International Coordination of Macroprudential Policy



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Abstract

Given cross-border spillovers from macroprudential policy, international policy coordination may improve global welfare. This paper's contribution is to investigate the benefits of policy coordination in a setting where some countries are "foreign-banked", meaning their bank sector assets are mostly held by foreign credit institutions. I use a two-country model where one of the countries is foreign-banked, and where each country has a borrower subject to a collateralised borrowing constraint. The constraint depends on the value of the borrower's housing and the country's loan-to-value ratio limit, which is the instrument of macroprudential policy. The main finding is that policy coordination is beneficial for welfare. Specifically, in the uncoordinated steady state, the foreign-banked country exerts direct control over net foreign assets through its loan-to-value ratio, which it uses to its advantage. In effect, in a foreign-banked country, borrower-targeting macroprudential policy doubles as a capital control. In reality, the effect is not limited to strongly foreign-banked countries. Rather, it may apply to any country whose bank sector assets are not purely domestically owned. Given that coordination is beneficial, I use the model to characterise the coordinated optimal policy response to a housing demand shock. Such policy consists of moving each country's loan-to-value ratio inversely proportionally to its house price, which nullifies the shock's effect on debt. Coordinated policy thereby maintains household consumption and welfare at their steady state levels, preventing any adverse effects of the shock.

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1 Introduction

Since the 1990s, many emerging economies have used macroprudential policy tools. More recently however, the popularity of macroprudential policy has soared (Galati and Moessner, 2018). In particular, the 2008/09 global financial crisis exposed severe financial system vulnerabilities in developed countries, which macroprudential policy tools appear well suited to address.

Macroprudential policy would not be needed if individual economic agents always internalised their contribution to systemic financial risk. Unfortunately, they do not, as Galati and Moessner (2018) explain. These authors divide the resultant externalities into two categories. The first is time-varying, where individual behaviour contributes to the procyclicality of the financial system. Deleveraging cycles caused by collateral-based borrowing constraints are an example. The second type is structural, arising from financial interconnectedness. For example, a bank may not internalise its effect on the interconnectedness of the financial system in deciding its credit exposure to other banks. Galati and Moessner (2018) also note that both types of externality may originate from either the supply-side (lenders) or demand-side (borrowers). For example, loan-to-value ratio limits address a time-varying demand-side externality.

As the use of macroprudential tools accelerates, so too do the associated international spillovers. Spillovers arise because, when a country adjusts its credit market regulations, lenders can respond by reallocating credit towards or away from that country. Buch and Goldberg (2016) conduct a meta-study of the effects of overseas macroprudential policy on credit growth. The constituent studies all apply the same method. The identification strategy relies on individual bank-level data to mitigate the endogeneity of macroprudential policy. The meta-study finds evidence of spillover effects, although the significance and even direction of the effect varies by country, instrument, and transmission channel.

With cross-border spillovers, theory indicates that international policy coordination may improve welfare. However, that is not guaranteed. For example, Jeanne (2014) develops a two-period model in which firms borrow from consumers, and many countries interact in a global credit market. The instrument of macroprudential policy is a borrowing tax. Like the loan-to-value ratio, the borrowing tax aims to rectify a time-varying demand-side externality. Each country controls its own tax. Optimal uncoordinated policy leads countries to competitively dampen their demand for debt, causing

a lower world interest rate. However, the interaction does not create a true externality, because it is mediated by a perfectly competitive pricing mechanism. Therefore, the first welfare theorem ensures Pareto efficiency of the decentralised equilibrium. International coordination offers benefits only with additional frictions, such as unemployment and a zero lower bound on the nominal interest rate. Therefore, cross-border spillovers are necessary but insufficient to warrant cross-border coordination.

Contrasting with Jeanne (2014), the contribution of this paper is to focus on the asymmetries generated when a country is "foreign-banked", meaning that foreign banks hold most of the country's bank sector assets through subsidiaries or local branches. I aim to investigate whether macroprudential policy coordination offers benefits in that setting, and if so, how to realise the benefits. The paper's analysis focuses on a currency union, as an environment where macroprudential spillovers may be greater and policy coordination may be more readily implementable. Among Euro area countries that are also members of the Organisation for Economic Co-operation and Development (OECD), examples of strongly foreign-banked countries include Estonia with 6% of bank sector assets held by domestic credit institutions, Luxembourg with 7%, Belgium with 36%, and Finland with 38% (European Central Bank, 2016, p. 71). However, the insights regarding foreign-banked countries apply to any country with materially less than 100% of bank sector assets held domestically.

Methodologically, I follow Ferrero et al. (2018). I use a model in which the home country has a borrower and a saver, whereas the foreign country has only a borrower. In this structure, the foreign borrower's debt is internationally supplied, but the home borrower's debt is domestically supplied. I use an endogenous borrowing constraint for each borrower, equal to the value of the country's housing stock multiplied by a loan-to-value ratio. The two loan-to-value ratios serve as the instruments of macroprudential policy. The borrowing constraints are the source of externality that necessitates macroprudential policy. Specifically, the borrowers do not internalise that they affect the borrowing constraint through their impact on the house price in their country. To characterise optimal policy in the coordinated case, I use the linear-quadratic framework. For optimal policy in the uncoordinated case, analysis of the steady state is sufficient to establish the paper's results.

The main result is that macroprudential policy coordination produces welfare benefits. In particular, the steady states in the coordinated and uncoordinated cases differ substantially. In the coordinated case, I use lump sum taxes to equalise the marginal utility of consumption between all

households in steady-state. In the uncoordinated case, I show that that is not possible. Rather, the foreign policymaker uses the loan-to-value ratio to select a steady state with arbitrarily low debt and interest payments made from the foreign to the home country. In so doing, he can increase the foreign borrower's consumption at the expense of the home country households. By contrast, the home policymaker's loan-to-value ratio has little bearing on the cross-border capital position. It instead determines the split of resources between the home borrower and saver. The uncoordinated steady state therefore favours the foreign-banked country.

Coordination is beneficial in the model because the foreign country's macroprudential policy doubles as a form of capital control, whereas that of the home country does not. The reason relates to the fact that macroprudential policy functions by restricting debt. In a country where debt is supplied by domestic institutions, the effect on net foreign assets is secondary. But in a foreign-banked country, debt is a foreign liability, and the act of restricting debt has a direct effect on net foreign assets.

The analysis therefore holds real-world implications. The greater the foreign-held portion of a country's bank sector assets, the stronger the capital control effects of demand-side macroprudential policy. Consequently, there is potential for beggar-thy-neighbour behaviour, and policy coordination improves welfare.

Given the benefits of coordination, I use the model to characterise optimal coordinated policy. I show that, in response to a housing demand shock, such policy achieves the first-best welfare outcome. The shock affects the model equilibrium by changing the house price, which affects the economy through the endogenous borrowing constraint. Fortunately, the loan-to-value ratio also operates through the borrowing constraint. Optimal coordinated policy moves the loan-to-value ratio inversely proportionally to the house price, nullifying the effect of the shock on borrower debt. The consumption of each household is held at the global welfare-maximising level.

The remainder of the paper is structured as follows. Section 2 develops the model of the effects of macroprudential policy in a two-country setting. Section 3 uses the model to compare coordinated and uncoordinated optimal policy. Section 4 discusses the results. Section 5 concludes.

