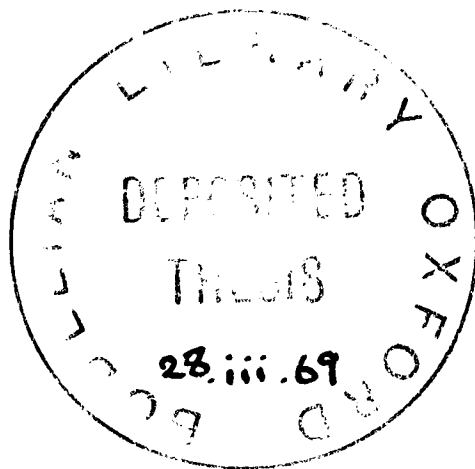


THE PROPAGATION OF ION WAVES IN THE POSITIVE COLUMN

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"When you can see a reason for doubting , at a point
where people have not so far seen a reason for doubting, then you
are making progress"

Chinese Taoist saying

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ABSTRACT

This thesis describes an experimental and theoretical investigation of equilibrium properties and low-frequency wave phenomena in the positive column of a low-pressure arc in mercury-vapour.

The structure of the positive column and the discharge kinetics were considered in detail. The conservation equations which govern them and the similarity laws which can be deduced therefrom were examined. The electron dynamics and in particular the ionization, mobility and energy processes were considered. The plasma sheath is of fundamental importance in determining the boundary conditions and mechanisms upon which many of the plasma phenomena depend. The properties of the plasma sheath in the actual conditions of a low-pressure arc were examined. Particular attention was paid to the generalization of Bohm's criterion in the presence of oscillations in the plasma as well as to the concept of sheath impedance and ways to measure this. Pressure effects in low-pressure arcs, which have been little studied and are little understood so far, are extensively analysed here and two distinct pressure effects are predicted.

The experimental investigation of the waves and oscillations was carried out with the help of phase correlation and spectral analysis techniques. Ion-acoustic waves propagating along the column and uniform oscillations of the column as a whole were found to occur spontaneously in a wide range of discharge regimes. Ion-acoustic waves could also be impressed on the plasma column by a suitable exciter.

The general properties of the ion-acoustic waves are reviewed, with special emphasis on the boundary conditions and propagating modes in a plasma column. The two-stream instability mechanism which operates in the actual conditions of a low-pressure arc is examined in detail. The properties of the observed ion-acoustic waves were found to be in

general good agreement with the available theories, although a few points still require further investigation.

In-phase and uniform oscillations have been reported to occur sometimes in plasmas. However, very little work exists in this particular field so far. The origin and nature of these oscillatory phenomena are examined and discussed. The specific case of a low-pressure positive column is considered. The theory of the purely radial ion-acoustic oscillation of a plasma column is given at first. Next, the electron temperature is allowed to vary as well, and the theory of the radial pulsation of a plasma column involving both electron temperature and density perturbations is developed. In the light of this theory, in-phase and even uniform oscillations of a plasma column are predicted. Moreover, when electron temperature and density perturbations coexist there is a net perturbation of the boundary losses experienced by the plasma column. This mechanism can give rise to the instability of the uniform oscillation of the column. It is believed that the observed large amplitude non-linear uniform oscillations of the positive column can be understood and identified in the light of this theory.

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Chapter I

INTRODUCTION

1.1. Of the different types of fluctuations which occur in a medium, the most fundamental of all are those which arise from the thermal motion of its particles. The investigation of the electron plasma waves in a plasma in thermodynamic equilibrium was done by Langmuir¹ and later by Gabor². They were able to evaluate the magnitude of the electric field of the thermal plasma waves, and to prove that the field acting on each individual electron due to all the electrons outside its Debye sphere is the field of the thermal plasma waves. Although the field of the wave is much weaker than the field produced by the electrons within the Debye sphere, the two fields are of nearly comparable importance as momentum and energy exchange processes in the plasma. The thermal plasma waves can in any case, not account for the magnitude of the commonly observed fluctuations and the strong scattering of electron beams in plasmas.

Fluctuations are generally subjected to bounds. In the case of an electron plasma whose state of lowest kinetic energy corresponds to a maxwellian distribution, the bounds are determined by the temperature and the "available energy" (which is the total energy less the minimum kinetic energy) (³ and ⁴).

Up to now we have only mentioned the magnitude and bounds of fluctuations in an electron plasma. Obviously this is still a very limited description of the fluctuations because their spectral distribution (in both space and time) as well as the role of the motion of the ions still remain to be taken into account.

The study of the thermal fluctuations in a medium can be attacked from a more radical point of view by investigating the response

of the medium to a random force. When this force is removed the response becomes a stochastic function of time whose spectral function distribution is determined by the temperature and by the admittance of the medium to that particular force. This method is based on the generalization of a Nyquist's original theorem (see Callen and Welton

⁵). In a two component and two temperature plasma the same method can be applied provided that the exchange of energy between particles of the same species is much faster than between particles of different species. This problem was studied by Salpeter ⁶ and Akhiezer et al.

⁷ (and by Dougherty and Farley in the case of a isothermal plasma ⁸). The relevant feature of the theoretical findings is that the spectral distribution of fluctuations has singularities at the same values of frequency and wave number which satisfy the plasma dispersion relation. In the range of phase velocity between the ion and the electron thermal speeds the level of charge density fluctuations increases with the electron to ion temperature ratio, and a sharp maximum is eventually defined which corresponds to the propagation of ion-acoustic waves. But even then the predicted level of fluctuations is still much lower than the commonly observed levels of ion-acoustic waves.

1.2. Among the wave phenomena observed in gas discharges the striations or waves of stratification of the positive column are the most widely known. In the Torr pressure range and at low anode current the positive column of a glow discharge is typically stratified. Striations can be seen moving towards either the cathode or the anode or just standing. They usually exhibit a backward character (when the phase and group velocities are directed on opposite directions) but other times they are forward (when the phase and group velocities have the same direction). Striations are large amplitude non-linear waves (up to 100 % emitted light modulation and 10 % anode current

modulation).

This phenomenon has been known for over one century, but it was studied in detail for the first time only in 1932 by Pupp ⁹. He established that there are critical values of gas pressure and anode current above which the striations cannot be observed, and that both the electron temperature and wall potential vary periodically along the plasma column but out of phase. More recently, it was shown that the ionization rate varies in the same way as the emitted light, and the importance of the role of the ionization in the whole wave process was stressed ¹⁰. A valuable contribution to the understanding of the striation phenomenon is due to Boyd and Twiddy ¹¹ who applied the Druyvestein method to measure the electron distribution function. They distinguished two main electron groups. The high-energy group is found to be attenuated from the head to the tail of each striation on account of inelastic collisions and wall loss, in such a way that the wall potential becomes less negative and the rate of ionization is slowed down. Then, a potential step, built up by a double charged layer at the head of the next striation, accelerates the former low-energy group of electrons producing a fresh high-energy group, and the cycle starts again.

In spite of the wide variety of distinct waves of stratification, the essential identity of the different sorts has sometimes been suggested. This identity was emphasized in a recent experiment in which an externally excited transient wave exhibited a continuous transition from a forward to a backward wave passing by a standing striation ¹².

An explanation of the stratification of the positive column was advanced by Robertson ¹³. By examining the mass balance equations of the different species one may find the equilibrium state of the uniform column to be unstable, and it was suggested that this instability

could lead to the stratification of the column. Introducing space dependent terms in the mass balance equations (axial diffusion and mobility) the wave motion is obtained and a few features of the stratification waves can be predicted. Gentle¹⁴ put forward a theory in line with Robertson's ideas, but where space charge was neglected and the electron gas was governed by Boltzmann's kinetic equation. He concluded that a backward wave would be unstable provided that the electron distribution function is far from maxwellian. He predicted the existence of an upper plasma density limit for the instability (possibly Pupp's limit), and that striations ought to occur in the inert gases and other gases with Ramsauer minima provided that the electron-neutral mean free path is smaller than the diameter of the plasma column.

However, the desintegration of the positive column into macroscopic domains is a phenomenon which has no apparent analogy with any other phenomenon of continuous media, and should not be regarded as a common wave motion. Pekarek¹⁵ advanced a new theoretical model based on this point of view. He assumed the stratification to be produced by the following chain of processes: (i) positive electron temperature perturbation, (ii) positive ionization rate perturbation, (iii) positive plasma density perturbation, (iv) positive space charge perturbation, (v) negative field strength perturbation, (vi) negative electron temperature perturbation. From the repetition of this chain of processes results the gradual macroscopic polarization of the plasma column with the creation of alternately positive and negative space charge domains. In a series of papers Pekarek and Krejci laid down the general basis of the theory of the waves of stratification¹⁶. They identified the fundamental processes necessary to the production of striations: the dependence of the rate of ionization on the electron temperature, the production of space charge due to the different

diffusion coefficients of electrons and ions, and the creation of an additional electric field. They also established that the displacement of the temperature relatively to the field perturbations is a necessary requirement for the oscillatory behaviour and amplification of the wave. Finally, a general equation was obtained to describe the wave of stratification. This, applied to the particular case of the stratification of a plasma column by a local pulse disturbance¹⁷ yielded correct predictions and shed further light into the nature of striations. The presence of metastables or existence of a non-maxwellian distribution of electrons were found to be not essential to the nature of the phenomenon (although they have been often considered to be). Besides, the oscillatory behaviour was found to be due to the perturbation of the ionization rate alone, which fact justifies the name of "wave of ionization" sometimes given to the striations. The theory of Pekarek and Krejci has been applied by a number of other authors to different experimental circumstances and found to be in general good agreement with the observed facts (see for instance Lee et al.¹⁸).

As far as low-pressure mercury-vapour arcs are concerned, a first observation was reported by Foulds¹⁹ . He found spontaneous positive striations at about 10 kHz frequency and 2×10^5 cm/s phase speed in the pressure range from 1 to 10 mTorr and up to 2 A anode current. Crawford and Pagels²⁰ , working in similar conditions, were unable to observe the stratification of the plasma column. Nevertheless, more observations of backward waves have been reported lately, down to 0.6 mTorr vapour pressure and up to 7.5 A anode current, although they generally are strongly damped.^{21, 22} and ²³ .

1.3. Ion-acoustic waves are now thought to be one of the most common types of wave motion in gas discharges. The existence of these

waves was predicted in 1929 by Tonks and Langmuir ²⁴ . The first definite observation of ion-acoustic waves was due to Revans ²⁵ who identified radial standing modes in both spherical and cylindrical discharges in mercury-vapour. But it was not until very recently that further and more extensive observations were reported. Then, a large amount of work has been produced in just a few years. Longitudinal standing waves in a magnetically supported reflex discharge ²⁶ and ²⁷ , radial standing waves in a plasma column ²⁸ , in a spherical discharge ²⁹ and in a spherical plasmoid ³⁰ have all been found. Externally excited propagating waves were observed by a few other authors ³¹ , ³² and ³³ at around the same time.

