Input Price Discrimination and Vertical Integration

Simon Cowan*

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Abstract

An upstream monopolist owns a downstream firm that competes with independent retailers and maximizes its own profit. The effects of allowing the integrated firm to discriminate in favor of its retail division, rather than requiring a uniform wholesale price, are assessed. When the retailers choose outputs discrimination raises social welfare, aggregate profits, and consumer surplus because it alleviates the double-margins problem. When there is upstream competition discrimination may operate via refusal to supply (if this is credible). This reduces welfare compared to uniform pricing. The main model is extended to price-setting retailers where the retailer owned by the upstream firm is less efficient than the rival retailer. Again wholesale price discrimination is good for consumers and social welfare. The elimination of double marginalization is the key.

Keywords: vertical integration, double marginalization, price discrimination.

JEL Classifications D42, D43, L12, L13, L42

*Department of Economics, University of Oxford, Manor Road Building, Oxford OX1 3UQ, UK. Email: simon.cowan@economics.ox.ac.uk. ORCID: 0000-0003-0704-2871. I am grateful to seminar participants at Compass Lexicon for their comments and questions.

1 Introduction

An upstream monopolist sells its product to downstream firms who compete in the retail market. It owns one downstream firm that maximizes its own profit. The upstream firm would like to discriminate when setting its wholesale prices. Alternatively the upstream firm may be required by a regulator to set the same wholesale price for its downstream division as for other firms. This paper assesses the implications for social welfare and consumer surplus of discrimination in wholesale pricing compared to a requirement to set identical wholesale prices. The assumption that the downstream division of the integrated firm maximizes its own profit implies that there is a difference between uniform pricing and discrimination, and also captures the anti-trust concern that discrimination may be problematic. When the downstream division, alternatively, maximizes integrated profits it sees through any wholesale price set by its upstream partner, and there is no difference between uniform pricing and discrimination.

In the two main models, with equally efficient Cournot retailers, and with price-setting retailers who differ in efficiency, price discrimination unambiguously raises social welfare and consumer surplus. The integrated firm induces a lower retail price. The common feature is that, compared to uniform pricing, discrimination enables the integrated firm to eliminate double marginalization. A requirement to price uniformly would introduce another margin and benefits the rival retailers but reduces social welfare. In the Cournot model the integrated firm forecloses the rival retailers. In contrast in the Bertrand model the rival retailer, which is more efficient, is the only retailer that operates. The Bertrand model is formally equivalent to one with vertical separation, and provides a contrast with models of input price discrimination with output-setting retailers with different efficiency levels.

A third model provides a case against discrimination. Two upstream firms compete in wholesale prices and the retailers are Cournot players. The integrated firm discriminates by refusing to supply the independent retailer. This retailer can only buy from the alternative upstream firm, which is induced to raise its wholesale price above marginal cost. Discrimination reduces total output and social welfare. This argument, however, has limitations: the upstream firm must be able to commit not to supply, it does not work if there are three or more upstream firms, and the independent upstream firm and retailer have an incentive to integrate vertically, which would unwind the negative effect of price discrimination.

The models have implications for anti-trust policies towards wholesale price discrimination and vertical integration. In the USA the Robinson-Patman Act of 1936 prohibits a firm from setting different prices for different customers for the same good when there is a reasonable possibility that an "injury to competition" will result. In particular a "secondary line injury" under the Act is when different retailers face different wholesale prices for the same good and the ones facing the higher prices are thereby harmed. The transfer of parts from a parent to a subsidiary is not considered a "sale" under the Act.[1] In 2024 and 2025 the Federal Trade Commission (FTC) initiated cases under the Robinson-Patman Act against Southern Glazer's Wine and Spirits, and against PepsiCo.

In 2018 the Court of Justice of the European Union (CJEU) provided general guidance on price discrimination in response to a Portuguese case where a pay-TV company was paying higher rates for licensed audiovisual content than a direct competitor. Firms with dominant positions are forbidden under Article 102 of the Treaty of the Functioning of the European Union from applying "dissimilar conditions to equivalent transactions with other trading parties, thereby placing them at a competitive disadvantage." The CJEU determined that price differences on their own were not sufficient for an anti-trust violation: EU competition law requires a full evaluation of all the circumstances of the case to determine whether there is a distortion in competition.

In 2020 the US Vertical Merger Guidelines were revised. The new guidelines recognized that a vertical merger can be pro-competitive if it eliminates double marginalization. In 2023 the Department of Justice and the FTC issued new unified guidelines which reverted to a more skeptical presumption about the efficiency effects of vertical mergers (and horizontal mergers). A recent vertical integration case with some price discrimination aspects concerned the planned purchase by biotech company Illumina of Grail, which makes cancer screening tests and previously was a subsidiary of Illumina. The *Financial Times* reported (14 July 2021) that "the FTC says that Illumina could hamper companies that want to compete with Grail's screening tests by charging them more or denying them access to Illumina's technology" and "to assuage the FTC, Illumina would commit to reducing its prices over time and charging external customers what Grail pays."

Although on occasion this paper will consider the difference between vertical integration and vertical separation the main focus is on pricing rather than on market structure. For the most part it is taken as given that a retailer is owned by an upstream firm. The model with price-setting retailers with vertical integration is formally equivalent to the model when there is vertical separation, and this enables comparison with an existing literature on wholesale price discrimination by a

[1] See https://www.ftc.gov/advice-guidance/competition-guidance/guide-antitrust-laws/price-discrimination-robinson-patman-violations.

vertically separated upstream monopoly when Cournot retailers differ in efficiency.

In Section 2 an upstream monopoly supplies Cournot retailers. Section 3 presents the model with upstream price competition and downstream output-setting. Section 4 analyzes an upstream monopoly supplying price-setting retailers that differ in efficiency. In Section 5 the results of Section 4 are compared with standard results on wholesale price discrimination when Cournot retailers have different efficiencies. Section 6 concludes.