2 Model

This section draws methodologically from Ferrero et al. (2018). In their paper, the authors investigate the interaction between monetary and macroprudential policy in a closed economy. The model includes nominal price stickiness, and a continuum of borrowers and of savers. Borrowers are more impatient than savers, as reflected by a lower discount factor. Borrowers are subject to a collateralised borrowing constraint, which follows that introduced by Kiyotaki and Moore (1997), and uses housing as collateral. A loan-to-value ratio moderates the borrowing constraint, and can be used either as an exogenous variable or a macroprudential policy instrument. The paper analyses optimal policy in a stylised scenario of a house price boom and then bust.

The model in this section has a similar structure. I maintain the borrower and saver construction, and the collateralised borrowing constraint, due to the widespread use and influence of loan-to-value ratio limits. For example, as at December 2017, that instrument was used by 17 out of 28 European Union countries (European Systemic Risk Board, 2018, p. 24). Since many households in developed countries borrow heavily to buy property, loan-to-value ratio limits wield broad influence. They also target an important pecuniary externality, created by the link between mortgage loans and house prices.

The model I develop nevertheless differs from Ferrero et al. (2018). I ignore monetary policy, effectively assuming that a central bank fully stabilises the output gap and inflation rate. To model lender capital allocation between countries, I add an extra borrower, with their own borrowing constraint, to represent the foreign country. Therefore there are two policy instruments in the model; a loan-to-value ratio for each borrower. I also assume that the saver need not own housing. Each borrower is then forced to own the entire domestic housing stock, so that only the price of housing, not the quantity, adjusts to shocks.

To focus strongly on the macroprudential properties of the loan-to-value ratio, I simplify further. First, I assume that the borrowers' borrowing constraints always bind. This sensitises the model to loan-to-value ratio changes. It may also reduce the accuracy of business cycle modelling, but that is not the objective of this study. Second, a regular endowment replaces the production side of the economy. Third, I assume no exchange rate between the countries, and normalise the price of the

internationally traded consumption good to one. This makes the model most applicable to currency unions, in which international policy coordination is more readily practicable. Fourth, I assume log utility in consumption and housing.

2.1 Households

The households are the only agents in the model, apart from the policymakers to be introduced in the next section. I assume the saver resides in the home country. In this section, where an equation is repeated for both home and foreign borrowers, I present the home borrower equation. Foreign variables are denoted with an asterisk (*).

The saver maximises

$$\mathbb{W}_0^s = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_s^t \ln C_t^s \tag{1}$$

The home borrower maximises

$$\mathbb{W}_0^b = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_b^t (\ln C_t^b + \chi_t \ln H_t) \tag{2}$$

where $\beta_b < \beta_s$, C_t^j is consumption (with j $\in$ {s, b, b*}, for saver, borrower, and foreign borrower), H_t is housing, and χ_t is an exogenous, time-varying housing preference, defined by $\chi_t = \chi e^{u_t}$ where u_t is an AR(1) shock. A separate housing preference parameter χ_t^* exists for the foreign country borrower.

2.1.1 Budget constraint

The saver's budget constraint is

$$C_t^s + D_t + D_t^* = Y + R_{t-1} D_{t-1} + R_{t-1}^* D_{t-1}^* - T_s \tag{3}$$

The home borrower's budget constraint is

$$C_t^b + R_{t-1} D_{t-1} + Q_t H_t = Y + D_t + Q_t H_{t-1} - T_b \tag{4}$$

where D_t is the borrower's debt at the end of the period, Q_t is the price per unit of housing, Y is the per-period endowment, R_t is the interest rate on domestic borrowing, and T_b and T_s are the borrower and saver lump sum taxes, which are negative for a subsidy.

2.1.2 Borrowing constraint

The home borrower collateralised borrowing constraint is

$$R_t D_t \leqslant Q_t H_t \Theta_t \tag{5}$$

where Θ_t is the loan-to-value ratio for the home borrower. The borrowing constraint is assumed to bind.

The borrowing constraint creates a role for macroprudential policy intervention. The general reasoning is given in Galati and Moessner (2018). Households do not internalise their influence on their borrowing constraint, which runs through their effect on Q_t. A pecuniary externality results, leading to overborrowing that reduces welfare. The borrowing constraint also creates an amplification mechanism for shocks to χ_t, the exogenous housing demand. For example, a negative shock will reduce Q_t, tightening the borrowing constraint, requiring the borrower to cut back on consumption and housing, and further reducing Q_t. Kiyotaki and Moore (1997) explore the amplification of shocks due to this type of mechanism.

2.2 First-order conditions

The saver Euler equations are

$$\begin{aligned} 1 &= \beta_s R_t \mathbb{E}_t \left(\frac{C_t^s}{C_{t+1}^s} \right) \\ 1 &= \beta_s R_t^* \mathbb{E}_t \left(\frac{C_t^s}{C_{t+1}^s} \right); \text{hence } R_t = R_t^* \end{aligned} \tag{6}$$

Since the two interest rates are equal, I use the home interest rate hereafter.

The borrower Euler equation is

$$1 = R_t \left[\beta_b \mathbb{E}_t \left(\frac{C_t^b}{C_{t+1}^b} \right) + \phi_t \right] \tag{7}$$

where ϕ_t is the multiplier on the following version of the borrowing constraint: $\frac{D_t}{C_t^b} \leqslant \frac{Q_t H_t \Theta_t}{C_t^b}$. It is thereby normalised by C_t^b.

The housing demand equation is

$$\frac{Q_t}{C_t^b} = \frac{\chi_t}{H_t} + \frac{Q_t}{C_t^b} \phi_t \Theta_t + \beta_b \mathbb{E}_t \left(\frac{Q_{t+1}}{C_{t+1}^b} \right) \tag{8}$$

The left-hand side shows the marginal cost of housing to the borrower, in utility terms. The right-hand side shows the marginal benefit of housing. It is composed of the marginal utility of an extra unit of housing today (first term), the marginal utility of relaxing the borrowing constraint by purchasing housing (second term), and the marginal, discounted utility of the value of housing in the next period (third term).

2.3 Market clearing

The following market clearing conditions are imposed.

$$C_t^s + C_t^b + C_t^{b*} = 3Y \tag{9}$$

$$H_t = H_t^* = H \tag{10}$$

where H is the total housing available in each country.

2.4 Equilibrium conditions

There are 12 endogenous variables in this model, which are C_t^s, C_t^b, C_t^{b*}, $H_t, H_t^*, D_t, D_t^*, Q_t, Q_t^*, R_t, \phi_t$, and ϕ_t^*. Their equilibrium is fully characterised by the numbered equations (3) to (10), where equations (4), (5), (7), and (8) are included twice, once for each borrower, and where Equation (9) can be omitted due to Walras' law.

2.5 Steady state

To implement the model, I derive a steady state in which all consumption levels are equal.

$$C^b = C^s = C^{b*} = Y \tag{11}$$

where the last equality arises from Equation (9). I set the household lump sum taxes T_s, T_b, and T_b^* to levels that ensure this condition holds. Later, I show that the condition leads to an efficient steady state, which is necessary for the derivation of optimal coordinated policy.