On the theoretical front, there first appeared the theory for the collisional dispersion and attenuation of the ion-acoustic waves by Hatta and Sato ³³ and Liperovskii ³⁴ , and also Fried and Gould's comprehensive paper treating this wave phenomenon from both the hydrodynamic and the Vlasov kinetic viewpoints ³⁵ . The analysis of the boundary conditions determined by the space-charge sheath at the plasma edge was done in a series of papers starting with Bertotti and Cavaliere ³⁶ . The propagating modes along a plasma column, taking in account the ionization and wall losses, were investigated by Woods ³⁷ and Davies ³⁸ .

The collisionless damping of ion-acoustic waves (Landau ³⁹) was first identified in a fully ionized gas by Wong et al. ⁴⁰ and later, in a weakly ionized gas, by Alexeff et al. ⁴¹ and ⁴² . Regarding the wave generation, the ion wave instability produced by the relative drift of ions and electrons (as in a current-sustained DC discharge) has been considered to be the most common of the generation mechanism. The theory of the ion wave instability has been extensively examined along the years starting with the works of Gordeyev ⁴³ ,

Pines and Schrieffer ⁴⁴ , Buneman ⁴⁵ and Fried and Gould ³⁵ . The experimental investigation of some of the features of this instability has been carried out by Tanaka et al. ⁴⁶ and Zaitsev and Shvilkin

⁴⁷ but much remains to be done in this field. New properties of the ion-acoustic waves are still being discovered and are open to further research, as are the cases of the amplitude oscillations exhibited by large non-linear waves ⁴⁸ , or the change of the wave character at large distances from the exciting source ⁴⁹ .

In a recent paper Ewald et al. ⁵⁰ offered a unified theory of the ion-acoustic waves and the so called "drift waves" propagating in a plasma column. These drift waves were reported to be, at low-pressure, backward waves with phase velocity far larger than the ion-acoustic speed. They were considered to be the low-pressure equivalent of the stratification waves and helical waves observed in the diffusion regime of the positive column (in the absence or presence of an axial magnetic field, respectively). Self et al. ⁵¹ carried out an experimental check of this theory by studying the propagation along a low-pressure mercury-vapour arc of externally impressed waves. They identified the principal mode of the drift wave, which exhibits no phase or amplitude radial dependence; they found this mode to be undamped (although its amplitude depends on the external discharge impedance), and to propagate at a phase velocity larger than 10^8 cm/s (far above the ion-acoustic speed). Higher non-azimuthal modes of the drift wave show poor agreement with experimental results, probably because the theory does not account for the oscillation of the electron temperature which is an essential feature of the waves of ~~the~~ stratification. However, the azimuthal modes of the drift wave show rather better agreement with the corresponding helical waves. As far as the ion-acoustic modes are concerned, the theoretical predictions of the unified theory of Ewald et al. ⁵⁰ are

in general agreement with the predictions of Woods³⁷ and Davies³⁸ theory, but it further incorporates the axial drift and an axial magnetic field which were neglected in this last theory.

A relationship between ion-acoustic and backward waves has also been suggested. An ion-acoustic wave, externally excited and propagation along a positive column with the electron drift, was found to change its character to a backward wave over a limited frequency range, provided that the electron drift speed exceeds its thermal speed

52 .

1.4. Oscillations of the positive column involving no phase variation throughout the whole column have been observed by a number of authors. An early report on the coherent oscillation of a plasma column was due to Thong Saw Pak⁵³. He found a wide band of low-frequency coherent noise in a DC gas discharge in both temperature limited and space-charge limited regimes. The oscillation was found to be propagating at a velocity far higher than 10^7 cm/s, and was not a simple modulation of the beam of primary electrons. In another experiment⁵⁴ in a magnetically supported plasma column both the light emitted throughout the plasma volume and the plasma boundary were observed to oscillate in-phase. Other authors⁵⁵ and ⁵⁶, when carrying out experiments on ion-waves in a non-magnetized plasma column, have detected a large oscillation of non-propagating nature on which the ion-wave would appear superimposed. These were constant-phase constant-amplitude oscillations which could be excited and could occur spontaneously as well²¹. Another low-frequency coherent oscillation sensitive to an applied magnetic field has been reported. The magnetic field would limit the region of correlation to a ellipsoidal volume with its long axis parallel to the field lines⁵⁷.

In a recent work Roth⁵⁸, when studying the light output

from a modified Penning discharge, found the whole plasma oscillating in unison. He was able to offer an explanation to the phenomenon in terms of periodic solutions of the continuity equations for electrons and neutrals in which the space dependent terms had been omitted. He assumed a time delay associated with the recirculation of the particles lost from the discharge which would not allow a detailed balance between the loss of plasma and the influx of neutrals to take place. In these circumstances a relaxation oscillation of the plasma density becomes possible. Frequency, wave-form and amplitude would be completely determined by the coefficients of the different terms in the continuity equations. Theoretical predictions and experimental data were found to agree in their main qualitative features. Although the explanation advanced by Roth is certainly limited to a particular type of in-phase oscillations ⁵⁹, it is, to our knowledge, the first attempt to explain the nature of in-phase oscillations in plasmas.

The importance of this type of oscillations has to be acknowledged because they seem to occur in such a variety of experimental conditions. Moreover, they could possibly account for an "instantaneous" feedback mechanism through the plasma as suggested by Tanaka et al.

⁴⁶ (if such a feedback has ever been really observed). The term "instantaneous" should not be misunderstood; it just means infinite phase velocity or, in other words, it can account for instantaneous periodic phenomena but not for transient ones.

1.5. The oscillation of a plasma column as a whole, exhibiting neither phase nor amplitude variations, have also been observed in a different context. It is well known that an electrical gas discharge can behave like a uniform impedance of lumped parameters which will necessarily be coupled to the external circuit impedance. The analysis of the impedance of a positive column was done by Granovski ⁶⁰, who

derived a law governing the discharge dynamics on the assumption that the ionization was by far the slowest of all processes in the column. The theory of the dynamics of an **univariant** and uniform positive column was done by Van Geel ⁶¹ on very general basis. According to this theory, an equivalent impedance can be defined in terms of the physical mechanisms going on in the discharge. Electron temperature and density perturbations were accounted for in a model of the discharge dynamics by Kracik ⁶² . Crawford ⁶³ ⁶⁴ studied the impedance of a low-pressure glow discharge and found it to behave like a uniform impedance to an external generator. Its impedance was analysed and its characteristic frequencies were related to the different regions and physical mechanisms operating in the discharge. If a discharge can behave like a uniform impedance, it can then be expected to oscillate uniformly either when excited by an external generator or when its impedance and the impedance of the external circuit match each other in such a way as to form an oscillating circuit ⁶¹ .

Chapter II

EXPERIMENTAL APPARATUS AND TECHNIQUES

2.1. THE EXPERIMENTAL APPARATUS

2.1.1. Our discharge tube is shown schematically in fig. 1 .

It is symmetrical and the discharge direction can be reversed when required. It is made in pyrex glass and, apart from the bulb at each end, it is uniform, 58 mm bore and 3.5 mm wall thickness. The two vertical sections of the tube are provided with 25 cm long water-cooling jackets (CJ) . At both ends there are identical electrode assemblies. These include: a mercury pool; a tungsten rod which carries the current and anchors the cathode spot (E1 and E6) ; a tungsten wire, ending close to the mercury surface, is the igniter; finally, an umbrella in Tantalum (E2 and E5) prevents the jet of mercury vapourized at the cathode spot from entering directly into the positive column (65). At the top of each vertical section there is an auxiliary anode in molybdenum (E3 and E4) which help in striking the discharge, in stages, from one end of the tube to the other. The horizontal section, between the two T-joints, is 98 cm long, and it is the useful experimental region. At both ends of the horizontal section there is a flange connection in order to allow for the interchange of different experimental facilities; this connection is sealed with an O-ring in Viton, greased with N-Apiezon, and held in position by a ring-shaped stainless spacer.

At both ends of the horizontal section of the tube there is a side arm leading to an ionization gauge (IG) ; also, but at one end only, there is a side arm leading through a cooled trap to another ionization gauge. The two former gauges give the total pressure (practically mercury only) and the latter gives the pressure of permanent gases (mainly residual air). Two mercury diffusion pumps operate at both ends

of the discharge tube (P), and are kept on all the time while the discharge is being run. Prior to pressure measurements the ionization gauges have to be outgassed; this process is facilitated by drawing a large filament current or applying a Tesla coil. Next, the gauges have to be calibrated; this has to be done in the absence of the discharge, by setting the vapour pressure by means of the automatic thermostatic system and noting the readings, and then repeating this procedure for a number of vapour pressure values.

2.1.2. The discharge is supplied by a large bank of electrolytic batteries, able to deliver up to 30 A current at 230 V . The discharge is earthed at the cathode. To start the discharge, all the electrodes are switched on to the power supply through limiting resistors. A Tesla coil is applied to the ignitor at first, and from there it is moved along the tube helping the discharge to run to the successive electrodes, till the anode is reached. At this point all the auxiliary electrodes (E2 to E5) can be switched off. When the current is large enough, say, above 3 A , the cathode spot is well anchored, but at lower current it tends to move around the anchoring tungsten rod and, at about 0.5 A , it switches off by itself. It is however possible to have a very low current discharge (down to 10 mA) by drawing a few amperes from the cathode to the umbrella (in order to stabilize the cathode spot) and then the required current to the anode. Usually, the external impedance of the discharge circuit is a resistance of the order of 10 to 100 Ω . When required, an inductive coil or capacitor can be connected in order to obtain either a very large or very small AC impedance.

2.1.3. A thermostatic system is necessary in order to control the mercury vapour pressure in the discharge. Cooling at the cathode and anode ends only is a relatively inefficient method for pressure control. The power released at the main electrodes is large, and given the low

thermal conductivity of the pyrex wall, one expects a substantial temperature difference between the external and the internal surface of the wall.

This is the reason why, besides the necessary cooling baths on the

cathode and anode bulbs (CB) we have made use of two cooling jackets on the

vertical sections (CJ) As the power released along the positive column is

small, the temperature drop across the pyrex wall is small as well, and the temperature of its inner surface can be more successfully controlled.

When the discharge is running, a liquid film of mercury is seen to develop on the surface of the cooling jackets, thick at the bottom, but becoming thinner and thinner upwards, until it eventually disappears half way along the jacket. This fact seems to suggest that a good control of the mercury-vapour pressure is being achieved.

The plan of the automatic thermostatic system is shown in fig. 2 . There are two heat circuits. The first one is operated by a compressor (C) , works with a suitable cooling fluid , and removes heat from the heat exchanger (HE) . The second circuit works with water, is driven by a centrifugal pump (P) and removes heat from the load (two cooling jackets CJ and two cooling baths CB) to the heat exchanger. The reservoir (R) contains 2 L. of water and proves to be very convenient to suppress temperature fluctuations. For the automatic control, there are two electrical circuits. One of them receives information from the contact thermometer (T1) at the inlet of the reservoir and switches on or off the compressor. The other electrical circuit receives information from the contact thermometer (T2) at the suction line of the pump and switches this one between two different adjustable speeds. When well adjusted, this system provides a reliable automatic control of the temperature. Some data about the technical features and performance of this system are listed below:

(i) Compressor: power removed, up to 3 kW ; temperature of the evaporator,

down to -18°C .

