

Towards HANK:
Investigating Interactions Between Labour and
Credit-Market Frictions in a New Keynesian Economy



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Abstract

How can the presence of unemployment affect the properties of economic fluctuations and the effectiveness of monetary policy when a fraction of households are also credit-constrained? This paper presents a New Keynesian model with limited asset market participation combined with a search & matching model of the labour market. The resulting framework uncovers a novel mechanism whereby aggregate shocks affect the hiring incentives of firms and the prevailing real wage through fluctuations in the total expected surplus from an employment relationship. In the presence of credit-constraints, this will have implications for the distribution of consumption across households. Quantitative evidence suggests that these effects are indeed significant, leading to a greater volatility in labour market tightness and aggregate labour income. In response to a contraction in monetary policy, the fall in output is found to be much greater due to the interaction between credit and labour frictions, whilst the response of inflation remains quantitatively close to the baseline NK model. *

1 Introduction

In assuming the absence of features such as unemployment and income heterogeneity among households, the standard New Keynesian framework outlined in Woodford (2003) by construction has very little to say regarding labour market and distributitional issues. This is potentially problematic at a time when large inequalities in income and financial wealth and a greater instability of work are pronounced features of advanced economies, particularly the US and UK. As highlighted recently by Kaplan and Violante (2018), theory strongly suggests that the effects of aggregate shocks may be amplified if there is heterogeneity in the ability of households to access financial markets or employment in order insure against income risk. Such concerns have led to many prominent figures within macroeconomics and central banking stressing the need for business cycle research to account for such heterogeneity, notably Bernanke (2015), Yellen (2016), Draghi (2016) and Stiglitz (2018)².

Responding to this challenge, recent research has tended to study the impact of incorporating *either* unemployment *or* household heterogeneity into the standard NK model³.

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²Christiano et al. (2018) provide a now well-known rebuttal of some of the Stiglitz critique of the DSGE paradigm.

³This literature is surveyed in detail in section 2.

This ignores the observation that household runs down savings during unemployment, and so this tends to be a time where households are more likely to become credit-constrained. This interaction between binding credit-constraints and unemployment is omitted from standard models, but may be highly relevant for the design and transmission of macroeconomic policy. The model presented in this paper is unique in that it brings together a search & matching model of the labour market with simple household heterogeneity in the form of limited asset market participation in order to study this interaction. These interaction effects have recently been studied in Gornemann et al. (2016) and Ravn and Sterk (2018) using larger and more complex models. However an advantage of the more tractable framework presented here is that it remains very close to the standard New Keynesian model without physical capital, allowing for a clearer characterisation of the main mechanisms at play as well as the use of standard solution techniques.

The main finding of the paper is the isolation of a novel mechanism whereby additional general equilibrium effects operate through the labour market to amplify the effects of aggregate shocks on household inequality. The mechanism in question is the effect of aggregate shocks on the expected total match surplus from an employment relationship. This leads to additional labour demand effects via the hiring decision and the determination of the real wage which are absent in models with a frictionless labour market where there is no employment or bargaining over wages. More explicitly, in response to a contraction in monetary policy the expected total surplus from an employment relationship reduces to the fall in consumption demand among the fraction of the population who can save through financial assets. This leads to a reduction in the demand from labour, which manifests itself in a greater fall in the real wage than otherwise, as well as a reduction in the employment rate, both of which amplify the fall in aggregate labour income and shift the distribution of aggregate consumption away from households who solely rely on labour income. As a result, contractionary monetary policy has a greater effect on output, whilst the inflation response remains close to the benchmark NK model.

Quantitative evidence suggest that these additional effects are indeed significant, leading to greater fluctuations in both output and labour market tightness in response to aggregate shocks compared to NK models capturing only unemployment or stylised household heterogeneity. Results from robustness analysis also suggest that this novel mechanism may play an even greater role in economies where workers have more bargaining power, when aggregate consumption is more highly dependent upon aggregate labour income, and when the transmission of production costs to inflation via a Phillips curve is relatively

weak. These results are of potential importance for the design of macroeconomic policy. In particular, they imply that an inflation targeting central bank may need to tolerate greater output fluctuations to stabilise inflation than is implied in standard models, which may leads to greater welfare losses from aggressive policy.

The remainder of the paper is organised as follows. Section 2 provides a review of the related literature and situates the contribution of the paper. Section 3 outlines the model, derives equilibrium conditions and log-linear equilibrium relationships. Section 4 presents the calibration strategy and solution method used, and derives the main predictions of the model using impulse response and simulation analysis. Section 5 reports the findings of some robustness analysis where we study the effects of varying key structural parameters on the model predictions. Section 6 summarises the main model findings and offers some suggestions for future work.

2 Related Literature

The model I present in this paper is motivated by three distinct strands of the New Keynesian literature: (i) models incorporating search & matching frictions into the standard New Keynesian framework, (ii) the tradition of “two-agent” models in macroeconomics, and (iii) a very recent literature on heterogeneous agent New Keynesian models, or ‘HANK’ models as coined by Kaplan et al. (2018). The model I develop in this paper is unique in that it is the first of its type to explicitly bring together (i) and (ii)⁴.

2.1 New Keynesian Models with Labour Market Frictions

There is a large existing literature which has sought to embed a more realistic description of the labour market into the standard New Keynesian model. This literature embeds standard Mortensen and Pissarides (1994) search & matching labour market frictions into the New Keynesian model. Seminal contributions in this direction were made by Chéron and Langot (2000), Walsh (2003) and Trigari (2006), all of whom incorporated labour market frictions into a business cycle model with Calvo (1983) price-setting firms.

The model I develop is closely related to the setups of Krause and Lubik (2007) , Thomas (2008) and Blanchard and Galí (2010). I abstract from convex hiring costs found in Thomas (2008) for simplicity, instead employing the linear specification for vacancy posting costs common in the Diamond-Mortsen-Pissardies tradition (hereafter DMP). I also maintain the DMP assumption that hiring costs are endogenous functions of labour

⁴Ravn and Sterk (2018) is most closely related paper in this sense, instead combining (i) and (iii).

market tightness. This is distinct from Blanchard and Galí (2010). I also follow Trigari (2006), Thomas (2008) and Thomas and Zanetti (2009) in allowing for intensive and extensive margins of adjustment for labour. This is done to nest the standard New Keynesian model as a special case where the extensive margin of labour adjustment is shut down.

Tractability is maintained by keeping heterogeneity as stylised as possible. As such I abstract from some more realistic features often modelled in the DMP literature. I do not allow for endogenous separations rates as found in Krause and Lubik (2007) and Thomas and Zanetti (2009), which rely on idiosyncratic productivity shocks leading to an endogenously evolving productivity distribution. I also abstract from on-the-job search, so only agents and firms outside of existing employment relationships engage in costly job search at any given time. These simplifications keep the supply side as standard as possible to facilitate comparison with the literature modelling simplified household heterogeneity.

2.2 The Two-Agent Tradition

The model I develop uses the traditional two-agent structure, usually with a focus on limited asset-market participation among a fraction of the population. This literature was motivated by the empirical work by Campbell and Mankiw (1989, 1990) and later Vissing-Jorgensen (2002), who find that a substantial fraction of US households (up to 40%) either save a very small fraction of their disposable income flow, or do not save at all. Instead, these households consume nearly all of their current income every month, living “hand-to-mouth”. Kaplan, Violante and Weidner (2014) have subsequently documented this type of behaviour using more recent US data. For the Euro area, Ampudia et al. (2018) have recently illustrated the vast heterogeneity in income composition across households, findings that labour income makes up around 70% of household income for households of working age⁵. This reflects the findings in Picketty (2014) that the distribution of financial wealth in most advanced economies is highly skewed.

These observations motivated a series of recent papers embedding limited asset-market participation into the benchmark New Keynesian model using a two-agent structure whereby some households behave hand-to-mouth in every period. This is typically done by imposing that some fraction $\psi \in (0, 1)$ of households in the economy are excluded from financial markets in every period and so consume hand-to-mouth. The remaining $1 - \psi$ households save in each period in order to smooth consumption over time in the Ricardian sense. These models were subsequently termed ‘TANK’ models by Kaplan et al. (2018).

⁵This is defined as households containing people between the ages of 16-54. This drops to less than 60% for households containing members up to the age of 64, after which there is a major drop.

Galí et al. (2007) use this setup with physical capital to study how the effects of government spending shocks in the New Keynesian model change in the presence of stylised heterogeneity, and find support for the presence of a positive multiplier on government spending. Bilbiie (2008) uses a similar setup abstracting from capital to study the implications of limited asset market participation for monetary policy. He shows that the dynamics of the New Keynesian model depend non-linearly on the degree of asset-market participation, and that for a high enough ψ in theory we could begin to see “inverted aggregate demand logic”. Nisticò (2016) studies optimal monetary policy in a stylised two-agent economy with discontinuous asset-market participation where an agents’ financial status is subject to a Markov-switching structure, and finds that ignoring heterogeneity of this type in policymaking can lead to non-negligible welfare losses. Finally, Broer et al. (2016) employ a novel two-agent setup, the difference being that some agents only earn income through labour and the rest only from capital gains.

2.3 Towards “HANK” Models

After Krusell and Smith (1998) it was generally acknowledged that the returns to increasing the dimensions of heterogeneity was small in business cycle models (Kaplan and Violante, 2018). However recent computational advances have allowed for more realistic heterogeneity in business cycle models. The resulting ‘Heterogeneous Agent New Keynesian’ framework - coined as ‘HANK’ by Kaplan et al. (2018) - has shown itself to be rather flexible, and is currently being used to address many research questions in macroeconomics⁶.

The HANK framework generally incorporates a standard incomplete markets (‘SIM’) model à la Aiyagari (1994) and Huggett (1993) into an otherwise standard New Keynesian setup with credit-constraints. This moves beyond TANK in allowing all agents to face idiosyncratic shocks to income. The seminal contribution in this area is Kaplan, Moll and Violante (2018) (hereafter KMV), who find that in a HANK model the transmission of monetary policy works mostly through general equilibrium effects, rather than through the direct intertemporal substitution implied by the aggregate Euler equation. Nevertheless, HANK and TANK models are closely related. Bilbiie (2018, 2019) transfers

⁶Examples are the “forward guidance puzzle” (McKay et al., 2016, 2017; Del Negro et al., 2012; Acharya and Dogra, 2018); optimal monetary policy (Challe, 2018; Bilbiie and Ragot, 2017; Nuño and Thomas, 2017; Rios-Rull, 1995); distributional consequences (Auclert, forthcoming; Gornemann et al., 2016); transmission of monetary policy (Werning, 2015; Kaplan et al., 2018) and rational inattention (Ellison and Macaulay, 2017)

the insights of KMV into a tractable “New Keynesian Cross” framework based on the standard TANK model⁷. Using this framework, Bilbiie (2018, 2019) shows that the slope of the Keynesian cross is increasing in the fraction of constrained households ψ ⁸. Debortoli and Galí (2018) identify three key distributional channels at work in HANK models: the ‘gap’ between constrained and unconstrained households, (ii) consumption dispersion within the set of unconstrained households, and (iii) the overall measure of constrained households in the economy. These authors suggest that we can understand a TANK model as allowing for (i) but shutting down (ii) and (iii).

Contributions by Ravn and Sterk (2018) and Gornemann et al. (2016) are most closely related to the model developed in this paper. Ravn and Sterk (2018) embed search & matching frictions into the standard HANK setup used by KMV and find that the introduction of labour market frictions can either amplify or dampen the effects of credit-market frictions depending on the cyclical nature of idiosyncratic income risk. Gornemann et al. (2016) build an even richer HANK model, allowing for heterogeneity in both preferences and skills, as well as in employment status and credit access. They find that consumption becomes much more responsive to monetary policy shocks in the presence of heterogeneous agents, and that contractionary shocks to policy lead to increases in inequality.

2.4 Contribution

The TANK framework has focused exclusively on asset-market participation as the only source of heterogeneity, whereas the contributions of Ravn and Sterk (2018) and Gornemann et al. (2016) have highlighted the potential importance labour and credit market interactions in the context of HANK. Given the quantitative evidence presented in Debortoli and Galí (2018) that TANK models approximate HANK models fairly well, it is a natural step to study this interaction in the context of a simpler model and then compare the findings to those from the HANK literature. To my knowledge, there is no other contribution which builds employment status into a standard TANK model as a second dimension of heterogeneity.

The main contribution of this paper is that it uncovers a novel amplification mechanism not present in the standard TANK model which operates through the interaction between

⁷For the purposes of exposition, Bilbiie (2018) initially abstracts from an economy with nominal rigidities.

⁸Demand amplification increases when the aggregate marginal propensity to consume is higher. This is a classic observation in Keynes (1936)

the labour market and the two-agent structure. More specifically, aggregate shocks have an effect on the total match surplus of employment relationships, which ultimately influences hiring decisions as well as the prevailing real wage. This can lead to greater amplification on output by amplifying the effect of shocks on the consumption gap between constrained and unconstrained households, which appears as a wedge in the aggregate Euler equation. The resulting model can therefore be used to understand how labour and credit market frictions interact in response to aggregate shocks, and to study the implications for policy.

3 Model

The model outlined below is based on Debortoli and Galí (2018) but extended to allow for the possibility of unemployment. This is done by embedding a DMP search & matching model of the labour market with an exogenous separation rate into the baseline TANK model. Labour market frictions are built into a NK economy in a way closely related to Krause and Lubik (2007) where firms are subject to quadratic costs of adjusting prices à la Rotemberg (1982). Compared to Ravn and Sterk (2018) and Gornemann et al. (2016) the model abstracts from physical capital, rigid wages and heterogeneity in productivity and degree of patience in order to stay closer to the textbook NK model for ease of comparison. I also allow for both the intensive and extensive margin of labour adjustment, meaning the model nests relevant special cases such as the standard TANK model and the standard the NK model with search frictions. The resulting framework therefore allows for the existence of constrained and unconstrained households, both of which will contain employed and unemployed workers.

