

**ASSESSMENT OF DESIGN PROCEDURES  
FOR STRUCTURAL GLASS BEAMS**

**by**

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**A Thesis submitted for the Degree  
of Master of Science  
at the  
University of Oxford**

## ABSTRACT

### **Assessment of Design Procedures for Structural Glass Beams**

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In structural design, ductile materials are usually preferred, because failure is rarely sudden and catastrophic. Once the material has yielded, large observable strains will occur before total collapse of a structure. This thesis is about the structural use of glass. Glass is probably the most perfectly brittle material that exists. It demonstrates linear elastic behaviour right up to the point of failure. When a piece of glass fails, this always happens suddenly, by the high-speed propagation of a crack through the entire element.

If glass is so different from the most common primary structural materials namely steel and reinforced concrete, then should we be using the same basic design approach for both? This thesis reviews current design methods tracing their development through this century. Current code formers are keen to bring all materials under the umbrella of Limit State Design. This philosophy has been developed for ductile materials and is shown to be somewhat inappropriate for materials where the main design criterion is not ultimate strength.

A further chapter looks at the statistical behaviour of multi-ply beams. This was highlighted, at the recent Glass in Buildings conference, as being an important area for further research. It has been shown that statistically two or more beams are always better than one of equivalent thickness. A method of reducing the thickness of a glass beam, without affecting its probability of failure, has been described. This could have significant cost implications in the design of glass beams.

It should be noted that this thesis is not about face-loaded glass elements such as windowpanes or structural glazing, where designs are governed by deflection and generally executed using rules of thumb, but about edge-loaded elements where strength governs and more rigorous design rules are required.

## **Preface**

I have very much enjoyed being a member of the Oxford Engineering Department. It has been a most friendly and stimulating place to work. I am indebted to the department for allowing me the opportunity to study here whilst employed in a teaching position. Professor Houlsby has been an excellent supervisor. Despite his many responsibilities as Professor of Civil Engineering, he has always made time to discuss and challenge my ideas for which I am most grateful.

I would also like to thank Mark Porter, who is also studying glass, for bringing a fresh approach to the subject, Byron Byrne, who was always willing to help me with any computing difficulties, and Nicola Houlston who typed parts of my thesis.

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## **Introduction**

Structural materials can be categorised in many different ways. One of the most common is by the nature of their failure. Materials, such as steel, after reaching their elastic limit, deform plastically before failing. Such materials, where reasonably large plastic strains are observed, are broadly categorised as ductile. On the other hand, those materials that exhibit little or no plasticity are in contrast termed brittle.

In structural design, ductile materials are usually preferred, because failure is rarely sudden and catastrophic. Once the material has yielded, large observable strains will occur before total collapse of a structure.

Steel, the most common primary structural building material, generally behaves in a ductile fashion. However, during the Second World War, over two hundred steel ships suffered serious brittle fracture, often at relatively low stress levels. It was observed that brittle behaviour, in normally ductile materials, almost always occurs in regions of elastic stress concentration, where some constraint exists to prevent plastic stress redistribution (McGuire, 1968).

Following these experiences, designers realised that brittle fracture in steel could be avoided by sensible detailing. This is also true of reinforced concrete. Concrete itself is a brittle composite, but with the addition of reinforcement it behaves in a ductile fashion. In earthquake-prone areas, where designing for ductility is paramount, reinforced concrete beam-column joints are highly reinforced to avoid the possibility of brittle behaviour.

Structural engineers, being concerned mainly with design in these two materials, have come to understand brittle fracture in structural elements to be unnecessary and avoidable, as expressed by Nethercott (1991):

“From the designers point of view, the most satisfactory way of dealing with brittle fracture, is to reduce the likelihood of its occurrence by sensible choice of materials.”

This thesis is about the structural use of glass. Glass is probably the most perfectly brittle material that exists. It demonstrates linear elastic behaviour right up to the point of failure. When a piece of glass fails, this always happens suddenly, by the high-speed propagation of a crack through the entire element. No amount of sensible detailing will prevent this fast fracture from occurring, when an element finally fails. This is not to say that sensible detailing will not delay the onset of failure, but merely that glass, by its very nature, will always fail by brittle fracture.

The question then arises as to why anyone would wish to design structural elements in glass. Examination of its mechanical properties appears to indicate that glass is the wrong material for structural applications. However, in prestige office developments of the eighties and nineties, glass alone stands out as the most prolific construction material. In the drive for total transparency, glass curtain walls are being supported by less and less structure. At the 1999 “Glass in Buildings” conference in Bath, numerous speakers presented glass-clad buildings with minimalist bracing provided by pretensioned steel wire trusses. How much more daring to remove the steel altogether, and make a wall totally transparent by bracing with glass fins? This, of course, has been done many times, mostly on a relatively small scale (the Sainsbury Centre being the

notable exception). Less common than glass fins are glass beams designed to carry gravity loads. Designers become considerably more cautious, not unreasonably, when it comes to designing floor and roof beams or even stairs. Horizontal, gravity-loaded beams pose a new problem. How will the glass behave under long term loading? “Static fatigue” in glass (described on page 31) is a well-known phenomenon, but how to account for it in a stress-based design is not so clearly understood.

Up until relatively recently, glass has primarily been used in simple load-carrying situations; such as its most common architectural uses for windowpanes and structural glazing, where it typically only carries its own weight and any loads directly applied to it. In these situations, the design is usually governed by deflection, and design methods usually involve ‘rules of thumb’. This thesis is about the use of glass for primary load-bearing structural elements, particularly beams. Here the elements under consideration are generally edge-loaded, rather than face-loaded, and the design is not governed by deflection but by strength. Such elements would usually have a higher consequence of failure and therefore more rigorous design rules are required.

The work of this thesis is threefold. The first part is an examination and review of current design methods, with reference to their application to common structural building materials. The second section describes the properties of glass in detail. This includes discussion on the nature of glass, brittle fracture, an examination of the strengths and weaknesses of glass as a structural material and a review of glass strength information currently available to design engineers.

In the third section, a specific design issue relating to glass beams is examined. The

strengths of multi-ply beams are predicted from the strength distribution found for single beams. This part aims to take a rational look at why glass designers almost always use multi-ply beams. Tim Macfarlane, of Dewhurst Macfarlane and Partners, one of the keynote speakers at the 1999 Glass in Buildings conference, cited the need for “a statistical approach to applying factors of safety in multi-ply construction” as an important area for further research and design development.

The final section of this report will briefly discuss the appropriateness and validity of stress-based design methods for glass.

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## 1. Design Methods

If we are to assess current structural design methods we need first to be clear about what "design" is in this context. The concept of "engineering design" has proved to be somewhat elusive to define. In trying to attribute a creative, ephemeral quality to what appears to many practitioners as mundane and mechanical has interested many engineers. This thesis is not concerned with what "engineering design" is, but, in fulfilling the aims of the title of this work "Assessment of design procedures for structural glass beams", it is obviously important to be clear about what a "design procedure" is.

An "engineering" design procedure really is a method of ensuring the safety and continued utility of whatever is being designed. In the case of a glass beam, the engineering design procedure aims to ensure adequate structural performance of the beam, under all possible applied loads, for as long as it is to be in service. Ultimately, if we could accurately assess the greatest possible load that is to be applied to an element of structure, and be utterly confident about the mechanical properties of the material throughout the element, there would be no need for a design procedure, as it is being defined here. Following this thesis, it becomes clear that engineering design procedure is essentially about the assessment of risk. In its most simple form of expression, it could be described by the following relationship.

$$\text{Design Actions} \geq \text{Safety Factor} \times \text{Material Response} \quad \text{Equation 1}$$

It should be stressed here that this safety factor includes an assessment of risks associated with both the expected design actions and possible irregularities in material

behaviour. It is therefore more broad than the safety factor used in permissible stress design.

It is important, therefore, in any design exercise, that the safety factor is intelligently and appropriately assessed for any given situation. The development of design procedures, this century, has been aimed at making the assessment of this safety factor a more logical and transparent process. This has led to the re-expression of the simple relationship above into a supposedly more defined, and certainly more complex, form that underlies the philosophy of limit state design. This philosophy, Professor Heyman once observed, "can be seen only rather partially and dimly, as if in a fog" (Heyman, 1980).

The philosophy of limit state design is generally considered, by the design community, to be the underlying basic procedure, which can and should form the basis for design in all engineering materials in any situation. This striving for an underlying process for engineering design has led to the formulation of Eurocode 1 - Basis of Design and Actions on Structures, which has as its primary objective "to ensure the consistency of design rules for a wide set of construction works made of various materials" (Eurocode 1, 1996). The question has to be posed at this stage, why attempt to make one set of design rules for two materials as fundamentally different as steel and sand, for example?

It is worth uncovering the mystique (a nicer word than fog) of just how limit state design has arisen. If we journey back to the beginning of this century, when modern theories of structural analysis and elasticity had gained wide acceptance, structural design was a relatively simple process, essentially based on the elastic, permissible

stress design method. Steel was the new material. Its behaviour under load was relatively well understood and provided the structural response remained elastic (which it never did, but was always assumed to), robust safe structures could easily be designed and built.

### **Permissible Stress Design**

The basis of the permissible stress design method was not dissimilar (as mentioned earlier) to the expression given in Equation 1. However, the safety factor was always the same for a given material. In the case of steel, although the material strength did not vary very much, a safety factor of about 0.7 was applied to the steel strength. This takes no account of the type of loading being applied or the likelihood of the design loading being exceeded. In other words, the permissible stress method allowed designers to ignore the risk associated with expected design actions. This method, although a simple and generally effective method of design, has been found to be inadequate in some situations. "Because it is based on an elastic stress distribution, it (the permissible stress method) [is unsuitable] when the deformations are not proportional to the load, as in slender columns. It has also been found to be unsafe when dealing with the stability of structures subject to overturning forces" (Mosley *et al*, 1996).

As a result of its inconsistencies, the permissible stress method came under scrutiny and attack during the middle of this century in what is sometimes described as "The Plastic Design Revolution". This is discussed in the next section. However despite some apparent inconsistencies, the permissible stress method is still used today, and in fact vehemently supported by some designers.

"However there are even today still many "elastic" designers who have not changed their allegiance" (Addis, 1990).

In 1979, Beal published a paper in the Proceedings of the Institution of Civil Engineers entitled "What's wrong with load factor design"<sup>1</sup>. In it he proposes a return to permissible stress design based on working loads, and shows that the attempt to define structural design strictly in terms of probability theory has serious limitations. These will be discussed later, under the heading of limit state design.

Furthermore the permissible stress method is still used for timber design in this country. Despite the commitment of European code committees to the limit state design method, the latest British timber design standard was published in 1996 using the "old" permissible stress method. This is an obvious statement by British designers of the inadequacies of the limit state philosophy for timber.

It is also interesting to note, that when BS5950, the limit state steel code, was first published, it adopted a title almost exactly the same as the existing permissible stress code, BS449. However, it did not supersede it, nor does it make any references to it; even in the list of publications referred to. The implication here being that BS449 should continue alongside BS5950 as an acceptable code of practice. The earlier code has now been all but withdrawn, as the limit state philosophy is ideally suited to steel. After all, this is the material for which it was developed, as can be seen in the next section.

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<sup>1</sup> Load Factor Design was the intermediate step between permissible stress design and limit state design. Partial

## **The Plastic Design Revolution**

By the late 1920's, the elastic permissible stress design method had been used in one form or another for well over fifty years, but there had been little or no development of it in this time. Designers were becoming increasingly dissatisfied with such a conservative approach. In 1929 the British Steelwork Association, with government support, set up the Steel Structures Research Committee (SSRC) with the following two-fold brief

- (i) To review present methods and regulations for the design of steel structures, and
- (ii) To investigate the application of modern theories to the design of steel structures, and to make recommendations for the translation to practice of such results as would appear to lead to more efficient and economical design. (Baker 1936/7).

The following quotation from Baker, discussing the commission's findings, is a covert attack on the elastic design process for steel.

"The designers task of producing a structure of adequate strength is complex. So complex is it that assumptions must be made, not only to make good deficiencies in the knowledge of conditions, but with the avowed object of simplifying strength calculations. Whatever advances in knowledge may be made, it is unlikely that this state of affairs will change radically; those of you engaged in design know that involved methods of calculation are usually impracticable. Simplification can, however, be bought at too great a price. There is in some branches of structural engineering, a tendency for assumptions to be chosen because they make some simple calculation possible, even though they are so sweeping that the true behaviour of the structure is

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safety factors are applied to the loads and then are compared with the unfactored material strength.

disguised. Under these conditions, while it may be possible with the safeguard of large load factors, to produce stable structures, any evolution of the method of construction is impossible" (Baker 1936/7).

This was the beginning of what would form the plastic design revolution. The main criticism of elastic design being that no information was discovered in the design about how, and at what level of load, the structure would collapse. Also there was concern that the stresses resulting from the design process were believed by many to be a true representation of the behaviour of the real structure. This was highly unlikely to be the case, because of local yielding in joints and movements in foundations. Heyman summed up this situation in 1973:

"In fact the elastic designer does not usually make a very good estimate of the working state of an actual frame, unless it happens to be statically determinate, and in most cases his estimate is so poor as to be virtually meaningless" (Heyman, 1973).

And Baker *et al* in 1956 along similar lines,

"Might it not be so that the whole basis of elastic design was at fault and that the path which designers had been following for nearly a century was nothing more than a blind alley" (Baker *et al*, 1956).

It may be worth, at this juncture, asking the question why is plastic design being examined at all, as part of a thesis which is about glass, a wholly elastic material? The reason is that plastic design was the fundamental basis out of which the philosophy of limit state design arose.

"Part of the difficulty actually lies in the changing use of words in the last thirty years. What used to be called simple plastic design, gradually became known as limit design, and was somewhat naturally assimilated into the philosophy of limit state design" (Heyman, 1980).

"Recently the phrase "plastic design" has tended to be replaced by the more general idea of "limit state" design" (Addis, 1990).

### **Limit State Design**

Out of plastic design theories, and the upper and lower bound theorems, arose an intermediate development of permissible stress design, called load factor design. The structural concrete code in its introduction describes this state in the development of structural design methods as follows.

"This method uses ultimate loads obtained by increasing the working loads by a safety factor, and it aims to ensure that the strength of the structure calculated using the expected actual material strengths is sufficient to support this ultimate loading" (BS8110, 1985).

The code goes on to explain that these two methods (permissible stress and load factor) are identical for materials which are fully elastic up to failure (except that obviously the focus is different). It also suggests that for most common materials very similar results can be obtained using both methods with an appropriate choice of coefficients.

The limit state design philosophy, combined and developed these two methods of design. It treats uncertainty more logically by the use of partial safety factors. The main argument for using different partial factors for different load types, such as live, wind and dead, was to prevent the unreasonable increase of a beneficial load. The most obvious example being dead load under wind uplift conditions. (In fact this problem does not arise with the permissible stress method, where all loads are unfactored).

To find a definition of limit state design *per se* is not easy. This is probably because the explicit reference to limit states was only one of several ideas that were simultaneously incorporated into this new generation of codes. The other two main ideas being rational statistical treatment of uncertainty and the partial safety factor idea as mentioned above. Even in Eurocode 1, no simple straightforward definition is expressed. However the following description is a summary of the work of the Joint Committee on Structural Safety acting for the Comite European du Beton (CEB) which carried out the ground work leading to the formulation of Eurocode 1.

The CEB proposed common design rules to form the basis of all codes. The general form of the equation to check structural safety was proposed as follows:

$$g_R \left( \frac{f_K}{\gamma_{m_1} \gamma_{m_2} \gamma_{m_3}} \right) \geq g_S (\gamma_{f_1} \gamma_{f_2} \gamma_{f_3} Q_k)$$

Where  $g_R$  and  $g_S$  are resistance and load effect functions which convert the terms in brackets to resistance and load effects respectively. The quantities  $\gamma_{m_i}$  and  $\gamma_{f_i}$  are the respective partial factors.

The partial factors,  $\gamma_m$ , on material strength, take account of the following factors (CEB, 1976):

- (a) The possibility of unfavourable deviations of the strengths of materials or elements from the specified characteristic value;
- (b) possible differences between the strength of the material or element in the structure from that derived from control test specimens.
- (c) possible local weaknesses in the structural material or element arising principally from, or in, the construction process;
- (d) possible inaccurate assessment of the resistance of elements derived from the strength of the material, including variations in dimensional accuracy achieved in construction as they affect the resistance.

The partial factors,  $\gamma_f$ , on actions take account of the following factors:

$\gamma_{f_1}$  deviation of actions from their characteristic value.

$\gamma_{f_2}$  (load combination factor) allows for the reduced probability that all actions are at their characteristic value.

$\gamma_{f_3}$  possible inaccurate assessment of the action effects, including dimensional inaccuracies.

In addition, either  $\gamma_m$  or  $\gamma_f$  may be modified to allow for low consequences of failure and/or the possibility of brittle fracture.

The characteristic values of the loads and resistances are considered as their 95 or 5 percentile values as appropriate, or their currently accepted values in lieu (Melchers,

1987).

Furthermore in defining "Limit State Design" it is necessary to include some discussion on limit states. These are generally divided into ultimate and serviceability limit states. The former being catastrophic states that require large safety factors to reduce their likelihood of occurrence to a very low level. Serviceability limit states on the other hand, are limits on acceptable behaviour in normal service. Examples of ultimate limit states for steel might include strength, buckling, stability against overturning or sway, fracture due to fatigue. Serviceability limit states would include deflection, vibration, repairable damage due to fatigue, corrosion (Dowling *et al*, 1988).

Limit state design, which includes the partial load factor philosophy, has come under criticism for three main reasons. Firstly the methodology of design has become so complex that the engineer's attention is diverted from the all-important question of details (Beal, 1979).

Secondly "most structural failures in practice are not collapses but are in fact serviceability failures. When structures do collapse, this is almost always caused by serious errors in design or construction, or else by extraordinary loads which bear little relation to normal imposed and permanent loads (e.g. collisions, typhoons). The case of structural collapse brought about by excessive statistical variation of normal dead and imposed loads is a rare one, and by concentrating on it, analysis based on the ultimate limit state is [given] excessive emphasis" (Beal, 1979).

And thirdly, "it is wrong to forget that the numerical work has been arranged

empirically, and to come to believe that the values of partial load factor found to give good practical results actually correspond to a real state of loading” (Heyman, 1973). This “forgetting” is because designers confuse plastic design theory with the load factor design method. Plastic design theory provides a method by which the collapse load of a structure can be calculated accurately. This is achieved by firstly allowing a structure not to be considered failed at first yield, but to be allowed to yield throughout its cross-section. Furthermore, this highly predictable fully plastic state will not cause collapse until a sufficient number of equivalent plastic hinges have formed, such that a mechanism is formed.

In load factor design, the predicted stress state of the structure will be at some proportion (the load factor) less than the accurately calculated collapse state. Although these stresses are based on the collapse state, they themselves, represent only one of an infinite number of possible equilibrium situations. In this way, they are no more “real” than a situation predicted by elastic design methods.

We now return to a question posed near the beginning of this chapter. Is it reasonable to try to formulate a unified design process for all construction materials, even though their natures vary enormously? The first step in answering such a question is to examine the latest generation of codes of practice, and see how apt this philosophy is for the most common construction materials - namely steel, reinforced concrete, masonry and timber.