2 Upstream Monopoly with Cournot Retailers

2.1 Demand Assumptions

Inverse demand depends on total output, Q, and is $p(Q)$, which is decreasing and three-times differentiable. The marginal revenue of the downstream industry, $MR(Q) = p(Q) + Qp'(Q)$, is assumed to be strictly decreasing: $MR'(Q) < 0$. The curvature of inverse demand is $\sigma(Q) \equiv -\frac{Qp''(Q)}{p'(Q)}$. Decreasing downstream marginal revenue is equivalent to the assumption that $\sigma(Q) < 2$ for all Q.

A single retailer whose only cost is the wholesale price, w, would set $MR(Q) = w$: downstream marginal revenue equals the wholesale price. Inverse demand for the upstream firm is then $MR(Q)$, with its revenue being $MR(Q)Q$ and upstream marginal revenue being $MR(Q)+QMR'(Q)$. Upstream marginal revenue is assumed to be strictly decreasing: $2MR'(Q) + QMR''(Q) < 0$. Define the curvature of upstream inverse demand as $\gamma(Q) \equiv -\frac{QMR''(Q)}{MR'(Q)}$, so upstream marginal revenue is strictly decreasing when $\gamma(Q) < 2$ for all Q.

These assumptions about marginal revenue hold widely, for example if $\sigma(Q)$ is constant, so $\gamma = \sigma$, and is below 2, or if $\sigma(Q) < 1$ and $\sigma'(Q) < 0$ (e.g., for demands based on the normal and logistic distributions). The assumptions imply that all the profit functions that follow in this section and the following one are strictly concave in output. First-order conditions are thus sufficient for the profit maximization problems.[2]

2.2 Cournot Retailers

An upstream monopolist supplies $n \geq 2$ downstream firms, the retailers, indexed by $i \in \{1, 2, \ldots, n\}$. Retailer 1 is owned by the upstream firm. The remaining retailers are independent. Retailer i pays a wholesale price w_i to the upstream firm and

[2]The assumptions are slightly more restrictive than necessary — what is needed is decreasing marginal revenue locally, for both the retailers and the upstream firm.

has no other costs. Each unit sold by the upstream firm is sold on by a retailer. The constant marginal cost of the upstream firm is $c \geq 0$. In stage 1 the upstream firm sets wholesale prices, $\{w_1, w_2, \ldots, w_n\}$, which may differ if discrimination is allowed. In stage 2 each retailer i chooses output q_i knowing all the wholesale prices and taking the outputs of the other retailers as given. Total output is $Q = \sum_{i=1}^{n} q_i$.

If the upstream monopolist were to own all the retail firms it would earn the unconstrained monopoly profit,

$$(p(Q^*) - c)Q^* \equiv \max_{Q}(p(Q) - c)Q,$$

which is characterized by the standard marginal revenue equals marginal cost condition: $MR(Q^*) = c$. Asterisks indicate values that apply when the unconstrained monopoly profit is earned.

The integrated firm's retail division maximizes its own retail profit, $(p(Q)-w_1)q_1$. This assumption captures the implicit reasoning behind the anti-trust policy that requires an integrated firm not to discriminate in favor of its own retailer. Retailer i's profit function is $(p(q_i + Q_{-i}) - w_i)q_i$ where $Q_{-i} = \sum_{j \neq i}^{n} q_j$ is the output of the other firms. This profit function is strictly concave in q_i by the assumption that industry marginal revenue is decreasing. When retailer i sets a positive output the Cournot first-order condition is

$$p(q_i + Q_{-i}) + q_i p'(q_i + Q_{-i}) - w_i = 0, \tag{1}$$

where Q_{-i} is taken as given. If the wholesale price is uniform for all n retailers the first-order condition (1) implies that each retailer chooses the same output in equilibrium. The assumption that $\sigma(Q) < 2$ for all Q implies that $n + 1 - \sigma > 0$, which is both the condition for uniqueness of the Cournot equilibrium when all retailers set the same output (Schlee 1993) and the Seade stability condition (Seade 1980). When wholesale prices differ some retailers may choose to set output to zero, so firm i's output in the Cournot equilibrium is, generally,

$$q_i = \max\left\{\frac{p(Q) - w_i}{-p'(Q)}, 0\right\}.$$

A retailer chooses output of zero if its margin, $p(Q) - w_i$, is zero or negative.

The integrated firm takes account of the fact that its downstream division maximizes its own profit rather than the profit of the integrated structure. The integrated firm achieves the unconstrained monopoly profit with price discrimination by favoring its downstream division and foreclosing the other retailers. It sells the product to its downstream division at a wholesale price equal to upstream marginal cost,

$w_1 = c$, so that double marginalization is avoided. The wholesale prices for the other retailers are set to induce their outputs to be zero: $w_i \geq p(Q^*)$ for $i \in \{2, \ldots, n\}$.

To confirm that the integrated firm chooses to discriminate in this way suppose alternatively that there is positive output of the rival retailers: $\sum_{i=2}^{n} q_i > 0$. The integrated firm's profit is its overall profit from its retail sales, $(p(Q) - c)q_1$, plus the wholesale profits from selling to the independent retailers, $\sum_{i=2}^{n}(w_i - c)q_i$. Rival retailer i sets a positive output if and only if $w_i < p(Q)$. With positive output of the independent retailers the integrated firm's profit is strictly below the unconstrained monopoly level:

$$\begin{aligned}(p(Q) - c)q_1 + \sum_{i=2}^{n}(w_i - c)q_i &< (p(Q) - c)q_1 + \sum_{i=2}^{n}(p(Q) - c)q_i \\ &= (p(Q) - c)Q \\ &\leq (p(Q^*) - c)Q^*.\end{aligned}$$

The strict inequality holds because positive output of some independent retailers requires that their wholesale prices are strictly below the retail price. The weak inequality is by the definition of unconstrained profit-maximization. This is intuitive. Suppose the integrated firm chooses not to sell one unit via its retail division and instead sells this wholesale to rival retailer i. The integrated firm loses retail revenue of $p(Q)$ and gains wholesale revenue of w_i. The rival is prepared to participate if and only if $p(Q) > w_i$, but this entails that the integrated firm makes lower profit on this unit than if it had sold it at the retail price.