3.1 Labour Market

The labour market takes the form of a standard DMP model with endogenous transition probabilities and an exogenous separation rate. The flow of workers into vacancies, referred to as *matches*, is determined by a standard Cobb-Douglas matching function:

$$m_t \equiv m(v_t, u_t) = \bar{m} v_t^{1-\alpha} u_t^\alpha \quad (1)$$

where v_t are vacancies, u_t is the unemployment rate, α is the elasticity of matches with respect to the stock of unemployed agents in the economy, and $\bar{m}$ is the (time-invariant) matching efficiency. We normalise the labour force to 1, so that every agent in the economy is either employed or unemployed:

$$e_t = 1 - u_t \quad (2)$$

where e_t is the employment rate. Labour market tightness is defined as standard: the ratio of firm vacancies to agents actively engaged in job search in the labour market. We abstract from on-the-job search and endogenous job separation, so in this case labour market tightness θ_t is the number of vacancies over the unemployment rate as in Thomas (2008):

$$\theta_t = \frac{v_t}{u_t} \quad (3)$$

This reflects the idea that an aggregate labour market with relatively low unemployment would be described as “tight” (a higher θ_t). This definition allows us to define the probability of finding a job and the probability of filling a vacancy for a firm as functions of labour market tightness:

$$\begin{aligned} p(\theta_t) &\equiv \frac{m_t}{u_t} = \bar{m}\theta_t^{1-\alpha} \\ q(\theta_t) &\equiv \frac{m_t}{v_t} = \bar{m}\theta_t^{-\alpha} \end{aligned}$$

The probability of finding a job $p(\theta)$ (the “job-finding rate”) is decreasing in the number of unemployed, as it becomes more unlikely to match with a firm when facing greater competition. Similarly, the probability of filling a vacancy $q(\theta)$ is decreasing in labour market tightness, as it becomes more difficult for firms to fill a vacancy when the number of job searchers in the labour market is relatively low compared to the number of vacancies. Finally we can define a law of motion for the aggregate employment rate:

$$e_t = (1 - \delta)e_{t-1} + p(\theta_t)u_t \quad (4)$$

where $\delta \in (0, 1)$ is the exogenous separation rate. In words, the stock of agents employed at the end of period t is equal to the proportion of yesterday’s stock who weren’t exogenously separated $(1 - \delta)e_{t-1}$, plus the proportion of unemployed agents who were matched in period t , $p(\theta_t)u_t$. Note that it must be the case in this economy that

$$p(\theta_t)u_t \equiv q(\theta_t)v_t \equiv m_t$$

This identity simply states that the number of unemployed workers who find a job is equal to the number of vacancies filled by firms, both of which give the number of matches in the economy in any period t .

3.2 Households

The economy is populated by a continuum of infinitely-lived heterogeneous households indexed by $i \in [0, 1]$. Due to the presence of labour market frictions, all households

contain both employed and unemployed agents. The head of a household maximises expected discounted utility defined over consumption and leisure as standard:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t^i, e_{k,t}^i N_{k,t}^i) \quad (5)$$

where $\beta \in (0, 1)$ is the individual discount factor, $C_t^i \equiv \left(\int_0^1 C_{k,t}^i \frac{\epsilon-1}{\epsilon} dk \right)^{\frac{\epsilon}{\epsilon-1}}$ denotes the usual Dixit-Stiglitz CES aggregator over a bundle of consumption goods and $\epsilon > 0$ is the elasticity of substitution among varieties, $N_{k,t}^i$ denotes hours worked by household i at firm k , and $e_{k,t}^i$ denotes the rate of employment of household i at firm k . For simplicity of exposition we use log-CRRA preferences⁹:

$$U(C, eN) \equiv \left(\log(C) - \chi \frac{eN^{1+\varphi}}{1+\varphi} \right)$$

where $\varphi > 0$ is the (inverse) Frisch elasticity of labour supply and χ is a weight on the disutility of labour. Each consumption good is indexed by k , where $k \in [0, 1]$ is a continuum of differentiated goods sold by monopolistically competitive firms. It can be shown as standard that the demand for each good across all households can be expressed as:

$$C_t(k) = \left(\frac{P_{k,t}}{P_t} \right)^{-\epsilon} C_t \quad (6)$$

where $P_t \equiv \left(\int_0^1 P_{k,t}^{1-\epsilon} dk \right)^{\frac{1}{1-\epsilon}}$ is the relevant price index and $C_t \equiv \int_0^1 C_t^i di$ is aggregate consumption.

Two-agent structure: We make the simplifying assumption that there are two types of household in the economy: those who have full access to all financial instruments available; and those (for reasons unspecified here) who are against an exogenous zero-borrowing limit and consume hand-to-mouth. The subset of households who are unconstrained in their financial market access is given by $\mathcal{U}_t \subset [0, 1]$ and is assumed to have measure $1 - \psi_t$ where $\psi_t \in (0, 1)$. The corresponding subset of constrained households is given by $\mathcal{K}_t \subset [0, 1]$ and has measure ψ_t . Hence in any period t , we can assign the superscript index to household $i \in \{U, K\}$ where $U \in \mathcal{U}_t$ are “unconstrained” and $K \in \mathcal{K}_t$ are “hand-to-mouth” (or “Keynesian”). Following Debortoli and Galí (2018), I abstract from allowing for the possibility of agents switching between types¹⁰. Consequently the composition and measure of the subsets $\mathcal{U}_t$ and $\mathcal{K}_t$ will be fixed (i.e. $\psi_t \equiv \psi$).

⁹The exogenous shock to preferences used in Debortoli and Galí (2018) is also omitted for ease of exposition.

¹⁰Nisticò (2016) allows for switching via a Markov structure, however when applied to this economy in earlier work the result was not quantitatively significant.

The period budget constraint for the representative unconstrained household is given by:

$$\frac{1}{P_t} \int_0^1 P_t(k) C_t^U(k) dk + \frac{B_t^U}{P_t} + Q_t S_t^U = \frac{B_{t-1}^U (1 + i_{t-1})}{P_t} + \int_0^1 \frac{W_{k,t}}{P_t} e_{k,t}^U dk + [Q_t + (1 - \varrho) D_t] S_{t-1}^U + T_t^U \quad (7)$$

where $P_t(k)$ is the price of good k at time t , B_t^U is the purchases of one-period bonds yielding the nominal interest rate i_t , S_t^U represents the holdings of shares in an equity fund which has some claim over a fraction $1 - \varrho$ of firm profits D_t where $\varrho \in (0, 1)$, and Q_t is the corresponding share price. These shares can be traded by unconstrained agents and therefore used to smooth consumption over time. The remaining fraction ϱ are assumed to constitute an *illiquid* component and will be redistributed according to some fiscal transfers as in Debrtoli and Galí (2018) (to be specified later).

The problem of unconstrained agents is to choose $\{C_t^U, e_t^U, B_t^U, S_t^U\}_{t=0}^\infty$ to maximise expected discounted lifetime utility (5) subject to the law of motion for aggregate employment given by (4) and the period budget constraint given by (7). Standard optimisation techniques yield the following conditions for unconstrained agents describing optimal consumption behaviour over time:

$$\frac{1}{1 + i_t} = \mathbb{E}_t \left\{ \Lambda_{t,t+1}^U \frac{1}{\Pi_{t+1}} \right\} \quad (8)$$

$$Q_t = \mathbb{E}_t \{ \Lambda_{t,t+1}^U (Q_{t+1} + (1 - \varrho) D_{t+1}) \} \quad (9)$$

where

$$\Lambda_{t,t+1}^U \equiv \beta \frac{C_t^U}{C_{t+1}^U}$$

is the stochastic discount factor of the unconstrained households. This will be the relevant discount factor for much of the analysis below, as only agents from unconstrained households price bonds and shares. $\Pi_{t+1} \equiv P_{t+1}/P_t$ is the gross rate of price inflation in the economy.

The remaining $i \in \mathcal{K}$ agents are assumed to face zero-borrowing constraints:

$$\frac{B_t^K}{P_t} = 0$$

$$S_t^K = 0$$

for $\forall t$, which is equivalent to being assumed to be excluded from participating in all asset markets. As such, the only source of income for constrained households in every period is earned labour income. The period budget constraint for $\forall i \in \mathcal{K}$ is therefore given by:

$$\frac{1}{P_t} \int_0^1 P_t(k) C_t^K(k) dk = \int_0^1 \frac{W_{k,t}}{P_t} e_{k,t}^K + T_t^K \quad (10)$$

Given that $U(C, N)$ has standard properties (i.e. $U_c > 0$, $U_{cc} < 0$), these agents will simply consume all their earned income (net of transfers) in each period.

Finally, we will assume that all firms are identical, that all agents are equally productive, and that all employed agents in each type of representative household choose to allocate an equal amount of time to each firm. This implies that labour supply will be allocated symmetrically across firms ($e_t = e_t^i = e_{k,t}^i$), that all agents employed will work the same number of hours at each firm ($N_t = N_t^i = N_{k,t}^i$)¹¹, and that all firms will pay an identical real wage to be determined via Nash bargaining ($\frac{W_t}{P_t} = \frac{W_{k,t}}{P_t}$). It is optimal for all households to work until they become indifferent between working and not working, i.e. when marginal benefit is equal to the marginal cost. This is described by the following condition:

$$\lambda_t^{1,i} \frac{W_t}{P_t} - \chi \frac{N_t^{1+\varphi}}{1+\varphi} + \beta(1-\delta)\lambda_{t+1}^{1,i} = \lambda_t^2(1+p(\theta_t)) \quad (11)$$

where $\lambda_t^{1,i}$ is the Lagrange multiplier on the budget constraint for $i \in \{U, K\}$, which is also the marginal utility of consumption, $\lambda_t^{1,i} \equiv \frac{1}{C_t^i}$, and λ_t^2 is the Lagrange multiplier on the law of motion for employment. I make the assumption that in steady state agents from unconstrained households consume more than constrained agents¹²:

$$C^U > C^K$$

This can be rationalised both by the ability of unconstrained agents to smooth consumption in response to aggregate shocks, as well by their holding of shares in firms which creates a positive relationship between firm profits and the consumption of unconstrained households. The marginal utility of unconstrained households will therefore be lower than that of constrained households. This implies that if the following holds

$$\lambda_t^{1,U} \frac{W_t}{P_t} + \beta(1-\delta)\lambda_{t+1}^{1,U} \geq \chi \frac{N_t^{1+\varphi}}{1+\varphi} \lambda_t^2(1+p(\theta_t))$$

this is enough to guarantee that all agents in this economy wish to work for the prevailing wage (to be determined), but there will be some fraction u_t who are unable to find work each period due to the presence of labour market frictions.

¹¹Hours worked will be determined separately, following Thomas (2008).

¹²Note that in this setup (as in the TANK setup) we are abstracting from the idea of “wealthy hand-to-mouth” consumers highlighted by Kaplan and Violante (2014). In the data there is a sizeable proportion of households who consume hand-to-mouth not because they have little personal wealth, but rather because their wealth is held in illiquid form.

3.3 Firms

Much of the New Keynesian literature with search & matching frictions employ the Calvo (1983) pricing assumption and assume the existence of two types of firms in the economy, usually intermediate goods and final goods producers. This is done to separate the hiring and pricing decisions to avoid any issues arising from strategic complementarities highlighted by Woodford (2005). More in keeping with the TANK literature and following Krause and Lubik (2007), I instead employ the Rotemberg (1982) assumption that firms face quadratic costs of adjusting prices¹³. This abstracts from issues relating to price dispersion and allows us to study the problem of a representative firm which now face costs of both posting vacancies and adjusting prices in any given period.

There exists a continuum of monopolistically competitive firms in the economy indexed by k who produce a differentiated good, which they then sell to consumers at price $P_{k,t}$. All firms employ workers $e_{k,t}$ who work for $N_{k,t}$ hours and are paid a real wage $\omega_{k,t} = \frac{W_{k,t}}{P_t}$. All producing firms have an identical constant returns to scale production function in labour input and hours:

$$Y_{k,t} = A_t e_{k,t} N_{k,t} \quad (12)$$

where A_t is an aggregate productivity shock, the log of which follows an AR(1) process:

$$\ln A_t \equiv a_t = \rho_a a_{t-1} + \varepsilon_t^a \quad (13)$$

where ε_t^a is an i.i.d. shock with distribution $\mathcal{N}(0, \sigma_a^2)$.

In every period each firm loses some fraction δ of its workers exogenously. Firms wish to replace these workers, but cannot do this costlessly due to the presence of search & matching frictions. In particular, firms must post vacancies $v_{k,t}$ at the cost per vacancy κ , which we assume is symmetric across firms. Without vacancy posting costs, firms could post as many vacancies as needed to guarantee that in expectation they perfectly replaced the workers they lost to separation each period. Recall that the probability of filling a vacancy is given by $q(\theta_t)$, which is also the fraction of total vacancies filled. The law of motion for employment in (4) can be written equivalently from the individual firm's perspective as:

$$e_{k,t} = (1 - \delta)e_{k,t-1} + q(\theta_t)v_{k,t} \quad (14)$$

A representative firm therefore chooses $\{e_{k,t}, v_{k,t}, P_{k,t}\}$ to maximise

$$\max_{\{e_{k,t}, v_{k,t}, P_{k,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t^{1,U}}{\lambda_0^{1,U}} \left[\frac{P_{k,t}}{P_t} Y_{k,t} - \kappa v_{k,t} - \omega_{k,t} e_{k,t} - \frac{\mathcal{C}}{2} \left(\frac{P_{k,t}}{P_{k,t-1}} - \Pi \right) \right]$$

¹³This is captured in the usual way, where deviations from steady state price changes are costly for firms.

subject to the production technology given by (12), the demand curves given by (6), and the law of motion given by (14). Note that the discount factor that matters here is that of the unconstrained households, who own the claims to the profits of the firms. As all firms are identical the first-order optimality conditions of this problem hold symmetrically across firms. The first-order conditions to this problem are:

$$\xi_t = \mu_t A_t N_t - \omega_t + \beta(1 - \delta) \xi_{t+1} \frac{\lambda_{t+1}^{1,U}}{\lambda_t^{1,U}} \quad (15)$$

$$\xi_t = \frac{\kappa}{q(\theta_t)} \quad (16)$$

$$\epsilon(1 - \mu_t) = 1 - \mathcal{C}(\Pi_t - \Pi)\Pi_t + \beta \frac{\lambda_{t+1}^{1,U}}{\lambda_t^{1,U}} \mathcal{C}(\Pi_{t+1} - \Pi)\Pi_{t+1} \frac{Y_{t+1}}{Y_t} \quad (17)$$

where μ_t is the Lagrange multiplier on the production function and ξ_t is the Lagrange multiplier on the law of motion for employment. Note in this setup μ_t gives the real marginal costs of production for the representative firm. Combining the first two conditions yields the usual job creation condition:

$$\frac{\kappa}{q(\theta_t)} = \mu_t A_t N_t - \omega_t + \beta(1 - \delta) \frac{\lambda_{t+1}^{1,U}}{\lambda_t^{1,U}} \frac{\kappa}{q(\theta_{t+1})} \quad (18)$$

which states that firms will hire up to the point at which the marginal cost of hiring an additional worker equals to marginal benefit. The remaining condition in (17) gives the optimal pricing condition for firms. Note that if price adjustment costs $\mathcal{C} = 0$ (i.e. prices are fully flexible) then it is optimal for the firm to ensure that marginal costs equal the usual price markup $\mu_t \equiv \mu = \frac{\epsilon-1}{\epsilon}$ ¹⁴.

