The Third Check: Surplus Extraction, Malthus, and the Origins of Agrarian Civilization

by

Robert C. Allen

Faculty of Social Science
New York University Abu Dhabi
Abu Dhabi, UAE

Senior Research Fellow
Nuffield College
Oxford, U.K.

email: bob.allen@nyu.edu

2025

Abstract

The paper analyzes the economic basis of agrarian civilizations. These had an agricultural part that produced food and a non-agricultural part that was supported by food from the countryside. The agricultural surplus was necessary for these cities but not sufficient. The technology (domestic seed, the plough) that generated the surplus was created by foragers and hoe cultivators who held the land in common. Their technology is modeled with engineering production functions derived with linear programming. Had they exploited the advanced technology, the result would have been a large population of cultivators living at subsistence and consuming the entire 'surplus.' This Malthusian nightmare was avoided by landlords who privatized land and organized it to maximize their income. Taxation could have a similar effect but less precisely. The rent proceeds supported the city. Theoretical analysis and simulation show that the effect of private property was to reduce the total population (the third Malthusian check) and to reduce the agricultural population even more. The difference was the urban population. In equilibrium the urban and rural labourers were at subsistence, the landowners were rich, Gini coefficients were high, and GDP per capita was greater than subsistence.

That duty—the duty for the performance of which I believe that Providence created landlords—is the keeping down population. If there were no one whose interest it was to limit the number of the occupants of land, it would be tenanted by all whom it could maintain, just as a warren is tenanted by all the rabbits that it can feed. Competition would force them to use the food that was most abundant—every failure of a crop would produce a famine ; they would have no surplus produce, and therefore no division of labour ; no manufactures, except the coarse clothing and furniture which each family must produce for itself; no separation of ranks, no literature -in short, no civilisation. This is a mere picture of a neglected estate held in rundale in Ireland. This is the state into which every common in England would fall, if the Lord of the Manor were to allow it to be occupied by squatters. To prevent all this, Providence created landlords--a class of persons whose interest it is that the land should produce as large as possible an amount of surplus produce, and for that purpose should be occupied by only the number of persons necessary to enable it to produce the largest possible amount beyond their own subsistence.

–Nassau Senior (1868), Drummond Professor of Political Economy, University of Oxford

V. Gordon Childe (1936)` identified two revolutions that transformed human life. The first was the neolithic revolution, which refers to the transition from mobile hunting and gathering to settled agriculture. In the middle east, which is our focus, agriculture consisted of the domestication of wheat, barley, legumes, and flax as well as animals including sheep, goats, cattle, and pigs. By 7500 BCE these domesticates were present in the Fertile Crescent. A millennium later, they were coming into general use, and they were joined by the plough, which was probably invented about that time and boosted the productivity of farming. This technology led to great increases in the human population, and its spread across Anatolia, Mesopotamia and beyond.

It took another three millennia for the second revolution to occur, and that was the urban revolution. By c. 4000 BCE small, 'statelets' were developing in northern Mesopotamia, and by 3000 BCE the first indubitable city-sate, Uruk, had appeared in southern Mesopotamia. Some people continued to live as mobile foragers or hoe cultivators, but for the next 4500 years, the majority probably lived in 'agrarian civilizations:' Babylon, Egypt, the Roman Empire, the Aztecs, the Incas, the Chinese and Mughal Empires, and many more. They had large agricultural sectors (the 'agrarian' part) and substantial nonagricultural components, often with large cities, (the 'civilization' part). They were also organized as states. While most recent research has focused on state formation, we concentrate on the organization of the economy that underlay the

In explaining the origins of agrarian civilizations, surplus extraction turns out to be critically important. Foragers generally have communal property, while most land in agrarian civilizations is private and subject to taxation. We argue that without private property or taxation, agrarian civilizations would never have developed. With communal property, improvements in agricultural productivity like domestic seed or the plough would simply have resulted in larger populations of impoverished farmers–the nightmare that Malthus feared. Surplus extraction prevented that. In the case of private property, the reason is that the landowners had an incentive to restrict the population on their estates to the level that maximized the rent they generated. This surplus was spent on craftsmen, soldiers, wenches, and priests–thus creating the urban economy. The population would adjust so that everyone had a subsistence diet, whatever sector they were in. With diminishing returns to labour in agriculture, private property also implies that the total population with 'civilization' would have been less than it would have been with common property (so private property is a third check to population growth). Private property reduces the total population, reduces the agricultural population even more, and the difference is the non-agricultural population that is created. That reorganization underpins all of the great civilizations of the past.

Taxation works similarly, although less exactly. Reducing the income of the rural population also reduces its size and the production of food so long as the marginal product of labour is positive. The tax proceeds are spent on employment in the non-agricultural sector similarly to private property. Taxation, therefore, also acts as a check to population growth. The big difference, we will argue, is that private property sets the upper limit to extraction. Taxation can never do better.

Between the domestication of crops and animals and the growth of Uruk, the middle east was transformed in many ways. The population was much bigger in 3000 than it had been in 7500, and settlement, which had largely been confined to the Fertile Crescent in 7500, had spread across Mesopotamia and Anatolia. The structural changes were even more dramatic. Before 7500 most everyone in the Fertile Crescent was engaged in securing their own food. By 3000 there was a division of labour with farmers specializing in food production and a variety of craftsmen specializing in the production of pottery, textiles, metals, and jewellery or providing services as builders, soldiers, servants, officials, or priests. While society was egalitarian before 7500, it was grossly unequal by 3000. Before 7500, people were likely organized in clans or tribes; by 3000 a state had emerged.

Why had all this happened? Technical progress in food production is a popular explanation. Domestic crops and animals were more productive than their wild progenitors (even though living standards probably fell during the emergence of agriculture). The invention of the plough and the development of irrigation pushed food production up more. As a result, agriculture yielded a surplus, which supported the division of labour and the state. In addition, population is also often invoked as an explanation. Although Malthus is mentioned, the implications of his model for the issues at hand are not fully worked out.

In this paper we clarify these issues by analyzing technical change in a Malthusian population model. In the *Principle of Population* (1798), Malthus argued that the population would increase indefinitely until something checked that growth. He identified two checks–the positive (the death rate would rise when food eventually became scarce) and the preventive (the birth rate would also fall under the same circumstance). What prevents this present exercise from being a regurgitation of Malthus's pessimistic argument is the introduction into the analysis of surplus extraction. Either private property with the land rented to tenants or taxation of agriculture would have sucked income out of the countryside to be used to feed the city. In either case, both the total and the farm populations would have been smaller than they would have been as untaxed common property, and the urban population would have been larger.

The argument of this paper interfaces with several lines of research. One concerns the origin of the state. Archeologists have been theorizing about this subject for a very long time. Recently, social scientists have joined the conversation. Most of the participants in this discussion begin with the assumption that there was an agricultural surplus and then go on to dispute whether states emerged to either (1) steal it,[1] (2) defend it,[2] or (3) create it through public investment.[3] But no one has measured the evolution of the surplus nor how its size was related to technology or the natural environment. When did the agricultural surplus appear? How large was it? Did it vary from location to location? How did it change with the domestication of plants and animals, the invention of the plough, improvements in tool design, the introduction of

[1]Scott (2017), Graeber and Wengrow (2022), Gallagher and McIntosh (2015), Cairnero (1970), Allen (1997).

[2]Dal Bó et al. (2021), Johnson and Earle (2000), and North, Wallis, and Weingast (2009)

[3]Engels (1853, p. 263), Marx (1853, p. 13), and Allen, Bertazzini Heldring (2023). Variants on these arguments include Mayshar (2022). Dow and Reed (2022, pp. 359-434) are dissenters arguing Uruk was based on a manufacturing surplus.

irrigation? And why was it that the surplus propelled the growth of the nonagricultural economy and an increase in per capita income rather than being dissipated as a larger population?

This paper also interfaces with the archaeological literature on inequality in ancient societies. The recent impetus for this research was the recognition that house sizes and grave goods could be used to calculate the Gini coefficient of wealth inequality (Fochesato, Bogaard, and Bowles 2019). Empirical studies followed to discover factors that explained differences in inequality between sites (Kohler et al 2017, Bogaard et al. 2018, 2019). These results promoted important theoretical investigations to understand the logic of the results (Bowles and Fochesato 2024, Borgerhoff Mulder et al. 2009)).

These early inquiries have proved so promising that an international collaboration has collected and analyzed data on a world scale. The first results have appeared as a Special Feature in the *PNAS* (Kohler, Bogaard, Ortman 2025, Kohler et al 2025, Bogaard et al 2025, Feinman et al. 2025). The eleven papers are based on samples of as many as 53000 houses in 4000 settlements. The statistical analyses highlight the importance of two variables in explaining high inequality in a location: high agricultural productivity and the site's being a large settlement in a society with a wide distribution of settlement sizes. Population growth plays a less fully analyzed role in the creation of inequality.

This paper is organized around the three themes that are central to these issues–the dynamics of population change, technical change in agriculture, and property rights. First, we use a graphical framework to develop Malthus's argument that technical progress in agriculture could not bring about a permanent increase in human well being since the technical progress would have provoked a population increase that would have swamped the increase in food production leaving everyone as hungry as they were at the outset. It can take a long time to traverse from one Malthusian equilibrium to the next, but time is one thing we have in abundance: there were many millennia between the origins of agriculture and the appearance of the first state (Kelley and O'Grada 2012, Madsen et al 2019).

Second, we introduce technical progress as a sequence of improvements. These include the domestication of plants and animals, the plough, and irrigated agriculture. The productivity changes wrought by these improvements are calculated using linear programming models of agriculture. Incorporating these productivity changes into the Malthusian model shows that they could bring short run improvements in living standards but that these increases disappear when the demographic consequences are worked out. Everyone ends up poor and equal. Per capita

GDP remains at subsistence.

Third, we note that the analysis thus far has followed Malthus' original argument in which food was divided equally among the population. This is the allocation that would have obtained had farming been organized as common property, i.e. the demand for labour equals its average product. In such a system, people might have been assigned particular plots, but they could not buy, sell, or rent them. Furthermore, the redistribution of land was carried out to accommodate changes in population and to equalize holdings. Such redistribution could not have occurred had there been private property. With landowners leasing out land in a system of private property (and we argue that would probably have been the situation), farm sizes would have been limited to those that maximized the rental value of land, i.e. that set the marginal product of labour equal to the prevailing wage. With the marginal product less than the average, there was an agricultural surplus which accrued as rent to landowners. Improvements in farm technology would have increased the surplus without its being diluted by population growth. The agricultural surplus, in turn, gave rise to the division of labour, greater inequality, and ultimately the state.

Private property in land was one basis for long run rises in per capita income (all of which accrued to landowners) and the trajectory that ended in agrarian civilizations. Taxation was a second, for, as will be shown, it also sucked food out of the countryside and fed cities– although not as comprehensibly as private property.

Demographic model

Malthus proposed his demographic model in the *Principle of Population* (1798). The exposition was verbal and the model was very simple involving only food and labour. Land was soon added to the analysis, and it was later formalized with simple graphs. Social scientists have also elaborated the model algebraically to model decisions like the number of children and to encompass more goods like 'manufactures' and children. Here we stick with food, labour, and land.

Figure 1 is a standard graphical representation of the basic Malthus model. The model has two parts: a demographic module (left quadrant) and an economic module (right quadrant).

The right quadrant plots the average productivity of labour in food production as a function of population. Large populations imply lower productivity–an example of the law of diminishing returns.

The left quadrant summarizes the demographic assumptions. In the usual formulation, the birth and death rates are plotted on the vertical axis as functions of per capita food consumption, which is on the horizontal axis.[4] Here, however, the usual graph has been tipped over backward to align the food consumption axis with the vertical axis of the productivity diagram, which has the same scale. The intersection of the birth and death rate functions determines the food consumption/production at which they are equal. In Figure 1 this value is projected across the labour productivity diagram, and its intersection with the labour productivity function determines the population size that produces the demographic-economic equilibrium. The dotted lines connect values that comprise that equilibrium: when population equals P, the birth rate equals the death rate so the population does not change. If the population had some other value (e.g. P* in Figure 2), then food per head would be lower and the death rate would be higher (M* instead of M), and the population would contract. Contraction would continue until P obtained again. The reverse would happen if P*<P.

The level of food consumption where births equal deaths, S, is known as the 'subsistence wage' since at that level of food intake, the population could just reproduce itself. This level of food consumption provides minimal nutrition for all members of the family. In practice, S can be specified with a diet meeting nutritional norms that guarantee minimally acceptable levels of calories, protein, fat, and iron, thiamine, folate, vitamin B1, niacin, and vitamin C. Calories are normally set as 2100 per person per day, averaged over the whole population. These nutrients protect people from the major deficiency diseases–anaemia, beri-beri, pelaggra, and scurvy. Subsistence diets can be calculated with linear programming (Allen xx). A grain-based diet for the average person during the neolithic might have consisted of 130 kg of barley, 25 kg of chickpeas, 30 kg of aurochs, 25 kg of gazelle, plus roots and leaves from a variety of plants.

Linear programming diets that select foods to minimize the cost of meeting the nutritional requirements used here provide a good description of the diets of poor people in poor countries and also are consistent with the incomes of poor labourers for the last two millennia (Allen 2017, 2020). The world's population of poor people does seem to gravitate to this income level. F ood consumption in this paper is designated simply as calories, but the related average

[4]The formulation shown here in which the birth rate is always at its maximum and independent of food availability, while the death rate is a declining function of food availability, is the positive check version of the model. In the preventive check version of the model, the birth rate is an increasing function of food availability at

and marginal products are based on the labour required to produce linear programming diets.

