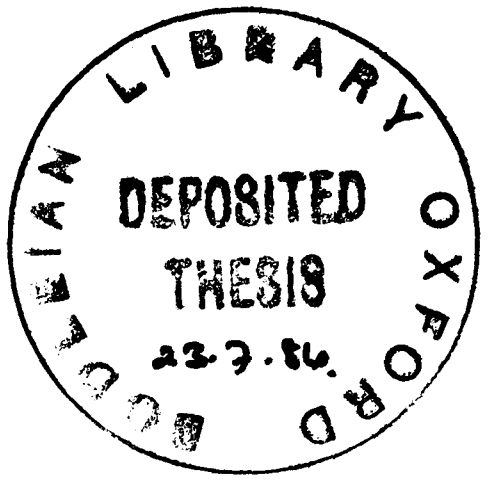


STOCHASTIC INVENTORY THEORY AND THE DEMAND FOR MONEY

Gregor W. Smith

A thesis submitted for the degree of Doctor of Philosophy in the University of Oxford.

Nuffield College

Hilary Term 1986

ABSTRACT

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This thesis describes an inventory-theoretic approach to the study of the demand for money. It aims to connect money demand theory with optimal inventory theory on the one hand and with time series empiricel evidence on the other. Thus it incorporates recent advances in inventory theory and extends these to allow the interest rate to follow a stochastic process. The problem of minimizing the expected, discounted sum of cash-management costs is ascribed to an agent. Through the use of continuous-time, stochastic, optimal control an optimal cash-management policy is shown to exist and be of a familiar target-threshold form. Closed-form expressions for the forward-looking, time-varying targets and thresholds are derived in special cases. The steady-state, Baumol-Tobin model, a further special case, also is examined in detail.

The theory implies that expected future interest rates may influence money holdings despite the absence of strictly convex adjustment costs. A distributed-lag expression for these holdings is

proposed in which the adjustment and expectations dynamics are derived from theory. Aggregation over time and, to a lesser extent, over agents is treated explicitly. The econometric issues involved in testing models of the demand for money with rational expectations are outlined and simulation evidence on the predictions of the theory is provided. The theory gives rise to new predictions concerning expectations effects and variable adjustment speeds. It can also account for the findings of empirical research. In particular, it largely resolves the problem of slow adjustment in empirical money demand equations.

ACKNOWLEDGEMENTS

For comments on some of the contents of this thesis I thank David Hendry, Ross Milbourne, Steve Schaefer, and seminar participants at the Bank of Canada, the Universities of Alberta, British Columbia, Toronto, and Western Ontario, the London Business School, McMaster University, McGill University, and Queen's University. For providing me with unpublished material I thank A.R. Bergstrom, Angelo Melino, and Hamish Taylor.

For two summers of employments which sparked my interest in the demand for money I thank the Department of Monetary and Financial Analysis of the Bank of Canada and John D. Murray in particular. For financial support in Oxford I thank the Rhodes Trust.

For much guidance I especially thank my supervisor, David Begg.

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1. INTRODUCTION.

1.1 Aims.

This thesis is concerned with the boundary between theory and empirics in the demand for money. Its aims are to answer the following questions, among others. First, why is there no testable, single-equation, rational expectations model of the demand for money? The techniques applied in examining other macroeconomic schedules seem conspicuously absent in most money demand studies. Second, can such a model be developed without invoking unrealistic convex costs of adjustment but while accounting for the empirical success of models traditionally rationalized with reference to such costs? Quadratic costs of adjustment, to take an example, seem very unrealistic in the decisions involved in managing cash yet a theory which does away with them must address the longevity of the partial-adjustment empirical models which can be derived from such a cost structure. Third, in particular, is there a resolution to the problem of slow adjustment in empirical studies of the demand for money? In empirical work the coefficient on the once-lagged, dependent variable is always highly significant and large. In quarterly M1 data from the United States, the United Kingdom, or Canada this coefficient typically takes a value near 0.9. This value implies a mean lag of nine quarters in response to changes in the independent variables. The problem is that such adjustment speeds are implausibly slow, yet the linear, econometric models involved seem to represent the data quite well.

One aim of this thesis is to propose a new way of thinking of the coefficient on this lagged, dependent variable and hence a resolution of the problem of slow adjustment. So the thesis begins with the problem that the time series of monetary aggregates typically are smoother (in frequency) than the time series of nominal interest rates or income.

The problem is one of theoretically interpreting the long lags on independent variables necessary to fit money demand equations. Are such lags due to adjustment costs? Or do we observe the outcome of instant adjustment to some target given by a moving average of these variables?

An infinite number of theories are consistent with the fact of long lags, but fortunately additional empirical evidence and economic theory can eliminate some. For example, models of partial-adjustment or error-correction form are inconsistent with the form which we know adjustment costs take. This thesis proposes another theory which does not share this failing, which can account for empirical evidence, and which also gives rise to predictions which cannot be derived from these familiar models.

Economic theory can eliminate other possible theories. For example, a dichotomy between explanations based on adjustment costs and those based on expectations clearly is a false one. Normally the values of independent variables will change between adjustments or while adjustments are being made, so that expectations are relevant. Then expectations formation must be modelled unless these variables follow random walks.

The suggestion that both types of explanation are relevant is not a startling insight. The aim of this thesis is to bring some theory to bear on the precise mix of backward (adjustment) and forward (expectations) dynamics in the demand for money. In other words, one interprets the data as arising from a process in which money is related to, say, a short-term interest rate with a short lag and a short lead rather than with a long lag. The aim is to parametrize this mixture to allow efficient tests of money demand theory.

The approach here is based on inventory theory, which has a long tradition in the theoretical study of the demand for money and in the normative work of operations researchers. It seems a realistic description of how individuals and firms hold money. Moreover, econometricians often interpret elasticities from linear, econometric models with reference to inventory theory. It seems worthwhile to see whether this sort of connection can be established explicitly.

It is straightforward to see how expectations become relevant to the demand for money through inventory theory. In this theory a money-holder faces an exogenous and uncertain net disbursements process in, say, cash. There are essentially two types of costs involved in managing cash balances. One is a holding or opportunity cost given by the product of the cash balance and the interest rate available on some alternative asset. The other is a linear transfer cost paid when the cash balance is adjusted. In this framework the optimal control rule is, roughly speaking. of the bang-bang variety, as would be expected with the linear cost structure. Costs are minimized if the cash balance is allowed to vary between two thresholds and returned to a target each time it reaches a threshold. The targets and thresholds depend on the holding and penalty costs. They also depend on expectations of these costs. Since the cash balance is adjusted only intermittently its manager must live for some time with the consequences of his most recent adjustment decision.

Section 1.3 below outlines how these notions can be formalized. Section 1.2 first describes the background to this study.

1.2 Background.

The tradition of treating money holdings as an inventory extends at least to Walras. Likewise Hicks argued that the 'cost-of-investment', that is the transfer cost, should be taken into account in studying the demand for money. Gilbert (1953) provides a brief discussion of this and other aspects of the history of money demand theory. Neither this tradition nor this thesis fully explains why money is held; the concern is rather with how it is held.

Although inventory theory can be associated with the transactions and precautionary motives for holding money, Keynes's links between motives and independent variables may not always be useful. Thus variables normally linked to the finance and speculative motives may influence money holdings in inventory-theoretic models even without risk aversion.

The formal inventory theory on which this thesis builds began with Arrow, Harris, and Marschak (1951) and Dvoretsky, Kiefer, and Wolfowitz (1952a, 1952b). These papers originated a large body of work concerned with establishing the existence and form of optimal policies for managing inventories. This work used discrete-time dynamic programming with the objective being the minimization of the expected sum (discounted or not) of holding, transfer, and penalty (or overdraft) costs. Sufficient conditions for the optimal monitoring policy to be of simple form were found to involve restrictions on the cost structure (an approach taken by Scarf and others) or on the distribution of net disbursements (an approach taken by Karlin and others). The form involved two targets and two thresholds (d,D,U,u) such that the inventory was replenished to a level D if it reached a lower threshold d and reduced to a level U if it reached an upper threshold u.

Operations researchers have extended this approach in a number of ways. They have studied problems with lead times before adjusting, more complicated cost structures, unknown demand distributions, and multiple inventories. Silver (1981) reviews these developments. One concern has been to find algorithms to solve for the optimal targets and thresholds or, where this is too difficult, to establish appropriate heuristic methods to reach a solution. These methods involve trial and error or rules-of-thumb typically of control-limit (that is, target-threshold) form. Silver, Vidal, and de Werra (1980) discuss such methods.

In describing how to set control limits for managing a cash balance heuristic methods often consider the expected future values of net disbursements and of costs. This consideration makes sense given the intermittent nature of adjustment in control-limit models yet formal, programming derivations of the control limits treat the holding cost, for example, as fixed. Money demand studies based on inventory theory so far do the same.

Proofs of optimal policy form are only worthwhile if the conditions under which they are obtained are realistic. Thus optimal inventory theory derived with the interest rate fixed cannot shed much light on time series data. To address this issue the analysis below begins with a more realistic environment and later considers no elements in the environment not taken into account in the proof.

The remedying of this flaw turns out to be most easily done by using continuous-time methods, another devleopment in inventory theory. Constantinides and Richard (1978) and Harrison, Sellke, and Taylor (1983), for example, apply these techniques. Sections 1.3 and chapter 2 discuss the appropriateness of continuous-time modelling. In general,

this development too seems not to have been absorbed into the theory of the demand for money.

Economists have contributed to the study of the existence and form of an optimal inventory policy in managing cash. For example, Eppen and Fama (1968, 1969) proved that with proportional transfer costs the optimal targets coincide with the thresholds so that the policy is of (d,u) form. Milbourne (1983) gives results for the more general (d,D,U,u) policy. In general, however, most papers in economics journals have been concerned with the second stage of the problem, namely the search for the actual values of the targets and thresholds. The tendency has been to establish these values through steady-state optimization (as in Miller and Orr (1966), for example) rather than through net-present-value optimization and hence to break the link with optimal inventory theory and preclude the derivation of forward-looking monitoring policies.

Well-known money demand papers that examine special cases of the general cash management problem include those by Baumol (1952), Tobin (1956), and, again, Miller and Orr and Eppen and Fama. These papers all treat the holding cost as fixed but they vary in their treatments of transfer costs. For example in Miller and Orr's work there are no proportional costs and in Eppen and Fama's no fixed ones.

Early models such as Baumol's treated net disbursements as being deterministic with negative drift. Miller and Orr and Orr (1970) took the opposite approach, representing disbursements by a driftless random walk. Barro (1974) argued for a mixture of deterministic and stochastic elements, a request which was first answered by Frenkel and Jovanovic (1980) in their stochastic version of the Baumol model. This thesis also allows for both elements.

Treating net disbursements as stochastic allows explicit treatment of the precautionary motive underlying so-called root laws of inventory holding. Whalen (1966) and Olivera (1971) have studied this motive. The inventory models in the money demand literature have also been extended by Tsiang (1969) to include lead-time requirements (as opposed to the usual assumption that transfers can be made instantly) and by Sastry (1970) and others to include cash-out or penalty costs rather than imposing a lower threshold at zero.

Allowing for overdrafts leads naturally to the many models with a third asset. The principal problem with this extension is that these models have corners; these make aggregation and comparative statics difficult. Barro (1976), Akerlof and Milbourne (1978), and Clower and Howitt (1978) examine the problems involved. The method in this thesis avoids the simplest of these problems; modelling in continuous time rids us of the artificial integer constraints of some discrete-time approaches.

One criticism of inventory-theoretic methods which goes unresolved here concerns their apparent inability to account for the absolute volume of cash or M1 in the economy. Orr (1974) discusses the debate on the role of compensating balances in aggregate U.S. M1. Inventory models have also had mixed success in microeconomic studies, for example those of Miller and Orr (1967) and Mullins and Homonoff (1976). However, most microeconomic studies have been explicitly normative. Aronson (1968) and Maldonado and Ritter (1971), for example, applied inventory models to the cash holdings of local governments in the U.S.

There are very few studies which use the theory in a positive, as opposed to normative, sense. Thus money demand theory has been disconnected not only from optimal inventory theory (as argued above)

but also from empirical studies. Derivations of the time series properties of money under inventory rules are rare. Chant (1976) and the seminal paper of Milbourne, Buckholtz, and Wasan (1983) are exceptions, but both use steady-state optimization.

Thus most empirical work in the demand for money has had little basis in theory. Moreover, popular rationalizations of successful empirical equations are inconsistent with inventory theory. For example, partial-adjustment models append quadratic costs of adjustment to a linear target. This device was introduced by Chow (1966) in imitation of his earlier work on consumer durables. Goldfeld (1973, p.580) appealed to transactions demand theory and mentioned the Baumol-Tobin model in positing a target linear in interest rates and income. Then he attributed the lagged dependent variable in demand functions to expectations of the independent variables or, more often, to partial adjustment. Goldfeld admitted (p.598) that 'despite the superficial plausibility of this mechanism, its theoretical foundation in the context of the demand for money is unclear.' Breen (1971) gave a concise statement of the inconsistency of partial adjustment with inventory theory. As Judd and Scadding (1983) remark in their survey of empirical work since Goldfeld, the implausibly slow adjustment implied by Goldfeld-type equations poses a further theoretical problem.

One answer to this problem is to argue that the partial-adjustment mechanism was purely instrumental and never intended to be taken as a realistic, microeconomic theory. There have also been attempts to rationalize the lagged, dependent variable (i) by noting that a gradual aggregate response to shocks may arise when individuals respond completely but with a delay that is smoothly distributed across money-holders, (ii) by attributing it to the gradual response of target

holdings to current interest rates and income, that is to expectations, or (iii) by attributing it to the gradual adjustment of independent variables to the money stock, that is to buffer stock effects.

None of these suggestions so far has a detailed microeconomic foundation. Without such a foundation the programme of seeking good empirical models seems excessivley modest (compared with research programmes elsewhere in macroeconomics) and perhaps even misguided if our goal is to understand fully the phenomena involved.

This thesis seeks to reconnect money demand theory with optimal inventory theory on the one hand and with empirical studies on the other. It ascribes to agents optimization problems in which all costs are taken into account simultaneously. It explicitly derives the effects of adjustment, expectations, and aggregation on the dynamics of money. One result of this derivation is a resolution of the problem of slow adjustment.

The approach is in the demand-determined tradition, since the evidence of simultaneity bias in empirical money demand models is scant. Aggregate demand-for-money equations are not regarded here as inverted price adjustment equations for example. Although the case for regarding historical data as having been generated with, say, M1 exogenous is not a strong one (see Smith (1985)) someone interested in regimes with exogenous money might still note two elements in the argument below. First, one result of the thesis is that models with fixed costs can give rise to smooth distributed lags of money on, say, interest rates. Unlike models with strictly convex costs of adjustment these models are conceptually reversible; they are at least consistent also with a smooth response of interest rates to changes in money where the latter is exogenous. Second, the resolution of the slow adjustment problem

proposed here does not rely on aggregation; it also applies for a single money-holder.

1.3 Outline.

Chapter 2 begins by extending some recent developments in inventory theory. It considers a scenario in which a cash-manager faces both fixed and proportional costs of altering the level of a cash balance. If uncontrolled, the balance follows a drifting random walk. The holding cost, or interest rate, also follows a stochastic process. This structure includes the Baumol-Tobin, Miller-Orr, and Eppen-Fama models, among others, as special cases.

In this structure a net-present-value objective function is attributed to the cash manager. Chapter 2 establishes the existence and form of the optimal policy in monitoring the balance. It also derives closed-form expressions for the parameters of this policy in special cases.

The methods used in chapter 2 are somewhat unfamiliar in macroeconomics. The problem is set in continuous time since (i) time is continuous, (ii) diffusion formulations are more tractable than traditional discrete review models, and (iii) this provides empirical realism; the agent's actions are intermittent but aperiodic and the frequency with which he transacts is endogenous.

The net disbursements process is a Wiener process, the continuous-time analogue to a drifting random walk in discrete time. This process is used because (i) more general mixed or Poisson processes are difficult to handle mathematically, (ii) it captures the Baumol-Tobin

(negative drift, non-stochastic) and Miller-Orr (zero drift) net disbursements processes as special cases, and (iii) extensions raise theoretical difficulties (discussed in chapter 4) which make this process a natural one at which to stop.

The future matters to the current decisions of the cash manager because of the persistence in the net disbursements process and the intermittent nature of adjustments. Thus future interest rates are worth knowing and current information helps forecast them if the interest rate does not follow a random walk. In chapter 2 the interest rate varies according to the simplest, stochastic process parameterized to include cases with and without independent increments. However, many possible empirical interest rate models are consistent with this process if written in state-space form.

Solving the stochastic control problem in this environment yields an optimal monitoring policy of target-threshold form. The innovation of chapter 2 lies in characterizing the targets and thresholds as explicitly forward-looking variables.

The Baumol-Tobin model is one interesting special case of the model of chapter 2, but no closed-form solution exists for the target in this model in the framework there. To develop such a solution chapter 3 adopts steady-state optimization, which is defensible if the interest rate, for example, follows a random walk. The steady-state format also allows simple treatments of aggregation and of the dynamics of money demand, foreshadowing developments in chapter 5. Thus chapter 3 derives the first dynamic, stochastic Baumol-Tobin model.

The models in this thesis involve two state variables, net disbursements and a spot interest rate. Chapter 4 looks at the problem of endogenizing the stochastic processes governing the evolution of

these variables. While there is no easy solution to the problem of characterizing the demand for money in general equilibrium models with transactions costs, this chapter generates the net disbursements process from consumption and portfolio decisions consistent with the sub-optimality assumed in the transactions demand literature. In an obviously related development, the model of chapter 2 is extended to allow the parameters of the net disbursements process to vary over time.

Chapter 5 turns to the problem of connecting optimal inventory theory with the time series properties of monetary aggregates and short-term interest rates. Previous approaches, such as that developed by Milbourne, Buckholtz, and Wasan and used in chapter 3, have considered transitions between steady states. The method of chapter 5 is not restricted to such cases, and can be used for any inventory policy though the analysis is done below only for the (0,u) case.

In inventory models the time series of the cash balance is simply the net disbursements process under control. There are two resons why this money time series behaves differently from the interest rate time series. One is that the interest rate affects money holdings through the optimal targets and thresholds, which will not vary over time in exactly the same way as the interest rate since, for example, they also depend on expected future interest rates. The other reason is that transactions are not made continuously so that the time series of money will depend both on the behaviour of the targets and thresholds and on the net disbursements process. For example, if the interest rate is high and transfer costs are low the region within which the balance is allowed to vary between adjsutments will be narrow so that adjustments will be made frequently and the money series will be much like the target and threshold series. Conversely, if the interest rate is low

and transfer costs high this continuation region will be wide so that adjustments will be made infrequently and the money series will resemble the net disbursements process. In general, the money series will be described by a probabilistic mixture of these two cases with weights varying over time with the state variables. Chapter 5 derives these weights and hence gives a theoretical money demand model with a variable adjustment speed. In aggregate this speed also depends on the distribution of cash balances across money holders; in chapter 5 the parameters of this distribution enter the dynamic model explicitly.

Chapter 5 also describes the econometric obstacles to rigourous tests of the theory. However, simulation evidence suggests that inventory theory can account for some of the findings of linear, econometric models. In particular, data generated through inventory control with rapid adjustment typically indicate a slow adjustment problem when fitted with stanadrd, linear models.

Chapter 6 contains a brief summary and some suggestions for further theoretical and empirical work.

2. FORWARD-LOOKING STOCHASTIC CASH MANAGEMENT.

2.1 Introduction.

The goals of this chapter are to derive the form of the optimal cash-management policy, to find closed-form expressions for the parameters of such a policy, and to assess the implications of this analysis for the demand for money. Net cash disbursements are taken to be stochastic, with drift, in continuous time. The transfers involved in adjusting the cash balance are subject to costs with fixed and proportional elements. These costs need not be symmetric. The cash manager is taken to have a net-present-value, objective function with an infinite horizon.

This setup is quite general compared to most previous studies in the money demand literature.[1] The generality is made possible by the use of continuous-time stochastic control techniques as described by Chow (1979), Fleming and Rishel (1975), and Malliaris and Brock (1982). Although the application is to the cash management problem, three generalizations in stochastic inventory theory are also made.

First, the interest rate or holding cost is allowed to be stochastic. This is a second state variable, in addition to the cash balance itself. There are some precursors to this extension. Three-asset inventory models, for example, contain two state variables. Constantinides's (1974) unpublished thesis allowed the holding cost to vary with the level of the inventory.[2] Several papers on the term structure of interest rates in money demand functions are also related to the approach here.[3]

Second, the interest rate model used contains a parameter that distinguishes between processes with and without independent increments. The random walk and stationary cases can be examined in the same framework. In the stationary case past interest rates help predict

future ones. The relevance of these to cash management depends on the persistence in the net disbursements process. It is this persistence (combined with the form of the optimal cash management policy), rather than some strict convexity in adjustment costs, that gives rise to interdependence between decisions taken at different times. This forward-looking aspect of the problem is mentioned often in heuristic inventory models[4] but has not been studied in connection with the demand for money.

Third, there may be a non-zero covariance between autonomous shocks to the cash balance and shocks to the interest rate. The money supply shocks contemplated by buffer stock theorists might be one source of such a covariance. To anticipate, however, these generalizations have no effect on the optimal policy form, the robustnesss of which is thus further established. But the economics of the solution is unlike that of previous studies in a number of ways.

The generalizations mentioned above are made in the inventory model of Constantinides and Richard (1978). This model is the most general one available, though it is not a familiar one in economics. One of the analytic solutions in section 10 is due to these authors, the other one is new. Taylor (1977), Harrison and Taylor (1978), and Harrison, Sellke, and Taylor (1983) consider some inventory models more restrictive than those studied by Constantinides and Richard but develop an alternative solution method which makes the calculation of optima easier. However, their solutions are not of closed form.

In discrete time the sufficient conditions for the optimal policy to be of a simple form have been progressively weakened, as in the proofs of Neave (1970), Roberts (1971), and Milbourne (1983). The present model requires only (i) that decisions are taken continuously,

and (ii) that net disbursements follow a Wiener process with drift.[5] Earlier studies using similar assumptions include Savage's (1962) continuous-time, one-sided inventory policy under a Poisson process, and subsequent models of the one-sided case by Antelman and Savage (1965) and Bather (1966). Sethi and Thompson (1970) treat the two-sided cash balance problem and find the optimal policy to be of bang-bang form. Their formulation is deterministic with a fixed holding cost. The two-sided problem has been studied also by Constantinides (1976), Puterman (1975), and Vial (1972). The latter assumed the existence of an optimal policy and showed it to be of a simple form. The impulse control theory developed by Bensoussan and Lions (1975) and Richard (1977) makes existence and form proofs considerably easier.

The appropriateness of assuming continuous monitoring depends on other time scales in the problem. The reader is referred to Magill and Constantinides (1976) and Merton (1982) for justifications of continuous-time modelling beyond those in section 1.3. Note in the present case assuming continuous trading is not involved since transfer costs are considered explicitly and the trading interval is endogenous.

Sections 2.2 and 2.3 below describe the interest rate and net disbursements processes respectively. The transfer, holding, and penalty costs are described in section 2.4. The control mechanism is given in section 2.5. Section 2.6 sets up the optimization problem which is solved subject to the boundary and corner conditions of sections 2.7 and 2.8. The objective is taken to be the minimization of the expected discounted sum of future transfer, holding, and penalty costs. Methods similar to those in this chapter can be applied to the case without discounting. In that case the problem can be simplified to one of solving an elliptic, second-order partial differential equation

(corresponding to the integral equation of discrete-time inventory theory) subject to boundary conditions. Whereas numerical methods are needed for this the present case has the virtue of providing some analytic solutions for the monitoring parameters of the optimal policy. The general solution is given in section 2.9 while section 2.10 gives these special cases. Section 2.11 begins to derive implications for money holdings, which depend on both the policy and the net disbursements process. One implication of the theory is that conventional, empirical money demand equations may underestimate the speed with which money holdings are adjusted to shocks. Section 2.12 provides a brief conclusion.

2.2 Interest Rate Process.

As a preliminary, assume throughout this thesis that there is a probability space (Ω,G,P) on which stochastic processes are defined, where G_t is a non-anticipating family of σ-fields. Similarly stochastic integrals are of the Itô type since these are based on sequences of non-anticipating functions. Expectations are conditioned on the current σ-field.

The interest rate is modelled as a diffusion process. Specifically, the interest rate r is the solution to

$$dr = \alpha(\gamma-r)dt + \sigma_r dz_r \qquad \alpha \geq 0 \qquad (1)$$

Here z_r is a standard, Wiener process (i.e. Markov and Gaussian) with incremental variance dt. In other words z_r is a process with independent, normal increments.

The instantaneous mean and variance of the interest rate are $\alpha(\gamma-r)$ and σ^2_r respectively. Note, however, that unlike the Wiener

process the interest rate process given in (1) does not have independent increments unless $\alpha=0$. For $\alpha>0$ it has a stationary distribution and, conditional on the current level, it is characterized by mean and variance as follows:

$$E_t\, r(s) = \gamma + (r(t)-\gamma)\exp(-\alpha(s-t))$$

$$\mathrm{var}_t\, r(s) = \frac{\sigma_r^2}{2\alpha}(1 - \exp(-2\alpha(s-t)) \qquad t \leq s \tag{2}$$

As α, the speed of adjustment, goes to infinity the conditional mean of the spot rate goes to the long-run value γ and the variance goes to zero. As α goes to zero the conditional mean goes to the current level and the variance to $\sigma_r^2 \cdot (s-t)$. As $s \to \infty$ the interest rate is distributed normally with mean γ and variance $\sigma_r^2/2\alpha$. If $\alpha>0$ the interest rate is said to follow an elastic random walk, tending to rise(fall) if it is below(above) the level γ.[6] The process is the analogue in continuous time to a first-order autoregression in discrete time. A unit root in the latter corresponds to the special case $\alpha=0$ in (1). Note finally that this process can be viewed as a modified Wiener process, by a transformation due to Doob (see Cox and Miller (1965) or Merton (1973)). It is known as the Ornstein-Uhlenbeck process.

The description in equation (1) is not the only one which could have been chosen for the interest rate. A number of more complex stochastic processes are used in options pricing theory, for example.[7] On the other hand, (1) has a history in financial theory as a spot rate process. Moreover, it is the simplest process which allows a check on the relation between the predictability of future interest rates and their relevance to current decisions. In any case, any ARIMA process can be written as (1) by an addition of states. Cox, Ingersoll, and Ross give examples of such additions.