3.3.1 Determining Hours Worked

Hours are determined in this economy to maximise the joint surplus of both the worker and a producing firm through an employment relationship following Thomas (2008). The firm's surplus is given by the job creation condition in (18):

$$\mathcal{J}_t = \mu_t A_t N_t - \omega_t + \beta(1 - \delta) \frac{\lambda_{t+1}^{1,U}}{\lambda_t^{1,U}} \frac{\kappa}{q(\theta_{t+1})} \quad (19)$$

The worker's surplus is given by the real wage net of labour disutility, plus the continuation value of the employment relationship¹⁵:

$$\mathcal{W}_t = \omega_t - \frac{1}{\lambda_t^{1,i}} \chi \frac{N_t^{1+\varphi}}{1+\varphi} + \beta(1 - \delta)(1 - p(\theta_{t+1})) \frac{\lambda_{t+1}^{1,i}}{\lambda_t^{1,i}} \mathcal{W}_{t+1} \quad (20)$$

¹⁴We will assume in steady state the usual government subsidy to ensure $\mu = 1$.

¹⁵Note that we abstract from unemployment benefits or the presence of any other outside options for the purposes of simplicity.

The total surplus is defined as:

$$\mathcal{S}_{tot} = \mathcal{J}_t + \mathcal{W}_t$$

Taking the first order condition with respect to hours N_t and rearranging gives the following condition:

$$N_t = \left(\frac{\mu_t A_t \lambda_t^i}{\chi} \right)^{\frac{1}{\varphi}} \quad (21)$$

Note that hours worked is technically a function of the type of household i through the presence of the marginal utility of consumption, λ_t^i . We will explicitly rule this out by imposing that $\lambda_t^i = \lambda_t^U$ to ensure that all workers work the same amount of hours.

3.3.2 Nash Bargaining

Real wages ω_t are determined by Nash bargaining between matched workers and producing firms in every period as is standard in the literature. The total surplus is split according to the Nash sharing rule:

$$(1 - \eta)\mathcal{W}_t = \eta\mathcal{J}_t$$

Using (19) and (20), together with:

$$\mathcal{W}_{t+1} = \frac{\eta}{1 - \eta} \frac{\kappa}{q(\theta_{t+1})}$$

It can be shown that the Nash bargained real wage in every period is equal to:

$$\omega_t = (1 - \eta) \left(\frac{\chi}{1 + \varphi} \right) \frac{1}{\lambda_t^{1,U}} N_t^{1+\varphi} + \eta \left[\mu_t A_t N_t + \beta(1 - \delta) \frac{\lambda_{t+1}^{1,U}}{\lambda_t^{1,U}} \frac{\kappa}{q(\theta_{t+1})} p(\theta_{t+1}) \right] \quad (22)$$

where again I assume $\lambda_t^{1,i} = \lambda_t^{1,U}$ in order to abstract from heterogeneity in the labour supply of different types of household¹⁶. This is done to keep the supply side of the model as standard as possible¹⁷. Substituting into the expression for the firm's job creation decision (18) gives the equilibrium job creation condition in this economy:

$$\frac{\kappa}{q(\theta_t)} = (1 - \eta) \left[\mu_t A_t N_t - \frac{1}{\lambda_t^{1,U}} \chi \frac{N_t^{1+\varphi}}{1 + \varphi} \right] + \beta(1 - \delta) \frac{\lambda_{t+1}^{1,U}}{\lambda_t^{1,U}} (1 - \eta p(\theta_{t+1})) \frac{\kappa}{q(\theta_{t+1})} \quad (23)$$

¹⁶This is imposed for simplicity to ensure that all workers are paid the same wage and work the same number of hours. Without this assumption we would have heterogeneity across hours and wages, as firms could pay different wages to workers from different types of households. Agents from constrained households would be willing to work longer hours for a lower wage due to the assumption that they have a higher marginal utility of consumption. Moreover, this is perhaps not an unreasonable assumption, given that we will assume that 70% households are unconstrained in steady state.

¹⁷The notion of heterogeneous labour supply and its role in a NK model could be incorporated into further work.

3.3.3 Marginal Costs

In a standard New Keynesian model without labour market frictions and allowing for both extensive and intensive margin of labour adjustment, marginal costs μ_t are given by

$$\mu_t = \frac{\omega_t}{A_t N_t}$$

In this setup with search & matching labour market frictions, we can rearrange the job creation condition given by (18) to find the following expression for marginal costs:

$$\mu_t = \frac{\omega_t}{A_t N_t} + \frac{1}{A_t N_t} \left[\frac{\kappa}{q(\theta_t)} - \beta(1 - \delta) \frac{\lambda_{t+1}^{1,U}}{\lambda_t^{1,U}} \frac{\kappa}{q(\theta_{t+1})} \right]$$

The presence of the second term illustrates clearly that the presence of search costs for firms leads to a wedge in the term for marginal costs relative to the frictionless benchmark, as shown in Krause and Lubik (2007). The second term represents the cost of hiring a worker net the future savings made from not having to hire in the next period. It can also be seen that if $\kappa = 0$ this collapses to the standard expression for marginal costs. It can be shown that if the following condition is satisfied

$$1 > \beta(1 - \delta)$$

then in steady state marginal costs are unsurprisingly higher in the presence of search costs. This condition will be satisfied for any standard calibration of these parameters.

3.4 Goods Market Equilibrium

We can express aggregate output in the economy Y_t as:

$$Y_t = \int_0^1 Y_{k,t} dk = A_t \int_0^1 e_{k,t} N_{k,t} dk = A_t e_t N_t$$

In equilibrium the total supply of goods net of the costs of posting vacancies must equal the total demand by households, given by $C_t \equiv \int_0^1 C_{k,t}^D dk$. This gives the following goods market clearing condition:

$$Y_t - \kappa v_t = C_t \tag{24}$$

3.5 Fiscal Transfers

In this model, the transfer scheme determines how the illiquid component of firm profits ϱD_t is distributed among households. I adopt the transfer/redistribution scheme presented in Debortoli and Galí (2018) for ease of comparison. Any transfer scheme has to satisfy the following balanced budget condition (in the absence of government debt):

$$\varrho D_t \equiv (1 - \psi) T_t^U + \psi T_t^K$$

where D_t are the aggregate profits of firms, given by:

$$D_t = Y_t \Delta^p(\Pi_t) - \omega_t e_t - \kappa v_t \quad (25)$$

where $\Delta^p(\Pi_t) = 1 - \frac{\varrho}{2}(\Pi_t - 1)^2$. Transfers $\{T_t^U, T_t^K\}$ are specified as follows:

$$T_t^U = \left(1 + \frac{\tau\psi}{1-\psi}\right) \varrho D_t \quad (26)$$

$$T_t^K = (1 - \tau) \varrho D_t \quad (27)$$

where $\tau \in [0, 1]$. It can be checked that this transfer scheme satisfies the balanced budget condition. If $\tau = 1$, it can immediately be seen that $T_t^K = 0$. This corresponds to redistributing profits equally across the unconstrained households only. This is similar to the “wealth-based” rule introduced in Gornemann et al. (2016). In the other extreme, $\tau = 0$ corresponds to uniformly distributing the illiquid profits across *all* households.

3.6 Heterogeneity

Again following Debortoli and Galí (2018) I define a measure of heterogeneity in this economy as ζ_t , which is the gap between the consumption of the two different types of household in this economy:

$$\zeta_t \equiv \frac{C_t^U - C_t^K}{C_t^U} = 1 - \frac{C_t^K}{C_t^U} \quad (28)$$

ζ_t therefore represents a simplistic notion of inequality in the model: the consumption gap between constrained and unconstrained households. A higher value of ζ_t indicates a higher level of inequality, and vice versa. $\zeta_t \in [0, 1]$ is bounded from below and above. Perfect equality between the two types gives $\zeta_t = 0$, corresponding to the standard NK model with search & matching frictions. The most extreme inequality corresponds to $\zeta_t = 1$, where the level of constrained consumption is $C_t^K = 0$. A more realistic value of ζ_t will lie somewhere in the interval between these two extremes. Given that in equilibrium bonds will be in zero net supply and the market for shares in the mutual fund must clear:

$$\int_0^1 B_t(i) di = 0$$

$$\int_0^1 S_t(i) di = 1$$

It follows that in equilibrium

$$C_t^U = e_t \omega_t + \frac{1 - \varrho}{1 - \psi} D_t + T_t^U$$

$$C_t^K = e_t \omega_t + T_t^K$$

Using the transfer specifications given by (26) and (27) and substituting into (28) yields:

$$\zeta_t = 1 - \frac{e_t \omega_t + (1 - \tau) \varrho D_t}{e_t \omega_t + \frac{1 - \psi(1 - \tau) \varrho}{1 - \psi} D_t}$$

Rearranging and using the definition of aggregate profits yields an expression for the heterogeneity index in terms of output, inflation and labour market variables:

$$\zeta_t = \frac{(1 - (1 - \tau) \varrho)(Y_t \Delta^p(\Pi_t) - \omega_t e_t - \kappa v_t)}{e_t \omega_t (1 - \psi) + (1 - \psi(1 - \tau) \varrho)(Y_t \Delta^p(\Pi_t) - \omega_t e_t - \kappa v_t)} \quad (29)$$

Comparative statics: One of the key mechanisms in this model is the relationship between the variables affecting the level of firm profits and the inequality between households expressed by ζ_t . Taking partial derivatives of ζ_t with respect to each component will help isolate how the frictional labour market and a simple measure of household inequality are related in this model.

For output it can be shown that

$$\frac{\partial \zeta_t}{\partial Y_t} = \Delta^p(\Pi_t)(1 - (1 - \tau) \varrho)(1 - \psi) e_t \omega_t \Upsilon_t \quad (30)$$

where $\Upsilon_t \equiv \frac{1}{[e_t \omega_t (1 - \psi) + (1 - \psi(1 - \tau) \varrho)(Y_t \Delta^p(\Pi_t) - e_t \omega_t - \kappa v_t)]^2}$. The sign of both the partial derivative and Υ_t depends on the sign of $\Delta^p(\Pi_t)$ ¹⁸. Under the calibration which we will use in chapter 4, $\mathcal{C} = 70$. It can then be shown that $\Delta^p(\Pi_t) > 0$ for any $\Pi_t < 1.17$, i.e. for inflation less than 17%. This will also be enough to ensure that $\Upsilon_t > 0$. Intuitively, increasing output holding prices and other costs fixed will increase the profits of firms. If we assumed $\tau = 1$ this will increase the level of consumption of unconstrained households only, and so shift the distribution of consumption towards unconstrained households and away from constrained households.

Proposition 3.1. *For low levels of inflation, increasing output contributes to increasing inequality between households in this environment (cet. par.).*

The direct effect of inflation is ambiguous, depending on the sign of the current *level* of inflation:

$$\frac{\partial \zeta_t}{\partial \Pi_t} = -\mathcal{C}(\Pi_t - 1) Y_t (1 - (1 - \tau) \varrho)(1 - \psi) e_t \omega_t \Upsilon_t \gtrless 0 \quad (31)$$

For any positive level of inflation $\Pi_t > 1$, then increasing inflation will reduce inequality. Intuitively, a higher level of inflation is more costly for firms due to the presence of price adjustment costs, which will decrease the profits of firms and reduce inequality between

¹⁸Note that we are also ruling out negative values for Y_t , e_t , and ω_t as standard.

households. Moreover, a higher level of *expected* inflation in the next period will also reduce the amount saved by unconstrained households today via bonds and shares, as the real expected return to wealth will be relatively smaller. This leads to an expected fall in the consumption gap in the next period. For any deflation $\Pi_t < 1$ these mechanisms will work in the opposite direction, so increasing inflation will actually *increase* inequality as firms do not change prices as frequently, which increases firm profits. When there is zero inflation (i.e. in the normal steady state analysed) then the direct effect of inflation will drop out of the model.

Proposition 3.2. *For positive levels of inflation a marginal increase in the level of inflation reduces the consumption gap (cet. par.). Conversely, when there is deflation a marginal increase in the inflation level will instead increase the consumption gap. In a zero inflation economy, a marginal increase in the level of inflation has no direct effect on inequality between households.*

The main mechanisms of interest in this model - which are not present in the TANK model - are the effects of labour market variables (the real wage, the aggregate employment rate and aggregate vacancy postings) on household inequality. The direct effects in all cases can be shown to be unambiguously negative for low levels of inflation:

$$\frac{\partial \zeta_t}{\partial \omega_t} = -(1 - (1 - \tau)\varrho)(1 - \psi)(Y_t \Delta^p(\Pi_t) - e_t \omega_t - \kappa v_t) e_t \Upsilon_t < 0 \quad (32)$$

$$\frac{\partial \zeta_t}{\partial e_t} = -(1 - (1 - \tau)\varrho)(1 - \psi)(Y_t \Delta^p(\Pi_t) - e_t \omega_t - \kappa v_t) \omega_t \Upsilon_t < 0 \quad (33)$$

$$\frac{\partial \zeta_t}{\partial v_t} = -\kappa(1 - (1 - \tau)\varrho)(1 - \psi) e_t \omega_t \Upsilon_t < 0 \quad (34)$$

The intuition behind these results is relatively straightforward, and rely on the fact that labour income is the only source of income for constrained households in this model due to the two-agent structure:

- *Wage effect:* Increasing the real wage earned by workers will increase labour income. This increases the consumption of constrained households relatively more, due to the fact that they solely rely on labour income for financing consumption and consume hand-to-mouth due to the credit-constraint. This effect is also present in the standard TANK model.
- *Employment effect:* Increasing the employment rate will ensure more agents from constrained households are employed. This has the same effect of increasing labour income in the economy, which again relatively benefits constrained households more

than unconstrained. This effect is absent in the standard TANK model due to the absence of an extensive margin of labour adjustment.

- *Vacancy posting effect:* An increase in the number of vacancies will increase hiring costs for firms, which means all else equal inequality will fall with firms profits. Note that this effect is increasing in the cost of posting vacancies κ . This effect is also absent in the TANK model, reflected in the fact that if there were no costs for firms posting vacancies ($\kappa = 0$) this effect disappears.

Proposition 3.3. *For low levels of inflation, marginal increases in the real wage, the aggregate employment rate and aggregate vacancy postings will reduce the consumption gap (cet. par.).*

Note that there are also interactions between these three effects. For instance, increasing the real wage will benefit constrained households even more when the employment rate is relatively higher and vice versa. Moreover, increasing vacancies will increase labour market tightness for a given number of job searchers which affects future hiring incentives of firms. Through Nash bargaining a tighter labour market leads to an increase in the real wage of workers in employment. This is due the fact that increasing tightness in the labour market makes it more costly for firms to higher labour, as the probability of filling a vacancy decreases $q(\theta)$. Conversely, from the perspective of unemployed agents the job finding rate $p(\theta_t)$ will increase with the number of vacancies, which reduces unemployment.