## **Steel**

Plastic design methods and therefore the limit state philosophy (as explained earlier),

were developed with structural steel in mind. None of the other materials listed above behave in such an ideally elastic-plastic way as steel; neither are they particularly ductile. Steel is unique amongst these materials in its isotropy, homogeneity and consistency of strength from one specimen to the next. It is an ideal material to develop a design philosophy around.

In the CEB publication referred to earlier, a suggestion is made that the partial factors  $\gamma_m$  and  $\gamma_f$  may be modified to allow for low consequences of failure and/or the possibility of brittle fracture (Melchers, 1987). This further emphasises that this unified design philosophy is essentially constructed around ductile materials (the reference to the possibility of brittle fracture, suggests that the materials for which this philosophy is concerned are fundamentally ductile, but can in unusual circumstances behave in a brittle fashion). As outlined in the introduction, brittle fracture is an occurrence that most designers have come to believe can be ruled out by sensible detailing, and sensible choice of materials. Steel certainly fits this mould perfectly.

### **Reinforced Concrete**

Reinforced concrete is a complex material, being comprised of a brittle composite which has been rendered ductile by the addition of reinforcement. It also fits reasonably well into the limit state method, because sensible detailing can ensure that an RC element behaves in a ductile fashion at failure. However, failure of reinforced concrete structures rarely occurs as a result of an inadequate design approach, but frequently occurs because of poor detailing or poor workmanship. Numerous examples of reinforcement corrosion problems could be cited, in which structures not much more than a few years old are showing serious signs of deterioration.

With this material, frequently too much emphasis is put on ensuring that statistical variations in loading are accounted for, such that the detailing receives too little attention. However, provided detailing is such that shear failures and compression failures are avoided, this material fits quite well into the ultimate limit state design process. This is because, for example, in an under-reinforced beam, the failure is essentially a ductile failure of the steel.

### **Masonry**

So far in this discussion the emphasis has been on stress. When we turn to masonry, stress-based design methods become all but irrelevant. This is because it is almost impossible for a masonry structure to fail completely as a result of over-stressing (except of course in tension). In compression, to crush an average sandstone with a strength of say  $40\text{N/mm}^2$ , would require a tower of similarly dense material to be 2km high (Gordon, 1978). Furthermore to discuss plastic behaviour of individual units of masonry would be nonsense, as the material displays little or no ductility. However, it is possible to "translate basic ideas stemming from plasticity theory into terms applicable to masonry" (Heyman, 1995).

The particular basic idea being referred to here is that of the design being based on the collapsed state of the structure rather than the working state. But here the collapse is not a result of material failure but of overall structural instability.

Heyman explains in "The Stone Skeleton" that in masonry design, it is not a question of strength but rather it is the shape of a structure that determines its integrity. Masonry

design, therefore, could possibly be moulded to fit the limit state philosophy, with the primary ultimate limit state under consideration being one of stability rather than strength.

Unfortunately, the limit state design philosophy is so strength-based (as it was developed for steel where material strength is of primary importance), that the masonry codes of practice based on it, still emphasise strength as the primary design criterion.

“The design of load bearing masonry members should be undertaken primarily to ensure an adequate margin of safety against the ultimate limit state being reached. Generally, this is achieved by ensuring that the design strength of a member is greater than or equal to the design load.” (BS5628, 1992).

This opening statement in the section of the code entitled "Basis of Design" categorically emphasises, the ultimate limit state being of the utmost importance. (This reinforces Beal's view, quoted earlier, that the ultimate limit state is given too much emphasis, when most structural failures are serviceability failures). It also puts the emphasis squarely on strength as the main design criterion. It seems obvious right from the outset, if we are to believe Heyman, that masonry design is being moulded to fit into a design procedure which is strength-based and to which it is not suited.

An example of how strength-based design is inappropriate for masonry comes from Benouville's analytical study of the structure of Beauvais Cathedral (a structure that, because of its height, would be pushing modern structural codes to their limit, if it would even be possible to construct it under their rules). Benouville could find no stresses greater than  $1.3\text{N/mm}^2$  in a material that would typically have a crushing

strength of about  $40\text{N/mm}^2$  (Heyman, 1995). Clearly in its current state it is nowhere near being over-stressed (apart from perhaps local crushing which can be easily tolerated by masonry). However, in its chequered history, Beauvais has suffered three major collapses. It is highly unlikely that the earlier structural forms of the cathedral induced stresses thirty times greater than its current form. Clearly these failures were from some other cause than over-stressing. The last collapse, of the main tower, in 1573, was almost certainly an instability failure. The unbraced columns were examined by the King's Architects thirteen days prior to the collapse. They discovered that some of the columns were over 3m out of plumb. (Cathedral Builders, Heyman, Straub, Cowan).

## **Timber**

Timber, because it comes from a natural living source is by its nature highly variable, anisotropic and inhomogeneous. This in itself suggests it would be a difficult material for construction. However, it is very widely used, because of its toughness, the ease with which it can be cut and joined and its natural attractiveness that is not so evident in other construction materials.

This high degree of variability is reflected in the strength of timber; not only between species but also within a species. The coefficient of variability<sup>1</sup> is typically between 10 and 30 percent depending on the species (see Fig. 1). The current British code for timber (BS5268:1996), defines the stresses that can safely be permanently sustained for particular species as grade stresses. These are calculated from the stress above which

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<sup>1</sup> Standard deviation/mean

95 percent of clear<sup>1</sup> test specimens should fall. This 5-percentile lower exclusion value is then reduced by a factor of about 2 to take account of the reduction in strength suffered by timber under sustained loading. Finally the grade stress is reached by reducing this further, to between 30 and 75 percent of its value by grading each piece of timber according to its number and distribution of defects. These defects are caused naturally during growth and artificially during seasoning. They include shakes, splits, waness and knots. The grade stresses are recorded in the code for different species and are the main parameter for design.

From this description, it is obvious that, unlike steel, the ultimate strength of a timber beam cannot be easily estimated. The plastic design method, that led to the limit state philosophy as described earlier, was so influential in steel design, because it describes a known state for a structure; that at collapse, whereas previously used elastic design methods had no true reference point. “True” here meaning one that can be calculated accurately and is representative of what is actually going on in the structure.

Returning to timber, the true collapse load of a particular timber beam can never be assessed with any degree of accuracy. The material is far too variable and so dependent on naturally formed, highly individual defects. This is one reason why the load factor philosophy may not be so appropriate for timber.

However, in a way timber design already incorporates some aspects of the limit state philosophy; that is the partial factor idea. Permissible stresses are calculated from the grade stresses described above. This is done by multiplying the grade stresses by various modification factors, or *K* factors, to take account of such criteria as load

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<sup>1</sup> Specimens without defects

duration, orientation of load in relation to grain direction and degree of load sharing. These factors have all been formulated empirically.

Returning to the grade stress, it is obvious that because of the high coefficient of variability on strength, this stress will be very much less than the mean strength for a given species (see Fig. 1). Furthermore, because of the way timber is used (e.g. floor joists, ceiling rafters and double or triple bearers) and because of its flexibility, loads are often shared between several members. These two factors, combined with its ability to sustain quite severe loads for short periods of time, mean that timber elements rarely fail by overloading<sup>1</sup>. It is therefore unnecessary to give much emphasis to statistical variations in ordinary dead and live loads which could lead to unexpected overloading. Whereas heavy emphasis is given to these when the partial load factor/limit state philosophy is applied to most materials (Dead load and Live load factors are generally set at 1.4 and 1.6 respectively).

Furthermore, because of the empirical basis of the code, it would be tedious, unnecessary and possibly risky to try to convert this empirically established information to a new format into which it would not necessarily fit well. This is probably why BS 5268 was reissued in 1996, (after the publication of draft Eurocode 1), retaining the permissible stress design method.

### **Timber and Glass**

It is worth noting that there are many similarities between timber and glass. This seems strange because timber is so forgiving to local damage and glass quite the opposite. However, closer investigation of their behaviour under load highlights some interesting

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<sup>1</sup> Most failures are in fact serviceability failures for example, warping, deterioration from exposure to

parallels. These are mentioned here briefly but will be discussed in more detail in Chapter 4, after the nature of glass has been described in detail in Chapters 2 and 3.

Firstly the span and location of timber and glass beams are often similar. They are both architectural materials, often exposed, and rarely used for large spans. Timber and glass both suffer from reduced strength under sustained loading. They both require visual inspection for defects prior to use. Timber behaves elastically right up to the point of failure and assessments of ultimate strength using a fracture mechanics approach assuming splits are cracks have shown close correlation to actual strengths. Finally, the tensile strength of timber, typically 35-55N/mm<sup>2</sup> (Ashby and Jones, 1996) is of a similar order of magnitude to glass.

### **Concluding Comments**

In this chapter the progression from permissible stress design to so-called limit state design has been documented. Permissible stress methods came under attack in the middle of this century, because of their over simplicity and inability to give a true representation of the real state of stress in a structure. Out of this dissatisfaction arose the plastic design method, in which design is based around the collapse state, for which realistic estimates of the loads causing it can be made. With this method, the way of calculating a safe margin between expected loading and material strength was shifted from a reduction in the estimated material strength to an enhancement of the expected design load. This was a very reasonable approach for steel, as its material properties do not vary very much.

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moisture or excessive deflection.

Out of this method limit state design, as we know it, emerged. This development of design methodology has been undertaken with steel and reinforced concrete in mind. However in recent years, a desire has arisen, amongst those who form codes of practice for the construction industry, to find an underlying method of design that could be adapted to suit any material. The method chosen is limit state design and its basis is described in Eurocode 1<sup>1</sup>. This method is essentially a stress-based design approach, that puts considerable emphasis on the ultimate limit state and on ensuring that the design strength of a member is greater than or equal to the design load.

It has been shown in this chapter that masonry does not fit comfortably into a stress-based design procedure. This is essentially because stress levels in masonry are a distraction from the more important ultimate condition of stability. Timber has been shown to exhibit such enormous natural variability in its strength, that estimation of collapse loads, and therefore use of the plastic design method, is impossible. Furthermore over-emphasis of normal statistical variations in gravity loads has been shown to be irrelevant in timber design, because of its ability to sustain short term overloading with ease.

The limit state/load factor method of design is therefore somewhat constraining and possibly inappropriate for masonry and timber. The all-important question in the context of this thesis, is how appropriate is it for glass. This question will be addressed in Chapter 4, after glass as a material, and some statistical work on an important aspect of design with glass, have been investigated in the next two chapters. However on this subject of the appropriateness of modern design methods to glass,

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<sup>1</sup> DD-ENV 1991-1:1996 Basis for Design and Actions on Structures

Heyman has this to say

“The so-called plastic design theory of structures applies therefore to steel and to reinforced concrete frames...but not to materials like cast iron or glass, which are brittle.” (Heyman, 1995)

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WESTERN HEMLOCK

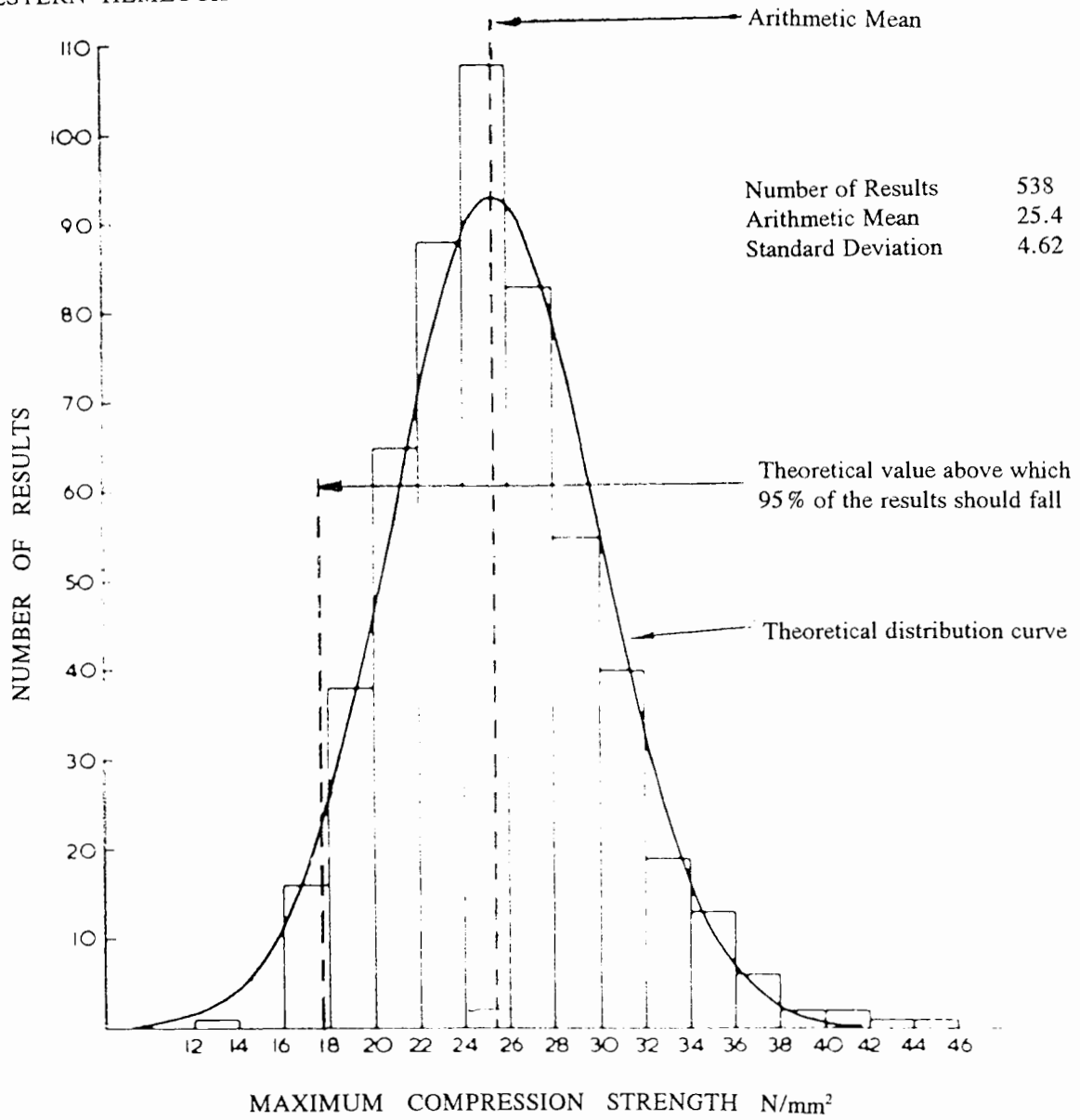


Figure 1 Frequency Distribution of Maximum Compressive Strength. (Building Research Establishment. © Crown copyright.) (From Illston *et al*, 1979).

## **2. Glass as a Structural Material**

### **History of Glass**

In one form or another, glass has been used for about 5000 years. The prolific use of glass dates back to Roman times, when, after the development of blowing techniques, glass lamps, tableware and ornaments became readily available. Its use for windows, albeit in small pieces, began in the Middle Ages, and by the sixteenth century many houses had glass windows. Larger panes of glass became available, at considerable expense, with the development of cylinder glass and crown glass. Finally with the invention of the float glass process, developed by Sir Alistair Pilkington in the 1950's, large sheets of glass can be produced relatively cheaply; such that glass with all its marvellous qualities can compete with other materials as a means of cladding modern framed buildings.

### **The Structure of Glass**

Glass is often described as a super-cooled liquid, partly because of a well-established myth that old windowpanes are usually thicker at the bottom because the "liquid" glass has flowed under gravity. This misleading description also arises because the atomic structure of glass has no long-range order, unlike most other solids. Glass is better described as an inorganic material that has been fused at high temperature and cooled at a carefully controlled speed to avoid crystallisation. Its solid structure at room temperature is therefore non-crystalline or amorphous, but is nevertheless solid; glass does not creep at room temperature.

Glass in its most pure form could be made entirely from silica,  $\text{SiO}_2$ , and indeed some specialist glasses, apart from unavoidable impurities, have no other ingredients. Most

other glasses, of which there are many available for different applications, are composed of about 70% silica. Sodalime glass in its most common form, contains about 73% silica, 15% soda and 10% lime. (Creyke *et al*, 1982). The soda is added to reduce the melting point of the mixture to a commercially acceptable level without sacrificing the clarity of the finished product. Lime assists with this process and also increases the hardness and chemical resistance of the finished product. Small quantities of other multivalent metal oxides are also added to improve the quality and durability of the glass.

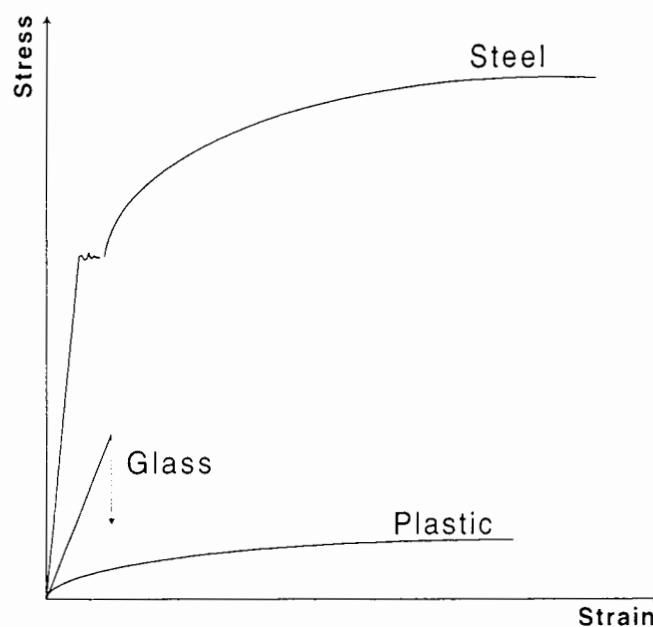
Glass for use in buildings, is produced by the float process, in which the ingredients are heated in a furnace with cullet (small pieces of broken glass). The molten glass is poured from the furnace onto a bed of molten tin in a carefully controlled atmosphere. Here the glass is cooled slowly to prevent crystallisation. It leaves the tin bath when the glass is hard enough not to be marked by the rollers. The carefully controlled cooling of glass is necessary to prevent any residual mechanical stresses from developing. This control is necessary for all glass produced by the float process and the product is termed annealed glass. The term annealed distinguishes standard float glass from glasses that have been heat-treated at a later stage. One of the most common forms of heat treatment is tempering.

Tempered glass or toughened glass is formed by applying further heat treatment to annealed glass. This process involves heating the glass to its softening point and then rapidly cooling the surface of the glass with air jets, such that the surface and edges solidify rapidly and contract while the centre is still in a fluid state. Further cooling to room temperature leaves the glass in a permanent state of stress. The centre goes into

tension causing the surface of the glass to be in compression to maintain equilibrium. Because glass almost always fails due to surface flaws being subjected to tension (see next section), this surface compression greatly increases the structural integrity. Toughened glass, however, cannot be cut or worked in any way. All mechanical working such as cutting, grinding and making holes needs to be carried out prior to toughening.

### **Glass as a Structural Material - Strengths and Weaknesses**

The stress-strain behaviour of glass differs from all other common materials. It exhibits almost perfectly linearity right up to the point of failure. However, unlike other linear elastic materials, when the limit of proportionality is reached, the atomic structure of glass allows no plastic deformation. Failure is therefore sudden (see fig 2a).



