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ABSTRACT
Disintegrations
of
Light Elements
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Abstract.

This work is divided into three parts. Part A discusses some of the focussing properties of magnetic fields used for the deflection and analysis of ion beams; part B describes the design and construction of magnets employing these theoretical principles; and part C is an account of some experimental work done using focussed beams of deuterons. The work was all done at the Clarendon Laboratory, between Michaelmas Term 1948 and Michaelmas Term 1953, and the three parts form a roughly chronological sequence.

Part A: The Focussing Properties of Magnetic Analysers.

The 1 M.V. and 500 KV. accelerators of the Clarendon Laboratory provide relatively high current beams of positive ions which are very homogeneous in energy. For separating the ions of different masses a magnetic analyser is clearly necessary, since it can be shown that the alternative, an electrostatic analyser, gives energy dispersion only.

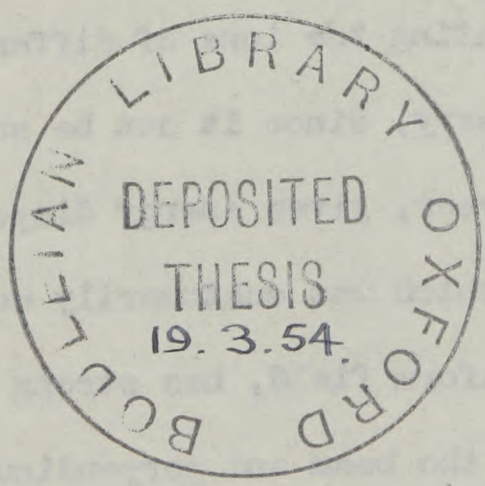
The simple magnetic analyser which was customarily used until recently, being a sector shaped uniform field, has strong focussing properties in the plane containing the beam and perpendicular to the field lines (the "perpendicular" plane), but none in a plane containing the emergent beam and parallel to the field lines (the "parallel" plane). This focussing becomes stronger as the sector angle ϕ is increased, until at 90° , which is frequently a convenient angle, an incident collimated beam of circular cross-section converges to a line focus at the exit boundary.

This is a great inconvenience in many cases where care has been taken with the accelerator to provide an intense ion beam for nuclear bombardment studies. The properties of such uniform sector fields are discussed

This work is divided into three parts. Part I describes the experimental method used for the collection and analysis of the data. Part II describes the design and construction of the apparatus used for the collection and analysis of the data. Part III describes the results of the experiments and the conclusions drawn from them.

Part II. The Apparatus

The apparatus used for the collection and analysis of the data is described in this part. It consists of a number of components, including a gas cylinder, a pressure gauge, a flowmeter, a gas analyzer, and a gas collector. The gas cylinder is filled with the gas to be analyzed. The pressure gauge is used to measure the pressure of the gas. The flowmeter is used to measure the flow rate of the gas. The gas analyzer is used to analyze the composition of the gas. The gas collector is used to collect the gas for analysis.



This is a general introduction to the work. The purpose of the work is to study the properties of the gas. The results of the experiments are given in the next part.

briefly, since they serve to demonstrate the formal analogy between ion trajectories in magnetic fields (ion optics) and light optics.

The power of the lens in the two planes can be changed by using a field which is non-uniform, varying with radial distance as $H \propto r^{-n}$. Such a field focusses in both planes, but it is shown that coincident foci can only be obtained at image distances ν too close to the magnet for practical convenience in many cases.

The inherent non-uniformity of the "fringe" field at the pole boundaries can be used to change the power of the lens in both planes if the boundaries make an angle with the beam at entrance or exit. Expressions are derived for the image distances in both planes and the conditions for equality are then discussed. The equation has a variety of possible solutions which are to some extent limited by practical considerations. In particular, the magnification of the system will in general increase with increase of image distance and this proves to be the principal limiting factor, since the object for the ion-optical system has a finite size (initially the ion source canal).

The theoretical results for a 90° analyser are compared with experiment and reasonable agreement is found. However it is found that there is a great loss of intensity in the analysed beam. The whole question of aberrations is discussed and it is found that the loss of intensity is likely to be due to spherical aberration in the electron lens system, a circumstance which can probably not be improved in the existing type of accelerator column, and which makes a 90° analyser of the type discussed of no great advantage over the uniform-sector field for many purposes.

For an accelerator with large spherical aberration the only way out of this difficulty is to sacrifice the large dispersion and mechanical

convenience of 90° deflection and use deflection through small angles. This is separately considered as a special case of a field with oblique non-uniform boundaries, and the optical properties of such a magnet as is used in Part C are discussed.

Part B: The Design of Magnets for Ion Analysers.

The general considerations which guide the design of an analysing electromagnetic are presented, and details are given of the construction of a magnet and pole-pieces designed to give 90° deflection of 1 M.V. deuterons in either of two opposite directions. The pole-pieces have faces which give the beam a 19 cm. radius path in the field, and freedom to choose the boundary angle (in accordance with the principles of Part A) is provided by adjustable segments at the pole tips.

The limitations on operation imposed by heating of the windings, non-uniformity of the field due to magnetic saturation and the proximity of the coils, and the effect of hysteresis on field stabilisation are mentioned.

The vacuum box which carries the beam between the magnet poles is made of brass wave guide with a rigid supporting structure of Yorkshire tubing which also serves as a high speed pumping by-pass.

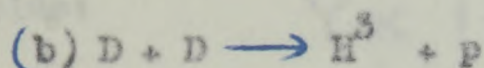
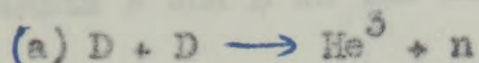
The field is stabilised by referring the voltage developed by the magnet current across a resistance to that of a standard cell. The unbalance current is amplified by means of a galvanometer, modulated light beam, photocell and AC amplifier, and corrects the magnet current by a motor driven variac in the power supply before rectification.

It is shown that a satisfactory magnet design can be based on triangular cores and windings. The dimensions of a magnet to deflect 1.4 M.V. tritons through 90° in either of two opposite directions are

given, and also details of the tests, on a $1/4$ scale model, to find the temperature rise in the windings and the expected uniformity of the field in the full-size magnet. The field at various points in space near a triangular winding has been worked out.

Part C: A Further Study of the D-D Reactions.

The angular distribution of the products of the two reactions:



has been investigated in the energy range 150 KV. to 450 KV. (deuteron bombarding energy). Although other workers have made measurements of (a) in this region, the technique used was different, and the present work partly bridges the gap between the work of Eliot (0 to 50 KV.) and that of Blair (1 to 315 KV.), both of whom used similar methods.

The apparatus consisted of a cylindrical target chamber filled with deuterium at a pressure of about 0.1 mm. Hg., and attached to the end of the accelerator column of a 500 KV Phillips generator. The beam was carefully collimated by a canal system, and the charged reaction products from a small volume at the centre of the target box were counted by ion counters operated in the proportional region. Two counters were arranged to swivel about an axis through the centre of the reaction volume, and a third, which was used to monitor conditions, was fixed to the lid of the box. The counter windows were of very thin (about 0.5μ) quartz. The counter gas was heptane vapour at a pressure (about 1 cm.) determined by a fixed temperature bath. Counts were made of the number of He^3 and H^3 particles penetrating the counter window at various angles. These particles were distinguished by a careful choice of bias applied to a pulse height discriminator, and by the use of absorbing mica screens

in front of the windows. The principle is that ^{The} three charged particles from the reaction, as well as scattered deuterons, give in general different amounts of ionization in the counter materials and absorbers, because of their difference of energy and charge. The results to be expected are discussed in detail.

The results are conveniently expressed in terms of the 'asymmetry' coefficients A and B in the expression for the angular distribution of the yields:

$$Y \propto 1 + A \cos^2 \phi + B \cos^4 \phi$$

There is a significant difference in the values of A and B for reactions (a) and (b), which is in agreement with the results of other workers.

The values are:

E(KV)	He ³		H ³	
	A	B	A	B
150	1.334 $\pm$ 0.267	-0.087 $\pm$ 0.294	0.773 $\pm$ 0.117	0.031 $\pm$ 0.130
200	1.355 $\pm$ 0.169	0.155 $\pm$ 0.183	0.687 $\pm$ 0.161	0.297 $\pm$ 0.186
260	1.718 $\pm$ 0.212	0.186 $\pm$ 0.245	0.978 $\pm$ 0.122	0.149 $\pm$ 0.150
350	1.427 $\pm$ 0.126	0.711 $\pm$ 0.145	0.637 $\pm$ 0.116	0.839 $\pm$ 0.130
450	1.342 $\pm$ 0.138	1.271 $\pm$ 0.157	0.949 $\pm$ 0.110	0.506 $\pm$ 0.130

The difficulties of interpretation of these results on the basis of the only published theoretical treatment of the D - D reaction is briefly discussed.

DISINTEGRATIONS OF LIGHT ELEMENTS

BY

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ST. CATHERINE'S SOCIETY

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Introduction.

The 1 MV and 500 KV cascade generators of the Clarendon Laboratory are well suited to nuclear disintegration studies involving light elements. The H.T. voltage on the accelerator column can be stabilised to much better than 1% so that when used with a Thonemann R.F. ion-source,² which introduces only a small voltage uncertainty, large beam currents of particles homogeneous in energy can be produced.

These beams have been successfully used for producing reactions which exhibit narrow resonances, as for example the radiative capture of protons in aluminium. Another use has been for the D - D reaction and the angular distribution of its product particles, a reaction which is of theoretical interest for the light it throws on nuclear interactions, and practical significance as a source of neutrons.

For many of these applications which involve ionization counters the statistical significance of the results can be increased if the density of the reactions is increased in a small volume. A considerable loss of beam intensity comes from the poor focussing properties of the usual magnetic analyser, so that an attempt has been made to overcome this restriction.

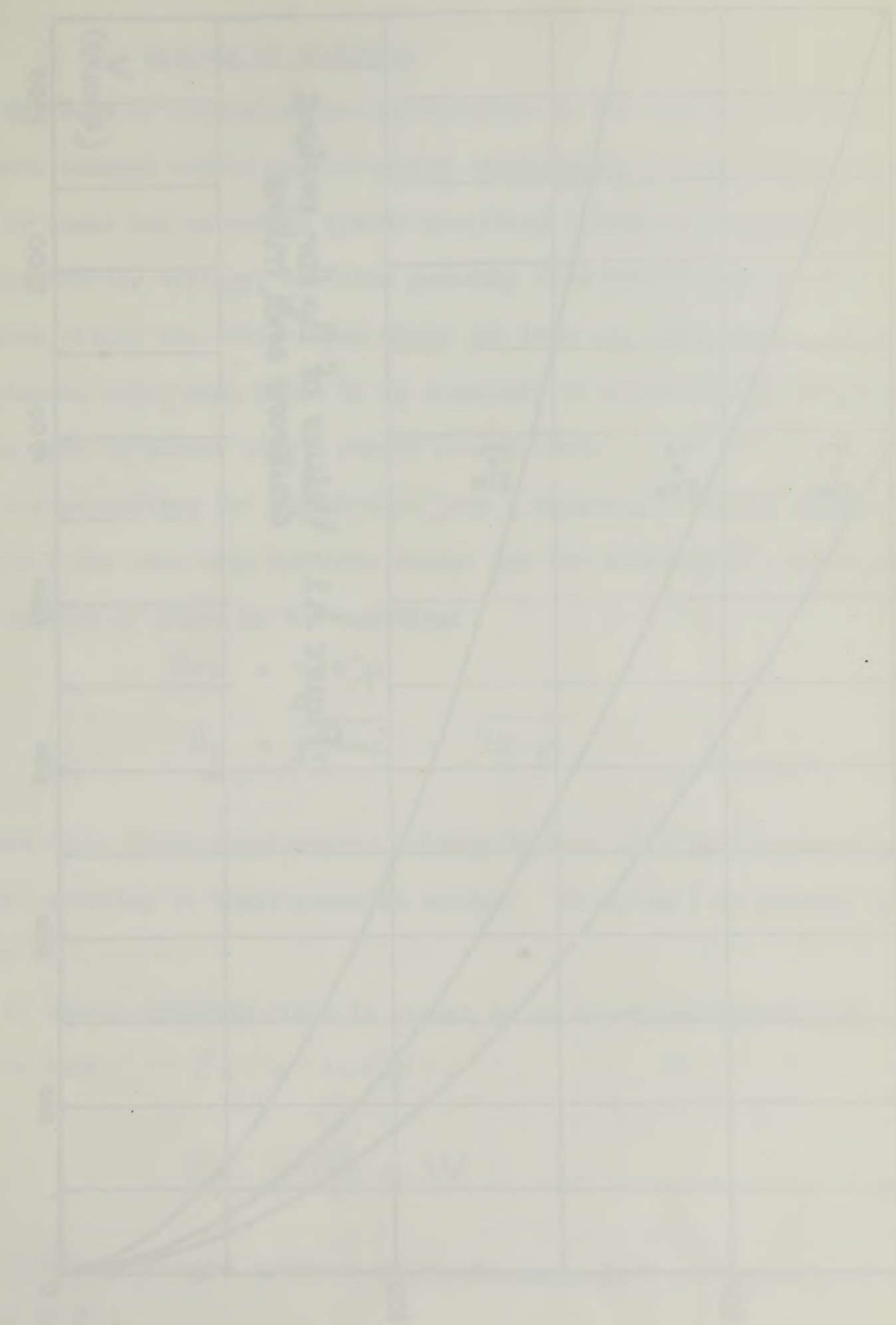
The work to be described can conveniently be divided into three parts. Part A discusses the theory of magnetic analysers suitable for focussing ion beams; part B is an account of the design and construction of electro-magnet analysers which embody the principles of part A; and part C contains the preparation and results of a nuclear disintegration experiment using an analysed and focussed deuteron beam.

The work on the properties of focussing magnetic fields was done during the time the analyser was being built; but the work on both

preceded the disintegration experiment, so that the division into parts A, B and C is roughly chronological.

PART A.

The Focussing Properties of Magnetic Analysers.



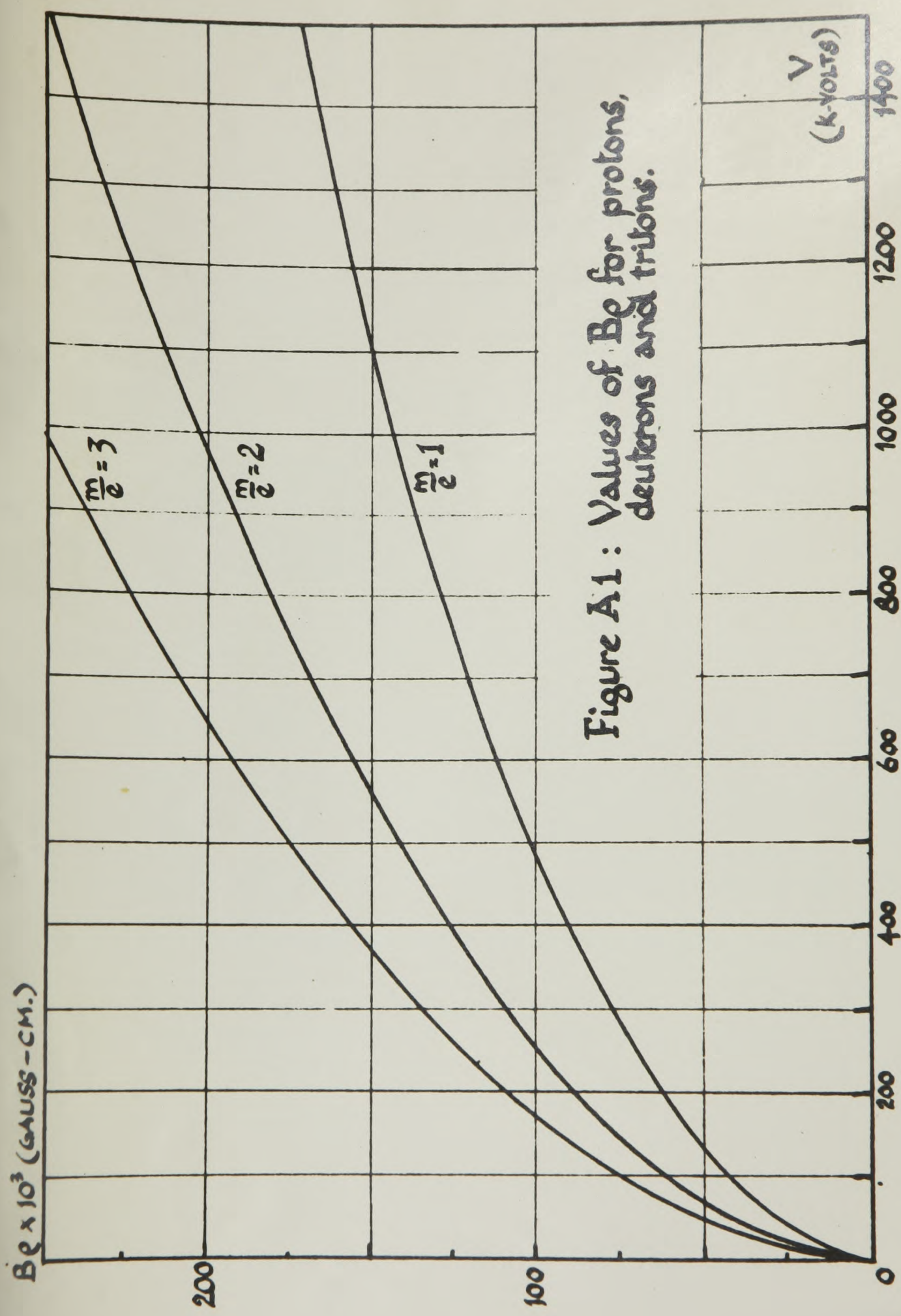


Figure A1 : Values of B for protons, deuterons and tritons.

A.

1. Methods of analysis.

The beam of particles accelerated from an ion source generally contains several atomic and molecular constituents formed in the source. Each of these has an energy spread resulting mainly from uncertainty in the accelerator voltage, but also possibly from uncertainty in the position within the source from which the ions are extracted. In most experiments using such beams it is necessary to separate the ion types and in some to select only a narrow energy band.

For separating the constituent ions a magnetic field is used. In a field B the ion, with specific charge e/m and velocity v , has a radius of curvature ρ given by the equation:

$$Bev = mv^2/\rho$$

Hence

$$\beta\rho = \frac{1}{e}\sqrt{2mE} = \sqrt{\frac{2m}{e}}V \quad (1)$$

so that for a given accelerating voltage V , ions will be separated into groups according to their specific charge. Equation 1 is plotted in figure 1.

An energy spectrum alone is formed by an electrostatic field $\mathcal{E}$.

In this case

$$\mathcal{E}e = mv^2/\rho$$

and

$$\mathcal{E}\rho = \frac{2E}{e} = 2V \quad (2)$$

so that energy changes produced by fluctuations in V will result in a change of ρ .

The procedure is often to use an electrostatic analyser to give

A.

precision in energy, followed by a magnetic one to separate the ions. The principle is similar to that underlying the design of some mass-spectrometers, except that in an analyser both velocity and mass dispersion are required.^{3,4} An alternative method which is in some cases suitable, although it gives coarser energy resolution and has other disadvantages,^{5,6} is to use crossed magnetic and electric fields, allowing the wanted ions to pass through undeviated.

In the case of the 500 kV and 1 Mev accelerators with which we are concerned, however, the R.F. ion source is of the Thonemann sort,² which produces particles of a very small energy spread, and the Phillips H.T. generator is so stable that each group of particles is homogeneous within the energy limits normally required. Hence magnetic analysis only is used.

The first magnetic analyser made used 90° deflection in a uniform field, with a radius of curvature of 10 cms: the construction is described by Price.¹ It is clear that such an analyser, in which the field boundaries are normal to the beam at entrance and exit, provides very strong focussing in the vertical plane, and to a first approximation none in the horizontal plane, so that a parallel incident beam will emerge wedge-shaped from the exit boundary. With this extreme astigmatism the beam intensity decreases inversely with the distance from the magnet. This is a great disadvantage for many experiments, since large beam intensities cannot be obtained except impractically close to the magnet. For special uses of the beam, as for example providing a point source of neutrons at a large distance from any considerable quantities of scattering material, or where a large current is to be put through

a small and distant aperture or canal, a method of beam analysis giving better focussing properties must be sought.

There are several ways in which this can be done. If the mechanical convenience of 90° deflection and its large resolution are sacrificed, the beam intensity can be preserved by using deflection through a smaller angle. Two-directional focussing can be got by using two uniform field sections with their field lines mutually perpendicular: this corresponds to the use of two cylindrical lenses. Two-directional focussing more nearly corresponding to a spherical lens system is got by using non-uniform fields: in one method the pole faces are cut to give a field changing with radial distances from the centre of curvature; in another the non-uniform stray field at an oblique boundary is used.

It must be pointed out that all the focussing properties to be described are first-order effects. Some of the second-order effects which are corrected to aid focussing in mass-spectrometers are considered later (section 12) when discussing aberrations.

2. Some factors limiting the choice of fields.

In the following sections the image forming properties of various types of possible magnetic field are discussed. It has been found that there is an optical analogy to a magnetic ion analyser⁷ that it can in general be represented by an arrangement of cylindrical lenses. This analogy is made use of in the following discussion, to find the cardinal points of the system and hence the position and size of the image.

The cases of fields which are non-uniform within the body of the field and those which are non-uniform only near the boundaries are

separately discussed, although later the properties of the general case - a non-uniform field with oblique boundaries - are briefly discussed.

Two cases have special practical significance: a simple field with parallel boundaries, in which the deflection of the ion beam is small - only about 10° - which has the merit of allowing analysed beams of large intensity without elaborate field shaping; and a field providing a deflection of 90° , which is mechanically very convenient.

There are several other limitations on the variety of solutions which are possible:

a) Alternative deflection: ~~It~~ is convenient if deflection in the opposite direction can be easily done by reversing the magnet current and hence the field direction. This increases the usefulness of the analyser, but makes certain demands on the type of field and the shape of the entrance boundary.

b) Magnification: The object provided for the analysing lens system is itself the image, formed by the accelerator, of the ion-source canal or aperture. The overall magnification by the electrostatic and magnetic lenses must be as small as possible, and in order to image the canal as a nearly circular spot, the magnetic magnification in two orthogonal planes must be as near equal as possible.

c) Cross-over: Where the ion paths cross the optic axis the charge density is likely to be large, and space-charge spreading may produce severe aberrations. Cross-overs are best avoided, so that if possible the object provided for the analyser should be a virtual one.

d) Boundary width: For the fields where focussing is provided by the fringe field at an oblique boundary, the obliquity must not be too

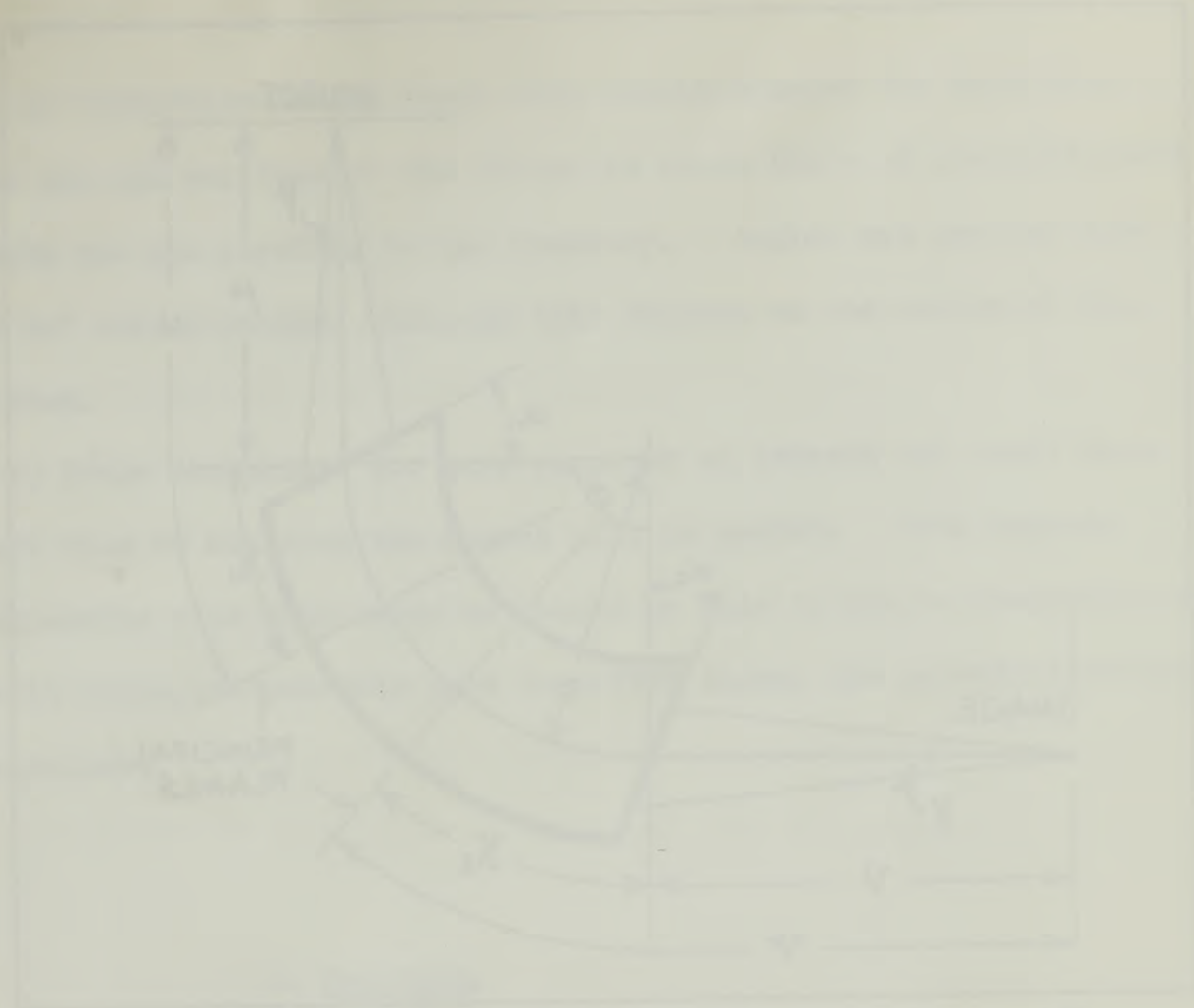


Figure 2: A graph showing a curve on a grid with axes labeled x and y . The curve starts at the origin and rises steeply, then levels off. A vertical line segment is labeled TAN and a horizontal line segment is labeled SEC .



Figure 3: A graph showing a curve on a grid with axes labeled x and y . The curve starts at the origin and rises steeply, then levels off. A vertical line segment is labeled TAN and a horizontal line segment is labeled SEC .

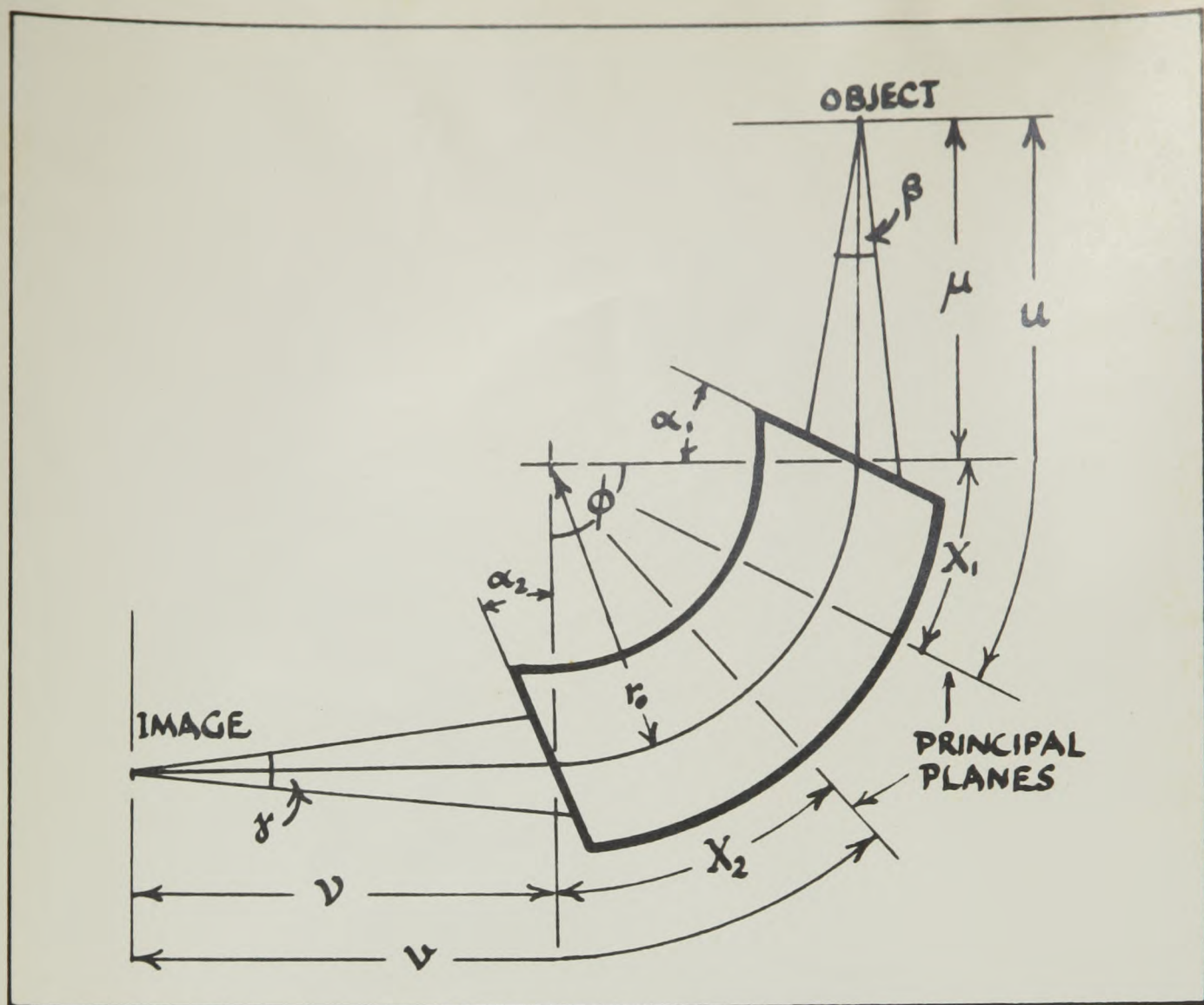


Figure A 2 : Notation

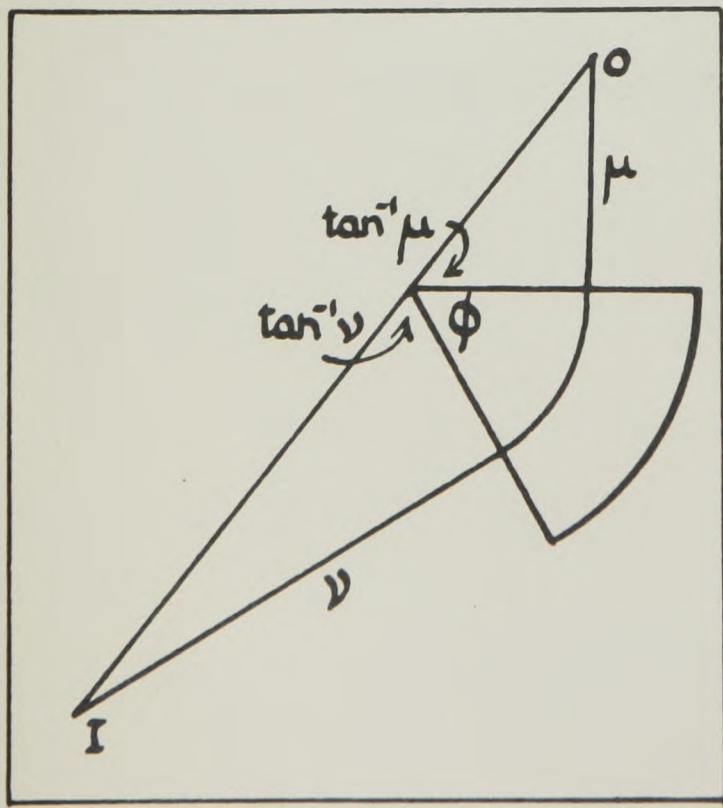


Figure A 3 : Sector geometry.

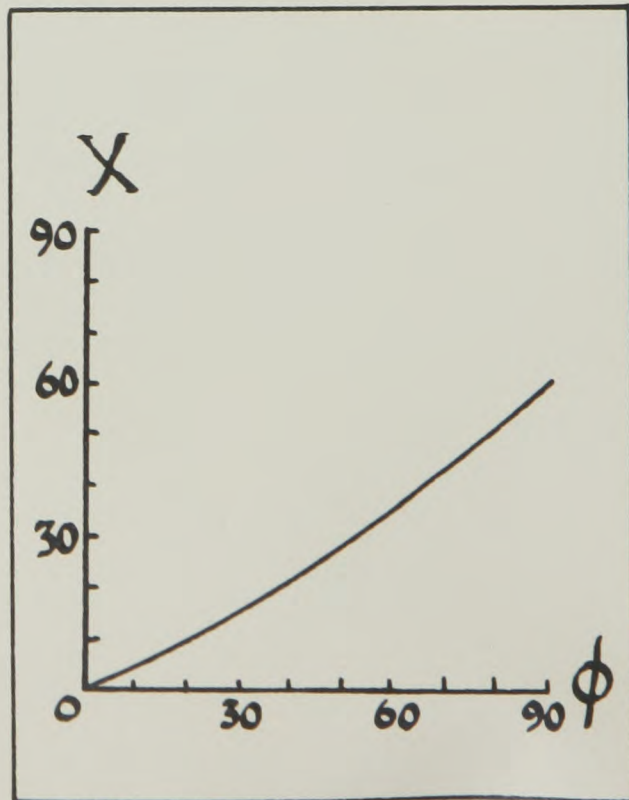


Figure A 4 : Position of the Principal Planes.

A.

great or distortion of the image will result because the beam then passes through portions of the fringe in which lines of constant field strength are not parallel to the boundary. Angles not greater than about 60° are allowable, although this depends on the design of the pole face.

e) Image distance: For many purposes an intense and small beam at more than 50 cms from the magnet will be needed. With magnets of reasonable size this means we should be able to use an image distance at least twice, and possibly more than five times, the principal radius of curvature.

3. Notation.

The notation used is shown in figure 2. Since there is an analogy with a cylindrical lens in an optical system, the image and object distances measured from the appropriate principal planes are useful and these distances are denoted by v and u . The unit of distance throughout is ρ , the radius of curvature of axial particles in the field; and with this same unit the focal length of the field is f . From a practical standpoint, however, the important distances are those of object and image measured from the geometrical field boundaries, and they are denoted by μ and ν .

We are concerned with focussing properties in two directions normal to the beam axis: that is, parallel and perpendicular to the direction of the field lines at the median plane. These two directions, in which the focussing properties differ, are denoted by the subscripts $\parallel$ and $\perp$; and the magnifications in these two planes are $M_{\parallel}$ and $M_{\perp}$.

A.

Cylindrical coordinates (r, z, θ) are used to refer the position of a particle in the beam to the centre of curvature of the beam axis. The radius of curvature of a particle at any point is ρ and that of the axial particles is ρ_0 . Thus on the beam axis we have $\rho_0 = r_0$; but otherwise, except in the case of a non-uniform field of the form $H \propto \frac{1}{r}$, we have $\rho \neq r$.

When the field boundaries are not normal to the beam at entrance and exit, the angles between the beam and the normal to the entrance and exit boundaries are denoted by α_1 and α_2 respectively, in such a sense that with the beam describing a clockwise circle the positive angle is measured clockwise from the beam at entry, and anti-clockwise at exit. We use $t = \tan \alpha$.

With this notation, the last four limiting conditions mentioned in section 2 above become:

$$b) \quad M_{\parallel} \sim 1 \quad M_{\perp} \sim 1$$

$$c) \quad \mu < 0$$

$$d) \quad |t_1| \text{ and } |t_2| < 2$$

$$e) \quad \nu_{\parallel} = \nu_{\perp} > 2$$

4. Uniform sector fields.

The theory of this type of field has been given often in connection with its use in mass spectrometers⁴, where the absence of focussing in one plane is unimportant because of the use of a slit source.

The beam axis enters and leaves the geometrical field boundaries normally, being deflected through the sector angle ϕ in the perpendicular plane. The object, image and sector apex lie on a straight line

(figure 3) as can be easily shown, so that the image distance (with the radius ρ as a unit of length) is given by

$$v_{\perp} = \frac{\tan \phi + \mu}{\mu \tan \phi - 1} \quad (3)$$

When $\mu \rightarrow \infty$ $v_{\perp} \rightarrow \cot \phi$ (4)

It is easy to show that the principal planes lie within the sector at an angle given closely by $X = \frac{1 - \cos \phi}{\sin \phi}$ (5)

This function is plotted in figure 4. Although the principal planes really cross within the magnet, for angles ϕ up to 60° , $X = \phi/2$ within 10%. For these angles the principal planes nearly coincide at the centre of the sector which can thus be considered as roughly equivalent to a thin (cylindrical) lens and the focal length is

$$f_{\perp} = \cot \phi + X = \frac{1}{\sin \phi} \quad (6)$$

When $\phi = \pi/2$, $v_{\perp} = 1/\mu$ and this simple relation shows that in this case a converging beam (for which μ is -ve) will always form the image within the magnet. In the case of a parallel incident beam, $v_{\perp} = 0$ and the beam diverges from the exit boundary with angle d/ρ where d is the width of the beam at entry.

The focal point can be moved away from the magnet by using a smaller sector angle ϕ . It is useful to consider the distance at which a beam (entering the magnet parallel and circular) has a circular cross-section of the same diameter as the incident beam. This will be at approximately twice the focal length, measured from principal plane i.e. $2/\sin \phi$ from the centre of the magnet. When the energy of the beam is changed so that the radius of curvature changes from ρ to $\rho + \Delta \rho$, it can be shown that the emergent beam turns about a point X distance $\rho(1 - \cos \phi)/\sin \phi$ from the exit boundary; i.e. about the principal point.

The angle through which the beam turns is

$$\Delta\phi = \frac{\Delta\rho}{\rho} \sin\phi \quad (7)$$

and the movement of the spot at the "circle of least confusion" is

$$\frac{2\Delta\rho}{\rho} = \frac{\Delta E}{E} \quad (8)$$

which is independent of ϕ .

It can be seen at once that in the case of a simple sector, one advantage of small-angle deflection is a small and circular beam cross section at larger distances from the magnet. However if a large dispersion at a fixed distance is needed, then small angle deflection is a disadvantage because dispersion diminishes with the angle ϕ .

In a given field B a change of ρ_0 will be made by a change in either specific charge or accelerating voltage. From equation (1)

$$\frac{d\rho}{\rho} = \frac{1}{2} \left[\frac{dV}{V} + \frac{d(m/e)}{(m/e)} \right]$$

We can expect $\frac{dV}{V} \sim \frac{1}{1000}$ (with a smoothed and stabilised H.T. supply) and $\frac{d(m/e)}{(m/e)} \sim 1$ (e.g. in separating deuterons from protons.)

At a target distance ν , i.e. $(\nu + X)$ from the principal plane, the distance in centimetres between two beams will be

$$y = \frac{\Delta\rho}{\rho} \left(\nu + \frac{1 - \cos\phi}{\sin\phi} \right) \sin\phi \cdot \rho$$

Thus for $\phi = \frac{\pi}{2}$ and $\rho_0 = 19$ cm (a case described in part B) and with $\nu = 2$, say the distance between proton and deuteron beams is $\frac{3}{2}\rho_0$ (28cm) and the spread of each spot due to voltage instability may be only about 0.1 m.m.

For the normal case of an incident beam converging to an object at a distance u below the 1st principal plane, the image position can be found by the optical lens formula. Thus the magnification is

$$M_{\perp} = \frac{v}{u} = \frac{1}{\cos\phi - \mu \sin\phi} \quad (9)$$



Figure 1.1: Schematic diagram of a mechanical system.

The diagram illustrates a mechanical system consisting of a horizontal beam supported by a central pivot. Two rectangular blocks, labeled '1' and '2', are positioned on the beam. Block '1' is on the left, and block '2' is on the right. A vertical line with an arrow points downwards from block '2' towards the beam, and another vertical line with an arrow points upwards from the beam towards block '1'. This setup likely represents a study of forces, torque, and equilibrium.

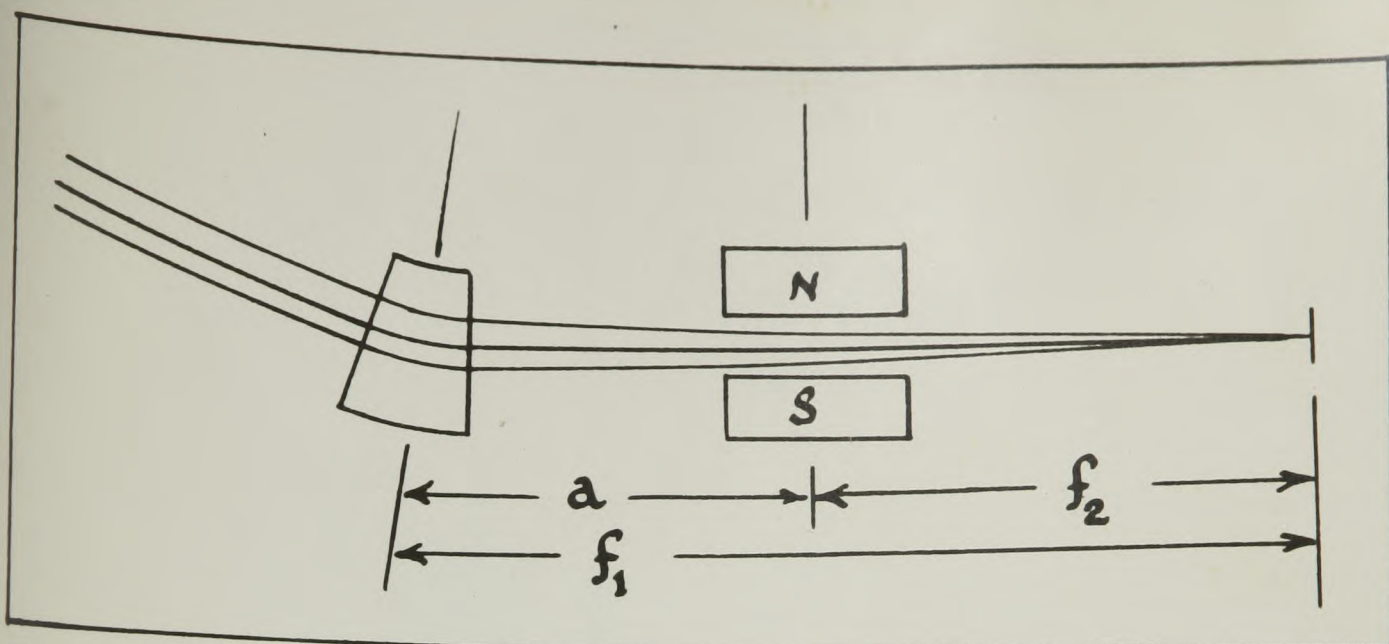


Figure A5 : Double focussing with two sector fields.

A.

(9)

Combinations of uniform sector fields could theoretically be arranged to give focussing in both planes. Thus we could put two sectors of angles ϕ_1, ϕ_2 ^{at} and a distance a apart and with their fields orthogonal (see figure 5). Then

$$\frac{1}{\sin \phi_1} = a + \frac{1}{\sin \phi_2} \quad (a < \frac{1}{\sin \phi_1}) \quad (10)$$

We might want an image distance of 100 cms. and $a\rho = 20$ cm. so using

$$\rho = 20 \text{ cm say,} \quad \frac{1}{\sin \phi_1} = 6 \quad \text{and} \quad \begin{matrix} \phi_1 \approx 9\frac{1}{2}^\circ \\ \phi_2 \approx 11\frac{1}{2}^\circ \end{matrix}$$

Such a combination of fields would be most inconvenient as an analyser, since it would involve the adjustment of both field strengths and the movement of the second magnet for every change of energy. However cases have been quoted in the literature, where sectors of small angle are combined in a similar way. ^{8.4}

5. Non-uniform fields.

The focussing action in the perpendicular plane of a sector field can be changed by introducing a radial variation of field strength. ¹⁰ If the field is a not too rapidly varying function of the radius it can be conveniently represented over the small portion in which the beam moves by $H = H_0 \left(\frac{r}{r_0} \right)^{-n}$ where H_0 is the field strength at the optic axis of the beam and r_0 is its radius of curvature. n is a positive or negative constant.

Since for normal entry and exit the centre of curvature of the optic axis coincides with the centre of the sector, we have $r_0 = \rho$, and the radius of curvature ρ appropriate to a radial distance r is given by



Figure 1: Wave propagation in a non-uniform medium.

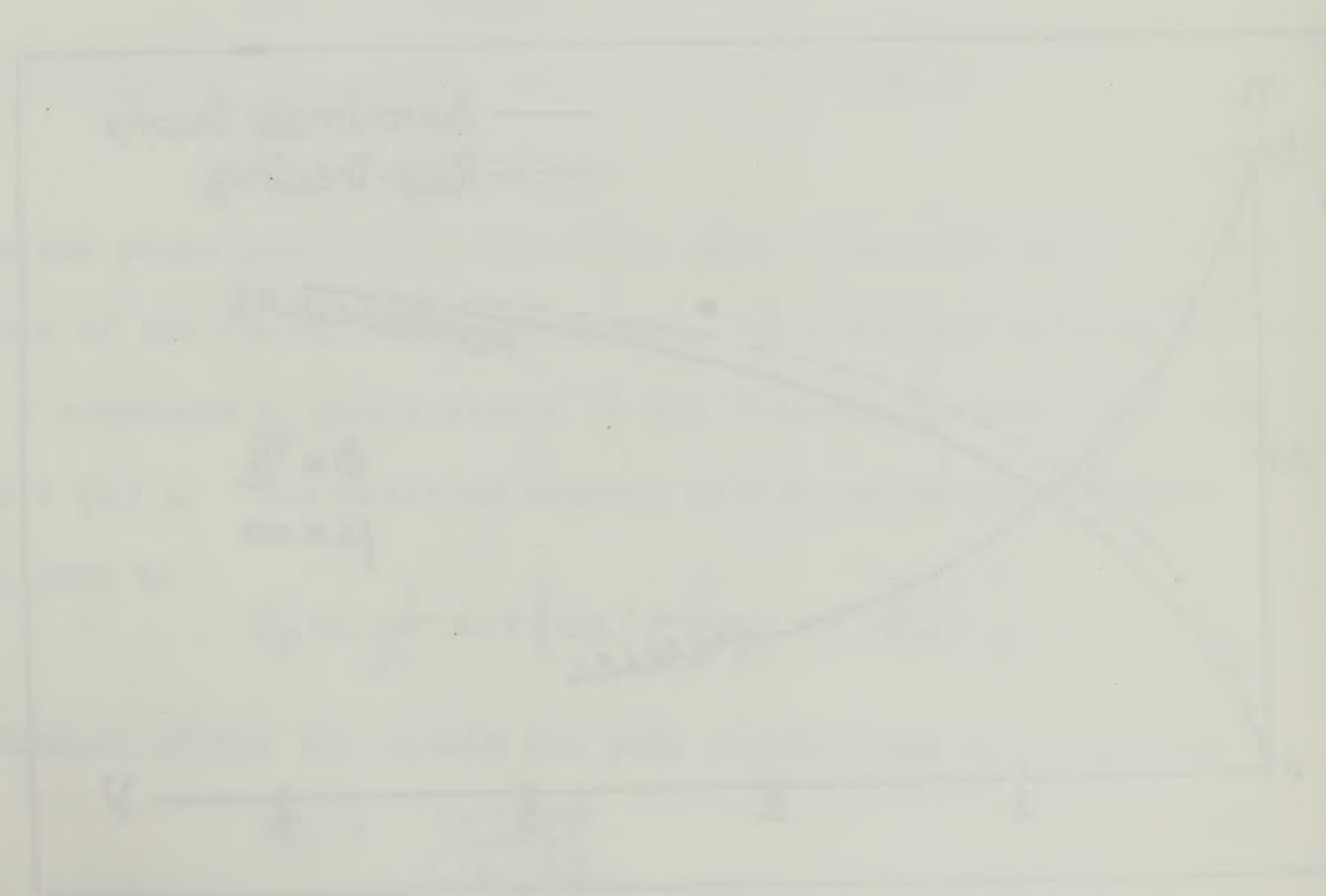
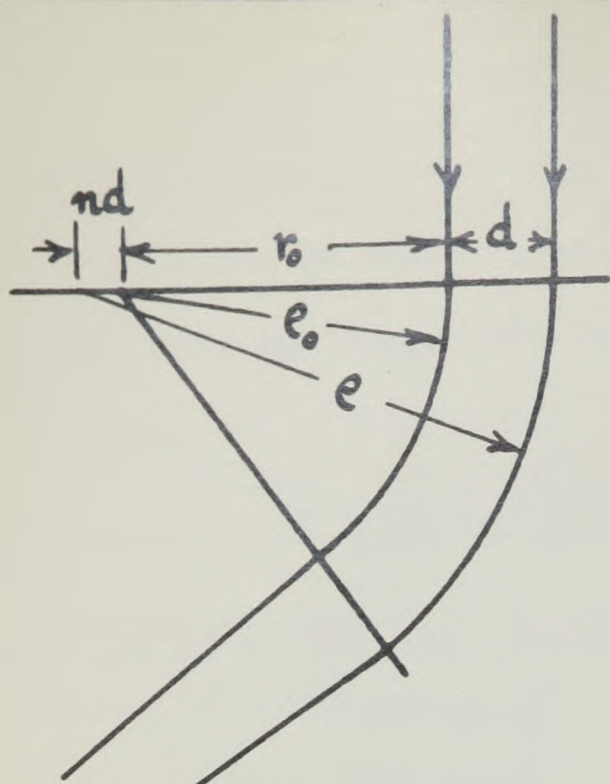
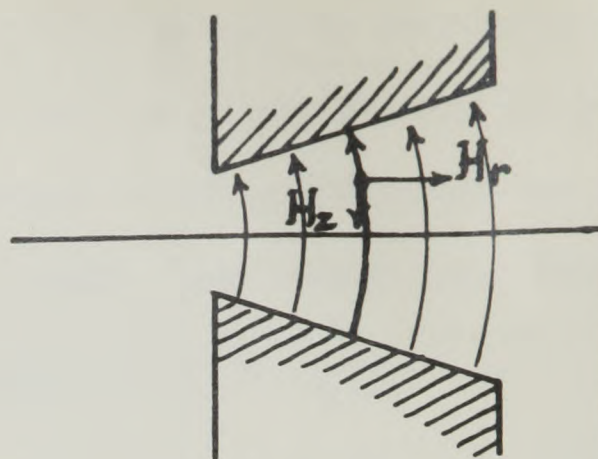


Figure 2: Plot of n versus y in a non-uniform medium.



(a) Perpendicular plane.



(b) Parallel plane.

Figures A6(a) and (b): Double focussing in a non-uniform sector field.

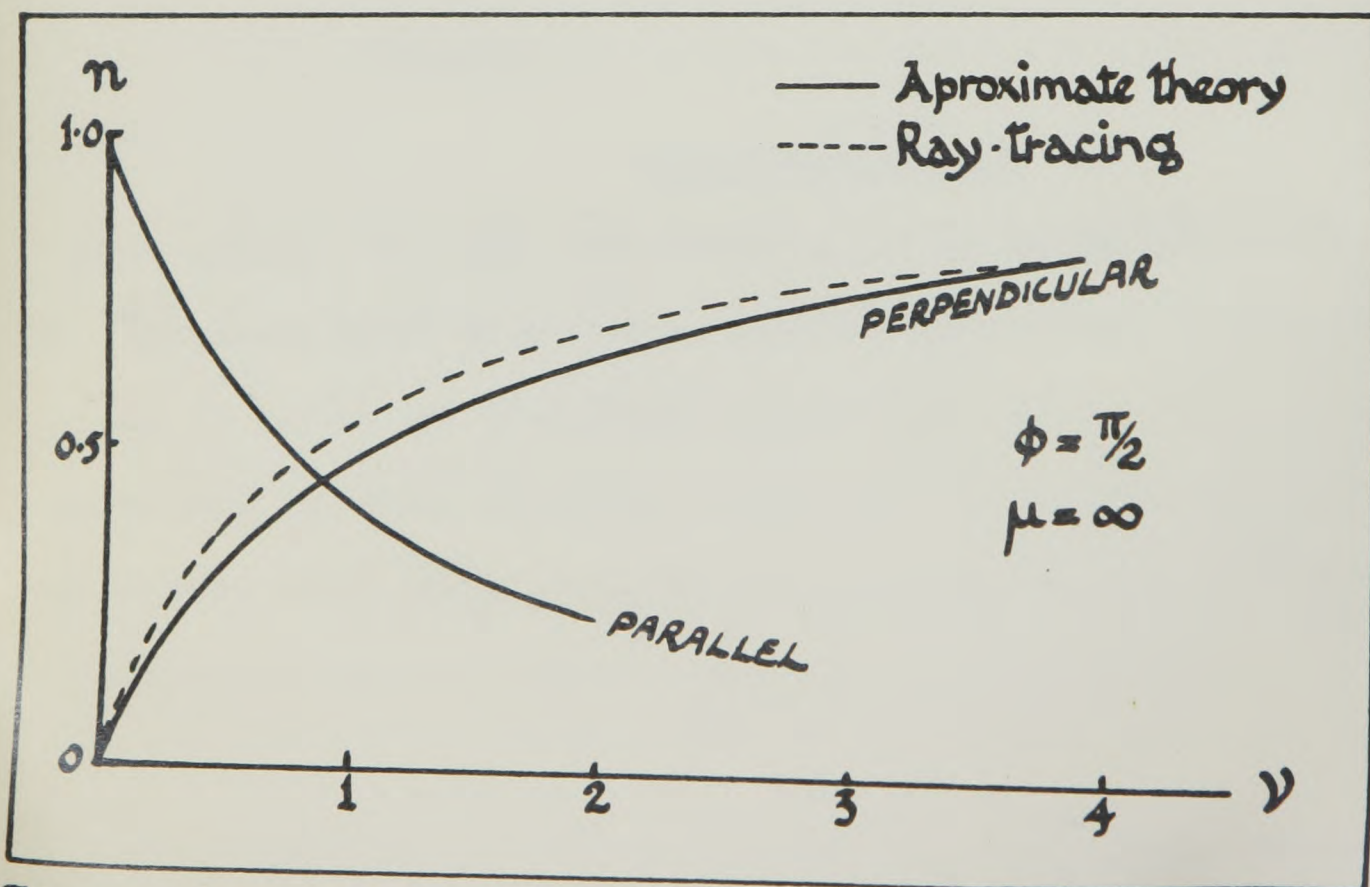


Figure A7 : Foci in a non-uniform field, with 90° deflection and parallel entry.

A.

$$\rho = \frac{H_0 \rho_0}{H} = \frac{r_0^{n+1}}{r^n}$$

For a particle at distance d (where $d \ll r_0$) from the optic axis we have (see figure 6 (a))

$$\rho = \frac{r_0^{n+1}}{(r_0 - d)^n} \approx r_0 \left(1 + \frac{nd}{r_0}\right) = r_0 + nd \quad (11)$$

By making the assumption (approximately true for large image distances) that a particle remains, throughout its path between the poles, in the same field as that appropriate to the position at which it entered, a relation between object and image distances can be found and thus an equivalent focal length for the magnetic lens.

The image distance is

$$v_{\perp} = \frac{\mu(n - n \cos \phi + \cos \phi) + \sin \phi}{\mu(1 - n) \sin \phi - \cos \phi} \quad (12)$$

The principal planes lie within the pole boundaries at a distance

$$x_{\perp} = \frac{1 - \cos \phi}{\sin \phi} \quad (13)$$

as for a uniform field sector, and the focal length is

$$f_{\perp} = \frac{1}{(1 - n) \sin \phi} \quad (14)$$

In the plane parallel to the field lines focussing is due to the curvature of the field, which introduces, for particles off the median plane, a component H_r proportional to the distance Z from this plane (figure 6 (b)). The relation between object and image distances in this case is

$$v_{\parallel} = \frac{1}{\sqrt{n}} \cot \left[\cos^{-1} \frac{\mu}{\sqrt{\frac{1}{n} + \mu^2}} + \phi \sqrt{n} \right] \quad (15)$$

The principal planes lie within the pole boundaries, at a distance

$$x_{\parallel} = \frac{1 - \cos \phi}{\sqrt{n} \sin \phi \sqrt{n}} \quad (16)$$

and the focal length is

$$f_{\parallel} = \frac{1}{\sqrt{n} \sin \phi \sqrt{n}} \quad (17)$$

A.

In the particular and important case of $\phi = \frac{\pi}{2}$, the image distances in the two planes are

$$v_{\perp} = \frac{n\mu+1}{\mu(1-n)} \quad (18)$$

$$v_{\parallel} = \frac{1}{\sqrt{n}} \cot \left[\cos^{-1} \sqrt{\frac{\mu}{\frac{1}{n} + \mu^2}} + \frac{\pi}{2} \sqrt{n} \right] \quad (19)$$

Since the form of the field is $H = ar^{-n}$, when $n = 0$ the field is uniform and a parallel incident beam ($\mu = \infty$) will converge on the exit boundary. If $n < 1$ the convergence will be even greater. The assumption was made (in deriving equations (12) to (19)) that because the convergence is small any particle in the beam remains in a constant field, so it is clear that the above equations will only apply to +ve values of n ; and indeed for the purposes of a focussing analyser these are the only values which are of use. When $\mu = \infty$, equations (18) and (19) simplify to

$$v_{\perp} = \frac{n}{1-n} \quad (20)$$

$$v_{\parallel} = \frac{1}{\sqrt{n}} \cot \frac{\pi}{2} \sqrt{n} \quad (21)$$

Under these conditions a true image will be obtained when $v_{\perp} = v_{\parallel}$. The two equations are plotted in figure 7, and the point common to both curves is

$$\begin{aligned} n &= 0.46 \\ v &= 0.8 \end{aligned} \quad (22)$$

Thus for a parallel incident beam a true focus is found at the short distance $0.8 f_0$ outside the exit boundary.

In fact, the two image distances remain nearly equal over a wide range of values of μ , so that in a field chosen to give $n = 0.46$ a true focus is also possible with non-parallel incident beams. This is shown in figure 8, where equations (18) and (19) are plotted. The abscissae

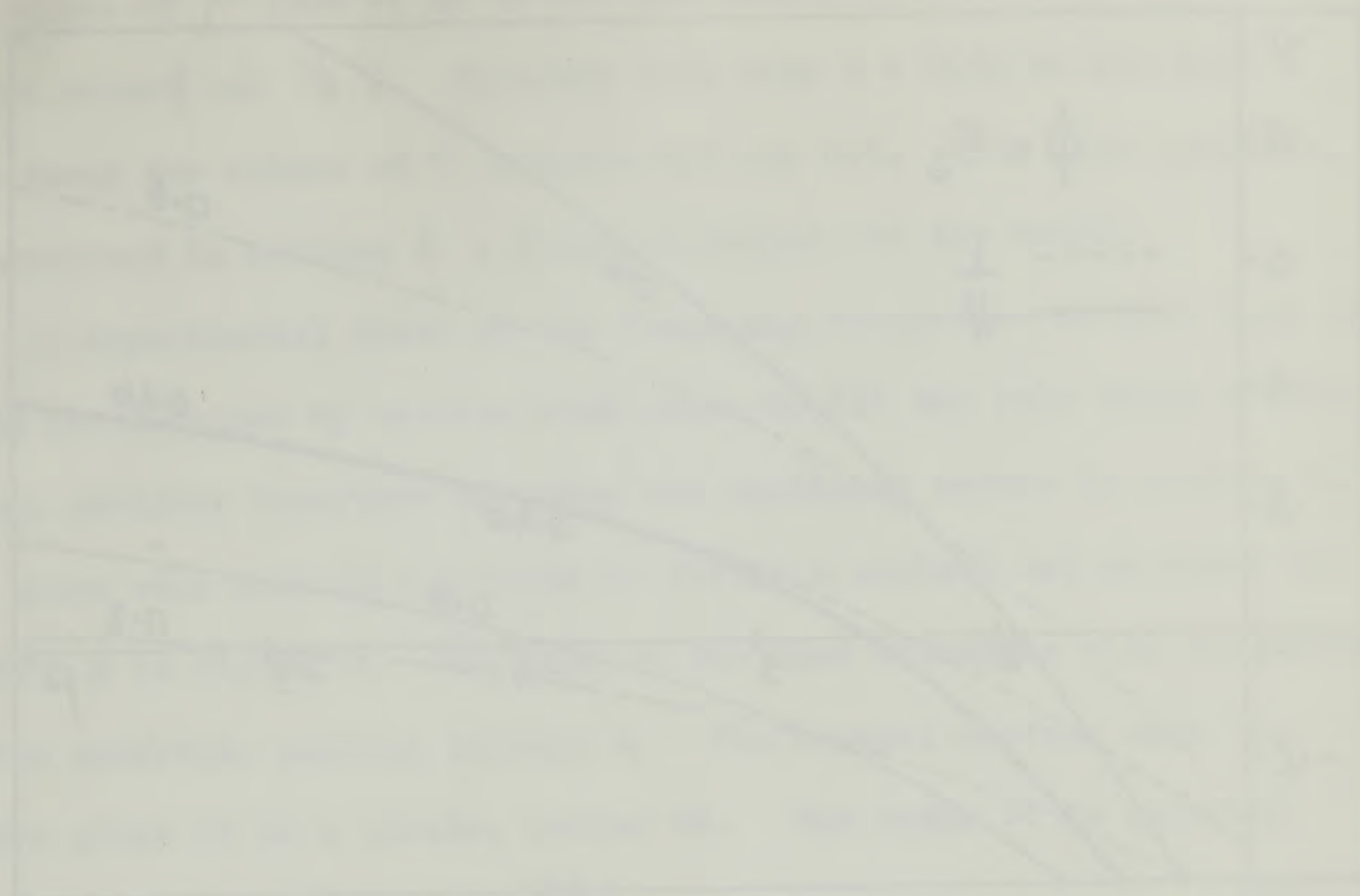


Figure 8B: Values of n and a for a constant c . The solid lines are for $c = 1$ and the dashed lines are for $c = 0.5$.



Figure 8C: Values of n and a for a constant c . The solid lines are for $c = 1$ and the dashed lines are for $c = 0.5$. The shaded area represents the region where $n < a$.

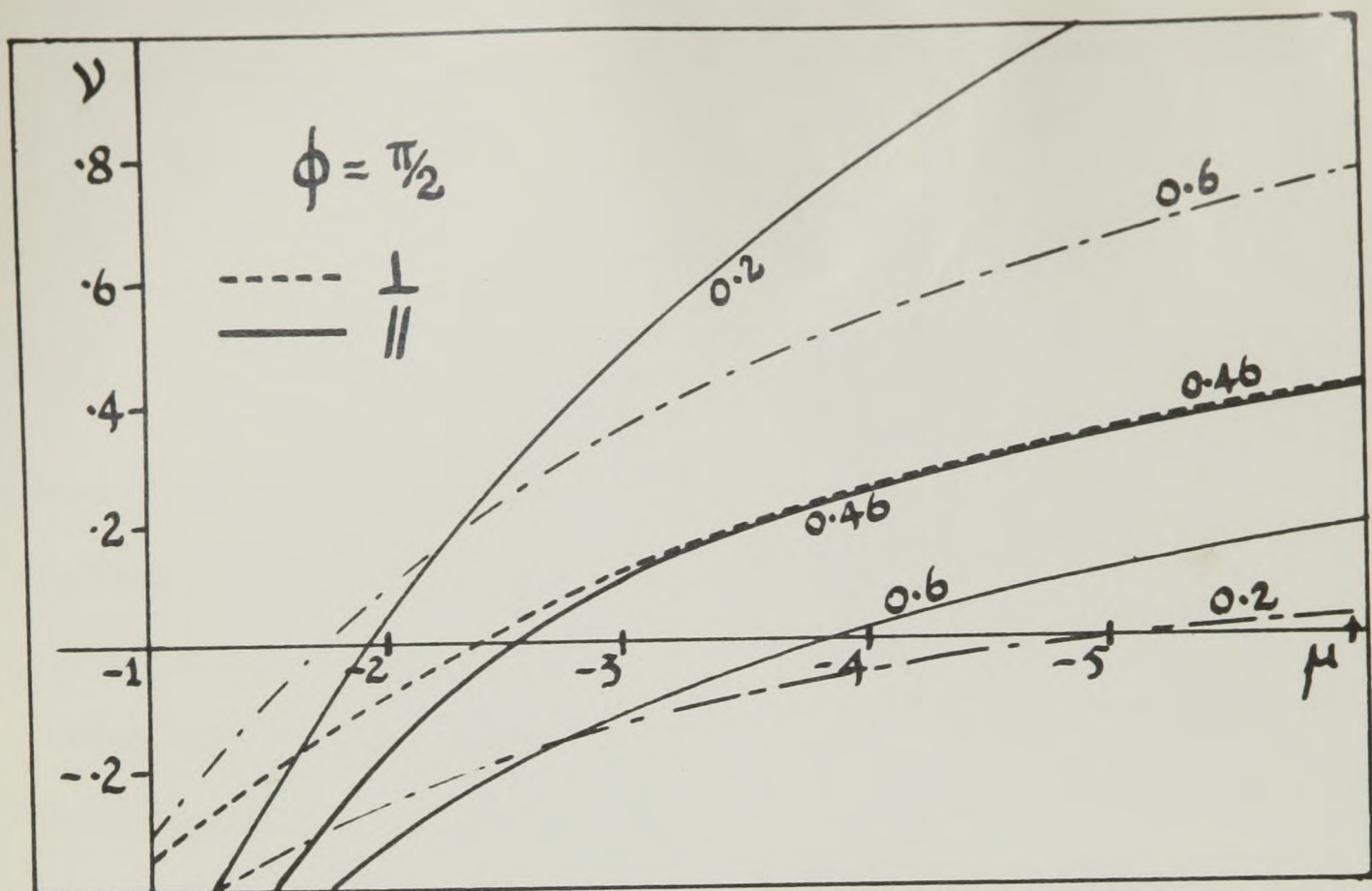


Figure A8 : Values of μ and ν in a non-uniform field $H \propto r^{-n}$ for several values of n .

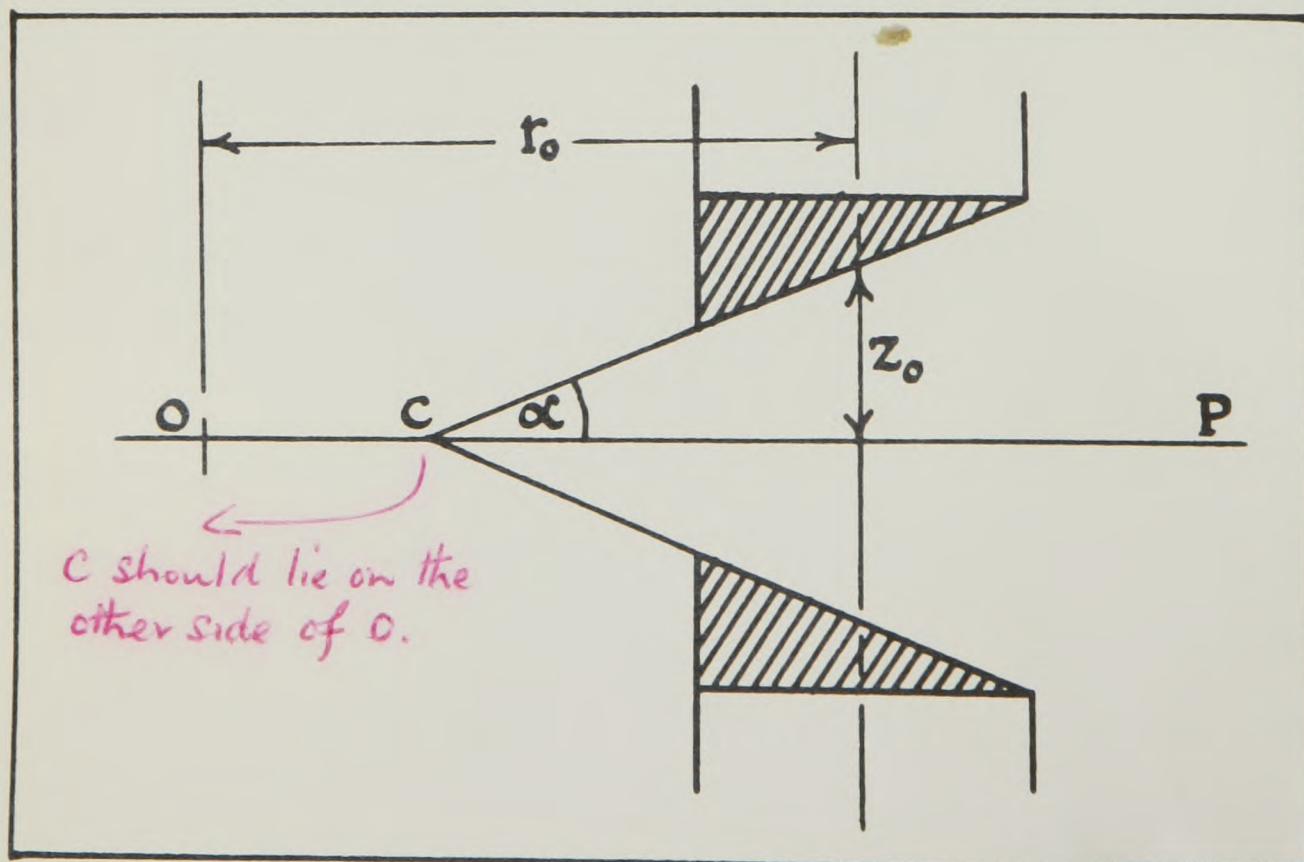


Figure A9 : Pole-face shims cut to generate a field $H \propto r^{-n}$.

A.

are values of μ from -1 to -3 and the ordinates are image distances y_1 (shown dotted) and y_{11} . It seems that with $n = 0.46$ we can have a true focus for values of y between 0.2 and 0.8. For many purposes, as mentioned in section 2, these distances are too small.

An experimental check of the focussing properties of this type of field was obtained by cutting steel shims to fit the pole faces of the 10 cm. analyser described by Price and mentioned before in section 1. The shims were made on the lathe by taking a conical cut as shown in section // in figure 9. The axis of the cone coincides with the Z-axis of the quadrant, passing through O. The conical surface cuts the median plane OP in a circle, radius OC. The angle α is given by

$$\tan \alpha = \left(\frac{\partial z}{\partial r} \right)_{r=r_0} \quad (23)$$

If we assume $H = \frac{b}{z}$, then since $H = ar^{-n}$

we have

$$z = \frac{b}{a} r^{+n}$$

and

$$\frac{dz}{dr} = + \frac{n}{r} \frac{b}{a} r^{+(n-1)}$$

$\therefore$

$$\tan \alpha = + \frac{n z_0}{r_0} \quad (24)$$

and

$$OC = \left(\frac{1-n}{n} \right) r_0 \quad (25)$$

The first shims to be cut were designed to give $n = 0.8$. Subsequently others to give $n = 0.6$ and 0.7 were made. Although the beam outline was at the time not sufficiently sharp to give quantitative results, there was reasonable qualitative agreement with the theoretical predictions.