3.7 Equilibrium

We can now formally characterise the competitive equilibrium in this economy.

Definition 3.1 (Competitive Equilibrium). *A competitive equilibrium in this economy is a sequence of prices and quantities such that: (i) the consumption and savings decisions of all households (constrained or otherwise) are fully characterised by the optimality conditions, (ii) firms maximise profits, and (iii) all markets in the economy clear.*

More explicitly, this requires that the consumption plans of all unconstrained households follow (8) and constrained households act hand-to-mouth. All firms maximise profits by creating vacancy openings according to (23), setting hours worked according to (21), and setting prices according to (17). Aggregate employment follows the law of motion (4). The goods market clears according to (24), and asset markets clear such that bonds are in zero net supply and each unconstrained household holds $\frac{1}{1-\psi}$ shares in firms. The model will be closed by a Taylor-type rule for monetary policy dictating how the nominal

interest rate i_t reacts endogenously to inflation and unemployment, which will be specified later.

It is worth noting that the model presented in this section nests three special cases analysed extensively in the literature reviewed in section 2. If $\psi = 0$, then there is no household heterogeneity and the model reduces to the standard representative agent NK model with search & matching frictions. If $\kappa = 0$ it is no longer costly for firms to post vacancies and so exogenous separations will be able to be perfectly replaced at no cost to firms. Hence there will be no unemployment, which implies $u = 0$ and $e = 1$. If we also assume that $\eta = 0$, then the model reduces to a standard TANK model with Rotemberg price adjustment costs presented in Debortoli and Galí (2017). When we have both $\psi = 0$ and $\kappa = \eta = 0$, the model reduces to the standard representative agent NK model with Rotemberg pricing.

3.8 Log-linear Equilibrium Dynamics

For the purposes of exposition in this section I present the log-linearised model¹⁹. The steady-state around which the model is log-linearised is the efficient zero inflation steady state (i.e. $\Pi \equiv 1$) where the standard Hosios' condition is satisfied ($\alpha = \eta$) and there is the steady state subsidy to firms to induce the efficient level of production²⁰.

From the labour market conditions we can write:

$$\hat{\theta}_t = \hat{v}_t - \hat{u}_t \quad (35)$$

$$\hat{u}_t = -\frac{e}{u} \hat{e}_t \quad (36)$$

$$\hat{e}_t = \left(\frac{1 - \delta}{1 + p(\theta)} \right) \hat{e}_{t-1} + \left(\frac{(1 - \alpha)p(\theta)u}{e(1 + p(\theta))} \right) \hat{\theta}_t \quad (37)$$

Log-linearisation of the unconstrained household's consumption Euler equation yields:

$$\hat{c}_t^U = \mathbb{E}_t \hat{c}_{t+1}^U - (\hat{l}_t - \mathbb{E}_t \pi_{t+1} - \rho) \quad (38)$$

where $\rho \equiv -\log \beta$. Combing the definition of the heterogeneity index ζ_t with the definition for aggregate consumption:

$$C_t \equiv \psi C_t^K + (1 - \psi) C_t^U$$

yields

$$C_t = (1 - \psi \zeta_t) C_t^U$$

¹⁹As is standard, for any variable Z_t we denote the lower case of the variable with a hat indicate the log-deviation of that variable from steady state, i.e. $\hat{z}_t = \ln \left(\frac{Z_t}{\bar{Z}} \right) \approx \frac{Z_t - \bar{Z}}{\bar{Z}}$.

²⁰The steady state is characterised explicitly in Appendix A.

which can be log-linearised to

$$\hat{c}_t = \hat{c}_t^U - \frac{\psi}{1 - \psi\zeta} \hat{\zeta}_t \quad (39)$$

and the heterogeneity index can be log-linearised to give

$$\hat{\zeta}_t = \Psi \left[Y \hat{y}_t - e\omega(\hat{e}_t + \hat{\omega}_t) - \kappa v \hat{v}_t \right] \quad (40)$$

where

$$\Psi = \frac{\omega e(1 - \psi)}{(Y - e\omega - \kappa v)(e\omega(1 - \psi) + (1 - \psi(1 - \tau)\varrho)(Y - e\omega - \kappa v))}$$

From the firm side, the log-linearised expression for aggregate output is

$$\hat{y}_t = \hat{a}_t + \hat{e}_t + \hat{n}_t \quad (41)$$

Log-linearising the expression for hours gives:

$$\hat{n}_t = \frac{1}{\varphi} (\hat{\mu}_t + \hat{a}_t - \hat{c}_t^U) \quad (42)$$

Following Krause and Lubik (2007) we can log-linearise the optimal pricing condition for firms to give a New Keynesian Phillips curve expressed in terms of marginal cost:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda \hat{\mu}_t \quad (43)$$

where $\lambda \equiv \frac{(\epsilon-1)}{\mathcal{C}}$. The log-linear expression of the real wage is given by

$$\hat{\omega}_t = \Gamma_0(\hat{a}_t + \hat{\mu}_t + \hat{n}_t) + \Gamma_1(\mathbb{E}_t \hat{\theta}_{t+1} - \mathbb{E}_t \Delta \hat{c}_{t+1}^U) + \Gamma_2(\hat{c}_t^U + \hat{n}_t) \quad (44)$$

where

$$\begin{aligned} \Gamma_0 &\equiv \frac{\eta \mu N}{\omega} \\ \Gamma_1 &\equiv \frac{\eta \beta (1 - \delta) \frac{\kappa}{q(\theta)} p(\theta)}{\omega} \\ \Gamma_2 &\equiv \frac{(1 - \eta) \left(\frac{\chi}{1 + \varphi} \right) C^U N^{1 + \varphi}}{\omega} \end{aligned}$$

The job creation condition (23) log-linearises to give:

$$\hat{\theta}_t = \Phi_0(\hat{a}_t + \hat{\mu}_t + \hat{n}_t) - \Phi_1 \hat{\omega}_t + \Phi_2(\alpha \mathbb{E}_t \hat{\theta}_{t+1} - \mathbb{E}_t \Delta \hat{c}_{t+1}^U) \quad (45)$$

where

$$\begin{aligned} \Phi_0 &\equiv \frac{\frac{\bar{m}}{\kappa} \mu N}{\Theta} \\ \Phi_1 &\equiv \frac{\frac{\bar{m}}{\kappa} \omega}{\Theta} \\ \Phi_2 &\equiv \frac{\beta(1 - \delta) \theta^\alpha}{\Theta} \end{aligned}$$

$$\Theta \equiv \alpha \frac{\bar{m}}{\kappa} (\mu N - \omega) + \alpha \beta (1 - \delta) \theta^\alpha$$

Log-linearising the goods market clearing condition gives:

$$\hat{y}_t = s_c \hat{c}_t + \kappa s_v \hat{v}_t \quad (46)$$

where $s_c = \frac{C}{Y}$ is the steady state consumption share of output and $s_v \equiv \frac{v}{Y}$ is the vacancy share of steady state output.

Finally, to close the model we need to make a stance on how monetary policy is set. I use a standard Taylor-type rule of the class analysed in Clarida, Galí and Gertler (1999, 2000), where the central bank sets the nominal rate to react to inflation and aggregate output:

$$\hat{i}_t = \phi_\pi \pi_t + \phi_y \hat{y}_t + \nu_t \quad (47)$$

where ν_t is a monetary policy shock which follows an AR(1) process:

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu$$

with $\varepsilon_t^\nu \sim \mathcal{N}(0, \sigma_\nu^2)$.

3.9 Deriving an Aggregate Euler Equation

A key relationship in this model is that between aggregate consumption and the consumption gap defined by the heterogeneity index. Specifically, it can be shown that household heterogeneity will appear as a wedge in the aggregate Euler equation.

Combining the Euler equation for unconstrained households (38) with the log-linearised relationship between aggregate and unconstrained consumption (39), it can be shown that

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} + \frac{\psi}{1 - \psi \zeta} \mathbb{E}_t \Delta \hat{\zeta}_{t+1} - (\hat{i}_t - \mathbb{E}_t \pi_{t+1} - \rho) \quad (48)$$

This equation illustrates how in a standard two-agent setup changes in the consumption gap can have effects on aggregate consumption. This equation holds for both the model presented here and the standard TANK model without labour frictions in Debortoli and Galí (2018) where household inequality is defined in the same way. The direct effect of a change in the policy rate is still present, however given that $\frac{\psi}{1 - \psi \zeta} > 0$ any contemporaneous increase in household inequality from an increase in monetary policy will amplify the effect of policy on aggregate consumption, and therefore the effects on output. Note also that if there are no constrained agents in the economy i.e. $\psi = 0$ the equation reduces to the usual aggregate Euler equation.

The difference between this model and the standard TANK model comes in the components of firm profits, which is ultimately what drives the consumption gap. In addition to the real wage, in this model profits are now also a function of the employment rate²¹. To highlight this difference, we can substitute in using (40) to express the aggregate Euler equation as

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \frac{1 - \psi\zeta}{1 + \psi(\Psi - \zeta)}(\hat{i}_t - \mathbb{E}_t \pi_{t+1} - \rho) - \frac{\psi\Psi\omega e}{1 + \psi(\Psi - \zeta)} \left[\mathbb{E}_t \Delta \hat{e}_{t+1} + \mathbb{E}_t \Delta \hat{\omega}_{t+1} \right] \quad (49)$$

whereas in a TANK economy the corresponding aggregate Euler equation is:

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \frac{1 - \psi\zeta}{1 + \psi(\Psi^T - \zeta)}(\hat{i}_t - \mathbb{E}_t \pi_{t+1} - \rho) - \frac{\psi\Psi^T\omega_T}{1 + \psi(\Psi^T - \zeta)} \mathbb{E}_t \Delta \hat{\omega}_{t+1} \quad (50)$$

where

$$\Psi^T = \frac{\omega_T(1 - \psi)}{(Y - \omega_T)(\omega_T(1 - \psi) + (1 - \psi(1 - \tau)\varrho)(Y - \omega_T))}$$

is a function of the steady state real wage in the TANK economy, ω_T . In both cases it can be seen that there is a direct effect of a change in the policy rate on aggregate consumption, as well as indirect effects which operate through the labour market channel.

Direct effect: The direct effect of a change in the interest rate on aggregate consumption is different in the presence of heterogeneous households. In fact, under the calibration which will be used to solve the model set out in Chapter 4, it can be shown that the direct effect of policy on aggregate consumption is less than one-for-one, which is what would prevail in a RANK model with log preferences:

$$\frac{1 - \psi\zeta}{1 + \psi(\Psi Y s_c - \zeta)} < 1$$

Using the calibration presented in section 4, it can be shown that the direct effect of a 1% increase in $\hat{i}_t$ approximately leads to a 0.3% fall in aggregate consumption. In particular note that the direct effect of monetary policy is inversely related to the fraction of constrained agents ψ . This result is found in Bilbiie (2008, 2018), as well as in KMV (2018). Interestingly, the direct effect with both labour and credit frictions is actually smaller than the response under the TANK specification, as $\Psi > \Psi^T$ ²².

Indirect effects: These effects operate through the respective wedges in the aggregate Euler equations, which are functions of labour market variables. Labour market variables

²¹Note that to a first-order approximation the vacancy posting effect on inequality given by (34) actually drops out of the model.

²²It can be shown under the calibration introduced in chapter 4 that $\omega > \omega_T$, which implies that $\Psi > \Psi^T$.

will also be affected by changes in the policy rate contemporaneously. This works indirectly through the effect on the consumption demand of unconstrained households, which will influence both labour supply through the fall in the marginal utility of consumption, and labour demand via the effect of falling aggregate demand on the total expected surplus from an employment relationship.

A common effect in both this model and the standard TANK model is that in response to a policy contraction which reduces the current consumption unconstrained households, these households become more willing to supply labour as their marginal utility of consumption is greater. This puts downward pressure on the real wage, which amplifies the fall in aggregate consumption via the aggregate Euler equations. This response of the wage rate to changes in the supply of labour will be slightly dampened by the bargaining power η of workers in this model, as illustrated by the coefficient Γ_2 in equation (44).

The presence of search & matching frictions in the labour market in this economy leads to further indirect effects which are absent in the TANK model. These operate through the effect of changes in the demand of unconstrained agents on the total expected surplus from an employment relationship. In particular, a monetary contraction leads to a fall in the total expected match surplus from an employment relationship, as demand in the economy is falling. This will influence both the real wage via Nash bargaining and labour market tightness through the job creation condition:

- *Wage effect:* Equation (44) illustrates that as well as the supply effect from a fall in $\hat{c}_t^U$ governed by the size of Γ_2 , there is also a demand effect governed by Γ_1 . Reducing the total match surplus reduces the expected saving from hiring workers, which puts further downward pressure on the real wage. Moreover, from the aggregate Euler equation, the magnitude of the amplification effect of the real wage on consumption is larger compared to TANK²³.
- *Employment effect:* Equation (45) illustrates that labour market tightness $\hat{\theta}_t$ is an increasing function of $\hat{c}_t^U$, where the response depends on Φ_2 . A reduction in the total expected surplus from a monetary contraction leads to firms hiring less labour and therefore posting fewer vacancies. This reduces the job-finding rate, and increases unemployment in the model.

Taken together, these additional indirect effects present in this model represent a novel mechanism suggesting that in the presence of labour and credit market frictions together,

²³For a high enough steady state employment rate e , the wage effect will be greater in the case of both credit and labour market frictions, as we have $\omega > \omega^T$ and $\Psi > \Psi^T$

in response to a monetary contraction we expect to see a greater increase in household inequality, manifesting in a greater fall in output. To see whether these additional effects matter quantitatively, we need to solve and simulate the model.

4 Quantitative Analysis

4.1 Calibration

The baseline calibration of the model is intentionally close to much of the literature discussed in chapter 2, and is summarised in Table 1. More specifically, I generally follow Debortoli and Galí (2018) in setting the parameters for the heterogeneous household side, and Krause and Lubik (2007) and Thomas and Zanetti (2009) for those relating to the labour market and production.

Each time period is assumed to be equal to a quarter. The individual discount factor β is set to 0.99 to achieve an annualised real interest rate of 4%, which is standard in the business cycle literature. The Frisch elasticity of labour supply φ is equal to 1, as standard in the literature. The leisure parameter χ is also set equal to 1. The elasticity among varieties ϵ is set equal to 6 following Krause and Lubik (2007), which is lower than that used in Debortoli and Galí (2018) where ϵ is set to 10. The quadratic price adjustment costs parameter $\mathcal{C}$ is set to equal 70, which lies in the middle of the range found in the literature, which goes from around $\mathcal{C} = 20$ (Lubik and Schorfheide, 2004) to about $\mathcal{C} = 105$ (Debortoli and Galí, 2018). This gives a value for the slope of the Phillips curve in terms of marginal costs $\lambda = 0.07$, which lies towards the lower end of the estimated range but is not inconsistent with the body of literature documenting a general ‘flattening’ of the slope of the Phillips curve in recent times (e.g. Blanchard, 2016).