When uniform pricing is required the integrated firm must set a single wholesale price, w, for both its own downstream division and the independent retailers. The Cournot first-order condition with symmetry implies that the profit of a retailer is

$$\frac{(p(Q) - w)Q}{n} = -\frac{Q^2 p'(Q)}{n^2}.$$

It helps to write the integrated firm's profits as a function of total output, which can be treated as the choice variable. The integrated firm's profits with uniform wholesale pricing are

$$(p(Q) - c)Q + Q^2 p'(Q)\left(\frac{n-1}{n^2}\right), \tag{2}$$

which is the industry profit, $(p(Q) - c)Q$, less the retail profits of the $n - 1$ independent retailers.

The first-order condition for the upstream firm's choice of Q, using the fact that $\frac{d}{dQ}Q^2 p'(Q) = QMR'(Q)$, is:

$$MR(Q^u) - c + Q^u MR'(Q^u)\left(\frac{n-1}{n^2}\right) = 0, \tag{3}$$

where Q^u is total output with uniform pricing. The retail first-order condition (1) and the upstream firm's first-order condition (3) imply that the uniform wholesale price may be written as a weighted average of marginal cost and the retail price:

$$w^u = \frac{nc + (n-1)(n+2-\sigma)p(Q^u)}{n + (n-1)(n+2-\sigma)}. \tag{4}$$

If $n = 1$, so the integrated firm is the only retailer, $w^u = c$. As n tends towards infinity the optimal wholesale price tends to $p(Q^u)$, which itself will tend to $p(Q^*)$.

With the number of retailers being above 1 and finite the first-order condition (3) and $MR'(Q) < 0$ imply that $MR(Q^u) - c > 0$ and thus that $Q^u < Q^*$: total output with uniform pricing is below the unconstrained monopoly level. Requiring the integrated firm to price uniformly, while its retail division behaves in the same way as the independent retailers, has negative welfare effects. Final consumers are better off with discrimination because the retail price is lower than with uniform pricing. Total profit with discrimination is higher because it equals the unconstrained monopoly level. Thus aggregate social welfare is higher with discrimination than with uniform pricing. The losers from discrimination are the independent retailers, whose profits go to zero. They are excluded from the market with price discrimination while they earn positive profits when uniform pricing is used. Putting this together gives the next result.

Proposition 1. *Suppose that the retail firms are Cournot competitors and the downstream division of the integrated firm maximizes its retail profit. Price discrimination by the integrated monopolist in favor of its downstream division yields higher consumer surplus, total profit and aggregate social welfare than uniform wholesale pricing.*

Uniform pricing and the assumption that the integrated firm's retail division maximizes retail profits generate double marginalization. Discrimination avoids this outcome. Figure 1 illustrates this with $n = 2$, $c = 0$ and $p(Q) = 100 - Q$. With discrimination the integrated retailer sets the monopoly output of $Q^* = 50$, implying a monopoly price of $p(50) = 50$. When a uniform price is required the integrated firm sets a wholesale price of 40, and each retailer has output of 20, yielding a retail price of 60. The integrated firm could have total output of 50 if it set a uniform wholesale price of 25. This would give it three-quarters of the monopoly profit. Its optimum, however, entails that industry profits are reduced, while the integrated firm has a larger share of total profits (five-sixths).

The effect of an increase in the number of retail firms, n, on output with uniform pricing can be determined. For $n \geq 2$ the expression $\frac{n-1}{n^2}$ in the first-order condition

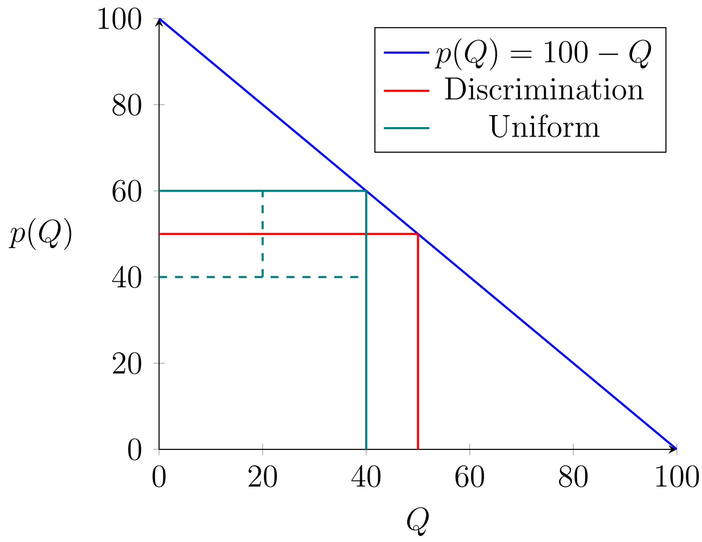

Figure 1: Discrimination and Uniform Pricing

in (3) defines a strictly decreasing sequence that takes its highest value when $n = 2$. Together with the fact that the integrated firm's profit is strictly concave in output this implies that output with uniform pricing is increasing in n. Thus the worst outcome for welfare occurs when $n = 2$. The best feasible outcome, with output at the unconstrained monopoly level, occurs either when $n = 1$, so there are no independent retailers, or when n tends to infinity, so the retail margin tends to zero. In both cases there is no double marginalization problem.

Four extensions follow. First, suppose that the downstream division of the integrated firm maximizes the integrated profit $(p(Q) - c)q_1 + (w_1 - c)Q_{-1}$ instead of its retail profit, $(p(Q) - w_1)q_1$. Suppose that a uniform wholesale price, w, is set. The integrated firm achieves the unconstrained monopoly profit by setting the uniform wholesale price equal to or above the monopoly price: $w \geq p(Q^*)$. In the second stage the retail division of the integrated firm sees that the true marginal cost for the integrated structure is c. It sets $q_1 = Q^*$ and the independent retailers set $q_i = 0, i = 2, \ldots, n$. With discrimination any value of w_1 works, including $w_1 = c$, because the retailer recognizes its marginal cost is c, while $w_i \geq p(Q^*)$ for $i = 2, \ldots, n$ ensures that the rivals are excluded. The requirement to have uniform pricing yields the same outcome as discrimination.