The Malthusian model implies causal separability. As is evident from what has just been said, food availability per head in the long run (i.e. when the model is in equilibrium) depends on the birth and death rate functions in the left quadrant. Factors like climate or technology affect the productivity of labour in the right quadrant, as we will discuss, and generally not the demographic functions.[5] Climate and technology affect the size of the population, while demography determines food per head.

A consequence of these results is Malthus' pessimistic belief that economic growth would not make humanity better off in the long run since per capita consumption would remain unchanged at 2100 calories per day and would only increase the population. This conclusion does not hold in more complicated versions of the model that distinguish social classes. In those models, it is usually only the incomes of labourers that remain at subsistence.

Technology

Technical change was a fundamental feature of agrarian history between the Neolithic and the Bronze Ages. Advances include the domestication of plants and animals, the shift from hand tools like hoes to oxen driven ploughs, and the development of irrigation. Technical change plays a leading role in both theoretical and empirical work on the origins of inequality, the division of labour, and the state.

I use linear programming models of agriculture to construct engineering production functions that imply the average and marginal revenue curves that appear in the Malthusian framework.[6] The particulars of the model vary from one technological configuration to another.

---

low levels of food availability. The dynamics of both versions of the model are similar.

[5]Exceptions are technologies that control fertility like contraceptive pills and those that affect mortality like medicine and sanitation. These were not important until recent times.

[6]A distinction that is commonly used to describe technical change is that between 'labour limited' technologies and 'land limited' technologies. These are often glosses for hoes versus ploughs, and the intuition is that hoes limit the size of a farm (which presumably could have been larger–ie there was available land–if the technology permitted) and ploughs which relax labour constraints so farms could have been bigger (although only to the point where harvest labour requirements bind). The concept of 'land limited' has been expanded to include increasing capital intensity involving terracing, irrigation, and drainage, sometimes quantified on a four point scale (Bogaard et al. 2025) Another approach distinguishes labour limited from land limited in terms of the ratio of the marginal product of labour to that of land. A high ratio indicates a labour limited technology, while a low ratio indicates land limited (when assessed at observed factor proportions) (Bowles and Fochesato 2024). While general

These configurations include cultivation with hoes and wild seed, hoes and domestic seed, ploughs and domestic seed and domestic animals, irrigation in southern Mesopotamia and so forth. The model incorporates experimental, historical, ethnographic, and scientific evidence to specify the hours of labour needed to obtain one kilogram of food in each technological and environmental configuration. The model includes an additional 'food' called 'surplus' that has the input requirements of barley but conveys no nutrients to the cultivator. The model maximizes the 'surplus' subject to the nutrient requirements of the cultivator's family–that is the Malthusian subsistence wage–and additional constraints describing the maximum hours of labour the family could work in each season of the year on the land available to it. The latter depends on the village population, and this provides the interface with the Malthusian model. Further details of the model are in the Appendix.

Figure 3, Part A, summarizes the history of output per worker in the middle east. Once domestic crops and animals became available c. 7500 BCE, the population grew and settlement spread beyond the Fertile Crescent across Anatolyia and the northern Mesopotamia steppe called the Jazira. The figures show what happened in a representative village as it was settled. The average product of labour (here measured in calories) is shown for the cultivation of the steppe with domestic seed with ploughs as well as hoes. (Cultivation of the steppe with wild seed and hoes is not shown since it was not viable: that technology generated less than 2100 calories per person per day, meaning that a family could not grow enough food to support itself at subsistence.) Plough cultivation was clearly more productive than hoe cultivation by a wide margin. Tell Brak, the leading city in the North could achieve agricultural productivity levels like those shown for plough cultivation on the Jazira.

Figure 3A also shows labour productivity for two more favoured (i.e. wetter) settlements in the middle east: Catalhoyuk in Anatoliya with a damp riverine environment (Ayala et al 2022) and Uruk with irrigated agriculture in southern Mesopotamia (Algaze 2008, Pournelle 2003). Output per worker was higher in both of these than on the Jazira. Both also show a decline in average product as population expanded.

There are noticeable kinks in the plough curves (and one that is not easily seen in the hoe curve) that correspond to the population at which the village is 'filled up,' and all farm land is arable.

---

formulations are useful, I focus here on the specific techniques that played key roles in the evolution of the middle east.

Figure 3, Part B, shows total production (here measured in kilograms) as a function of the number of farms. The curves are linear with a kink. The graph has a common sense interpretation. We are used to thinking that there is a definite quantity of land that a family relying on its own labour could cultivate. In this case, that was about 5 hectares per family. As the region is settled by families each of which cultivates 5 hectares, total production increases in proportion to the number of farms. This is the rising straight line to the left of the kink. Eventually, the region is filled with 5 hectare farms. The result is the kink in the graph. Increases in the number of farms to the right of the kink lead to smaller and smaller farms. Farmers may compensate for the reduction in their land by undertaking low productivity activities that they spurned previously. Total output continues to go up but at a slower rate than before. Hence the graph rises less steeply to the right of the kink than it did to the left.

Figure 3, Part C, shows the total surplus (in kilograms) generated by the four technologies as they would have played out in the same hypothetical village. Each follows an inverted V pattern as the population expands. The rising half reflects the filling up of the village with productive farms. On the Jazira and Chatalhoyuk, there was also a shift of land from steppe pasture to cultivation as the population expanded. The maximum is reached when pasture disappears. Thereafter, surplus declines as greater population leads to smaller farms and more mouths to feed. Around Uruk, the area of the village simply becomes full of identical farms with no pasture since the climate was too dry in that region for grass to grow without irrigation. The maximum supportable population of the village in each case is reached when the average product drops to 2100 calories. A larger population could not have been fed with the available technology.

<u>Institutions: communal property</u>

Figure 3B indicates that technical progress raised labour productivity in the middle east and with it the agricultural surplus that would have manifested itself as a high Gini coefficient if it was unequally distributed. Did that happen? The answer depends on the system of property rights–a variable that is not usually considered.

Consider the case of communal property. I model communal property along the lines of the repartitional communes of Imperial Russia or musha tenure in the middle east. In both cases, the lands of the village are owned by the village, and members of the village have assigned plots for their use. These plots cannot be sold or rented, and they are periodically reassigned to equalize holdings among the people present. Members had security of tenure of their plot–so they had the certainty they could harvest what they had planted (North and Thomas 1977, p. 235, Bowles and Choi 2019). Chiefs adjudicated disputes between tribal members, and the tribe as a whole defended its land from incursions. However, these plots could not be sold or rented, and, if they were unused, they would have reverted to community control for reassignment. Land could not serve as a source of rental income or a means of accumulating wealth.

Figure 4 describes the situation with the appropriate average productivity curve in Figure 3A replacing the generic labour productivity curve in Figure 1. The lowest curve in Figure 3A shows the productivity of planting domestic seed on land tilled with hoes on the Jazira. At the outset this technology would have generated a surplus of 77 kg per person, and the model predicts rapid population growth. As settlement expanded, population growth would have slowed until the population stabilized at 74 farms with 333 people. All of the food produced would have been eaten by the producers with everyone subsisting on 2100 calories per day. There would have been no inequality.

Now suppose the plough was invented and available. Every farmer would have had an incentive to adopt the plough since its use would have raised output per farm by 403 kg of barley per year. If everybody used the plough, there would have been no change in inequality since everyone's income would have gone up by the same amount. But that is only the beginning of the story since the higher income would have cut the mortality rate, and the population would have expanded as the left quadrant in Figure 4 came into play. The result would have been population expansion to 216 farms when the entire product would have been eaten by the now enlarged number of cultivators. Everyone would again have been at subsistence, there would have been no rise in per capita GDP. Technical progress would have caused no increase in inequality–even with an agricultural revolution. Avoiding this outcome was the Malthusian challenge

<u>Institutions: Private Property</u>

Surplus extraction makes all the difference. I begin with private property and consider taxation later.

To see how the property system matters, we first ask how the village of plough cultivators just described would have changed, had an individual managed to acquire ownership of all of the land and leased it to tenants. This would have been like the enclosure of a village in early modern England. With the land divided into 216 farms, all of the output would have been eaten by the cultivators, and there would have been no rent. Rent could have been created by evicting tenants and consolidating farms. 141 farms would have maximized the rent, and that is what an income maximizing landlord would have tried to effect. Figure 5 summarizes relevant features of the production function. Surplus as a function of the number of farms is shown on the bottom of the page, and its maximum is at 141 farms. Above it is shown the marginal production of labour. This equals 3220 if there were less than 141 farms and 756 if there were more than 141. It would pay landlords to increase the number of farms to 141 but not to go higher. The same break point occurs in the total product curve shown in the upper right. Its slope is the marginal product curve which declines in value at 141 farms as the curve becomes less steep.

The result of this policy would have been substantial unemployment. Reducing the number of farms required evicting 75 families of cultivators and turning them into proletariat. What was their fate? The landowners would have taken in 56 tons of barley as rent. They would have spent this on employees–soldiers, builders, potters, priests–and they could have hired 59 at a wage that included the subsistence of their dependents. There would not have been enough, however, to support 16 families since total food production would have fallen with the decline in farm employment. The decline in food output equals the area under the marginal product curve DE between 141 and 216. So long as the marginal product of labour is positive for those losing their jobs in agriculture, total food production and the total population would fall. This is how privatization acts as a third check on population.

The new equilibrium is shown Figure 6. There are three panels separated by vertical lines at 0 and 141 on the horizontal axis. The left panel plots birth and death rates as functions of food consumption on the vertical axis and increasing to the left. The equilibrium income (subsistence wage = 2100 calories/person/day) is determined by the intersection of the birth and death rate functions. This extends across the other quadrants defining the equilibrium level of food consumption.

The center panel shows the marginal and average products of labour. NBF is the average product of labour in a village with each family owning its plough team. 216 families is the Malthusian equilibrium population for common property. NBCF is the marginal product of labour. NBCw equals total rent, the difference between output and labour cost. A rent maximizing land owner would cut the population to 141 families.

The rent generated would have been spent on non-farm employees. The right panel shows the non-farm labour market and the number of families employed in it. The curved line, the demand for labour, equals the rent NBCw divided by the wage. This is a rectangular hyperbole drawn with respect to the horizontal axis and the vertical line at 200. The curve goes left from the line at 200 and the distance from it to BD at 141 is the number of families employed outside of agriculture by the spending of the rent. The hyperbola intersects the marginal product of labour at C, the subsistence wage. This is an equilibrium condition that implies food consumption in the two sectors are the same (so no one has an incentive to shift sectors) and everyone in both sectors is at subsistence, so births equal deaths.[7] The total population is the free variable: it adjusts in size to maintain those equilibria by intersecting the vertical line at 141 families in agriculture. After the shift to private property there was enough to support 200 families, which became the population.

What happened to the 16 families that could not be supported? How social disruptive was this decline? The answer depends on how it was accomplished and the circumstances in which it occurred. If private property was created early in the settlement of northern Mesopotamia and Anatolia, when population density was low, disruption would have been minimal since the population would have been below the rent maximizing size and there would have been considerable unoccupied farm land. Of course, rents would also have been minimal, so the incentive to privatize land would not have been great. On the other hand, if these regions were fully populated and population was greater than the rent maximizing size, then disruption would have been substantial. The gains to land owners would have also been at their greatest–so

[7]The fertility of the landowning families was probably not much of an issue. The was Malthus' assumption. When he wrote 80% of the farm land in England was owned by 15,000 families, so they were too few to have an important demographic response–whatever it might have been. What if landowners were a bigger fraction of the population? It is still reasonable to assume that the landowners' rate of natural increase would not have risen unusually for two reasons. First, later history tells us that much land is owned by institutions: cathedrals, temples, monasteries, colleges, waqfs, and the state. In addition, the state can tax the rental income of private landowners. In all of these cases, the recipients of the income were restricted, and, in some cases, the recipients were celibate. Institutions like these emerged in the Chalcolithic and Bronze Ages. Second, individual owners have usually acted to prevent subdivision of their properties. Entails and strict settlements were the norm in pre-industrial Europe. Failing these arrangements, the heirs themselves could act to prevent subdivision of estates by fraud and murder. Rebekah and Jacob's plot to steal Esau's birthright probably had a familiar ring.

this is a likely scenario.

In this context, a state would have been useful to would-be landlords and those engrossing more farms. A state could define private property and the right to privatize as well as providing military force to advance and protect privatization. The late fifth millennium saw the appearance of statelets in northern Mesopotamia, among them Tell Brak, Hamoukar, al-Hawa, Samsat, and Grai Reshi. Spending rents may have supported their workforces, and the same process would have deranged the countryside. Tell Brak was the largest of these sites. It was first settled in the sixth millennium. By c. 4000 BCE it had grown to 55 hectares and may have had a population of roughly 11000 people. c.. 3700 BCE, it contained 130 hectares and its population may have reached perhaps 26000. Tell Brak contained industrial districts with workshops mass producing standardized pottery, stone tools, and jewellery. There were also bakeries. Tell Brak was possibly home to the farmers who cultivated the fields around the city. It was also the home of rich people (witness its Gini coefficient of 0.66). It had a city wall built 4200-3900. Tell Brak contained a temple, assembly rooms, and a highly decorated communal dining area (Al Quntar et al 2014).

