Geometry of Gains: How Competitive Markets Keep Everyone on Board

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At the heart of economics is a scientific mystery ... a scientific mystery as deep, fundamental and inspiring as that of the expanding universe or the forces that bind matter ... How is order produced from freedom of choice?

—Vernon L. Smith,
"Microeconomic Systems as an Experimental Science"

Since at least the time of Adam Smith, economists have extolled the benefits of open trade through a system of prices. The classic claim, in short, is that allowing more participants, from local farmers to entire nations, to buy and sell freely increases overall welfare. In game-theoretic terms, any smaller group tends to do better within the all-inclusive "grand coalition" than on its own. This idea rests on the key concept of the core of a market game, the binding force that holds market economies together. Core stability means that no splinter of participants can profit by leaving; thus the entire coalition (the large open market) remains the most beneficial arrangement.

In slightly more mathematical language, a set of agents (buyers and suppliers) each has something to gain from trade by forming a coalition, and the total gains depend on who is in the coalition. A distribution of payoffs is said to be in the core if no subset of agents can profit by leaving and trading solely among themselves. When the entire economy is competitive, meaning that prices are free to move so that total supply meets total demand, the competitive equilibrium not only maximizes the total gains from trade but also ensures that no proper subset (no splinter group of countries or agents) can do better on its own. In other words, the competitive solution binds the entire group together by ensuring that neither individuals nor groups have incentive to exit. This stands as one of the most remarkable achievements of economic science.

Classical fixed-point proofs underpin modern general equilibrium [2, 5, 10], but they are algebraic and nonconstructive, so the economic intuition can be hard to see. Under quasilinear preferences with a money numéraire, Marshallian and Hicksian demands coincide for non-numéraire goods [9]. The assumption is narrower, yet it sharpens the geometry. In fact, consumer and producer surplus are read directly as areas under the demand curve and areas above the supply curve, making the geometry of gains transparent. The supply-and-demand diagrams then show why the competitive equilibrium is core-stable in a one-commodity economy, while Legendre–Fenchel duality lifts the argument to many goods [16]. We also verify Edgeworth's conjecture by demonstrating that in large replicated economies, the core converges to competitive equilibrium. This result affirms competitive equilibrium as a sound theoretical construct and a guiding principle for the organization of efficient markets.

While competitive equilibrium and core stability embody the ideal of efficient markets, real-world frictions—such as weak enforcement of property rights, market power, strategic misrepresentation of private information, and externalities—erode this vision. These factors distort prices and allocations, increase transaction costs, and promote rent-seeking, rendering competitive equilibrium a fragile benchmark rather than a guaranteed outcome. As a result, strong institutions, including antitrust policies and competition laws, are essential to counteract these frictions and guide markets closer to theoretical efficiency. In what follows, we discuss these challenges further, with a particular focus on the geometry of informational frictions and the Vickrey–Clarke–Groves mechanism [3, 7, 18].

The Production Economy

We consider a monetary production economy with n buyers and m suppliers, denoting the sets of buyers and suppliers by, respectively, $N = \{1, 2, \dots, n\}$ and $M = \{1, 2, \dots, m\}$. There are ℓ perfectly divisible commodities available for trade. The buyers and suppliers have quasilinear payoff functions. For buyer i, the consumption payoff is defined by the value function $v_i : \mathbb{R}^{\ell}_{+} \to \mathbb{R}_{+}$, giving final payoff $v_i(\mathbf{x}_i) - p_i$ on consuming $\mathbf{x}_i$ and paying p_i. Similarly, for supplier j, the profit, determined by the cost function $c_j : \mathbb{R}^{\ell}_{+} \to \mathbb{R}_{+}$, is $p_j - c_j(\mathbf{y}_j)$ on producing $\mathbf{y}_j$ and earning p_j. Money acts as the numéraire, allowing utilities and profits to be measured in monetary terms.

We assume that each value function $v(\cdot)$ is strictly increasing with $v(\mathbf{0}) = 0$, strictly concave, twice continuously differentiable, and that it satisfies the Inada conditions: for each coordinate k and a given $\mathbf{x}_{-k} > \mathbf{0}$, $\partial v(x_k, \mathbf{x}_{-k})/\partial x_k \to \infty$ as $x_k \downarrow 0$ and $\partial v(x_k, \mathbf{x}_{-k})/\partial x_k \to 0$ as $x_k \uparrow \infty$. The Inada conditions ensure that for every price vector $\boldsymbol{\lambda} \in \mathbb{R}^{\ell}_{++}$, the consumer's problem

$$L(\boldsymbol{\lambda}) = \max_{\mathbf{x} \in \mathbb{R}^{\ell}_{+}} \{ v(\mathbf{x}) - \boldsymbol{\lambda}^{\mathrm{T}} \mathbf{x} \}$$

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has a unique solution in $\mathbb{R}^{\ell}_{++}$. Likewise, each cost function $c(\cdot)$ is strictly increasing with $c(\mathbf{0}) = 0$, strictly convex, twice continuously differentiable, and it satisfies the reverse Inada conditions: for each k and $\mathbf{y}_{-k} > \mathbf{0}$, $\partial c(y_k, \mathbf{y}_{-k})/\partial y_k \to 0$ as $y_k \downarrow 0$ and $\partial c(y_k, \mathbf{y}_{-k})/\partial y_k \to \infty$ as $y_k \uparrow \infty$. Hence for every $\lambda \in \mathbb{R}^{\ell}_{++}$, the producer's problem

$$K(\lambda) = \max_{\mathbf{y} \in \mathbb{R}^{\ell}_{+}} \{\lambda^{\mathrm{T}}\mathbf{y} - c(\mathbf{y})\}$$

also has a unique solution in $\mathbb{R}^{\ell}_{++}$. The functions $L(\lambda)$ and $K(\lambda)$ are the indirect utility function and the profit function, respectively. For a classic and detailed treatment of the Inada conditions, see [8].

This trading environment can be studied from three distinct perspectives: (1) a competitive market in which prices drive individual choices; (2) a coalitional game with transferable utility that focuses on how the surplus is divided; and (3) an efficient mechanism-design viewpoint that confronts private information and strategic misrepresentation.