From equation (6), for savers

$$R = \frac{1}{\beta_s} \tag{12}$$

Therefore, from equation (7) for borrowers

$$1 = R(\beta_b + \phi)$$

$$\phi = \beta_s - \beta_b \tag{13}$$

The steady state of the saver's budget constraint, Equation (3), is

$$C^s + D + D^* = Y + RD + RD^* - T_s \tag{14}$$

Setting $C^s = Y$ gives

$$\begin{aligned} T_s &= (R-1)(D+D^*) \\ &= \left(\frac{1}{\beta_s} - 1\right)(D+D^*) \end{aligned}$$

Similarly, the steady state of the borrower's budget constraint, Equation (4), with $C^b = Y$, gives

$$T_b = -\left(\frac{1}{\beta_s} - 1\right) D \tag{15}$$

The steady state of the housing demand equation, Equation (8), is

$$Q = \frac{\chi C^b}{H(\omega - \beta_b)} = \frac{\chi Y}{H(\omega - \beta_b)} \tag{16}$$

where $\omega = 1 - \phi\Theta = 1 - (\beta_s - \beta_b)\,\Theta$. Lastly, D is pinned down by the borrowing constraint.

$$\begin{aligned} RD &= QH\Theta \\ D &= \frac{\chi C^b \Theta \beta_s}{\omega - \beta_b} = \frac{\chi Y \Theta \beta_s}{\omega - \beta_b} \end{aligned} \tag{17}$$

The steady state values of the foreign country variables are determined similarly.

2.6 Calibration

For β_s and β_b, there are two relevant considerations. Firstly, β_s determines the real interest rate. Secondly, β_b must be less than β_s, consistent with the borrower being the more impatient of the two. I adopt a yearly conversion of the calibration used by Ferrero et al. (2018).

For Θ and Θ^*, as at December 2017, of the 17 European Union members with loan-to-value ratio limits (European Systemic Risk Board, 2018, p. 24), nine were in the Euro area. The average of the

currency union-member limits was 91%, which I adopt for the steady state loan-to-value ratio policy in both countries.

For calibration of the steady state housing preference parameters, χ and χ^*, Equation (17) gives the steady state debt to income ratio as $\frac{D}{Y} = \frac{\chi\Theta\beta_s}{\omega-\beta_b}$. Given parameters β_b, β_s, and Θ (and thus ω), calibrating χ and χ^* is equivalent to calibrating the debt to income ratio for each borrower. To proceed, I note that the overall ratio of residential mortgages to GDP in the Euro area was 41.5% in 2017 (European Mortgage Federation, 2018, p. 116). In the model, the total GDP is the income of all three households, but the total household debt is split among only the two borrowers. Therefore, I assume that the borrowers evenly share the total debt, leading to a borrower debt/income ratio of $0.415 \times 3/2 = 0.6225$.

For the persistence of housing demand shocks, ρ, as per Ferrero et al. (2018), I use a high value, setting $\rho = 0.9$, consistent with slow adjustment of housing preferences.

2.7 Results

Figure 1 shows the response of model variables to a 1% increase (loosening) of the loan-to-value ratio of the home country, Θ_t. Here, I use the loan-to-value ratio not as a policy instrument, but an exogenous variable. Through its effect on the domestic borrowing constraint, the shock leads to higher domestic debt. The saver, facing more demand for domestic lending, responds by lending less to the foreign borrower, to maintain her consumption levels. The model replicates the anticipated international credit rebalancing by lenders in response to macroprudential policy. In addition, the model makes a prediction for the direction of the spillover; credit is reallocated towards the country with looser macroprudential policy.

The figure also illustrates the amplification role of the house price. The extra debt strengthens the domestic borrower's short-term demand for housing. The resulting higher house price further relaxes the borrowing constraint, leading to even more debt. The overall greater demand for debt following the relaxation of Θ_t is reflected by the higher interest rate.

In the next section, coordinated optimal policy modelling will focus on a shock to χ_t, the exogenous demand for housing. Figure 2 shows the effect of such a shock when the policy variables, Θ_t and Θ_t^*, remain at their steady-state levels. The demand shock elevates the house price, Q_t, which loosens the

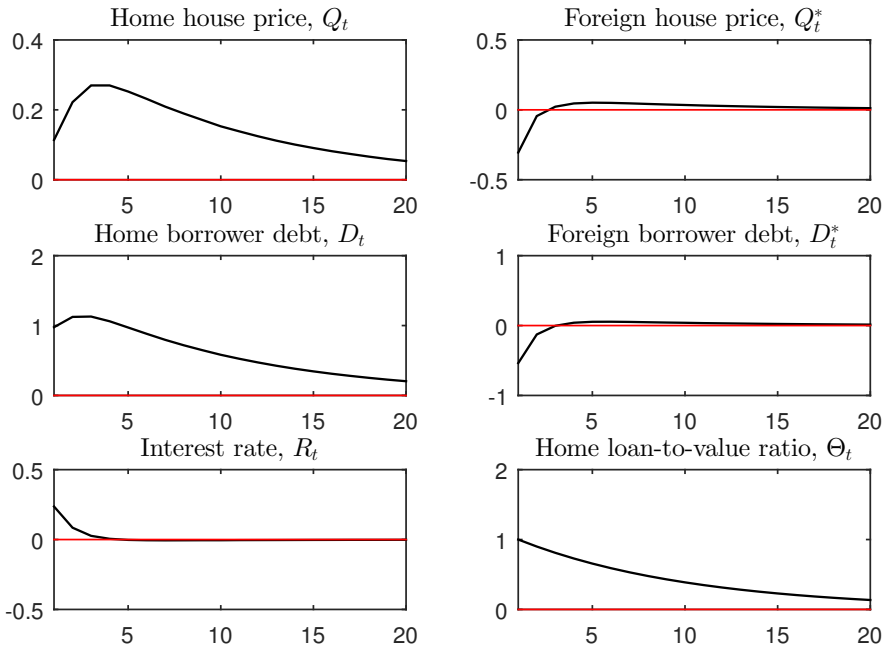


Figure 1: Responses of model variables to a 1% shock to Θ_t

In units of % deviation from steady state. Although not shown, ϕ_t and ϕ_t^* remain above zero.

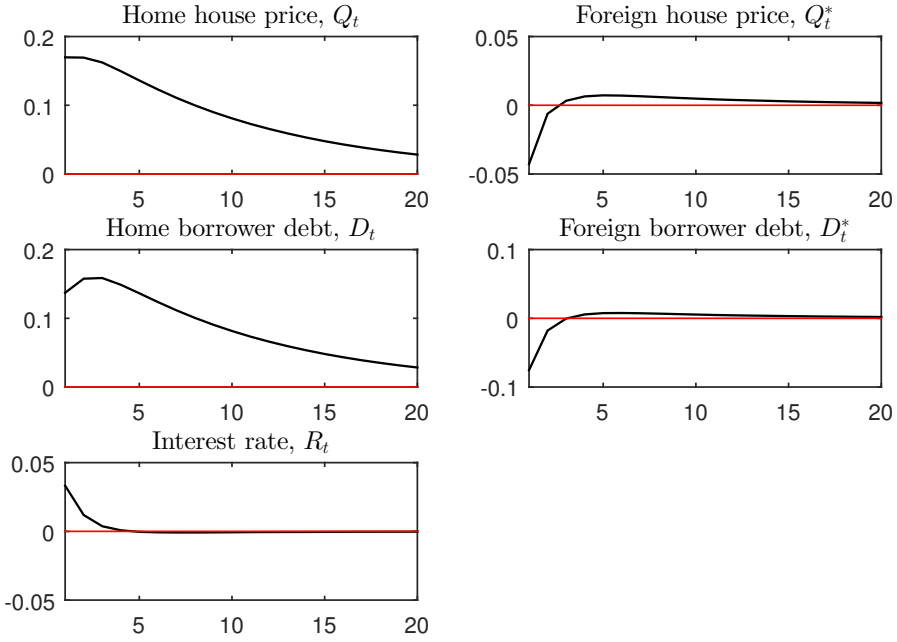


Figure 2: Responses of model variables to a 1% shock to χ_t

In units of % deviation from steady state. Although not shown, ϕ_t and ϕ_t^* remain above zero.

borrowing constraint as before, with the same subsequent effects. It is telling that both χ_t and Θ_t operate through the borrowing constraint. Evidently, the loan-to-value ratio is a powerful policy tool for responding to housing demand shocks.