It looks like Tell Brak experienced considerable internal conflict. Its excavation has revealed a half dozen large mass graves of poorly nourished people, and the best interpretation is that they were killed in a civil war (McMahon et al 2011). Emberling (2015, p. 262) speculated that Tell Brak was governed by a collection of lineage heads, who dined together in the communal hall. No minutes of their meetings survive, but dealing with the unemployment caused by privatizing land may have been a recurring point of discussion

Simulating the effects of property systems

Simulation provides another approach to visualizing the different implications of communal and private property. We use a modified version of the model in Figures 7 to compute trajectories of the principal variables as the hypothetical village is settled. Simulation allows some realistic additions to the model that are not easily incorporated into diagrams.

We assume that there are 500 hectares of farmland in the village and that a farm family could cultivate 5 hectares, so the village could accommodate 100 five hectare farms. When there are 100 farms or less, the average and marginal products of labour are set at 3000 calories per person per day. Above 100 farms, farm size shrinks and the marginal product of farms drops to

300 calories per person per day. The average product of labour declines until it reaches 2100 calories at xx farms, and population expansion ceases.

Population growth is governed by a difference equation in which the rate of growth depends on per capita calories consumption relative to 2100. Consequently, the population changes until per capita consumption reaches the subsistence value of 2100.

Per capita food consumption equals production (3000 calories) minus rent and/or taxes. The maximum rent equals production minus the subsistence cost of labour. It is assumed that the possibility of extracting any rent was very low when population density was low and rose to the maximum when the land was fully settled. This view was operationalized by multiply the maximum rent by the proportion of farms in the village that were occupied by tenants. In addition, food consumption per person was also capped by assuming that any available calories after payment of rent or taxes that exceeded 2600 calories was spent on manufactures.

The number of landlords in the village was fixed at 1% of the number of farmers. Landlords are assumed to demand 1.5 times the food consumed by a farm family, and landlords and rent payments are only introduced when enough surplus is produced to meet this demand. In addition, landlords receive all rent generated in the village.

It is assumed that the cost of living in the 'city' is 20% greater than in rural villages, so 'urban' workers are assumed to receive 2520 calories per day. The number of urban workers equals rent plus any peasant spending on manufactures divided by the urban wage. Many simulations of the natural increase of urban workers have been made assuming the same demographic function as governs the growth of the rural population. These simulations produce growth trajectories for urban workers that are very close to the trajectories implied by dividing surplus by the wage. Experiments have been made to integrate the rural and urban labour markets, but they have little bearing on the simulations and often introduce instability into the simulations, so no such links are included in these results.

Communal and private property: simulation results

With these assumptions, the rural, urban, and total population of the village can simulated as well as the net food consumption of the rural population, GDP per head, and the Gini coefficient of inequality. The simulations trace out what happens when a village is settled. Hence, the starting point is zero population. The model is out of equilibrium at the outset with a rural income substantially above subsistence. The horizontal axis measures ‘time,’ and years on the graph, it is hoped, have some correspondence to calendar years. The simulation begins sometime in the fifth or sixth millennium. As time passes, the curves flatten out and remain constant. The constant values correspond to equilibrium values in the underlying model, which is along the lines of Figures 6.

The simulations establish six main points. These are illustrated in the six graphs in Figure 7, which contrast the behaviour of communal and private property systems as a region like northern Mesopotamia was settled.

(1) Panel A shows the evolution of the total population. It is less with private property than in the communal system. This shows private property functioning as as the Third Check to population growth.

(2) Panel B traces the history of the agricultural population. It is much less in the private system than in the communal system. This is because private landowners have an interest in restricting the agricultural population to the level that maximizes surplus, while the agricultural population expands in the communal system until all of the surplus has been dissipated and farm income equals the average product of labour.

(3) Panel C. The difference between the total and the agricultural populations is the ‘urban’ (non-agricultural) population, which includes the landlords in my accounting. The urban population is fed with the agricultural surplus as it is spent by landlords. There is also a second source of demand for manufactures in the early years since I assume that peasant income in excess of 2600 calories was spent on manufactures. The ‘urban’ population is a persistent and equilibrium feature of the private property system, but the urban sector disappears in the communal system once farm incomes drop below 2600 calories.

(4) Panel D. The property rights system has no effect on the income of workers in the long run. The income of farmers always go to subsistence (2100 calories). In this respect, my model is Malthusian and displays the characteristic equilibrium condition.

(5) Panel E shows Gini coefficients of inequality. The Gini is bounded between zero and one. ‘One’ indicates maximum inequality (one person gets everything), while ‘zero’ indicates complete equality– everyone has the same income. Inequality is always low and transitory in the communal system, but large and enduring in the private property system. Private property leads to enduring inequality.

(6) GDP equals food production plus the goods and services produced by the ‘urban’ population–

all valued in calories. Per capita GDP equilibrates at subsistence (2100 calories) under the communal property system but stabilizes at 3549 (69% more) with private property.

An important question–especially for a Malthusian–is how the two systems of property react to an increase in productivity. We explored this issue with the comparative statics diagram (Figure 6). We can also explore it with simulation. Figure 8 contrasts the trajectories in Figure 7 with trajectories when agricultural labour productivity is increased by 50%. This increase roughly corresponds to the productivity advantage of Catalhoyuk and Uruk had over Tell Brak. In these diagrams the reddish lines are private property and the blueish lines are communal. Light colours indicate the original level of productivity, while dark colours indicate results when productivity is raised. I review the same six variables as previously:

(1) Panel A traces out the population history of a hypothetical village in northern Mesopotamia as it its settled. An increase in productivity raises the population in both common and private property villages. Whatever the productivity level, the population is always greater in common property villages than in private property, as before.

(2) Panel B traces the history of the agricultural population. This is capped at 100 families in private property villages irrespective of the productivity level. Higher productivity raises the population in common property villages. In equilibrium the agricultural population equals the total population in the common property villages.

(3) Panel C traces the history of the urban population. In common property villages, there are upward blips in the urban population, but they disappear once peasant incomes fall below 2600 calories since there is no longer any purchase of manufactures. The blips are greater, the higher the level of productivity. In private property villages, the rise in productivity translates into a large increase in the urban population since the agricultural population is fixed, as just noted.

(4) Panel D shows trajectories of income per head in the farm population. The rise in productivity increases the initial level of food consumption in both private and common property villages. In all cases the trajectory eventually declines and reaches the same terminal value of 2100 calories–the subsistence income.

(5) Panel E shows Gini coefficients of Inequality. In common property villages, the Gini is always zero or virtually so since all families have the same income except for the minor and transitory premium for a higher cost of living in the city. This is not true in private property villages since landlords reap very high incomes. As productivity increases, their incomes increase, and the Gini coefficient increases as well.

(6) Panel F traces GDP per head. In both common and private property villages, the initial value is higher, the higher is productivity. In common property villages, GDP per head declines to subsistence as population increases irrespective of the starting value. In private property villages, however, it declines as population increases but always remains much above subsistence.

<u>The adoption of the plough</u>

The plough probably played a more pivotal role in social evolution than the analysis thus far has revealed. It is likely that the introduction of the plough precipitated the shift from common to private property.

The adoption and diffusion of the plough cannot be traced with excavated remains since ploughs were made of wood and have long since rotted away. The best approach is to focus on the oxen that pulled the plough since their bones survive and reveal the story.

The widespread use of the plough probably began in the seventh millennium BCE. Helmer et al (2018) found pathologies on the phalanxes (toes) of cattle in PPNB (8800-6500 BCE) sites in the middle east that suggest they were used for traction–as pullers of heavy loads and possibly of ploughs, although there is no evidence specifically on that question. The deformities discussed by Helmer et al were not widespread in their data, moreover, suggesting that oxen were not widely used for traction in the PPNB. In addition, genetic evidence shows that domestication began c. 8000 BCE and involved only about 80 female arurochs. About 2000 years would have been required before smaller, docile domestic cattle were bred. (Bollongino et al 2012). This chronology is confirmed by Conolly et al. (2012), who report on bone assemblages from hundreds of archaeological sites. They found that sites dated to approximately 8500 BCE contained the remains of a negligible number of domestic cattle, while aurochs amounted to about 5% of the animals disinterred. By 7000 BCE aurochs and domestic cattle were equally likely to have been found with each amounting to 2% of all animals. By 6000 BCE, however, the share of domestic cattle had shot up to 17% of the animals unearth, while the aurochs remained stuck at 3%. Sometime between 7000 and 6000 BCE, therefore, looks like the period when domestication had been completed, and the breeding of cows and oxen as farm animals was under way. This was also when settlement began to spread from the Fertile Crescent across northern Mesopotamia and Anatolia. It may also have been the time when the plough came into widespread use, and larger estates were first established.

The plough increased productivity by allowing a farmer to till more land than he could have managed with a hoe. A plough team (two oxen and a plough) was an indivisible input that could till the land of half a dozen family farms. Consolidation of landownership into 30-40 hectare blocks or more would have made land more valuable than a system of 5 hectare family farms. This is illustrated by Figure 9, which shows the surplus per person when land in the Jazira was organized as family farms, each with its own plough team, versus estate organization with centralized ploughing. (Hoe returns are included to show the vast improvement effected by the plough.) The land of the village was worth 30% more, if it was cultivated by an estate with tenants rather than as individual family farms.

Bowles and Fochesato (2024) have argued that this saving of labour led to an increase in inequality. By 6500 BCE there could not have been many oxen in the Fertile Crescent since the breeding of domestic cattle was just beginning. Even if everyone had recognized the productivity boosting effect of ox driven ploughs, there was only a limited supply. Those who had some savings would have been the ones who could buy plough teams. These purchasers would also have had a great incentive to acquire the 30-40 hectares that their team could plough. Property accumulation on that scale was incompatible with communal ownership and would have led to its demise.[8]

Figure 10 shows the implications of adopting the plough in a Malthusian framework. At the outset, supposing there to have been a demographic equilibrium of hoe cultivators, the population in a representative village would have been 74 families. It would have been highly profitable for individual farmers to acquire a plough team, for they could earn their subsistence plus a surplus of 90 kg per person per year. This was true in communal property villages and in those with private property. Farmers taking this option would have had high net reproduction rates (according to left quadrant in the figures), so their numbers would have expanded. This

[8]Conceivably ploughs and oxen could have been owned by the community and shared among its members or private individuals who owned ploughs and oxen could have hired out their services to those who did not have their own. In practice, however, these arrangements are often not viable since farmers prefer owning inputs they might need at a critical time. For this reason, medieval manors with large demesnes used their own teams to plough them, while the other tasks were performed by servile tenants on small holdings. Economics of scale in farm operations also led to consolidation of small properties in early modern England, so that the operations could be conducted at a large scale (Allen 1992xx). The implications of the economies of large scale production for the disappearance of small family farms was a theme in the late nineteenth century debates on peasant differentiation (Kautsky 1902 and many others).

was a ‘free land’ era in the Domar (1970) sense. Eventually, the average product of labour would have dropped to 2100 calories per person per day when the population hit 141 families, and population growth ceased.

This outcome was avoided by privatizing the land. Bowles and Fochesato (2024) propose that more prosperous people bought out less prosperous people to accumulate land to form estates since more efficient use of the plough team would have allowed them to earn an extra (approximately) 30 kg per person in surplus–the difference in Figure 9A between the surplus in ‘estate & tenants’ and ‘family & 2 oxen’. The latter is what they would have had to allow their tenants as earnings since it would have equaled the opportunity cost of their labour. Land accumulation could only happen where private property was allowed.

The high income era would have ground to a halt once the population reached 141 farms per village in communal villages. This point was reached around 4000 BCE–two millennia after the plough was coming into general use. Villages in the Jazira had roads called hollow ways that radiated into the fields. Traces of these roads can be seen in satellite photos, and they indicate that northern Mesopotamia was fully settled by the early Bronze Age (Wilkinson and Tucker 1995, p. 25, Ur 2010, Widell, Hritz, Ur, and Wilkinson 2013, Palmisano et al. 2021).

Further population growth, which was inevitable given the high income of family farms, would have forced down farm sizes and incomes. In private property villages, 141 farms was the number that maximized the total rent from the land. (Six or more of these farms would have been operated as a consolidated unit for ploughing under the estate system.) This is shown directly in Figure 9B. This value can also be determined from the marginal product of labour curve, which is shown in Figure 5. The marginal product is the increased output produced by an additional farm in the village.[9] The discontinuity at 141 farms marks the shift from labour’s having a marginal product greater than the subsistence wage to one where it is less. Landowners would have had a great incentive not to expand the number of farms in private property villages beyond 141 but to keep them at the rent maximizing size. Once farm incomes were falling in common property villages, landowners could have raised their rents as tenants’ alternative earnings shrank. In this situation, population growth would have been driving an increase in rents and inequality.

The processes described here would have proceeded to Malthusian equilibria that depended on the property system in play. In common property villages, the result would have

[9]This can be calculated directly from a table of output as a function of the number of farms.

been 216 farms and an impoverished peasantry in complete equality. In private property villages, there would have been 140 farms supporting families all of whom were at subsistence and a class of landowners living off by their rents.

The story might not have ended here, however. The common property system would probably have collapsed in the villages where it still existed. The commoners would have been at subsistence and their land would have become worthless in that configuration. Rich land owners could have paid them something for their land and made big money by renting it to tenants in farms of the rent maximizing size. The peasants who sold out would have been proletarianized.