(ii) Centrifugal pump: pumping speed up to 120 gl/h .

(iii) Performance: power removed, 0.5 kW ; temperature drop across the load, 1°C ; temperature drop across each cooling jacket, 0.1°C ; temperature fluctuations at the cooling jackets, 0.1°C ; temperature range of operation, from 0.5 to 26°C

2.2. THE EXPERIMENTAL TECHNIQUES

2.2.1. The discharge tube is provided with two double electrostatic planar probes, directed radially, the first couple at 36 cm from the cathode end of the horizontal section and the second couple at 40 cm from there towards the anode. There is also a planar single probe on the same cross section and diameter as the second double probe, and facing it. Finally, there is a longitudinal planar single probe directed along the axis of the discharge. Each probe is supported by a stainless-steel slug which can be driven magnetically from the outside, the whole being completely sealed off.

The two components of each double-probe are parallel and their axis are 3.0 mm apart. Each component is made of tungsten wire, 0.6 mm in diameter, shielded by silica tubing, 1.0 mm O.D. and 0.8 mm bore. The metal and its shield are fixed together at the back end and flushed at the front end which faces the plasma.

Prior to probe measurements, a large positive bias was applied to the probes, so that they would be maintained red hot for about 15 min. in order to clean the metal surface and to vaporize any conducting film deposited around the metal tip on its shield. When this procedure was not followed, the double-probe characteristic would normally be rather asymmetrical and the measurements not reproducible.

The use of the double-probe method (see Johnson and Multer ⁶⁶

and Chen ⁶⁷) in a non-quiescent plasma is very convenient. Although the Langmuir probe method can be applied in the presence of fluctuations in such a way as to give useful information (see Garascadden and Eneleus ⁶⁸ and Kenjo and Hatta ⁶⁹), the double probe method has the advantage of being directly applicable in the presence of space-potential fluctuations and of being less disturbing to the plasma (the collected current being limited by the ion saturation current). To use this method, it is required to use a perfectly floating probe circuit (in particular, no mains-operated instruments should be used) to avoid DC and AC currents flowing to earth. When the probe characteristic is perfectly symmetrical (which requires both components to have the same area and the same contact potential), the characteristic can be easily interpreted graphically by the "equivalent resistance" method, yielding both the electron density and temperature.

Other probing methods, also employed in the present work, will be described now.

The magnetic field produced by the discharge current can be picked up by external magnetic probes. Each magnetic probe is a small low inductance coil (of the order of 0.1 mH). This is normally connected through a coaxial cable (with about 60 pF capacitance) to a high input impedance instrument (the preamplifier of an oscilloscope, typically 1 M plus 20 pF, or of a wave analyser, 100 k plus 20 pF); in these circumstances one measures the derivative of the magnetic field in a frequency range up to about 1 MHz which covers the range of interest to us. The direct measurement of the magnetic field itself would require either a low impedance connected across the coil output or an integrating circuit; in both cases one would have a substantial loss of signal level; as the signal level is already very small one would then require an amplifier; however, even a low-noise amplifier proves to be too disturbing for the

actual signal level.

Of the axial, azimuthal and radial components of the field, the azimuthal is the largest one and the one which shows the richest spectrum. The attenuation of the azimuthal field with the radial distance was checked, and this showed that the magnetic signal is a genuine measure of the current in the discharge. The influence of an electrostatic shield was checked as well, and it was established that the pick up is not of electrostatic nature in any substantial extent.

External electrodes placed upon the wall of the discharge tube are used for electrostatic pick up. The measured capacitance across the pyrex wall per unit area is 1.0 pF/cm^2 . These electrodes are narrow (1.0 cm wide) copper strips wound around the tube and can easily be moved along it. They are normally connected to a high input impedance instrument, so that the detected potential closely follows the internal wall potential itself, and it is also expected that the AC current drained by the electrode is small enough not to upset the plasma sheath dynamics in any appreciable extent. This probing device is commonly called a "capacitive probe".

Light measurements were carried out using a photomultiplier whose spectral response ranges from 1500 to 6500 $\text{\AA}$. Interference filters can be used to select particular spectral bands (100 $\text{\AA}$ width). For light measurements in the ultra-violet region, a fused silica (Vitreosil) window can be placed at the end of the horizontal section of the discharge, looking along its axis.

2.2.2. In the study of oscillatory and wave phenomena, it is common to face a situation in which the signal/noise ratio is small. It is then required to increase that ratio in some way. In some cases, as it is also ours, it is possible to use a suitable exciter to increase the amplitude of the signal (see Little ³²). Techniques based on the

electronic averaging of a large number of repetitive signals can also be applied (Alexeff and Jones ⁷⁰). However, a most widely known technique is the phase-sensitive-detection (Smith ⁷¹) which has been extensively applied in the study of ion waves. The phase-sensitive-detection technique is based on the averaging of the product of two signals, and it yields information on the phase and amplitude of an unknown signal within a very narrow band-width of a reference one; this band-width is determined by the actual time constant of the averaging device, and it is typically of the order of a few Hz .

The experimental set up for the phase correlation study of spontaneous waves in the positive column as reported in this thesis is shown in fig. 3 . It will be analysed in some detail now. Let the two filters be tuned to the same frequency ω_0 , and let $G(\omega, \omega_0)$ denote the transference function of each filter-amplifier set; let τ be the time delay introduced by the phase-shifter. The two signals coming from the two probes are $X_1(\omega)$ and $Y_1(\omega)$; the two signals at the input of the phase-sensitive-detector are

$$X_2(\omega, \omega_0) = G(\omega, \omega_0) X_1(\omega)$$

$$Y_2(\omega, \omega_0, \tau) = \exp(-j\omega\tau) G(\omega, \omega_0) Y_1(\omega)$$

The phase-sensitive-detector performs then the product of the two signals; the frequency spectrum of its output will therefore be the convolution of the Fourier transforms of the two inputs:

$$v(\omega, \omega_0, \tau) = \frac{1}{2\pi} \int X_2(s, \omega_0) Y_2(\omega-s, \omega_0, \tau) ds$$

The continuous component (which is the output of the averaging circuit) is obtained by taking $\omega = 0$, and one arrives at

$$v(\omega_0, \tau) = \frac{1}{2\pi} \int |G(s, \omega_0)|^2 X_1(s) Y_1^*(s) \exp(js\tau) ds$$

Now notice that $G(\omega, \omega_0)$ is non-zero in a narrow interval around ω_0 only. Introduce the filter band-width $\Delta\omega$. The integral above can be approximated by:

$$v(\omega_0, \tau) = |G|^2 |X_1| |Y_1| \cos(\phi + \omega_0 \tau) \Delta\omega / 2\pi$$

where all quantities refer to ω_0 and ϕ is the difference of phase between X_1 and Y_1 . When one of the two signals is known and the other one is unknown, the latter's phase and amplitude can be determined by adjusting the time delay τ to obtain maximum output.

The experimental arrangement used in the phase-correlation study of externally excited waves is shown in fig. 4. Without going into the mathematical details of its analysis, it will be enough to say that, once again, it is possible to determine the phase and amplitude of the detected signal relatively to the exciter.

The phase-sensitive-detector and the phase-shifter of our experiments have a band-width from 1 to 300 kHz (Brookdeal). The screening and earthing of the whole circuit requires a careful attention. The cases of all the instruments are earthed through their power cables, and the commons and screens are connected all together and earthed at one point only; cases and commons ought to be not short-circuited.

2.2.3. Two types of sources have been employed for the excitation of waves. First a grid, 40 mm in diameter, made of a star in platinum supported by a circular nickel frame. The grid could either be biased or floating, and the AC current drawn by it was nearly 50 mA. The mechanism of excitation is thought to be two-fold: on one hand a local density modulation, on the other hand the oscillation of the plasma boundary (the plasma-sheath interface). Another source which has also been used is a coil of 120 turns and 4 cm long wound around the discharge tube. This is fed simultaneously by a DC and a AC current which produce a field of the order of 3 and 0.5 Gauss respectively. The presence of a large DC component ensures that the field direction is never reversed; otherwise the main excited frequency would be twice the applied one, because the excitation mechanism depends on the magnitude of the field alone (not

on its direction). The excitation mechanism here is supposed to be two-fold as well: a density modulation due to the compression by the field, and a local longitudinal electric field at the end of the coil due to the transition between two different potential profiles. The exciting coil is provided with an inner and an outer screen to minimize the direct pick up by the detector; this aspect proved to be essential for the successful use of this type of exciter.

The sources were supplied by a power-amplifier (up to 5 W power output and up to 200 kHz band-width) driven by a low distortion oscillator (both Hewlett Packard).