Following Thomas and Zanetti (2009) and conforming with empirical findings in the US which estimate the job finding rate to be around 30% and the separation rate to be around 3.5%, I set the steady state job finding rate $p(\theta)$ equal to 0.3 and the exogenous job separation rate δ is set equal 0.035. This corresponds to a steady state unemployment rate of around 10%. The elasticity of matches with respect to unemployment α is set equal to 0.5, which is reasonable in the DMP literature. The bargaining power of workers η is also set to 0.5 to ensure the Hosios’ condition is satisfied for efficiency of the steady state. Linear vacancy posting costs κ are set equal to 0.05.

The share of illiquid profits distributed by fiscal policy ϱ is set equal to 0.92, following Kaplan et al. (2018). We initially set τ equal to 1, corresponding to a “wealth-based”

redistribution rule. This is chosen as it is more realistic, given the wealth distributions of most advanced economies. The share of constrained households ψ is set to 0.3, which falls in the middle of the range of values used in the TANK literature, which ranges from as low as 0.13 in McKay et al. (2016) up to an implied value of 0.41 in KMV.

The steady state share of vacancies s_v is equal to 0.03, and so $s_c = 0.97$. The persistence of the monetary policy shock ρ_v is set to 0.5 and the standard deviation σ_v is set to 0.02, which are standard values used in the New Keynesian literature. For the productivity shock we use the estimated labour productivity process in Shimer (2005) for the purposes of comparison, which estimates $\rho_a = 0.004$ and $\sigma_a = 0.0165$. The Taylor coefficient on inflation ϕ_π is set to the usual value of 1.5, and that on output ϕ_y to 0.125, which are also standard values used for Taylor-type rules in the literature (e.g. Taylor (1993), Clarida, Galí and Gertler, 1999).

Table 1: Calibrated Parameter Values

Parameter	Description
$\beta = 0.99$	Discount factor
$\varphi = 1$	Frisch elasticity of labour supply
$\chi = 1$	Leisure parameter
$\epsilon = 6$	Elasticity of substitution among varieties
$\mathcal{C} = 40$	Quadratic price adjustment parameter
$p(\theta) = 0.3$	Job finding probability
$q(\theta) = 0.9$	Vacancy filling probability
$\delta = 0.035$	Job separation rate
$\alpha = 0.5$	Matching function elasticity
$\kappa = 0.05$	Vacancy posting costs
$\eta = 0.5$	Workers' bargaining power
$\varrho = 0.92$	Fraction of illiquid assets
$\tau = 1$	Wealth-based fiscal rule
$\psi = 0.3$	Share of constrained households
$s_c = 0.97, s_v = 0.03$	Steady state shares of output expenditure
$\rho_a = 0.004, \rho_v = 0.5$	Persistence of shocks
$\sigma_v = 0.02, \sigma_a = 0.0165$	Standard deviation of shocks
$\phi_\pi = 1.5, \phi_y = 0.125$	Taylor rule coefficients

4.2 Solution Method

The model is solved using standard perturbation techniques. The log-linearised model in the previous section in equations (35) - (47) presents the model approximated to the first-order around an efficient steady state. This version of the model can then be expressed as a system of linear expectational difference equations in the usual state-space form:

$$A\mathbb{E}_t\mathbf{Y}_{t+1} = B\mathbf{Y}_t + C\epsilon_t$$

where $\mathbf{Y}_t$ is a vector of endogenous variables and $\epsilon_t = (\varepsilon_t^a, \varepsilon_t^v)^\top$ is a vector of structural shocks. Policy rules for choice variables as functions of the endogenous state variables $(\hat{\theta}_t, \hat{\zeta}_t)$ and exogenous state variables $(\hat{a}_t, \nu_t)$ can then be obtained by solving for A and B ²⁴, provided the usual Blanchard-Kahn (1980) conditions for equilibrium determinacy are satisfied.

4.3 Impulse Response Analysis

I first present and discuss the dynamics of the model conditional on a 1% shock to both monetary policy and technology separately, before moving onto simulation analysis. “SAMTANK” refers to the model presented in this paper and calibrated according to Table 1. “SAMRANK” refers to the version of the model where $\psi = 0$ (i.e. no household heterogeneity) but search & matching frictions still present ($\kappa \neq 0$). “TANK” refers to the case where we preserve the two-agent structure but remove search & matching frictions ($\kappa = 0$), which shuts down the extensive margin for labour adjustment, and $\eta = 0$ which shuts down worker’s bargaining power in the Nash bargaining process determining the real wage. Finally, “RANK” refers to the standard representative New Keynesian model with no household heterogeneity ($\psi = 0$), no search & matching frictions ($\kappa = 0$) and no Nash bargaining over the prevailing real wage ($\eta = 0$).

4.3.1 Model Dynamics in Response to a Monetary Contraction

Figure 1 presents the responses of the model conditional on a 1% monetary policy shock, where the units are in percentage deviation from an efficient steady state. The main finding is that the introduction of labour market frictions to the standard TANK model appears to lead to contractionary monetary policy having a greater impact on the consumption gap, which then leads to a greater response of output relative to the TANK and RANK models.

²⁴This is done in Dynare using the standard linear solver which makes use of the Jordan decomposition to solve for policy rules given the state-space form.

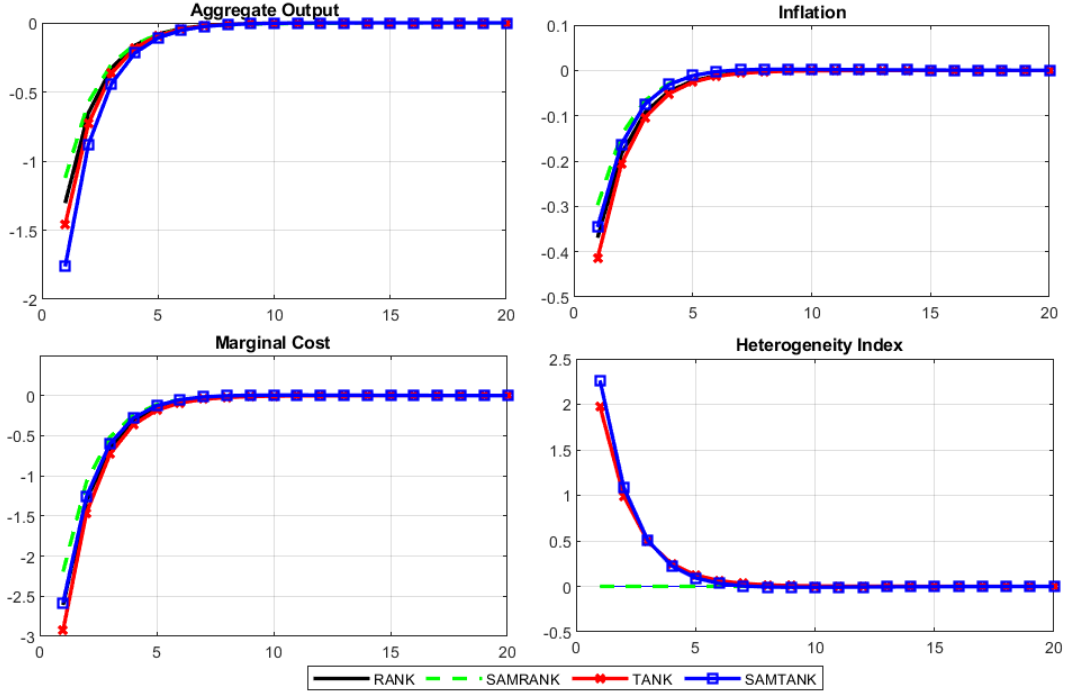


Figure 1: Impulse responses to a 1% monetary policy shock

This conforms with the findings in Gornemann et al. (2016) that contractionary monetary policy has a greater impact on inequality when we account for labour market frictions. As discussed in the previous section, this additional interaction relative to the TANK model works through the general equilibrium effects in the labour market in response to a policy shock. Specifically we isolated a wage effect and an employment effect as the key mechanisms in this model. Figure 2 presents the responses of these variables to a monetary contraction.

- *Wage Effect.* Figure 2 shows that the real wage falls by around 4.3% in the presence of both credit and labour frictions, compared to only a 3% fall under TANK. This is due to the presence of the additional demand effect, which works through the effect of an increase in the marginal utility of consumption for unconstrained households directly on the total expected match surplus from an employment relationship. This occurs due to the reduction of the expected savings from hiring workers, which reduces firm demand for labour and adds further downward pressure on the real wage. This demand effect can only occur if wages are determined via bargaining and so relies on some form of labour frictions, whereas the supply effect is present in both SAMTANK and TANK economies. The greater fall of the real wage in the SAMTANK model suggests that this additional demand effect is quantitatively

significant.

- *Employment Effect.* In this setup firms can also adjust labour on the extensive margin by choosing to hire fewer workers. Figure 2 illustrates that vacancies fall on impact by around 7.5%. This again comes about due to the reduction in the total expected match surplus in response to a fall in the consumption demand of unconstrained households, which leads to firms reducing their demand for hiring labour today. This produces greater slackness in the labour market with the employment rate decreasing by about 0.09%. Higher unemployment further increases the consumption gap due to the presence of hand-to-mouth households, who rely exclusively on employment to finance consumption. This leads to a further amplification on aggregate consumption, an effect which is absent in the TANK economy where firms cannot adjust on the extensive margin.

Taken together, these two effects lead to a 2.25% increase in the consumption gap on impact, compared to a 2% increase under TANK. Quantitatively, we can say that the presence of both labour and credit market frictions leads to a larger fall in the consumption of constrained households in response to a monetary policy tightening, driven by a larger fall in wages and a reduction in the employment rate. This larger shift in the distribution of consumption away from constrained households - who have a higher marginal propensity to consume - leads to a greater amplification of monetary policy on output via the aggregate Euler equation.

Interestingly the response of inflation in this economy is slightly muted compared to the TANK economy, and lies much closer to that of the RANK. This is because the fall in the surplus from an employment relationship increases the expected cost of hiring, which works in the opposite direction to the fall in real wages. As such, the presence of hiring costs means that marginal costs do not fall by as much in this economy, muting the fall in inflation via the Phillips curve relationship.

Overall, these findings illustrate that labour market frictions interact with household heterogeneity in response to aggregate shocks in ways that are relevant for policy, and which are not captured by existing models in the literature. The presence of both labour and credit market frictions gives rise to much greater amplification of monetary policy on output compared to RANK and TANK models, which works via the impact on the distribution of consumption through the labour market. At the same time, the inflation response to policy is quantitatively similar to a RANK model, and slightly muted compared to the TANK model. For a central bank which cares about the volatility of both output

and inflation, these results suggest that achieving a given reduction in inflation using monetary policy will lead to a significantly greater fall in output, potentially resulting in much larger welfare losses than those implied by a RANK model.

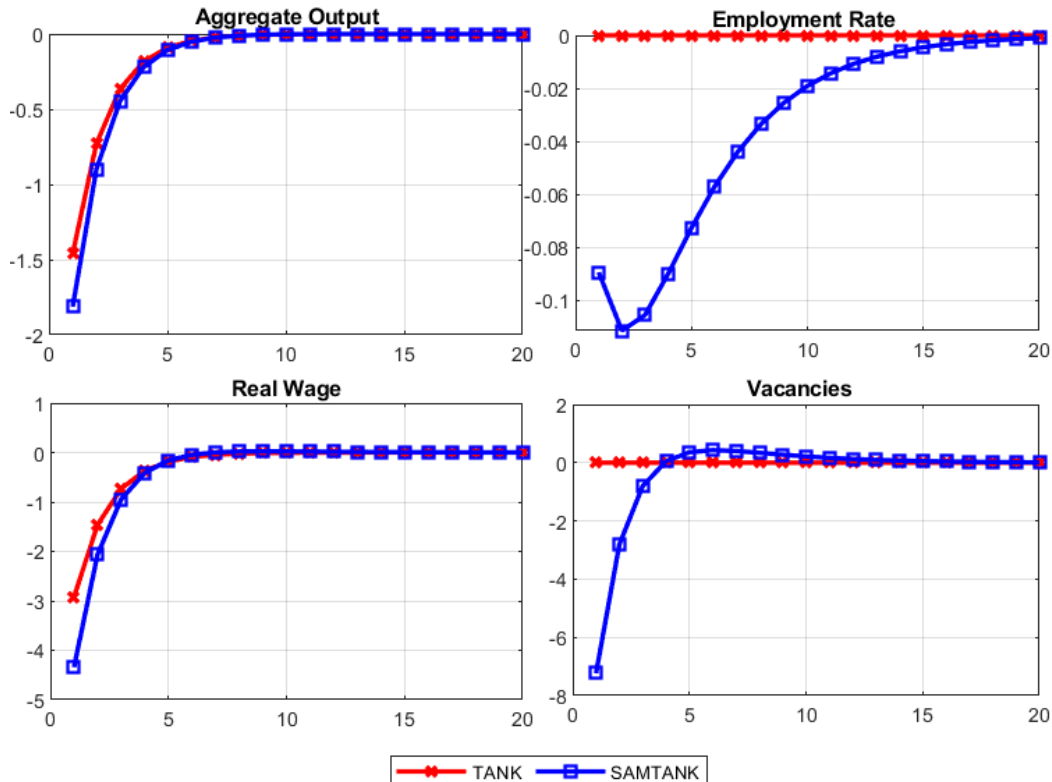


Figure 2: Heterogeneity determinants (monetary policy shock)

4.3.2 Model Dynamics in Response to a Productivity Increase

Figure 3 presents the response of the model conditional on a 1% productivity shock. In the presence of both labour and credit market frictions, the output response to an increase in productivity is muted compared to the TANK model²⁵. This is due to the fact that firms cannot costlessly acquire additional labour in order to fully utilise the increase in productivity, and because the distribution of aggregate consumption shifts towards unconstrained households who have a lower marginal propensity to consume.

²⁵As the supply side of the model is identical in RANK and TANK, the effect of a supply shock in both cases is identical.

