UNCERTAINTY, INVESTMENT AND PRODUCTIVITY WITH RELATIONAL CONTRACTS

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Abstract
With relational contracts, increased uncertainty with no change in factor prices is shown to reduce investment in the long run even if the parties are risk-neutral. This contrasts with models based on the impact of financial risk on the cost of capital and on the option value arising from irreversible investment. For the latter, Bloom et al. (*Econometrica*, 2018) find that a negative first-moment shock, in addition to increased uncertainty, best matches the data. This paper develops a relational contract model to demonstrate the impact of uncertainty on investment, depending on whether investment is general or specific. It then uses a specification calibrated with parameters from Bloom et al. (*Econometrica*, 2018) to show that this model can generate effects on productivity and investment of the magnitude of the negative aggregate shock in that paper purely with an increase in uncertainty. (JEL: C73, D82, D86, E22, E32)

Teaching Slides
A set of Teaching Slides to accompany this article is available online as Supplementary Data.

1. Introduction

Relational contracts, arrangements between parties for which the ongoing relationship between them plays an essential role in determining outcomes, have proved an

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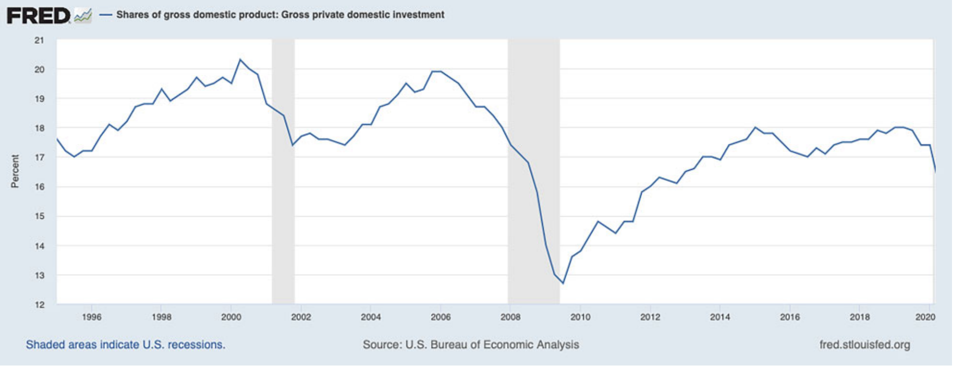

FIGURE 1. US real gross private domestic investment, percentage of GDP.
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insightful way to view a variety of economic relationships.[1] This paper investigates what they can contribute to understanding the relationship between uncertainty, capital investment, and productivity. It first shows that, even with risk-neutral parties and no change in factor prices, uncertainty affects long-run equilibrium capital investment. It then uses a calibrated specification based on Bloom et al. (2018) to illustrate how relational contracts can help resolve empirical issues in the relationship between them.

Capital investment falls in recessions. Figure 1, in which shaded areas show designated periods of recession, illustrates this. Bloom (2009) provides evidence that uncertainty has an impact on production and employment; Bloom (2014) documents the evidence that commonly used measures of uncertainty are higher in recessions. Bloom et al. (2022) document that such measures correspond to managers' subjective assessments of uncertainty and that investment is negatively related to those subjective measures. Two recent approaches to understanding these relationships are based on the option value of delaying irreversible investment and on the impact of financial risk on the cost of capital. For examples, see Bloom et al. (2018) and Fernández-Villaverde and Guerrón-Quintana (2020) for the former, Caballero and Simsek (2020) and Pflueger, Siriwardane, and Sunderam (2020) for the latter. In the option value approach in Bloom et al. (2018), firms are risk neutral and so unconcerned about the extent of productivity and demand uncertainty in the long run. However, with irreversible investment, the option value to delay increases with uncertainty, as demonstrated by Dixit and Pindyck (1994). In the cost of capital approach, perception of high financial risk results in a low value for risky assets and thus a high cost of capital for risky firms. Earlier literature stemming from Kydland and Prescott (1982) emphasized the time taken to build capital equipment.

The time-to-build and option value approaches are essentially ones of adjustment costs. With risk-neutral firms, investment eventually returns to the same share of GDP even if increased uncertainty persists. With the calibration in Bloom (2009), nearly all

1. See Malcomson (1999) for applications of relational contracts to employment and Malcomson (2013) for applications to supply relationships.

the adjustment is completed after 36 months. However, in a general equilibrium setting, Bloom et al. (2018) show that, on data up to 2010, the increase in uncertainty itself is insufficient to capture the drop in investment and productivity in recessions. A negative aggregate shock to total factor productivity (a first-moment shock, not just a second-moment shock) is required for that, which Bloom et al. (2018, pp. 1033–4) describe as "controversial, as it suggests that recessions are times of technological regress." This is illustrated for the 2008–2009 recession in Figure 1. Although the investment share increased after 2009, it remained below its average for 1995–2007 up to 2020 which, with a pure option value effect, would require continually increasing uncertainty. The data for 2010–2019 in Ahir, Bloom, and Furceri (2022, p. 104) do not indicate that.

Pflueger, Siriwardane, and Sunderam (2020) measure perceptions of financial risk by the inverse of the price of volatile stocks, defined as the book-to-market ratio of low-volatility stocks minus the book-to-market ratio of high-volatility stocks, and show that this is inversely related to real investment. However, in their formulation, a higher-risk distribution again has only a short-run effect on investment and so would not substitute for the first-moment shock in Bloom et al. (2018).

What the present paper adds is to show that, in the presence of relational contracts, greater output or revenue uncertainty *per se* (i.e. with no first-moment shock and no change in the cost of capital) affects long-run equilibrium investment even when the parties are jointly risk-neutral. Moreover, with a production framework similar to Bloom et al. (2018) but with the option value of delaying investment replaced by the constraints arising from a relational contract and the calibrated parameters in that paper, it is straightforward to generate effects on general investment of the magnitude of the negative first-moment shock in that paper purely with greater uncertainty. This raises the possibility that these real business cycle models could be freed from unattractive negative aggregate technology shocks.

In the standard model of relational incentive contracts between a principal and an agent, for example, MacLeod and Malcomson (1989), the agent chooses a level of performance each period (typically referred to as *effort* in the literature) that affects the payoff to the principal. However, agent performance is non-contractible in the sense that payment cannot be conditioned on performance in an enforceable legal agreement. This may be either because performance cannot be verified by third parties in court or because it is too complicated to describe in a legally enforceable way for that to be worthwhile. Whatever the reason, conventional performance-related payments are not available. Instead, incentives for performance are provided by a combination of bonuses related to output that, while not legally enforceable, the principal nevertheless finds it worthwhile to pay and the future payoff from continuation of the relationship. A central question in the literature concerns what payment and performance the parties will deliver even though they are not legally obliged to. To that standard model, the present paper adds two things. The first is that the value of the agent's performance to the principal is uncertain because it is affected by an *iid* shock each period. The second is that the parties may invest in capital that enhances the productivity of the relationship.

The main theoretical results in the paper are the following. The non-contractibility of performance, and the consequent use of a relational contract, places an upper limit on the agent's performance which is more restrictive with favorable shocks that increase the productivity of effort. Under plausible conditions, an increase in risk as measured by second-order stochastic dominance lowers this upper limit and thus, even with risk-neutral parties, affects the expected return on capital investment. With investments that are general in the sense of Becker (1975), this unambiguously reduces investment. Investments that are specific in the sense of Becker (1975), however, increase the upper limit on performance, which mitigates the negative impact of an increase in risk. It may even increase investment, a result of particular interest given the observation in Bloom (2014) that uncertainty stimulates some types of investments.

The paper is organized as follows. The next section discusses related literature. Section 3 sets out the model and the assumptions used for the analysis. Section 4 analyzes optimal effort in a relational contract in this setting. Section 5 studies the effect of risk on the returns to the relationship for given capital stock. Section 6 studies the effect of risk on optimal investment in capital. Section 7 develops a specification of the model suitable for calibration and provides results from that. Section 8 contains concluding remarks. Proofs of results are in Appendix A. Online appendices provide details of derivations and calculations for the calibration exercise.

2. Related Literature

The literature on the option value associated with uncertainty and irreversible investment stems from Dixit and Pindyck (1994). The theoretical implications for aggregate investment are set out in Abel and Eberly (1996). The empirical implementations closest to the present paper are in Bloom, Bond, and Van Reenen (2007), Bloom (2009), and Bloom et al. (2018). Many other contributions on capital adjustment costs are discussed in the survey by Bloom (2014). Fernández-Villaverde and Guerrón-Quintana (2020) provide a more recent review of the mechanisms that researchers have postulated to link uncertainty shocks and business cycles. Among the papers not discussed there are Bond, Söderbom, and Wu (2011), Sedláček (2020), and Berger, Dew-Becker, and Giglio (2020). The last of these argues that innovations in realized stock market volatility, but not in forward-looking uncertainty, are robustly followed by economic contractions but these are not distinguishable in the model used here. Angeletos, Collard, and Dellas (2020) appraise different business-cycle drivers.

In the relational contracts literature, Malcomson (2015b) analyzes the effect of productivity shocks, and Li and Matouschek (2013) the effect of shocks to the opportunity cost to the principal, but the models used there have no investment. Malcomson (2015a) studies investment but without productivity shocks, as does Garicano and Rayo (2017) for investment in human capital in the form of knowledge transfer. Ramey and Watson (1997) study the impact of productivity shocks on specific (but not general) investment with productivity restricted to just two possible levels. Halac (2015) similarly considers only (implicitly) specific investment with binary productivity levels and the principal-agent example in Goldlücke and Kranz (2018) only binary productivity levels in a very special setting. Fahn, Merlo, and Wamser (2017) and Fahn, Merlo, and Wamser (2019) consider the implications of relational contracts for firm capital structure. Englmaier and Fahn (2019) consider their implications for investments in liquidity-generating capital on the ability of firms to meet their financial commitments. None of these investigate the impact of uncertainty on investment of the type illustrated in Figure 1.

3. The Model

The model used here is that of MacLeod and Malcomson (1989) with the two additions specified in the Introduction. First, to allow for risk, the productivity of the agent's effort is subject to an *iid* shock each period. Second, the parties may make a capital investment that enhances the productivity of the relationship.

A principal uses an agent to perform a specified task each period. Both are risk-neutral and discount the future with the same discount factor $\delta \in (0, 1)$. The relationship between the two can, in principle, continue indefinitely. The value of output from the match in period t is $y(e_t, K, \theta_t)$, where $e_t \in [0, \bar{e}]$ is the agent's effort at t, $K \in [0, \bar{K}]$ is the capital stock, and $\theta_t \in [\underline{\theta}, \bar{\theta}]$ is an *iid* random shock distributed according to the distribution $F(\theta, \sigma)$ that admits an everywhere positive density, with $\sigma \in \Sigma$ a parameter that determines its riskiness. Both parties observe the shock θ_t at the start of period t, as well as observing e_t and K; there is no asymmetric information. The shock can be to productivity or to the revenue from given output. Effort e_t in period t is chosen at cost $c(e_t)$ to the agent after θ_t is revealed and so can be conditioned on the shock. Neither the value of output nor effort is contractible in the sense that payment can be conditioned on performance in a formal legal agreement. This may be either because they cannot be verified by third parties or because it is too complicated to describe them in a legally enforceable way to be worthwhile. Effort can be thought of as anything unverifiable the agent may do that affects the payoff to the principal. In the context of employment, it could be literal effort. In the context of a supply chain, it could be the quality of the intermediate products supplied. In principle, it can be multidimensional.

For reasons given in the adjustment cost literature, it is implausible that capital can be fully adjusted to shocks in the short run so, because the concern here is with long-run equilibrium properties, investment in capital is assumed to take place at the beginning of the relationship, before any shock has been revealed, at a one-off cost $C(K)$. This cost is to be thought of as the present discounted cost of using capital K, including replacement investment.[2]

ASSUMPTION 1. The functions y, c and C have the following properties:

(1) $y(e, K, \theta)$ is strictly concave in (e, K) for all $\theta \in [\underline{\theta}, \bar{\theta}]$, is three times differentiable in its arguments with $y_1(e, K, \theta) > 0$, $y_2(e, K, \theta) > 0$, $y_3(e, K, \theta) > 0$, $y_{12}(e, K, \theta) > 0$ and $y_{13}(e, K, \theta) > 0$ for all $(e, K, \theta) \in (0, \bar{e}] \times [0, \bar{K}] \times [\underline{\theta}, \bar{\theta}]$;

2. For the calibration in Section 7, $C(K)$ is taken to be linear so, even if capital can be added each period, it is not optimal to build it up slowly over time.

it also has $y(0, K, \theta) = 0$ for all $(K, \theta) \in [0, \bar{K}] \times [\underline{\theta}, \bar{\theta}]$ and $y(e, 0, \theta) \geq 0$ for all $(e, \theta) \in [0, \bar{e}] \times [\underline{\theta}, \bar{\theta}]$;

(2) $c(e)$ is twice differentiable, with $c'(e) > 0$ and $c''(e) \geq 0$ for $e \in [0, \bar{e}]$ and $c(0) = 0$;

(3) $C(K)$ is twice differentiable, with $C'(K) > 0$ and $C''(K) \geq 0$ for $K \in [0, \bar{K}]$ and $C(0) = 0$;

(4) $y(e, K, \theta) - c(e) - C(K)$ is strictly increasing in e for $e > 0$ sufficiently small and strictly decreasing in e for e sufficiently close to $\bar{e}$ for $(K, \theta) \in (0, \bar{K}] \times [\underline{\theta}, \bar{\theta}]$; it is also strictly increasing in K for $K > 0$ sufficiently small and strictly decreasing in K for K sufficiently close to $\bar{K}$ for all $(e, \theta) \in (0, \bar{e}] \times [\underline{\theta}, \bar{\theta}]$.

Assumption 1 specifies conventional concavity/convexity conditions on functions to ensure that effort and capital maximizing the joint payoff to principal and agent are interior for all θ and thus avoid the complications of handling corner solutions. The signs of the cross-partial derivatives involving θ in the function y essentially order shocks so that a higher θ corresponds to a more favorable environment.