**Fig 2a Stress-strain comparison with other materials**

This lack of plasticity has the following implications for its structural behaviour:-

- (i) There is no creep or fatigue because these phenomena occur when a material deforms plastically.

- (ii) There is no permanent deformation. When the load is removed from a piece of glass it returns to its original shape.
- (iii) The absence of plastic deformation leads to glass being highly susceptible to local over-stressing.

It is this third characteristic that is the most significant in terms of the usefulness of glass as a structural material. The approximate theoretical strength of glass based on its atomic bond strength is of the order of 15-20 kN/mm<sup>2</sup>. Although glass has never been shown to exhibit this strength, glass fibres when incorporated in a matrix of resin have usable strengths of greater than 2 kN/mm<sup>2</sup>. (Gordon, 1976). However this figure is more than twenty times the typical breaking strength of bulk glass.

### **Griffith Flaws**

The huge disparity between the theoretical strength and the actual strength of bulk glass was first observed by A.A. Griffith in the 1920's. Griffith proposed that the surfaces of bulk glass contain tiny flaws, that form points of local stress concentration. These invisible cracks now bear his name, and his theory for the fracture of brittle solids (see Appendix 1) is universally accepted to this day.

The size of these flaws appears to be highly variable, but remains below some maximum value. Their distribution on any piece of glass appears to be random. This gives rise to a wide range of strengths observed if glass is randomly tested. However, deliberate damage to the glass surface leads to much more consistent failure strengths, and importantly the lower limit of both the undamaged and the pre-damaged sets of data, appears to converge to the same minimum value. (See Fig. 2b). It appears that

“unabraded” glass will not fail below this value (unless of course there is some other visible damage). This increased consistency of pre-damaged specimens of glass is also demonstrated in data published by Schott Glaswerke, a German float glass manufacturer. (See Fig. 2c ).

These observations are consistent with the Griffith flaw theory, that suggests the probability of a particular piece of glass failing, under a certain load, is directly related to the probability of a flaw greater than some limiting size being present at the point of maximum stress. There is also, of course, the possibility that there is an even larger flaw at a point of lower stress that reaches a critical combination of flaw size and stress first. Therefore, if we pre-damage the glass surface so that there are large flaws all over its surface, then it is highly likely to break at the point of maximum stress. This is because the largest flaws are now distributed evenly all over the surface of the glass, such that the point of failure initiation is now wholly dependant on the applied stress and not on the condition of the glass surface.

The presence of Griffith flaws affects the structural behaviour of glass in two other important ways.

### **Volume/Area Effects**

Obviously it is not practical and indeed unnecessary to go about pre-damaging the surface of panes of glass before we put them to any structural use. There is no need to have a pre-damaged surface as long as we assume the glass behaves as if it was pre-damaged (i.e. assume that there are large flaws everywhere). Then we can safely design a glass pane such that the design stress is kept well below a safe minimum value for

failure. However, the fact that the glass is undamaged does introduce a statistical advantage to the survival probability of smaller areas of glass over larger ones. It has been observed experimentally that larger test specimens are more likely to fail at lower stress levels than smaller ones. (See Fig. 2d).

### **Static Fatigue of Glass**

The other important effect of Griffith flaws on the structural integrity of glass is static fatigue (as it is misleadingly called). A better name would be stress corrosion as it is the loss of strength in glass with static load duration that is being referred to here. No cyclic loading normally associated with “fatigue” is necessary for this process to occur in glass (See Fig. 2e).

Most structural applications of glass involve long term loading, whereas windowpanes (the most common use for glass) generally do not. Therefore it is necessary for the designer wishing to make use of glass for primary load bearing elements to take account of this.

The relationship between load duration and strength is generally accepted to be as follows:-

$$\sigma^n t_f = \text{Constant}$$

where  $\sigma$  = stress applied,  $t_f$  = time to failure,  $n$  = constant.

The value of  $n$  is all-important, and most researchers have found  $n$  to lie somewhere between 12 and 20. Pilkington use a value of 16 that was found by experiment (see Fig. 2f), and also happens to lie in the middle of this range. Schott Glaswerke have

published the results of their experimental work on stress corrosion (see Fig. 2g), that concur with the relationship given above but suggest a value of  $n$  nearer 20.

It is important to note that the relationship between stress and time to failure described here suggests that after some time the strength will effectively reduce to zero. However, it is generally agreed among glass manufacturers that this is not the case, and that there is some limiting value below which glass will not fail. John Colvin, of Pilkington, has suggested that this figure seems to be of the order of  $7 \text{ N/mm}^2$ . This figure is also suggested by Schott Glaswerke (see Fig. 2h), where, if we assume the minimum short term strength of glass to be  $25 \text{ N/mm}^2$  (see Fig. 2j)<sup>1</sup>, then after 3170 years the long term strength would be about  $7 \text{ N/mm}^2$ . Saint Gobain, another leading multinational glass manufacturer, also use this figure for in-house design. (see Table 2).

### **Information Currently Available to Structural Engineers**

There is a dearth of published data and a general lack of understanding of glass in the public domain. This has led to a very conservative approach to structural design in glass. It generally involves an extensive testing programme usually requiring a model similar in size to the prototype, where the specimens are subjected to every kind of loading they are likely to meet in service. This method of design is both expensive and time consuming. Expertise and previous experience become vital.

This attitude is reflected in the available information to be found in codes of practice.

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<sup>1</sup> Note: This figure is to be adopted in the new European standard on glass strength, which is expected to be issued in 1999. Further experimental evidence for the adoption of this value is demonstrated in Fig. 2k, where the points represent batches of samples and taking the line of average mean strength and standard deviation (as drawn), we see the intersection of this with the standard deviation axis is at  $25 \text{ N/mm}^2$ . This mean strength at which the standard deviation is zero is assumed to be the value below which the short-term strength of glass never goes.

Most standards relating to glass are written in such a form that they avoid providing any generally applicable information, but are full of tables, giving the allowable loads for glass in very specific situations, mainly four edge supported glazed panels. All of this is not of much use to the structural glass designer wanting fundamental data on glass. The next few sections of this thesis aim to summarise the information the writer has found on the structural properties of glass. Some of the information is published data and some has been given verbally.

The density, Young's Modulus and Poisson's Ratio of glass are relatively easy parameters to find and the values quoted are fairly consistent (see Table 1, below). These parameters do not vary with the toughening process. Therefore, in investigating information available in both codes of practice and from manufacturers, the main focus of attention has been on strength rather than stiffness.

Density, $\rho = 2500 \text{ kg/m}^3$
Young's Modulus, $E = 70 \text{ GN/m}^2$
Poisson's Ratio, $\nu = 0.2$

Table 1 – Typical Mechanical Parameters for Sodaglass

## **Codes of Practice**

### **(i) British Standards**

There are several British Standards giving guidelines on the use of glass in building. However only three of these contain numerical values relevant to this discussion. Although these three do not provide specific information on glass strength, they do contain tables which can be used to back-calculate allowable strengths for glass. The particular tables used and the means of back-calculation employed are explained in the

notes following Table 1a.

## **(ii) Foreign Codes of Practice**

The German standard DIN 1249 provides some useful information on toughened glass in the form of minimum values expected of suppliers. These are quoted and explained in Table 1b. Some parts of this standard have been superseded by the European Standard EN 572 - Glass in Building - Basic Sodalime Silicate Glass Products. This new European standard, however does not contain any bending strength data. It is mainly concerned with dimensional tolerances and optical quality requirements.

The Canadian General Standards Board has produced the most useful documents for an engineer looking for information on the structural use of glass. The code of practice, entitled “Structural Design of Glass for Buildings” published in 1989, is a landmark document, in that its purpose is to provide a designer with general guidelines for the structural use of glass. It follows the limit state design procedure adopted for most other structural building codes around the world.

This standard is particularly aimed at providing information on the strength of glass plates when they are being loaded normal to the surface. However, it does not preclude the possibility of glass being loaded in plane, and provides some discussion on glass fins and all-glass structures, but the information is in the form of general guidelines rather than specific failure stresses or loading limits. This is because the specific strength information that is provided in the code is based on Beason and Morgan’s paper published in 1984, which presents a failure prediction model for the probability of failure of laterally loaded glass plates.

There are two American standards that provide information relevant to this discussion. ASTM E1300-94 Practice for Determining the Minimum Thickness and Type of Glass Required to Resist a Specified Load, provides charts of allowable loads on glass plates supported on all four edges. There are charts for different thicknesses of glass, and the values within each chart vary with the aspect ratio and span. The charts are similar to those provided in the aforementioned Canadian standard. However, this standard does not have an equivalent section for applications other than windowpanes, so correspondingly there are no specific strengths quoted. The values given in Table 1 of this document are achieved by back-calculation from the charts provided. To carry out this exercise it has been assumed that for high aspect ratios such as one to four or five, the glass will span one way across the shorter span<sup>1</sup>.

The other American standard which provides information relevant to this discussion is C1048-90 Specification for Heat Treated Flat Glass. The values provided are quoted and discussed in Table 1.

**NB Tables 1a and 1b need to be read in conjunction with the explanatory notes following them.**

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<sup>1</sup> When this method is used for the tables in the Canadian standard, it gives rise to values similar to those quoted in the section of the code entitled Application to Other Types and Configurations.

Property	Standard	BS 6262 <sup>2</sup>	BS 5516 <sup>3</sup>	BS 7449 <sup>4</sup>	DIN 1249 <sup>5</sup>	CGSB 12.20 <sup>6</sup>	ASTM C1048 <sup>7</sup>	ASTM E1300 <sup>8</sup>
Bending Strength								
- Remote from cut edge								
- Near (or on) cut edge								
Allowable Strength <sup>1</sup>								
- Remote from cut edge			20 - 30 <sup>3a</sup>		30 <sup>5a</sup>	(15-25)/1.5 <sup>6a</sup>		15 - 25 <sup>8a</sup>
- Near (or on) cut edge		20 - 30 <sup>2a</sup>				20/1.5 <sup>6a</sup>		
Variation with Area		Not assessed	None apparent	None apparent		Yes <sup>6b</sup>		Yes <sup>8b</sup>
Variation with Thickness		Not assessed	Yes <sup>3a</sup>	None apparent		None apparent		None apparent
Allowable Strength under Sustained Loading								
- Remote from cut edge			7.5 - 11.25 <sup>3b</sup>	6 - 7 <sup>4a</sup>		(6-10)/LF <sup>6c</sup>		(9 - 15) <sup>8c</sup>
- Near (or on) cut edge				6 - 7 <sup>4a</sup>		8/LF <sup>6c</sup>		

Table 1a - Strength Properties of Annealed Glass from various codes of practice (N/mm<sup>2</sup>).

Property	Standard	BS 6262 <sup>2</sup>	BS 5516 <sup>3</sup>	BS 7449 <sup>4</sup>	DIN 1249 <sup>5</sup>	CGSB 12.20 <sup>6</sup>	ASTM C1048 <sup>7</sup>	ASTM E1300 <sup>8</sup>
Bending Strength		1982	1991	1991	1988	1989	1990	1994
- Remote from cut edge					120 <sup>5b</sup>		(69) <sup>7a</sup>	
- Near (or on) cut edge					120 <sup>5b</sup>		(67) <sup>7a</sup>	
Allowable Strength <sup>1</sup>								
- Remote from cut edge			55 <sup>3c</sup>		50 <sup>5a</sup>	(60-100)/1.5 <sup>6d</sup>		60 - 100 <sup>8d</sup>
- Near (or on) cut edge					50 <sup>5a</sup>	80/1.5 <sup>6d</sup>		
Variation with Area			Minimal <sup>3c</sup>	Yes <sup>4b</sup>		Yes <sup>6b</sup>		Yes <sup>8b</sup>
Variation with Thickness			Minimal <sup>3c</sup>	Yes <sup>4b</sup>		None apparent		None apparent
Allowable Strength under Sustained Loading								
- Remote from cut edge				8 - 57 <sup>4b</sup>		(48-80)/LF <sup>6e</sup>		54 - 90 <sup>8e</sup>
- Near (or on) cut edge				8 - 57 <sup>4b</sup>		64/LF <sup>6e</sup>		

Table 1b - Strength Properties of Toughened Glass from various codes of practice (N/mm<sup>2</sup>).

## Explanatory Notes for Table 1

1. Allowable strengths in these rows are for short term wind loading.
2. BS 6262: 1982 Glazing for Buildings
  - 2a. Strengths back-calculated from charts giving fin width for given thickness and level of wind loading. It is not specified whether the glass needs to be toughened. The thicknesses are based on sealant strengths. Curiously, the tables do allow the possibility of even higher stresses, if the width of window supported by the fin is greater than the height of the fin.
3. BS 5516 : 1991 Design and Installation of Sloping and Vertical Patent Glazing.
  - 3a. Charts are only provided for 4mm, 6mm and 10mm thick glass up to spans of 2m. Back-calculation for 10mm thick panes consistently gives strengths of about  $20 \text{ N/mm}^2$ , but for 4mm and 6mm thick panes, stresses are consistently about  $30 \text{ N/mm}^2$ .
  - 3b. In an earlier section of this standard, it states that the relative strength of glass subject to sustained loading is about  $3/8$  of that for short term loading.
  - 3c. For toughened glass, charts provide strengths for 4mm, 5mm, 6mm and 10mm thicknesses only. Back calculation using these charts consistently gives a strength of about  $55 \text{ N/mm}^2$  irrespective of the thickness.
4. BS 7449 : 1991 Specification for the inclusion of glass in furniture other than tables or trolleys, including cabinets, shelving systems and wall hung or free standing mirrors.
  - 4a. This figure comes by back-calculation from tables for allowable loads of simply supported shelves support on two sides only. The spans vary between 300mm and 1500mm and the thicknesses vary from 4 to 12mm. Bending strengths for one way spanning shelves would develop across the full width which would

obviously include edge zones. All back-calculated values lie between 6 and 7 N/mm<sup>2</sup>.

- 4b. A table is provided for toughened glass shelves in the same format and covering the same spans and thicknesses, as described in note 4a. However the back-calculated values for the strength of toughened glass vary enormously both with thickness (values increase with increasing thickness) and with span (values decrease with increasing span).
5. DIN 1249 Glass for use in Building Construction.
  - 5a. This value is quoted from the paper by Sedlacek *et al*, published in The Structural Engineer, volume 73, No 2, 17 January 1995.
  - 5b. These values come from Part 12:1990 of DIN 1249 - Toughened Glass, Dimensions, Working and Requirements. This part gives minimum bending strengths for toughened glass. This is a requirement for manufacturers and is defined in the standard as “the minimum bending stress resulting in a probability of failure of 5% for a 95% confidence level”. It is not specified whether this value refers to areas remote from or near to edges.
6. CAN/CGSB-12.20-M89: Structural Design of Glass for Buildings.
  - 6a. In the section of this code entitled *Application to other types and configurations*, a limiting tensile stress of 25 N/mm<sup>2</sup> is recommended away from the edges of plates and 20 N/mm<sup>2</sup> for clean cut edges. The 25 N/mm<sup>2</sup> value is the maximum attainable value (for a pane of area 1.0m<sup>2</sup>) by back-calculation from the charts provided in the code. According to the procedures set out in Section 5 of the code entitled Loads and safety criteria, these values obtained from the charts need to be compared with factored loads. Therefore to make a comparison between these values and the other values quoted in Table 1, we need to convert

them to allowable loads by dividing by the recommended load factor, which is quoted earlier in the code as being 1.50 for wind loading.

- 6b. Factored failure loads are tabulated for different areas of plate glass, ranging from  $0.2\text{m}^2$  to  $15.0\text{m}^2$ . For other applications than window glass, it is recommended that the stresses used for a  $1.0\text{m}^2$  plate (which the stresses given in Table 1 are based on) should be multiplied by a factor of  $\text{Area}^{-1/7}$  for other areas.
- 6c. These strengths are based on the allowable strengths quoted in Table 1a. The code suggests that the standard  $1.0\text{m}^2$  stresses should be multiplied by coefficients for different loading terms. The recommended value for long term loading (one year - ten years) is 0.4. The code allows a slightly higher value for less sustained loadings such as snow or ponding (one week to one month) of 0.5. This value then needs to be divided by the appropriate Load Factor (LF) which is 1.25 for dead loads and 1.50 for live loads.
- 6d. The recommended strengths for toughened glass are achieved by multiplication of the standard strengths for annealed glass by a coefficient, which in this case is 4.0.
- 6e. The recommended coefficients that the standard strength should be multiplied by for sustained loadings is much higher for toughened glass than annealed glass. They amount to 0.80 for long term loading and 0.90 for ponding or snow loading (see note 6c).
- 7. ASTM C1048-90 Specification for Heat Treated Flat Glass
- 7a. These values are not actually bending strength values, but are quoted as surface compression requirements. To obtain bending strength values, the bending strength of the annealed glass being toughened would have to be added to these

values. As we will see from Table 2, this value is quoted as being between 40 and 50 by manufacturers. This brings the bending strength in line with the minimum requirement quoted in DIN 1249 (see note 5b).

8. ASTM E1300-94 Standard Practice for Determining the Minimum Thickness and Type of Glass Required to Resist a Specified Load.
  - 8a. This standard contains design charts which are virtually the same as those presented in the Canadian standard (see note 6a). This standard however considers the loads presented in the charts to be *allowable* loads, whereas the Canadian standard requires that they be compared with factored loads. The values provided in this standard are for a failure probability of 8 in 1000.
  - 8b. The values obtained by using these charts decrease with increasing area in exactly the same fashion as the charts in the Canadian standard.
  - 8c. This standard only gives values for medium term (up to one month) sustained loading. The values quoted here are obtained by multiplying the values for short term loads by the recommended factor of 0.6. This value is higher than the equivalent value for medium term sustained loading given in the Canadian standard of 0.5.
  - 8d. These values were obtained by multiplying the values for the allowable strength of annealed glass (given in Table 1a) by the recommended factor of 4.0 for fully tempered glass.
  - 8e. These values are also for medium term (up to one month) sustained loading and are obtained by multiplying the values for short term loads by the recommended factor for medium term loads of 0.9. This factor is the same as that quoted for medium term loads in the Canadian standard. (The values given in Table 1b for the Canadian standard are lower because these are for longer term sustained

loading (see note 6c)).

It is worthy of note here that nowhere, in any of the standards consulted, is there any reference to risk of failure. Furthermore all the stresses provided in these tables, relate to glass in a load-carrying situation, such as a windowpane; where the risk of failure is much lower than in a load-transfer situation, such as a structural glass beam.

### **Float Glass Manufacturers Product Data**

This section deals with strengths of glass provided by manufacturers. Some published data from glass manufacturers has been referred to in Chapter 2. This, with additional data from other manufacturers, is presented in Table 2. Although by no means exhaustive, Table 2 includes leading manufacturers from Europe, North America and S E Asia.

Some of the information in Table 2 is published data and some has been given verbally. The sources of the information are in the explanatory notes following the table.

