEMYSŁOWA (OKUNIN)	45.076
ZDOBYWCÓW KOSMOSU (NOWY DWÓR MAZ.)	43.8525
URZĄD MIASTA (NOWY DWÓR MAZ.)	42.161
WILEŃSKA (WARSZAWA)	1
WARSZAWA ZACISZE-WILNO (WARSZAWA)	1

Table C.3: Warsaw public transportation network stop rankings according to harmonic closeness.

Line	Ranking
401	4.35047
ZS1	3.68147
409	3.61188
M1	3.50998
517	3.49688
521	3.36766
N03	3.33716
509	3.2192
N22	2.70241
503	2.64231
414	2.59019
N50	2.38181
523	2.37918
N01	2.36027
411	2.34274
N72	2.29072
L13	2.27243
525	2.15284
N83	2.13044
520	2.07329
700	2.01761
N61	2.01174
527	1.92521
727	1.83742
807	1.71199
514	1.67385
25	1.66269
190	1.63287
N38	1.61578
805	1.5987
730	1.59303
722	1.56798
N13	1.54438
326	1.4856
708	1.47096
500	1.44412
718	1.4384
709	1.4289
501	1.41879
N35	1.41703
E-2	1.41585
33	1.4019
15	1.36844
519	1.36531
E-7	1.36334
23	1.36047
131	1.34244
426	1.33844
N24	1.3238
191	1.31462
N02	1.3134
24	1.31137
185	1.29989
731	1.29734
N46	1.28003
26	1.27832
T	1.2717
507	1.25848
511	1.23368
N33	1.22475
E-8	1.20256
101	1.17306
182	1.16374
N43	1.15247
N25	1.1509
L18	1.1145
114	1.10136
22	1.09597
504	1.09073
N66	1.07263
721	1.07002
121	1.05591
518	1.04689
160	1.04555
209	1.0403
E-1	1.03322
N88	1.03039
512	1.0143
N71	1.00933
734	1.00896
724	0.972912
N56	0.952879
7	0.937092
178	0.932683
28	0.903967
N12	0.890005
171	0.874612
809	0.861386
E-5	0.8411
218	0.834441
N14	0.828095
N91	0.827792
728	0.824459
522	0.821367
35	0.817313
3	0.81358
169	0.813529
11	0.80292
707	0.79479
77	0.794674
706	0.758868
27	0.727172
1	0.702478
186	0.700158
S1	0.692667
N11	0.681276
N45	0.672968
14	0.603664
723	0.594802

109	0.591096
N44	0.590175
N31	0.569074
N32	0.566378
2	0.538147
N41	0.536084
736	0.534033
N42	0.530452
13	0.51659
143	0.504835
300	0.49819
KML	0.483561
N52	0.459932
713	0.459932
9	0.455747
N21	0.45167
175	0.446969
L-3	0.432324
L-7	0.43097
116	0.410153
502	0.397033
L-2	0.395066
10	0.39149
183	0.384421
KM7	0.380763
KM2	0.368743
M2	0.363438
N95	0.361363
S3	0.358813
L11	0.33221
711	0.32976
742	0.329303
N36	0.294772
149	0.282989
125	0.277929
222	0.27259
227	0.260211
155	0.257377
317	0.222002
112	0.204111
245	0.195553
KM1	0.163372
ZS7	0.156494
710	0.141965
800	0.139661
N85	0.127137
KM8	0.114902
733	0.10031
900	0.0881525
S2	0.0717041
17	0.0589656
184	0.0583735
105	0.0565447
132	0.044861
168	0.0309684
207	0.0256227
117	0.0169455
141	0.0121768
719	0.00127768
KM6	2.29022e-009
Z-1	-0.00809814
217	-0.0165596
154	-0.0171265
KM3	-0.0222183
716	-0.0361148
142	-0.0484641
18	-0.0601849
Z-2	-0.073679
151	-0.094889
181	-0.114161
KM9	-0.114633
L-9	-0.123766
735	-0.129167
110	-0.158915
189	-0.178722
N37	-0.182719
187	-0.182942
199	-0.183354
212	-0.186558
139	-0.204213
145	-0.235944
720	-0.236116
385	-0.237979
192	-0.261704
N81	-0.261874
703	-0.275979
N16	-0.281374
345	-0.31368
193	-0.325325
201	-0.335569
196	-0.337695
365	-0.338589
L22	-0.35257
L12	-0.368515
136	-0.406199
N62	-0.411004
L17	-0.413176
S9	-0.417411
L21	-0.426902
104	-0.442973
147	-0.464307
162	-0.476928
WKD	-0.477194
133	-0.499129
206	-0.51171
225	-0.523687
705	-0.53438
216	-0.558637
738	-0.562083
188	-0.585811

L-5	-0.586495
732	-0.606564
717	-0.62402
331	-0.635955
516	-0.650986
L20	-0.654681
N34	-0.67755
118	-0.678184
6	-0.704111
338	-0.704209
704	-0.705213
148	-0.776107
129	-0.797645
174	-0.804957
195	-0.832908
172	-0.840943
166	-0.841668
305	-0.851855
128	-0.907298
203	-0.93542
170	-0.939287
729	-0.941711
N64	-0.980179
167	-0.983604
L25	-1.02047
156	-1.02146
L23	-1.0222
741	-1.02682
314	-1.03157
134	-1.04921
712	-1.07991
126	-1.12885
180	-1.13615
204	-1.14318
L15	-1.17432
306	-1.18695
740	-1.21117
725	-1.24131
208	-1.26849
107	-1.29043
111	-1.34195
L-6	-1.35661
164	-1.36174
L24	-1.37793
311	-1.41688
4	-1.42111
739	-1.43055
161	-1.4713
153	-1.51675
176	-1.604
127	-1.61705
135	-1.61788
L19	-1.68133
715	-1.69002
158	-1.69311
163	-1.73619
20	-1.74684
N63	-1.75616
198	-1.77621
173	-1.78069
165	-1.81731
N58	-1.86238
L16	-1.91761
179	-2.07059
102	-2.08588
702	-2.14397
123	-2.15705
701	-2.17288
L10	-2.18952
L-4	-2.24055
122	-2.24767
L-1	-2.25119
202	-2.26294
L-8	-2.35974
152	-2.4618
303	-2.50686
138	-2.52038
115	-2.53934
194	-2.54181
124	-2.56839
205	-2.6973
L14	-2.73759
743	-2.83661
159	-2.88362
119	-2.93832
219	-2.94148
714	-2.95981
103	-3.16267
197	-3.41835
146	-3.45569
177	-3.70823
120	-4.00671
213	-4.46367
157	-4.96148
211	-5.0642
108	-5.28704

Table C.4: Warsaw public transportation network bus, tram, metro and train line rankings.

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