The payoff to the principal in period t of the match is $y(e_t, K, \theta) - W_t$, where W_t is the payment to the agent in period t. The payoff to the agent is $W_t - c(e_t)$. Because both are risk neutral, the joint payoff to them both from being matched in a period is just the sum of their individual payoffs. For a period in which the principal and agent are not matched, the principal's payoff is $\underline{v}(K, \sigma) \geq 0$ and the agent's payoff $\underline{u}(K, \sigma) \geq 0$, with $\underline{s}(K, \sigma) := \underline{u}(K, \sigma) + \underline{v}(K, \sigma) > 0$, the inequalities holding for all $K \in [0, \bar{K}]$. Conditional on (e, K, θ), the joint payoff to principal and agent from being matched in a period is $s(e, K, \theta) := y(e, K, \theta) - c(e)$. As a benchmark, if effort were contractible, it would be set at the first-best level for given (K, θ), denoted $e^*(K, \theta)$, that maximizes this joint payoff and is given by

$$y_1(e^*(K, \theta), K, \theta) = c'(e^*(K, \theta)). \tag{1}$$

Because investment in capital is decided at the beginning of the relationship before any shock is revealed, the first-best benchmark for capital stock if effort were contractible is

$$K^*(\sigma) \in \arg \max_{K \in [0, \bar{K}]} \frac{1}{1 - \delta} \int_{\underline{\theta}}^{\bar{\theta}} s(e^*(K, \theta), K, \theta) dF(\theta, \sigma) - C(K). \tag{2}$$

If $y(e_t, K, \theta_t)$ or e_t were contractible, it would be straightforward for the parties to agree a contract that would deliver effort schedule $e^*(K, \theta)$ and investment $K^*(\sigma)$. However, with neither contractible, effort above the minimum level $e = 0$ can be sustained only by a relational contract. Following MacLeod and Malcomson (1989), payment W_t to the agent has two components, a fixed component w_t that is guaranteed independent of performance in period t and a bonus component b_t that can be conditional on performance in period t. The bonus cannot be legally enforceable because performance is not contractible, so the relational contract must ensure that it is in the principal's interest to pay it.

4. Optimal Effort

Optimal effort in the model is a special case of that in Malcomson (2015b) so this section (but only this section) draws on results from there.[3] By an argument in (Levin 2003, Theorem 2) and extended in Goldlücke and Kranz (2018) that applies here, if an optimal relational contract exists, there are stationary contracts that are optimal. A stationary contract depends only on current payoff-relevant information, so effort and payment have the forms $e_t = e(K, \theta_t, \sigma)$, $w_t = w(K, \theta_t, \sigma)$ and $b_t = b(e_t, K, \theta_t, \sigma)$.

PROPOSITION 1. *An effort schedule $e(K, \theta, \sigma)$ that generates expected joint payoff $S(K, \sigma)$ each period with capital stock K and distribution σ can be implemented by a stationary contract if and only if*

$$\frac{\delta}{1-\delta}[S(K,\sigma) - \underline{s}(K,\sigma)] \geq c(e(K,\theta,\sigma)), \quad \textit{for all } \theta \in [\underline{\theta}, \bar{\theta}]. \tag{3}$$

The proof of Proposition 1 is a special case of that in Proposition 1 in Malcomson (2015b). In Proposition 1, $S(K, \sigma)$ is the expected joint payoff from one period of the relationship, given investment in capital K at the start, before the realization for that period of the shock θ from a distribution parameterized by σ. Thus, the left-hand side of (3) is the joint payoff *gain* to both parties from continuing the relationship in future periods. So (3) states that, for effort $e(K, \theta, \sigma)$ to be implementable, this joint gain must be at least as great as the cost to the agent of delivering effort $e(K, \theta, \sigma)$ in a period. This is a standard result for relational contracts with risk-neutral parties originally derived in MacLeod and Malcomson (1989). For further discussion of this condition and the intuition behind it, see Malcomson (2013). The following corollary is a straightforward consequence of Proposition 1.

COROLLARY 1. *Suppose, for given $S(K, \sigma)$, first-best effort $e^*(K, \theta)$ does not satisfy (3) for some $\theta' \in [\underline{\theta}, \bar{\theta}]$. Then first-best effort $e^*(K, \theta)$ does not satisfy (3) for any $\theta \in [\theta', \bar{\theta}]$.*

Corollary 1 is a special case of Proposition 2 in Malcomson (2015b). It follows from Proposition 1 because first-best effort given by (1) is increasing in θ and, hence, so is $c(e^*(K, \theta))$. However, because θ is an *iid* shock, the left-hand side of (3) is independent of θ because it is an expectation over future θ. So, with the left-hand side of (3) independent of θ and the right-hand side increasing in θ when effort is first best, the result in the corollary follows directly. To avoid complicating the exposition with less interesting cases, the following assumptions, which are satisfied by the calibrated version of the model in Section 7, are used in what follows.

3. In Malcomson (2015b), the agent has a type $a \in [\underline{a}, \overline{a}]$ that is unknown, but $\underline{a}$ and $\overline{a}$ are known, to the principal. The present model corresponds to $\underline{a} = \overline{a}$.

ASSUMPTION 2.

(1) For each $\sigma \in \Sigma$, there exists some $K \in (0, \bar{K}]$ and some $e(K, \theta, \sigma)$ with $e(K, \underline{\theta}, \sigma) = e^*(K, \underline{\theta})$ for which (3) is satisfied with strict inequality for some $\theta \in [\underline{\theta}, \bar{\theta}]$.
(2) For all $K \in [0, \bar{K}]$ and all $\sigma \in \Sigma$, (3) is not satisfied when $e(K, \theta, \sigma) = e^*(K, \theta)$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

Part 1 of this assumption rules out optimal effort involving full pooling over θ, Part 2 rules out optimal effort being first best for all θ. Those cases are covered in Malcomson (2021).

PROPOSITION 2. *If, for given (K, σ), there exists an effort schedule $e(K, \theta, \sigma)$ that implements $S(K, \sigma)$ satisfying (3), an optimal stationary effort schedule for (K, σ) takes the form that there exists $\hat{\theta}(K, \sigma) \in (\underline{\theta}, \bar{\theta})$ such that*

$$e(K, \theta, \sigma) = \begin{cases} e^*(K, \theta), & \text{for } \theta \in [\underline{\theta}, \hat{\theta}(K, \sigma)], \\ e^*(K, \hat{\theta}(K, \sigma)), & \text{for } \theta \in (\hat{\theta}(K, \sigma), \bar{\theta}), \end{cases}$$

with $\hat{\theta}(K, \sigma)$ the highest $\tilde{\theta} \in (\underline{\theta}, \bar{\theta})$ satisfying

$$\frac{\delta}{1-\delta} \left\{ \int_{\underline{\theta}}^{\tilde{\theta}} [s(e^*(K, \theta), K, \theta) - \underline{s}(K, \sigma)] dF(\theta, \sigma) + \int_{\tilde{\theta}}^{\bar{\theta}} [s(e^*(K, \tilde{\theta}), K, \theta) - \underline{s}(K, \sigma)] dF(\theta, \sigma) \right\} - c(e^*(K, \tilde{\theta})) = 0. \quad (4)$$

Proposition 2 establishes that optimal effort is the same for all θ above some cutoff value $\hat{\theta}(K, \sigma)$ and first-best for all θ below $\hat{\theta}(K, \sigma)$.[4] A proof is given in Appendix A because, although the result corresponds to Proposition 3 in Malcomson (2015b), the relationship is not straightforward. Intuitively, because shocks are *iid*, the left-hand side of (3), which is the joint payoff gain from continuing the relationship in the future, is independent of the current shock, as well as of current effort. So, for all current shocks for which first-best effort satisfies (3), first-best effort is optimal. For all other shocks, the highest effort that satisfies (3) is optimal and thus independent of the shock. In view of Corollary 1, this implies that there is a critical shock $\hat{\theta}(K, \sigma)$ such that effort is first best for all $\theta \leq \hat{\theta}(K, \sigma)$ and is independent of θ for all $\theta > \hat{\theta}(K, \sigma)$ at $e^*(K, \hat{\theta}(K, \sigma))$. (Because it is optimal to have first-best effort for as many shocks as

4. It can be shown that the result in Proposition 2 holds even if only the principal observes θ. In that case, the principal will reveal θ truthfully provided the expected payoff from doing so is non-decreasing in θ and this property is satisfied by optimal effort in the proposition. To show that formally, however, complicates the exposition and so is not done here.

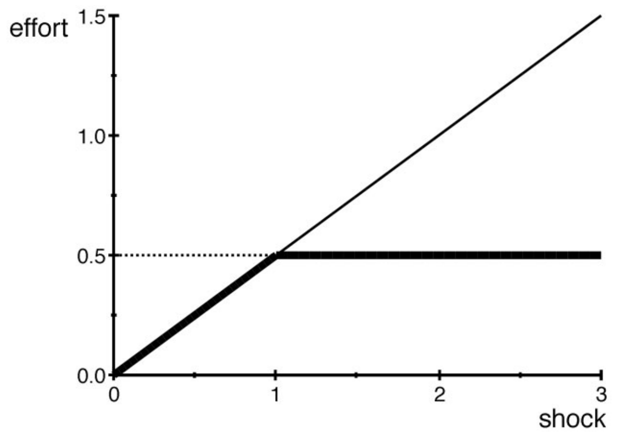

FIGURE 2. Effect of shock on effort.

possible, $\hat{\theta}(K,\sigma)$ is the highest θ that satisfies (4) if there is more than one.) Figure 2 illustrates this. In that, the thin solid line corresponds to first-best effort for given capital stock, the dashed line to the highest effort sustainable given total future payoff gain. So optimal effort is given by the bold line, with $\hat{\theta}(K,\sigma) = 1$. It follows from (4) that $\hat{\theta}(K,\sigma)$ satisfies

$$\frac{\delta}{1-\delta}\left\{\int_{\underline{\theta}}^{\bar{\theta}} s(e^*(K,\theta),K,\theta)dF(\theta,\sigma) - \underline{s}(K,\sigma)\right.$$
$$\left. - \int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}} [s(e^*(K,\theta),K,\theta) - s(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)]dF(\theta,\sigma)\right\}$$
$$= c(e^*(K,\hat{\theta}(K,\sigma))). \tag{5}$$

The second integral in (5) is strictly positive because $e^*(K,\theta)$ uniquely maximizes $s(e,K,\theta)$.[5]

5. The relational contract models in Baker, Gibbons, and Murphy (1994), Schmidt and Schnitzer (1995) and the literature stemming from these include a verifiable performance measure z on which legally enforceable payment can be conditioned in addition to unverifiable effort. The resulting relational contract constraint corresponding to (3) then permits, at least on average, higher effort for given $S(K,\sigma)$ and $\underline{s}(K,\sigma)$ which, in turn, results in higher $S(K,\sigma)$. However, the verifiable performance measure also increases $\underline{s}(K,\sigma)$ when it permits higher effort with an alternative principal, so the net result may be higher or lower unverifiable effort. See Malcomson (2013) for a fuller explanation. Intuitively, as long as the right-hand side of the constraint corresponding to (3) constrains effort further below first best for higher θ, there will still be a cutoff $\hat{\theta}(z,K,\sigma)$ with effort at first best for $\theta \le \hat{\theta}(z,K,\sigma)$ and effort at $e^*(z,K,\hat{\theta}(z,K,\sigma))$ for $\theta > \hat{\theta}(z,K,\sigma)$.

5. The Effect of Risk on Effort

A novel contribution of this paper concerns the effect of risk on investment via its impact on the relational contract constraint and hence on effort. A conventional way to compare riskiness of distributions is in terms of second-order stochastic dominance. The standard definition of second-order stochastic dominance specifies the sign of the integral of the difference between two distributions for all values of the *upper* limit of integration. However, comparison of the expression in (5) for different σ requires comparison of the second integral for variable *lower* limit of integration. This requires a modified definition of second-order stochastic dominance.

DEFINITION 1. *$F(\theta, \sigma_L)$ dominates $F(\theta, \sigma_H)$ in the second-order stochastic sense for $\tilde{\theta} \in [\underline{\theta}, \bar{\theta}]$* if

$$\int_{\theta}^{\bar{\theta}} [F(x, \sigma_L) - F(x, \sigma_H)] dx \geq 0, \qquad \text{for all } \theta \in [\underline{\theta}, \bar{\theta}],$$

with strict inequality for a set of values of $x \in [\tilde{\theta}, \bar{\theta}]$ with positive probability.

This definition corresponds to the standard definition when $F(\theta, \sigma_L)$ has the same mean as $F(\theta, \sigma_H)$ and $\tilde{\theta} = \underline{\theta}$ (in which case the integral in Definition 1 equals zero for $\theta = \underline{\theta}$, see Laffont (1989, p. 25)). It then corresponds to a mean-preserving spread in the sense of Rothschild and Stiglitz (1970). The extension to $\tilde{\theta} > \underline{\theta}$ is required to handle the second integral in (5) that has lower limit of integration greater than $\underline{\theta}$.

The first result here is a straightforward adaptation to Definition 1 of the result for the standard definition of second-order stochastic dominance that a risk-averse agent prefers $F(\theta, \sigma_L)$ to $F(\theta, \sigma_H)$ if the former stochastically dominates the latter in the second-order sense. The proof is a straightforward adaptation of the standard proof for the case $\tilde{\theta} = \underline{\theta}$ in Laffont (1989, pp. 32–33).

PROPOSITION 3. *Suppose a twice-differentiable function $g(\theta)$ is non-increasing and concave on $[\tilde{\theta}, \bar{\theta}]$ for some $\tilde{\theta} \in [\underline{\theta}, \bar{\theta})$ and $g(\tilde{\theta}) = 0$ if $\tilde{\theta} > \underline{\theta}$. Then, if $F(\theta, \sigma_L)$ dominates $F(\theta, \sigma_H)$ in the second-order stochastic sense for $\tilde{\theta} \in [\underline{\theta}, \bar{\theta}]$ and the two distributions have the same mean,*

$$\int_{\tilde{\theta}}^{\bar{\theta}} g(\theta) dF(\theta, \sigma_L) \geq \int_{\tilde{\theta}}^{\bar{\theta}} g(\theta) dF(\theta, \sigma_H),$$

with the inequality strict if $g(\theta)$ is strictly concave on $[\tilde{\theta}, \bar{\theta}]$.

To apply the result in Proposition 3, define

$$h(e, K, \theta) := s(e, K, \theta) - s(e^*(K, \theta), K, \theta), \qquad \text{for } e \in [0, e^*(K, \theta)]. \tag{6}$$

This is the difference in the joint payoff in a period for effort e less than first best and that for first-best effort for given K and θ, the negative of the integrand in the second integral in (5). Clearly, $h(e, K, \theta)$ is zero for $e = e^*(K, \theta)$. Lemma A.1 in Appendix A shows that it is decreasing in θ, and gives conditions under which it is strictly concave in θ, for every K and θ. Under those conditions, Proposition 3 can be used to compare the second integral in (5) for $\sigma = \sigma_H$ with that for $\sigma = \sigma_L$. Since comparison of the first integral gives the effect when effort is contractible, this provides a measure of the difference in the response to risk when effort is not contractible from when it is.

With relational contracts, it is important to distinguish between different ways in which changes in risk affect the relationship. To define these formally, let $S^*(K,\sigma)$ be the expected joint payoff each period before the realization for that period of the shock θ from adopting the optimal effort schedule in Proposition 2.