Manufacturer	Pilkington <sup>1</sup>	Saint Goban <sup>2</sup>	Schott <sup>3</sup>	PPG <sup>4</sup>	Hankuk <sup>5</sup>	Nippon <sup>6</sup>
<b>Property</b>						
Bending Strength						
- Remote from cut edge			50 - 90 <sup>3a</sup>	41.4 <sup>4a</sup>	37 - 49 <sup>5a</sup>	49 <sup>6a</sup>
- Near (or on) cut edge			50 - 90 <sup>3a</sup>	41.4 <sup>4a</sup>	35 <sup>5a</sup>	49 <sup>6a</sup>
Allowable Strength						
- Remote from cut edge		28 - 41 <sup>2a</sup>				
- Near (or on) cut edge		18 - 28 <sup>2b</sup>				
Variation with Area		None suggested	Yes <sup>3b</sup>	None suggested	None suggested	None suggested
Variation with Thickness		Yes <sup>2c</sup>	None suggested	None suggested	None suggested	None suggested
Allowable Strength under Sustained Loading						
- Remote from cut edge		7.5 <sup>2d</sup>	15 - 28 <sup>3c</sup>		12 <sup>5b</sup>	
- Near (or on) cut edge		7.5 <sup>2d</sup>	15 - 28 <sup>3c</sup>		9 <sup>5b</sup>	

**Table 2a - Strength Properties of Annealed Glass from the Float Glass Industry (N/mm<sup>2</sup>).**

Manufacturer	Pilkington <sup>1</sup>	Saint Goban <sup>2</sup>	Schott <sup>3</sup>	PPG <sup>4</sup>	Hankuk <sup>5</sup>	Nippon <sup>6</sup>
<b>Property</b>						
Bending Strength						
- Remote from cut edge					147 <sup>5a</sup>	
- Near (or on) cut edge					108 <sup>5a</sup>	
Allowable Strength						
- Remote from cut edge		59 <sup>2e</sup>				
- Near (or on) cut edge		59 <sup>2e</sup>				
Variation with Area		None suggested			None suggested	
Variation with Thickness		Yes <sup>2e</sup>			None suggested	
Allowable Strength under Sustained Loading						
- Remote from cut edge		35 <sup>2f</sup>			49 <sup>5b</sup>	
- Near (or on) cut edge		35 <sup>2f</sup>			34 <sup>5b</sup>	

**Table 2b - Strength Properties of Toughened Glass from the Float Glass Industry (N/mm<sup>2</sup>).**

## **Explanatory Notes for Table 2.**

1. Pilkington Glass Ltd. (Head office, St Helens, UK)
2. Saint Goban (Head Office, France). These figures were quoted over the phone, as values used for in-house design, by a representative for Solaglas, a wholly owned subsidiary of Saint Goban, which uses float glass manufactured in France by the parent company. I was told the company do not publish any bending strength data for their glass.
  - 2a. This is the range of values I was quoted for four edge supported glazing.
  - 2b. This range of values was quoted for two edge supported glazing. I have therefore made the assumption that these stresses could occur at the edges and have therefore placed them in this row corresponding to edge strengths.
  - 2c. There is a range of values quoted above as these vary with thickness. The maximum values are for glass less than or equal to 6mm thick, and the smallest values are for glass greater than or equal to than 10mm thick.
  - 2d. It was not specified whether these values were for points away from or near edges.
  - 2e. For toughened glass Saint Goban use one value for allowable strength irrespective of thickness or support arrangement.
  - 2f. Again for sustained loading on toughened glass, as with annealed glass, no distinction is made between four edge and two edge support conditions.
3. Schott Glaswerke (Head office, Mainz, Germany). Schott Central Research and Development Division have produced a booklet entitled Schott Technical Glasses. Under the heading of Mechanical Properties are graphs of the results of strength tests carried out. The test method is not mentioned, but the values are described as tensile strengths. These graphs have been included in the main body

of this chapter, and the strength values included in Table 2a have been estimated from these plots.

- 3a. The range of values given here are the average (50% probability of failure) breaking stresses for different stressed areas. The lowest value is for a stressed area of 10000 mm<sup>2</sup> and the highest value for 10mm<sup>2</sup>. As the method of testing is not specified, it is unclear whether these values are edge strengths or not.
- 3b. These values do obviously vary with stressed area as described above, but this is not quite the same as the variations with area described in Table 1, which are variations in surface area rather than stressed area.
- 3c. These values come from a plot of time related strengths against experimentally found short term strengths. The particular values given in Table 2 correspond to experimentally found short term strengths of 50 and 90 N/mm<sup>2</sup> for a ten year sustained load. It is also stated in the booklet that “only about 1/2 to 1/3 of the experimental tensile strengths can be expected for constant loading over a period of years”. This ties in well with the statement in BS 5516 which suggests 3/8 (see Table 1).
4. PPG Industries Inc. (Head office, Pittsburgh, USA)
  - 4a. This value is published in PPG’s performance data information sheets for their various float glasses. It is described as a “tensile strength (determined as Modulus of rupture, ultimate)”, with the caveat that “the properties given are applicable to specific test samples under specific testing conditions”. It is not specified whether this strength is applicable near edges or not. This value does not vary for PPG’s different types of float glass such as tinted or coated glass.
5. Hankuk Glass Industries Inc. (Head Office, Seoul, Korea). These figures are quoted from Hankuk’s information booklet Hanglas Architectural Glass.

- 5a. Average breaking stresses are tabulated in the booklet for both annealed and toughened glass. Different values are given for the face and the edges.
- 5b. These figures for sustained loading are described in Hankuk's data sheets as "long term allowable stresses". I have assumed that the long term here refers to the loading, and that these values are not merely for short term loads on old glass.
6. Nippon Sheet Glass (Head Office, Osaka, Japan).
- 6a. This value is described as the average ultimate bending stress for flat glass. There is no mention as to whether it applies to four or two edge supports. I have therefore included the value as an edge strength as well.
- In the same section in the data sheets, the compressive stress of the glass is given as being between 588 and 1177 N/mm<sup>2</sup> (values in the original are given in round numbers in kg/cm<sup>2</sup>).

### **Glass Tests carried out in this Department**

This is a summary of three point bend tests carried out on glass beams as part of a final year project carried out by Fair and Williams in 1996 under the supervision of the author.

#### **Aims**

This project set out to ascertain the bending strength and stiffness of glass experimentally, and to ascertain whether this differs when loaded laterally or in plane. Also included in the project were tests carried out on bonded and unbonded double beams, principally to determine whether the strengths of multiple beams could be predicted from the results of strength tests carried out on single beams, but also to see if

the bonding material affected the bending strength.

### Tests

Testing was carried out in a loading rig described in detail in the project reports (Fair, 1996 & Williams 1996). The beams were simply supported on smooth curved steel supports 500mm apart. A central point load was applied by a steel dowel with a soft piece of rubber hosing between it and the glass. The glass samples were 600mm long x 100mm wide x 10mm thick. Table 1 is a summary of all the tests carried out.

	Annealed Glass		Toughened Glass	
	In Plane Bending	Out of Plane Bending	In Plane Bending	Out of Plane Bending
Single	30	20	20	15
Double (bonded)	48*			
Double (unbonded)	32*			

**Table 1 - Summary of Tests**

\* These tests were in batches of 16. This was to take account of different loading arrangements and support conditions that were observed to be affecting the results.

### Results

The testing of the double beams posed some difficulties. The first set were tested without using a linkage piece to provide even loading to both beams, so these results are difficult to interpret. The remainder of the tests gave some unusual results, particularly in that the failure loads seemed to be lower when the beams were supported on soft rather than hard supports. Only those that are judged the most reliable sets of results are presented in this summary.

Table 2 shows the results of the single samples both annealed and toughened, and Table 3 shows the most reliable sets of results for the double beams.

Failure Stress Statistics	Annealed Glass		Toughened Glass	
	In Plane Bending	Out of Plane Bending	In Plane Bending	Out of Plane Bending
Mean	39.2	47.3	136.7	171.8
Std Deviation	3.4 (4.5)*	7.7	9.9	13.2
Range	33.4 - 45.9 (33.4 - 56.0)*	37.7 - 62.2	117.4 - 155.7	143.3 - 186.0

**Table 2 - Test results for Single Beams ( N/mm<sup>2</sup>)**

Failure Stress Statistics	Annealed Glass	
	Unbonded	Bonded
Mean	37.2 (37.1)	39.8
Std Deviation	2.0 (3.1)	3.8
Range	33.6 - 40.5 (29.6 - 43.1)*	32.9 - 46.1

**Table 3 - Test Results for Double Beams ( N/mm<sup>2</sup>) (halved)**

\* The figures in brackets are the results if outlying values, that should be rejected as statistical freaks (see discussion), are included.

The Young's Modulus was calculated from measured deflections near the centre of the beams. The results from three sets of measurements gave fairly consistent answers that are fairly close to generally accepted values. However, the values obtained for the toughened samples loaded in plane were significantly lower, suggesting that deflection of the rig was taking place under the much higher loads being applied. The results for the other three sets showed little variation gave a mean Young's Modulus of 68 GPa with a range of 66.0 to 70.74 GPa.

Some rig deflection was probably taking place here that would account for these values being slightly lower than generally accepted values.

## **Discussion**

### 1. Single Panes

The results for the average face strengths of both annealed and toughened glass of 47.3 and 171.8 N/mm<sup>2</sup> agree well with generally accepted values. However, both sets of results suggest that the strength of glass is greater on the face than on clean cut edges. The average values quoted above suggest that the strength on the edge is about 80% of the face strength.

The high strength outlier which broke at a stress 10 N/mm<sup>2</sup> higher than the next highest can be rejected statistically using Chauvenet's criterion (Fair, 1996). This principle provides the basis for rejecting outlying data in a small sample. Such data cannot be ignored completely, but can be rejected as anomalous when trying to fit the data to a probability distribution curve. However, the reasons for such an outlier occurring do need to be investigated. If unusually strong or weak specimens of glass can be expected from time to time, then this stresses the importance of always providing alternative load paths in design of structural glass elements.

The other interesting fact about these results is the relatively small range and standard deviation of the failure stresses. The coefficient of variability (= std deviation/mean) is only about 8 % for three out of the four sets of results<sup>1</sup>. This 8% figure is considerably less than the figure for timber, which typically has a coefficient of variability of between

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<sup>1</sup> It is 16% for the flat float specimens (probably because they were failing at much lower levels of load).

10% and 30% within a species. Glass is not therefore as unpredictable a construction material as is generally believed.

## 2. Double Panes

The double pane tests had some outliers that were rejected using Chauvenet's principle, but these were not as extreme as the outlier in the single pane tests. The bonded samples (using 3M tape) showed marginally higher strengths than the unbonded ones. However, this is not significant enough to draw any strong conclusions, particularly as the spread of the bonded results was much larger.

It was interesting to note, however, that the average failure strength of the unbonded double beams was less than the twice that for the single beams. The possible reasons for this will be discussed in the next chapter on comparisons between multi-pane beams and single ones.

### **Conclusions Drawn from Information Gathered in this Chapter**

From all the evidence that has been assimilated in the production of this report, it appears that there is some lower limit (of about 25 N/mm<sup>2</sup>) for the strength of undamaged glass under short term loading. "Damaged" here refers to visible damage to the glass surface that would render it unsuitable for architectural use.

The particular parts in this chapter that support this hypothesis are as follows

- (i) Fig 2j. Here from over 700 glass specimens, from nine leading European glass manufacturers, not a single specimen broke at less than 30 N/mm<sup>2</sup>. (NB The point at 25 N/mm<sup>2</sup> is not a data point).

(ii) Fig 2k. Here the mean stress from each batch of twenty specimens (from the 700 described in (i) above) is plotted against standard deviation. The line of best fit through this data crosses the zero standard deviation line at  $25 \text{ N/mm}^2$ , suggesting that if a batch of samples could have a mean strength of  $25 \text{ N/mm}^2$ , then all the samples in that batch would break at exactly this strength. Based on this data the minimum strength is therefore estimated at  $25 \text{ N/mm}^2$ .

(iii) Fig 2c. This graph shows how the strength of glass, as expected, diminishes with increased deliberate surface damage. Clearly damaging the surface of the glass, even with the finest grit used (600), dramatically increases the consistency of the failure strength. This goes some way to suggest that “naturally occurring” flaws are smaller than would be caused by a 600 grit mesh. However, damage caused by a 600 grit mesh would probably be invisible. More significant are the results of the coarsest grit (100), which is equivalent to a grit size of  $156\mu\text{m}^1$ . This would cause visible flaws. The 100 grit mesh line on the graph suggests that glass roughened to this extent is “unlikely” to fail below  $25 \text{ N/mm}^2$ .

(iv) Finally the 130 tests on annealed glass carried out here in the Department of Engineering Science. None of these had a strength of less than  $25 \text{ N/mm}^2$ .

An interesting exercise at this stage would be to calculate the theoretical flaw size that corresponds to the strengths proposed here. The Fracture Toughness of glass is typically quoted as  $0.7 \text{ MPa}\cdot\text{m}^{1/2}$ , therefore the crack size for a strength of  $25 \text{ N/mm}^2$  would be

$$c = K_c^2 / \sigma^2 \pi = 250 \mu\text{m}$$

This figure correlates well with the 156 $\mu\text{m}$  flaws associated with a 100 mesh grit described in (iii) above. The thesis being proposed so far in this chapter relies totally on this size of crack (a quarter of a mm) being unacceptably visible on a piece of architectural glass.

Only the short-term strength of glass has been considered here. The issue that is more relevant to structural engineers wishing to use glass, is its long-term strength. On this issue, there is a consensus in the various codes of practice that the long-term strength of glass is about 3/8 of the short-term strength. This reduction in strength is thought to be caused by slow crack growth. Cracks slowly grow due to the presence of water or water vapour reaching the crack tips and reacting chemically with the molecules there to form a hydroxide (Ashby & Jones, 1998).

However, if there is a minimum strength for short-term loading, and the long-term strength is 3/8 of the short-term figure, then slow crack growth either does not go on indefinitely, or it slows down to such an extent that it never drops below a certain value in the expected maximum possible lifetime of a structure. The third possibility, that is generally accepted as the true explanation, is that for sufficiently low stresses the stress intensity factor is low enough that no growth occurs. For a crack of about 0.25 mm, this is at about 7 MPa. If we take 3/8 of 25 N/mm<sup>2</sup>, we get about 9 N/mm<sup>2</sup> as the minimum possible long-term load. The manufacturers consulted all quote the long-term strength to be of this order; the minimum quoted allowable long-term strength being 7.5 N/mm<sup>2</sup>. Furthermore in Fig 2c (discussed earlier), Schott have even quoted 8 N/mm<sup>2</sup> as being a

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<sup>1</sup> Norton Abrasives Ltd

“safe calculation value for normal glass constructions”.

In conclusion two hypotheses are being proposed here

(i) That glass, under short term loading (say  $d\sigma/dt > 5 \text{ N/mm}^2 \cdot \text{s}$ ), has a minimum strength of  $25 \text{ N/mm}^2$ , unless it is visibly damaged to such an extent that it would be unsuitable for architectural use.

(ii) That the above statement holds true for long term loading with the minimum strength being  $7 \text{ N/mm}^2$  rather than  $25 \text{ N/mm}^2$ .

Both of these hypotheses would be interesting to study experimentally. In such a study it would also be useful to include the effect of the presence of water on the strengths proposed. However experimental work is not part of this MSc.

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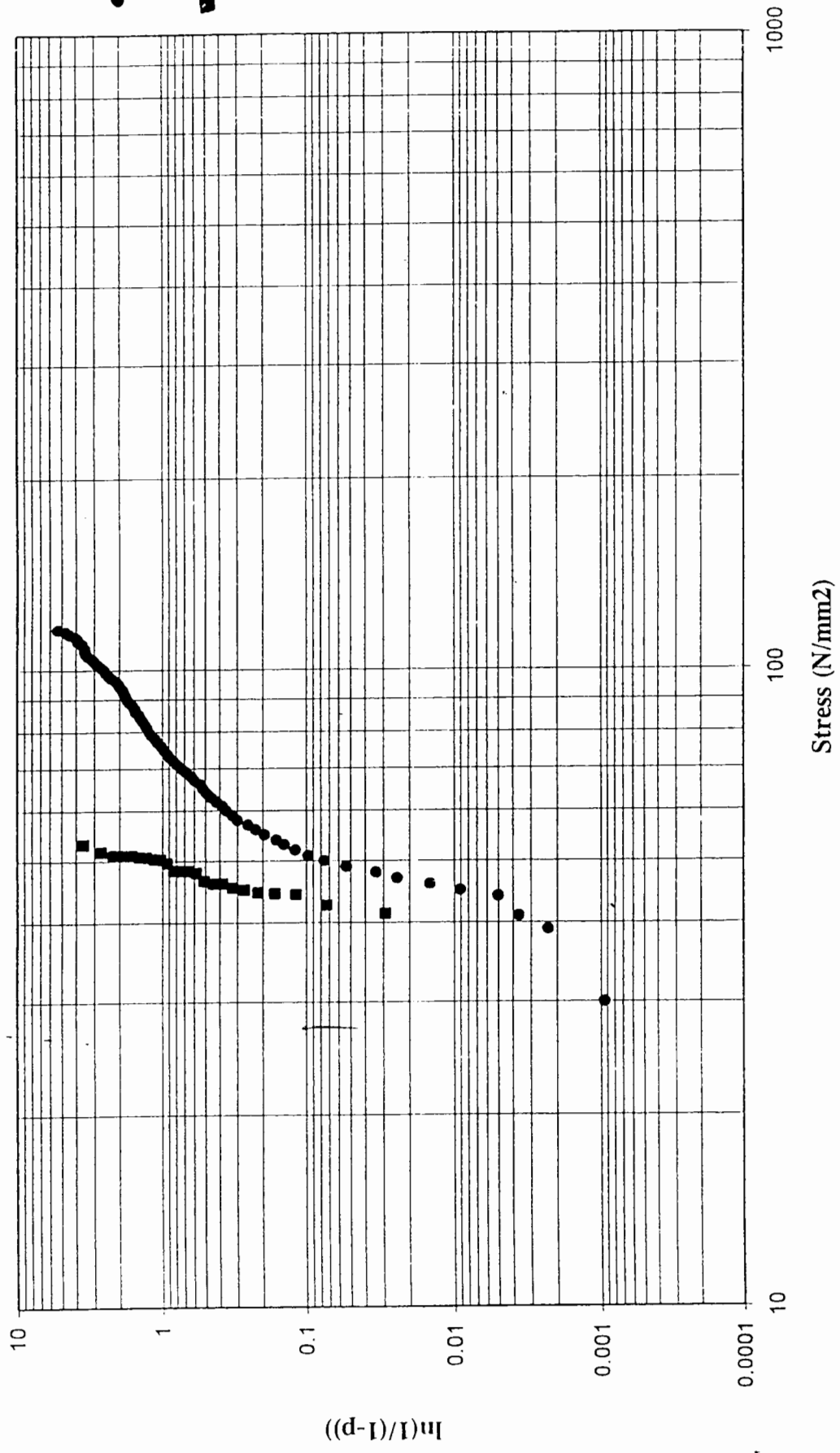
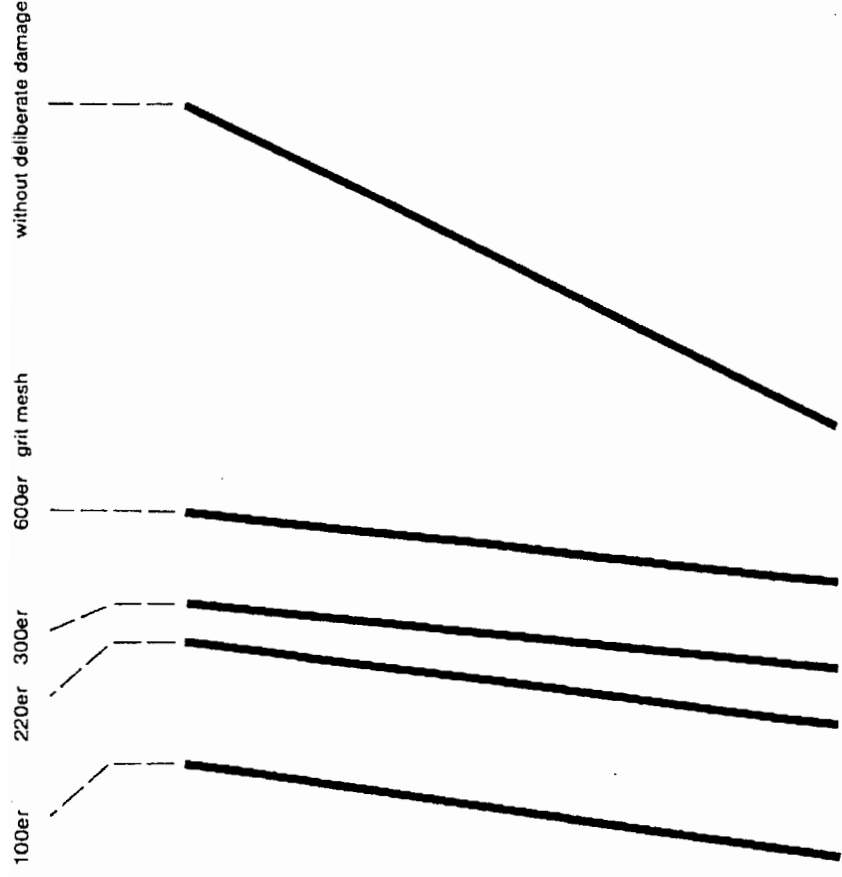