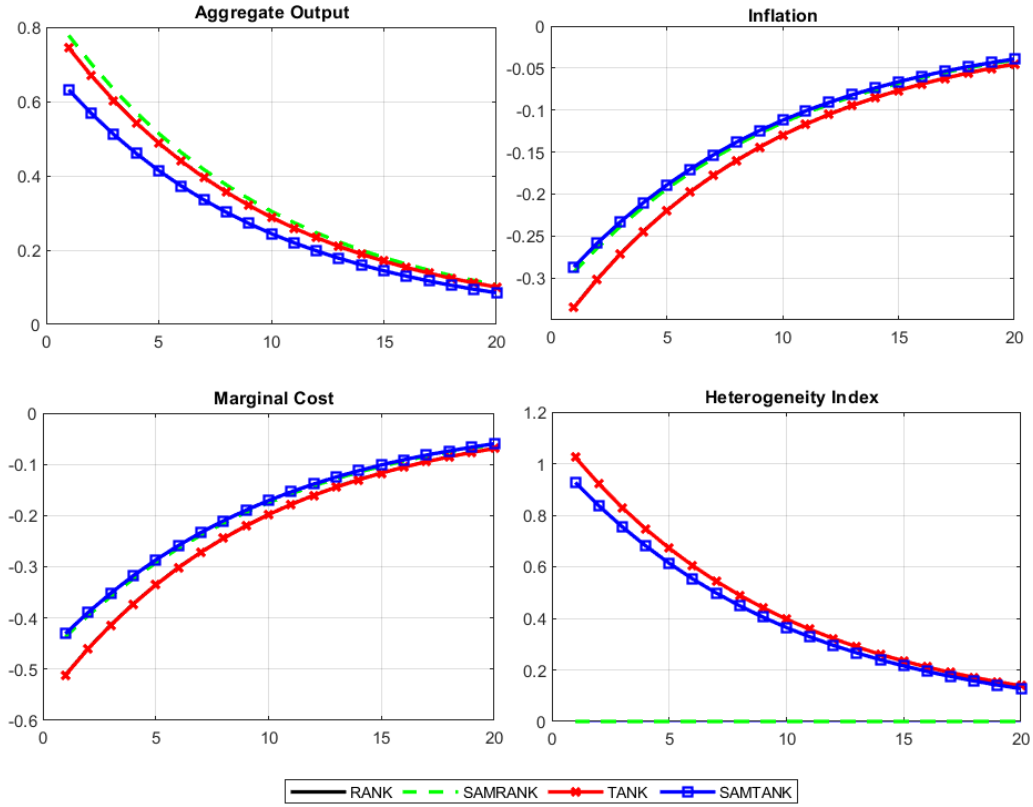


Figure 3: Impulse responses to a 1% technology shock

Initially, an increase in productivity increases the marginal return to labour, which reduces marginal costs. In this model, this reduction works through two distinct channels. Firstly, there is a reduction in the marginal cost of currently employed labour for firms, which leads to an increase in output for the same amount of workers employed. Secondly, a higher level of productivity reduces the cost of hiring for firms. Falling marginal costs then increase firm profits. However, the very presence of hiring costs means that marginal costs do not fall by as much in response to the supply shock compared to models with a frictionless labour market. This then leads to a lower fall in the inflation rate through the Phillips curve. The increase in firm profits from reduced production costs leads to greater household inequality, as only unconstrained households own the profits of firms and so benefit disproportionately from the productivity shock. This mutes the overall response of output relative to the SAMRANK model via the aggregate Euler equation, an effect that is quantitatively significant.

Compared to the TANK model we see that the increase in household inequality from a supply shock is actually muted. As with the monetary policy shock, this can be explained through the general equilibrium effects on labour market variables, whose responses are presented in Figure 4. The reduction in hiring costs leads to an increase in the total expected match surplus, which increases labour demand from firms as workers are more productive. This increases both the real wage and vacancy postings, leading to an additional increase in the employment rate. The result is that aggregate labour income in the economy increases by more than in the TANK economy, which counteracts the increase in firm profits from the positive productivity shock. However, Figure 3 illustrates that the response of output is actually *greater* in the TANK model despite there being a larger increase in household inequality. This is because in the absence of labour market frictions, marginal costs fall further in the TANK model. The fact that the output response is larger despite a greater increase in household inequality suggests the effect on marginal costs actually dominates in the event of a positive supply shock. Overall, the main finding is that the interaction between credit and labour market frictions mutes the output response to aggregate productivity shocks, which operates through both the costly acquiring of additional labour preventing a greater fall in production costs, and the increase in household inequality that results from greater firm profits.

4.4 Simulation Results

We first analyse the results for labour market variables before going on to look at those for standard business cycle moments. Following Shimer (2005), for all simulation results reported the moments are detrended using a standard HP filter with the smoothing coefficient equal to 10^5 . Simulations are repeated 10,000 times in order to generate reasonable estimates of moments and correlations implied by the model.

4.4.1 Labour Market Moments

Tables 2 and 3 report the simulation results for labour market variables in the presence of technology shocks and monetary policy shocks respectively. The first column in each table reports the empirical moments and correlations of labour market variables used in Shimer (2005), which are calculated using quarterly US data between 1951-2003. As a theoretical benchmark, the second column in the first table reports Shimer’s (2005) results from the same quantitative exercise obtained instead in the context of a standard DMP search & matching model. These results illustrate the so called “Shimer puzzle”: the

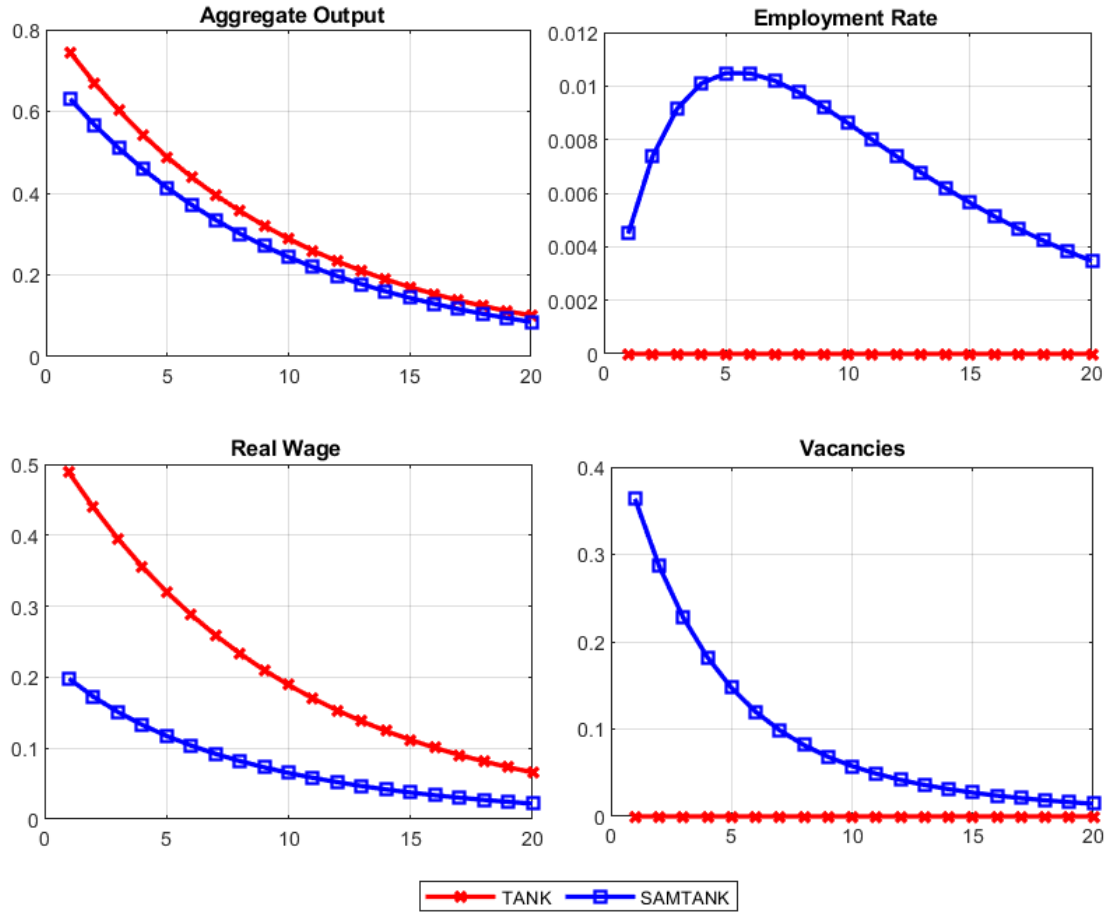


Figure 4: Heterogeneity determinants (technology shock)

inability of a standard DMP model to generate realistic business cycle volatility in both the unemployment rate and vacancies. The third and fourth columns of the table report the results obtained using the SAMTANK and SAMRANK economies respectively.

The main finding is that the SAMTANK model generates significantly more volatility in vacancies compared to the benchmark DMP model and the SAMRANK model in the presence of aggregate shocks, whilst generating a quantitatively similar volatility of the unemployment rate to Shimer's (2005) model. Moreover, despite this marginal improvement in increasing the volatility of vacancies, the model implied volatilities are still far away from those estimated in the data. As such, vacancies in the SAMTANK are approximately 3 times less volatile than the data compared to 10 times less in Shimer's (2005) baseline model, whereas the unemployment rate remains around 10 times less volatile. Note also that the model-implied correlation between vacancies and unemployment - the Beveridge

Table 2: Technology Shocks: Labour Market

	Data	Shimer (2005)	SAMTANK	SAMRANK
<i>Standard deviations:</i>				
σ_v	0.202	0.027	0.070	0.037
σ_u	0.190	0.009	0.0109	0.006
σ_θ	0.382	0.035	0.076	0.041
<i>Quarterly autocorrelations:</i>				
ρ_v	0.940	0.835	-0.068	-0.065
ρ_u	0.936	0.939	0.716	0.721
ρ_θ	0.941	0.878	-0.014	-0.01
<i>Cross-correlations:</i>				
$Corr(v, u)$	-0.894	-0.927	-0.542	-0.540
$Corr(v, \theta)$	0.975	0.996	0.993	0.993
$Corr(u, \theta)$	-0.971	-0.958	-0.639	-0.638

curve relationship - is actually lower in the SAMTANK model at -0.542 compared to -0.894 in the data. However, the similar quantitative performance of SAMRANK compared to Shimer's (2005) baseline DMP model in Table 2 indicates that the increase in the volatility of vacancies comes from the interactions between labour and credit frictions, suggesting that this interaction effect is quantitatively significant but not enough on its own to resolve the Shimer puzzle.

Table 3 illustrates a similar pattern of findings in the presence of monetary policy shocks instead. Interestingly, these results suggest that the model's quantitative performance is better when allowing for a shock process with greater persistence, given that the variance of the shocks is very similar. In particular, the volatility of vacancies and labour market tightness are much closer to those estimated in the data, although the volatility of unemployment is quantitatively unaffected, and the quarterly autocorrelations now have the correct sign and are significantly different from zero. However, it is not clear that the volatility in aggregate shocks required to achieve this significant improvement in the quantitative performance of the model is empirically plausible.

Table 3: Monetary Policy Shocks: Labour Market

	Data	SAMTANK	SAMRANK
<i>Standard deviations:</i>			
σ_v	0.202	0.152	0.117
σ_u	0.190	0.037	0.028
σ_θ	0.382	0.174	0.134
<i>Quarterly autocorrelations:</i>			
ρ_v	0.940	0.3869	0.392
ρ_u	0.936	0.8861	0.888
ρ_θ	0.941	0.4609	0.466
<i>Cross-correlations:</i>			
$Corr(v, u)$	-0.894	-0.517	-0.517
$Corr(v, \theta)$	0.975	0.984	0.984
$Corr(u, \theta)$	-0.971	-0.662	-0.663

4.4.2 Business Cycle Moments

Table 4 reports the simulation results for the relative volatility of business cycle variables for the SAMTANK, SAMRANK and TANK economies relative to the volatilities generated by the baseline RANK model²⁶. Panel (a) shows the results for technology shocks and panel (b) gives the results for monetary policy shocks.

Consistent with the impulse response analysis, the main findings is that in response to aggregate shocks output is significantly more volatile in the presence of both labour and credit market frictions relative to the representative agent model compared to TANK and SAMRANK, whilst movements in inflation and marginal costs are slightly less volatile and seem to be quantitatively similar to those generated in the RANK. The additional volatility in output in the SAMTANK model compared to the standard TANK model is generated by aggregate labour income being more risky. This is driven by the volatility the total expected surplus from an employment relationship, represented by hours worked²⁷, which directly affects both the hiring decision and the prevailing real wage in the economy in the presence of labour frictions. More specifically, this leads to greater volatility in both real wage and vacancy postings, which then amplify output fluctuations via the wedge in the aggregate Euler equation. Interestingly the magnitude of the relative moments

²⁶Superscript *R* indicates the result generated by the RANK model.

²⁷Recall that hours are set to maximise the match surplus in the model.

Table 4: Business Cycle Moments

	SAMTANK	SAMRANK	TANK
(a) <i>Technology shock:</i>			
σ_y/σ_y^R	9.98	2.48	8.45
σ_c/σ_c^R	8.73	2.57	8.45
σ_n/σ_n^R	1.65	0.88	1.60
σ_π/σ_π^R	1.16	0.87	1.60
σ_μ/σ_μ^R	1.20	0.89	1.60
$\sigma_\omega/\sigma_\omega^R$	3.16	1.68	2.29
σ_ζ	0.04	0.00	0.04
(b) <i>Monetary policy shock:</i>			
σ_y/σ_y^R	1.35	0.87	1.12
σ_c/σ_c^R	1.27	0.88	1.12
σ_n/σ_n^R	1.26	0.79	1.12
σ_π/σ_π^R	0.92	0.80	1.12
σ_μ/σ_μ^R	0.98	0.84	1.12
$\sigma_\omega/\sigma_\omega^R$	1.61	1.24	1.12
σ_ζ	0.05	0.00	0.04

is very different between the two aggregate shocks. Again this seems to be driven by the persistence of the process describing the aggregate shocks. In particular we see that the relative differences in volatility compared to the baseline RANK model appear to be decreasing in the persistence of the aggregate shocks, although this has no effect on the relative differences in the moments between the models being compared.

Overall, simulation evidence suggests that aggregate labour income becomes significantly more risky in the presence of both credit and labour market frictions which leads to a greater volatility in aggregate output, whilst the volatility of inflation appears to be quantitatively similar to a RANK model. This result is largely driven by the volatility in the total match surplus, which leads to greater volatility in the real wage. There is also evidence this increases the volatility of vacancy postings and labour market tightness relative to the standard DMP model through the hiring decision, although this effect is not large enough quantitatively to explain the data. From a policy perspective, these findings suggest that a central bank which cares about both inflation and output will wish to act more aggressively against output fluctuations than is implied in either a RANK or

a TANK model.

5 Robustness

In this section I present the findings of changes in the environment. In particular, I present and compare the impulse responses of the model for alternative values of key structural parameters conditional on a 1% monetary policy shock. Given the model setup, there are some key parameters to vary to further understand the robustness of the results described in the previous section to alternative calibration choices. Results for the bargaining power of workers η , the fraction of constrained agents ψ , and the slope of the Phillips curve λ are presented here. Analysis was also done for the fiscal specification indicated by τ , steady state level of consumption inequality ζ and vacancy posting costs κ , however varying these parameters within a plausible range do not appear to quantitatively affect the model²⁸.