DEFINITION 2. Risk is *specific* if $\underline{s}(K,\sigma)$ is independent of σ for all $K \in [0, \bar{K}]$. Risk is *systemic* if $S^*(K,\sigma) - \underline{s}(K,\sigma)$ is independent of σ for all $K \in [0, \bar{K}]$.

The definition of specific risk captures changes in risk that affect only a particular relationship and not the opportunities $\underline{s}(K,\sigma)$ available if the parties separate. An example would be a change in the distribution of prices for the product of employee skills that are valuable only with the current employer. However, some changes in risk will affect opportunities available if the parties separate. For example, a change in the distribution of prices in a competitive industry that affects an employer with both the current and an alternative employee. The definition of systemic risk captures a form of this that is analytically valuable for illustrating essential insights. In particular, it captures the idea that, if the parties respond to a change in risk by optimally changing the equilibrium effort schedule, the change affects their joint payoff the same whether they continue the relationship or separate. There can obviously be changes in risk for which neither $\underline{s}(K,\sigma)$ nor $S^*(K,\sigma) - \underline{s}(K,\sigma)$ is independent of σ (and these are allowed for in the calibration exercise in Section 7) but the two cases in Definition 2 are helpful for clarifying the channels through which changes in risk affect a relational contract.

PROPOSITION 4. *Consider two distributions for θ, $F(\theta,\sigma_L)$ and $F(\theta,\sigma_H)$.*

(1) *When risk is systemic: $e(K,\theta,\sigma_H) = e(K,\theta,\sigma_L)$ for all $(K,\theta) \in [0,\bar{K}] \times [\underline{\theta},\bar{\theta}]$.*
(2) *When risk is specific, $F(\theta,\sigma_L)$ dominates $F(\theta,\sigma_H)$ in the second-order stochastic sense for $\hat{\theta}(K,\sigma_H) \in [\underline{\theta},\bar{\theta}]$, and the two distributions have the same mean: for all $K \in [0,\bar{K}]$ such that $h(e^*(K,\hat{\theta}(K,\sigma_H)),K,\theta)$ is strictly concave in θ for $\theta \in [\hat{\theta}(K,\sigma_H),\bar{\theta}]$ and*

$$\int_{\underline{\theta}}^{\bar{\theta}} s(e^*(K,\theta),K,\theta)dF(\theta,\sigma_L) \geq \int_{\underline{\theta}}^{\bar{\theta}} s(e^*(K,\theta),K,\theta)dF(\theta,\sigma_H), \qquad (7)$$

then $\hat{\theta}(K,\sigma_H) < \hat{\theta}(K,\sigma_L)$ and the joint payoff is lower for σ_H than for σ_L; moreover, there is a larger difference for σ_H than for σ_L between the joint payoff when effort is contractible and when it is not.

Proposition 4 establishes that systemic risk has no impact on optimal effort for given capital stock. For specific risk, this is not the case even when (7) holds with equality, which corresponds to the parties being jointly risk neutral in the following sense. When (7) holds with equality, the sum of their payoffs would be the same for σ_H as for σ_L if effort were verifiable, as a result of which they could implement first-best effort for every θ. The constraints required to make the contract self-enforcing then result in the parties receiving a higher joint payoff from a less risky distribution. In this respect, it is as if they were risk averse. Because with specific risk $\hat{\theta}(K,\sigma_H) < \hat{\theta}(K,\sigma_L)$, effort is at first-best level for a larger interval of θ for σ_L than σ_H. Moreover, because first-best effort is strictly increasing in θ, $e^*(\hat{\theta}(K,\sigma_L),K) > e^*(\hat{\theta}(K,\sigma_H),K)$, so those θ for which effort is below first-best under σ_L have higher effort than under σ_H. Thus, for all θ with effort below first-best under σ_H, effort is strictly higher under σ_L.

6. Optimal Capital

Optimal capital when effort is contractible is $K^*(\sigma)$ given by (2). When effort is non-contractible, optimal capital is the solution to

$$\max_{K\in[0,\bar{K}]} \frac{1}{1-\delta}\left\{\int_{\underline{\theta}}^{\hat{\theta}(K,\sigma)} s(e^*(K,\theta),K,\theta)dF(\theta,\sigma) + \int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}} s(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)dF(\theta,\sigma)\right\} - C(K)$$

$$\text{with } \hat{\theta}(K,\sigma) \text{ being the highest value of } \tilde{\theta} \text{ satisfying (4).} \tag{8}$$

Denote optimal capital with non-contractible effort by $\widehat{K}(\sigma)$.

Becker (1975) made the distinction between investment in specific capital that is valuable only with a specific agent and investment in general capital that is equally valuable with other agents. In the context of relational contracts, this distinction has the additional importance, noted by Klein and Leffler (1981), that investment in specific capital has the effect of relaxing the relational contract constraint (3). For the model here, the distinction is captured by the following definition.

DEFINITION 3. Capital is *general* if the optimal effort schedule in Proposition 2 generates $S^*(K,\sigma)$ such that $S^*(K,\sigma) - \underline{s}(K,\sigma)$ is independent of K for all $\sigma \in \Sigma$. Capital is *specific* if $\underline{s}(K,\sigma)$ is independent of K for all $\sigma \in \Sigma$.

There are, of course, intermediates between these two extremes in which $\underline{s}(K,\sigma)$ increases with K but not by as much as $S^*(K,\sigma)$. The present paper follows the literature in focussing on the two cases identified by Becker (1975).

6.1. General Capital

For general capital, the next proposition shows that optimal capital stock with non-contractible effort, denoted $\widehat{K}^G(\sigma)$, is lower than with contractible effort.

PROPOSITION 5. *When capital is general,* $\widehat{K}^G(\sigma) < K^*(\sigma)$.

This result is perhaps not surprising. When effort is not contractible, it is constrained below the first-best schedule for given capital for some values of θ. Thus, given the complementarity of effort and capital in producing output in Assumption 1, the marginal product of capital averaged over θ is lower than when effort is contractible and capital is, therefore, less valuable. The next result concerns the effect of risk on general capital.

PROPOSITION 6. *Suppose capital is general, the conditions of Proposition 4 hold, and the derivative* $h_2(e^*(K, \hat{\theta}(K, \sigma_H)), K, \theta)$ *is decreasing and strictly concave in* θ *for* $\theta \in [\hat{\theta}(K, \sigma_H), \bar{\theta}]$. *Then with either systemic or specific risk:*

(1) $K^*(\sigma_H) - \widehat{K}^G(\sigma_H) > K^*(\sigma_L) - \widehat{K}^G(\sigma_L)$;
(2) *when the parties are jointly risk neutral* ((7) *holds with equality),* $\widehat{K}^G(\sigma_H) < \widehat{K}^G(\sigma_L)$.

This result establishes that, under the conditions specified, a more risky distribution increases the difference between optimal general capital when effort is contractible and when effort is not contractible. This applies even to increases in systemic risk which, by Proposition 4, do not affect effort for given capital stock. The intuition can be seen from Figure 2, in which the thin solid line corresponds to first-best effort and the bold line to optimal effort with a relational contract. For given capital, the latter results in a lower return to investment for shocks $\theta > 1$ and greater risk corresponds to a higher probability of such shocks. That lowers the joint payoff to continuation of the relationship in the future which, with the relational contract constraint (3), lowers the highest effort sustainable in the present and thus moves the kink point in the bold line in Figure 2 down the thin solid line. With effort and capital complementary in producing output, this reduces the return on capital. Formally, the second integral in the maximand in (8) makes the objective function more concave than when effort choice is unconstrained, which makes the parties jointly more risk averse. This effect is exacerbated with increases in specific risk that, again by Proposition 4, also lower $\hat{\theta}(K, \sigma)$ so that the interval of θ for which effort is first best is reduced. Lemma A.1 in Appendix A gives a sufficient condition for $h_2(e, K, \theta)$ everywhere decreasing in θ that holds for the specification in the calibration in Section 7.

6.2. Specific Capital

By Definition 3, additional specific capital increases $S(K, \sigma)$ for any given effort schedule while leaving $\underline{s}(K, \sigma)$ unchanged. It thus relaxes the relational contract constraint (3). Malcomson (2015a) showed that, in the absence of productivity shocks, the optimal level of capital is higher when capital is specific than when it is general whenever the relational contract constraint is binding. The formal result for the present model with productivity shocks, with $\hat{K}^S(\sigma)$ denoting optimal specific capital, is as follows.

PROPOSITION 7. *$\hat{K}^S(\sigma) > \hat{K}^G(\sigma)$ and, for $y_{12}(e, K, \theta)$ sufficiently small for all $(e, K, \theta) \in (0, \bar{e}] \times [0, \bar{K}] \times [\underline{\theta}, \bar{\theta}]$, $\hat{K}^S(\sigma) > K^*(\sigma)$.*

Proposition 7 establishes not only that, with non-contractible effort, optimal specific capital is higher than optimal general capital but also that, under certain circumstances, it is higher even than the optimal level when effort is contractible. This occurs when the reduced return that results from the non-contractibility of effort is more than offset by the relaxation of the relational contract constraint. The next result concerns the effects of riskiness on specific capital.

PROPOSITION 8. *Suppose the conditions of Proposition 4 hold. Then with either systemic or specific risk, for $y_{133}(e, K, \theta) \geq 0$ for all $(e, K, \theta) \in (0, \bar{e}] \times [0, \bar{K}] \times [\underline{\theta}, \bar{\theta}]$, $\hat{K}^S(\sigma_H) - \hat{K}^G(\sigma_H) > \hat{K}^S(\sigma_L) - \hat{K}^G(\sigma_L)$ and, for $y_{12}(e, K, \theta)$ also sufficiently small for all $(e, K, \theta) \in (0, \bar{e}] \times [0, \bar{K}] \times [\underline{\theta}, \bar{\theta}]$, $\hat{K}^S(\sigma_H) - K^*(\sigma_H) > \hat{K}^S(\sigma_L) - K^*(\sigma_L)$.*

Proposition 8 gives conditions under which a more risky distribution results in a greater difference between optimal specific and optimal general capital. It may even result in a greater difference between optimal specific capital when effort is not contractible than when effort is contractible, in which case greater uncertainty results in more investment when the parties are jointly risk neutral. As with Proposition 7, the reason is that additional specific capital relaxes the relational contract constraint (3). When a change in σ reduces the joint payoff $S^*(K, \sigma)$ as in Proposition 4, it tightens the constraint (3) and so increases the return to specific capital that relaxes that constraint. It thus results in higher specific capital. The condition $y_{133}(e, K, \theta) \geq 0$ is sufficient but not necessary. It holds for the example $y(e, K, \theta) = \theta \hat{y}(e, K)$.

The overall conclusions from this section are as follows. With general capital, optimal capital when effort is non-contractible is less than when effort is contractible. Moreover, increased risk increases the difference. However, with specific capital, increased capital relaxes the relational contract constraint and so optimal capital is higher than if capital were general and, in some cases, higher even than if effort were contractible. Moreover, the difference can actually be greater with a more risky distribution than with a less risky one.

7. A Calibrated Model

The previous section has shown that, with a relational contract, a more risky distribution can have lower optimal capital than a less risky one even when the parties are jointly risk neutral. The purpose of this section is to show that the relational contract framework has the potential to resolve a puzzle raised by the results in Bloom et al. (2018) for which there seems, as yet, no other satisfactory resolution. It is not intended as an empirical test of the underlying relational contract framework, for which there is an extensive evidence discussed in Malcomson (1999), Malcomson (2013), and subsequent literature.

For their model without a relational contract constraint, Bloom et al. (2018) find that a long-run fall in productivity attributed to a negative first-moment shock (specifically, a fall in the mean of total factor productivity (TFP)) that accompanies a negative second-moment shock best fits the data. This section shows that addition of a relational contract constraint enables this fall in productivity to be accounted for entirely by the negative second-moment shock, without a negative first-moment shock. It first applies the functional forms in Bloom et al. (2018) to the relational contract model used here. It then derives theoretical conditions under which the relational contract specification with different risk distributions with the same mean can match the productivity effects of a change in risk distributions with different means in the absence of the relational contract constraint. It ends by using the parameter calibrations in Bloom et al. (2018) to investigate the characteristics of the relational contract specification that match the long-run fall in TFP in Bloom et al. (2018).

The comparisons of the effects of different levels of risk in the previous section are for long-run equilibrium values, not for the adjustment path after a change in the level of risk (which is far from straightforward under relational contracts). So the comparison here is purely with the long-run characteristics of the model in Bloom et al. (2018), using a specification with the same long-run functional forms and calibrated parameter values. Moreover, in the absence of the relational contract constraint, it has the same long-run characteristic that higher risk with the same mean would have no long-run effect on capital and average productivity.

The precise specification, referred to here as the *Cobb–Douglas specification*, is as follows:

$$y(e, K, \theta) = \theta^{\gamma} K^{\alpha} e^{\beta}, \quad \alpha, \beta, \gamma > 0, \quad \alpha + \beta \leq 1; \tag{9}$$

$$c(e) = ce^{n}, \quad c > 0, n \geq 1, n > \beta/(1-\alpha); \tag{10}$$

$$C(K, \sigma) = C(\sigma)K^{k}, \quad C(\sigma) > 0, k \geq 1. \tag{11}$$

The value of output function in (9) is that in Bloom et al. (2018) with the labor hours input substituted by the agent's effort. In the absence of the relational contract constraint, agent effort and labor hours play exactly the same role, so this substitution makes no difference. Bloom (2009) shows how the exponents γ, α, and β relate to those in an underlying Cobb–Douglas production function and an isoelastic demand curve for the product. Bloom et al. (2018) use labor-hours cost and capital cost functions that are linear in long-run equilibrium, both with the addition of adjustment costs. Here the former is replaced by the effort cost function (10), the latter by (11), which also allow for convex costs. The restriction $n > \beta/(1-\alpha)$ ensures that optimal capital is strictly interior to $[0, \bar{K}]$ and is satisfied by the calibrated parameter values in Bloom et al. (2018). In previous sections, the concern was with an individual relationship for which the market price of capital was independent of σ. However, the analysis in Bloom et al. (2018) is for the whole economy, in which case the cost of capital term C may well change with σ. In the comparison of long-run equilibria, there is no role for adjustment costs.