Second, suppose that the upstream firm is vertically separated. It has no motive for discrimination and thus sets a uniform wholesale price. The upstream firm's profit from wholesaling is $(p(Q) - c)Q + Q^2 p'(Q)/n$, which is industry profit less total retail profit. The first derivative is $MR(Q) - c + \frac{Q}{n} MR'(Q)$. Evaluating this at at the level of output that maximizes the integrated firm's profit, Q^u, defined by

(3), gives:

$$MR(Q^u) - c + Q^u MR'(Q^u)\frac{1}{n} = Q^u MR'(Q^u)\frac{1}{n^2} < 0.$$

With vertical separation total output is lower than with integration and uniform pricing. The double marginalization problem is more severe. In fact moving from vertical separation to integration with uniform pricing yields a Pareto improvement. Retailers benefit from a lower wholesale price.[3] Consumers pay a lower retail price, and the integrated firm earns higher profits than its separate parts.

Third, suppose that with discrimination the wholesale price for the independent retailers is capped by the one that applies with vertical separation. The integrated firm may set a different wholesale price for its own retailer. Suppose that inverse demand has constant curvature. The optimal wholesale price with vertical separation equals the unconstrained monopoly price, i.e. $w = p(Q^*)$, when σ is constant. To show this write direct demand as $Q(p)$ so the monopoly problem is to choose p to maximize $(p-c)Q(p)$.[4] With constant curvature demand for the upstream firm may be written as $k(n)Q(w)$ where $0 < k(n) < 1$: see the Appendix. The wholesale pricing problem with vertical separation is to choose w to maximize $(w-c)k(n)Q(w)$. Because $k(n)$ is a multiplicative constant the value of w that solves the wholesale pricing problem equals the value of p that solves the monopoly problem. When there is vertical integration the wholesale price for the independents, w, may be no higher than the monopoly price. The integrated firm achieves the unconstrained monopoly profit by setting $w_1 = c$ and $w = p(Q^*)$. Its downstream division sets $q_1 = Q^*$, while each independent retailer sets $q_i = 0$. The independent retailers are not helped by the wholesale price cap when inverse demand curvature is constant.

Fourth, suppose that there is vertical separation and the upstream firm may use nonlinear pricing. The upstream firm can earn the unconstrained monopoly profit without the need for integration and discrimination. There are two versions. First, the upstream firm can use quantity forcing, with each retailer purchasing Q^*/n for a total payment of $p(Q^*)Q^*/n$. Second, a two-part tariff, with a common wholesale price and fixed fee, can be set. The wholesale price, with superscript "t",

$$w^t = \frac{c}{n} + \frac{(n-1)}{n}p(Q^*),$$

is a weighted average of marginal cost and the monopoly price. This induces each retailer to set output of Q^*/n. The fixed fee $(p(Q^*)-w^t)\frac{Q^*}{n}$ ensures that the upstream firm obtains the unconstrained monopoly profit.

[3] The derivative of retail profit, $-Q^2p'(Q)/n^2$, with respect to Q has the same sign as $2-\sigma > 0$.

[4] Decreasing marginal revenue implies that $(p-c)Q(p)$ is strictly quasi-concave in p and thus there is a unique profit-maximizing price.

3 Wholesale Competition and Cournot Retailers

Now there is upstream competition. The upstream firms compete in wholesale prices while downstream competition remains in outputs. The demand assumptions are strengthened slightly. Demand is log-concave, so $\sigma(Q) \leq 1$.[5] This guarantees that outputs at the retail stage are strategic substitutes. The integrated firm, now called A, has a single upstream competitor, B, which is equally efficient and has marginal cost of c. There are no upstream fixed costs. Firm A owns retailer 1, which maximizes its own profit. Retailer 2 is independent. The assumption that there is a single rival upstream firm is important and will be reconsidered later.

The main result is stated informally first. With uniform pricing the upstream firms set their wholesale prices at marginal cost and the downstream retailers earn symmetric Cournot profits. The integrated firm discriminates by refusing to supply the independent retailer. This retailer now can only buy its input from the rival upstream firm, which raises its wholesale price above marginal cost. The integrated firm's retail division benefits from the fact that its retail rival faces a higher input cost. Discrimination reduces social welfare because total output is lower than with uniform pricing.

Before the game starts a regulator determines whether the upstream firms must price uniformly or may discriminate. If discrimination is allowed the integrated firm decides at stage 0 whether or not to supply the independent retailer. In stage 1 the upstream firms choose their wholesale prices. If uniform pricing is required the integrated firm sets w_A and the rival upstream firm's price is w_B. If discrimination is allowed upstream firm $i \in \{A, B\}$ sets prices $\{w_{i1}, w_{i2}\}$ for retailer 1 and 2 respectively. Retailer 1 maximizes its retail profit.

The equilibrium with uniform pricing has pure Bertrand competition at the wholesale stage, with both upstream firms pricing at marginal cost: $w_A = w_B = c$. In stage 2 each retailer chooses the Cournot output that applies when both firms have a marginal cost of c. The integrated firm obtains no advantage from being integrated and earns zero profits from its wholesale sales.

Now consider discrimination. First, suppose that in stage 0 the integrated firm decides it will supply the independent retailer. The equilibrium in the sub-game starting at stage 1 has the same outcome as uniform pricing. Firm A sets $w_{A1} = c$ and $w_{A2} = c$, and B sets $w_{B1} = c$ and $w_{B2} = c$. The logic of Bertrand competition with identical goods entails that each wholesale price equals marginal cost. Second, assume that in stage 0 firm A refuses to supply retailer 2. This makes B the

[5] Log-concavity of demand is itself a weak assumption: many analyses of Cournot competition have this assumption.

monopoly supplier to 2 so B sets its wholesale price above marginal cost. The effect is to disadvantage retailer 2 and to increase retailer 1's profits compared to when each wholesale price equals marginal cost. Thus the overall equilibrium has the integrated firm deciding in stage 0 not to supply the independent retailer. In stage 1 the wholesale prices are $w_{A1} = c, w_{A2} = \infty$, i.e., A refuses to supply 2, $w_{B1} = c$ and $w_{B2} > c$. There is pure Bertrand competition in the market to supply retailer 1 but market power is created for the supply to retailer 2. In stage 2 retailer 1 sets output $q(c, w_{B2})$ where it pays c for its input and its rival pays w_{B2}, while 2 sets the lower output $q(w_{B2}, c)$ because it pays a higher wholesale price than its rival. Total output is below the level when all wholesale prices equal c because $w_{B2} > c$.