3 Optimal policy

Having developed the baseline model capable of simulating the economy's reaction to loan-to-value ratio policy, I now examine the coordinated policy regimes that optimise global welfare, alongside the Nash equilibrium policies in an uncoordinated regime.

3.1 Steady state

3.1.1 Conditions for an efficient steady state in the coordinated case

Similarly to Ferrero et al. (2018), in the coordinated case I consider a social planner who maximises the following weighted sum of households' per-period welfare in steady state:

$$\mathbb{U}^c = \tilde{\xi}_s U^s(C^s) + \tilde{\xi}_b U^b(C^b, H) + \tilde{\xi}_{b*} U^{b*}(C^{b*}, H) \tag{18}$$

where $\tilde{\xi}_s, \tilde{\xi}_b$, and $\tilde{\xi}_{b*}$ are the Pareto weights (welfare weights) placed on the utility of each household by the social planner, and U^s, U^b, and U^{b*} are the single period utility functions for the saver and borrower households.

In this optimisation, the social planner is subject only to the market clearing constraints as per Equations (9) and (10). That is, to establish a first-best benchmark for policy, I do not subject the social planner to the borrowing constraint. After all, the goal of policy is to overcome the inefficiency caused by the constraint.

The planner maximises Equation (18) by choosing households' consumption only, as housing is determined by the market clearing constraints in Equation (10). The first-order conditions are:

$$\tilde{\xi}_b U^b_{C^b} = \tilde{\xi}_s U^s_{C^s} = \tilde{\xi}_{b*} U^b_{C^{b*}} = \mu_1$$

This equation shows that, for the efficient steady state to be reached, the weighted marginal utility of consumption must be equal between the three households.

I set the Pareto weights $\tilde{\xi}_s = \tilde{\xi}_b = \tilde{\xi}_{b*} = 1$ for convenience, and note that each household's marginal utility of consumption is the same function of its consumption:

$$U^j_{C^j} = \frac{1}{C^j} \quad \forall\, j \in \{s, b, b^*\}$$

Thus in the efficient steady state consumption must be equal between all households. Combined with the market clearing condition for consumption, Equation (9), this identifies Equation (11) as the condition for an efficient steady state.

3.1.2 Conditions for a Nash equilibrium steady state in the uncoordinated case

Contrary to the coordinated case, the uncoordinated case involves two national policymakers. In steady state, each maximises their domestic households' single-period welfare, taking as given the choices of the foreign household(s) and policymaker, and subject to the market clearing constraints (or, equivalently, budget constraints). As before, the borrowing constraints are not considered.

I maintain the assumption that $\beta_p = \beta_s$ for the home policymaker and $\beta^*_p = \beta_s$ similarly for the foreign policymaker. While setting $\beta^*_p = \beta_b$ may seem appropriate, it would lead to infinite borrowing. For example, setting $\beta^*_p = \beta_b$ is indistinguishable from making Θ^*_t a choice variable in the foreign borrower's optimisation problem. Therefore, the foreign household models a net borrowing country where the policymakers are more long-sighted than households; where citizens are privately impatient but support public policies that look after their collective long-term future.

The single-period welfare of the foreign borrower is:

$$\mathbb{U}^{u*} = U^{b*}(C^{b*}, H) \tag{19}$$

with terms defined in the same way as in Equation (18). Since H cannot change, and since the marginal utility of consumption is positive, the foreign policymaker would like to increase C^{b*} as much as possible. Consider the foreign borrower's budget constraint in steady state:

$$C^{b*} + (R-1)D^* = Y - T^*_b \tag{20}$$

The home saver's Euler equation determines $R = \frac{1}{\beta_s}$, and Y and T^*_b are fixed. The equation therefore shows that, the higher the foreign borrower's debt D^*, the lower will be his steady state consumption. The foreign policymaker therefore lowers D^* as much as possible. The foreign borrowing

constraint (Equation 17) shows that the policymaker would implement this by lowering Θ^*, all the way to the point where foreign consumption is zero, if allowed. Therefore, the foreign policymaker is able to implement beggar-thy-neighbour policy, leading to a Pareto-inefficient outcome and scope for international policy coordination.

To formalise, I obtain C^{b*} in terms of Θ^* and the model parameters, and show that C^{b*} increases with Θ^* despite any secondary effect through the house price Q^*. I also show that the foreign policymaker is able to raise C^{b*} independently of home country policy, Θ.

From Equation (20), and substituting in for D^* using Equation (17):

$$C^{b*} + \left(\frac{1}{\beta_s} - 1\right) \frac{\beta_s \Theta^* \chi^*}{1 - (\beta_s - \beta_b)\Theta^* - \beta_b} C^{b*} = Y - T_b^*$$

$$\frac{Y - T_b^*}{1 + \left(\frac{1}{\beta_s} - 1\right) \frac{\beta_s \Theta^* \chi^*}{1 - (\beta_s - \beta_b)\Theta^* - \beta_b}} = C^{b*}$$

Since this expression is independent of Θ, the home country's policy does not interfere. Differentiating $\frac{\Theta^*}{1-(\beta_s-\beta_b)\Theta^*-\beta_b}$ with respect to Θ^*:

$$\frac{\partial \left(\frac{\Theta^*}{1-(\beta_s-\beta_b)\Theta^*-\beta_b}\right)}{\partial \Theta^*} = \frac{1 - \beta_b}{(1 - (\beta_s - \beta_b)\Theta^* - \beta_b)^2} > 0$$

Since the inverse of the differentiated expression appears in the formula for C^{b*}, it follows that:

$$\frac{\partial C^{b*}}{\partial \Theta^*} < 0$$

The foreign policymaker has the power to increase the foreign borrower's consumption using Θ^*. Importantly, this is irrelevant of the choice of lump sum taxes. No lump sum taxes will equalise steady state consumption, unlike in the coordinated case.

The only constraint preventing Θ^* being lowered infinitely is the goods market clearing constraint, which requires $C_t^{b*} \leqslant 3Y$. However, to ensure that the foreign borrower does not become a saver, I assume that $\{\Theta_t, \Theta_t^*\} \geqslant 0$. This constraint will bind for the foreign policymaker in the steady state, so that $\Theta^* = 0$ and $D^* = 0$. Then $C^{b*} = Y - T_b^* > Y$, meaning that the foreign policymaker manages to increase his borrower's steady state consumption above the cooperative equilibrium by an amount equal to the lump sum subsidy, $-T_b^*$. Without the arbitrary restriction that $\{\Theta_t, \Theta_t^*\} \geqslant 0$, or with a less constraining restriction, the increase in consumption would be greater.

Move now to the home policymaker, who maximises the following in the steady state:

$$\mathbb{U}^c = \tilde{\xi}_s U^s(C^s) + \tilde{\xi}_b U^b(C^b, H) \tag{21}$$

Again, since housing is fixed for the borrower, it is the first-order conditions with respect to consumption that matter here. As in the coordinated case, after setting $\tilde{\xi}_s = \tilde{\xi}_b = 1$, the optimal target for policy is to set $C^s = C^b$.