2.3 Cash Flow or Net Disbursements Process.

The net disbursements process, taken as given by the cash manager, is given by

$$dx = \mu dt + \sigma_x dz_x \qquad (3)$$

Disbursements follow a diffusion process of the Itô class. Here $z_x(t)$ follows a Wiener process with no drift and unit variance. The $x(t)$ process in (3) has instantaneous mean μ, instantaneous variance σ^2_x, and independent normal increments; thus it is itself a Wiener process with $x(t) \sim N(x(0)+\mu t, \sigma_x{}^2 t)$. It is continuous and non-differentiable almost surely. It may be regarded as the limit of a drifting random walk in discrete time. Note that μ and σ^2_x are parameters, though they could be modelled as variables (see section 4.2).[8] The description in (3) may be justified to some extent by regarding x as the difference between two other variables, say income and expenditure (see Harrison and Taylor). Chapter 4 discusses the problems raised by trying to decompose (3).

Although in some circumstances a Poisson process may seem a more appropriate representation of net disbursements the stochastic calculus pertaining to equations like (1) and (3) turns out to be particularly helpful below. In particular Itô's lemma is invoked several times.

2.4 Costs.

There are three costs involved in managing the cash balance. The holding and penalty costs are summarized in a piecewise linear function $C(x,r)$ where

$$C(x(t),r(t)) = \max[r(t)x(t), -gx(t)] \qquad (4)$$

and r and g are positive.[9] If cash balances are positive the holding cost is the usual foregone interest cost. If they are negative the penalty cost is the cost of credit. There is only one interest-bearing asset.

Transfer costs are paid each time the cash balance is controlled. These costs have fixed and proportional elements which need not be symmetric. If a volume ξ is transferred <u>into</u> cash the cost is

$$B(\xi) = \begin{cases} K^+ + k^+\xi & \xi \geq 0 \\ K^- - k^-\xi & \xi < 0 \end{cases} \qquad (5)$$

$$K^+, K^-, k^+, k^- \geq 0$$

For example, a negative ξ indicates a transfer out of cash into an interest-bearing asset. The fixed cost of this adjustment downward from the upper threshold is K^- and the proportional cost is k^-.

Transfer, holding, and penalty costs are assumed not to be charged to the cash balance. Porteous (1972) and Porteous and Neave (1972) show that this simplification is not vital.

2.5 Control.

Let $0 \leq \tau_1 \leq \tau_2 \leq \ldots \leq \tau_i \leq \ldots$ be an increasing series of stopping times only a finite number of which will occur in a bounded interval. These are the times at which the cash level will be adjusted. If there are fixed costs $\tau_i < \tau_{i+1}$ since with $\tau_i = \tau_{i+1}$ the fixed cost would be incurred twice.

Note that the continuous time approach is not only analytically useful but also empirically so.[10] In it both opportunities and behaviour can be represented in a realistic way. The stopping times τ_i

are random; there is no fixed period between adjustments as in discrete-time models.

At τ_i an impulse control ξ_i is applied. Both τ_i and ξ_i are non-anticipating; that is, they are independent of future states though they may of course depend on expectations of those states. A policy v is a set of stopping times and impulse controls

$$v = (\tau_1, \xi_1; \ldots ; \tau_i, \xi_i; \ldots) \tag{6}$$

Under policy v the path of cash is as follows. Let $x(\tau_i^-)$ be the cash level just before control ξ_i is applied and $x(\tau_i)$ the level immediately after the control is applied. Then the cash level follows

$$\begin{aligned} dx(t) &= \mu dt + \sigma_x dz_x \qquad 0 \le t < \tau_1 \qquad x(0^-) = x_0 \\ x(\tau_i) &= x(\tau_i^-) + \xi_i \\ dx(t) &= \mu dt + \sigma_x dz_x \qquad \tau_i \le t < \tau_{i+1}^- \qquad i \ge 1 \end{aligned} \tag{7}$$

2.6 Optimization.

The total cost functional involved in policy v is

$$J_{x,r}(v) = E\{\sum_{i=0}^{\infty} e^{-\rho\tau_i} B(\xi_i) + \int_0^{\infty} e^{-\rho s} C(x(s), r(s)) ds\} \tag{8}$$

where the expectation is conditional on $x(0)$, $r(0)$. We seek to minimize the expectation subject to the state equations (1) and (3). In other words we seek a control v' such that $J_{x,r}(v') = \inf_v J_{x,r}(v)$. The total cost is the sum of the expected transfer, holding, and penalty costs, discounted at rate ρ.

It may seem odd to discount using β, a subjective, constant rate of time preference, rather than r(t), especially when the process in (1) seems half-way to an endogenous term structure of interest rates. Indeed, Vasicek (1977) and Cox, Ingersoll, and Ross (1985) derive the term structure under general conditions with the spot rate, the only state variable, following the Ornstein-Uhlenbeck process. So far, however, equilibrium models of the term structure all involve perfect capital markets. The stress here on transfer costs makes the Cox-Ingersoll-Ross solution to the problem of discounting with respect to a stochastic rate of return inapplicable. Nevertheless it seems likely that the effects of a decline in the long-run interest rate, γ, on the demand for money will be enhanced by the effect of this change on the term structure by which future costs are discounted. On the other hand the effect of high future interest rates on current money demand will be offset by their effect on discount rates.

Now suppose that for this dynamic programme there exists a value function $V(x,r) = J_{x,r}(v')$ corresponding to the minimized expected total cost. Before establishing the existence and form of the optimal policy note some necessary conditions the function V must satisfy. As in any dynamic programme a small change in time is examined. This examination gives rise to two complementary inequalities, rather than a single equality, because it may or may not begin at a time when an asset transfer occurs.

Assume first that one can examine a time period (t,t+dt) small enough so that no stopping time is encountered.[11] However t itself may be a stopping time. Apply Bellman's Principle of Optimality to get

$$V(x(t^-),r(t^-)) = \min\{ \begin{array}{l} \inf_{\xi}[B(\xi_i) + E(C(x(t),r(t))dt + \\ e^{-\rho dt}V(x(t)+dx,r(t)+dr))] \\ E(C(x(t^-),r(t^-))dt + \\ e^{-\rho dt}V(x(t^-)+dx,r(t^-)+dr)) \end{array}$$

This can be expressed as two complementary inequalities, the first corresponding to t's being a stopping time.

$$V(x(t^-),r(t^-)) \leq \inf_{\xi}[B(\xi) + E(C(x(t),r(t))dt + e^{-\rho dt}V(x(t)+dx,r(t)+dr)) \qquad (9)$$

The second inequality is

$$V(x(t^-),r(t^-)) \leq E(C(x(t^-),r(t^-))dt + e^{-\rho dt}V(x(t^-)+dx,r(t^-)+dr)) \qquad (10)$$

One of (9) and (10) holds as an equality.

If V has continuous partial derivatives of all orders less than three in some open set containing the line segment connecting the two points $(x(t^-),r(t^-))$, $(x(t^-)+dx,r(t^-)+dr)$ then Itô's lemma can be applied in (10). Thus

$$\rho V(x,r) - \mu V_x(x,r) - \alpha(\gamma-r)V_r(x,r) - \qquad (10')$$

$$-\frac{\sigma^2_x}{2}V_{xx}(x,r) - \frac{\sigma^2_r}{2}V_{rr}(x,r) - \sigma_{xr}V_{xr}(x,r,) - C(x,r) \leq 0$$

where the time arguments are suppressed and subscripts on V indicate partial derivatives. In deriving (10') use has been made of the rules:

$$dt^2 = 0 \qquad dtdz = 0$$

$$dz^2_x = dz^2_r = dt \qquad dz_x dz_r = \rho_{xr} dt$$

Letting $dt \to 0$ in the first inequality gives

$$V(x,r) \leq \inf_{\xi}[B(\xi) + V(x+\xi,r)] \tag{9'}$$

Suppose there exists a value function that satisfies (9') and (10') complementarily and also satisfies the following regularity conditions.[12]

$$V(x,r) \geq 0;\ V_x(x,r) \text{ absolutely continuous and bounded;}$$
$$V_{xx}(x,r) \text{ square integrable on } R^1 \tag{11}$$

If such a function exists one can use the following theorem.

Theorem 1. If there exists a solution to (9'), (10'), and (11) then $V(x,r) = J_{x,r}(v') \leq J_{x,r}(v)$ $\forall$ v and v' defines the optimal impulse control.

Proof. The proof is given by Richard (1977).

Richard's theorem simplifies the problem to that of finding a solution to the differential inequalities, subject to regularity conditions. The next several sections show that an optimal policy exists and is of a simple form and characterize it in a set of nonlinear equations. The procedure involves three steps, showing that (i) the value function must satisfy certain boundary conditions and a partial differential equation, (ii) there exists a value function with these properties, and (iii) the same value function satisfies the conditions of Theorem 1.

In sum, if an optimal policy exists it can be shown informally that it must satisfy boundary conditions. A policy exists which does satisfy these boundary conditions. This policy also satisfies the inequalities and regularity condition of Theorem 1. Therefore this policy is an optimal one.

2.7 Boundary Conditions.

Assume first that the optimal policy is of control-limit form, involving parameters d, D, U, u with $d \leq D \leq U \leq u$. If the cash level is x there is a transfer to reach y(x) where

$$y(x) = \begin{cases} U & u \leq x \\ x & d < x < u \\ D & x \leq d \end{cases} \qquad (12)$$

This candidate policy has two targets and two thresholds.

At stopping times x=d or x=u so that (9') is an equality. If x=d $\xi = D-d$ while if x=u $\xi = U-u$. Substituting these impulses in (9') gives

$$\begin{aligned} V(d,r) &= V(D,r) + K^{+} + k^{+}(D-d) \\ V(u,r) &= V(U,r) + K^{-} + k^{-}(u-U) \end{aligned} \qquad (13)$$

In addition to these consistency conditions there are marginal conditions which transfers must satisfy. The optimal transfer from the threshold d is to the target D only if the marginal reduction in expected future costs equals the marginal cost of transfer. Thus

$$\begin{aligned} V_x(D,r) + k^{+} &= 0 \qquad & V_{xx}(D,r) &\geq 0 \\ V_x(U,r) - k^{-} &= 0 \qquad & V_{xx}(U,r) &\geq 0 \end{aligned} \qquad (14)$$

Now consider a situation in which the cash balance is initially just below d by an amount δ. From (12) the optimal policy is to move the balance to the target D. Thus $\xi = D-d+\delta$. Then in (9')
$V(d-\delta,r) = V(D,r) + K^+ + k^+(D-d+\delta)$, or $V(d,r) - V(d-\delta,r) = -k^+\delta$ from (13). Letting $\delta \to 0$ gives

$$V_x(d,r) + k^+ = 0 \qquad V_{xx}(d,r) \leq 0$$
$$V_x(u,r) - k^- = 0 \qquad V_{xx}(u,r) \leq 0 \qquad (15)$$

Finally if x is in the continuation region (i.e. $x \in (d,u)$) then t is not a stopping time and $\xi=0$. Now V must satisfy (10') as an equality, which requires

$$\rho V(x,r) - \mu V_x(x,r) - \alpha(\gamma-r)V_r(x,r)$$
$$- \frac{\sigma^2_x}{2} V_{xx}(x,r) - \frac{\sigma^2_r}{2} V_{rr}(x,r) - \sigma_{xr} V_{xr}(x,r) = C(x,r) \qquad (16)$$

This is the Hamilton-Jacobi-Bellman equation, a deterministic, partial differential equation.

2.8 Corners.

A final restriction concerns the corner conditions familiar in inventory theory. The unconditional expected present value of the unit holding cost from now to infinity is γ/ρ. If $\gamma/\rho \leq k^-$ it will never be optimal to reduce the cash level. A similar argument at the lower threshold gives the following four possibilities:

Inequalities		Policy
$\gamma - \rho k^- \geq 0$	$g - \rho k^+ \geq 0$	Adjust both ways.
$\gamma - \rho k^- < 0$	$g - \rho k^+ \geq 0$	Adjust up only.
$\gamma - \rho k^- \geq 0$	$g - \rho k^+ < 0$	Adjust down only.

(17)

$\gamma - \beta k^- < 0$ $\quad g - \beta k < 0$ $\qquad$ Never control.

Note that the inferences may also depend on the fixed costs. For example, negative drift and a large K^- can give rise to the degeneracy in the second of the four possibilities above. The precise policy form will also depend on the presence of fixed costs.

This chapter deals with the two-sided inventory problem. The one-sided problem (caused by one inequality being violated as in the second and third rows of the table) is examined in a similar way in chapter 3. Vial (1972) discusses corners at length.

2.9 Solution.

Lemma. Suppose the inequalities in the first row of (17) hold. There exist parameters $d_L D_L U_L u$ and a twice continuously differentiable solution $V(x,r)$ of (16) satisfying the regularity conditions (11) and the boundary conditions (13), (14), and (15).

Proof. The proof is given by Constantinides and Richard (1978).

The generalizations in this chapter do not complicate the proof, which is omitted since it is quite long. Most of the new economic insights follow from the next theorem, an extension of one due to Constantinides and Richard.

Theorem 2. Suppose the inequalities in the first row of (17) hold. Then an optimal policy exists and is of a simple form given by (12).

Proof. Define the function V(x,r) as

$$V(x,r) = \begin{cases} V(u,r) + (x-u)k^- & u \leq x \\ \dfrac{xr}{\beta+\alpha} + \dfrac{\alpha\gamma x}{\beta(\beta+\alpha)} + \dfrac{\mu r}{(\beta+\alpha)^2} + \dfrac{\mu\gamma\alpha}{\beta^2(\beta+\alpha)}\left(1 + \dfrac{\beta}{(\beta+\alpha)}\right) + \dfrac{\sigma_x r}{\beta(\beta+\alpha)} + & \\ c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x} & d \leq x \leq u \\ -\dfrac{gx}{\beta} - \dfrac{g\mu}{\beta^2} + c_3 e^{\lambda_1 x} + c_4 e^{\lambda_2 x} & d \leq x \leq 0 \\ V(d,r) + (d-x)k^+ & x \leq d \end{cases} \tag{18}$$

In (18) $\lambda_1, \lambda_2 = \dfrac{-\mu}{\sigma^2 x} + \dfrac{(\mu^2 + 2\beta\sigma^2_x)^{1/2}}{\sigma^2 x}$,

and c_1, c_2, c_3, and c_4 are undetermined coefficients.

It is straightforward to show that V(x,r) in (18) satisfies the partial differential equation (16). By the lemma d, D, U, u, c_1, c_2, c_3, c_4 can be chosen so that the boundary and regularity conditions are satisfied. Thus V(x,r) must be chosen to satisfy the complementary differential inequalities of Theorem 1.

Consider (9'), the inequality which is strict in the continuation region and an equality at stopping times.

If $x \leq d$

$$\begin{aligned} \inf_\xi [B(\xi)+V(x+\xi,r)] &= [B(\xi)+V(x+\xi,r)]_{\xi=D-x} \\ &= K^+ + k^+(D-x) + V(D,r) \quad \text{from (5)} \\ &= K^+ + k^+(D-x) + V(d,r) - K^+ - k^+(D-d) \\ &= k^+(d-x) + V(d,r) \quad \text{from (13)} \\ &= V(x,r) \end{aligned}$$

If $d < x < u$

$$\inf_\xi [B(\xi)+V(x+\xi,r)] = [B(\xi)+V(x+\xi,r)]_{\xi=0}$$

$$= B(0) + V(x,r) > V(x,r)$$

since a zero control has a cost.

If $x \geq u$

$$\inf_{\xi}[B(\xi)+V(x+\xi,r)] = [B(\xi)+V(x+\xi,r)]_{\xi=U-x}$$

$$= K^- + k^-(U-x) + V(U,r)$$

$$= K^- + k^-(x-U) + V(u,r) - K^- - k^-(u-U)$$

$$= k^-(x-u) + V(u,r)$$

$$= V(x,r)$$

Thus (9') is satisfied since

$$\inf_{\xi}[B(\xi)+V(x+\xi,r)] \geq V(x,r)$$

with strict inequality if $x \in (d,u)$.

Consider (10'). If $x \in (d,u)$ it is satisfied as an equality.

If $x \leq d$

$$\beta V(x,r) - \mu V_x - \alpha(\gamma-r)V_r - \frac{\sigma^2_x}{2}V_{xx} - \frac{\sigma^2_r}{2}V_{rr} - \sigma_{xr}V_{xr} - C(x,r)$$

$$= \beta k^+(d-x) + \beta V(d,r) + \mu K^+ - C(x,r)$$

$$= \beta V(d,r) - \mu V_x(d,r) - \frac{\sigma^2_x}{2}V_{xx}(d,r) + gd + \frac{\sigma^2_x}{2}V_{xx}(d,r)$$

$$+ \mu(k^+ + V_x(d,r)) + (g-\beta k^+)(x-d) < 0 \qquad \text{using (17).}$$

Strict inequality also holds if $x \geq u$.

Thus

$$\beta V(x,r) - \mu V_x - \alpha(\gamma-r)V_r - \frac{\sigma^2_x}{2}V_{xx} - \frac{\sigma^2_r}{2}V_{rr} - \sigma_{xr}V_{xr} - C(x,r)$$

$$= 0 \qquad x \in (d,u)$$

$$< 0 \qquad x \leq d \text{ or } x \geq u$$

Thus (9') and (10') are satisfied. Therefore $V(x,r)$ is the value function and (d,u) the continuation region.

Cross-sections of the function V drawn for constant r are u-shaped as, for example, in the study of Milbourne (1983). These are shown in figure 2.1a. The shape is asymmetric since in general $K^+ \neq K^-$, $k^+ \neq k^-$. This chapter adds a dimension by allowing the interest rate to vary. The function V is now an asymmetric trough as in figure 2.1b.

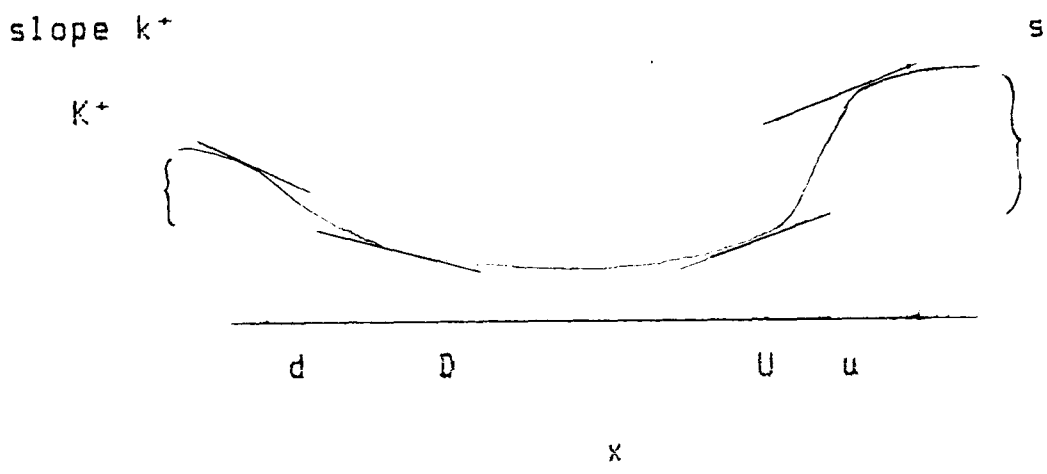


Figure 2.1a

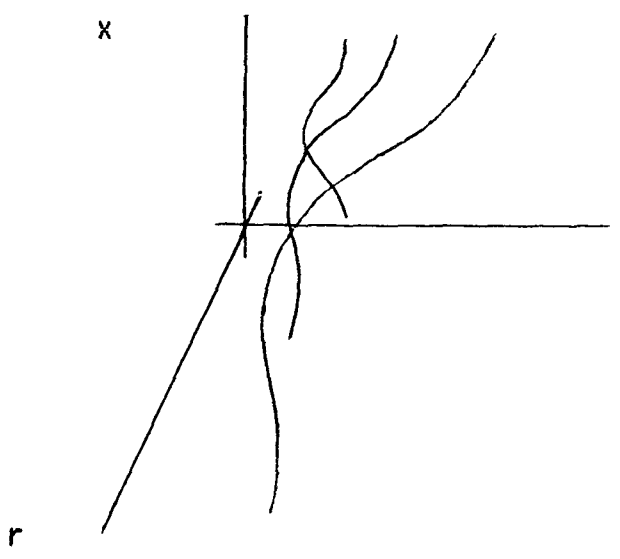


Figure 2.1b

As the interest rate rises the trough rises in the x-V plane and twists open towards the r-V plane. The twisting lowers the targets and thresholds, as would be expected.

2.10 Analytic Solutions.

Providing analytic solutions to the policy problem has been a stumbling block for inventory theory in general and for discrete-time money demand studies in particular. Most closed-form solutions in the latter literature are based on steady-state rather than present-value optimization. The present structure gives at least one closed-form solution more general than those in existing money demand theory but still very few of the possible cases can be solved. Specifically one finds that fixed costs must be zero to obtain expressions for the monitoring parameters.[13] These expressions are of interest because they allow a general examination of the partial effects of the independent variables.

The problem is to find the unknowns d, D, U, u, c_1, c_2 by using the value function and the boundary conditions. Analytic solutions can be obtained in certain cases by restricting the state processes and the costs. Two such cases are described below.

Case 1.

This example is due to Constantinides and Richard with an extension to include a stochastic interest rate. The restrictions are[14]

(i) $\mu=0$ (ii) $K^+=K^-=0$

(iii) $k^+=k^-$ (iv) $r=g$

The agent in this scenario might be a large firm able to ignore fixed costs with easy access to credit and to overnight money substitutes. The model may also describe holdings in a U.S. money market mutual fund account. Such accounts are interest-bearing so that r must be regarded as an interest rate differential. Although there is an entry fee for

this type of account a cheque can be written on this immediately and the average balance can be maintained near zero thereafter.

This case seems a good one in which to test for the effect of future interest rates on money holding. On the one hand the manager of this account is very flexible in cash. Less flexible agents are more likely to be affected by future rates. On the other hand, the manager may be seen as being well-informed enough to take future interest rates into account if they matter.

With the restrictions given it is simple to show u=U, d=D, u=-d, and $\lambda_1=-\lambda_2=\lambda$. The appendix derives the solution for u which is

$$u = \frac{\sigma_x}{(2\beta)^{1/2}} \ln\left\{\frac{-A - (2Ak - k^2)^{1/2}}{k - A}\right\} \qquad (19)$$

where $A = \frac{r}{\beta+\alpha} + \frac{\alpha\gamma}{\beta(\beta+\alpha)}$

Note that α, γ, and r completely summarize conditional expectations in this model. The optimal threshold depends on all future interest rates but in (19) these are already projected on current information.[15] One can learn the signs of several of the partial derivatives of the threshold u, although the expressions involved are unwieldy. For example, $\delta u/\delta r<0$ and $\delta u/\delta k>0$ as one would expect. It is also the case that $\delta u/\delta\sigma_x>0$; greater cash flow variance leads to a greater distance between the thresholds. Finally, $\delta u/\delta\gamma\lessgtr 0$, $\delta^2 u/\delta\gamma\delta\alpha\lessgtr 0$ with equality if $\alpha=0$. To the extent that future interest rates are predictable increases in them will lead to lower thresholds now. Not much can be said about the behaviour of u over time in this case. The following case is more revealing in this respect.

Case 2.

In the second case the restrictions are

(i) $K^{+}=K^{-}=0$ (ii) $d=0$

As in a number of theoretical money demand studies the lower threshold is set at zero, perhaps due to high credit costs. Eppen and Fama (1969) studied a similar case; this chapter adds proofs of existence and optimality, a varying and stochastic interest rate, drift in the net disbursements process, and asymmetry in the proportional adjustment costs.

The cash manager in this second scenario again disregards fixed costs. Unlike his predecessor in Case 1, who could borrow at his lending rate, he finds negative cash balances prohibitively costly. His balance may drift up or down.

Again the targets coincide with the thresholds so that the net disbursements process is allowed to run between two instantaneously reflecting barriers. One barrier is at zero, the other is derived in the appendix as

$$u = \frac{1}{\lambda_2} \ln\left(\frac{k^{-} - A}{-k^{+} - A}\right) \qquad (20)$$

where A and λ_2 are defined as before.[16]

The partial derivatives of u have signs as follows. First, $\delta u/\delta k^{-}>0$, $\delta u/\delta k^{+}>0$; an increase in the cost of adjusting at either threshold leads to a wider continuation region. Second, $\delta u/\delta|\mu|<0$; this seems to be the only sign which is not general. Third $\delta u/\delta\sigma_{\alpha}>0$; the more predictable the net disbursements process the less cash need be held.

The interest rate effects can be quite complicated. First, $\delta u/\delta\gamma\leq 0$ and $\delta^2 u/\delta\gamma\delta\alpha\leq 0$ with equality in both cases when $\alpha=0$. If the

interest rate is not a random walk then the current threshold is responsive to expected future interest rates; the responsiveness is greater the less the interest rate behaves like a random walk. Second, $\delta u/\delta\alpha > (<) 0$ as $r > (<) \gamma$. Roughly speaking, an increase in interest rate predictability increases the threshold if the interest rate is expected to fall and decreases the threshold if the interest rate is expected to rise. Finally, $\delta u/\delta r < 0$ and $\delta^2 u/\delta r^2 > 0$; the higher the current interest rate, the lower the threshold. Although the interest rate variance does not appear to influence the optimal target and threshold in (20) in chapter 4 it is shown to have an indirect role through the disbursements variance. This role does not depend on risk aversion.

The dynamics in this case are fairly easy to establish. The target/threshold u is a non-anticipating control the value of which is found by solving backwards from a terminal condition $u=u(\gamma)$ since only expected interest rates, and not past ones, matter for its setting. Assume that the expectations held by money-holders are rational in that the expected interest rate process coincides with the actual interest rate process almost surely. The interest rate is the only state variable so its conditional expectation is simply its deterministic component:[17]

$$dr^e = \alpha(\gamma - r)dt \tag{21}$$

Then Itô's lemma gives simply

$$du = u_r \alpha(\gamma - r)dt \tag{22}$$

which can be regarded as a right-hand time derivative. Here the subscript denotes the partial derivative of u with respect to r.

Equation (22) is quite general; only u_r depends on the case being considered.

A rational expectations equilibrium exists at $r=\gamma$, $u=u(\gamma)$. From (21) and (22) when $u_r<0$ this equilibrium is a regular saddlepoint (of one dimension in a system of two dimensions) so that there is a unique path for u in the absence of bubbles. In these circumstances the twist in the trough of the function V is determinate and it is meaningful to formalise the intuition that despite the possibility of instantly adjusting the cash balance persistence in x(t) should lead agents to economise by considering expected future interest rates. Moreover, the economic sense of the roots in (21) and (22) is easily checked. The equilibrium is shown in figure 2.2.