1. Competitive market: In this setting, a prevailing price vector $\lambda \in \mathbb{R}^{\ell}_{++}$ dictates self-interested decisions. Each buyer i chooses consumption $\mathbf{x}_i$ to maximize his utility $v_i(\mathbf{x}_i) - \lambda^{\mathrm{T}}\mathbf{x}_i$, and each supplier j selects production $\mathbf{y}_j$ to maximize her profit $\lambda^{\mathrm{T}}\mathbf{y}_j - c_j(\mathbf{y}_j)$. The optimal consumption and production in response to prices are represented by the individual demand function $\mathbf{D}_i(\lambda)$ and individual supply function $\mathbf{S}_j(\lambda)$. Market equilibrium is reached at the price λ_{CE} where total demand equals total supply:

$$\sum_{i \in N} \mathbf{D}_i(\lambda_{\mathrm{CE}}) = \sum_{j \in M} \mathbf{S}_j(\lambda_{\mathrm{CE}}).$$

Competitive consumption and production are given by $\mathbf{x}_i^{\mathrm{CE}} = \mathbf{D}_i(\lambda_{\mathrm{CE}})$ and $\mathbf{y}_j^{\mathrm{CE}} = \mathbf{S}_j(\lambda_{\mathrm{CE}})$ for each $i \in N$ and $j \in M$, provided that the market-clearing price exists.[1]

2. Coalitional game with transferable utility: The bargaining over the surplus from trade between buyers and suppliers can be modeled as a coalitional game $(N \cup M, w)$. Here, N represents the set of n buyers, and M the set of m suppliers. The set $\mathcal{C}$ consists of all possible coalitions $C^{\mathrm{CE}}_{\cup} C_{\mathrm{S}}$, where $C_{\mathrm{B}} \subseteq N$, $C_{\mathrm{S}} \subseteq M$, and $C_{\mathrm{B}} \cup C_{\mathrm{S}}$ is nonempty. The characteristic function w assigns to each coalition $C_{\mathrm{B}} \cup C_{\mathrm{S}} \in \mathcal{C}$ a real number $w(C_{\mathrm{B}} \cup C_{\mathrm{S}})$ (the worth of $C_{\mathrm{B}} \cup C_{\mathrm{S}}$), derived from

$$\begin{aligned} &\underset{(\mathbf{x}_i)_{i \in C_{\mathrm{B}}}, (\mathbf{y}_j)_{j \in C_{\mathrm{S}}}}{\text{maximize}} \sum_{i \in C_{\mathrm{B}}} v_i(\mathbf{x}_i) - \sum_{j \in C_{\mathrm{S}}} c_j(\mathbf{y}_j) \\ &\text{subject to} \sum_{i \in C_{\mathrm{B}}} \mathbf{x}_i = \sum_{j \in C_{\mathrm{S}}} \mathbf{y}_j. \end{aligned}$$

Inada's conditions ensure that the worth of each nontrivial coalition, containing at least one buyer and one supplier, is well defined with a unique interior solution, determined by first-order conditions. Meanwhile, trivial coalitions (only buyers, suppliers, or the empty set) have worth zero.

The payoff profile $((u_i)_{i \in N}, (u_j)_{j \in M})$ is in the core of the transferable utility game $(N \cup M, w)$ if it satisfies

$$\begin{aligned} \text{Feasibility: } & \sum_{i \in N} u_i + \sum_{j \in M} u_j = w(N \cup M), \\ \text{Coalitional Rationality: } & \sum_{i \in C_{\mathrm{B}}} u_i + \sum_{j \in C_{\mathrm{S}}} u_j \geq w(C_{\mathrm{B}} \cup C_{\mathrm{S}}), \\ & \forall C_{\mathrm{B}} \cup C_{\mathrm{S}} \in \mathcal{C}. \end{aligned}$$

3. Efficient mechanism design: Let V be the space of all value functions that meet Inada's conditions and let C be the space of all cost functions that meet the reverse Inada's conditions. It is assumed that the value functions of buyers and the cost functions of suppliers are private. To implement efficient allocation amid private information, the market maker incentivizes truth-telling by charging each agent the (reported) externality imposed on the remaining participants. This payment scheme along with the efficient allocation rule is the Vickrey–Clarke–Groves (VCG) mechanism.

In particular, each buyer i pays

$$p_i^{\mathrm{VCG}} = w(N \cup M \setminus i) - (w(N \cup M) - v_i(\mathbf{x}_i^{\mathrm{CE}})),$$

while each supplier j earns

$$p_j^{\mathrm{VCG}} = (w(N \cup M) + c_j(\mathbf{y}_j^{\mathrm{CE}})) - w(N \cup M \setminus j),$$

where $\mathbf{x}_i^{\mathrm{CE}}$ and $\mathbf{y}_j^{\mathrm{CE}}$ represent the competitive consumption and production levels, respectively, for buyer i and supplier j in calculating the grand coalition's worth $w(N \cup M)$. The market maker runs an ex post deficit if $R = \sum_{i \in N} p_i^{\mathrm{VCG}} - \sum_{j \in M} p_j^{\mathrm{VCG}} \leq 0$, across all admissible value and cost functions. Under the VCG mechanism, truth-telling becomes a dominant-strategy equilibrium, and each agent's equilibrium payoff equals its marginal contribution. Specifically, for every buyer or supplier k, the equilibrium payoff is $w(N \cup M) - w(N \cup M \setminus k)$. We will examine the VCG mechanism in a later section, focusing on how adverse selection reshapes the efficiency–revenue tradeoff relative to the complete-information benchmark.

Existence, Uniqueness, and Core Stability of Competitive Equilibrium

To demonstrate the existence and uniqueness of the market-clearing price, we consider the planner's problem of finding a feasible allocation that maximizes the trade surplus. This involves solving the optimization problem

$$\begin{aligned} w(N \cup M) = \max_{(\mathbf{x}_i), (\mathbf{y}_j)} & \left\{ \sum_{i \in N} v_i(\mathbf{x}_i) - \sum_{j \in M} c_j(\mathbf{y}_j) \right\} \\ \text{subject to } & \sum_{i \in N} \mathbf{x}_i = \sum_{j \in M} \mathbf{y}_j. \end{aligned} \tag{1}$$

[1] Under the Inada conditions [8], we later prove in Theorem 1 that a market-clearing price exists and is unique.