In order to justify the use of the assumptions mentioned above, a geometrical ray-tracing method was used to find the position of the image (in the perpendicular plane) formed by a parallel incident beam after passage through a field with n of various values.

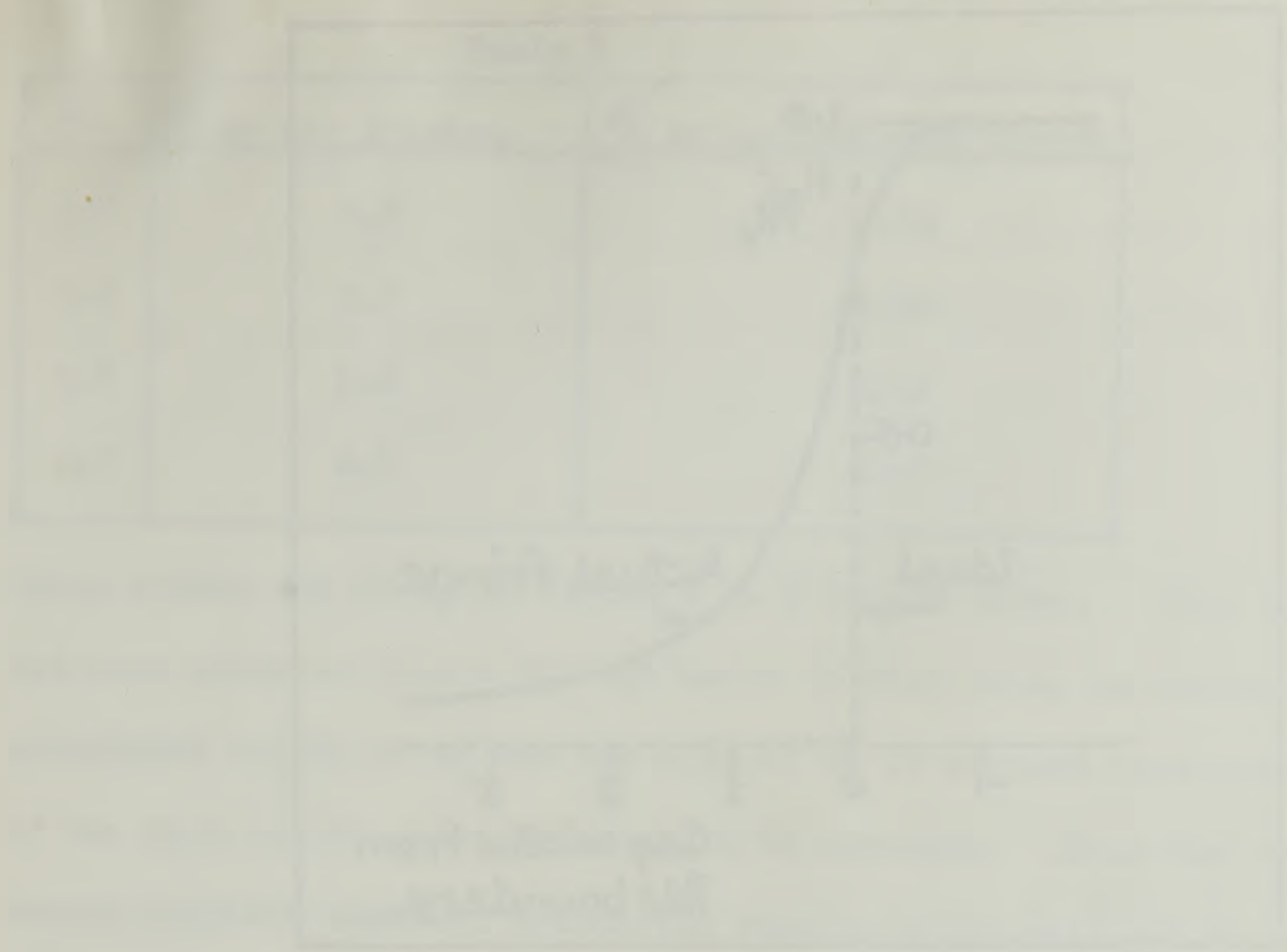


Figure 10: Form of the fringe field.



Figure 11: (a) Form of the fringe field; (b) Form of the fringe field.

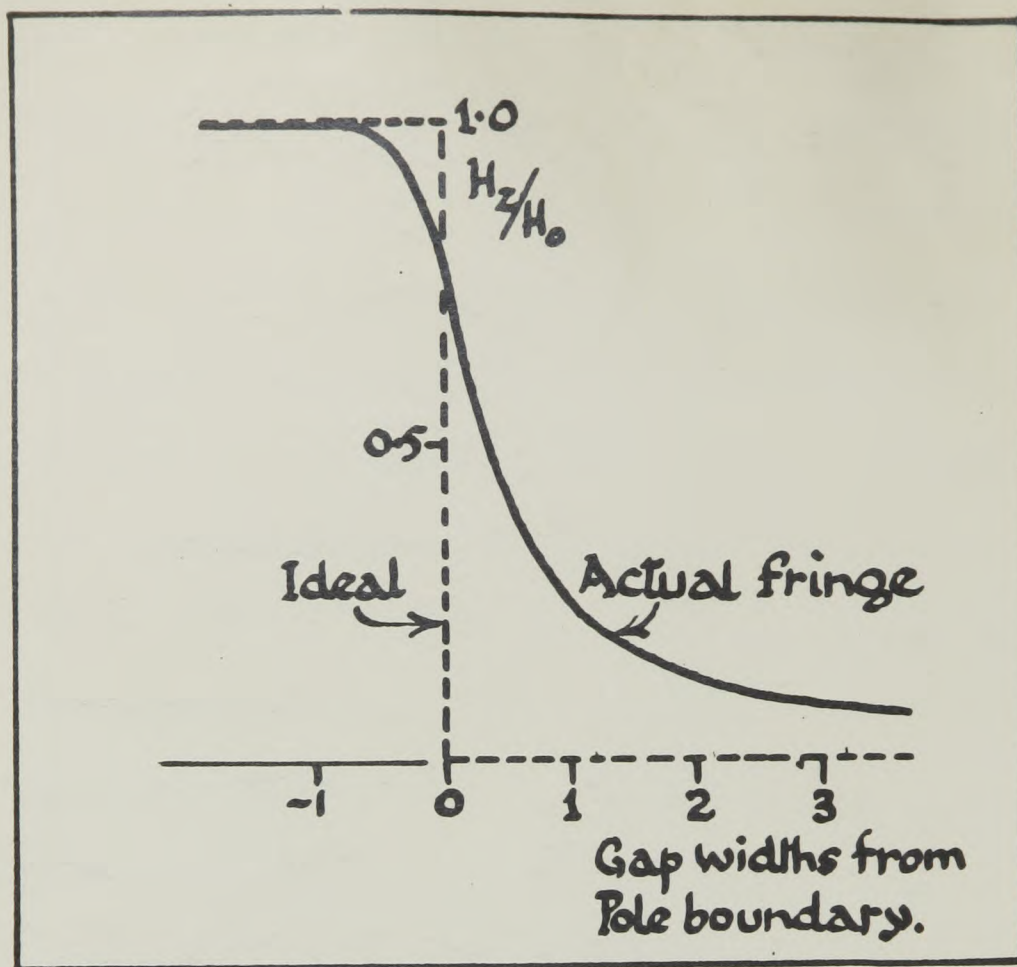


Figure A10: Form of the fringe field.

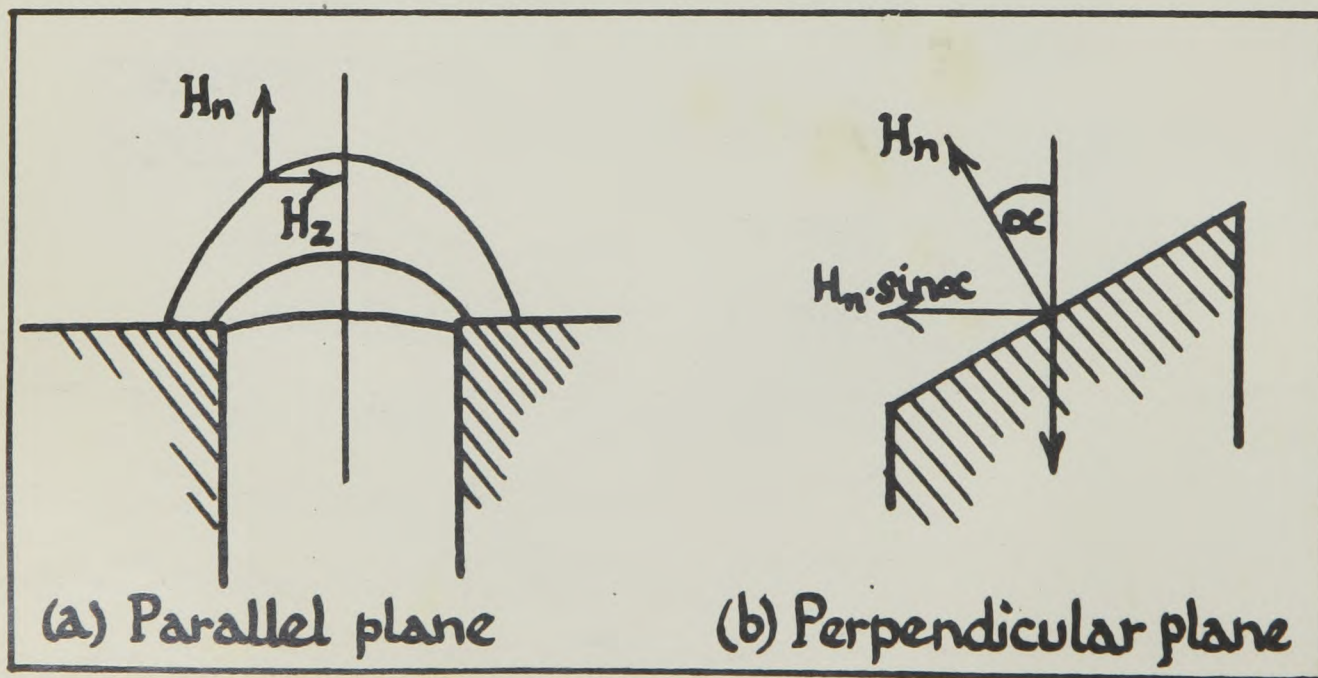


Figure A11: Stray field components at an oblique boundary.

The results were

Table 1

n	ν_1 by ray tracing	ν_1 by approximate theory
0.2	0.2	0.25
0.5	0.7	1.00
0.7	1.9	2.33
0.8	4.0	4.00

These values are shown in figure 7 as a dotted curve. They are probably not more accurate than $\pm 5\%$, but serve to show that, as expected, the calculated values of ν_1 are too high if it is assumed that each particle of the beam remains in a constant field strength. Thus the approximate theory predicts somewhat optimistic results which will not be achieved in practice.

6. The fringe-field at an oblique boundary.

The assumption that a field is uniform within the area between parallel pole faces and falls sharply to zero outside requires correction, and ways of applying the corrections have been given in detail by Coggeshall and others.^{9,12} Figure 10 shows the form of the field H_z in the median plane near the boundary, and also a typical form of the ideal field of the assumption above. If the pole boundary is wide compared with the beam width in the median plane, then lines of constant field strength are parallel to the boundary and to a first approximation (extent of fringe small, deflection of beam within it small) all parts of the beam pass through the fringe at normal incidence. At points in the fringe off the median plane the component H_z is less and a component H_y increases

$$\delta\psi = \frac{r\delta\theta}{\rho} = \frac{r}{\rho} \sin\alpha \cdot H_z \cdot r\delta\theta$$

A.

with increasing Z but there is no deflection in the parallel plane because $H_r = 0$. If the width of the boundary cannot be ignored because it is not very much greater than the beam width, or if the departure from normal incidence resulting from deflection (in the perpendicular plane) by the fringe must be taken into account, there will be a deflection in the parallel plane due to the small component H_r which has been introduced. Such // second-order focussing effects and their corrections have been discussed by Ploch and Walcher¹² and others.

For the case of normal incidence, where $H_r = 0$, the correction (to simulate the beam path in an ideal field) is made by removing iron with a cut parallel to the boundary, as to be described in section B.2.

It is clear that deliberate focussing effects in the parallel plane may be achieved by using oblique boundaries so that $H_r \neq 0$. This makes possible the simultaneous first-order focussing of a beam in both planes.

Consider an ion entering the field at an angle α with the normal to the pole boundary, and at a distance Z off the median plane, as shown in figures 11(a) and (b). The curvature of the field lines introduces a component H_n along the normal to the boundary, and hence at an angle α with the particle velocity. Thus normal to the velocity there is a field $H_n \sin \alpha$ (figure 11 (b)), which hence produces a force $H_n \sin \alpha \cdot e v$ urging the ion to move in the parallel plane (the plane of figure 11 (a)). The curvature in this plane will thus be

$$1/\rho_{||} = H_n \sin \alpha \cdot e/mv$$

The angular deflection in passing through a small distance of the fringe is

$$\delta \psi = \frac{r \delta \theta}{\rho_{||}} = \frac{e}{mv} \sin \alpha \cdot H_n r \delta \theta \quad (26)$$

A.

If we make the assumption that the fringe is short so that while in it the ion path retains its distance Z from the median plane, the total deflection in passing through the fringe is

$$\psi = \frac{e}{mv} \sin \alpha \int_{-\infty}^{+\infty} H_n r \delta \theta \quad (27)$$

where the integration is done between positions remote from the fringe i.e. from a place outside the boundary where the total field H (and hence also the component H_n) is effectively zero, to a place inside the boundary where the resultant field H is effectively uniform and hence again H_n is zero.

Now the integration is to be done along the path of the ion, and we transform by the relation

$$H_n = H_\theta \sec \alpha \quad (28)$$

Also
$$r \int_{-\infty}^{+\infty} H_\theta d\theta = - \int_{-\infty}^{+\infty} \frac{\partial V}{\partial \theta} \cdot d\theta = V_{(-\infty)} - V_{(+\infty)} \quad (29)$$

where V is the magnetic scalar potential.

Now
$$\begin{aligned} V_{(+\infty)} &= 0 \\ V_{(-\infty)} &= \frac{Z}{Z_0} V_0 \end{aligned} \quad (30)$$

(where Z is the semi-gap width, and V_0 is the potential of the pole when the potential at the median plane is chosen to be zero). Equation (27) becomes

$$\psi = \frac{e}{mv} V_0 \frac{Z}{Z_0} \tan \alpha = \frac{1}{\rho_0} Z \tan \alpha \quad (31)$$

Hence the deflection produced by the fringe is proportional to the distance of the ion off the median plane, so that the fringe acts as a cylindrical lens, the focal length in terms of ρ_0 as unit of length being

$$\frac{\rho_0}{f_{||}} = \frac{Z}{\psi} = \cot \alpha \quad (32)$$

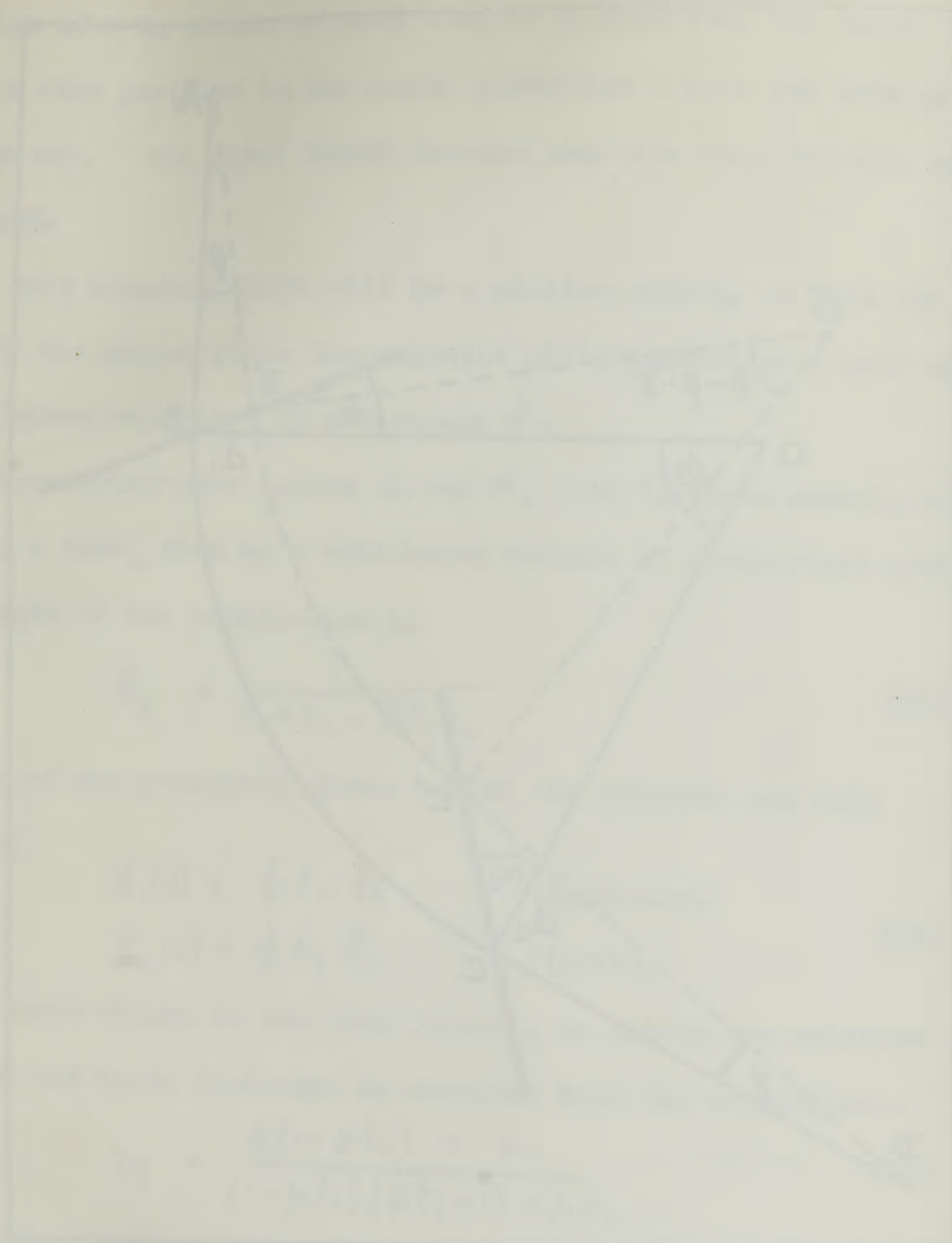


Figure 11.3: Geometric boundary focusing in the perpendicular plane.

$$d_1 = d_2 = \frac{1}{2} \left(\frac{1}{\sin \theta} - \frac{1}{\cos \theta} \right) = \frac{1}{2} \left(\frac{1}{\sin \theta} - \frac{1}{\cos \theta} \right)$$

A.

The sign of α is chosen so that when it is positive, the focal length f is also positive in the usual convention - i.e. the lens is a converging one. The focal length becomes negative when the sign of α is changed.

At the exit boundary there will be a similar effect, so that for particles off the median plane the magnetic field behaves as a pair of cylindrical lenses separated by a distance ϕ .

If the boundaries make angles α_1 and α_2 with the beam normal, and $t_1 = \tan \alpha_1$, $t_2 = \tan \alpha_2$ then by a well-known formula of geometrical optics, the focal length of the combination is

$$f_{||} = \frac{1}{t_1 + t_2 - \phi t_1 t_2} \quad (33)$$

The distances of the principal planes behind the entrance and exit boundaries are

$$\begin{aligned} \chi_1(II) &= \phi t_2 f_{||} & (\text{entrance}) \\ \chi_2(II) &= \phi t_1 f_{||} & (\text{exit}) \end{aligned} \quad (34)$$

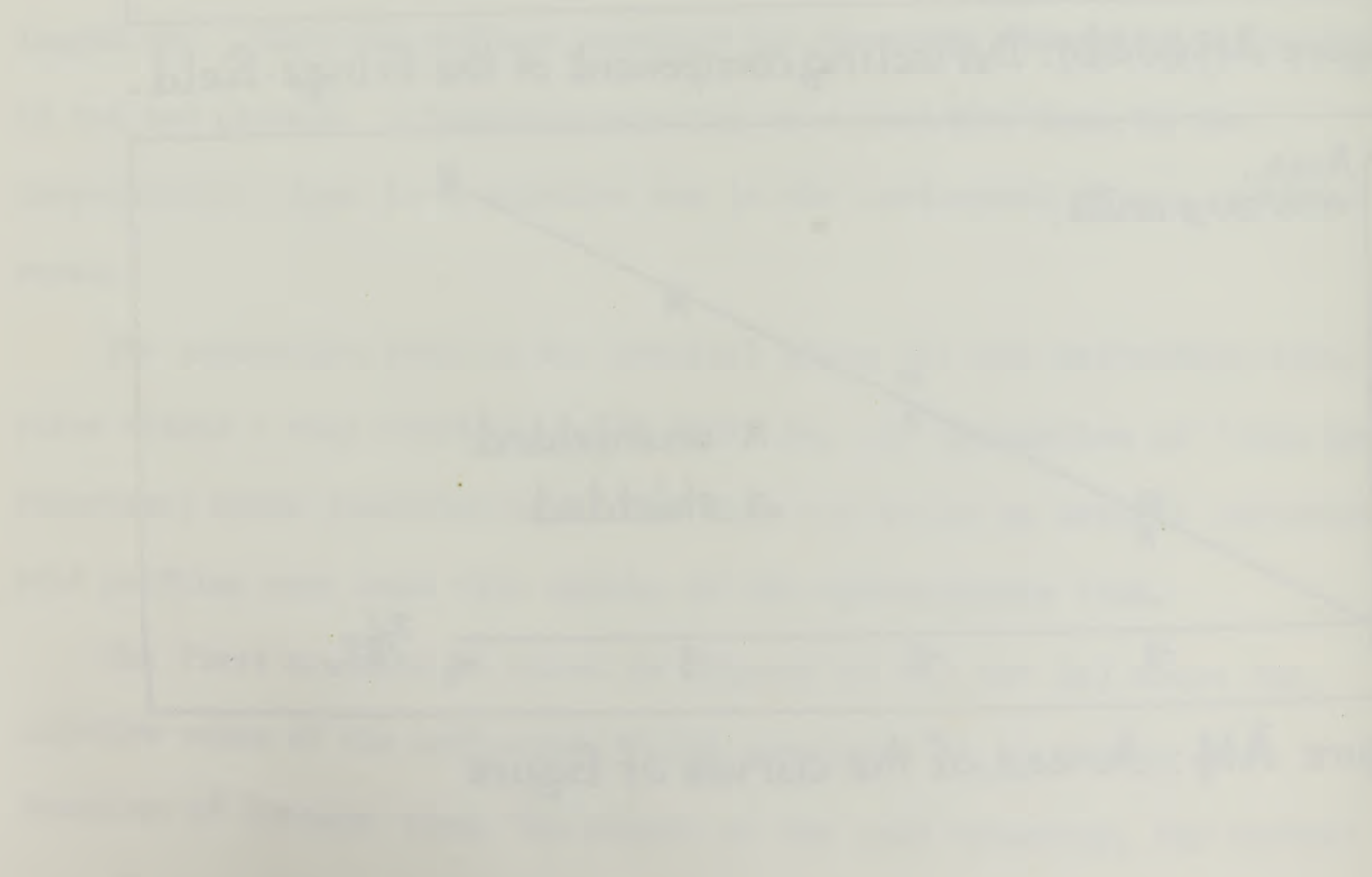
Substituting these values in the lens formula, we obtain the relation between object and image distances as measured from the boundaries:-

$$v_{||} = \frac{\phi(1 - \mu t_1) + \mu}{(1 - \mu t_1)(\phi t_2 - 1) + \mu t_2} \quad (35)$$

In the perpendicular plane the equation (3) giving the object-image relationship for a uniform sector field must be modified to include the effect of the oblique boundaries. Refer to figure 12. Two ion paths ABCD and A B' C' D from an object A to its image D are shown. ABCD is the optic axis, and A B' C' D is separated from it by a distance d_1 at the entrance boundary and d_2 at the exit, both d_1 and d_2 being measured normal to the optic axis.

Resolving the distance BC horizontally we have

$$d_1 + \rho_0 \cos \beta - \rho_0 \cos(\phi - \gamma) + \frac{d_2}{\cos \alpha_2} \sin(\phi - \alpha_2 - \frac{\pi}{2}) = \rho_0 - \rho_0 \cos \phi \quad (36)$$



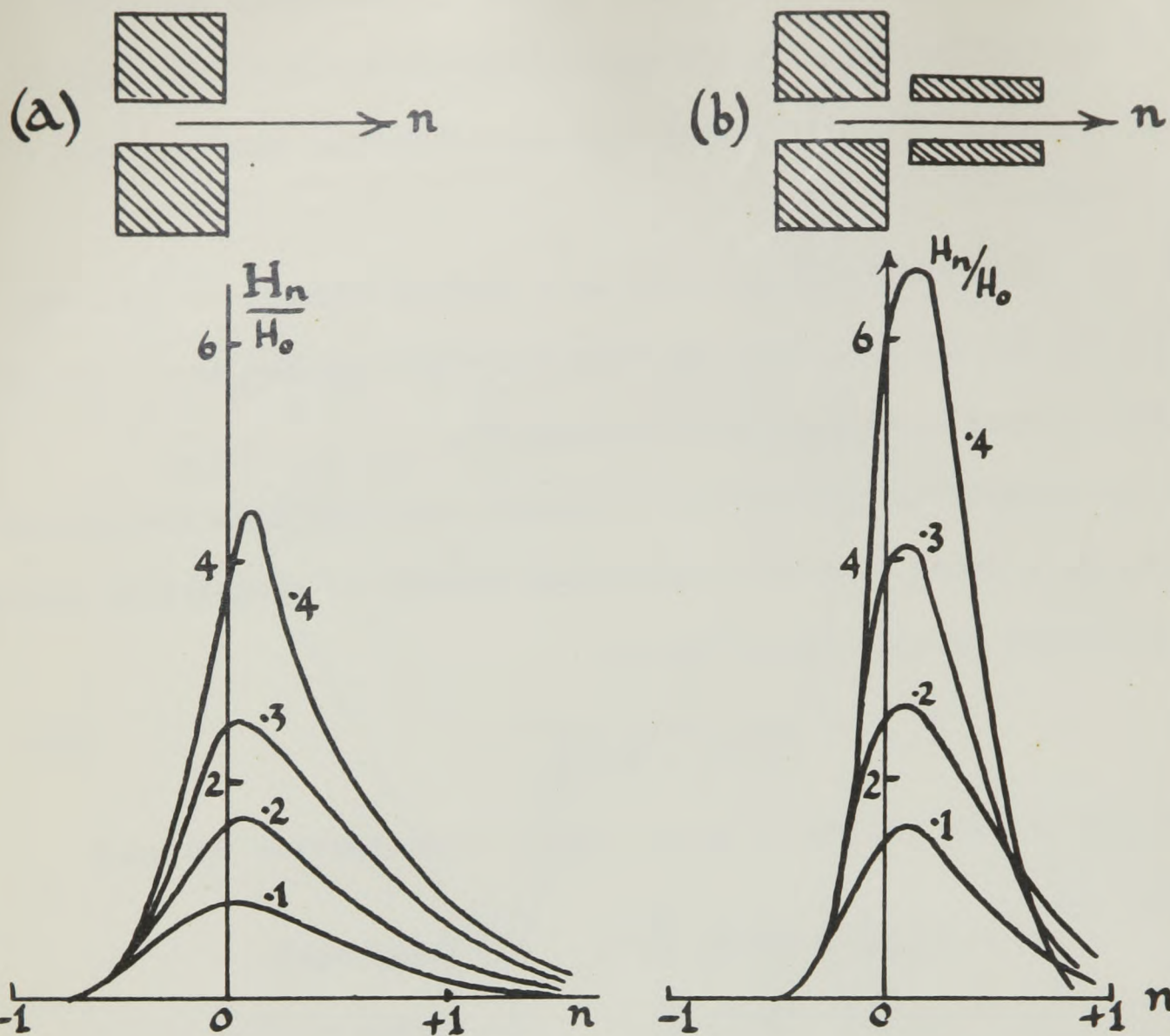


Figure A13(a) & (b): Deflecting component of the fringe field.

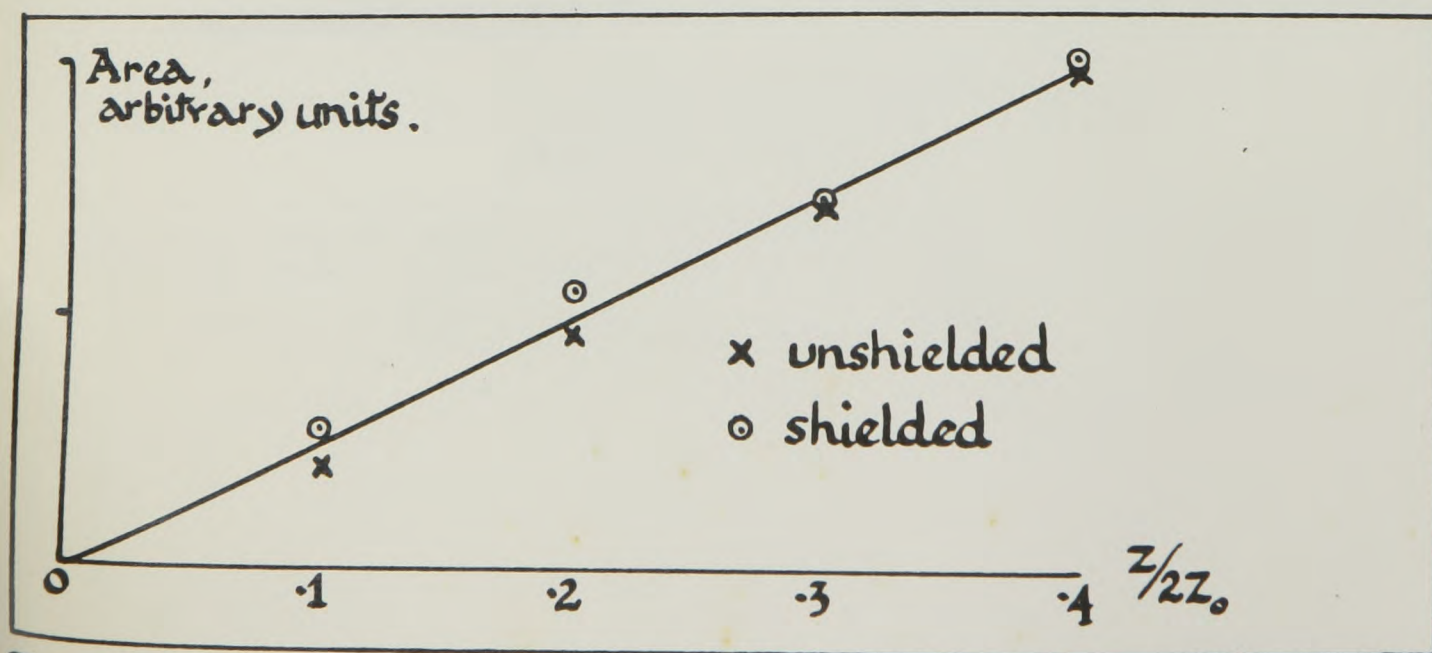


Figure A14: Areas of the curves of figure

A,

or, since β and δ are small, and $\nu \rho_0 \delta = d_2$ and $\mu \rho_0 \beta = d_1$,

$$d_1 = d_2 \left[t_2 \sin \phi + \cos \phi + \frac{1}{\nu} \sin \phi \right] \quad (37)$$

Resolving vertically,

$$d_1 t_1 + \rho_0 \sin \beta - \rho_0 \sin(\phi - \delta) - \frac{d_2}{\cos \alpha_2} \cos(\phi - \alpha_2 - \frac{\pi}{2}) = -\rho_0 \sin \phi \quad (38)$$

or
$$d_1 \left(t_1 + \frac{1}{\mu} \right) = d_2 \left[-t_2 \cos \phi + \sin \phi - \frac{\cos \phi}{\nu} \right] \quad (39)$$

Combining equations (37) and (39)

$$\nu_1 = \frac{\mu [1 + t_1 \tan \phi] + \tan \phi}{\mu \tan \phi (1 - t_1 t_2) - (t_1 + t_2) - t_2 \tan \phi - 1} \quad (40)$$

When $t_1 = t_2 = 0$

$$\nu_1 = \frac{\tan \phi + \mu}{\mu \tan \phi - 1} \quad (41)$$

as is to be expected (see section 4).

By comparing the focussing properties of a uniform field both with and without a single oblique boundary, it can be shown that in the perpendicular plane an oblique boundary acts as a thin lens of focal length $-t$. Thus the oblique boundary has opposite focussing properties in the two planes: a boundary behaving as a positive lens in the perpendicular plane is a negative one in the horizontal plane, and vice versa.

The assumption that in the parallel plane all the deflection takes place within a very short distance (that is, the assumption of 'thin lens' behaviour) needs justification. To this end tests on several different pole profiles were done with models in the electrolytic tank.

The first of these is shown in figures 13 (a) and (b) where the relative value of the deflecting field component H_n is shown, as a function of distance along the normal to the pole boundary, for various

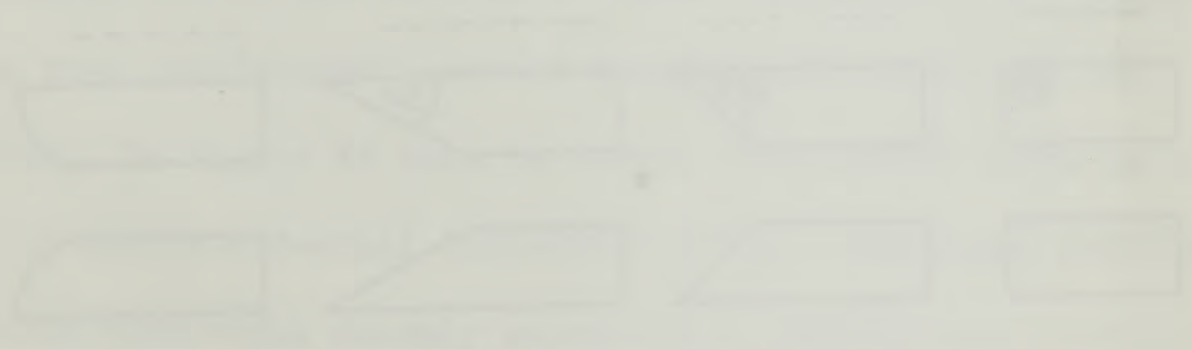
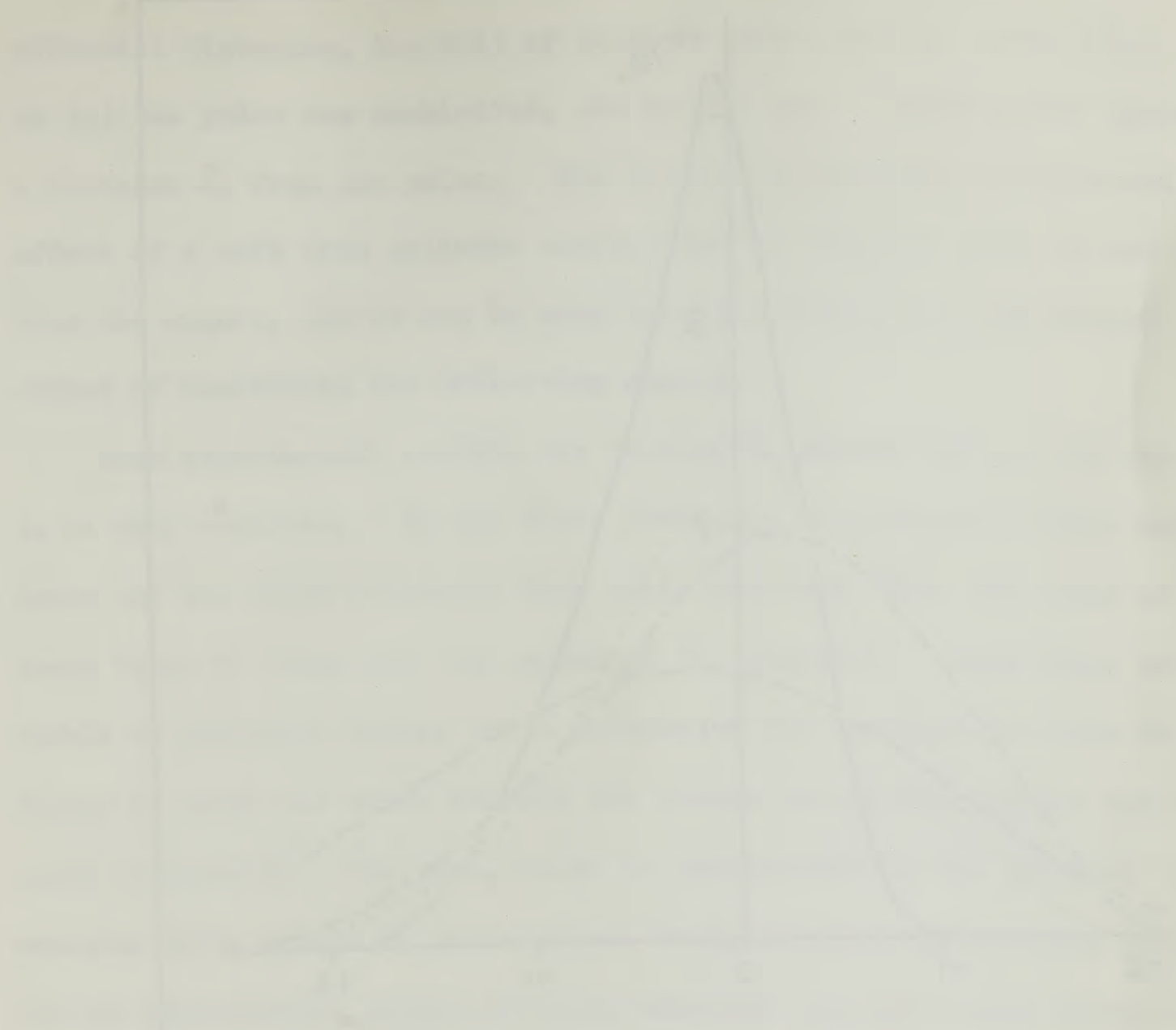


Figure 11: The effect of concentration on the rate of reaction.

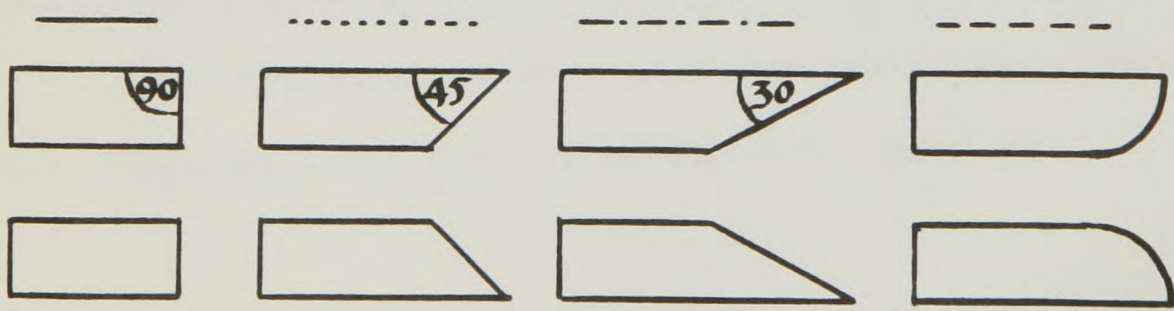
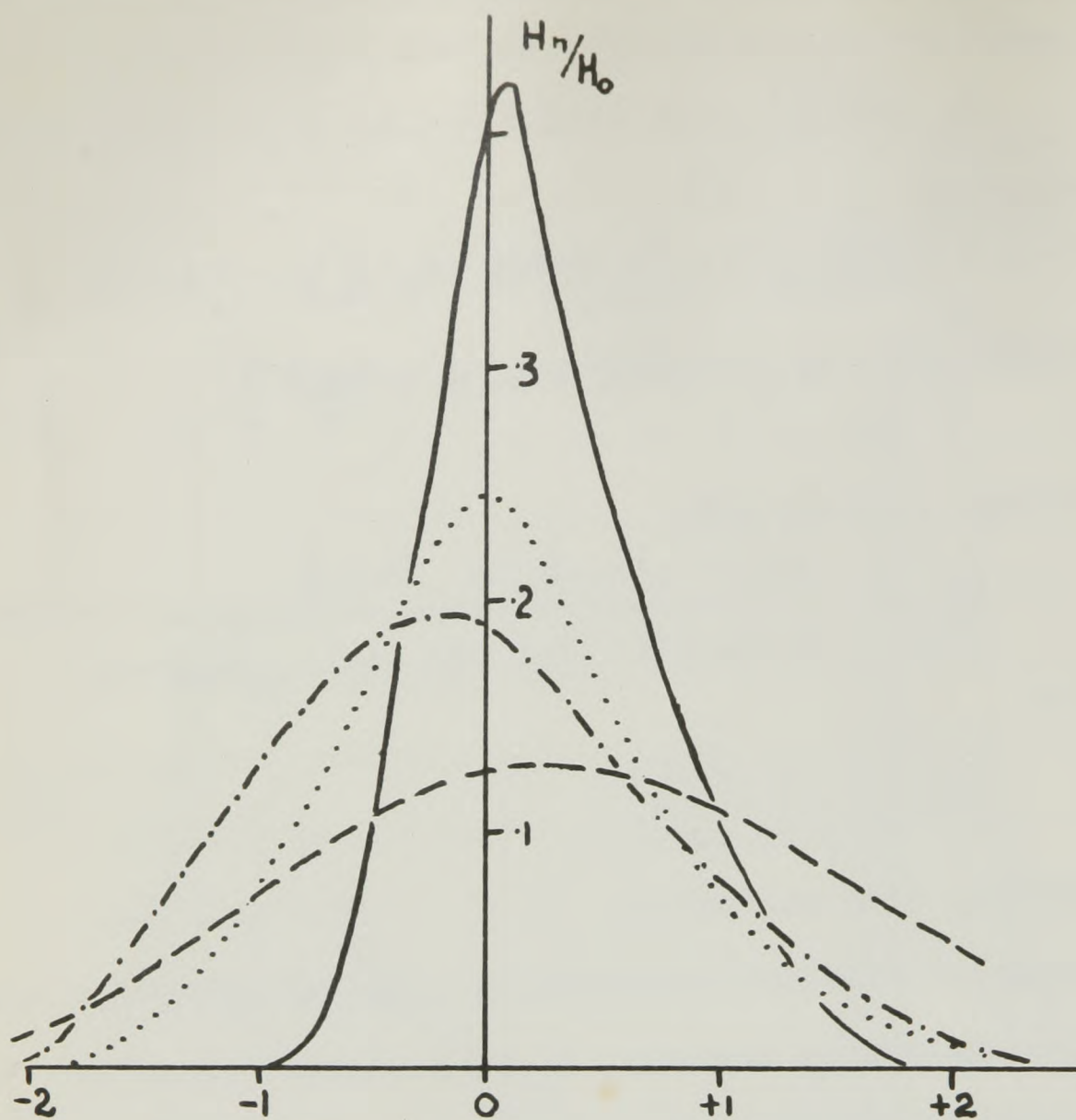


Figure A15 : The effect of a change of pole profile.

A.

off-axial distances, the unit of distance being the gap width $2Z_0$.

In (a) the poles are unshielded, and in (b) have a thick shield spaced a distance Z_0 from the poles. The latter was intended to represent the effect of a soft iron cylinder surrounding the beam at entry to and exit from the magnet, and it can be seen that the shield has the desired effect of shortening the deflecting region.

Such experimental results are tedious to obtain and are not likely to be very accurate. In the first place the equipotential lines must be drawn and the field estimated from their spacing: then the angle of these lines is found and the component H_n computed. Both steps are liable to graphical errors and a measure of the accuracy is shown in figure 14 where the areas beneath the curves are plotted against the off-axial distance Z . The area, which is represented by the integral of equation (27), should be proportional to Z , as shown by equation (29); and the experimental points for both shielded and unshielded poles are found to be within 10% of the line of the best fit through the origin.

A change from the 'square' profile broadens the extent of the fringing field. In figure 15 this is shown, the plotted curve in each case being that for an off-axial distance $0.4 Z_0$. The round profile was included because this might roughly represent a virtual pole profile at high field strength when the iron becomes saturated.

When $\mu = \infty$, we get for the focal distance from the exit boundary

$$v_{\perp} = \frac{1 + t_1 \tan \phi}{\tan \phi (1 - t_1 t_2) - (t_1 + t_2)} \quad (42)$$

To find the magnification, we need the position of the principal planes.

These are at distances X_1 and X_2 behind the pole boundaries. The value of X_2 is given by

$$(v + x_2) \rho_0 \delta = d, \quad (43)$$

A.

(where ν has the value corresponding to $\mu = \infty$ given in equation (42) above)

$$\therefore X_2(\perp) = \frac{1 - \cos \phi - t_1 \sin \phi}{\sin \phi (1 - t_1 t_2) - \cos \phi (t_1 + t_2)} \quad (44)$$

Similarly
$$X_1(\perp) = \frac{1 - \cos \phi - t_2 \sin \phi}{\sin \phi (1 - t_1 t_2) - \cos \phi (t_1 + t_2)} \quad (44a)$$

A point image of a point object will theoretically be formed when

$$\nu_{\perp} = \nu_{\parallel} \quad (= \nu)$$

the two values being given by equations (35) and (40).

7. 90° Deflection.

For the important case $\phi = \frac{\pi}{2}$, the two image distances are

$$\nu_{\perp} = \frac{1 + \mu t_1}{\mu (1 - t_1 t_2) - t_2} \quad (45)$$

$$\nu_{\parallel} = \frac{\frac{\pi}{2} (1 - \mu t_1) + \mu}{(1 - \mu t_1) (\frac{\pi}{2} t_2 - 1) + \mu t_2} \quad (46)$$

so that to find the conditions for a point image these two equations must be solved simultaneously. The great variety of solutions is limited by the various conditions mentioned in section 2. Eliminating between equations (45) and (46) we have a quadratic in μ or t_1 :-

$$\mu^2 \left[t_1^2 (\pi + \nu) - t_1 \left(2 + \frac{\pi}{2} \nu \right) + \nu \right] + \mu \left(\frac{\pi}{2} \nu - 2 \right) - (\pi + \nu) = 0. \quad (47)$$

For a selected value of ν the two allowable values of t_1 and the corresponding values of t_2 can be plotted as a function of μ .

The value of t_2 is found from e.g. equation (45), which transforms to

$$t_2 = \frac{\mu}{(\mu t_1 + 1)} - \frac{1}{\nu} \quad (48)$$

The calculations for $\nu = 2, 5$ and 10 are shown in figure 16(a)(b)

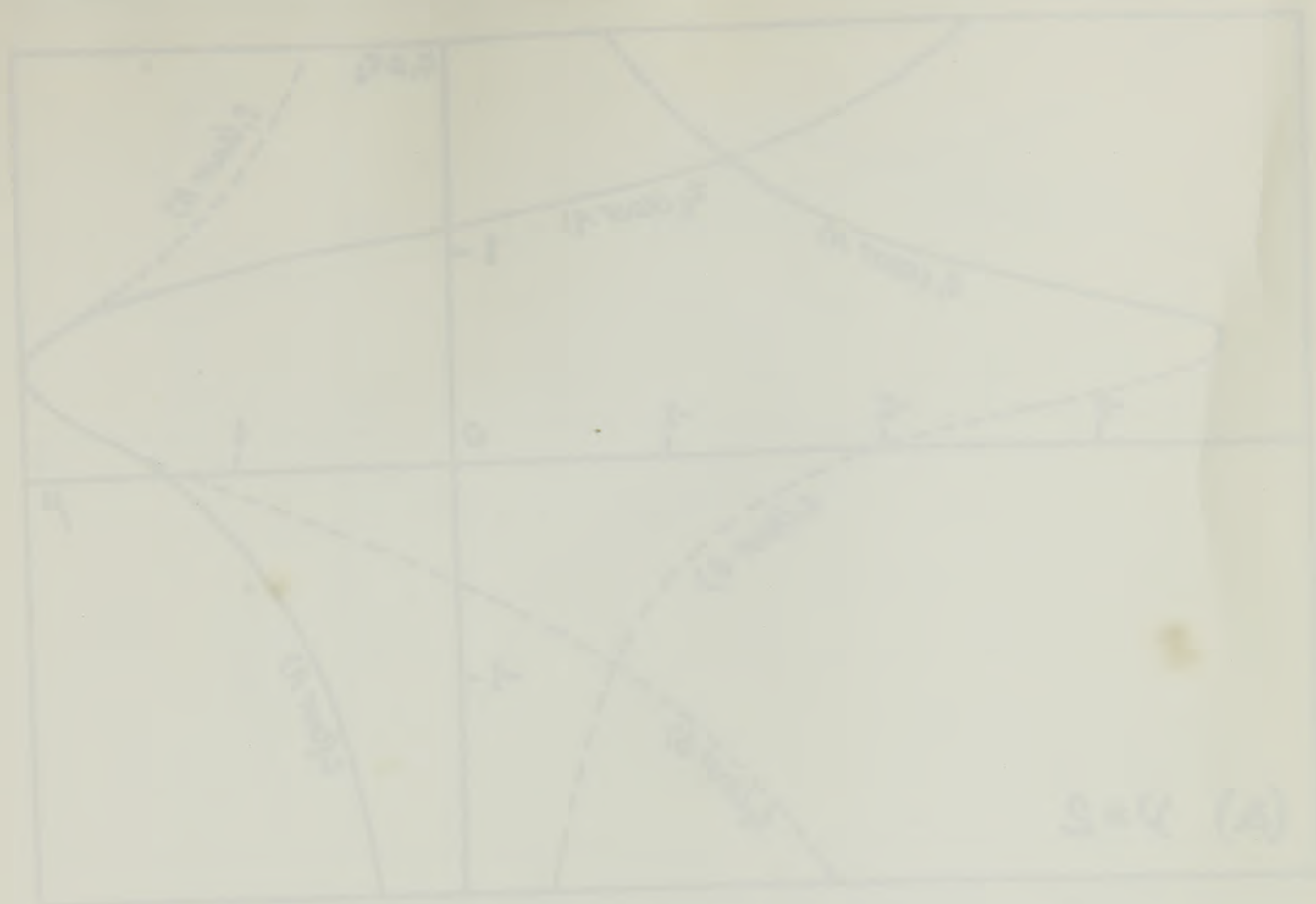
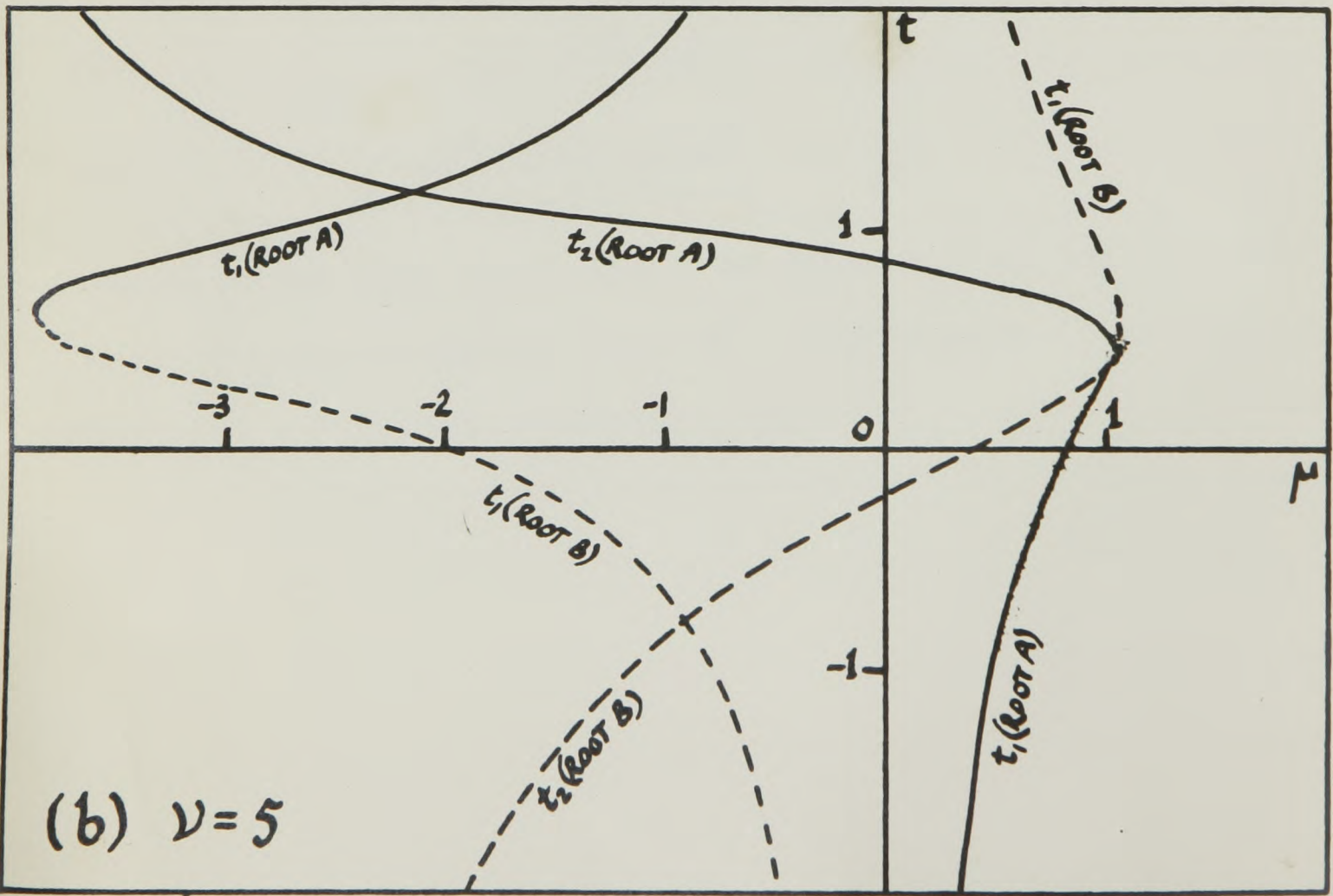
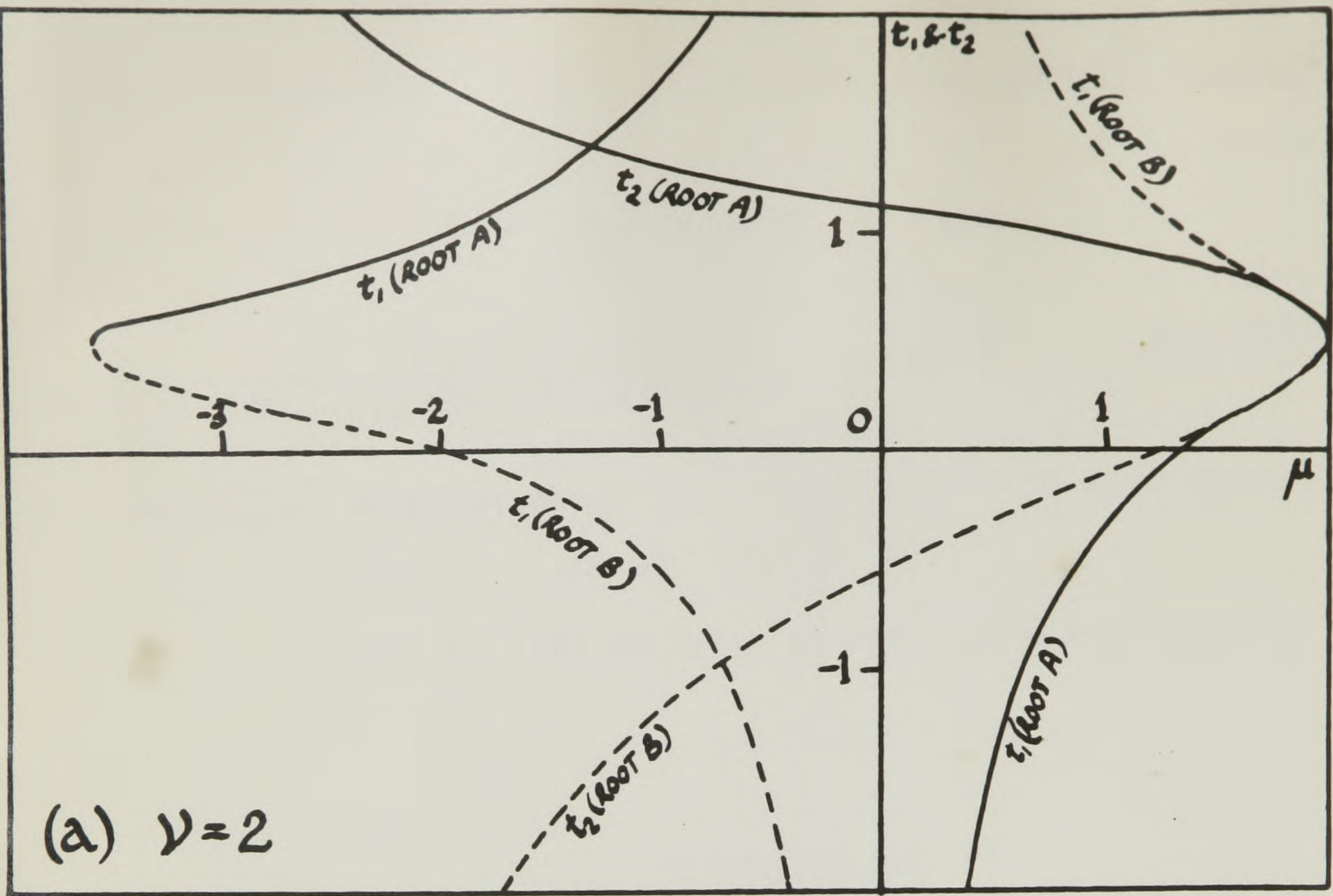


Figure 1(a) and (b): Values of p, t, q for considered.



Figures A16 (a) and (b): Values of μ, t_1, t_2 for coincident foci.

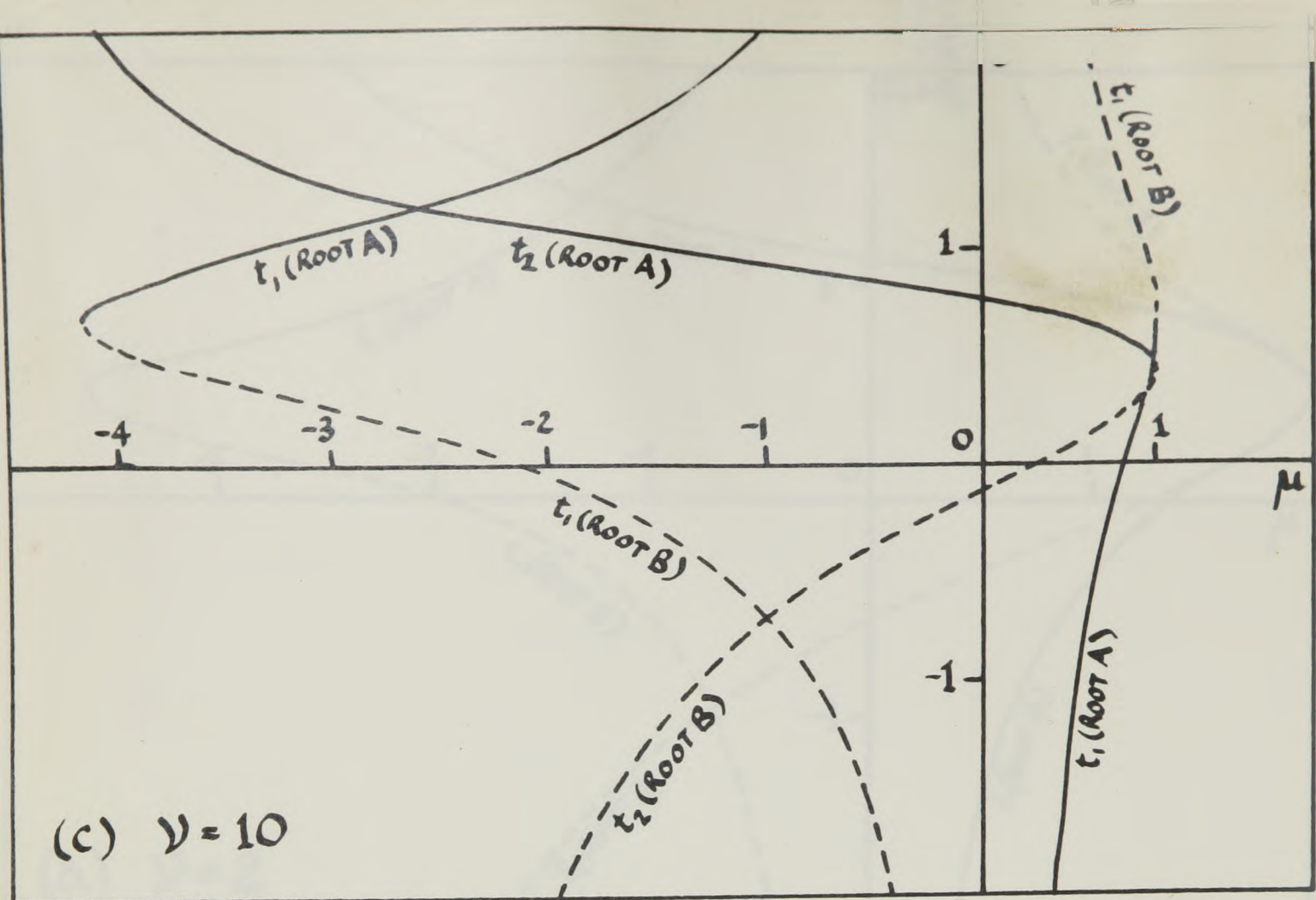


Figure A16(c): Values of μ , t_1 , t_2 for coincident foci.



Figure 1: A graph showing several curves on a coordinate system. The curves appear to be hyperbolas or similar functions, with some labeled with 'a' and 'b'.

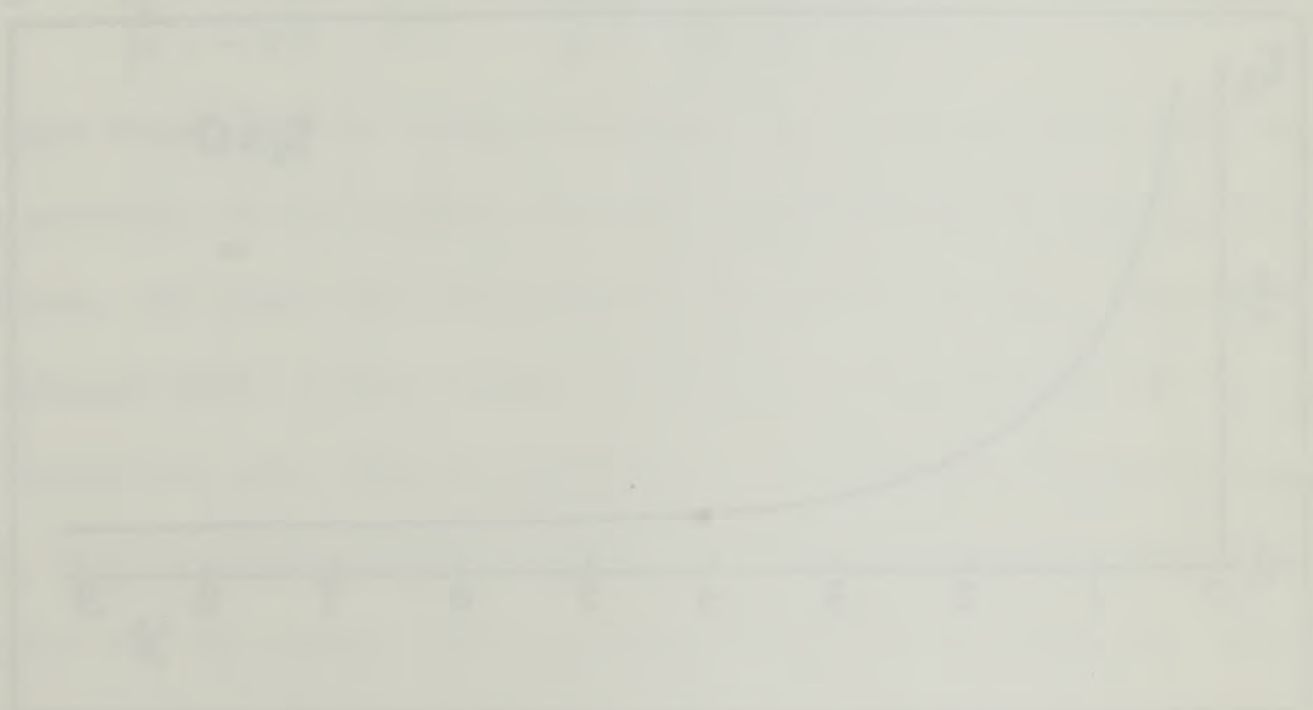


Figure 2: A graph showing a single curve on a coordinate system. The curve is a smooth, increasing function, possibly a logarithm or an exponential function.

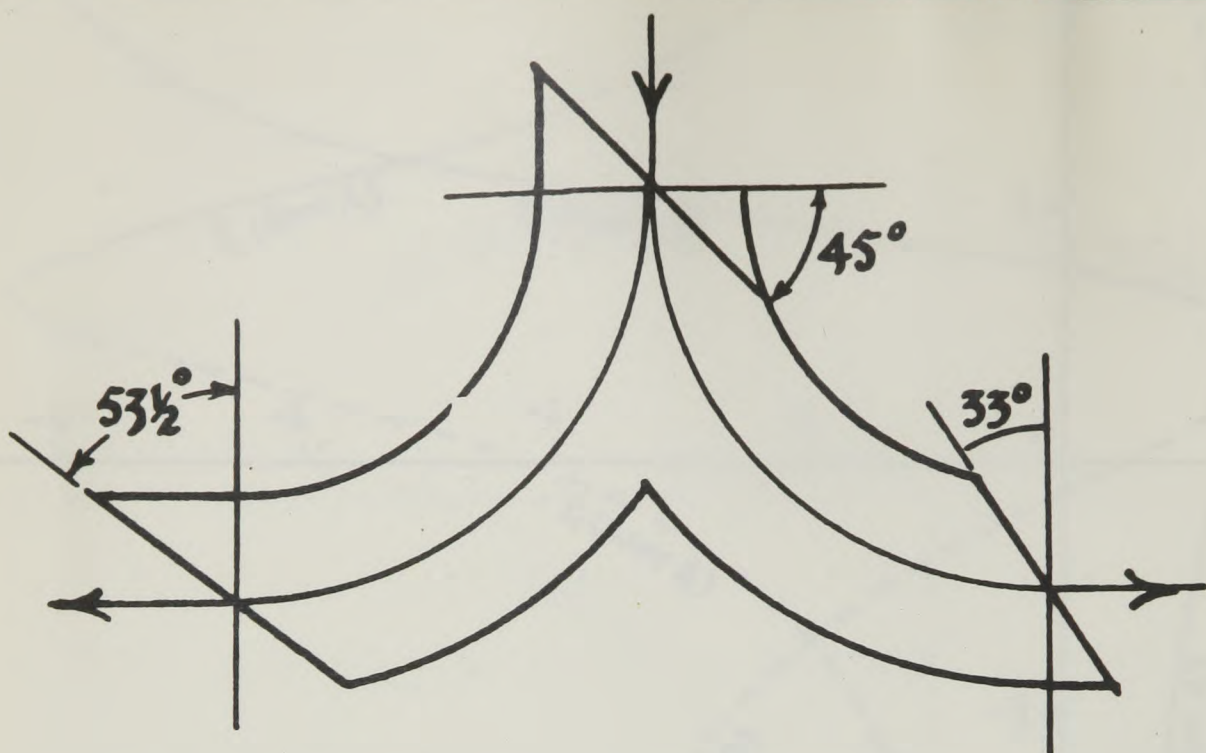


Figure A17 : Pole-face shape giving $v=5$ in both directions.

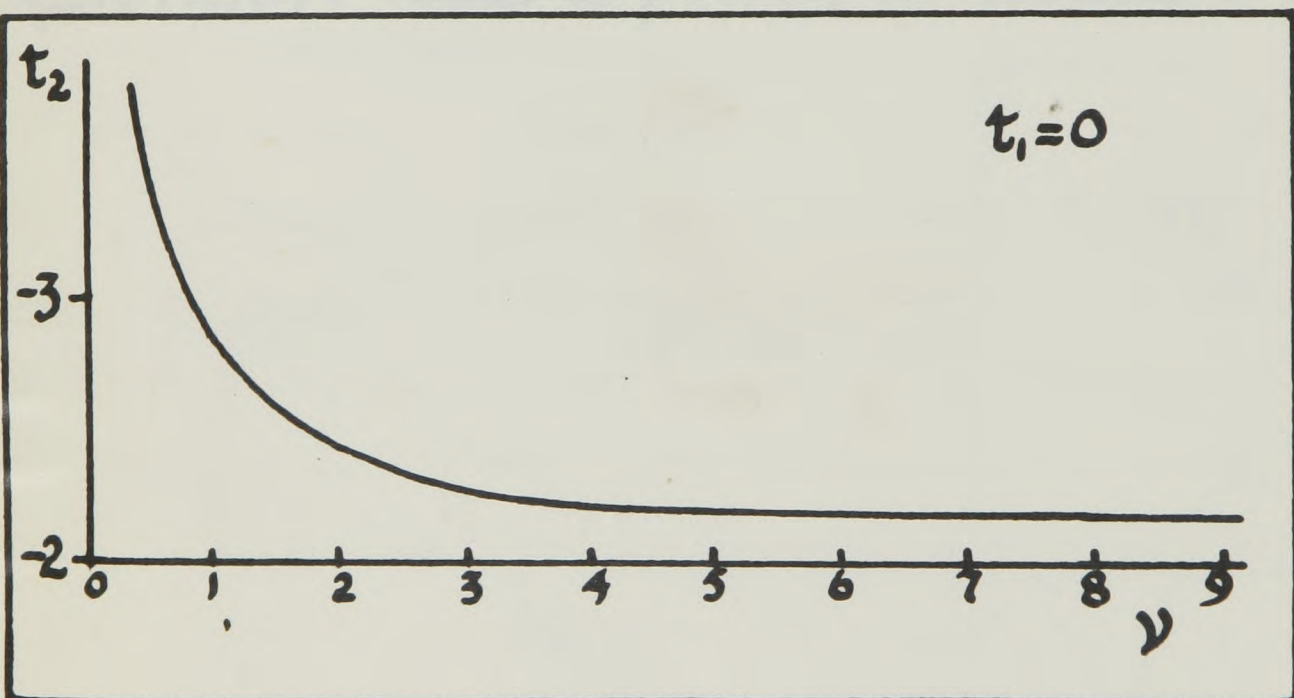


Figure A18 : Values of t_2 to give coincident foci when $t_1=0$.