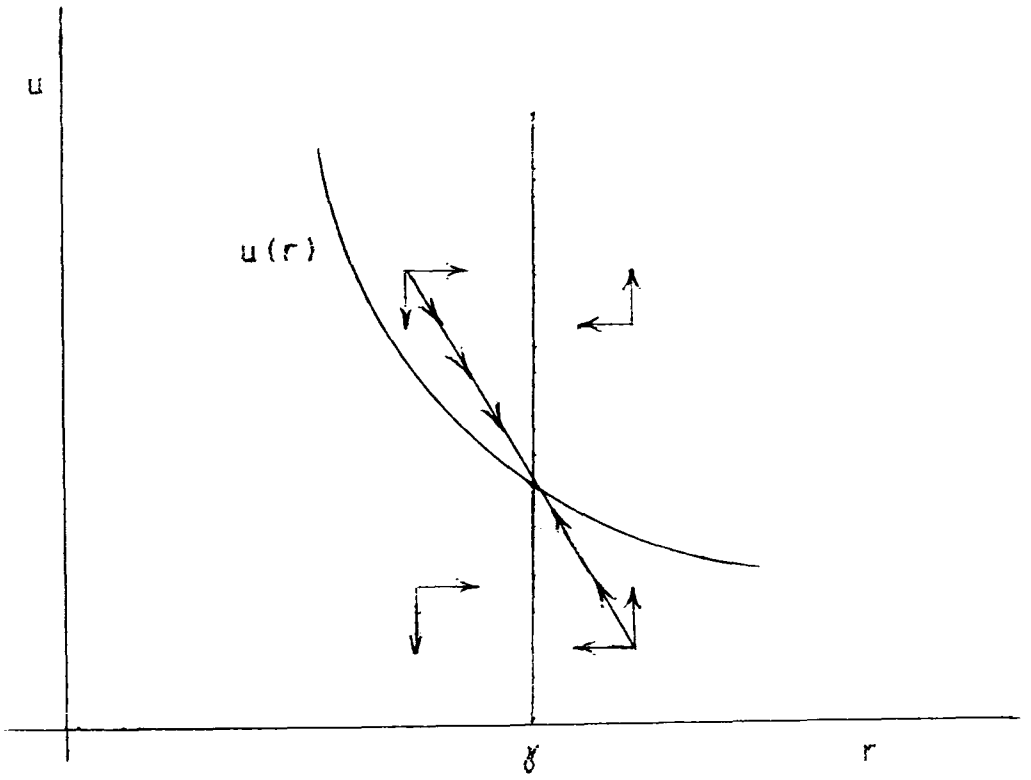


Figure 2.2

A number of comparative dynamics exercises could be performed with this model. For example, if it is learned that at some point in the future all transfer costs will fall permanently then targets and

thresholds will jump upwards and agents may find themselves outside their new thresholds. It is true that as the first passage time becomes small the distant future becomes unimportant. But the situation here is no different from elsewhere in macroeconomics where practical interest centers on the path rather than the long-run destination. This consideration reduces the need to be concerned with the possibility that the optimal policy form could change along the path as the corner inequalities (17) change.

Attempting to provide forward-looking, non ad hoc dynamics for money complicates the interpretation of time series. For example, an unexpected increase in the interest rate decreases u whereas an expected increase causes u to jump upwards at the time when the increase is foreseen. If y is expected to be higher in the future then typically r will be expected to follow a rising path. But an increasing spot rate is consistent with rational expectations equilibrium only if the threshold u is falling. This condition on the path of u affects its current level; since the spot rate process has a continuous sample path u must jump up so that decisions are intertemporally consistent. That these effects are what an economist would expect adds credence to the new predictions below about the dynamics of money. As usual, these sorts of stories are complicated in reality by reaction functions; lower than expected interest rates today may signal higher ones tomorrow.

To illustrate the usefulness of the model consider equation (22) with $|u_r(r)|<1$ (though this partial derivative could be a large negative number). This implies that the spectrum of u will have power at lower frequencies than that of r. The threshold will be more autoregressive and less volatile than the interest rate.

This restriction on u_r is not directly testable through examining these time series predictions (for example in volatile U.S. interest rate data since 1979) since u is unobservable, but the simulation evidence discussed in chapter 5 suggests that it will hold. This is one reason why the money time series may not resemble the interest rate time series, but there is another. Since transactions are not undertaken continuously the money time series will be influenced also by the net disbursements process. Thus money holdings can be more autoregressive in turn than the monitoring parameters; although these decision variables are adjusted instantly the money time series will resemble one generated with actual money holdings as a decision variable and with very slow adjustment.

2.11 Money.

The next step in the analysis is to seek the path of x(t) under controls which alter its initial conditions. The mixture of net disbursements and target dynamics in this path, called M(t), will depend on the frequency of absorption or adjustment. This mixture of backward and forward-looking elements of M(t) varies over time. One reason for the past neglect of the study of the implications of inventory theory for the behaviuor of M(t) has been the normative, microeconomic stance of the theory. Here I assume that the net disbursements process and the stopping times are unobservable to an econometrician.

The problem is to characterize the relationship between current money holdings and current and past targets and thresholds. Again consider two extreme cases. In one (say a hyperinflation), the interest rate is very high and transfer costs are very low so that the continuation region is narrow. Cash-holders go to the bank very

frequently in this case and cash balances will behave much as targets and thresholds do. In another case, the interest rate is low and transfer costs are high so that the continuation region is very wide. Here the time series of money will closely resemble the net disbursements process.

As these two extremes are defined by values of the independent variables in the model so the mixture of the two cases which applies more generally will depend on these variables. Thus inventory theory gives rise to money demand functions with variable speeds of adjustment. Chapter 3 gives a worked-out example while chapter 5 discusses what can be learned about dynamics in general.

2.12 Conclusion.

This chapter has found an optimal policy form for the cash management problem under weak conditions, derived closed-form expressions for the monitoring parameters in special cases, and begun to discuss implications for observed money holding. Previous studies for the most part have addressed the issues involved in a piecemeal fashion. The progress here has been based on the work of Constantinides and Richard; a contribution to stochastic inventory theory is made by extending their analysis to include a stochastic, time-varying interest rate. The use of net-present-value rather than steady-state optimization allows the consistent derivation of a rational expectations model of the demand for money. Although this model involves adjustment costs which are non-convex the analysis leads to a distributed-lag expression for money holdings with a variable adjustment speed.

Alternate objective functions, transfer costs, and stochastic processes could be considered. The survey evidence on transfer costs is

contradictory. For the U.S. Daellanbach (1974) found proportional costs to be negligible while fixed costs were significant. Sethi's (1971) survey of treasurers in large firms found the reverse. Although the first part of this chapter allows asymmetric fixed and proportional costs, closed-form solutions require some restrictions the appropriateness of which would be worth examining for various holders of money.

One objection to some previous money demand theories has held that the disbursements process x(t) is neither perfectly predictable, as in the Baumol-Tobin model, nor perfectly random, as in the Miller-Orr model. Stone (1972), for example, claims that these polar cases are unrealistic. The present model obviously accomodates this criticism. A desirable extension would be to study mixed processes, with intermittent jumps in the balance representing wage receipts, rent cheques, or dividend payouts. A more dramatic but necessary extension is to reconsider the separability involved in taking x(t) as exogenous. Chapter 4 begins this extension by generating this process endogenously in a portfolio-consumption model.

Appendix.

Case 1. In this case the boundary conditions are

$V_x(u,r) = k$ $\quad V_x(-u,r) = -k$

$V_{xx}(u,r) = 0$ $\quad V_{xx}(-u,r) = 0$

$V(u,r) = V(-u,r)$ $\quad V_x(0,r) = 0$ $\quad$ with $\lambda=(2\beta)^{1/2}/\sigma_u$

These give

$$A + \lambda(c_1-c_2) = 0 \qquad \text{A.1}$$

$$A + \lambda(c_1e^{\lambda u} - c_2e^{-\lambda u}) = k \qquad \text{A.2}$$

$$\lambda^2(c_1e^{\lambda u} + c_2e^{-\lambda u}) = 0 \qquad \text{A.3}$$

These can be rewritten as

$$c_1 = c_2 - A/\lambda \qquad \text{A.1'}$$

$$e^{2\lambda u} - \frac{(k-A)}{\lambda c_1}e^{\lambda u} - \frac{c_2}{c_1} = 0 \qquad \text{A.2'}$$

$$e^{2\lambda u} + \frac{c_2}{c_1} = 0 \qquad \text{A.3'}$$

Subtract A.2' from A.3'.

$$2c_2 + \frac{(k-A)}{\lambda}e^{\lambda u} = 0$$

$$c_2 = -\frac{(k-A)}{2\lambda}e^{\lambda u}$$

From A.1' $\quad c_1 = -\frac{(k-A)}{2\lambda}e^{\lambda u} - \frac{A}{\lambda}$

Replacing c_1 and c_2 in A.3', using the quadratic formula, and taking natural logarithms gives the expression in the text.

Case 2. The boundary conditions are

$V_x(0,r) = -k^+$ $\quad V_{xx}(0,r) = 0$

$V_x(u,r) = k^-$ $\quad V_{xx}(u,r) = 0$

These give

$$A + c_1\lambda_1 + c_2\lambda_2 = -k^+ \qquad A.4$$

$$c_1\lambda_1^2 + c_2\lambda_2^2 = 0 \qquad A.5$$

$$A + c_1\lambda_1 e^{\lambda_1 u} + c_2\lambda_2 e^{\lambda_2 u} = k^- \qquad A.6$$

$$c_1\lambda_1^2 e^{\lambda_1 u} + c_2\lambda_2^2 e^{\lambda_2 u} = 0 \qquad A.7$$

From A.4 and A.5

$$c_2 = \frac{-k^+ - A}{\lambda_2 - \lambda_2^2/\lambda_1}$$

$$c_1 = \frac{\lambda_2^2(k^+ + A)}{\lambda_1^2(\lambda_2 - \lambda_2^2/\lambda_1)}$$

From A.6 and A.7

$$e^{\lambda_2 u} = \frac{k^- - A}{c_2(\lambda_2 - \lambda_2/\lambda_1)}$$

$$= \frac{k^- - A}{-k^+ - A}$$

Notes.

1. The setup includes the continuous-time equivalents of the models of Baumol, Tobin, Miller and Orr, Eppen and Fama, and Akerlof and Milbourne.

2. With $r=r(x)$ there is still one state variable.

3. See Friedman (1977), Heller and Khan (1979), and Khan (1980).

4. For example, see Stone (1972).

5. These are the only requirments given the adjustment cost structure characteristic of cash management models. See section 4 below.

6. The process thus includes as special cases processes for which the regressive expectations discussed by Keynes are rational. It is quite general, though, since longer lags and multiple interest rates can be stacked in companion form in (1). See Huang (1985) on environments which give rise to processes such as (1).

7. For example σ^2_r might itself follow some stochastic process.

8. The division of disbursements into expected and unexpected components is an empirical difficulty but not a theoretical one. The drift and variance may be modelled as time-varying.

9. Equation (1) does not rule out $r \leq 0$ but it can easily be replaced by $dr = \alpha(\gamma-r)dt + \sigma_r\sqrt{r}dz_r$ where the origin is inaccessible if $2\alpha\gamma \geq \sigma^2$ and negative interest rates are precluded.

10. See Magill and Constantinides (1976) for the elaboration of this idea.

11. Although in the Wiener-Itô framework the mean time between shocks is zero this is a weak assumption since with a fixed cost the mean time between stopping times must be non-zero for policies with finite costs.

12. Differentiability of the value function is not guaranteed in general without this condition being imposed. Since $V(x,r)$ is the minimum of two functions it is not continuously differentiable in general. The right- and left-hand derivatives may differ where the two arguments of the min function are equal. The regularity condition overcomes this difficulty. See also Benveniste and Scheinkman (1979).

13. For example it can be proved that there is no closed-form solution of the net present value version of the problem studied by Miller and Orr (1966). Target-threshold monitoring is still optimal with proportional costs but without fixed costs. Eppen and Fama (1969) and Magill and Constantinides (1976) provide well-known examples.

14. In Case 1 the penalty cost thus follows an Ornstein-Uhlenbeck process that is the mirror image of that followed by the interest rate. In general g can follow an independent stochastic process. In the cases considered here there are no roles for the parameters of this process; in the first case because they coincide with those of the holding cost process and in the second case because the lower threshold is at zero.

15. In discrete time if r follows an nth order autoregression this dependence and projection leave n-1 lagged interest rates in the control. The expressions in (19) and below in (20) are general because the autoregressive reprsentation of r can be written in companion form.

16. The argument in (20) can be shown to be always positive, using (17).

17. One of the properties of stochastic Ito integrals is that E and ∫ may be reversed. The expectation is just the deterministic part. See Chow (1979).

3. A DYNAMIC BAUMOL-TOBIN MODEL OF MONEY DEMAND.

3.1. Introduction.

This chapter examines a special case of the general model of chapter 2. The intention is to trade generality for precision in order to clarify that chapter's claims about dynamics. For example, the lag weights in the money demand equation are determined explicitly.

The example is based on three approximations.[1] First, in the interest rate process $\alpha = 0$ so that the holding cost follows a Wiener process without drift. In this case myopia is rational since changes in the interest rate are unpredictable. Although this approximation is empirically relevant if the interest rate does follow a Wiener process its principle justification is that it allows the Kolmogorov equation to be solved giving a precise expression for the path of money holdings.

Second, this chapter considers only models with corner solutions (as described in section 2.8). Section 3.2 deals with the case in which the cash balance is never adjusted downwards, section 3.3 with that in which it is never adjusted upwards. As section 2.8 makes clear, these two policies can coexist only if agents face different transfer costs. Thus the treatment below is an approximation in the sense that where drift is strongly (relative to standard deviation) towards one threshold the other threshold is ignored.

This one-sided policy (also referred to by operations researchers as an inventory as opposed to a cash-management policy) can be analysed by the methods of chapter 2. In particular, with downward drift Theorem 1 is valid under the restriction that $\beta \geq 0$, and an optimal policy can be shown to exist and be of a simple form. This form is given by a transfer to reach $y(x)$ where

$$y(x) = \begin{cases} x & d < x \\ D & x \leq d \end{cases} \qquad (1)$$

This adjustment rule is known as an (S,s) policy where S=D and s=d. While the value function can be found there are no closed-form solutions available using the methods of chapter 2. Of course, the monitoring parameters could be studied with numerical methods.

For the stochastic, one-sided policy Scarf (1960) and Boylan (1967) have given optimality proofs in discrete time with a fixed interest rate. Since Arrow, Harris, and Marschak (1951) many papers have examined closed-form solutions under these conditions. As noted in chapter 2 continuous-time closed-form solutions have been given by Savage (1962) for a Poisson disbursements process, and by Antelman and Savage (1965), Bather (1966), and Constantinides for a Wiener process with no discounting in the objective function. Puterman (1975) also assumes a policy form and obtains expressions for monitoring parameters with fixed costs only and a steady-state objective function. Constantinides and Richard (1978) adopt the discounted net-present-value criterion and provide the first existence and form proofs in continuous time.

The third approximation adopted in this chapter is to use a steady-state objective function. This is motivated partly by the absence of closed-form solutions along discounted net-present-value lines and partly by the use of this objective by Milbourne, Buckholtz, and Wasan (henceforth MBW) (1983). In general solutions obtained under the two objective functions will differ even if the interest rate is fixed. Most importantly, the steady-state formulation allows an aggregation theorem of Caplin (1984) to be applied below.

Section 3.2 uses the techniques developed by MBW in their study of the Miller-Orr model to derive the dynamic, stochastic, Baumol-Tobin model. In this model the net disbursements process has negative drift

and a (0,D) policy is observed. There are no proportional transfer costs; the fixed cost is K. Although this combination of process and policy is of interest partly for historical reasons this model is somewhat different from the usual Baumol-Tobin one.[2] In the latter the payments period is given, the rate of disbursement is constant, and the agent chooses the number of cash-management transfers between bonds and money. In the model below the disbursements process is again exogenous but the agent chooses the return point, D. An irregular sawtooth pattern results. In the case where the variance of the net disbursements process is zero, choosing the return point is equivalent to choosing the number of cash-management transfers. Thus the model nests the Baumol-Tobin one or at least that part corresponding to induced increases in money between autonomous ones (such as lump-sum wage receipts).

Section 3.2 describes how the evolution of money holdings is determined by the disbursements process and the choice of return point. The path of money is traced for the case in which independent variables change to new, steady-state values. The (S,s) inventory policy is shown to give rise to a partial-adjustment equation with a variable adjustment speed. Section 3.3 discusses aggregating over agents and over time. The possibility that some agents follow an inverted Baumol-Tobin rule is addressed also. A brief summary concludes the chapter.

3.2. The Dynamic, Stochastic, Baumol-Tobin Model.

The agents's cash holdings follow a simple diffusion process

$$dx(t) = -\mu dt + \sigma dz(t) \qquad X(0) = X_0 \qquad (2)$$

where $z(t) \sim N(0,t)$ is a temporally independent, standard, Wiener process and the subscript on σ is omitted. Here μ, the downward drift parameter, embodies the deterministic component of net disbursements $x(t)$. The agent follows a (0,D) policy such that the cash balance is returned to the target D whenever it reaches the theshold or absorbing barrier at zero.[3]

MBW consider a restricted version of the process in (2) with no drift, $\mu=0$. This chapter deals with the more general process with each agent following a less general policy with one threshold. Section 3.3 notes that different agents may follow different policies depending on whether drift is positive or negative. The economy may then be viewed as having a general two-threshold policy with empirical properties different from those of an economy with identical Miller-Orr-type agents each facing zero drift.

Let $f(x,t)$ be the density function of money holdings x at time t. The steady-state distribution is $\Phi(x) = \lim_{t\to\infty} f(x,t)$. It exists under the assumed cash-management policy for $\mu > 0$. In the steady-state (and not otherwise) $X_o=D$ and the mean of the steady-state distribution of money holdings is given by (see Frenkel and Jovanovic (1980) pp. 35-36)

$$\int_0^\omega x\Phi(x)dx = \frac{X_o}{2} + \frac{\sigma^2}{2\mu} \qquad (3)$$

The optimal target can be found by minimizing steady-state management costs. These are the sum of expected opportunity costs and expected transfer costs. The former are simply the product of the interest rate and expected mean money holdings. The latter are the

product of the fixed transfer cost K and the inverse of the expected duration of the process, which is X_o/μ. Total costs are denoted TC with

$$TC = \frac{K\mu}{X_o} + \frac{r}{2}\left(X_o + \frac{\sigma^2}{\mu}\right) \qquad (4)$$

whence, in the steady state

$$X_o = D = \sqrt{(2K\mu/r)} \qquad (5)$$

as in the Baumol model. Substitution in (3) gives

$$E(X) = \sqrt{(K\mu/2r)} + \sigma^2/2\mu \qquad (6)$$

Next consider the change in x following changes in the exogenous variables. For this the transition probability density function for x(t) is required. Since {x} can be shown to be a normal, Markov process it satisfies the Chapman-Kolmogorov equation that defines the transition probability of x(s) from x(t). The Chapman-Kolmogorov equation describes how the transition probability density is connected over time.

First, write the distribution as

$$F(x,t,y,s) = \text{prob}(x(t) \leq x \mid x(s) \leq y)$$

Then for a diffusion process (i.e. one in continuous time and continuous state space) the density satisfies

$$f(x,t,y,s) = \int_{-\infty}^{\infty} f(z,v,y,s)f(x,t,z,v)dz \qquad t<v<s$$

This equation simply represents total probability in the Markov case.

Given regularity conditions, the transition probability density function can be expanded in a Taylor series. Letting y→x and s→t gives the forward Kolmogorov or Fokker-Planck equation

$$\frac{\delta f}{\delta t} = \frac{\sigma^2}{2}\frac{\delta^2 f}{\delta x^2} + \mu\frac{\delta f}{\delta x} \qquad \mu>0 \qquad (7)$$

This must be solved subject to

$$f(0,t) = 0 \qquad t>0 \qquad (8)$$

and to the initial condition

$$f(x,0) = \delta(x-X_o) \qquad (9)$$

Equation (8) holds for absorption at the zero threshold. Equation (9) in the Dirac delta function gives probability in a 'spike' as the diffusion process begins.

Equation (7) is a second-order, partial differential equation which can be solved by the method of separation of variables. Writing

$$f(x,t) = \psi(x)\Theta(t) \qquad (10)$$

and substituting in (7) gives

$$\psi(x)\theta'(t) = \frac{\sigma^2}{2}\psi''(x)\theta(t) + \mu\psi'(x)\theta(t) \qquad (11)$$

The general solution is given by

$$f(x,t) = \exp\left(-\frac{\mu x}{\sigma^2} - \eta t\right)\sin(n\pi x) \qquad (12)$$

for integer values of n. Clearly equation (12) satisfies the boundary condition (8). To satisfy the forward equation requires

$$\eta = (\mu^2/2\sigma^2) + (n^2\pi^2\sigma^2/2) \qquad (13)$$

The complete solution can be found by Kelvin's method of images or by using Fourier series. In the latter approach a linear combination of terms gives

$$f(x,t) = \exp(-\mu(x-X_o)/\sigma^2)\sum_{n=1}^{\infty} A_n \exp(-\eta t)\sin(n\pi x) \qquad (14)$$

The problem now is to find coefficients A_n such that equation (14) satisfies the initial condition (9) and also the steady-state density for $t \to \infty$. These are treated in turn.

First, from Cox and Miller (1965, chapter 5.7) the initial condition is satisfied by

$$f(x,0) = \exp(-\mu(x-X_o)/\sigma^2)\sum_{n=1}^{\infty} 2\sin(n\pi X_o)\exp(-\eta t)\sin(n\pi x) \qquad (15)$$

Equation (15) simply represents equation (9) as a Fourier series.[4]

Second, again citing the results of Frenkel and Jovanovic, the steady-state density is given by

$$\Phi(x) = \begin{cases} (1/D)(1-\exp(-2x\mu/\sigma^2) & 0 \leq x \leq z \\ (1/D)(\exp(-2(x-D)\mu/\sigma^2) - \exp(-2x\mu/\sigma^2)) & z < x \end{cases} \quad (16)$$

where D is given by equation (5) and the mean of the distribution by equation (3) with X_o=D. For use in equation (14) the Fourier coefficients pertaining to the steady-state density are given by

$$B_n = 2\int_0^{\infty} \Phi(x)\sin(n\pi x)dx \quad (17)$$

Substituting for $\Phi(x)$ from (16) and integrating by parts yields

$$B_n = \frac{4\mu}{D\sigma^2((n\pi)^2+(2\mu/\sigma^2)^2)}\left(\frac{2\mu}{\sigma^2 n\pi} - \frac{2\mu}{\sigma^2 n\pi}\cos(n\pi D) - \sin(n\pi D)\right) \quad (18)$$

The constants A_n such that equation (14) satisfies both (15) for $t=0$ and $f(x,\infty)=\sum_0^{\infty} B_n\sin(n\pi x)$ are thus given by the principle of superposition in

$$f(x,t|X_o) = \exp(-\mu(x-X_o)/\sigma^2)\sum_{n=1}^{\infty}\{2\exp(-\eta t)\sin(n\pi X_o) +$$

$$B_n\exp(\mu(x-X_o)/\sigma^2)(1-\exp(-\eta t))\}\sin(n\pi x) \qquad (19)$$

The expression for $f(x,t|X_o)$ in equation (19) satisfies the forward equation, the initial and absorption conditions, and the steady-state density.

Average money holdings are given by

$$E(X(t)|X(0)=X_o) = \int_0^{X_o+\sigma^2/2\mu} x f(x,t|X_o)\,dx \qquad (20)$$

Two things should be noted about this integral. First, the process has a finite supremum almost surely.[5] The mean of the supremum (Cox and Miller, p. 213) is $X_o + \sigma^2/2\mu$. This is used as the upper limit of integration, the effect of this approximation being outweighed by others below. Second, sufficient conditions to allow term-by-term integration inside the summation sign can be shown to hold.[6]

Solving for the expectation in (20) requires the evaluation of two integrals, as can be seen in equation (19). The first of these is of the form

$$\int_0^{X_o+\sigma^2/2\mu} x\exp(-\mu x/\sigma^2)\sin(n\pi x)\,dx = C_n \qquad (21)$$

Integrating equation (21) by parts in two different ways and collecting terms gives as a solution

$$C_n = \frac{1}{(\mu/\sigma^2)+(n\pi)^2}\{\exp(\frac{\mu X_o}{\sigma^2} + \frac{1}{2})\sin(n\pi x)[1 - \frac{2(\mu/\sigma^2)^2}{(\mu/\sigma^2)+(n\pi)^2}$$

$$- (\frac{\mu X_o}{\sigma^2} + \frac{1}{2})] - \exp(\frac{\mu X_o}{\sigma^2} + \frac{1}{2})\cos(n\pi x)[n\pi x - \frac{2n\pi\mu/\sigma^2}{(\mu/\sigma^2)^2+(n\pi)^2}]$$

$$- \frac{2n\pi\mu/\sigma^2}{(\mu/\sigma^2)^2+(n\pi)^2}\} \qquad (22)$$

The second of the two integrals required for the evaluation of the expectation is of the form

$$\int_0^{X_o+\sigma^2/2\mu} x\sin(n\pi x)dx = D_n$$

$$= -(X_o+\frac{\sigma^2}{2\mu})\cos(n\pi(X_o+\frac{\sigma^2}{2\mu}))\frac{1}{n\pi} + \frac{1}{(n\pi)^2}\sin(n\pi(X_o+\frac{\sigma^2}{2\mu})) \qquad (23)$$

The expectation can now be represented as

$$\int_0^{X_o+\sigma^2/2\mu} xf(x,t|X_o)dx = \sum_{n=1}^{\infty}\{2\exp(-\eta t)\sin(n\pi X_o)\exp(\frac{\mu X_o}{\sigma^2})C_n$$

$$+ B_n(1-\exp(-\eta t)D_n\} \qquad (24)$$

Here η is given by (13), B_n by (18), C_n by (22), and D_n by (23).