The first-order conditions require that the marginal value $\nabla v_i(\mathbf{x}_i)$ for each buyer i, and the marginal cost $\nabla c_j(\mathbf{y}_j)$ for each supplier j, must equal the Lagrange multiplier λ of the feasibility constraint. By expressing $\mathbf{x}_i$ and $\mathbf{y}_j$ as functions of λ and then determining λ through the feasibility constraint, this approach mirrors the process of establishing the market-clearing price. Consequently, competitive allocations at this price are efficient. This result is concisely presented in the following theorem.

Theorem 1 (First welfare theorem). *The competitive allocation is the solution to the planner's problem* (1). *Furthermore, the Lagrange multiplier associated with the feasibility constraint corresponds to the competitive price vector.*

Proof. Let $L_i(\lambda)$ be the indirect utility function for buyer i and let $K_j(\lambda)$ be the profit function for supplier j, both well defined over the domain $\mathbb{R}^{\ell}_{++}$ due to Inada's assumptions. We begin by showing that $w(N \cup M)$, calculated from the surplus maximization problem (1), is finite because of weak duality. Specifically, for every price $\lambda \in \mathbb{R}^{\ell}_{++}$ and for all feasible consumption and production profiles $(\mathbf{x}_i)_{i\in N}$ and $(\mathbf{y}_j)_{j\in M}$ that satisfy the condition $\sum_{i\in N} \mathbf{x}_i = \sum_{j\in M} \mathbf{y}_j$, the following inequality holds:

$$\begin{aligned}\sum_{i\in N} v_i(\mathbf{x}_i) - \sum_{j\in M} c_j(\mathbf{y}_j) &= \sum_{i\in N}(v_i(\mathbf{x}_i) - \lambda^{\mathrm{T}}\mathbf{x}_i) + \sum_{j\in M}(\lambda^{\mathrm{T}}\mathbf{y}_j - c_j(\mathbf{y}_j)) \\ &\leq \sum_{i\in N} L_i(\lambda) + \sum_{j\in M} K_j(\lambda).\end{aligned}$$

Thus, $w(N \cup M)$ is finite, and due to Inada's conditions, a unique interior solution $((\mathbf{x}_i^*), (\mathbf{y}_j^*), \lambda^*)$ that achieves $w(N \cup M)$ exists. This solution is characterized by first-order conditions $\nabla v_i(\mathbf{x}_i^*) = \lambda^*$ for $i \in N$ and $\nabla c_j(\mathbf{y}_j^*) = \lambda^*$ for $j \in M$, ensuring feasibility with $\sum_{i\in N} \mathbf{x}_i^* = \sum_{j\in M} \mathbf{y}_j^*$. These conditions confirm that λ^* is the market-clearing price vector, implementing the surplus-maximizing outcome. □

Figure 1 demonstrates how the market achieves equilibrium at the point A for the case of a single commodity, through the interaction of demand and supply, establishing a unique equilibrium price λ_{CE} where total demand matches total supply. The market-clearing price optimally coordinates economic activities, maximizing the total surplus, which is geometrically represented by the triangular region (OAB).[2] Any production or consumption beyond this equilibrium level would lead to a reduction in social surplus.

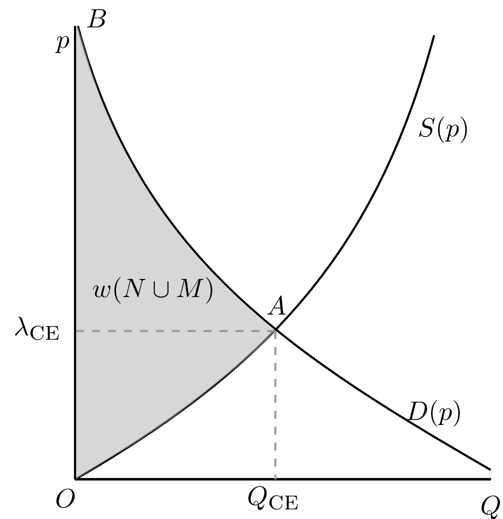

Figure 1. Market equilibrium—the intersection of demand and supply.

Having established the existence and uniqueness of a competitive equilibrium, we now focus on its stability by viewing the market as a cooperative game $(N \cup M, w)$. Consider the competitive price λ_{CE} and the competitive payoff profile $((u_i^{\mathrm{CE}})_{i\in N}, (u_j^{\mathrm{CE}})_{j\in M})$, where $u_i^{\mathrm{CE}} = v_i(\mathbf{x}_i^{\mathrm{CE}}) - \lambda_{\mathrm{CE}}^{\mathrm{T}}\mathbf{x}_i^{\mathrm{CE}}$ for each buyer $i \in N$, and $u_j^{\mathrm{CE}} = \lambda_{\mathrm{CE}}^{\mathrm{T}}\mathbf{y}_j^{\mathrm{CE}} - c_j(\mathbf{y}_j^{\mathrm{CE}})$ for each supplier $j \in M$. The competitive payoff profile is feasible, since it matches the grand coalition's worth by the first welfare theorem. To show that it meets coalitional rationality, we use the following lemma from convex optimization theory.

Lemma 2 (Legendre–Fenchel duality). *The problem of maximizing surplus through feasible allocations is dual to minimizing the total of indirect utilities and profits*:

$$w(N \cup M) = \min_{\lambda \in \mathbb{R}^{\ell}_{++}} \left\{ \sum_{i\in N} L_i(\lambda) + \sum_{j\in M} K_j(\lambda) \right\},$$

where the competitive price λ_{CE} is the unique solution.