A.

and (c). Although negative values of μ (i.e. a virtual object) are required, the curves have been completed for positive μ also because it has been found that apart from space-charge distortions of the beam, satisfactory solutions are also possible for these cases.

It is seen that there is a considerable choice of pairs of angles α_1 and α_2 for which t_1 and t_2 both lie between ± 2 . We are seeking in the first place a fixed value of t_1 , which will allow deflection through 90° in opposite directions by reversing the field. Hence a value $+t_1$ chosen for deflection in one direction will become $-t_1$ for the other, and the values of t_2 must be chosen appropriately. Thus a possible set of values would be

$$\nu = 5 \quad \left. \begin{array}{l} t_1 = +1 \\ t_2 = +1.35 \\ \mu = -2.7 \end{array} \right\} \quad \left. \begin{array}{l} t_1 = -1 \\ t_2 = -0.65 \\ \mu = -0.75 \end{array} \right\} \quad (47)$$

This pole face shape is shown in figure 17. Whether such sets of values are suitable or not depends on the magnifications $M_\perp$ and $M_\parallel$ in the two planes, and these are discussed in the next section. The graph for $\nu = 5$ (figure 16(b)) also shows that there is a solution with $t_1 = 0$ so that a symmetrical pole face is possible. However the corresponding value of t_2 is -2.2 which is inconveniently large and moreover does not change rapidly for the better with increase of ν , as is shown in figure 18. A further disadvantage, as will be seen, is that the magnifications only become reasonably small when t_2 is large.

8. Magnifications in 90° deflection.

The magnifications depend only on the distances u and v of the object and image from their principal planes, so that for given values of μ and ν the magnifications are functions of the distances X_1 and X_2



Figure 1: The curves y_1, y_2, y_3, y_4 are plotted in the xy -plane as a function of x .

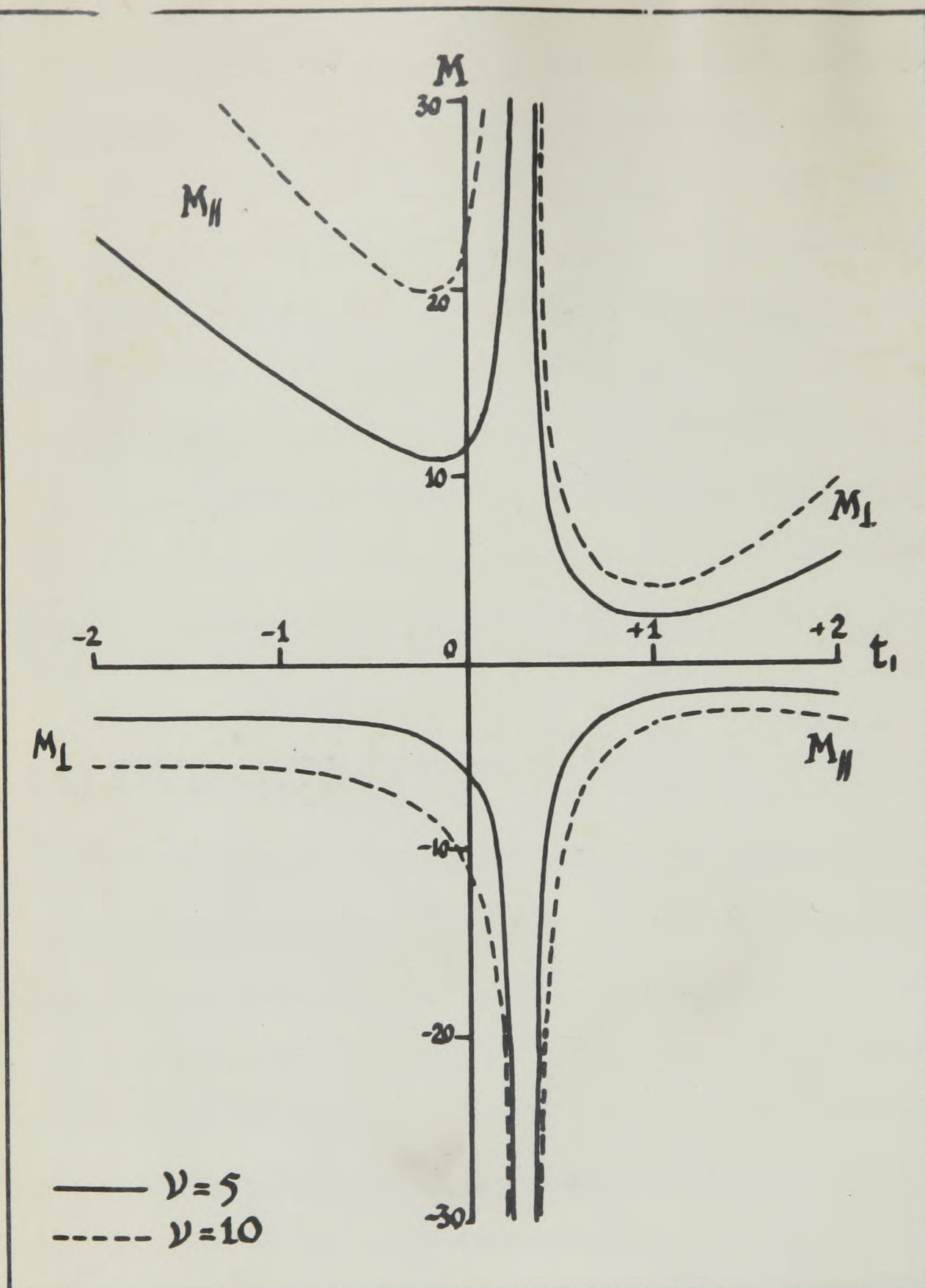


Figure A19 : Magnifications in both planes as a function of t_1 .

A.

of these planes from the field boundaries. As shown in equations (33) and (34), (44) and (45), these distances depend greatly on the signs and magnitudes of t_1 and t_2 .

In the parallel plane the magnification is simply found by considering the system as a pair of thin lenses, at the boundaries.

Thus

$$M_{||} = \frac{\nu t_2 - 1}{\mu t_1 - 1} \quad (50)$$

This expression can be reduced to fewer variables by using equation (46) which transforms to

$$\mu t_1 - 1 = \frac{\mu(\nu t_2 - 1)}{\frac{\pi}{2}(\nu t_2 - 1) - \nu} \quad (51)$$

Hence

$$M_{||} = \frac{\frac{\pi}{2}(\nu t_2 - 1) - \nu}{\mu} = \frac{\nu}{\frac{\pi}{2}(\mu t_1 - 1) - \mu} \quad (52)$$

In the perpendicular plane the magnification, being the ratio of image to object distances measured from their principal planes, is

$$M_{\perp} = - \frac{\nu_{\perp} + X_2(\perp)}{\mu + X_1(\perp)} \quad (53)$$

$X_1(\perp)$ and $X_2(\perp)$ are given by putting $\phi = \frac{\pi}{2}$ in equations (44) and (44a) so that

$$M_{\perp} = - \frac{\nu_{\perp}(1 - t_1 t_2) + (1 - t_1)}{\mu(1 - t_1 t_2) + (1 - t_2)} \quad (54)$$

Using now equation (45), this becomes

$$M_{\perp} = - \frac{\nu_{\perp}}{\mu t_1 + 1} = - \frac{\nu_{\perp} t_2 + 1}{\mu} \quad (55)$$

The calculated values of $M_{\perp}$ and $M_{||}$ for $\nu = 5$ and $\nu = 10$ are shown in figure 19. It is seen that the smallest magnifications in both planes are given by a value of $t_1 \sim 1$, and although different in sign they are not very different in magnitude. However the magnifications for all negative values of t_1 are large, particularly in the parallel plane, so that a pole face with $t_1 = -1$ could not conveniently be used for

A.

deflection in the opposite direction (where $t_1 = -1$).

If alternative deflection with a fixed entrance boundary were to be used, the best compromise would probably have $t_1 = \pm 1$. The values of t_2 , $M_{\perp}$, and $M_{\parallel}$ have been calculated for these two values of t_1 and with values of ν between 0 and 10. They are shown in table 3.

Table 2.

ν	$t_1 = +1$				$t_1 = -1$			
	μ	t_2	$M_{\perp}$	$M_{\parallel}$	μ	t_2	$M_{\perp}$	$M_{\parallel}$
0	-1.00	—	1.80	0	-0.61	—	0	5.0
1	-1.50	2.00	2.00	-0.41	-0.67	-1.40	-0.60	7.8
2	-1.91	1.60	2.20	-0.75	-0.71	-0.91	-1.17	9.6
3	-2.20	1.45	2.40	-1.10	-0.72	0.78	-1.70	11.5
4	-2.50	1.40	2.70	-1.35	-0.73	0.69	-2.30	13.5
5	-2.70	1.38	2.94	-1.60	-0.73	-0.62	-2.88	15.6
6	-2.90	1.36	3.30	-1.90	-0.74	-0.58	-3.42	18.0
7	-3.05	1.35	3.60	-2.20	-0.74	-0.55	-4.00	20.0
8	-3.15	1.35	3.90	-2.50	-0.75	-0.54	-4.55	22.2
9	-3.20	1.34	4.2	-2.75	-0.75	-0.53	-5.10	24.2
10	-3.27	1.34	4.4	-2.90	-0.76	-0.53	-5.65	26.3

Thus supposing the beam was to be deflected onto two targets at different distances, say $\nu = 2$ and $\nu = 5$; then using $t_1 = +1$ for the longer distance, the values would be:

$$\begin{array}{ll}
 t_1 = +1.0 & \nu = 5 \\
 \mu = 2.7 & \\
 t_2 = 1.38 & \\
 M_{\perp} = 2.94 & \\
 M_{\parallel} = -1.60 & \\
 t_1 = -1.0 & \nu = 2 \\
 \mu = -0.71 & \\
 t_2 = -0.91 & \\
 M_{\perp} = -1.77 & \\
 M_{\parallel} = 9.6 &
 \end{array} \quad (56)$$

Approximate values of $M_{\perp}/M_{\parallel}$, which is the ratio of the axes of the elliptical image of a circular object, are shown for $\nu = 5$ on a logarithmic scale in figure 20. The curve is very similar for other

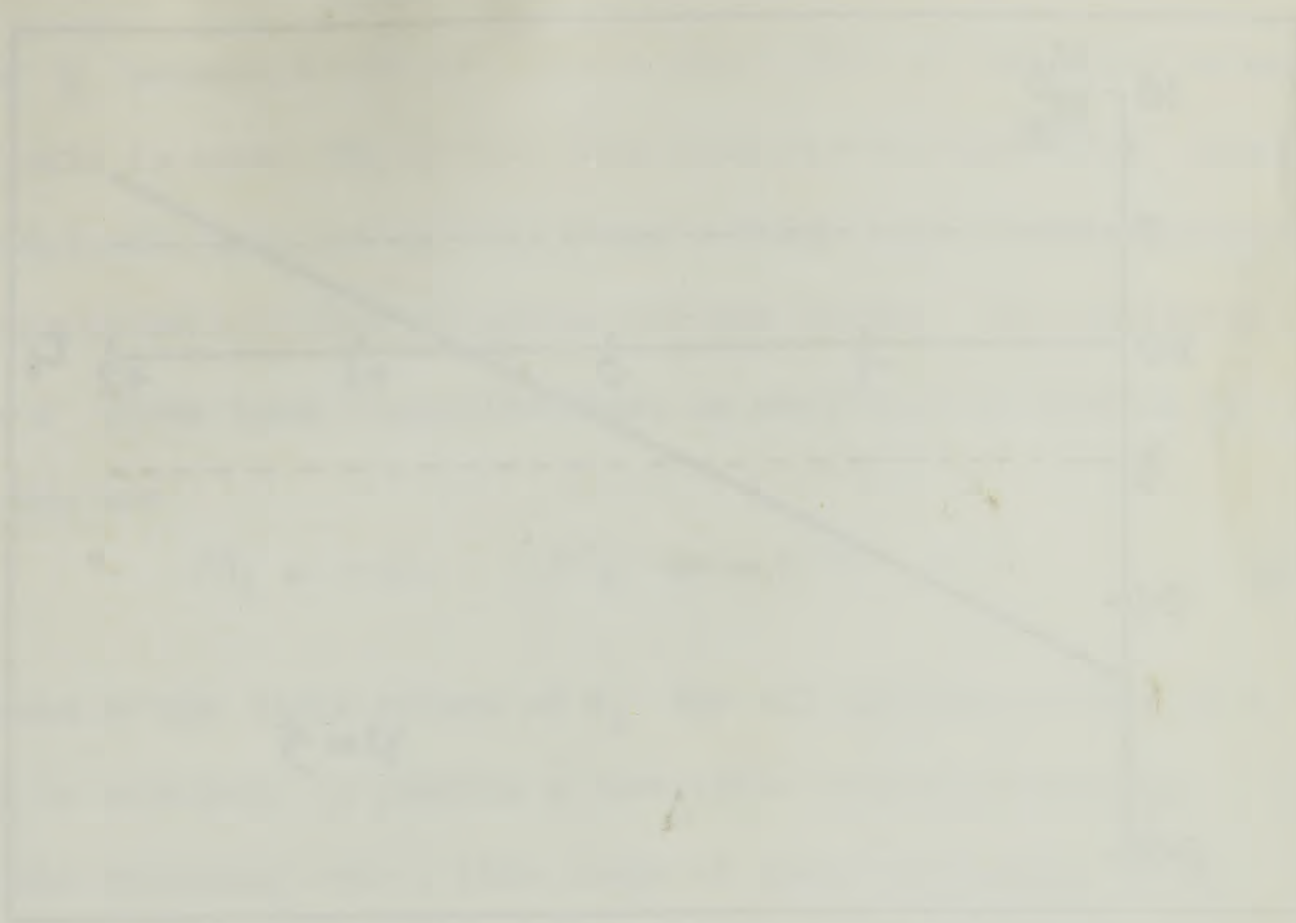


Figure 3: Relationship of variables

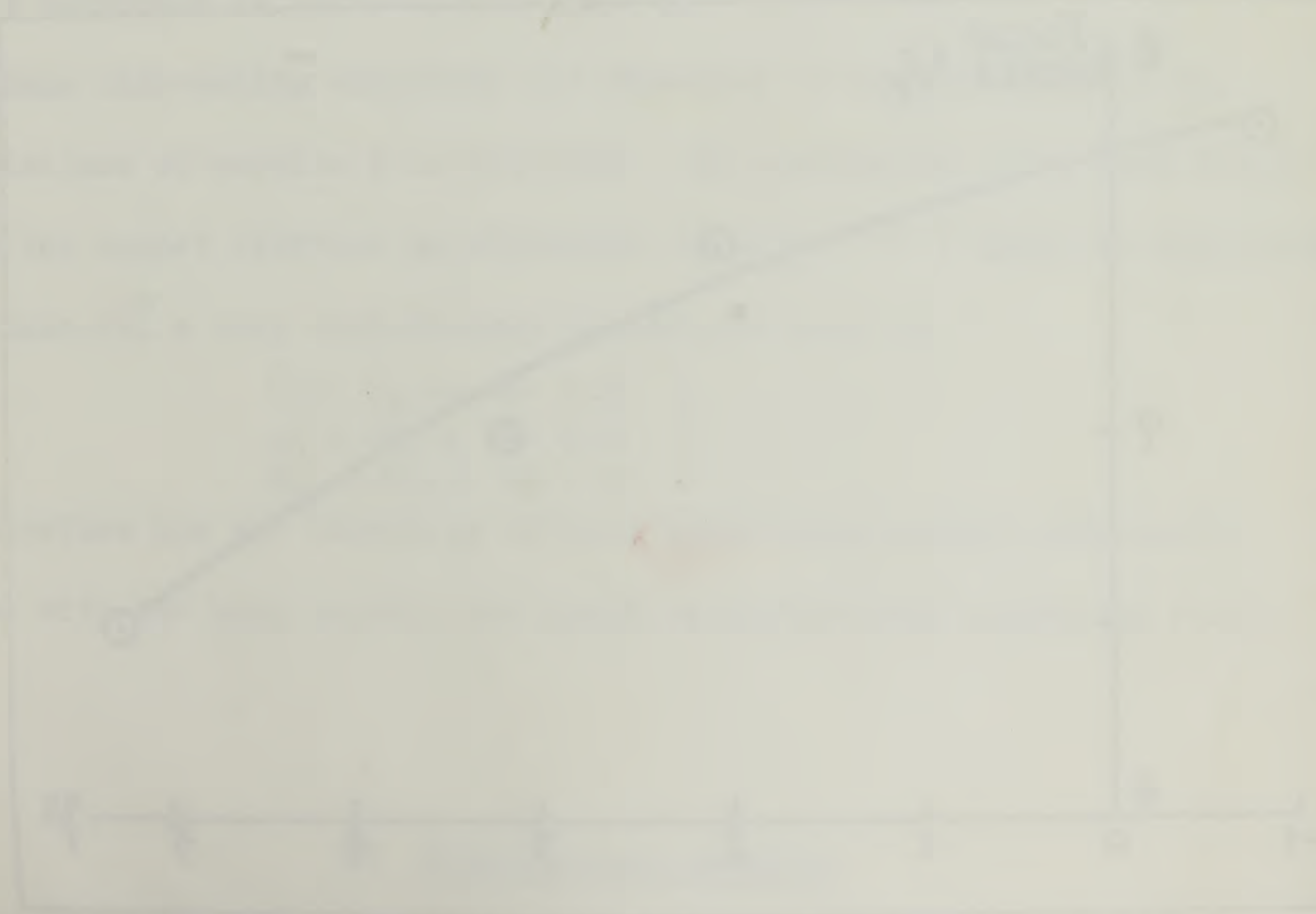


Figure 4: Relationship of variables

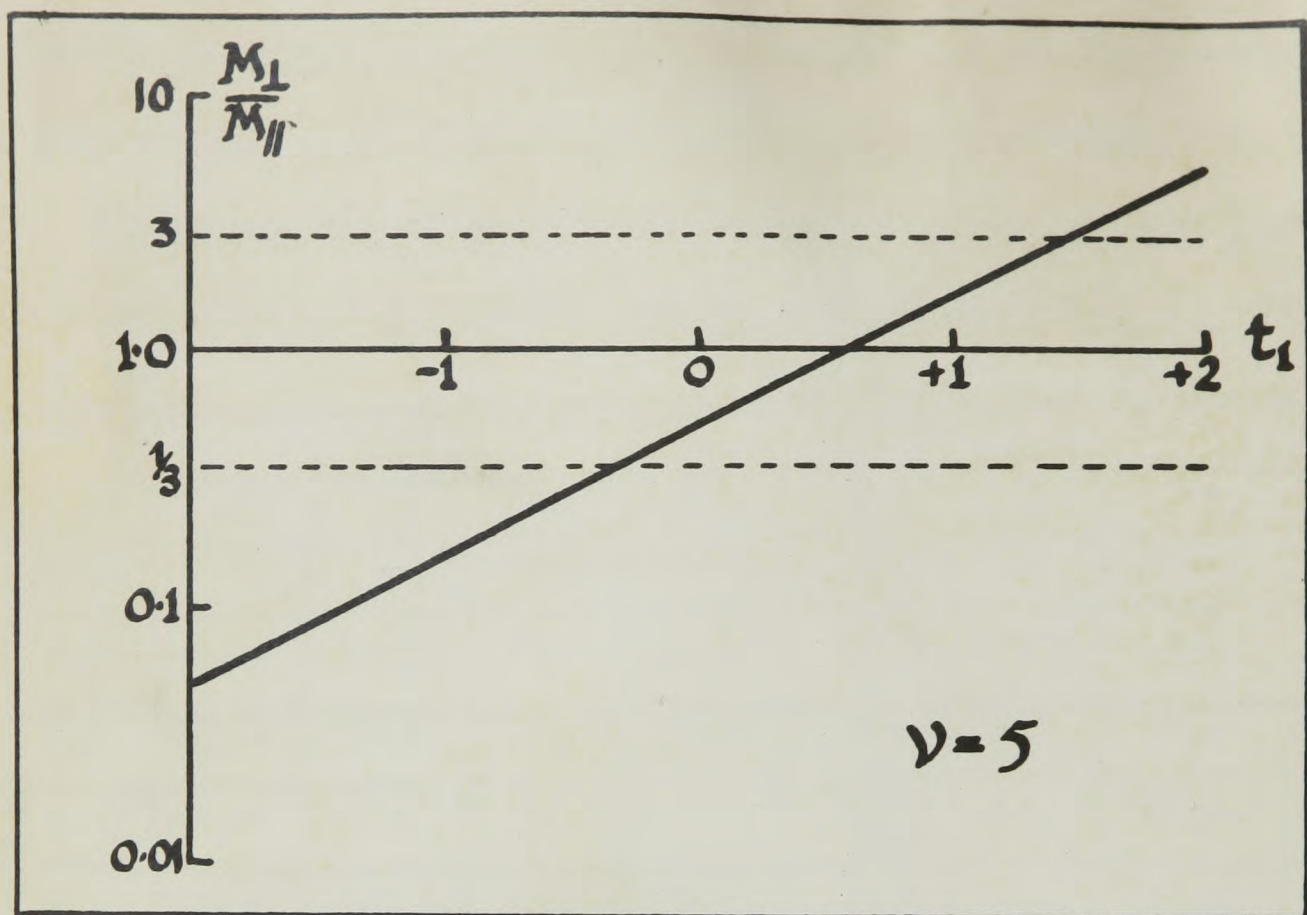


Figure A20: Ratio of magnifications.

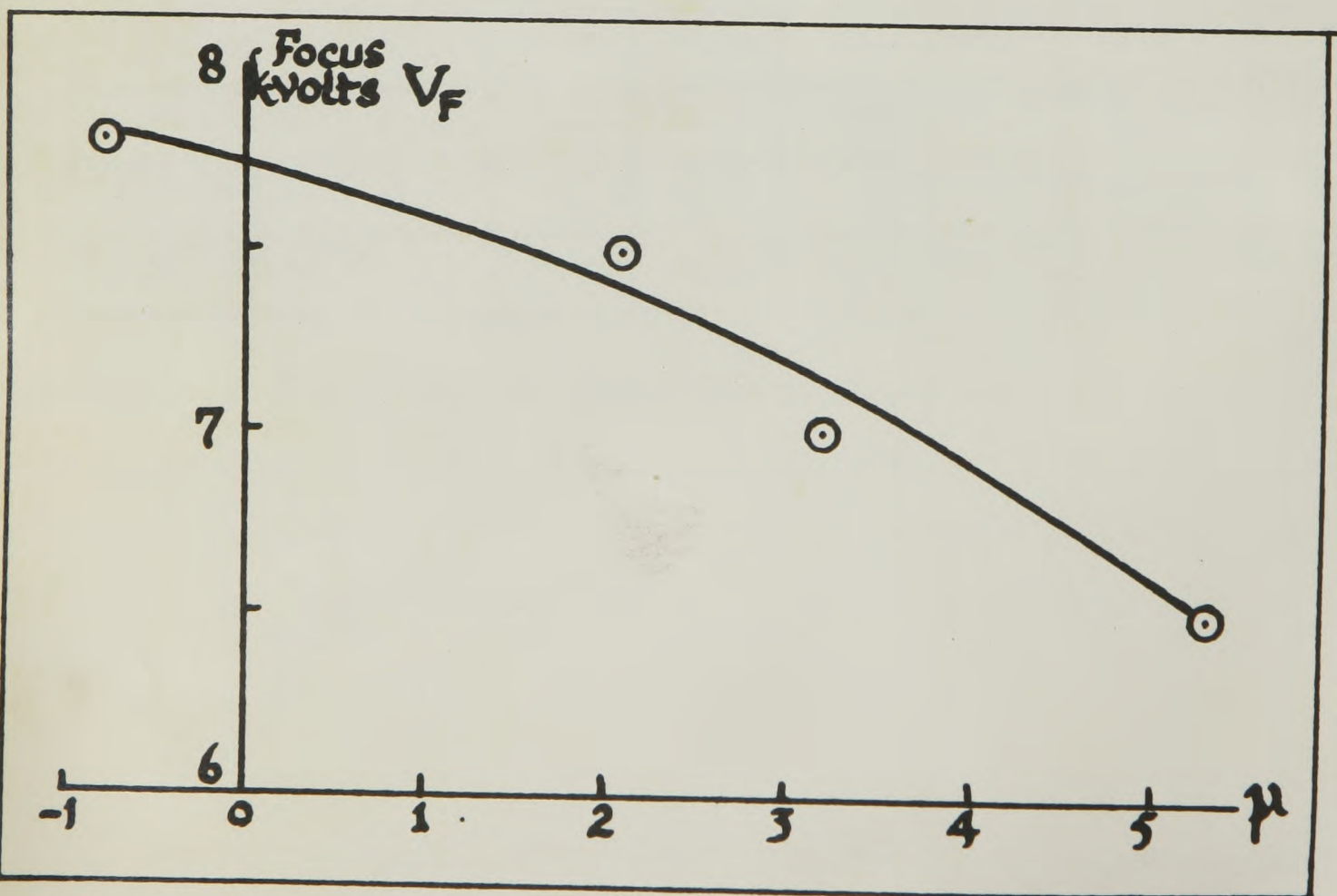


Figure A21: Position of object at various focus voltages.

A.

values of ν between 2 and 10. It is clear that if the ratio of major to minor axis is to be for example less than 3, the value of t_1 must be between -0.3 and +1.5, and of this range a large part is unacceptable because the individual magnifications are too large. In particular the case $t_1 = 0$ gives large magnifications, as mentioned in section 7; for $\nu = 5$ they are

$$M_{\perp} = -5 \quad M_{\parallel} = +11.9 \quad (57)$$

Because of the large values of $M_{\parallel}$ for all negative values of t_1 and μ it is necessary to provide a reversible entrance boundary. In order to aid focussing over a wide range of image distances, it is useful for all three boundaries to be gradually adjustable for angle. These facts formed the basis for the pole-face design of the 19 cm. magnet described in section 2 of part B.

Some interesting solutions are obtained if one or other of the restrictions of section 2 is relaxed. In particular, if a real object above the magnet entrance is allowable (i.e. $\mu > 0$) then, as explained by Cockcroft,¹³ a very satisfactory symmetrical case is

$$\left. \begin{aligned} t_1 &= t_2 = +0.5 \\ \nu &= \mu = +2.0 \\ M_{\perp} &= M_{\parallel} = +1.0 \end{aligned} \right\}$$

It therefore has the advantage of pole boundaries making only small angles with the beam normal, and small magnifications which are equal.

9. Experimental results.

(i) Trial Analyser.

Before the 19 cm. analysing magnet with adjustable boundaries had



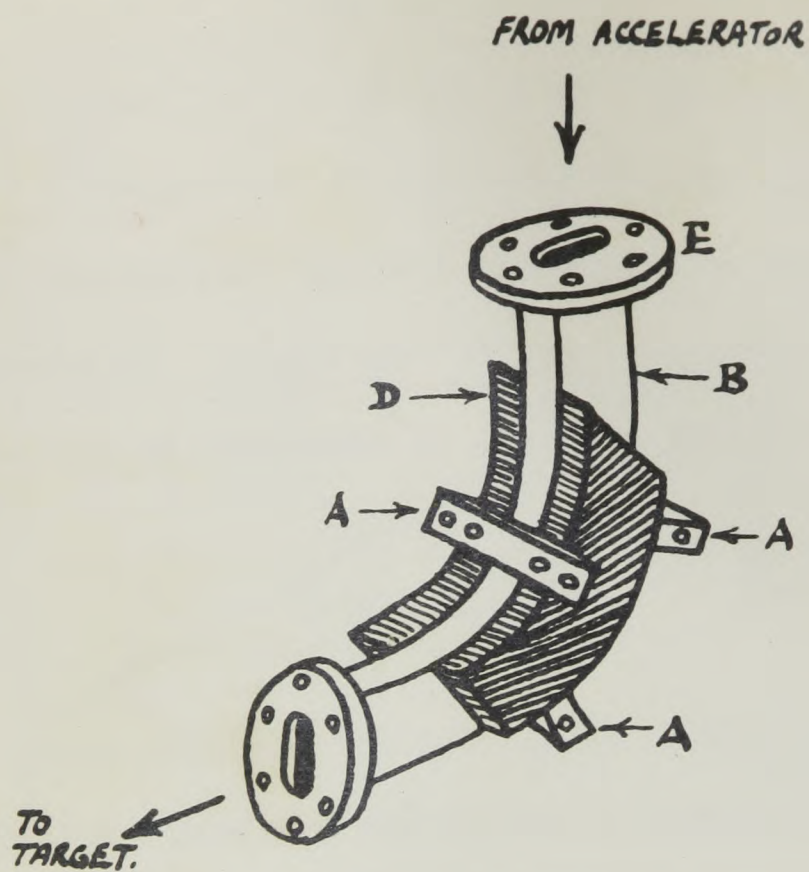


Figure A22: Trial analyser, 10 cm. radius.

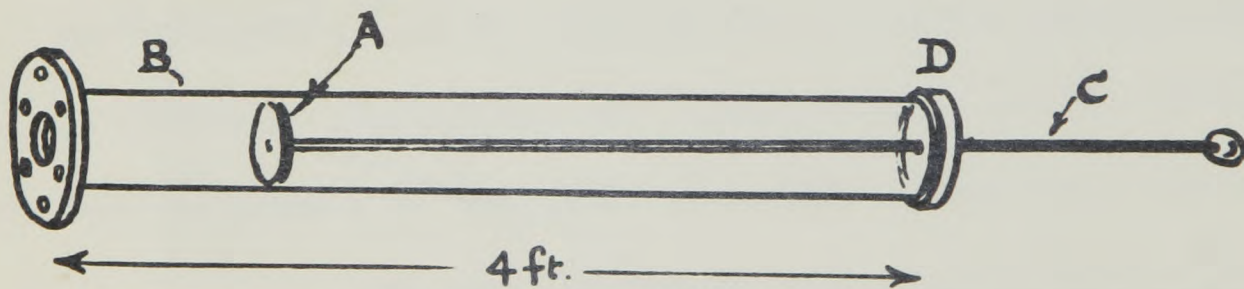


Figure A23: Movable target.

A.

been made a trial analyser was built, based on a small magnet with 4" cores. The form of the vacuum box and pole pieces is shown in figure 22. As described by Price, the magnet had already been fitted with pole pieces to give one-way 90° deflection of deuteron beams of several hundred electron kilovolts energy. The vacuum box B (consisting of bent brass wave-guide) was clamped (by means of the brass blocks A soldered to it) between the pole pieces, so that the shaped mild steel cheeks D could be slipped between the box and the poles and clamped in position by the grub screws shown at A.

Above the flange E there was a shutter which could be moved to cut the beam in the parallel plane, so that a cross-over in the perpendicular plane could be detected by the direction from which the shadow of this shutter moved.

The image and shape of the beam was investigated by using the movable target shown in figure 23. The brass disc target A could be moved within the glass tube B by the brass rod C which passed through a vacuum "O ring" seal in the flange D. Three pairs of pole cheeks were made, the angles being:

$$\begin{array}{lll} \text{(a)} & \alpha_1 = +45^\circ & \text{(b)} \quad \alpha_1 = +45^\circ \quad \text{(c)} \quad \alpha_1 = -45^\circ \\ & \alpha_2 = +55^\circ & \alpha_2 = +58^\circ \quad \alpha_2 = -42^\circ \end{array}$$

It was intended that case (a) should produce an image at $\gamma = 4$ and

(b) and (c) at $\gamma = 2$. Using the figures in table 2 the theoretical characteristics of the poles are

$$\begin{array}{lll} \text{(a)} & t_1 = +1 & \text{(b)} \quad t_1 = +1 \quad \text{(c)} \quad t_1 = -1 \\ & t_2 = +1.4 & t_2 = +1.6 \quad t_2 = -0.9 \\ & \mu = -2.5 & \mu = -1.9 \quad \mu = -0.7 \\ & \gamma = +4 & \gamma = +2 \quad \gamma = +2 \\ & M_{\perp} = +2.7 & M_{\perp} = +2.2 \quad M_{\perp} = -1.17 \\ & M_{\parallel} = -1.35 & M_{\parallel} = -0.75 \quad M_{\parallel} = +9.6 \end{array}$$

A.

The results were disappointing: in cases (a) and (b) the image had a distorted "Y" shape as if parts of the beam nearer the poles had been through a stronger field than the middle parts. It was insufficiently well defined to check values of the magnifications, although as expected these were found to be larger in (a) than in (b). With change of accelerator focussing (i.e. value of μ) the image shape changed from roughly a vertical to a horizontal line in the way expected.

It was later realised that no allowance had been made for the extent of the fringing field by cutting back the faces, so that when the beam was arranged to enter and leave the analyser centrally it had an effective principal radius of curvature greater than 10 cms. by an amount which depended on the angles α . For the angles and pole gap used, the effective radius is about 15 cms. It is easily shown that the path of this beam axis passes near to the inside wall of the wave guides so that it is then in a less uniform part of the field. Thus the severe aberrations of the image actually found should have been expected.

In case (c) no image at all could be formed, only a very diffuse scattered beam emerging from the exit aperture. This was put down to the large value of $M_{\parallel}$, resulting at the exit boundary in a large width of the beam, which consequently was scattered from the vacuum box walls.

Because of the unsatisfactory nature of the injected beam, which was very far from stigmatic, it was decided to leave further tests until a new accelerator column had been built, and the new analysing magnet became at the same time available.

(ii) 19 cm. magnet.

Using the new accelerator column and analysing magnet, a considerable

A.

number of experimental measurements were made. Of these, the most reliable were made by Dr. F.V. Price¹⁶, and are set out here for comparison with theory.

Under various conditions, the beam cross-section at various places was found by using the movable target already described in this section and shown in figure 23. The target was inscribed with a series of circles so that the size of the region of intense fluorescence, where the beam hits the target, could be estimated. By removing the analysing magnet the target tube could be placed vertically beneath the accelerator and the position of the "object" found. Using fixed values of the overall H.T. voltage and probe voltage and for a fixed beam current, this object position is a function only of the focussing voltage V_F , which can thus be used to find the value of μ . V_F and μ are plotted in figure 21.

The H.T. voltage used in this and all the subsequent measurements was 150 KV, and the probe voltage 2.5 KV. Unfortunately, these values produce large spherical aberrations in the lenses, as will be shown in section 12. This was not realised at the time, but the effect could be seen on the target, where in most cases the central bright spot was surrounded by a less intense region. Measurements were made of the size of both parts of the beam, so that it is possible to check the spot sizes calculated both with and without spherical aberration. In this section only the central spot sizes are recorded, but it is worth noting that most of the distortion in the form of a diffuse region of smaller intensity gave a vertical extension to the spot, very little being in the horizontal plane. As will be shown in section 12, this is in accord with the suggestion that the accelerator lenses are responsible for a large spherical aberration.

A.

Measurements were made by setting the entrance boundary angle α_1 , changing the exit angle α_2 in steps of 5° , and recording the spot sizes in the perpendicular and parallel planes at several different image distances. The process was repeated at different focus voltages V_F and for different entrance angles. Throughout, the probe and overall HT voltages, and as far as possible the beam current, were constant. The range of conditions in which measurements were made is shown below.

$\alpha_1 =$	40 , 45 , 50°
($t_1 =$	0.84 , 1.00 , 1.19)
$\alpha_2 =$	35° , 40° , 45° , 50° , 55° , 60° , 65°
($t_2 =$	0.70 , 0.84 , 1.00 , 1.19 , 1.43 , 1.73 , 2.14)
Image dist. =	40 , 50 , 60 , 80 , 100 cms.
($\nu =$	2.1 , 2.6 , 3.2 , 4.2 , 5.3)
$V_F =$	6.5 , 7 , 7.5 , 8 KV
($\mu =$	-5 , -3 , -2 , +1)

The experimental survey was done to find the best focussing circumstances in practice, so that in general a set of conditions chosen from the above will not give coincident foci in the two planes, and a direct comparison with the theoretical curves, for example in figure 16, is not possible. Also the formulae derived from the Gaussian optical analogy do not give the beam cross-section at any place except in the focal plane, as this involves the position and size of aperture stops in the actual accelerator and magnet systems.

It is therefore necessary to calculate the position of the focal planes $\nu_{\perp}$ and $\nu_{\parallel}$ with equations (45) and (46) and then, using equations (52) and (55), to find the magnifications $M_{\perp}$ and $M_{\parallel}$ at these

positions. In many cases the focal plane is found to be well outside the range investigated, or even negative. However where a direct comparison with the calculated values is possible, the agreement is found to be fairly good. As an example, table 3 below shows values obtained with an entrance angle of 45° and for two object distances.

Table 3.

V_F (KV)	μ	α_1	γ_L	M_L		$\gamma_{ }$	$M_{ }$	
				Calc.	Observ.		Calc.	Observ.
7	-3	35	1.25	-0.63	-0.4	-	-	-
		40	1.5	-0.75	-0.6	-	-	-
		45	2.20	-1.00	-0.8	-	-	-
		50	3.2	-1.60	-1.7	-	-	-
		55	-	-	-	4.9	-1.5	-1.3
		60	-	-	-	1.95	-0.6	-0.9
		65	-	-	-	1.05	-0.3	-0.4
7.5	-2	50	1.25	-1.25	-1.2	-	-	-
		55	1.75	-1.75	-1.9	3.3	-1.2	-1.5
		60	3.70	-3.70	-3.1	1.6	-0.6	-1.1
		65	-	-	-	0.96	-0.35	-0.3

For estimating the 'observed' magnification it was assumed that the object diameter was 0.5 cm. as had been found by previous measurement. The poor definition of the image formed on the target did not allow estimates to better than about 0.1 cm (0.2 in M) so that with this possible error observations are seen to be in good quantitative agreement with the calculations. In all cases the sign of the magnification could be checked by obscuring part of the beam in either plane with a shutter placed before the entrance to the magnet.

10. Combined non-uniform and oblique boundary fields.

The ion-optical properties of a non-uniform field which is given oblique boundaries can be deduced approximately from the lens constants of the separate elements of the system. Such a combination of focussing methods introduces complications but will be of use if it can for example provide a smaller magnification at large image distances by bringing the principal planes nearer to the desired position of the image.

The focal lengths and positions of the principal planes of a non-uniform field have been given in section 5. This element must be combined with two thin lenses at the field boundaries which have focal lengths $+t_1$ and $+t_2$ in the parallel plane and $-t_1$ and $-t_2$ in the perpendicular plane. The combined focal length is found by using the formulae of geometrical optics, and the results for the simple case of 90° deflection are:

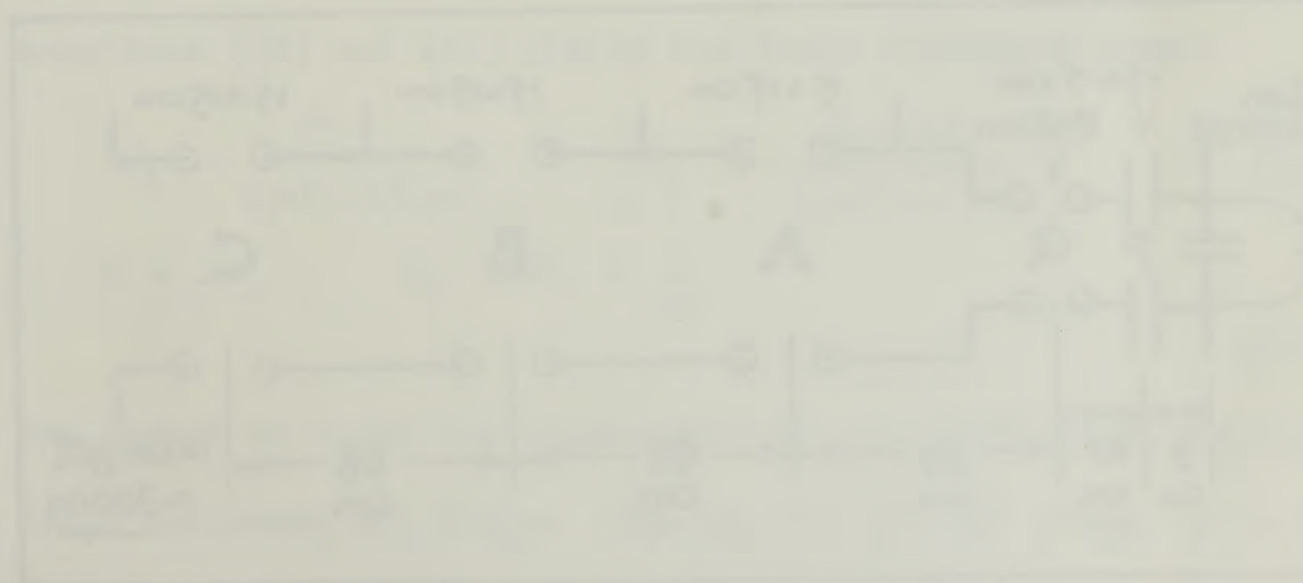
$$f_L = \frac{1}{[(1-n-nt_1)(1+t_2) + t_2(1+t_1)]}$$

$$f_H = \frac{\sqrt{n} \sin P}{\sqrt{n} \sin P (\sqrt{n} \sin P + t_1 \cos P + t_2) + (1 - \cos P)(\sqrt{n} \sin P + t_1 \cos P - t_1) t_2}$$

(where $P = \frac{\pi}{2} \sqrt{n}$)

It is likely that for such a magnetic system the aperture aberrations resulting from distortion of the fringe field by the non-parallel pole faces (an effect not considered above) would be severe. However the optimum values of the variables have not been worked out and the possible advantages not properly assessed.

It is suggested that for this complex case the simplest way of investigating might be to use the light-optical analogy and set up two focussing systems. Each plane would be represented by a pair of thin lenses containing between them a thick lens of appropriate power. It would be an advantage to use lenses of continuously variable power to



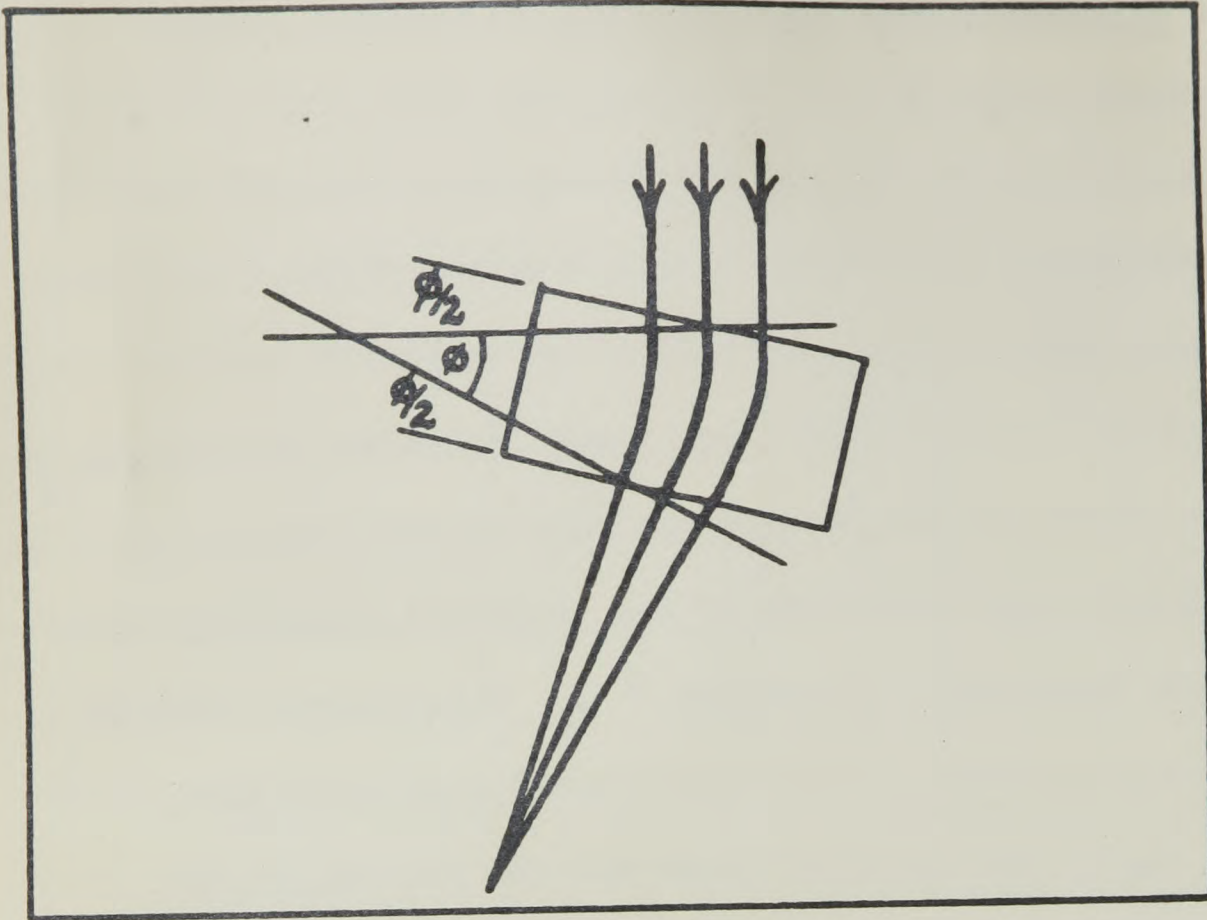


Figure A24: Small angle deflection between parallel boundaries.

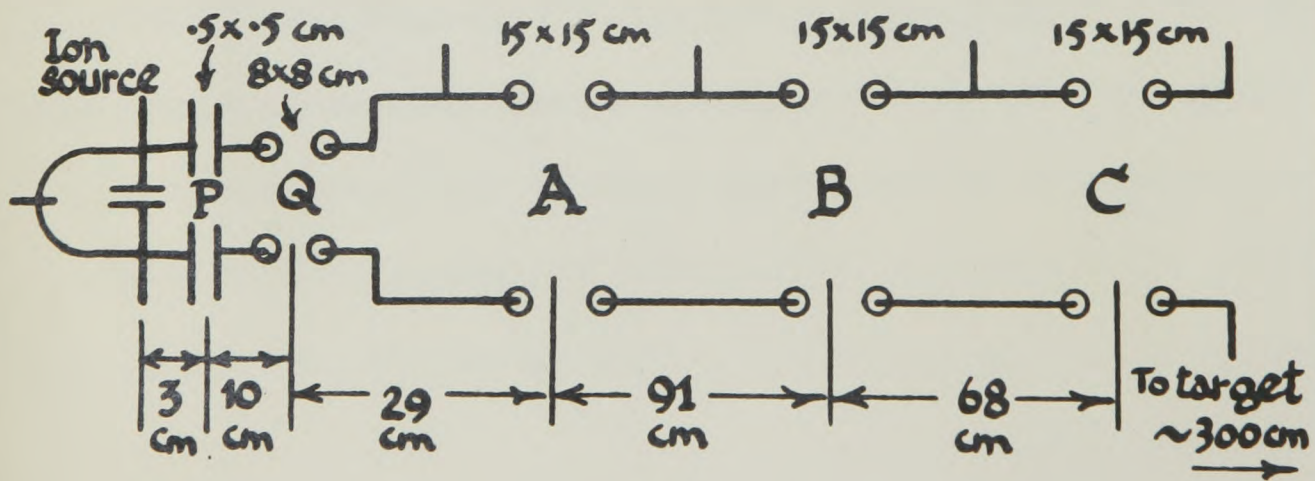


Figure A25: Accelerating lens system.

represent the effect of a change of pole face angle, and in the Appendix a description is given of such a lens built for the purpose.

11. Small Angle Deflection.

It will be shown in the next Section that unless the first image formed by the electrostatic lens system of the accelerator is free from aberrations, a large loss of ion intensity occurs when the beam is deflected through 90° . The intensity can be preserved, despite quite severe aberrations, by using very much smaller deflecting angles.

A particularly useful case is that of a parallel beam passing symmetrically through a parallel sided magnetic field (see figure 24).

Thus $\alpha_1 = \alpha_2 = \phi/2$ ($= \alpha$, say)

and $\tan \phi = \tan 2\alpha = \frac{2t}{1-t^2}$ (58)

If also t is so small that terms in t can be neglected in comparison, then the equations (35) and (40) giving the image distances become

$$v_L = \frac{\mu}{2\mu t - 2t - 1} \quad v_{||} = \frac{\mu + 2t}{2\mu t - 1} \quad (59)$$

When

$$\mu = \infty, \quad v_L = v_{||} = \frac{1}{2t} \quad (60)$$

Thus for the region in which the approximations are valid (i.e. $\phi < 15^\circ$, say) the image distances are large: for example $\phi = 11^\circ$ gives $v = 5$. This image distance is probably much greater than needed, because the radius ρ_0 can now be so much larger whilst still using a small magnet. However because the beam is narrow a large range of working distances is possible.

An analyser of this type has been used in several experiments,

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in particular the one described in Part C. Here a permanent magnet was used, the field being adjusted by means of a mild steel shunt. The angle of deflection ϕ was 11° and the effective radius of curvature about 40 cms.

The incident beam was extracted from a deuterium ion source and probably contained about 80% deuterons. It entered the magnet as a roughly parallel beam about 3 mm diameter, and it was found possible to pass about 50% of the total beam current of about $300 \mu A$ (i.e. about 60% of the deuterons) through the system of 4 mm diameter canals at about 40 cms ($\gamma = 1$) below the magnet.

The reason for the large increase in beam intensity given by 11° deflection over that given by 90° deflection is largely the fact that spherical aberration of the accelerator lens system will have a much smaller effect on the emergent beam in the former case. This subject is discussed in some detail in the next section.

12. Aberrations.

(i) Aperture aberrations.

Following the light-optical analogy we can expect in our combined electrostatic and magnetic focussing system all the aberrations normally associated with an uncorrected optical system consisting of centered spherical surfaces followed by two thick cylindrical lenses with their axes orthogonal.

Thus we can expect chromatic aberration and the related chromatic difference of magnification if the ions from the source have a spread of velocities. The primary effect is a shift of the beam in the perpendic-

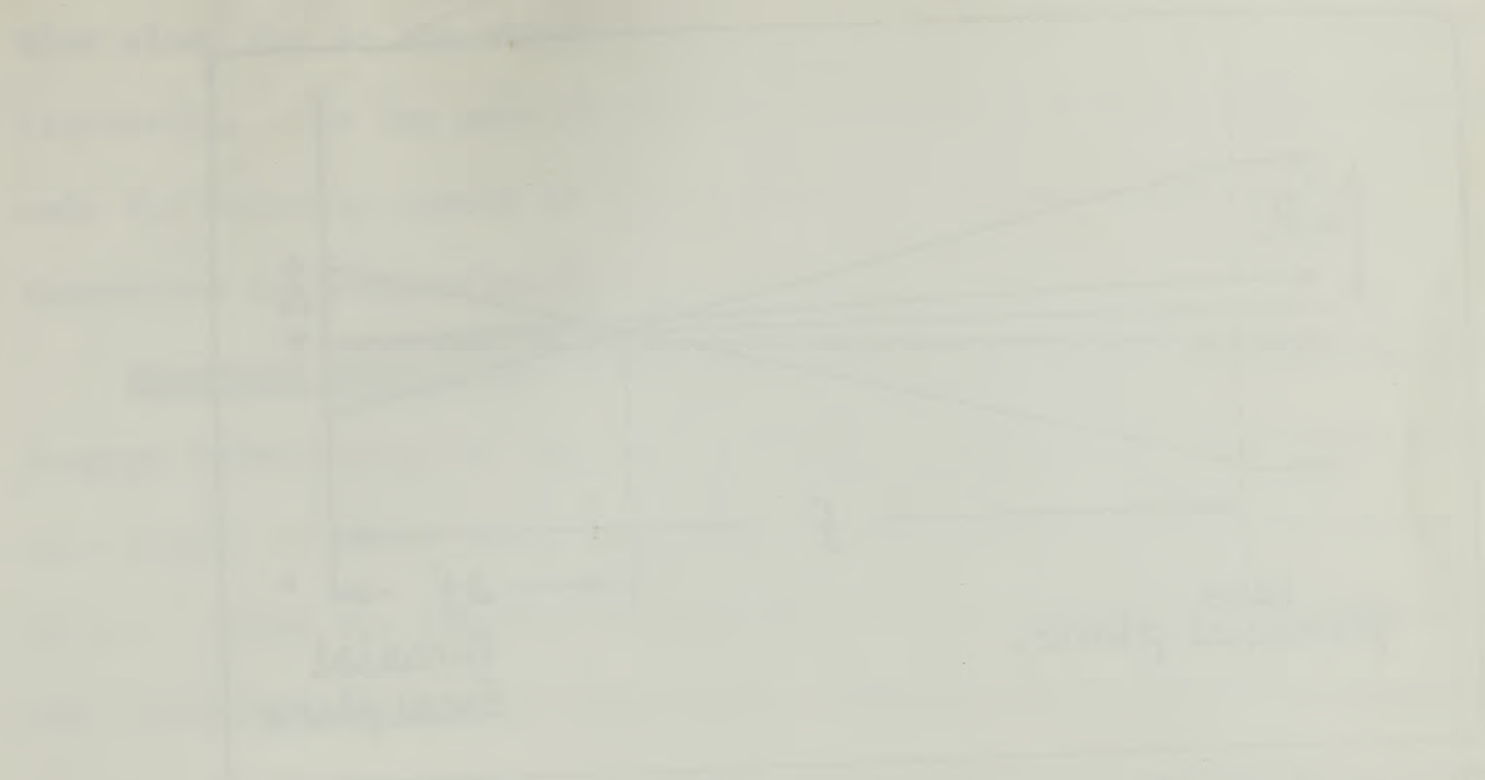


Figure 1: The effect of a shift in the right position.



Figure 2: The effect of a shift in the right position.

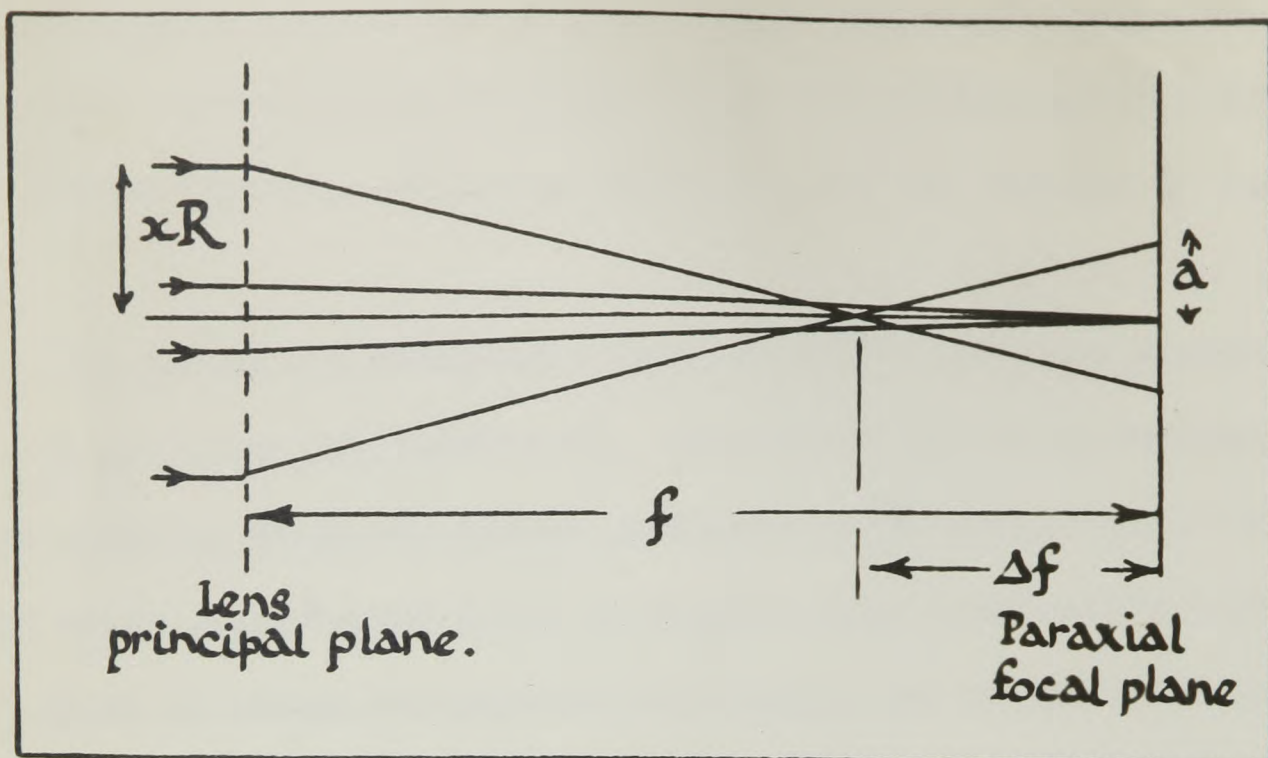


Figure A26: Spherical aberration.

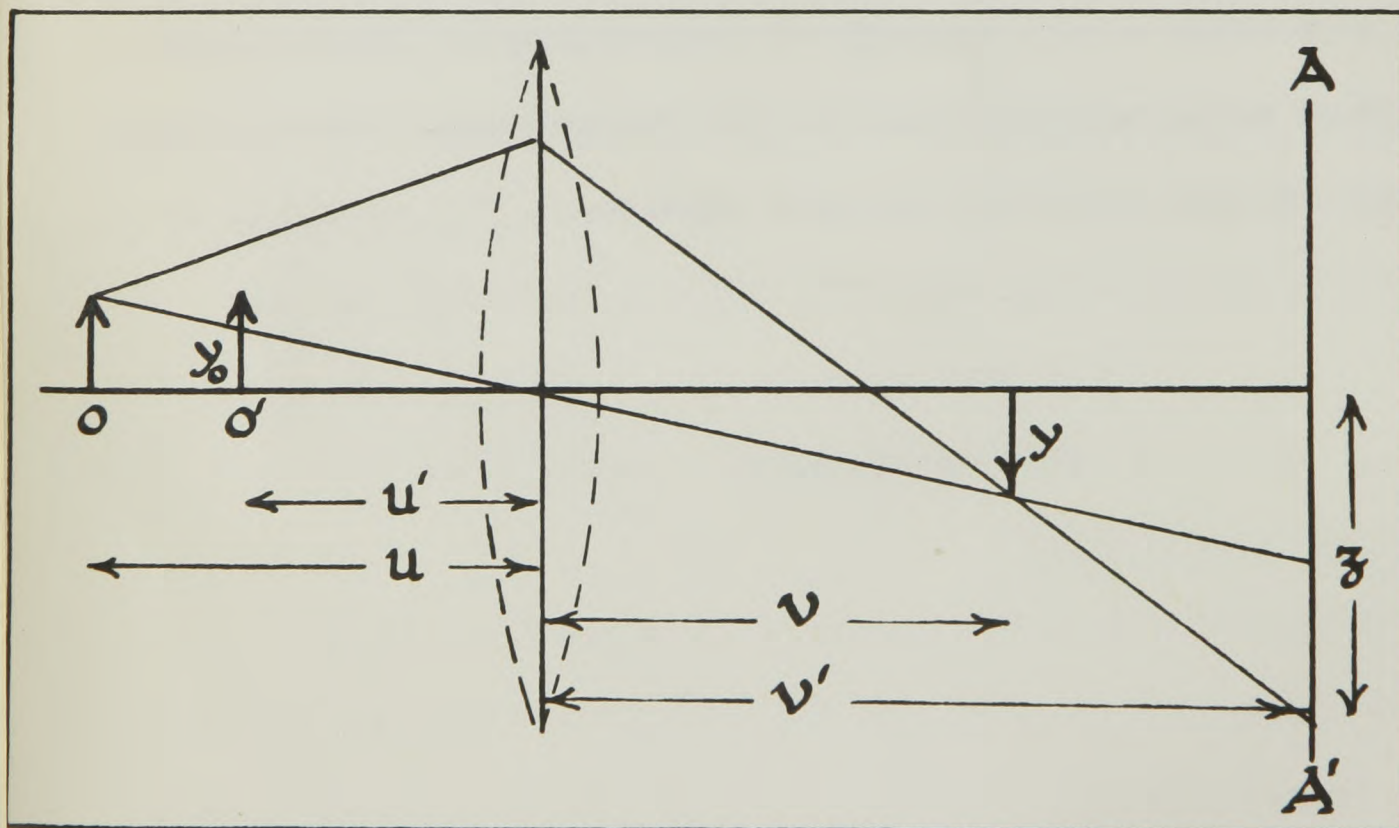


Figure A27: The effect of a shift of the object position.

A.

ular plane due to the magnetic lens, and this was treated in section 4 as dispersion. In the case of the 500 KV accelerator with which tests were made the velocity spread is very small so that the effect of the aberration is insignificant.

Spherical aberration in the electrostatic lenses, enhanced by space-charge defocussing, is the most important effect, and this gives rise to a spread of first image positions - i.e. uncertainty in the value of μ . Since the inherent astigmatism of the magnetic lens system has been corrected (by use of the results obtained in the previous sections) for only one value of μ , the gross effect will be an astigmatic image.

Spherical aberration cannot easily, if at all, be eliminated from electrostatic lenses, and indeed in the design of the 500 kv accelerator very little attention had been given to reducing it. Theoretical figures given by Price¹ show that the beam has its largest cross-section within the intermediate lenses A and B (figure 25), where it may occupy fractions of the lens diameter ranging from one-half at the lowest overall voltages (~ 50 kV), to one-seventh at the highest (~ 550 kV). In the aperture lens P the fractional diameter is more than one-half at all voltages.

A specific value for the aberration of these lenses has not been found in the literature, but it can be assumed that for a weak immersion lens and a parallel incident beam $\Delta f \approx 0.3 x^3$ ^{14,15} where a is the beam radius in the paraxial focal plane and x is the fraction of the lens diameter R occupied by the beam (figure 26). This gives for the fractional change in focal length,

$$\frac{\Delta f}{f} = \frac{a}{xR} = 0.3 \frac{x^2}{R} \quad (\text{when } \frac{\Delta f}{f} \ll 1) \quad (61)$$

All the five lenses in the accelerator may have an appreciable effect on the aberration. Not only will each lens contribute its own aberration, but also its longitudinal magnification, if greater than unity, will enhance the contribution of all the lenses which precede it. It is necessary to consider the cumulative effect of spherical aberration in such a system of lenses.

If, in the usual notation, u and v are the object and image distances measured from their principal planes and f is the focal length in the image space, then for each lens

$$\frac{1}{v} - \frac{1}{nu} = \frac{1}{f} \quad (62)$$

where $n^2 = \frac{V'}{V}$, the ratio of voltages on the two sides of the lens.

The shift of image position, at a single lens, due to a change from paraxial to marginal rays will be

$$dv = \frac{\partial v}{\partial f} df + \frac{\partial v}{\partial u} du \quad (63)$$

$$\text{Now } \frac{\partial v}{\partial u} = \frac{nf^2}{(nu+f)^2} \quad ; \quad \frac{\partial v}{\partial f} = \frac{n^2 u^2}{(nu+f)^2} \quad (64)$$

$$\text{so that } dv = \frac{nf^2 du + n^2 u^2 df}{(nu+f)^2} \quad (65)$$

For two adjacent lenses a and b, $du_b = dv_a$ and the complete expression for e.g. three lenses is

$$dv_3 = \frac{n_3 f_3^2 \left[n_2 f_2^2 \left[\frac{n_1 f_1^2 du_1 + n_1^2 u_1^2 df_1}{(n_1 u_1 + f_1)^2} \right] + n_2^2 u_2^2 df_2 \right] + n_3^2 u_3^2 df_3}{(n_3 u_3 + f_3)^2} \quad (66)$$

Multiplying out and simplifying to include quantities already known for the electron lenses, we have finally for a system of l lenses:

$$dv_e = NM^2 du_1 + \frac{N}{n_1} \left(\frac{M}{m_1} \right)^2 (m_1 - 1)^2 df_1 + \frac{N}{n_1 m_2} \left(\frac{M}{m_1 m_2} \right)^2 (m_2 - 1)^2 df_2 + \dots + (m_l - 1)^2 df_l \quad (67)$$

A.

In this expression

$$N = n_1 \times n_2 \times \dots \times n_l$$

$$M = m_1 \times m_2 \times \dots \times m_l$$

The values of n , and the magnifications m , are known and f can be found by extrapolating to low voltage ratios the figures given by Terman.

$\frac{df}{f}$ is found by putting the known values of x into the approximate expression (61) above.

For an object with lateral extension only, $du_1 = 0$ and the first term in (67) is zero. This is assumed to be the case although in fact space-charge shaping of the ion beam within the canal source may subtract a small amount from the aperture aberration. (It should be noted that an ion source and extracting canal of appropriate design might prove to be a way of correcting a lens system for aberrations).

On substituting values it immediately becomes clear that in most cases the second term is the largest so that the major aberration is that of the first lens P and the size of the resulting defect is due to the magnifications of the subsequent lenses.

Some values for the ultimate spread of image positions are shown in table 4, for several different voltages (at which different combinations of the lenses are used),

Table 4

Energy, KV	Lenses used	$\Delta \mu (= \frac{\Delta v_c}{p_o})$
52.5	A B C	0.15
100	P A B C	0.25
153	P A B C	4.5
540	P Q A B C	1.5

A.

If the reasonable assumption is made that the ions emerge from the canal source with the same intensity throughout any cross-section of the cone in which they are confined, it can be shown that the uncertainty in image position will show itself as a line with longitudinal extension of uniform intensity. For the number of ions entering an annular region of a lens, of radius xR and width Rdx will be proportional to the area of the annulus; i.e. $dn = k_1 2\pi R^2 x dx$. All these ions will re-cross the axis within a small region $d(\Delta v)$ at a distance Δv from the paraxial image position, and from the expression (61) above, (since for a given lens and object distance $dv/df = \text{constant}$),

$$\frac{d(\Delta v)}{dx} = k_2 \cdot \frac{0.6 f \cdot x}{R}$$

so that the linear density of ions on the axis at the image is

$$\frac{dn}{d(\Delta v)} = \frac{k_1 2\pi R^3}{0.6 f} = \text{constant.}$$

A simple optical analogy shows that this uncertainty in the value of μ has a very considerable effect on the size and shape of the image. In figure 27 where to avoid confusion a simple converging lens is shown, y_0 is the height of the object, assumed to be constant at whatever distance u it is put. Unprimed quantities refer to rays which were marginal in the system forming y_0 and primed quantities to the paraxial rays. AA' is the paraxial image plane for an object at O' . When the object is shifted to O the spot size at AA' is given by

$$z = \gamma(v' - v) + y \cdot \frac{v'}{v} \quad (65)$$

γ is the angle of convergence of rays to points on the image and by the Lagrange relation it is given by $\gamma = \beta/m$.

Further,

$$y = y_0 m$$

so that in terms of quantities which



Figure 1: Values of y and x in both planes for the values of x and y for the 2D plane.



Figure 2: Values of y and x in both planes for the values of x and y for the 2D plane.

The values of x and y in both planes for the values of x and y for the 2D plane are given in the table below.

x	y
0	1
1	2
2	3
3	4
4	5
5	6
6	7
7	8
8	9
9	10
10	11

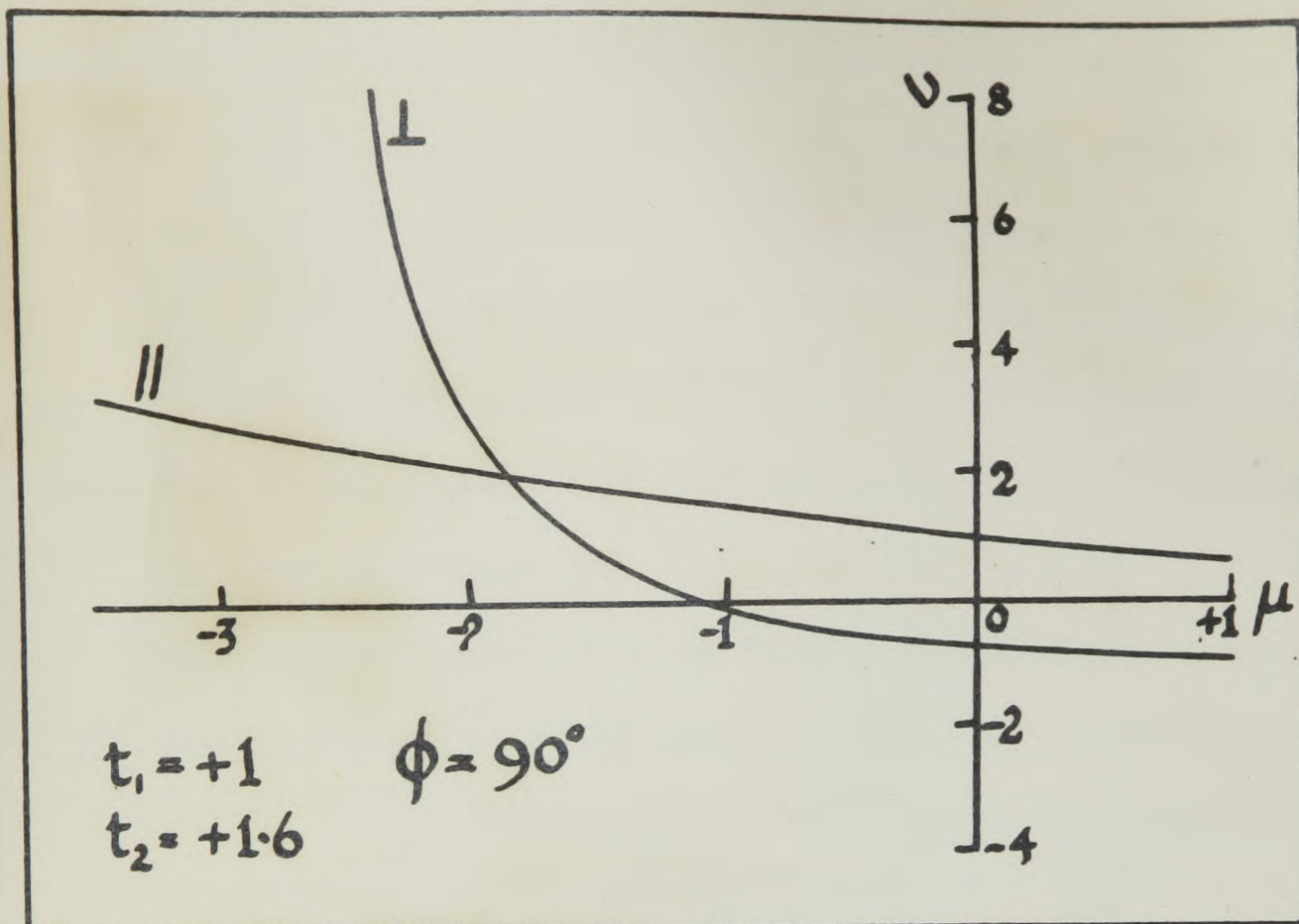


Figure A28: Values of μ and v in both planes for fixed values of t_1 and t_2 ; $\phi = 90^\circ$

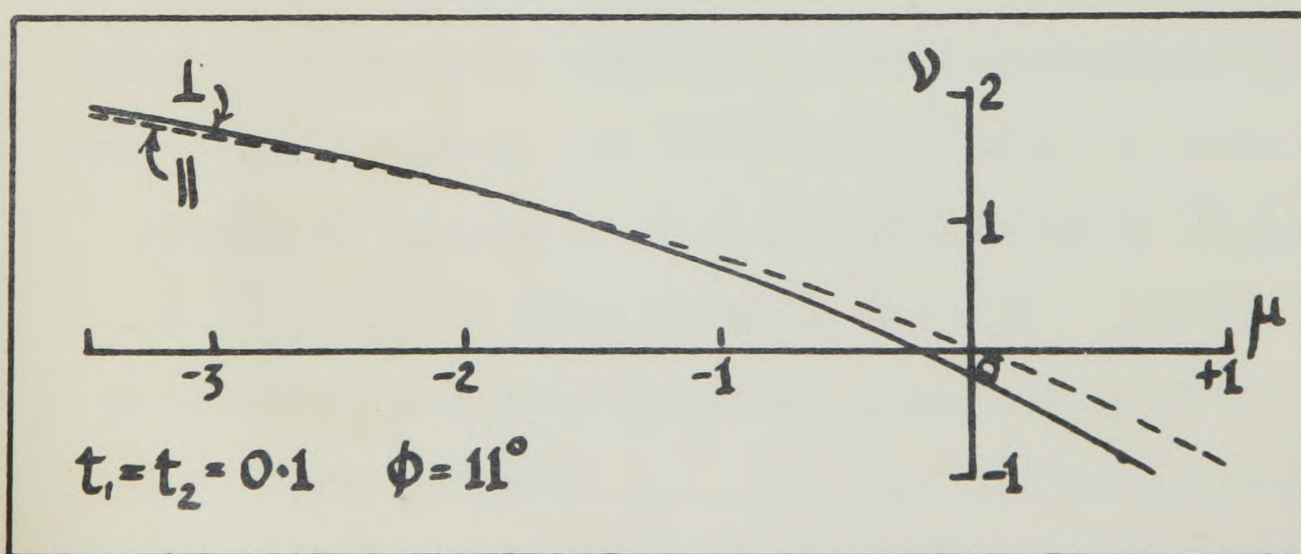


Figure A29: Values of μ and v in both planes for fixed values of t_1 and t_2 ; $\phi = 11^\circ$.