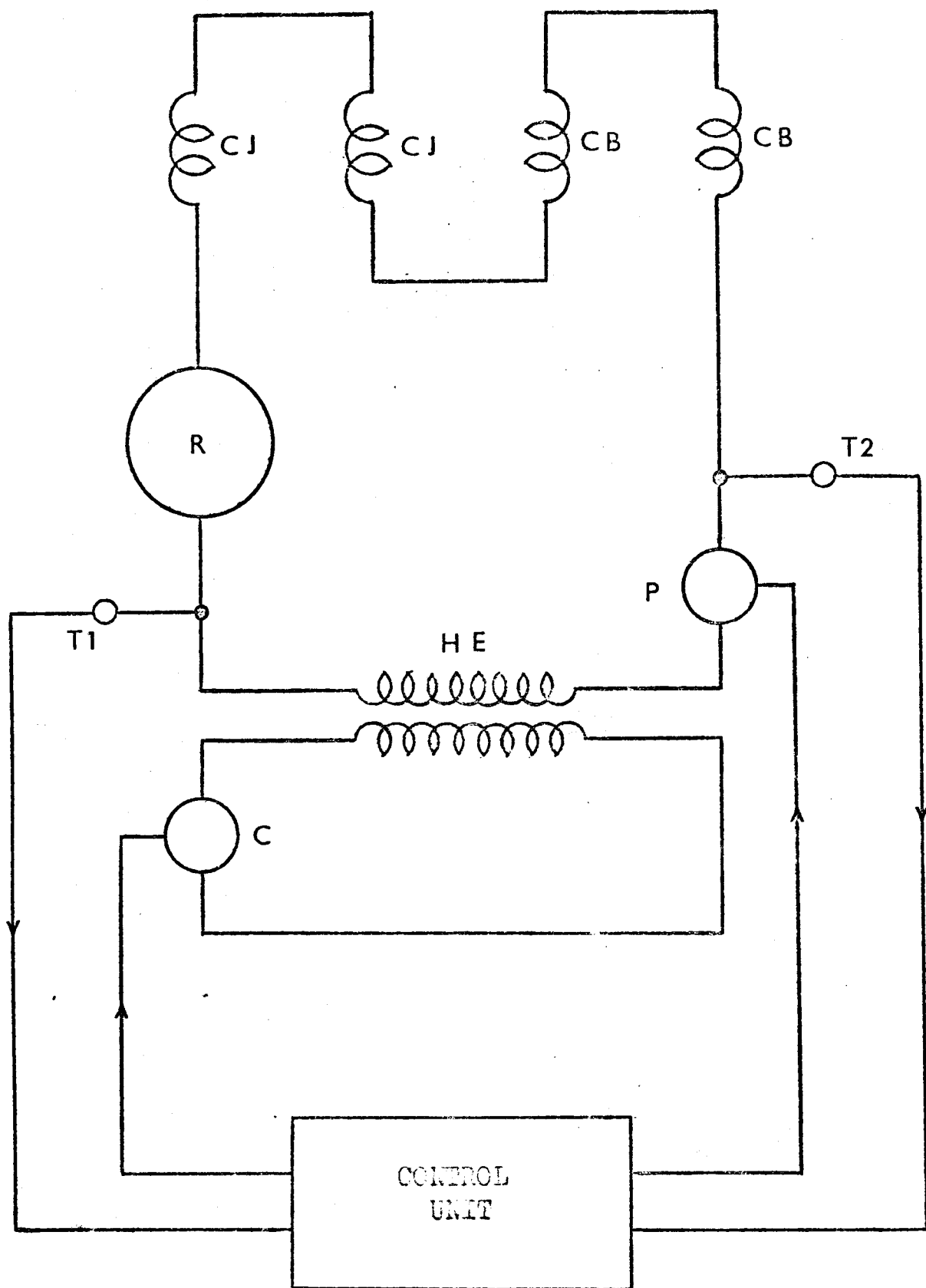


Fig . 2. Automatic thermostatic system .

CJ : Cooling jackets ; CB : Cooling baths ; P : Water pump
R : Reservoir ; C : Compressor ; HE : Heat exchanger
T : Contact thermometer ;

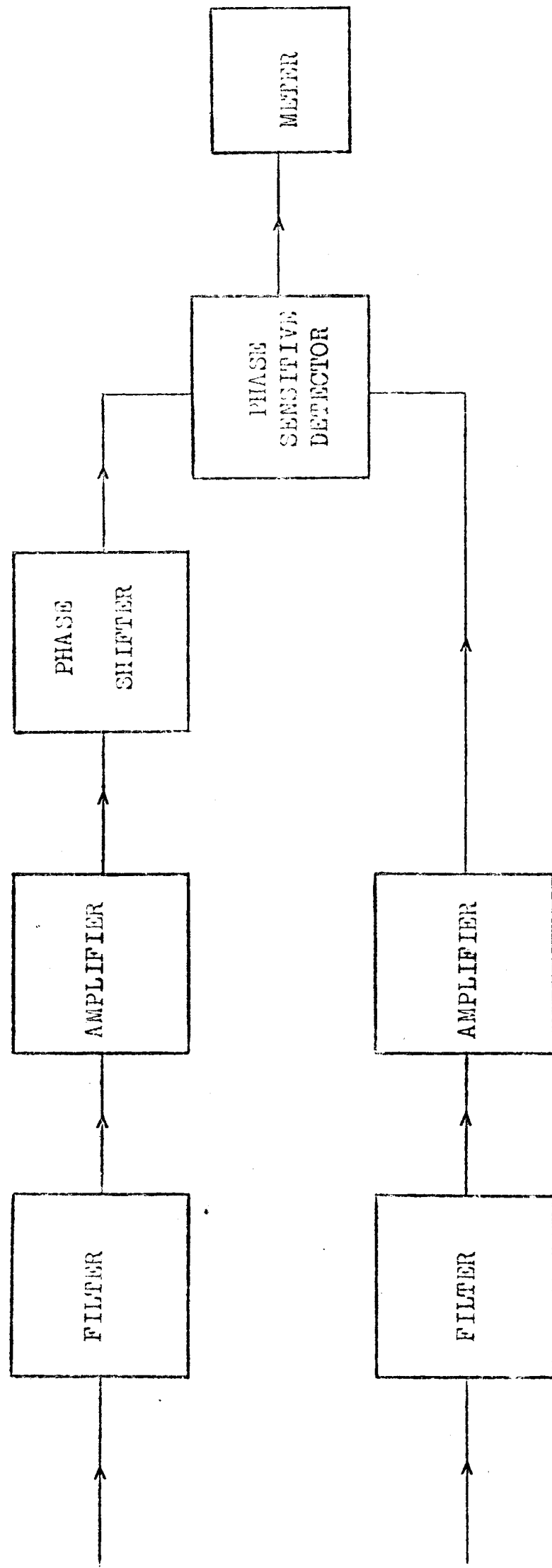


Fig. 3. EXPERIMENTAL ARRANGEMENT FOR THE STUDY OF SPONTANEOUSLY OCCURRING WAVES

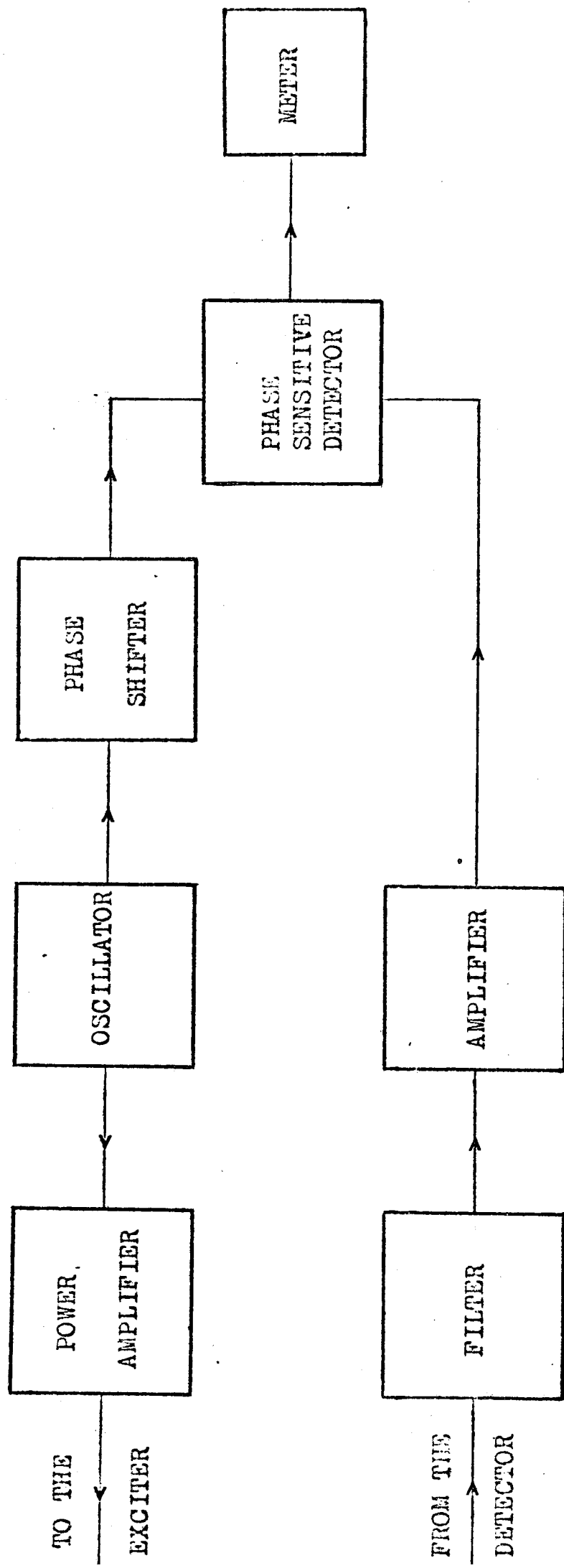


Fig. 4. EXPERIMENTAL ARRANGEMENT FOR THE STUDY OF EXTERNALLY EXCITED WAVES

Chapter III

EXPERIMENTAL RESULTS

3.1. VAPOUR-PRESSURE AND ELECTROSTATIC PROBE MEASUREMENTS

3.1.1. In a low-pressure current-sustained DC discharge the gas kinetic stress tensor will in general be anisotropic. The anisotropy is associated with the neutral gas flow from the cathode to the anode due to the pumping effect of electron-neutral collisions. Besides, there may also exist pressure gradients, associated with either temperature gradients or the above mentioned pumping effect. In view of these facts one might expect to obtain different readings from two pressure gauges connected to different points along the discharge tube or differently orientated towards the discharge current.

Our pressure measurements were taken as the ratios of the reading increment produced by the discharge to the readings as obtained in its absence: $\Delta p/p$. The measurements of the two pressure gauges were then combined to give the semi-sum and the ~~semi~~-difference of the relative increments. In this way one obtains a measure of the isotropic and the anisotropic components of the stress tensor, or of the average pressure rise and its gradient if the column proves to be not uniform.

The influence of the discharge current upon the pressure can be appreciated in table 1. The data recorded here was obtained with a discharge in which the mercury-vapour pressure in the temperature controlled sections was maintained at $p = 0.50$ mTorr. The temperature of the glass wall of the horizontal section of the tube was also recorded, and this is thought to give a good estimate of the gas temperature there. One observes that, when the discharge is switched on, the pressure rises at both ends. It is interesting to notice that, while the pressure at both ends rises rapidly with current, their difference rises rather

more slowly. Having in mind that the horizontal section of the tube is several times longer than the distance from the T-joints to the temperature controlled jackets, one is led to admit that the observed pressure rise cannot be due to a mere pressure gradient built up. The pumping effect can possibly account for the differential pressure because one of the gauges faces the cathode while the other one faces the anode. But the large rise of the ^{average} pressure is not accounted for in an obvious way.

Pressure measurements at other values of the vapour-pressure in the temperature controlled jackets are shown in table 2 . It is clear that the lower the gas density the larger the pressure effects are, specially so with the isotropic component of the stress tensor (or the average pressure). The rise of average gas pressure is a very substantial effect, and it ought to be taken in consideration when interpreting data obtained in experiments in the positive column.

3.1.2. Our data on electron temperature and density in the positive column was obtained with double electrostatic probes. An example of the double-probe characteristic is shown in fig. 5 . The plot is rather symmetrical and can therefore be quickly interpreted graphically by the "equivalent resistance" method (Johnson and Walter⁶⁶). In the range of plasma parameters used here, the ratio of the probe diameter to the Debye length lies between 10 and 30 , and the interpretation of the ion-current from the thin-sheath point of view seems acceptable. All the ions entering the sheath are assumed to be collected by the probe. However, the variation of the ion-current with the sheath thickness has been taken in account and it was suitably extrapolated from the saturated regions towards the zero probe current point. The collecting area at zero current has been taken to be the probe area. The ion-current density has been equated to en_1c where n is the local plasma density and $c = (kT_e/m_i)^{1/2}$ has been taken as Bohm's speed .