Being constrained by the behaviour of the foreign policymaker, the home policymaker has less than 2Y of the consumption good to allocate between the home borrower and saver. The optimal allocation is therefore:

$$C^s = C^b = \frac{2Y - T_s - T_b}{2} = Y + \frac{T_b^*}{2}$$

Substituting this into the borrower's budget constraint gives an expression for steady state debt D:

$$D = -\frac{T_b + \frac{T_b^*}{2}}{\frac{1}{\beta_s} - 1}$$

A target value for D does imply a specific value of Θ, due to the borrowing constraint. However, the calculation must account for Q's dependence on Θ. Substituting for steady state Q gives:

$$-\frac{T_b + \frac{T_b^*}{2}}{\frac{1}{\beta_s} - 1} = \frac{\chi\Theta\left(Y + \frac{T_b^*}{2}\right)}{1 - (\beta_s - \beta_b)\Theta - \beta_b}$$

where I have used $C^b = Y + \frac{T_b^*}{2}$ instead of $C^b = Y$ as in the coordinated case. Re-arranging for Θ produces:

$$\Theta = (1 - \beta_b)\left[\frac{\chi(1 - \beta_s)\left(Y + \frac{T_b^*}{2}\right)}{-\left(T_b + \frac{T_b^*}{2}\right)} + \beta_s - \beta_b\right]^{-1}$$

With the parameter values in Section 2.6, the formula gives $\Theta = 1.05$. This exceeds the calibration of $\Theta = 0.91$ in the coordinated case. After all, interest payments need to be higher to offset the saver's missing interest payments from abroad.

3.2 Optimal policy responses to a housing demand shock

To compute the optimal coordinated policy, I employ the linear-quadratic framework. In the coordinated case, this method finds the policy that minimises deviations from the efficient steady state, subject to model agents' behaviour as constraints. In the uncoordinated case, it is not well defined, as I will explain.

3.2.1 Quadratic approximation to the welfare loss function in the coordinated case

The internationally coordinated policymaker uses the following as the social welfare function.

$$\mathbb{W}_0^u = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_p^t \mathbb{U}_t^c \tag{22}$$

where β_p is the policymaker's discount rate, and where $\mathbb{U}_t^c$ is the coordinated policymakers' utility function:

$$\mathbb{U}_t^c = \tilde{\xi}_s U^s(C_t^s) + \tilde{\xi}_b U^b(C_t^b, H_t) + \tilde{\xi}_{b*} U^{b*}(C_t^{b*}, H_t)$$

This is a time-subscripted version of the social planner objective in Section 3.1.1.

Let $\{U_C^j, U_{CC}^j\}$ be the first and second derivatives with respect to consumption of U^j at the steady state; these are the same for all $j \in \{s, b, b^*\}$. Setting $\tilde{\xi}_j = 1 \; \forall \; j$ and approximating to second-order:

$$\begin{aligned}
\mathbb{U}_t^c - \mathbb{U}^c \simeq\; & U_C^s(C_t^s - C^s) + \frac{1}{2} U_{CC}^s (C_t^s - C^s)^2 \\
& + U_C^b(C_t^b - C^b) + \frac{1}{2} U_{CC}^b (C_t^b - C^b)^2 + U_H^b(H_t - H) + \frac{1}{2} U_{HH}^b (H_t - H)^2 \\
& + U_C^{b*}(C_t^{b*} - C^{b*}) + \frac{1}{2} U_{CC}^{b*} (C_t^{b*} - C^{b*})^2 + U_H^{b*}(H_t^* - H) + \frac{1}{2} U_{HH}^{b*} (H_t^* - H)^2
\end{aligned} \tag{23}$$

The market clearing condition (9) removes all first-order consumption terms, since $U_C^s = U_C^b = U_C^{b*} = \frac{1}{Y}$. The market clearing condition (10) removes all housing terms. Therefore, since $U_{CC}^j = -\frac{1}{C^{j^2}}$:

$$\mathbb{U}_t^c - \mathbb{U}^c \simeq -\frac{1}{2} \left[\left(\frac{C_t^s - C^s}{C^s} \right)^2 + \left(\frac{C_t^b - C^b}{C^b} \right)^2 + \left(\frac{C_t^{b*} - C^{b*}}{C^{b*}} \right)^2 \right]$$

Noting that $\frac{Z_t - Z}{Z} \simeq z_t + \frac{1}{2} z_t^2$ for any variable Z_t, and ignoring terms above second-order:

$$\mathbb{U}_t^c - \mathbb{U}^c \simeq -\frac{1}{2} \left(c^{s^2} + c^{b^2} + c^{b*^2} \right) \tag{24}$$

3.2.2 Reactions to shocks in the coordinated case

The international policymaker maximises Equation (22), using the approximation of single-period welfare given in Equation (24). In equation (22) I assume the policymaker uses $\beta_p = \beta_s$. The constraints in the linear-quadratic welfare optimisation problem are the log linear equilibrium conditions derived in Appendix A. After all, optimal policy is required to be implementable given equilibrium behaviour of model agents.

I optimise policy under discretion, calculating the Markov-perfect equilibrium. Unlike models without state variables, such as the basic New Keynesian model, this model has two debt variables d_t and d_t^*, and the equilibrium depends on the interest rate r_t from the previous period. In optimising policy, the policymaker takes account of the effect of policy on future periods' welfare through the effect on state variables.

The Lagrangian function is as follows.

$$\begin{aligned}
\mathcal{L}(d_{t-1}, d^*_{t-1}, r_{t-1}, u_t, u^*_t) = \min_{\{c^s_t, c^b_t, c^{b*}_t, d_t, d^*_t, \varphi_t, \varphi^*_t, \theta_t, \theta^*_t, r_t, q_t, q^*_t\}} & \frac{1}{2}({c^s_t}^2 + {c^b_t}^2 + {c^{b*}_t}^2) + \beta_s \mathbb{E}_t \mathcal{L}(d_t, d^*_t, r_t, u_{t+1}, u^*_{t+1}) \\
& - \lambda_{xt}\left[\mathbb{E}_t c^s_{t+1} - c^s_t - \frac{\beta_b}{\beta_s}(\mathbb{E}_t c^b_{t+1} - c^b_t) + \left(\frac{\beta_s}{\beta_b} - 1\right)\varphi_t\right] \\
& - \lambda^*_{xt}\left[\mathbb{E}_t c^s_{t+1} - c^s_t - \frac{\beta_b}{\beta_s}(\mathbb{E}_t c^{b*}_{t+1} - c^{b*}_t) + \left(\frac{\beta_s}{\beta_b} - 1\right)\varphi^*_t\right] \\
& - \lambda^s_{ct}\left[\frac{D}{Y}d_t + \frac{D^*}{Y}d^*_t + c^s_t - \frac{1}{\beta_s}\left(\frac{D}{Y}(d_{t-1} + r_{t-1}) + \frac{D^*}{Y}(d^*_{t-1} + r_{t-1})\right)\right] \\
& - \lambda^b_{ct}\left[\frac{D}{Y}d_t - c^b_t - \frac{1}{\beta_s}\frac{D}{Y}(d_{t-1} + r_{t-1})\right] \\
& - \lambda^{b*}_{ct}\left[\frac{D^*}{Y}d^*_t - c^{b*}_t - \frac{1}{\beta_s}\frac{D^*}{Y}(d^*_{t-1} + r_{t-1})\right] \\
& - \lambda_{dt}[r_t + d_t - q_t - \theta_t] \\
& - \lambda^*_{dt}[r_t + d^*_t - q^*_t - \theta^*_t] \\
& - \lambda_{qt}\left[q_t - c^b_t - \frac{1}{\omega}\left((\omega - \beta_b)u_t + \beta_b \mathbb{E}_t(q_{t+1} - c^b_{t+1}) + \Theta\left(1 - \frac{\beta_b}{\beta_s}\right)(\varphi_t + \theta_t)\right)\right] \\
& - \lambda^*_{qt}\left[q^*_t - c^{b*}_t - \frac{1}{\omega^*}\left((\omega^* - \beta_b)u^*_t + \beta_b \mathbb{E}_t(q^*_{t+1} - c^{b*}_{t+1}) + \Theta^*\left(1 - \frac{\beta_b}{\beta_s}\right)(\varphi^*_t + \theta^*_t)\right)\right]
\end{aligned}$$