A.

are known for the ion beam, and when $\phi = \frac{\pi}{2}$,

$$z = \frac{\beta p_0}{M} (\nu' - \nu) + y_0 M \frac{\nu' + 1}{\nu + 1} \quad (69)$$

Figure 28 shows the way in which ν can be expected to change with μ when the values of t_1 and t_2 are fixed. In this case $t_1 = -t_1$ and $t_2 = +1.6$ (values appropriate to a coincident focus in both planes at $\nu = 2$ when $\mu = 1.91$). It can be seen at once that a change in μ of about 1 reduces $\nu_{\perp}$ to zero although it makes only a small difference to $\nu_{\parallel}$.

We can substitute values in equation (69). The angle of convergence on the object, β , depends on the accelerating voltages, but at about 100 kv overall, $\beta \approx \frac{1}{100}$. The object size is 0.5 cm. diameter. Considering first the ^{perpendicular} ~~parallel~~ plane, for $\nu_{\perp} = 0$ and $\mu = -0.9$, $M_{\perp} = 1$

Hence

$$z_{\perp} = \frac{19 \times 2.1}{100 \times 1} + 0.5 \times 1 \times \frac{(2+1)}{(-0.1+1)} = 2.07 \quad (70)$$

In the parallel plane the same change of μ brings $\nu_{\parallel}$ from +2 to +1.5.

For $\mu = -1$ and $\nu_{\parallel} = +1.5$, $M_{\parallel} = -0.7$

Hence $z_{\parallel} = 0.57$

These values of the spot dimensions should be compared with those of the paraxial image. Using the magnifications as given in table 3, the paraxial image has a size 1.1×0.38 cm.

Thus the ratio of areas of images with and without aberration is

$$\frac{2.07 \times 0.57}{1.1 \times 0.38} = 2.8$$

The effect changes slowly as the image distance ν is increased, as shown in table 5, which is calculated for a constant uncertainty $\Delta\mu = 1$ a constant object diameter $y_0 = 0.5$ cm, and a constant angle of

A.

convergence

$$\beta = 0.01.$$

Table 5.

ν	Ideal Image Size (cm)		Defective Image Size (cm)		Ratio of Areas Defective/ideal
	$\perp$	$\parallel$	$\perp$	$\parallel$	
2	1.10	0.33	2.07	0.57	2.8
4	1.35	0.68	2.44	0.97	2.6
5	1.47	0.80	2.56	1.17	2.4
10	2.20	1.45	4.46	2.13	3.0

The areas of the images formed under ideal and defective conditions are those of ellipses with the major and minor axes given in the table above. It is possible here to check the results by experiment. In one arrangement a 4 mm. canal was placed at about 70 cms. from the exit of the magnet and the pole boundaries were set at 45° ($t_1 = +1$) and 55° ($t_2 = 1.4$), values appropriate to an ideal image formation at $\nu = 4$. Although a large target placed at $\nu = 4$ collected 250 μ A of beam current at 150 KV, the most which passed through the canal (at roughly the same distance) was 10 μ A, or 4%. According to the above results the spot area might be 1.86 cm², so that through a 4 mm. canal, with area 0.125 cm², we could in any case expect only about $6\frac{1}{2}\%$. The agreement in this example, and in several other similar direct measurements, is fairly good but indicates that other aberrations also are probably having an effect.

When spherical aberration is present, the advantages of small angle deflection (with large ρ_0) is very apparent. Firstly, since the radius of curvature ρ_0 is so much greater, a given shift in position of the

A.

electrostatic lens image corresponds to a very much smaller fraction of ρ_0 ; and secondly this given shift results in a much smaller change of image position because for small angle deflection the longitudinal magnifications are smaller.

For comparison with the 90° case, the variation of ν with change of μ is shown in figure 29 for the case $t_1 = t_2 = 0.1$ (11° deflection). It can be seen that the value of $\frac{d\nu}{d\mu}$, which is the longitudinal magnification, is comparatively small and very nearly equal in the two planes over a wide range of μ . Aperture aberration in the magnetic lens will produce image defects in both planes. In the perpendicular plane one result is the formation of an asymmetrical image which will be further considered from the point of view of space charge effects. A second defect is due to the different angles with which the ions cross the field boundary, and the sort of corrections which can be made by shaping the boundaries (to achieve "second order" focussing) have been discussed by Kerwin.¹⁷

Focussing in the parallel plane will be affected for the same reason, since then the values of t_1 and t_2 may not be the same for all ions in the beam.

A further effect in both planes is distortion due to non-uniformity of the field near the edges. At the entrance and exit boundaries corrections have already been applied but within the field the beam should preferably take only a small fraction (say $\frac{1}{3}$) of the pole gap and width, unless correcting "lips" are provided on the magnet poles to flatten the field-lines in the important region.

These aperture aberrations will however produce only a small image distortion compared with that due to the aperture aberration of the

...the latter is given by the formula...

10.7. Theorem 10.7.1

Let $\mathcal{A}$ be a $\mathcal{C}^*$ -algebra and let $\mathcal{B}$ be a $\mathcal{C}^*$ -subalgebra of $\mathcal{A}$. Then the following conditions are equivalent:

(i) $\mathcal{B}$ is a $\mathcal{C}^*$ -ideal of $\mathcal{A}$.

(ii) $\mathcal{B}$ is a $\mathcal{C}^*$ -ideal of $\mathcal{A}$.

(iii) $\mathcal{B}$ is a $\mathcal{C}^*$ -ideal of $\mathcal{A}$.

Proof. (i) $\Rightarrow$ (ii). Let $\mathcal{B}$ be a $\mathcal{C}^*$ -ideal of $\mathcal{A}$. Then $\mathcal{B}$ is a $\mathcal{C}^*$ -ideal of $\mathcal{A}$. (ii) $\Rightarrow$ (i). Let $\mathcal{B}$ be a $\mathcal{C}^*$ -ideal of $\mathcal{A}$. Then $\mathcal{B}$ is a $\mathcal{C}^*$ -ideal of $\mathcal{A}$. (iii) $\Rightarrow$ (i). Let $\mathcal{B}$ be a $\mathcal{C}^*$ -ideal of $\mathcal{A}$. Then $\mathcal{B}$ is a $\mathcal{C}^*$ -ideal of $\mathcal{A}$.

Figure 10.7.1: A diagram illustrating the relationship between $\mathcal{A}$ and $\mathcal{B}$.

...the latter is given by the formula...

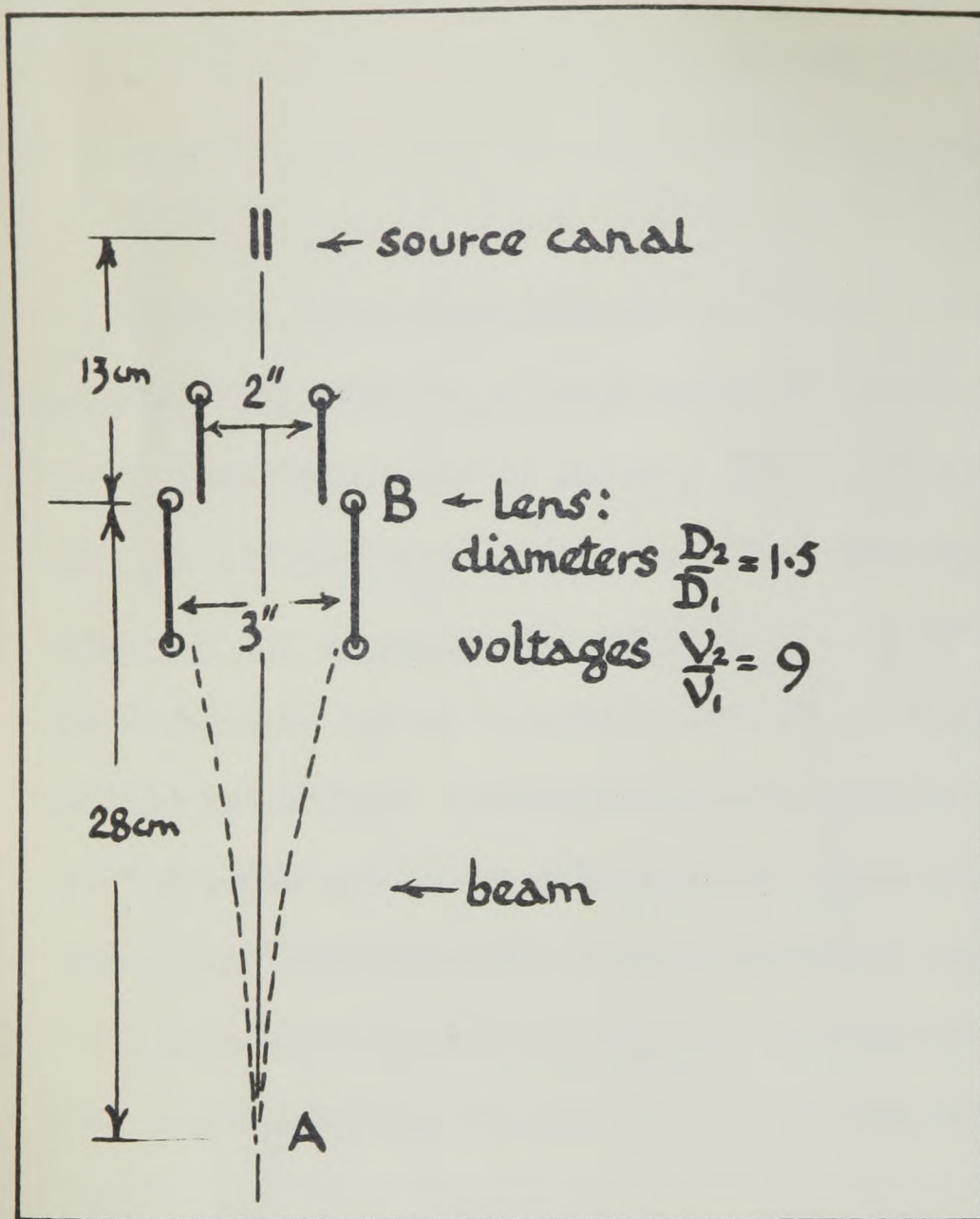


Figure A30: 50 KV. accelerating lens.

accelerator lenses, and for this reason are not worth correcting unless the latter is greatly reduced first.

(ii) Space charge effects.

At small energies - i.e. about 50 kv, the effects of space charge may become very noticable. Of the three possible effects of space charge, neutralisation by secondary electrons will generally be small and the constricting effect of the interaction of each ion with the magnetic field of the others is negligible, so that the main effect is a defocussing one due to mutual repulsion.

The method of calculating the effect has been given by Bedford who, following Watson gives solutions for the case where the ion streamlines do not cross. The general solution in any other case is presumably too complex, yet the accelerated ion beams, as has been shown, are far from stigmatic. It seemed possible that when the minimum beam diameter due to space charge repulsion alone is about the same size as the minimum beam diameter due to spherical aberration alone, deviation of ions through large angles might result.

This was thought to be the explanation of the effects which had been observed in a simple accelerating lens used by Dr. P.F.D. Shaw to provide deuteron beams with energies up to 50 kv. The two-cylinder lens shape was that shown in figure 30, where a typical outline of the emergent beam is also shown. When the beam just above the point A was intercepted by a target, currents of about 100 μ A could be collected. When the point A was raised above the target by increasing the focussing power of the lens the current quickly dropped to half its former value; and below A the clear outline of the beam could no longer be seen. The general shape of the beam and its disappearance at A had been put down to space-charge

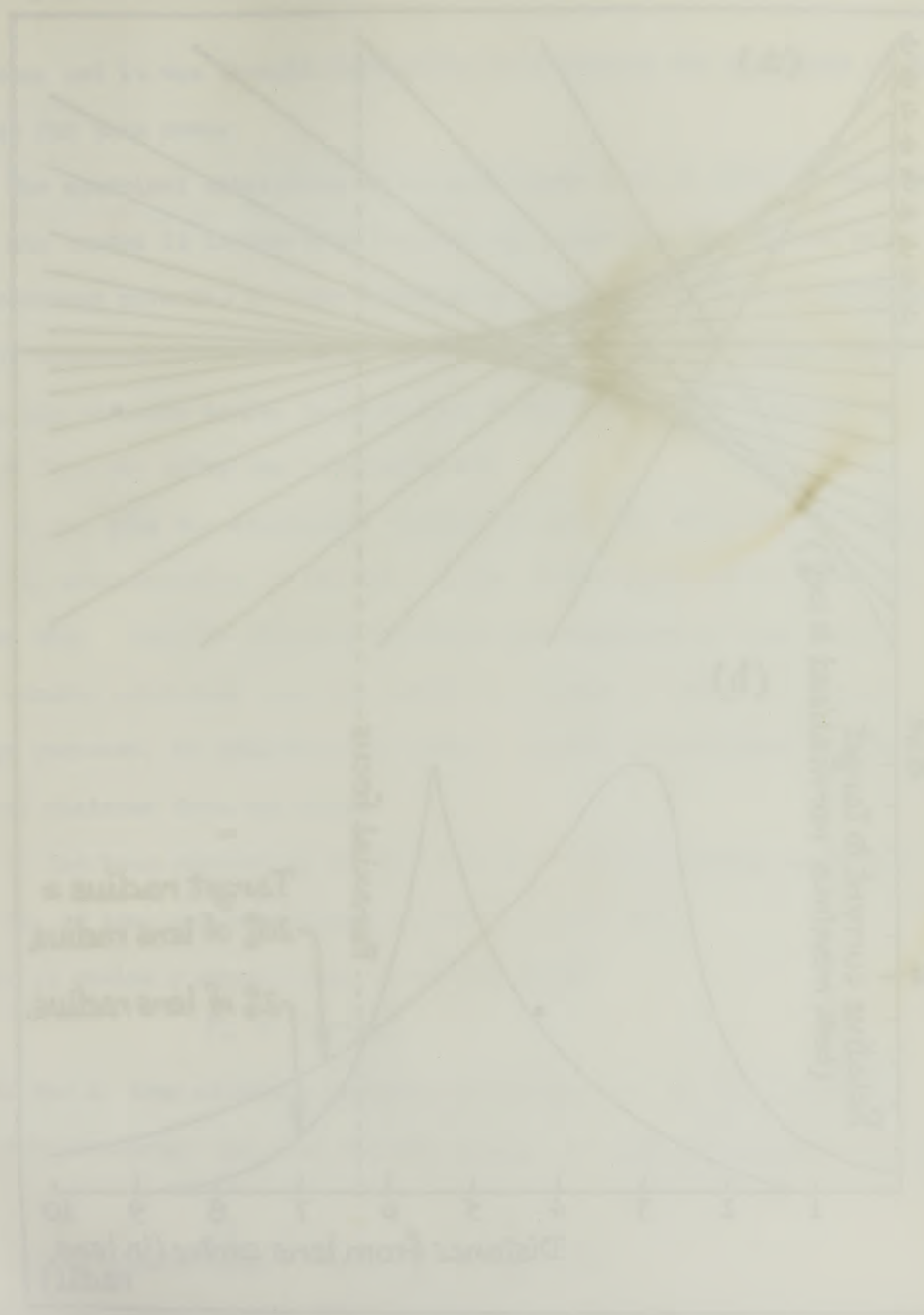


Figure 1: (a) Absorption pattern of a two-point source. (b) Curves plotted for large and small targets. (c) Curves plotted for large and small targets. (d) Curves plotted for large and small targets.

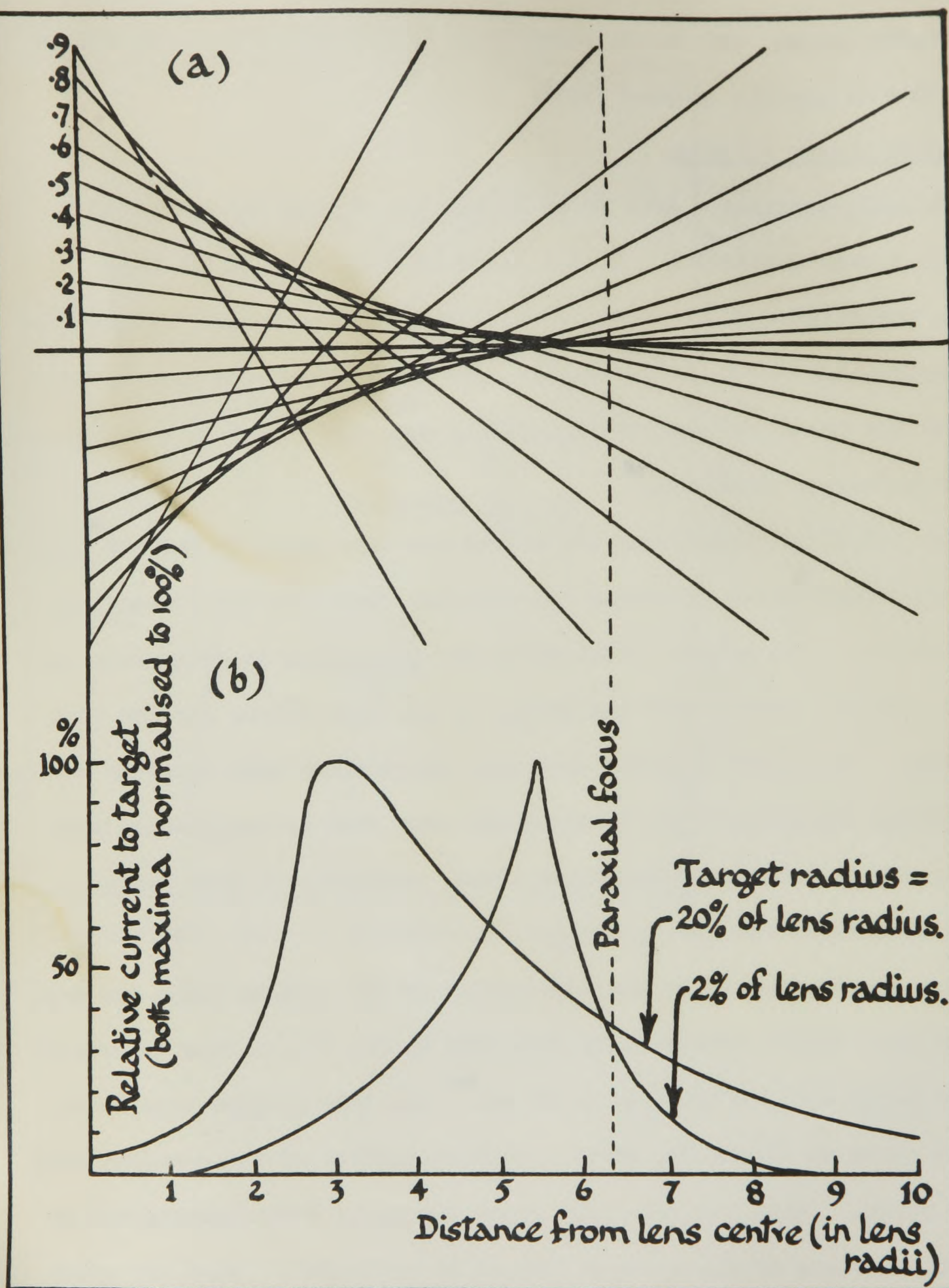


Figure A31: (a) Abberation pattern of a two-cylinder lens.
 (b) Currents collected by large and small targets.

A.

effects, and it was thought worthwhile to calculate the magnitude of the effect for this case.

The spherical aberration of an equivalent lens is given by Coslett.¹⁵ From his curves it is possible to draw the paths (in the absence of space-charge effects) of ions focussed by parts of the lens at different radial distances from the axis: the pattern is shown in figure 31(a) where the ordinate scale, representing fractional radial distance up to 0.9 from the axis, has been expanded.

We can make the reasonable assumption that the intensity is uniform over the cross-section of the beam at the middle plane of the lens (B, figure 30). Thus in dividing the beam cross-section at this point into annular positions each ray marked in figure 31 must, for space-charge purposes, be considered to carry a charge proportional to its initial distance from the axis.

If the beam current is $100 \mu A$ and the deuteron energy is 50 Kev, then

$$\text{No. of ions per unit length of beam} = 2.8 \times 10^6.$$

An ion at radius r experiences an outward force

$$F_r = 2\pi e^2 \frac{n}{r}$$

due to the n ions within a cylinder of radius r . If the beam has cylindrical symmetry the ions outside radius r make no contribution to F_r .

$$\therefore \text{Radial acceleration } a = \frac{F_r}{m} = \frac{2e^2}{m} \left(\frac{n}{r} \right)$$

Starting in a plane remote from the paraxial focal plane, a step-by-step calculation was done to find the positions of each group of ions at 1 cm. intervals. At each point the change of radial distance

$$\Delta r = \frac{v_r}{v_z} + \frac{a}{2v_z^2} \quad (\text{where } v_z = 2.25 \times 10^8 \text{ cm/sec})$$

and the change of radial velocity

$$\Delta v_r = \frac{a}{v_z}$$

A.

were found.

Thus the paths of ions through a fine focus in the presence of space-charge could be followed. Contrary to initial expectation, there was no evidence for possible deflection through large angles.

However it is possible to explain the observed effects quite simply by spherical aberration alone. The pattern of figure 31 (a) gives an indication of the shape of the beam which will be seen in side view, since the density of lines drawn is a rough measure of the ionization intensity in profile. This gives the reason for both the curved outline and the apparent disappearance at A.

By plotting the current falling on a target of considerable size as a function of position along the beam, it can be seen (figure 31(b)) that the maximum lies nearer the lens than the apparent (paraxial) focus A. The position of this maximum is where the envelope giving the beam outline has the radius of the target. Thus with a smaller target the maximum current will be taken when this is nearer to the paraxial focus. A very small target collects currents proportional to the intensity of the "central spot" by which such a beam is commonly focussed, and the very sharp maximum is also shown. It is possible to explain in this way the large changes in apparent beam current which were found either side of the paraxial focus.

It appears, then, that in the presence of considerable spherical aberration the effects of space-charge repulsion at the focus will not be important at even low voltages.

Within the magnet the ion beam will cross over in one or other plane: when t_1 and t_2 are positive, the crossover is in the parallel plane; and

A.

when they are negative, it is in the perpendicular plane. From the point of view of space-charge spreading the crossover in the perpendicular plane is better, since at no place does the beam come to an intermediate line focus. The intensity distribution in this crossover can easily be found: in the absence of space-charge it would be proportional to $\sqrt{\frac{P_0}{y}}$ where y is the distance, measured along a radius of curvature of the beam, from the beam axis. Thus there must be an accumulation of charge on the axis (where $y = 0$) which will lead to spreading of the beam, although here also this is not likely to be serious.

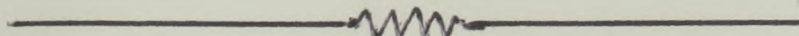
13. Conclusion.

The investigation of the focussing properties of analysing fields, outlined in the sections above, has shown that in the ideal case of an aberration-free accelerating system it is possible to produce small images at large distances after deflection through 90° . Unfortunately, however, the types of field which must be chosen to give this result are suitable only for use with a closely stigmatic incident beam so that with a multiple lens accelerator of the type at present used with the Phillip's generators in the Clarendon Laboratory, large losses of beam intensity are unavoidable. For experiments where large beam currents are important, deflection through a small angle must be adopted, for then the image size is only slightly affected by quite severe aberrations. This is the reason for using 11° deflection in the experiment to be described in Part C.

90° Deflection using oblique boundaries should however prove to be of greater use eventually if a new accelerator tube for the 1 Mev. set

A.

can be made with much reduced aberrations. The method is presumably more applicable to uniform field accelerating tubes of the type common in van de Graaf accelerators and it is understood that a magnet with oblique boundaries made at A.E.R.E., Harwell, is now being used on such a machine there.



PART II.

The Design of Magnets for Ion Accelerators.

PART B.

The Design of Magnets for Ion Analysers.

Figure 81: Double quadrant plot face

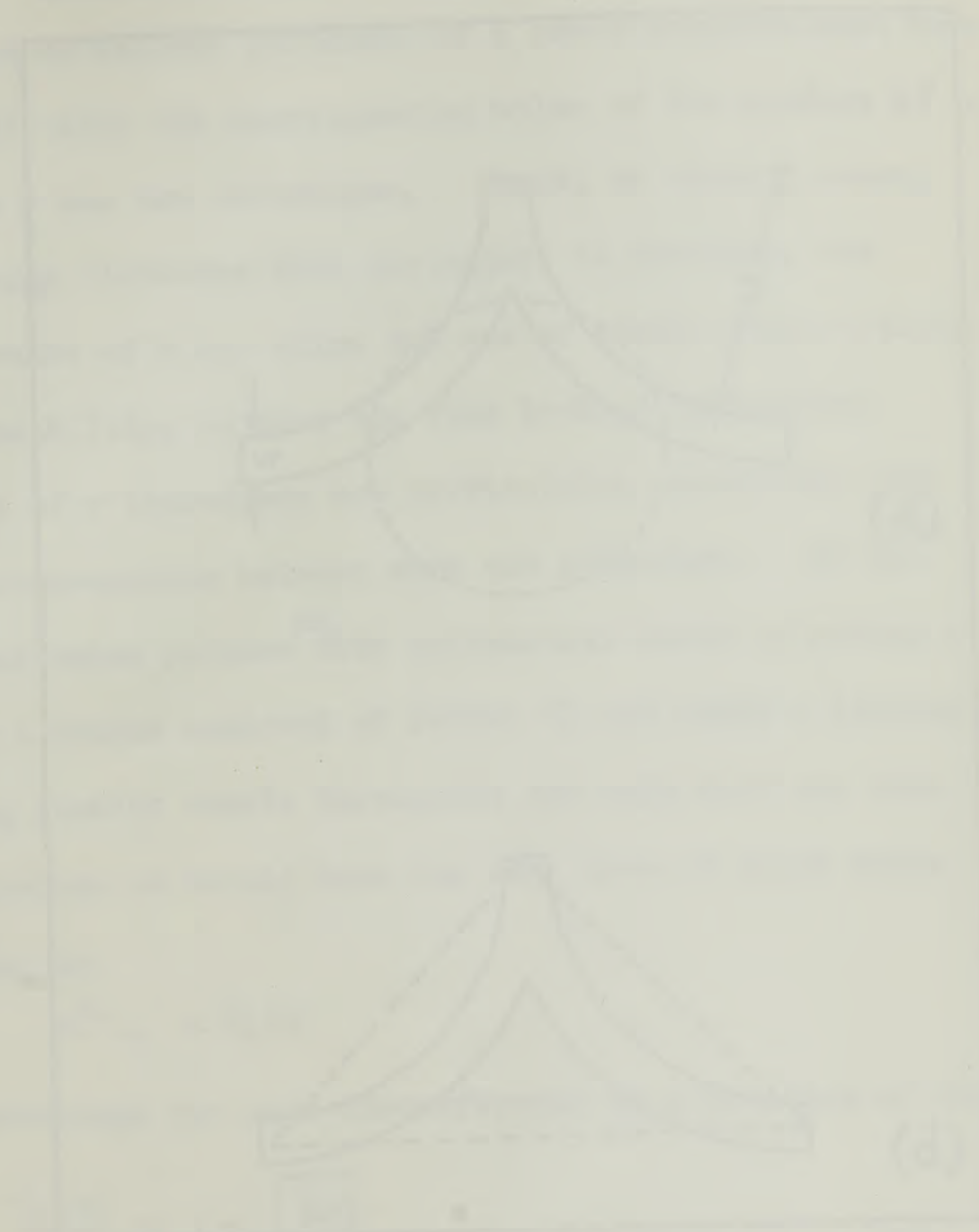


Figure 81: Double quadrant plot face
based on circles and triangles
one of the same size.

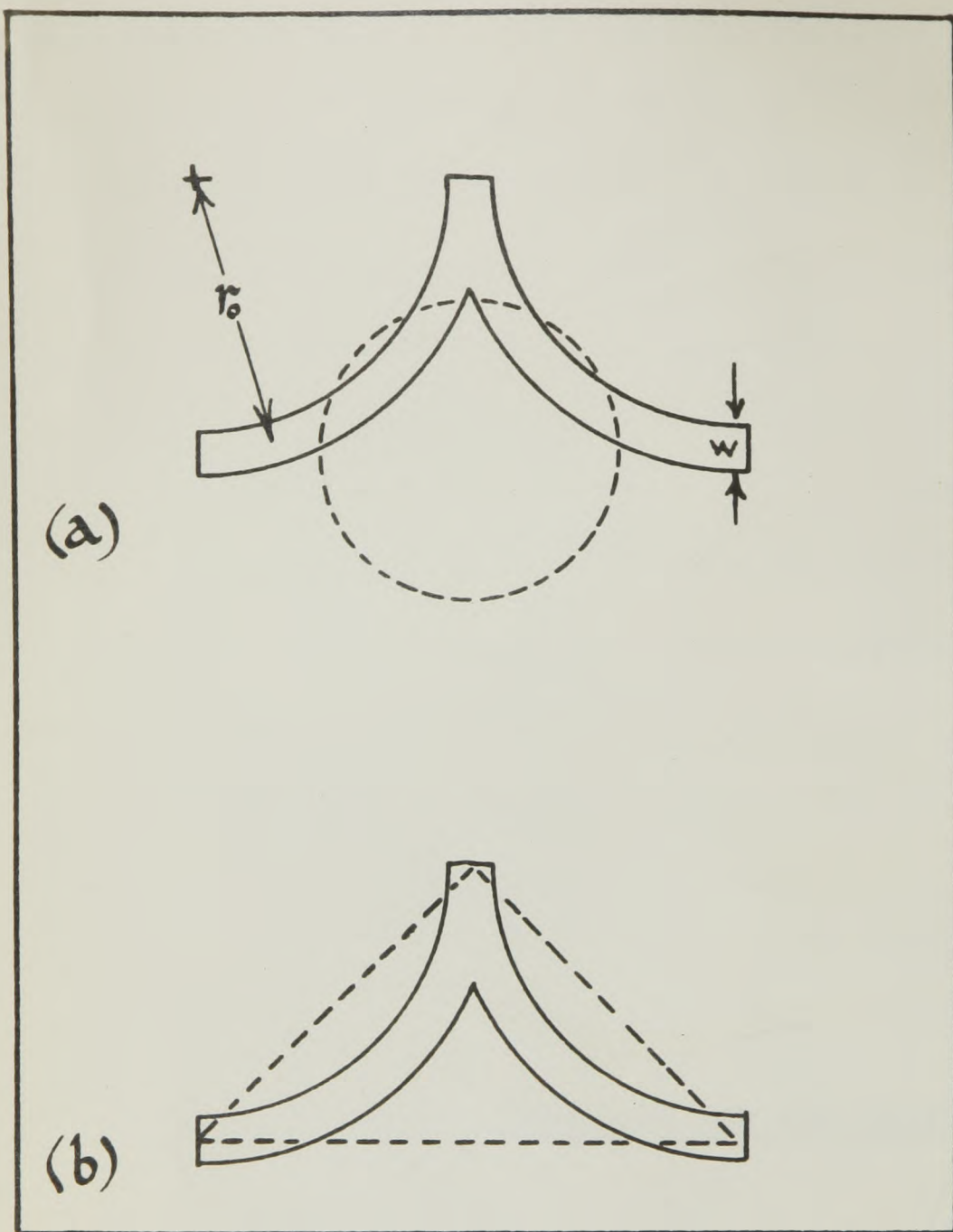


Figure B1 : Double quadrant pole-face based on circular and triangular cores of the same area.

B.

1. General Remarks.

A magnet designed to deflect particles of a given momentum must have a pole-face which will allow the corresponding value of the product $B \rho$. The choice of a large ρ has two advantages. First, as already shown, a smaller image at large distances from the magnet is possible, and second, the smaller value of B may allow the use of steels near to their point of maximum permeability, so that the flux leakage is smaller. However a large value of r introduces new difficulties associated with the change of iron cross-section between core and pole-face. If the magnet is of the usual Weiss pattern with cylindrical cores of radius a and the pole-face is a double quadrant of radius r_0 and width w (figure 1(a)), then assuming similar steels throughout and only half the core flux used at the pole-face we should have the core area at least twice that of the pole-face, or

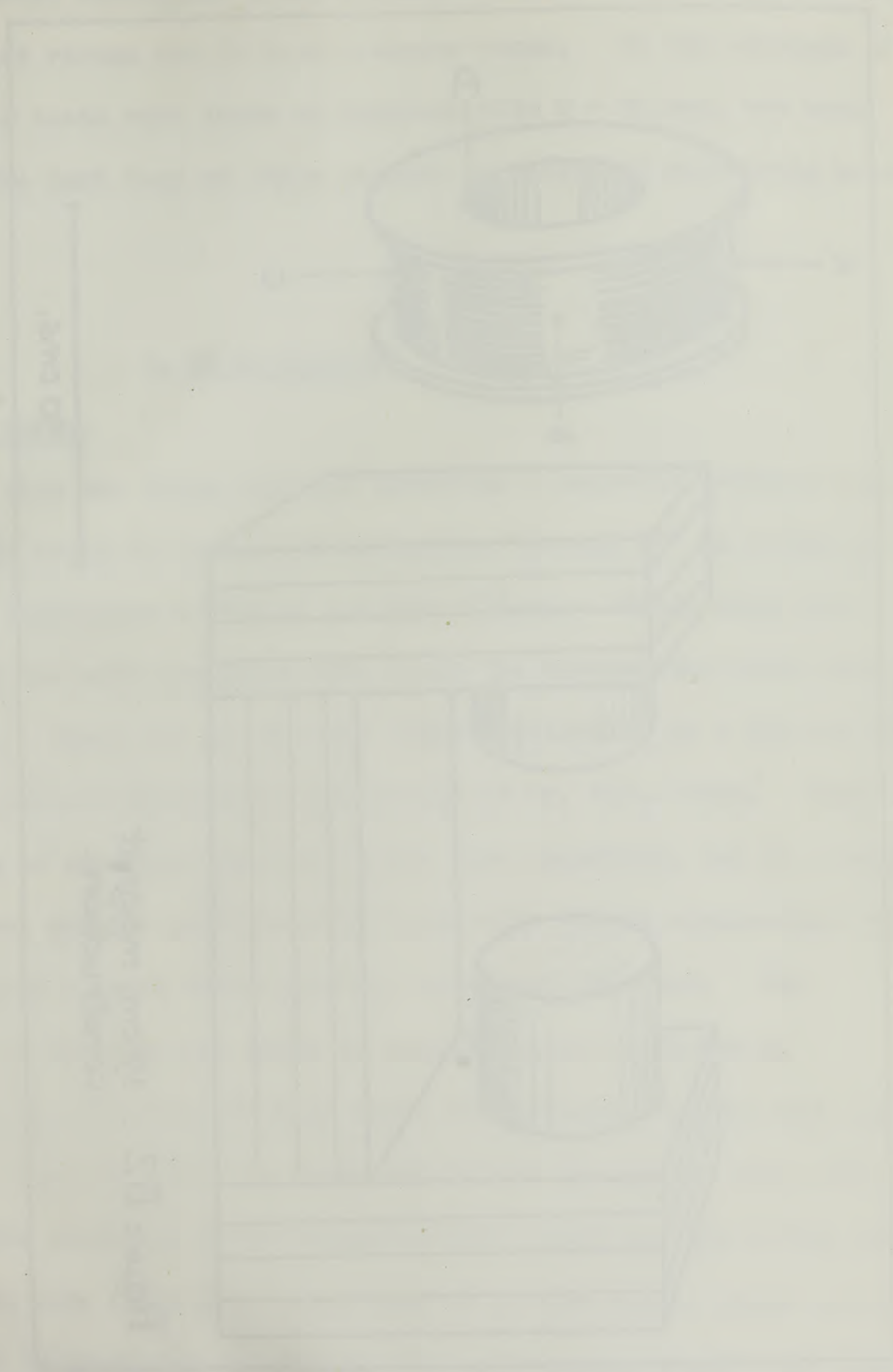
$$a^2 = 2 r_0 w$$

The pole face then overhangs the core circumference by a fraction of the radius

$$\frac{r_0 - a}{r_0} = 1 - \sqrt{\frac{2w}{r_0}} \quad (1)$$

Usually w must be as small as possible to keep down the size of the whole magnet, yet large enough to maintain a uniform field strength within the used part of the pole face boundaries, and the overhang will be large for large values of r_0 . This may produce a serious degradation in the field strength at the ends of the quadrants. Because of the limitations it is usual for analysers designed to deflect particles of a few million volts maximum energy to have a radius of 50 cms. or less.

Experience with the design of analysing magnets was gained with one



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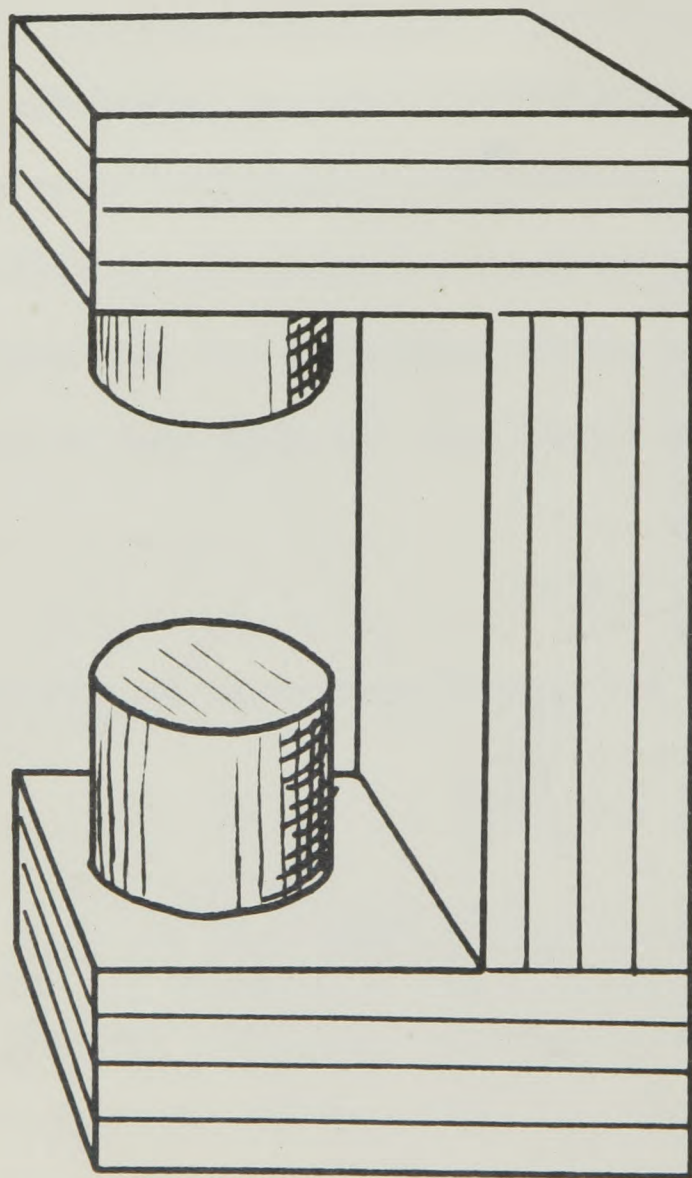
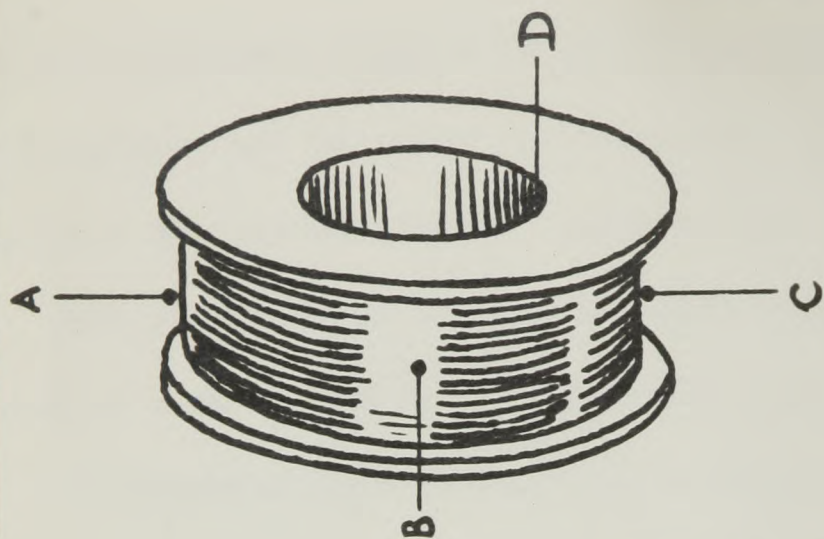


Figure B2 : 19 cm. magnet construction.

50 cms.

B.

of 10 cm quadrant radius and 4 inch diameter cores, and a larger one with 19 cm. quadrant radius and 10 inch diameter cores. On the strength of the results of tests with these an analyser with $r_c = 30$ cms. has been designed. The work done on these magnets is described separately below.

2. 19 Cm Magnet.(i) Yoke and Coils.

For use with the large Phillips generator a magnetic analyser was required which would be capable of deflecting through 90° in either of two opposite directions a beam of 1.4 MeV. T^+ ions. These ions were thought to be the most energetic with which the accelerator would ever be concerned. There was no suitable magnet available, so a new one had been built by Messrs Tickford to the design of Dr. F.V. Price. Ease and quickness of construction were at the time important, and as a result of this several details were included which with slight elaboration and the advantage of a model could probably have been improved. The construction of the yoke and coils is broadly shown in figure 2.

The yoke is built from 2" mild steel laminations, and the bed and the upright faces against which the cores are bolted are ground flat after assembly. The cores are of 10" diameter mild steel and the coils, on brass formers, have about 1700 turns each of 10 SWG single glass covered wire. There is provision for cooling the coil former cheeks with water, but this has not proved necessary and is not shown in the figure 2. The weight of the coils is taken by the bed of the yoke, on which they rest.

Tests were done to find how the power supplied to the coils was

B.

dissipated. In prolonged runs it is important to know the temperature which will be reached by the coils, both because this must not exceed the safe maximum, and because this will affect their resistance and the type of long period stabilisation needed. Temperature measurements were made both by resistance measurements on the coils, and with four copper - constantan thermocouples attached with adhesive tape to positions ABCD on the coil (see figure 2). The cold junctions of the couples were attached to the bulb of a mercury thermometer touching the bed of the yoke.

The temperature distribution throughout the winding is known for certain types of coil. It has been shown^{22,23} that if

t_o = surface temperature,

t_R = temperature by ^{measuring} resistance of winding,

t_{max} = temperature of hottest spot in coil,

$\bar{t}$ = mean temperature measured by embedded thermocouples,

then we can expect

$$\bar{t} \sim 1.4 t_o$$

$$t_{max} \sim 1.65 t_o$$

$$t_R = 1.06 \times t_o \text{ to } 1.2 t_o \text{ increasing with thermal resistance of the insulation.}$$

Thus we can expect $t_{max} \sim 1.3 \times t_R$ and this has to be borne in mind when finding the maximum current at which the coils can safely be run.

The steel coil formers have been left with a rough cast finish at the inside surface where they touch the cores, in an attempt to avoid large temperature changes in the cores; so it is assumed that most of the heat is lost by radiation and convection from the outside and sides of the winding. For glass covered wire the heat dissipated is at the

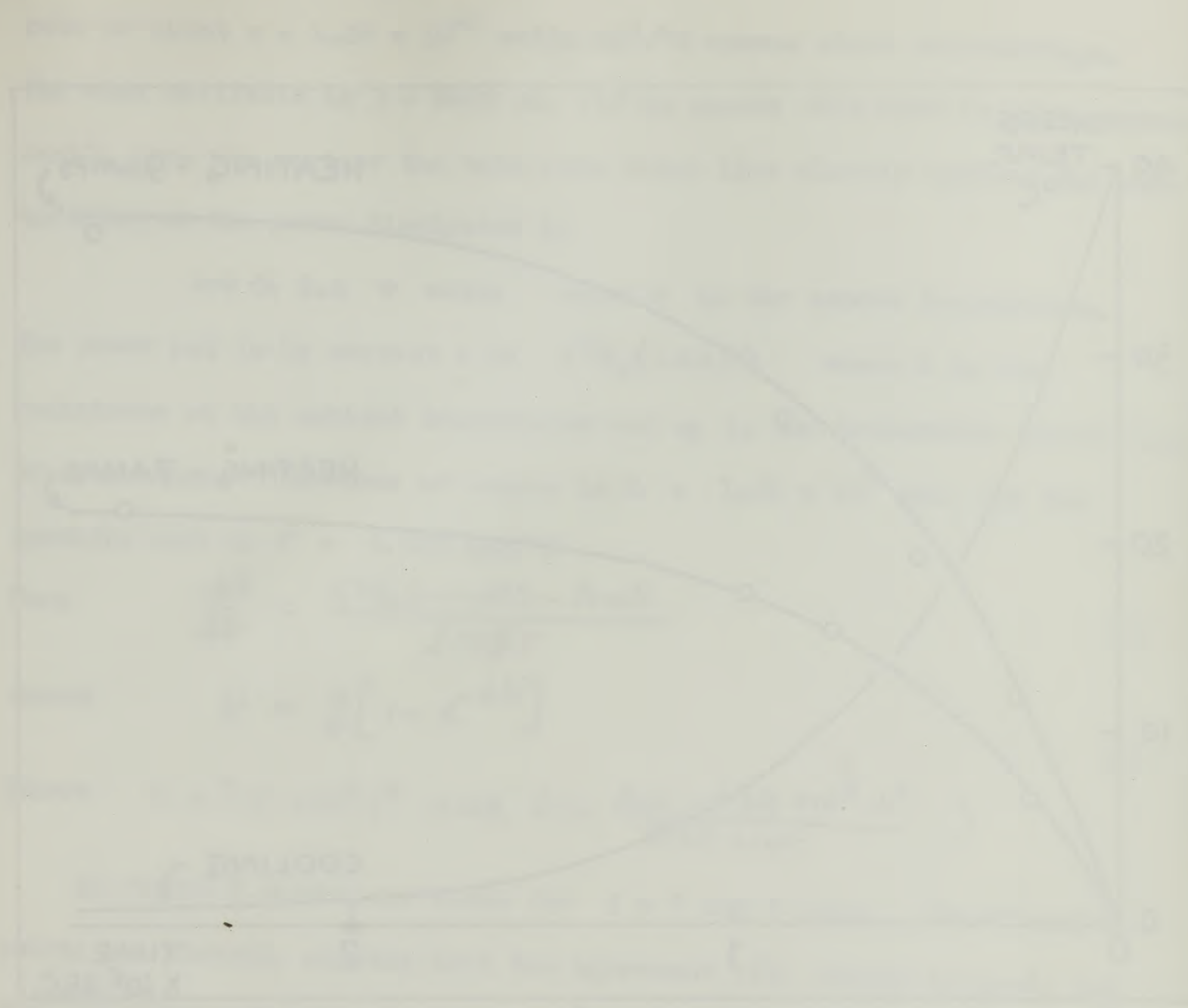


Figure 1.1: Cooling curve of a body.

$$\frac{dT}{dt} = -k(T - T_{\infty})$$

The cooling curve of a body is a graph showing the relationship between the temperature of the body and the time taken for it to cool down. The temperature of the body decreases as time increases, and the curve approaches a horizontal asymptote at the ambient temperature. The rate of cooling is highest when the temperature difference between the body and the surroundings is largest.

$$T = T_{\infty} + (T_0 - T_{\infty})e^{-kt}$$

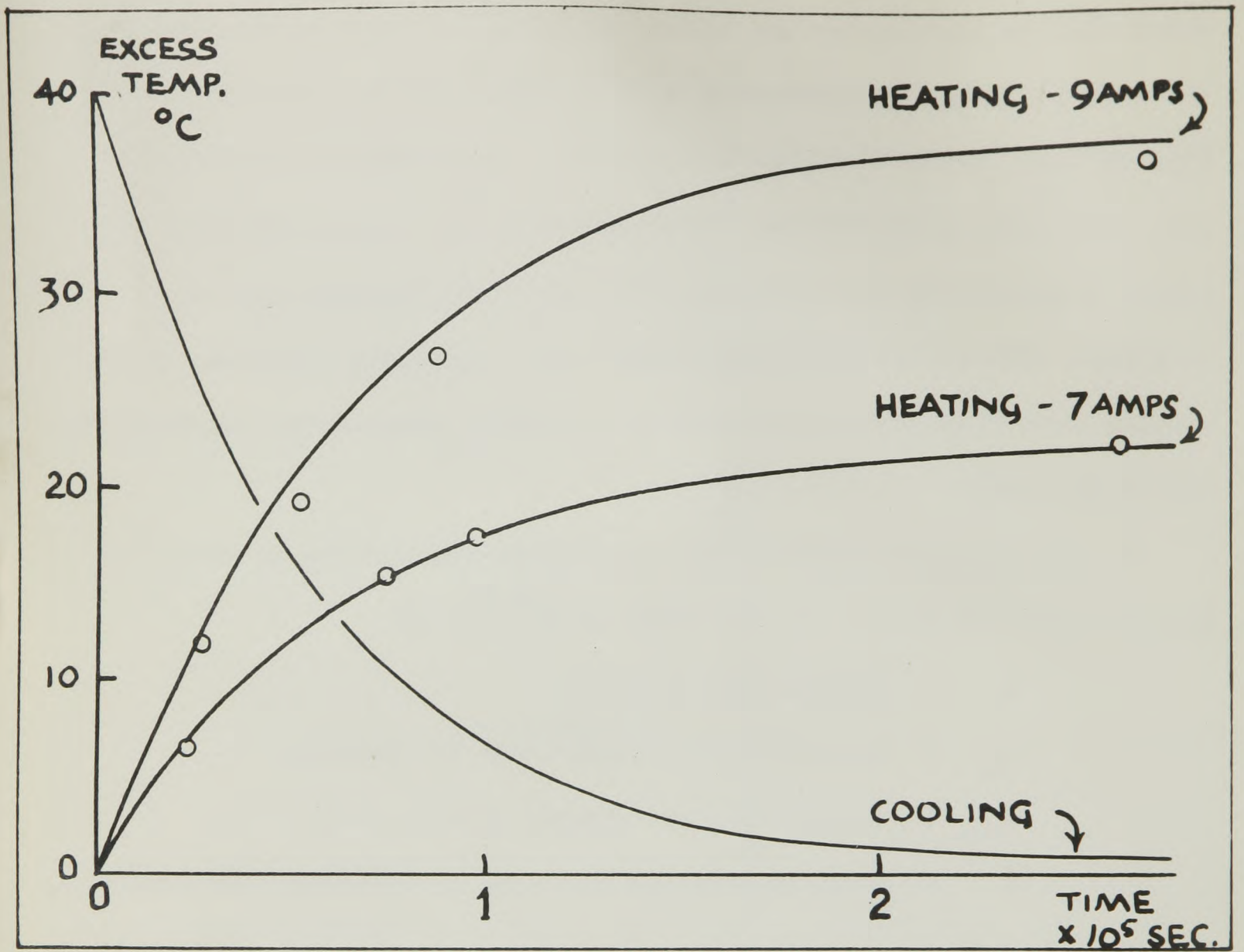


Figure B3 : 19 cm. magnet - heating and cooling curves.

B.

rate of about $w = 1.35 \times 10^{-3}$ watts/cm²/°C excess above surroundings.

The area available is $A = 6960$ cm. (if we assume that heat is also removed easily from the part of the coil side which lies closely against the yoke upright) so the power dissipated is

$Aw\theta \approx 9.3 \theta$ watts where θ is the excess temperature.

The power put in by current i is $i^2 R_0(1 + \alpha\theta)$ where R is the resistance at the ambient temperature and α is the temperature coefficient of resistance. The mass of copper is $M = 1.35 \times 10^5$ gms. and the specific heat is $\sigma = 0.093$ cal/°C

Then
$$\frac{d\theta}{dt} = \frac{i^2 R_0(1 + \alpha\theta) - Aw\theta}{J M \sigma} \quad (2)$$

whence
$$\theta = \frac{a}{b} [1 - e^{-bt}] \quad (3)$$

(where $a = 7.21 \times 10^5 i^2$ and $b = \frac{Aw - 1.63 \times 10^3 i^2}{5.27 \times 10^4}$)

In figure 3 curves are shown for $i = 7$ and 9 amps. Experimental points are plotted, showing that the agreement with theory is good, and that the value assumed for Aw is reasonable. ~~This is so since the fractional changes in Aw and θ are given by~~

$$\frac{d\theta}{\theta} = \frac{d(Aw)}{Aw} \left[a - \frac{kt}{k} - 1 \right] \quad (k = 5.27 \times 10^4) \quad (4)$$

~~and for values of t less than 10^4 seconds (2 hours 40 minutes) the function in brackets has a value between -1 and -2. Thus at the times and temperatures of the experimental points considered a 2% or 3% increase in the assumed value of Aw would be sufficient to put the theoretical curve for 9 amps through the experimental points.~~

The cooling curve is

$$\theta = \theta_0 e^{-1.76 \times 10^{-4} t} \quad (5)$$



Figure 54: 1000 magnet - equilibrium
 temperature.

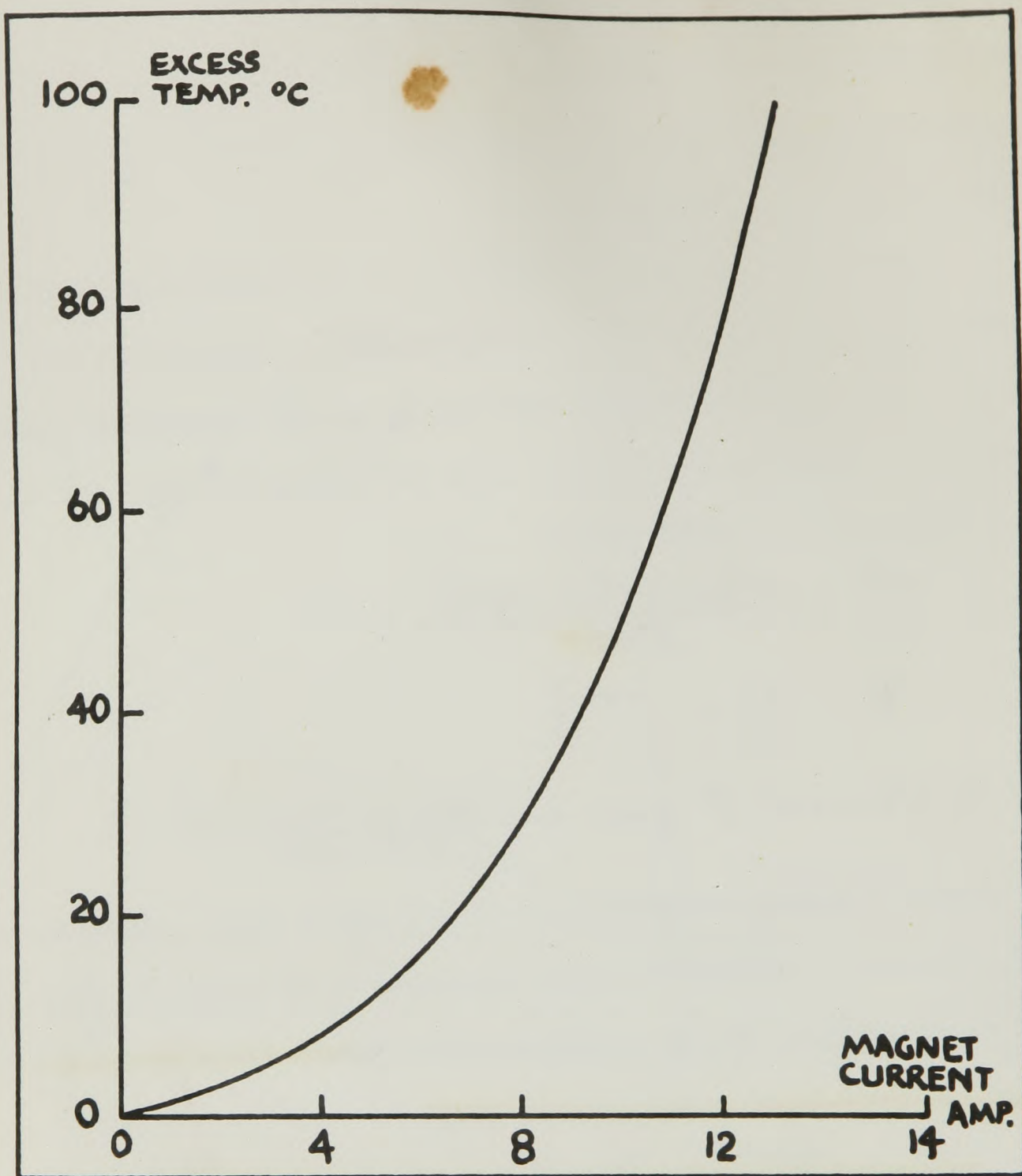


Figure B4 : 19 cm. magnet ~ equilibrium temperatures.



Fig. 1. A cross-section of the pump mechanism. The upper part shows the internal components, and the lower part shows the large circular component. The drawing is a pencil sketch on a light-colored paper.

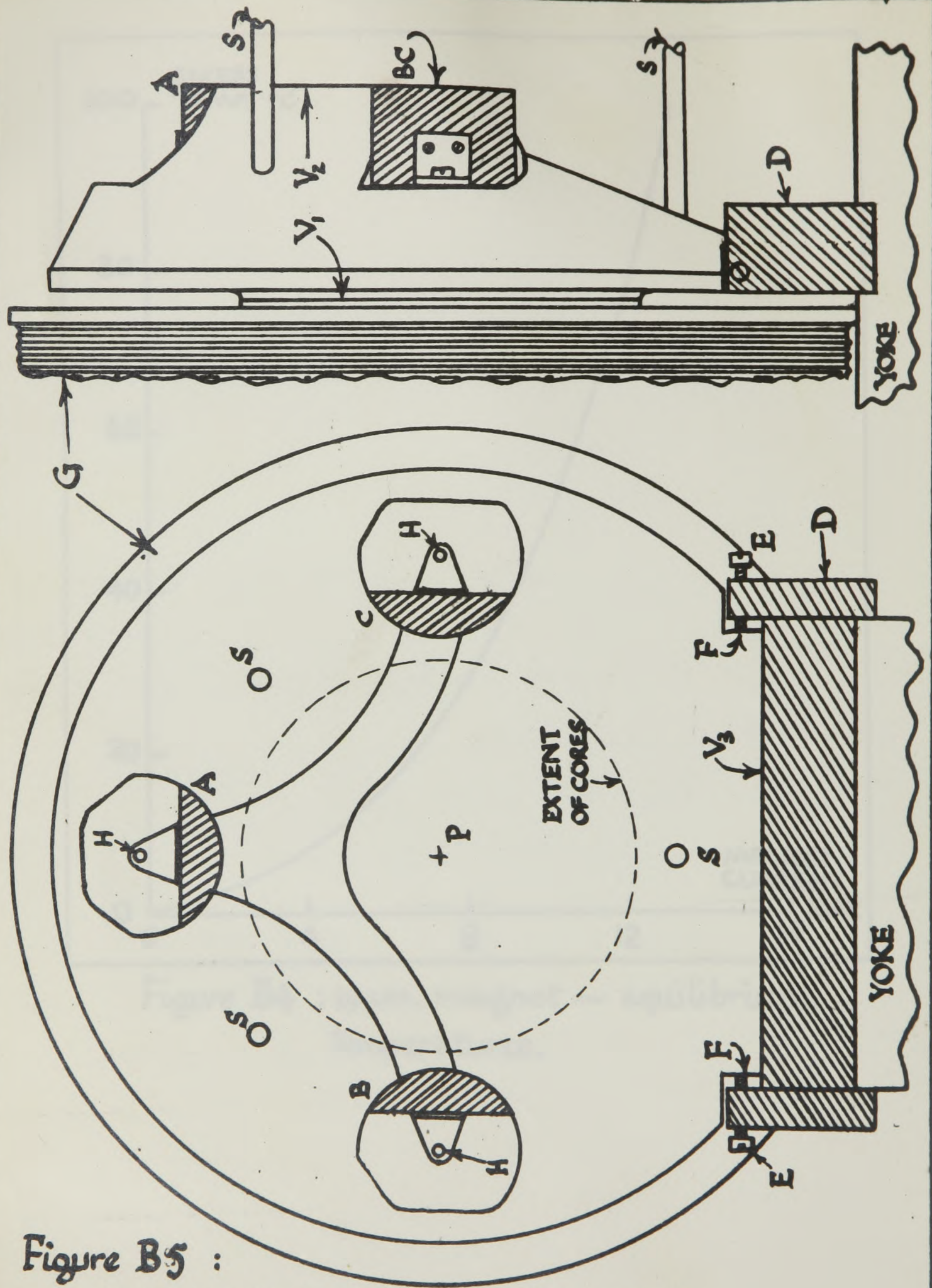


Figure B5 :
19 cm. magnet pole-pieces ($\frac{1}{4}$ scale).

B.

and this is also plotted.

The equilibrium temperatures will be given by

$$\theta_{t=\infty} = a/b$$

and this is shown in figure 4. It will be seen that at 12 amps the equilibrium temperature excess is about 80° C. If $t_{\max} = 1.3 t_R$ and the surroundings are at about 16°C parts of the winding would then be at 120°C, which is probably the maximum which can be safely allowed. Hence the coils cannot be run at over 12 amps for long periods without forced cooling.

(ii) Pole Pieces.

For the deflection of 1.4 MeV T^+ ions $B_p = 3 \times 10^5$ g.cm. Experience with the 10 cm. magnet had shown that with the cast magnet steel used the leakage flux becomes large when $B > 12,500$ gauss. Since the yoke and coils had been designed assuming only 10% leakage this required $\rho_0 \geq 24$ cm. But this in turn meant that the pole piece had to flare out from the core to pole tip at an angle of 40°. It was thought that the large maximum field would fall off at the pole tips with so large an angle as this, and as a compromise a design with $\rho_0 = 19 \frac{\text{cm}}{(\text{flare angle } 23^\circ)}$ was prepared. In this design (see figure 5) the iron of the pole-piece extends to nearly the full diameter of the coils G so as to collect as much as possible of the field which would otherwise remain as leakage flux outside the pole face.

The merit of this design is not certain. Although additional flux lines are collected from the coil, some are lost from the pole face and distributed over the neighbouring iron surface where there is a competing air gap in parallel whose reluctance is not very large. The extent of this leakage has been calculated as carefully as possible.

B.

It was assumed that all flux lines in air between the poles travelled parallel to the core axis, and that the reluctance of all paths in iron was small enough to be neglected. The areas ΔA between successive contour lines on a plan of the pole-piece were found with a planimeter and divided by the distance l between poles at the mean contour line. Then the total reluctance of the air gap $R = \sum \frac{l_x}{\Delta A_x}$, where $R_x = \frac{l_x}{\Delta A_x}$. The reluctance of the pole gap ($A = 250 \text{ cm}^2$, $l = 2 \text{ cm}$) = R

Then the ratio $\frac{\text{useful flux}}{\text{total flux}} = \frac{R}{R_g} = 0.396$ by calculation.

Hence however many flux lines enter the pole piece at the back, only 39.6% leave the front surface at the pole gap. This result was checked by a model in the electrolytic tank (figure 7). The model was two-dimensional and made of paraffin wax blocks to simulate half the pole gap. The lengths a, b and c are chosen so that a and $(b+c)$ are in the ratio of the areas of pole gap and the rest of the pole face, but otherwise the model is to scale. The positions of the electrodes are shown by thick black lines, and the numbers are the values of the potential found at points on the surface of the pole-piece. Equipotentials in this tank model represent field lines in the magnet so that at any point on the boundary of the wax the value of the potential represents the number of flux lines which have left the surface between this point and the reference electrode. Then from experiment

$$\frac{\text{useful flux}}{\text{total flux}} = 0.425 \quad \text{which agrees well}$$

with the calculated result above, considering the crudity of the models used. It is not known what the ratio would have been if the pole pieces had dropped sharply back from the double quadrant pole face to the

B.

circular core.

The amount of flux contributed by the coil has been estimated. It is assumed for this that the flux lines within the coil travel parallel to the coil axis (in fact it is known that a proportion is lost through the curved surfaces of the coil). The coil has radii r_1 and r_2 and length l , and the number of turns per cm. is n . At radius r the air path between back of pole piece and yoke is roughly $l + 2 + 1.5(r - 13)$ where $1.5(r - 13)$ is an estimate of the length of path from the back of the coil to the yoke. For an annular region of thickness dr the area of the air path is $2\pi r dr$

The reluctance of this bit is $dR = \frac{2[l + 2 + 1.5(r - 13)]}{2\pi r dr} + R_0$ (6)

where R_0 is the reluctance between pole pieces, already found above.

The number of turns beyond radius r is $2l(r_2 - r)n$, so the magnetomotive force producing flux in this circuit is $8\pi l(r_2 - r)n^2 \frac{i}{10}$ where i is in amps.

The flux is then $dN = (MMF)_r / dR$ (7)

and the flux due to all turns is

$$N_{\text{coil}} = \int_{r_1}^{r_2} dN \quad (8)$$

For the flux produced in the core, the whole reluctance is R_0 and the magnetomotive force is $4\pi n \frac{i}{10}$ where n is the total number of

turns. Then $N_{\text{core}} = \frac{4\pi n i}{R_0}$

By substituting values, $N_{\text{coil}} = 5.58 \times 10^4 i$ lines

$$N_{\text{core}} = 1.35 \times 10^6 i \text{ lines}$$

$$N_{\text{total}} = 1.41 \times 10^6 i \text{ lines}$$

Therefore $\frac{N_{\text{coil}}}{N_{\text{total}}} = 0.041$

i.e. the part of the pole piece covering the end of the coil only collects

B.

about 4% of the field lines and, of these, 60 % are not useful. Assuming that 40% of the total flux is used

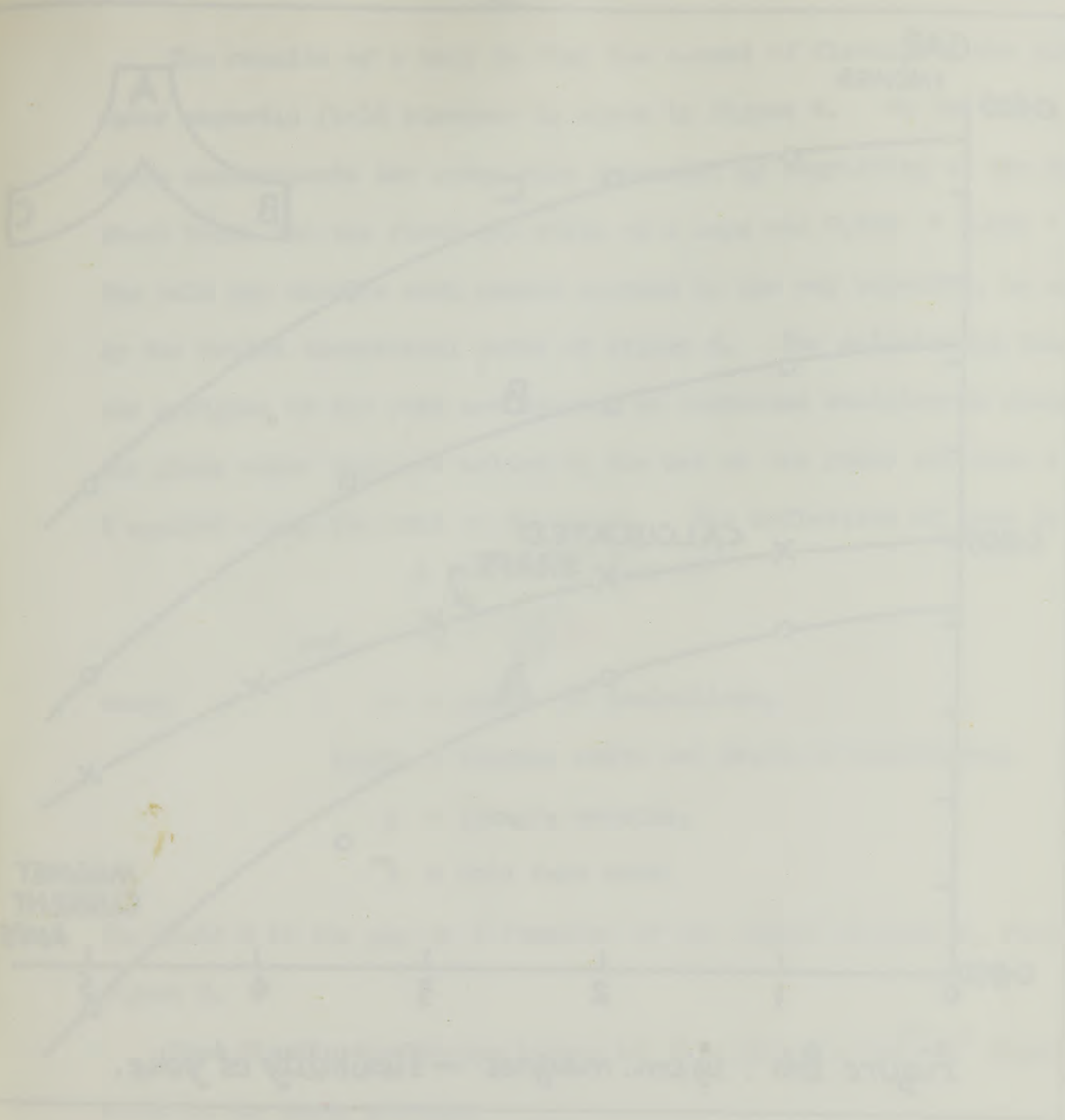
$$\text{Useful flux} = 5.64 \times 10^5 \text{ i lines}$$

This can be compared with the experimental measurements shown in figure 8.