In stage 1 firm B chooses w_{B2} to maximize its wholesale profit from supplying the independent retailer, $(w_{B2} - c)q(w_{B2}, c)$. B's optimal wholesale price satisfies $c < w_{B2} < p(Q^*)$. B sets w_{B2} above c to earn a positive profit, and this works because it faces no wholesale price competition. At the same time it chooses a wholesale price strictly below the unconstrained monopoly price, $p(Q^*)$. A wholesale price at or above $p(Q^*)$ would entail that the integrated retailer has a drastic cost advantage: its monopoly price would be at or below the wholesale price that the rival retailer pays so $q_1 = Q^*$ and $q_2 = 0$, and thus B would earn zero profit.

The effect of the increase in w_{B2} on A's profit, $(p(Q) - c)q_1$, is positive. The rival retailer 2 reduces its output. In response the integrated firm raises its output as outputs are strategic substitutes. In addition the reduction in total output ensures that the market price, $p(Q)$, increases.[6] Firm A benefits from both higher output and a higher margin.

To illustrate suppose that, as in Figure 1, $p(Q) = 100 - Q$ and $c = 0$. Call $w_{B2} = w$. In stage 2 the outputs are $q_1(w) = (100 + w)/3$ and $q_2(w) = (100 - 2w)/3$. The optimal wholesale price of the rival upstream firm maximizes $wq_2(w)$ and is $w = 25$, yielding $q_1 = 125/3$ and $q_2 = 50/3$. Total output is $Q = 175/3$, which is half-way between monopoly output of 50 and the aggregate Cournot output of 200/3 that obtains when all the wholesale prices equal marginal cost.

The welfare effects of discrimination, which involves refusal to supply the rival retailer, are clear because total output is below the level with uniform wholesale pricing.

Proposition 2. *Suppose the two retail firms are Cournot competitors, there is a single rival upstream firm that has the same marginal cost as the integrated firm and*

[6] Adding the two retail first-order conditions yields the fact that total output is a decreasing function of the average of the marginal costs that the retailers face. This average with discrimination is $\frac{c+w_{B2}}{2} > c$.

upstream competition is in prices. The integrated firm discriminates by refusing to supply the independent retailer. Price discrimination yields lower consumer surplus, higher total profit, and lower aggregate social welfare than uniform wholesale pricing.

The losers from price discrimination are consumers, the independent retailer and society. The winners are the integrated firm and its upstream rival. This is the first time in this paper that price discrimination has been shown to yield lower welfare than uniform pricing. The argument, however, is delicate. There are several criticisms of the argument that can be made. First, it depends on the integrated firm being able to commit not to supply the rival retailer. In the absence of such a commitment the logic of pure Bertrand competition would entail that all wholesale prices would equal marginal cost. Second, with two or more upstream competitors Bertrand competition between the independent upstream firms would keep the wholesale price equal to marginal cost. Refusal to supply would have no effect. Third, the upstream rival and the independent retailer have an incentive to integrate, and this would offset the effects of refusal to supply by the integrated firm. The symmetric Cournot outcome would apply again.[7]

Ordover, Saloner & Salop (1990) have a closely related analysis. Their main model has differentiated products, price-setting at the retail stage and a retail division of the integrated firm that maximizes the profits of the combined firm. Their main focus is on the incentives to integrate vertically. In an extension to cover a Cournot game similar to ours they argue, as we have above, that the rival upstream and downstream firms would have an incentive to integrate in response to refusal to supply. In summary Proposition 2 yields an argument against price discrimination, but depends on rather specific assumptions.

4 Price-Setting Retailers

Two retailers compete in prices and sell identical products. As in Section 2 there is a monopoly upstream firm which owns retailer 1. Retailer 2 is independent. Retailer 1 has direct marginal cost $r > 0$, while retailer 2 is more efficient and has no direct marginal cost. Assume that monopoly profit, as a function of price, is single-peaked for all values of marginal cost.[8] Denote the maximized monopoly

[7]When only A is integrated the derivative of the joint profit of B and retailer 2 with respect to B's wholesale price w is $(w-c)\frac{\partial q_2}{\partial w} + q_2 p'(Q)\frac{\partial q_1}{\partial w}$. This is negative: a rise in w causes q_2 to fall and q_1 to increase, and $w \geq c$. It follows that if B integrates with retailer 2, with w equal to c, their joint profit increases.

[8]Sufficient conditions are that (i) $\sigma(Q) < 2$ for all Q so that monopoly marginal revenue is strictly decreasing in output or (ii) the curvature of direct demand is less than 2, $-\frac{pQ''(p)}{Q'(p)} < 2$,

profit for a constant marginal cost, k, by $\Pi(k)$, a decreasing and convex function, and the monopoly price for cost k by $p(k)$.

To set the scene consider three counter-factual scenarios. First, suppose that $r = 0$ so the retailers are equally efficient. There is no need for price discrimination. The upstream firm sets the uniform wholesale price equal to the monopoly price for marginal cost c, i.e., $w = p(c)$. Bertrand competition between the retailers then gives $p_1 = p_2 = w = p(c)$ and the upstream firm's wholesale profit is $(p(c)-c)Q(p(c)) = \Pi(c)$. With retail profit margins equaling zero the upstream firm captures all the industry profit, and the wholesale price choice ensures that profit is at its maximum possible level. A difference in efficiency is needed to create a role for price discrimination, so hereafter $r > 0$.

Second, suppose that retailer 2 is not in the market. Retailer 1 faces a wholesale price of w and chooses its price, as a monopolist, to maximize $(p - w - r)Q(p)$. Its price is $p(w + r)$ and the integrated firm's profit is $(p(w + r) - c - r)Q(p(w + r))$. The best wholesale price for the integrated structure, as is standard, is $w = c$. By profit maximization,

$$\begin{aligned}\Pi(c + r) &\equiv (p(c + r) - c - r)Q(p(c + r)) \\ &\geq (p(w + r) - c - r)Q(p(w + r))\end{aligned}$$

with strict inequality for $w \neq c$, so it is optimal to set $w = c$. By avoiding double marginalization the integrated firm achieves the maximum monopoly profit, $\Pi(c+r)$.