Once the plasma density and the discharge current are known, the electron drift speed can be calculated, and this has been done assuming a Tonks-Langmuir density profile. The axial electric field was obtained from the variation of the floating potential along the axis. For this purpose, two radial electrostatic probes, 40 cm apart, were employed. The drop in potential was measured by means of a high input impedance voltmeter connected to either one or the other probe, or by applying a voltage source between them and adjusting for zero current.

The DC probe measurements were found to be insensitive to large changes of the external discharge circuit impedance provided that the current was kept the same. Having in consideration that the level of oscillations in the column is substantially affected by large changes of the external impedance, one is led to admit that the DC probe measurements are not being affected by the presence of oscillations in the column.

Our probe measurements are summarized in table 3 .
from $p = 0.25$ to 2.0 mTorr . One can notice the general trend shown by the electron temperature to decrease and by the axial field to increase with the discharge current. The values of the plasma density obtained at low current (1 A) may be considered to be relatively too large. This could be due to an effective probe collecting area larger than the assumed one, because the ratio of the probe diameter to the Debye length is smaller at lower plasma density.

TABLE 1

Discharge parameters		Pressure increments		$\Delta T / T$
p (mTorr)	I(Amp)	$\Delta p / p$		
		Isotropic	Anisotropic	
0.50	2.5	0.06	0.03	
	3.0	0.05	0.04	0.11
	5.0	0.15	0.08	0.15
	6.0	0.20	0.11	
	7.0	0.24	0.15	0.19
	7.5	0.26	0.19	
	8.0	0.30	0.19	
	9.0	0.30	0.26	
	10.0	0.42	0.31	0.25
	12.0	0.48	0.28	0.28
	15.0	0.75	0.64	0.34

TABLE 2

Discharge parameters		Pressure increments	
p (mTorr)	I (Amp)	$\Delta p / p$	
		Isotropic	Anisotropic
0.20	2.5	0.17	0.01
	5.0	0.32	0.15
	7.5	0.37	0.20
	10.0	0.52	0.31
0.50	2.5	0.06	0.03
	5.0	0.15	0.08
	7.5	0.26	0.19
	10.0	0.42	0.31
0.80	2.5	0.03	0.07
	5.0	0.13	0.04
	7.5	0.23	0.10
	10.0	0.36	0.24

TABLE 3

Discharge parameters		Electrostatic probe data				
p (mTorr)	I (Amp)	kT (eV)	n (10^{11}) cm ⁻³	u (10^7) cm/s	a _o (10^5) cm/s	X (V/cm)
0.25	1.0	2.72	0.255	1.32	1.14	0.042
	3.0	2.88	0.75	1.35	1.18	
	5.0	2.85	1.20	1.40	1.17	
	7.0	2.52	1.60	1.48	1.10	0.052
	10.0	2.40	2.10	1.60	1.07	
0.50	1.0	2.30	0.280	1.20	1.05	0.063
	3.0	2.40	0.66	1.54	1.07	0.065
	5.0	2.48	1.01	1.67	1.09	0.068
	7.0	2.25	1.43	1.65	1.04	0.072
	10.0	2.20	2.02	1.67	1.03	0.082
	13.0	2.05	2.65	1.66	0.99	0.088
1.00	1.0	1.82	0.265	1.28	0.94	0.105
	3.0	1.90	0.68	1.49	0.96	
	7.0	1.80	1.59	1.48	0.93	0.120
	10.0	1.92	2.33	1.45	0.96	
1.50	5.0	1.85	1.20	1.40	0.95	
	7.0	1.80	1.70	1.39	0.93	
2.00	5.0	1.65	1.32	1.28	0.89	
	7.0	1.45	2.08	1.14	0.83	

kT - electron thermal energy
 n - plasma density on the axis
 u - electron drift speed
 a_o - ion-acoustic speed
 X - axial electric field

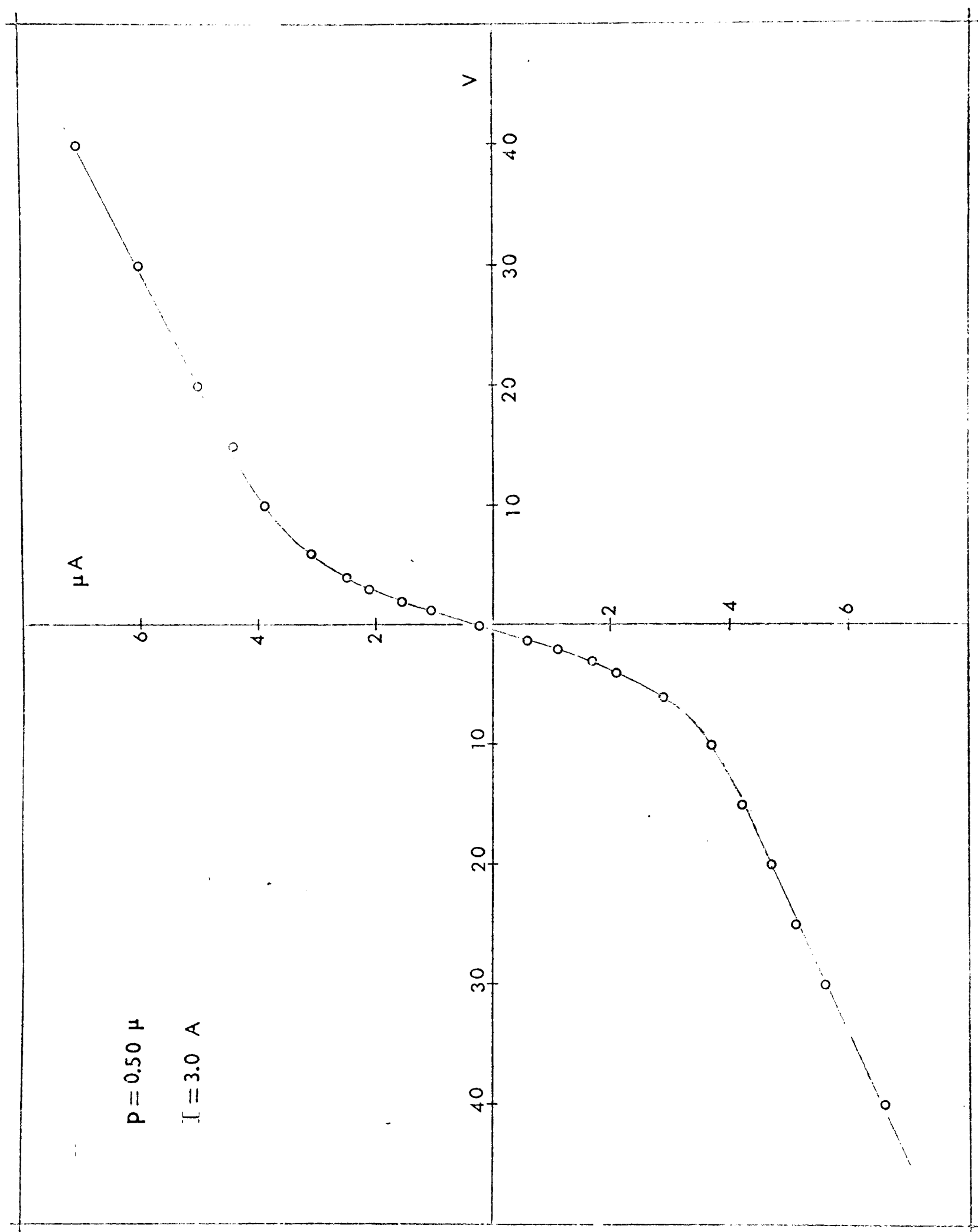


Fig. 5. Example of double-probe characteristic

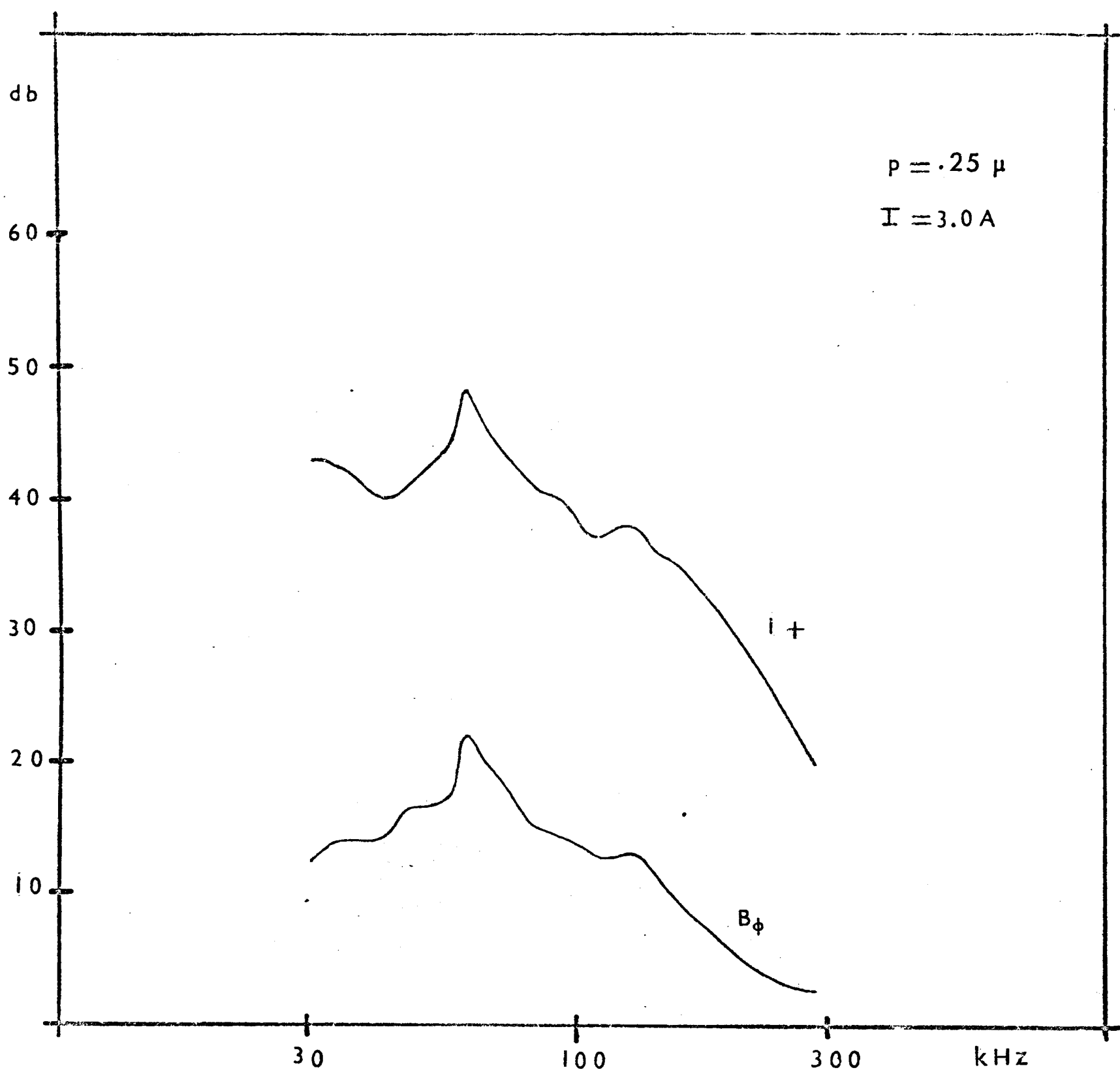


Fig. 6a. Spectra of fluctuations ; $p = 0.25 \frac{\text{mTorr}}{\mu}$ $I = 3.0 A$.