The value of $\frac{N(\text{expt})}{N(\text{calc})}$ as a function of i is given in figure 9.

This shows that at low currents we find in the gap about 90% of the flux predicted by calculations. At larger currents the ratio falls due to approaching saturation of the iron, which causes a spreading of the flux outside the useful area.

In the drawings made for Messrs. David Brown Ltd. the pole face had a width of 3.6 cms and the depth was $5\frac{1}{2}$ inches, giving a pole gap of 1.2 cms., but it was planned that subsequent grinding and possible shaping of the faces should increase the gap to 2 cms. and the face width to about 4.5 cms. The pieces were made by a centrifugal casting method, to avoid blow holes, in a steel known to the foundries as Mark IA magnet steel. At the same time a square section ring was cast and put through the same annealing process for hysteresis tests. All the parts were given a radiographic examination and found satisfactory. The three surfaces on each pole piece marked V in figure 5 were given a rough ground finish so that preliminary tests could be made. The surface V3 sits on a light alloy block D which spaces the iron 6 cms. from the bed of the yoke, and screws E in this block bear on steps F for alignment. The pole faces are braced back against the cores by brass struts S which fit into recesses in the iron. Surfaces VI and V2 were ground parallel by the Pressed Steel Company, so that the parallelism of the gap depends on that of the core faces only.



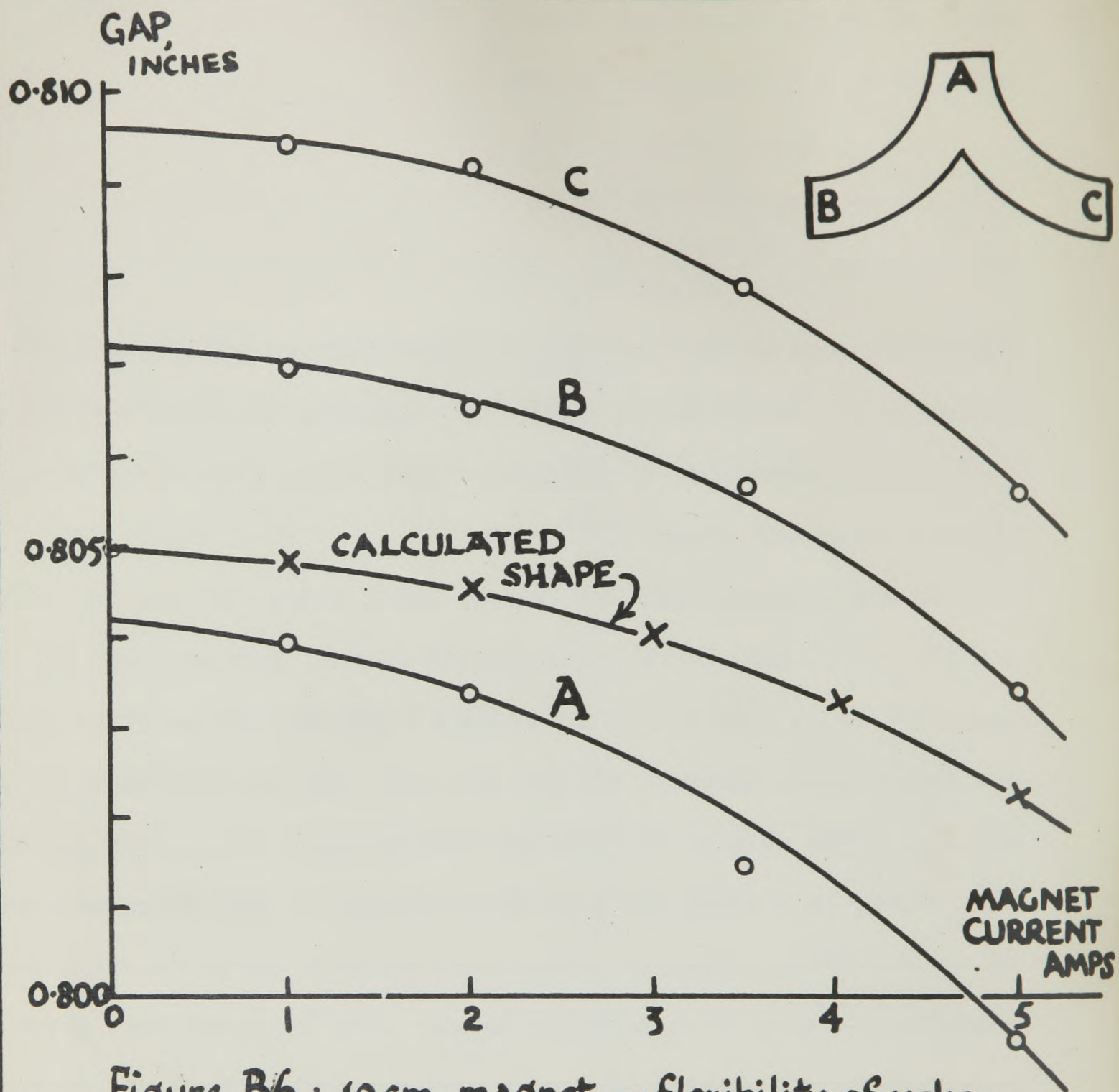


Figure B6 : 19 cm. magnet - flexibility of yoke.

B.

The results of a test to find the amount of flexing of the yoke under magnetic field stresses is shown in figure 6. On the basis of these measurements the cores were corrected by regrinding at the Pressed Steel Works and the final gap width at 2 amps was 0.832 ± 0.002 ". The pole gap changes with magnet current in the way expected, as shown by the dotted theoretical curve of figure 6. For calculating this the uprights of the yoke are treated as laminated cantilevers clamped at the place where they are bolted to the bed of the yoke, and with a load W applied along the axis of the cores. The deflection of each is then

$$\delta = 4Wl^3n^2/qab^3$$

and
$$W = \frac{AH^2}{8\pi} \mu_0$$

where

n = number of laminations,

l, a, b = length, width and depth of cantilever,

q = Young's modulus,

A = Pole face area.

The field H in the gap is a function of the magnet current i , found from figure 8.

Then finally the change in gap is $\Delta = 2\delta = 9 \times 10^{-5} i^2$ inches (9) which is the curve plotted.

Since $n = 4$, it is clear from the expression for δ above that the deflections would be reduced by a factor 16 if the laminations lay in the plane of b and l . This arrangement would have the further advantage of reducing the circuit reluctance and flux losses from the yoke.

(iii) B-H. Measurements on the Pole-Piece Steel.

The steel ring mentioned in paragraph (ii) above, was taken through various magnetising processes in an attempt to predict the way in which



Figure 17: Elasticity of pole pieces.

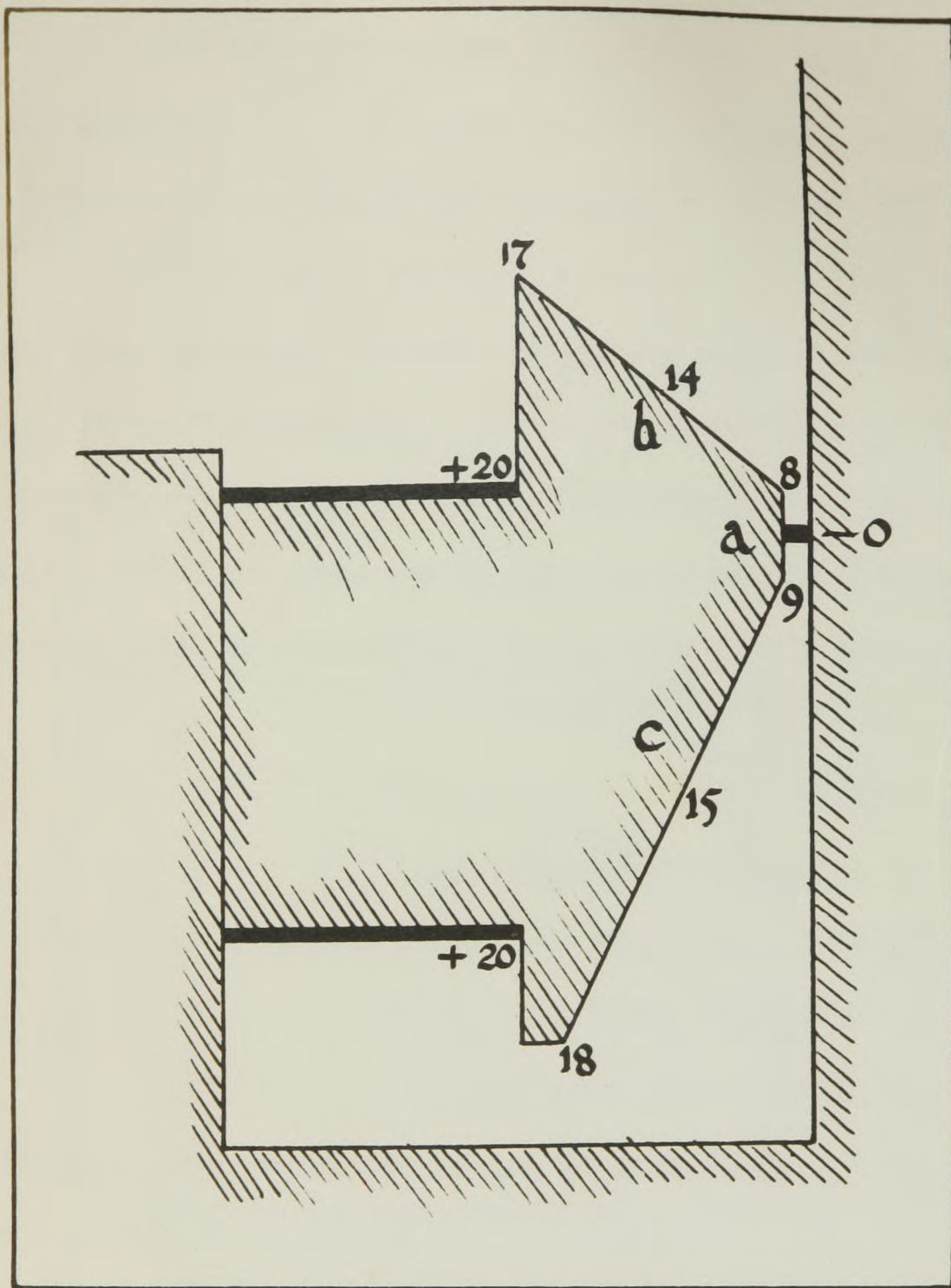


Figure B7 : Electrolytic tank representation of pole-pieces.

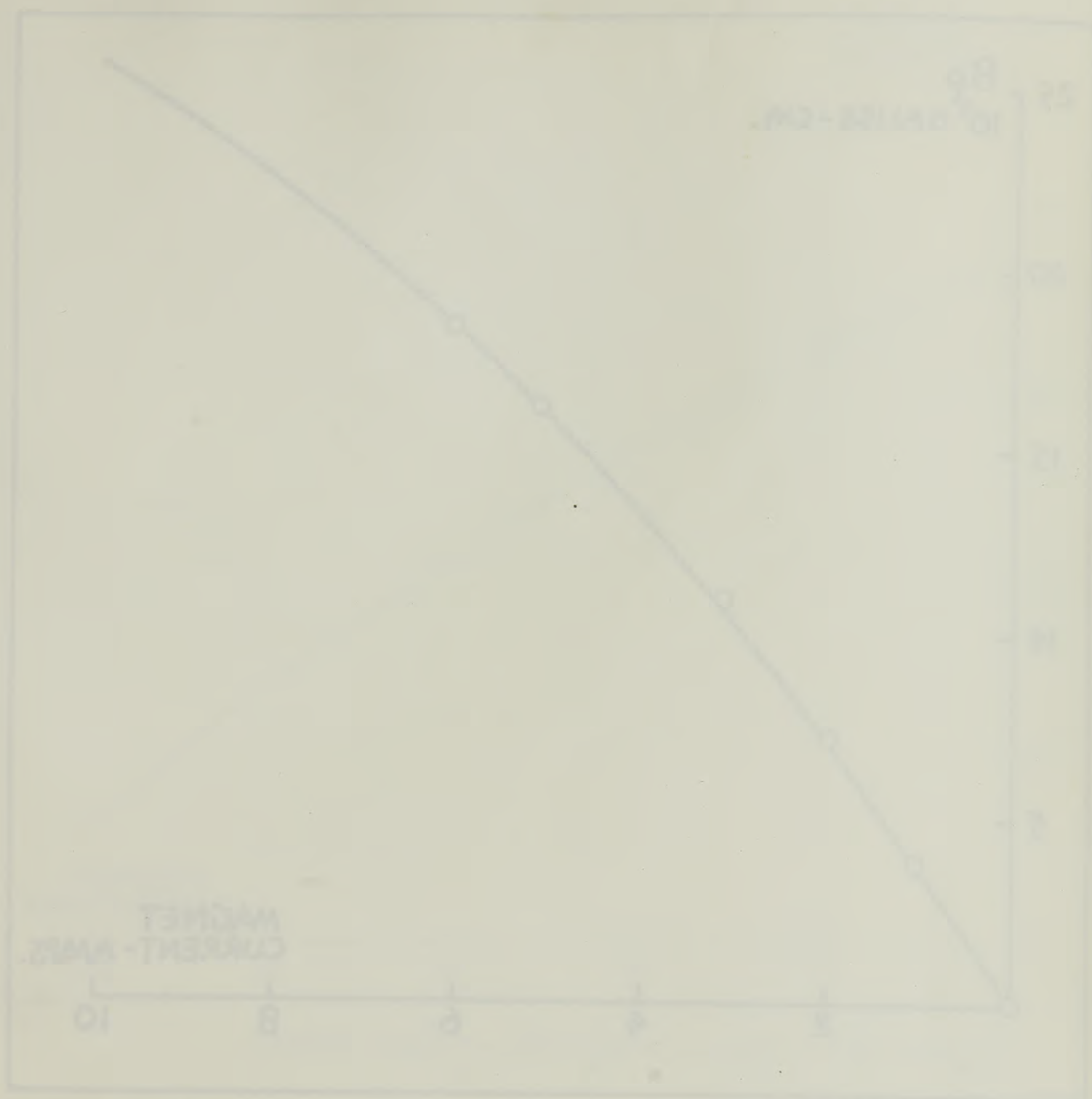


Figure B8: Field in gap of 19 cm. magnet.

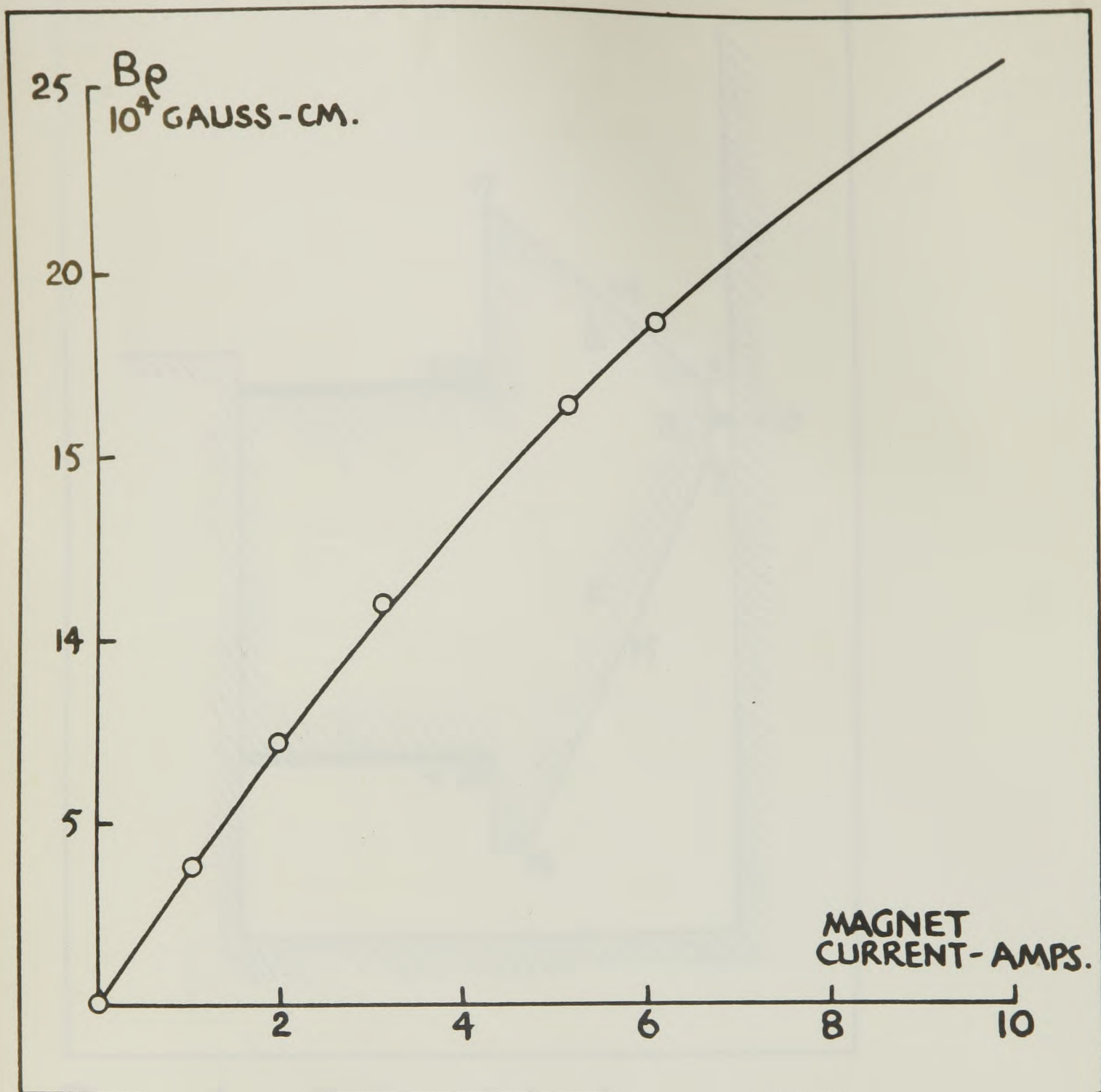


Figure B8 : Field in gap of 19 cm. magnet.

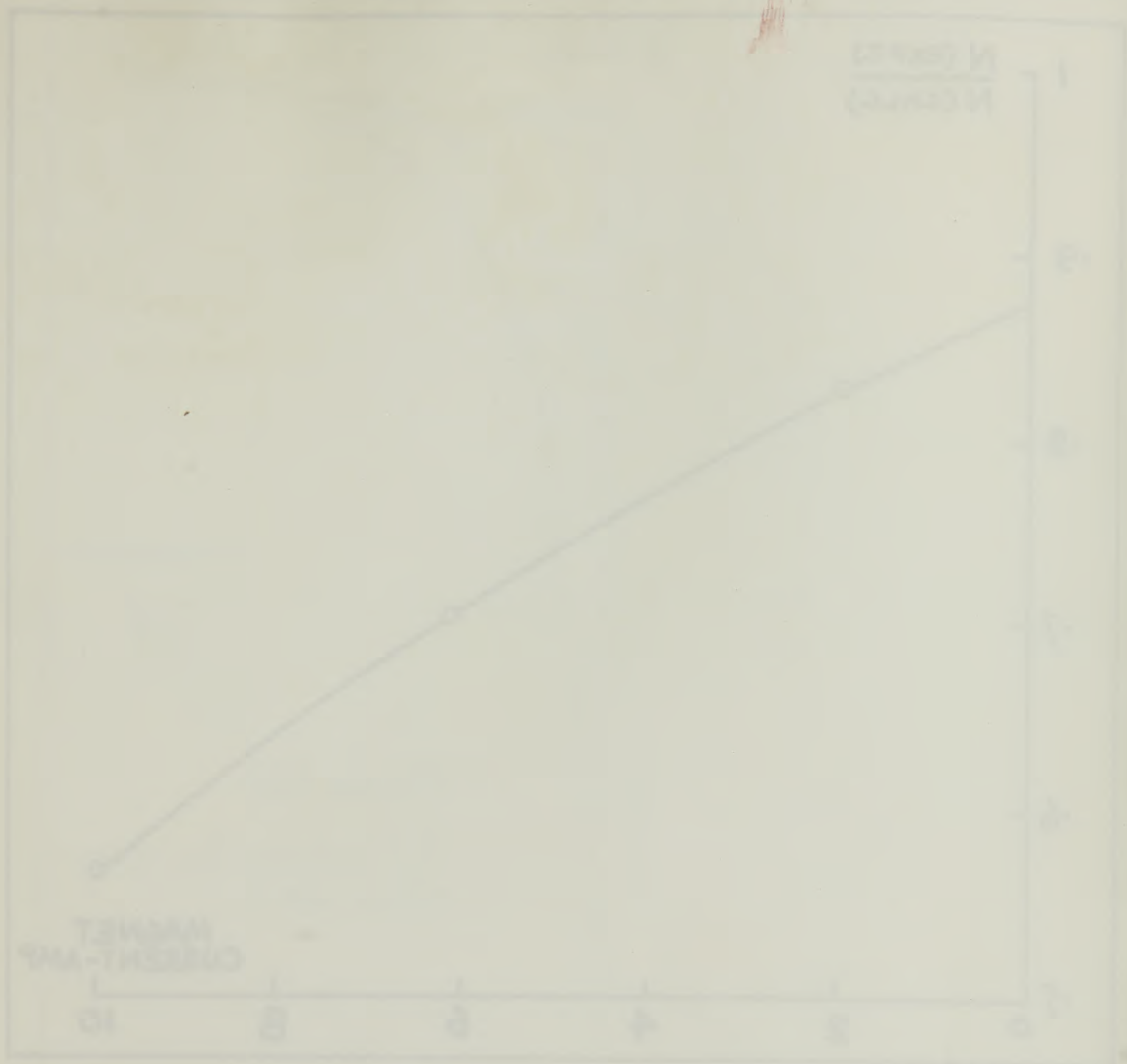


Figure 5: 10 ohm magnet - 100 turns.

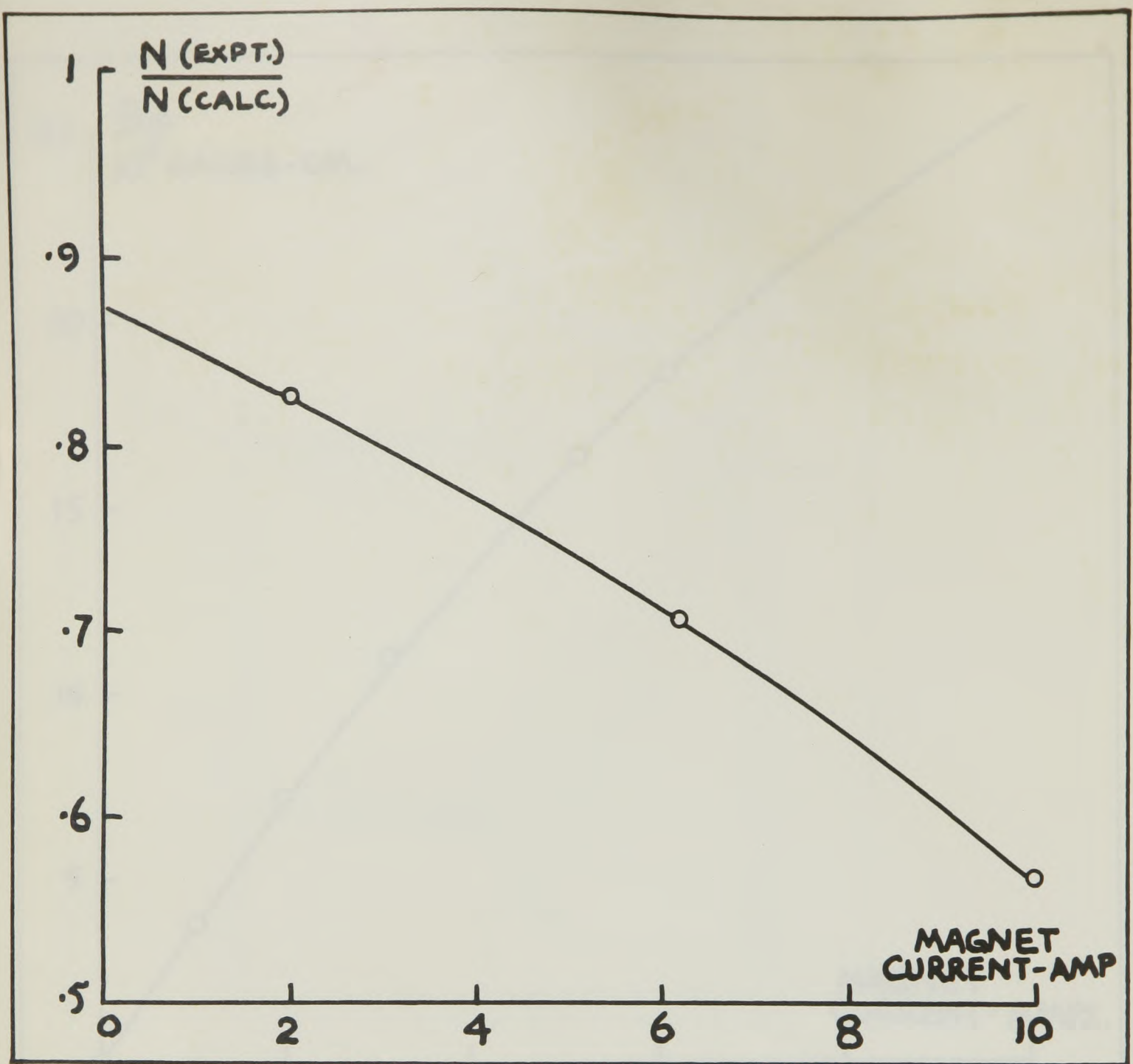


Figure B9 : 19 cm. magnet — flux losses.

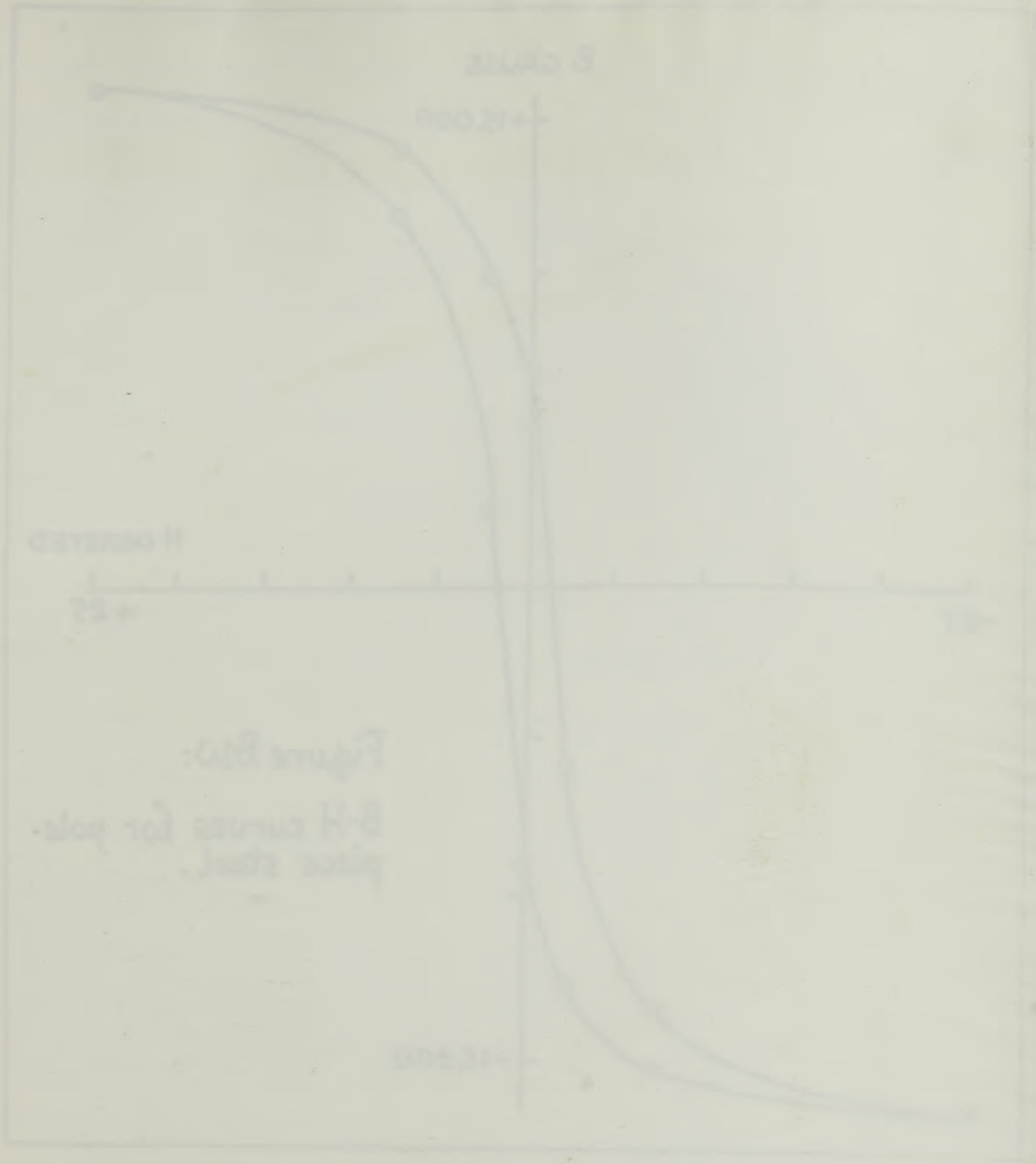


Figure 10:
B-H curves for pole
piece steel.

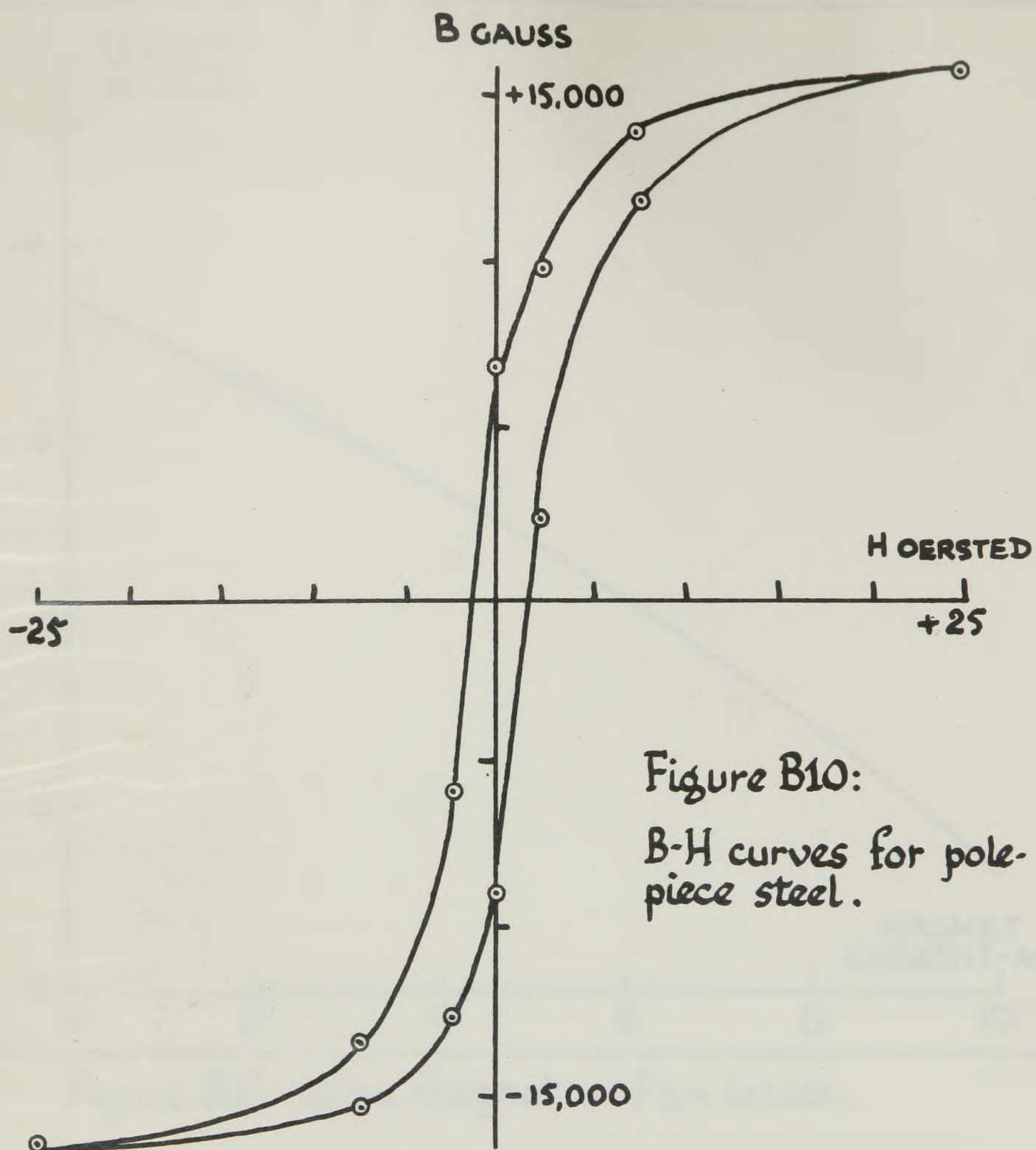
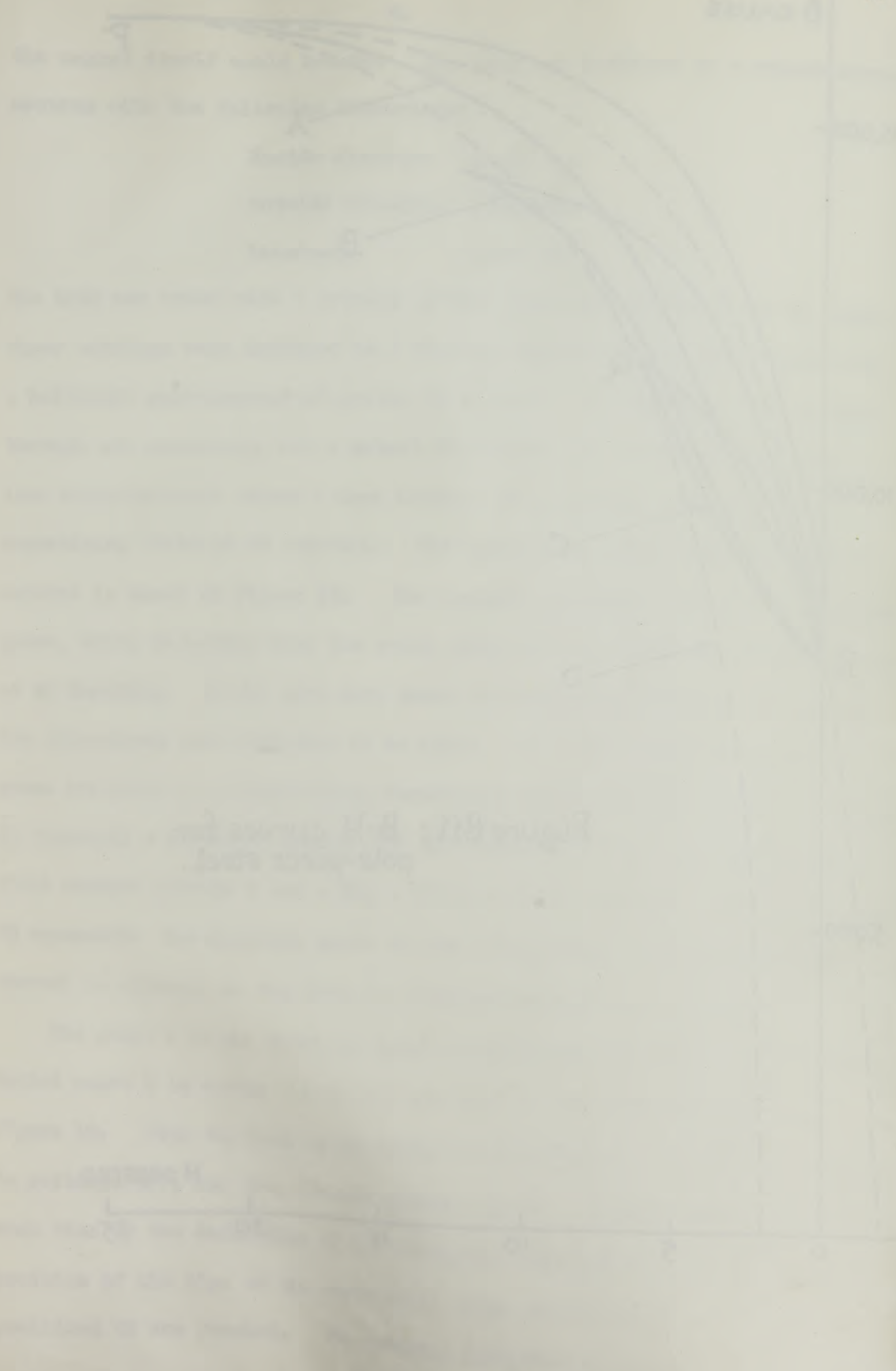


Figure B10:
B-H curves for pole-
piece steel.

0.0002



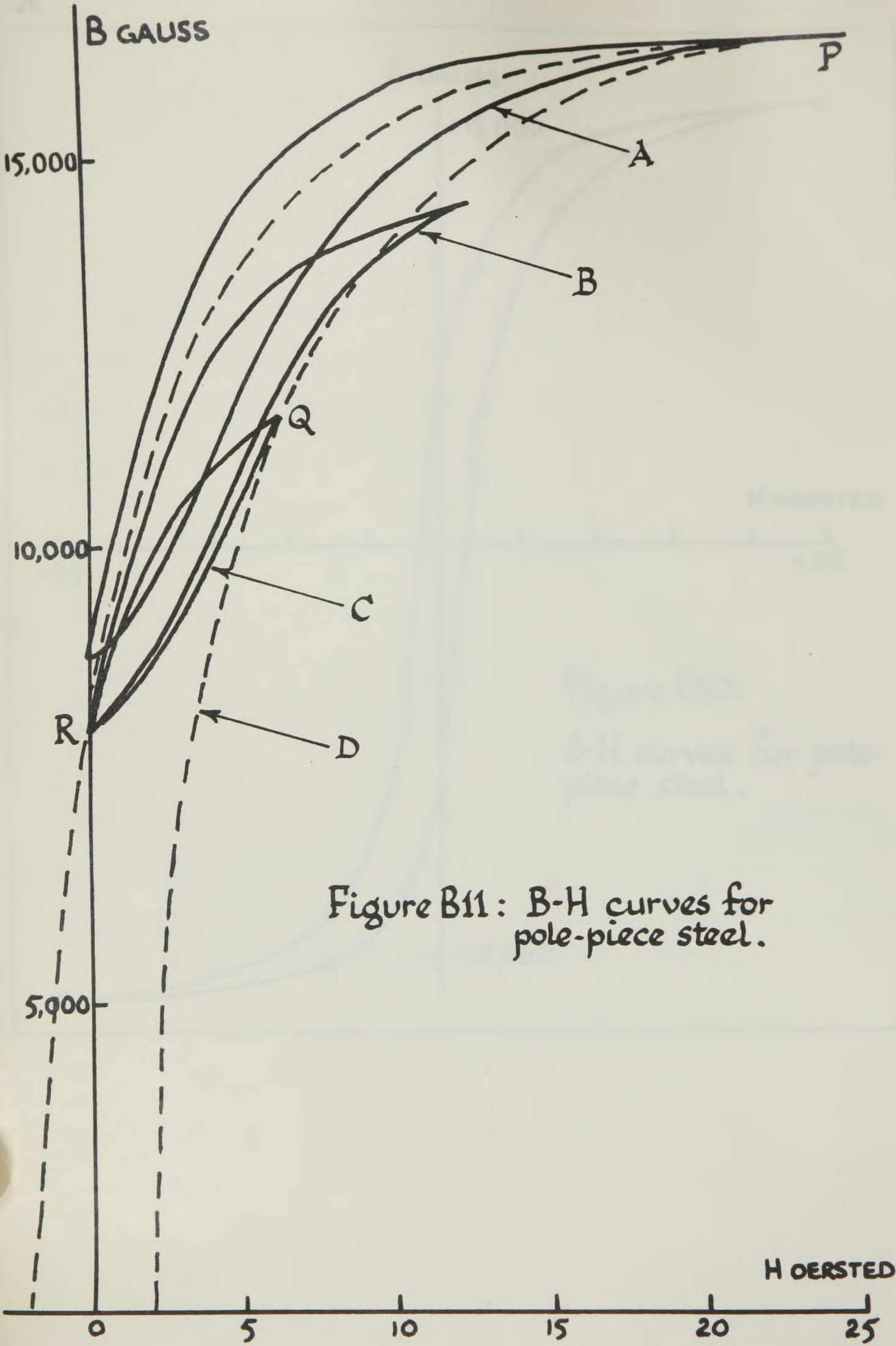


Figure B11: B-H curves for pole-piece steel.

B.

the magnet itself would behave. The ring was machined to a square cross-section with the following dimensions:

inside diameter	15.27 cm.
outside diameter	19.78 cm.
thickness	2.405 cm.

The iron was wound with a primary of 550 turns and secondary of 12 turns. These windings were included in a testing circuit of the usual type with a ballistic galvanometer of period 10 seconds for detecting flux changes through the secondary, and a mutual inductance for calibration. ²⁴ The iron saturated with about 2 amps through the primary, representing a magnetising field of 25 oersted. The hysteresis curve between $H = \pm 25$ oersted is shown in figure 10. The maximum induction appears to be 16450 gauss, which is better than the value given by the foundries (14,500 gauss at 25 oersted). It is also much above the maximum induction with which the pole-pieces were intended to be used: the curve shows that 12,000 gauss are given by a magnetising current of about 0.5 amps (6.25 oersted). In figure 11 a series of curves ABC is shown in which the magnetising field changes between 0 and + 25, + 12.5, + 6.25 oersted. The small loop AB represents the magnetic state of the pole-piece iron as the magnet current is switched to and from the maximum designed to be used.

The point R on the curve is found as follows: the form of the large dotted curve D is known; it is the top half of the complete cycle of figure 10. With the iron in the state represented by point P the current is switched off, and then through cycles $+\frac{1}{2}$, 0, $+\frac{1}{2}$ amps etc., recording each time by the deflection of the ballistic galvanometer the shift in the position of the tips of the curve until after several cycles the stable positions QR are reached. The remanent induction OR is then known. The

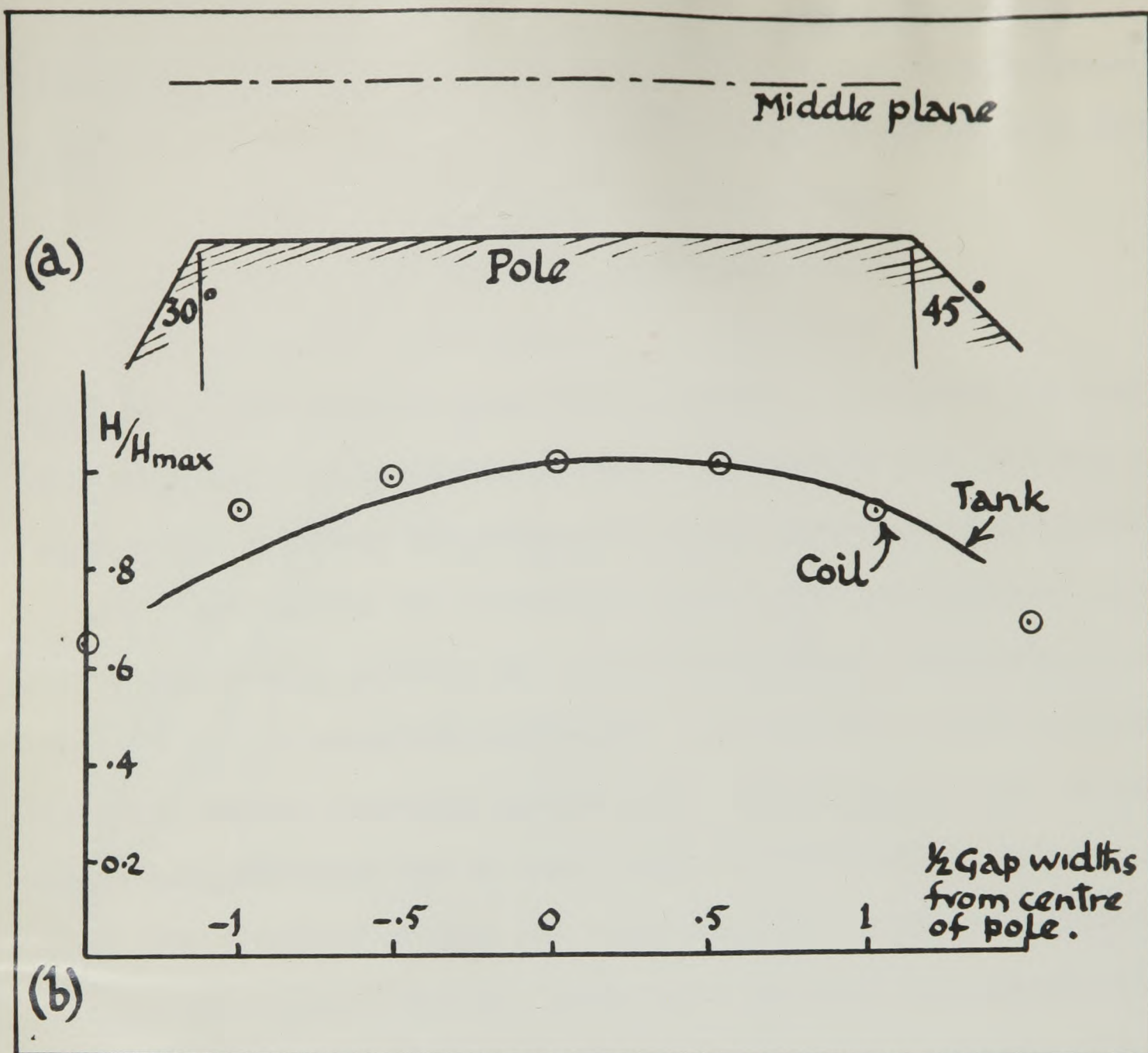


Figure B12: Electrolytic tank and search-coil measurements of field variation across length of pole gap.

B.

form of this loop is important, as it shows that for circuits of small air reluctance quite a high induction can be maintained with small magnetising currents if the current is first taken up to the maximum and then adjusted by reduction to the value wanted. However in the case of the analysing magnet, the reluctance of the magnetic circuit consists almost entirely of that of the air gap so that changes in the permeability of other parts make very little difference, and the magnet current to maintain a given flux in the gap is almost independent of the history of the iron.

(iv) Pole-Faces.

The pole-piece steel slopes away from the pole-face at angles between 20° and 50° to the normal. A representative cross-section is that shown in figure 12 (a), which is the wax model set up in the electrolytic tank to find the corrections necessary to make a uniform field over as wide a region as possible. The gap is 2 cm. and the face width only 4.5 cms. so that without correcting lips the uniformity is not expected to be good. The tank measurements gave a pessimistic picture of the field as is shown in figure 12 (a) where the solid curve is the relative field strength found from the density of equipotentials in the tank, and the circles are the measurements with search coil and ballistic galvanometer. At the edges of the gap the field has dropped to 80% of its maximum. The loss is made good by correcting lips of $1/8$ " soft-iron welding rod clipped to the edges of the vacuum box which fits in the gap (see paragraph (v) below). The alternative method, shaping the pole faces, was not convenient here because the pole gap cannot be closed up to regain the field strength lost by removing metal,

B.

and also it lacks the flexibility of empirical correction by wires. Correction of some sort is essential because a reduced field and the accompanying curvature of the field lines produce defocussing effects in both planes, as pointed out in section 12 of Part A.

The theoretical results of Part A require pole-face boundaries with variable angles. This was provided by rotatable segments A, B and C (figure 5). A first trial was made with segments of David Brown Mk. 1A steel 5/8" thick. These failed to produce the focussing effects expected because the field change at the boundary was not abrupt enough, and segments of greater depth were needed. The pole-pieces were sent back to Pressed Steel for milling and a cylinder of cast steel was supplied by David Brown. This was machined into segments cut to fit the cylindrical surfaces in the pole-pieces.

The best size and shape of the segments were calculated as carefully as possible and checked by experiment. The effect on an ion beam entering normally the fringing field at the boundary of a uniform field area is discussed briefly in section A 6. The premature bending which occurs increases the effective radius of curvature by d where

$$d = (\rho_0 - |\bar{x}|) \quad (10)$$

$$\text{and } \int_{x_0}^{\bar{x}} h(x) dx = \pm \rho_0 \quad (11)$$

x_0 is the distance from the pole boundary at which the ion beam starts ($x_0 = \infty$ effectively for most applications) and $\bar{x}$ is defined by the integral. ρ_0 is the principal radius of curvature and $h(x)$ is the ratio of field H_x at x to the uniform field H_0 well within the boundary.

Theoretical curves of $h(x)$ have been given by Coggeshall¹¹ and others. These can be deduced by satisfying Laplace's equation either by a relaxation method or, within its limits of accuracy, by the electrolytic

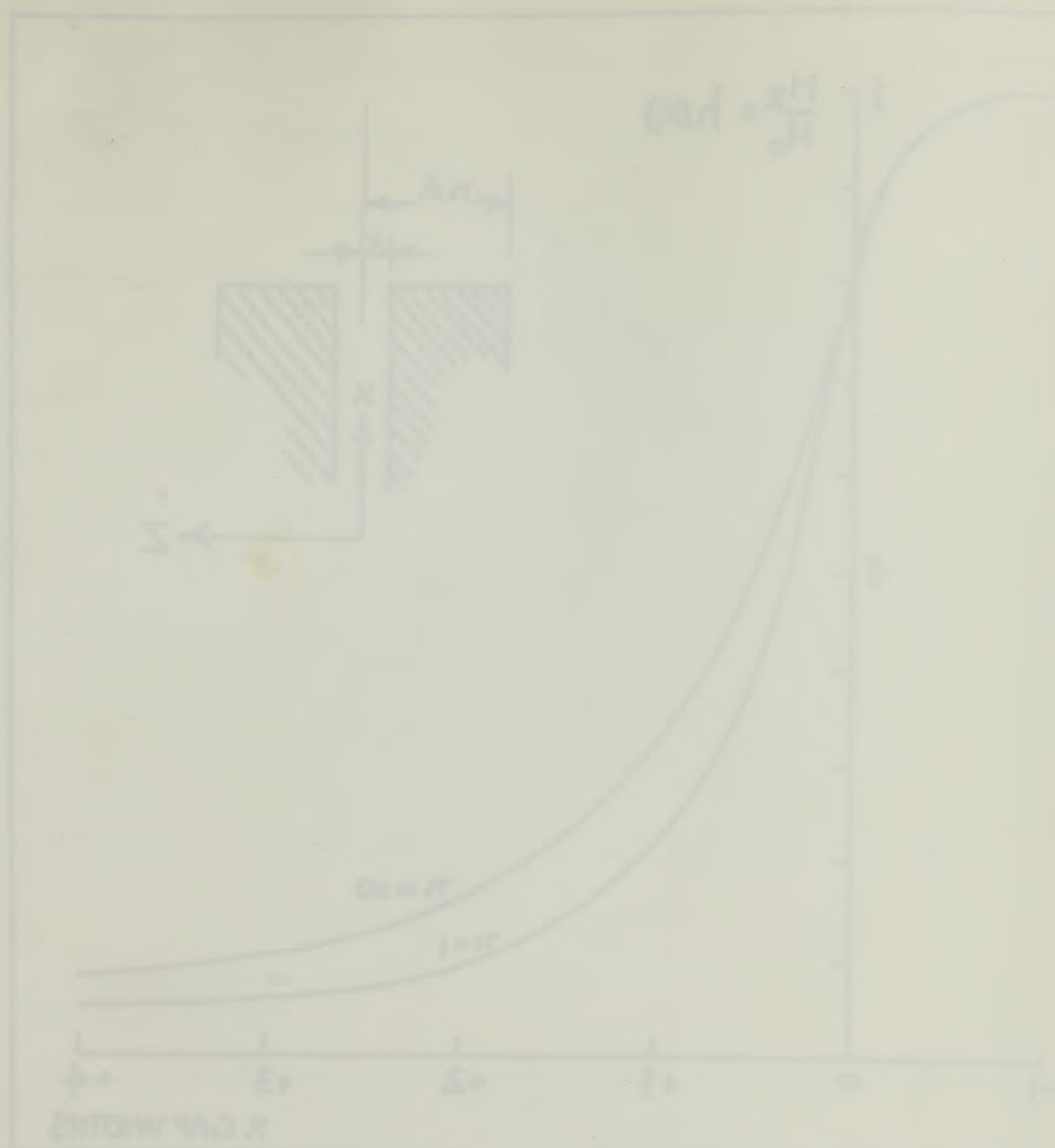


Figure 83: Ideal shape of fringe field.

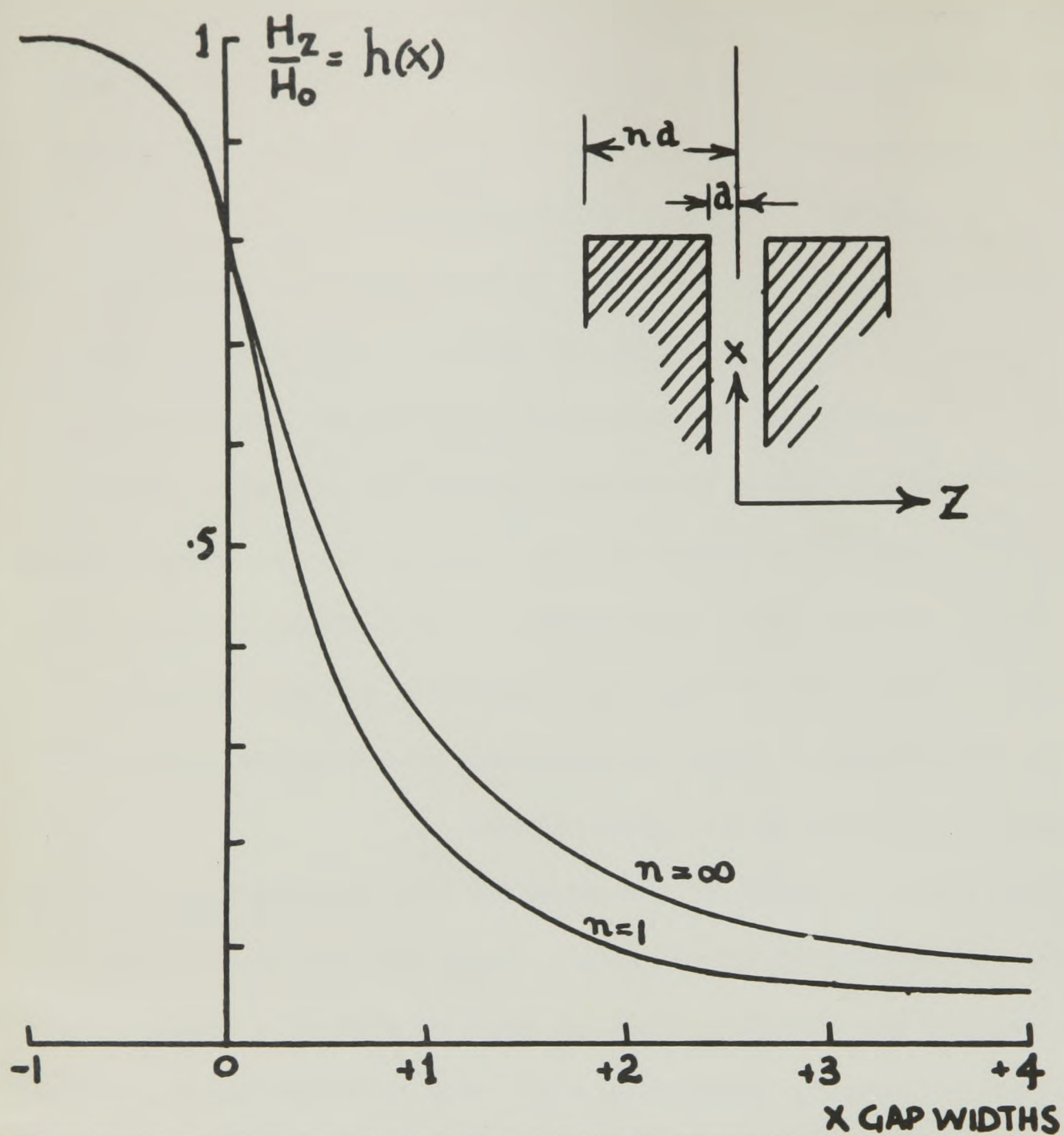


Figure B13: Ideal shape of fringe field.

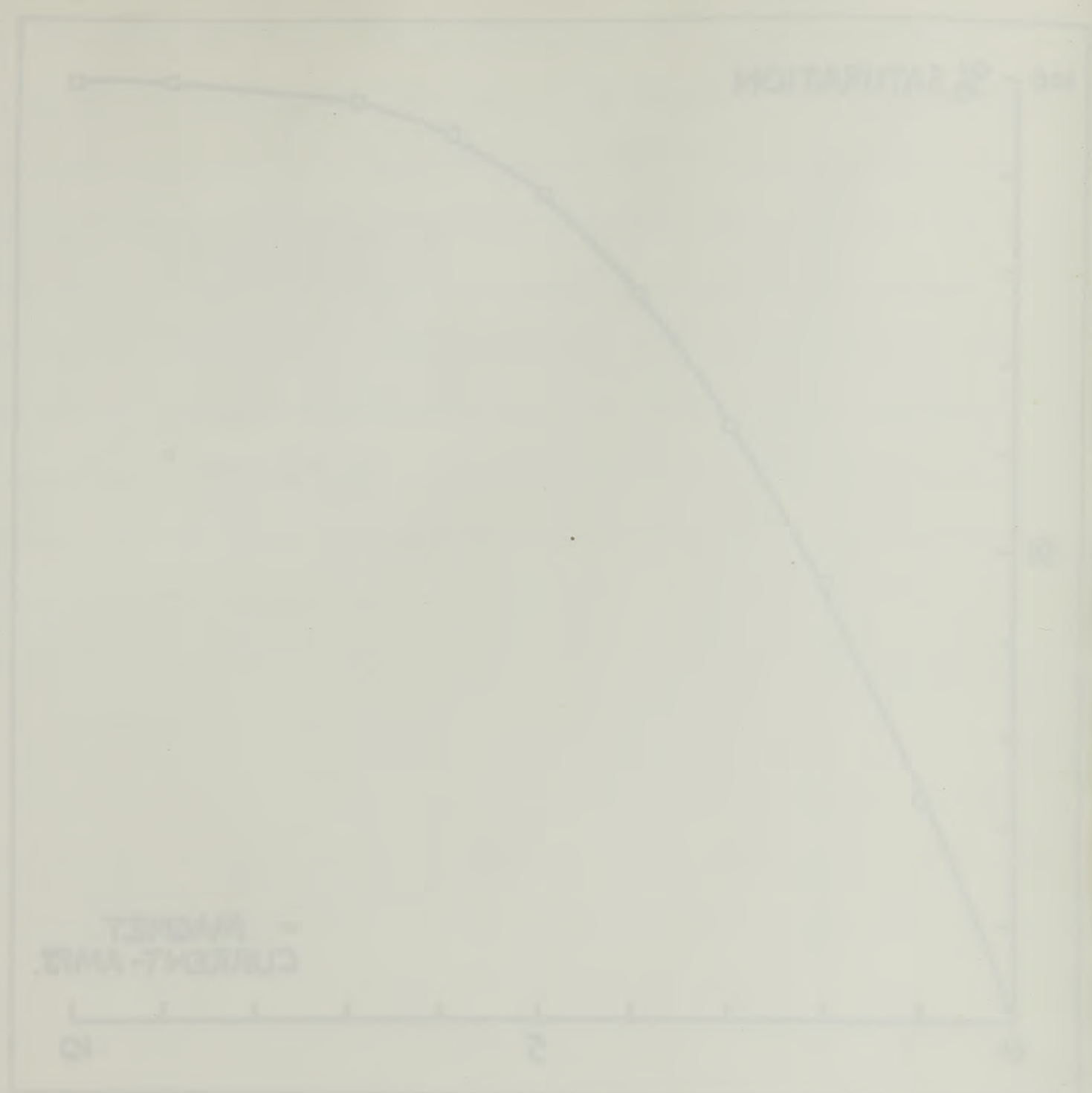


Figure 14: Saturation curve - mild steel polar.

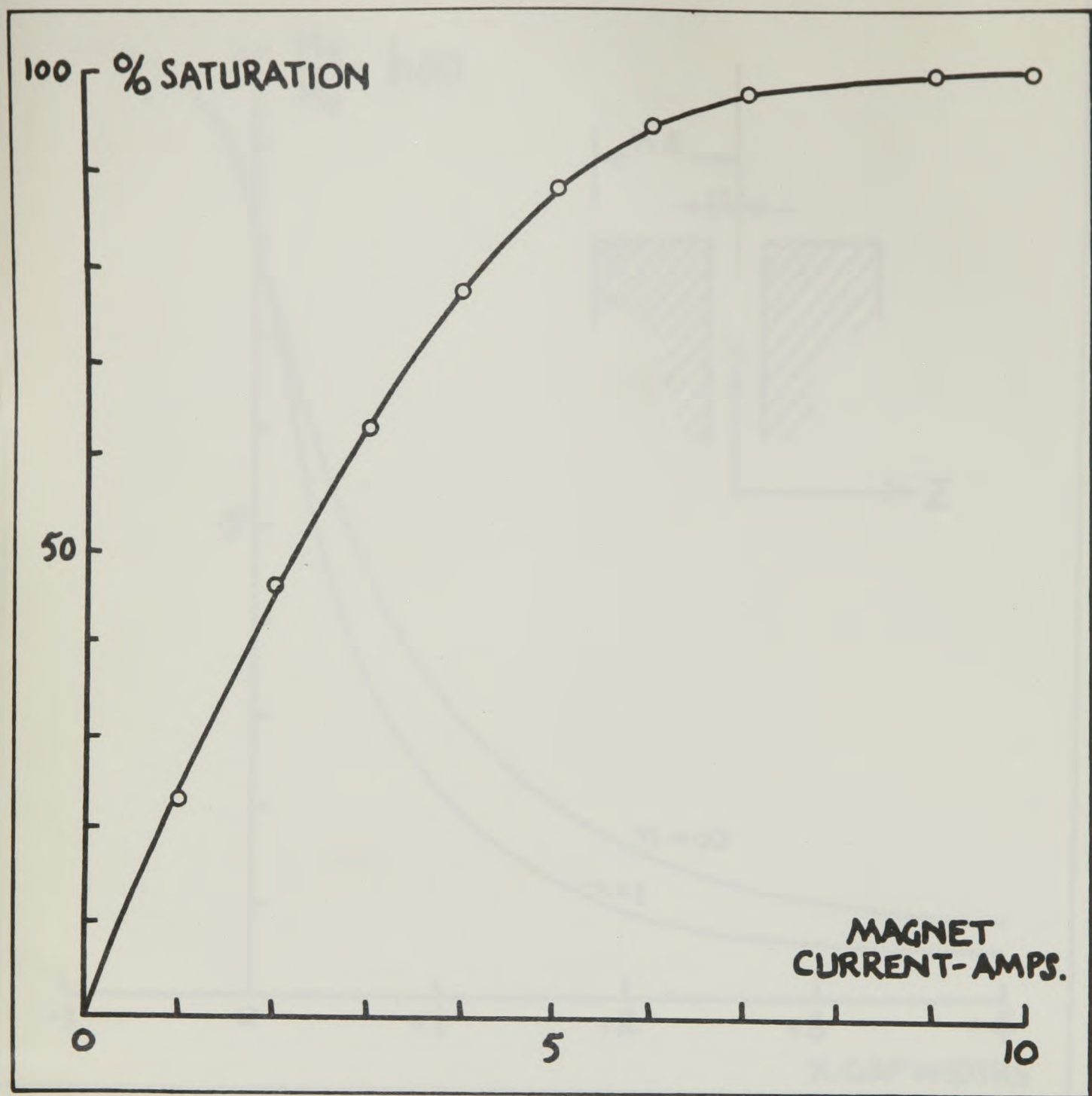


Figure B14: Saturation curve - mild steel poles.



Figure 10: The field component H_y of a plasma jet.

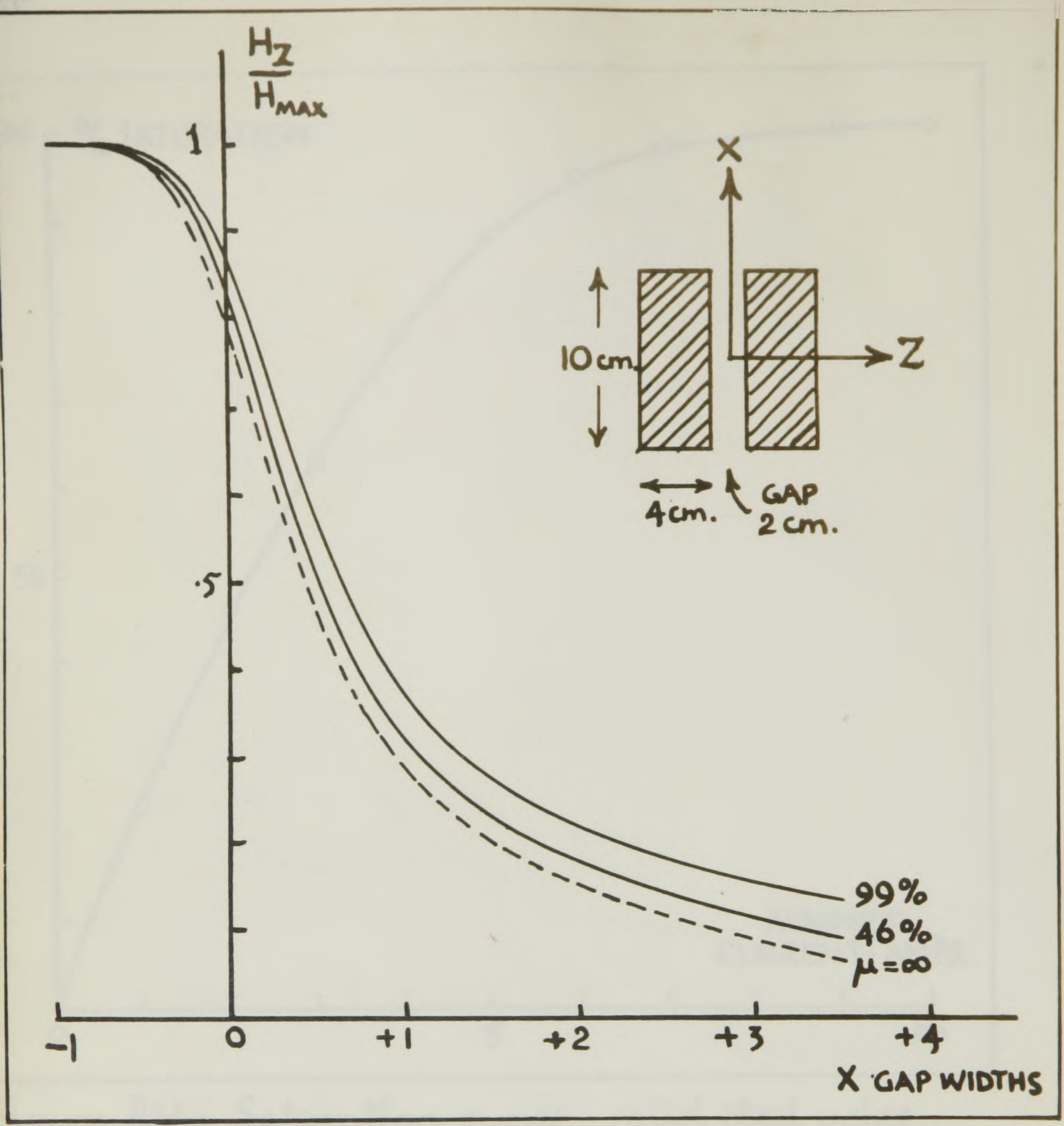


Figure B15: The field component H_z of 'square' poles.

B.

tank. Results for "thin" and "thick" poles are shown in figure 13. The unit of distance is the pole gap. Such calculations give the form of the field for iron of infinite permeability. The changes produced as the iron approached saturation were required, and poles were set up on a small magnet with 3" cores to test this. Mild steel was chosen partly because it was hoped saturation would then be attained at smaller fields before the contribution from the coils alone became too large. The poles were rectangular 3" x 4" and 4 cms deep. The gap chosen was 2cms., and the field measurements made were of H_z as a function of x . The iron was near saturation at about 8 amps (see figure 14) and the curve for this current together with curves for 2 amps and the theoretical one for infinite permeability are shown in figure 15.