Third, suppose that only retailer 2 operates, and thus sets the monopoly price $p(w)$. The upstream firm chooses its wholesale price to maximize wholesale profit, $\pi(w) = (w - c)Q(p(w))$. Maximized upstream profit is $\pi(w^s)$ where w^s is the optimal wholesale price. The independent retailer's margin is $p(w^s) - w^s$ and the upstream firm's margin is $w^s - c$. Assume that $\Pi(c + r) \geq \pi(w^s)$. In other words r is small enough that the upstream firm's monopoly profit when retailing itself is larger than its wholesale profit when selling to the independent retailer with double marginalization.

Now the rival retailer is in the market. The upstream firm chooses its wholesale prices and uses no additional vertical restraints. When there is a uniform wholesale price, w, retailer 2 is the lower-cost firm. Assume that $p(w) \geq w + r$ so that the cost difference is non-drastic: see Appendix B for more details. Retailer 2 sets its price at the marginal cost of retailer 1, i.e., $p = w + r$ and does all the sales. Here the equilibrium of Blume (2003) is used. He shows that a duopoly with asymmetric costs and price-setting has an equilibrium with the more efficient firm pricing at

which implies that monopoly profit is strictly concave in p.

the less efficient firm's marginal cost and with the less efficient firm selling nothing. This equilibrium does not rely on dominated strategies. Retailer 2's margin is r. The resulting profit for the upstream firm is

$$(w - c)Q(w + r) = (p - c - r)Q(p).$$

It is as if the upstream firm is a monopoly retailer with marginal cost $c + r$, so the maximum feasible profit is $\Pi(c + r)$, which is earned with retail price $p(c + r)$. By setting the wholesale price at $w = p(c + r) - r$ the integrated firm achieves the maximum feasible profit, which also is the profit earned when there is no rival retailer. Consumers face the same price. Retailer 2 earns profit of $rQ(p(c+r)) > 0$, which equals the reduction in total cost and is the gain in social welfare compared to when the efficient retailer is not in the market.

If demand is log-concave, so $\sigma(Q) \leq 1$, the monopoly profit margin is decreasing in marginal cost. It follows that $p(c+r) - c - r \leq p(c) - c$. Adding c to both sides gives an upper bound for the uniform wholesale price: $w = p(c + r) - r \leq p(c)$: the wholesale price is at most equal to the monopoly price for cost c. If demand is log-convex the wholesale price is bounded below by $p(c)$.

What happens when the upstream firm discriminates? The integrated firm has three objectives: to ensure that the efficient retailer does the retailing, to determine the retail price so that industry profit is at the highest possible level and to leave the rival retailer with minimal profit. It can achieve all of these with discrimination. It gives the rival a positive, but small, margin. The wholesale price for its retailer, combined with a higher wholesale price for the rival retailer, determines the appropriate retail price while also providing its retailer with an incentive not to operate.

Let the wholesale prices satisfy

$$w_1 + r = w_2 + m \tag{5}$$

for $m \in (0, r)$. The instruments for the upstream firm are the two wholesale prices, w_1 and w_2, and m, which will be the margin of retailer 2. The marginal cost for retailer 1 is $w_1 + r$, and that for retailer 2 is $w_2 < w_1 + r$, so retailer 2 sets $p = w_1 + r = w_2 + m$ and earns a margin of $m > 0$. The inefficient retailer faces a lower wholesale price, $w_1 - w_2 = m - r < 0$, but does not operate.

Suppose that instead the integrated firm sets $m = 0$ so that the effective marginal costs for the retailers are equal. A standard Bertrand pricing equilibrium would obtain, with $p_1 = w_1 + r = w_2 = p_2 = p$. The inefficient retailer would sell half of the quantity demanded. The resulting profit function for the integrated firm would be $(p - c - 0.5r)Q(p)$, and its maximized profit would be $\Pi(c + 0.5r)$, the monopoly profit with marginal cost $c + 0.5r$. If it set $m < 0$ all the retailing would be done by its inefficient retailer and it would earn profit $\Pi(c + r)$. The firm does better in both cases by setting a small positive value of m, as is now shown.

With (5) applying and $p = w_2 + m$, the integrated firm's profit is

$$(w_2 - c)Q(w_2 + m) = (p - c - m)Q(p).$$

Now it is as if a retail monopolist with a marginal cost of $c + m$ chooses its price. Every unit of output costs the integrated firm c, its production cost, plus m, the margin of retailer 2. The upstream firm induces the retail price to be $p(c + m)$, yielding profit $\Pi(c + m)$. By setting m very close to zero the integrated firm makes its profit arbitrarily close to the maximum feasible level $\Pi(c)$. The profit of retailer 2 is $mQ(p(c + m))$.

The procedure for the integrated firm is as follows. First, fix the smallest possible $m > 0$. Second, calculate the monopoly price for marginal cost $c + m$, $p(c + m)$. Third, set the wholesale prices at $w_1 = p(c + m) - r$ and $w_2 = p(c + m) - m$. The result is that retailer 2 sets its price at $p(c + m)$.

The welfare comparison of discrimination with uniform pricing is clear. Consumers benefit as the price is reduced from $p(c + r)$ to $p(c + m)$. Industry profit increases. Define industry profit for a general margin of retailer 2, x, as $\Pi(c + x) + xQ(p(c + x))$. This decreases as x increases.[9] With uniform pricing $x = r$, while with discrimination $x = m < r$, so profit is strictly higher with discrimination. Thus social welfare increases. This is summarized in the following proposition.

Proposition 3. *Suppose that the retailers are price-setting duopolists selling identical goods, and the retailer owned by the upstream firm is inefficient. When the integrated firm sets discriminatory wholesale prices, social welfare is higher than with uniform pricing. The retail price is lower, so consumers benefit, and industry profit is higher.*

The difference between uniform pricing and discrimination can be summarized in terms of the margin that the rival retailer earns. With uniform pricing the margin is fixed at r, while with discrimination the margin is reduced to m, which is chosen by the integrated firm to be as close to zero as possible. With a smaller margin the retail price is lower.