To simplify equation (24) the assumption that independent variables change once to new constant values can be made. As in MBW, making this assumption allows an approximation based on large values of t so that only the first exponential term need be considered. Thus equation (24) becomes

$$E(X(t)|X(0)=X_o) \simeq k_1 + k_2\exp(-(\frac{\mu^2}{2\sigma^2} + \frac{\pi^2\sigma^2}{2})t) \qquad (25)$$

Now, as in MBW the boundary conditions are

$$X(0) = X_o$$

$$E(X(\infty)) = X^* \qquad (26)$$

since we assume that some or all of the independent variables change. Using the requirements of (26) to solve for the constants in (25) gives

$$E(X(t)|X(0)=X_o) = X_o + (1 - \exp(-(\frac{\mu^2+\pi^2\sigma^2}{2\sigma^2})t))(X^*-X_o) \qquad (27)$$

Equation (27) serves as the basis for an aggregate Baumol-Tobin model after aggregation over agents and over time.

3.3 Aggregation.

The difficulties involved in aggregating (S,s) inventory policies traditionally have been the largest obstacles to their macroeconomic application. This chapter makes use of an aggregation theorem due to Caplin (1984) which holds that the net disbursements of several agents following (S,s) rules can be interdependent without affecting steady-state inventories. Under regularity conditions the long-run behaviour of inventories is insensitive to the correlations between the net disbursements processes of different agents. The aggregate holding is thus the sum of individual holdings, an outcome which may be surprising to readers of Edgeworth (1888). Caplin's result is a multi-agent analogue to the multi-item independence theorem of Bellman et al (1955).

The simplest assumption with which to begin is that agents may have different initial balances although their net disbursements all have the same instantaneous mean and variance. The restoration of cash

inventories from the absorbing barrier at zero to the re-order point at D on the part of some agents can account for the downward drift experienced by the rest.[7]

Writing $\underline{X}_o$ for the average of initial balances and substituting for X^* from equation (6) gives

$$E(X(t)) = \underline{X}_o + (1-\exp(-(\frac{\mu^2}{2\sigma^2} + \frac{\pi^2\sigma^2}{2})t))(\sqrt{(K\mu/2r)+\sigma^2/2\mu}-\underline{X}_o) \quad (28)$$

Equation (28) can be estimated with discrete data after time aggregation.

Again following MBW take the interval over which money holdings are measured to be [0,1]. The variables are averages over time though point-in-time (skip sample) data could be studied also. Section 3 of chapter 5 below discusses the consequences for the error term of this distinction. Then with unit-averaged data the variable M_t is defined by

$$M_t = \int_0^1 E(X(t))dt$$

$$= M_{t-1} + (M^*_t - M_{t-1})(1 + \frac{1}{\frac{\mu^2}{2\sigma^2} + \frac{\pi^2\sigma^2}{2}}(\exp(-(\frac{\mu^2}{2\sigma^2} + \frac{\pi^2\sigma^2}{2}))-1)) \quad (29)$$

To replace M^*_t in (29) again note that $M^*_t=\sqrt{(r\mu/2r)+\sigma^2/2\mu}$ since the optimal return point is also the initial balance in the steady state. Also approximate the exponential by the first three terms in its Maclaurin expansion to give

$$M_t = M_{t-1} + (\sqrt{(K\mu/2r)} + \frac{\sigma^2}{2\mu} - M_{t-1})(\frac{\mu^2}{4\sigma^2} + \frac{\pi^2\sigma^2}{4}) \quad (30)$$

This function can be shown to be homogeneous of degree one in the price level. Of course, while targets and thresholds adjust instantly to price changes observed money holdings do not.

MBW suggest that the Baumol model generates the same functional form as their model and (30) shows this suggestion to be essentially correct. Thus although it gives the first dynamic, testable, Baumol-Tobin model it is of a familiar form. Note the presence of M_{t-1}; money balances can change gradually although the transfer cost is lump-sum and the target and threshold adjust instantly. As chapter 2 noted, bang-bang control can give rise to a smooth lag distribution at the microeconomic level. A smooth lag distribution at the macroeconomic level need not represent a distribution of delay times (a possibility mentioned by Trivedi (1985)). The money adjustment speed is non-constant and depends on the drift and variance of the net disbursements process.[8]

An objection to the use of the Baumol-Tobin model as an aggregate empirical one is that it only describes spenders or purchasing agents. Rather than accounting for the downward drift of most agents by the upward adjustments of a few it seems worthwhile to model some as experiencing upward drift.

An obvious step is to allow a general stochastic process and a policy characterized by two absorbing barriers. Unfortunately, there is no closed-form expression for the upper threshold available from either the net present value or the steady-state approach, although a limiting distribution exists (see Feller (1971), volume II, Theorem 8.1). The Baumol-Tobin model can be complemented, however, by a model of a cash accumulator or earner, who periodically transfers accumulated balances

into an interest-bearing asset.[9] Akerlof and Milbourne (1980) have examined a deterministic version of this sort of model with the target and threshold fixed. A stochastic, dynamic version can be constructed as the mirror-image of the Baumol-Tobin case.

Thus suppose that the net disbursements process is

$$dx'(t) = \mu' dt + \sigma' dz'(t) \qquad \mu' > 0 \qquad (31)$$

with instantaneous mean and variance μ' and $\sigma^{2\prime}$. Each time the balance reaches a threshold u' the accumulator reduces it to zero. The transition probability density function must now satisfy

$$\frac{\delta f}{\delta t} = \frac{\sigma^{2\prime}}{2}\frac{\delta^2 f}{\delta x'^2} - \mu'\frac{\delta f}{\delta x'} \qquad x' < u' \qquad (32)$$

as well as the boundary conditions

$$f(x',0) = \delta(x') \qquad (33)$$

$$f(u',t) = 0 \qquad t > 0 \qquad (34)$$

The steady-state density is of the same form as previously. The method of separation of variables gives terms like

$$\exp\left(\frac{\mu' x'}{\sigma^{2\prime}} - \eta' t\right)\sin\left(\frac{n\pi x'}{u'}\right) \quad \text{where } \eta' = \frac{\mu'^2}{2\sigma^{2\prime}} + \frac{n^2\pi^2\sigma^{2\prime}}{2u'^2} \qquad (35)$$

The analysis proceeds exactly as above and yields the following money demand function for accumulators:

$$M_t = M_{t-1} + \left(\sqrt{(K'\mu'/2r) + \frac{\sigma^2}{2\mu'}} - M_{t-1}\right)\left(\frac{\mu'^2}{4\sigma^{2\prime}} + \frac{r\sigma^{2\prime}\pi^2}{8K'\mu'}\right) \qquad (36)$$

In (36) the speed of adjustment depends not only on the mean and variance of net disbursements but also on the interest rate and the transfer cost. With a low transfer cost and a high interest rate the threshold is low so that absorption is frequent and adjustment is rapid. Thus the demand equation (36) illustrates the general dynamic principles set out in section 2.11.

Now say that the economy contains a proportion ω of spenders and a proportion $1-\omega$ of accumulators. From (30) and (36) the aggregate money demand function is[10]

$$M_t = M_{t-1} + \omega\left(\sqrt{(K\mu/2r)} + \frac{\sigma^2}{2\mu} - M_{t-1}\right)\left(\frac{\mu^2}{4\sigma^2} + \frac{\pi^2\sigma^2}{4}\right)$$

$$+ (1-\omega)\left(\sqrt{(K'\mu'/2r)} + \frac{\sigma^{2\prime}}{2\mu'} - M_{t-1}\right)\left(\frac{\mu'^2}{4\sigma^{2\prime}} + \frac{r\sigma^{2\prime}\pi^2}{8K'\mu'}\right) \qquad (37)$$

The 'target' money holding and adjustment speed in (37) are different from those in the MBW case, yet (37) can describe an economy with zero net drift. By Itô's Lemma total net disbursements follow the stochastic process

$$dy = (\mu' - \omega\mu' - \omega\mu)dt + \omega\sigma dz + (1-\omega)\sigma' dz' \qquad (38)$$

Equation (38) shows zero net drift is possible if the category with the larger proportion of the group also has lower drift.[11]

3.4 Summary.

This chapter has derived an aggregate money demand function for an economy in which agents follow (S,s) or Baumol-Tobin cash-management

rules. It has shown that zero net drift can result not just from identical agents following Miller-Orr rules but also if there are two groups, one following the stochastic Baumol-Tobin rule and the other this rule on its head. Since the rules considered are based on steady-state optimization note that the argument that myopia is not costly because the absorption frequency is high relative to the frequency of interest rate changes backfires. If the absorption frequency is high relative to a third frequency - that of time series observations - then the case for a lagged dependent variable based on the target-threshold approach is weakened.

A claim of Milbourne, Buckholtz, and Wasan that dynamic demand equations with variable adjustments speeds can be derived from inventory theory is substantiated for the (S,s) case. These equations resemble familiar ad hoc empirical specifications (and may thus fit data) but require a different theoretical interpretation.

Chapter 5 discusses transitions more general than those between steady-states, but many of the speed-of-adjustment effects illustrated in this chapter turn out to be general. First, chapter 4 discusses a number of reinterpretations of time series evidence in the light of the approach of this and the previous chapter.

NOTES

1. To simplify the problem the lower threshold, d, is taken to be zero. The usual assumptions of two assets and instant transfers are made.

2. The historical reasons contemplated include the popularity of the Baumol-Tobin model in textbooks. Barro (1974) makes a case for the general use of this model.

3. The threshold is an absorbing barrier rather than a reflecting barrier since the process is controlled there.

4. This representation is valid since the Dirac delta function is absolutely integrable.

5. See Cox and Miller p. 212.

6. For the sum to be integrable and for equality between its integral and the sum of integrals dominated convergence of the sum is sufficient. This can be shown to hold in equation (24) below, bearing in mind that η is a function of n. I am grateful to Ulrich Zachau for clarifying this point.

7. The first group could also include the government and foreigners whose holdings are not counted in the money stock. Net autonomous payments can also be non-zero since one agent's induced payment is another's autonomous one; my withdrawal from the bank forms part of its exogenous net disbursements process.

8. Chant noted that changes in drift would affect adjustment more directly than would changes in the interest rate. This distinguishes the model from the conventional, partial-adjustment specification in which money holdings respond in the same way no matter which independent variable changes.

9. Saving is consistent with drift in either direction.

10. There are no aggregation results for the case in which different agents follow different policies. Aggregate holdings may be less or greater than those given by (37). Cannan (1921 p.456) noted that the demand for currency rose in Britain in the 1914-1918 War when families were separated:

> The calling up of men for military service, and subsequently the large removal of women from their homes for munition-making and other purposes during the recent war, greatly increased for a time the demand for currency, because the members of families, when separated, found it convenient to keep much more currency by them in the aggregate than when they were living at home and together.

11. One must bear in mind that the policy only makes sense for high ratios of drift to variance.

4. INTERPRETING INVENTORY-THEORETIC MODELS OF THE DEMAND FOR MONEY.

4.1 Introduction.

This chapter returns to the general problem of chapter 2. It discusses the interpretation of the model of that chapter as a prologue to discussing empirical work. The aims are fairly modest. The partial equilibrium approach is maintained but the empirical content of the theory is enhanced through several extensions.

The first of these allows for the state variable processes to change over time. Section 4.2 examines one of these state variables, net disbursements, and allows the stochastic process governing its evolution to be non-homogeneous; that is, μ and σ_x^2 may be time-dependent and stochastic. Once these drift and variance measures are regarded as variables the way is clear to dissect them. Section 4.3 derives μ and σ_x^2 as functions of the variables in a model of portfolio, consumption, and money demand behaviour. This extension adds to the observable implications of the theory for money demand. Some of these are drawn explicitly in section 4.4. The final two sections present brief notes on the consumption function with target-threshold monitoring and on some further extensions to state variable behaviour respectively.

4.2 A Time-Dependent Disbursements Process.

In this section the model of chapter 2 is generalized by allowing the instantaneous mean and variance of the net disbursements process to be stochastically time-varying. There are three reasons for this added complexity. First, a number of empirical studies use variance terms as variables.[1] Thus σ^2_x must be treated in this way from the start in order to try to explain the results of these studies.

Second, section 4.3 generates μ and σ^2_x endogenously in a model of consumption and portfolio behaviour. Under a target-threshold policy in

this model the cash manager takes μ and σ^2_x as given but not fixed. Moreover the consumption function turns out to depend on the target threshold dynamics which in turn requires the extension of this section.

Third, forecasting disbursements variance is treated as an important part of finding a sensible monitoring ploicy in the heuristic approach to cash management. Stone (1972) discusses the problem of forming expectations about the variance of cash needs. Schmalensee and Trippi (1978) invert the Black-Scholes option pricing model to obtain estimates of expected stock price volatility. They find some evidence that changes in volatility are predictable (in other words that K below is not zero). Again foreshadowing later sections this evidence is relevant because volatility in asset prices can contribute to volatility in cash disbursements.

The state variables now obey

$$dx(t) = \mu(t)dt + \sigma_x dz_x \qquad (1)$$

$$dr(t) = \alpha(\gamma - r)dt + \sigma_r dz_r \qquad (2)$$

$$d\mu(t) = \psi(\mu^* - \mu)dt + \sigma_\mu dz_\mu \qquad (3)$$

$$d\sigma_x(t) = K(\sigma_x^* - \sigma_x)dt + \sigma_\sigma dz_\sigma \qquad (4)$$

Ornstein-Uhlenbeck processes have ben chosen for $\mu(t)$ and $\sigma_x(t)$ since they include the following two cases:

(i) stationary case. As σ_μ and σ_σ approach zero and ψ and K become large numbers the model of chapter 2 results.

(ii) nonstationary case. For ψ and K both zero the instantaneous drift and variance in the disbursements process follow random walks.

Between these polar cases interest centers on the possibility of forecasting μ and σ_x.

Chapter 2 found no role for covariances between the state variables in the optimal monitoring rules, though they do affect the expected

minimized cost. Ignoring these terms, then, the solution to the Hamilton-Jacobi-Bellman equation in the interval (0,u) is given by[2]

$$V(x,r,\mu,\sigma_x) = \frac{rx}{\beta+\alpha} + \frac{r\mu}{(\beta+\alpha)(\beta+\alpha+\psi)} + \frac{\alpha\gamma x}{\beta(\beta+\alpha)}$$

$$+ \frac{\psi\mu^* r}{(\beta+\alpha)^2(\beta+\alpha+\psi)} + \frac{\mu\alpha\gamma}{(\beta+\alpha)(\beta+\psi)}\left(\frac{1}{\beta} + \frac{1}{\beta+\alpha+\psi}\right)$$

$$+ \frac{\alpha\gamma\psi\mu^*(3\beta^2 + 3\alpha\beta + \alpha^2 + 2\beta\psi + \alpha\psi)}{\beta(\beta+\alpha)^2(\beta+\psi)(\beta+\alpha+\psi)}$$

$$+ c_1\exp(\lambda_1 x) + c_2\exp(\lambda_2 x) \qquad (5)$$

Here λ_1 and λ_2 are defined as before. A perhaps surprising result here is the absence of a role for K in V. The target-threshold monitoring of chapter 2 is robust in the sense that, even when σ^2_x can be forecasted (that is, when K>0), its variation can be ignored in setting the monitoring parameters.

The value function still satisfies the regularity condition (2.11) and the weaker regualrity requirements of Itô's lemma. By the methodsd used earlier it is straightforward to show that in case 2 the expression for the optimal target/threshold u is unchanged from chapter 2. Thus the dynamics are given by

$$du = u_r dr^* + u_\sigma d\sigma^* + u_\mu d\mu^*$$

$$= (u_r\alpha(\gamma-r) + u_\sigma\psi(\sigma^*-\sigma) + u_\mu K(\mu^*-\mu))dt \qquad (6)$$

where as previously the stochastic terms can be ignored in the expectation.

Note the $u_r<0$, $u_\mu<0$, and $u_\sigma>0$ in the case 2 model. Thus there is a rational expectations equilibrium at $u(\gamma,\mu^*,\sigma^*)$ which is a regular saddlepoint since one stable and one unstable root have been added to the system.

The results also coincide with those of the heuristic approach. If the cash manager learns that disbursements variance will be higher in

the future he makes the target/threshold jump upwards now. This effect corresponds to what Keynes called the finance motive for holding money.[3]

4.3 Structure.

The net disbursements process can be examined in detail now that μ and σ^2_x are regarded as variables. The examination begins with a set of assumptions:

Assumption 1. Assets.

The capital market involves trading in m+1 assets, indexed by i where i = 0,1,2,...,m. The 0th is a nominally riskless security, the next m-1 are risky securities, and the mth is money. The securities can be bought and sold at current prices in unlimited amounts and are perfectly divisible. The market is open continuously.

Assumption 2. Perfect Information.

All information on current prices and distributions is available continuously and costlessly.

Assumption 3. Dynamics.

The price of the ith security is denoted by p_i, i = 0,...,m-1. It is held in amount $x_i(t)$ so that the value of holdings in security i at time t is $s_i(t)=x_i(t)p_i(t)$. The investment opportunity set is described by a vector diffusion process of the Itô type:

$$dp_i = f_i(p_i,t)dt + \sigma_i(p_i,t)dz_i \quad i = 0,...,m-1 \qquad (7)$$

Here $dz_i(t)$ is the increment of a standard Brownian motion. As noted earlier, a process of this type is continuous and non-differentiable almost surely. The Markov property is not restrictive since the set of state variables can be enlarged to fulfill it.

There is one consumption good with an uncertain future price in terms of money. Its price obeys

$$dp = \pi(p,t)dt + \sigma_p(p,t)dz_p \qquad (8)$$

The return on a security is represented as an increase in the price of a single unit. Besides this income from capital gains the investor earns other nominal income y(t) which obeys

$$dy = h(y,t)dt + \sigma_y(y,t)dz_y \qquad (9)$$

In (7), (8), and (9) all instantaneous variances are positive and all instantaneous correlations less than one in absolute value.

In equation (7) $f_i(p_i,t)$ is the instantaneous mean rate of change of the price of the ith asset. Suppose that this can be written $p_i r_i(t)$ so that r_i is the instantaneous mean rate of return. As is well-known, if r_i is a constant then the asset price evolves according to a Merton-Samuelson geometric Brownian motion and is lognormally distributed. However, as Merton (1973) has remarked, the rate of interest is a source of non-stationarity. Thus the Ornstein-Uhlenbeck description of the behaviour of r_i, say, can be thought of as characterizing the time-varying and stochastic drift of the asset price process (7). If interest centers on the description of money holding over time then interest rates should be treated in this way. One implication of this is that studies in which asset prices follow geometric Brownian motions (for example Magill and Constantinides (1976) and Poncet (1983)) must be somewhat suspect in their conclusions about the demand for money. Another is that Merton-Samuelson-Ross-type sufficient conditions (in the form of restrictions on asset returns) for separation are also incompatible with this interest in money demand.

Assumption 4 Capital Market Imperfections.

Trnsactions costs are incurred each time a security is bought or

sold. As in Magill and Constantinides these costs are proportional to the values transacted. Let v_i denote the instantaneous value of a purchase ($v_i>0$) or sale ($v_i<0$) of security i. The function giving the cost of transacting in the m securities is

$$k(v_0,v_1,v_2,\ldots,v_{m-1}) = \sum_{i=0}^{m-1} k_{v_i} v_i \qquad (10)$$

where

$$k_{v_i} = \begin{cases} k_i^- & v_i>0 \\ -k_i^+ & v_i<0 \end{cases}$$

and where $0 \leq k_{v_i} \leq 1 \quad i = 0,1,2,\ldots,m-1.$

The proportional cost may be thought of as a bid-ask spread but may also include brokerage fees and the costs of information (about the probability of default for example) more generally. Zabel (1973) notes the undesirability of excluding fixed costs. Such costs could explain the relativley few assets held by most investors and also preferences for investing through specialized financial institutions. Although the scenario with proportional costs only is a special one fortunately it is precisely that in which chapter 2 found closed-form solutions for the cash-management parameters.

There is no cost in adjusting money per se. Indeed it is held in this model for this reason since the nominally riskless asset has a higher rate of return. The formulation and notation in (10) are consistent with those in chapter 2. There a transfer of size ξ into cash (a purchase of money) incurred a proportional transfer cost k^+; this corresponds to an asset sale ($v_i<0$).

A final capital market imperfection makes the closed-form solution in case 2 of chapter 2 applicable here. It is assumed that a very large penalty cost g is paid on negative cash balances so that the lower threshold is at zero.

On the basis of these assumptions the net disbursements process can be derived. The instantaneous wealth constraint is

$$dW(t) = \sum_{i=0}^{m} x_i(t)dp_i(t) + \sum_{i=0}^{m} p_i(t)dx_i(t) + \sum_{i=0}^{m}\sum_{j=0}^{m} dp_i dx_j \quad (11)$$

The derivation of (11) using Itô's lemma is valid on subintervals where x_i, dx_i, p_i, and dp_i are continuous for all i; that is, everywhere, given that asset prices as in (7) and the quantities transacted (see (13) below) follow Itô processes.

As in Merton (1971) changes in wealth can be attributed to capital gains or to saving from disposable income. Thus

$$dW = \sum_{i=0}^{m} x_i(t)dp_i(t) + y(t) - p(t)c(t) \quad (12)$$

where y(t) and c(t) denote nominal income and real consumption respectively.

From (11) and (12)[4]

$$\sum_{i=0}^{m} v_i = \sum_{i=0}^{m} p_i dx_i$$

$$= dW - \sum_{i=0}^{m} x_i dp_i - \sum_{i=0}^{m}\sum_{j=0}^{m} dp_i dx_j$$

$$= [y(t) - p(t)c(t) - \sum_{i=0}^{m}\sum_{j=0}^{m} \sigma_{ij}]dt - \sum_{i=0}^{m} x_i \sigma_i dz_i \quad (13)$$

The last term is the stochastic element in capital gains income, retained for symmetry with the other stochastic term in the disbursements process below. There are no strong conventions here, but this decomposition of wealth changes seems most useful empirically. The innovation of Markose (1984) is to permit the quantities transacted, the dx_i, to be stochastic. Thus discrepancies between plans and realizations can arise though they are taken to be of a particular form,

namely increments of a Brownian motion. Thus the last term in (11) and (13) does not vanish.

One can write

$$dx_i = f_{xi}\,dt + \sigma_{xi}\,dz_{xi} \qquad (14)$$

This allows the $v_i = p_i dx_i$ to be written

$$v_i = p_i f_{xi} + p_i \sigma_{xi}\,dz_{xi} \qquad (15)$$

Substituting (15) in (13) and deducting transactions costs from the cash balance gives

$$dx_m = v_m = \Big[y - pc - \sum_{i=0}^{m-1}(1+k_i)p_i f_{xi} - \sum_{i=0}^{m-1}\sum_{j=0}^{m-1}\sigma_{ij}\Big]dt - \sum_{i=0}^{m-1}(1+k_i)p_i\sigma_{xi}\,dzxi - \sum_{i=0}^{m-1} x_i\sigma_i\,dz_i \qquad (16)$$

This is the net disbursements process consistent with the instantaneous budget or wealth constraint. The form of (16) is similar to that in Magill and Constantinides with the added stochastic terms attributable to Markose's distinction between planned and realized transactions and to the stochastic element in capital gains respectively. Unlike Markose, I do not assume that all capital gains on the nominally riskless asset accrue as cash.[5]

Cash balances tend to increase when income receipts exceed nominal consumption outlays. The first two terms in the drift expression thus represent money's role as a buffer when income and consumption are not synchronized. In the third term purchases (sales) of assets decrease (increase) cash holdings. This term also allows for the deduction of transactions costs which are paid in cash. As noted in chapter 2 Porteous and Neave have shown that this deduction makes no difference to the optimal policy form and little difference to the optimal monitoring

parameters. The fourth term allows for covariance between asset purchases and their prices. For example if σ_{i,x_i} is positive then a larger than planned acquisition of asset i will be associated with a higher than expected price for that asset and hence a larger cash outlay.

There are several reasons for the omission of penalty costs, besides that due to Porteous and Neave. First, the lower threshold, d, is taken to be zero so that the penalty cost is never paid. A sufficient condition for this is that the penalty cost is prohibitive; with a small penalty cost d will be negative which seems an unnecessary theoretical complication when the case 2 closed-form solution can be used otherwise.

Second, in the drift term the penalty cost would be multiplied by the probability of falling below zero at time t. This probability may not be defined explicitly with two thresholds. Moreover it is a function of the drift and variance in the net disbursements process, as well as contributing to the former. These complexities seem worth avoiding too.

In the stochastic term in equation (16) shocks to cash holdings arise from shocks in asset transactions or asset prices. Unsurprisingly, these have larger effects on the cash balance the higher the unit price, quantity held, and transaction cost of each asset.

The net disbursements process under control is precisely the Clower (1967) cash-in-advance constraint.[6] The constraint is in its 'illiquid assets' version since it applies to asset trades. A simple version of this constraint in continuous time is

$$dx(t) = -p(t)c(t)dt$$

$$x(t) \geq 0$$

The model here cannot be said to provide foundations for this constraint in that necessary conditions have not been found for the lower threshold to be zero. Nevertheless it is worth noting some implications of general equilibrium models incorporating this constraint, though different models invoke different trading rules and timing conventions.

One intutitive result is that of Grossman and Weiss (1983), Rotemberg (1984), and Akerlof (1973) on open-market operations. Imagine an equally distributed increase in the money supply in an economy with target-threshold monitoring. The increase in money holdings will cause some agents to adjust downwards (bidding the interest rate down, say) since their upper thresholds will have been met immediately. As others reach their upper thresholds more rapidly than in the absence of the change, the interest rate will continue to adjust. Interest rates will adjust gradually to money supply changes because people do not visit the bank continuously or simultaneously. Thus this theory of money holding leads to a theory of the transmission mechanism (as long as agents are heterogeneous); the periodic bank trips act like overlapping contracts. Furthermore, with one adjustment cost it can explain the gradual adjustment of the interest rate (and desired money holdings) to the supply of money and the gradual adjustment to the interest rate of the demand for money, unlike the partial-adjustment mechanism for example.

The papers mentioned above all undertake steady-state analysis. Grossman and Weiss also keep fixed the period between trips to the bank. This period is endogenous here, and the goal is to account for empirical evidence.