For this specification, first-best effort conditional on K and θ is

$$e^*(K,\theta) = \left(\frac{\beta}{nc}\theta^{\gamma} K^{\alpha}\right)^{\frac{1}{n-\beta}}. \tag{12}$$

(Derivations for this section are given in Online Appendix C.) Then

$$s(e^*(K,\theta),K,\theta) = \left(1-\frac{\beta}{n}\right)\left(\frac{\beta}{nc}\right)^{\frac{\beta/n}{1-\beta/n}} \theta^{\frac{\gamma}{1-\beta/n}} K^{\frac{\alpha}{1-\beta/n}}, \tag{13}$$

which is concave in θ for $\gamma \leq 1-\beta/n$, and

$$s(e^*(K,\tilde{\theta}),K,\theta) = \left(\frac{\beta}{nc}\right)^{\frac{\beta/n}{1-\beta/n}} \tilde{\theta}^{\frac{\gamma}{1-\beta/n}} K^{\frac{\alpha}{1-\beta/n}} \left[\left(\frac{\theta}{\tilde{\theta}}\right)^{\gamma} - \frac{\beta}{n}\right]. \tag{14}$$

Expected revenue per agent (average labor productivity in the case of employment) is

$$K^{\frac{\alpha}{1-\beta/n}} \left(\frac{\beta}{nc}\right)^{\frac{\beta/n}{1-\beta/n}} \left[\int_{\underline{\theta}}^{\tilde{\theta}} \theta^{\frac{\gamma}{1-\beta/n}} dF(\theta,\sigma) + \tilde{\theta}^{\gamma\frac{\beta/n}{1-\beta/n}} \int_{\tilde{\theta}}^{\bar{\theta}} \theta^{\gamma} dF(\theta,\sigma)\right]. \tag{15}$$

Bloom (2009, footnote 17), following a discussion in Abel and Eberly (1996), argues that it is appropriate to calibrate the model to avoid long-run effects of uncertainty reducing or increasing output, which corresponds here to setting $\gamma = 1-\beta/n$. In the absence of the relational contract constraint (in which case $\tilde{\theta} = \bar{\theta}$), average productivity given by (15) is, for given K, the same for different distributions of θ that have the same mean.

Rather than solve the relational contract model directly for optimal capital, $\widehat{K}^G(\sigma)$ and $\widehat{K}^S(\sigma)$, it is convenient to solve for the optimal cutoff values $\hat{\theta}(\widehat{K}^i(\sigma),\sigma)$, $i = G, S$, below which effort, by Proposition 2, is first best when capital is chosen optimally. (Among other things, it makes it straightforward to impose the constraint $\hat{\theta}(\widehat{K}^i(\sigma),\sigma) \in [\underline{\theta}, \bar{\theta}]$.) Denote these cutoff values by $\hat{\theta}^G(\sigma)$ and $\hat{\theta}^S(\sigma)$ for general and specific capital, respectively.

When capital is general, $S^*(K,\sigma) - \underline{s}(K,\sigma)$ in Proposition 1 is by Definition 3 independent of K, so it is convenient to define

$$\hat{S}(\sigma) = \frac{\delta}{1-\delta}[S^*(K,\sigma) - \underline{s}(K,\sigma)]. \tag{16}$$

It can then be shown that the first-order condition for optimal capital to yield $\hat{\theta}^G(\sigma)$ interior to $[\underline{\theta}, \bar{\theta}]$ is

$$(1-\delta)\frac{k}{\alpha}\left(\frac{\beta}{n}\right)^{1-\frac{k}{\alpha}} c^{\frac{k}{\alpha}\frac{\beta}{n}} C(\sigma) - \left[\frac{\hat{\theta}^G(\sigma)}{\hat{S}(\sigma)}\right]^{\left(1-\frac{\beta}{n}\right)\frac{k}{\alpha}-1} \times \left[\int_{\underline{\theta}}^{\hat{\theta}^G(\sigma)} \theta dF(\theta,\sigma) + \hat{\theta}^G(\sigma)^{\frac{\beta}{n}} \int_{\hat{\theta}^G(\sigma)}^{\bar{\theta}} \theta^{1-\frac{\beta}{n}} dF(\theta,\sigma)\right] = 0 \tag{17}$$

and that any such $\hat{\theta}^G(\sigma)$ corresponds to a maximum. It can also be shown that

$$\hat{K}^G(\sigma) = \left(\frac{\hat{S}(\sigma)}{c\hat{\theta}^G(\sigma)}\right)^{\frac{1-\beta/n}{\alpha}} \left(\frac{\beta}{nc}\right)^{-\frac{1}{\alpha}}. \tag{18}$$

See Online Appendix C for details. For systemic risk as specified in Definition 2, $\hat{S}(\sigma)$ is independent of σ.

When capital is specific, $\underline{s}(K,\sigma)$ is by Definition 3 independent of K and so is here written $\underline{s}(\sigma)$. To simplify the algebra for this case, let

$$\psi(\tilde{\theta},\sigma) = \left(1-\frac{\beta}{n}\right)\int_{\underline{\theta}}^{\bar{\theta}} \theta dF(\theta,\sigma) - \frac{1-\delta}{\delta}\frac{\beta}{n}\tilde{\theta} - \int_{\tilde{\theta}}^{\bar{\theta}} \left[\left(1-\frac{\beta}{n}\right)\theta - \tilde{\theta}^{\frac{\beta}{n}}\theta^{1-\frac{\beta}{n}} + \frac{\beta}{n}\tilde{\theta}\right] dF(\theta,\sigma). \tag{19}$$

It can then be shown that the first-order condition for optimal capital to yield $\hat{\theta}^S(\sigma)$ interior to $[\underline{\theta}, \bar{\theta}]$ is

$$\frac{\psi_1(\hat{\theta}^S(\sigma),\sigma)}{\psi(\hat{\theta}^S(\sigma),\sigma)}\left[\hat{\theta}^S(\sigma) - \delta k \frac{1-\beta/n}{\alpha}\left(\frac{\beta}{n}\right)^{-k\frac{\beta/n}{\alpha}-1} c^{k\frac{\beta/n}{\alpha}} \times C(\sigma)\left(\frac{\underline{s}(\sigma)}{\psi(\hat{\theta}^S(\sigma),\sigma)}\right)^{k\frac{1-\beta/n}{\alpha}-1}\right] = 1 \tag{20}$$

and that any such $\hat{\theta}^S(\sigma)$ corresponds to a maximum if $\psi(\hat{\theta}^S(\sigma),\sigma)$ and $\psi_1(\hat{\theta}^S(\sigma),\sigma)$ are both strictly positive. It can also be shown that

$$\hat{K}^S(\sigma) = \left[\frac{\underline{s}(\sigma)}{\psi(\hat{\theta}^S(\sigma),\sigma)}\right]^{\frac{1-\beta/n}{\alpha}} \left(\frac{\beta}{nc}\right)^{-\frac{\beta/n}{\alpha}}. \tag{21}$$

Again, see Online Appendix C for details. For specific risk as specified in Definition 2, $\underline{s}(\sigma)$ is independent of σ.

In the absence of the relational contract constraint, $\tilde{\theta} = \bar{\theta}$. Optimal capital is then $K^*(\sigma)$ defined in (2). With a change in the distribution of θ from σ_L to σ_H with means $E(\theta \mid \sigma_L)$ and $E(\theta \mid \sigma_H)$, respectively, it can be shown that the first-order condition for optimal capital when $\gamma = 1 - \beta/n$ yields

$$\frac{K^*(\sigma_H)}{K^*(\sigma_L)} = \left[\frac{E(\theta \mid \sigma_H)/C(\sigma_H)}{E(\theta \mid \sigma_L)/C(\sigma_L)}\right]^{\frac{1}{k-\alpha/(1-\beta/n)}}. \tag{22}$$

Again, see Online Appendix C for details.

PROPOSITION 9. *Consider the Cobb–Douglas specification with $\gamma = 1 - \beta/n$. There exist specifications with the relational contract constraint and $\hat{\theta}^i(\sigma_L), \hat{\theta}^i(\sigma_H) \in (\underline{\theta}, \bar{\theta})$, for $i = G, S$, such that a change in the distribution of θ from $F(\theta, \sigma_L)$ to $F(\theta, \sigma_H)$ exactly matches the ratios of productivity and capital resulting from a change in the distribution of θ from $F(\theta, \sigma'_L)$ to $F(\theta, \sigma'_H)$ in the specification with no relational contract constraint with $C(\sigma_L) = C(\sigma'_L)$ and $C(\sigma_H) = C(\sigma'_H)$ only if*

$$\frac{\int_{\underline{\theta}}^{\hat{\theta}^i(\sigma_H)} \theta dF(\theta, \sigma_H) + \hat{\theta}^i(\sigma_H)^{\beta/n} \int_{\hat{\theta}^i(\sigma_H)}^{\bar{\theta}} \theta^{1-\beta/n} dF(\theta, \sigma_H)}{\int_{\underline{\theta}}^{\hat{\theta}^i(\sigma_L)} \theta dF(\theta, \sigma_L) + \hat{\theta}^i(\sigma_L)^{\beta/n} \int_{\hat{\theta}^i(\sigma_L)}^{\bar{\theta}} \theta^{1-\beta/n} dF(\theta, \sigma_L)} = \frac{E(\theta \mid \sigma'_H)}{E(\theta \mid \sigma'_L)}, \quad \text{for } i = G, S, \tag{23}$$

and

(1) *for general capital:*

$$\frac{\hat{S}(\sigma_H)/\hat{\theta}^G(\sigma_H)}{\hat{S}(\sigma_L)/\hat{\theta}^G(\sigma_L)} = \left[\frac{E(\theta \mid \sigma'_H)/C(\sigma_H)}{E(\theta \mid \sigma'_L)/C(\sigma_L)}\right]^{\frac{\alpha/(1-\beta/n)}{k-\alpha/(1-\beta/n)}}; \tag{24}$$

(2) *for specific capital:*

$$\frac{\hat{\theta}^S(\sigma_H) - \frac{\psi(\hat{\theta}^S(\sigma_H),\sigma_H)}{\psi_1(\hat{\theta}^S(\sigma_H),\sigma_H)}}{\hat{\theta}^S(\sigma_L) - \frac{\psi(\hat{\theta}^S(\sigma_L),\sigma_L)}{\psi_1(\hat{\theta}^S(\sigma_L),\sigma_L)}} = \frac{E(\theta \mid \sigma'_H)}{E(\theta \mid \sigma'_L)}. \tag{25}$$

For general capital, these necessary conditions are sufficient if

$$\frac{\bar{\theta}^{\left(1-\frac{\beta}{n}\right)\frac{k}{\alpha}-1}}{\underline{\theta}^{\left(1-\frac{\beta}{n}\right)\left(\frac{k}{\alpha}-1\right)}} > \max\left\{\frac{\int_{\underline{\theta}}^{\bar{\theta}} \theta^{1-\beta/n} dF(\theta, \sigma_L)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dF(\theta, \sigma_L)}, \frac{\int_{\underline{\theta}}^{\bar{\theta}} \theta^{1-\beta/n} dF(\theta, \sigma_H)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dF(\theta, \sigma_H)}\right\}. \tag{26}$$

Condition (23) follows directly from (15) when the ratio of capital is required to be the same whether the relational contract constraint is imposed (with the cutoff value $\tilde{\theta} < \bar{\theta}$) or not (with $\tilde{\theta} = \bar{\theta}$). The right-hand side of (23) is the ratio of TFP under σ'_H and σ'_L when the relational contract constraint is not imposed. This condition applies whether capital is general or specific. Condition (24) follows directly from requiring the ratio of (18) for σ_H to σ_L to match (22) for σ'_H and σ'_L. Condition (25) likewise follows from requiring the ratio of (21) for σ_H to σ_L to match (22). For general capital, condition (26) ensures that there exists c for which both $\hat{\theta}^G(\sigma_L)$ and $\hat{\theta}^G(\sigma_H)$ are interior to $[\underline{\theta}, \bar{\theta}]$. For specific capital, it is not straightforward to establish general conditions that ensure sufficiency but that can be checked for particular cases.

Proposition 9 shows that the general capital relational contract specification can, under a wide variety of circumstances, match the average productivity and capital changes resulting from a change in the distribution from σ'_L to σ'_H in a specification without a relational contract constraint even for given σ_L and σ_H. To do that requires only finding $\hat{\theta}^G(\sigma_L)$ and $\hat{\theta}^G(\sigma_H)$ that satisfy (23) and then, for that $\hat{\theta}^G(\sigma_L)$ and $\hat{\theta}^G(\sigma_H)$, finding $\hat{S}(\sigma_H)/\hat{S}(\sigma_L)$ that satisfies (24). The latter can always be done because the only restriction on $\hat{S}(\sigma)$ implied by the theory is that it is positive. The former is always possible if $\underline{\theta} = 0$ because then the left-hand side of (23) can range over the whole positive real line and (26) is always satisfied. For specific capital, there is less flexibility because the left-hand side of neither (25) nor (23) depends on $\underline{s}(\sigma_L)$ or $\underline{s}(\sigma_H)$, so it is not always possible to find values of these that ensure both are satisfied for the same values of $\hat{\theta}^S(\sigma_L)$ and $\hat{\theta}^S(\sigma_H)$. Proposition 9 is not restricted to the case with $C(\sigma_H) = C(\sigma_L)$ that corresponds to the effect of a change in risk analyzed in previous sections.

Proposition 9 applies to any σ_L, σ_H, σ'_L, and σ'_H. The change in TFP in Bloom et al. (2018) corresponds to σ'_H having a lower mean than σ'_L. The particular interest here is with circumstances in which the relational contract specification captures the implications of that fall when σ_L and σ_H have the same mean. To explore this, the Cobb–Douglas specification is calibrated using parameters based on those in Bloom et al. (2018), as follows. Long-run changes in labor hours are at constant marginal cost, implying $n = 1$ in (10). The long-run cost of capital is the going market rate, which corresponds to $k = 1$ in (11). The parameters α and β are calibrated from factor shares adjusted for an isoelastic product demand function with a 33% markup. The resulting parameter values are given in the top part of Table 1. Note that results calculated from (17) and (20) depend on β and n only through their ratio β/n, so they are robust to changes in those parameters that leave this ratio unchanged.

For the distribution of θ, it fits with Bloom et al. (2018) to take $\theta/E(\theta \mid \sigma)$ as log-normally distributed with risk parameterized by σ the standard deviation of $\ln \theta/E(\theta \mid \sigma)$. (For more on this, see below.) Because this distribution has $\underline{\theta} = 0$ and $\bar{\theta} = \infty$, Assumption 2 holds for all σ. Full pooling is never an optimal relational contract because it would require $\tilde{e}(K, \sigma) \leq e^*(K, 0) = 0$, so continuing the relationship would not be worthwhile. The same applies to an optimal relational contract with first-best effort for all θ as long as the future joint payoff is finite because then, with first-best effort given by (12), (3) cannot be satisfied as θ goes to infinity. Moreover, (26)

TABLE 1. Parameter values for calibration.