Fig. 2 b Weibull Cumulative Data of GEPVP Float Test Results



**Fig 2c** Probability of breakage  $F$  for samples variously abraded by defined mesh grits, with a stressed area of  $100\text{mm}^2$  and a rate of stress increase  $\sigma = 10 \text{ Nmm}^{-2} \text{ s}^{-1}$

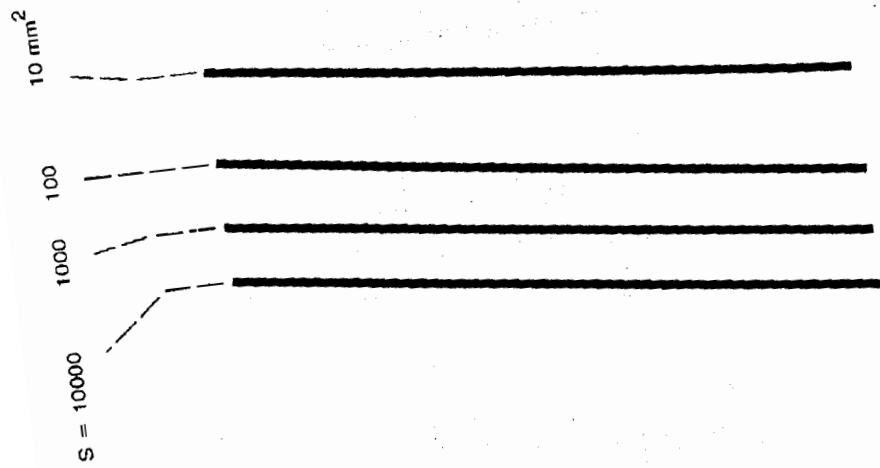


Fig 2d Probability of breakage  $F$  for different magnitudes of stressed area  $S$ . All samples abraded with 600 mesh grit, stress rate  $\sigma = 10 \text{ Nmm}^{-2} \text{ s}^{-1}$ .

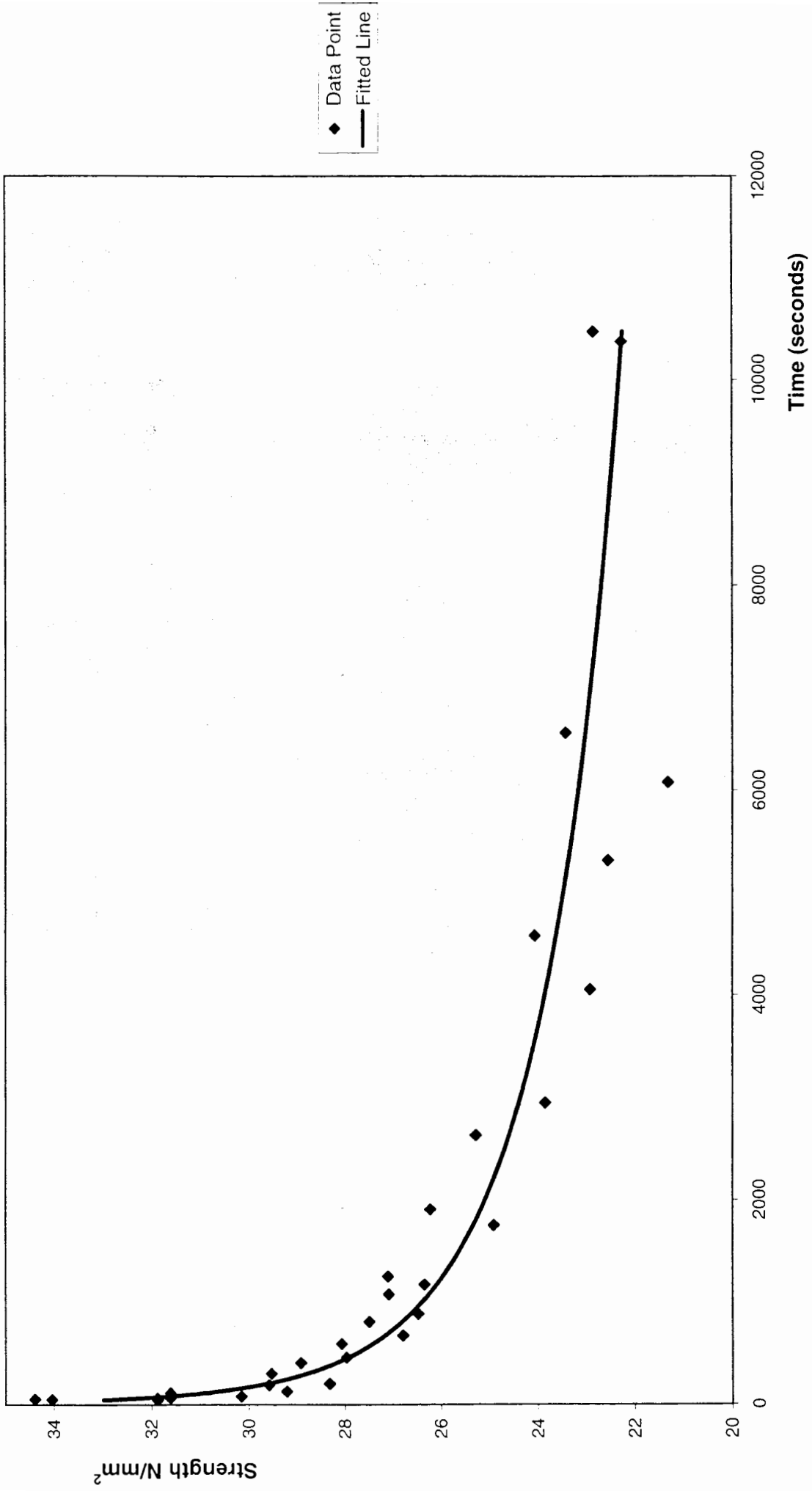


Fig 2(e) Decrease in Glass Strength with Load Duration (after Sglavo *et al*)

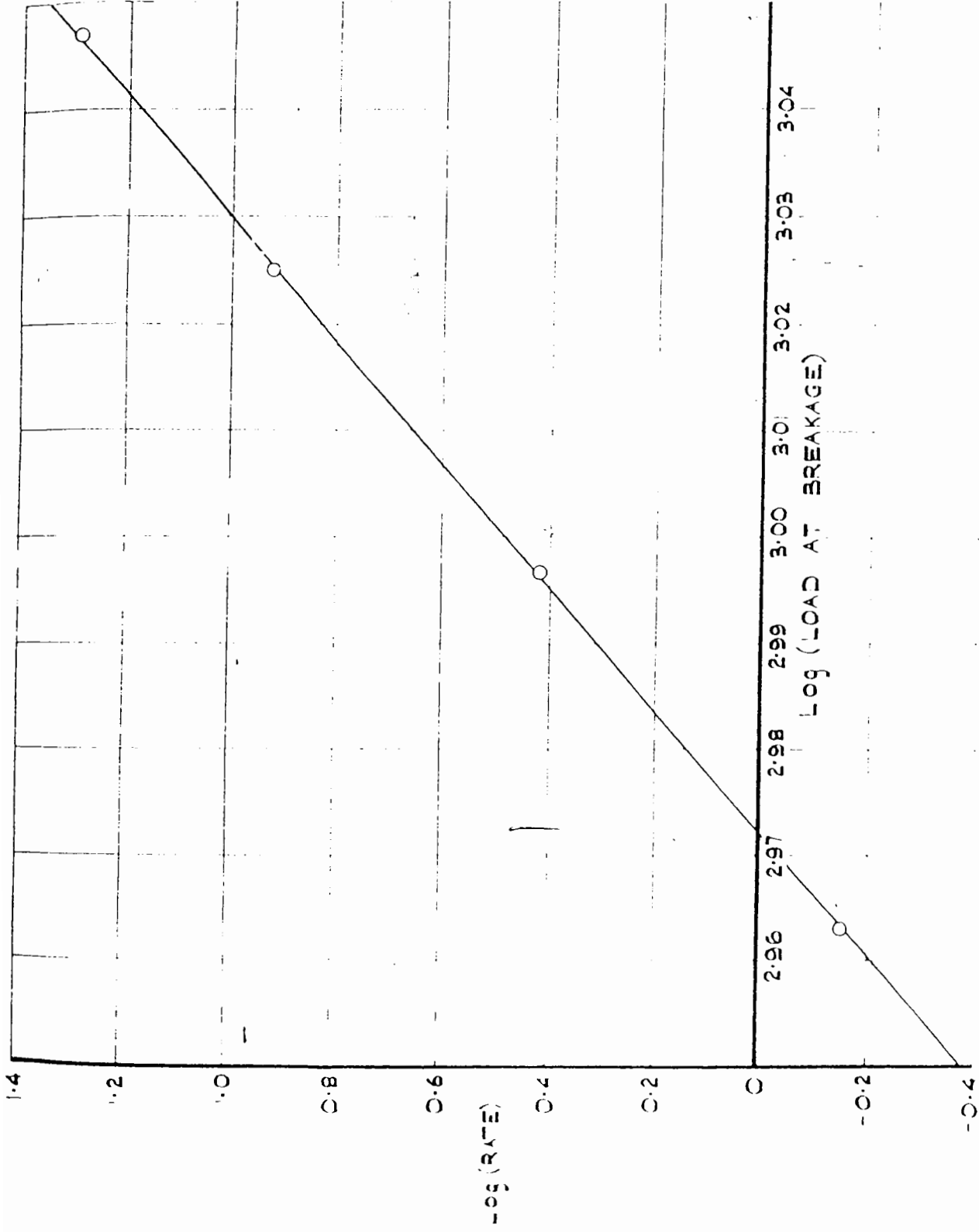
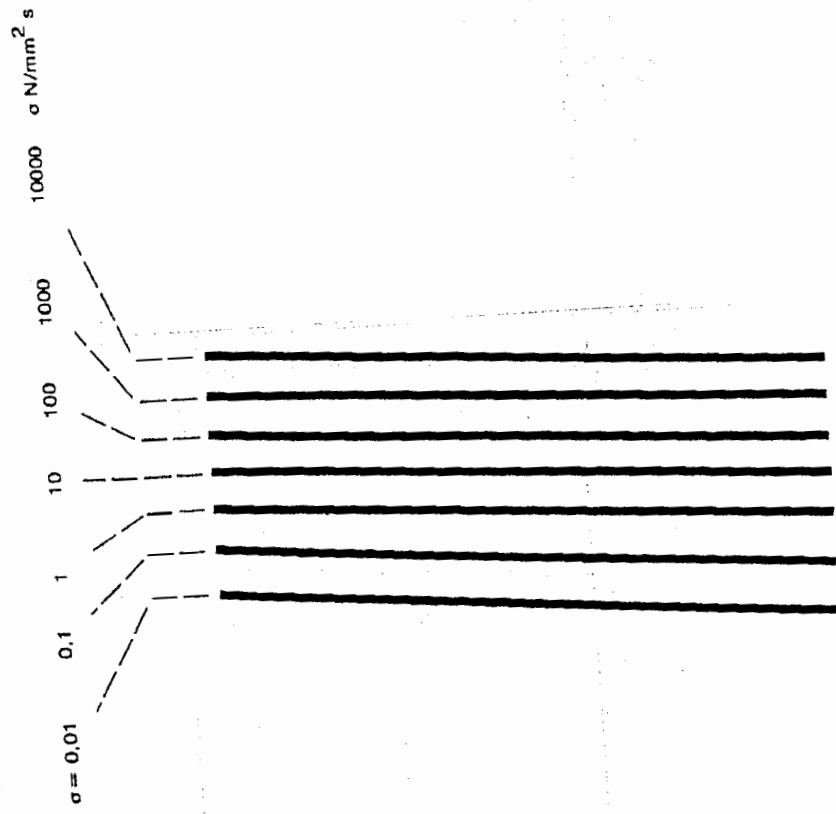
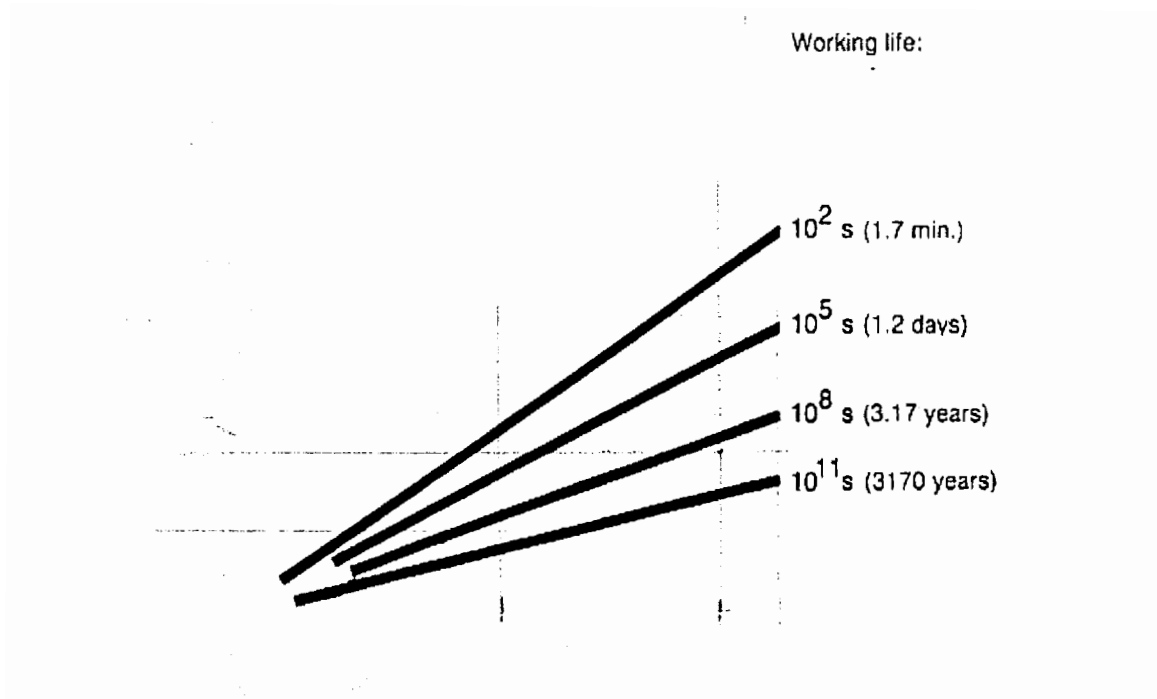


Fig 2f - Glass Strength vs Time (Experimental Results)



**Fig 2g Probability of breakage  $F$  for various rates of stress increase  $\sigma$  using a sample abraded with 600 mesh grit and a stressed area of  $100\text{mm}^2$ .**



**Fig 2h** Time-related strength  $\sigma_D$  (strength under constant loading) compared to experimental strength  $\sigma_E$  (at  $10 \text{ Nmm}^{-2} \text{ s}^{-1}$  stress increase) in a normally humid atmosphere (soda-lime glass).