The first-order conditions are as follows.

$$0 = c^s_t - \lambda^s_{ct} + \lambda_{xt} + \lambda^*_{xt} \tag{25}$$

$$0 = c^b_t + \lambda^b_{ct} - \frac{\beta_b}{\beta_s}\lambda_{xt} + \lambda_{qt} \tag{26}$$

$$0 = c^{b*}_t + \lambda^{b*}_{ct} - \frac{\beta_b}{\beta_s}\lambda^*_{xt} + \lambda^*_{qt} \tag{27}$$

$$\begin{aligned}
0 = \frac{D}{Y}\left[\mathbb{E}_t(\lambda^s_{c,t+1} + \lambda^b_{c,t+1}) - \lambda^s_{ct} - \lambda^b_{ct}\right] - \lambda_{dt} - (B_{c^s} - \frac{\beta_b}{\beta_s}B_{c^b})\lambda_{xt} \\
- (B_{c^s} - \frac{\beta_b}{\beta_s}B_{c^{b*}})\lambda^*_{xt} + \frac{1}{\omega}\beta_b(B_q - B_{c^b})\lambda_{qt} + \frac{1}{\omega^*}\beta_b(B_{q*} - B_{c^{b*}})\lambda^*_{qt}
\end{aligned} \tag{28}$$

$$0 = \frac{D^*}{Y}\left[\mathbb{E}_t(\lambda^s_{c,t+1} + \lambda^{b*}_{c,t+1}) - \lambda^s_{ct} - \lambda^{b*}_{ct}\right] - \lambda^*_{dt} - (B^*_{c^s} - \frac{\beta_b}{\beta_s}B^*_{c^b})\lambda_{xt}$$

$$- (B^*_{c^s} - \frac{\beta_b}{\beta_s} B^*_{c^{b*}})\lambda^*_{xt} + \frac{1}{\omega}\beta_b(B^*_q - B^*_{c^b})\lambda_{qt} + \frac{1}{\omega^*}\beta_b(B^*_{q*} - B^*_{c^{b*}})\lambda^*_{qt} \tag{29}$$

$$0 = \ - \lambda_{xt} + \frac{\Theta}{\omega}\lambda_{qt} \tag{30}$$

$$0 = \ - \lambda^*_{xt} + \frac{\Theta^*}{\omega^*}\lambda^*_{qt} \tag{31}$$

$$0 = \lambda_{dt} + \frac{\Theta}{\omega}\left(1 - \frac{\beta_b}{\beta_s}\right)\lambda_{qt} \tag{32}$$

$$0 = \lambda^*_{dt} + \frac{\Theta^*}{\omega^*}\left(1 - \frac{\beta_b}{\beta_s}\right)\lambda^*_{qt} \tag{33}$$

$$0 = \mathbb{E}_t\left[\frac{D + D^*}{Y}\lambda^s_{c,t+1} + \frac{D}{Y}\lambda^b_{c,t+1} + \frac{D^*}{Y}\lambda^{b*}_{c,t+1}\right] - \lambda_{dt} - \lambda^*_{dt} - (B^r_{c^s} - \frac{\beta_b}{\beta_s}B^r_{c^b})\lambda_{xt}$$
$$- (B^r_{c^s} - \frac{\beta_b}{\beta_s}B^r_{c^{b*}})\lambda^*_{xt} + \frac{1}{\omega}\beta_b(B^r_q - B^r_{c^b})\lambda_{qt} + \frac{1}{\omega^*}\beta_b(B^r_{q*} - B^r_{c^{b*}})\lambda^*_{qt} \tag{34}$$

$$0 = \lambda_{dt} - \lambda_{qt} \tag{35}$$

$$0 = \lambda^*_{dt} - \lambda^*_{qt} \tag{36}$$

where, due to the envelope theorem,

$$\frac{\partial}{\partial d_t}\mathbb{E}_t\beta_s\mathcal{L}_t = \frac{D}{Y}\mathbb{E}_t(\lambda^s_{t+1} + \lambda^b_{t+1})$$

and

$$\frac{\partial}{\partial r_t}\mathbb{E}_t\beta_s\mathcal{L}_t = \mathbb{E}_t\left[\frac{D + D^*}{Y}\lambda^s_{c,t+1} + \frac{D}{Y}\lambda^b_{c,t+1} + \frac{D^*}{Y}\lambda^{b*}_{c,t+1}\right]$$

where $\mathcal{L}_t = \mathcal{L}(d_{t-1}, d^*_{t-1}, r_{t-1}, u_t, u^*_t)$. In the above I make use of the fact that the derivative of the future expectation of a variable with respect to a state variable is constant in the linear-quadratic framework. These constants are represented as follows.

$$\frac{\partial \mathbb{E}_t x_{t+1}}{\partial d_t} = B_x \qquad \frac{\partial \mathbb{E}_t x_{t+1}}{\partial d^*_t} = B^*_x \qquad \frac{\partial \mathbb{E}_t x_{t+1}}{\partial r_t} = B^r_x$$

Substituting (35) and (36) into (32) and (33) gives

$$\lambda_{dt} = \lambda_{qt} = \lambda^*_{dt} = \lambda^*_{qt} = 0 \tag{37}$$

Substituting (37) into (30) and (31) gives

$$\lambda_{xt} = \lambda^*_{xt} = 0 \tag{38}$$

Therefore, many of the model equations are non-binding for the optimal policy solution. Substituting (37) and (38) into (25), (26), and (27) gives

$$c^s_t = \lambda^s_{ct} \qquad c^b_t = \lambda^b_t \qquad c^{b*}_{ct} = \lambda^{b*}_{ct} \tag{39}$$

Substituting (39) into (28) and (29) gives

$$\mathbb{E}_t c_{t+1}^s - c_t^s = \mathbb{E}_t c_{t+1}^b - c_t^b = \mathbb{E}_t c_{t+1}^{b*} - c_t^{b*} \tag{40}$$

Since $\mathbb{E}_t c_{t+1}^s - c_t^s = r_t$, it follows that

$$3r_t = \mathbb{E}_t(c_{t+1}^s + c_{t+1}^b + c_{t+1}^{b*}) - (c_t^s + c_t^b + c_t^{b*})$$

$$r_t = 0 \tag{41}$$

due to the market clearing condition (47). From (53), (40), and (41), it also follows that $\varphi_t = \varphi_t^* = 0$.

From (40) and (41), it follows that $\mathbb{E}_t c_{t+1}^j = c_t^j$, where $j \in \{s, b, b^*\}$. Iterating forward, it is clear that as long as the economy eventually converges to steady state, then

$$c_t^j = \lim_{T \to \infty} \mathbb{E}_t c_T^j = 0 \tag{42}$$

Therefore, the policymaker is able to attain the efficient (first-best) welfare level, where the deviation of agents' consumption from steady state is zero in each period.