Parameter	Value	Source
δ	$0.95^{1/4}$	Bloom et al (2018), annual discount factor of 95%
α	0.25	Bloom et al (2018), factor share with isoelastic demand, 33% markup
β	0.5	As α with labor share 2/3, capital share 1/3
n	1	Implied by Bloom et al (2018) model
k	1	Implied by Bloom et al (2018) model
σ_L^A	0.67	Bloom et al (2018) estimate, %
σ_H^A/σ_L^A	1.6	Bloom et al (2018) estimate
σ_L^Z	5.1	Bloom et al (2018) estimate, %
σ_H^Z/σ_L^Z	4.1	Bloom et al (2018) estimate
σ_L	0.10	Calculated combined σ_L^A and σ_L^Z for θ
σ_H/σ_L	4.07	Calculated from combined σ_H^A and σ_H^Z for θ

holds for all σ_L and σ_H. Because Bloom et al. (2018) are concerned with business cycles, TFP in their model (corresponding to θ^γ) is time varying. It is the product of separate aggregate (A) and firm (Z) components that switch between two regimes, low risk (L) and high risk (H), all of which follow autoregressive processes with normally distributed logs of innovations. The counterpart to these processes in the stationary long-run framework in the model used here is that θ is log-normally distributed, with the low risk and high risk regimes the two different long-run equilibria corresponding to σ_L and σ_H, respectively, in the theoretical model of the previous sections. Because the estimated standard deviations in Bloom et al. (2018) apply to θ^γ, not θ itself, they need to be adjusted for use here. Moreover, for the purpose of showing the effects of changes in long-run uncertainty, the aggregate and firm components are combined using the standard formula for the product of log-normal distributions to give the low risk and high risk parameters σ_L and σ_H, respectively. For the relational contract specification, the resulting low and high risk distributions for θ are specified to have the same mean to ensure that the parties are jointly risk neutral in the absence of the relational contract constraint (which corresponds to the expression in (13) being linear in θ with the calibration $\gamma = 1 - \beta/n$). The resulting distribution parameter values are given in the lower part of Table 1. (Details of calculations for this section are given in Online Appendix D.) Note that the aggregate component of the change in risk is essentially swamped by the firm component because $\sigma_H/\sigma_L \approx \sigma_H^Z/\sigma_L^Z$.

In their analysis of the empirical evidence, Bloom et al. (2018) find that, to capture the effects of periods of increased uncertainty as measured by second-moment shocks in their specification (in which the long-run properties are independent of second-moment shocks), they require a simultaneous negative first-moment shock to TFP, which implies $E(\theta \mid \sigma'_H) < E(\theta \mid \sigma'_L)$. To generate empirically more realistic simulations from their model with their calibrated parameter values, they use a 2% negative aggregate first-moment shock to TFP. To capture that in a specification with a relational contract constraint without a negative first-moment shock (i.e.

TABLE 2. General capital relational contract specifications with same mean θ for σ_H as for σ_L matching capital and productivity changes in specification without relational contract constraint with 2% lower TFP for σ_H than for σ_L with parameter values in Table 1.

	Matching specifications						
$\frac{\hat{\theta}^G(\sigma_L)}{E(\theta)}$	0.01	0.25	0.50	0.75	1.00	1.25	1.57507
$\frac{\hat{\theta}^G(\sigma_H)}{E(\theta)}$	0.01008	0.2502	0.504	0.809	1.312	1.570	1.57507
$\frac{\hat{S}(\sigma_H)C(\sigma_H)}{\hat{S}(\sigma_L)C(\sigma_L)}$	0.98078	0.9808	0.988	1.057	1.286	1.231	0.980
$\frac{\hat{S}(\sigma_H)}{\hat{S}(\sigma_L)}$[a]	0.9207	0.9207	0.927	0.992	1.207	1.805	0.920

a. Note: $\hat{S}(\sigma_H)/\hat{S}(\sigma_L)$ for $C(\sigma_L)/C(\sigma_H) = 0.939$—see text.

with $E(\theta \mid \sigma_H) = E(\theta \mid \sigma_L)$ denoted $E(\theta)$) requires satisfying the conditions of Proposition 9 with $E(\theta \mid \sigma'_H)/E(\theta \mid \sigma'_L) = 0.98$. The columns in Table 2 give specifications for the case of general capital that do this for the parameter values in Table 1.[6]

The top two rows in Table 2 give values of $\hat{\theta}^G(\sigma_L)/E(\theta)$ and $\hat{\theta}^G(\sigma_H)/E(\theta)$ that satisfy (23) for $E(\theta \mid \sigma'_H)/E(\theta \mid \sigma'_L) = 0.98$. There is a continuum of such values for $\hat{\theta}^G(\sigma_L)/E(\theta)$ between 0 and 1.57507 (and thus values both below and above the mean of 1) but, because $\hat{\theta}^G(\sigma_L)/E(\theta) = 0$ corresponds to full pooling under the relational contract constraint, the lowest value given in Table 2 is for $\hat{\theta}^G(\sigma_L)/E(\theta) = 0.01$. Table 2 illustrates with values of $\hat{\theta}^G(\sigma_L)/E(\theta)$ interspersed between these. In each case, $\hat{\theta}^G(\sigma_H)/E(\theta)$ is at least as high as $\hat{\theta}^G(\sigma_L)/E(\theta)$, which corresponds to a cutoff of θ below which effort is first best being at least as high for σ_H as for σ_L. The difference is greatest for $\hat{\theta}^G(\sigma_L)/E(\theta)$ somewhat above its mean of 1 and essentially negligible for $\hat{\theta}^G(\sigma_L)/E(\theta)$ at each end of its acceptable range. The third row of Table 2 gives the ratio of $\hat{S}(\sigma)C(\sigma)$ for $\sigma = \sigma_H$ to that for $\sigma = \sigma_L$ for the cutoff values of $\hat{\theta}^G(\sigma)$ in the top two rows, calculated from (24). If capital costs were the same for σ_H as for σ_L (i.e. $C(\sigma_H) = C(\sigma_L)$), this would correspond to the ratio of the joint payoff gain to principal and agent from continuing the relationship in the future over what they would get by ending it for σ_H to that for σ_L. Bloom et al. (2018) do not report capital costs corresponding to $C(\sigma)$. One could infer a ratio $C(\sigma_L)/C(\sigma_H)$ from numbers for the long-run effects on capital and productivity using (22) and (15) with $\tilde{\theta} = \infty$ but the simulations in Bloom et al. (2018) are for only 12 quarters, which may not be sufficient to capture long-run impacts, and for changes in investment, not capital. If one assumes that 12 quarters *are* sufficient to capture long-run impacts and that the proportionate change in capital is the same as the proportionate change in investment

6. All calculations were carried out using Scientific WorkPlace 6.

at the end of this period, the ratio is $C(\sigma_L)/C(\sigma_H) = 0.939$, which is used to calculate the bottom row in Table 2.

If shocks are entirely systemic, $\hat{S}(\sigma_H) = \hat{S}(\sigma_L)$. In that case, the bottom row of Table 2 would indicate $\hat{\theta}^G(\sigma_L)/E(\theta)$ either between 0.75 and 1 or near the highest value that satisfies (23). If, however, one were to take the measures in Table 1 of the firm component of risk as specific shocks and of the aggregate component as systemic shocks, shocks would be primarily specific.[7] If they were entirely specific, the relational contract specification would result in a lower joint payoff to the parties from continuing the relationship because of the adverse impact on capital but no impact on the joint payoff to separating. That would imply $\hat{S}(\sigma_H) < \hat{S}(\sigma_L)$, in which case the bottom row of Table 2 would indicate $\hat{\theta}^G(\sigma_L)/E(\theta)$ further from the mean than with purely systemic shocks but still consistent with the model and the calculated $C(\sigma_L)/C(\sigma_H)$ as long as $\hat{S}(\sigma_H)/\hat{S}(\sigma_L) \geq 0.92$—an 8% reduction in the joint gain from continuing the relationship over ending it.[8]

There is, though, evidence of at least some capital specificity, though how much there is in the sense used here in practice is unclear.[9] For the relational contract specification with specific capital, the same pairs of cutoff values of θ satisfy (23) as for general capital, so the values in the top two rows in Table 2 are also candidates for $\hat{\theta}^S(\sigma_L)$ and $\hat{\theta}^S(\sigma_H)$. However, for the specific capital specification, the same pair of cutoff values must also satisfy (25). A grid search over pairs of $\hat{\theta}^S(\sigma_L)$ and $\hat{\theta}^S(\sigma_H)$ that satisfy (23) reveals a single pair that satisfies (25), approximately (to two decimal places) $\hat{\theta}^S(\sigma_L)/E(\theta) = 0.93$ and $\hat{\theta}^S(\sigma_H)/E(\theta) = 1.15$. For these values, the ratio of $\underline{s}(\sigma)C(\sigma)$ for $\sigma = \sigma_H$ to that for $\sigma = \sigma_L$ can be calculated by equating the capital ratios implied by (21) and (22). This is in the third column of Table 3. For a change in risk that is entirely specific, $\underline{s}(\sigma_H) = \underline{s}(\sigma_L)$. If some risk is systemic, one might expect $\underline{s}(\sigma_H) < \underline{s}(\sigma_L)$ because of the higher risk adversely affecting payoffs if the relationship ends. For the value $C(\sigma_L)/C(\sigma_H) = 0.939$ calculated from Bloom et al. (2018), the ratio $\underline{s}(\sigma_H)/\underline{s}(\sigma_L) = 0.94$ as in the final column of Table 3, which is consistent with some risk being systemic, as implied by the calibrated parameters in Table 1.

The most important conclusions from investigation of the calibrated model are that, under a wide variety of circumstances, a general capital relational contract model

7. It is not necessarily appropriate to do that. According to Definition 2, a firm component that applied equally if the firm replaced one employee with another would be systemic, whereas specific risk applies to a risk that is specific to a particular *relationship* between a principal and an agent.

8. The calculations underlying Table 2 are based on all firms using relational contracts, which will presumably not apply to the whole US economy. However, it follows from Proposition 9 that the relational contract specification with general capital could match larger percentage reductions in productivity and capital, which would be consistent with only a proportion of the US economy operating under relational contracts.

9. Kermani and Ma (2023) report that the liquidation value of fixed assets of non-financial firms is only 35% of net book value in the average US industry. However, specific capital as specified by Definition 3 is concerned with value as a going concern with a different agent, not liquidation value.

TABLE 3. Specific capital relational contract specification with same mean θ for σ_H as for σ_L matching capital and productivity changes in specification without relational contract constraint with 2% lower TFP for σ_H than for σ_L with parameter values in Table 1.

	$\hat{\theta}^S(\sigma_L)/E(\theta)$	$\hat{\theta}^S(\sigma_H)/E(\theta)$	$\frac{\underline{s}(\sigma_H)C(\sigma_H)}{\underline{s}(\sigma_L)C(\sigma_L)}$	$\underline{s}(\sigma_H)/\underline{s}(\sigma_L)$[a]
Matching specification	0.93	1.15	0.97	0.94

a. Note: $\underline{s}(\sigma_H)/\underline{s}(\sigma_L)$ for $C(\sigma_L)/C(\sigma_H) = 0.939$—see text.

with the same mean TFP for higher as for lower uncertainty can capture the effects that Bloom et al. (2018) attribute to a fall in mean TFP that accompanies the higher uncertainty. For the calibrated parameter values in Bloom et al. (2018), there is also a specification of the specific capital relational contract model that does so. There is no need for a negative first-moment shock to accompany the second-moment shock to capture those effects.

8. Conclusion

This paper has analyzed the effect of changes in the riskiness of the distribution of output on capital investment in a relational contract in which agent performance is not contractible. Unlike when performance is contractible, the effect on investment depends on whether capital is general or specific in the sense of Becker (1975). Investment in general capital is lower with a relational contract than with contractible performance and, under plausible conditions, is more adversely affected by an increase in uncertainty. Investment in specific capital is less adversely affected by reliance on a relational contract and may even increase with an increase in uncertainty. The reason is that, as Klein and Leffler (1981) note, specific capital relaxes the incentive constraints in a relational contract because it increases the payoff to the parties staying together relative to the payoff from separating and it is this difference that constrains performance.

An implication of these theoretical results is that, in the presence of relational contracts, an increase in uncertainty alone reduces long-run equilibrium investment in general capital and productivity when it would not otherwise do so—even, that is, if the parties are risk neutral. This is potentially important given that relational contracts are widely seen as an insightful way to view economic relationships in a variety of contexts.

To illustrate the potential empirical significance, the paper uses a calibrated production framework similar to that in the real business cycle model in Bloom et al. (2018) with the addition of the constraints arising from a relational contract. Bloom et al. (2018) add a negative first-moment shock to aggregate TFP in addition to a higher second moment to match the data on recessions. They comment (pp. 1033–4) that "(t)he reliance on negative technology shocks has proven to be controversial, as it suggests that recessions are times of technological regress". The calibration exercise here shows that, in the presence of relational contracts, the same long-run equilibrium impact on productivity and capital as their negative first-moment shock can arise from the higher second moment alone. It is hard to think of alternative mechanisms that would deliver this with risk-neutral firms: mechanisms such as adjustment costs, the option value arising from irreversibility and the time taken to build capital have short-run, not long-run equilibrium, effects. This is suggestive that recessions of the magnitudes in the data might arise solely from a second-moment shock to aggregate TFP, with no negative first-moment shock. It is also suggestive that adding relational contracts to perception of capital risk models might account for movements that cannot be accounted for by the cost of high-risk capital alone. To verify these obviously requires a relational contract model that, unlike the one used here, takes full account of dynamics—a far from straightforward task. However, if verified in that way, it would open up attractive possibilities.