4.4 Money Demand.

This section establishes connections between the structure above and the analysis of chapter 2. The parameters μ and σ^2_x of the net disbursements process are reinterpreted as the drift and variance in equation (16). The theoretical aim of this section is modest in the sense that while μ and σ^2_x are no longer taken as fixed they are still taken as given. Thus suboptimality or separation is imposed, the intent being to explore the basis of the partial approach so widely used in empirical work. A number of studies (Feige and Parkin (1971), Grossman and Policano (1975)) examine steady-state multiple inventory problems, attempting to push back the boundary of separation. Related studies (including those of currency substitution) add state variables and controls for greater realism; firms get cash from bank credit, issuing paper, float, and the extension of accounts payable and hold it in Treasury Bills, certificates of deposit, and commercial paper. Clower and Howitt (1978) note the difficulties in drawing comparative statics conclusions with this added generality. I avoid the inconclusiveness of this approach by imposing separation. Crane (1971) found that banks, for example, tend to treat liquidity as a residual. However, the separability assumption is not supported by theory or empirical evidence in the same way as the structure of chapter 2 for example. It is justified as taking the partial approach to its logical conclusion; in doing this I argue that the theory accounts for the results of a wide variety of empirical studies.

The extension begins with a reconsideration of the problem of chapter 2 in the light of the general structure of section 4.3.

(i) Interest Rate Process.

There are now m alternatives to holding money and their prices follow the diffusion processes in (7). The rates of return r_i obey

$$r_i dt = \frac{f_i(p_i,t)dt + \sigma_i(p_i,t)dz_i}{p_i} \qquad (17)$$

where the time-dependence of f_i and σ_i accords with the interest rate dynamics of chapter 2. These are given by Ornstein-Uhlenbeck processes

$$dr_i = \alpha_i(\gamma_i - r_i)dt + \sigma_{r_i}dz_{r_i} \qquad \alpha_i \geq 0 \quad \sigma_{r_0}=0 \qquad (18)$$

The opportunity cost of a unit of cash is the marginal return on the security portfolio:

$$r = \sum_{i=0}^{m-1} \omega_i r_i \qquad (19)$$

where ω_i are the shares in the portfolio (not including cash) and sum to one.

For (19) to be a valid representation of the opportunity cost requires that a marginal unit of cash be spread across the portfolio in the current proportions. Portfolio separation would provide a warrant for this assumption but will not normally hold. Note instead the result of Magill and Constantinides that with transactions costs the portfolio share will only be altered in response to large changes in the exogenous variables such as relative rates of return. The intuition here is that with these costs the portfolio is managed like a multi-item inventory. The investor operates target-threshold policies for the proportion of wealth in each security. These proportions are required to be strictly within their respective continuation regions.[7]

With the ω_i treated as fixed Itô's lemma applied to (18) and (19) yields

$$dr = \Big[\sum_{i=0}^{m-1} \omega_i \alpha_i (\gamma_i - r_i)\Big]dt + \sum_{i=0}^{m-1} \omega_i \sigma_{r_i} dz_{r_i} \qquad (20)$$

Equation (20) thus corresponds to equation (2.1).

(ii) Cash Flow Process.

Equating terms in equations (16) and (2.3), net disbursements follow an Itô process with instantaneous mean and variance as follows:

$$\mu = y(t) - p(t)c(t) - \sum_{i=0}^{m-1} (1+k_{v_i})(p_i f_{x_i}) - \sum_{i=0}^{m-1}\sum_{j=0}^{m-1} \sigma_{i\times j} \qquad (21)$$

$$\sigma^2_x = \Big\{\sum_{i=0}^{m-1} [(1+k_{v_i})p_i \sigma_{x_i} dz_{x_i} + x_i \sigma_i dz_i]\Big)^2 \qquad (22)$$

There is no ready simplification of the expression in (22) but it is increasing in σ^2_i, $\sigma^2_{x_i}$, $\sigma_{i,j}$, $\sigma_{i\times j}$, and $\sigma_{x_i\times j}$.

(iii) Costs.

Costs of adjusting the cash balance must be present for the optimal policy to be of target-threshold form rather than one in which the balance is continuously at its target level. In the structure of section 4.3 there are no costs to altering x_m, strictly speaking, but as noted the costs of transacting in all other assets serve the same purpose. Thus

$$k_m^+ = \sum_{i=0}^{m-1} \omega_i k_i^+$$

$$k_m^- = \sum_{i=0}^{m-1} \omega_i k_i^- \qquad (23)$$

By the argument of (i) above, with only proportional costs a unit of cash transferred from the cash balance will be distributed over the portfolio in accordance with existing proportions.

A zero cost to adjusting cash may be an oversimplification, but all that is required for the theory to have force is that cash has transfer costs low enough relative to those of other assets to overcome its zero rate of return. This cost advantage for a particular asset is used by Magill (1976) to explain the existence of mutual funds. Separation theorems explain why investors might be indifferent between holding the market portfolio and holding the stock of appropriate mutual funds; some cost advantage may be necessary (and is sufficient) to account for a positive preference for the latter.[8] Money acts as a similar institution.

Using the correspondence with chapter 2 the signs of partial derivatives of the optimal threshold can be found with respect to the state variables. Table 4.1 gives the signs, with the portfolio shares unchanging.[9]

Table 4.1 Signs of Partial Derivatives of Target/Threshold

Variable	Sign
k_i^+, k_i^-	+
r_i	-
$\|y-pc\|$	-
σ_i	+
σ_{mi}	+
σ_{ij}	+
$\sigma_{i\times j}$	+
$\sigma_{mi\times j}$	+

A change in transactions cost affects u directly and also through its effect on μ. These signs have been established only for the case 2 model under suboptimality.

From the net disbursements variance term cash holdings are larger the larger the variance in the value of asset transactions and the larger the positive correlation between the unexpected elements in these transactions. Conversely, if $\rho_{x_i x_j}<0$ a larger than expected transaction in asset i is associated with a smaller than expected transaction in asset j so that cash needs are lower than otherwise.

It should be noted that, as in the illustrative model of chapter 3, any variable with a positive effect on the target has a negative effect on the speed of adjustment in money holdings. For example, increases in transactions variances increase u and by expanding the continuation region lengthen the memory in the time series for money. So the theory's predictions about dynamics are linked with its predictions about comparative statics.

Some of these predictions can now be summarized. First, the theory predicts that money holdings will adjust slowly to changes in exogenous variables. If the data generation process is that used here then money's being regarded as a decision variable can only be reconciled with linear regressions of money holdings on current and past interest rates by an appeal to strictly convex costs of adjustment. Chapter 5 discusses this issue more fully.

Second, the model can obviously accomodate growth rates of narrow money s_m and broad money s_m+s_o that diverge, as in the U.K. in the early 1980's. The model also holds that the entire term structure of interest rates will be relevant to narrow money holdings. It can thus underpin the empirical results of Heller and Khan.

Third, the model makes some new, strong predictions relating money to the second moments of exogenous variables. Recall that an increase in σ^2_x increases targets and lengthens the lag in regressions with money

as the dependent variable. With the decomposition of net disbursements variance increases in $\sigma^2_{x_i}$, the transactions variance, σ^2_i the asset price variance, or $\sigma_{x_i x_j}$, σ_{ij} the covariances between unexpected transaction volumes or prices, are predicted to have these effects.

Dealing with the variance of asset prices first, in discrete time the variance of the change in the price of asset i corresponds to the variance in the rate of return. While time aggregation will be discussed further in chapter 5 this correspondence follows from the properties of the multivariate normal distribution. Thus $p_i(t)-p_i(t-h) \sim N(f_i(p_i,t)\cdot t - f_i(p_i,t-h)\cdot(t-h), \sigma_i^2(p_i,t)\cdot t-\sigma_i^2(p_i,t-h)\cdot(t-h))$ or, in the homogeneous case, $\sim N(f_i\cdot h, \sigma_i^2\cdot h)$. Setting h=1 shows that σ_i^2 in discrete data is the variance of the change in the ith security's price or its rate of return. One finds that recent empirical evidence for the U.S. supports both the level and the adjustment speed effects of this variable. Starr's (1983) maturity shift hypothesis holds that increased long rate volatility under the Fed's new operating procedures led to increased demand for the money market mutual fund shares counted in M2. This could account for its having grown faster than M1B in the early 1980's.

More strikingly, recent work by Baba, Hendry, and Starr (1985) finds both effects. These authors construct a measure of volatility in holding period yields and find that it has a positive effect on money holding and a negative effect on adjustment speed in a linear, error-correction-mechanism model. This evidence is particularly important since the testing methodology precludes these effects being artifacts of the 1979-1982 period; stability tests reject this possibility.

Theoretical papers by Buiter and Armstrong (1978) and Turk (1984), based on Tobin (1958), also predict that increases in interest rate

variance will increase the demand for money. Both take variances of money in the static, deterministic, Baumol model and assume that utility is decreasing in this variance. This framework seems inappropriate, not least because risk averse individuals may value variance in money as a buffer stock reducing variance in consumption.

The prediction about trading value effects also finds some support. In examining the origins of the Great Depression in the U.S. Field (1984a,1984b) suggests that an increase in the transactions demand for money short-circuited the Fed's feedback rule for interest rates and went unaccomodated. This view was held by some observers in the late 1920's. Field supports his conjecture by estimating demand functions for the period 1919-1929 for six measures of money ranging from demand deposits in New York to M3. He adds an NYSE transactions value measure to partial adjustment equations and finds a significant positive effect. Though this specification can be criticized, stability tests show the effect was not confined to 1928-1929. Nor does the value measure simply proxy for wealth since a series on deposit turnover has a similar effect.

The theory may also reconcile the monetary and portfolio approaches to the exchange rate. In the monetary approach the exchange rate between two currencies under uncovered interest parity depends on a weighted average of future differences between the incomes and money supplies of the two economies. With the money demand function developed here the right-hand variables should include current and future asset price risks, variables normally associated with the portfolio approach.

In general the formulation here erodes the connections between specific variables and specific motives for holding money. The demand for narrow money is not a speculative or asset demand in the usual sense

yet it is affected by the variances of interest rates. These variances correspond to risks given that the state variables follow diffusion processes, which are characterized completely by their first two moments. They play a role even in the absence of risk aversion.

On the other hand it is perhaps not surprising that the variance effects here are similar to those in models that treat money as a capital asset. The target-threshold structure contains a safety-first requirement that in other applications has generated choice sets equivalent to those generated by the mean-variance criterion. Kira and Ziemba (1977) survey this equivalence.

4.5 A Note on the Consumption Function with Target-Threshold Monitoring.

This section reconsiders the suboptimality assumed so far. The question addressed is whether a wider optimization problem is tractable. Unlike most general equilibrium theory, financial theory makes strong assumptions (one might except arbitrage-based theories) about general optimization problems and generates strong predictions as a result. Unfortunately this approach is not well-developed with the demand for money included in these optimizations.

Consider the problem of finding optimal consumption, portfolio, and money demand plans. There are three approaches to this problem. First, ideally one would solve simultaneously for these plans. In this approach the objective function would be a utility functional while the sum of expected future cash-management costs would be absorbed in the budget constraint rather than constituting the objective function. Also the form of the optimal cash-management policy would emerge in the solution. This approach is analytically challenging.

In a second approach, necessary and sufficient conditions are sought for the separation of the problem into suboptimizations. These conditions could then be understood as having been implicit in still narrower partial equilibrium studies. Cass and Stiglitz (1970) and Ross (1978) summarize restrictions on preferences and beliefs respectively that give separation in consumption-portfolio problems. With these restrictions chapter 2's proof of the optimality of target-threshold monitoring would be valid in the general problem. But this way is also blocked since there are no such restrictions in the case of money demand. Magill and Constantinides have shown that the consumption plan depends on the portfolio and money plans. Since their framework is less general than that of this chapter their result holds here. Conversely, inspection of equation (16) shows that the money plan depends on the consumption and portfolio plans.

A third approach begins without separation but with target - threshold monitoring imposed. First, the cash-management problem is solved, then money is treated as a state variable in the consumption problem. Breaking the wider problem down avoids the analytical difficulties of the first approach while allowing for some interdependence avoids the theoretical objections to the second one. Now the optimality of target-threshold monitoring cannot be guaranteed, but the implications of this policy for consumption can be examined.

Begin then with

Assumption 5.

The investor's preference ordering can be represented by a functional

$$J(S,t) = \max_{f_{x_1},c} E \int_0^{\infty} e^{-\rho t} U(c(t))dt \qquad (24)$$

where the expectation is conditional on the initial values of a set of state variables $S = \{p, y, s_i, M\}$ $i=0,1,2,\ldots,m-1$. The concave function $U(c)$ is the instantaneous utility function.

Under regularity conditions Bellman's Principle and Itô's lemma allow the maximization to be expressed as

$$\max\{ U(c) + D(J(S,t)) \} = 0 \qquad (25)$$

Here D is the differential generator or Dynkin operator (see Kushner (1967, p. 15)) which indicates that $J(S,t)$ is stochastically differentiated.

For further simplicity, ignore the portfolio allocation. Expanding (25) and differentiaong with respect to c gives

$$U_c + J_M \cdot \frac{\delta}{\delta c} (dM) = 0 \qquad (26)$$

The rest of this section discusses what can be learned about consumption from the first-order condition (26).

There are two methods available for examining consumption. One, followed in the theoretical literature, is to specify a functional form for the utility function (typically of the HARA family or its isoelastic subset) then to invert the first-order conditions, substitute in the Hamilton-Jacobi-Bellman equation and thus solve for $J(S,t)$. This method has the disadvantage that any tests of the model's predictions are also tests of the assumed representation of preferences.

The second method of exploiting the first-order conditions avoids this representation by deriving the stochastic Euler equation for consumption. Stockman (1981) derives and studies this under a cash-in-advance constraint. The model here provides a new version of this constraint and the second term in equation (26) could be examined for case 2 of chapter 2 or for the model of chapter 3. As usual, the

existence of this constraint can account for apparent violations of the life-cycle model of consumption behaviour. Since it is more appropriate to study the effect of liquidity on consumption not as an exogenous influence but as the result of joint optimization (as in the first approach mentioned above) I do not pursue this subject here.

4.6 A Note on Mixed Processes and Inflation.

This section makes two further comments on the interpretation of target-threshold models of money demand. The first comment is about the use of mixed processes for the state variable dynamic equations. It would be desirable to represent the net disbursements process, in particular, by

$$dx(t) = \mu(t)dt + \sigma_x(t)dz_x + m(t)dq_x \qquad (27)$$

where (z) is a standard Wiener process and (q) a Poisson process independent of (z) and with parameter λ. The probability of one event in an interval of size Δt is $\lambda\Delta t + o(\Delta t)$, of no event $1 - \lambda\Delta t + o(\Delta t)$, and of multiple events $o(\Delta t)$, where λ may be time-varying. An event is a jump in $x(t)$ with size given by a random variable. Jumps in the net disbursements process could reflect wage payments as in Merton (1971) or other changes in income and consumption. Although a maximum principle and a differential generator exist for processes like (33) (see Kushner, pp. 18-22) the difficulty of using these techniques in chapter 2 precludes my extending the model in this way.

The situation is rather different when the targets and thresholds can jump. This cannot be the case if all state variables follow diffusion processes since this formulation implies that new information accrues continuously and in small amounts. But the controls depend on expected future values of the state variables and transfer costs; they

can jump in the model of chapter 2 if γ and k do. Thus in equations describing target-threshold dynamics $\delta u/\delta t$, for example, must be interpreted as a right-hand time derivative. Jumps of this kind (which have zero probability ex ante and hence cannot be represented as Poisson processes) can be interpreted as arising from 'news' about the future of variables on which the targets and thresholds depend. With this extension the model is more difficult to reject since a wide range of target-threshold behaviour is possible. Put differently, the unobservability of transactions costs is more worrying for the modeller if they vary over time..

The second comment concerns the role of inflation in dynamic target-threshold models. Several channels by which inflation affects the demand for money are already included in the portfolio influences. For example, equity asset demands (which are real demands) depend on the variance in the rate of inflation, σ_p^2, essentially due to Jensen's Inequality. Fischer (1975) and Boonekamp (1978) demonstrate this effect. The phenomenon that inflation has a depressing effect on equity values is also captured. This effect could be due to the time-wedge created by the cash-in-advance constraint (see Kohn(1984)).

More generally with this constraint inflation encourages substitution out of financial or market activities altogether. Thus the rate of inflation can be considered a rate of return and added to the vector of these rates. A negative coefficient on inflation could reflect this substitution or the non-indexation of the tax system. In any case the discussion above suggests possible roles for the rate of inflation and its variance in both the long-run demand for money and in the dynamic specification (as in Milbourne (1983), and Baba, Hendry, and Starr (1985)).

4.7 Summary.

This chapter has dealt with a number of topics in the interpretation and application of inventory models in economic time series. The chief innovation is the derivation of the net disbursements process, equation (16), in a model which includes consumption and portfolio decisions. While remaining in the partial equilibrium framework the analysis shows that a number of economic variables (such as certain variances) are no less admissible than income or the price level as candidate regressors in empirical money demand functions..

Notes

1. Section 4.4 examines these terms and studies.

2. The proofs of chapter 2 for the existence and form of the optimal policy go through. Jagannathan (1978) gives an inventory model in which changes in the net disbursements distribution do not affect the policy form.

3. In reply to Ohlin in Keynes (1937, p.667) wrote

> I allowed, it is true, for the effect of an increase in actual activity on the demand for money. But I did not allow for the effect of an increase in planned activity, which is superimposed on the former, and may sometimes be the more important of the two, because the cash which it requires may be turned over so much more slowly.

4. To derive equation (13) notice that
$$dp_i dp_{n-j} = \sigma_i dz_i \sigma_{n-j} dz_{n-j} = \sigma_i \sigma_{n-j} \rho_{i\,n-j} dt = \sigma_{i\,n-j} dt.$$

5. Nor can I derive the cash process in the form at which Markose arrives. I am grateful to her for providing me with her unpublished work on this subject.

6. Lucas (1984) observes that the cash-in-advance constraint gives rise to a precautionary demand for money.

7. Equation (25) in a sense resolves the debate between Barro (1974) and Orr (1974a) concerning the correct opportunity cost. Orr claimed the highest alternative rate was relevant; Barro claimed the rate on the adjacent asset (r_0 here) was. The argument in the text based on transactions costs gives the simple weighted average result.

8. See also Benston and Smith (1976).

9. Complete variability in the portfolio shares would partly offset these effects.

5. ADJUSTMENT, EXPECTATIONS, AND THE TIME SERIES PREDICTIONS OF STOCHASTIC INVENTORY THEORY: REINTERPRETING LINEAR, MACROECONOMETRIC MODELS OF THE DEMAND FOR MONEY.

5.1 Introduction.

This chapter begins to assess the empirical usefulness of stochastic, inventory-theoretic models in macroeconomic data. It focuses on the dynamic implications of such models for several reasons.

First, much controversy surrounds the interpretation of dynamics in existing, empirical models. But a constructive research programme must not ignore previous empirical work. Such a programme must account for the successes as well as the failures of traditional, linear models. Chapter 3 illustrated a case in which target-threshold monitoring gave rise to a demand equation of partial-adjustment form but with a variable adjustment speed. Section 5.3 below generalizes this result.

Second, there are many papers which add regressors to the standard, partial-adjustment equation. Proxy variables for financial innovation, inflation effects, and variability measures are examples of additional regressors. Chapter 4 argued that the presence of some of these can be justified through inventory theory. But the lagged, dependent variable plays a dominant role irrespective of the presence of these regressors. Thus this chapter attempts to account for this role, essentially by interpreting traditional empirical models as misspecified versions of inventory-theoretic models.

Section 5.2 reviews some previous attempts (not based on inventory theory) to do this misspecification analysis. In particular, it examines proposed solutions to the problem of slow adjustment. Most of these proposals are unsatisfactory theoretically and empirically.

Section 5.3 turns to the dynamic predictions of inventory theory, which are just as detailed as its target predictions but easier to derive. The method used to derive an aggregate, adjustment model is

completely general, although the analysis is done for the simple (0,u) policy form. The theory is based on explicit aggregation over time and, to a lesser extent, agents. It combats arbitrariness in empirical modelling in two ways. First, the mix of forward and backward-looking dynamics in money demand is derived from the theory. This derivation leaves the modeller no scope to mix these elements to, say, increase the implied speed of adjustment. Second, in the case of linear models the decision on how to enrich an empirical model by generalizing the dynamics or by letting the parameters vary is also made by the theory.

I argue that stochastic inventory theory can account for traditional empirical results in a theoretically sensible way. It also makes new predictions. Section 5.4 examines some inefficient tests of these predictions and describes the obstacles to rigourous econometric testing. It also gives a long-run aggregation theorem for cash-management policies.

Section 5.5 discusses simulations in which traditional, linear models are fitted to data generated by means of optimal cash-management rules. This evidence suggests that interpreting data from the viewpoint of inventory theory provides a resolution of the problem of slow adjustment. In section 5.6 a brief summary concludes the chapter.

5.2 The Problem of Slow Adjustment.

The problem of slow adjustment in demand-for-money equations is a theoretical one that arises as follows. Empirical models that fit data well involve lagged dependent variables and, for example, the coefficient on a once-lagged dependent variable is usually large and significant. The theoretical problem is that the rationalizations of these successful empirical models use adjustment costs to explain the

dynamics and the adjustment speeds implied by the models are typically very low.

To see the problem precisely write a Markov model of money demand in discrete time as

$$M_t = \rho M_{t-1} + T(\underline{v}_t) + \varepsilon_t \qquad (1)$$

where $\underline{v}_t$ is a vector of independent variables and the target function T (which in general will not coincide with the inventory-theoretic target) can involve the lag operator. The estimate of ρ clearly depends on the choice of $\underline{v}_t$ and the choice of the generalized function T.

Say one writes $\underline{v}_t = (r_t\ y_t\ p_t)'$ and $T(\underline{v}_t) = \alpha r_t + \beta y_t + \gamma p_t$. With these choices the estimated ρ will be large, generally well above 0.8 in quarterly M1 data. The most influential rationalization of ρ's not being zero is that of Chow (1966) based on the partial-adjustment mechanism. In this interpretation $\rho = 1 - \lambda$ where λ is the rate at which balances are adjusted in response to deviations from a target in this case given by $(\alpha r_t + \beta y_t + \gamma p_t)/\lambda$.

In monthly Canadian data from 1962 to 1979 for the logarithms of seasonally-adjusted real M1, real GNE, and the ninety-day corporate paper rate the estimate of ρ in this model is 0.934.[1] This result implies a mean lag , $\rho/(1-\rho)$, of 14.2 months, which seems implausibly long. Models using more general error-correction mechanisms do not constrain the dynamic responses to be the same for all independent variables but give similarly slow adjustment.[2] The theoretical problem arises because these models fit the data very well.

There are some alternatives to the rationalizations that involve only adjustment costs. By their choice of T or, more rarely, $\underline{v}$ these alternative approaches try to eliminate or reduce the coefficient on the lagged, dependent variable. One can use the fact that (1) fits better

that most of the alternatives to see what they would have to do to fit as well. Write

$$M_t = \rho M_{t-1} + T(\underline{v}_t) + \varepsilon_t$$

$$T(\underline{v}_t) = \alpha r_t + \beta y_t + \gamma p_t \qquad (2)$$

where ε_t is now an innovation with respect to $(M_{t-1}, \underline{v}_t)$ and I include a constant term implicitly in T.

The obvious way to set up a model in which H: $\rho = 0$ cannot be rejected is to use as the target $S(\underline{v}_t) = T(\underline{v}_t)/(1-\rho L)$. The data will suggest that adjustment to this target is immediate:

$$M_t = S(\underline{v}_t)$$

More generally (and ignoring the error term) in the stationary case the spectral density function for M_t is

$$S_M(e^{-i\omega}) = \left(\frac{1}{1-\rho e^{-i\omega}}\right)\left(\frac{1}{1-\rho e^{i\omega}}\right)\sigma_T^2$$

$$= \frac{1}{1-\rho(e^{i\omega}+e^{-i\omega})+\rho^2}\,\sigma_T^2$$

$$= \frac{1}{1-2\rho\cos\omega+\rho^2}\,\sigma_T^2$$

$$\text{So that } \frac{\delta S_M}{\delta \omega} = -\frac{(2\rho\sin\omega)}{(1-2\rho\cos\omega+\rho^2)}\,\sigma_T^2$$

which is negative on $(0,\pi)$ for ρ positive. The spectrum also declines more rapidly as $\rho \to 1$. If (2) is the data generation process the problem of slow adjustment can be reduced by applying a low-pass filter to $T(\underline{v}_t)$ and eliminated if this filter has a one-sided, infinite moving average representation $\rho^s L^s$, $s = 0, 1, 2, \ldots$. Smoothing $\underline{v}_t$ increases the speed of adjustment.

Studies which attempt to do this can be classified in three groups, which I now list with examples. In the first group are studies which attempt to smooth $\underline{v}_t$ to match the properties of M_t but fail. For example, Lieberman (1979) writes $T(\underline{v}_t) = \alpha(L)r_t + \beta(L)y_t$ where $\alpha(L)$ and

$\beta(L)$ are one-sided polynomials in the lag operator based on Shiller's smoothness priors. With these priors the second differences of the lag weights are normal variates with mean zero and variance chosen by the modeller.

I estimate the model over 1962 to 1979 for the natural logarithms of monthly Canadian real M1, real GNE, and the ninety-day rate with the smoothness priors chosen by Shiller's rule-of-thumb and with twelve lags on each regressor. This yields mean lags on interest rates and income of 2.8 months and 5.8 months respectively. However, $R^2 > dw$, indicating misspecification. This outcome is similar to that obtained by Lieberman using quarterly U.S. data. If, like Lieberman, I 'correct' for first-order autocorrelation the estimated AR(1) coefficient in the residuals is 0.96. This simply means that the restriction $\rho = 0$ has been rejected. A significant lagged, dependent variable is still required and hence adjustment is slow.

In the second group are studies which succeed in appropriately smoothing $\underline{v}_t$ but give rise to further restrictions which are rejected. Chief among these are adaptive expectations/permanent $\underline{v}$ models in which $S(\underline{v}_t) = T(\underline{v}_t)/(1-\rho L)$. The added restriction in this case is that the residual term in the partial-adjustment specification should follow a first-order moving average process with parameter ρ if the permanent or expectations theory is correct. Since this is typically not found the adjustment interpretation is preferred to the expectations one.

A third group of studies nest the partial adjustment equation and empirically are reducible to it. These studies leave the lagged dependent variables in the model but try to reinterpret its coefficient. Examples are the Keynes-Friedman demand function of Meyer and Neri (1975) and partial-adjustment-adaptive expectations (PAAE) models of

Feige (1967).[3] In Feige's formulation $\rho = [(1-\eta) + (1-\lambda)]$ where η is the adaptive expectations parameter. The point here is that this extension increases the λ we associate with an estimated ρ. The problem is that it is difficult to reject H: $\eta = 0$. Also further restrictions implied by PAAE models, such as low-order moving averages in the residuals, generally are not found to hold.