Proof. Using the envelope theorem (see, e.g., [11]) for the indirect utility function $L_i(\lambda) = \max_{\mathbf{x}_i}\{v_i(\mathbf{x}_i) - \lambda^{\mathrm{T}}\mathbf{x}_i\}$ and profit function $K_j(\lambda) = \max_{\mathbf{y}_j}\{\lambda^{\mathrm{T}}\mathbf{y}_j - c_j(\mathbf{y}_j)\}$, the gradients are given by $\nabla L_i(\lambda) = -\mathbf{x}_i(\lambda)$ and $\nabla K_j(\lambda) = \mathbf{y}_j(\lambda)$ for each buyer i and supplier j. In addition, $L_i(\lambda)$ and $K_j(\lambda)$ are strictly convex, and thus the objective function of the dual problem is strictly convex. Because the first-order

[2]The notation $(P_1, P_2, \dots, P_n)$ represents the area enclosed by the points $P_1, P_2, \dots, P_n$, traced along the corresponding curves.

condition for the dual problem zeros out at the competitive price λ_{CE},

$$\sum_{i\in N}\nabla L_i(\lambda_{\mathrm{CE}})+\sum_{j\in M}\nabla K_j(\lambda_{\mathrm{CE}})=-\sum_{i\in N}\mathbf{x}_i(\lambda_{\mathrm{CE}})+\sum_{j\in M}\mathbf{y}_j(\lambda_{\mathrm{CE}})=\mathbf{0},$$

the competitive price vector, λ_{CE}, uniquely solves the dual problem. Moreover, at the competitive prices, the dual achieves the minimum of $w(N\cup M)$, because

$$\sum_{i\in N}L_i(\lambda_{\mathrm{CE}})+\sum_{j\in M}K_j(\lambda_{\mathrm{CE}})=\sum_{i\in N}u_i^{\mathrm{CE}}+\sum_{j\in M}u_j^{\mathrm{CE}}=w(N\cup M),$$

completing the proof. □

Figure 2 illustrates the consumer surplus (CS), producer surplus (PS), and the consequences of deviating from the market-clearing price, λ_{CE}. At a given price λ, the market may experience either excess demand or excess supply. The region (CEB) represents the consumer surplus, whereas the producer surplus is shown by the region (ODC). Together, they surpass (OAB), which is the total indirect surplus at the equilibrium price. Lemma 2 enables a simple argument showing that competitive equilibrium is in the core of the production economy, presented in the following theorem.

Theorem 3 (The core and competitive equilibrium). *The competitive payoff profile satisfies feasibility and coalitional rationality, thereby belonging to the core of* $(N\cup M, w)$.

Proof. The feasibility of the competitive payoff profile is confirmed as $\sum_{i\in N}u_i^{\mathrm{CE}}+\sum_{j\in M}u_j^{\mathrm{CE}}=\sum_{i\in N}L_i(\lambda_{\mathrm{CE}})+\sum_{j\in M}K_j(\lambda_{\mathrm{CE}})=w(N\cup M)$, according to Lemma 2.

Verifying coalitional rationality is straightforward for trivial coalitions comprising only buyers or suppliers, since $w(C_{\mathrm{B}})=w(C_{\mathrm{S}})=0$ for any subsets $C_{\mathrm{B}}\subseteq N$ and $C_{\mathrm{S}}\subseteq M$. For nontrivial coalitions $C_{\mathrm{B}}\cup C_{\mathrm{S}}$, coalitional rationality is maintained by applying Lemma 2 again:

$$\begin{aligned}\sum_{i\in C_{\mathrm{B}}}u_i^{\mathrm{CE}}+\sum_{j\in C_{\mathrm{S}}}u_j^{\mathrm{CE}}&=\sum_{i\in C_{\mathrm{B}}}L_i(\lambda_{\mathrm{CE}})+\sum_{j\in C_{\mathrm{S}}}K_j(\lambda_{\mathrm{CE}})\\&\geq\sum_{i\in C_{\mathrm{B}}}L_i(\lambda_{C_{\mathrm{B}}\cup C_{\mathrm{S}}})+\sum_{j\in C_{\mathrm{S}}}K_j(\lambda_{C_{\mathrm{B}}\cup C_{\mathrm{S}}})\\&=w(C_{\mathrm{B}}\cup C_{\mathrm{S}}),\end{aligned}$$

where $\lambda_{C_{\mathrm{B}}\cup C_{\mathrm{S}}}$ is the competitive price vector for the market confined to buyers and suppliers in $C_{\mathrm{B}}\cup C_{\mathrm{S}}$. Thus, the competitive payoff profile is in the core. □

Figure 3 illustrates the coalitional rationality of the competitive payoff profile for a specific coalition $C_{\mathrm{B}}\cup C_{\mathrm{S}}$. In both diagrams, the supply curves $S_{C_{\mathrm{S}}}(p)$ and the demand curves $D_{C_{\mathrm{B}}}(p)$ are confined to buyers in C_{B} and suppliers in C_{S}, respectively. The shaded region (OCB) in the left panel quantifies the worth $w(C_{\mathrm{B}}\cup C_{\mathrm{S}})$, which is less than the shaded region $(ODEB)$ in the right panel, representing the aggregate competitive payoffs of the coalition, $\sum_{i\in C_{\mathrm{B}}}u_i^{\mathrm{CE}}+\sum_{j\in C_{\mathrm{S}}}u_j^{\mathrm{CE}}$. This diagram showcases how the interplay between the demand and supply curves reveal deep insights into the relationship between the core and competitive equilibrium.

Edgeworth's Conjecture

In this section, we examine a captivating insight from [6]. Picture a bustling marketplace that continually grows as more buyers and sellers join. As the market expands, the range of possible core-stable outcomes narrows until only

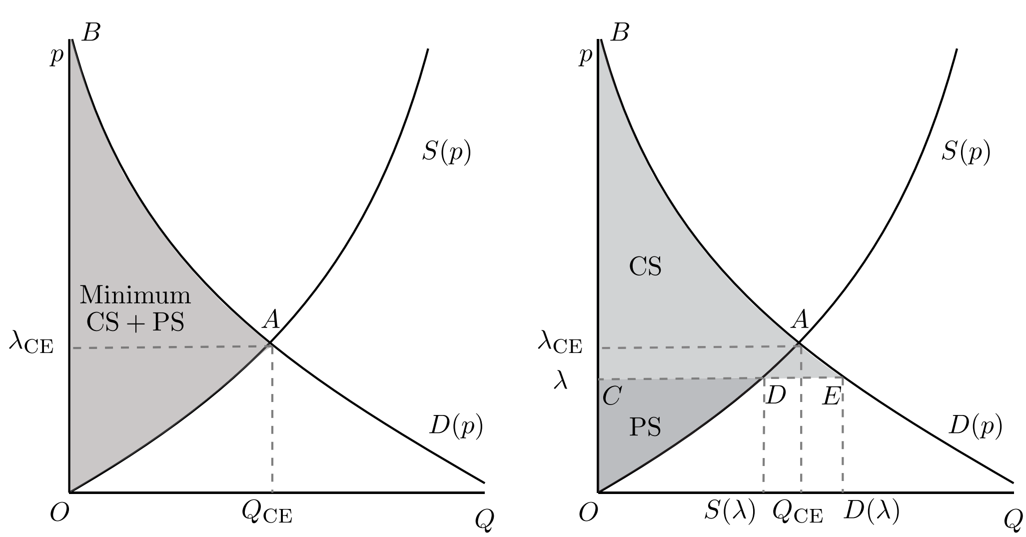

Figure 2. Illustration of Legendre–Fenchel duality: the market-clearing price minimizes the dual objective (total indirect utilities plus profits), giving the maximum social surplus.