To derive optimal policy, substituting (41) and (42) into the equation of motion for debt, (50), gives

$$d_t = \frac{1}{\beta_s} d_{t-1} \tag{43}$$

where this and following equations in this section hold for the foreign country as well. Since $\beta_s < 1$, unless debt grows exponentially and indefinitely, $d_t = 0 \ \forall \ t$. Therefore, since debt levels do not respond to housing demand shocks under optimal policy, there is no international transmission of the shock. Setting $d_t = 0$ in the borrowing constraint (51) gives

$$\theta_t = -q_t \tag{44}$$

When shocks to housing demand occur, optimal policy reacts by moving each country's loan-to-value ratio inversely proportionately to the domestic house price. This leaves the borrowing constraint in each country unchanged, thus maintaining a constant debt level, and constant consumption level for every household.

Finally, substituting the above variable values into equation (54), I derive the behaviour of house prices under optimal policy:

$$q_t = (\omega - \beta_b) u_t + \beta_b \mathbb{E}_t q_{t+1}$$

$$= (\omega - \beta_b) \sum_{s=0}^{\infty} \beta_b^s \mathbb{E}_t u_{t+s} \tag{45}$$

Under optimal policy, a country's house price is a discounted sum of current and expected future domestic housing demand shocks. The house price depends only on domestic housing demand shocks, given the absence of international shock transmission. Since the loan-to-value ratios θ_t and θ_t^* depend only on their respective domestic house price, a shock to one country (borrower) will not affect the loan-to-value ratio in the other country.

Since a shock is not transmitted beyond the country of impact, one may suspect that policy under autarky is isomorphic to the internationally optimal policy derived above. This is indeed the case; see Appendix B. However, this isomorphism result provides little specific insight regarding the potential additional gains from cooperation in response to shocks. Specifically, in addition to a beggar-thy-neighbour steady state, uncoordinated Nash equilibrium policy may cause inefficient reactions to housing demand shocks, causing further welfare losses. I turn to this question next.

3.2.3 Limitations of the linear-quadratic framework in the uncoordinated case

Unfortunately, the linear-quadratic framework does not offer a straightforward way to derive the equilibrium policy in the uncoordinated case. This is because the second-order approximation of the welfare function normally includes first-order terms. Recall that to derive the welfare function approximation in Equation (24) I relied on the first-order consumption terms cancelling due to the market clearing condition. In the uncoordinated case, the key difference is that the national policymakers' welfare functions include fewer agents' welfare, relative to the global policymaker's. However, the market clearing condition remains unchanged. Therefore, the first-order terms remain in the welfare function. Specifically, for the home country, take $\mathbb{U}_t^u = \tilde{\xi}_s U^s(C_t^s) + \tilde{\xi}_b U^b(C_t^b, H_t)$ as the uncoordinated single-period welfare function that excludes the foreign borrower's utility. Its second-order approximation is

$$\mathbb{U}_t^u - \mathbb{U}^u \simeq c_t^s + c_t^b - \frac{1}{2}\left(c_t^{s^2} + c_t^{b^2}\right) \tag{46}$$

The negative quadratic terms from Equation (24) for the domestic country remain. However, positive linear consumption terms also enter for both borrower and saver, indicating that the national policymaker would like to increase domestic consumption at the expense of foreign consumption. The

market clearing condition, which aggregates both domestic and foreign consumption, cannot be used to eliminate the two first-order terms.

To see why first-order terms become problematic, consider that the constraints are in effect substituted into the objective function. Since the constraints are approximated to first-order, they contain second-order errors. If there are only second-order terms in the objective function, then second-order errors in the constraints are not a problem. They are transformed into fourth-order errors by the squaring, and the objective function remains accurate to second-order, as assumed. However, with first-order terms in the objective function, the second-order errors are not transformed into higher order errors, and the objective function is no longer accurate to second-order.

Other ways to derive equilibrium reactions to shocks are beyond the scope of this paper. Nevertheless, the analysis up to this point is sufficient to answer the research question; namely, whether benefits to coordination exist, and how to realise them. Specifically, this section has established the Pareto inefficiency of the uncoordinated steady state, and characterised optimal policy reactions to shocks in the coordinated case.

4 Discussion

The benefits from coordination arise from the home-foreign asymmetry. In the steady state, the foreign policymaker prefers to minimise interest payments by reducing debt, at the expense of the home country households' consumption. The home policymaker would love to retaliate. However, her loan-to-value ratio primarily controls domestic debt. The foreign policymaker takes advantage of his strong influence on net foreign assets.

The model highlights a circumstance in which demand-side macroprudential policy doubles as a capital control. To see why, consider that tighter macroprudential policy reduces indebtedness. When the debt is supplied by overseas credit institutions, tighter macroprudential policy directly increases net foreign assets. The effect is not limited to strongly foreign-banked countries. It exists to the extent that a country's bank sector assets are foreign-owned. By contrast, when debt is supplied by domestic credit institutions, macroprudential policy does still generate substitution away from foreign debt. However, that effect is smaller, as illustrated by Figure 1 in which D_t^* moves by less than 1%

in response to a 1% shock.

Ultimately, the model captures some phenomena and not others. It draws attention to foreign-banked countries' desire to cheapen foreign credit, and their incentive to promote their borrowers' interests above foreign bank owners' interests. It highlights the circumstances that cause macroprudential policy to double as a capital control. However, unlike the model's uncoordinated steady state, foreign-banked countries will not legislate their mortgage markets out of existence. Many of their citizens could not enter the housing market without mortgage loans. Further work on the model could more accurately capture the long-term value of debt to foreign-banked countries. One approach may be to impose $\beta_p^* < \beta_s$, and add an endogenous risk premium to prevent infinite borrowing.

5 Conclusion

This paper investigates whether there are benefits from international coordination of macroprudential policy, and if so, how to realise them. Its contribution is to examine an asymmetric setting where some countries are foreign-banked, meaning that they have few domestic lenders.

The model developed uses collateralised borrowing constraints in a two-country setting, with the loan-to-value ratio limit as the instrument of macroprudential policy in each country. In the model's equilibrium, if a country loosens macroprudential policy relative to others, the lender moves more credit to that country. The effects of a change in macroprudential policy are very similar to those of a housing demand shock. After all, both operate through their effect on the borrowing constraint.

The model's main result is that there are benefits to coordination. In particular, the steady state of the model differs markedly between the coordinated and uncoordinated cases. The coordinated global policymaker seeks to equalise consumption between households. By contrast, the uncoordinated national policymakers each prefer to increase their own citizens' consumption at others' expense. Given the asymmetric structure of the model, the foreign policymaker has much more power over the net foreign asset position between the countries. In the steady state, he uses his advantage to minimise his household's interest payments. In effect, his macroprudential policy tool doubles as a capital control instrument, which the home policymaker lacks. His behaviour results in Pareto-inefficient steady state consumption, and incurs welfare losses relative to the coordinated steady state.

In light of the benefits to coordination, the paper calculates the optimal coordinated policy response to housing demand shocks. Optimal policy moves the loan-to-value ratio inversely proportionally to the house price. It cancels the effect of the shock on the borrowing constraint, such that the borrower's debt is unaffected. All households' consumption levels remain at their Pareto-efficient steady state levels, and the shock is not internationally transmitted.

The inefficiency of uncoordinated policy has real-world implications. Wherever international lending activity is substantive, international coordination is beneficial. In particular, the capital control effects of demand-side macroprudential policy are not limited to strongly foreign-banked countries. They apply in proportion to a country's extent of foreign ownership of bank sector assets.