The empirical success of partial-adjustment and error-correction is a concern for a further theoretical reason in addition to slow adjustment. Laidler's (1985) brief history of modelling in this area shows that there has been a dichotomy between the expectations and adjustment approaches. This dichotomy is artificial since if adjustment is spread over time the values of $\underline{y}_t$ will change while adjustment is taking place so that expectations will be relevant. The matter of concern is that little support has been found in the data for the addition of expectations terms to adjustment equations.

My suggestion is that this lack of support may be due to a neglect of some other theoretical information. While expectations and adjustment are complementary rather than alternative sources of dynamics neither does this idea provide a warrant for arbitrarily mixing them to, say, increase the adjustment speed. Instead, one can use target-threshold theory to provide restrictions on mixtures of forward (expectations) and backward (adjustment) dynamics and to account (without convex adjustment costs) for the empirical success of equations with slow adjustment .

These mixtures have faster adjustment than pure adjustment models and in this sense partly solve the slow adjustment problem.[4] They do this by undertaking the necessary smoothing of $\underline{y}_t$ not by long lags but

by a short lag and a related short lead so that the filter is two-sided. The theory underlying this suggestion is developed in section 5.3.

5.3 Aggregate Adjustment Theory.

This section derives some properties of short-run, aggregate adjustment in the demand for money. The discussion is not confined to transitions between steady states since initial balances are described by an arbitrary density function. Although a discrete-time approximation is made, the time aggregation from the continuous-time theory is explicit, so we can know where the approximation may be inappropriate. I stress that there need be no assumption that agents are identical or that they all follow the same type of monitoring policy. In this sense the dynamics are fairly general.

The analysis resumes with a simple demonstration that smooth distributed lags can arise at the microeconomic level when stochastic, bang-bang, control rules are followed. Take the discrete-time, data generation process to be

$$M_t = \beta U_{t-\delta} + \varepsilon_t \qquad (3)$$

where M responds to a target U with a variable delay. Assume ε_t has zero mean and is strictly exogenous: $E(\varepsilon_t U_s) = 0 \ \forall s$. Assume that δ has discrete density q on [0,n]. Then say that the econometrician cannot observe δ. He conditions on δ by projecting M_t on past targets as follows:

$$M_t = b_0 U_t + b_1 U_t + \ldots + b_i U_{t-i} + \ldots + b_n U_{t-n} \qquad (4)$$

so that there is no truncation problem. Estimation by ordinary least squares is based on n^2 normal equations (since $i = 0,\ldots,n$ and $\delta = 0,\ldots,n$) which yield $E(b_i) = \beta\, q(\delta=i)$. Thus if q is smooth the distribution of lag weights will be smooth. The coefficient b_0, for

example, is not the fraction of adjustment made instantly but the full adjustment multiplied by the sample frequency with which it occurs immediately. So this demonstration provides a simple counter-example for the reader who suspects that bang-bang control is inconsistent with time series evidence.

Given that other theories also give rise to models like (4) what can target-threshold theory contribute to our understanding? Two implications of target-threshold theory are (i) that future values of the independent variables will enter the targets, with cross restrictions involving the b_i, and (ii) that the b_i will be variables.

To see these implications and derive the b_i explicitly begin at time 0 with cash balances $x(0) = x_o$ distributed on $(0,u(0))$ with density function $h_o(x(0))$ or h_o. Assume that a $(0,u)$ policy is in effect though the method to be followed is completely general. Net disbursements are taken to follow the Wiener process of chapter 2 so that from each point on h_o probability mass spreads out according to a time-homogeneous and normal transition density:

$$f(x_o;x,t) = \frac{1}{\sigma\sqrt{2\pi t}} \exp\left\{-\frac{[x(t) - x_o - \mu t]^2}{2\sigma^2 t}\right\} \qquad (5)$$

which is the solution to the unrestricted forward Kolmogorov equation.[5] One is interested in the probability that the process has not yet hit a barrier at time t:

$$F_o(t) = \text{prob}(x(t)\varepsilon(0,u(t)) \mid x_o \sim h_o)$$

$$= \int_0^{u(t)} f(x_o,h_o;x,t)\,dx$$

$$= \frac{1}{\sigma\sqrt{2\pi t}} \int_0^{u(t)} \int_0^{u(0)} \left\{1 - \frac{[x(t)-x_o-\mu t]^2}{2\sigma^2 t}\right\} h_o\,dx_o\,dx \qquad (6)$$

by taking the first two terms in the McLaurin series expansion of the exponential. Thus

$$F_o(t) = \frac{1}{\sigma\sqrt{2\pi t}} \int_0^{u(t)} \{ 1 - \frac{1}{2\sigma^2 t}(x^2-2x\mu t+\mu^2 t^2)$$

$$- \frac{1}{2\sigma^2 t} \int_0^{u(0)} [x_o^2 h_o + 2(\mu t - x)x_o h_o] dx_o \} \, dx$$

$$= \frac{1}{\sigma\sqrt{2\pi t}} \int_0^{u(t)} \{1 - \frac{1}{2\sigma^2 t}[x(t)^2 - 2x(t)\mu t + \mu^2 t^2$$

$$+ V_o + \Omega_o^2 + 2\mu t\Omega_o - 2x(t)\Omega_o]\} \, dx$$

assuming Ω_o and V_o, the mean and variance of h_o, exist. Thus

$$F_o(t) = \frac{1}{\sigma\sqrt{2\pi t}} \{u(t) - \frac{u(t)}{2\sigma^2 t}[\mu^2 t^2 + V_o + \Omega_o^2 + 2\mu t\Omega_o$$

$$+ \frac{u(t)^2}{3} - \mu t u(t) - \Omega_o u(t)]\} \qquad (7)$$

As shown below, $F_o(t)$ is essentially the weight on the lagged, dependent variable in the money demand equation. The adjustment speed in this equation is a monotone decreasing function of F. Figure 5.1 shows how $F_o(t)$ is found by simply counting the probability mass still 'alive' at time t.

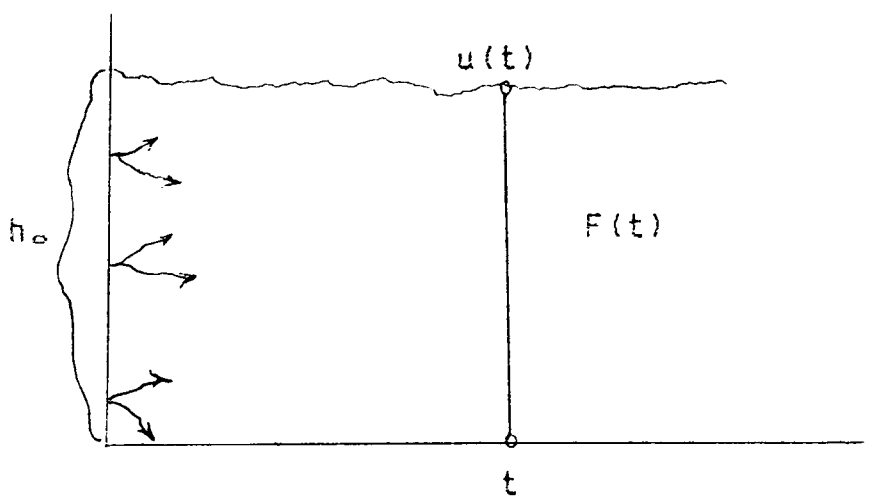


Figure 5.1

In (7) observe that

(i) $\delta F_o(t)/\delta u(t) > 0$; the wider the continuation region the slower the

adjustment,

(ii) $\delta F_o(t)/dV_o < 0$; the greater the variance of the initial distribution the more probability mass overflows between 0 and t, hence the faster adjustment, and

(iii) $\delta F_o(t)/\delta \Omega_o < (>)\ 0$ as $\mu > (<)\ 0$; a high Ω_o gives fast (slow) adjustment if drift is towards the upper (lower) threshold.

Starting the process from a density h_o rather than the special case of the Dirac delata function allows study of an ensemble of stochastic processes. In an alternative interpretation agents are identical except for their initial cash balances; equation (7) shows how the distribution of balances across agents affects the dynamic behaviour of aggregate money.

In (6) h_o can be any density that integrates to unity and has two moments. However the steady-state density $\pi(x)$ is of interest since it reveals further properties of F(t) (which contains all adjustment speed information since adjustment is mechanical once targets and thresholds are selected). For the (S,s) policy chapter 3 derived some of these properties. For a proof that the steady-state density exists in the general problem see Gihman and Skorohod (1974) or Mandl (1978). In the general case, writing the target as U, the density is given by the solution to an integral equation:

$$\pi(x(t)) = F(x_o,\pi;x,t) + \int_0^t (1-F(\tau))F(U(\tau);x,t)d\tau$$

$$+ \int_\tau^t (1-F(s))F(U(s);x,t-\tau)ds + \ldots \qquad (8)$$

where the possibility of multiple absorptions in (0,t) accounts for the added terms. The aliasing problem refers to our inability to identify

these multiple absorptions in data with observation intervals of length t.[6]

A fixed point in (8) can be found numerically. An alternative is to use the method of Feller (1957) as applied in Milbourne, Buckholtz, and Wasan (1982,1983). Feller's method is to add a term to the forward equation (no longer the formal adjunct of the backward equation with absorbing barriers) to reflect the non-local characteristics of transitions to U(t) from the boundaries. For example, in discrete time and state space with probability p of a unit step up and q of a unit step down the augmented forward equation includes

$$f(U,t) = pf(U-1,t-1) + qf(U+1,t-1) + pf(u-1,t-1) + qf(1,t-1)$$

Rather than solve this forward equation I use the same method used to find the value function in chapter 2. The method has three steps. First I obtain necessary conditions the density $\pi(x)$ must satisfy. Then I show that there exists a function that satisfies these conditions. Finally I show that the same function satisfies the integral equation in (8).

Using this method for a (0,U,u) policy the necessary conditions are, first, $\pi(0) = 0$ and $\pi(u) = 0$. Then write

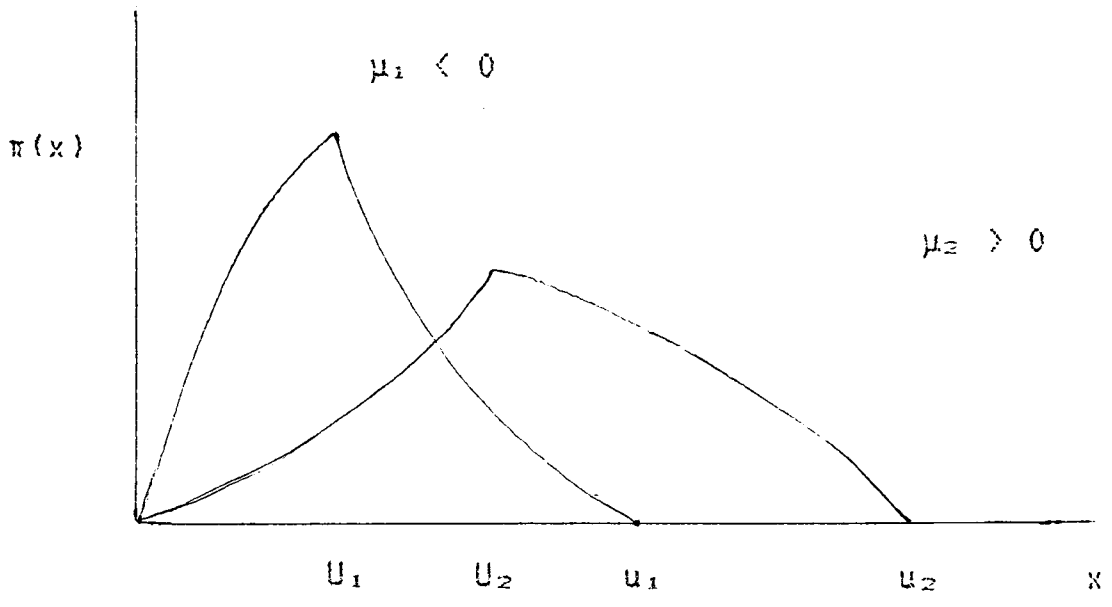
$$\pi(x) = \begin{cases} g(x) & 0 \leq x \leq U \\ j(x) & U \leq x \leq u \end{cases}$$

One requires $g'(x) \geq 0$ and $g''(x) \geq (\leq)\ 0$ as $\mu \geq (\leq)\ 0$. Similarly $j'(x) \leq 0$ and $j''(x) \leq (\geq)\ 0$ as $\mu \geq (\leq)\ 0$. The density must also integrate to unity. Finally when $\mu = 0$ MBW (1983) have solved for $\pi(x)$; we must get their density as a special case.

A function which satisfies these conditions is

$$\pi(x) = \begin{cases} \dfrac{2x}{uU} - \dfrac{\mu x(U-x)}{\sigma^2 U^3} & 0 \leq x \leq U \\ \dfrac{2(u-x)}{u(u-U)} + \dfrac{\mu(x-U)(u-x)}{\sigma^2 (u-U)^3} & U \leq x \leq u \end{cases} \qquad (9)$$

which can be shown also to satisfy equation (8).[7] This is thus the steady-state density. It looks like this:



As $\mu \to 0$ the density is triangular. As σ^2 increases the density is elongated since U and u increase. The shape is intuitive; probability appears to pile up between the target and the threshold towards which the cash balance drifts.

From (9) tedious integration gives:

$$\Omega_w = \frac{u+U}{3} + \frac{\mu}{12\sigma^2}(u+U-1)$$

$$V_w + \Omega_w^2 = \int_0^u \pi(x)x^2 dx$$

$$= \frac{u^2+uU+U^2}{6} + \frac{\mu}{\sigma^2}\left(\frac{u^2}{20} - \frac{uU}{15}\right)$$

So far we have considered a policy of (0,U,u) form. Policies of (0,u) form (i.e. with proportional transfer costs only) are of interest in particular since a closed-form expression for u is known from chapter

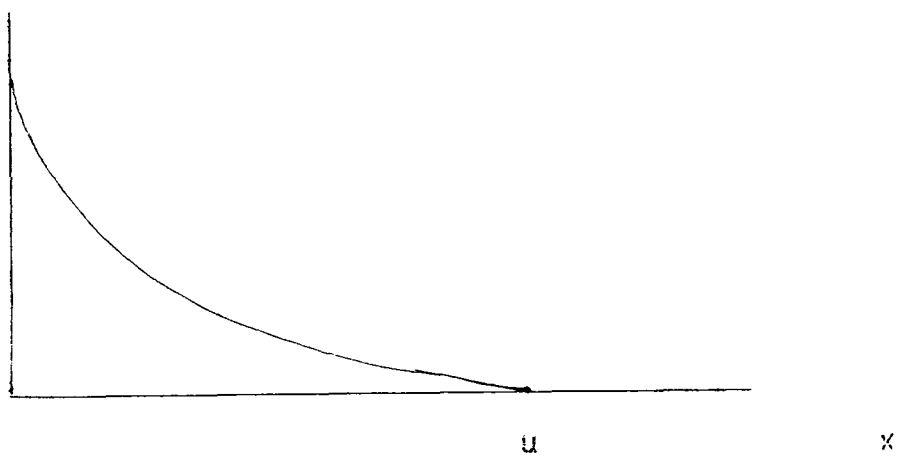
2. For small μ the same methods as above give the steady-state density with this policy as:

$$\frac{2(u-x)}{u^2} + \frac{\mu}{\sigma^2}\left(u - \frac{3x}{2}\right)x \qquad \mu < 0 \qquad (10)$$

$$\pi(x) = \frac{1}{u} \qquad \mu = 0 \qquad (11)$$

$$\frac{2x}{u^2} - \frac{\mu}{\sigma^2}\left(u - \frac{3x}{2}\right)x \qquad \mu > 0 \qquad (12)$$

The threshold u in (10)-(12) could be replaced by the optimal value from chapter 2. The steady-state density would then be a function of the interest rate, the discount rate, the transfer cost, and the parameters of the stochastic processes followed by the interest rate and net disbursements. For $\mu < 0$ the density looks like this:



The threshold away from which the process drifts is termed transient.

For illustrative purposes consider the transition from the steady state given by (11). Taking (11) first,

$$\Omega_\infty = \frac{u}{2}$$

$$V_\infty + \Omega_\infty^2 = \frac{u^2}{3}$$

Thus in equation (7)

$$F_0(t) = \frac{u(t)}{\sigma\sqrt{2\pi t}}\left\{1 - \frac{1}{2\sigma^2 t}\left[\frac{u_0^2}{3} + \frac{u(t)^2}{3} - \frac{u_0 u(t)}{2}\right]\right\} \qquad (13)$$

From (13) one can see an asymmetry in adjustment speeds. For equal changes in the threshold, adjustment will be more rapid ($F_o(t)$ smaller) in the case of a decrease than in that of an increase. To see this find the transition within a steady state by setting $u_o = u(t) = u$ in (13) and then note that $\delta^2F/\delta u^2 > 0$. The intuitive explanation of this asymmetry is that when thresholds decline some holders of cash will find themselves outside their new thresholds and will adjust immediately. Of course, the adjustment speed is not simply a function of the change in the threshold; there is a levels effect also.

To make use of the adjustment theory developed so far one needs a timing convention for discrete data. The most natural is that all probability which overflows in (0,t) multiplies u(t). This convention ignores multiple reflections or absorptions (the aliasing problem) and variation in the targets and thresholds between observations. The approximation is less misleading the smoother the paths of these controls. Since large changes in monitoring behaviour between observations become less likely as the observation frequency increases the usefulness of the approximation obviously depends on this frequency. From (7) $\delta F/\delta t < 0$; as time passes it becomes less likely that a barrier has not yet been reached. Of course, 'faster adjustment' (a low coefficient on the lagged, dependent variable in annual data, say) is misleading here since ideally we should weight each absorption or reflection by the appropriate u_τ, $\tau \in [0,t]$. To put the matter differently note that $\lim_{t\to\infty} F_o(t) = 0$. This result can be taken to show that (i) in a steady state the target is the best guess of where the cash balance is or (ii) one cannot learn much about dynamics from highly time-aggregated data. In other words, the effect of the approximation depends on the behaviour of the independent variables. Formally, the approximation is

$$\int_{t-1}^{t} [1 - F(t-1;\tau)]\, u(\tau)\, d\tau = [1 - F(t-1;t)]\, u(t) \qquad (14)$$

where the integral is stochastic.[8]

At this point a digression on the properties of the error term in discrete data is required also. To describe the cash balance drifting between thresholds one should write

$$x(t) = F(x(t-1);x(t))\,[x(t-1) + \int_{t-1}^{t} \mu\, ds + \int_{t-1}^{t} \sigma_\kappa dz_\kappa(s)] \qquad (15)$$

where the first integral is a Riemann-Stieljes integral. Since, strictly, $dz_\kappa(t)$ does not exist interpret the second integral as a stochastic integral of the Itô type. For proofs of the existence and uniqueness of this solution to the stochastic differential equation see Malliaris and Brock (1982). Using results there we can write $\varepsilon_t \sim IIN(0,\sigma_\kappa^2)$ for the second integral. The independence of successive discrete-time error terms arises from the use of point-sampled data, which are available for monetary aggregates and interest rates and are used in the simulations of section 5.5. Bergstrom (1984) and Melino (1985) discuss the problems involved in aggregating unit-averaged series, as is done in chapter 3. They show that if {x} is unit-averaged then ε_t will follow a first-order moving average process. Thus if the theory is correct ε_t will be serially uncorrelated in point-sampled data and MA(1) in unit-averaged data. Further, it is simple to show $E(F_t\varepsilon_t) = E(F_t)E(\varepsilon_t) = 0$ using equation (7).

From the way in which ε_t arises it should be clear how the Hansen-Sargent-type problem of accounting for the error term is resolved. In many empirical models derived through optimization it is difficult to explain the error term's presence since decision rules are nonstochastic functions of information on the state variables. In the

case of stochastic inventory theory money is not directly a control variable so that even if controls are non-stochastic functions of this information M_t will not be so. However, one could also describe this resolution by noting that the econometrician can observe neither the net disbursements process nor the stopping times although he knows μ and σ_x and so can calculate the targets and thresholds.

With the timing convention, set $\Delta t = 1$ in (7) and write

$$M_t = F_t(M_{t-1} + \mu + \varepsilon_t) + (1 - F_t)u_t$$

$$= \mu F_t + F_t M_{t-1} + (1 - F_t)u_t + F_t\varepsilon_t \qquad (16)$$

$$F_t = \frac{u(t)}{\sigma\sqrt{2\pi}}\{1 - \frac{1}{2\sigma^2}[\mu^2 + V_o + \Omega_o^2 + 2\mu\Omega_o$$

$$- \frac{u(t)^2}{3} - \mu u(t) - \Omega_o u(t)]\} \qquad (17)$$

In equation (16) money holdings are given by the net disbursements process with probability F_t and by the target[9] with probability $1-F_t$. Equation (17) gives an expression which parameterizes this time-varying mixture. The two equations can be rewritten to express M_t as an infinite, distributed lag on the past targets from which it may have come, as in equation (4). The lag is infinite since there is a small probability of the cash balance's drifting forever between the thresholds. As in chapter 3 the Markov property in (16) arises from this property in the net disbursements process.[10]

Equation (16) resembles the partial-adjustment equation and so may have some success in fitting aggregate data. At the same time equation (17) and theoretical restrictions on u_t provide guidance in exploring the failures of the traditional approach. For example, (17) implies that the error in (16) will be heteroskedastic. The heteroskedasticity may be of the autoregressive conditional form discussed by Engle (1982).

The distinction between (16) and (17) also provides a logical framework in which to study departures from traditional empirical models.

Normally in empirical work the decision to improve a model by letting the parameters vary over time rather than allowing richer dynamics in the variables is a somewhat arbitrary one. The assignment of dynamics to variables in (16) and to parameters in (17) removes this arbitrariness in linear models. Thus if an equation such as (16) does not fit the data well the modeller should draw on information in (17) (giving rise, upon substitution, to a non-linear, Markov model) rather than, say, adding M_{t-2} to the equation.

The theory also prevents the modeller from arbitrarily mixing forward and backward filters to combat the problem of slow adjustment.[11] In (16) u_t operates a forward filter on the independent variables. This filter was derived in chapter 2 through the same theory that accounts for the backward or adjustment filter in (16). Ideally, this theoretical consistency would allow efficient tests of the hypothesis that money depends on a short lag and a short lead in, say, the interest rate rather than on a long lag only.

As an illustrative device, drop all distributional information contained in F_t and set $F_t = \nu\ \forall t$. Thus there is no conditioning on the previous period's density of cash balances in determining the adjustment speed. This is another way of saying that I study (16) and temporarily ignore (17).

This fixed-parameter approach gives rise to a strong restriction on empirical models. To see this recall that at time t probability $(1 - F_t)$ is repiled at U_t. Thus the net disbursements process relevant to choosing this target begins from a Dirac delta function that integrates to $(1 - F_t)$. Ignoring distributional information means that without

further loss of generality

$$\text{prob}(x_{t+1} \in (0, u_{t+1}) \mid x_t = U_t) = F(x_{t+1} \mid x_t, U_t) = \nu \qquad (18)$$

Since I ignore h_i $\forall i$ to be consistent I shall also ignore it when it is known exactly. Again for illustration suppose that the target is linear in current and expected future spot rates. Thus a linear target can be written

$$U_t = \alpha E_t(\underline{z}_t + \nu \underline{z}_{t+1} + \nu^2 \underline{z}_{t+2} + \ldots) \qquad (19)$$

In this model ν appears in the forward filter in (19) and in the backward one in (16). The pattern of past targets relevant to current money holdings is the same as the pattern of future independent variables relevant to the current target. In the model including (17) a weaker restriction would hold; the two patterns would still be similar. For example if the continuation region is narrow both the past and the expected future have little effect on current money holdings.

Combining (16) and (19) gives a money demand function which could be estimated jointly with the interest rate process.[12] Even with a fixed adjustment speed and a linear target the theory is more restrictive than the intertemporal partial-adjustment mechanism since the two filters have the same parameter.

Now that this section has derived and discussed the restrictions involved in (16) and (17) the next task is to investigate tests of these restrictions.

5.4 Econometric Evidence.

This section first considers some obstacles to efficient tests of the theory summarized in equations (16) and (17). The main (but not mutually exclusive) problems arise due to (i) aggregation, (ii) non-

linearity, and (iii) the unobservability of some variables. I begin by discussing these, then turn to some inefficient tests.

The first obstacle is the strength of the case for agnosticism about the functional form of targets and thresholds to be adopted in macroeconomic models. The first rationale for this stance stems from aggregation. Modelling exercises can ignore the question of aggregating over agents and still yield well-fitting equations, but the interpretation of results requires some awareness of aggregation problems. These problems have posed the greatest obstacles to the macroeconometric use of inventory models.[13]

In chapter 3 (section 3.3) agents were assumed to be identical but to hold differing initial balances. In these circumstances exact linear aggregation was possible and the parameters of the aggregate demand equation could be thought of as those of an average agent. In general money-holders have different initial balances, transfer and opportunity costs, and disbursements processes. There is thus no point in seeking necessary conditions for aggregate demand to be interpretable as that of an average or representative agent. Such conditions would be extrememly stringent since optimal targets and thresholds are non-linear in the independent variables.

This point can be illustrated by the second part of section 3.3 where there are two groups of agents each following a different monitoring policy. Akerlof and Milbourne (1978) also study a case with two types of policy. Now in the equation for aggregate holdings the usual elasticities are not those of individuals.

Aggregation considerations also affect the interpretation of macroeconomic, distributed-lag responses. Obviously a lagged response cannot be interpreted as if it were that of a single agent. Moreover,

bang-bang control with a constant delay time for each agent can give rise to smooth, slow adjustment at the macroeconomic level; the lag distribution may represent the distribution of delay times.[14] Depending on the model it may be impossible to distinguish between (i) all agents adjusting to a proportion λ of a shock and (ii) a proportion λ of all agents adjusting fully to a shock.[15]

The second justification of agnosticism about functional form is more fundamental. The targets and thresholds derived in chapter 2 are highly non-linear in the independent variables. Normally this would at least suggest a misspecification test of linear models but there are no closed-form expressions for cases with fixed costs. Conceivably, one could use the steady-state Miller-Orr and Baumol-Tobin targets as approximate counterparts to the forward-looking, Eppen-Fama-type targets and thresholds of chapter 2.[16] This returns us to the first justification: aggregate holdings depend on some unknown mixture of these mutually exclusive policies.