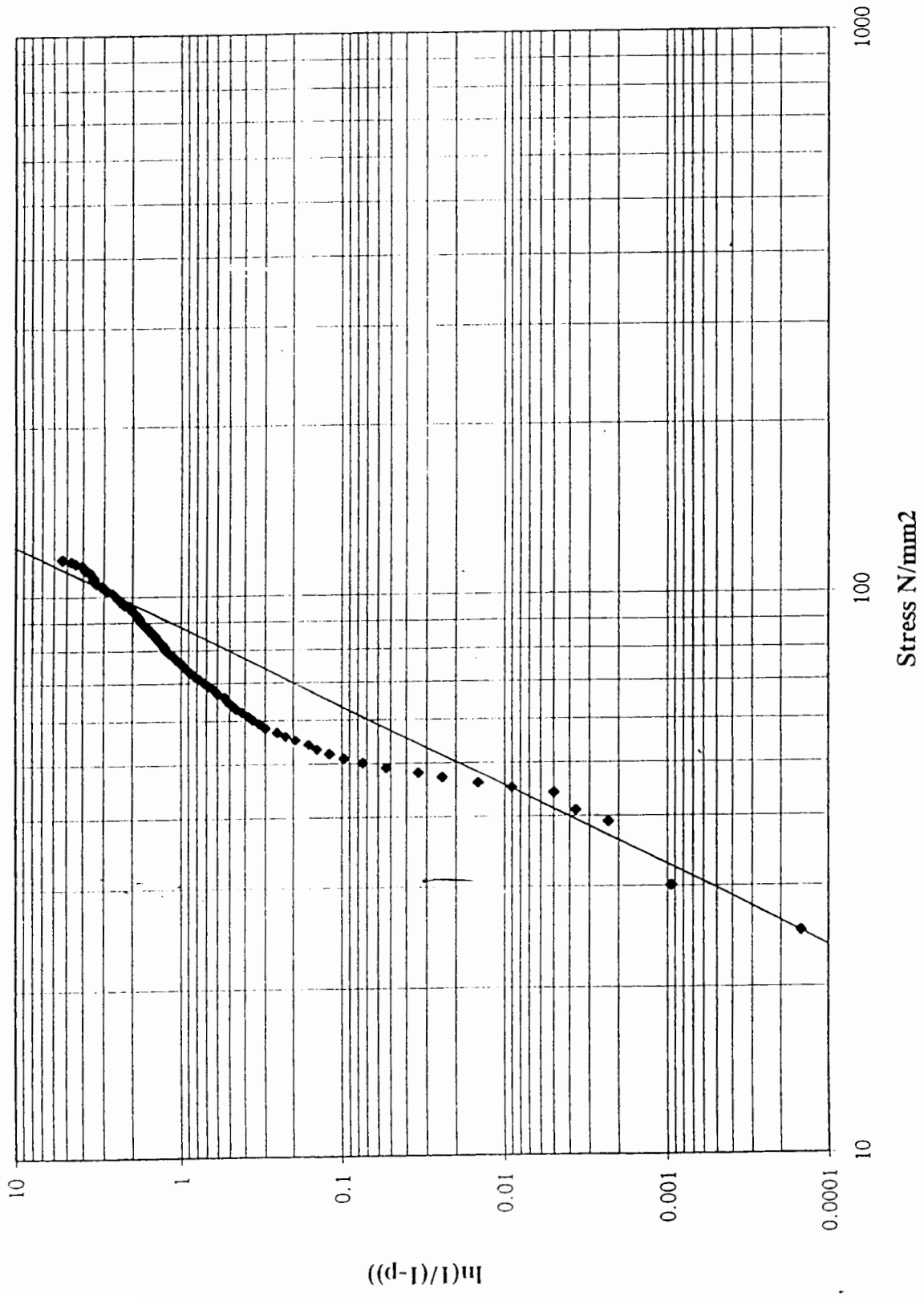


Fig. 2 j Estimation of Low Probability Point

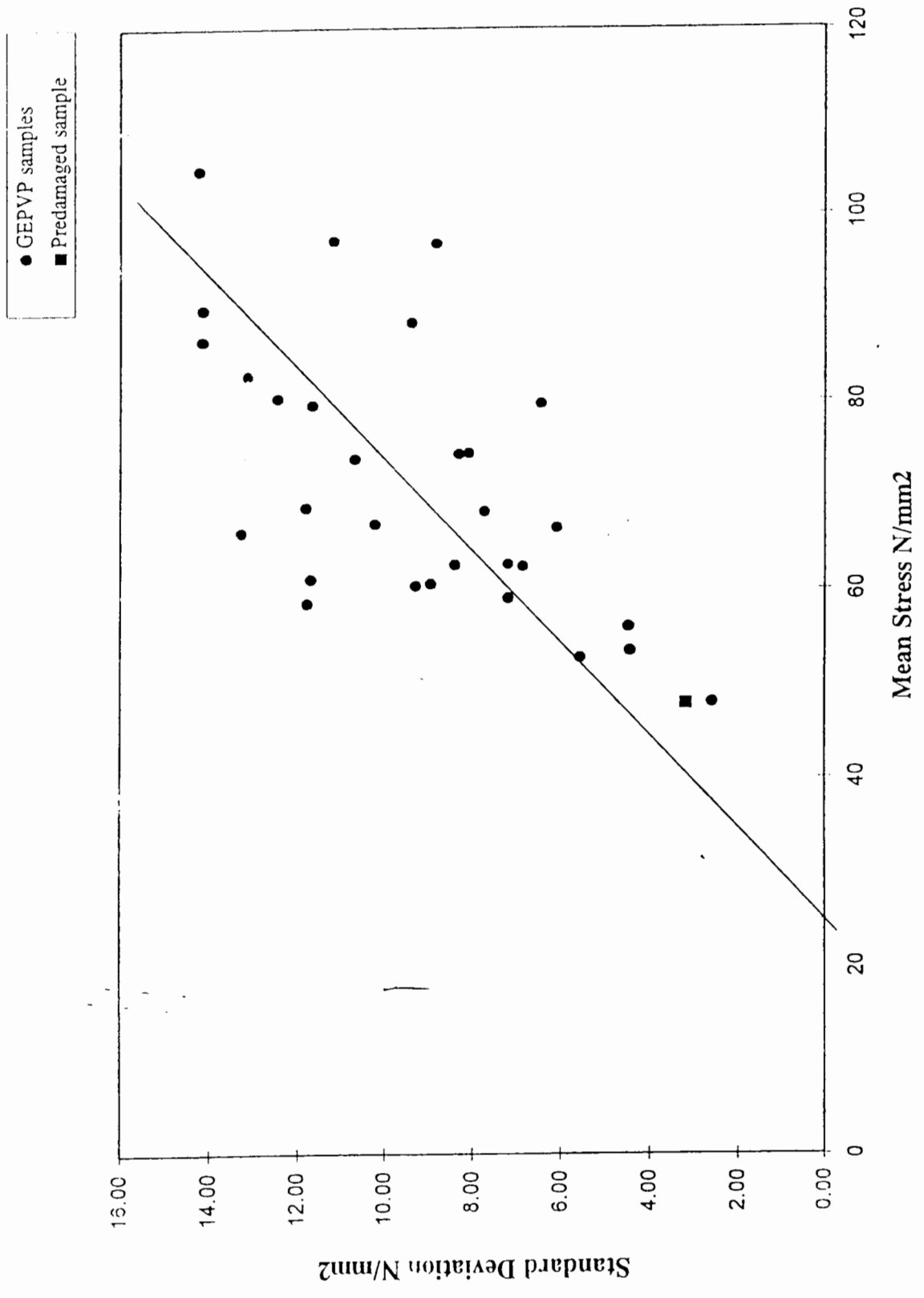


Fig 2 k EN 1288-2 Test Samples

### **3. The Statistics of Alternative Load Paths**

This chapter is an investigation into the statistical advantages of using multi-ply beams instead of single ones. As mentioned in the introduction, this issue was cited by one of the keynote speakers at the recent “Glass in Buildings” conference in Bath, as being an important area for further research.

To ameliorate the risk of sudden and catastrophic failures in glass beams, designers have adopted the approach of providing an alternative load path. In other words, if one 10mm thick piece of glass is sufficient to withstand a particular loading, with an adequate factor of safety, then two are provided. If one of the elements then fails for an unpredicted reason, the other should remain intact as it has been designed to support the full load.

A recent example of this philosophy is the floor of the top storey of the Blackpool Tower, which is made of two thick pieces of toughened glass laminated together. Should one fail then the other will apparently have enough strength to take the load of a baby elephant. Furthermore, we often see more than two elements being provided. For example in the work of Dewhurst Macfarlane and Partners, a residential glass pavilion in Hampstead has three 12mm interleaving glass beams and columns to make a portal structure (Architects Journal, June 96). They have also designed a glass box extension to a house in Powys Terrace in west London with seven beams laminated together (Architects Journal Oct 95).

There are three main reasons why designers have adopted the idea of using multiple beams in preference to one.

Firstly, being a brittle material, the failure of any part of the element (such as in an area of high local stress) will cause the failure of the whole element. Therefore, if an unexpected stress concentration, from say an alien element lodged between two contact surfaces, causes one beam to fail completely, the second beam will continue to support the structure.

Secondly, the consequences of a structural element failing are catastrophic both in that the element would offer no residual support whatsoever in its failed state, and that having failed, it would shower shards of glass on any unsuspecting people beneath. This is prevented by providing multiple beams and laminating them to each other.

Thirdly, glass as a material is generally perceived to be of unpredictable strength.

This third reason is a red herring. It is not normal to provide an alternative load path just because the strength of a material varies significantly. More typically the material safety factor is increased. This procedure is used with timber, for example, where within a species there is a wide variation in strength. However, with glass, it is generally believed that there is a possibility that a randomly selected, visibly undamaged, piece of glass can actually have zero strength. This belief stems from a spate of spontaneous failures of toughened glass elements under very low, or even zero, load. These failures almost all occurred in curved plate glass elements as a result of either differential thermal expansion between the outer and inner face of the pane, or because of nickel sulphide inclusions in the glass. These inclusions can increase in size over time in the presence of uv light, however they have now been all but eradicated by glass

manufacturers (Brantley, 1996). These spontaneous failures therefore are highly specific to curved glass and are not relevant in a discussion on edge-loaded structural beams.

### **The statistical advantage of multi-ply construction for annealed glass beams**

Setting aside the need for an alternative load path and simply looking at the material strength of annealed glass, it can be shown that statistically two beams should always have a strength greater than or equal to twice the strength of one beam. It follows therefore that two beams, each of half the thickness of the required single beam, would in fact be better than one beam on its own.

To demonstrate this, we need to start with some, perhaps arbitrary, probability distribution for the failure of a single pane of annealed glass. For this purpose it seems most reasonable to use the Weibull Distribution. This statistical distribution function, which was developed from empirical observations in the 1940's (Weibull, 1951), is the one most frequently used to determine the probability of failure of an element under stress. In its simplest form, it is described by two parameters, a normalising factor,  $\sigma_0$ , and a measure of the spread of the distribution called the Weibull modulus, usually denoted by the letter  $m$ . It is slightly asymmetrical, but otherwise provided  $m > 3$ , it is not very dissimilar to the normal distribution curve (Doremus, 1994).

Usually used in its cumulative probability form the Weibull distribution is described as follows:-

$$P_f = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right] \quad \text{Equation 1}$$

where  $P_f$  is probability of failure,  $m$  is the Weibull modulus and  $\sigma_0$  is a normalising factor that bears no relation to any obvious measurable property, but can be related to the mean stress using the standard gamma function (Stanley et al, 1973).

To find out how well a set of data fits the Weibull distribution, and to calculate the parameters  $\sigma_0$  and  $m$ , the above expression can be rearranged to

$$1 - P_f = \exp \left[ - \left( \frac{\sigma}{\sigma_0} \right)^m \right]$$

Then taking logarithms of both sides gives

$$\ln(1 - P_f) = - \left( \frac{\sigma}{\sigma_0} \right)^m$$

and taking logarithms again gives

$$\log(-\ln(1 - P_f)) = m \log \sigma - m \log \sigma_0 \quad \text{Equation 2}$$

To use this expression, the probability of failure of each value in a series of failure stresses needs to be known. This can be calculated by putting the  $N$  test values in ascending order of strength, and then assigning the  $n$ th ranked sample a probability of failure,  $P_f = (n-0.5)/N$  (Hahn, 1967). A plot of  $\log(-\ln(1-P_f))$  vs  $\log \sigma$  can be made, and goodness of fit ascertained by finding the linear regression correlation coefficient. Provided this is suitably high, then the Weibull modulus will be the slope of the fitted line, and the normalising stress can be found from the y-intercept.

As mentioned earlier, the actual probability distribution used for a single beam can be somewhat arbitrary for the exercise here, which is to investigate the benefits of using multiple beams of equivalent thickness instead of one beam. Therefore the data used here, have been taken from a textbook (Creyke et al, 1982), where they are simply described as “typical results for strength measurements on glass”. There are twenty samples, which is the minimum number recommended for the Weibull distribution, and they are listed in Appendix 2.

Figure 3a shows a log-log plot of this set of data. The line of best fit is drawn through the data points. From this plot, which shows good linearity ( $R^2 = 0.96$ ), the slope  $m = 10.0$  and the normalising stress,  $\sigma_0 = 63.55 \text{ N/mm}^2$  can be calculated. These two parameters are then used in the Weibull expression (Equation 1), and the Weibull cumulative probability distribution can be plotted (Figure 3b).

To plot the probability of failure curve for two or more beams, the Weibull distribution described by Equation 1, needs to be modified to include a volume term. It is well known that the strength of a brittle element is dependent on its size (Dieter, 1991 & Stanley et al, 1973). The bigger the element is, the lower its strength is likely to be. The reason for this has been discussed in detail in Chapter 2.

Equation 1 can be modified to take account of volume as follows

$$P_f = 1 - \exp \left[ - \left( \frac{V}{V_0} \right) \left( \frac{\sigma}{\sigma_0} \right)^m \right]$$

where  $V$  is the volume of the material, and  $V_0$  is a reference volume. The derivation of this expression is given in Appendix 3 (after Stanley et al, 1973).

When comparing the strength of two beams of thickness,  $0.5t$ , with a single beam of thickness,  $t$ , then  $(V/V_0)$  simply becomes one half. However, to calculate the probability of failure of a two-beam system, all the possible routes to failure must be identified.

Figure 3c shows a probability tree for a two-beam system.

<b>Beam 1</b> (under stress= $\sigma_1$ )	<b>Beam 2</b> (under stress= $\sigma_1$ )	<b>Surviving Beam</b> (under stress= $2\sigma_1$ )	
S	S		(System Survives)
S	F	S	(System Survives)
		F	(System Fails)
F	S	S	(System Survives)
		F	(System Fails)
F	F		(System Fails)

**Figure 3c Probability Tree for a two Beam System**

Three routes lead to survival of the system, and three to failure. To find the probability of failure of the system, the individual probabilities of all three routes to failure need to be added. For this to be done the following definitions are used

$Pf_1$  = Probability of failure of a single beam, of relative thickness one half, at any particular applied stress,  $\sigma_1$ .

$Pf_2$  = Probability of failure of a single beam, also of relative thickness one half,

at an applied stress of  $2\sigma_1$ .

Then the probability of failure of a two-beam system can be described by the following equation .

$$Pf = Pf_1 \times Pf_1 + Pf_1 \times (1 - Pf_1) \times Pf_2 + (1 - Pf_1) \times Pf_1 \times Pf_2$$

The last two terms in this expression are equivalent and therefore the algorithm can be simplified to:-

$$Pf = Pf_1^2 + 2 \times Pf_1 \times (1 - Pf_1) \times Pf_2$$

Similarly an algorithm for the probability of failure of three beams can be formulated from the probability tree shown in Figure 3d.

For this,  $Pf_1$  becomes

$Pf_1$  = Probability of failure of a single beam, of relative thickness one third, at any particular applied stress,  $\sigma_1$ .

And two more definitions are required

$Pf_{1.5}$  = Probability of failure of a single beam, of relative thickness one third, at an applied stress of  $1.5\sigma_1$ .

$Pf_3$  = Probability of failure of a single beam, of relative thickness one third, at an applied stress of  $3\sigma_1$ .

Summing all the routes in Figure 3d that lead to a system failure gives the probability of

failure of a three-beam system to be

$$Pf = Pf_1^3 + 3 \times Pf_1^2 \times (1 - Pf_1) \times Pf_3 + 3 \times Pf_1 \times (1 - Pf_1)^2 \times Pf_{1.5}^2 + 6 \times Pf_1 \times (1 - Pf_1)^2 \times Pf_{1.5} \times (1 - Pf_{1.5}) \times Pf_3$$

A similar expression can be formulated for four beams

$$Pf = Pf_1^4 + 4 \times Pf_1^3 \times (1 - Pf_1) \times Pf_4 + 6 \times Pf_1^2 \times (1 - Pf_1)^2 \times Pf_2^2 + 12 \times Pf_1^2 \times (1 - Pf_1)^2 \times Pf_2 \times (1 - Pf_2) \times Pf_4 + 4 \times Pf_1 \times (1 - Pf_1)^3 \times Pf_{1.33}^3 + 12 \times Pf_1 \times (1 - Pf_1)^3 \times Pf_{1.33}^2 \times (1 - Pf_{1.33}) \times Pf_4 + 12 \times Pf_1 \times (1 - Pf_1)^3 \times Pf_{1.33} \times (1 - Pf_{1.33})^2 \times Pf_2^2 + 24 \times Pf_1 \times (1 - Pf_1)^3 \times Pf_{1.33} \times (1 - Pf_{1.33})^2 \times Pf_2 \times (1 - Pf_2) \times Pf_4$$

Using the following definitions

$Pf_1$  = Probability of failure of a single beam, of relative thickness one quarter, at any particular applied stress,  $\sigma_1$ .

$Pf_{1.33}$  = Probability of failure of a single beam, of relative thickness one quarter, at an applied stress of  $1.33\sigma_1$ .

$Pf_2$  = Probability of failure of a single beam, of relative thickness one quarter, at an applied stress of  $2\sigma_1$ .

$Pf_4$  = Probability of failure of a single beam, of relative thickness one quarter, at an applied stress of  $4\sigma_1$ .

The probability tree for four beams is large and complex and has therefore not been included here.

The results of applying these algorithms to the probability distribution for a single beam

(plotted in Figure 3b), are plotted in Figure 3e. However it is very difficult to see the differences between the probabilities of failure for multiple beams from those for a single beam. This is because it is only at very low probabilities that any differences are actually observed. In order to see these differences more clearly, the ordinate can be plotted on a logarithmic scale (Figures 3f & 3g).

These last figures demonstrate that two beams are always at least as strong as a single beam of equivalent total thickness, for a given probability of failure.

### **Tests**

Having produced the plots illustrated in Figures 3f & 3g, which clearly indicate the benefits of multi-ply beams over single ones, the obvious next step would be to verify these statistical findings by physical test. However this is problematic, because of the very large numbers of test specimens that would be required. In Figure 3f, the bifurcation of the line for two beams from the single beam line occurs at a probability of failure of just under 1 in 100. Clearly any number of tests below one hundred of each type (i.e. two hundred tests with three hundred beams) would not reveal any significant difference. Even at a probability of 1 in 1000, the difference in predicted failure stress for a two-beam system is only about 7% more than for one beam. So even three thousand test specimens would be unlikely to produce results that would verify the reduced probability of failure of two-beam systems over equivalent single beams.

In fact some tests on single and multi-ply annealed glass beams were carried out, under the supervision of the author, by Fair and Williams in 1996, as part of their final year project. These are described in detail in Chapter 2. In all thirty single beams, 10mm

thick, and sixteen double beams, each 10mm thick, were tested. The mean strength of the single beams was  $39.2 \text{ N/mm}^2$  and of the double beams  $37.2 \text{ N/mm}^2$ . This seems an unusual result in the light of what is being proposed in this chapter. Although the sample is very small, and therefore really not much can be concluded from results, one would expect the mean of the two-ply beams to be at least as high as the mean of the single ones. However there is an explanation for this.

If we were to reconstruct the graphs shown in Figures 3f and 3g, without taking into consideration the volume term in the Weibull expression, it would look slightly different. (See Figure 3h. – only up to three plies have been plotted here). In this figure, the probability of failure of two beams is in fact greater than one beam for stresses above about  $32 \text{ N/mm}^2$ . This explains why the mean stress of the two-ply beams in the tests was lower than for the single beams, as they were all breaking at stresses above  $32 \text{ N/mm}^2$  and were of the same volume. Had the two-ply test specimens had an equivalent overall thickness as the single beams (rather than double) then their mean stress would be expected to be higher as demonstrated in Figures 3f and 3g.

Therefore, even this reasonably large number of test samples gives absolutely no indication of the advantage of multi-ply beams over single ones, as expected. However Figures 3f & 3g clearly indicate an indisputable advantage, at probabilities of failure below 1 in 10,000. (At 1 in 1,000,000 the strength of a two-beam system is nearly 50% stronger than an equivalent single beam).

### **Conclusions**

The information presented in this chapter can be practically applied to annealed glass

beam design in the following ways. Firstly, if the thickness of glass is fixed for a given design stress, say  $25 \text{ N/mm}^2$ , then the probability of failure can be found from Figure 3f for a single beam to be about  $1\text{E-}5$ . If two beams are used, then the probability of failure reduces by just over 20 times. More than two beams, of the same equivalent thickness, would give a probability of failure of more than three hundred times less than for one beam.