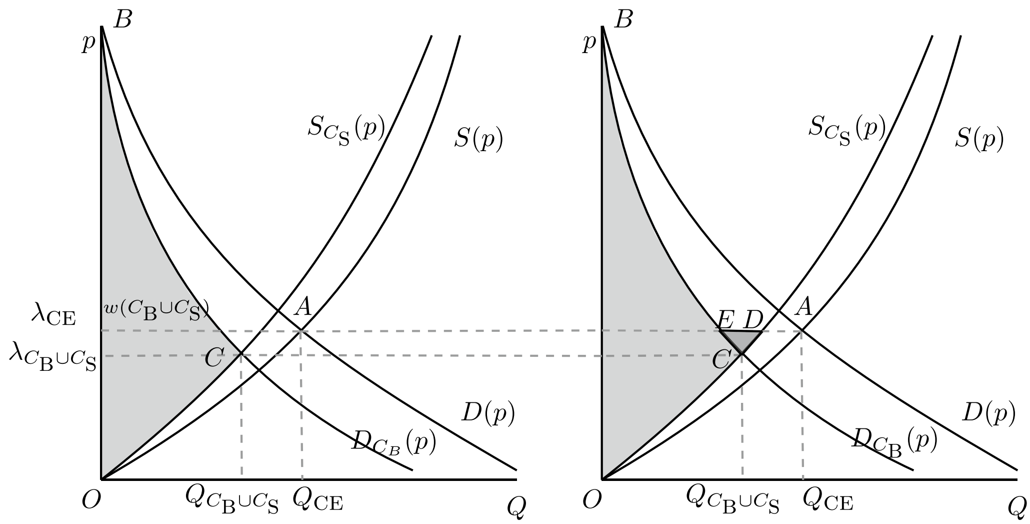

Figure 3. Graphical proof of Theorem 3, showing coalitional rationality and the inclusion of the competitive equilibrium in the core.

one remains—the competitive equilibrium. In this scenario, no subgroup can improve its situation by breaking away on its own. Instead, the entire market naturally settles into one stable surplus-maximizing outcome. This phenomenon beautifully illustrates how even in a complex and ever-growing economy, the forces of competition drive the market toward a single, optimal state.

To formalize the concept of a bustling economy, let us construct an r-fold replica of the original production economy consisting of rn buyers and rm suppliers, with each type of buyer and supplier replicated exactly r times. Let $N^{(r)}$ be the set of those rn buyers and let $M^{(r)}$ be the set of those rm suppliers, inducing a new transferable utility game $(N^{(r)} \cup M^{(r)}, w)$. First, we prove the principle of equal treatment within the core.

Lemma 4 (Equal treatment within the core). *If the payoff profile $((u_i)_{i \in N^{(r)}}, (u_j)_{j \in M^{(r)}})$ belongs to the core of the r-fold replica economy, then agents of each type receive an identical payoff.*

Proof. The proof rests on the key observation that $w(N^{(r)} \cup M^{(r)}) = rw(N \cup M)$, since competitive prices remain unchanged under replication. Assume that the payoff profile $((u_i)_{i \in N^{(r)}}, (u_j)_{j \in M^{(r)}})$ lies in the core, yet some identical agents receive strictly different payoffs. Form a coalition C consisting of n buyers and m suppliers, chosen as those within each type who are treated the least favorably. Because of this unequal treatment, the aggregate payoff of C falls strictly below $w(N \cup M)$. But C replicates the structure of the original market, so $w(C) = w(N \cup M)$. This contradiction shows that coalitional rationality is violated. Identical agents must receive equal payoffs. □

Lemma 4 allows for a convenient description of the core using payoff profiles from the original market, rather than specifying payoffs for the entire r-fold market. The main question now turns to which payoff profiles from the original market survive repeated market replication. Theorem 3 shows that the competitive payoff vector is part of the core of $(N^{(r)} \cup M^{(r)}, w)$ for every r, since the market-clearing prices stay the same, $\lambda_{CE}^{(r)} = \lambda_{CE}$. However, proving that it is the only profile to survive is more challenging. The following result asserts that every feasible payoff profile differing from the competitive profile will eventually be excluded from the core as the market thickens.

Theorem 5 (Core convergence). *The core of an r-fold replica economy converges to a singleton, consisting of the competitive payoff profile, as $r \to \infty$.*

Proof. We first show that for every buyer $i_0 \in N$ and for every supplier $j_0 \in M$, the differences $\Delta_r^{i_0}$ and $\Delta_r^{j_0}$, defined as $\Delta_r^{i_0} = w(N^{(r)} \cup M^{(r)}) - u_{i_0}^{CE} - w(N^{(r)} \cup M^{(r)} \setminus i_0)$ and $\Delta_r^{j_0} = w(N^{(r)} \cup M^{(r)}) - u_{j_0}^{CE} - w(N^{(r)} \cup M^{(r)} \setminus j_0)$, converge to zero as $r \to \infty$. It suffices to prove the convergence for buyers, since the suppliers' case follows analogously.