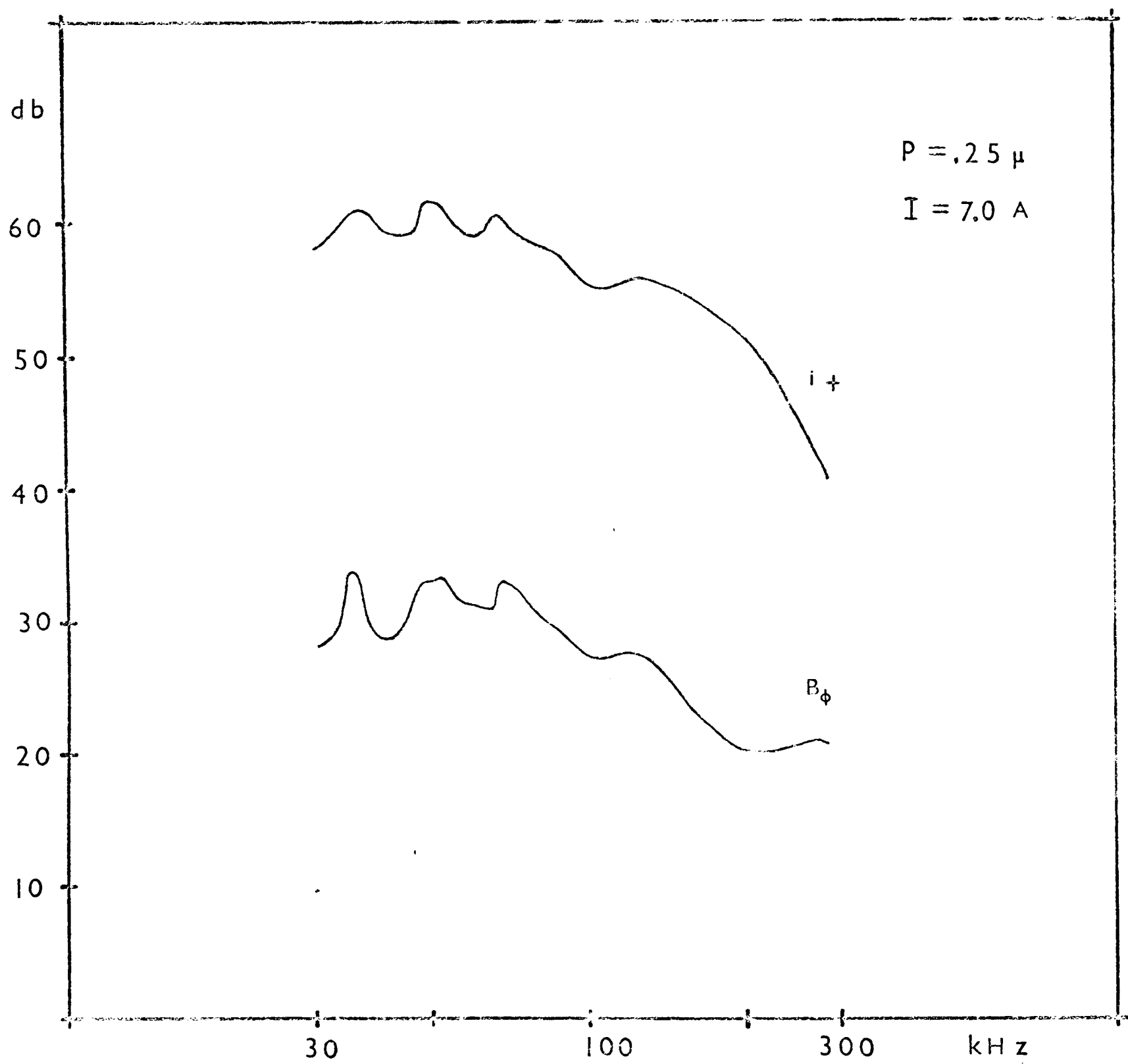


Fig. 6b. Spectra of fluctuations ; $p = 0.25 \frac{\text{mTorr}}{\text{A}}$ $I = 7.0 A$.

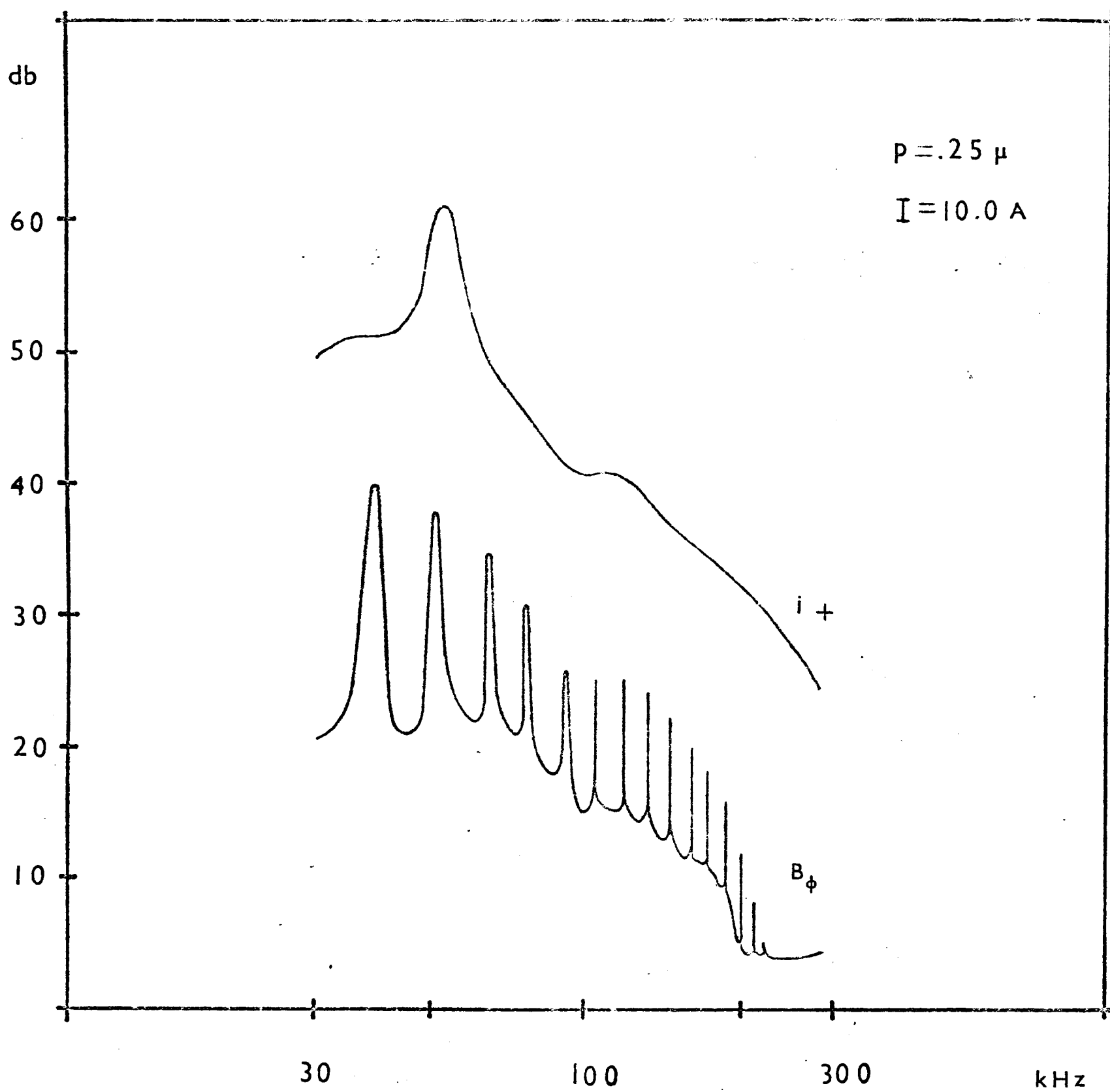


Fig. 6c. Spectra of fluctuations ; $p = 0.25 \frac{\text{mTorr}}{\text{A}}$ $I = 10.0 A$.