If we knew this mixture, could we simply add up individual monitoring policies, neglecting interaction between net disbursements processes, to obtain some aggregate policy? This question about the aggregation of inventories was raised by Edgeworth (1888) in his parable of the club managers. He noted, first, that money-holders can hold less than otherwise if unexpected demands are independent over time. This independence, the basis of 'root laws', is incorporated here via the Markov property of the net disbursements process.

Edgeworth's second suggestion was that one agent's transfers may offset another's so that acting together they may economize on reserves.[17] Similarly, in his parable reserves can be reduced if one club manager can borrow from clubs whose members are guests at his club.

The question here is, taking the degree of financial intermediation as given, whether understanding of aggregate demand is lost by neglecting the covariance between the disbursements shocks of different agents, that is, the extent to which one's unexpected payment is another's unexpected receipt.[18] Intuition might suggest that groups whose members experience negatively correlated net disbursement shocks might hold less money than groups for whose members these correlations are non-negative. This intuition turns out to be incorrect; in the steady state, target-thresholds policies shield inventory holdings from any interdependence in demand. This result is summarized in the following theorem.

Theorem.

In the cash-management problem as formulated in chapter 2 the optimal targets and thresholds and the steady-state money holdings of separate cash-managers are independent of the correlations between their respective net disbursements processes.

Proof.

(a) If two agents are denoted i and j then agent i's value function V^i has terms in $\sigma_{x^i x^i}$, $\sigma_{x^i x^j}$, and $\sigma_{x^j x^j}$. Since it is the solution to a linear, partial differential equation by the principle of superposition there are no cross-product terms. Thus the covariances do not appear in the derivatives of the value function used in the boundary conditions to determine (d,D,U,u).[19] (Recall from chapter 2 that the covariance between the state variables x(t) and r(t) has no role in the targets and thresholds. This result holds with the additional state variable $x^j(t)$.)

(b) Computing steady-state money holdings requires no information beyond

that contained in the targets and thresholds. Examples are given in section 5.3 above. []

This theorem generalizes that of Caplin (1985) who considers the (S,s) policy in discrete time.[20] The proof is made simple by the use of continuous-time techniques. The gist of the theorem is that targets and thresholds do not depend on net disbursements interaction and hence that steady-state money holdings share this quality since they depend only on information used to calculate targets and thresholds. It is an aggregation theorem since it allows the individual demands of chapter 2, say, to be added up, ignoring net disbursements interaction.

Of course, in the short run money holdings are not determined solely by targets and thresholds. For example, say that interest rates have just risen sharply and unexpectedly so that targets and thresholds have shifted down and that agent j is observed reducing his money holdings. These two pieces of information, along with $\sigma_{x_i z}$ and $\sigma_{x_i x_j}$, provide information on the location of agent i's balance in the continuation region.

The problem is how to interpret information about money out of steady states. The second major obstacle here stems from the non-linearity of targets and thresholds. Even if a linear target is adopted for empirical work the substitution of (17) in (16) leads to a non-linear model due to the adjustment speed's being a variable. This non-linearity hinders rigourous aggregation theory, but there is a further problem. Consider a discrete-time, macroeconomic model. In such a model the interest rate h periods hence should enter the current target weighted by the probability that if the balance were reset at the current target a further adjustment would not be made within h periods. Thus the relevance of future interest rates to the current target

depends on forecasts of the future width of the continuation region but this width itself depends on future interest rates. The modeller's problem is that there is no way to form conditional expectations of products of non-independnt random variables like $F_{t+1} \cdot F_{t+2} \cdot \ldots \cdot F_{t+h} \cdot r_{t+h}$.

The third obstacle stems from the macroeconomic unobservability of the drift and variance of net disbursements and of transfer costs. While these measures can be learned in microeconomic data such data are rarely found except in highly time-aggregated form. In any case the implicit goal is to account for the properties of macroeconomic time series, as well as to perform tests of the theory.

There is a further difficulty with unobservability. In continuous time the steady-state density $\pi(x)$ is differentiable since there is continuous overflow and repiling. That is, someone goes to the bank at each instant. With delayed repiling due to time aggregation the initial density h_{t-1} has a Dirac pulse at U_{t-1} while the rest of the density integrates to F_{t-1}. Now the process's memory is contained in h_{t-1} and due to the use of Wiener processes Ω_{t-1} and V_{t-1} are sufficient statistics for this memory.

The problem is that while V_{t-1} affects the adjustment speed it is unobservable. In theory this problem can be addressed by attributing that part of h_{t-1} not at U_{t-1} to h_{t-2} and so on successively so that all probability at t is viewed as having come from some past target. In this case the model would contain lagged, forward-looking variables involving expectations of each future interest rate conditional on various past information sets. Unfortunately in 'withholding equations' of this type long lags in the state variable processes also are necessary to identify the parameters. An alternative is to begin from a

steady-state density which by definition has no memory. The paper of Milbourne et al (1983) also made use of the steady-state density to derive an adjustment speed. The assumption that transitions are from one steady state to another is unpalatable because adjustment generally may not be completed within one period.

Despite this rather pessimistic review there are several ways to use the theory to interpret empirical studies. One is to adopt simulation methods. Section 5.5 does this and shows that the theory is consistent with the records of some traditional, empirical models. There are also some extensions to these empirical models that so far have no theoretical support. The remainder of this section suggests that stochastic inventory theory (unlike partial-adjustment theory) can account for these extensions or, alternately, that they can be regarded as inefficient tests of the model in (16) and (17).

The first extension is that made by Swamy, Tinsley, and Moore (1982). These authors estimate a money demand function for U.S. M1 of real, partial-adjustment form. Their extension is to let the parameters in this model vary over time according to driftless random walks. Normally this decision to let the paramters vary rather than to generalize the dynamic specification would be somewhat opaque. In this case, though, the model can be regarded as a misspecified version of (16) and (17) with a linear target in the former and the latter written as $F_t = F_{t-1} + e_t$, where e_t is an error term. Although most of the theoretical information in (17) is lost the authors find that F_t is inversely related to the level of U.S. short rates; adjustment is faster when interest rates are high. This outcome is predicted by inventory theory.

In a second extension the dependence of the speed of adjustment on the interest rate is explicit. Koot (1975) writes down an adaptive expectations model of the demand for money in which the rates at which income and interest rate expectations are updated depend on the levels of these variables. The result is a model like Tinsley's (1967) variable adjustment speed model; if interpreted in this way it can be rationalized through inventory theory.

A more direct test is provided by Ouliaris and Corbae (1985). These authors make the adjustment speed in the partial-adjustment equation an arbitrary logistic function of the short-term interest rate. For quarterly U.S. M1 data from 1959 to 1983 they find strong support for the hypothesis that the adjustment speed is an increasing function of the short rate.

These relatively atheoretical studies are mentioned to emphasize that stochastic inventory theory gives rise to predictions not made by other theories. While testing in this area is only beginning so far these predictions have found support. It remains to show that inventory theory also can account for more familiar results. The next section turns to this task, using simulation to avoid inefficiency.

5.5 Simulation Evidence.

Section 5.2 used the fact that the partial-adjustment equation often fits M1 data well to deduce the properties of other specifications. The properties of inventory-theoretic models cannot be found this way since (as in any rational expectations model) they depend on the properties of the state variable processes. That is, the interest rate is not weakly exogenous in (16) and (17). Since the model

is non-linear and non-stationary it also is difficult to use spectral methods to assess its effect on the estimated speed of adjustment.[21]

In this section the viewpoint is the reverse of that in section 5.2. The data generation process is inventory-theoretic. Earlier sections have discussed the problem faced by an econometrician who fits an inventory-theoretic model but cannot observe stopping times or levels of individual cash balances. This section studies a second econometrician, one who fits linear models to data. It uses simulation methods to find what he will find.[22]

The method is to generate a net disbursements process under optimal control. The agent follows the monitoring policy of section 2.10, case 2, under conditions in which it is optimal to do so.[23] The reflecting barriers are at zero and at $u = u(r(t),\alpha,\gamma,\beta,k^+,k^-,\mu,\sigma_z)$ with the functional form derived in chapter 2.

In calculating the optimal thresholds the first three arguments are based on actual economic variables. The interest rate is the quarterly average maximum of the passbook, bank certificate of deposit, and money market mutual fund rates for the U.S.. The data are described more fully in the section on data sources at the end of the thesis. These rates refer to alternatives to M1. The series runs from 1959II to 1983IV and so contains one hundred observations. For simplicity α and γ are taken to be fixed; they are estimated from a first-order autoregression in r_t over the entire sample. Since α and γ are parameters of a putative, underlying, continuous-time process Sargan's (1976) approximation is reversed to derive them from discrete-time estimates.[24] In the data generation process r_t is measured in dollars (not basis or percentage points) at quarterly rates.

The discount rate is fixed at a quarterly rate of 0.15, which is approximately the sample average of the interest rate. The transfer cost is the same at each threshold but varies across simulations. The net disbursements process is a discrete-time, driftless random walk with independent, pseudo-normal increments with zero mean and variance σ_x^2.[25] The parameters of this process also enter the optimal threshold.

In each simulation the net disbursements process begins at the mid-point of the continuation region, a choice which affects the results in general since the process is non-stationary. Each simulation includes a count of the proportion of times the balance does not hit a threshold. This measure is denoted ν; it cannot be found by linear, least-squares regression on the threshold since the structure is non-linear and heteroskedastic.

Table 5.1 lists the simulation results. The first three columns describe the data generation process. Here k = (1 3 5 7) and σ_x = (1 4 7 10 13) with a full factorial design. The measure ν, which can be thought of as the true coefficient on the lagged, dependent variable, is given with a binomial standard error.

The next six columns present estimates from two models in each case. In the first model the generated money series is projected on a constant (c) and on its lagged value and the interest rate (at an annual rate in percentage points) with coefficients ρ and a respectively. In the second model the constant term is omitted. Heteroskedastic-consistent standard errors are given in parentheses.

Table 5.1 Simulation Results

	DGP		MODEL				
k	σ_u	v	c	ρ	a	ser	$\bar{x}$	dw
1	1	0.08(0.027)	0.119(0.029)	-0.02(0.1)	0.0012(0.003)	0.12	0.12	1.97
			...	0.156(0.89)	0.013(0.0002)	0.13		2.01
1	4	0.12(0.03)	0.43(0.11)	0.023(0.1)	-0.0004(0.01)	0.49	0.43	1.97
			...	0.17(0.09)	0.05(0.008)	0.52		1.99
1	7	0.12(0.03)	0.75(0.19)	-0.001(0.1)	0.02(0.02)	0.86	0.88	1.99
			...	0.15(0.09)	0.09(0.02)	0.92		2.07
1	10	0.05(0.02)	1.97((0.40)	-0.05(0.1)	-0.074(0.04)	1.4	1.43	1.99
			...	0.26(0.12)	0.12(0.03)	1.6		2.05
1	13	0.11(0.03)	1.62(0.38)	0.06(0.1)	-0.04(0.05)	1.6	1.48	1.97
			...	0.25(0.09)	0.13(0.03)	1.7		2.05
3	1	0.3(0.045)	0.27(0.07)	0.28(0.09)	-0.003(0.009)	0.33	0.36	2.04
			...	0.44(0.09)	0.02(0.006)	0.35		2.17
3	4	0.27(0.44)	0.96(0.3)	0.148(0.09)	0.02(0.04)	1.37	1.30	1.97
			...	0.27(0.09)	0.13(0.02)	1.43		2.03
3	7	0.3(0.045)	2.26(0.63)	0.23(0.09)	0.0006(0.07)	2.38	2.94	2.02
			...	0.42(0.08)	0.21(0.05)	2.54		2.15
3	10	0.35(0.47)	2.94(0.78)	0.34(0.09)	-0.02(0.08)	3.14	4.34	1.91
			...	0.54(0.09)	0.24(0.07)	3.34		2.04
3	13	0.32(0.046)	5.6(1.1)	0.26(0.10)	-0.39(0.09)	4.1	4.2	2.0
			...	0.56(0.08)	0.18(0.05)	4.55		2.23
5	1	0.38(0.048)	0.32(0.11)	0.50(0.07)	0.002(0.013)	0.47	0.67	2.14
			...	0.62(0.07)	0.03(0.009)	0.49		2.27
5	4	0.50(0.05)	1.77(0.47)	0.20(0.09)	0.07(0.05)	2.02	2.77	2.02
			...	0.36(0.08)	0.23(0.04)	2.14		2.16
5	7	0.43(0.049)	2.7(0.87)	0.29(0.09)	0.12(0.10)	3.6	5.0	2.0

				0.43(0.086)	0.38(0.083)	3.8		2.11
5	10	0.39(0.048)	7.3(1.7)	0.36(0.1)	-0.28(0.14)	4.6	8.7	1.86
				0.72(0.06)	0.28(0.1)	5.1		2.14
5	13	0.44(0.049)	6.5(1.6)	0.40(0.09)	-0.47(0.15)	5.8	9.9	2.1
				0.64(0.07)	0.20(0.07)	6.3		2.3
7	1	0.39(0.049)	0.33(0.14)	0.56(0.08)	0.014(0.17)	0.64	0.96	2.2
				0.63(0.07)	0.46(0.013)	0.65		2.3
7	4	0.44(0.49)	1.92(0.55)	0.14(0.11)	-0.067(0.06)	2.31	1.76	2.0
				0.29(0.11)	0.14(0.035)	2.44		2.2
7	7	0.49(0.05)	3.9(1.2)	0.32(0.09)	0.19(0.12)	4.7	7.5	2.1
				0.48(0.08)	0.52(0.11)	4.9		2.2
7	10	0.51(0.05)	14.3(2.2)	0.15(0.12)	-0.27(0.13)	4.7	14.8	1.9
				0.78(0.06)	0.38(0.15)	5.8		2.4
7	13	0.44(0.49)	5.9(1.8)	0.52(0.01)	-0.44(0.17)	6.9	16.4	2.2
				0.69(0.07)	0.19(0.08)	7.2		2.4

One problem with simulations with this data generation process is that invariance or homogeneity relations are difficult to establish because of the non-linearity. So in general experiments should be conducted all across the parameter space. Nonetheless, some generalizations can be drawn from the table. For example, the results are insensitive to small deviations of μ from zero. Also calculating α and γ on-line, so that the interest rate process can change, has little effect.

Note also that the choice of σ_x depends on the degree of time aggregation one has in mind. The effect of σ_x on the adjustment speed is non-linear; a higher variance widens the continuation region but also may increase the reflection frequency.

It is difficult to summarize the interest rate effects. In a number of cases the coefficient is positive though in some in which it is statistically significant it is not economically so. Cooley and Leroy (1981) note some similar surprises in U.S. M1 data.

Like all inventory theory, the simulations cannot account for very much money being held.[24] This is not a new problem, but I note that taking an implicit (or otherwise) interest rate on M1 into account would not change this outcome; in the simulations the threshold clearly smoothes the interest rate, which section 2.10 showed is consistent with $|u_r| < 1$. Thus the optimal threshold would increase less than one-for-one with a decrease in the opportunity cost of holding M1.

The claim that inventory theory resolves the problem of slow adjustment stems from the observation that $\rho > \nu$ for the second model in most cases. In this model the constant term is omitted, as if the modeller knew the true constant term (drift in the balance) were zero. Estimating a constant term has a large effect on the coefficient ρ essentially due to multicollinearity. Particularly if σ_x is small and the interest rate is stationary the net disbursements process has a long memory when uncontrolled. If money holdings tended to rise over time, perhaps because of additional state variables, this problem would be reduced.

The fast adjustment in the data generation process results from (i) interest rate expectations effects and (ii) the functional form of the optimal threshold-target. These two causes cannot be separated since they are derived through a single optimization problem.

In the parameter space explored in the simulations the ratio ρ/ν does not depend on ν or k. This suggests using this ratio to predict ν for $\rho = 0.9$, say. No exact evidence on this value of ρ can be found

without higher transfer costs, additional state variables, or aggregation (see below). The predicted ν is 0.45 (from the average ratio); the simulation result is that the true coefficient on the lagged, dependent variable is half the value estimated in a traditional, linear model. This reduction in the coefficient grealty reduces the mean lag since the relation between the two is non-linear. If money-holders follow inventory rules and transact every second period (with a mean lag of 1) an econometrician will estimate the mean lag at 9 periods.

The cash balance histories described in table 5.1 could be summed to give aggregate holdings for a group across which transfer costs and disbursements variances vary. The behaviour of this aggregate would be dominated by that of agents with large balances. It is a prediction of the theory (and evident in the table) that these agents also would transact infrequently and hence have high ν's and ρ's. However, the effect of aggregation on the money time series also depends on the covariances between the various net disbursements processes, since the environment is not a steady-state one. For example, negative covariances would tend to make the aggregate series smoother than its componenets. Since these covariances are endogenous in a more general model I have not explored this area in simulations.

The simulations do not describe twenty-five years of M1 history. Ideally one would add variables such as nominal income and the price level and estimate richer models, testing these in various ways. To add nominal state variables is to add decision variables; since there are few theoretical guidelines here the present results seem a natural stopping point. They provide some idea of the quantitative effects of

variable adjustment to a non-linear, forward-looking target while remaining true to the theory.

5.6 Summary.

This chapter has discussed a general approach to modelling the dynamics of asset holding subject to linear transfer costs. The approach is based on the same theory used to find optimal targets; hence the variable speed of adjustment and the mixture of forward and backward dynamics carry theoretical restrictions. In dynamic demand equations the coefficient on the lagged, dependent variable arises not from gradual, continuous adjustment to shocks but from 'chattering' between zero (at a threshold) and one (in the continuation region) which varies with the state variables and with the degree of time aggregation.

In steady states more information can be extracted from the theory. In particular, the derivation of steady-state densities under various policies strengthens the predictions for the speed of adjustment (section 5.3) and allows stronger aggregation results (section 5.4).

More generally, simulation evidence suggests that inventory theory can account for the slow-adjustment properties of empirical, linear, money demand equations. There are some obstacles to further econometric and theoretical exploration in this area.

NOTES

1. Details of several examples of slow adjustment are given more fully in an appendix.

2. For example, Hendry's (1980) study of M1 in the U.K. found mean lags of 7.9 quarters and 1.4 quarters on interest rates and income respectiveley. The corresponding median lags are 4 and 0.6 quarters.

3. PAAE models are discussed by Waud (1968) and Doran and Griffiths (1978).

4. The solution does not rely on the possibility that the balance simply drifts for a long time without reaching a threshold.

5. Equation (5) is the solution to the unrestricted forward equation. I ignore probability mass which overflows the continuation region, begin at an arbitrary initial density, and do not require that F satisfy any restrictions from the steady-state density. For these reasons the method is different from that of MBW (used in chapter 3) who have two boundary conditions on each of x and t. I also consider a more general net disbursements process than MBW and derive the dynamic effects of the distribution of cash balances.

6. More formally, aliasing refers to the difficulty in inferring parameters uniquely from a discrete spectrum, that is, distinguishing between data generation processes with time series whose frequencies differ by integer multiples of the observation period. Concerned at this, one might seek high frequency data. Sprenkle (1972) objects even to monthly-averaged data based on daily skip-sampling since end-of-day balances may be very different from average daily balances.

7. The proof uses the method of Frenkel and Jovanovic (1980, section IV.A). Where T is the mean first passage time they show that the steady-state density satisfies

$$\pi(x) = T \int_0^{\infty} F(x,t)\,dt$$

Cox and Miller (1965, chapter 5) show that T does not exist where $\mu = 0$. Fortunately, MBW (1983) have derived $\pi(x)$ by Feller's method in precisely this case, and (12) gives their result when there is no drift.

One can also show that $\pi(x)$ given in (12) approximates the solution to (11) arbitrarily closely by making the interval (0,t) small so that the probability of multiple absorptions in the interval is arbitrarily small.

8. I have also ignored the technical problems of integrating to a stochastic upper limit u(t), which could give F_t an error term..

9. In fact some reflections will occur at the lower threshold; $1-F_t$ is the probability of a reflection at either threshold.

10. If {x} is not a Wiener process (making the analysis of chapter 2 more difficult) F_t could have an upper limit less than one.

11. See Laidler (1985) for a history of some of these mixtures.

12. In chapter 2 α and γ were assumed to be known. Using the Wiener-Kolmogorov prediction formulae α, γ, and r can be shown to be sufficient statistics for the expectations in a forward-looking target.

13. See, for example, Blinder (1981).

14. Trivedi (1985) mentions this possibility. Section 5.3 shows that with a variable delay time a smooth lag distribution may arise for a single agent.

15. Breen (1971) mentions this difficulty.

16. The steady-state approach makes sense in linear models if the independent variables follow random walks, as Nickell (1985) shows. In inventory theory the results of steady-state and net-present-value optimization normally differ even under this condition. For examples, see Arrow et al (1958) or Frenkel and Jovanovic (1980).

17. This was Cannan's (1921) point in the citation in chapter 3.

18. An unexpected change in one's balance also could arise from an induced change in another's.

19. Covariances do not affect the choice of policy but they do affect the costs. For example, the value function in chapter 2 is increasing in $\sigma_{x,r}$ as would be expected. This effect could create an incentive to pool holdings to reduce the number of sources of shocks.

20. Caplin obtains a uniform steady-state density for an (S,s) policy in discrete state-space.

21. A Volterra series expansion could be used to study the evolutionary bispectrum.

22. Blinder uses simulation with an (S,s) DSP to examine the signs of partial derivatives (which I find by analysis) and the effects of aggregation.

23. For example the corner restrictions of chapter 2 are passed.

24. The approximation is $r_t = (\alpha\chi/(1+\alpha/2)) + (1-\alpha/2)/(1+\alpha/2)r_{t-1}$. The coefficients also could be found by maximum likelihood methods.

25. The pseudo-normal variates are generated by the multiplicative congruential method. The results may vary slightly with the seed.

26. Explaining the volume of money holding is a problem for other theories too. In the U.S. the stock of cash held by the public includes several one hundred dollar bills for each citizen.

6. CONCLUSIONS AND EXTENSIONS.

In macroeconomics much of our knowledge of the time series involved in the demand for money is summed up in linear, econometric models. The alternatives considered in this thesis are in some ways more general than many of these models. For example, their adjustment speeds are variables rather than constants. In these circumstances they obviously can find support; the more general alternatives can account both for, say, the partial-adjustment equation's fitting data and for some of its failures.

What makes the alternatives interesting is that they are not reached through examining data. Instead they are based on a long tradition in money demand theory and on realistic microeconomic assumptions. Several times in its history money demand theory has drawn fruitfully on advances in operations research. This thesis has tried to repeat this and to begin the task of connecting inventory theory with time series evidence. To make this connection, the inventory problem is set in a dynamic, stochastic environment. The result is a model with rich dynamics but without the unpalatable myopia, convex adjustment costs, and slow adjustment involved in many other interpretations of data.

While allowing rich models optimal inventory theory also leads to additional predictions. The rational expectations model of the demand for money to which it gives rise contains strong restrictions on expectations and adjustment dynamics as well as on the form of targets and thresholds. The dynamic and static restrictions follow from the same theory; the approach thus answers for the money demand schedule the problem in investment theory resolved through the flexible accelerator in the 1960s.

There seem to be two principle paths for future research. The first involves setting the inventory problem in a wider optimization. This extension is necessary so that further state variables (such as income and prices) can be linked to money holding. The findings of linear, econometric models depend to some extent on their inclusion of these variables.

It also seems important to study models in which the interest rate process, as well as the net disbursements process, is endogenous. Conditions under which the type of process for the interest rate considered here arise with linear transfer costs are not known.

The second path involves empirical work. The theory considered above has applications to holdings of commodity buffer stocks, to the real (as well as financial) inventories of firms, to the pricing policies of firms facing menu costs, and to central bank holdings of international reserves. Simulation can provide some evidence on these topics but eventually progress will require econometric tests.

Data Sources and Definitions.

Section 5.2 uses the following monthly Canadian data, provided by the Bank of Canada. Variables are seasonally adjusted. Cansim numbers are given in parentheses.

(1) Money: M1. Currency plus demand deposits, average of Wednesdays. (B2033)

(2) Income: GNE at market prices, interpolated from quarterly.

(3) Prices: GNE deflator.

(4) Interest Rate: Rate on ninety-day corporate paper. (B14017)

Section 5.5 uses the maximum of the quarterly averages of three U.S. interest rates, seasonally adjusted. The data originate at the Federal Reserve Board of Governors and were provided generously by David Hendry.

(1) Passbook interest rate, quarterly average.

(2) Commercial bank small CD rate, quarterly average.

(3) Money market mutual fund rate, quarterly average, starting 1974III.

Examples of Slow Adjustment.

Section 5.2 cites the adjustment speeds of two empirical models in Canadian data. The first of these is the real, partial-adjustment equation in the logarithms of real M1, real GNE, and the interest rate on ninety-day corporate paper. Estimating this model using data from 1962M1 to 1979M9 yields:

$$m_t = \underset{(4.11)}{-0.248} + \underset{(44.8)}{0.934}\, m_{t-1} + \underset{(5.83)}{0.055}\, y_t - \underset{(3.93)}{0.0185}\, r_t$$

$$R^2 = 0.995 \quad LLF = 697.9 \quad dw = 1.87$$
$$z_1 = 1.766 \quad z_2 = 8.06 \quad z_3 = 0.709$$

with t-statistics in parentheses. The test statistic z_1 gives a stability test based on re-estimation to 1981M6. It is distributed as F(21,204) under the null. The diagnostic test statistics z_2 and z_3 give LM tests for heteroskedasticity and first-order residual serial correlation respectively. They are distributed as $X^2(4)$ and $X^2(1)$ under the null.

The second model is of the following form:

$$m_t = c_0 + \sum_{i=0}^{n} b_i\, y_{t-i} + \sum_{i=0}^{n} d_i\, r_{t-i} + v_t$$

where all variables are in natural logarithms. Shiller's (1973) smoothness priors are imposed on the coefficients b_i and d_i. The first degree priors used require that the second differences of the lag weights be normal variates with mean zero and variance a function of the degree of tightness chosen by the modeller. In the absence of end-point priors implementing the Shiller procedure simply involves augmenting the ordinary least squares data matrices with certain dummy variables.