The value of the integral in the equation (11) above differs for the three curves, and the amount d by which the pole boundary should be set back to correct for the effect of the fringing field changes correspondingly. For example, it is found from the curves that

$$d(\infty \text{ permeability}) = 0.84 \text{ gap widths}$$

$$d(46\% \text{ saturation}) = 1.00 \text{ " "}$$

$$d(99\% \text{ saturation}) = 1.25 \text{ " "}$$

Ions off the median plane will be affected by a fringing field component H_z the strength of which depends on the distance from the pole surface. With the same set of poles measurements were made of the fringing field in several planes between the median plane and the pole surface. In figure 16 the big increase just within the boundary for distances beyond 0.3 gap widths from the middle is shown. A different value of d is required for each off-axis position. Since this is not possible, there results a distortion of the deflected image

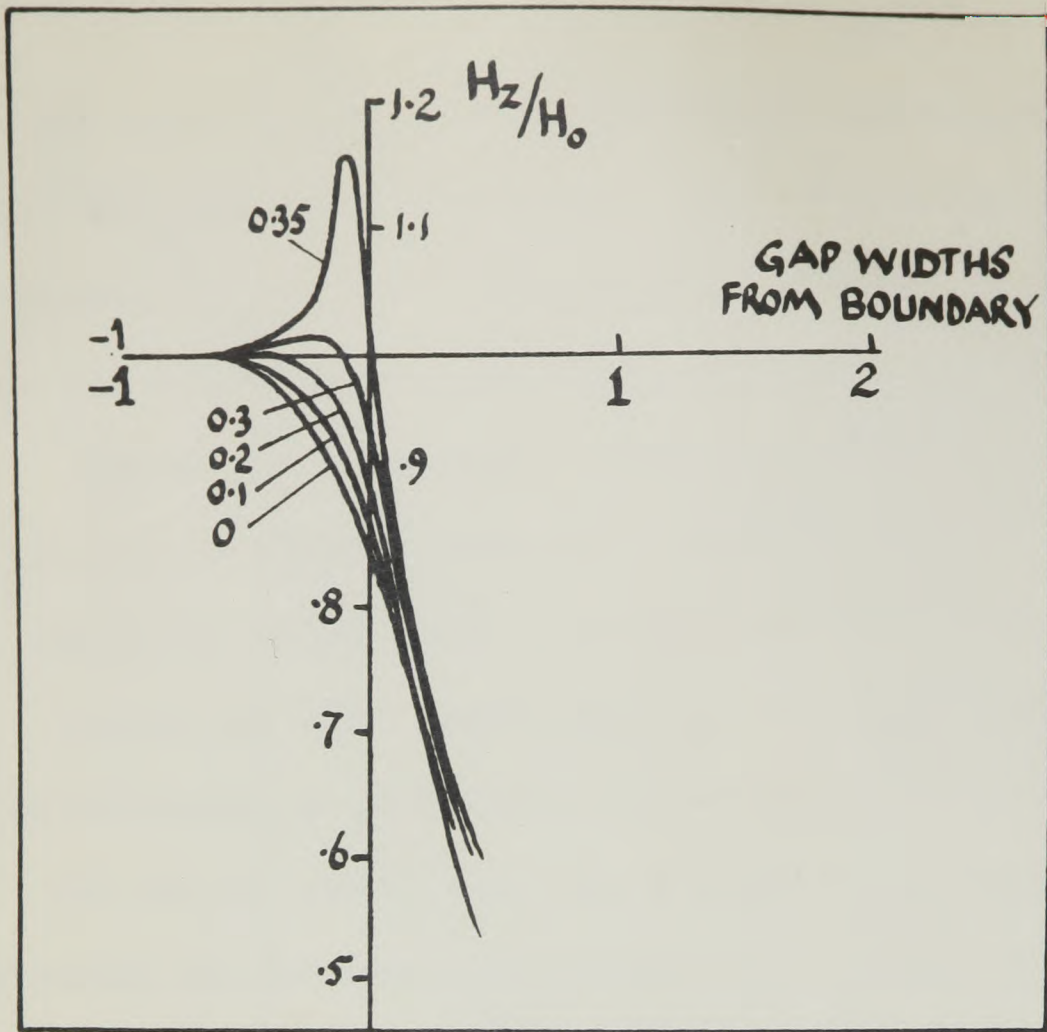


Figure B16: Fringe-field component H_z for several distances off the median plane.

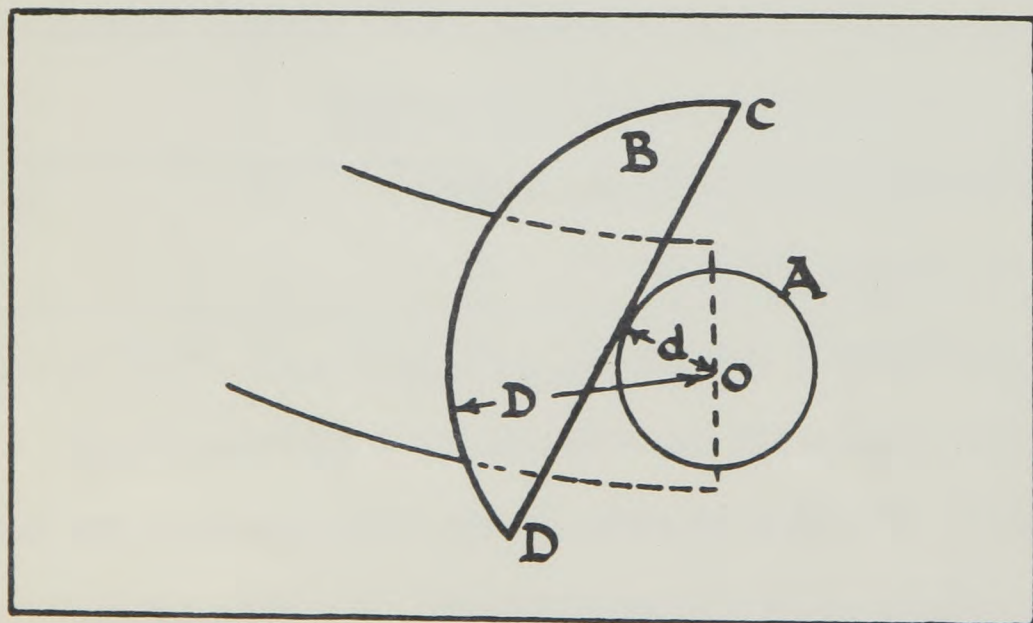


Figure B17: Geometry of the rotatable segments.

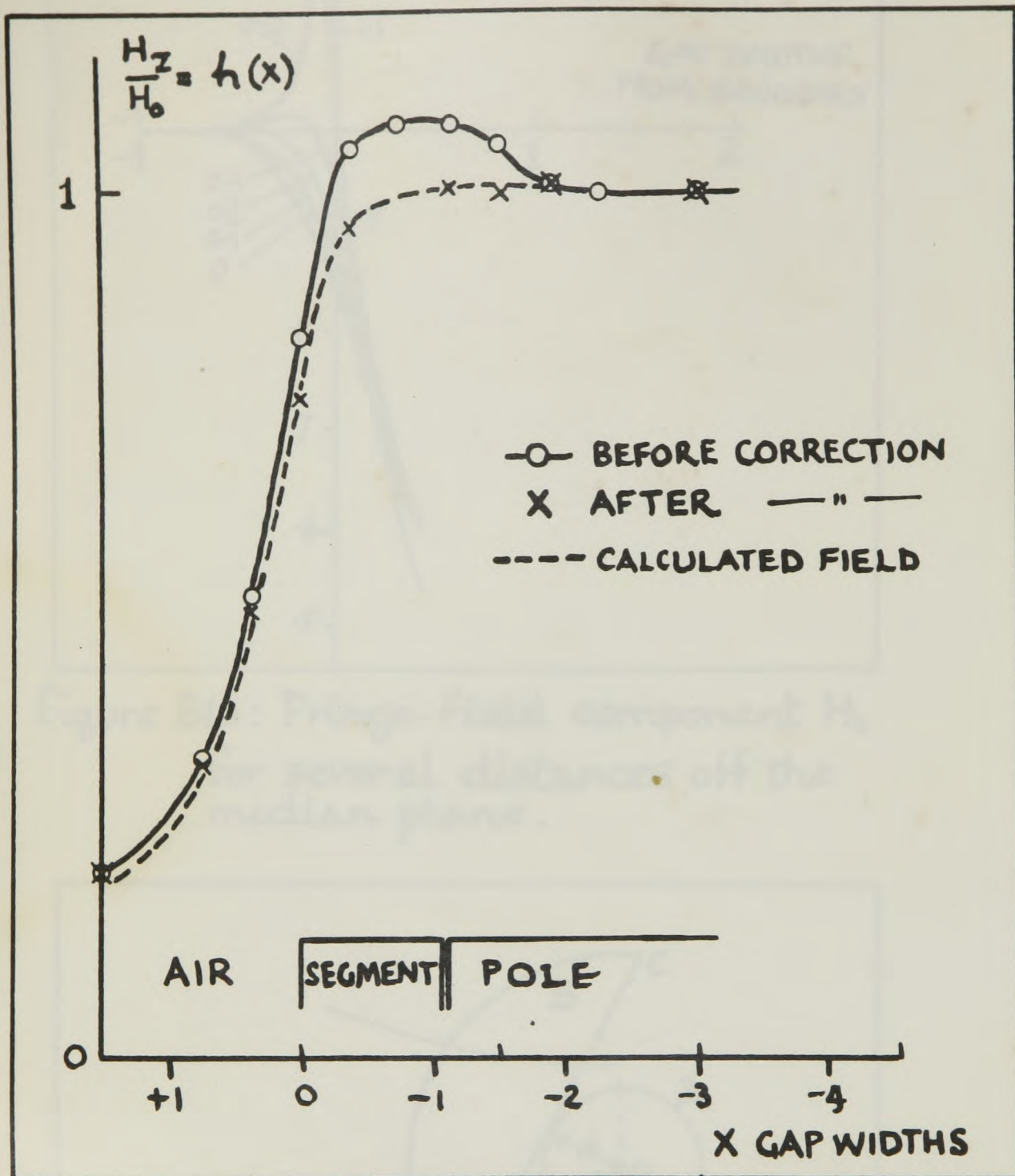


Figure B18: Correction of field between 'segments'.

B.

which must be added to the aberrations discussed in more detail in section A 12.

A value $d = 1.25$ gap widths ($= 2.5$ cms.) was used. The geometry of the rotatable segments is shown in figure 17. The extent of the uncut pole-face (i.e. without any correction for fringing) is shown as a dotted line. A circle A of radius d is drawn about the centre O of this boundary and the cylindrical segment B is pivoted so that its axis passes through O, and the plane surface touches A. The curved surface has a radius $\neq D = 5.4$ cms. chosen so that at the greatest obliquity needed (angle $\theta = 55^\circ$, see section A 8), the boundary CD still stretches right across the pole-face, and is wide enough to avoid as much as possible the defocussing effects which result when the lines of constant field strength are not parallel to the boundary over the portion of the fringe through which the beam passes. When the normal to the field boundary makes an angle θ with the beam at entry, the latter still passes through the entire significant stray field, but in the integral of equation (11) above, used for finding the position of the virtual boundary, the lengths x are increased by a factor $\sec \theta$, and thus so also must be required shift of the boundary, which with the geometry described is automatically provided.

In field measurements on a similar arrangement used for a different purpose, Ploch and Walcher¹² found an abrupt drop of several per cent at the joint between pole and rotatable segments. To cope with this if necessary the segments were left proud of the face by 2.5 % of the gap. The gap between segments was thus 5% small. The field measurements at low current (2.35 amps) are shown in figure 18. Between segments the field rises to 7.5% above the uniform portion between the faces. The

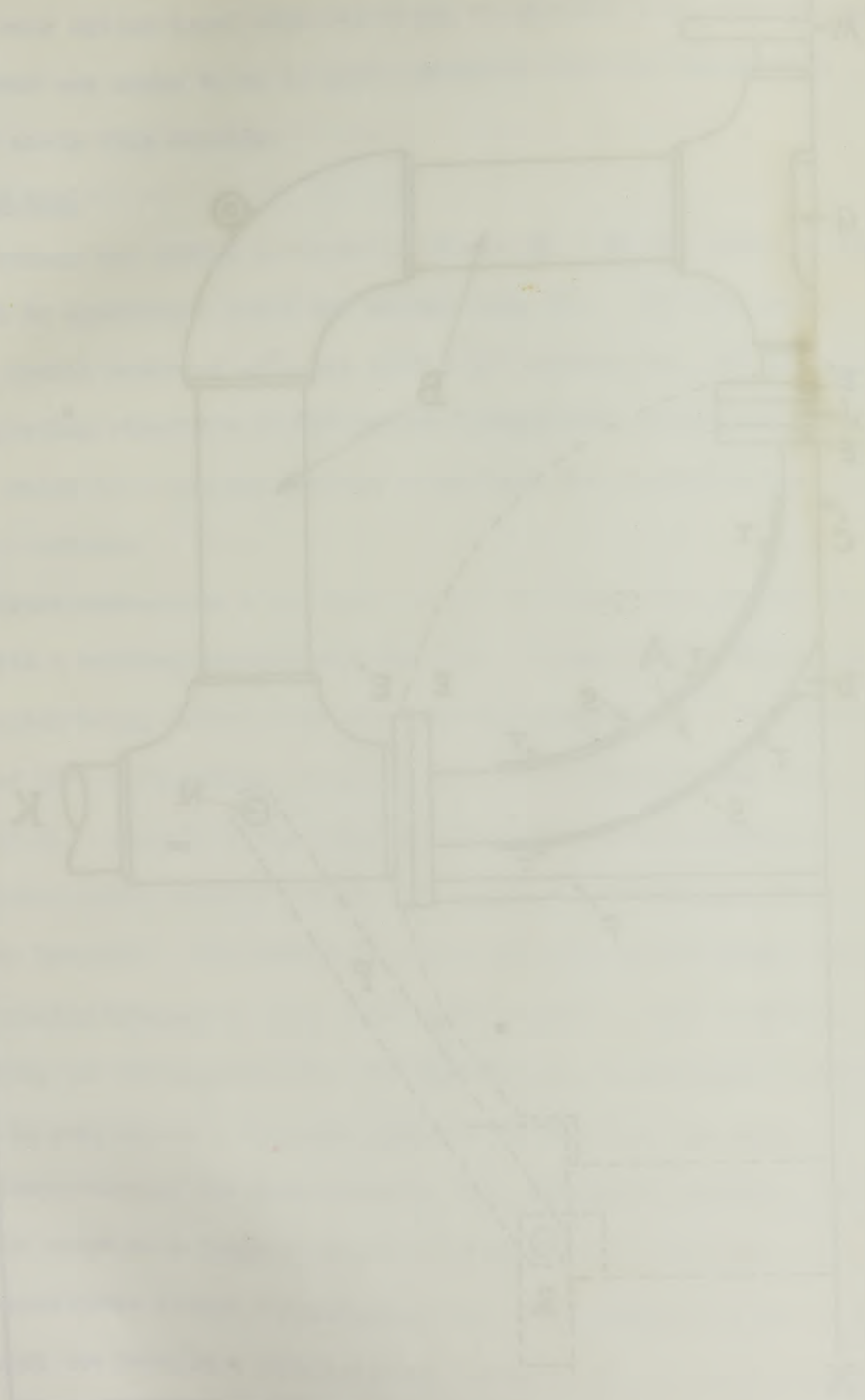


Figure 10: 10 inch diameter - minimum for and
bearing (general) - 10 inch diameter

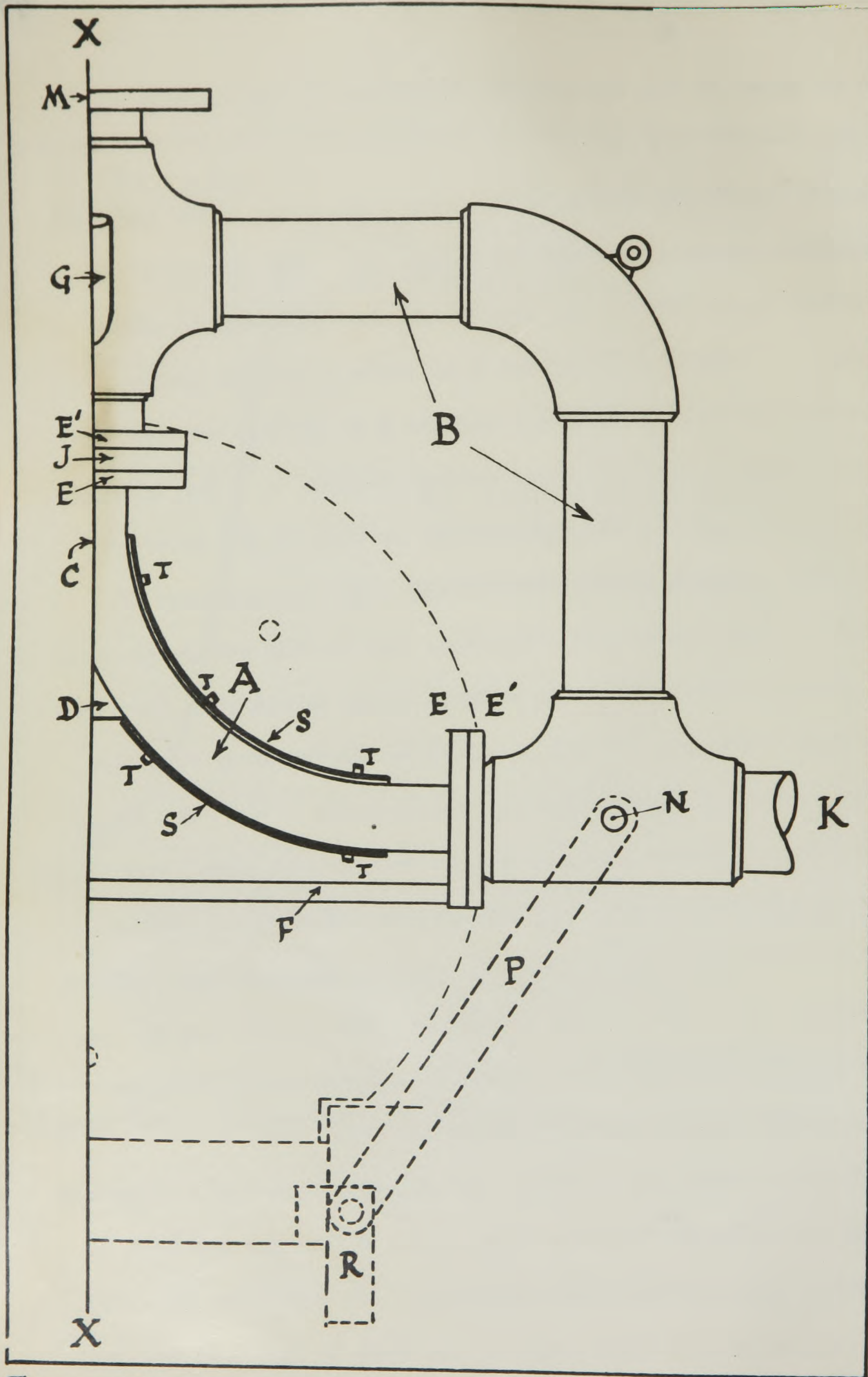


Figure B19: 19 cm. magnet - vacuum box and pumping by-pass. ($\frac{1}{4}$ scale.)

segments were milled level with the faces to correct this, and the new measurements are shown to be in good agreement with the theoretical curve for small flux density.

(v) Vacuum Box.

The vacuum box design is shown in figure 19. In the plane of the drawing it is symmetrical about the centre line XX. The main parts are (A) a double quadrant of brass $3/4"$ x $1\frac{1}{2}"$ wave-guide, and (B) a rigid supporting structure of $2\frac{1}{2}"$ copper "Yorkshire" tubing and brass fittings, which is a pumping by-pass route from the target to the accelerator column.

The brass wave-guide A was bent on the flat to a mean radius of 19 cms. with a bending machine made for the purpose in the workshops, the wave-guide being filled with lead and then bent cold. Two quadrants were joined with hard solder along the seam C and strengthened with the brass fillet D. Flanges E are fitted to the ends of the wave-guide to make vacuum seals through Edwards "O" rings to the flanges E' of the pumping by-pass. The structure is braced with a $5/8"$ brass rod F.

The pumping by-pass is made from short lengths of $2\frac{1}{2}"$ Yorkshire copper tubing and the appropriate fittings (cross, right-angle bends and T-pieces) in cast brass. A glass window G fitted into the cross is used for inspection of the beam entering the wave-guide section, and to observe its shape on a tungsten shutter which can be moved from outside the half-inch brass flange J containing it. This shutter moves so as to intersect the beam in a line parallel to the plane of the drawing. Beyond the T-piece at K there are shut-off vacuum taps. These were especially made by the Saunders Valve Company with aluminium bodies for lightness, and with non-ferrous components inside to avoid magnetic

B.

distortion of the beam. They are the $1\frac{1}{2}$ " diameter straight-through pattern normally used for treacle and other viscous liquids, but in this case fitted with diaphragms suitable for making a vacuum seal. The outside flanges of the taps and the top flange M of the cross are of a type previously made standard in the Laboratory for components to be joined to the high tension accelerator columns.

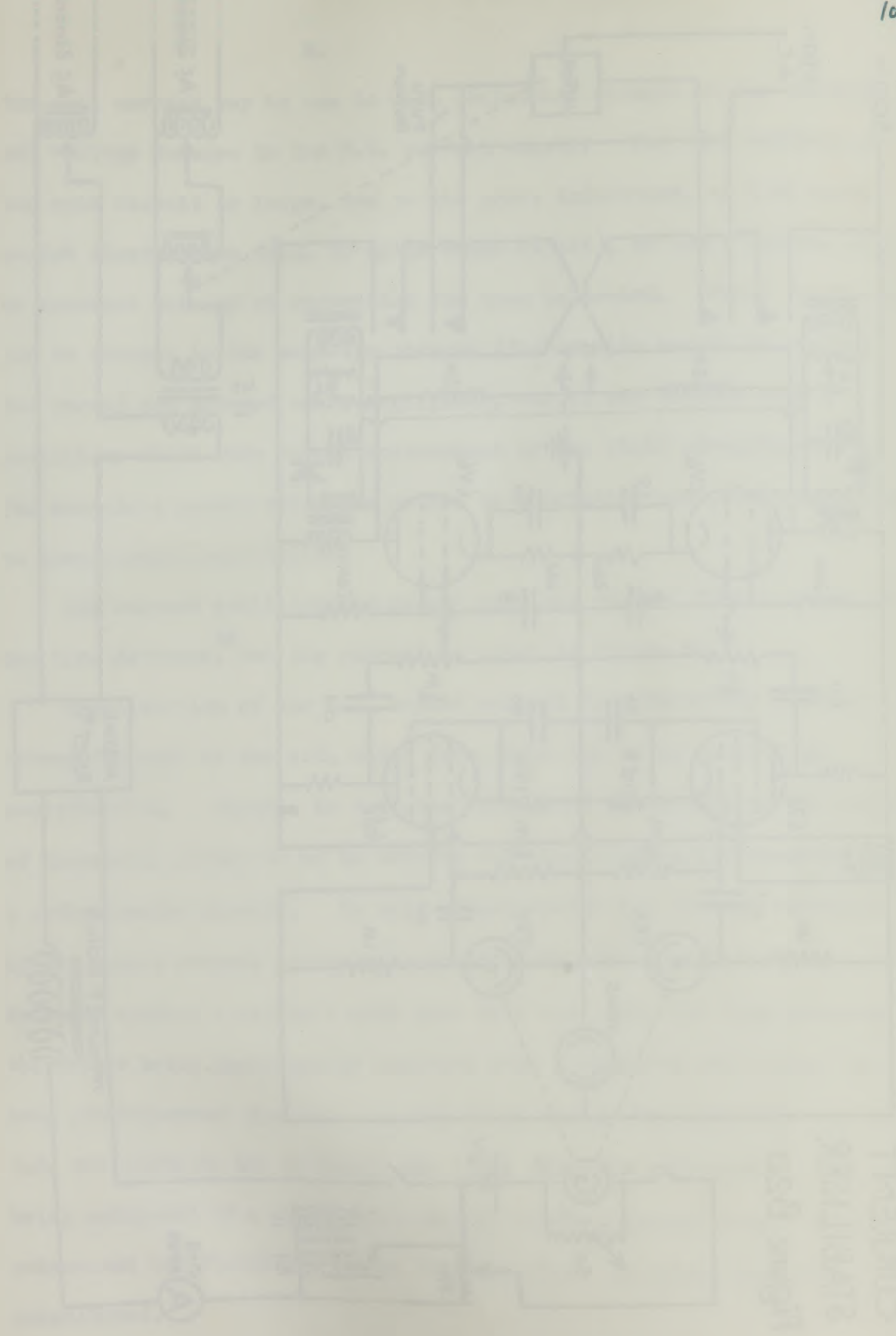
From the T-piece to the centre of the cross there are two pumping routes in parallel. The one through the by-pass, has a speed of about 85 litres per second for air, and the wave-guide has a speed of 6 litres per second, so that in clearing gas from a target about 93% is taken through the by-pass.

The whole vacuum box is supported by brass trunions N and struts P from blocks R bolted to the bed of the magnet yoke. (In figure 19 the pole pieces, bed and supports are shown by dotted lines.) Brass strips $3\frac{3}{4}$ " wide are spaced off from the curved sides of the wave-guide and can be fastened down by nuts T to clamp the pieces of $1/8$ " soft iron wire used as an empirical correction to the non-uniformity of the field.

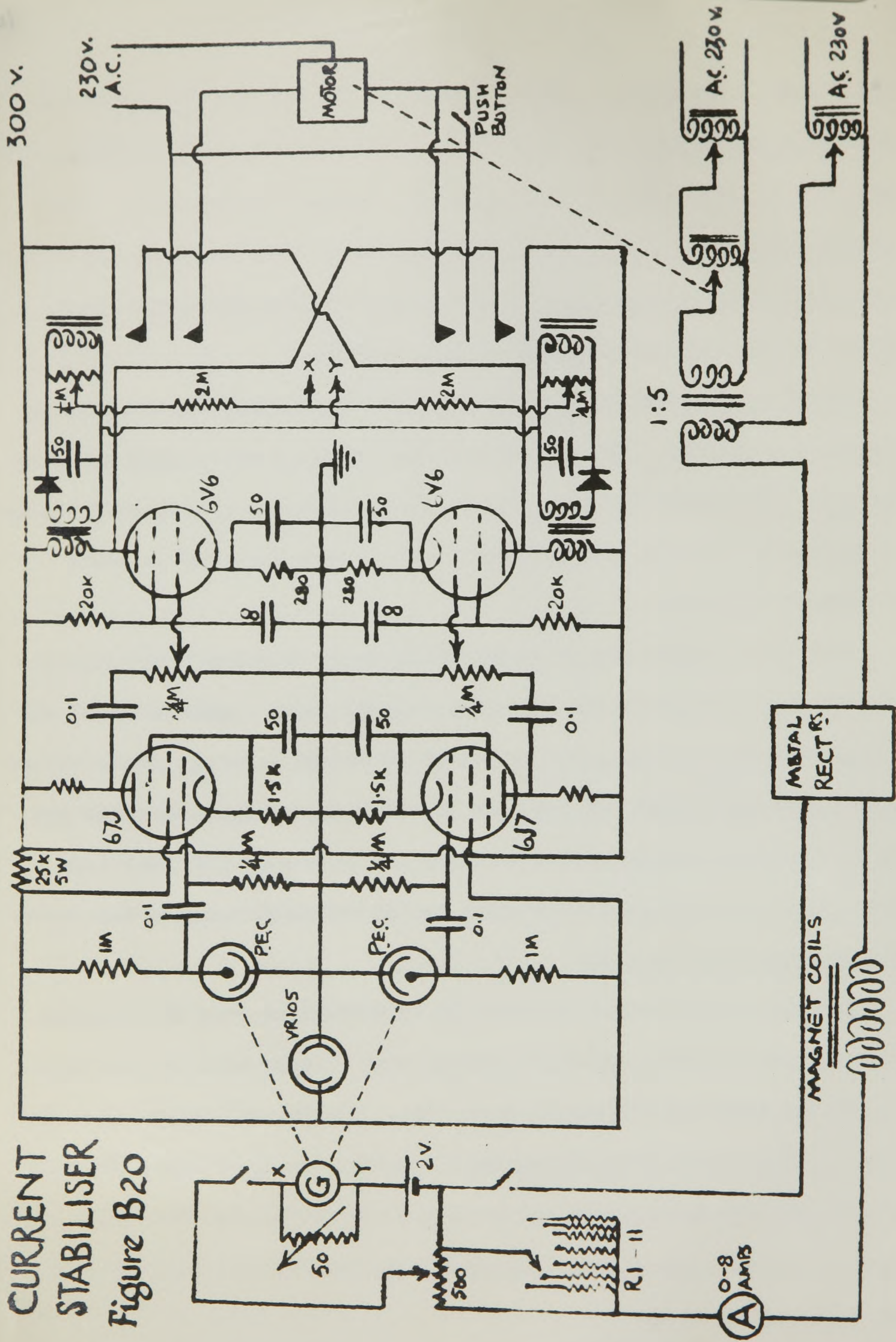
(vi) Current Stabilisation.

For most experimental purposes it is sufficient that the magnetic field should be stable enough to ensure that a large beam current passes through the limiting diaphragms or canals. Better stabilisation would be needed if accurate beam energy measurements were to be made, but such measurements are in any case unlikely to be achieved with the types of magnetic analyser described, largely due to uncertainty about the exact path taken by the beam.

Fluctuations in the magnetic field strength caused by variations in



CURRENT
STABILISER
Figure B20



B.

the coil current may be due to both resistance changes in the winding and voltage changes in the D.C. current supply. The time constant of the coil circuit is large, due to the great inductance, so that short period fluctuations (e.g. 50 cycle mains ripple), or short pulses, will be smoothed out and no correction for them is needed. Field fluctuations due to changes in the magnetic circuit (for example caused by heating the cores) are assumed to be negligible, and it was thought that a stabilizer which made direct measurement of the field strength, such as for example a proton resonance method or a rotating coil, would introduce an unnecessary complication.

The current stabilisation method used was adapted from a design of Mr. T.O. Jeffreys,²⁵ and the circuit is shown in figure 20.

Stabilisation of the D.C. magnet current is effected by a motor driven 'variac' in the A.C. mains from which the DC is derived by rectification. Signals to the motor are given by amplifying the output of photocells placed so as to receive the light from a galvanometer in a potentiometer circuit. In this potentiometer the voltage developed by the magnet current across an appropriately chosen resistance is balanced against a stable 2 volt lead cell used only for this purpose, the latter being occasionally compared with a standard cell using the same potentiometer circuit. In the first design the amplifier was the A.C. one shown in the diagram; the light from the galvanometer lamp being modulated by a small electrically driven chopping disc. In a subsequent modification a steady light and D.C. amplifier have been substituted.



Fig. 1. Design for a magnet with triangular cross.

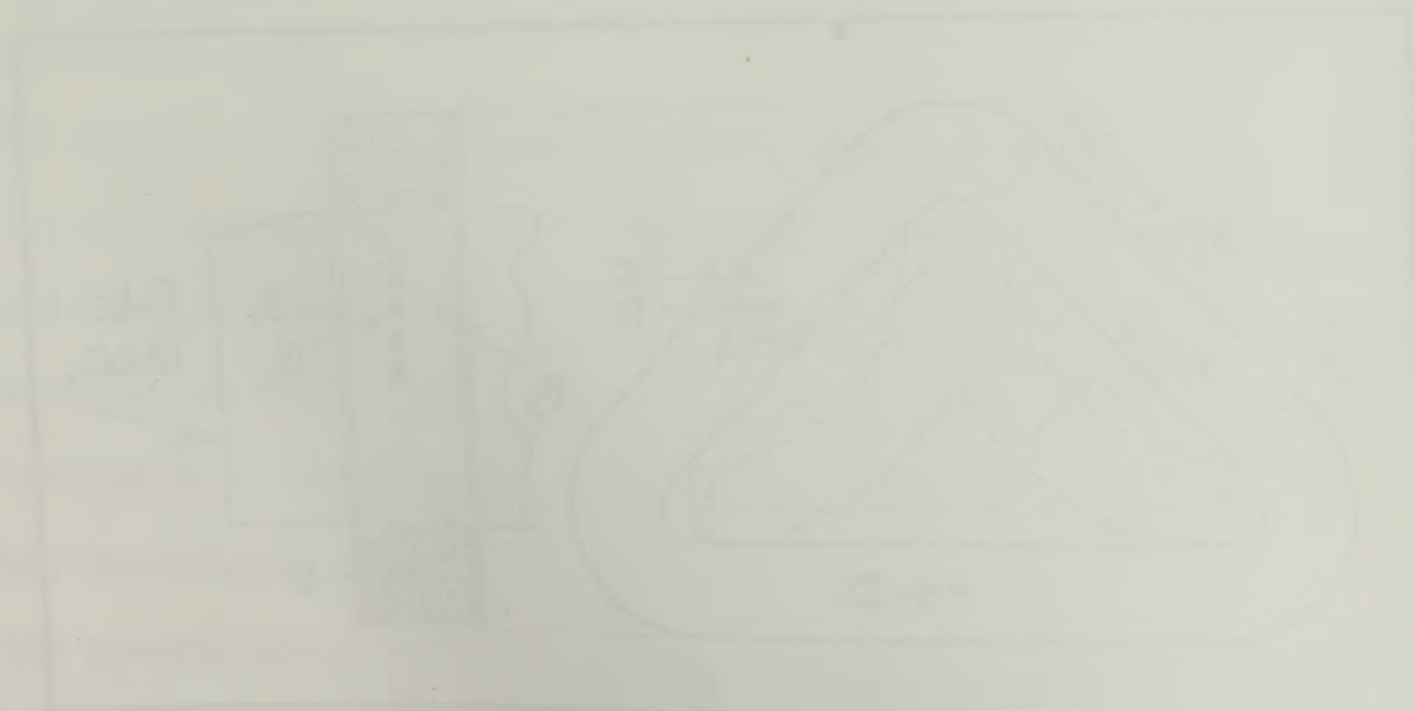


Fig. 2. Dimensions of core and coils, and position of thermopiles on its surface.

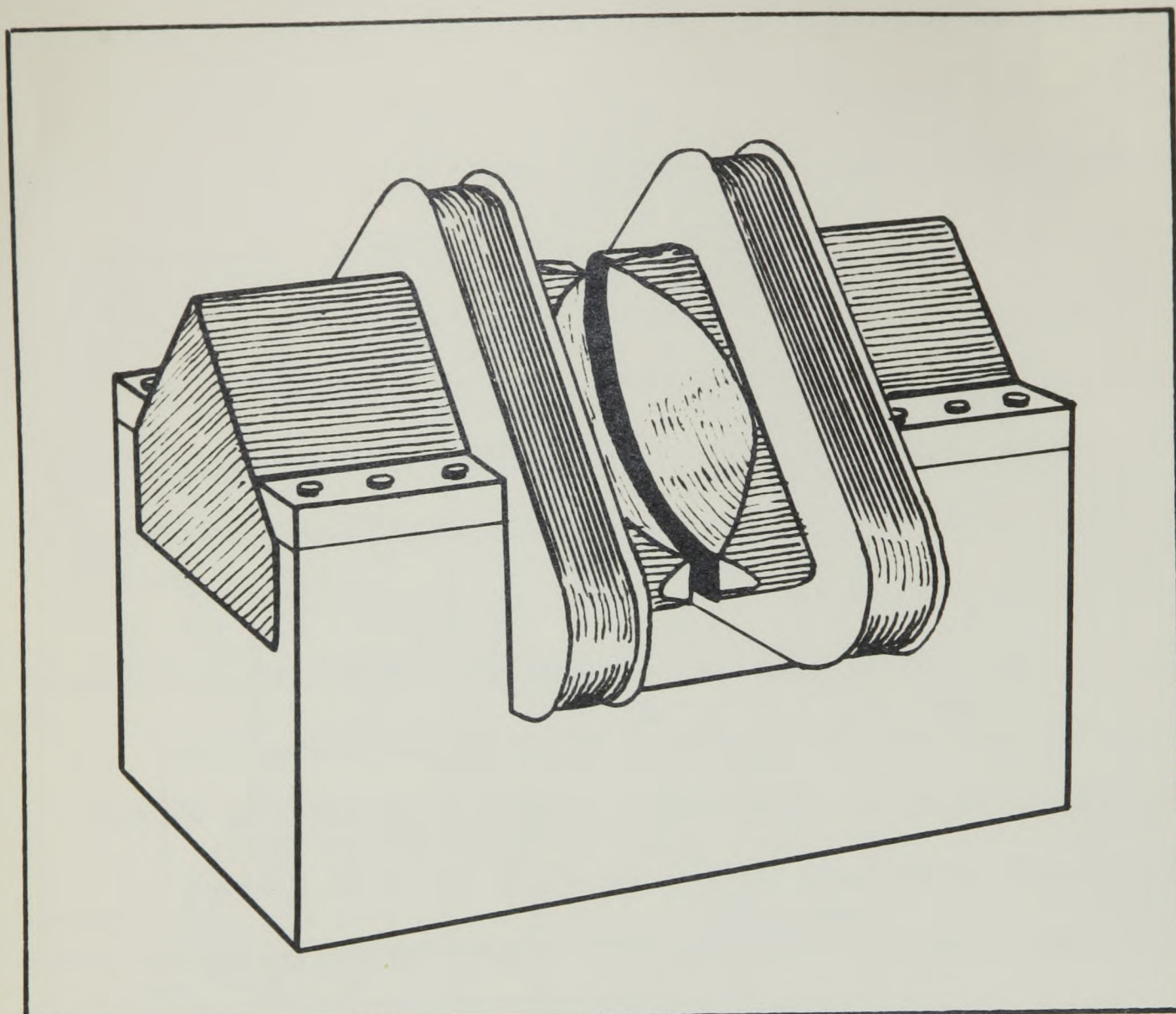


Figure B21: Design for a magnet with 'triangular' cores.

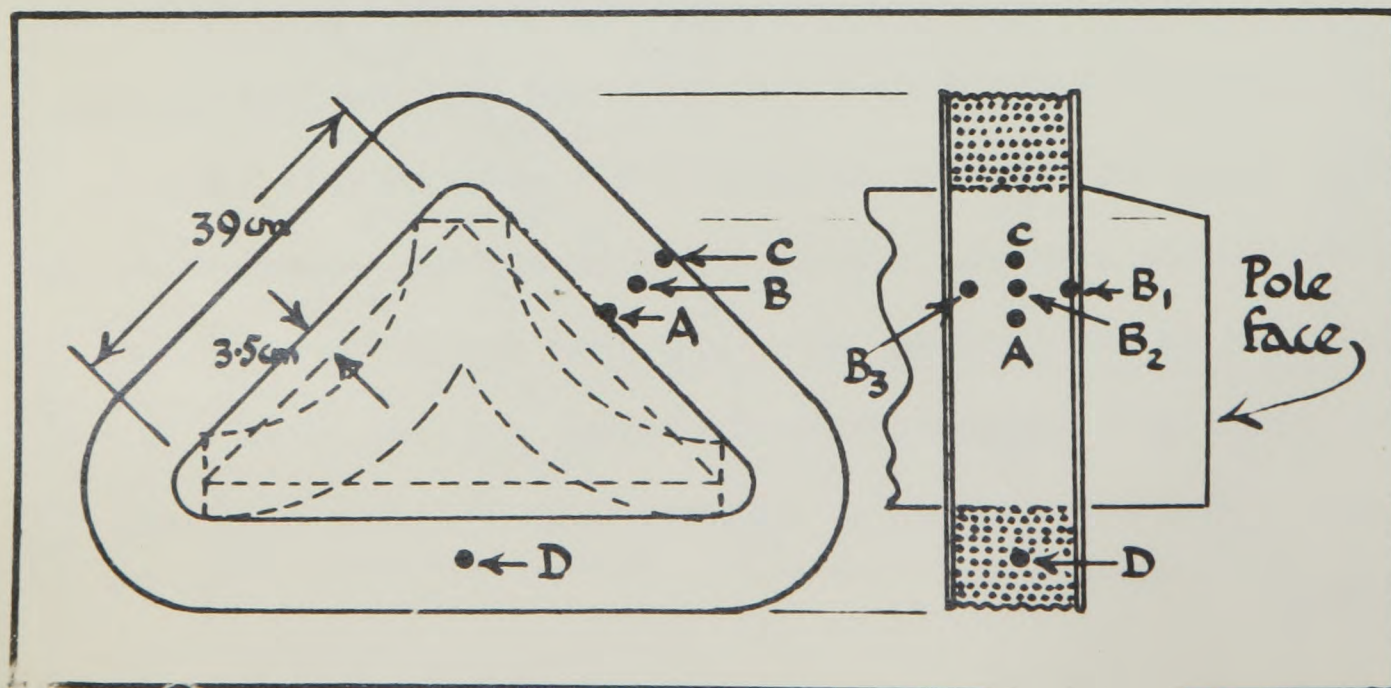


Figure B21a: Geometry of cores and coils, and position of thermocouples.

B.

3. 30 Cm. (triangular) magnet design.

(1) Design Data.

In Part A it was shown that there are advantages in a large quadrant radius. In section 1 however it was mentioned that too large a radius introduces difficulties associated with flux losses when the magnet coils have cylindrical cores. For a single quadrant a core and winding which closely follow the shape of the pole-face is better, but for the double quadrant type of pole-face this cannot be conveniently arranged. A triangular core and windings seem^a satisfactory compromise, since the method of calculation for a cylindrical-core magnet can be applied without such change and the core cross-section follows fairly closely the pole-face shape.

If the quadrant radius is r then the double quadrant will fit entirely within a triangle of area r^2 (refer back to figure 1 (b)). If we were to use a cylindrical core of the same area, the radius of the circular cross-section would be $a = \sqrt{\frac{r^2}{\pi}}$, and the pole-face would overhang the core by $r_0(1 - \frac{1}{\sqrt{\pi}}) = 0.45 r_0$. That is, nearly half the pole-face would lie outside the core.

The ratio of triangular core to pole-face area will be $\frac{r_0}{\pi w}$ where w is the width of the pole face. This ratio must not be too large if the pole-face and core are of the same steel or the flux density in the core will not be near the value for maximum permeability. Allowing for flux leakage from the core sides, a value of the ratio about 2 or 3 is probably reasonable.

The general shape and construction proposed for a magnet of this type is shown in figure 21. The details of the magnet designed were arrived at in the way outlined in the next paragraphs.

B.

The basic figure chosen for the design of the magnet is $(B\rho)_{\max} = 3 \times 10^5 \text{ g.cm}$

From an extension of the curves of figure A1 this is seen to be sufficient to deflect beams of singly charged ions of mass 3 and energy 1.4 Mev.

To avoid saturation of the poles

$$B = 10,000 \text{ gauss}$$

and hence

$$Y_0 = \rho_0 = 30 \text{ cm.}$$

A convenient width is

$$w = 5 \text{ cm.}$$

with $w = 5 \text{ cms}$ a vacuum box formed from $1\frac{1}{2}'' \times 3/4''$ wave guide can be used,

so that the gap length should be about

$$l = 2 \text{ cm.}$$

With a ratio $w/l = 2.5$ it should be fairly simple to maintain a uniform field across the vacuum box, since there will be portions of the face 0.6 cms wide on either side of the wave guide which can be machined to form correcting lips of the appropriate shape.

A double quadrant pole-face with $Y_0 = 30$, $w = 5 \text{ cms}$. will have an area

$$A_{\text{gap}} = 4,20 \text{ cm.}^2$$

The leakage flux near the pole-face is estimated roughly as $1/3$ of the useful flux.

Then, useful flux gap

$$N_{\text{gap}} = 4.2 \times 10^6 \text{ lines}$$

and flux in core

$$N_{\text{core}} = 5.6 \times 10^6 \text{ lines}$$

The core cross-section, as shown in figure 21(a), is based on a right-angled triangle of short side 39 cm. and with a 3.5 cm border.

The core area is

$$A_{\text{core}} = 1260 \text{ cm}^2$$

and the flux density is

$$B_{\text{core}} = 4500 \text{ gauss}$$

The magnetisation curves show that using the David Brown magnet steel proposed for the cores, the permeability is then

$$\mu_{\text{core}} \approx 4000$$

For mild steel the permeability has a maximum at about $B = 4000$, so for a mild steel yoke we should have

$$A_{\text{yoke}} \approx 1.4 \times 10^3 \text{ cm}^2$$

The yoke dimensions chosen are 60 cms. x 25 cms (or 24" X 10" if it is to be built from 2" or 3" laminations). The length of the yoke uprights is determined by the coil sizes.

The reluctance of the magnetic circuit is almost entirely due to that of the airgap, which is $R_{\text{gap}} = 5 \times 10^3$ pract. units.

Then assuming a perfect magnetic circuit the magneto-motive force required is

$$ni = 2.2 \times 10^4 \text{ amp-turns}$$

If the current is

$$i = 10 \text{ amps}$$

the number of turns is

$$n = 2200$$

or 1100 turns per coil.

Calculation shows that 12 SWG is a convenient wire size. Using this, with single glass insulation, a winding of 1085 turns in 31 layers each of 35 turns has a resistance of about 4.5 ohms. The winding then has a cross-section 9 cms. deep by 10 cms. wide and the area available for cooling is about 5500 cms². If the effective emissivity is 1.6×10^{-3} watts/cm/^oC temperature excess, the coils should run at a maximum temperature rise of about 65° centegrade and no forced cooling is needed.

The total weight of the magnet would be about 2 tons, made up as follows:

yoke: 3,100 lbs.

cores : 1,300 lbs.

coils: 300 lbs.

(ii) Scale Model.

A quarter-scale model was made to test some features of the design. If the voltage applied to the windings of the model is the same as for the full-sized magnet, then the same number of turns is needed to provide the same field in the gap. The total length of wire, its diameter, and

the current are divided by four, so that the resistance and power dissipated per unit area are multiplied by four. Some form of forced cooling is thus needed if the model is to be a useful representation at high fields, and a water cooling channel is soldered to the inner coil cheeks. The yoke and cores are both made from mild steel. This tends to saturate at smaller fields than the soft steel which would be chosen for the cores of the full-sized magnet, but it is possible to allow for this difference in making field measurement.

a) Heating of coils. Copper-constantan thermocouples were buried in the windings at the positions A, B₁, B₂, B₃, C and D shown in figure 21(a). The results, figure 22, show that the centre of the winding on any side of the triangle is some 10% hotter than the outside of the windings, and about 50% hotter than the turns adjacent to the inside of the former.

Measurements of the mean temperature were also made by including the coil in a bridge circuit and calculating from the resistance change. The temperature derived in this way is close to that found by the couples in positions B₁ B₃ C - i.e. the outside turns of the coil. This is in agreement with the measurements of Windred and others.

The equilibrium temperatures on the outsides of the coil are consistent with an effective emissivity of about 2×10^{-3} watts/cm²/°C. This is larger than given in the literature for glass covered wire or matt copper surfaces and is probably explained by heat conduction from the inside of the coil-former to the core itself. This seems likely in view of the low temperature recorded by thermocouple A.

It is important to know what temperatures would be recorded at similar positions on a full-sized coil. Consider 2 similar coils 1 and 2 with

with linear dimensions in the ratio $l_1/l_2 = a$ and assume as a good approximation for each that the surface temperature is the same as that found using the winding as a resistance thermometer.

For the same field in the gap the currents are in the ratio $i_1/i_2 = a$ and for these currents let the equilibrium temperatures be t_1 and t_2 °C above the common ambient temperature, when the cooling is by conduction and natural convection and considered proportional to t .

The resistances are then

$$R_1(t_1) = R_1(0)(1 + \alpha t_1) \quad (12)$$

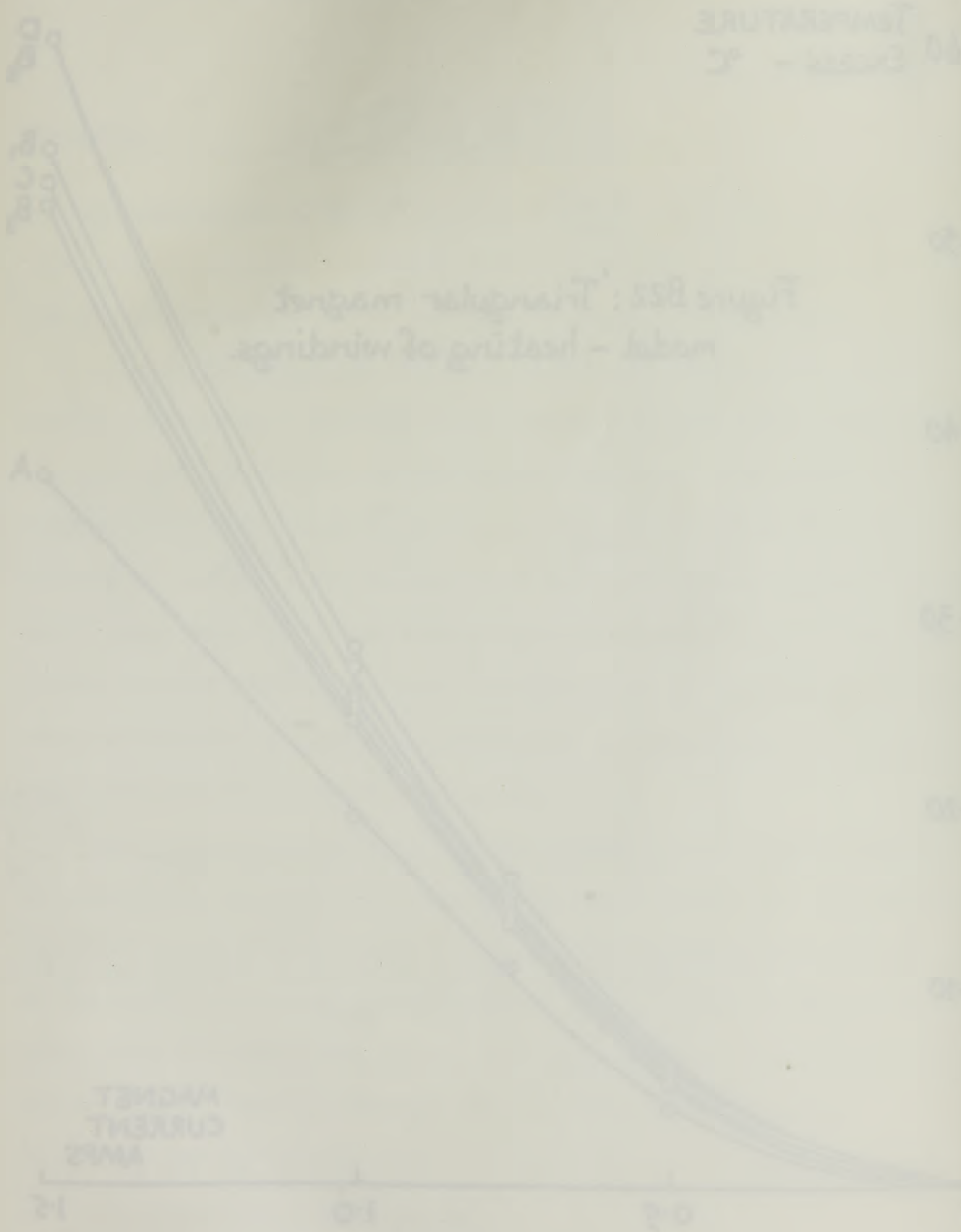
$$R_2(t_2) = R_2(0)(1 + \alpha t_2)$$

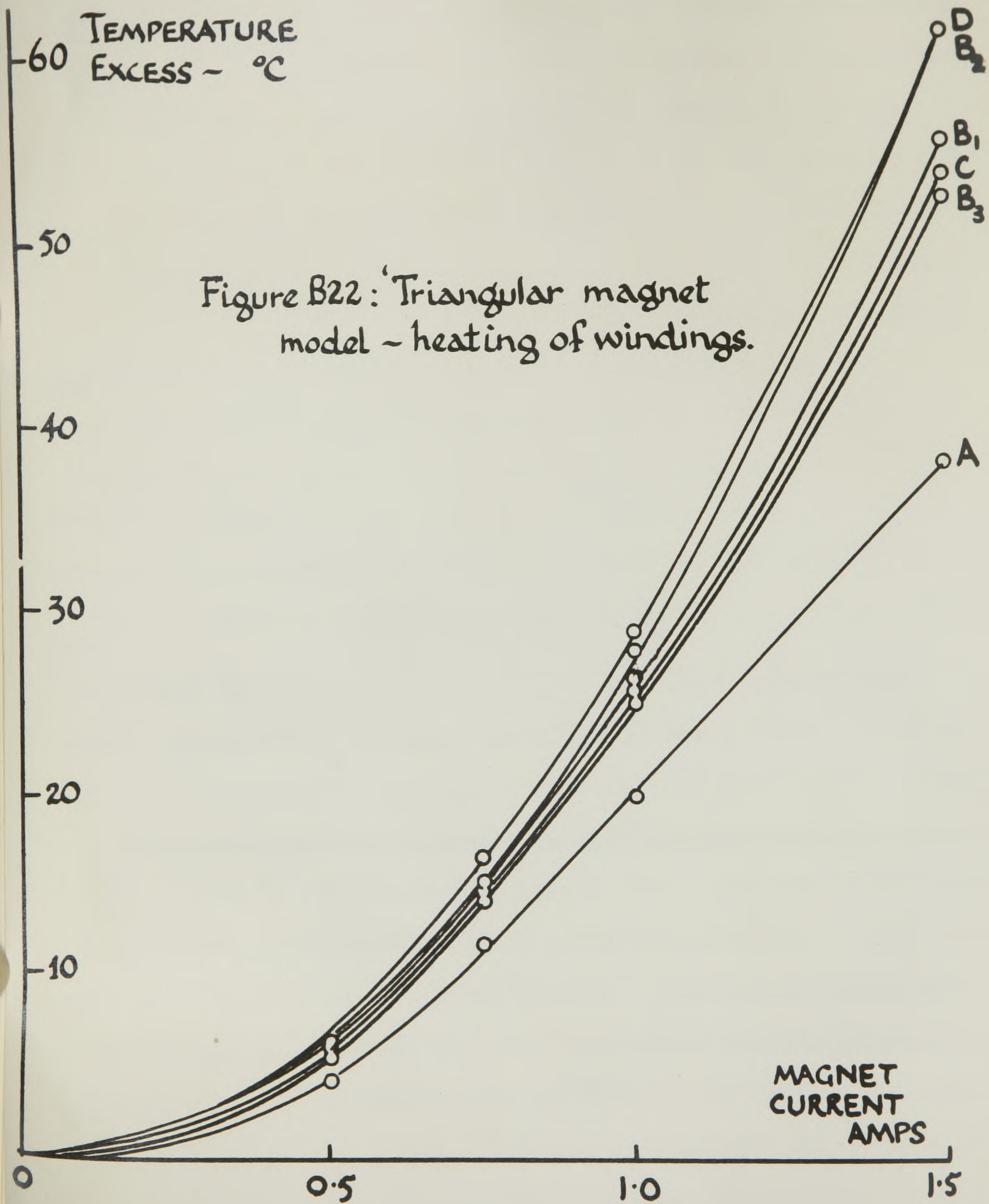
Also
$$R_1(0)/R_2(0) = 1/a \quad (13)$$

In the steady state
$$t = i^2 R(0)/EA - i^2 R(0)\alpha \quad (14)$$

hence
$$\frac{t_1}{t_2} = \frac{EA_2 - i_2^2 R_2(0)\alpha}{aEA_2 - i_2^2 R_2(0)\alpha} \quad (15)$$

Thus for very small currents the temperatures, for the same field, are in the inverse ratio of the linear dimensions of the coils. Since the current is being kept constant despite increases in resistance, the ~~current is being kept constant despite increases in resistance, the~~ temperature of a coil rises toward infinity when $EA = i^2 R(0)\alpha$. $\frac{EA}{\alpha i^2}$ has the same value for both coils, but $R(0)$ is inversely proportional to the linear dimensions. Thus the smaller the model the more rapidly the limiting current is reached. With the quarter-scale model of the triangular magnet, the limiting current is about 2.5 amps corresponding to about 10 amps (the maximum design current) on the full-sized magnet. However currents up to about 1.5 amps can safely be used, so that full-size conditions up to 6 amps can be represented. Without a bigger model these circumstances cannot be improved.





For the following figure (1) and (2) are the same

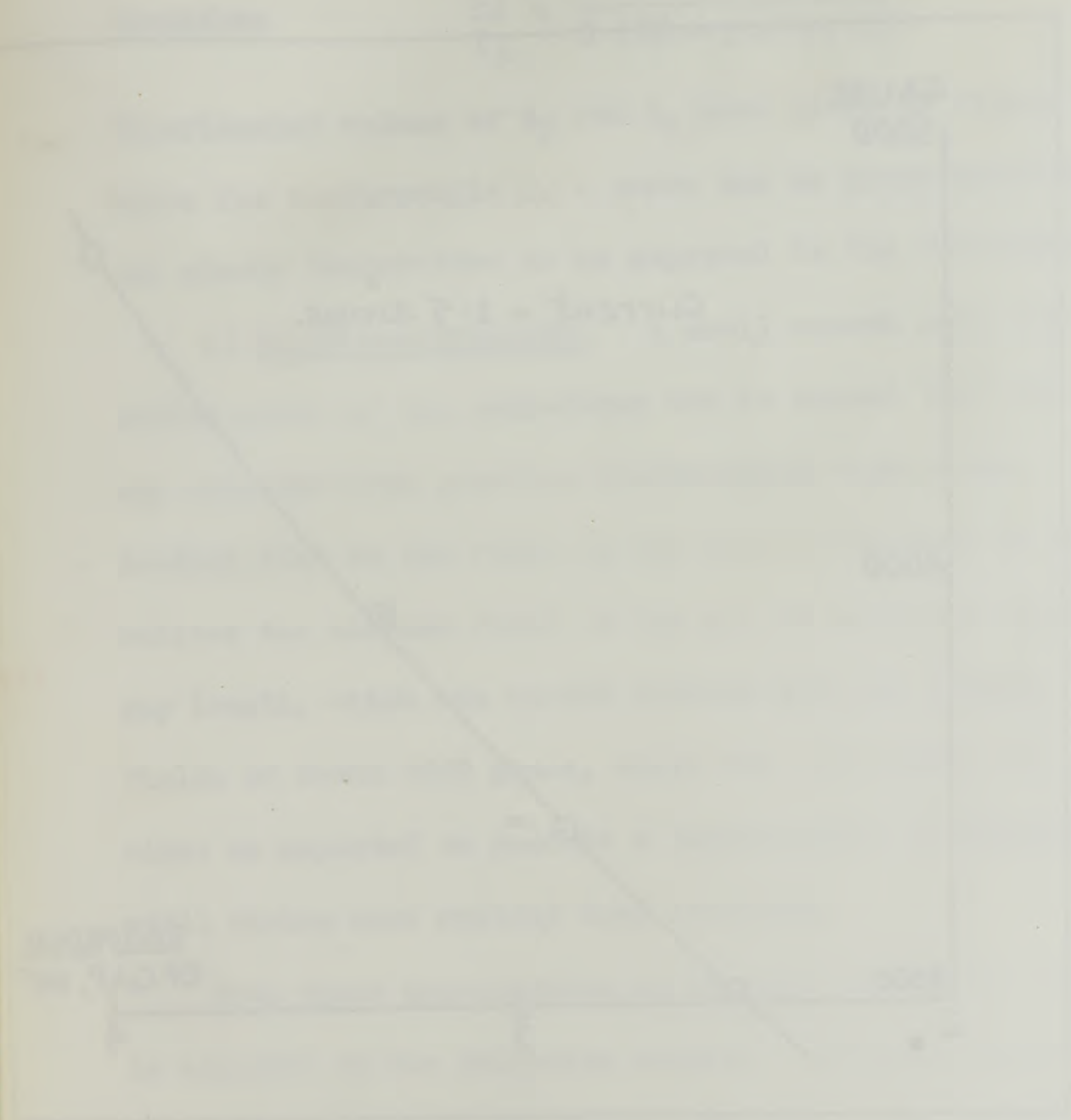


Figure 1.1: Graph of Current (A) versus Voltage (V) for a resistor.

The graph shows a linear relationship between Current (A) and Voltage (V). The line passes through the origin (0,0) and the point (10, 2.0). The equation of the line is $I = 0.2V$.

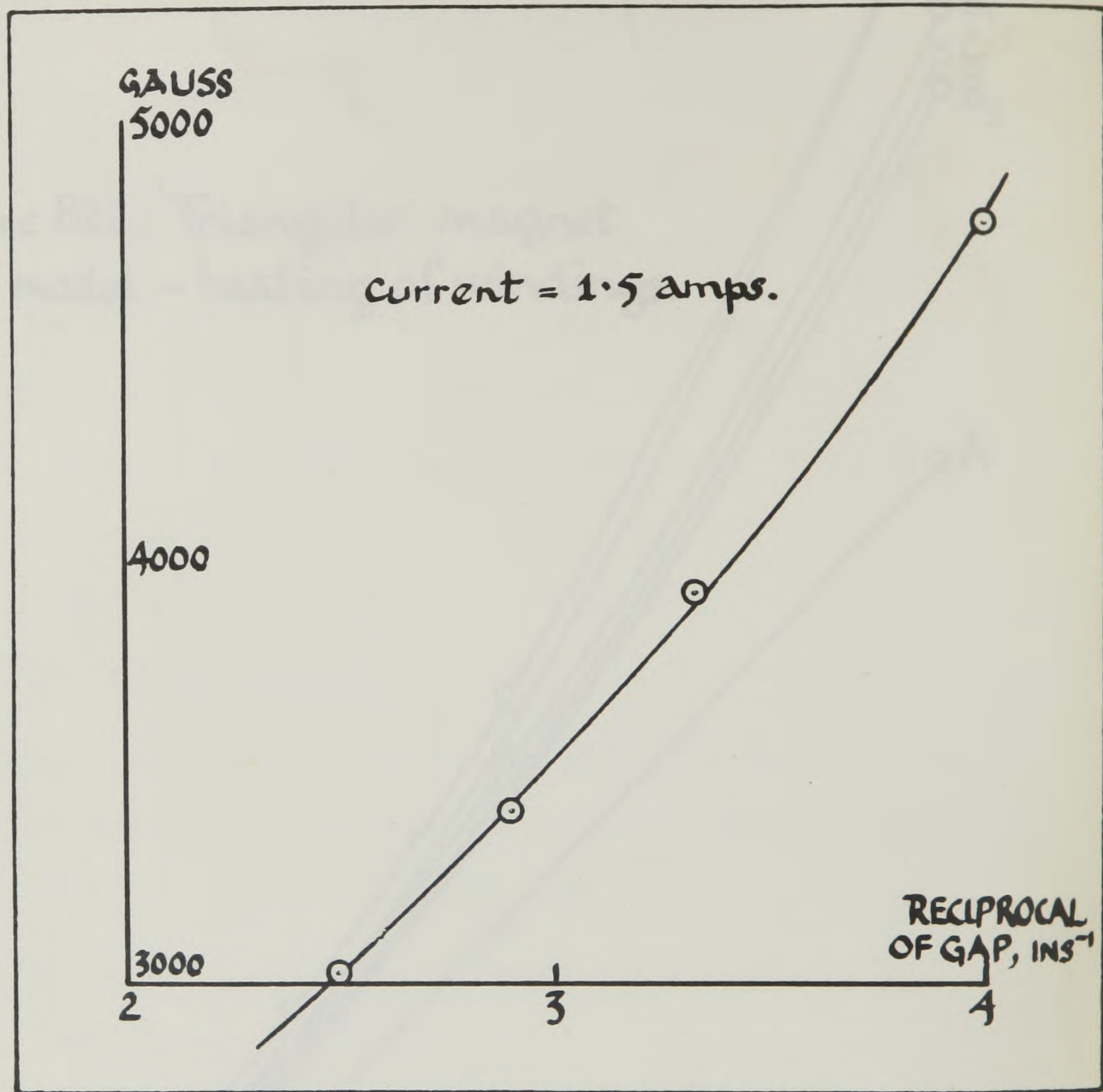


Figure B23 : Magnet model - field as a function of pole-gap.

B.

For the triangular magnet (1) and model (2), we have $a = 4$

$$eA_2 = 0.547$$

$$R_2(o)\alpha = 0.0775$$

therefore
$$\frac{t_1}{t_2} = \frac{0.547 - 0.0775 i_2^2}{2.188 - 0.0775 i_2^2} \quad (16)$$

Experimental values of t_2 and i_2 were given in figure 22. Using the curve for thermocouple C, a curve can be drawn relating the current and steady temperature to be expected in the full-sized magnet.

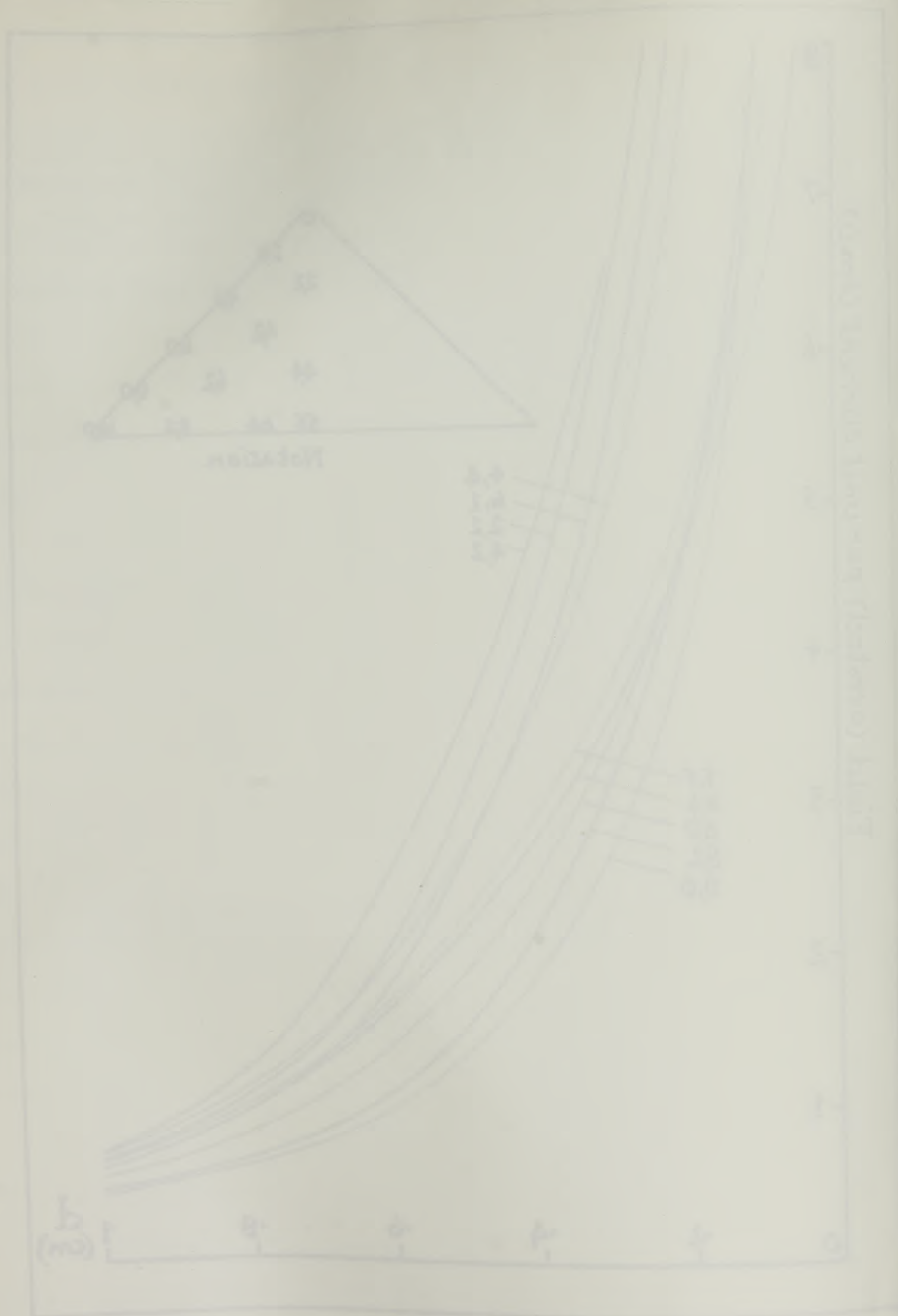
b) Field measurements. A small search coil was moved in the median plane of the pole-faces and it showed that the field varied in the way expected from previous electrolytic tank plots. The effect of leakage flux on the field in the gap can be seen in figure 23 which relates the maximum field in the gap at 1.5 amps to the reciprocal of the gap length, which was varied between $1/4$ and $1/2$ inch. Even at the higher fields of about 5000 gauss, where for mild steel the approach to saturation might be expected to produce a considerable counteraction, the field is still rising more rapidly than linearly.

From these measurements we can get an idea of the leakage flux to be expected on the full-size magnet. Without allowing for any leakage, the field would be 7900 gauss at 6 amps. The corresponding current in the model is 1.5 amps, which extrapolating the curve of figure 23 gives about 6000 gauss for the gap 0.2" corresponding to the full-size gap of 2 cms. Thus the leakage flux is nearly 30% of the total, justifying the use of this figure in the design of the magnet.

(iii) The Field of a Triangular Coil.

A small fraction of the field in the magnet gap is due to the coils alone. In the case of the 19 cm. analyser magnet it amounted to about

Figure 8-1: The field of a triangular coil of short side 10 cm as a function of the distance d from the coil.