Another way of using Chart 6, and probably more akin to usual design practice, would be to have a fixed probability of failure and a fixed design stress; the variable being the thickness of the glass. Using multi-ply beams in such a scenario would allow the overall thickness, and hence the material cost, to be reduced compared with that required for a single beam in the following way. If a beam of thickness,  $t$ , is required to carry a stress of say  $20 \text{ N/mm}^2$ , with a probability of failure of approximately  $1\text{E-}5$ , then following the argument for an alternative load path described in the first page of this chapter, the designer would provide two beams each of thickness,  $t$ . However, should the designer provide three beams, and still require a probability of failure for the alternative load path of  $1\text{E-}5$ , then the overall thickness could be reduced as follows.

If one failed for an unpredicted reason, then two beams would be left. The stress a two-beam system could withstand, to give a failure probability of  $1\text{E-}5$ , is approximately  $25 \text{ N/mm}^2$ . The overall thickness of the two beams would therefore only need to be  $4/5$  of  $t$ . The third beam would obviously need to be the same thickness as the other two (i.e.  $2/5 t$ ). Hence the three-beam system would have an overall thickness  $6t/5$ , and would now have a probability of failure of much less than  $1\text{E-}5$ . Now taking account of the need for an alternative load path, should one beam fail unexpectedly, there would still

only be a  $1E-5$  probability of the remaining system failing. The reduction in thickness (and therefore material cost) is from  $2t$  to  $6/5t$ , a saving of 40%.

It should be stressed here, that the calculations presented in this chapter are based on an arbitrary set of strength values which approximate to those commonly found for ordinary annealed glass. In practice, because of its additional strength, toughened glass may be used for glass beam construction. It may appear to the reader, on examination of Figures 3f and 3g, that there is no advantage in using multi-ply toughened glass beams over single ones. This is not true for two reasons. Firstly, the results presented here are for annealed glass only. To discover the advantages of multi-ply construction using toughened glass beams would require a separate study. This has not been undertaken as part of this thesis. Secondly, although toughened glass has a much higher “minimum” strength than annealed glass, and therefore under some circumstances a second ply may seem unnecessary, it is required because of the threat of vandalism. Any toughened glass element can be relatively easily shattered by impact with a sharp metal object near a corner.

### **References for Chapter 3**

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Stanley P, Fessler H & Sivill A D (1973) An Engineers Approach to the Prediction of Failure Probability of Brittle Components, Proc. Brit. Ceram. Soc., 1973, vol. 22.

Weibull W (1951) A Statistical Distribution Function of Wide Applicability, Journal of Applied Mechanics, September, 1951.

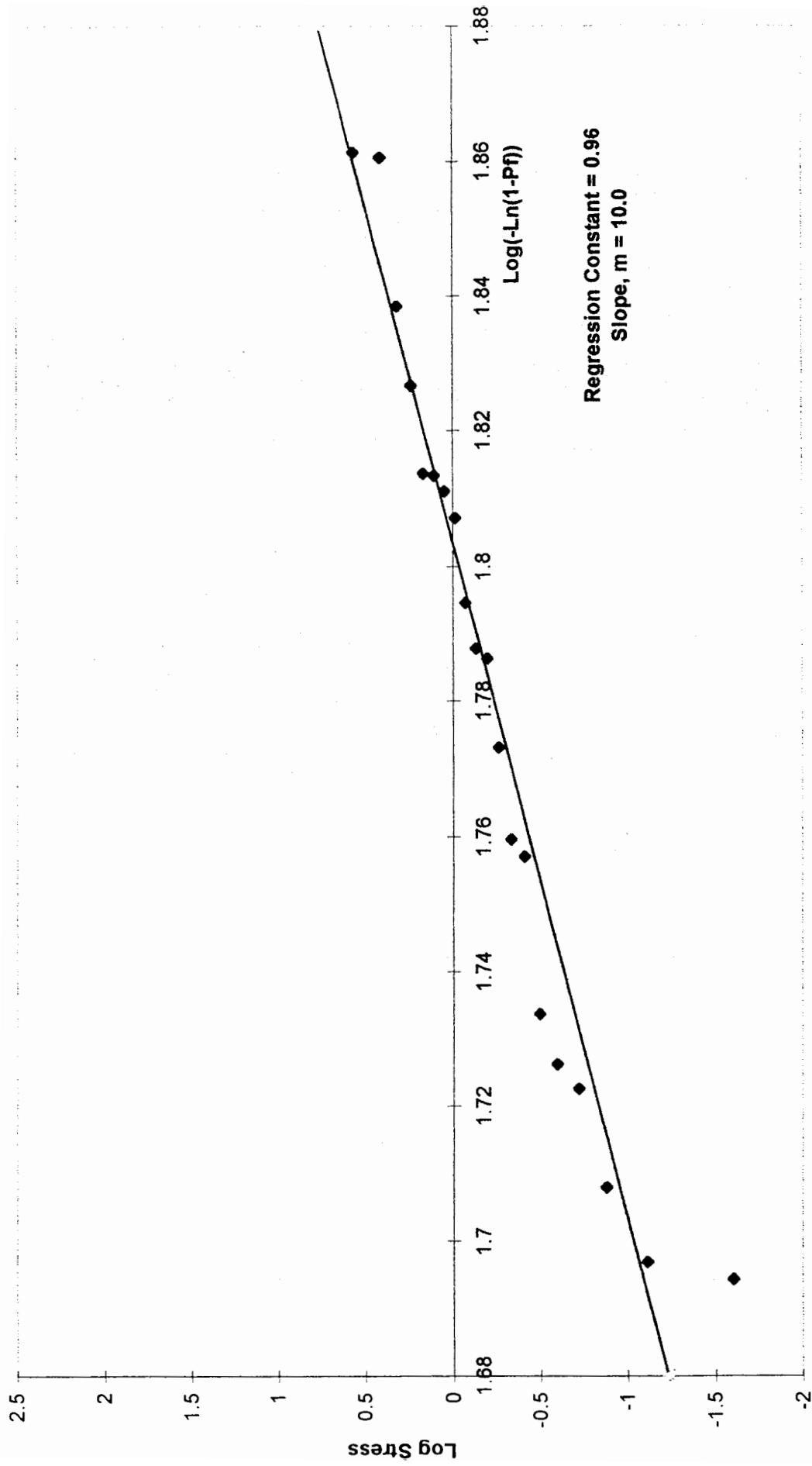


Figure 3a Log log plot of Creyke data to establish Weibull Modulus, m, and normalising stress, So.

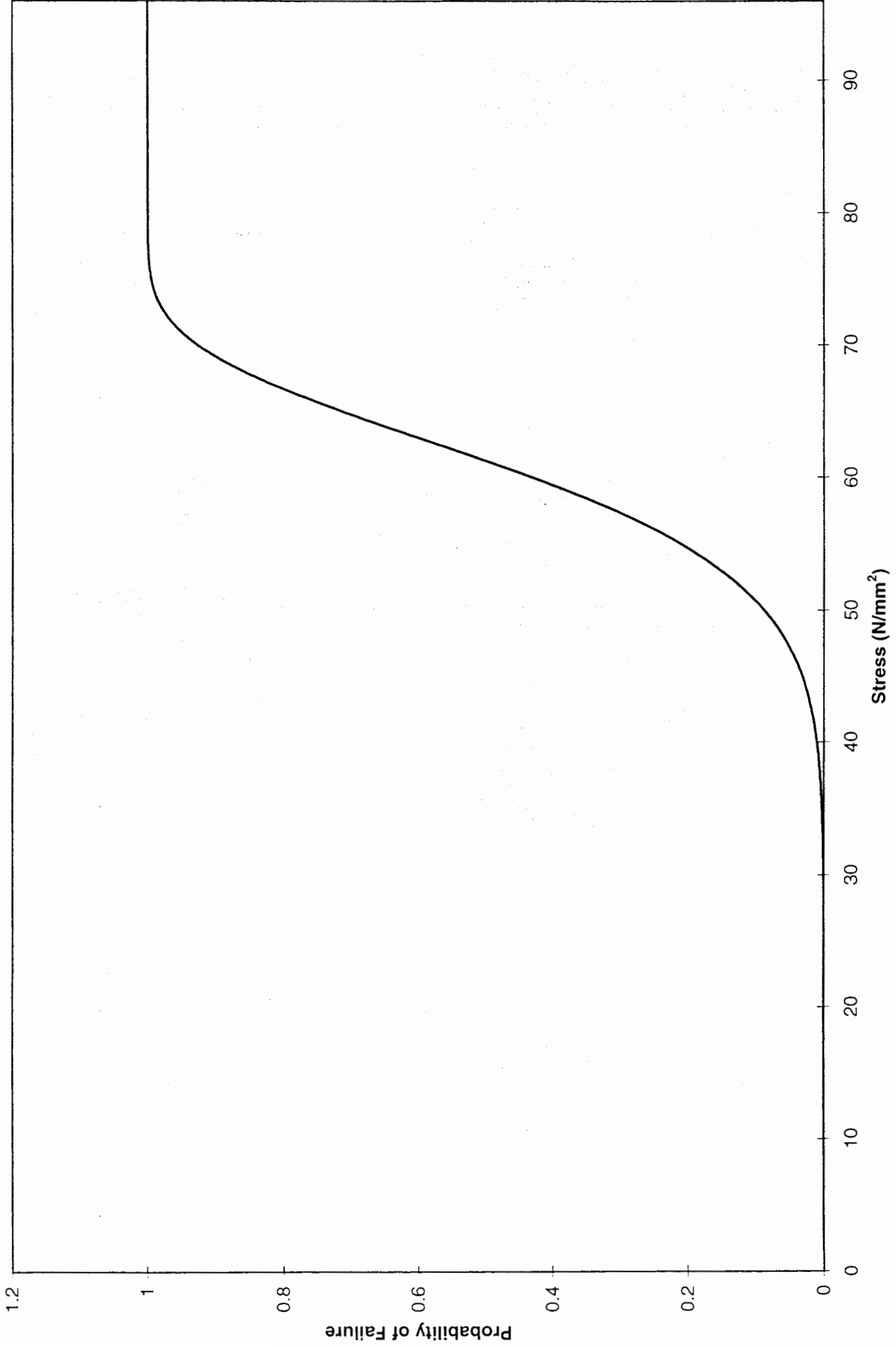


Fig. 3b Weibull Cumulative Probability Distribution, using Creyke Data.



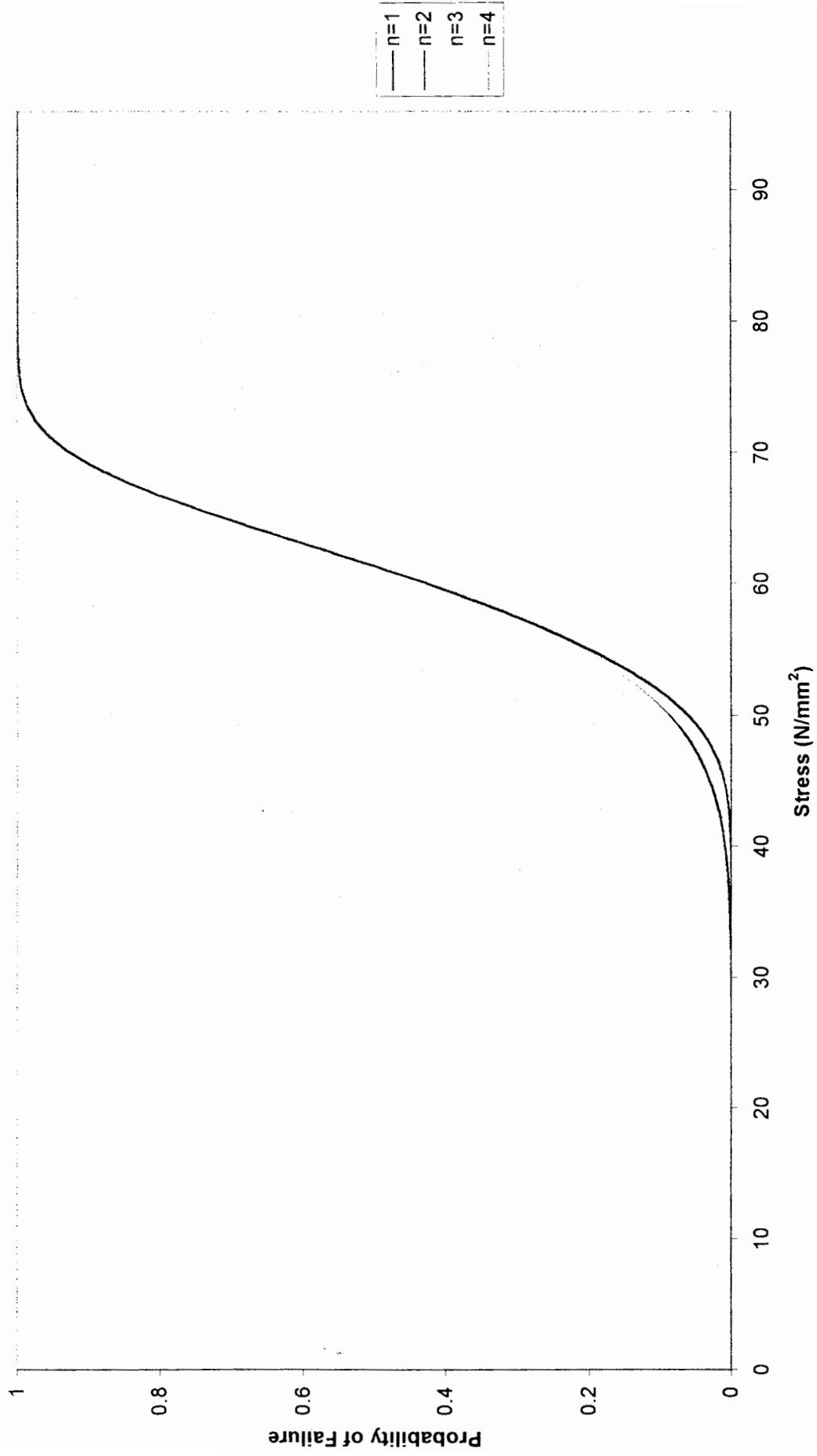
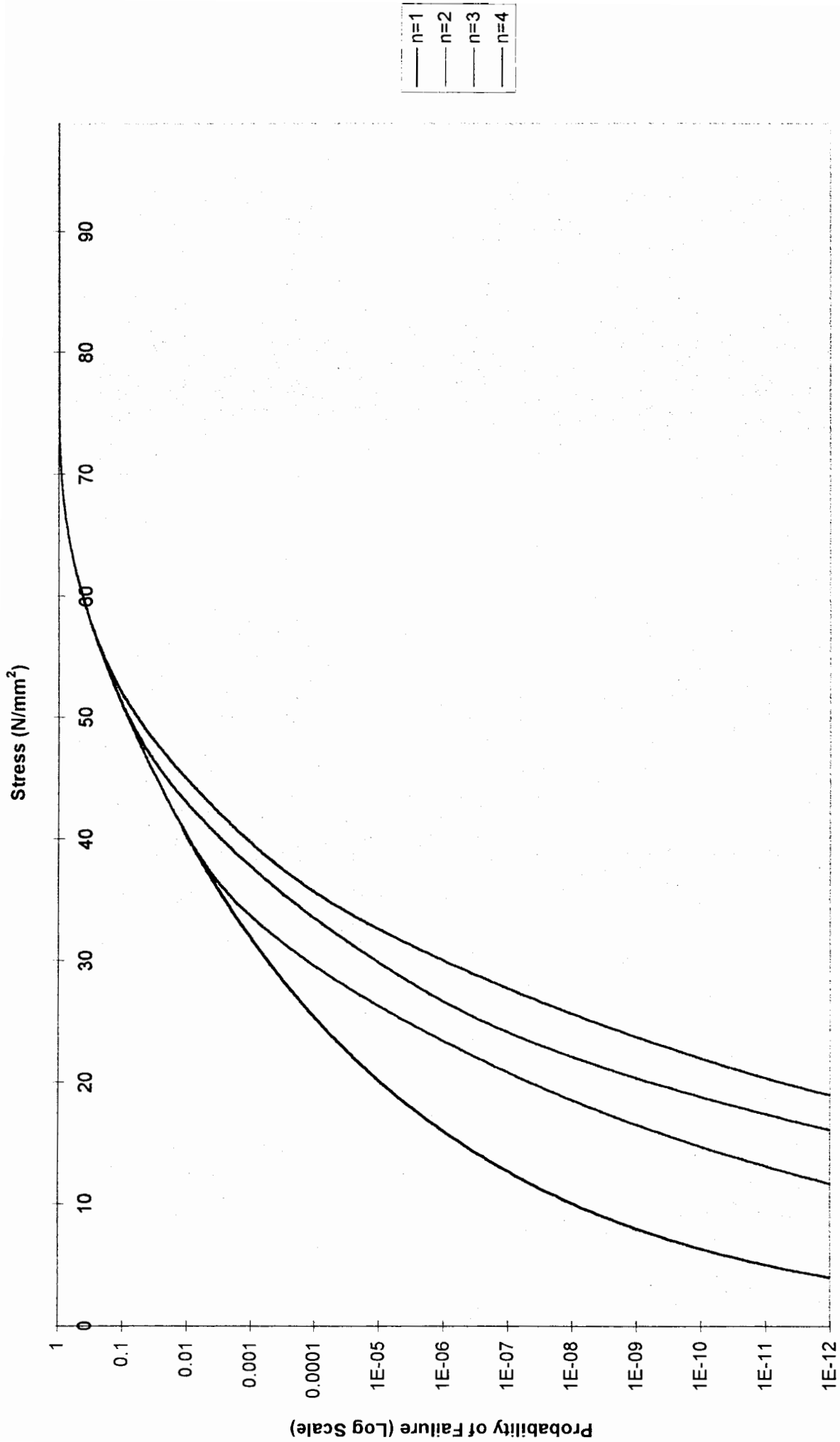


Figure 3e Weibull Cumulative Probability Distribution for Multi-pane Beams using Creyke Data