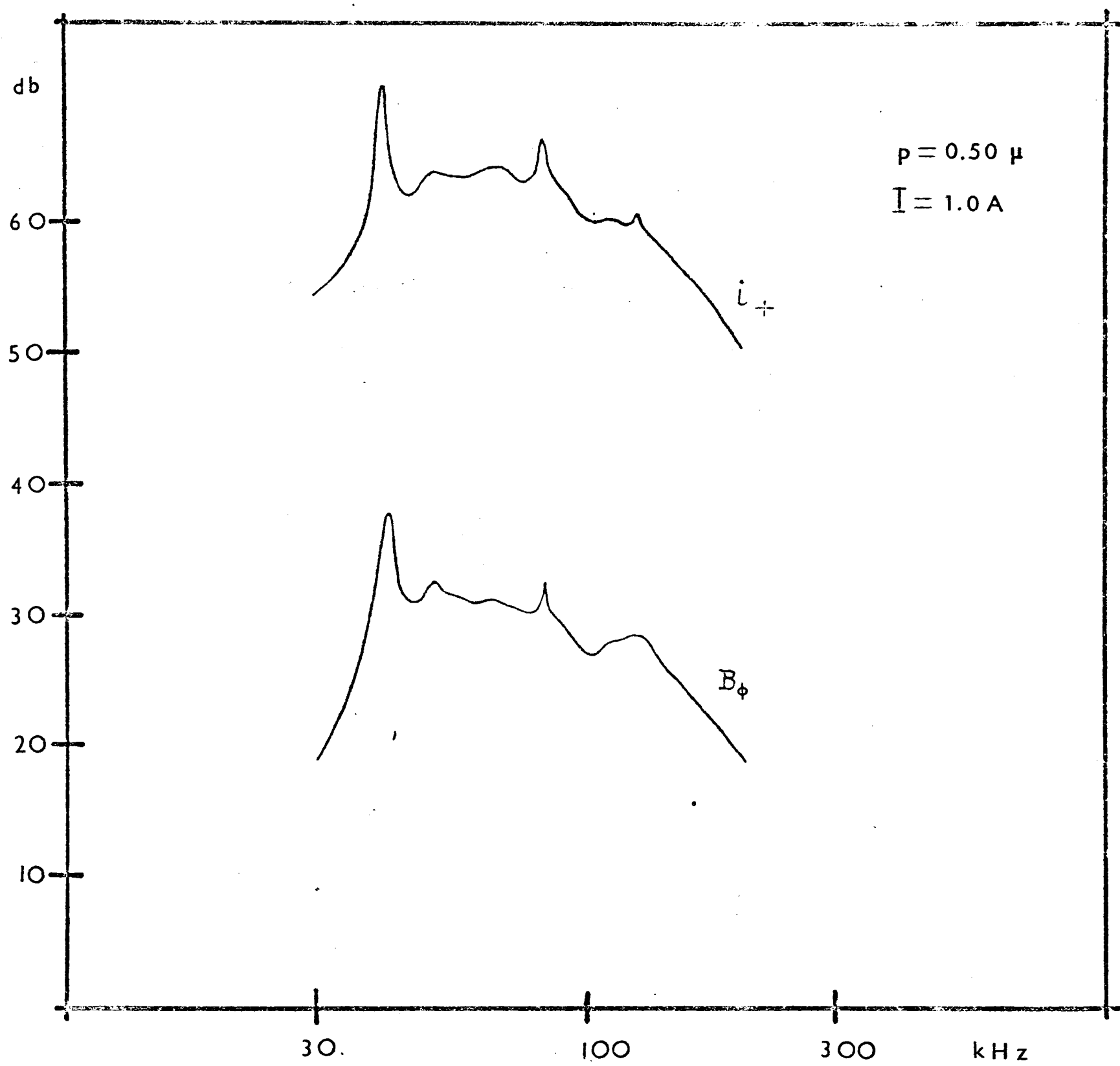


Fig. 7a. Spectra of fluctuations ; $p = 0.50$ $\frac{\text{mTorr}}{\text{A}}$ $I = 1.0 A$.

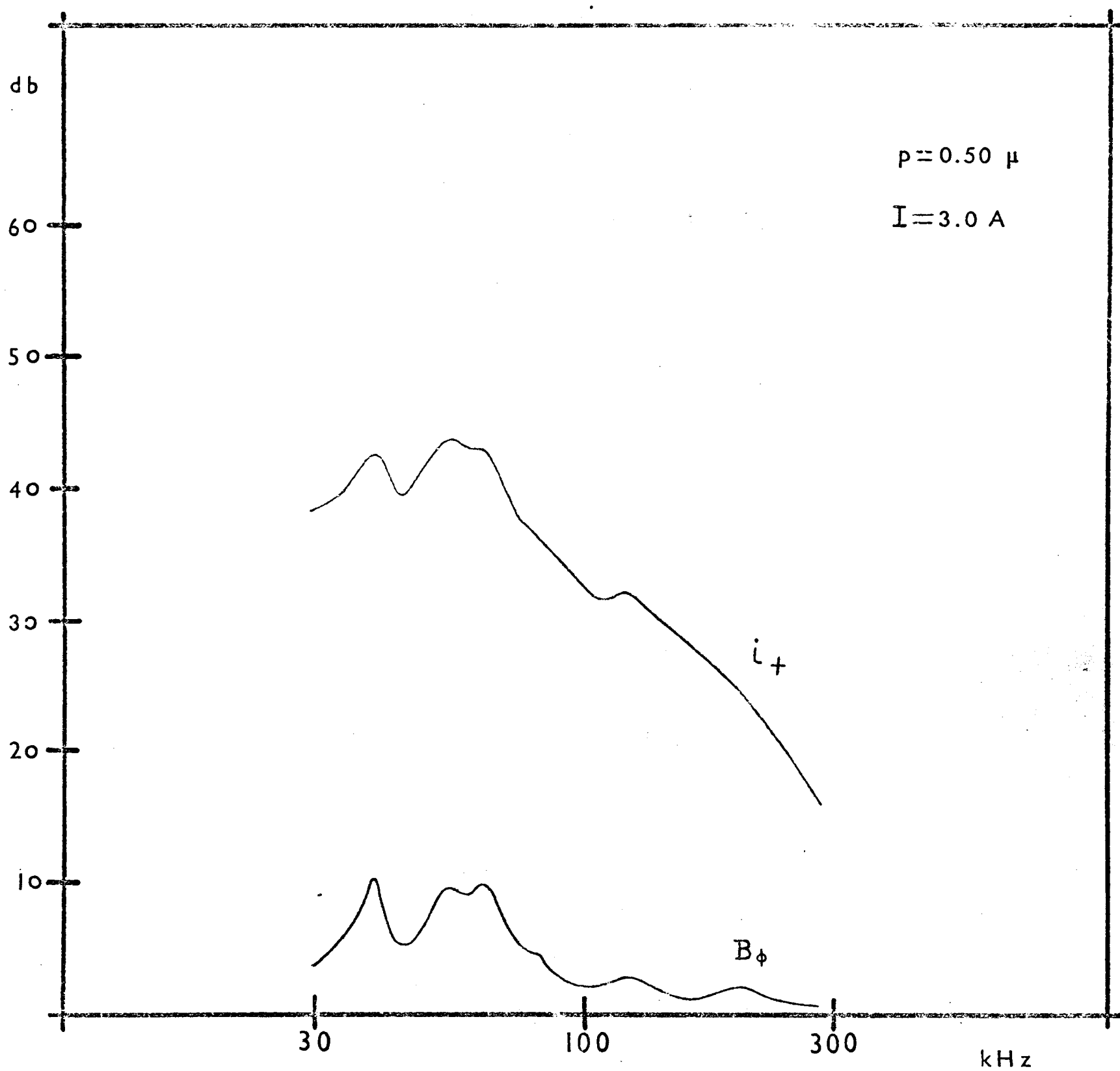


Fig. 7b. Spectra of fluctuations ; $p = 0.50 \frac{\text{mTorr}}{\mu}$ $I = 3.0 A$.

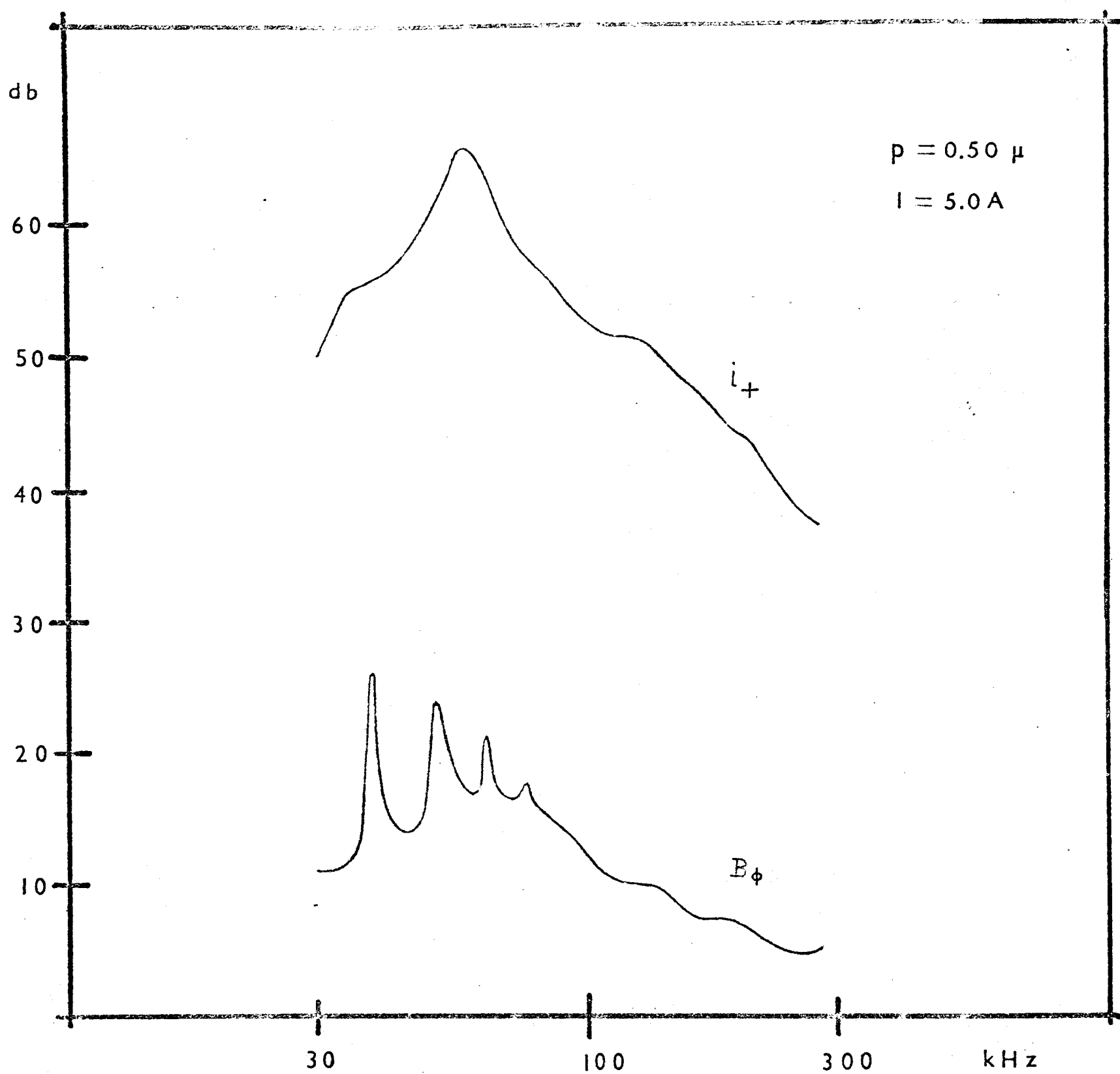


Fig. 7c. Spectra of fluctuations ; $p = 0.50 \mu$ $I = 5.0 A$.