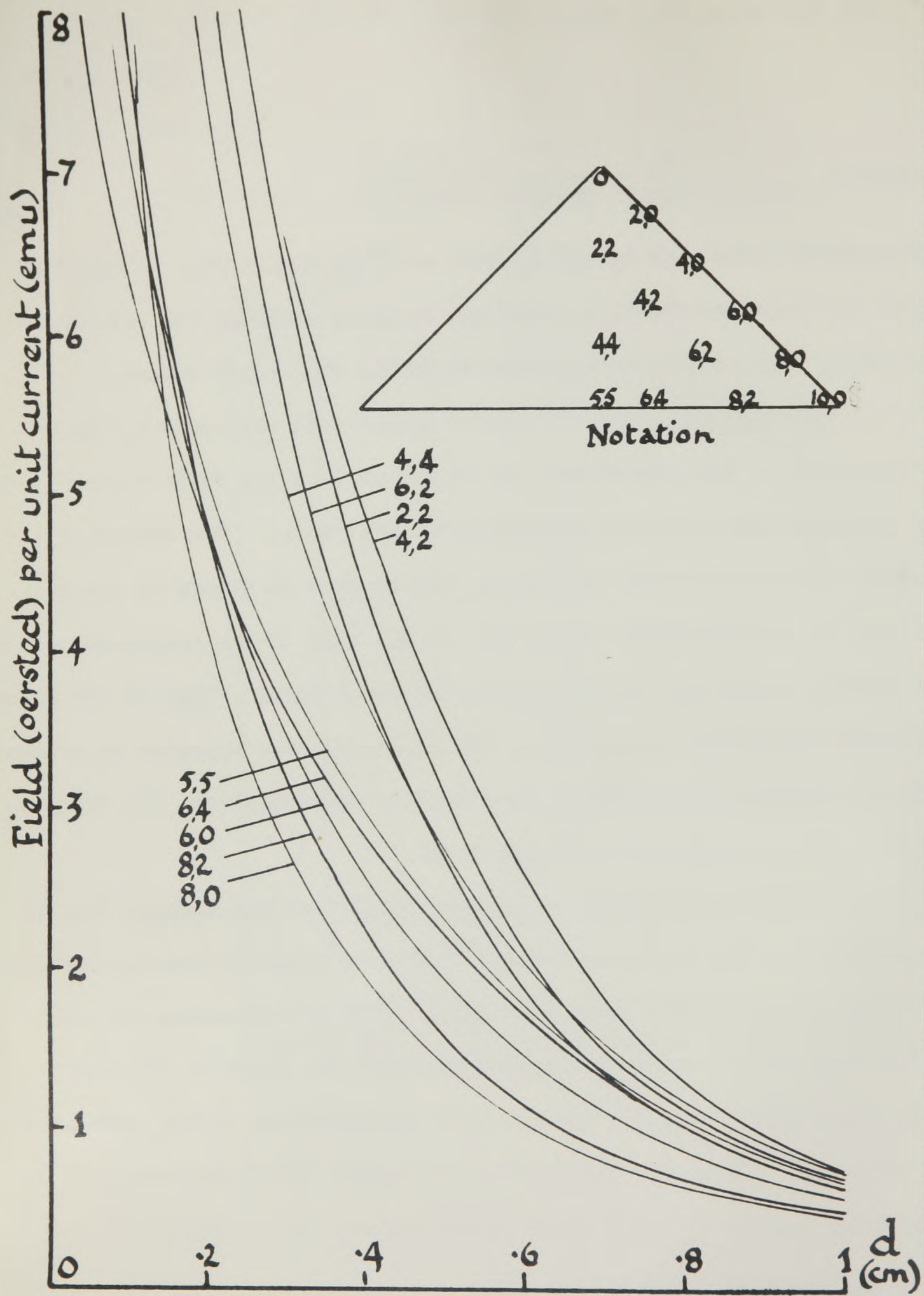
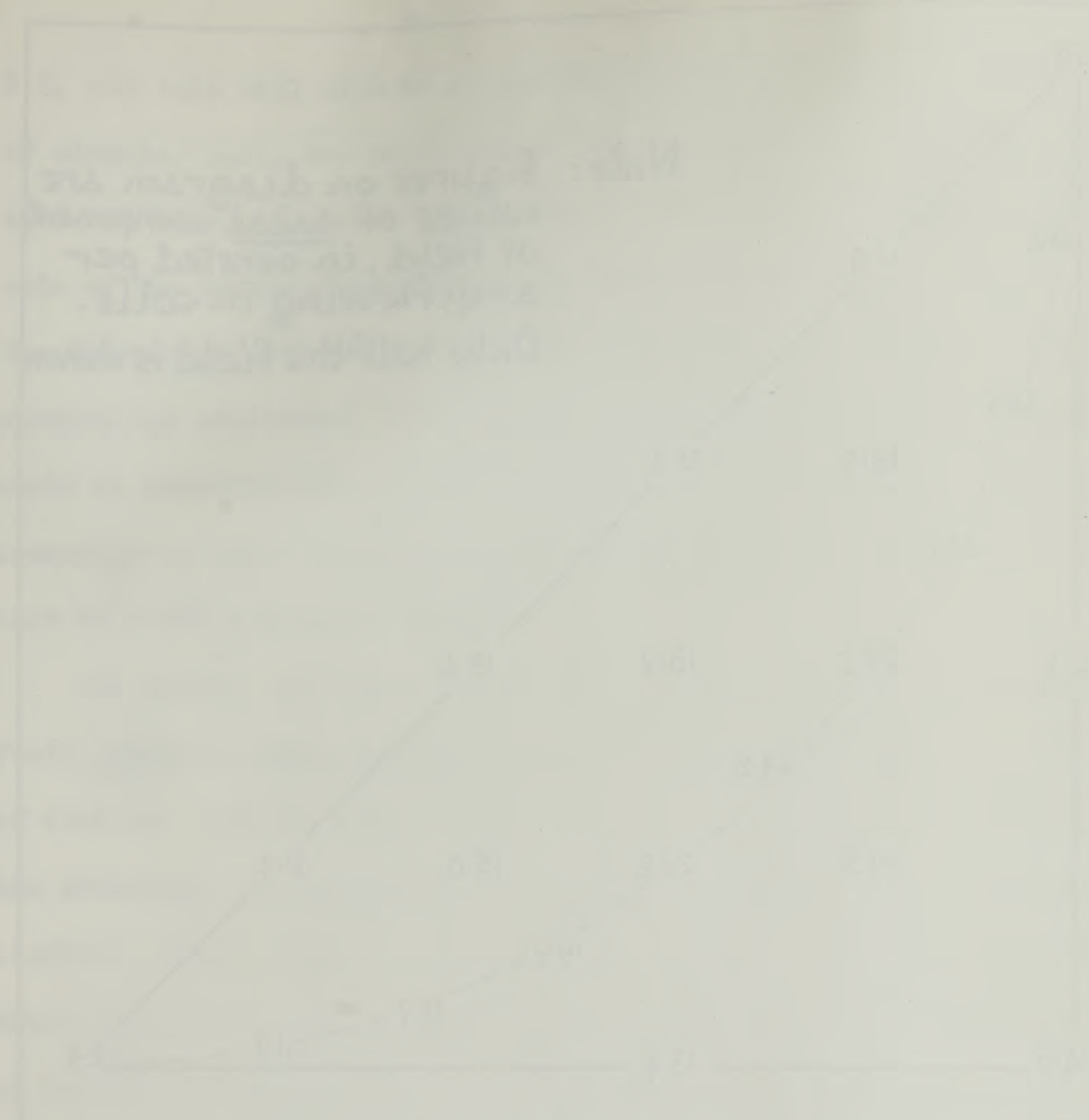


Figure B24: The field of a triangular coil of short side 1 cm. as a function of the distance d from the coil.



The rate of reaction of A with B was measured at various temperatures. The results are shown in Figure 1. The rate of reaction decreases as the temperature increases. This is because the reaction is exothermic. The rate of reaction is highest at 0°C and lowest at 10°C. The rate of reaction is approximately 1.0 M/min at 0°C and 0.45 M/min at 10°C. The rate of reaction is approximately 0.8 M/min at 0°C and 0.3 M/min at 10°C. The rate of reaction is approximately 0.75 M/min at 4°C and 0.6 M/min at 4°C. The rate of reaction is approximately 0.65 M/min at 6°C and 0.5 M/min at 6°C. The rate of reaction is approximately 0.55 M/min at 8°C and 0.4 M/min at 8°C.

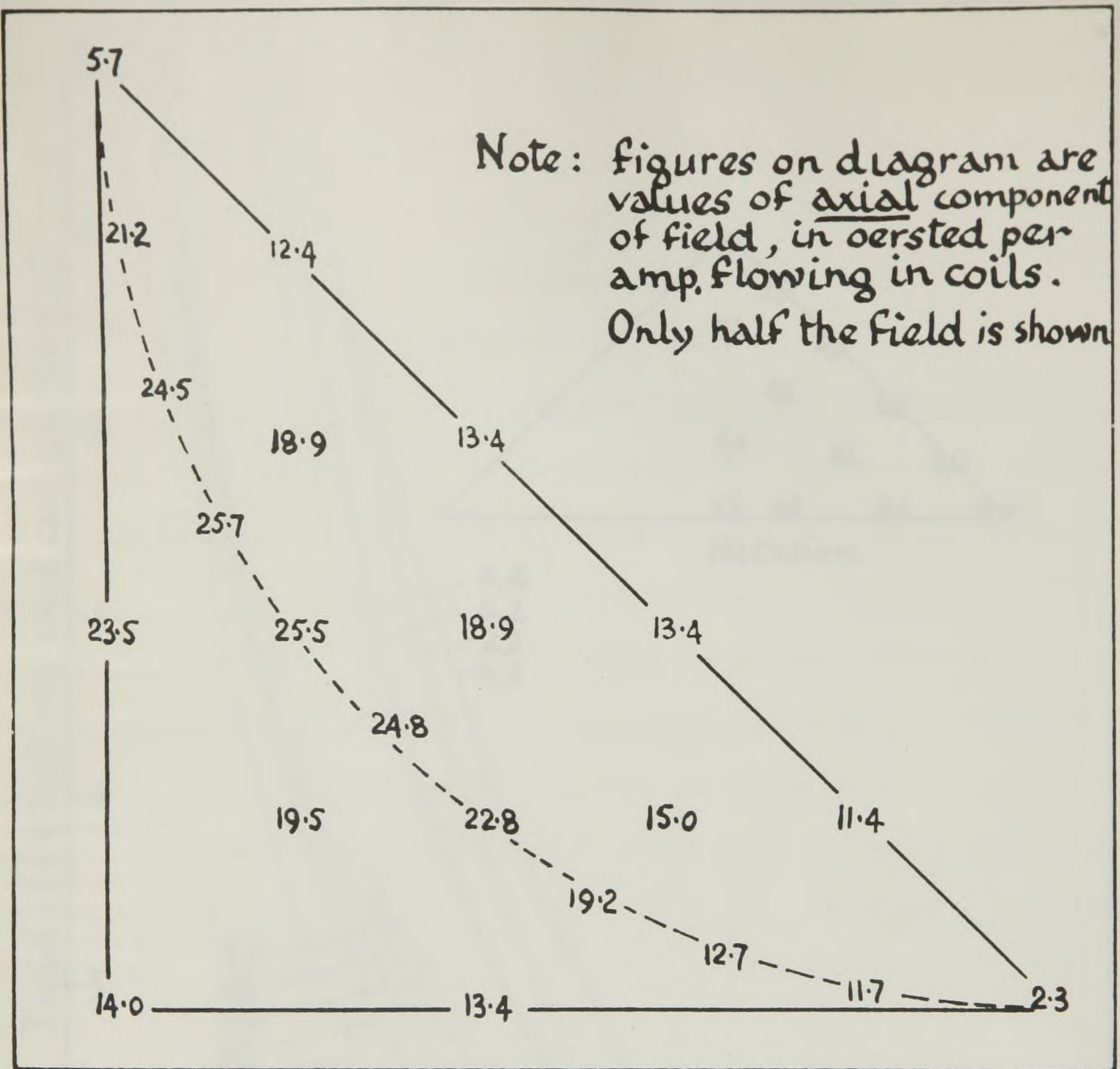


Figure B25: Proposed triangular magnet - field in gap due to coils alone.

B.

2 %, and this will also be so for the triangular magnet. In the case of circular coils the variation of field with radial distance off the axis has been calculated and is known to be small. Moreover the three ends of the double quadrant are all equi-distant from the axis. But the triangular coil has no axis of symmetry and it was not known whether the additional field in the gap at the outlet ends of the quadrant would be significantly different from that at the inlet. It seemed necessary to check this by calculating the field in a plane parallel to that of a 45° triangular coil.

The results are shown in figures 24 and 25. The former gives the field due to a single triangular turn of short side 1 cm. as a function of distance from the plane of the coil and for different positions within the triangle; the current in the turn is 1 e.m.u. For a larger, but similar, winding with n turns of short side S and with current i amps, these field values must be multiplied by $(\frac{ni}{10} S)$ so that for the coils of the 30 cm. triangular magnet the field distribution is as shown in figure 25. Over a considerable region the field is about 22 oersted per amp (about 2% of the total field) which is also the value on the axis of the circular coils of the 19 cm. magnet. From the dotted quadrant paths indicated, it can be seen that the beam enters the more uniform region more rapidly than it leaves, but the differences are small and unimportant.

The coil field in the part below the fork of the double quadrant, which might be the most convenient place for a field-monitoring or stabilising head of the proton resonance sort, has a gradient of about 2% per cm., or about 0.04 % per centimeter of the total field in this region. In this respect the triangular coil is worse than a circular one, since with a double quadrant based on cylindrical cores

B

a proton resonance head could be placed nearly on the axis of the coils and the total variation of field due to the coils alone would be certainly less than 0.005% per cm. of the total field.



Introduction.

The reactions produced by the bombardment of deuterons by deuterons have been the subject of many investigations.²⁶ It has been said that the processes are sufficiently simple to be given theoretical treatment in detail and there is in fact considerable agreement with early experiments. However subsequent experimental work has shown discrepancies which cannot be explained without making new assumptions. One of these discrepancies is the subject of the present investigation.

The reaction proceeds with nearly equal probability in two ways:



The features of interest which provide the common ground for theoretical and experimental investigations are

- (1) The total cross-sections for (a) and (b).
- (2) The large asymmetry in the angular distribution of the reaction products.
- (3) The difference in the angular distributions of (a) and (b).

Although cross-section measurements have been made with the present apparatus, the main interest has been the angular distributions of the reaction products. This distribution has been measured for both reactions with deuteron bombarding energies from 150 to 450 KV, so that the present work is a continuation of that done recently by Eliot, Roaf and Shaw,²⁷ who made measurements in the region below 50 KV.

Previously Moffatt, Roaf and Sanders⁵ had made measurements of cross-sections below 50 KV using a thin gas target with nuclear photographic

C.

plates to record the particles, and their work was continued, using the same technique from 50 to 430 KV, by Davenport et al.²⁸ The use of a gas target in this work is considered to be an improvement on the thick targets (heavy ice and other compounds) of other workers,³¹ since it eliminates large uncertainties in the correction for the energy loss of deuterons and other particles in the target.

The work of Eliot et al., mentioned above showed that the asymmetry of the angular distribution differed in the two reactions. Measurement of both reactions in the same apparatus was made possible, as described below, by a careful choice of absorbing windows in front of the proportional counters used to record the particles, and a suitable choice of bias on the pulse amplifying system. Since the double measurements were always made under closely similar conditions, the difference found in the angular distributions was considered to be very significant.

The angular distribution of the reaction products is conventionally given in the form

$$n(\phi) = n(90) \left[1 + A \cos^2 \phi + B \cos^4 \phi + \dots \right] \quad (1)$$

Where $n(\phi)$ is the number of particles emitted per unit solid angle at the angle ϕ , $n(90)$ is the number at 90° , all angles being measured in a system of co-ordinates where the centre of mass is at rest (CM system) $A, B, C \dots$ are functions of the relative kinetic energy E of the deuterons.

It is clear that from a measurement of the counts made at only two angles (in the case of Eliot et al. $\phi =$ nearly 90° and nearly 125°) the "asymmetry coefficient" A , only, can be evaluated. The indications were that A increased linearly with energy up to 50 KV, within experimental error. This agreed with theoretical expectations,^{29,30} but the additional

c.

finding that A differed for reactions (a) and (b) did not, and a more careful estimate of asymmetry was called for. Moreover, the theoretical predictions, and also the experiments of others, indicated that the coefficient B should be significantly large below 500KV. For these reasons, a counter system was designed which could record particles throughout a large fraction of 360° , and this was used with deuterons accelerated by a Philips generator capable of energies up to 500KV.

A significant difference in the distributions had been found by Blair et al.³², who made measurements by a similar gas target technique in the higher energy range from 1 to 3.5 MeV. Their apparatus had been designed primarily for investigation of the elastic scattering of deuterons by deuterons and they recorded pulses produced by He^3 from reaction (a) and H^1 from reaction (b). As pointed out in their paper the proton measurements were difficult to obtain with sufficient accuracy due to large corrections for the effect of neutrons from reaction (a). It was not possible to measure the H^3 because at the energies used the scattered deuterons had very similar energies and discrimination was not possible.

In the energy range of the present experiments, however, discrimination is possible by the same technique used in the earlier work of Eliot, and so the reactions (a) and (b) are measured by counting He^3 and H^3 .

2. Experimental Apparatus.

(1) Gas Target.

The target arrangement is shown schematically in figure 1, which is

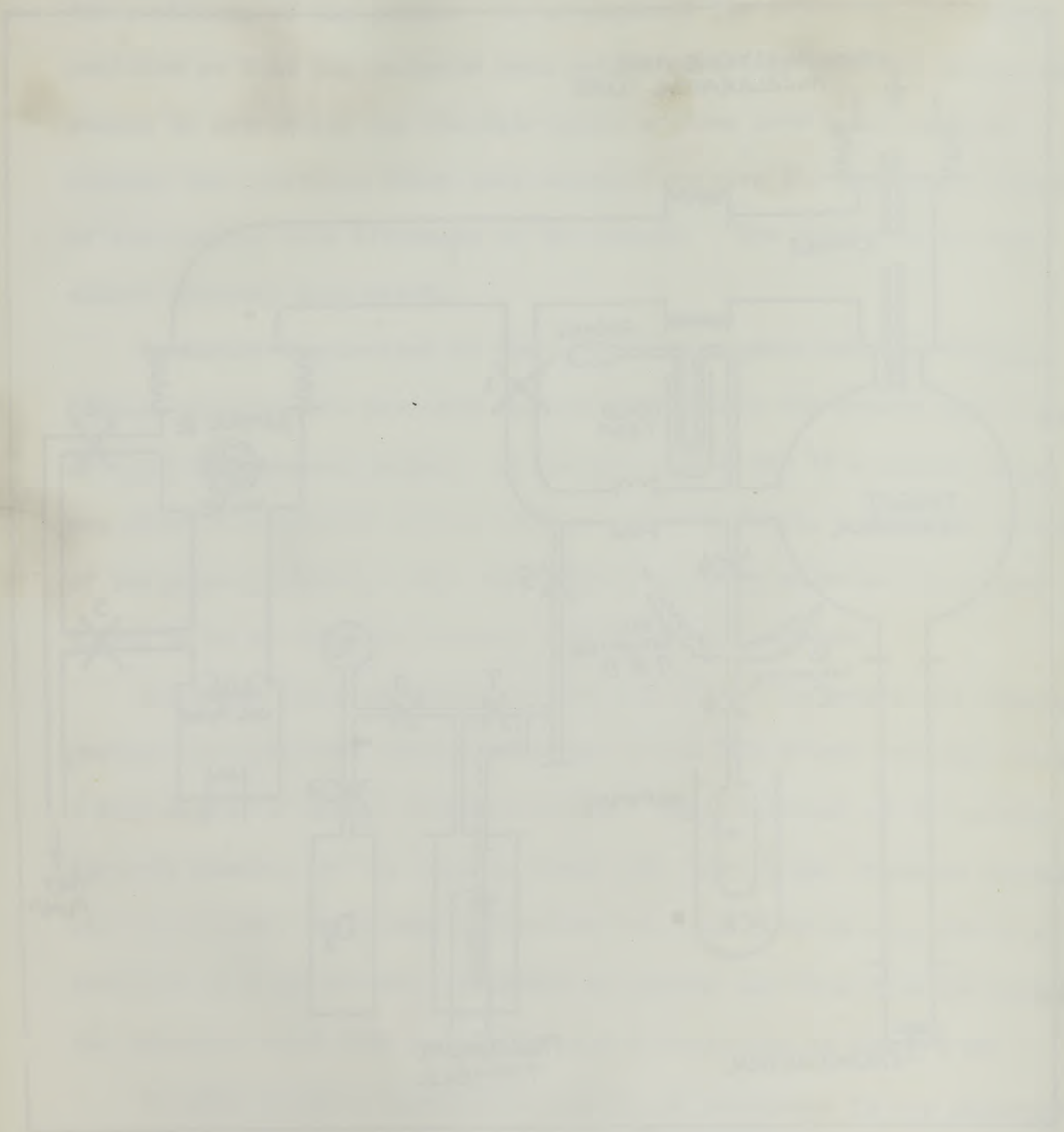


Figure 1 - Pumping system for the gas engine.

The diagram illustrates a pumping system for a gas engine. It shows a central vertical shaft connected to a pump mechanism. The pump is driven by a motor or engine, which is represented by a large, rounded shape on the right. The pump draws fluid from a reservoir or chamber at the bottom and discharges it into a pipe or duct that leads to the gas engine. The diagram also shows various valves and control mechanisms that regulate the flow of fluid through the system.

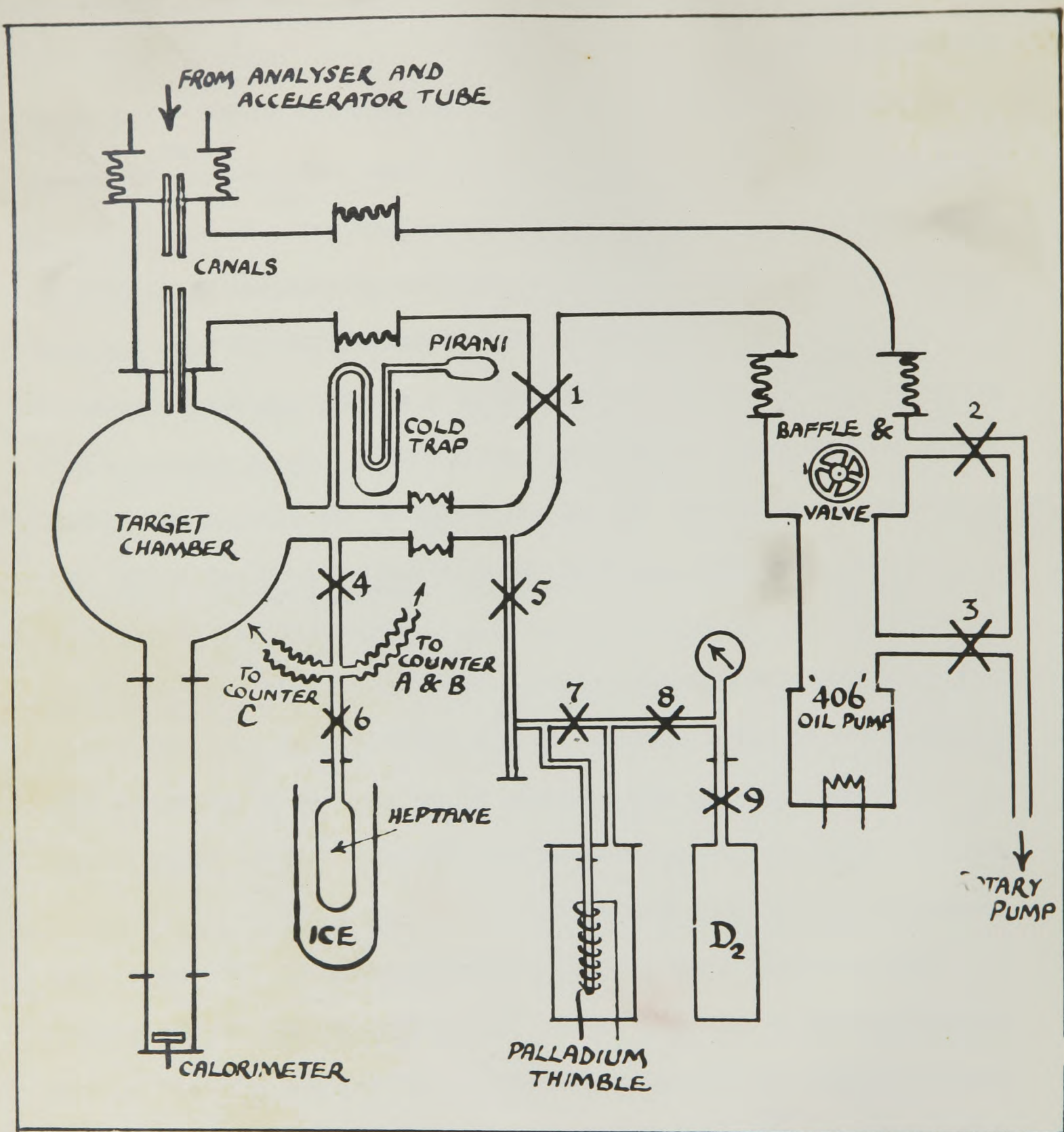


Figure C.1 : Pumping system for the gas target.

C.

sufficiently closely to scale to indicate the relative sizes and roughly the positions of the parts. An arrangement for adjusting the target position so that the deuteron beam passes centrally down the collimating canals is not shown, but flexible brass bellows have been drawn at roughly the positions which they occupied to give the necessary degrees of freedom for this alignment of the canals. The apparatus is made almost entirely from metal.

Deuterons accelerated by the high voltage generated in a Phillips cascade machine were deviated through 11° between the square pole faces of a small permanent magnet: at energies above 250 KV a second magnet was placed in parallel and in contact with the first. For fine control of the magnetic field a mild steel shunt could be adjusted on a screw thread so as to vary the leakage flux between the poles.

The small angle of deflection was sufficient to remove all unwanted protons and molecular complexes formed in the ion source and also allowed a fair degree of energy discrimination. At a distance of 40 cm. below the exit boundary of the magnet, where the first 4 mm. diameter aperture stop was placed, the energy separation was such that at e.g. 250 KV a variation of ± 12 KV was sufficient to remove the beam from the target: the intensity would fall noticeably with a variation of about 3 KV.

In order to get a sufficient density of reactions it was necessary to use a target pressure of about 0.1 mm. Hg; but to avoid both electrical breakdown and too great a loss of beam by scattering in the accelerator column, the pressure there had to be less than 5×10^{-5} mm. Hg. This was made possible by passing the beam through two canals separated by a space evacuated by a high speed pump, as shown.

The canals are each 4 mm. diameter and 12 c.m. long, separated by

C.

5 cm. This gap is pumped by an Edwards type 406 oil diffusion pump (speed about 250 litres/sec) through a 4" bore tube (speed about 300 litres/sec), and the pressure in the gap is kept down to about 10^{-5} m.m. Hg. At the target gas pressure used the mean free path of the gas molecules is about 1 m.m., so over most of the length of the canals the mass of gas removed by the pump can be considered to be governed by Knudsen flow, for which the tube has a speed of about 0.2 litres/sec. The gas reservoir (containing 98% deuterium) is capable of supplying much greater quantities of gas than this through the heated palladium thimble via tap 5, taps 1 and 4 being closed.

To prepare the gas target, the target chamber was first evacuated through the by-pass tap 1. The residual pressure could be read with the Pirani gauge connected to the target chamber via a liquid air trap. At the position of the Pirani gauge a calibrated McLeod gauge could be used when precise measurements of pressure were needed (as for example in finding cross-sections). The vessel containing the palladium thimble was filled to about 2 atmospheres with D_2 as measured on a Bourdon gauge, and then released into the target chamber by heating the thimble. The thimble current was adjusted until the target gas reached an equilibrium pressure of about 90 - 100 microns Hg.

(ii) Proportional Counters.

In figure 2 (a) is shown the construction of the type of counter used in all the measurements. The body was made of brass and the internal dimensions were $7\frac{1}{2}$ cm. long by about $1\frac{1}{2}$ cm. square. It was made with a removable lid and the central tungsten wire 4 cm. long was supported in a brass frame by means of 'cover' metal-glass seals. This frame could be accurately located in the body and easily removed

The following are the
the same components
standing in the same
relative positions
as in the previous
figure.



Fig. 1. Mechanical assembly.
a) Top view of assembly.
b) Front view of assembly.
c) Side view of assembly.

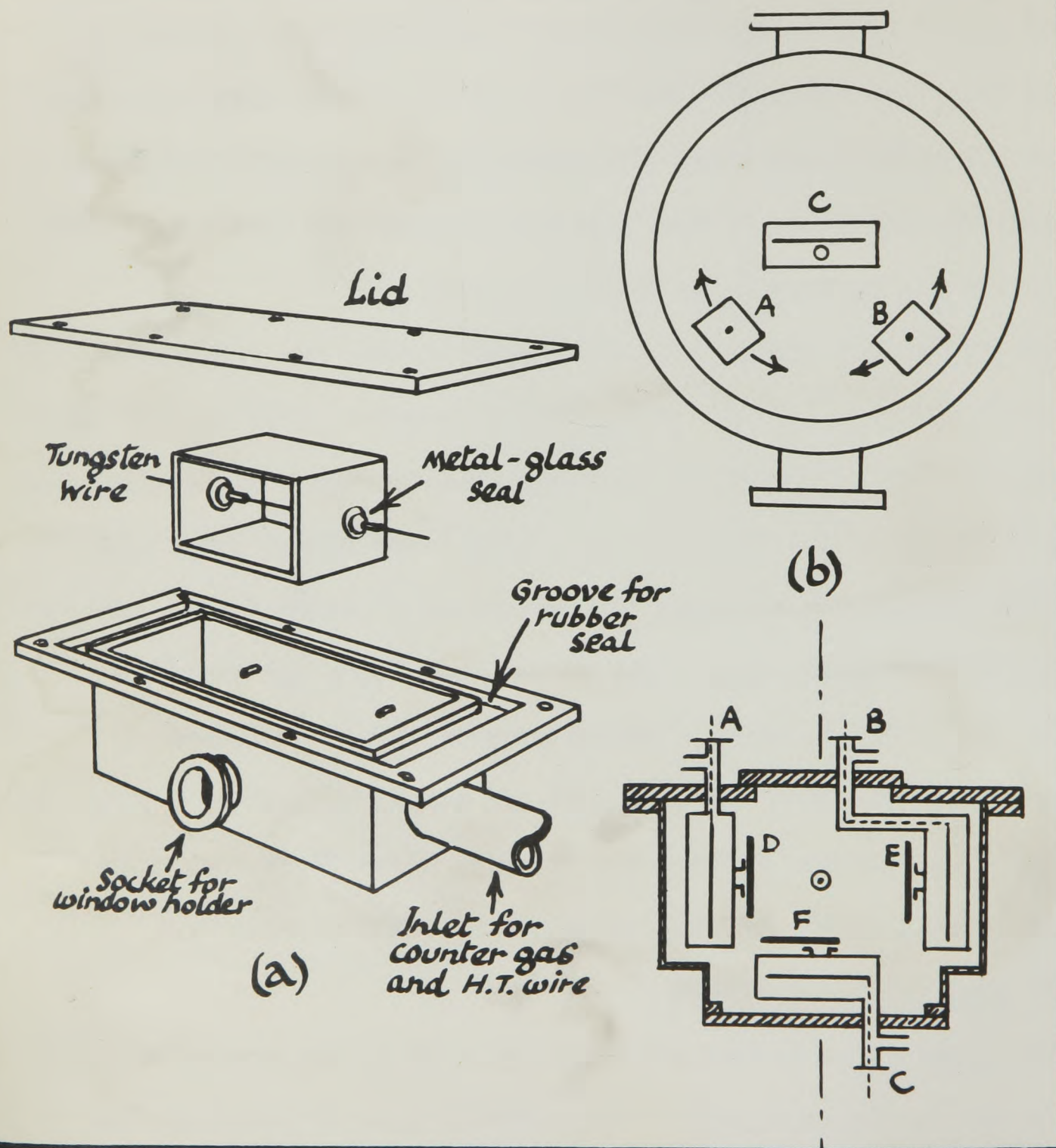


Figure C 2: Proportional counters.

(a) Construction of counter.

(b) Position of counters in target chamber.

C.

for inspection. The design was adopted because previous experience had shown the convenience of a counter which could be dismantled for cleaning in the event of electrical breakdown or other fault. The electrical connection to the wire was made by soldering to a nickel tab which had been spot welded to the wire.

The counting gas was heptane, chosen for several reasons. Firstly, it is known to be a stable counting gas in the pressure range of interest here, and had been used with success in the previous work of Elliott et al. The He^3 particles have such a short range that very thin windows must be used, and as will be shown these will not safely withstand pressure differences greater than about a few cm. Hg. Other counting gases such as argon break down at this pressure. Secondly, heptane has a saturated vapour pressure of the right magnitude at convenient temperatures (about 1 cm. at 0°C) so that the counter pressure can be kept very constant in spite of gas absorption into the walls. This is done by allowing the counting gas vapour to come into equilibrium with a small reservoir of the liquid which is surrounded by a bath at the appropriate temperature. During these experiments a bath of crushed ice was used. Thirdly, the gas has a reasonably large stopping power for protons or alpha particles and gives larger pulses than, for example, methane at the same pressure.

The windows were made of quartz waxed onto the end of small brass holders which were sealed into the counters by means of 'O' rings so that they could be easily removed if necessary. The window was prepared by blowing a large bubble on the end of a silica tube which had been softened in an oxygen-gas flame. After many trials it was found that pieces of quartz of the right thickness and larger than the area needed

C.

could be blown. These were selected by the interference colours shown in daylight and then sealed to the holders with a thin ring of soft wax, great care being taken that no wax should run on to the unshielded window surface. The unsupported region of the window was circular, 2 m.m. diameter. The thickness was measured with a split-amplitude interferometer of the Jamin type, using white light to find the shift of the central black fringe when the window was placed in one of the parallel light beams. For the windows used in the three counters, and assuming light of a mean wavelength $5500\text{\AA}$ the thicknesses (which so far as could be seen were uniform) were

A -	0.57 μ
B -	0.55 μ
C -	0.52 μ

We can roughly estimate the bursting pressure-difference of these windows by considering them as thin plates clamped at the edges. In the absence of better information we assume the values of the elastic constants for drawn quartz fibres. Using a formula given by Strong, the bursting pressure is found to be 250 cm Hg., but with possible uneven stresses in the windows the bursting pressure is likely to be very much less than this: 50 cm. Hg. is probably a reasonable estimate.

Figure 2 (b) shows the positions of the three counters within the target chamber. Counters A and B can be rotated in a plane containing the beam about a horizontal axis through the centre of the target chamber as shown in the upper drawing. Counter A revolves with the lid of the chamber and B with a smaller flange which is concentric and in contact with this lid, but can be rotated relative to it. The angular position of both counters is indicated by a vernier scale which moves



Figure 3: Geometry of the counting system

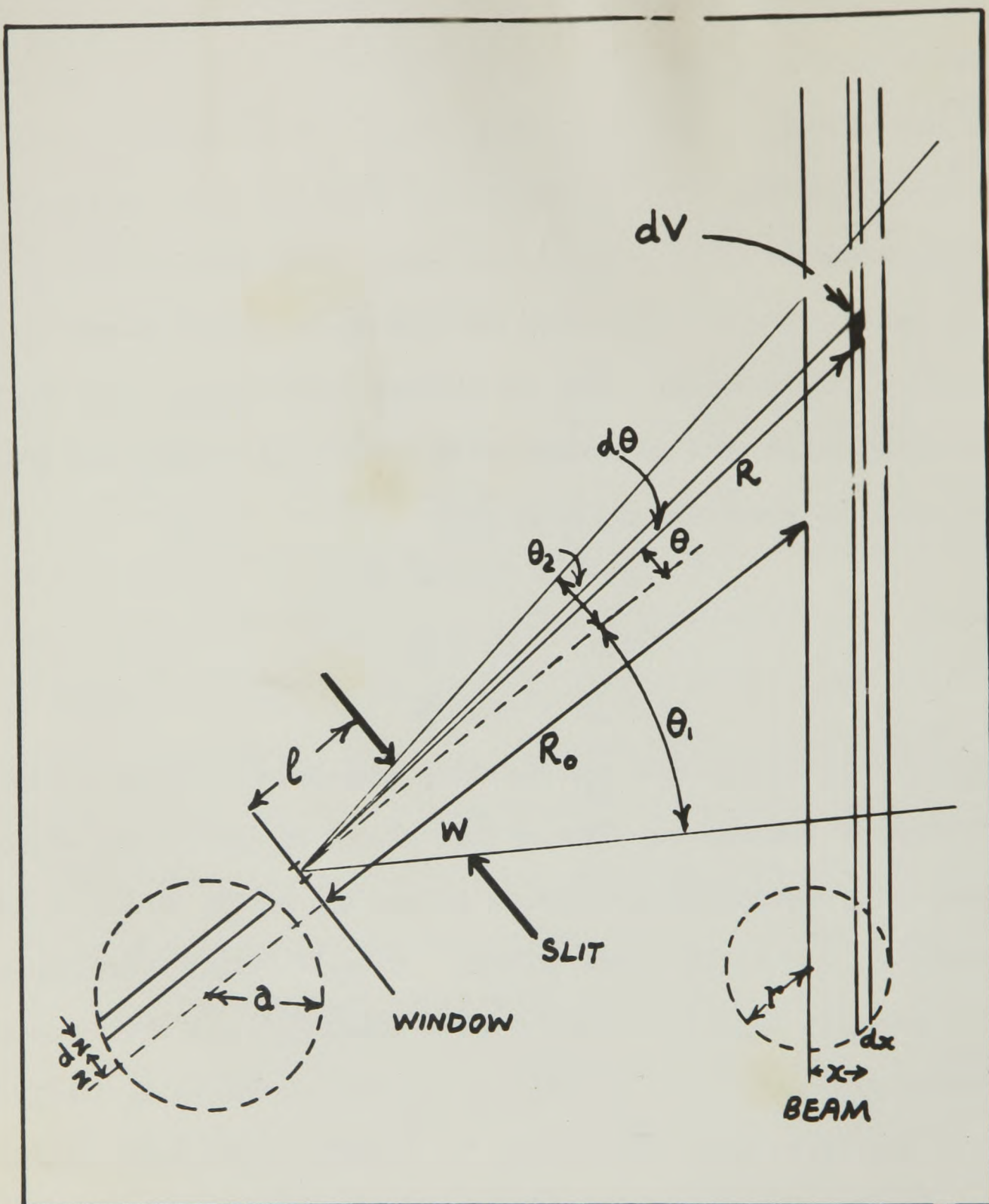


Figure C 3 : Geometry of the counting system.

C.

over graduations scribed on part of the chamber body. At all positions of the counters the radial distance of the windows from the centre of the chamber is the same for A and B. Either counter A or B could be moved to all angles through 360° , but in the experiment forward angles of greater than 13° had to be used to avoid the beam hitting the outside ~~of~~ ^{wall} the counter wall. Counter C remains fixed to the back of the chamber with its window at the same distance from the beam and is in every respect similar to A and B.

Electrical connection to the counter wires is made by polythene covered copper wire which is threaded through $3/8"$ pipe supplying the counter gas and emerges through an 'O' ring seal in one arm of a T-junction as shown by the dotted lines in the lower drawing of figure 2(b). The polythene covering and the vacuum seal is covered with a soft wax of low vapour pressure to inhibit percolation of air or the outgassing of impurities which might affect the counting.

The gas is supplied to the counters through $\frac{1}{2}"$ diameter flexible brass tubing with helical convolutions as indicated in figure 1.

The target chamber and especially the counter system was most exhaustively tested for leaks. This was considered to be of prime importance in the counters, since in use they form a closed system and any impurities in the gas will lead to a change in counter efficiency.

Between each counter window and the reaction volume is a slit with jaws normal to the beam axis. The geometry of this system is shown (not to scale) in figure 3. It will be seen that any point on the window sees a length of beam defined by the separation of the slit jaws (W), and a width of beam defined by the diameter of the collimating

C.

canals ($2r$). At any point in the beam, particles from a disintegration there will enter the counter if they are within a solid angle defined by the area of the part of the window (radius a) seen through the slit jaws and the perpendicular distance R_0 of the window centre from the beam axis. All these distances were measured with great care for future estimation of absolute cross-sections. It can be said that the actual dimensions used provide "good geometry" in the sense that, as shown later, the errors introduced are negligible if it is assumed that the mean number of particles counted per ^{unit} solid angle is the same as the number which would have been recorded if a 'point source' of reactions were viewed with a counter of very small aperture.

Absorbing films could be moved between the slits and the beam in the positions shown by D, E, F in figure 2 (b). This was the principal way in which pulses due to H^3 ^{were} differentiated from those due to scattered deuterons. In the case of counters A and B each slit was covered by a brass slide with two frames covered with thin mica: either frame could be placed in front of the slit by moving a knob outside the chamber. With the brass portion of the slide, between the frames, in front of the slit all charged particles could be stopped and 'background' counts due to neutron collisions, x-rays etc., could be recorded. Mica was split down from sheets for the purpose and the thickness found by weighing measured areas. The thicknesses required were usually about 1 mgm/cm^2 : details are given in the next section.

The absorbers in front of the slit of the monitoring counter C were arranged on a rotatable disc. There were five positions, one of which was left blank for background measurements, three were occupied by films of different thicknesses of mica and the fifth had no absorber. By

Figure 2-4-1 Pulse counting equipment

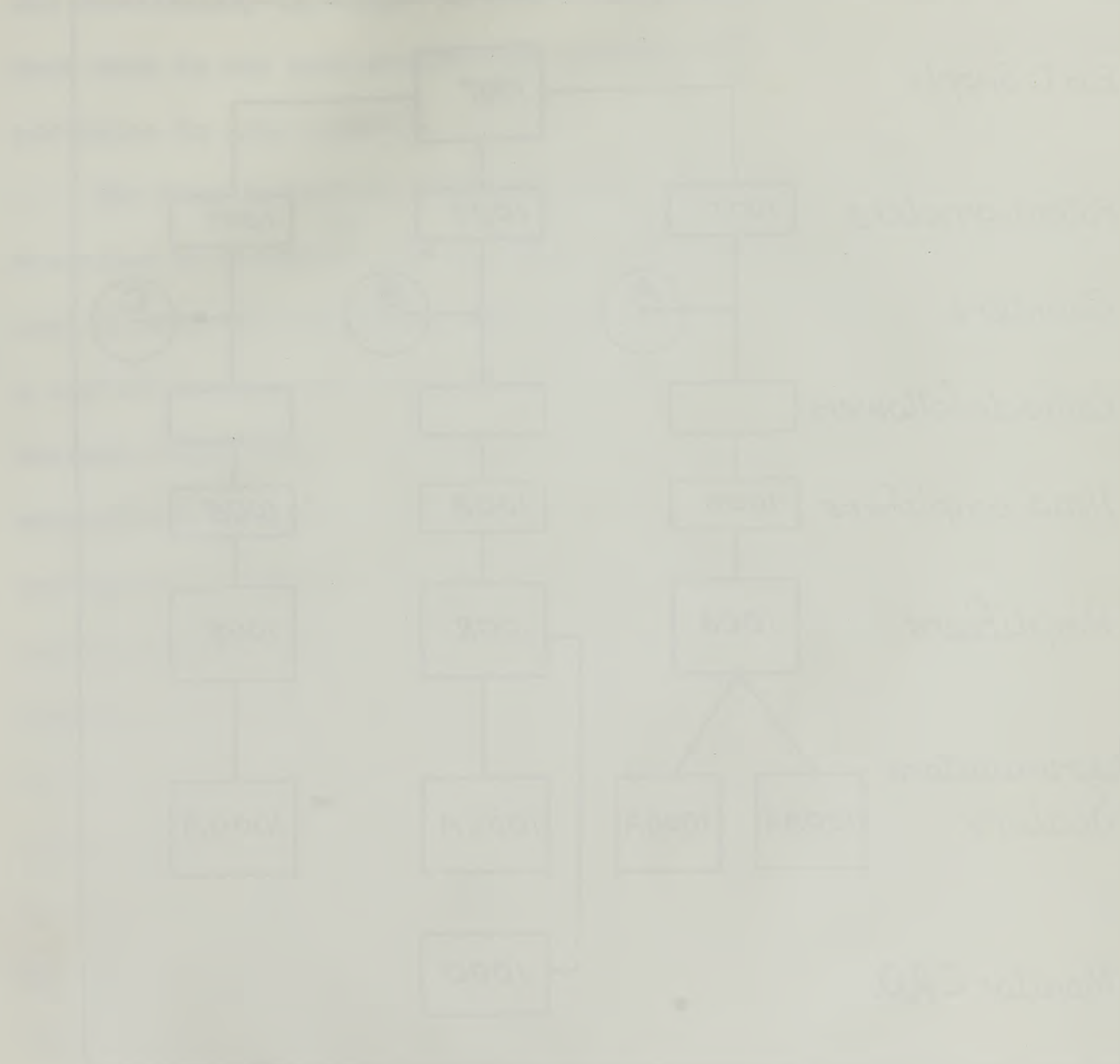


Figure 2-4-1 Pulse counting equipment

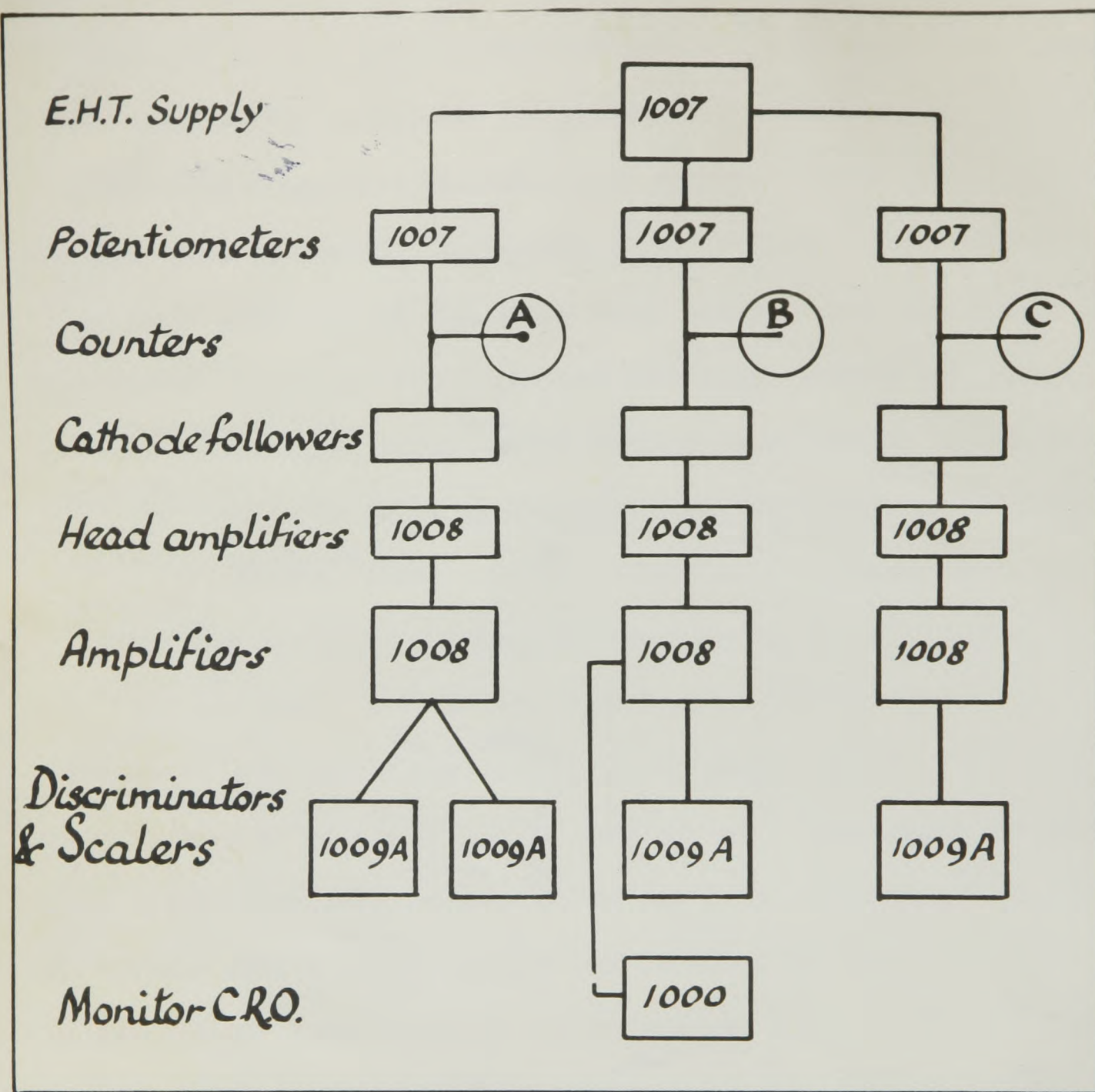


Figure C4 : Pulse counting equipment.

having available several different absorbers, the appropriate thickness for use on the counters A and B could be more easily found by trial without dismantling the target system. Four absorbers of increasing thickness were in any case needed for counting He^3 , H^3 , H^1 and background particles in this counter.

The beam current is measured with a calorimeter similar to that described by Sanders.³³ In the angular distribution measurements it was used to keep the current as near constant as possible although, because a monitor counter was used, no first order errors were introduced by the current variations. For later cross-section determinations an accurate measurement of the current was necessary, and the calorimeter was calibrated by heating it under the experimental conditions of pressure and temperature with a small nichrome heater built into it. The temperature gradient along the copper stem between the copper block which received the beam, and a water cooled base, was measured for known power inputs. The calorimeter was used successfully with beam powers up to about 60 watts, the beam current being deduced from a knowledge of the accelerating voltage.

(iii) Electronic Counting Equipment.

Conventional methods with standard items of electronic equipment were used for amplifying and counting the pulses from the proportional counters. The layout is shown in figure C 4, where the figures on the rectangles representing pieces of equipment are the Harwell designations. The cathode followers were made in the laboratory in a way which allowed them to be fitted directly onto the part of the counter from which the H.T. wire emerged, the copper box containing the circuit elements acting as an effective electrostatic screen. Screened cable was used throughout and great care was taken to avoid electromagnetic pick-up in

C.

partially grounded loops by using thick copper strip for earth connections wherever possible. On the whole the equipment was fairly free from induced effects, although severe disturbances from the nearby synchrotron were occasionally found at high amplifications.

The amplification system was the same for all three counters, except that pulses from A were fed in parallel into two discriminators and scalars with different bias settings, as explained below. The oscilloscope could be used to inspect the pulse spectrum from any of the counters.

Experimental Method.

The charged particles which are potentially able to produce pulses in the counters are:

H^1 from reaction (b)

H^3 " " (b)

He^3 " " (a)

n (recoils) from (a)

H^2 scattered by deuterons

H^2 scattered by heavier elements.

Because of the finite window thickness, not all these particles will be able to enter the counters under all conditions of angle and bombarding energy. For example, deuterons elastically scattered from deuterium will have an energy

$$E_s = E_1 \cos^2 \phi \quad (2)$$

Where E_1 is the bombarding energy and ϕ is the counting angle, both measured in the Lab. system. Now the quartz windows have a thickness of about

C.

$$0.5 \mu = 0.15 \text{ mgm/cm}^2 = 0.1 \text{ cm Air at S.T.P.}$$

Using curves given by Bethe,³⁴ this thickness is found to just absorb deuterons of energy 50 KV. Thus no singly scattered deuterons will be counted when

$$E_1 \cos^2 \phi < 50 \text{ KV.}$$

(The limits are: $\phi = 55^\circ$ for $E_1 = 150 \text{ KV.}$,
 $\phi = 70^\circ$ for $E_1 = 450 \text{ KV.}$)

In experimental runs counts were recorded at angles ϕ between 18° and 110° . No deuterons can be scattered from deuterium at angles greater than 90° , and none of those singly scattered at angles greater than 70° (55° for the low energies) can penetrate the counter windows. However at smaller angles penetration will be possible and because the intensity is large absorbing foils must be placed in front of the windows.

Deuterons may also be scattered from heavy atoms which may be present as impurities. These can be scattered through angles larger than 90° and may even then retain most of their energy. This was the case with scattering from oxygen when it was attempted to use a gas target of heavy water vapour in the early stages of the same experiment. Because these scattered deuterons kept such a large part of their energy at large angles where the energy of the reaction products was small, discrimination between particles became impossible, and the method was abandoned.

The scattered deuterons may become a source of D - D reactions outside the collimated beam and if the disintegrations take place near the plane of figure 3, it is possible for some of the charged reaction products to enter the counter: but the chance of this occurrence is extremely small.

However, neutrons produced in this way, as well as from primary

C.

disintegrations, penetrate the counter walls and charged atomic recoils then given rise to counts which must be regarded as 'background' since they are unaffected by the presence of absorbing foils. The energy of a recoil scattered through lab. angle θ is

$$E_r = \frac{4A}{(1+A)^2} E_n \cos^2 \theta \quad (3)$$

where A = mass number of recoil nucleus

E_n = neutron energy.

The likely recoil atoms are carbon and hydrogen from the counter gas $C_7 H_{16}$. The maximum recoil energy for these cases is

$$E_H (\text{max}) = E_n$$

$$E_C (\text{max}) = 0.28 E_n$$

The size of the pulse produced by these recoils is not easy to predict, since it is not known to what extent the atom is ionized by the collision, or how long is its path in the counter.

In order to indicate the way in which the absorbers were chosen, the Table 1 below has been drawn up to show the ranges, in the counter materials, of the reaction products and scattered deuterons.

The energy E_2 of a reaction product, mass M_2 emitted at a lab. angle ϕ is given by the Conservation laws in the equations:

$$E_3 M_3 = E_1 M_1 + E_2 M_2 - 2 \sqrt{E_1 E_2 M_1 M_2} \cos \phi \quad (4)$$

$$Q = E_2 + E_3 - E_1 \quad (5)$$

where suffix refers to the bombarding particle, suffix 3 to the other reaction product, and Q is the reaction energy release. According to

recent work $Q(\text{reaction a}) = 3.27 \text{ MeV.}$ (6)

$Q(\text{reaction b}) = 4.04 \text{ MeV.}$ (7)

The ranges in standard air were found from Bethe's data for protons, deuterons and alpha particles.³⁴ The ranges of H^3 and He^3 are found

from the proton and alpha-particle curves by the relations:

$$R(H^3, E) = 3R(H^1, \frac{1}{3}E) \quad (8)$$

$$R(He^3, E) = \frac{3}{4}R(He^4, \frac{4}{3}E) \quad (9)$$

The two bombarding energies shown are the largest and smallest used: the figures for the angles 0° and 90° are to be taken as representative of values for the small angles (say $< 30^\circ$) where energies are high, and large angles where the scattered deuteron energy is very small.

Table 1.

(a) H ² Energy KV.	(b) Lab. Angle	(c) Part- icle	(d) Init. Energy MeV.	(e) Air Range Mgm ₂ cm	(f) Ab- sorber Mgm ₂ cm	(g) Ab- sorber + window Mgm ₂ cm	(h) Resid- ual Range Mgm ₂ cm	(i) Relat- ive Pulse Height. (ion pairs)
450	0	H ¹	4.16	36.7				
		H ³	2.08	5.93	1.5	1.65	4.28	1500
		He ³	1.78	1.43	0.85	1.00	0.43	7200
		H ^{2*}	0.45	1.00	0.85	1.00	0	0
		H ^{2**}	0.45	1.00	0.85	1.00	0	0
		n	3.88					
	90	H ¹	3.13	22.80	17.2	17.35	5.45	700
		H ³	0.89	2.02	0.9	1.05	1.15	2100
		He ³	0.69	0.58	0	0.15	0.43	7200
		H ^{2*}	0	0	0	0.15	0	0
		H ^{2**}	0.45	1.00	0	0.15	0.85	2000
		n	2.56					
150	0	H ¹	3.66	29.7				
		H ³	1.56	4.05	1.22	1.37	2.68	1600
		He ³	1.32	1.05	0.36	0.51	0.54	7200
		H ^{2*}	0.15	0.30	0.36	0.51	0	0
		H ^{2**}	0.15	0.30	0.36	0.51	0	0
		n	2.94					
	90	H ¹	3.07	22.00	17.2	17.35	4.65	800
		H ³	0.97	2.25	0.9	1.05	1.20	2000
		He ³	0.78	0.65	0	0.15	0.5	7200
		H ^{2*}	0	0	0	0.15	0	0
		H ^{2**}	0.15	0.30	0	0.15	0.15	2700
		n	2.49					

* Scattered from deuterium

** Scattered from heavy elements

C.

In column (e) the ranges in standard air are given in mgm/cm^2 and it can be assumed that in these units the ranges in the materials of absorbers, counterwindows and gas will not be very different.

Column (f) gives the thicknesses of typical mica absorbers which were used during the work. A thick absorber used for counting protons was only fitted to counter C, where there was room for it, so proton figures are only given for 90° .

In column (g) the window thickness $0.15 \text{ mgm}/\text{cm}^2$ has been added to that of the absorber. It is seen that in each case the He^3 are counted with an absorbing material sufficient to remove deuterium-scattered deuterons, and H^3 are counted with neither He^3 nor deuterons. Although in counting He^3 it is possible to discriminate by biasing against pulses from deuterons scattered from heavy atoms, this is no longer possible with those scattered from deuterium, which must be stopped from entering the counter if possible, since the high intensity affects the operation of the counter by increasing the abnormally high pulses due to "pile up".

Column (h) shows the estimated residual range in the counter gas. Using Bethe's data we find the counter gas at 1 cm Hg. pressure has a thickness equivalent to an air range of about $0.1 \text{ mgm}/\text{cm}^2$ per cm. The width of the counter is about 1.5 cm., so the gas thickness is about $0.15 \text{ mgm}/\text{cm}^2$. It can be seen that all the particles will traverse the counter and be stopped in the back wall. Although larger initial ionization and hence larger pulses might result if the particle is stopped in the counter gas, because of the peak in the Bragg curve, there are disadvantages in arranging for this to happen. Firstly, since the slope of the Bragg curve increases near the end of the range, the pulse size due to the transmitted particles will be more sensitive to inhomogeneities in the

C.

window and absorber with a consequent increase in the part of the 'plateau' slope of the bias curve which is due to this cause (see below). Secondly, the number of absorbers needed would be greatly increased, and this would entail more frequent dismantling of the target vacuum box.

We can estimate the energy lost by the particle to the counter gas. If a constant energy of 30 e.v. is assumed for the formation of an ion pair, the pulse heights (expressed as the initial ionisation) can be roughly estimated. This is done in column (i), using again Bethe's data and equations (8) and (9).

The next paragraphs show the scheme which was followed in making most of the angular distribution measurements.

(i) The counter C was used to monitor the beam current and other possibly variable conditions: it was arranged to count (He^3 + background) at angle 90° in the lab. system.

(ii) The rotating counters A and B were set at C.M. angles ϕ on either side of the beam axis and recorded (He^3 + background) or (H^3 + background) according as reaction (a) or (b) was being investigated. The output from either counter A or B was fed after amplification into two scalers with the discriminator bias set at levels 5 volts apart, a procedure which checked that the slope of the proportional counter 'plateau' was not greater than the tolerable level, as described below.

(iii) The windows of counters A and B were then masked by a thickness of brass sufficient to stop all the charged reaction products, and another count made for one-third to one-half of the original number of monitor counts. By subtraction of (iii) ^{from} (ii) after conversion to the same number of monitor counts, the number of H^3 or He^3 counts (devoid of background) per unit monitor count was found. These relative counts were then corrected for the background in the monitor counter, and finally the

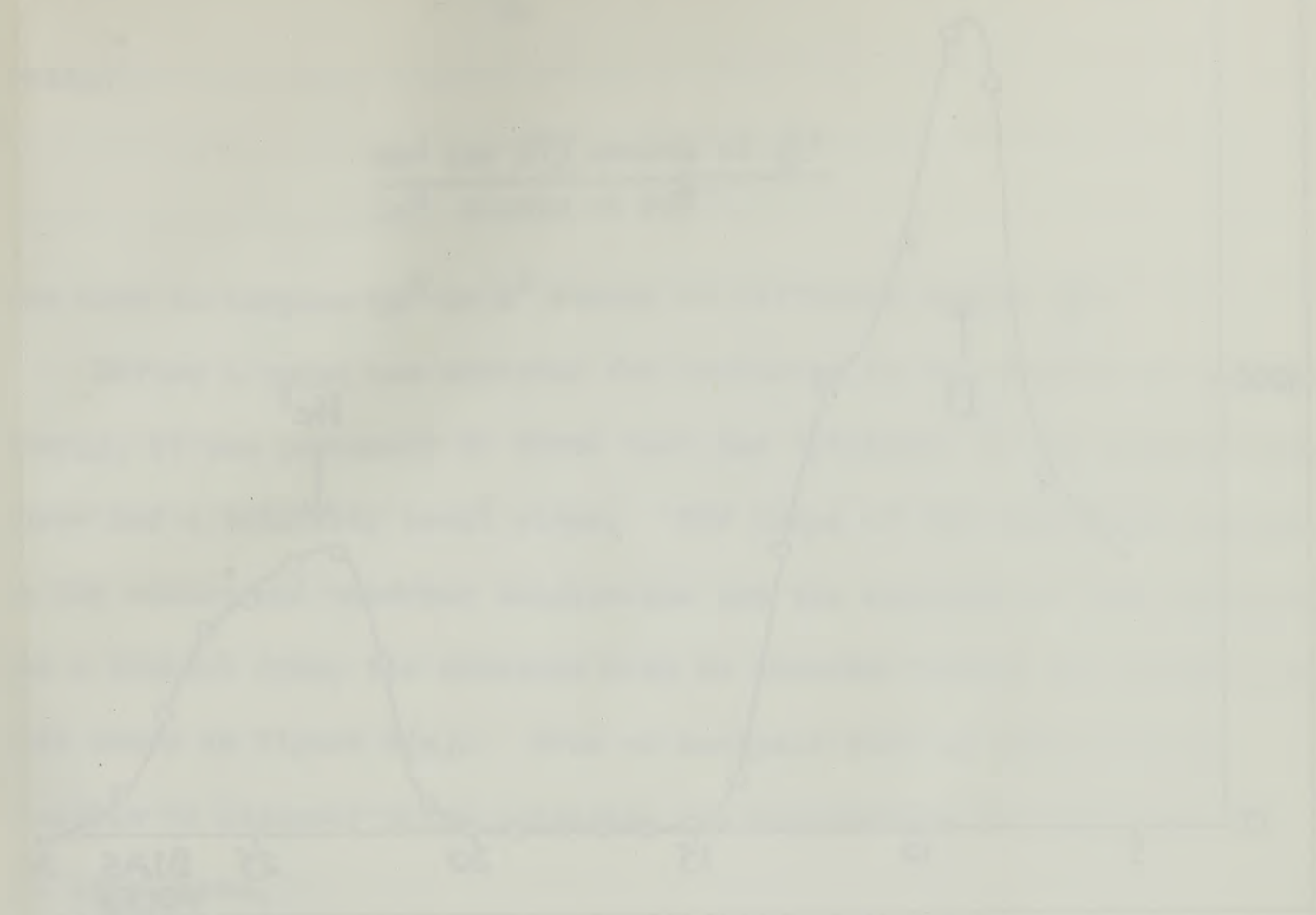


Figure C-3: 25°C thermal analysis curves.

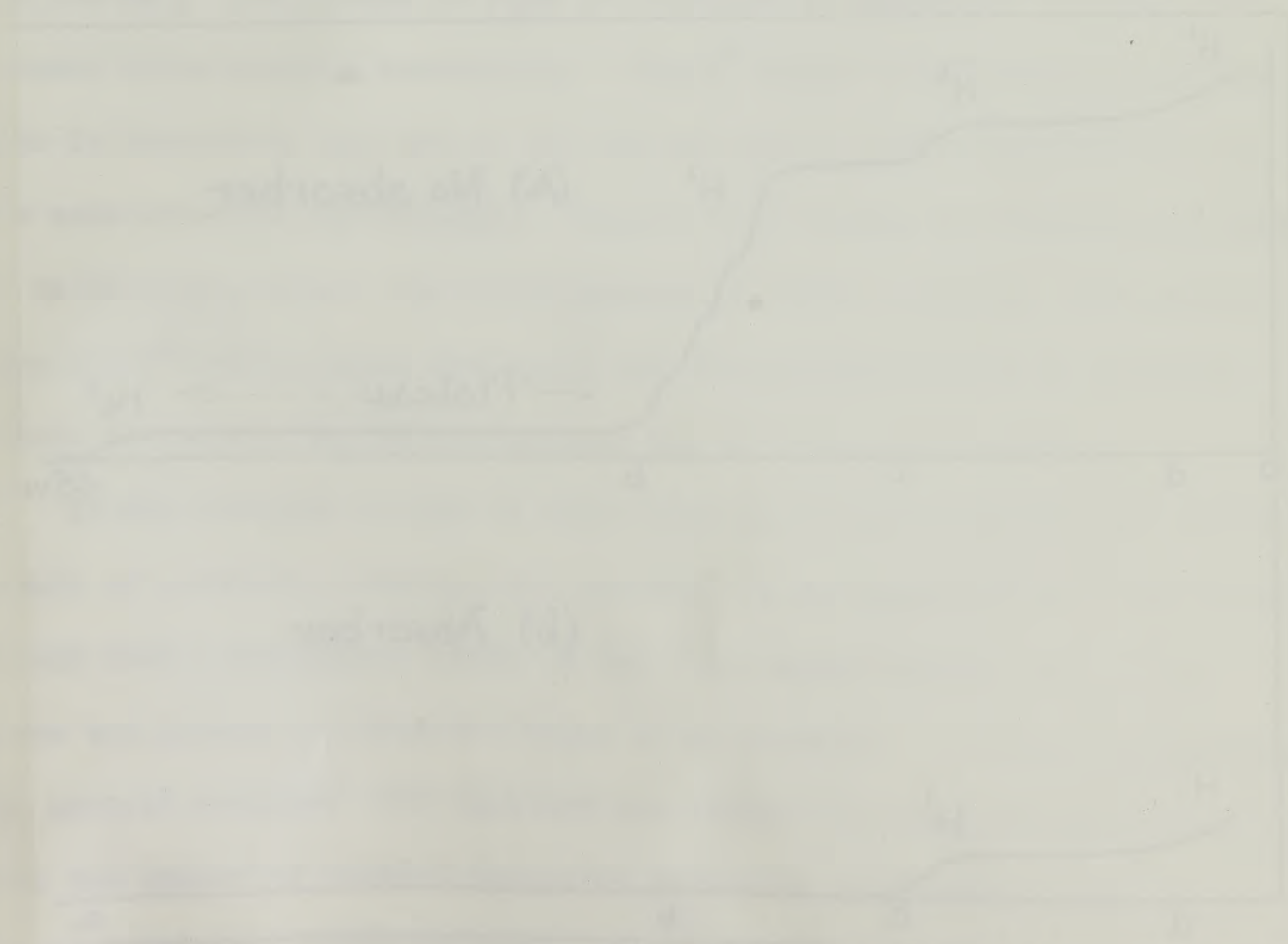


Figure C-4: Thermal analysis curves of samples arranged in

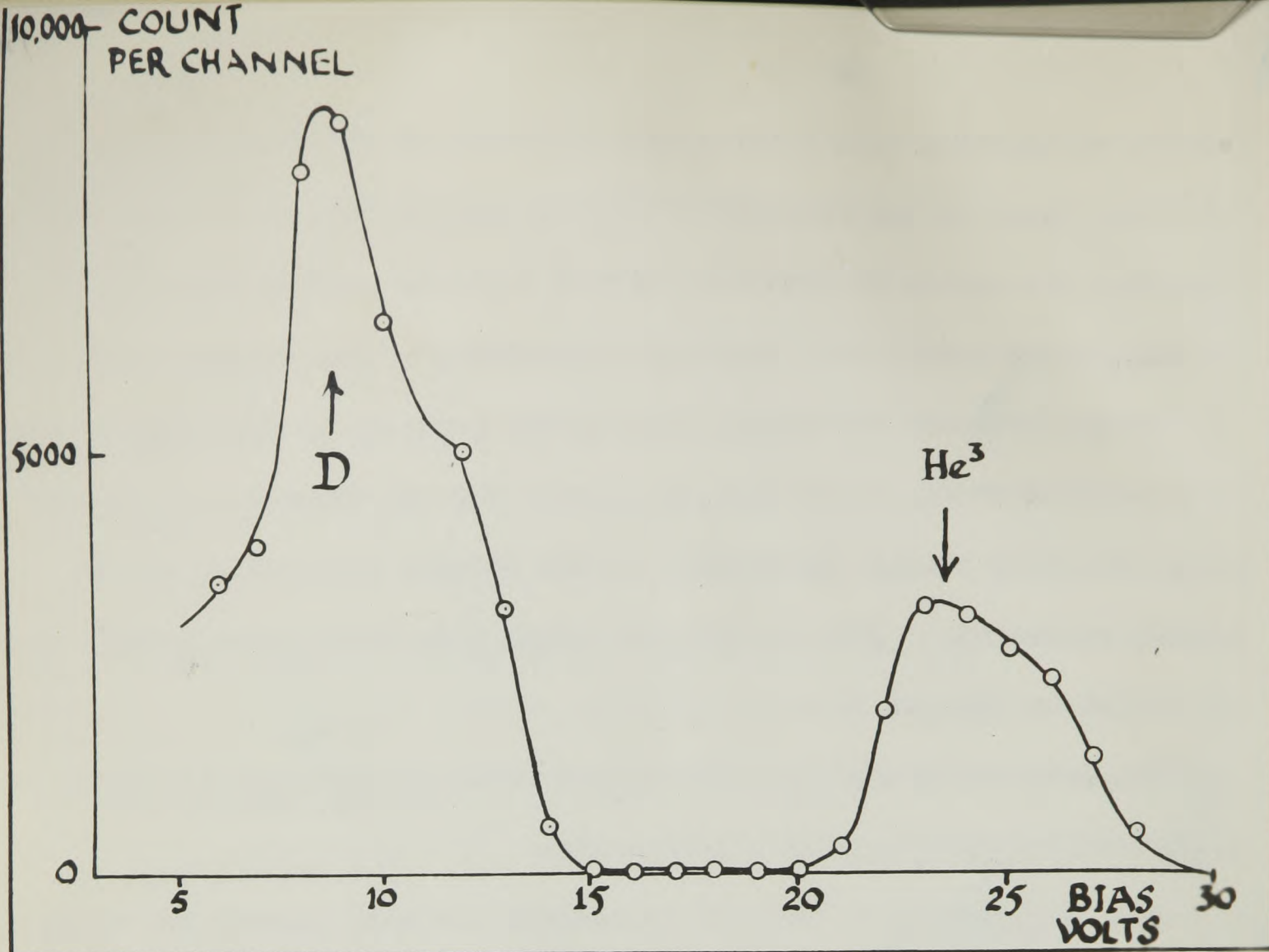


Figure C5 : 25-Channel pulse discriminator curve.