What would have happened to them? A landlord reorganizing a communal village with 216 tenants, would have cut the number to 140, leaving 76 families (342 people) without farms. They would not have remained unemployed, however. The landowners would not have eaten the rent they extracted from their tenants. Instead, they would have spent it to hire people (craftsmen, servants, soldiers, philosophers) or to purchase their wares or services. Indeed, the rent (35% of agricultural output) would have supported the 342 people through non-agricultural work.[10] There is no necessary reason that the job gains in the city should exactly balance the job losses in agriculture as they do here, but the productivity boosting effect of the plough would have increased the urban population, offsetting the declines in farm employment.

Taxation and Private Property

Taxation was another way by which surplus food could have been removed from agriculture and used to support the urban population. What were the demographic implications of taxation? Did it also provide a check to population? How effective was taxation in producing an agrarian civilization?

On the theoretical plain, there is an argument that taxation could never improve on private property. Consider the planner's problem: how should the countryside be organized to maximize the urban population? In the very simple framework we are using here, agriculture only produces 'food.' Whether it accrues to the state or landlords, the surplus is dispensed to support the urban population. Their number equals the total rent divided by the income of a farm family. Consequently, the urban population is maximized by maximizing the agriculture surplus.

[10]There is no necessary reason that the loss in farm jobs should equal the increase in non-farm employment.

Maximizing their rental income is the objective of private landowners, so, if they are successful, they maximize that surplus and the urban population. Taxation could not do better.

To investigate this hypothesis, we simulate the model of a common property village subjected to taxation by a state and compare the results with those already computed for a village with private landowners. To do this, we must specify the tax system. Two possibilities were considered. One was imposing a head tax on the farming population. This might have been imposed on the village, charging the local leader with collecting it. The problem with this system is administrative. How can tax collection be enforced? The history of the redemption payments levied by the Imperial Russian government on villages of freed serfs after 1861 is a cautionary example, for villages generally defaulted on payments, they could not be forced to pay, debts were eventually forgiven, and by the twentieth century the system was abolished as unworkable.

The second system was a tax on farm output. In medieval Europe and in the Ottoman empire, tithes (a 10% tax on grain output) was the method used to tax agriculture. It was administratively feasible since the grain was harvested in a short period at the end of summer. In Europe, the institution with the right to tithe a village sent wagons and crews to the harvest and took one tenth of the sheaves as they were carried off the fields. Tax evasion was harder with this arrangement. Tithes were a system that could be implemented, so we use it as our model of taxation.

The question is: what rate to set? Was there a rate that would beat private property? What was the impact of tithing agriculture on the population, income, and inequality variables?
To answer this question, the behaviour of communal property villages was simulated with varying tithe rates–0%, 10%, 20%, 30%., and more The results are in Figure 11. The higher the tax rate, the darker trajectory. The private property simulation with the same values for background variables is shown in blue. An important feature of the graphs is that the line corresponding to 30% is generally unobservable since it is overlain by the private property trajectory–in other words, a 30% tax on farm output has identical results to private property.

The graph also contains a yellow curve. This corresponds to a 32% tax rate. This is distinguishable for 30% and points to the essential conclusion: Tax rates higher than 30% produce worse results than private property in so far as the population variables are concerned. A ruler could not do better than private property if the aim was to maximize the urban population or, indeed, population overall.

(1) Panel A shows simulations for total population. The highest curve is for a zero tax rate. This corresponds to the common property case simulated in Figure 7. As the tax rate is increased, the equilibrium population declines, and the curve for a 30% tax is identical to the private property tax. Any further increase (e.g. 32%) reduces the population further.

(2) Panel B reports simulations for the farm population. Increases in the tax rate reduce the farm population in common property villages. A 30% produces the same farm population as in private property villages, and a higher tax a smaller population.

(3) Panel C shows the simulations for the urban population. A tithe rate of zero produces a blip in urban population as rich peasants buy manufactures, but they stop doing that as they get poorer. Higher tax rates increase the equilibrium size of the urban population. Taxation is, thus, a viable basis for agrarian civilization. A rate of 30% produces a result that is virtually identical to the private property rate. Higher rates produces smaller city populations. Thus, taxation cannot do better than private property in creating cities.

(4) Panel E shows the income per head of the farm population. The trajectories are all similar and all end up with equilibrium incomes of 2100 calories–the Malthusian subsistence wage.

(5) Panel E reports Gini coefficients. The Gini coefficients are large and persistent for private property–it produces enduring inequality. The situation is more complicated for communal property. If the coefficient is computed over the farmers and urban workers, Gini's are very close to zero, irrespective of the tax rate. This would be the normal way of proceeding; the state does not usually figure in these calculations as an income recipient. Theorists of early states who see them as kleptocracies–as little more than mafia bands–might want to include the chief mafios along with the farmers and manufacturers in computing the Gini. I have done this for the case where the tax rate is 30%. That curve is almost identical to the curve for private property, and they converge to the same value.[11]

(6) Panel F traces the history of GDP per head. A tithe of 30% produces the same curve as the private property system. Lower tithe rates yield less revenue, and a higher rate is also marginally lower. (The yellow line in the figure barely visible on the low side of the blue private property trajectory.)

[11] The difference at approximately 250 years is due to a difference in modeling when landlords or the ruler (as the case may be) entered the simulation: there was no point including them in the model unless they could take in more from letting land or taxing it than they could by working.

Conclusion

This paper highlights the importance of private property and taxation in creating the urban societies of the Bronze Age. Most analyses of the importance of private property emphasize its role in improving resource allocation by aligning individual incentives with social productivity. The present analysis emphasizes its impact on the structure of the economy. Private property and taxation were the syphons that sucked food out of the countryside and piped it to the cities where it fed their populations. Surplus extraction did not lead to long term improvements in the standard of living of the working population, but it did allow for persistent inequality and the development of non-agricultural industries, activities, and the state. Private property and taxation were a third check to the growth of the population, and they limited the stagnation implied by the usual Malthusian population dynamics. Without private property or taxation, there would have been no ancient agrarian civilizations.

Figure 1
Simplest Malthusian model

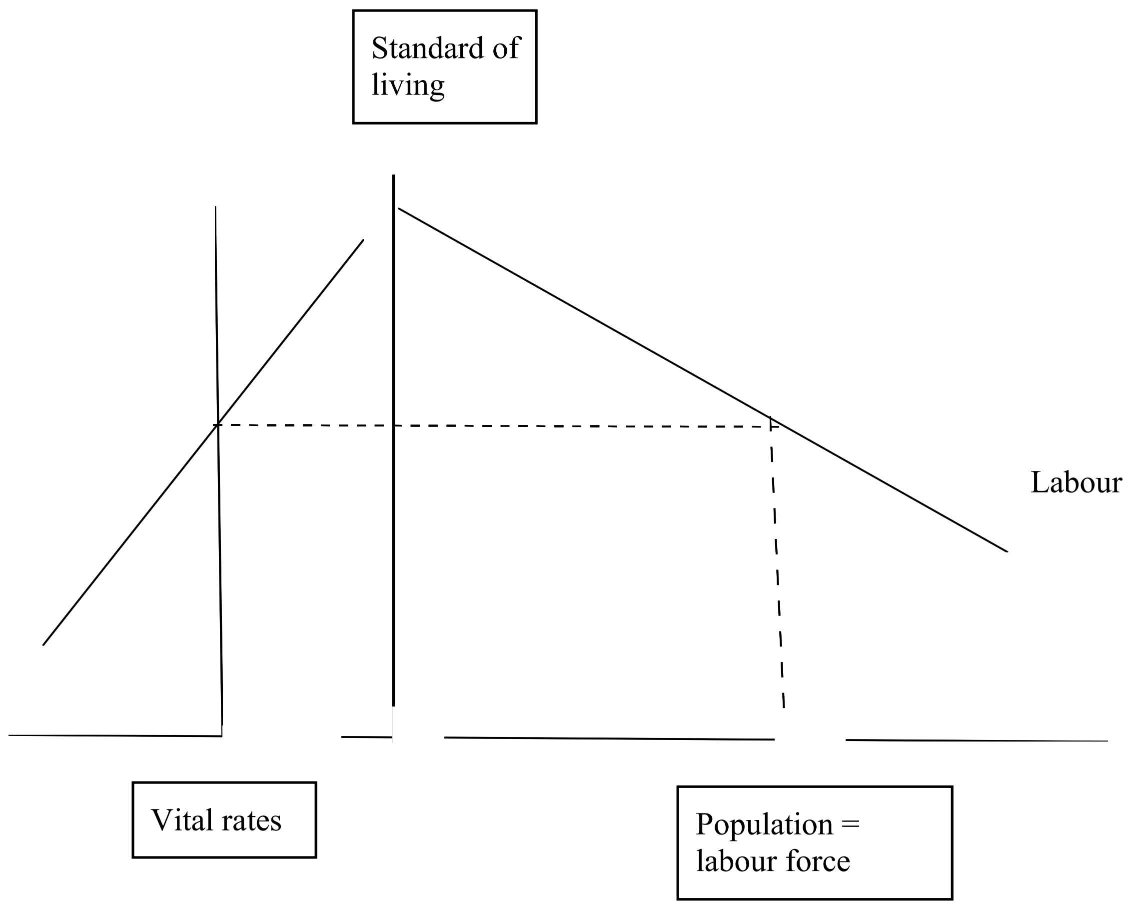


Figure 2

Malthus: out of equilibrium

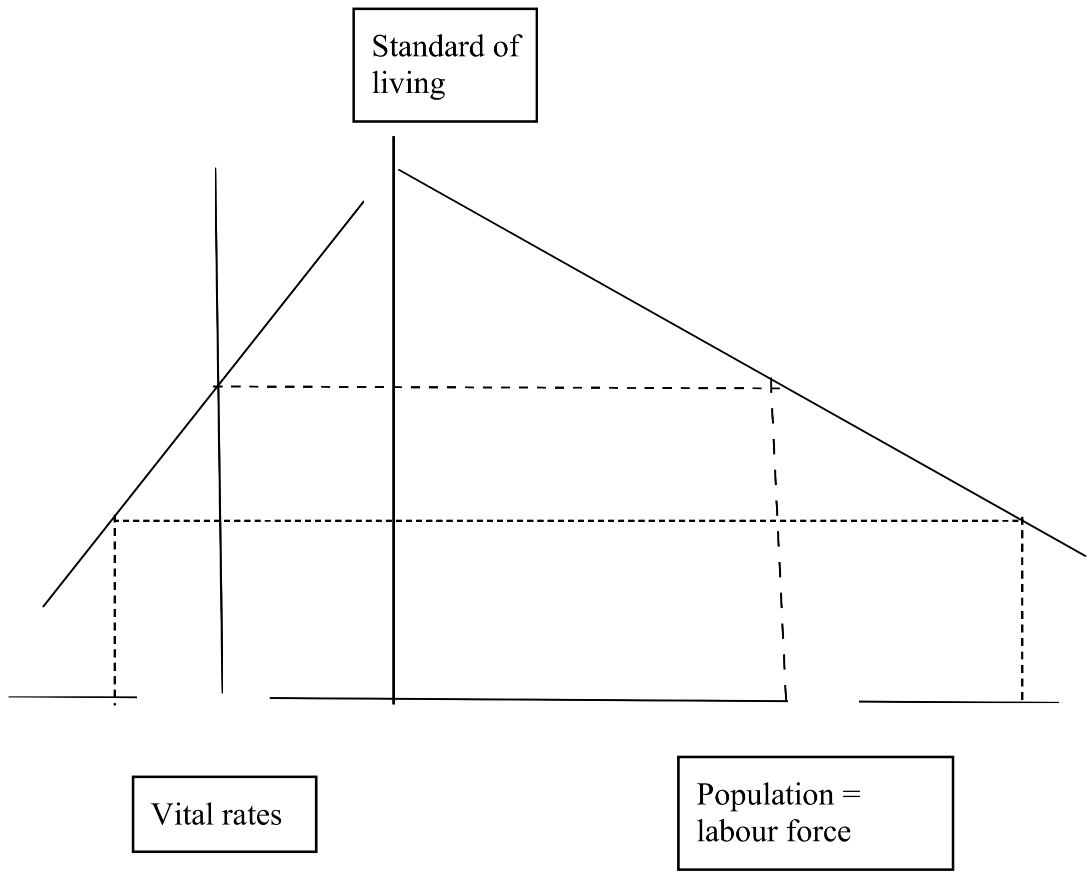
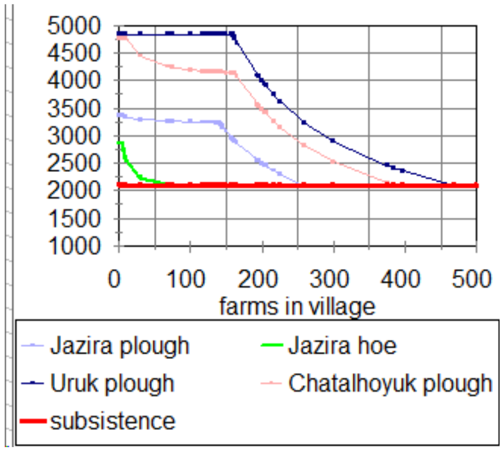


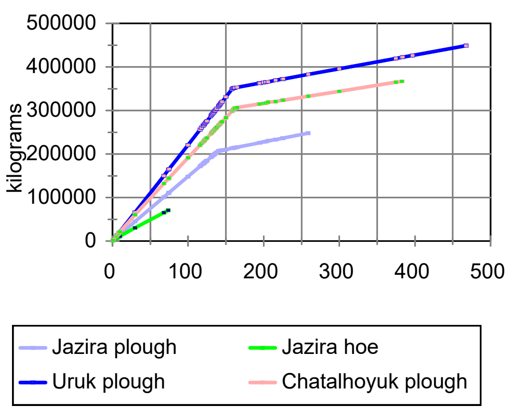
Figure 3

The productivity of hoes and ploughs with domestic seed on the Jazira

Part A Output/person in agriculture (calories/day)