5.1 Bargaining Power

The bargaining power reflects the share of the match surplus going to employed workers. Figure 5 plots the responses of the model variables under different values for η , namely 0.2, 0.5 and 0.6. The Figure shows that a higher value of η (reflecting greater bargaining power of workers) leads to an even greater amplification of monetary policy on labour market variables and output. This works via the effect on the expected total surplus in response to a monetary contraction outlined earlier in the paper.

In response to the fall in the marginal utility of consumption of unconstrained households, a larger value of η reduces the magnitude of the labour supply effect on the real wage governed by Γ_2 , but increases the demand effect dictated by Γ_1 . The greater fall in real wages shown in Figure 5 suggests that it is the demand effect which dominates. Increasing η also increases the steady state real wage ω , which increases the magnitude of the effect on labour market tightness via Φ_2 . Intuitively a higher bargaining power of workers will benefit those workers who are employed, but increases the cost of hiring for employers in terms of wages that will be paid, meaning that hiring costs are increasing in η . Hence we also see a greater fall in employment in response to monetary contraction when the bargaining power of workers is greater.

The overall result is a much larger increase in household inequality, translating into a larger output response via the aggregate Euler equation. These results suggest that the novel mechanism outlined in this model is magnified when bargaining power is greater.

²⁸The findings of this additional analysis are reported in Appendix B.

One implication of this result is that monetary policy has a potentially greater effect on output in economies with a higher level of labour unionisation compared to economies like the US and UK where the bargaining power of labour is much weaker.

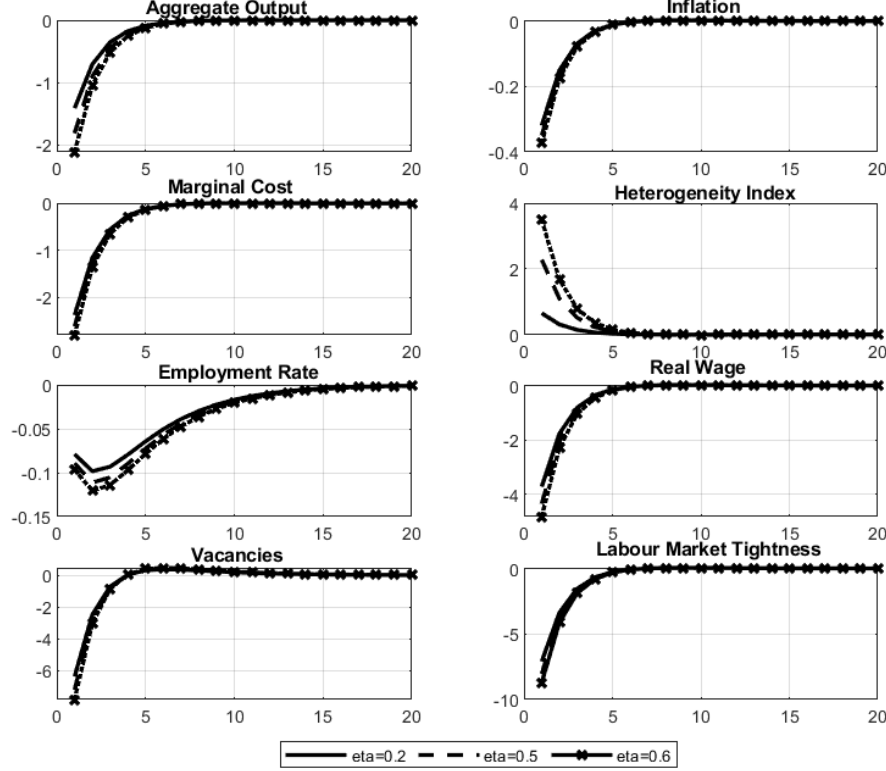


Figure 5: Changes in environment: Bargaining power

5.2 Fraction of Constrained Agents

The fraction of constrained households ψ in this model is a crucial parameter given the two-agent structure. Figure 6 illustrates the dynamics of the model for increasing values of ψ , from 0.2 at the lower end of those values used in the literature up to 0.8, which lies considerably outside the plausible range. These results support the finding in Bilbiie (2019) that a higher fraction of constrained agents increases the amplification of monetary policy²⁹. Intuitively, increasing the fraction of constrained agents ψ reduces the influence of the consumption decisions made by unconstrained households on the aggregate level. This can be seen in the aggregate Euler equation, where the direct effect of a change in $\hat{c}_t$ is decreasing in ψ . Correspondingly, increasing ψ means that aggregate consumption is more sensitive to changes in the real wage and the employment rate.

²⁹Bilbiie (2019) casts his findings in terms of a “New Keynesian cross”, where the slope of the usual planned expenditure curve is increasing in the fraction of constrained agents.

Interestingly the relationship between ψ and the effect on output seems to be increasing at a decreasing rate, as we see quantitatively the responses where ψ is 0.5 and 0.8 are very similar. Moreover, as ψ is decreasing the effect of changes in employment and the real wage on the heterogeneity index itself is reducing in the fraction of constrained agents. This is because a larger ψ implies that constrained households make up a greater proportion of households in the economy, so a contractionary policy shock is benefitting relatively fewer households as ψ increases. Hence a large enough ψ begins to dampen the effect of increasing household inequality on aggregate consumption. These results overall suggest that the degree of access to wealth and financial markets in an economy has important effects on the effectiveness of policy. In particular, we may expect monetary policy to be more effective in economies where households are more reliant on labour income.

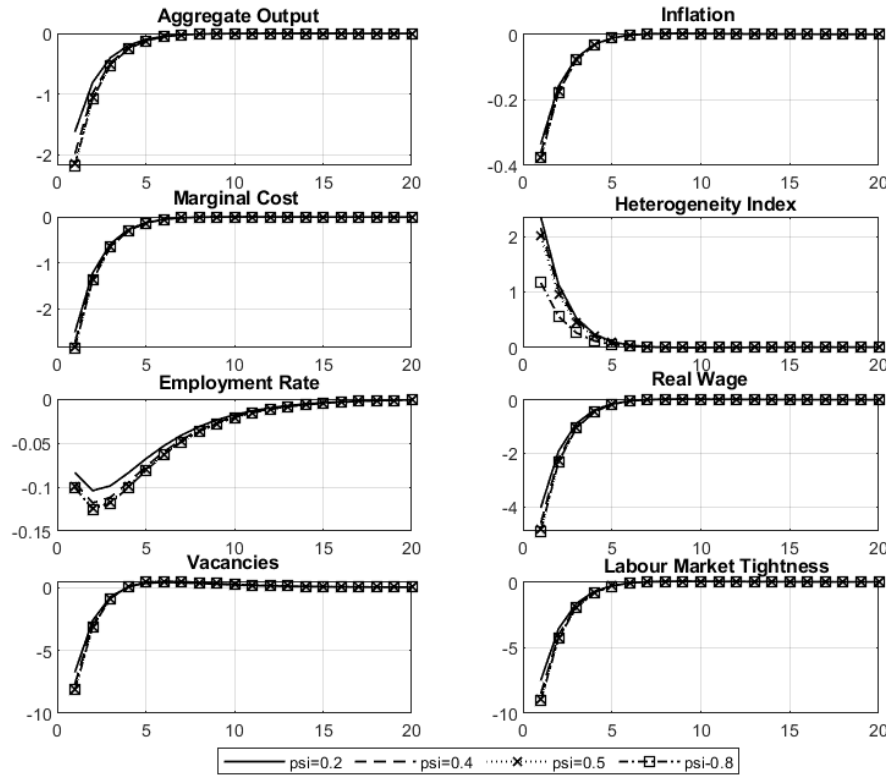


Figure 6: Changes in environment: Fraction of unconstrained agents

5.3 Slope of the Phillips Curve

Recall that the slope of the Phillips curve $\lambda \equiv \frac{\epsilon-1}{\mathcal{C}}$ is fully pinned down by two structural parameters - the elasticity of substitution ϵ and the Rotemberg price adjustment costs parameter $\mathcal{C}$. There is much uncertainty surrounding these parameters in the empirical

literature, and values are usually chosen to target a certain value of λ . The model is initially solved for $\epsilon = 6$, which is a standard value used in the New Keynesian literature with search & matching frictions but is significantly lower than those values often used in the TANK literature, which usually set ϵ to much higher values. $\mathcal{C}$ is initially set to 70, but ranges from as low as 20 (Lubik and Schorfheide, 2004) to as high as 105 in the TANK literature (e.g. Debortoli and Galí, 2017). The model is first solved varying ϵ whilst holding $\mathcal{C}$ fixed at the originally calibrated value, and then vice versa. The impulse responses are plotted in Figure 7 and Figure 8 respectively.

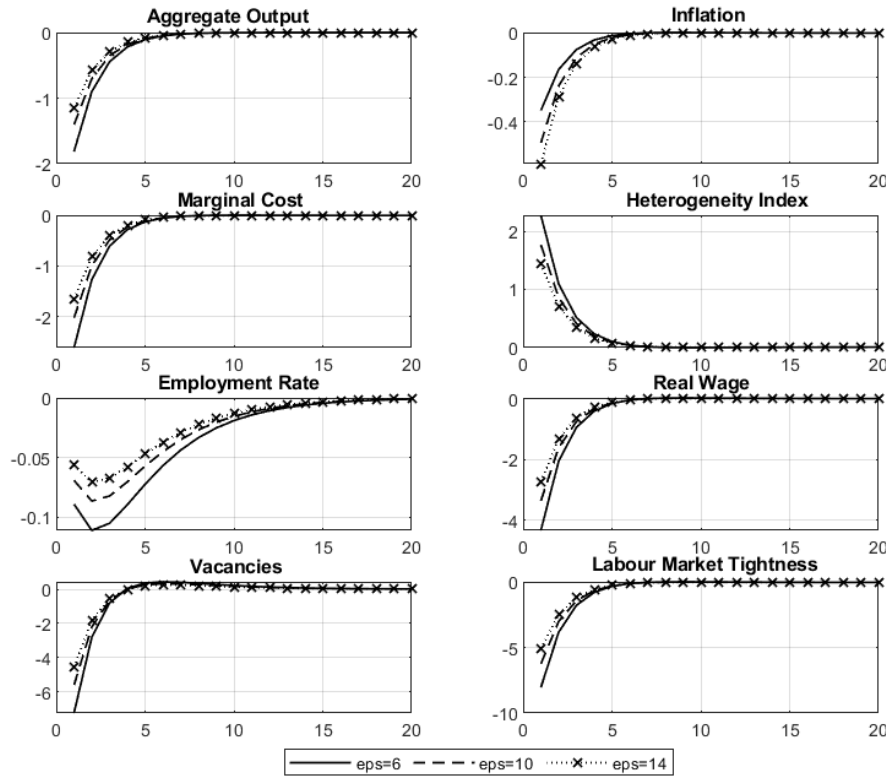


Figure 7: Changes in environment: Elasticity of substitution

Figure 7 illustrates that higher values of ϵ - implying a larger Phillips curve slope λ - are unsurprisingly associated with larger falls in inflation in response to a reduction in marginal costs which works through the Phillips curve. Interestingly, a larger value of λ implies a slightly dampened response of both the marginal cost and labour market variables. This leads to smaller increase in the consumption gap and a more muted response of output. In Figure 8 unsurprisingly we see almost the reverse picture of Figure 7, where higher values of $\mathcal{C}$ - implying a lower value of λ - are associated with a smaller fall in inflation and a larger response of marginal cost, labour market variable and output.

These results suggest that the amplification of monetary policy in the presence of

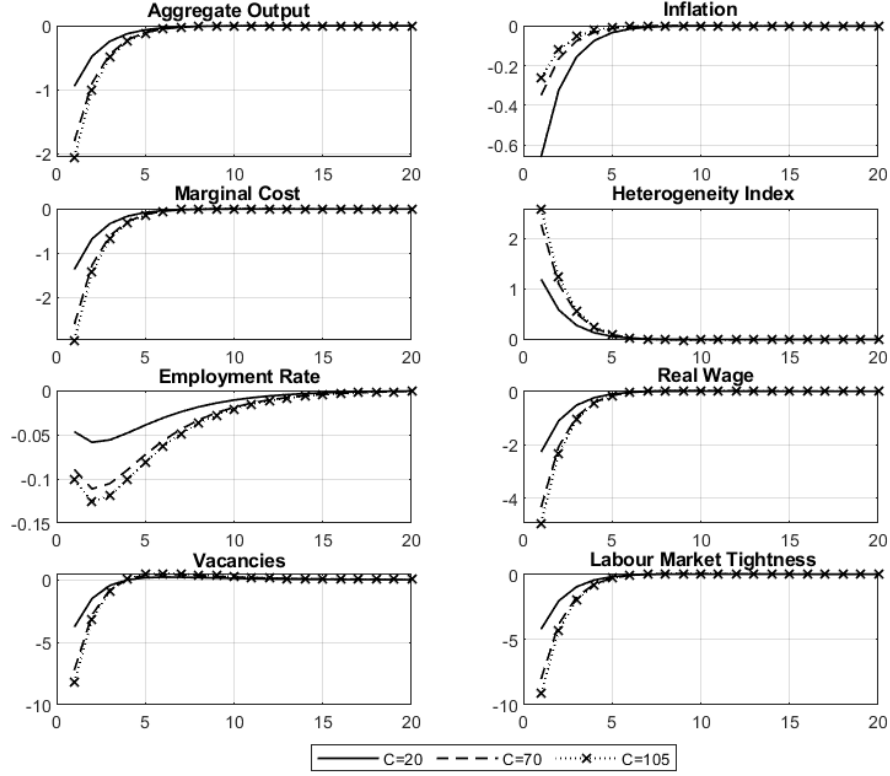


Figure 8: Changes in environment: Price adjustment costs

labour and credit market frictions is greater when the slope of the Phillips curve is flatter. This works through the effect of marginal costs on unconstrained households' expectations of inflation. As unconstrained households are forward-looking they know how inflation will respond to a monetary policy shock. In the event of such a shock these households know that inflation will be higher in the next period and therefore face a trade-off between saving more in the current period in response to a higher interest rate or consuming more today whilst prices are lower. Under standard calibrations it is the first effect which dominates. However, a greater slope of the Phillips curve leads to a greater expected change in inflation and increases the strength of the second effect, leading to a smaller fall in $\hat{c}_t^U$. This leads to smaller general equilibrium effects by dampening the response of the total expected match surplus and the subsequent effects on labour market variables discussed above. This eventually leads to a smaller effect of policy on output.