Uniform and discriminatory pricing are illustrated in Figure 2 with the same demand function, $Q(p) = 100 - p$, as in Figure 1 and with $c = 0$. Here $r = 20$

[9] The derivative of industry profit with respect to x, using the envelope theorem result that $\frac{d}{dx}\Pi(c + x) = -Q(p(c + x))$, is $xQ'(p)p'(c + x) < 0$.

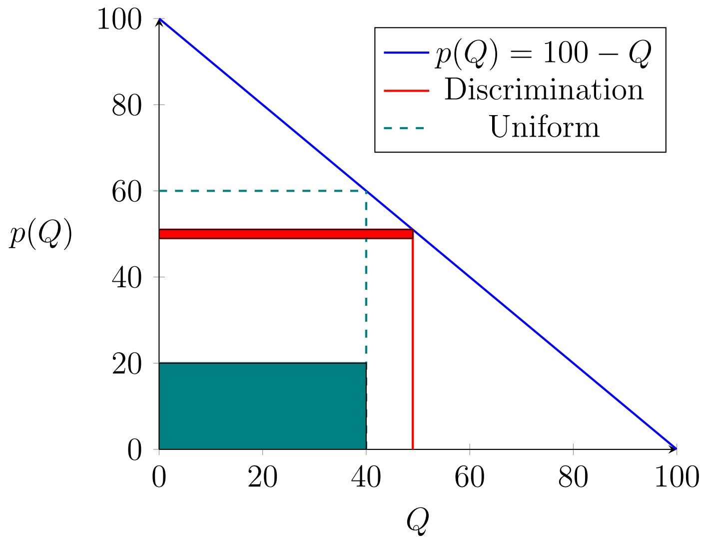

Figure 2: Price-setting retailers

so the retail price with uniform pricing is $\frac{100+20}{2} = 60$ and output is 40. The uniform wholesale price is $w = 40$ and the profit of the rival retailer is the shaded green rectangle, with area 20×40. The integrated firm's profit is 40×40, the square bounded by the dashed green lines above the green shaded rectangle. With discrimination suppose the upstream firm sets $m = 2$. The retail price is $\frac{100+2}{2} = 51$, and output is 49. The wholesale prices are $w_1 = 31$ and $w_2 = 49$. The integrated firm's profit is 49×49 and the rival retailer's profit is the red rectangle with area 2×49.

A notable, and perhaps surprising, feature is that if demand is sufficiently convex both wholesale prices are below the uniform price.

Proposition 4. *If demand is log-convex then both wholesale prices with discrimination are below the uniform price.*

Proof. Demand is log-convex if and only if $\sigma(Q) \geq 1$ for all Q. For marginal cost k the monopoly price is $p(k)$ and the cost pass-through rate is $p'(k) = \frac{1}{2-\sigma} \geq 1$. The monopoly margin, $p(k) - k$, is increasing in k as $p'(k) - 1 \geq 1$. The uniform wholesale margin is $w - c = p(c + r) - c - r$. The margin on sales to retailer 2 with discrimination is $w_2 - c = p(c+m) - c - m$. Because $r > m$ and demand is log-convex $w - c \geq w_2 - c$ and thus $w \geq w_2$. The wholesale price for 1 is $w_1 = w_2 + m - r < w_2$, so discrimination reduces both prices: $w \geq w_2 > w_1$. The weak inequality is strict if demand is strictly log-convex. □

To apply Proposition 4 suppose that demand has a constant elasticity of 2, so $Q(p) = p^{-2}$. For marginal cost k the monopoly price is $p(k) = 2k$. The uniform wholesale price is $w = 2c + r$, the discriminatory price for retailer 2 is $w_2 = 2c + m$, and the discriminatory price for 1 is $w_1 = 2c + m - (r - m)$. Proposition 4 does not apply in the linear demand case of Figure 2 because demand is log-concave.

Two extensions follow. First, let the integrated firm use additional vertical restraints. When nonlinear pricing is feasible the integrated firm sets $w = c$ and a franchise fee of $\Pi(c) = (p(c) - c)Q(p(c))$. Only the efficient retailer can afford the franchise fee, and it prices at $p(c)$. Alternatively with resale price maintenance the integrated firm sets each retail price equal to $p(c)$. The wholesale price is equal to $p(c)$, so there is a full margin squeeze on the efficient retailer. Retailer 1 does not operate: its margin would be $-r < 0$. In both cases the integrated firm earns the maximum feasible profit of $\Pi(c)$. There is no need to use discrimination to achieve the maximum profit level.

Second, suppose retailer 1 is more efficient, with zero direct marginal cost, while retailer 2 has a marginal cost of $r > 0$. The maximum available profit for the integrated firm is $\Pi(c)$. Neither discrimination nor additional vertical restraints are needed to achieve this. With a uniform wholesale price of $w = p(c) - r$ retailer 1 prices at its rival's cost, i.e., $w + r = p(c)$ and retailer 2 sells nothing. Retailer 1 earns $rQ(p(c))$ and the upstream firm's wholesale profit is $(p(c) - c - r)Q(p(c))$. Thus the combined profit of the integrated firm is $\Pi(c)$. The foreclosure of the rival retailer is beneficial for society because the rival is inefficient.

In the main model here discrimination increases welfare and benefits consumers. This is consistent with the results in the Cournot model with equally efficient firms. In both cases double marginalization is eliminated by price discrimination, and this is good for the discriminating firm, for consumers, and for society.

5 Vertical Separation and Discrimination

The model in Section 4 is framed as one with vertical integration to facilitate comparison with the Cournot model of Section 2. Nothing, however, depends on retailer 1 being vertically integrated with the upstream firm. Retailer 1 chooses its retail price to maximize its own profit, and the integrated firm earns all its profit from wholesaling to the independent retailer. The same outcomes apply if there is vertical separation, with retailer 1, as well as retailer 2, being independent.