Part B total production in village (kg)



Part C Total Ag Surplus in village (kg)

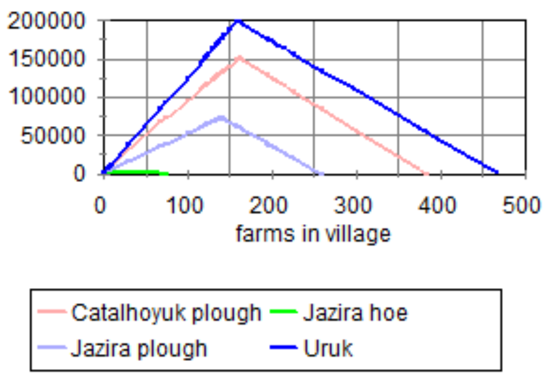


Figure 4

Common property equilibrium

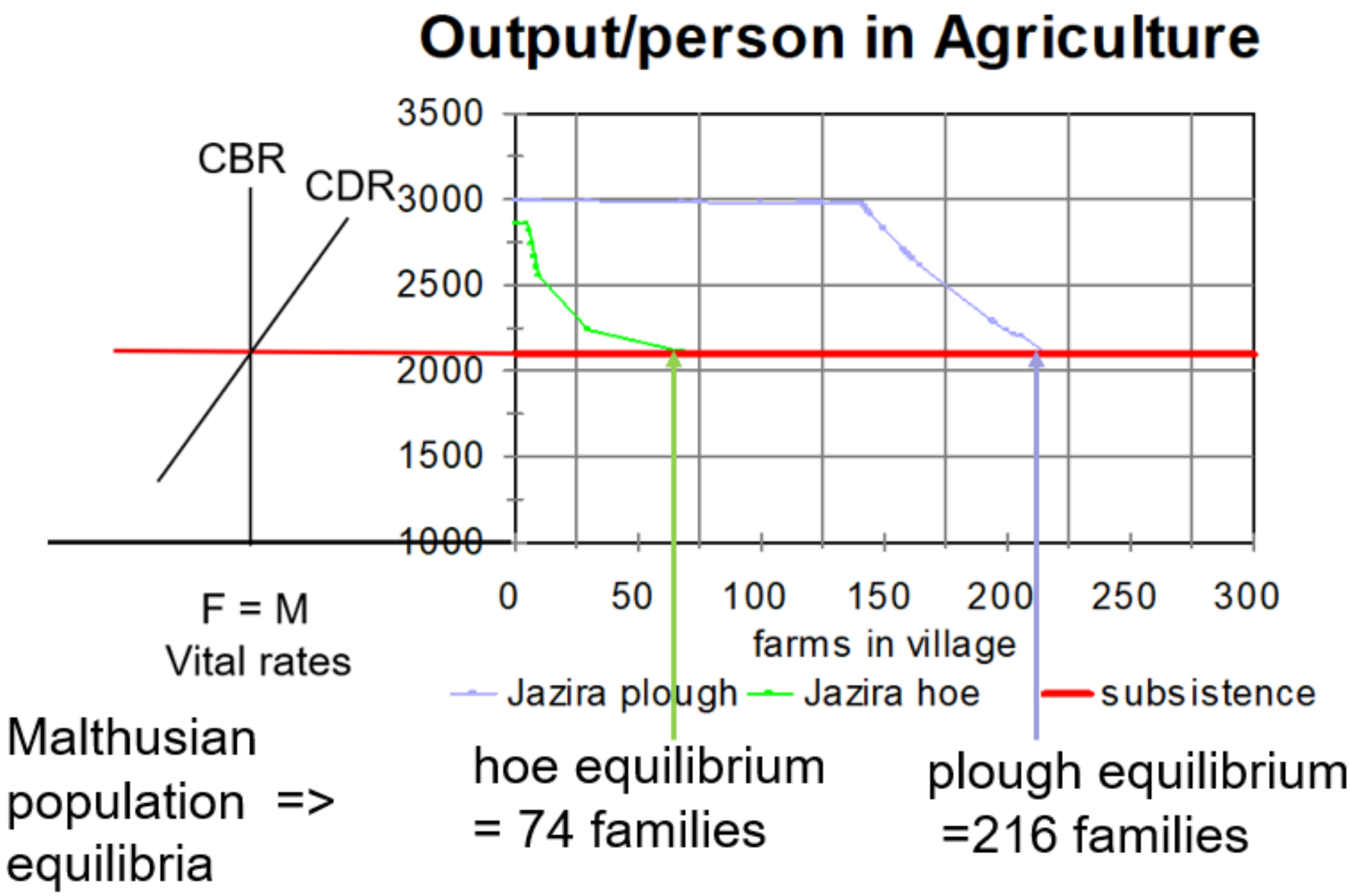
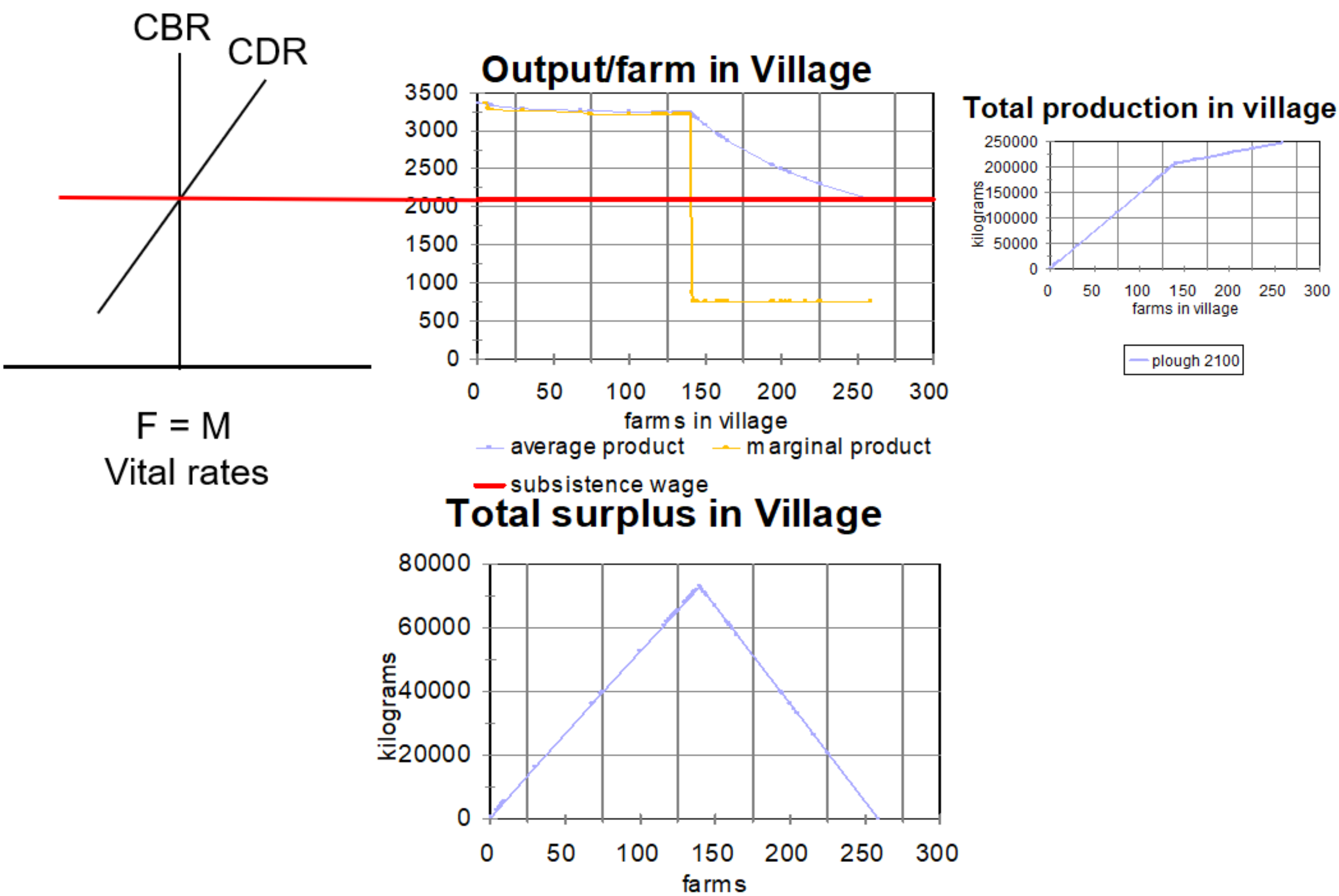


Figure 5

Rent maximization with the Plough Technology



Note: Total village rent is maximized at 141 farms. In 'Output/farm in Village,' marginal product equals subsistence wage at 141 farms, in 'Total Surplus in Village' the surplus is maximized at 141 farms, and in 'Total production in village' the kink is at 141 farms.

Figure 6

Privatizing a Fully Populated Plough Village

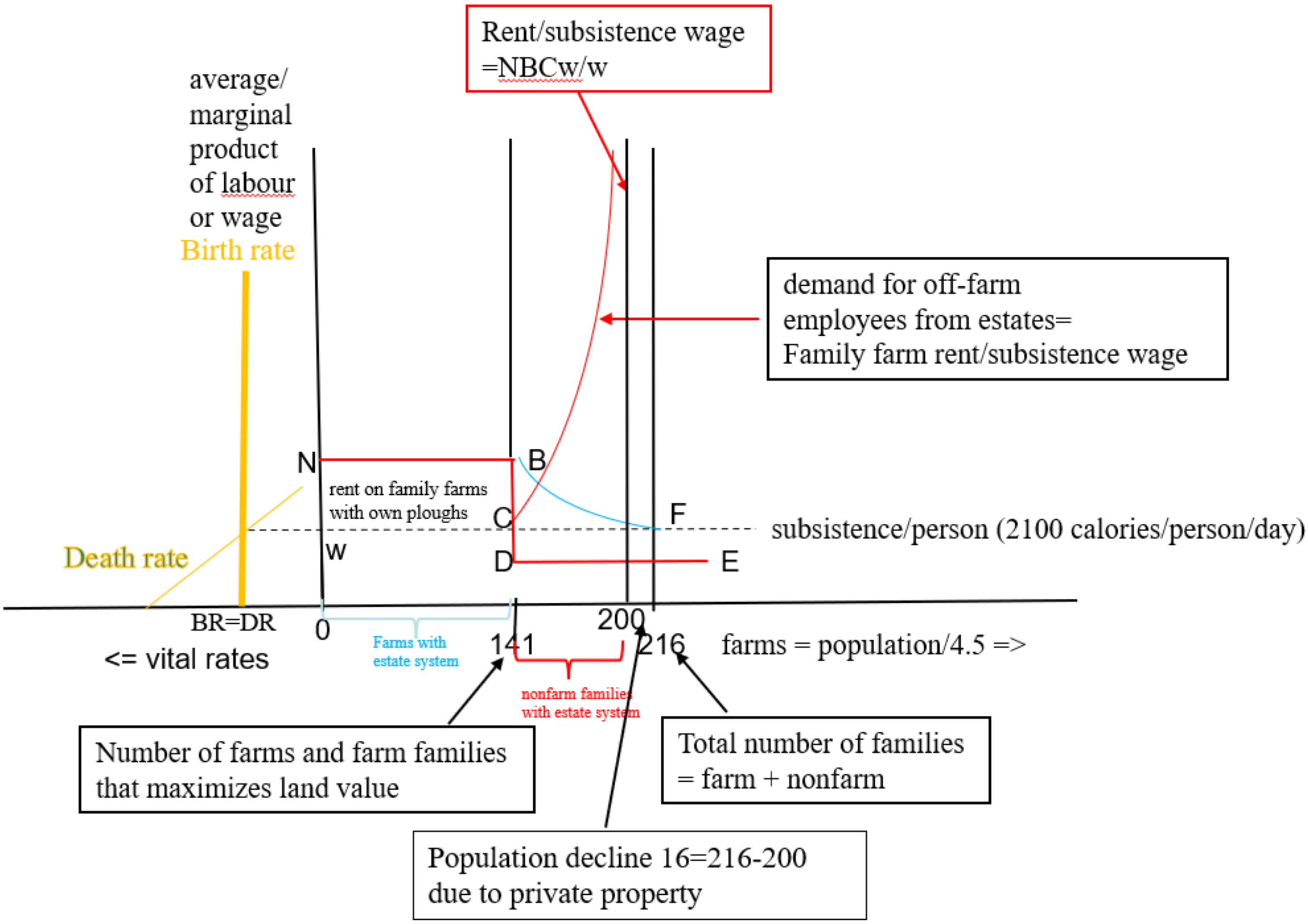
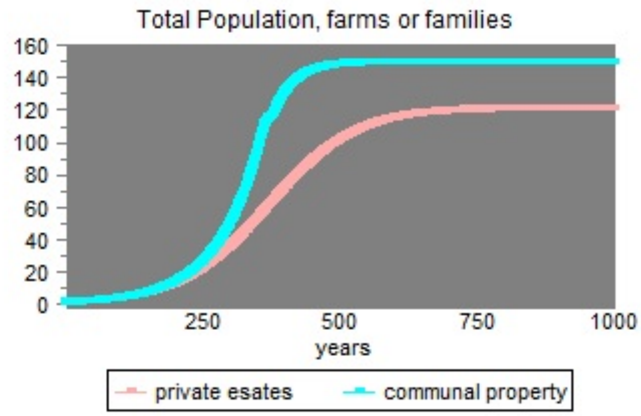
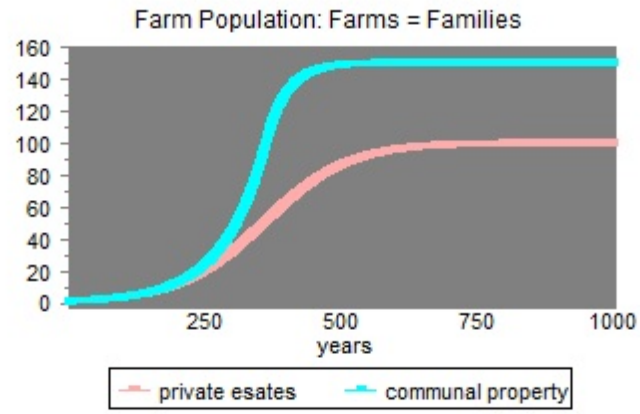