Appendix A: Proofs

Proof of Proposition 2. By Proposition 1, (3) is necessary and sufficient for an effort schedule to be implementable. Thus, an optimal effort schedule for given (K, σ) is a solution to

$$\max_{e(K,\cdot,\sigma)\in[0,\bar{e}]} \quad \frac{1}{1-\delta}\int_{\underline{\theta}}^{\bar{\theta}} [s(e(K,\tilde{\theta},\sigma), K, \tilde{\theta}) - \underline{s}(K,\sigma)]dF(\tilde{\theta},\sigma)$$

$$\text{subj. to:} \quad \frac{\delta}{1-\delta}\int_{\underline{\theta}}^{\bar{\theta}} [s(e(K,\tilde{\theta},\sigma), K, \tilde{\theta}) - \underline{s}(K,\sigma)]dF(\tilde{\theta},\sigma) \geq c(e(K,\theta,\sigma)), \text{ for all } \theta \in [\underline{\theta}, \bar{\theta}].$$

By Assumptions 1 and 2, optimal effort for all θ is interior to $[0, \bar{e}]$. For any $\theta' \in [\underline{\theta}, \bar{\theta}]$ for which the constraint is not binding, it is immediate that it is optimal to set $e(K, \theta', \sigma) = e^*(K, \theta')$, the first-best effort for which $s_1(e^*(K, \theta'), K, \theta') = 0$, because that both maximizes the objective function for θ' and relaxes the constraint the most for $\theta \neq \theta'$. By Part 2 of Assumption 2, this cannot though apply to all $\theta \in [\underline{\theta}, \bar{\theta}]$. For any $\theta' \in [\underline{\theta}, \bar{\theta}]$ for which the constraint binds, it is optimal to set $e(K, \theta', \sigma)$ at the highest level that satisfies the constraint as this is the closest to first best and, as the left-hand side of (3) is independent of θ, so is $e(K, \theta', \sigma)$. From Corollary 1, if the constraint binds for θ', it also binds for all $\theta > \theta'$. By Part 1 of Assumption 2, the constraint does not bind for $\underline{\theta}$. That implies there is a cutoff type $\hat{\theta}(K, \sigma) \in (\underline{\theta}, \bar{\theta})$ such that, for all $\theta \leq \hat{\theta}(K, \sigma)$, effort is at the first-best level and, for all $\theta > \hat{\theta}(K, \sigma)$, effort is constant at $e^*(K, \hat{\theta}(K, \sigma))$, as specified in the proposition. □

Proof of Proposition 3. Integration by parts gives

$$\int_{\tilde{\theta}}^{\bar{\theta}} g(\theta)dF(\theta,\sigma_L) - \int_{\tilde{\theta}}^{\bar{\theta}} g(\theta)dF(\theta,\sigma_H)$$
$$= [g(\theta)F(\theta,\sigma_L)]_{\tilde{\theta}}^{\bar{\theta}} - \int_{\tilde{\theta}}^{\bar{\theta}} g'(\theta)F(\theta,\sigma_L)d\theta$$
$$-\left[[g(\theta)F(\theta,\sigma_H)]_{\tilde{\theta}}^{\bar{\theta}} - \int_{\tilde{\theta}}^{\bar{\theta}} g'(\theta)F(\theta,\sigma_H)d\theta\right]$$
$$= -\int_{\tilde{\theta}}^{\bar{\theta}} [F(\theta,\sigma_L) - F(\theta,\sigma_H)]g'(\theta)d\theta,$$

the last line following because $F(\bar{\theta},\sigma_H) = F(\bar{\theta},\sigma_L) = 1$, $F(\underline{\theta},\sigma_H) = F(\underline{\theta},\sigma_L) = 0$ and the statement of the proposition specifies that $g(\tilde{\theta}) = 0$ if $\tilde{\theta} > \underline{\theta}$. Integration of that last line by parts gives that

$$\int_{\tilde{\theta}}^{\bar{\theta}} g(\theta)dF(\theta,\sigma_L) - \int_{\tilde{\theta}}^{\bar{\theta}} g(\theta)dF(\theta,\sigma_H)$$
$$= -\left[g'(\theta)\int_{\underline{\theta}}^{\theta} [F(x,\sigma_L) - F(x,\sigma_H)]dx\right]_{\tilde{\theta}}^{\bar{\theta}}$$
$$+\int_{\tilde{\theta}}^{\bar{\theta}} \left[\int_{\underline{\theta}}^{\theta} [F(x,\sigma_L) - F(x,\sigma_H)]dx\right] g''(\theta)d\theta. \tag{A.1}$$

Now,

$$\int_{\underline{\theta}}^{\theta} [F(x,\sigma_L) - F(x,\sigma_H)]dx$$
$$= \int_{\underline{\theta}}^{\bar{\theta}} [F(x,\sigma_L) - F(x,\sigma_H)]dx - \int_{\theta}^{\bar{\theta}} [F(x,\sigma_L) - F(x,\sigma_H)]dx$$

and, with $F(\theta,\sigma_L)$ and $F(\theta,\sigma_H)$ having the same mean,

$$\int_{\underline{\theta}}^{\bar{\theta}} [F(x,\sigma_L) - F(x,\sigma_H)]dx = 0,$$

see Laffont (1989, p. 25). Use of these in (A.1) gives that

$$\int_{\tilde{\theta}}^{\bar{\theta}} g(\theta)dF(\theta,\sigma_L) - \int_{\tilde{\theta}}^{\bar{\theta}} g(\theta)dF(\theta,\sigma_H)$$
$$= -g'(\tilde{\theta})\int_{\tilde{\theta}}^{\bar{\theta}} [F(x,\sigma_L) - F(x,\sigma_H)]dx$$
$$-\int_{\tilde{\theta}}^{\bar{\theta}} \left[\int_{\theta}^{\bar{\theta}} [F(x,\sigma_L) - F(x,\sigma_H)]dx\right] g''(\theta)d\theta \geq 0,$$

the sign following from $g'(\theta) \leq 0, g''(\theta) \leq 0$ and, from second-order stochastic dominance, $\int_{\theta}^{\bar{\theta}}[F(x,\sigma_L) - F(x,\sigma_H)]dx \geq 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$, with strict inequality for a set of values of $x \in [\tilde{\theta}, \bar{\theta}]$ with positive probability, as specified in Definition 1. If $g''(\theta) < 0$, the inequality is strict. □

LEMMA A.1. *$h(e, K, \theta)$:*

(1) *is decreasing in θ for all $(e, K, \theta) \in (0, e^*(K, \theta)] \times (0, \bar{K}] \times [\underline{\theta}, \bar{\theta}]$;*
(2) *is strictly concave in θ for all $(e, K, \theta) \in (0, e^*(K, \theta)] \times (0, \bar{K}] \times [\underline{\theta}, \bar{\theta}]$ if $y_{133}(e, K, \theta) \geq 0$ for all $(e, K, \theta) \in (0, e^*(K, \theta)] \times (0, \bar{K}] \times [\underline{\theta}, \bar{\theta}]$;*
(3) *has $h_2(e, K, \theta)$ strictly decreasing in θ for all $(e, K, \theta) \in (0, e^*(K, \theta)] \times (0, \bar{K}] \times [\underline{\theta}, \bar{\theta}]$ if $y_{123}(e, K, \theta) \geq 0$ for all $(e, K, \theta) \in (0, e^*(K, \theta)] \times (0, \bar{K}] \times [\underline{\theta}, \bar{\theta}]$.*

Proof. *Part 1.* From the definition of h in (6),

$$h_3(e, K, \theta) = s_3(e, K, \theta) - s_1(e^*(K, \theta), K, \theta)e_2^*(K, \theta) - s_3(e^*(K, \theta), K, \theta). \tag{A.2}$$

The definition of $e^*(K, \theta)$ in (1) implies

$$s_1(e^*(K, \theta), K, \theta) = 0, \quad \text{for all } \theta \in [\underline{\theta}, \bar{\theta}], \tag{A.3}$$

so

$$h_3(e, K, \theta) = s_3(e, K, \theta) - s_3(e^*(K, \theta), K, \theta) \leq 0 \text{ for } e \leq e^*(K, \theta),$$

the sign following from $s_{13} = y_{13} > 0$ by Assumption 1, so h is decreasing in θ for $e \leq e^*(K, \theta)$.

Part 2. From (A.2),

$$\begin{aligned} h_{33}&(e, K, \theta) \\ &= s_{33}(e, K, \theta) - s_{11}(e^*(K, \theta), K, \theta)e_2^*(K, \theta)^2 - s_{13}(e^*(K, \theta), K, \theta)e_2^*(K, \theta) \\ &\qquad - s_1(e^*(K, \theta), K, \theta)e_{22}^*(K, \theta) - s_{13}(e^*(K, \theta), K, \theta)e_2^*(K, \theta) \\ &\qquad\qquad - s_{33}(e^*(K, \theta), K, \theta) \\ &= s_{33}(e, K, \theta) - s_{33}(e^*(K, \theta), K, \theta) \\ &\qquad - s_{11}(e^*(K, \theta), K, \theta)e_2^*(K, \theta)^2 - 2s_{13}(e^*(K, \theta), K, \theta)e_2^*(K, \theta), \end{aligned}$$

the last equality making use of (A.3). Differentiation of (A.3) gives

$$e_2^*(K, \theta) = -\frac{s_{13}(e^*(K, \theta), K, \theta)}{s_{11}(e^*(K, \theta), K, \theta)}.$$

Thus,

$$\begin{aligned} h_{33}(e,K,\theta) = s_{33}(e,K,\theta) - s_{33}(e^*(K,\theta),K,\theta) \\ - s_{11}(e^*(K,\theta),K,\theta)\left(-\frac{s_{13}(e^*(K,\theta),K,\theta)}{s_{11}(e^*(K,\theta),K,\theta)}\right)^2 \\ - 2s_{13}(e^*(K,\theta),K,\theta)\left(-\frac{s_{13}(e^*(K,\theta),K,\theta)}{s_{11}(e^*(K,\theta),K,\theta)}\right) \\ = s_{33}(e,K,\theta) - s_{33}(e^*(K,\theta),K,\theta) + \frac{s_{13}(e^*(K,\theta),K,\theta)^2}{s_{11}(e^*(K,\theta),K,\theta)}. \end{aligned}$$

By the definition of $s(e,K,\theta)$, $s_{ij}(e,K,\theta) = y_{ij}(e,K,\theta)$ for $i,j = 1,2$, so the first two terms in this are non-positive for $e \le e^*(K,\theta)$, as specified in the definition of $h(e,K,\theta)$ in (6), if $y_{133}(e,K,\theta) \ge 0$ and the final term is strictly negative because, by Assumption 1, $y_{11}(e,K,\theta) < 0$ and $y_{13}(e,K,\theta) > 0$ for all $(e,K,\theta) \in (0,\bar{e}] \times [0,\bar{K}] \times [\underline{\theta},\bar{\theta}]$. This implies $h_{33}(e,K,\theta) < 0$ and thus $h(e,K,\theta)$ strictly concave in θ for given (e,K) for $y_{133}(e,K,\theta) \ge 0$.

Part 3. From the definition of h in (6),

$$h_2(e,K,\theta) = s_2(e,K,\theta) - s_2(e^*(K,\theta),K,\theta) - s_1(e^*(K,\theta),K,\theta)e_1^*(K,\theta),$$

so

$$\begin{aligned} h_{23}(e,K,\theta) = s_{23}(e,K,\theta) - s_{23}(e^*(K,\theta),K,\theta) - s_{12}(e^*(K,\theta),K,\theta)e_2^*(K,\theta) \\ - s_{13}(e^*(K,\theta),K,\theta)e_1^*(K,\theta) - s_1(e^*(K,\theta),K,\theta)e_{12}^*(K,\theta) \\ - s_{11}(e^*(K,\theta),K,\theta)e_1^*(K,\theta)e_2^*(K,\theta). \end{aligned}$$

With the use of (A.3), this can be written

$$\begin{aligned} h_{23}(e,K,\theta) = s_{23}(e,K,\theta) - s_{23}(e^*(K,\theta),K,\theta) - s_{13}(e^*(K,\theta),K,\theta)e_1^*(K,\theta) \\ - [s_{12}(e^*(K,\theta),K,\theta) + s_{11}(e^*(K,\theta),K,\theta)e_1^*(K,\theta)]e_2^*(K,\theta). \end{aligned} \quad \text{(A.4)}$$

Differentiation of (A.3) gives

$$e_1^*(K,\theta) = -\frac{s_{12}(e^*(K,\theta),K,\theta)}{s_{11}(e^*(K,\theta),K,\theta)} > 0,$$

from which the expression in square brackets in (A.4) is zero so

$$\begin{aligned} h_{23}(e,K,\theta) = s_{23}(e,K,\theta) - s_{23}(e^*(K,\theta),K,\theta) \\ -s_{13}(e^*(K,\theta),K,\theta)e_1^*(K,\theta). \end{aligned}$$

Because $s_{23}(e,K,\theta) = y_{23}(e,K,\theta)$ and $s_{123}(e,K,\theta) = y_{123}(e,K,\theta)$, the first two terms are non-positive for $e \le e^*(K,\theta)$, as specified in the definition of $h(e,K,\theta)$ in (6), if $y_{123}(e,K,\theta) > 0$ and the final term is strictly negative because $e_1^*(K,\theta) > 0$ and, by Assumption 1, $y_{13}(e,K,\theta) > 0$ for all $(e,K,\theta) \in (0,\bar{e}] \times [0,\bar{K}] \times [\underline{\theta},\bar{\theta}]$. □

Proof of Proposition 4. Part 1. For systemic risk, the constraint (3) is by Definition 2 unaffected by changes in σ for given K, so the optimal effort schedule in Proposition 2 is also unaffected by changes in σ for given K.