There is a significant literature on wholesale price discrimination when downstream firms with different efficiency levels compete à la Cournot and all are independent of the upstream monopolist: see Katz (1987), DeGraba (1990), Yoshida (2000), Valletti (2003) and Li (2014). Standard results are that: (i) with linear demand total output is the same whether or not there is discrimination, and (ii) the less efficient retailers face lower wholesale prices than the more efficient firms under discrimination. Valletti (2003) shows that (ii) does not depend on the functional form of the retail demand function. Welfare in the linear demand case is lower with discrimination because the given total output is distributed less efficiently across the buyers than with a uniform wholesale price. Li & Zhang (2024) assess the welfare effects of such discrimination for general demand functions. They show that if demand is sufficiently convex (and curvature is non-decreasing in output) the increase in total output can be large enough to offset the inefficient distribution effect, and thus social welfare rises. An example is when the demand function has a constant elasticity that is at most 3.

In contrast the welfare effects of wholesale price discrimination when the retailers choose prices are unambiguous: social welfare increases and consumers benefit. The welfare effects do not depend on functional forms. As in the Cournot model the less efficient firm is offered a lower wholesale price, but this does not generate inefficiency because the output of this firm is zero.

6 Conclusion

In the two models with upstream monopoly discrimination is good for the integrated firm, consumers and social welfare. Double marginalization is avoided. Only in the model with upstream price competition and downstream Cournot retailers is there a case against price discrimination, and this argument is delicate. The key assumption in the paper is that the integrated retailer maximizes its own profit. This allows for differences in outcomes between uniform pricing and discriminatory pricing and explains why there may be anti-trust concerns about differential wholesale pricing.

Future research might model an upstream monopoly selling an input to price-setting retailers who supply differentiated products. Differentiation at the retail level will entail positive profit margins and thus double marginalization. Without additional vertical restraints the upstream firm will be unable to capture all of the downstream profit. Another direction is to allow for imperfect information. The models in this paper have wholesale prices and marginal costs being fully observed: in practice business-to-business contracts are secret.

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Appendices

A Vertical Separation and Cournot

Direct demand when inverse demand curvature is constant may be written as

$$Q(p) = \left(\frac{(1-\sigma)(a-p)}{b} + 1 \right)^{\frac{1}{1-\sigma}}$$

for $\sigma \neq 1$ and $b > 0$, and $Q(p) = e^{\frac{a-p}{b}}$ when $\sigma = 1$. The exponential version is the limit of the general expression as σ goes to 1. Using the Cournot first-order condition, $p(Q) + \frac{Q}{n} p'(Q) - w = 0$, demand for the upstream firm, as a function of the wholesale price, is

$$\left(\frac{n}{n+1-\sigma} \right)^{\frac{1}{1-\sigma}} \left(\frac{(1-\sigma)(a-w)}{b} + 1 \right)^{\frac{1}{1-\sigma}}$$

for $\sigma \neq 1$ and $e^{-\frac{1}{n}} e^{\frac{a-w}{b}}$ for $\sigma = 1$. In each case this is a multiplicative term $k(n) \in (0,1)$ times $Q(w)$. The wholesale price, w, that maximizes $(w-c)k(n)Q(w)$ equals the retail price that maximizes $(p-c)Q(p)$, which is the unconstrained monopoly price $p(Q^*)$. The problems differ by the multiplicative term. Tyagi (1999) shows that with this class of demand functions the optimal wholesale price is independent of the number of firms. This can be seen here because n only enters wholesale demand through $k(n)$.

B The size of the retail cost difference

In the price-setting model of Section 4 the results for uniform pricing require two conditions to hold that together limit the size of the retail cost difference, r. Condition 1, that $\Pi(c+r) \geq \pi(w^s)$, has already been assumed to hold. The integrated firm would prefer using its own retailer, despite its inefficiency, to supplying the rival retailer when the latter acts as a monopolist and generates double marginalization. Condition 2 is that at the given wholesale price $p(w) \geq w + r$, so the cost difference is non-drastic. Because $w = p(c+r) - r$ this is the same as $p(w) \geq p(c+r)$, which in turn is equivalent to $w \geq c + r$ given that $p(k)$ is a strictly increasing function. Finally the condition may be written as $p(c+r) - c - r \geq r$, i.e., the margin of the integrated firm when it uses its own retailer is higher than the cost difference, r, which is also the margin of the rival retailer.

When inverse demand has constant curvature Condition 2 is

$$(1-\sigma)(a-c) + 1 \geq (3-2\sigma)r. \tag{6}$$

If demand is exponential, $Q(p) = e^{-p}$, then $\sigma = 1$ and condition (6) is $1 \geq r$. For this demand function Conditions 1 and 2 are identical. For linear demand consider the functional forms used in Figures 1 and 2 (for which $\sigma = 0$, $c = 0$ and $a = 99$ so the intercept of demand is $a + 1 = 100$). Condition 1 is $r \leq 29.3$, while Condition 2 is $r \leq 33.33$, so if the former holds so does the latter.

If demand is log-convex with $\sigma \geq 1.5$ then the right-hand side of (6) is non-positive. Since the left-hand side is $(2-\sigma)(p(c)-c) > 0$ the condition automatically holds. Demand functions with constant elasticities that are at most 2 are examples: for the constant elasticity class with price elasticity $\eta > 1$ condition (6) is $c \geq (\eta - 2)r$. The general statement of Condition 1 for this class of log-convex demands is

$$\left(\left(\frac{\eta}{\eta-1}\right)^{\frac{\eta}{\eta-1}} - 1\right) c \geq r,$$

which is $3c \geq r$ when $\eta = 2$.

Suppose that Condition 2 does not hold so retailer 2 would price at the monopoly price $p(w)$. The upstream firm's profit would be $(w-c)Q(p(w))$, which is maximized by setting $w = w^s$ yielding profit of $\pi(w^s)$, which is wholesale profit with double marginalization. If Condition 1 holds then the upstream firm prefers to do its own retailing, and thus forecloses the rival retailer. The retail price will be $p(c+r)$, as in the main model, though this time a real resource cost of $rQ(p(c+r))$ will be incurred.