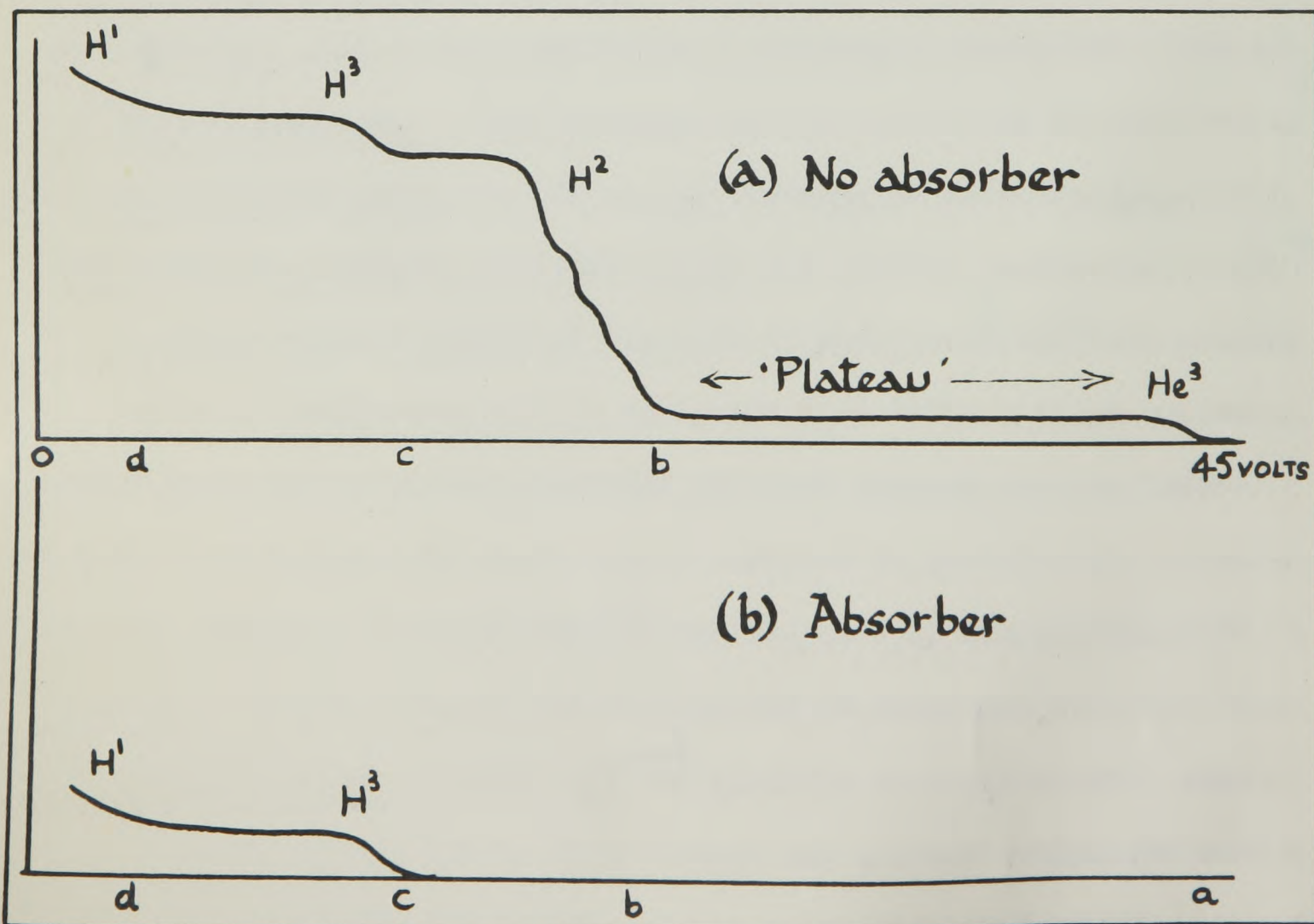


Figure C6: Bias curves of counters arranged for counting (a) He³ and (b) H³

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ratio:

$$\frac{\text{He}^3 \text{ (or H}^3\text{) counts at } \phi^*}{\text{He}^3 \text{ counts at } 90^\circ}$$

was used to compare He^3 or H^3 counts at different angles ϕ^* .

Before a count was accepted for inclusion in the results at a given energy, it was necessary to check that the 'plateau' in the counter bias curve had a tolerably small slope. The shape of the bias curve depends on the window and absorber thicknesses and the energies of the particles, but a typical form, for counters with no absorber before the windows, is that shown in figure 6(a). From an analysis such as Table 1 it is possible to discover which particles are responsible for the rises in the bias curve.

The pulses counted at the highest discriminator bias (about 45 volts) are the He^3 : the groups at bias (b) are due to deuterons entering the counter after elastic scattering. The H^3 begin to be counted when the bias is lowered to (c) and at (d) and the small pulses from protons in the same reaction are counted. Figure 6 (b) shows the filtering effect of an absorbing film: the only charged particles entering the counter then are H^3 and protons, and so if the discriminator bias is properly chosen to exclude the proton pulses, the H^3 alone are counted.

If the ordinate height of this curve is to be a good measure of the number of particles entering the counter, it is important that the curve should have a negligible slope in the bias region between the places where two groups of particles begin to be counted. A slope is expected for several reasons: for instance the statistical uncertainty associated with the number of initial ion pairs formed in the counter leads to a 'streggle' in the pulse heights; also at high counting rates 'pile up'

C.

of pulses will occur because of the finite time which the counter takes to be cleared. Again, because of the finite angle of acceptance of the particles by the counter window, particles of the same energy may pass through different lengths of counter gas and so produce a 'straggle' in the pulse heights.

In order to estimate the slope of the 'plateau' the pulses from one of the counters (say A) were fed in parallel to two scalers with discriminators set to give biases differing by 5 volts as shown in figure 4. The criterion for accepting a count was that the slope should be not greater than 0.3% change in counts per 5 volts bias, and by careful choice of bias levels a place could usually be found where the slope was much smaller than this. For He^3 , the mean value from 24 experimental runs was a slope of 0.04% per 5 volts; for H^3 the mean of 19 runs was 0.06%. Provided this 'plateau' slope were satisfactory, the count from the counter A (with the divided scaler channel) was given by the mean of these two scaler counts, and the bias of the scaler on counter B was set to the mean of those on counter A. Figure 5 is drawn from the counts recorded in a 25 channel pulse discriminator. The curve is effectively a frequency diagram of pulse heights, or the differential of such bias curves as those of figure 6.

It was important that the discriminator bias on all four scalers should be properly related each one to the others, in particular so that the bias difference of 5 volts could be set on the scalers recording the 'plateau' slope. This was done by feeding pulses from a pulse generator through two channels into two scalers simultaneously. The bias on each was then raised just sufficiently to stop the counting. This was done for various pulse heights, and by plotting the one scaler bias against

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the other, the expected linear relationship was found. These lines were then used to refer bias settings on three of the scalers to the equivalent setting on the fourth.

By a proper choice of conditions, it was possible to count H^1 pulses. Since each H^3 of reaction (b) is produced in company with a H^1 , a count of H^1 is a useful check on the efficiency of the counter. Since the specific ionization of protons is small the gas amplification in the counters was raised by increasing the H.T. to about 800v. An absorber could then be chosen to cut out the He^3 , H^2 and H^3 , and with the appropriate discriminator bias, only H^1 were counted.

With the increase in voltage, the background count rose greatly owing mainly to the increase in pulses due to neutron recoils, and was in fact nearly $\frac{1}{2}$ the total count. This lead to uncertainties in the proton count larger than for the H^3 . In order to compare the H^1 counts with those of H^3 at the same angle, two adjustments to the recorded counts are necessary. First, the particles have different energy, so the factors used to bring angles and solid angles from the lab. to CM system of coordinates are different. Second, the asymmetry of the angular distribution products must be taken into account to bring counts of both particles to the values which would have been recorded at the same C.M. angle: rough values for the coefficients A and B were taken from the work of Davenport et al.

After making these corrections, the final figures were:

$$\text{Ratio } \frac{H^1 \text{ counts}}{\text{Monitor counts}} = 0.855 \pm 0.017$$

$$\text{Ratio } \frac{H^3 \text{ counts}}{\text{Monitor counts}} = 0.847 \pm 0.013$$

In the circumstances, the agreement is seen to be very good.

Results.

The experimental results for the angular distribution can be displayed in several ways. Conventionally, the points are fitted by curves of the form

$$n(\phi^*) = n(90^\circ)(1 + A \cos^2 \phi^* + B \cos^4 \phi^* + C \cos^6 \phi^* + \dots) \quad (10)$$

where $A, B, C, \dots$ are functions of the deuteron energy E . This is an extension of the analysis of earlier results at low energies, where the asymmetry could be most conveniently represented by a single term $A \cos^2 \phi^*$. It also gives an indication of the extent to which partial waves of different angular momentum contribute their anisotropic distributions to the isotropic component.

From this latter point of view the curve can be better represented by a linear expansion in Legendre polynomials:

$$F(\cos \phi^*) = A_0 P_0 + A_2 P_2 + A_4 P_4 + \dots \quad (11)$$

where $P_n(\cos \phi^*)$ is the Legendre polynomial of the first kind of order n , and $A_0, A_2, A_4, \dots$ are now proportional to the intensities of outgoing particle waves of angular momentum quantum number $n = 0, 2, 4, \dots$

Despite the theoretical convenience of this representation, it was considered that there would be an advantage in calculating the results so that the curve fitted could be written in the form

$$n(\phi^*) = b_0 P'_0 + b_1 P'_1 + b_2 P'_2 + \dots \quad (12)$$

where b_0, b_1 and b_2 are numerical coefficients and $P_0, P_1, P_2, \dots$ are orthogonal polynomials. Such a polynomial P_n contains power terms in $x (= \cos^2 \phi^*)$ up to x^n and the coefficient of x^n itself is unity. The particular property which is of value to the present analysis is that the coefficients $b_0, b_1, b_2, \dots$ are independent in the sense that if a set of points is fitted with a curve containing terms in x

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up to x^n represented by equation (12) above, and it is then desired to add a term in $x^{(n+1)}$, this can be done simply by calculating, and then adding on, the polynomial P'_{n+1} and its coefficient b_{n+1} . That is, the other coefficients $b_0, b_1, \dots, b_n$ then remain unchanged.

The advantage would become apparent if, for example, the present series of results were extended by measurements at higher energies, e.g. up to 1 MeV. It would then become necessary to consider a term in x^3 . At higher energies the coefficient of such a term would be large, and it might be necessary to determine whether it first became significant in the lower energy region up to 450 KV. where terms up to x^2 only had previously been fitted. A representation

$$n(\phi^x) = 1 + Ax + Bx^2$$

to which a term Cx^3 was added, would be less convenient, since the coefficient A and B would then have to be recalculated.

Reduction of the formula (12) to the form (10) is easily done by substitution, since if

$$a_0 = (b_0) + \beta_0 (b_1) + \gamma_0 (b_2)$$

$$a_1 = b_1 + \gamma_1 b_2$$

$$a_2 = b_2$$

where β and γ are given below,

then $A = \frac{a_1}{a_0}$ and $B = \frac{a_2}{a_0}$

β_0, β_1 and γ_1 , are computed from the experimental results as

$$\left. \begin{aligned} \beta_0 &= -\bar{x} \\ \gamma_0 &= [\Sigma x \Sigma x^3 - (\Sigma x^2)^2] / [n \Sigma x^2 - (\Sigma x)^2] \\ \gamma_1 &= [\Sigma x \Sigma x^2 - n \Sigma x^3] / [n \Sigma x^2 - (\Sigma x)^2] \end{aligned} \right\} \quad (13)$$

Values were obtained for the ratio



$$y = \frac{H^3 \text{ counts at angle } \phi^*}{\text{Monitor counts}} \times \sin \phi$$

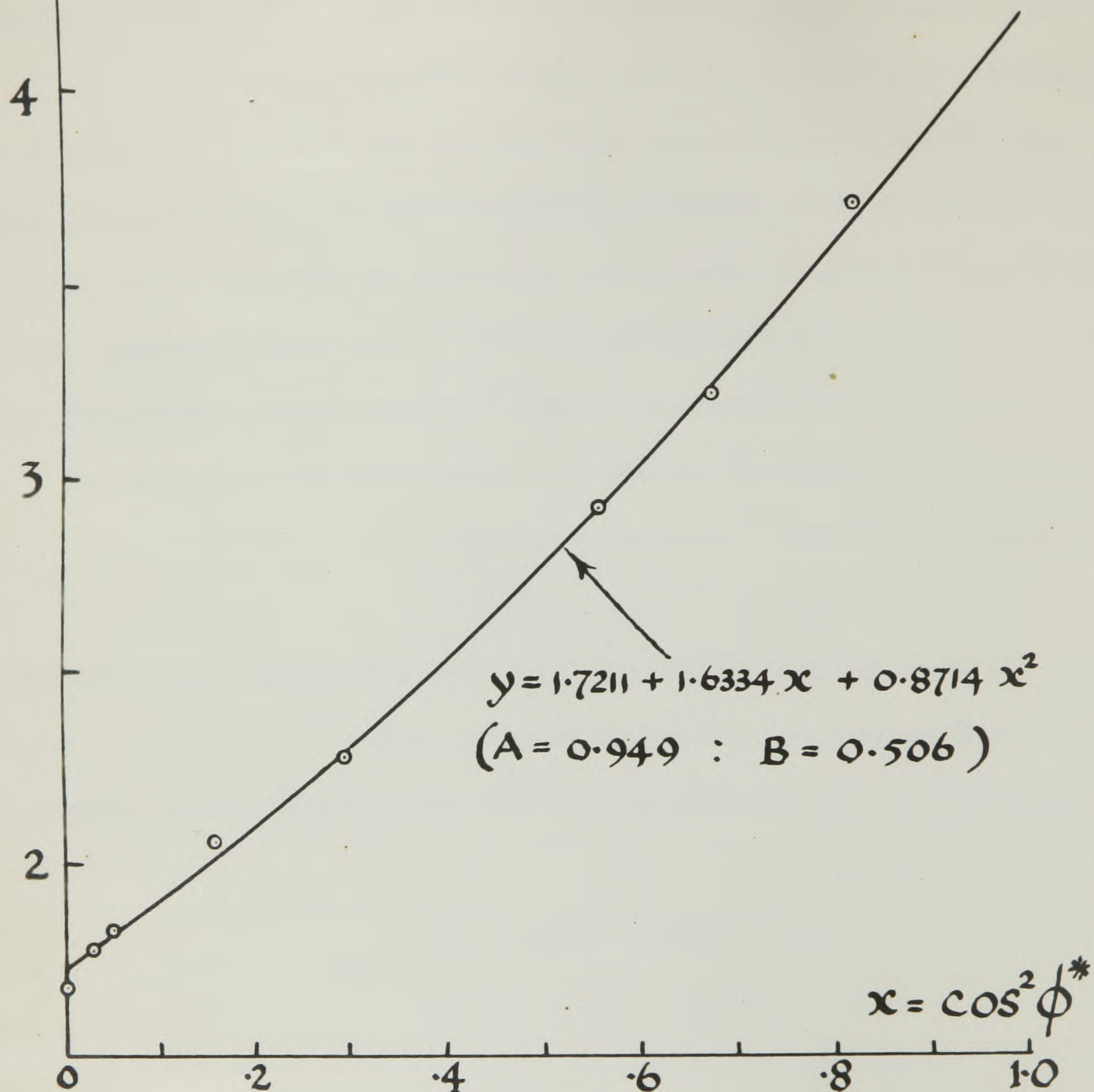


Figure C7 : Angular distribution of tritons from $D(d,p)H^3$ at 450 kV.

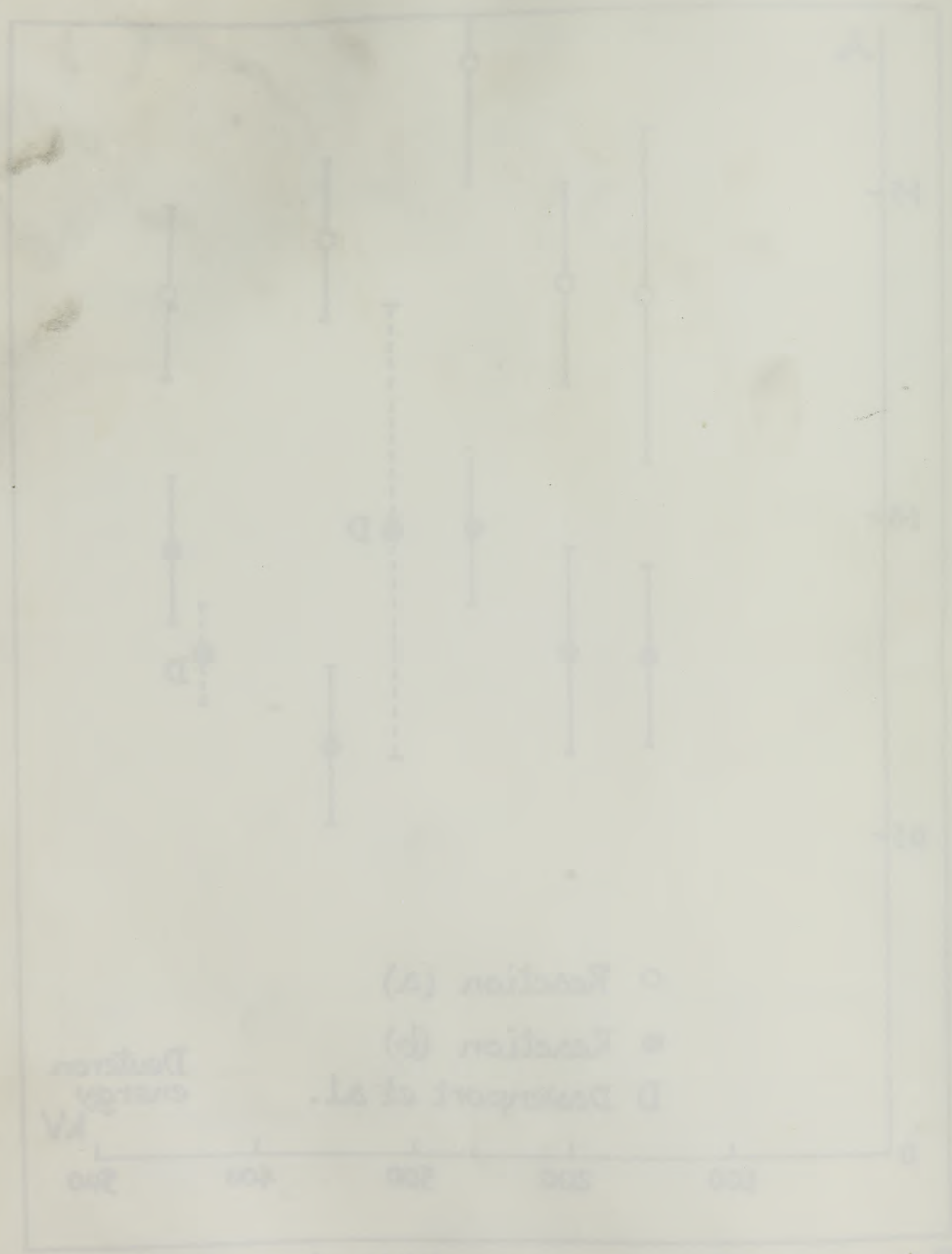


Figure C8: Dependence of coefficient A on deuteron energy for reactions (a) and (b).

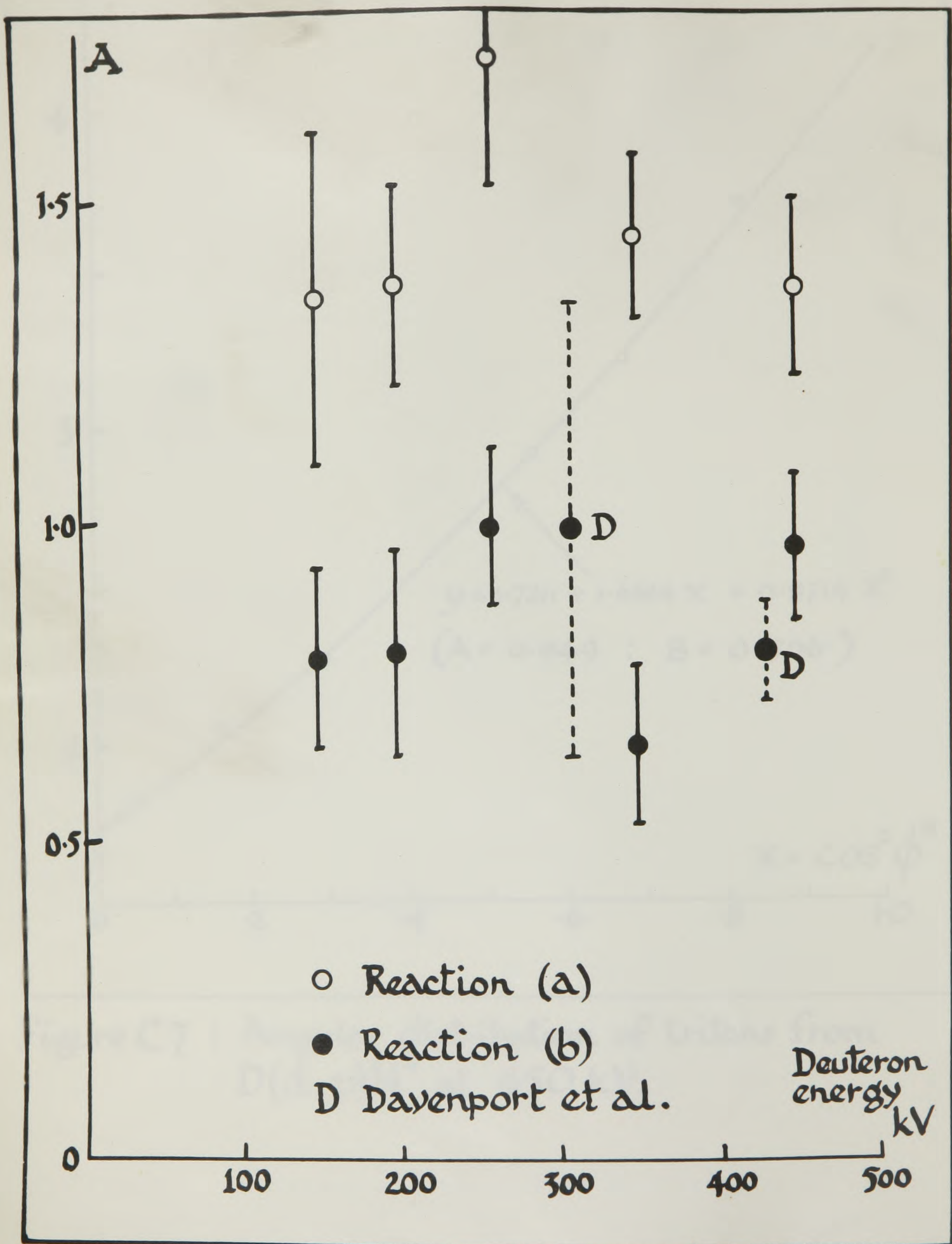


Figure C8: Reactions (a) and (b): dependence of coefficient A on deuterium energy.



Figure C.91: Dependence of coefficient B on activation energy.

○ Reaction (a)
 ● Reaction (b)
 D - Debye-Hückel eqn.

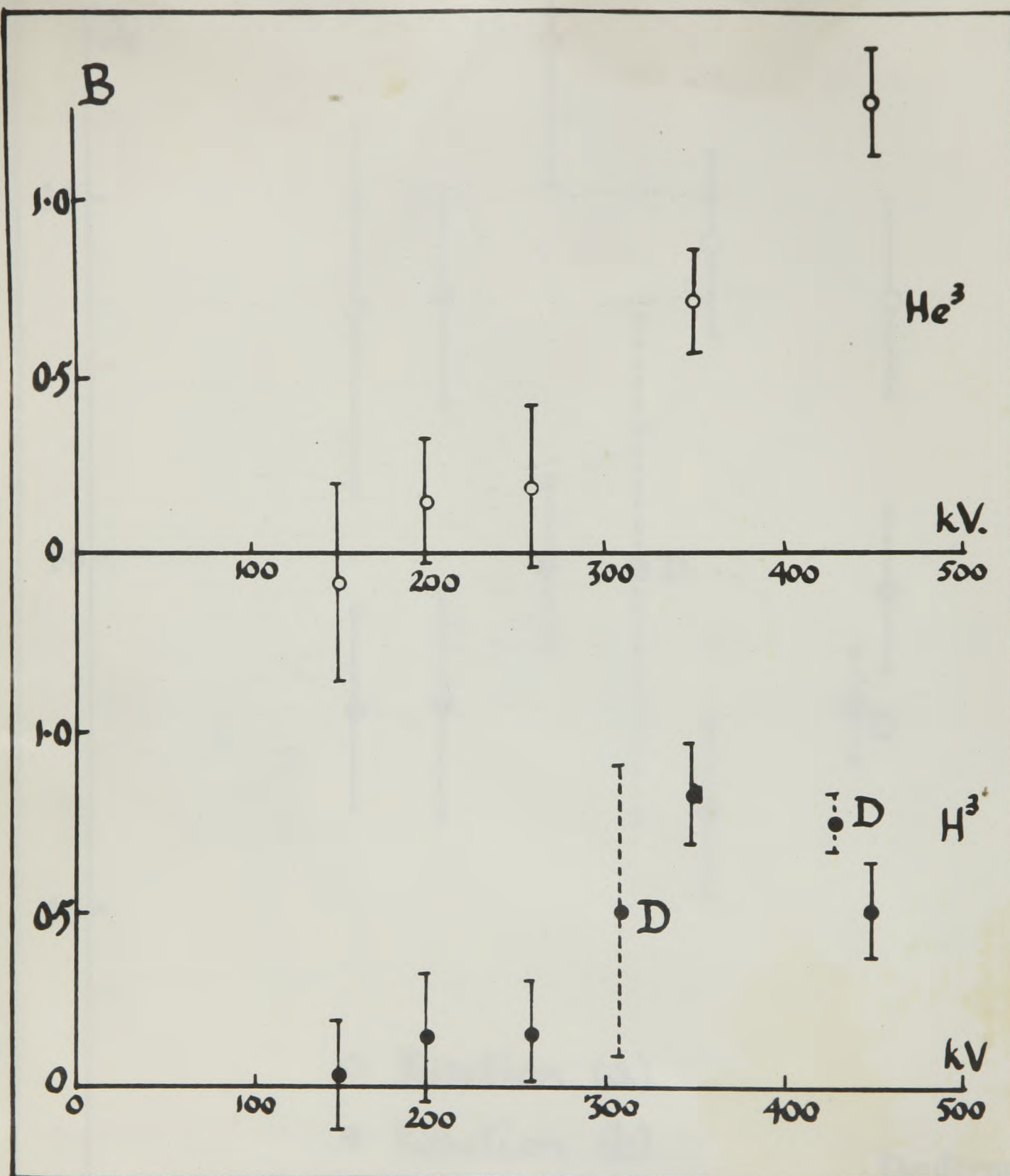


Figure C 9 : Reactions (a) and (b) : dependence of coefficient B on deuteron energy.

- Reaction (a)
- Reaction (b)
- D- Davenport et al.

Table 2.

Values of the Asymmetry Coefficients.

E(KV)	He^3		H^3	
	A	B	A	B
150	1.334 ± 0.267	-0.087 ± 0.294	0.773 ± 0.117	0.031 ± 0.130
200	1.355 ± 0.169	0.155 ± 0.183	0.687 ± 0.161	0.297 ± 0.186
260	1.718 ± 0.212	0.186 ± 0.245	0.978 ± 0.122	0.149 ± 0.150
350	1.427 ± 0.126	0.711 ± 0.145	0.637 ± 0.116	0.839 ± 0.130
450	1.342 ± 0.138	1.271 ± 0.157	0.949 ± 0.110	0.506 ± 0.130



Figure 10. Energy dependence of reaction (a) and (b):
energy dependence of (A+B)

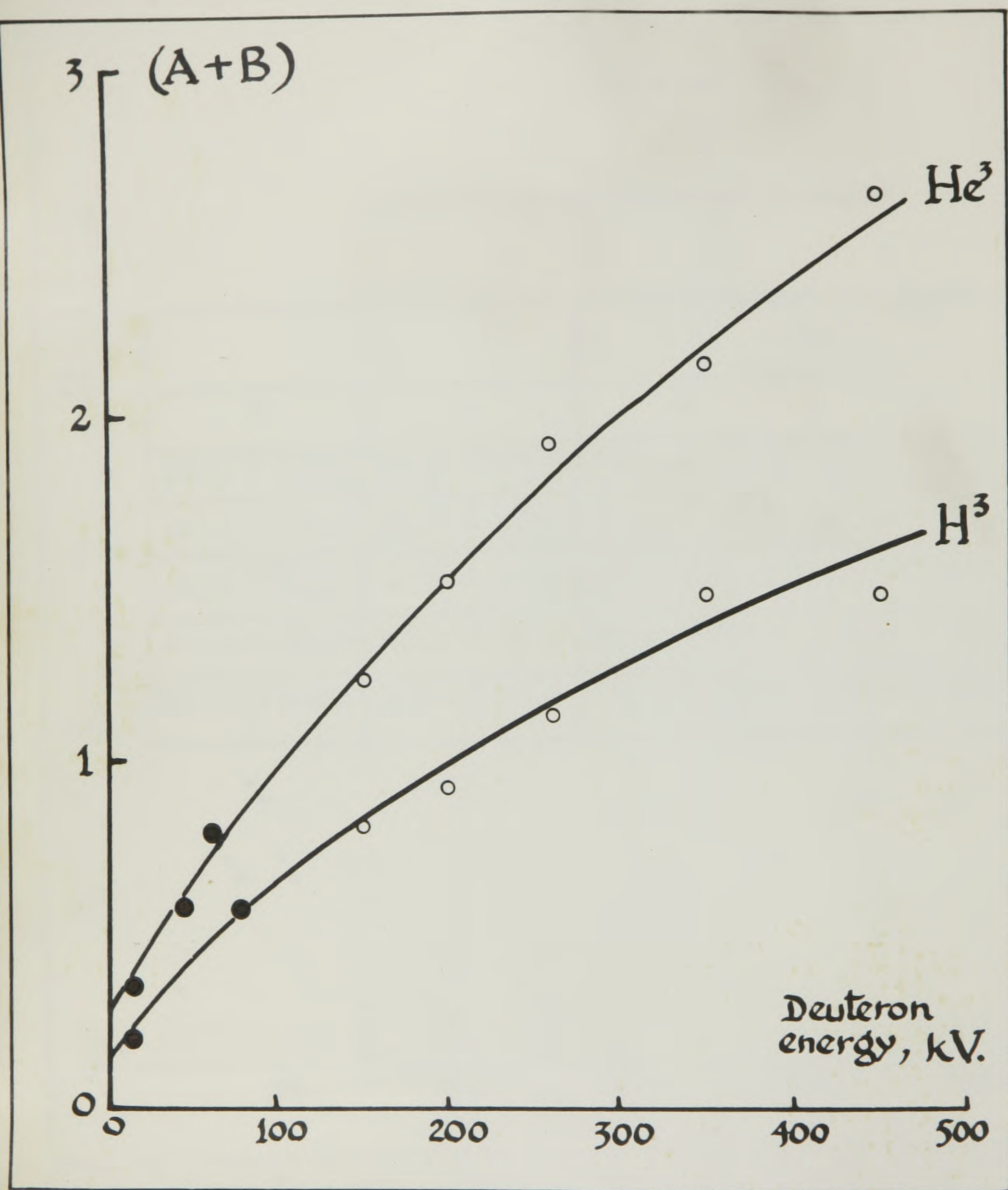
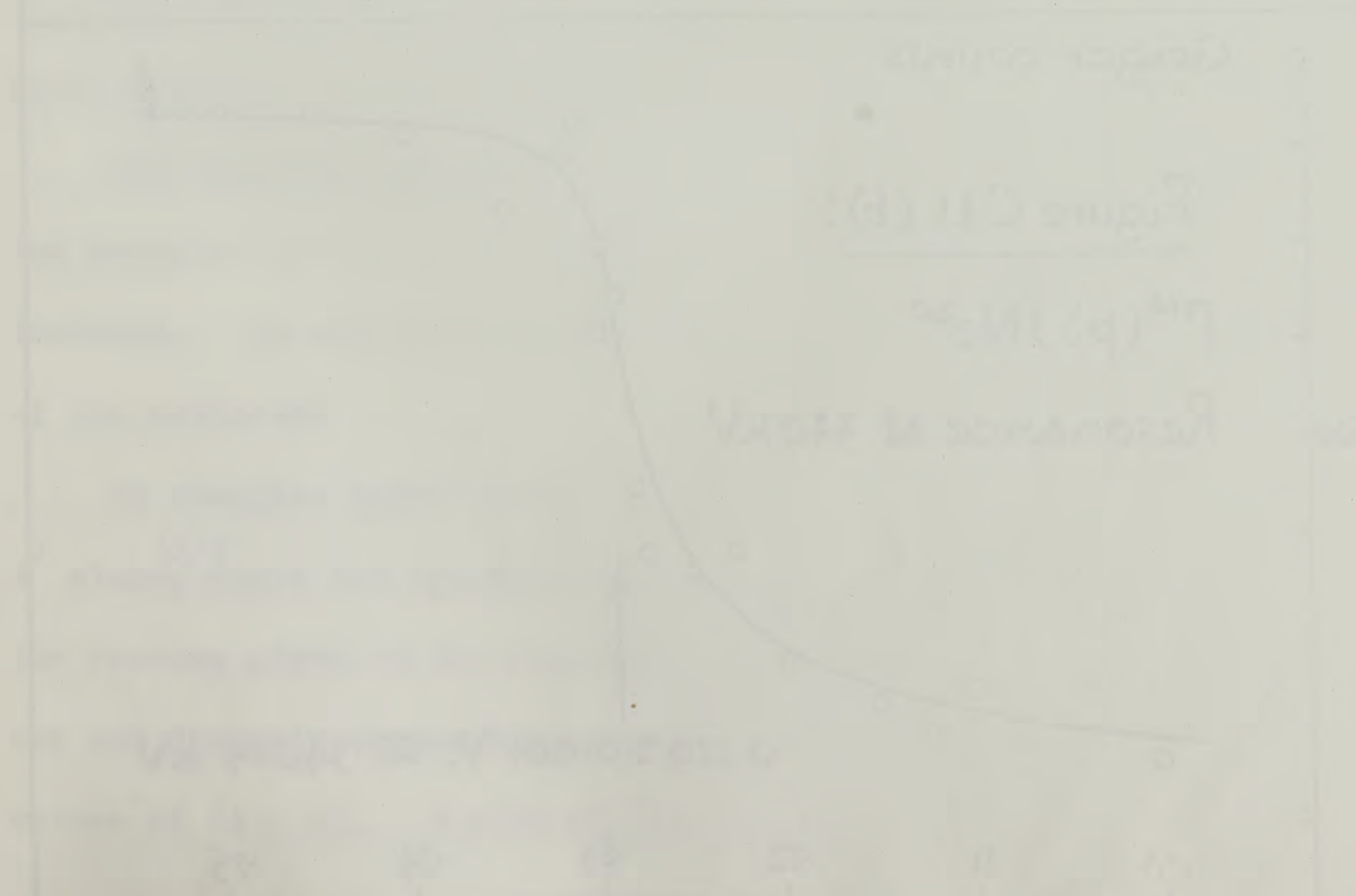
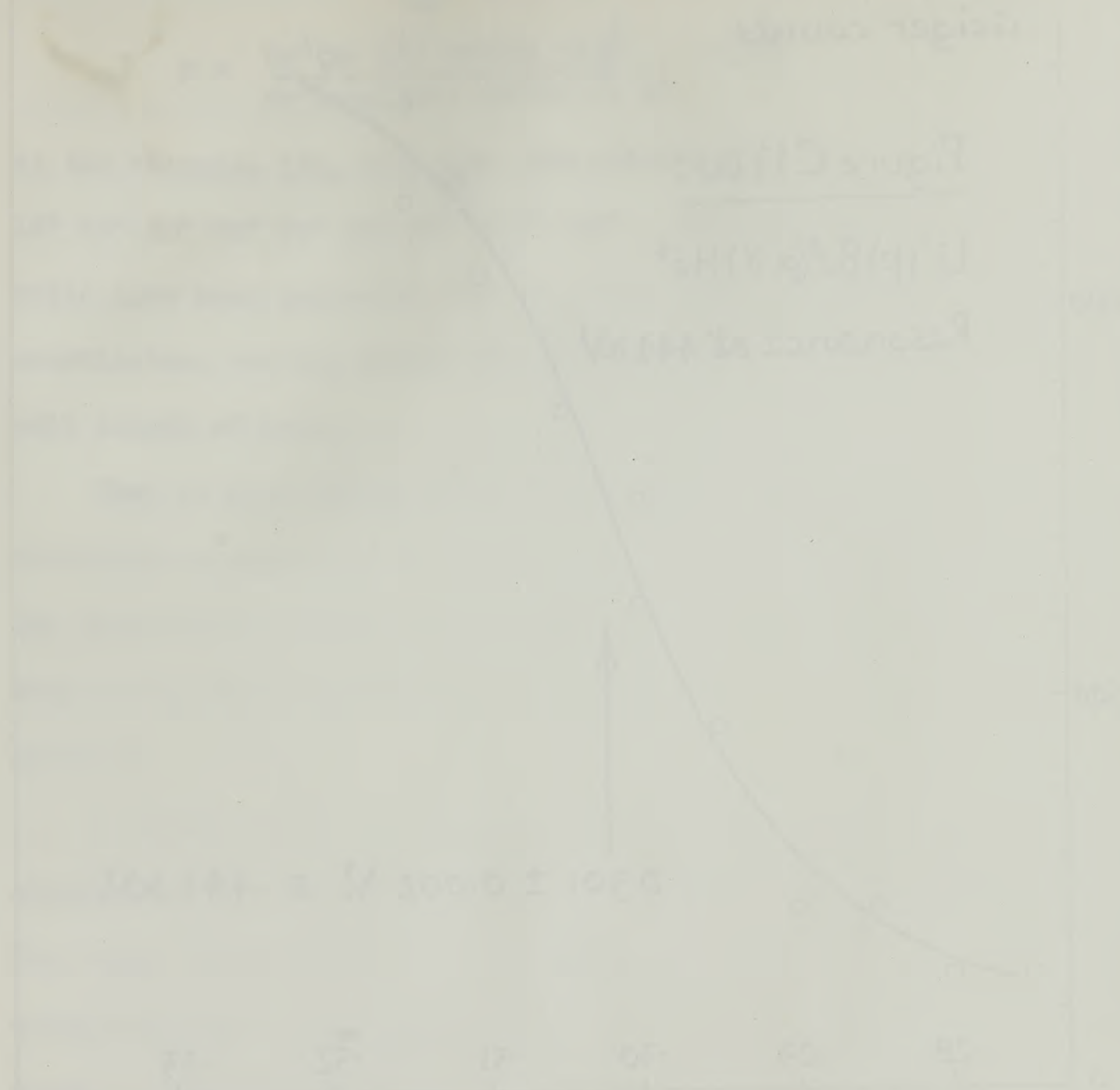


Figure C10: Asymmetry of reactions (a) and (b):
energy dependence of $(A+B)$

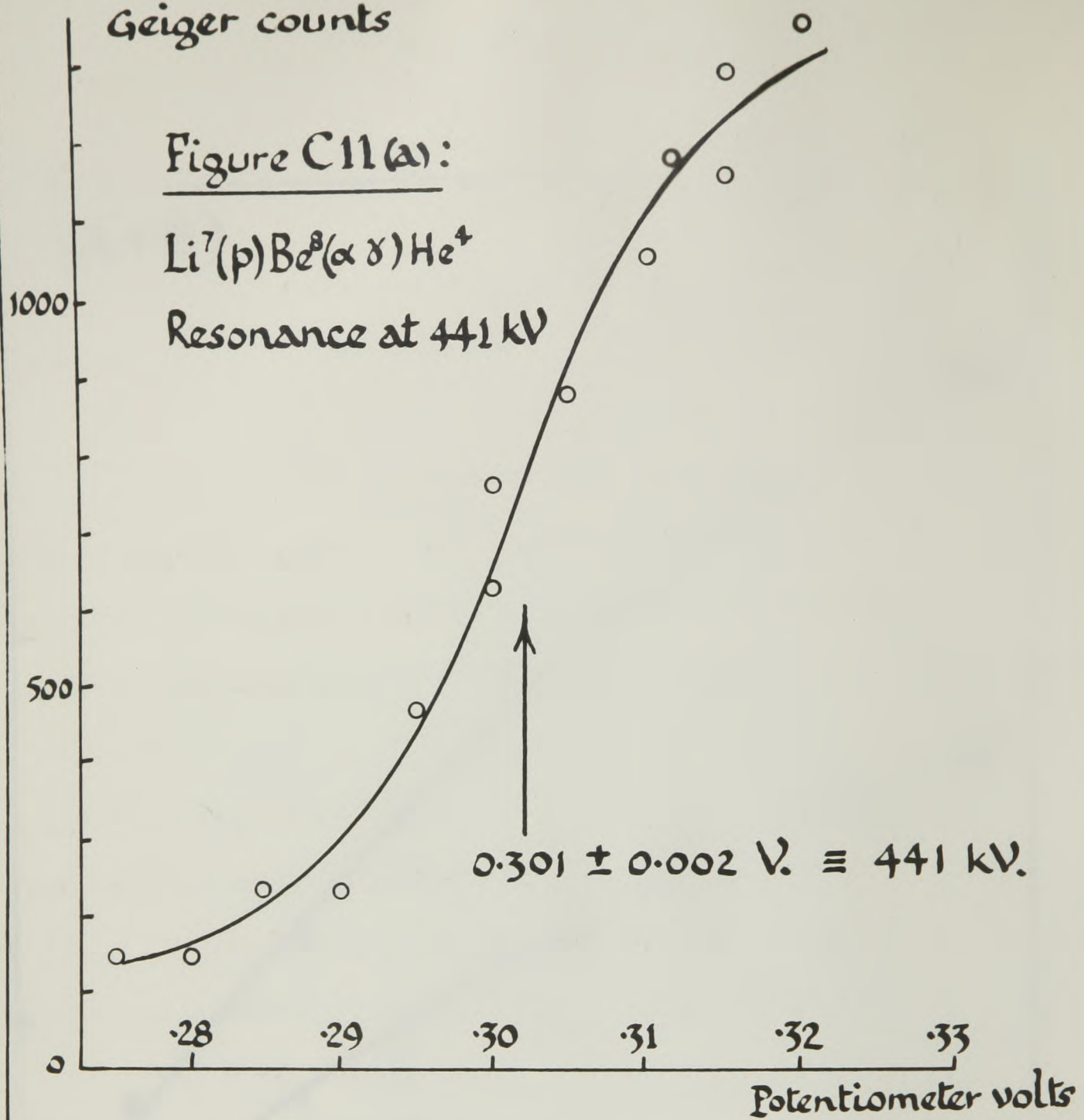


Geiger counts

Figure C11 (a):

$\text{Li}^7(\text{p})\text{Be}^8(\alpha\gamma)\text{He}^4$

Resonance at 441 kV

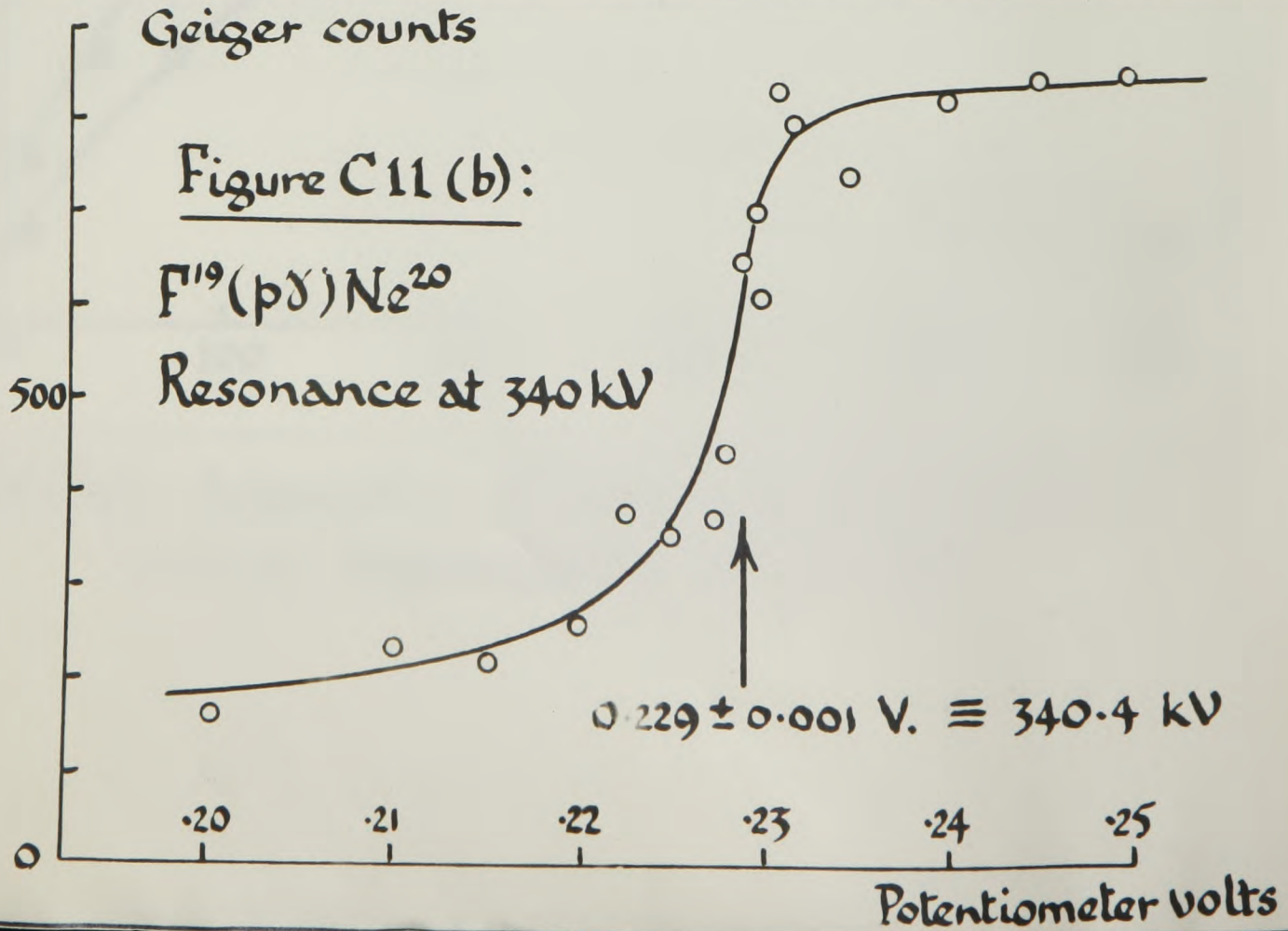


Geiger counts

Figure C11 (b):

$\text{F}^{19}(\text{p}\gamma)\text{Ne}^{20}$

Resonance at 340 kV



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$$y = \frac{\text{He}^3(\text{or H}^3) \text{ counts at } \phi^*}{\text{He}^3(\text{monitor}) \text{ counts at } 90} \cdot \sin \phi \quad (14)$$

at the energies 150, 200, 260, 350 and 450 KV., and at the lab. angles $18^\circ 25^\circ 30^\circ 60^\circ 70^\circ 80^\circ 90^\circ 100^\circ 110^\circ$. The number of counts in this ratio have been corrected for the solid angle conversion from lab. to C.M. coordinates, and the factor $\sin \phi$ converts the yield to the value for unit length of beam.

Thus at each energy there were 9 points through which an angular distribution curve had to be fitted. At each point the count was made for about 10,000 counts on the monitor, so that each point can be given very nearly the same statistical weight, and the standard deviation is about 1%.

A typical plot of y against $x(=\cos^2 \phi^*)$ is shown in figure 7. The standard deviation of each point lies within the circle drawn round it. The curve fitted has been determined by a type of least squares method, using orthogonal polynomials as the independent variable as described above, and then converting to the form of equation (10).

The values of the coefficients A and B , determined in this way for both reactions at each energy, are shown in Table 2 and figures 8 and 9.

For reaction (b) Davenport et al. have estimated A and B values for two energies in the range considered here, and their points are also included. On all these points the vertical spread is the standard error of the estimate.

At energies below 150 KV. there are many estimates of the coefficient A' alone,^{27,28} where the analysis has not been carried to terms in $\cos^4 \phi^*$. For the reasons given at the beginning of this Section, these values of A' are not directly comparable with the present results, but roughly with our values of $(A + B)$. A plot of $(A + B)$ including a few low energy values

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of A is given in figure 10. The low energy results are due to Eliot et al., Davenport et al.,²⁸ and Preston.³⁵

reading at resonance could then be used to find a value for the resistance stack.

The value of this resistance was also found at the same

1000 volt power. Corrections and Errors.

Energy Calibration.

The voltmeter on the control desk of the H.T. Generator was checked by using known nuclear resonances. The reactions were:

$F^{19}(p, \gamma)Ne^{20}$ resonance at 340.4 KeV.

$Li^7(p, \gamma)Be^8(\gamma)_2He^4$ resonance at 441 KeV.

These energies are known with an estimated error of about 500 volts.^{1, 36, 37}

Thick targets of calcium fluoride and metallic lithium were used in turn, and these were bombarded with constant proton currents at various voltages. The voltages were measured by balancing a small fraction of the total potential against a standard cell on a Tinsley potentiometer. This small voltage was generated across a small resistance in series with a stack of about 1300 1.1 M Ω carbon resistors which was placed in parallel with the accelerator column, although its value was not at the time accurately known.

The γ - quanta from the reactions were recorded by a Geiger counter and scaler, and at each energy the background counts, due mostly to neutrons, but partly to X-rays produced in the accelerating lens system, was found by moving a shutter in front of the target to intercept the beam. The beam current was kept low to avoid dispersing the target, and varied from 10 to 50 μA .

The potentiometer readings were plotted against the geiger counts (corrected for background), as shown in figure 11 for the fluorine and

C.

lithium resonances. Using the measured value of 971.3^{Ω} for the small resistor, and assuming an energy for the resonances, the potentiometer reading at resonance could then be used to find a value for the high resistance stack.

The value of this resistance was also found at low energies by using a 1000 volt power pack. The results are

$$R(4412KV) = 1430 \pm 18 \text{ M}\Omega$$

$$R(340KV) = 1448 \pm 6 \text{ M}\Omega$$

$$R(0.8KV) = 1440.7 \pm 6 \text{ M}\Omega$$

weighted mean

$$R = 1443.6 \pm 4 \text{ M}\Omega$$

During angular distribution experiments, the beam voltage was kept to values selected on the control desk voltmeter. In cross-section determinations, greater precision was necessary and the standard cell and Tinsley potentiometer (with a galvanometer to indicate the balanced state) was used as a reference.

For this reason the two voltages need to be related one to the other. In Table 3 below the potentiometer readings have been translated, by means of the nuclear resonance measurements, into a scale of true kilovolts. It seems that the voltmeter is in error by about 10 KV. at the full scale deflexion.

Table 3.

Control Desk Voltmeter Kilovolts.	True Kilovolts.
100	103
200	205
300	308
400	410
500	512

Finite Angle of Acceptance.

In the plane of rotation of the counters, the counter window slit system subtends an angle of about 1.5° at the beam and each point on the window has a 'field of view' of about 6° , so that a considerable length of beam is seen. For purposes of comparison with theory, the counts recorded at each position must be reduced to those which would have been recorded if all the disintegrations had taken place on the axis of the counter system, and if the window slit width were negligible.

The geometry is shown in figure 3. The constants independent of counter angle ϕ are:

Window radius	$a = 0.102$ cm
Window slit width	$w = 0.150$ "
Window distance from counter axis	$R_0 = 5.760$ "
Slit distance from window	$= 1.524$ "
Beam radius	$r = 0.200$ "

In the plane of the diagram:

x = Perpendicular distance from beam axis.

z = Distance (in plane of window) from centre of window.

R = Distance from area of window dA at Z to volume dV of beam at x

θ = Angle between sectors R and R_0 .

θ_1, θ_2 = Limits of θ , defined by slit jaws.

Angles and solid angles measured in a co-ordinate system in which the centre of mass of the reaction particles is stationary ("C.M." system)

are indicated with an asterisk. Otherwise angles are measured in a co-ordinate system stationary in the laboratory ("lab." system). We

write the ratio of angles and ratio of solid angles in these systems as

$$\phi^*/\phi = F(\phi) \quad \omega^*/\omega = G(\phi) \quad (15)$$

Let the number of disintegrations/cm³/sec = N . Considering a given

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disintegration product, the number/sec/unit solid angle (CM) at angle ϕ^* is

$$\frac{dN(\phi^*)}{d\omega^*} = \frac{dN(90^*)}{d\omega^*} [1 + A \cos^2 \phi^* + B \cos^4 \phi^*]$$

The number in a solid angle (lab.) $\Delta \omega$ at angle ϕ^* is

$$\frac{dN(\phi^*)}{d\omega} \cdot \Delta \omega = \frac{dN(90^*)}{d\omega^*} [1 + A \cos^2 \phi^* + B \cos^4 \phi^*] G(\phi) \Delta \omega$$

Consider the disintegrations in a volume $dV [= f(\phi, \theta, x, z)]$.

The number/sec reaching a strip ΔA of window from dV is

$$\begin{aligned} \frac{dN(90^*)}{d\omega^*} [1 + A \cos^2(\phi+\theta) F(\phi+\theta) + B \cos^4(\phi+\theta) F(\phi+\theta)] G(\phi+\theta) \frac{\Delta A}{R^2} dV \\ \equiv n(\phi+\theta)^* \frac{\Delta A}{R^2} dV \end{aligned} \quad (16)$$

Now
$$dV = \frac{R \cdot 2\sqrt{r^2 - x^2} dx \cdot d\theta}{\sin(\phi+\theta)}$$

and
$$\Delta A = 2\sqrt{a^2 - z^2} \Delta z$$

∴ Total number/sec reaching the strip is

$$N_{dA} = \iint n(\phi+\theta)^* \cdot \frac{2\sqrt{a^2 - z^2}}{R} \Delta z \cdot 2\sqrt{r^2 - x^2} d\theta dx$$

Since $R = (R_0 \sin \phi + z \cos \phi + x) / \sin(\phi+\theta)$ this becomes

$$N_{dA} = 4\sqrt{a^2 - z^2} \Delta z \int_{\theta_1}^{\theta_2} n(\phi+\theta)^* d\theta \int_{-r}^{+r} \frac{\sqrt{r^2 - x^2}}{(R_0 \sin \phi + z \cos \phi + x)} dx \quad (17)$$

The factor containing x can readily be integrated, but the one containing θ is best solved by numerical summation. The two parts were considered separately.

We have
$$\int_{-r}^{+r} \frac{\sqrt{r^2 - x^2}}{k+x} dx = \pi [k - \sqrt{k^2 - r^2}]$$

where

$$k = R_0 \sin \phi + z \cos \phi$$

The summation of the factor in θ was done for 5 strips of window of equal area, for which

$$\sqrt{a^2 - z^2} \Delta z = \text{constant}$$

The resulting correction which must be applied to the counts to correct

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them for the finite angle of acceptance of the counter are shown in

Table 4 below, and are seen to be negligible in all cases.

Table 4

	ϕ	Correction %
He^3 at 450 KV	18°	.01
	90°	.04
260 KV	90°	-.25
150 KV	90°	-.28

Discussion.

The results of the present experiment, together with the work of Eliot et al.²⁷ at lower energies and that of Blair et al.³² at higher energies, leave little doubt that there is a pronounced difference in the angular distribution of the products of the two reactions.

In the absence of any other published theoretical treatment of the D-D reactions it is necessary to rely on the work of Konopinski and Teller²⁹ and that of Beiduk et al.³⁰ These authors do not expressly predict any difference in the asymmetry of the reactions, but it may be convenient to summarise very briefly the steps which lead to their expression for the differential cross-section, since if their theory be accepted it is in terms of differences in the parameters contained in this expression that the different asymmetry must be explained.

In order to explain the general behaviour of the coefficient A for the proton reaction (which rises to a maximum of about 1 at about 400 KV. and has negative values after about 1 MV.) it was necessary to assume some degree of spin orbit coupling. That spin-orbit interaction does exist has been recently demonstrated by measurements of the degree of polarization of the product protons and neutrons.^{9,21}

The transitions from initial to final state of the compound nucleus are limited by considerations of parity, the exclusion principle, and the symmetry of the reaction. The differential cross-section for each allowed transition is a sum of products, each of which includes a weighting factor determined by the spin configuration, an "intrinsic reaction probability" which determines the magnitude of the outgoing waves, the cross-section for compound nucleus formation σ_c (which includes the penetrability of the nuclear barriers by the incoming waves), and a normalized spherical harmonic.

The value of σ_e was found using the WKB approximation to the conditions at the surface of the nucleus, although this approximation is of doubtful validity here. In the notation used by Beiduk, the final expression for the differential cross-section becomes

$$\frac{d\sigma}{d\omega} = K_0 \sigma_0 + K_1 \sigma_1 + A_1 \sigma_1 \cos^2 \phi^* + K_{02} (\sigma_0 \sigma_2)^{1/2} (Q_0 Q_2)^{1/2} (1 - 3 \cos^2 \phi^*) \quad (18)$$

where σ_e has been defined, Q_l is the probability of particle emission with angular momentum l , and K_0 , K_1 , K_{02} , and A_1 are parameters fixed by comparison with experiment, and in theory related to the matrix elements of the perturbations responsible for a transition. This expression is valid for energies up to about 300 KV. where a $\cos^4 \phi^*$ component of the angular distribution is effectively zero. To include a $\cos^4 \phi^*$ variation (i.e. first-order D-wave interaction) we must add the term

$$+ A_2 \sigma_2 \cos^2 \phi^* + B_2 \sigma_2 \cos^4 \phi^* + (\sigma_0 \sigma_4)^{1/2} K_{04} (Q_0 Q_4)^{1/2} (1 - 10 \cos^2 \phi^* + \frac{35}{3} \cos^4 \phi^*) \quad (19)$$

Below 53 KV., Eliot et al. had measured the total cross-sections and found, within 1% that

$$\sigma(a) = \sigma(b)$$

Since the last term in expression (18) vanishes on integration they concluded that a sufficient explanation of the asymmetry difference was that K_0 , K_1 and A_1 were the same but that K_{02} inexplicably differed.

The results of the present experiments suggest however that this cannot be true, for a calculation of the cross-sections of both reactions from measurements made subsequent to the angular distribution runs has shown that in this energy region

$$\sigma(a) > \sigma(b)$$

although apparently

$$\sigma(a) \rightarrow \sigma(b) \text{ as } E \rightarrow 0$$

The figures are:

E(KV)	σ (a) m.barns	σ (b) m. barns
155.30	29.2	25.7
205.74	40.1	38.2
270.40	47.3	42.4
361.56	60.5	52.5
465.94	73.5	61.8

The standard errors of these cross-section figures are estimated to be less than 10%.

It would therefore seem that even without introducing differences in the extra terms of expression (19), we must assume that some of the parameters $K_0 K_1 A_1 K_{02}$ differ.

In view of the freedom of choice offered by the extra parameters of (19) the values of these parameters obtained by fitting our results to the Beiduk and Konopinski theory may not have significance.

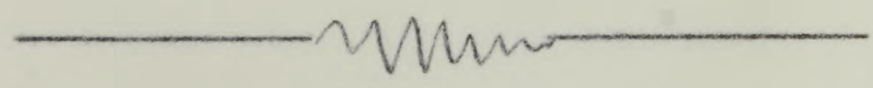




Figure 1 - Electrolytic tank

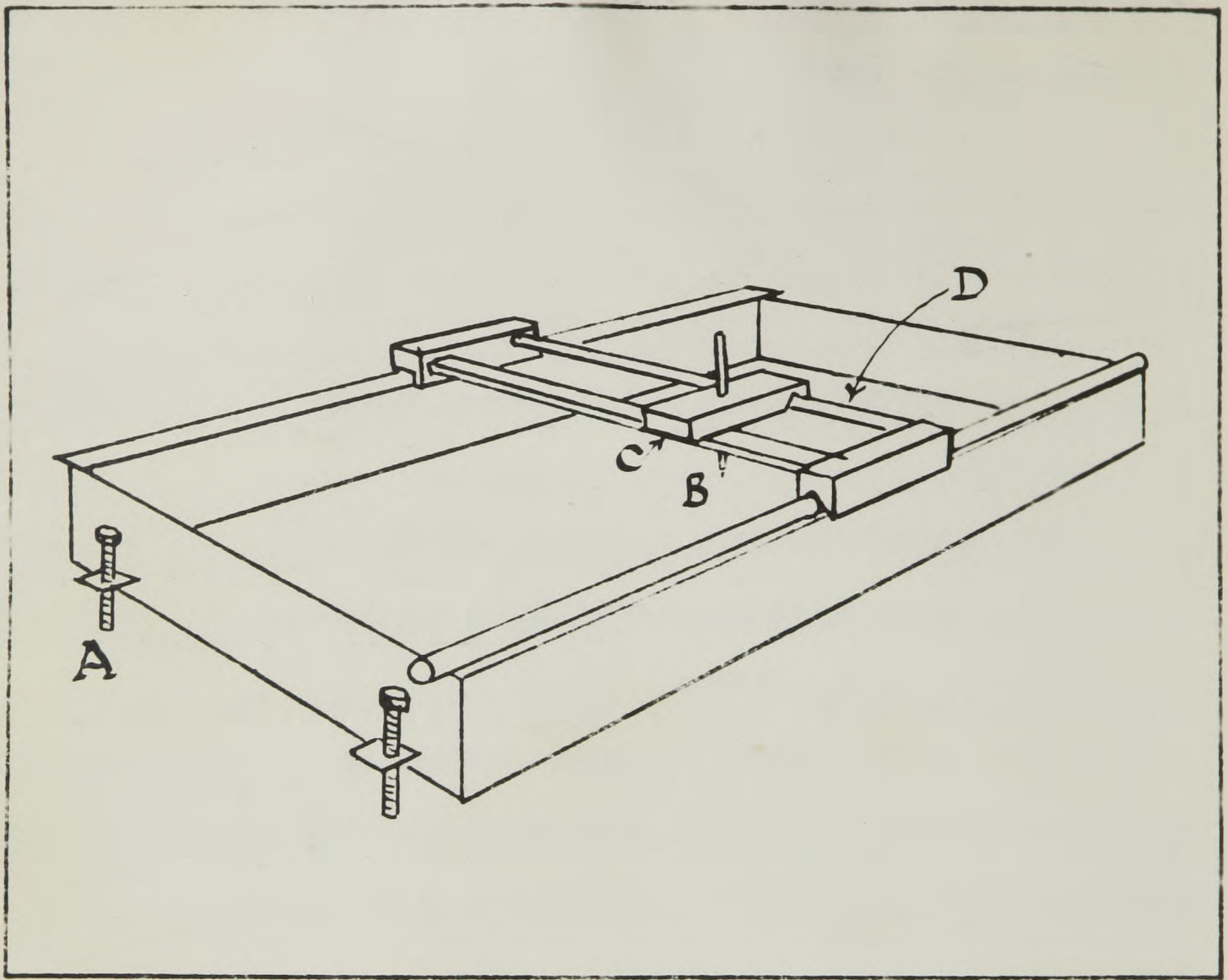


Figure App.1: Electrolytic tank.

Appendix.

Some apparatus used in the investigation of magnetic focussing fields.

1. Electrolytic Tank.

A tank of area 105 cm. x 75 cm. and 7 cm. deep was made for investigating two-dimensional potential distributions. This is shown in figure 1. By adjustment of the four levelling screws A the tank can be tilted to give a wedge of electrolyte, so extending the use to the three-dimensional survey of fields which have cylindrical symmetry.

The tank is made from 'paxolin' sheet in a steel frame. The wire probe in its holder B is carried in the frame C which travels across the width of the tank on rails D. These rails form part of the structure of a carriage which can be moved along the length of the tank also on rails. Each set of rails consists of a rod and parallel plane on which its carriage is supported at five points by steel ball contacts. Thus accurate motion of the probe is set by the truth of rails. For the latter, pieces of light alloy rod and T-section girder were carefully selected. Tap water was used as the electrolyte, and the probe was a piece of copper wire about $1/64$ " diameter immersed to a depth of about $1/4$ " below the surface. (A thinner probe was not justified by the accuracy of the tank measurements, and might have introduced some of the troubles associated with corrosion and saturation at high current densities.) The position of the probe was found by reference to scales attached to the rails.

Electrodes representing either equipotentials or field lines of the magnet were fed with about 30 volts at 500 cycles, and the out-of-balance current of the resistance bridge network used was most easily detected with a cathode ray oscilloscope or headphones.



Figure 1. A. Sectional view of the pump.

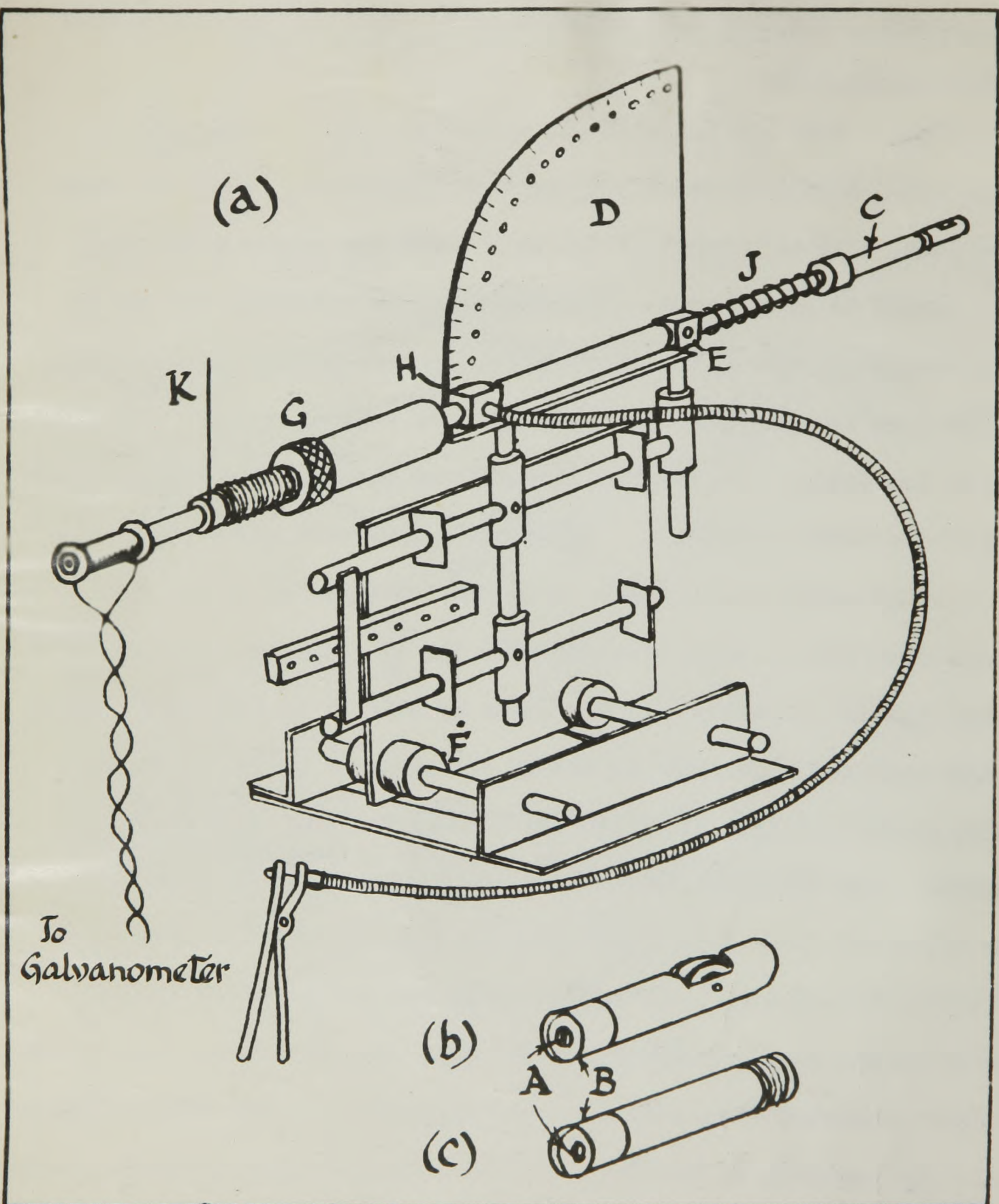


Figure App.2 : Search coil.

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It was estimated that the positions of the model equipotentials corresponded to those of the real case within about 5% if care was taken to avoid distortion by the tank walls.

In general it was found that models could be most easily set up in the tank by representing magnetic pole surfaces with insulating walls. For these, bricks of paraffin wax were cast and planed to size. Iron sheet electrodes were shaped roughly to correspond with actual field lines in a part of the tank remote from the area under investigation. Thus 'model' equipotentials represented 'real' field lines, and vice versa. An advantage of this method of working is that flux-leakages from magnetic surfaces are easily estimated from measurements of the potential at the corresponding places on the model.

2. Search Coil.

The field variation along the path of the beam and at the pole boundaries was found accurately with the search coil arrangement shown in figure 2(a). The coil was made as small as possible, a small plastic bobbin being wound with about 100 turns of 40 SWG enamelled copper wire, the overall outside diameter being only $3/16$ " . Coils were mounted in holders either as shown in figure 2 (b) or (c) according as the direction of the field component to be measured. The ends of the wire were taken to a nut A and ring B which made contact with corresponding parts of the long probe C of figure (a). Possible disturbing induction in loops formed by the coil leads was avoided by making the probe effectively a coaxial cable.

The probe has freedom to rotate through 90° (marked on scale D) about a horizontal axis E. By clamping the whole assembly to the magnet yoke, this axis is normally set to coincide with the axis about which the

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ion beam turns. The probe axis can also be shifted through about 10 cms. vertically and horizontally and the coil can thus be easily and accurately placed on any part of the ion path before, during or after passage through the field. Motion across the length of the pole gap is provided by the slide and screws F.

It is possible to move the probe forward in small predetermined steps by winding back the nut G on a millimetre thread attached to the probe; and then by releasing a friction trigger H the probe is jerked forward by the phosphor bronze spring J. To avoid shaking the probe the trigger is released by a lever remotely attached by a length of bowden cable.

The probe can be rotated about its own axis, and field measurements are normally made by turning the coil rapidly through 180° , the pointer K indicating its position.

For mapping regions of a field where the strength changes very rapidly with distance, as for instance near the boundary, the simplest method is to use a differential method, jerking the coil in steps of a millimetre or more. The absolute level of the curve so obtained is then found by using the normal coil reversal method.

3. Variable Power Lens.

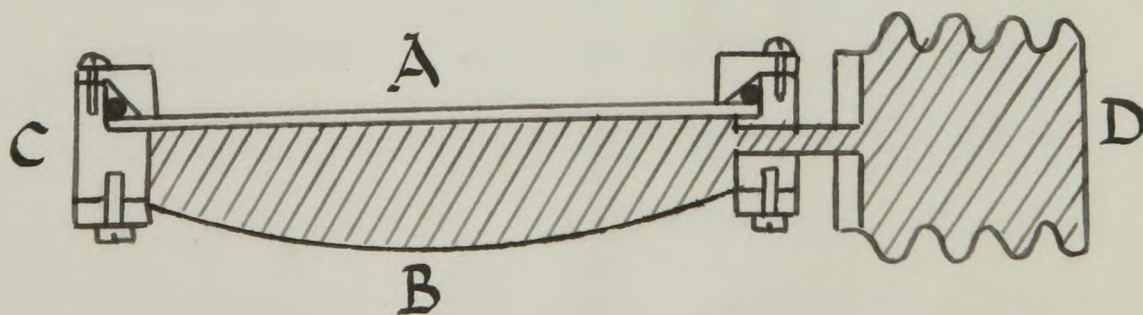


Figure App.3: Variable-power lens.

This lens was designed to test the practicability of an optical analogue of a magnetic lens system, as described in section 10 of Part A. The lens has one fixed glass surface A, and one surface of variable curvature made by a thin transparent rubber membrane B. A brass ring C forms the wall of the cell, which is filled with water. The curvature of B, and hence the power of the lens, can be changed by compressing or expanding the sylphon bellows D, which is provided with a screw adjustment (not shown).

The quality of the image is of course poor, yet it is sufficiently good for the purpose contemplated. In the single lens so far made, the radius of curvature of the membrane could be varied between ± 9 cm., which gave a focal length between ± 27 cm. There would be some advantage in using a liquid of greater optical density.

It is interesting to note that with the lens in the position shown, i.e. as a plano-convex lens with the plane surface uppermost, the increased hydrostatic pressure at the centre will cause an increase in curvature which can in theory give an aspheric surface devoid of spherical aberration if the membrane is given the right initial tension.

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