Part 2. For specific risk, (4) and thus (5) must hold for both σ_L and σ_H. From (6), $h(e^*(K,\theta),K,\theta)=0$ and, from Lemma A.1, $h(e,K,\theta)$ is decreasing in θ. Thus, by Proposition 3 for $h(e^*(K,\hat{\theta}(K,\sigma_H)),K,\theta)$ strictly concave in θ for $\theta \in [\hat{\theta}(K,\sigma_H),\bar{\theta}]$, $F(\theta,\sigma_L)$ stochastically dominating $F(\theta,\sigma_H)$ in the second-order sense for $\tilde{\theta}=\hat{\theta}(K,\sigma_H)$ implies

$$\int_{\hat{\theta}(K,\sigma_H)}^{\bar{\theta}} h(e^*(K,\hat{\theta}(K,\sigma_H)),K,\theta)dF(\theta,\sigma_L)$$
$$> \int_{\hat{\theta}(K,\sigma_H)}^{\bar{\theta}} h(e^*(K,\hat{\theta}(K,\sigma_H)),K,\theta)dF(\theta,\sigma_H).$$

Because h is the negative of the integrand in the second integral in (5) and $\underline{s}(K,\sigma)$ is by Definition 2 unaffected by changes in σ for given K for specific risk, the left-hand side of (5) would be lower for σ_H than for σ_L if $\hat{\theta}(K,\sigma_L)=\hat{\theta}(K,\sigma_H)$ when (7) holds. So, with (5) holding for σ_H, its left-hand side would be greater than its right-hand side for σ_L if $\hat{\theta}(K,\sigma_L)=\hat{\theta}(K,\sigma_H)$, which implies that the left-hand side of (4) would be positive for σ_L if $\tilde{\theta}=\hat{\theta}(K,\sigma_H)$. The left-hand side of (4) is continuous in $\tilde{\theta}$ and negative for $\tilde{\theta}=\bar{\theta}$ because otherwise Proposition 2 would be violated. Because, by definition, $\hat{\theta}(K,\sigma_L)$ is the highest value of $\tilde{\theta}$ that satisfies (4) for σ_L, the left-hand side of (4) must be lower for all $\tilde{\theta}>\hat{\theta}(K,\sigma_L)$ than for $\tilde{\theta}=\hat{\theta}(K,\sigma_L)$—otherwise, by continuity, there would be a $\tilde{\theta}>\hat{\theta}(K,\sigma_L)$ for which equality holds in (4), which is a contradiction. So, it cannot be that $\hat{\theta}(K,\sigma_L)\leq\hat{\theta}(K,\sigma_H)$. It must, therefore, be that $\hat{\theta}(K,\sigma_L)>\hat{\theta}(K,\sigma_H)$. From (5), this implies a larger difference for σ_H than for σ_L between the joint payoff when effort is contractible and when it is not. When (7) holds, it also implies a lower joint payoff for σ_H than for σ_L for given K. □

LEMMA A.2. *First-best capital $K^*(\sigma)$ is given by*

$$\frac{1}{1-\delta}\int_{\underline{\theta}}^{\bar{\theta}} s_2(e^*(K^*(\sigma),\theta),K^*(\sigma),\theta)dF(\theta,\sigma)-C'(K^*(\sigma))=0. \tag{A.5}$$

When capital is general, optimal capital $\widehat{K}^G(\sigma)$ satisfies

$$\frac{1}{1-\delta}\left\{\int_{\underline{\theta}}^{\bar{\theta}} s_2(e^*(\widehat{K}^G(\sigma),\theta),\widehat{K}^G(\sigma),\theta)dF(\theta,\sigma)\right.$$
$$+\int_{\hat{\theta}(\widehat{K}^G(\sigma),\sigma)}^{\bar{\theta}}[s_2(e^*(\widehat{K}^G(\sigma),\hat{\theta}(\widehat{K}^G(\sigma),\sigma)),\widehat{K}^G(\sigma),\theta)$$
$$\left.-s_2(e^*(\widehat{K}^G(\sigma),\theta),\widehat{K}^G(\sigma),\theta)]dF(\theta,\sigma)\right\}-C'(\widehat{K}^G(\sigma))=0. \tag{A.6}$$

The left-hand sides of (A.5) and (A.6) are everywhere strictly decreasing in $\widehat{K}^G(\sigma)$.

Proof. Assumption 1 ensures first-best capital $K^*(\sigma)$ is interior to $[0, \overline{K}]$ and so must satisfy the first-order condition for the maximization in (2) which, given the definition of s, is just (A.5). Strict concavity of y in Assumption 1 ensures that the left-hand side of (A.5) is everywhere strictly decreasing in $K^*(\sigma)$.

For general capital, optimal capital $\widehat{K}^G(\sigma)$ is given by (8) with $\hat{\theta}(K,\sigma)$ the highest $\tilde{\theta}$ satisfying the constraint (4) whatever K is chosen. The integral terms in the constraint (4) are just $S^*(K,\sigma) - \underline{s}(K,\sigma)$ which is independent of K for general capital by Definition 3. So, total differentiation of (4) with respect to K for $\tilde{\theta} = \hat{\theta}(K,\sigma)$ gives, with $de^*(K,\hat{\theta}(K,\sigma))/dK$ the derivative of $e^*(K,\hat{\theta}(K,\sigma))$ with respect to K taking into account the impact through $\hat{\theta}(K,\sigma)$,

$$-c'(e^*(K,\hat{\theta}(K,\sigma)))\frac{de^*(K,\hat{\theta}(K,\sigma))}{dK} = 0$$

which, with $c'(e) > 0$ for all e, implies $de^*(K,\hat{\theta}(K,\sigma))/dK = 0$. Because the two integrands in (8) take the same value for $\theta = \hat{\theta}(K,\sigma)$, the derivative of the maximand in (8) with respect to K subject to the constraint is

$$\frac{1}{1-\delta}\left\{\int_{\underline{\theta}}^{\hat{\theta}(K,\sigma)}\left[s_1(e^*(K,\theta),K,\theta)\frac{\partial e^*(K,\theta)}{\partial K} + s_2(e^*(K,\theta),K,\theta)\right]dF(\theta,\sigma)\right.$$
$$+\int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}}\left[s_1(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)\frac{de^*(K,\hat{\theta}(K,\sigma))}{dK}\right.$$
$$\left.\left.+s_2(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)\right]dF(\theta,\sigma)\right\} - C'(K). \quad \text{(A.7)}$$

Because $de^*(K,\hat{\theta}(K,\sigma))/dK = 0$ and (1) implies $s_1(e^*(K,\theta),K,\theta) = 0$, (A.7) can be re-arranged to give the first-order condition (A.6) for $\widehat{K}^G(\sigma)$. The proof that the left-hand side of (A.6) is everywhere strictly decreasing in $\widehat{K}^G(\sigma)$ is in Online Appendix B. □

Proof of Proposition 5. From Lemma A.2, $\widehat{K}^G(\sigma)$ is uniquely determined by (A.6). To have $\widehat{K}^G(\sigma) = K^*(\sigma)$, the second integral would have to be zero in view of (A.5). However, $s_{12} = y_{12} > 0$ by Assumption 1, so the second integral in (A.6) is negative because $e^*(K,\theta) > e^*(K,\hat{\theta}(K,\sigma))$ for $\theta > \hat{\theta}(K,\sigma)$. By Lemma A.2, the left-hand side of (A.6) is strictly decreasing in K, which implies $\widehat{K}^G(\sigma) < K^*(\sigma)$. □

Proof of Proposition 6. Part 1. From Lemma A.2, $\widehat{K}^G(\sigma)$ is uniquely determined by (A.6). With the definition of h in (6), (A.6) can be written

$$\frac{1}{1-\delta}\int_{\underline{\theta}}^{\bar{\theta}} s_2(e^*(\widehat{K}^G(\sigma),\theta),\widehat{K}^G(\sigma),\theta)dF(\theta,\sigma) + \frac{1}{1-\delta}\int_{\hat{\theta}(\widehat{K}^G(\sigma),\sigma)}^{\bar{\theta}} h_2(e^*(\widehat{K}^G(\sigma),\hat{\theta}(\widehat{K}^G(\sigma),\sigma)),\widehat{K}^G(\sigma),\theta)dF(\theta,\sigma) - C'(\widehat{K}^G(\sigma)) = 0. \quad \text{(A.8)}$$

In view of (A.5), the second integral term would have to be zero to have $\widehat{K}(\sigma) = K^*(\sigma)$. By Proposition 3, with $h_2(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)$ decreasing and strictly concave in θ for $\theta \in [\hat{\theta}(K,\sigma),\bar{\theta}]$,

$$\int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}} h_2(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)dF(\theta,\sigma_L) > \int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}} h_2(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)dF(\theta,\sigma_H), \quad \text{for given } K, \hat{\theta}(K,\sigma).$$

From Proposition 4, $\hat{\theta}(K,\sigma_H) \leq \hat{\theta}(K,\sigma_L)$ for given K for both systemic and specific risk. By Assumption 1, $s_{12} > 0$ which implies $h_{12} > 0$ and h_2 negative for $e < e^*(K,\theta)$ implies $h_2(e^*(K,\hat{\theta}(K,\sigma)),K,\theta) < 0$ for $\theta > \hat{\theta}(K,\sigma)$. Moreover, from (1), $e^*(K,\theta)$ is increasing in θ, so $e^*(K,\hat{\theta}(K,\sigma_L)) \geq e^*(K,\hat{\theta}(K,\sigma_H))$ and thus, with $h_{12} > 0$,

$$\int_{\hat{\theta}(K,\sigma_L)}^{\bar{\theta}} h_2(e^*(K,\hat{\theta}(K,\sigma_L)),K,\theta)dF(\theta,\sigma_L) > \int_{\hat{\theta}(K,\sigma_H)}^{\bar{\theta}} h_2(e^*(K,\hat{\theta}(K,\sigma_H)),K,\theta)dF(\theta,\sigma_H).$$

From (2), (A.8) and, by Assumption 1, the strict concavity of $s(e,K,\theta) - C(K)$, this implies $K^*(\sigma_H) - \widehat{K}^G(\sigma_H) > K^*(\sigma_L) - \widehat{K}^G(\sigma_L)$.

Part 2. With the parties jointly risk-neutral, (7) holds with equality. It then follows from (2) that $K^*(\sigma_H) = K^*(\sigma_L)$, so the result follows from the argument in the previous paragraph. □

LEMMA A.3. *When capital is specific, optimal capital* $\widehat{K}^S(\sigma)$*, if interior to* $[0, \bar{K}]$*, satisfies*

$$\frac{1}{1-\delta}\left\{\int_{\underline{\theta}}^{\bar{\theta}} s_2(e^*(\widehat{K}^S(\sigma),\theta),\widehat{K}^S(\sigma),\theta)dF(\theta,\sigma) + \int_{\hat{\theta}(\widehat{K}^S(\sigma),\sigma)}^{\bar{\theta}} [s_2(e^*(\widehat{K}^S(\sigma),\hat{\theta}(\widehat{K}^S(\sigma),\sigma)),\widehat{K}^S(\sigma),\theta) - s_2(e^*(\widehat{K}^S(\sigma),\theta),\widehat{K}^S(\sigma),\theta)]dF(\theta,\sigma)\right\}$$
$$\times \frac{1}{1-\dfrac{\frac{\delta}{1-\delta}\int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}} s_1(e^*(\widehat{K}^S(\sigma),\hat{\theta}(\widehat{K}^S(\sigma),\sigma)),\widehat{K}^S(\sigma),\theta)dF(\theta,\sigma)}{c'(e^*(\widehat{K}^S(\sigma),\hat{\theta}(\widehat{K}^S(\sigma),\sigma)))}} - C'(\widehat{K}^S(\sigma)) = 0, \quad \text{(A.9)}$$

in which

$$c'(e^*(\widehat{K}^S(\sigma),\hat{\theta}(\widehat{K}^S(\sigma),\sigma))) > \frac{\delta}{1-\delta}\int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}} s_1(e^*(\widehat{K}^S(\sigma),\hat{\theta}(\widehat{K}^S(\sigma),\sigma)),\widehat{K}^S(\sigma),\theta)dF(\theta,\sigma).$$

The left-hand side of (A.9) is everywhere strictly decreasing in $\widehat{K}^S(\sigma)$.

Proof. For specific capital, optimal capital $\widehat{K}^S(\sigma)$ is given by (8) with $\hat{\theta}(K,\sigma)$ the highest $\tilde{\theta}$ satisfying the constraint (4) whatever K is chosen. With specific capital, $\underline{s}(K,\sigma)$ is independent of K by Definition 3. So, because the two integrands in (4) take the same value for $\theta = \hat{\theta}(K,\sigma)$, total differentiation of (4) with respect to K for $\tilde{\theta} = \hat{\theta}(K,\sigma)$ gives, with $de^*(K,\hat{\theta}(K,\sigma))/dK$ the derivative of $e^*(K,\hat{\theta}(K,\sigma))$ with respect to K taking into account the impact through $\hat{\theta}(K,\sigma)$,

$$\frac{\delta}{1-\delta}\left\{\int_{\underline{\theta}}^{\hat{\theta}(K,\sigma)}\left[s_1(e^*(K,\theta),K,\theta)\frac{\partial e^*(K,\theta)}{\partial K} + s_2(e^*(K,\theta),K,\theta)\right]dF(\theta,\sigma) + \int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}}\left[s_1(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)\frac{de^*(K,\hat{\theta}(K,\sigma))}{dK} + s_2(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)\right]dF(\theta,\sigma)\right\}$$
$$- c'(e^*(K,\hat{\theta}(K,\sigma)))\frac{de^*(K,\hat{\theta}(K,\sigma))}{dK} = 0. \quad \text{(A.10)}$$

Because $s_1(e^*(K,\theta),K,\theta)=0$ for all (K,θ) by definition of $e^*(K,\theta)$ in (1), this gives

$$\frac{de^*(K,\hat{\theta}(K,\sigma))}{dK} = \frac{\delta}{1-\delta}\left\{\int_{\underline{\theta}}^{\hat{\theta}(K,\sigma)} s_2(e^*(K,\theta),K,\theta)dF(\theta,\sigma) + \int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}} s_2(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)dF(\theta,\sigma)\right\} \div \left\{c'(e^*(K,\hat{\theta}(K,\sigma))) - \frac{\delta}{1-\delta}\int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}} s_1(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)dF(\theta,\sigma)\right\} > 0, \quad \text{(A.11)}$$

the sign following because $s_2 > 0$ and, if the denominator were not positive, $e^*(K,\hat{\theta}(K,\sigma))$ would not be the highest effort satisfying the relational contract constraint (3). Because the two integrands in (8) are the same for $\theta = \hat{\theta}(K,\sigma)$, the derivative of the maximand in (8) with respect to K is

$$\frac{1}{1-\delta}\left\{\int_{\underline{\theta}}^{\hat{\theta}(K,\sigma)}\left[s_1(e^*(K,\theta),K,\theta)\frac{\partial e^*(K,\theta)}{\partial K} + s_2(e^*(K,\theta),K,\theta)\right]dF(\theta,\sigma) + \int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}}\left[s_1(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)\frac{de^*(K,\hat{\theta}(K,\sigma))}{dK} + s_2(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)\right]dF(\theta,\sigma)\right\} - C'(K).$$

Because $s_1(e^*(K,\theta),K,\theta)=0$ by definition of $e^*(K,\theta)$ in (1), this can, with the use of (A.11), be re-arranged as

$$\frac{1}{1-\delta}\left[\int_{\underline{\theta}}^{\hat{\theta}(K,\sigma)} s_2(e^*(K,\theta),K,\theta)dF(\theta,\sigma) + \int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}} s_2(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)dF(\theta,\sigma)\right] \times \frac{c'(e^*(K,\hat{\theta}(K,\sigma)))}{c'(e^*(K,\hat{\theta}(K,\sigma))) - \frac{\delta}{1-\delta}\int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}} s_1(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)dF(\theta,\sigma)} - C'(K), \quad \text{(A.12)}$$

which can be re-written to give the first-order condition (A.9) for interior $\widehat{K}^S(\sigma)$. The proof that the left-hand side of (A.9) is everywhere strictly decreasing in $\widehat{K}^S(\sigma)$ is in Online Appendix B. □

Proof of Proposition 7. For specific capital, Lemma A.3 applies. Assumption 1 does not rule out $\widehat{K}^S(\sigma) = \overline{K}$ but, since it ensures both $\widehat{K}^G(\sigma)$ and $K^*(\sigma)$ are strictly less than $\overline{K}$, the proposition certainly holds if $\widehat{K}^S(\sigma) = \overline{K}$.