Figure 7
Contrasting Communal and Private Property

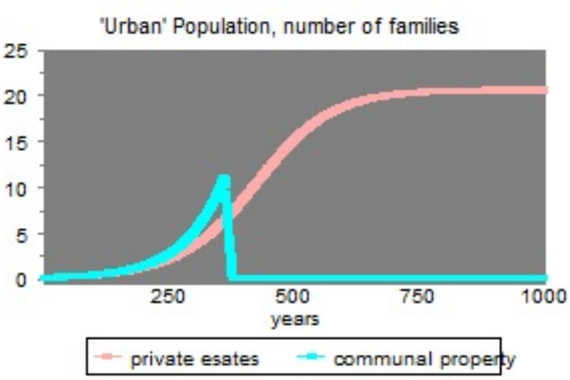
Panel A



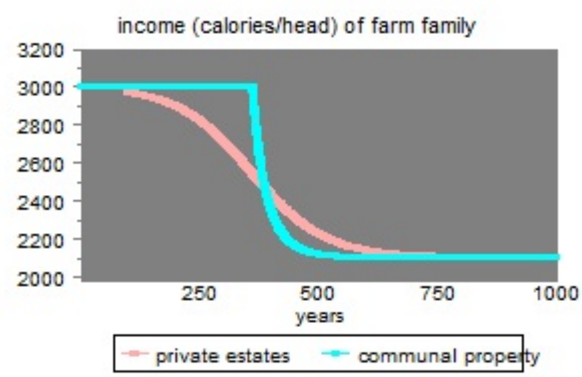
Panel B



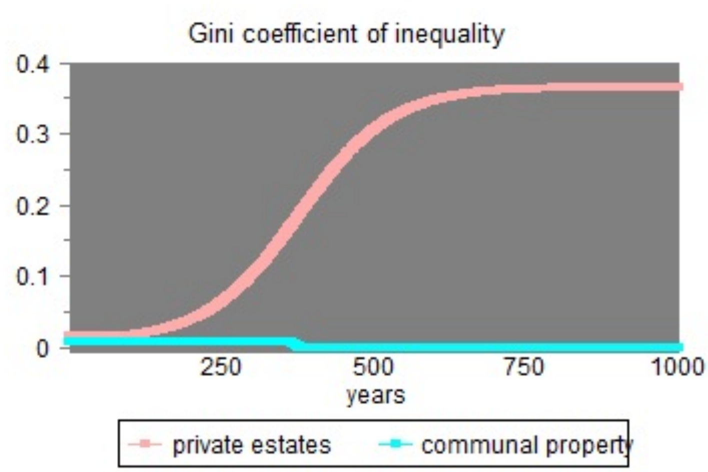
Panel C



Panel D



Panel E



Panel F

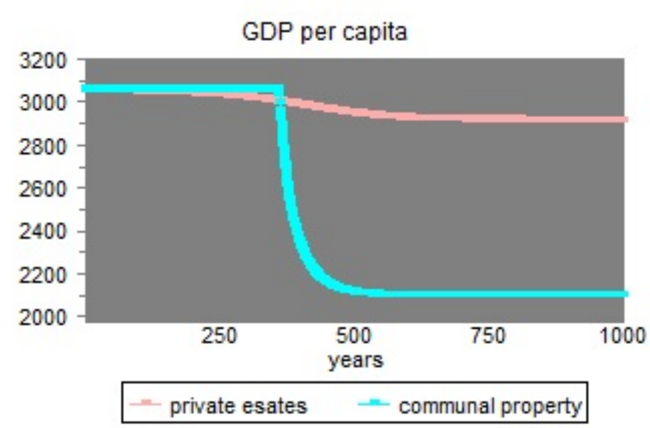


Figure 8
Effect of Productivity Growth with Communal and Private Property

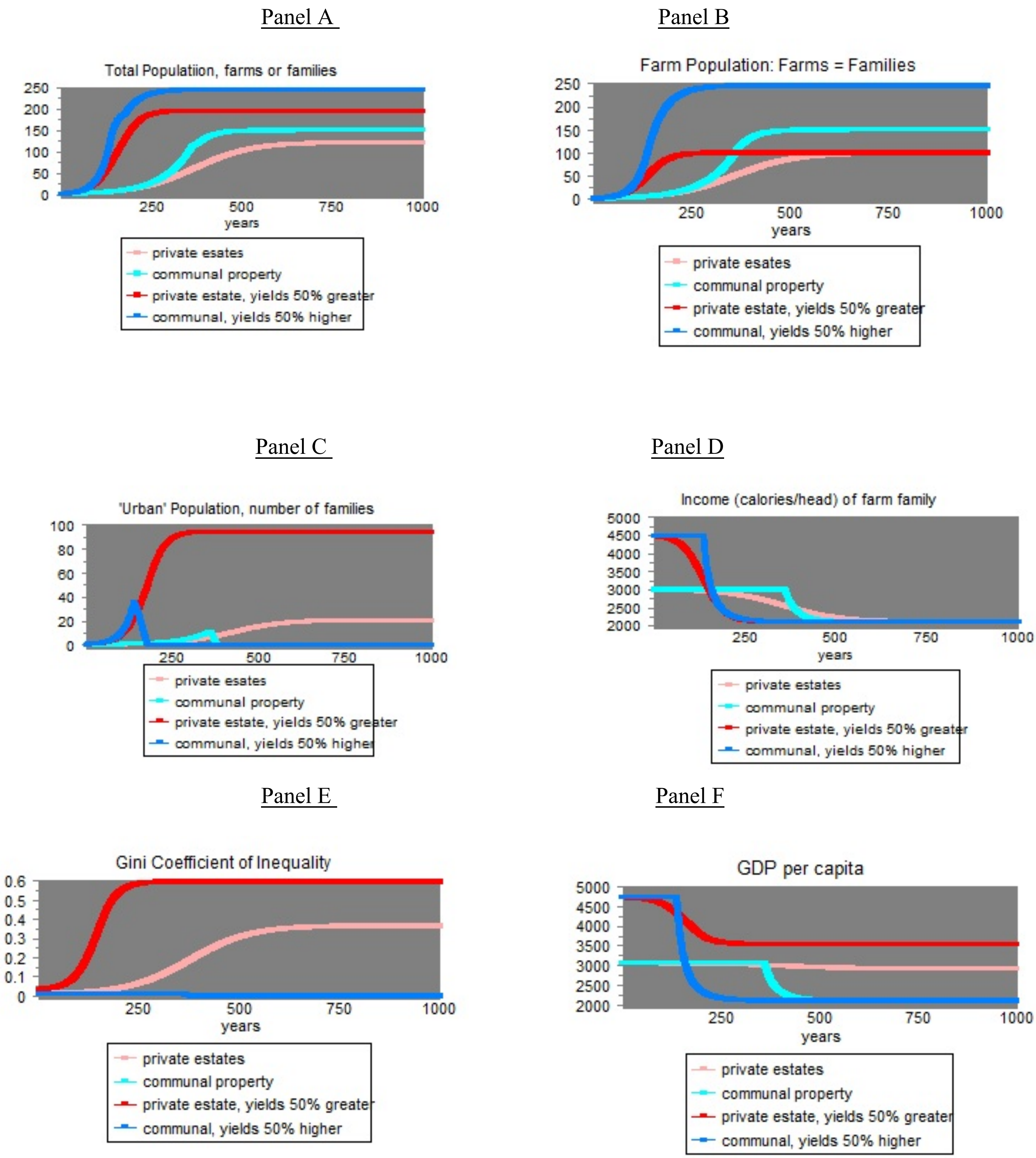
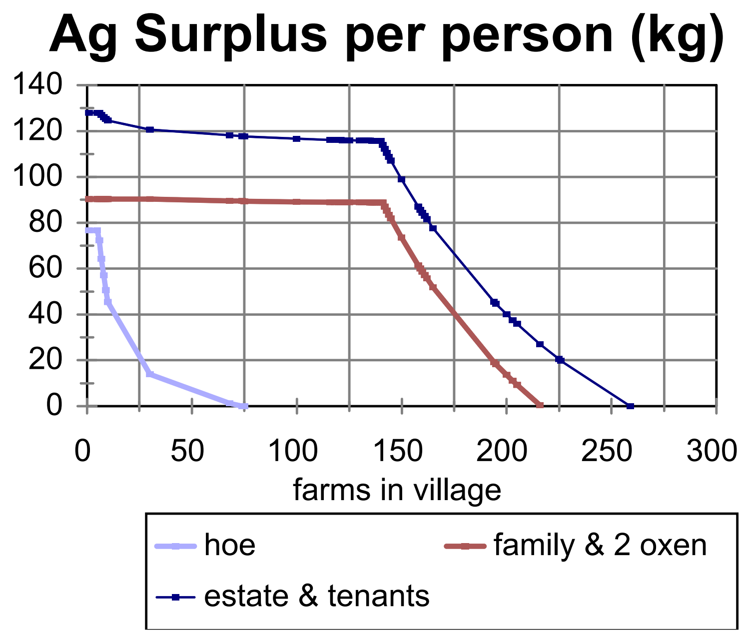


Figure 9

Family Farms versus Estates

Part A



Part B

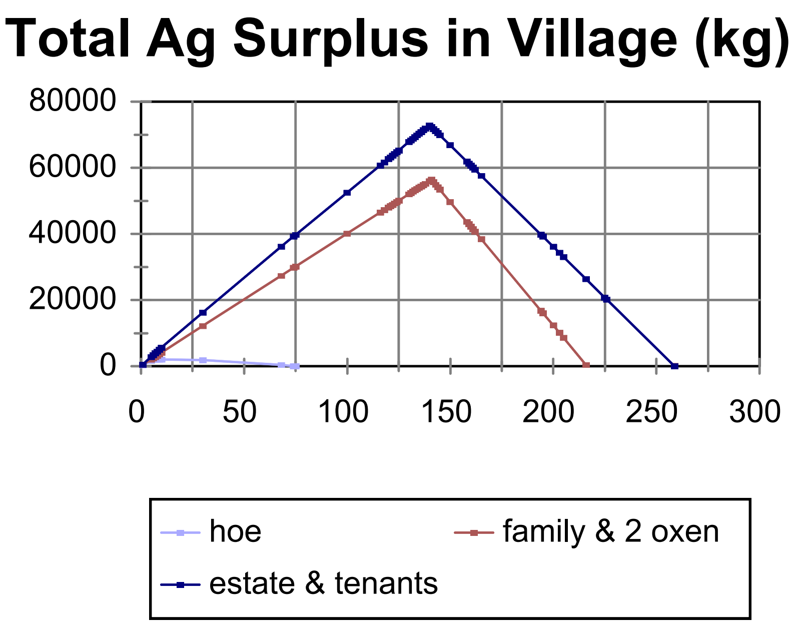


Figure 10

Privatizing a Village to Obtain the Productivity Advantage of Estate Plough Teams