Let λ_r be the competitive prices in an r-fold replica market excluding i_0. Using a second-order Taylor expansion around λ_r and λ_{CE}, we obtain

$$\begin{aligned}\Delta_r^{i_0} &= r\left(\frac{r-1}{r}L_{i_0}(\lambda) + \sum_{i \in N \setminus i_0} L_i(\lambda) + \sum_{j \in M} K_j(\lambda)\right)\Bigg|_{\lambda=\lambda_r}^{\lambda=\lambda_{CE}} \\ &= \frac{r}{2}(\lambda_{CE} - \lambda_r)^{\mathrm{T}} H_r(\lambda_r)(\lambda_{CE} - \lambda_r) + rO(\| \lambda_{CE} - \lambda_r \|^3)\end{aligned} \tag{2}$$

$$
\begin{aligned}
&= \mathbf{x}_{i_0}(\lambda_{\mathrm{CE}})^{\mathrm{T}}(\lambda_{\mathrm{CE}} - \lambda_r) \\
&\quad - \frac{r}{2}(\lambda_{\mathrm{CE}} - \lambda_r)^{\mathrm{T}} H_r(\lambda_{\mathrm{CE}})(\lambda_{\mathrm{CE}} - \lambda_r) \qquad (3) \\
&\quad + rO(\| \lambda_{\mathrm{CE}} - \lambda_r \|^3),
\end{aligned}
$$

where $H_r(\lambda)$ is the Hessian of $\frac{r-1}{r} L_{i_0}(\lambda) + \sum_{i \in N \setminus i_0} L_i(\lambda) + \sum_{j \in M} K_j(\lambda)$.

Since $\lambda_r \to \lambda_{\mathrm{CE}}$, the Hessians $H_r(\lambda_r)$ and $H_r(\lambda_{\mathrm{CE}})$ converge to a fixed matrix $H_\infty(\lambda_{\mathrm{CE}})$. Given these convergences, from equations (2) and (3), the norm $\| \lambda_r - \lambda_{\mathrm{CE}} \|$ is $O(1/r)$, ensuring that $\Delta_r^{i_0}$ converges to zero as r approaches infinity. This convergence must occur from above, since the competitive payoff profile remains within the core (Theorem 3). If an alternative payoff profile $((u_i^*)_{i \in N}, (u_j^*)_{j \in M})$ survives the sequence of cores generated by the indefinite replication of the economy, there must exist a buyer $i_0 \in N$ or a supplier $j_0 \in M$ who obtains a strictly higher payoff than its competitive payoffs. The coalitional rationality of the grand coalition minus this particular individual, say i_0, is violated for sufficiently large r. In fact, $\lim_{r \to \infty}[w(N^{(r)} \cup M^{(r)}) - u_{i_0}^* - w(N^{(r)} \cup M^{(r)}) - u_{i_0}^* - w(N^{(r)} \cup M^{(r)} \setminus i_0)] = u_i^{\mathrm{CE}} - u_i^* < 0$. This completes the proof. □

Figure 4 demonstrates the convergence of Δ_r^i, a slackness measure for coalitional rationality within the r-fold coalition excluding buyer i while retaining the $r-1$ "twins" of buyer i within the coalition. Originally, the market operates with demand $D(p)$ and supply $S(p)$. On market duplication, both demand and supply shift to the right, becoming $D^{(2)}(p)$ and $S^{(2)}(p)$, effectively doubling their initial quantities. Despite this expansion, the efficient consumption level for buyer i remains unchanged, since the market-clearing price λ_{CE} stays constant (thus, $AE = A_2E_2$ remains). However, the impact of buyer i's absence on the competitive price is reduced by half, $\lambda_{\mathrm{CE}} - \lambda_{-i}^{(2)} \approx \frac{1}{2}(\lambda_{\mathrm{CE}} - \lambda_{-i})$. This is due to the doubling of the slopes of the demand and supply curves, which lessens buyer i's influence on price changes. The triangle (AEC) represents Δ_1^i, and the triangle $(A_2E_2C_2)$ represents Δ_2^i. Since the bases AE and A_2E_2 remain constant at x_i^{CE} and the price effect is halved, the area of $(A_2E_2C_2)$ is approximately half of (AEC), visually confirming the convergence.

Why International Trade Delivers Big Gains

A central implication of core stability and core convergence is that as markets expand or are replicated, the competitive equilibrium remains stable. This stability persists even as the number of potential breakaway coalitions grows. In practice, this helps explain why large trade blocs, such as the European Union and cross-continental free-trade areas, can still support stable outcomes with little incentive for members to withdraw.

A small country contemplating tariffs or exit from a larger alliance typically finds that outside the collective market, it faces higher costs, thinner demand, and less reliable supply, and hence a lower surplus. This is the coalition argument scaled to the international arena.

Real-World Policy Examples

Regional trade agreements. Consider the North American Free Trade Agreement (NAFTA, now USMCA). Each member (Canada, the United States, and Mexico) brings distinct demand (consumption patterns) and supply (productive capacities). Committing to open trade effectively pools these at a common "regional price." If a member operated alone, it would forgo the scale and scope of the

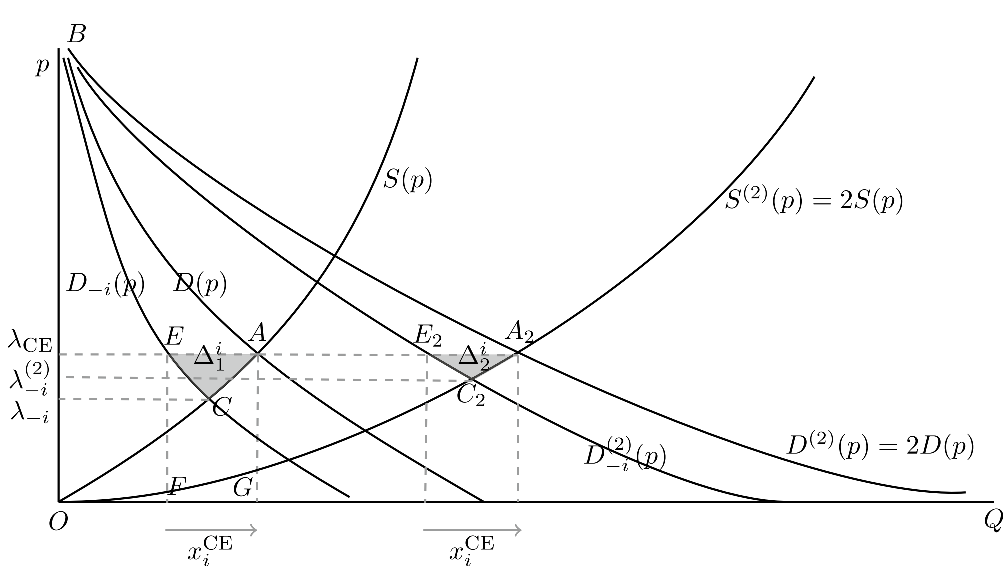

Figure 4. Graphical proof of Theorem 5, showing that the slackness in coalitional rationality converges to 0 as market replication increases.

pooled market, reducing its surplus relative to membership in the grand coalition.