Further work could enhance the model to better simulate the effects of asymmetry. For example, the foreign policymaker could be made more impatient than his home counterpart. Another line of enquiry might investigate the benefits to coordination under symmetry. Such benefits may exist due to market power.

6 References

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7 Appendix A: Log-linear model equations

In the below, variables that are expressed log (%) deviations from their steady state appear in lower case.

The market clearing conditions, from Equations (9) and (10), are:

$$c_t^b + c_t^s + c_t^{b*} = 0 \tag{47}$$

$$h_t = h_t^* = 0 \tag{48}$$

Saver budget constraint, from Equation (3) and substituting for C^s and R:

$$c_t^s + \frac{D}{Y} d_t + \frac{D^*}{Y} d_t^* = \frac{1}{\beta_s} \frac{D}{Y} (r_{t-1} + d_{t-1}) + \frac{1}{\beta_s} \frac{D^*}{Y} (r_{t-1} + d_{t-1}^*) \tag{49}$$

Similarly, the borrower budget constraint, from Equation (4) and substituting for C^b, R, and using $h_t = 0$:

$$\frac{D}{Y} d_t = c_t^b + \frac{1}{\beta_s} \frac{D}{Y} (r_{t-1} + d_{t-1}) \tag{50}$$

Binding borrowing constraint for borrowers, from Equation (5) and using $h_t = 0$:

$$r_t + d_t = \theta_t + q_t \tag{51}$$

Saver Euler equation, from Equation (6):

$$\mathbb{E}_t(c_{t+1}^s) - c_t^s = r_t \tag{52}$$

Borrower Euler equation, from Equation (7):

$$\frac{\beta_b}{\beta_s} \left(\mathbb{E}_t(c_{t+1}^b) - c_t^b \right) = r_t + \left(1 - \frac{\beta_b}{\beta_s} \right) \varphi_t \tag{53}$$

where φ_t is the log-linearisation of ϕ_t, the Lagrange multiplier on the borrowing constraint. Also note that r_t is the log deviation of the interest rate from its steady state, which is $\frac{1}{\beta_s}$. It is not the interest rate expressed as a percentage.

Housing first-order condition, from Equation (8):

$$q_t = c_t^b + \frac{1}{\omega}\left[(\omega - \beta_b)u_t + \beta_b \mathbb{E}_t(q_{t+1} - c_{t+1}^b) + (1-\omega)(\varphi_t + \theta_t)\right] \tag{54}$$

where u_t is the AR(1) housing demand shock defined in Section 2.1.

8 Appendix B: Optimal policy under autarky

This appendix proves that optimal national policy under autarky is isomorphic to optimal internationally coordinated policy. To calculate autarkic policy, I use the same equilibrium conditions as in Section 3.2.2, while leaving out the foreign borrower and using the following updated goods market clearing condition.

$$C_t^s + C_t^b = 2Y \tag{55}$$

Define $\tilde{c}_t := c_t^b - c_t^s$ as the consumption gap, in log linear terms. Log linearising (55) gives

$$\begin{aligned} 0 &= c_t^s + c_t^b \\ \frac{1}{2}\tilde{c}_t &= c_t^b = -c_t^s \\ \frac{1}{2}\tilde{c}_t^2 &= {c_t^s}^2 + {c_t^b}^2 \end{aligned}$$

Optimal policy under autarky may be derived as follows.

$$\begin{aligned} \mathcal{L}(d_{t-1}, r_{t-1}, u_t) = \min_{\{\tilde{c}_t, d_t, \varphi_t, \theta_t, r_t, q_t\}} & \frac{1}{4}\tilde{c}_t^2 + \beta_s \mathbb{E}_t \mathcal{L}(d_t, r_t, u_{t+1}) \\ & - \lambda_{xt}\left[-\frac{1}{2}\left(1 + \frac{\beta_b}{\beta_s}\right)(\mathbb{E}_t\tilde{c}_{t+1} - \tilde{c}_t) + \left(1 - \frac{\beta_b}{\beta_s}\right)\varphi_t\right] \\ & - \lambda_{ct}\left[\frac{D}{Y}d_t - \frac{1}{2}\tilde{c}_t - \frac{1}{\beta_s}\frac{D}{Y}(d_{t-1} + r_{t-1})\right] \\ & - \lambda_{dt}[r_t + d_t - q_t - \theta_t] \\ & - \lambda_{qt}\left[q_t - \frac{1}{2}\tilde{c}_t - \frac{1}{\omega}\left((\omega - \beta_b)u_t + \beta_b\mathbb{E}_t(q_{t+1} - \frac{1}{2}\tilde{c}_{t+1}) + \Theta\left(1 - \frac{\beta_b}{\beta_s}\right)(\varphi_t + \theta_t)\right)\right] \end{aligned}$$

The first-order conditions are

$$0 = \frac{1}{2}\tilde{c}_t - \frac{1}{2}\left(1 + \frac{\beta_b}{\beta_s}\right)\lambda_{xt} + \frac{1}{2}\lambda_{ct} + \frac{1}{2}\lambda_{qt} \tag{56}$$

$$0 = \frac{D}{Y}(\mathbb{E}_t\lambda_{c,t+1} - \lambda_{ct}) - \lambda_{dt} + \frac{1}{2}\left(1 + \frac{\beta_b}{\beta_s}\right)B_{\tilde{c}}\lambda_{xt} + \frac{\beta_b}{\omega}\left(B_q - \frac{1}{2}B_{\tilde{c}}\right)\lambda_{qt} \tag{57}$$

$$0 = -\lambda_{xt} + \frac{\Theta}{\omega}\lambda_{qt} \tag{58}$$

$$0 = \lambda_{dt} + \frac{\Theta}{\omega}\left(1 - \frac{\beta_b}{\beta_s}\right)\lambda_{qt} \tag{59}$$

$$0 = \frac{D}{Y}\mathbb{E}_t\lambda_{c,t+1} - \lambda_{dt} + \frac{1}{2}\left(1 + \frac{\beta_b}{\beta_s}\right)B_{\tilde{c}}^r\lambda_{xt} + \frac{\beta_b}{\omega}\left(B_q^r - \frac{1}{2}B_{\tilde{c}}^r\right)\lambda_{qt} \tag{60}$$

$$0 = \lambda_{dt} - \lambda_{qt} \tag{61}$$

Proceeding as before, I substitute (61) into (59) to show that

$$\lambda_{dt} = \lambda_{qt} = 0 \tag{62}$$

Substituting (62) into (58) gives

$$\lambda_{xt} = 0 \tag{63}$$

Three of the constraints—the Euler equation, the borrowing constraint, and the housing demand condition—do not bind at the optimal policy equilibrium, as before.

Applying the results to (56) gives

$$\tilde{c}_t = -\lambda_{ct} \tag{64}$$

Finally, substituting these results into (57) gives

$$\mathbb{E}_t\tilde{c}_{t+1} = \tilde{c}_t \tag{65}$$

As before, as long as the model eventually attains its steady state, (65) implies that $\tilde{c}_t = -\frac{1}{2}c_t^s = \frac{1}{2}c_t^b = 0 \ \forall \ t$. In addition, note that $r_t = \mathbb{E}_t c_{t+1}^s - c_t^s = 0$. Substituting these expressions into the equation of motion for borrower debt (50) gives

$$d_t = \frac{1}{\beta_s}d_{t-1} \tag{66}$$

Since debt returns to steady state in equilibrium and therefore does not grow indefinitely, $d_t = 0 \, \forall \, t$. Substituting into (51) therefore gives the optimal loan-to-value ratio.

$$\theta_t = -q_t \tag{67}$$

as before.