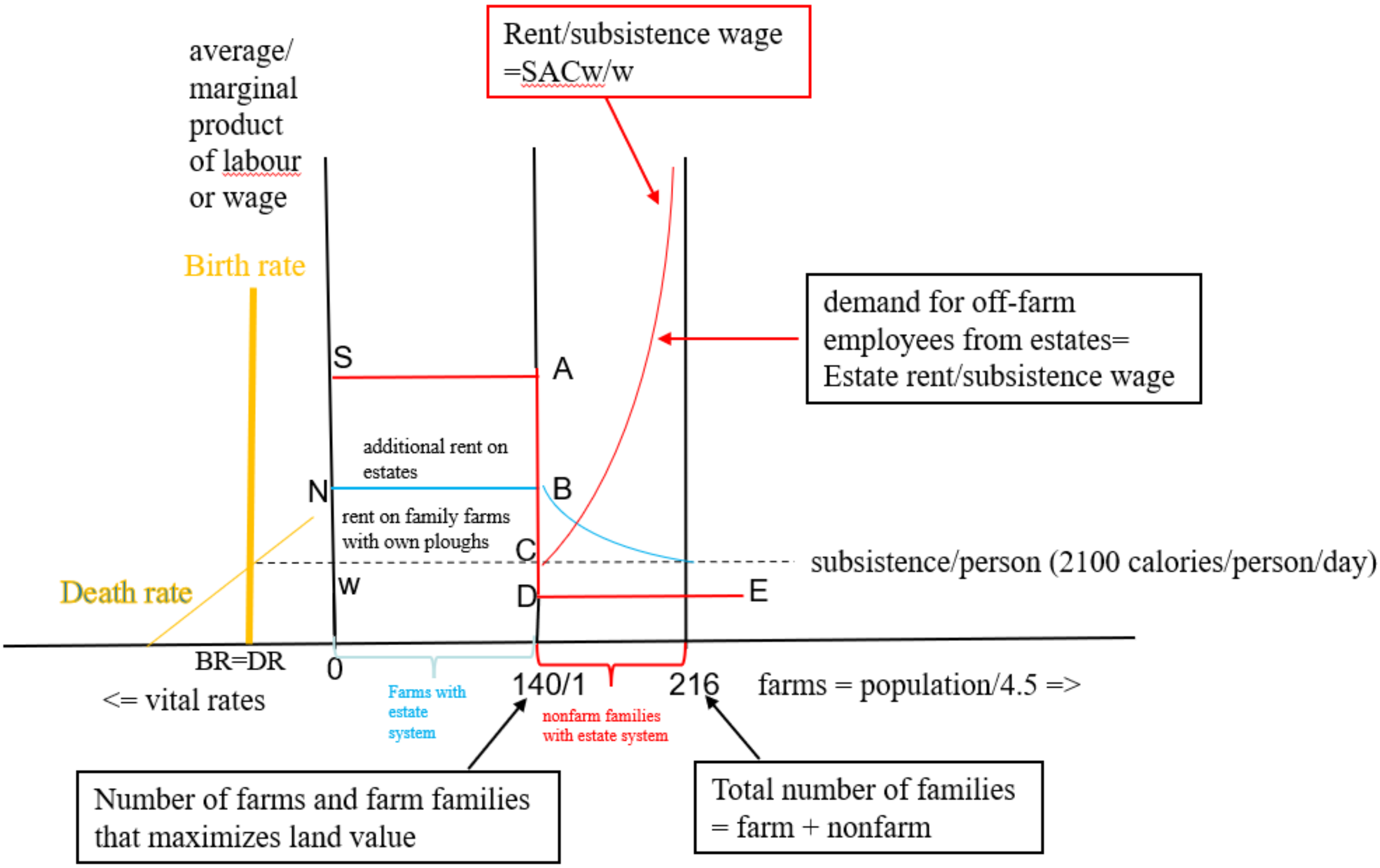
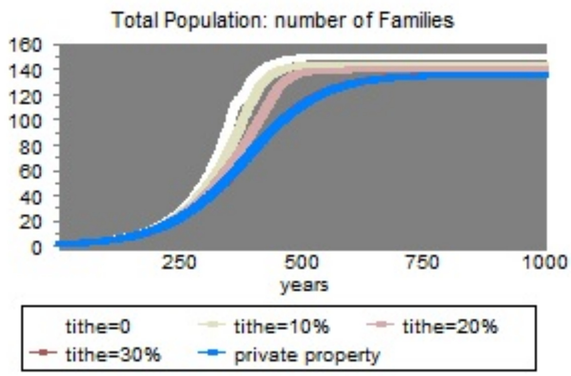
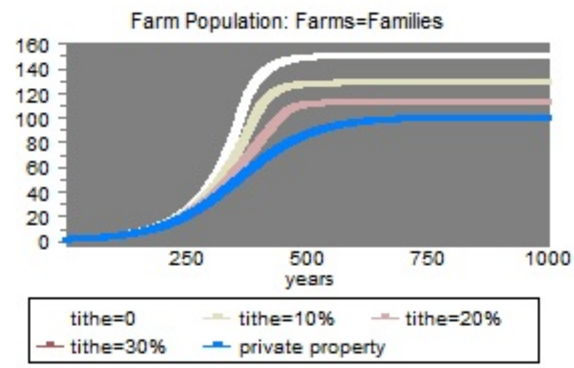


Figure 11
Comparing Private Property with Communal Property under Alternative Tithe Rates

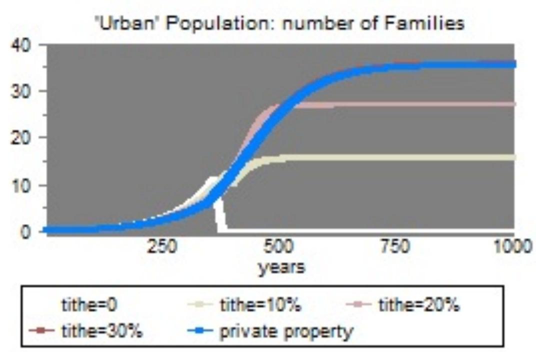
Panel A



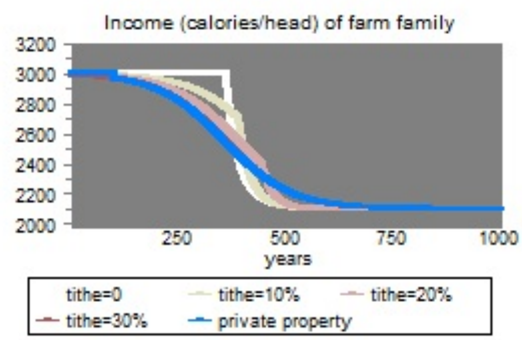
Panel B



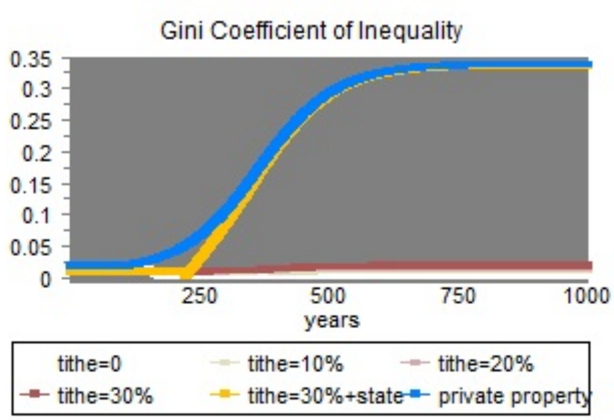
Panel C



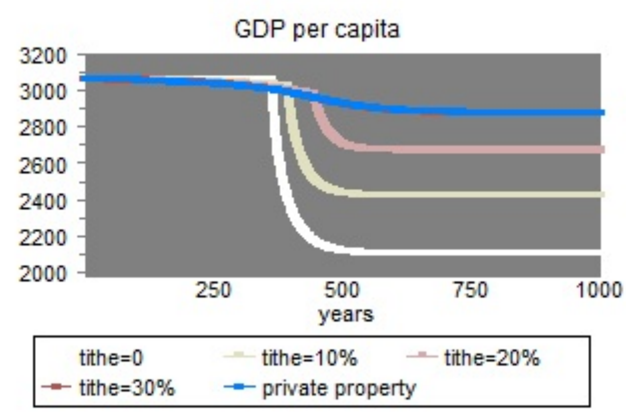
Panel D



Panel E



Panel F



Appendix

The linear programming model of agriculture

Linear Programming Model of food procurement

In the first instance, we model agriculture as a family enterprise whose members produce their own food. (Later we introduce classes and states.) Production beyond the family's subsistence needs is the agricultural surplus. The LP models are structured in these terms.

The first component models the subsistence of the forager or farm family. The classic LP diet problem is purposed for this task. Linear programming is a procedure for optimizing a linear objective function subject to linear constraints. The diet problem is to choose a diet from a list of foods that minimizes the cost of meeting a stipulated set of nutritional requirements (Stigler 1945). The objective function to be minimized is the cost of the diet:

$$\text{Cost} = \sum p_i F_i \qquad (1)$$

where p_i is the price of a food and F_i is the quantity of the food consumed. The summation can extended over a list of t foods $F_1...F_t$, many of which will not be selected in the solution.

The nutritional requirements are specified with a set of inequalities, each of which sets the requirement for a single nutrient.

$$\sum n_{ji} F_i \geq R_j \qquad (2)$$

Here the summation also runs over all of the foods indexed with i. R_j is the required amount of nutrient j. n_{ji} is the quantity of nutrient j per unit of food F_i. For instance, n_{ji} might be the number of calories per kilogram of wheat flour, F_i the kilograms of wheat flour in the diet, while R_j is the overall requirement of calories, e.g. 2100 per day. Each nutrient required in the diet has an inequality specifying that requirement.

Finally, the consumption of each food has to be at least zero:

$$F_i \geq 0 \quad \text{for all } i \qquad (3)$$

Most of the foods are simple to define and measure–kilograms of barley flour or gazelle meat–but three are composites in my model. 'Sheep' and 'Goats' yield the amount of meat and milk produced per animal in a self-sustaining flock, as simulated in a model of sheep or goat breeding and husbandry. A second is 'Cattle,' which is the amount of meat and milk produced per draft animal, taking into account the corresponding number of cows and calves. As with sheep, these values are derived from a simulation model of a cattle herd.[12]

In this paper, the model applies to an 'average' individual over a year. The 'average' is across both sexes and all ages and follows the age structure of India. Since there is no record of

[12]The models were developed from the data recorded and summarized by Swift (1979), My models are based on his work, although the results differ in detail. The cattle model, in particular, was varied to yield herds with proportions of draft animals in line with 'traditional' herds in Africa (Barrett 1991, Fall 1995).

market prices in the Neolithic, the 'prices' are taken to be the hours of labour required to obtain and process (to the point of cooking) one kilogram of food. Hence, the LP is minimizing the time required to obtain the stipulated nutrients. The technology of food procurement is summarized in the hours required to produce the various foods, so the model is a model of food production as well as consumption. Thus, my model describes a family that produces its own food.

The required nutrients are calories, protein, fat, niacin, vitamin B-1 (thiamine), vitamin C, iron, vitamin B12, and folate. The micro-nutrients prevent pellagra, beri-beri, scurvy, and anaemia, life threatening diseases. The required intake of each nutrient other than calories is set equal to the weighted average of the recommended daily requirement of that nutrient for each age/sex group where the weights are population shares in India.[13] Calories range from 1800 per person per day (close to the lowest practicable energy intake for a human society like we are describing) to about 2400 if cultivators are allowed to maximize their calorie intake. LP diets with these requirements and calories set at 2100 per day are close to the consumption pattern of poor people in poor countries in the 1960s (Allen 2017).

Additional restrictions on diet choice are of two kinds. First, the model includes an inequality to limit protein intake to the maximum level that humans can metabolize.[14] Second, maximum intakes of many plants and animals are also set to account for seasonal availability and patch limitations. At this level of abstraction, these levels are arbitrary but plausible. They broaden the diet.

<u>labour requirements</u>

Inequalities are included in the LP to limit hours worked to be no greater than hours available. The population is assumed to consist of families, each with one adult male and one adult female and 2-3 children depending on food availability.[15]

Two tasks must be performed every day of the year: grinding the family's grain (done by the woman) and tending animals (done by a man). Other farm tasks are assumed to be done in 300 days divided between soil preparation (70 days), harvest (40 days) and the rest (190 days) (USDA, n.d.). Each adult is assumed to work up to 10 hours per day, so 20 hours of labour is available each day for the production of food. Children certainly assisted adults, but child labour is not explicitly budgeted. Labour available must be greater than or equal to the labour required.

For foragers, the labour constraints are also usually irrelevant since the time required to obtain a family's subsistence was considerably less than the time available–a feature long noted by anthropologists (Sahlins 1972, Graeber and Wengrow 2012). The situation was different for early cultivators, especially those using wild seed. These cultivators used hoes and other hand tools to till the land and process the harvest. The labour constraints were binding for these people. Labour constraints were also binding for all later farmers.

[13]Further details in Appendix.

[14]This assumes that the average person in the Neolithic weighed 78% of the average adult who weighed 50 kilograms.

[15]When calories are set at 1800, only 2 children are assumed present, 2100 calories has 2.5 children, and higher levels of calories have 3 children. This means that population growth is zero at 1800 and highest when calories are highest.

Land utilization

Land requirements are not modeled for neolithic foragers since their number was low and land was probably treated as an open access common with unrestricted entry. Many of the larger animals they hunted migrated over long distances making private control impossible (North and Thomas 1977).

The situation of farmers was different. We focus on northern and southern Mesoptoamia. Settlement in these regions was in nucleated villages surrounded by fields. Archaeologists have mapped ancient sites in Mesopotamia by walking the ground and, more recently, from satellite photographs. Settlements were arranged at roughly regular intervals in a grid pattern. Typically, villages were 3-5 kilometres apart. The pattern is sharply etched in late Assyrian sites surveyed by Wilkinson and Tucker (1995, p. 184), and it serves as the empirical counterpart to the more abstract model of this paper. I imagine the Mesopotamia as a flat, featureless plain covered with a regular hexagonal tiling. The centres of the hexagons are the village sites and the area of the hexagon is the land at their disposal. The distance between villages is twice the apothem (the length of a perpendicular from the center to the center of any of the sides of the hexagon). Presumably, that distance was determined by balancing the advantages to farmers of living in close proximity to each other against the travel costs to their farmland: the greater the distance, the higher the cost. In practice, that distance was determined by georeferencing Tucker and Wilkinson's map of the Assyrian sites and then dividing the Jazira survey region into voronoi polygons. The mean area of these polygons was computed, and it was taken to be the mean area of a hexagon in the theoretical spacial model. The apothem of the hexagon could then be calculated. Each hexagon contained 873.99 hectares, and its apothem was 1.739 km. implying a distance between villages of 3.478 km. Deductions for the village site, holloways (roads) radiating through the fields, and nonagricultural land left 783.9225 hectares of agricultural land.