**Figure 3f Multi-beam Failure Model, using Creyke Data, (min Pf = 1E-12)**

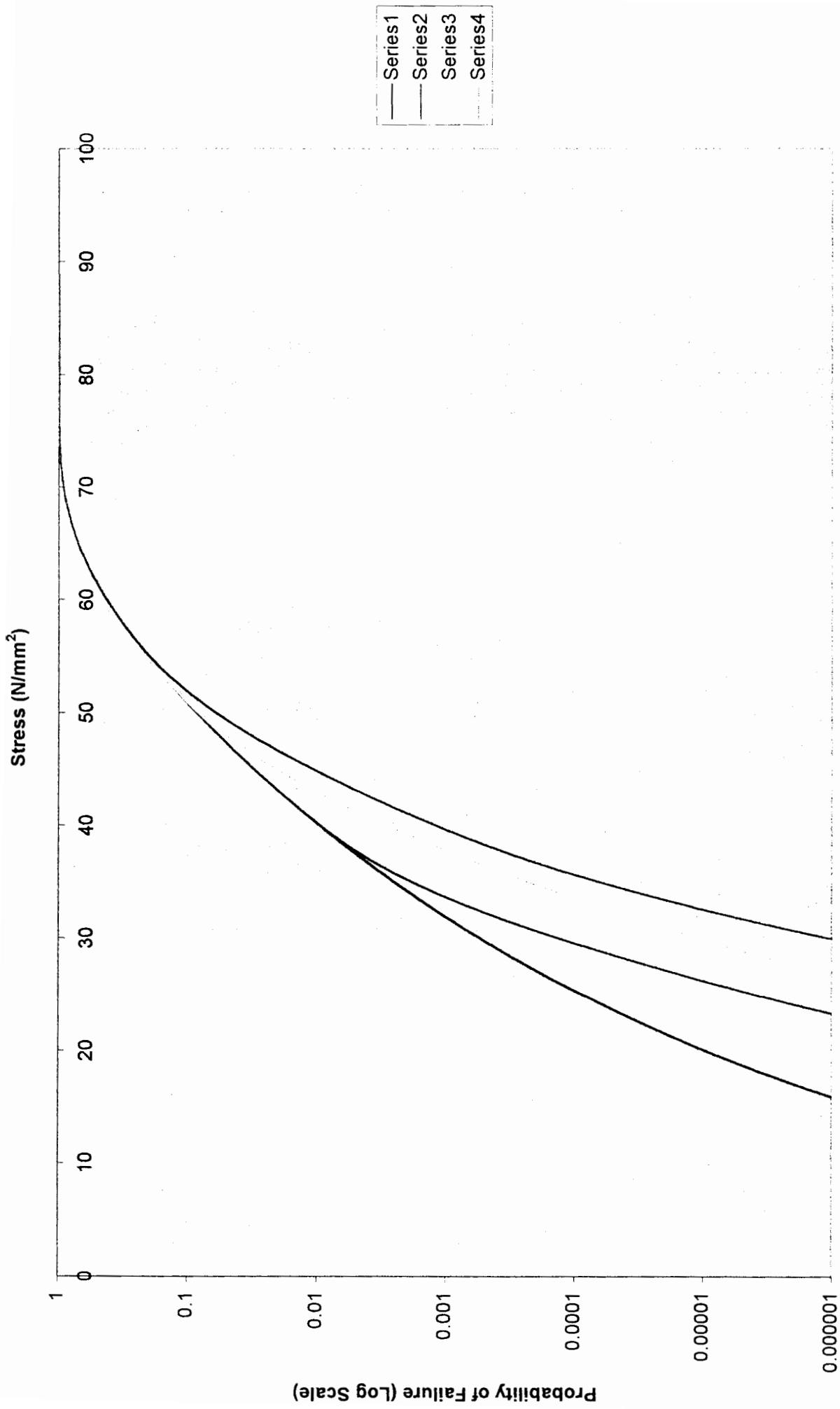


Figure 3g Multi-beam Failure Model, using Creyke Data, (min Pf = 1E-6)

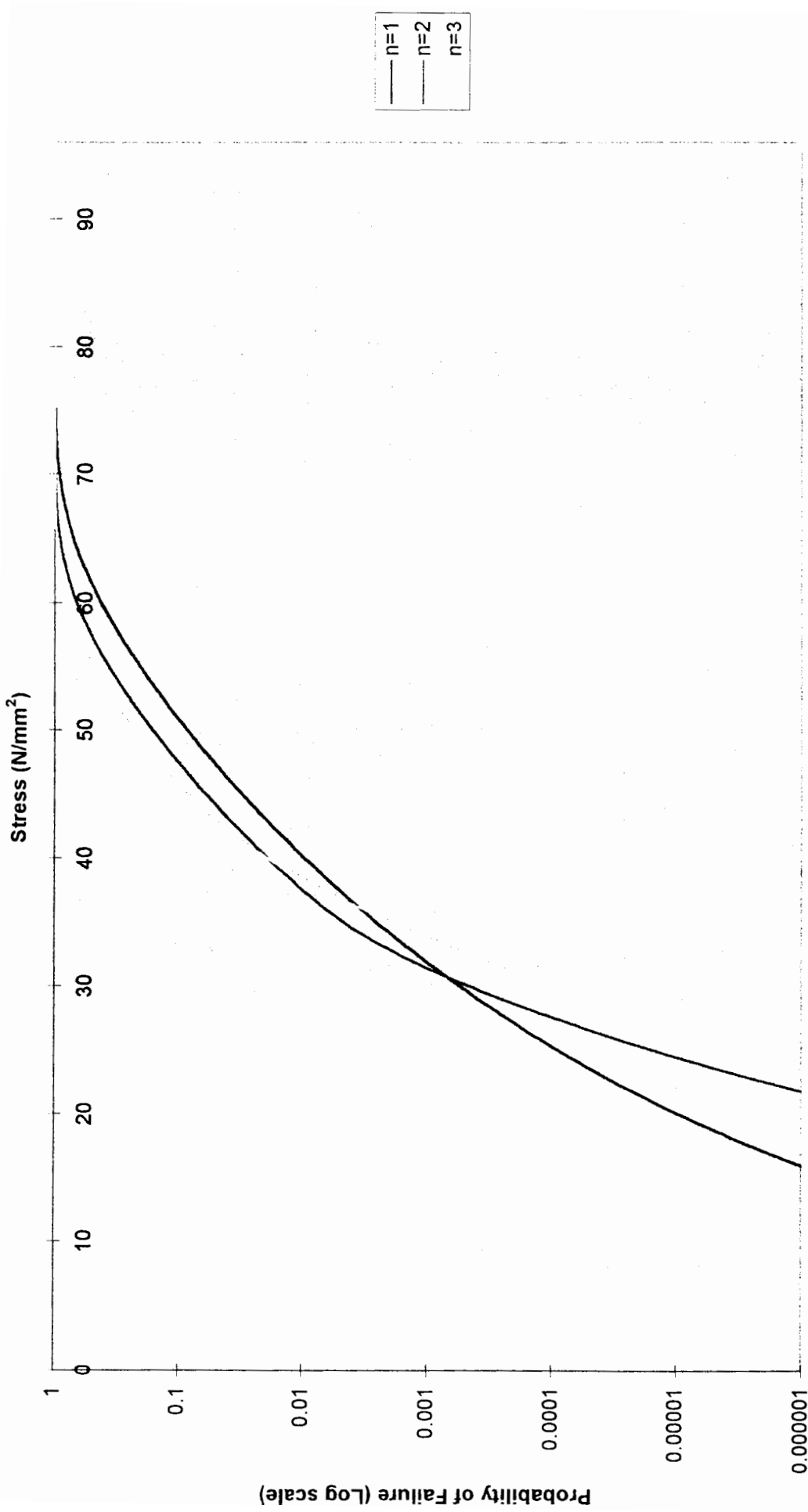


Fig. 3h - Multi-beam Failure Model, using Creyke data, and ignoring volume effects.

## 4. Conclusions and Discussion

Engineering design is essentially about the reasonable assessment of risk; risk of expected design actions being exceeded and risk of expected material response being inadequate. This assessment is a continually evolving process. In Chapter 1 of this thesis, the evolution of design procedure over this century has been outlined – from the simplest form usually called Permissible Stress Design to the more complex procedures that have been collected together under the title of Limit State Design.

Limit State Design, although it has proved elusive to define, encapsulates two very simple ideas – that of designing within several well defined limit states and the treatment of uncertainty more logically by the use of partial safety factors. Unfortunately, however, because this limit state philosophy was developed from plastic design theory, the two have become somewhat confused.

Plastic design methods are appropriate for “only one of a large number of possible design criteria, that of strength.” (Heyman, 1973). Therefore, as has been outlined in Chapter 1, too much emphasis is given in limit state design to the limit state of strength. Over-stressing as a result of normal statistical variations in dead and live loads accounts for only a very small percentage of structural failures.

This over-emphasis on strength, has caused masonry to be forced into a design procedure for which it is unsuited. Masonry could fit quite well into a limit state model, if the primary limit state being considered was one of overall stability and not strength. However, current masonry codes of practice have adopted strength as the primary

design criterion. As a result, designers using these codes have very little understanding of how close to failure their structures are. This is because their design is based not on the probable true failure mode of overall instability, but instead the designer must base his or her calculations on the strength of a masonry unit, an all but irrelevant parameter. It is not being suggested here that a design based on a limit state masonry code such as BS 5628 will be inadequate, but merely that the designer carrying out the calculations is quite divorced from the real design criterion of overall stability. Masonry design to current codes is therefore an artificial process based on an artificial ultimate limit state.

The other material that has been demonstrated not to sit comfortably into a stress-based limit state code is timber. The key advantage of plastic design theory, which has been incorporated into the limit state philosophy for a material such as steel, is that predictions of loads that are likely to cause collapse can be calculated with a relatively high degree of accuracy. This is not the case with timber. Timber is a highly variable material with each piece requiring individual grading. The ultimate strength of any particular piece of timber can never be predicted with any degree of accuracy, because it is dependent on the number of defects and the use to which the structural element is to be put in service.

In Chapter 1 several parallels were drawn between timber and glass. One interesting commonality is in the nature of their failure. The strength of a piece of timber, as with glass, cannot be sensibly estimated by thinking in terms of the theoretical strength and attempting to identify reasons why the theory is never satisfied. Rather, a more practical approach considering the present state of the timber and ignoring its theoretical strength is necessary. This approach is already current practice. Any piece of timber for

structural use is individually graded, and its utility determined solely by the presence of some defect, which will be a point of crack initiation that will propagate through the specimen causing its ultimate failure (Illston *et al*, 1979). The failure of timber is in fact brittle in nature, though we rarely think of timber as a brittle material. However, one only needs to compare the behaviour in bending of a piece of kindling with say an aluminium moulding to verify this. The latter could be bent into a tight radius, whereas the former, once its elastic strain limit had been reached would fracture suddenly with little or no prior plastic deformation.

Let us now address glass itself. Glass definitely cannot be made to conform with a design method created for ductile materials. The statement quoted in the Chapter 1 is worthy of repetition here: “The so-called plastic design theory of structures applies therefore to steel and to reinforced concrete frames ... but not to materials like cast iron and glass which are brittle” (Heyman, 1995).

Glass needs to be looked at in an entirely different way. As outlined in Chapter 2, glass is a brittle material and therefore fails by brittle fracture. For any brittle material, failure is not the result of stress alone, but occurs as in timber when a critical combination of stress and crack size is reached. Therefore the assessment of the likely performance of a piece must be highly individual. As with timber, each piece of glass would need to be graded to assess its defects, and from this the maximum stress that could be applied to a particular piece of glass could be estimated.

Measuring the maximum size of crack on a piece of glass is not a trivial matter. Unlike timber, where the defects are highly visible, in glass the Griffith flaws are generally

invisible to the naked eye. However, if the flaws were large enough to cause the short-term strength of the piece of glass to be of the order of  $25 \text{ N/mm}^2$ , then they would almost certainly be visible at a quarter of a millimetre in width. Therefore, as long as we visually inspect each piece of glass, we can be relatively certain of its minimum load carrying capacity.

An immediate question that presents itself, is what about the possibility of the glass being damaged between assessment and it being installed in service. This problem does not arise with timber, because the defects are on a larger scale, and minor damage is unlikely to create a crack large enough to challenge a knot or seasoning split as a site for fast fracture initiation. Glass on the other hand would need to be inspected for any defects in situ. However, this problem is not as serious as it at first appears. Glass is known by all to require extra special care in handling, and where it is being used for structural purposes it is generally transported and erected by specialist contractors, who would understand the implications of surface damage.

Chapter 3 of this thesis has dealt with an important issue identified by Tim Macfarlane at the 1999 “Glass in Buildings” conference, that is to further the understanding of the statistical behaviour of multi-ply beams. In this thesis it has been clearly shown that stochastically multi-ply beams are less likely to fail than single beams of equivalent thickness. There are two main reasons for this. Firstly, the failure of any glass element is probabilistic. This means that if there is two elements supporting a load and one fails, there is always some probability that the second element will survive at twice the stress it was designed for. Secondly, it is a well-known phenomenon that a smaller volume of glass is statistically less likely to fail, under a given load, than a larger one. A

description has been given of how the use of multi-ply beams can reduce the overall thickness, and therefore cost, of a glass element without any increase in the risk of failure. Interestingly, in this respect glass demonstrates another similarity to timber. Because of the stochastic nature of timber strength assessment, higher design strengths are permitted in timber beams when loads are shared between two or more structural elements.

Finally if one was to try to fit glass into any established design procedure, then it would seem that a permissible stress approach, similar to that used for timber would be the most appropriate.

### **Future Work**

This MSc thesis has been a broad look at design methods in general, and at the nature of glass as a structural material. There are many areas that could be carried forward. However, probably the most important area is in finding a suitable fundamental design approach for glass. Mark Porter in this department is currently working on formalising a design method for glass that does not focus on stress as the main design criteria, but instead concentrates on the development of cracks. It would also be useful to extend this work on beams to other structural forms.

## Appendix 1 - Fracture of Brittle Solids.

A brittle solid can be defined as one that remains elastic right up to the point of failure. It fails by the tearing apart of atomic bonds across a fracture plane. The theoretical strength of a brittle solid (or any solid for that matter) is therefore based on its bond strength. Most current theoretical models for brittle fracture lead to the following expression

$$\sigma_{\theta} = (\gamma_s E / a_0)^{1/2}$$

where  $\sigma_{\theta}$  is the theoretical strength,  $\gamma_s$  is the energy required to create a new surface (bearing in mind that there are two new surfaces created when a crack is formed),  $E$  is the Young's modulus and  $a_0$  is the original bond length.

This theoretical strength is never achieved in practice, because the surfaces of all solids are covered with tiny cracks. At the tips of these cracks the stress is much higher than elsewhere in the specimen. This leads to progressive failure, with the bonds (or lines of bonds) failing one by one from the crack tip. Inglis showed that the stress at a crack tip is dependent on the radius of curvature of the tip of the crack,  $\rho$ , and the crack length,  $c$ .

$$\sigma_{\text{tip}} \cong 2\sigma(c/\rho)^{1/2}$$

This equation, however, is not of much use for calculating the strength of a material, as the radius of curvature of the crack tip is difficult to define (it depends on the distance the bonds need to separate before they break). An alternative theoretical approach is

therefore required.

Griffith, using the work of Inglis, approached this problem by considering the balance of energy. He proposed that if more energy was released, from the elastic strain relaxation around the crack tip, than was absorbed in breaking the atomic bonds to create new surfaces, then the crack would propagate unstably through the material. Griffith also essentially showed that the elastic strain energy associated with a surface crack of length,  $c$  is

$$U_{el} = \pi c^2 \sigma^2 / 2E$$

where  $\sigma$  is the applied stress and  $E$  is the Young's modulus. The energy absorbed in creating new surfaces is

$$U_s = 2\gamma_s c$$

Fast fracture will occur, if when a crack advances,

$$dU_{el}/dc \geq dU_s/dc$$

which can be rearranged to give Griffith's classic expression for the fracture stress of a brittle solid.

$$\sigma_f = (2\gamma_s E / \pi c)^{1/2}$$

The  $2\gamma_s$  is normally expressed as  $G_c$ , the energy absorbed per unit area of crack, which is a material property that can be established by experiment.  $G_c$  is often termed the toughness of the material and the equation above is expressed as follows

$$\sigma(\pi c)^{1/2} = (G_c E)^{1/2}$$

This equation shows that the failure of a brittle solid occurs when the combination of crack length and applied stress reaches a critical level. The LHS of the above equation is sometimes termed the Stress Intensity Factor and has the symbol,  $K$ . The RHS is constant for any given material, and is termed the Critical Stress Intensity Factor or, more commonly, the Fracture Toughness,  $K_c$ . It has units  $\text{MPa}\cdot\text{m}^{1/2}$ .

The invisible cracks, which abound on the surfaces of brittle materials as mentioned earlier are commonly termed Griffith flaws. Griffith also proposed that the size and distribution of these flaws is essentially random.

The random nature of these flaws means that, for a brittle material, the strength of a particular sized test specimen will be highly variable. Any attempt to estimate it must therefore be stochastic. If the stress was constant throughout the material, such as in a direct tension test, then the prediction of failure would be wholly dependent on the largest flaw present. However, direct tension tests on brittle materials are not very easy to carry out; because it is difficult to grip the specimens without causing damage. The most common form of test for brittle materials is therefore a three or four point bend test. This introduces an additional source of error, in that the point of maximum stress is highly localised, and therefore one must take account of the possibility that the largest flaw does not coincide with the location of highest stress. In any stochastic analysis of bend test results therefore, not only must the random distribution of flaws be considered but also the size of the area under stress.

The most common statistical distribution that is used for the prediction of the failure strength of brittle materials is the Weibull distribution. This was empirically developed during the 1930's. Ignoring area effects (as described above), it can be expressed in the following form

$$P_f = 1 - \exp\{-(\sigma/\sigma_0)^m\}$$

where  $P_f$  is the probability of failure,  $\sigma_0$  is a normalising stress that bears no direct resemblance to any measurable quantity, and  $m$  is the Weibull modulus which gives a measure of the spread of the data.

A high Weibull modulus indicates that the strength of the material shows minimal variability. Mild steel for example has a Weibull modulus of about 100. This is because mild steel is a ductile material and being so, its microstructure allows plastic flow to take place around crack tips. This process absorbs energy and prevents fast fracture occurring. Consequently the failure of mild steel is not dependent on the random presence of flaws. Glass on the other hand having almost no ability to plastically flow, invariably fails by fast fracture. Its failure is wholly dependent on a random event (the presence of a particular sized flaw being coincident with the required stress level). It therefore has a relatively low Weibull modulus; of the order of about 10.

## Appendix 2 - Glass strength data from Creyke *et al.*

<b>Specimen No.</b>	<b>Flexural Strength</b>
1	72.65
2	67.08
3	53.23
4	62.31
5	61.13
6	72.53
7	59.30
8	49.76
9	49.46
10	65.07
11	64.72
12	68.94
13	64.14
14	51.04
15	57.48
16	52.78
17	65.11
18	61.34
19	54.15
20	57.15

### Appendix 3 - Derivation of three parameter Weibull expression

Beginning with the standard two parameter Weibull Equation (Weibull)

$$P_f = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right]$$

and since  $P_s = 1 - P_f$ , this can be written

$$P_s = \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right]$$

Therefore

$$\text{Ln}P_s = -\left(\frac{\sigma}{\sigma_0}\right)^m$$

Then for a small unit volume,  $v$ , with constant stress throughout, this expression could be rewritten

$$\text{Ln}P_{sv} = -\left(\frac{\sigma}{\sigma_{0v}}\right)^m$$

Now taking a sample of volume,  $V$ , which is made up of  $N$  elements of equal volume,  $v$ , the overall survival probability could be written in terms of the product of the survival probabilities of the separate elements.

$$\text{Ln}P_s = -\left(\frac{\sigma}{\sigma_{0v}}\right)^m \times N$$

This value can also be obtained direct from the original Weibull expression

$$\text{Ln}P_s = -\left(\frac{\sigma}{\sigma_0}\right)^m$$

Equating these two expressions gives

$$\left(\frac{\sigma}{\sigma_0}\right)^m = -\left(\frac{\sigma}{\sigma_{0v}}\right)^m \times N$$

Since  $N = V/v$ , this can be rewritten

$$\left(\frac{\sigma_0}{\sigma_{0v}}\right)^m = v/V$$

Now substituting into the original Weibull expression gives

$$Pf = 1 - \exp\left[-\frac{V}{v}\left(\frac{\sigma}{\sigma_{0v}}\right)^m\right]$$