Optimal specific capital $\widehat{K}^S(\sigma)$ satisfies (A.9), whereas optimal general capital $\widehat{K}^G(\sigma)$ satisfies (A.6), which would imply $\widehat{K}^S(\sigma) = \widehat{K}^G(\sigma)$ if the fraction multiplying the term in braces in (A.9) were equal to 1. However, it follows from Lemma A.3 that this fraction is strictly greater than 1. Moreover, because specific capital relaxes the constraint (3), $\hat{\theta}(K,\sigma)$ is higher for given (K,σ) when capital is specific than when it is general. Furthermore, $s_{12} = y_{12} > 0$ from Assumption 1, which implies that the second integral in (A.9) is positive given $e^*(K, \hat{\theta}(K,\sigma)) < e^*(K,\theta)$. Thus, the left-hand side of (A.9) is larger for given K when capital is specific than when it is general so, because that left-hand side is everywhere strictly decreasing in K, it must be that $\widehat{K}^S(\sigma) > \widehat{K}^G(\sigma)$. For contractible effort, $K^*(\sigma)$ satisfies (A.5), so (A.9) implies that $\widehat{K}^S(\sigma)$ would converge to $K^*(\sigma)$ as $s_{12} = y_{12}$ converges to zero if the fraction multiplying the term in braces in (A.9) were equal to 1. Because from Lemma A.3 this fraction is strictly greater than 1, this implies $\widehat{K}^S(\sigma) > K^*(\sigma)$ for $y_{12}(e,K,\theta)$ sufficiently small for all e, K, and θ. □

Proof of Proposition 8. For the purpose of the proof, let

$$g(\theta) = 1 - \frac{y_1(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)}{c'(e^*(K,\hat{\theta}(K,\sigma)))}, \qquad \text{for given } (K,\sigma).$$

From the definition of $e^*(K,\theta)$ in (1), $g(\theta) = 0$ for $\theta = \hat{\theta}(K,\sigma)$ and, with $y_{13} > 0$ everywhere by Assumption 1,

$$g'(\theta) = -\frac{y_{13}(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)}{c'(e^*(K,\hat{\theta}(K,\sigma)))} < 0,$$

$$g''(\theta) = -\frac{y_{133}(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)}{c'(e^*(K,\hat{\theta}(K,\sigma)))} \leq 0, \qquad \text{for } y_{133} \geq 0.$$

Thus, under the conditions of the proposition, $g(\theta)$ satisfies the conditions of Proposition 3 and

$$\int_{\tilde{\theta}}^{\bar{\theta}} g(\theta)dF(\theta,\sigma_L) \geq \int_{\tilde{\theta}}^{\bar{\theta}} g(\theta)dF(\theta,\sigma_H), \qquad \text{for given } (K,\tilde{\theta}). \tag{A.13}$$

In Proposition 2, $\tilde{\theta}$ corresponds to $\hat{\theta}(K,\sigma)$ for both σ_H and σ_L. Moreover, $g(\theta) < 0$ for $\theta > \hat{\theta}(K,\sigma)$ and, from Proposition 4, $\hat{\theta}(K,\sigma_L) \geq \hat{\theta}(K,\sigma_H)$ for given

K for both systemic and specific risk. Furthermore, from (1), $e^*(K,\theta)$ is increasing in θ so $e^*(K,\hat{\theta}(K,\sigma_L)) \geq e^*(K,\hat{\theta}(K,\sigma_H))$ and it follows from Assumption 1 that $y_1(e,K,\theta)/c'(e)$ is decreasing in e. Together with (A.13) these imply

$$\int_{\hat{\theta}(K,\sigma_L)}^{\bar{\theta}} \left[1 - \frac{y_1(e^*(K,\hat{\theta}(K,\sigma_L)),K,\theta)}{c'(e^*(K,\hat{\theta}(K,\sigma_L)))}\right] dF(\theta,\sigma_L)$$
$$> \int_{\hat{\theta}(K,\sigma_H)}^{\bar{\theta}} \left[1 - \frac{y_1(e^*(K,\hat{\theta}(K,\sigma_H)),K,\theta)}{c'(e^*(K,\hat{\theta}(K,\sigma_H)))}\right] dF(\theta,\sigma_H), \quad \text{for given } K,$$

so

$$\int_{\hat{\theta}(K,\sigma_H)}^{\bar{\theta}} \left[\frac{y_1(e^*(K,\hat{\theta}(K,\sigma_H)),K,\theta)}{c'(e^*(K,\hat{\theta}(K,\sigma_H)))} - 1\right] dF(\theta,\sigma_H)$$
$$> \int_{\hat{\theta}(K,\sigma_L)}^{\bar{\theta}} \left[\frac{y_1(e^*(K,\hat{\theta}(K,\sigma_L)),K,\theta)}{c'(e^*(K,\hat{\theta}(K,\sigma_L)))} - 1\right] dF(\theta,\sigma_L), \quad \text{for given } K. \tag{A.14}$$

Moreover, (A.9) applies with either systemic or specific risk. With $s_1(e,K,\theta) = y_1(e,K,\theta) - c'(e)$,

$$\frac{\int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}} s_1(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)dF(\theta,\sigma)}{c'(e^*(K,\hat{\theta}(K,\sigma)))}$$
$$= \int_{\hat{\theta}(K,\sigma)}^{\bar{\theta}} \left[\frac{y_1(e^*(K,\hat{\theta}(K,\sigma)),K,\theta)}{c'(e^*(K,\hat{\theta}(K,\sigma)))} - 1\right] dF(\theta,\sigma),$$

so (A.14) implies

$$\frac{\int_{\hat{\theta}(K,\sigma_H)}^{\bar{\theta}} s_1(e^*(K,\hat{\theta}(K,\sigma_H)),K,\theta)dF(\theta,\sigma_H)}{c'(e^*(K,\hat{\theta}(K,\sigma_H)))}$$
$$> \frac{\int_{\hat{\theta}(K,\sigma_L)}^{\bar{\theta}} s_1(e^*(K,\hat{\theta}(K,\sigma_L)),K,\theta)dF(\theta,\sigma_L)}{c'(e^*(K,\hat{\theta}(K,\sigma_L)))}.$$

Thus, for given $\widehat{K}^S(\sigma)$, the fraction multiplying the term in braces in (A.9) is larger for σ_H than for σ_L. If this fraction had been equal to 1, (A.6) and (A.9) would have implied $\widehat{K}^S(\sigma) = \widehat{K}^G(\sigma)$. However, this fraction is greater than 1, so it is larger for σ_H than for σ_L. This, along with the left hand side of (A.9) being strictly decreasing in $\widehat{K}^S(\sigma)$, implies $\widehat{K}^S(\sigma_H) - \widehat{K}^G(\sigma_H) > \widehat{K}^S(\sigma_L) - \widehat{K}^G(\sigma_L)$. Moreover, (A.6) in Lemma A.2 implies that $\widehat{K}^G(\sigma)$ approaches $K^*(\sigma)$ as $s_{12} = y_{12}$ approaches zero. So, for $s_{12} = y_{12}$ sufficiently small for all e, K, and θ, $\widehat{K}^S(\sigma_H) - K^*(\sigma_H) > \widehat{K}^S(\sigma_L) - K^*(\sigma_L)$. □

Proof of Proposition 9. Necessity. To match the ratio of productivity in the specification with the relational contract constraint to the specification without that constraint when $\gamma = 1 - \beta/n$, it is necessary that the ratio of (15) should be the same for $\tilde{\theta} = \hat{\theta}^G(\sigma)$ or $\tilde{\theta} = \hat{\theta}^S(\sigma)$ for σ_H to σ_L as for $\tilde{\theta} = \bar{\theta}$ for σ'_H to σ'_L when $\gamma = 1 - \beta/n$. For the proportionate change in capital also matched, that ratio reduces to (23). For general capital, (24) follows immediately from requiring the ratio of (18) for σ_H to σ_L to match (22). For specific capital, the first-order condition (20) can be re-arranged as

$$\hat{\theta}^S(\sigma) - \frac{\psi(\hat{\theta}^S(\sigma), \sigma)}{\psi_1(\hat{\theta}^S(\sigma), \sigma)} = \delta k \frac{1 - \beta/n}{\alpha} \left(\frac{\beta}{n}\right)^{-k\frac{\beta/n}{\alpha} - 1} c^{k\frac{\beta/n}{\alpha}} C(\sigma) \left(\frac{\underline{s}(\sigma)}{\psi(\hat{\theta}^S(\sigma), \sigma)}\right)^{k\frac{1-\beta/n}{\alpha} - 1}.$$

The ratio of this for σ_H to σ_L is

$$\frac{\hat{\theta}^S(\sigma_H) - \frac{\psi(\hat{\theta}^S(\sigma_H),\sigma_H)}{\psi_1(\hat{\theta}^S(\sigma_H),\sigma_H)}}{\hat{\theta}^S(\sigma_L) - \frac{\psi(\hat{\theta}^S(\sigma_L),\sigma_L)}{\psi_1(\hat{\theta}^S(\sigma_L),\sigma_L)}} = \frac{\delta k \frac{1-\beta/n}{\alpha} c^{k\frac{\beta/n}{\alpha}} \left(\frac{\beta}{n}\right)^{-k\frac{\beta/n}{\alpha} - 1} C(\sigma_H) \left[\frac{\underline{s}(\sigma_H)}{\psi(\hat{\theta}^S(\sigma_H),\sigma_H)}\right]^{k\frac{1-\beta/n}{\alpha} - 1}}{\delta k \frac{1-\beta/n}{\alpha} c^{k\frac{\beta/n}{\alpha}} \left(\frac{\beta}{n}\right)^{-k\frac{\beta/n}{\alpha} - 1} C(\sigma_L) \left[\frac{\underline{s}(\sigma_L)}{\psi(\hat{\theta}^S(\sigma_L),\sigma_L)}\right]^{k\frac{1-\beta/n}{\alpha} - 1}}.$$

Use of (25) to substitute for the ratio of $\underline{s}(\sigma_H)/\psi(\hat{\theta}^S(\sigma_H), \sigma_H)$ to $\underline{s}(\sigma_L)/\psi(\hat{\theta}^S(\sigma_L), \sigma_L)$ in order to match the capital ratio for the specification without a relational contract gives

$$\frac{\hat{\theta}^S(\sigma_H) - \frac{\psi(\hat{\theta}^S(\sigma_H),\sigma_H)}{\psi_1(\hat{\theta}^S(\sigma_H),\sigma_H)}}{\hat{\theta}^S(\sigma_L) - \frac{\psi(\hat{\theta}^S(\sigma_L),\sigma_L)}{\psi_1(\hat{\theta}^S(\sigma_L),\sigma_L)}} = \frac{C(\sigma_H)}{C(\sigma_L)} \left\{ \left[\frac{E(\theta \mid \sigma'_H)/C(\sigma_H)}{E(\theta \mid \sigma'_L)/C(\sigma_L)}\right]^{\frac{\alpha}{k(1-\beta/n)-\alpha}} \right\}^{k\frac{1-\beta/n}{\alpha} - 1}$$

or

$$\frac{\hat{\theta}^S(\sigma_H) - \frac{\psi(\hat{\theta}^S(\sigma_H),\sigma_H)}{\psi_1(\hat{\theta}^S(\sigma_H),\sigma_H)}}{\hat{\theta}^S(\sigma_L) - \frac{\psi(\hat{\theta}^S(\sigma_L),\sigma_L)}{\psi_1(\hat{\theta}^S(\sigma_L),\sigma_L)}} = \frac{C(\sigma_H)}{C(\sigma_L)} \left\{ \left[\frac{E(\theta \mid \sigma'_H)/C(\sigma_H)}{E(\theta \mid \sigma'_L)/C(\sigma_L)}\right]^{\frac{\alpha}{k(1-\beta/n)-\alpha}} \right\}^{\frac{k(1-\beta/n)-\alpha}{\alpha}},$$

which can be written as (25).

Sufficiency. Establishing sufficiency requires that there exist specifications of the relational contract model that satisfy the incentive constraint (4) and conditions for optimal capital choice for both σ_L and σ_H to be interior to $[\underline{\theta}, \bar{\theta}]$. The conditions in the proposition are derived to ensure that (4) is satisfied.

For general capital, the left-hand side of (17) is positive for $\hat{\theta}^G(\sigma) = \underline{\theta}$ if

$$(1-\delta)\frac{k}{\alpha}C(\sigma)\left(\frac{\beta}{n}\right)^{1-\frac{k}{\alpha}} c^{\frac{k}{\alpha}\frac{\beta}{n}} > \left[\frac{\underline{\theta}}{\hat{S}(\sigma)}\right]^{\left(1-\frac{\beta}{n}\right)\frac{k}{\alpha}-1} \underline{\theta}^{\frac{\beta}{n}} \int_{\underline{\theta}}^{\bar{\theta}} \theta^{1-\frac{\beta}{n}} dF(\theta,\sigma)$$

and negative for $\hat{\theta}^G(\sigma) = \bar{\theta}$ if

$$\left[\frac{\bar{\theta}}{\hat{S}(\sigma)}\right]^{\left(1-\frac{\beta}{n}\right)\frac{k}{\alpha}-1} \int_{\underline{\theta}}^{\bar{\theta}} \theta dF(\theta,\sigma) > (1-\delta)\frac{k}{\alpha}C(\sigma)\left(\frac{\beta}{n}\right)^{1-\frac{k}{\alpha}} c^{\frac{k}{\alpha}\frac{\beta}{n}}.$$

When both these hold, the optimal solution for $\hat{\theta}^G(\sigma)$ must be interior to $[\underline{\theta}, \bar{\theta}]$. There exists $c > 0$ for which both hold if

$$\bar{\theta}^{\left(1-\frac{\beta}{n}\right)\frac{k}{\alpha}-1} \int_{\underline{\theta}}^{\bar{\theta}} \theta dF(\theta,\sigma) > \underline{\theta}^{\left(1-\frac{\beta}{n}\right)\left(\frac{k}{\alpha}-1\right)} \int_{\underline{\theta}}^{\bar{\theta}} \theta^{1-\frac{\beta}{n}} dF(\theta,\sigma)$$

which, under the conditions of the proposition, reduces to

$$\frac{\bar{\theta}^{\left(1-\frac{\beta}{n}\right)\frac{k}{\alpha}-1}}{\underline{\theta}^{\left(1-\frac{\beta}{n}\right)\left(\frac{k}{\alpha}-1\right)}} > \frac{\int_{\underline{\theta}}^{\bar{\theta}} \theta^{1-\beta/n} dF(\theta,\sigma)}{\int_{\underline{\theta}}^{\bar{\theta}} \theta dF(\theta,\sigma)}. \tag{A.15}$$

When (26) holds, there exists $c > 0$ such that (A.15) is satisfied for both σ_L and σ_H. □

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Supplementary Material

Supplementary data are available at *JEEA* online.

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