Dividing the agricultural land by the number of families in the village gives the average size of a farm. The land cultivated with grain on each farm is computed by dividing the family consumption of grain (family size * kg of each grain per person) plus any grain taken as rent or taxes by the yield per hectare of grain. Likewise, the family's need for pasture equals the requirement for each 'sheep' and 'goat' kept by the family divided by pasture grazed per 'sheep' or 'goat.' These needs are grossed up to village requirements by multiplying the family needs by the number of families in the village. Required land must be less than or equal to available land. Ultimately, the available land sets a limit on the size of the village population.

Maximizing the Agricultural Surplus

Using LP to minimize the equation (1) subject to the inequalities (2) and (3) generates the least cost diet that satisfies the constraints. The value of the minimized function equals the time it would have taken a family to produce its subsistence. There is no surplus in this solution.

To expand the model to include the agricultural surplus, we create a new 'food' called Surplus. It is barley that was not eaten by the people who grew it. We model surplus in this way since we regard barley as money: it was intrinsically valuable, divisible, storable, and transportable, so it could function as a store of value, a medium of exchange, and a unit of account. Barley could support local craftsmen and soldiers posted far from where it was grown. In the Old Babylon period (1894-1595 BCE) and even as late as the New Babylon period (626-539 BCE), prices were quoted in barley as well as in silver (Faber 1978, Jursa 2010). In the model, Surplus has the same labour requirements as barley but is of no nutritional value to the cultivator. The simplex algorithm can maximize the surplus subject to all of the constraints previously discussed. The solution includes the production and consumption patterns, the allocation of land and labour between arable and steppe pasture and the division among foods, and the total labour time.

References

Algaze, Guillermo (2008). *Ancient Mesopotamia at the Dawn of Civilization*, Chicago, University of Chicago Press.

Álvarez-Nogal, Carlos, and Leandro Prados de la Escosura. (2013). "The Rise and Fall of Spain, 1270–1850." *Economic History Review* Vol. 66, no. 1, pp. 1–37.

Allen, Robert C. (1992). *Enclosure and tjhe Yeoman: The Agricultural Development of the South Midlands, 1450-1850*, Oxford, Clarnedon Press.

Allen, Robert C. (1997). "Agriculture and the Origins of the State in Ancient Egypt," *Explorations in Economic History*, Vol. 34, 1997, pp. 134-54.

Allen, R. C. (2017) "Absolute Poverty: When Necessity Displaces Desire,"*American Economic Review*, Vol. 107 , pp. 2690-3721.

Allen, Robert C. (2020). "Poverty and the Labour Market: Today and Yesterday," *Annual Review of Economics*, Vol. 12, pp. 107–34.

Ayala, Gianna, Amy Bogaard, Michael Charles & John Wainwright (2022). "Resilience and adaptation of agricultural practice in Neolithic Çatalhöyük, Turkey," *World Archaeology*, DOI: 10.1080/00438243.2022.2125058

Bogaard,, Amy, Michael Charles, Katheryn C. Twiss, Andrew Fairbairn, Nurcan Yalman, Dragana Filipovi´c, G. Arzu Demirergi, F¨usun Ertuĝ, Nerissa Russell, and Jennifer Henecke (2009). "Private pantries and celebrated surplus: storing and sharing food at Neolithic Çatalhöyük, Central Anatolia," *Antiquity*, Vol. 83, pp. 649-688.

Bogaard, Amy, Robin Allaby, Benjamin S. Arbuckle, Robin Bendrey, Sarah Crowley, Thomas Cucchi, Tim Denham, Laurent Frantz, Dorian Fuller, Tom Gilbert, Elinor Karlsson, Aurélie Manin, Fiona Marshall, Natalie Mueller, Joris Peters, Charles Stépanoff, Alexander Weide & Greger Larson (2021) "Reconsidering domestication from a process archaeology perspective," *World Archaeology*, DOI: 10.1080/00438243.2021.1954990

Bogaard, A., Fochesato, M.,and Bowles., S. "The farming-inequality nexus: new insights from ancient Western Eurasia," *Antiquity*, 93 371, (2019), pp. 1129-43.

Bogaard, A., Styring, A., Whitlam, J., Fochesato, M., and Bowles, S. "Farming, Inequality, and Urbanization: A Comparative Analysis of Late Prehistoric Northern Mesopotamia and Southwestern Germany," in T.A. Kohler, and M.E. Smith, eds, *Ten Thousand Years of Inequality: The Archaeology of Wealth Differences*, (Tucson: 2018), pp. 201-29.

Bogaard, A., et al. (2025). \"Labor land, and the global dynamics of economic inequality," *PNAS*, Vol. 122, No. 16.

Bollongino, Ruth, and Joachim Burger, Adam Powell, Marjan Mashkour, Jean-Denis Vigne, and Mark G. Thomas (2012). "Modern Taurine Cattle Descended from Small Number of Near-Eastern Founders," *Mol. Biol. Evol. Letter*, Vol. 29(9), pp. 2101–2104. doi:10.1093/molbev/mss092

Borgerhoff Mulder, Monique et al. (2009). "Intergenerational Wealth Transmission and the Dynamics of Inequality in Small-Scale Societies," *Science,* Vol. 326(5953), p. 682–688. doi:10.1126/science.1178336.

Bowles, S. and Choi, J.-K. (2019). "The Neolithic Agricultural Revolution and the Origins of Private Property," *Journal of Political Economy*, Vol. 127(5), pp. 1993-2568.

Bowles, Samuel and Mattia Fochesato (2024). "The Origins of Enduring Economic Inequality," *Journal of Economic Literature*, vol. 62, no. 4, pp. 1475–1537.

Broadberry, Stepheny, Hanhui Guan and David Daokui Li (2018). "China, Europe, and the Great Divergence," *The Journal of Economic History*, Vol. 78, No. 4, pp. 955-1000.

Campbell, Bruce M.S. (2016). *The Great Transition: Climate, Disease and Society in the Late-Medieval World*, Cambridge, Cambridge University Press.

Childe, V. G. (1936). *Man Makes Himself*,

Conolly, James, and Katie Manning, Sue Colledge, Keith Dobney and Stephen Shennan (2012). "Species distribution modelling of ancient cattle from early Neolithic sites in SW Asia and Europe," *The Holocene*, Vol. 22, pp. 997-1010. DOI: 10.1177/0959683612437871

Domar, Evsey (1970). "The Causes of Slavery or Serfdom: A Hypothesis." *The Journal of Economic History*, Vol. 30, No. 1, *The Tasks of Economic History*, pp. 18-32.

Engels, R. (1953). "Letter to Marx, 6 June 1853," in Iqbal Husain, ed., *Karl Marx on India*, New Delhi, Tulika Books, 2006, pp. 263-5.

Feinman Gary M. et al. (2025). "Assessing grand narratives of economic inequality across time," *PNAS*, Vol. 122, no. 16

Fochesato, M., Bogaard, A. and Bowles, S. "Comparing ancient inequalities: the challenges of comparability, bias and precision," *Antiquity*, 93 370, (2019), pp. 853-69.

Forde, C. Daryll. (1931). "Hopi Agriculture and Land Ownership," *The Journal of the Royal Anthropological Institute of Great Britain and Ireland*, Vol. 61, pp. 357-405

Fryxell, John M., David J. T. Hussell, Anne B. Lambert, Peter C. Smith (1991). "Time Lags and Population Fluctuations in White-Tailed Deer," *The Journal of Wildlife Management*, Vol. 55, No. 3, pp. 377-385. https://doi.org/10.2307/3808963

Graeber, D. and Wengrow, D. (2022). *The Dawn of Everything: A New History of Humanity*, London, Allen Lane.

Helmer Daniel, Blaise Émilie, Gourichon Lionel, Sana Segui Maria. (2018). “Using cattle for traction and transport during the Neolithic period. Contribution of the study of the first and second phalanxes,” Bulletin de la Société préhistorique française, Vol. 115(1), pp. 71-98; doi : https://doi.org/10.3406/bspf.2018.14861

Kautsky, Karl (1902). *Die Agrarfrage*, Stuttgart , J. H. W. Dietz nachf. (g. m. b. h.).

Kelly, Morgan, and Cormac O’Grada (2012). “The preventive check in medieval and preindustrial England,” *Journal of Economic. History*, Vol. 72 (4), pp. 1015-1035

Koenig, W.D., Carmen, W.J., Stanback, M.T., Mumme, R. L. (1991). *Determinants of Acorn Productivity Among Five Species of Oaks in Central Coastal California*, USDA Forest Service General Technical Report PSW-126.

Kohler, T.A., Smith, M.E., Bogaard, A., Feinman, G.M., Peterson, C.E., Betzenhauser, A., Pailes, M., Stone, E.C., Prentiss, A.M., Dennehy, T.J., Ellyson, L.J., Nicholas, L.M., Faulseit, R.K., Styring, A., Whitlam, J., Fochesato, M., Foor, T.A., Bowles, S.. “Greater post-Neolithic wealth disparities in Eurasia than in North America and Mesoamerica,” *Nature*, 551, (2017), pp. 619-23.

Kohler, Timothy A., Bogaard, Amy, and Ortman, Scott G. (2025). “Introducing the special Feature on housing differrences and inequality over the very long term” *PNAS*, Vol. 122, no. 16

Kohler, Timothy A., et al. (2025). “Economic inequality is fueled by population scale, land-limited production, and settlement hierarchies across the archaeological record” *PNAS*, Vol. 122, no. 16

Kuijt, I., and N. Goring-Morris (2002). . “Foraging, farming, and social complexity in the Pre-Pottery Neolithic of the Southern Levant: a review and synthesis,” *Journal of World Prehistory, Vol.* 16, pp. 361-440.

Kuijt, I. & B. Finlayson (2009). “Evidence for food storage and predomestication granaries 11,000 years ago in the Jordan Valley,” *Proceedings of the National Academy of Sciences* Vol. 106: 10966-70.

Madsen, Jakob B., Peter E. Robertson, Longfeng Ye (2019). “Malthus was right: Explaining a millennium of stagnation,” *European Economic Review*, Vol. 118, pp. 51-68.

Malanima, Paolo. (2011). “The Long Decline of a Leading Economy: GDP in Central and Northern Italy, 1300–1913.” *European Review of Economic History* Vol. 15, no. 2, pp.169–219.

Malthus, Thomas Robert (1798). *An Essay on the Principle of Population*.

Marx, K. (1853). "The British Rule in India," New York Daily Tribune, 25 June, 1853 in Iqbal Husain, ed., *Karl Marx on India*, New Delhi, Tulika Books, 2006, pp. 11-17.

Mayshar, Joram, Omar Moav, and Luigi Pascali (2022). "The Origin of the State: Land Productivity or Appropriability," *Journal of Political Economy*, Vol. 130 (4), pp 1091-1144.

Mizuki, Inoue, Hiroki Itô, Michimasa Yamasaki, Shigeru Fukumoto, Yuuki Okamoto, Masaya Katsuki, Keitaro Fukushima, Masaru Sakai, Shota Sakaguchi, Daisuke Fujiki, Hikaru Nakagawa, Masae Iwamoto Ishihara, Atsushi Takayanagi (2020). "Seasonal and annual fluctuations of deer populations estimated by a Bayesian state–space model," *PLoS One*; Vol. 15(6):e0225872. doi: 10.1371/journal.pone.0225872

Palmisano, A., Dan Lawrence, D., de Gruchy, M.W., Bevan, A., Shennan, S. (2021). "Holocene regional population dynamics and climatic trends in the Near East: A first comparison using archaeo-demographic proxies," *Quartenary Science Review*, 252.

Pournelle, Jennifer R. (2003). *Marshland of Cities: Deltaic Landscapes and the Evolution of Early Mesopotamian Civilization,* University of California, San Diego, PhD. Dissertation.

Senior, Nassau William (1868). *Journals, Conversations and Essays Relating to Ireland*, London, Longmans, Green, and Company, Volume 1

Scott, James C. (2017). *Against the Grain: A Deep History of the Earliest States*, New Haven, Yale University Press.

Smith, S.J., McCarthy, B.C., Hutchinson, T.F. Snell, R.S. (2022). "Individual-level variation in reproductive effort in chestnut oak (Quercus montana Willd.) and black oak (Q. Velutina Lam.)," *Forest Ecology and Management*, Vol. 508, pp. 1-11.

Stewart, Guy R. (1940). "Conservation in Pueblo Agriculture: II. Present-Day Flood Water Irrigation," *The Scientific Monthly* , Vol. 51, No. 4, pp. 329-340.

Swift, Jeremy (1979). *The Economics of Traditional Nomadic Pastoralism: The Twareg of the Adrar N Foras (Mali)*, University of Sussex, PhD. Dissertation.

Ur, Jason (2010). *Urbanism and Cultural Landscapes in Northeastern Syria: The Tell Hamoukar Survey, 1999-2001*, Oriental Institute Publications, Vol. 137, Chicago, Oriental Institute.

Widell, M., C. Hritz, J.A.Ur, and T.J. Wilkinson (2013). "Land Use in the Model Communities," in T.J. Wilkinson, McG. Gibson, M. Widell, eds., *Models of Mesopotamian Landscapes*, BAR International Series 2552, Oxford, BAR Publishing..

Wilkinson, T.J. and D.J. Tucker (1995). *Settlement Development in the North Jazira, Iraq*, Warminster, Wiltshire, Aris & Phillips Ltd.