The European Union as a large coalition. A similar logic applies to the EU single market. Many small states remain in the broader union because the "grand coalition" enlarges the trading zone, deepens specialization, and lowers average costs. Even if a small region believes it has a strong comparative advantage in certain goods, it is not necessarily better off forming a smaller union. The single EU market's scale confers gains in purchasing power and cost reductions for everyone. This logic has motivated countries such as Ukraine and Turkey to seek accession in order to capture these gains.

Historical precedent: the Hanseatic League. In medieval northern Europe, the Hanseatic League coordinated merchants and city-states to secure safer routes and broader market access. A lone city trading in isolation would have lost common infrastructure, access to further markets, and the protective scope provided by the larger group. Here, too, broad "price-making" and scale effects help explain why the coalition remained stable for centuries.

The message is simple and powerful. when open competition prevails, no splinter can reproduce the total surplus it can achieve within the larger market. Core theory thus clarifies why open trade is good: the largest feasible coalition tends to yield the greatest net gains by widening trading opportunities and muting market power.

Market Frictions

Protectionist measures increasingly distort outcomes away from the competitive benchmark. Under President Trump's "America First" agenda, the United States threatened withdrawal from NAFTA, imposed broad tariffs, and ultimately renegotiated the agreement as USMCA—moves intended to redress perceived imbalances but that risk fragmenting the grand coalition and inviting retaliation. At the same time, China's market interventions (e.g., industrial subsidies, state-directed finance, and episodes of exchange-rate management) tilt competitive conditions and weaken the price-taking assumption. Both dynamics shift bargaining power, alter effective prices, and can move outcomes away from the competitive core.

While competitive equilibrium and core stability offer a compelling benchmark, real markets operate within legal, informational, and strategic environments that theory abstracts from. Without credible enforcement of property rights, contracts are unreliable and transaction costs rise; institutions that secure property and contracts (courts, registries, bankruptcy procedures) are essential [14, 20]. Market power further erodes price-taking: when a few buyers or suppliers command large shares, they can influence prices or quantities, reallocating surplus in their favor and pushing outcomes outside the core; competition policy addresses these risks [12, 17, 19].

Private information creates a tension between efficiency and truthfulness. Mechanisms must elicit accurate reports of values and costs; Vickrey–Clarke–Groves implements the surplus-maximizing allocation in dominant strategies but generally entails budget imbalances, precluding costless implementation at scale [3, 7, 18]. In bilateral trade, the impossibility of efficient trade shows that under private information and voluntary participation, no mechanism can be simultaneously efficient, balanced, and incentive compatible [13]. Even where markets are otherwise competitive, externalities break the link between private choices and social surplus; Pigouvian instruments and liability rules aim to realign incentives [4, 15].

These frictions are not anomalies but the normal conditions of markets. Trade liberalization enlarges the feasible surplus set, while property-rights enforcement, competition policy, and externality pricing protect that surplus from erosion by collusion, dominance, misreporting, and unpriced spillovers. Among these frictions, the one most amenable to the demand–supply geometry developed above is informational friction, in particular, adverse selection [1]. Eliciting truth while preserving voluntary participation places systematic pressure on budget balance. The next section makes this precise.

Private Information and Revenue-Efficiency Tradeoff

One striking instance of market friction arises in designing mechanisms that deliver efficient trade despite hidden private preferences. The celebrated impossibility theorem in mechanism design highlights that aligning individual incentives with social welfare is inherently costly. In the Vickrey–Clarke–Groves mechanism [3, 7, 18], the market maker must induce honest revelation of private information while ensuring voluntary participation, often at the expense of running a deficit. This tradeoff between efficiency and revenue reflects the inherent cost of overcoming informational frictions. The competitive outcome, residing in the core, necessitates that each buyer receive a discount relative to his competitive payment, and each supplier, a corresponding subsidy.

Theorem 6 (Impossibility of efficient trade). *In the VCG mechanism, each buyer $i \in N$ pays no more than $\boldsymbol{\lambda}_{\mathrm{CE}}^{\mathrm{T}} \mathbf{x}_i(\boldsymbol{\lambda}_{\mathrm{CE}})$, and each supplier $j \in M$ earns at least $\boldsymbol{\lambda}_{\mathrm{CE}}^{\mathrm{T}} \mathbf{y}_j(\boldsymbol{\lambda}_{\mathrm{CE}})$.*

Proof. The coalitional rationality of the competitive payoff profile at coalitions $N \cup M \setminus i$ and $N \cup M \setminus j$ establishes the following inequalities:

$$w(N \cup M) - (v_i(\mathbf{x}_i^{\mathrm{CE}}) - \boldsymbol{\lambda}_{\mathrm{CE}}^{\mathrm{T}} \mathbf{x}_i^{\mathrm{CE}}) \geq w(N \cup M \setminus i),$$
$$w(N \cup M) - (\boldsymbol{\lambda}_{\mathrm{CE}}^{\mathrm{T}} \mathbf{y}_j^{\mathrm{CE}} - c_j(\mathbf{y}_j^{\mathrm{CE}})) \geq w(N \cup M \setminus j),$$

where $\mathbf{x}_i^{\mathrm{CE}} = \mathbf{x}_i(\boldsymbol{\lambda}_{\mathrm{CE}})$ and $\mathbf{y}_j^{\mathrm{CE}} = \mathbf{y}_j(\boldsymbol{\lambda}_{\mathrm{CE}})$. These inequalities can be rearranged to derive the bounds for VCG payments:

$$p_i^{\mathrm{VCG}} = w(N \cup M \setminus i) - (w(N \cup M) - v_i(\mathbf{x}_i^{\mathrm{CE}})) \leq \boldsymbol{\lambda}_{\mathrm{CE}}^{\mathrm{T}} \mathbf{x}_i^{\mathrm{CE}},$$
$$p_j^{\mathrm{VCG}} = (w(N \cup M) + c_j(\mathbf{y}_j^{\mathrm{CE}})) - w(N \cup M \setminus j) \geq \boldsymbol{\lambda}_{\mathrm{CE}}^{\mathrm{T}} \mathbf{y}_j^{\mathrm{CE}},$$

as desired. This implies an unavoidable ex post deficit $\sum_{i\in N} p_i^{\mathrm{VCG}} - \sum_{j\in M} p_j^{\mathrm{VCG}} \leq \lambda_{\mathrm{CE}}^{\mathrm{T}}(\sum_{i\in N} \mathbf{x}_i^{\mathrm{CE}} - \sum_{j\in M} \mathbf{y}_j^{\mathrm{CE}}) = 0$, completing the proof. □

Theorem 6 strengthens the conclusion of [18]. It shows that the discounts received by buyers and the subsidies granted to suppliers precisely capture the incremental gains from competitive trade when an additional participant joins the market. This observation has significant implications for Walrasian auctions, which are commonly used in financial markets. In these settings, the efficiency–revenue tradeoff is critical.