For data from 1962M1 to 1979M9, n=12, and k, the degree of tightness, chosen by Shiller's rule-of-thumb estimation yields $R^2 > dw$,

indicating misspecification. This outcome is similar to that obtained by Lieberman using quarterly U.S. data. Correcting for first-order autocorrelation gave rise to the following estimates:

$$m_t = \underset{(2.0)}{-0.13} + y_t \underset{(0.6)}{(0.065} + \underset{(1.0)}{0.066L} + \underset{(1.9)}{0.078L^2} + \underset{(1.6)}{0.055L^3}$$

$$+ \underset{(1.4)}{0.049L^4} + \underset{(1.4)}{0.048L^5} + \underset{(1.5)}{0.053L^6} + \underset{(0.5)}{0.014L^7} + \underset{(0.2)}{0.009L^8}$$

$$+ \underset{(0.3)}{0.012L^9} - \underset{(0.08)}{0.0003L^{10}} - \underset{(0.7)}{0.014L^{11}} \underset{(1.9)}{-0.0018L^{12})}$$

$$- r_t \underset{(1.79)}{(0.0167} + \underset{(1.7)}{0.0166L} + \underset{(2.8)}{0.027L^2} + \underset{(0.47)}{0.0044L^3} + \underset{(0.28)}{0.0027L^4}$$

$$+ \underset{(0.92)}{0.0087L^5} + \underset{(2.5)}{0.024L^6} + \underset{(0.20)}{0.0019L^7} + \underset{(0.29)}{0.0027L^8} + \underset{(1.6)}{0.015L^9}$$

$$+ \underset{(0.71)}{0.0065L^{10}} - \underset{(0.64)}{0.0054L^{11}} \underset{(1.64)}{+0.015L^{12})} + \underset{(1.99)}{0.00064Time}$$

$$R^2 = 0.9992 \quad LLF = 743.6 \quad dw = 1.88 \quad rho = 0.96$$

Here L is the lag operator and the absolute values of t-statistics are given in parentheses. The variable Time is a linear trend, included in imitation of Lieberman, to capture financial innovations. Its coefficient is economically insignificant. The long-run elasticities with respect to real income and the interest rate are 0.43 and 0.11 respectively. The high value of the first-order serial correlation coefficient in the residuals, rho, shows that adjustment is slow. In addition, the relativley large terminal t-statistics suggest too early truncation of the lags. Diagnostic testing, involving artificial linear regressions, is difficult because the same smoothness and common-factor restrictions must be applied.

BIBLIOGRAPHY

Akerlof, G.A. (1973) The Demand for Money: A General-Equilibrium Inventory-Theoretic Approach. Review of Economic Studies. 40, 115-130.

_____ (1978) The Microeconomic Foundations of a Flow of Funds Theory of the Demand for Money. Journal of Economic Theory. 18, 190-215.

_____ (1982) The Short-Run Demand for Money: A New Look at an Old Problem. American Economic Review.(P) 72, 35-39.

Akerlof, G.A. and Milbourne, R.D. (1978) New Calculations of Income and Interest Elasticities in Tobin's Model of the Transactions Demand for Money. Review of Economics and Statistics. 60, 541-546.

_____ (1980) The Short Run Demand for Money. Economic Journal. 90, 885-900.

Antelman, G.R. and Savage, I.R. (1965) Surveillance Problems: Wiener Processes. Naval Research Logistics Quarterly. 12, 35-55.

Aronson, J.A. (1968) The Idle Cash Balances of State and Local Governments: An Economic Problem of National Concern. Journal of Finance. 23, 499-508.

Arrow, K.J., Harris, T., and Marschak, J. (1951) Optimal Inventory Policy. Econometrica. 19, 250-272.

Arrow, K.J., Karlin, S. and Scarf, H. eds. (1958) Studies in the Mathematical Theory of Inventory and Production. Stanford University Press.

_____ (1962) Studies in Applied Probability and Management Science. Stanford University Press.

Arrow, K.J., Karlin, S. and Suppes, P. (1960) Mathematical Methods in the Social Sciences, 1959. Stanford University Press.

Baba, Y., Hendry, D.F., and Starr, R.M. (1985) U.S. Money Demand, 1960-1984. mimeo.

Barro, R.J. (1970) Inflation, the Payments Period, and the Demand for

Money. Journal of Political Economy. 78, 1228-1263.

_____ (1974) Review of Cash Management and the Demand for Money. Journal of Money, Credit, and Banking. 6, 123-135.

_____ (1976) Integral Constraints and Aggregation in an Inventory Model of Money Demand. Journal of Finance. 31, 77-88.

Bather, J.A. (1963) Control Charts and the Minimization of Costs. JRSS(B). 25, 49-70.

_____ (1966) A Continuous Time Inventory Model. Journal of Applied Probability. 3, 538-549.

Baumol, W.J. (1952) The Transactions Demand for Cash: An Inventory-Theoretic Approach. Quarterly Journal of Economics. 66, 545-556.

Bergstrom, A.R. ed. (1976) Statistical Inference in Continuous Time Economic Models. Amsterdam: North Holland.

Bergstrom, A.R. (1984) Continuous Time Stochastic Models and Issues of Aggregation Over Time. Chapter 20, 1145-1212 in Z. Griliches and M.D. Intriligator, eds. Handbook of Econometrics. volume II. Amsterdam: North Holland.

Bellman, R., Glicksberg, I., and Gross, O. (1955) On the Optimal Inventory Equation. Management Science. 2, 83-104.

Bensoussan, A. and Lions, J.L. (1975) Nouvelles Methods en Controle Impulsionnel. Applied Mathematical Optimization. 1, 289-312.

Benston, G.J. and Smith, C.W., Jr. (1976) A Transactions Cost Approach to the Theory of Financial Intermediation. Journal of Finance. 31, 215-231.

Benveniste, L.M. and Scheinkman, J.A. (1979) On the Differentiability of the Value Function in Dynamic Models of Economics. Econometrica. 47, 727-732.

Blinder, A.S. (1981) Retail Inventory Behavior and Business Fluctuations. Brookings Papers on Economic Activity.(2), 443-505.

Boonekamp, C.F.J. (1978) Inflation, Hedging, and the Demand for Money. American Economic Review. 68, 821-833.

Boylan, E.S. (1967) Multiple (s,S) Policies and the n-Period Inventory Problem. Management Science. 14, 196-204.

Breen, W. (1971) A Note on the Demand for Cash Balances and the Stock-

Adjustment Hypothesis. International Economic Review. 12, 147-151.

Buiter, W.H. and Armstrong, C.A. (1978) A Didactic Note on the Transactions Demand for Money and Behavior Towards Risk. Journal of Money, Credit, and Banking. 10, 529-538.

Cannan, E. (1921) The Application of the Theoretical Apparatus of Supply and Demand to Units of Currency. Economic Journal. 31, 453-461.

Caplin, A.S. (1985) The Variability of Aggregate Demand with (S,s) Inventory Policies. Econometrica. 53, 1395-1409.

Caplin, A.S. and Spulber, D.F. (1985) Inflation, Menu Costs and Endogenous Price Variablity. Discussion Paper No. 1181, Harvard Institute of Economic Research.

Cass, D. and Stiglitz, J.E. (1970) The Structure of Investor Preferences and Asset Returns, and Separability in Portfolio Allocation: A Contribution to the Pure Theory of Mutual Funds. Journal of Economic Theory. 2, 122-160.

Chant, J. (1976) Dynamic Adjustments in Simple Models of the Transactions Demand for Money. Journal of Monetary Economics, 2, 351-366.

Chow, G.C. (1966) On the Long-Run and Short-Run Demand for Money. Journal of Political Economy. 74, 111-131.

_____ (1979) Optimum Control of Stochastic Differential Equation Systems. Journal of Economic Dynamics and Control. 1, 143-175.

Clower, R.W. (1967) A Reconsideration of the Microfoundations of Monetary Theory. Western Economic Journal. 6, 1-8.

Clower, R.W. and Howitt, P.W. (1978) The Transactions Theory of the Demand for Money: A Reconsideration. Journal of Political Economy. 86, 449-466.

Constantinides, G.M. and Richard, S.F. (1978) Existence of Optimal Simple Policies for Discounted-Cost Inventory and Cash Management in Continuous Time. Operations Research. 26, 620-636.

Constantinides, G.M. (1976) Stochastic Cash Management with Fixed and Proportional Transaction Costs. Management Science. 22, 1320-1331.

_____ (1974) Transaction Costs in Portfolio and Cash Management. unpublished doctoral dissertation, Indiana University.

Cooley, T.F. and Leroy, S.F. (1981) Identification and Estimation of Money Demand. American Economic Review. 71, 825-844.

Cox, J.C., Ingersoll, J.E., Jr., and Ross, S.A. (1981) A Re-Examination of Traditional Hypotheses about the Term Structure of Interest Rates. Journal of Finance. 36, 769-799.

_____ (1985a) An Intertemporal General Equilibrium Model of Asset Prices. Econometrica. 53, 363-384.

_____ (1985b) A Theory of the Term Structure of Interest Rates. Econometrica. 53, 385-407.

Cox, D.R. and Miller, H.D. (1965) The Theory of Stochastic Processes. London: Methuen.

Crane, D.B. (1971) A Stochastic Programming Model for Commercial Bank Bond Portfolio Management. Journal of Financial and Quantitative Analysis. 6, 955-976.

Daellanbach, H. (1974) Are Cash Management Optimization Models Worthwhile? Journal of Financial and Quantitative Analysis. 9, 607-626.

Demsetz, H. (1968) The Cost of Transacting. Quarterly Journal of Economics. 82, 33-53.

Doran, H.E. and Griffiths, W.E. (1978) Inconsistency of the OLS Estimator of the Partial Adjustment-Adaptive Expectations Model. Journal of Econometrics. 7, 133-146.

Dvoretsky, A., Kiefer, J., and Wolfowitz, J. (1952a) The Inventory Problem: I. Case of Known Distributions of Demand. Econometrica. 20, 187-222.

_____ (1952b) The Inventory Problem: II. Case of Unknown Distributions of Demand. Econometrica. 20, 450-466.

_____ (1953) On the Optimal Character of the (s,S) Policy in Inventory Theory. Econometrica. 21, 586-596.

Edgeworth, F.Y. (1888) The Mathematical Theory of Banking. JRSS. 51, 113-127.

Engle, R.F. (1982) Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation.

Econometrica. 50, 987-1007.

Eppen, G.D. and Fama, E.F. (1968) Solutions for Cash Balance and Simple Dynamic Portfolio Problems. Journal of Business. 41, 94-112.

_____ (1969) Cash Balance and Simple Dynamic Portfolio Problems with Proportional Costs. International Economic Review. 10, 119-133.

Feige, E.L. (1967) Expectations and Adjustment in the Monetary Sector. American Economic Review (P). 57, 462-473.

Feige, E.L. and Parkin, M. (1971) The Optimal Quantity of Money, Bonds, Commodity Inventories, and Capital. American Economic Review. 61, 335-349.

Feller, W. (1957) On Boundaries and Lateral Conditions for the Kolmogorov Differential Equations. Annals of Mathematics. 65, 527-570.

_____ (1968,1971) An Introduction to Probability Theory and Its Applications. volume 1 3rd ed., volume 2 2nd ed., New York: Wiley.

Field, A.J. (1984a) Asset Exchanges and the Transactions Demand for Money, 1919-1929. American Economic Review. 74, 43-59.

_____ (1984b) A New Interpretation of the Onset of the Great Depression. Journal of Economic History. 44, 489-498.

Fischer, S. (1975) The Demand for Index Bonds. Journal of Political Economy. 83, 509-534.

Fleming, W.H. and Rishel, R.W. (1975) Deterministic and Stochastic Optimal Control. New York: Springer-Verlag.

Frenkel, J.A. and Jovanovic, B. (1980) On Transactions and Precautionary Demand for Money. Quarterly Journal of Economics. 95, 25-43.

Friedman M. (1977) Time Perspective in Demand for Money. Scandinavian Journal of Economics. 79, 397-416.

Geweke, J. (1976) The Use of Point Sampled and Unit Averaged Time Series in Regression. Workshop Series 7605, University of Wisconsin-Madison.

_____ (1978) Temporal Aggregation in the Multiple Regression Model. Econometrica. 46, 643-661.

Gihman, I. and Skorohod, A.V. (1974) The Theory of Stochastic

Processes. New York: Springer-Verlag.

Gilbert, J.C. (1953) The Demand for Money: The Development of an Economic Concept. Journal of Political Economy. 61, 144-159.

Goldfeld, S.M. (1973) The Demand for Money Revisited. Brookings Papers on Economic Activity. 577-645.

Grimmett, G.R. and Stirzaker, D.R. (1982) Probability and random processes. Oxford: Clarendon Press.

Grossman, H.I. and Policano, A.J. ((1975) Money Balances, Commodity Inventories, and Inflationary Expectations. Journal of Political Economy. 83, 1093-1112.

Grossman, S. and Weiss, L. (1983) A Transactions-Based Model of the Monetary Transmission Mechanism. American Economic Review. 73, 871-880.

Hansen, L.P. and Sargent, T.J. (1980) Formulating and Estimating Dynamic Linear Rational Expectations Models. Journal of Economic Dynamics and Control. 2, 7-46.

_____ (1981) A Note on Wiener-Kolmogorov Prediction Formulas for Rational Expectations Models. Research Department Staff Report 69. Federal Reserve Bank of Minneapolis.

Harrison, J.M. and Taylor, A.J. (1978) Optimal Control of a Brownian Storage System. Stochastic Processes and Their Applications. 6, 179-194.

Harrison, J.M., Sellke, T.M., and Taylor, A.J. (1983) Impulse Control of Brownian Motion. Mathematics of Operations Research. 8, 454-466.

Heller, H.R. and Khan, M.S. (1979) The Demand for Money and the Term Structure of Interest Rates. Journal of Political Economy. 87, 109-129.

Hendry, D.F. (1980) Predictive Failure and Econometric Modelling in Macroeconomics: The Transactions Demand for Money. in P. Ormerod, ed., Economic Modelling. London: Heinemann.

Huang, C.-F. (1985) Information Structure and Equilibrium Asset Prices. Journal of Economic Theory. 35, 33-71.

Jagannathan, R. (1978) A Minimax Ordering Policy for the Infinite Stage Dynamic Inventory Problem. Management Science. 24, 1138-1145.

Jazwinski, A.H. (1970) Stochastic Processes and Filtering Theory. Academic Press, New York.

Judd, J.P. and Scadding, J.L. (1983) The Search for a Stable Money Demand Function: A Survey of the Post-1973 Literature. Journal of Economic Literature. 20. 993-1023.

Keynes, J.M. (1973) The General Theory of Employment Interest and Money. Royal Economic Society edition. London: Macmillan. (1st edition 1936)

Keynes, J.M. (1937) The "Ex-Ante" Theory of the Rate of Interest. Economic Journal. 47, 662-669.

Khan, M.S. (1980) The Term Structure of Interest Rates in Money Demand Models. Economics Letters. 6, 261-266.

Kira, D. and Ziemba, W.T. (1977) Equivalence Among Alternative Portfolio Selection Criteria. 151-161 in H. Levy and M. Sarnat, eds. Financial Decision Making Under Uncertainty. New York: Academic Press.

Kohn, M. (1984) The Finance (Cash-In-Advance) Constraint Comes of Age: A Survey of Some Recent Developments in the Theory of Money. mimeo, Dartmouth College.

Koot, R.S. (1975) Non-Constant Coefficients of Expectation and the Recent Demand for Money. Journal of Monetary Economics. 1, 375-382.

Kushner, H.J. (1967) Stochastic Stability and Control. New York: Academic Press.

Laidler, D. (1985) Expectations and Adjustment in the Monetary Sector Revisited: A Comment. 243-254 in K. Brunner and A. Meltzer, eds. Understanding Monetary Regimes. Carnegie-Rochester Conference Series on Public Policy. volume 22. North-Holland.

Lieberman, C. (1979) Structural and Technological Change in Money Demand. American Economic Review. (P) 69, 324-329.

Lovell, M.C. (1981) Comments on Blinder. Brookings Papers on Economic Activity.(2) 506-513.

Lucas, R.E. (1984) Money in a Theory of Finance. 9-45 in K. Brunner and A. Meltzer, eds. Essays on Macroeconomic Implications of Financial and Labor Markets and Political Processes. Carnegie-Rochester Conference Series on Public Policy. volume 21. North-Holland, Amsterdam.

Magill, M.J.P. (1976) The Preferabilty of Investment Through a Mutual Fund. Journal of Economic Theory. 13, 264-271.

Magill, M.J.P. and Constantinides, G.M. (1976) Portfolio Selection with Transactions Costs. Journal of Economic Theory. 13, 245-263.

Maldonado, R. and Ritter, L.S. (1971) Optimal Municipal Cash Management: A Case Study. Review of Economics and Statistics. 53, 384-388.

Malliaris, A.G. and Brock, W.A. (1982) Stochastic Methods in Economics and Finance. Amsterdam: North-Holland.

Mandl, P. (1968) Analytical Treatment of One-Dimensional Markov Processes. New York: Springer-Verlag.

Markose, S.M. (1984) Non-Separability of Consumption and Portfolio Choice with a Precautionary Demand for Money. mimeo. London Business School.

Melino, A. (1985) Estimation of Unit Averaged Diffusion Processes. Working Paper 8502, Department of Economics, University of Toronto.

Merton, R.C. (1971) Optimum Consumption and Portfolio Rules in a Continuous-Time Model. Journal of Economic Theory. 3, 373-413.

_____ (1973) Erratum. Journal of Economic Theory. 6, 213-214.

_____ (1973) An Intertemporal Capital Asset Pricing Model. Eonometrica. 41, 867-887.

_____ (1980) On Estimating the Expected Return on the Market: An Exploratory Investigation. Journal of Financial Economics. 8, 323-361.

_____ (1982) On the Mathematics and Economics Assumptions of Continuous-Time Models. 19-51 in W.F. Sharpe and C.M. Cootner, eds. Financial Economics: Essays in Honor of Paul Cootner. Englewood Cliffs: Prentice-Hall.

_____ (1982) On the Microeconomic Theory of Investment under Uncertainty. chapter 13 in K.J. Arrow and M.D. Intriligator, eds. Handbook of Mathematical Economics. volume II. Amsterdam: North-Holland.

Meyer, P.A. and Neri, J.A. (1975) A Keynes-Friedman Money Demand Function. American Economic Review. 65, 610-623.

Milbourne, R.D. (1983a) Optimal Money Holding Under Uncertainty.

International Economic Review. 24, 685-697.

_____ (1983b) Price Expectations and the Demand for Money: Resolution of a Paradox. Review of Economics and Statistics. 65, 633-638.

Milbourne, R.D., Buckholtz, P., and Wasan, M.T. (1982) Cash Balances as a Random Walk. Paper presented to the second Canadian conference on applied statistics.

_____ (1983) A Theoretical Derivation of the Functional Form of Short Run Money Holdings. Review of Economic Studies. 50, 531-541.

Miller, J. and Orr, D. (1966) A Model of the Demand for Money by Firms. Quarterly Journal of Economics. 80, 413-435.

_____ (1967) An Application of Control-Limit Models to the Management of Corporate Cash Balances. 133-151 in A.A. Robichek, ed. Financial Research and Management Decisions. New York: Wiley.

_____ (1968) The Demand for Money by Firms: Extensions of Analytic Results. Journal of Finance. 23, 735-759.

Mullins, D.W. and Homonoff, R.B. (1976) Application of Inventory Cash Management Models. ch. 27 in S.C. Myers, ed. Modern Developments in Financial Management. New York: Praeger.

Neave, E.H. (1970) The Stochastic Cash-Balance Problem with Fixed Costs for Increases and Decreases. Management Science. 16, 472-490.

Nickell, S.J. (1985) Error Correction, Partial Adjustment and All That: An Expository Note. Oxford Bulletin of Economics and Statistics. 47, 119-130.

Olivera, J.H.G. (1971) The Square-Root Law of Precautionary Reserves. Journal of Political Economy. 79, 1095-1104.

Orr, D. (1970) Cash Management and the Demand for Money. New York: Praeger.

_____ (1974a) Cash Management and the Demand for Money. A Comment on Robert J. Barro's Version. Journal of Money, Credit, and Banking. 6, 136-140.

_____ (1974b) A Note on the Uselessness of Transactions Demand Models. Journal of Finance. 29, 1565-1572.

Duliaris, S. and Corbae, D. (1985) The Demand for Money: A Variable Adjustment Model. Economics Letters. 18, 197-200.

Poncet, P. (1983) Optimum Consumption and Portfolio Rules with Money as an Asset. Journal of Banking and Finance. 7, 231-252.

Porteous, E.L. (1972) Equivalent Formulation of the Stochastic Cash Balance Problem. Management Science. 19, 250-253.

Porteous, E.L. and Neave, E.H. (1972) The Stochastic Cash Balance Problem with Charges Levied Against the Balance. Management Science. 18, 600-602.

Puterman, M.L. (1975) A Diffusion Process Model for a Storage System. Logistics. 1, 143-159.

Richard, S.F. (1977) Optimal Impulse Control of a Diffusion Process with Both Fixed and Proportional Costs of Control. SIAM Journal of Control and Optimization. 15, 79-91.

Roberts, S.M. (1971) A Stochastic Cash Balance Inventory Model with Non-Zero Fixed and Proportional Transfer Costs and Proportional Opportunity and Penalty Costs. Special Studies Paper Number 13, Division of Research and Statistics, Federal Reserve Board, Washington, D.C.

Rosenberg, B. and Ohlson. J.A. (1976) The Stationary Distribution of Returns and Portfolio Separation in Capital Markets: A Fundamental Contradiction. Journal of Financial and Quantitative Analysis. 393-402.

Ross, S.A. (1978) Mutual Fund Separation in Financial Theory - The Separating Distributions. Journal of Economic Theory. 17, 254-286.

Rotemberg, J.J. (1984) A Monetary Equilibrium Model with Transactions Costs. Journal of Political Economy. 92, 40-58.

Rothschild, M. (1971) On the Cost of Adjustment. Quarterly Journal of Economics. 85, 605-622.

Sargan, J.D. (1976) Some Discrete Approximations to Continuous Time Stochastic Models. chapter 3 in A.R. Bergstrom, ed. (1976).

Sastry, A.S.R. (1970) The Effect of Credit on Transactions Demand for Cash. Journal of Finance. 25, 777-781.

Savage, I.R. (1962) Surveillance Problems. Naval Research Logistics Quarterly. 9, 187-209.

Scarf, H. (1960) The Optimality of (S,s) Policies in the Dynamic Inventory Problem. 196-204 in K.J. Arrow et al eds., Mathematical Methods in the Social Sciences, 1959. Stanford University Press.

Schmalensee, R. and Trippi, R.R. (1978) Common Stock Volatility Expectations Implied by Option Premia. Journal of Finance. 33, 129-147.

Sethi, S.P. (1971) A Note on a Planning Horizon Model of Cash Management. Journal of Financial and Quantitative Analysis. 6, 659-664.

Sethi, S.P. and Thompson, G.P. (1970) Applications of Mathematical Control Theory to Finance: Modeling Simple Dynamic Cash Balance Problems. Journal of Financial and Quantitative Analysis. 5, 381-394.

Shiller, R.J. (1973) A Distributed Lag Estimator Derived from Smoothness Priors. Econometrica. 41, 775-787.

Silver, E.A. (1981) Operations Research in Inventory Management: A Review and Critique. Operations Research. 29, 628-645.

Silver, E.A., Vidal, R.V.V., and de Werra, D. (1980) A Tutorial on Heuristic Methods. European Journal of Operations Research. 5, 153-162.

Sims, C.A. (1971a) Discrete Approximations to Continuous Time Distributed Lags in Econometrics. Econometrica. 39, 545-563.

_____ (1971b) Approximate Specification in Distributed Lag Models. Bulletin of the International Statistical Institute. 44, 285-294.

Smith, G.W. (1984) Dynamic Target-Threshold Models of the Demand for Money. unpublished M.Phil. thesis, University of Oxford.

_____ (1985) An Allusive Critique of the Buffer-Stock Notion in Monetary Economics. mimeo.

Sprenkle, C.M. (1969) The Uselessness of Transactions Demand Models. Journal of Finance. 24, 835-847.

_____ (1972) On the Observed Transactions Demand for Money. Manchester School. 40, 261-267.

Starr, R.M. (1983) Variation in the Term Structure of Asset Holding and Behavior of the Monetary Aggregates: The Maturity Shift

Hypothesis. Discussion Paper 82-9 Revised. University of California, San Diego.

Stockman, A.C. (1981) Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy. Journal of Monetary Economics. 8, 387-393.

Stone, B.K. (1972) The Use of Forecasts and Smoothing in Control-Limit Models for Cash Management. Financial Management. 1, 72-84.

Swamy, P.A.V.B., Tinsley, P.A., and Moore, G.R. (1982) An Autopsy of a Conventional Macroeconomic Relation: The Case of Money Demand. Special Studies Paper 167. Federal Reserve Board.

Taylor, A.J. (1977) Optimum Impulse Control of a Brownian Storage System. Working Paper 77-22. School of Business, Queens's University, Kingston.

Tinsley, P.A. (1967) An Application of Variable Weight Distributed Lags. Journal of the American Statistical Association. 62, 1277-1289.

Tobin, J. (1956) The Interest Elasticity of the Transactions Demand for Cash. Review of Economics and Statistics. 38, 241-247.

_____ (1958) Liquidity Preference as Behavior Towards Risk. Review of Economic Studies. 25, 65-86.

Trivedi, P.K. (1985) Distributed Lags, Aggregation, and Compounding: Some Econometric Implications. Review of Economic Studies. 52, 19-35.

Tsiang, S.C. (1969) The Precautionary Demand for Money: An Inventory Theoretical Analysis. Journal of Political Economy. 77, 99-117.

Turk, C. (1984) Uncertainty and the Demand for Money. Working Paper Number 644. Centre for Labour Economics, London School of Economics.

Vasicek, O. (1977) An Equilibrium Characterization of the Term Structure. Journal of Financial Economics. 5, 177-188.

Vial, J.-P. (1972) A Continuous Time Model for the Cash Balance Problem. 244-291 in G.P. Szegö and K. Shell, eds. Mathematical Methods in Investment and Finance. Amsterdam: North-Holland.

Vind, K. (1967) Control Systems with Jumps in the State Variables. Econometrica. 35, 273-277.

Waud, R.N. (1968) Misspecification in the 'Partial Adjustment' and 'Adaptive Expectations' Model. International Economic Review.

9, 204-217.

Wei, W.W.S. (1978) The Effect of Temporal Aggregation on Parameter Estimation in Distributed Lag Models. Journal of Econometrics. 8, 237-246.

Whalen, E.L. (1966) A Rationalization of the Precautionary Demand for Cash. Quarterly Journal of Economics. 80, 314-324.

Zabel, E. (1973) Consumer Choice, Portfolio Decisions, and Transaction Costs. Econometrica. 41, 321-335.

Ziemba, W.T. and Vickson, R,G. (1975) Stochastic Optimization Models in Finance. Academic Press, New York.