These findings suggest that in the presence of credit and labour frictions, a lower slope of the Phillips curve makes output more responsive to policy whilst inflation becomes less responsive. Put another way, in order to achieve more stable inflation a central bank in this economy would have to tolerate greater fluctuations in output than compared to a RANK or TANK model, leading to higher welfare losses. This coheres with the quantitative

evidence produced in the previous section. This case is perhaps particularly interesting, given that econometric evidence on the slope of the Phillips curve suggests that λ has been quite close to zero during both the Great Moderation and the post-crisis period.

6 Conclusions

The model presented in this paper is a version of the standard TANK model found in Debortoli and Galí (2017) extended to allow for interactions between unemployment and credit-constraints. The resulting model identifies a novel mechanism whereby additional general equilibrium effects operating through the labour market can amplify the effect of policy on aggregate output relative to the standard TANK model due to a larger increase in household consumption inequality. This works through the effects of aggregate shocks on the expected total match surplus from an employment relationship, which ultimately influences the hiring decision and the bargained real wage. The overall effect is to make aggregate labour income more risky relative to the standard TANK model, due to the fact agents now face the risk of becoming unemployed. Quantitative evidence suggests that this greater riskiness of labour income significantly increases the volatility of output relative to the standard TANK model, as well as increasing the volatility of labour market tightness relative to the standard DMP model of the labour market.

The tractability of this model allows for a simple analytical characterisation of the role of the expected total surplus from an employment relationship in the transmission of aggregate shocks. The findings suggest that even when captured in a stylised way, the interaction between credit and labour market frictions lead to non-negligible effects which are omitted in standard RANK and TANK models that are of potential interest for the design and transmission of policy. For instance, one implication of these findings is that an inflation targeting central bank will have to tolerate greater welfare losses through larger fluctuations in output in order to achieve the same level of inflation stability compared to those implied by a TANK model. Moreover, the model predicts that the effect of aggregate shocks on the expected match surplus from employment relationships may play a greater role in the transmission of monetary policy in economies where labour is more highly unionised, aggregate consumption is more dependent on aggregate labour income, and when the relationship between inflation and production costs is weaker.

The framework presented in this paper naturally raises further questions which could be addressed in future research. A natural next step would be to study optimal policy formally in this framework. The preliminary findings in this paper suggest that in the presence

unemployment and credit-market constraints it will be optimal to place a greater weight on stabilising output fluctuations relative to the TANK findings in Debortoli and Galí (2017), reinforcing the results of Gornemann et al. (2016). The number of policy instruments could also be extended to allow for unemployment benefits and hiring subsidies, which would allow for a comparison between the optimal monetary-fiscal mix in this simpler environment and richer models such as Jung and Kuester (2015). Several extensions would take the model in the direction of the HANK literature. Endogenising credit-constraints would allow for the observation that being unemployed and credit-constrained are not mutually exclusive. Another extension would be to allow for idiosyncratic shocks in order to allow for a more realistic wealth distribution. It would be interesting to see whether these extensions greatly change the quantitative performance of the model, which would test the finding in Debortoli and Galí (2017) that the simpler TANK models approximate HANK models fairly well.

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A Analytical Steady State

We consider a zero inflation steady state where prices are flexible and there exists the usual steady state subsidy to firms in order to induce efficient pricing. We also assume that the Hosios'-type condition holds, so that we have efficiency in the steady state generally. In steady state we therefore impose $A \equiv \Pi \equiv 1$. From the labour market conditions we can derive the usual Beveridge relation for the steady state unemployment rate:

$$u = \frac{\delta}{p(\theta) + \delta} \quad (51)$$

where u is a function of the separation rate and the job finding rate only. Using the identity $e = 1 - u$ we can then back out the steady state value for e .

As standard in New Keynesian models, the Euler equation for consumption of the unconstrained agents pins down the value for the steady state interest rate via the individual discount factor:

$$i = \frac{1}{\beta} - 1 \quad (52)$$

Steady state aggregate output is pinned down by the employment rate and hours worked:

$$Y = eN \quad (53)$$

and the steady state level of unconstrained consumption can be expressed in terms of employment hours:

$$C^U = \frac{\chi}{\mu} N^\varphi \quad (54)$$

The steady state goods market clearing condition is given by

$$Y = C + \kappa v \quad (55)$$

where

$$C = (1 - \psi\zeta)C^U \quad (56)$$

The steady state real wage is given by

$$\omega = (1 - \eta) \left(\frac{\chi}{1 + \varphi} \right) C^U N^{1+\varphi} + \eta \left[\mu N + \beta(1 - \delta) \frac{\kappa}{q(\theta)} p(\theta) \right] \quad (57)$$

and the steady state job creation condition gives

$$\frac{\kappa}{q(\theta)} = \mu N - \omega + \beta(1 - \delta) \frac{\kappa}{q(\theta)} \quad (58)$$

Combining these expressions yields an equation for N in terms of the structural parameters of the model:

$$\mu N - \left(\frac{\chi}{1 + \varphi} \right) \frac{\chi}{\mu} N^{1+2\varphi} = \frac{\kappa}{q(\theta)(1 - \eta)} \left(1 - \beta(1 - \delta)(1 - \eta p(\theta)) \right) \quad (59)$$

Using the parameterisation of the model given in section 4, this expression reduces to a cubic polynomial in terms of N that admits three solutions. I impose that $N > 0$ in steady state, which rules out the negative solution and further rules out another solution where $N \approx 0$. The remaining solution is $N \approx 1$, which is used to solve the model. I also admit the approximation that $C \approx C^U$, which is admissible for smaller values of ψ and ζ .

B Additional Robustness Checks

B.1 Fiscal Policy

One finding from the TANK and HANK literature is that the design of fiscal policy can have important implications for aggregate macroeconomic dynamics in the presence of even stylised, simplistic heterogeneity (see Bilbiie, 2018). In this model the fiscal transfer rule is initially calibrated to $\tau = 1$, which implies that the illiquid component of firm profits is only redistributed to those households which already own shares in the firms (i.e. unconstrained households). This is generally thought to be more realistic. To understand the extent to which the properties of the model are driven by this assumption, I solve and simulate the model for different values of τ : $\tau = 0$, $\tau = 0.5$ and $\tau = 1$. The findings are presented in Figure 9. It can be seen that in this environment where $\varrho = 0.92$, changes in the fiscal transfer specification have very little impact on the responses of the variables in the model to changes in monetary policy. There is some evidence that moving towards uniform redistribution of profits leads to a marginally greater response in the employment rate which feeds into the heterogeneity index.

B.2 Inequality

Initially, the model is solved for $\zeta = 0.3$, which implies that steady state constrained household consumption is 70% of that of unconstrained households. Testing how variation in the steady state level of inequality affects the model's properties is therefore crucial, given that there is no obvious way to calibrate this parameter. I solve and simulate the model setting ζ to 0.2, 0.5 and 0.8. The findings are presented in Figure 10, and show that the level of inequality in steady state does not appear to affect the results of chapter 4. Similarly to variations in the fiscal transfer scheme, we find that in more unequal steady states the employment effect is slightly larger, which feeds through to the effect on the consumption gap, but this is not a significant result.

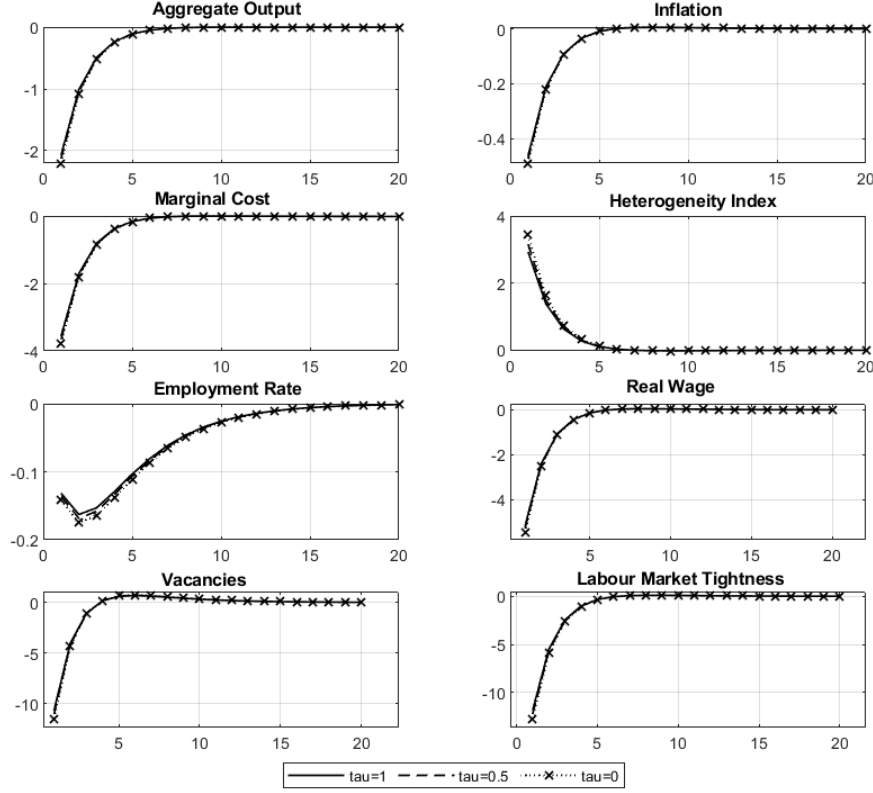


Figure 9: Changes in environment: Fiscal policy

B.3 Vacancy posting costs

Given the central role of the presence of vacancies in the interaction between labour and credit market frictions, analysing the effect of changing the value of the vacancy posting costs parameter κ . Vacancy posting costs in the linear specification used in this setting are difficult to calibrate, often chosen to target moments or to avoid issues of indeterminacy. The model is initially solved for $\kappa = 0.05$, which is higher than the targeted value of 0.01 used in Thomas and Zanetti (2009), but less than has been used elsewhere. Figure 11 presents the impulse responses conditional on a monetary shock for values of 0.01, 0.05 and 0.1. Again interestingly we see that the value of κ seems to have little effect on the results presented in chapter 4.

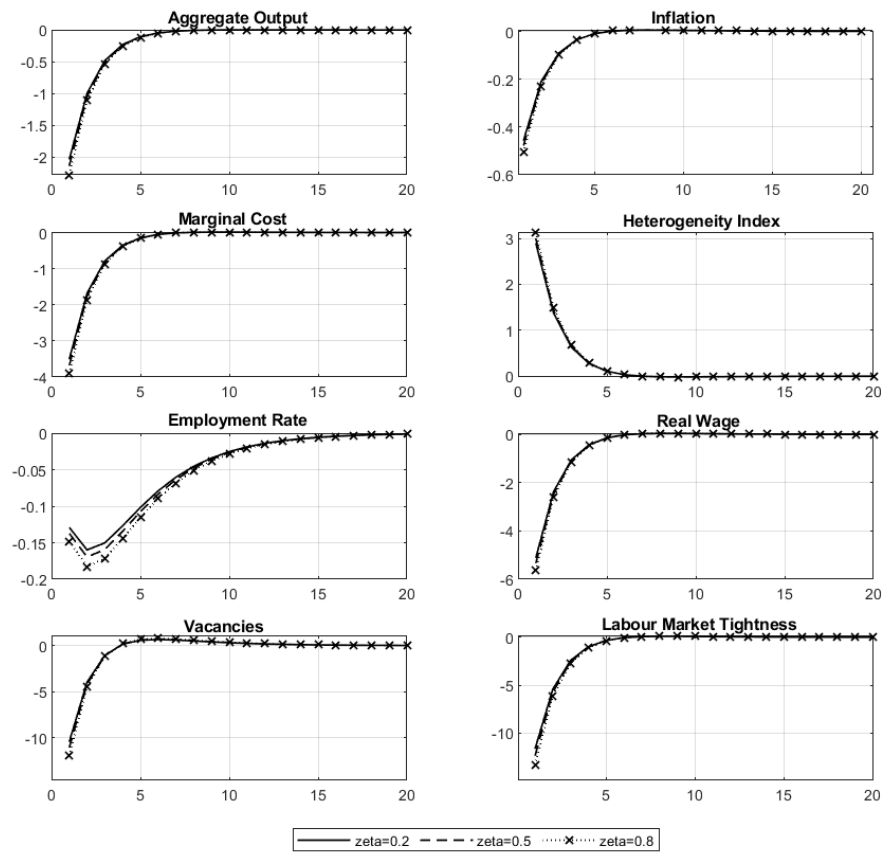


Figure 10: Changes in environment: Inequality

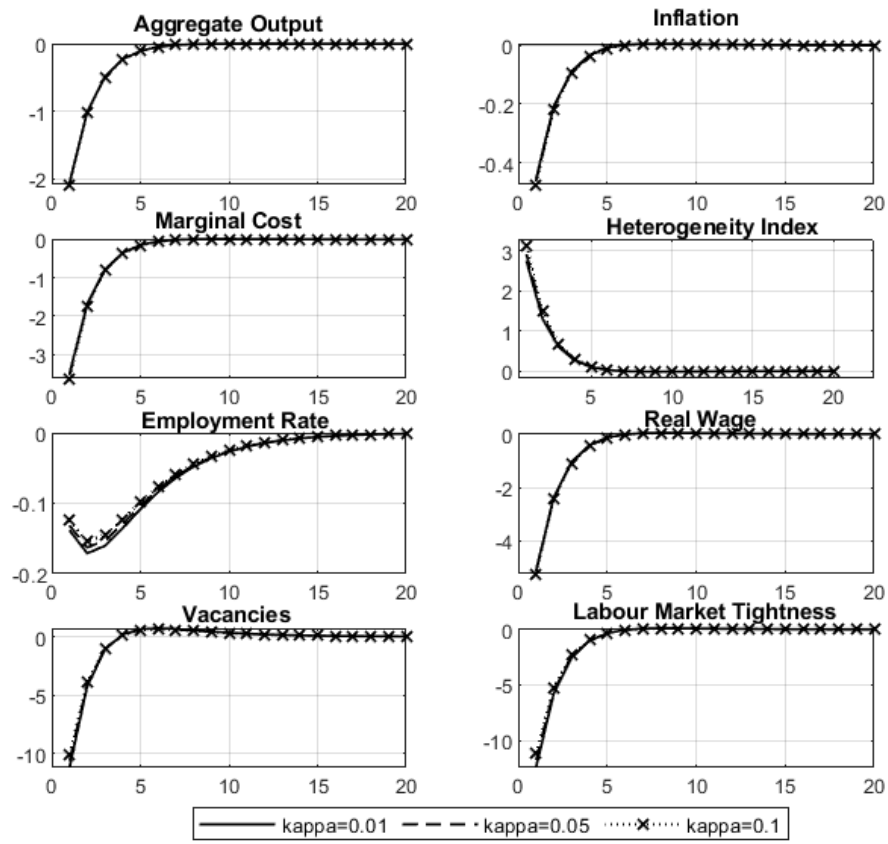


Figure 11: Changes in environment: Vacancy posting costs