When buyers and suppliers must report their private values and costs to the market maker for competitive equilibrium implementation, they have incentives to misreport. Buyers tend to understate their true valuations to secure a lower payment, while suppliers are inclined to overstate their costs to command a higher transfer. This strategic behavior distorts the competitive outcome and leads to deviations from the theoretically efficient allocation. Consequently, the discounts and subsidies observed in the VCG mechanism serve as a precise measure of the extent to which informational frictions distort the competitive equilibrium. This situation highlights the inherent tension between achieving allocative efficiency and maintaining revenue neutrality in practice.

Competition Law and Antitrust Policy

Real markets are pervaded by frictions, especially informational frictions that generate adverse selection, so prices and quantities often drift from the competitive benchmark. Competition (antitrust) policies work as market maintenance: legal rules that raise the cost and reduce the payoff of profitable blocking coalitions, helping to keep allocations near the competitive outcome.

Cartels and horizontal collusion, including price-fixing, bid-rigging, and market allocation, replicate a coalition of suppliers or buyers that elevates price above the competitive level, and per se prohibitions together with leniency programs deter such deviations.[3] Abuse of dominance or monopolization captures unilateral strategies such as predation, margin squeezes, tying and bundling, exclusivity, and strategic refusals to deal that foreclose rivals and entrench market power; U.S. monopolization rules and EU abuse-of-dominance rules target these strategies.[4] Merger control then operates to screen, block, or remedy horizontal transactions, and, where effects warrant, vertical or conglomerate transactions that would substantially lessen competition or create a monopoly. This prevents structural changes that enable profitable blocking coalitions ex post.[5]

Vertical restraints and most-favored-nation clauses, including resale-price maintenance, parity clauses, and exclusivity, can facilitate coordination or raise rivals' costs; effects-based analysis scrutinizes such restraints where they dampen pass-through or amplify market power, and the European Union's updated Vertical Block Exemption Regulation provides a structured framework.[6] Enforcement increasingly addresses buyer power and labor markets as well, where no-poach and wage-fixing agreements and monopsonistic purchasing depress input prices and quantities, shifting effective demand left and reducing total surplus.[7] Public policy likewise disciplines state aid, subsidies, and trade remedies: preferential finance, tax relief, and direct subsidies that tilt competition are constrained by ex ante EU state aid control and multilateral subsidy rules so that surplus reflects market rather than fiscal coalitions.[8] In digital markets, network effects, self-preferencing, and data-driven barriers can facilitate tacit coordination or foreclosure at scale; ex post antitrust and, in the EU, the ex ante *Digital Markets Act* impose conduct obligations on designated gatekeepers to preserve contestability.[9]

Viewed through the lens of the core, these instruments make profitable deviations costlier and less effective; in the familiar demand and supply diagram, they help preserve market-clearing price and quantity and the associated surplus area. In practice, the regime is anchored in prohibitions on cartels and monopolization or abuse, merger control, and the review of vertical restraints, under statutory frameworks including the U.S. Sherman and Clayton Acts and, in the EU, TFEU arts. 101 and 102 and the EU Merger Regulation.

Conclusion

In a production economy with a numéraire, demand–supply geometry makes market stability transparent: the aggregate intersection maximizes total surplus. The intersection of any coalition's own demand and supply shows what it can achieve in autarky, and that outcome is dominated by the grand-market allocation. Thus any proper subgroup trading alone generates a smaller surplus and cannot gain by breaking away. This conclusion does not depend on the number of participants.

[3] Sherman Act §1, 15 U.S.C. §1; Treaty on the Functioning of the European Union (TFEU) art. 101(1). See also U.S. Department of Justice, *Corporate Leniency Policy* (1993, as revised); European Commission, *Notice on Immunity from Fines and Reduction of Fines in Cartel Cases*, OJ C 298, 8.12.2006, p. 17.

[4] Sherman Act §2, 15 U.S.C. §2; TFEU art. 102.

[5] Clayton Act §7, 15 U.S.C. §18; U.S. Department of Justice & Federal Trade Commission, *Merger Guidelines* (Dec. 2023); Council Regulation (EC) No 139/2004 on the control of concentrations between undertakings (EU Merger Regulation).

[6] *Leegin Creative Leather Products, Inc. v. PSKS, Inc.*, 551 U.S. 877 (2007) (RPM under rule of reason in the U.S.); Commission Regulation (EU) 2022/720 of 10 May 2022 (Vertical Block Exemption Regulation) and accompanying Guidelines.

[7] U.S. DOJ & FTC, *Antitrust Guidance for Human Resource Professionals* (Oct. 2016). See also TFEU art. 101 for buyer-side collusion in the EU framework.

[8] TFEU arts. 107 to 109 (EU state aid control); *Agreement on Subsidies and Countervailing Measures* (WTO, 1994).

[9] Regulation (EU) 2022/1925 of 14 September 2022 on contestable and fair markets in the digital sector (Digital Markets Act).

Applied to international trade and market integration, as markets widen from national to regional to global, the set of trading partners, the variety of goods, and economies of scale expand, raising efficiency. Viewed through the core, no collection of countries benefits from seceding so long as the broader market remains competitive. In large economies, Edgeworth's core-convergence insight implies that competitive equilibrium is the only stable limit. That said, frictions can derail this outcome: market power, search and transaction costs, and information asymmetries. Competition and antitrust policies aim to steer real markets back toward, or at least closer to, the competitive benchmark.

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