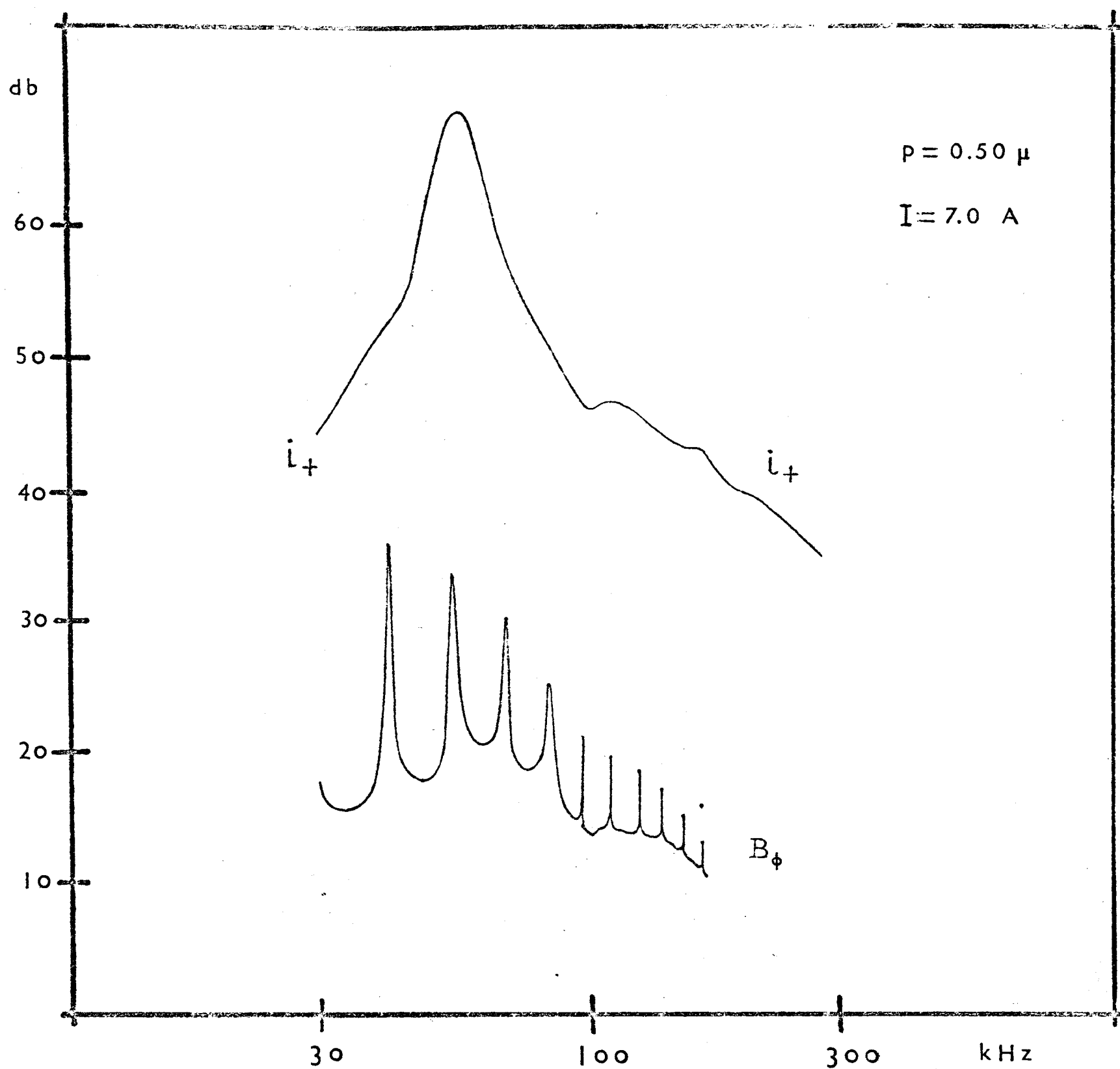


Fig. 7d. Spectra of fluctuations ; $p = 0.50$ $\frac{\text{mTorr}}{\text{A}}$ $I = 7.0 A$.

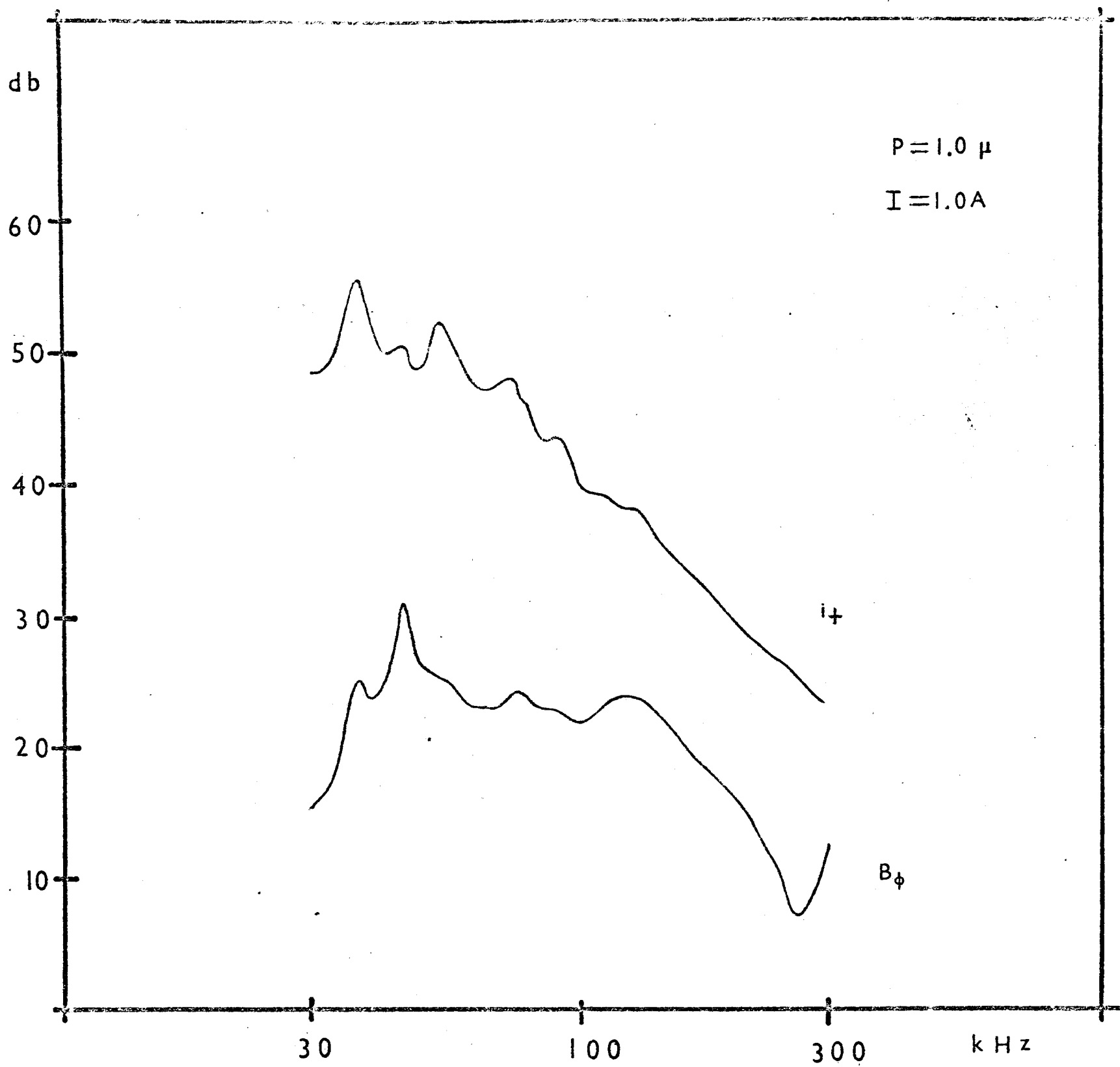


Fig. 8a. Spectra of fluctuations ; $p = 1.0 \frac{\text{mTorr}}{\text{A}}$ $I = 1.0 A$.

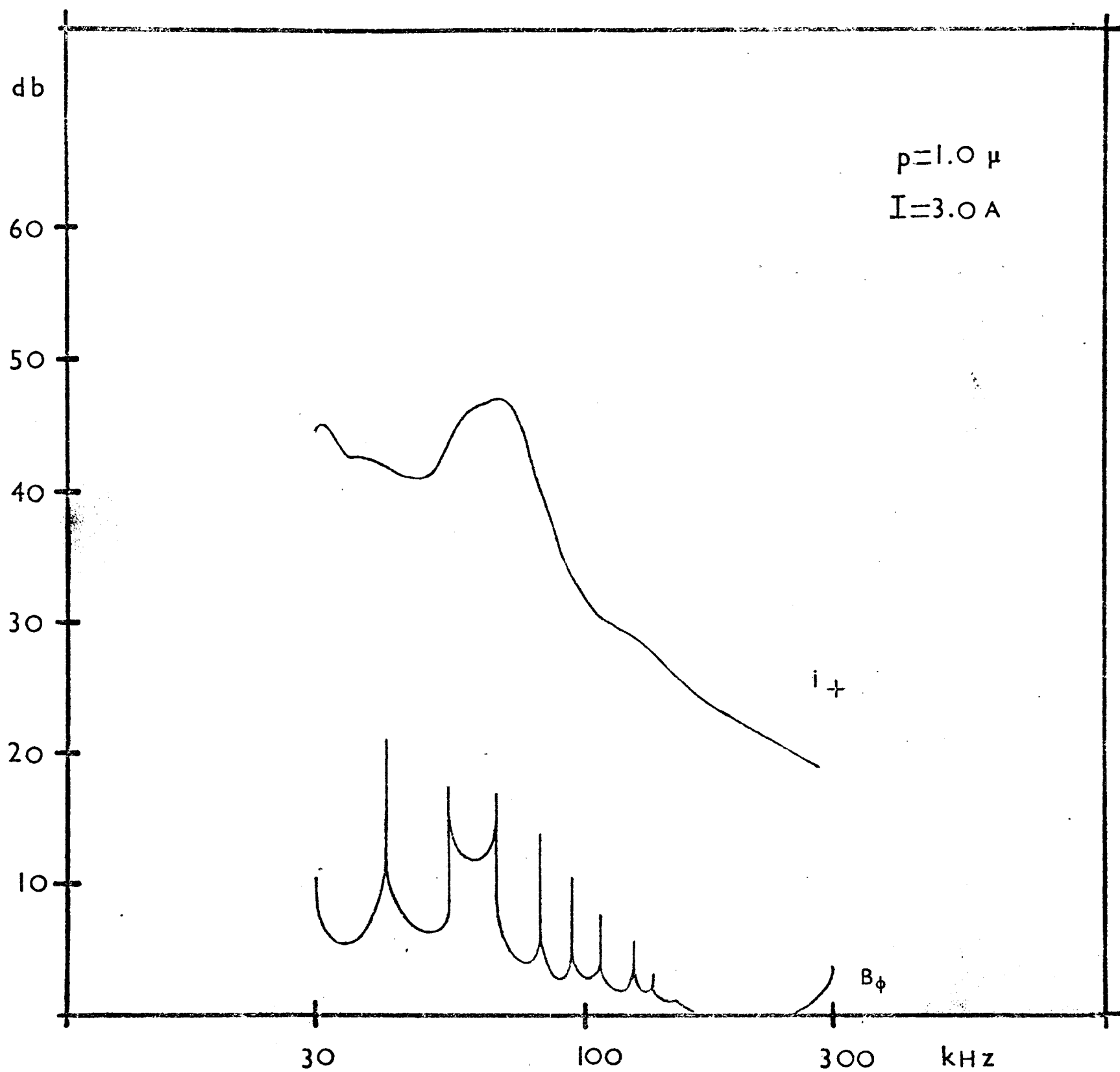


Fig. 8b. Spectra of fluctuations ; $p = 1.0 \sqrt{\text{mTorr}}$ $I = 3.0 A$.

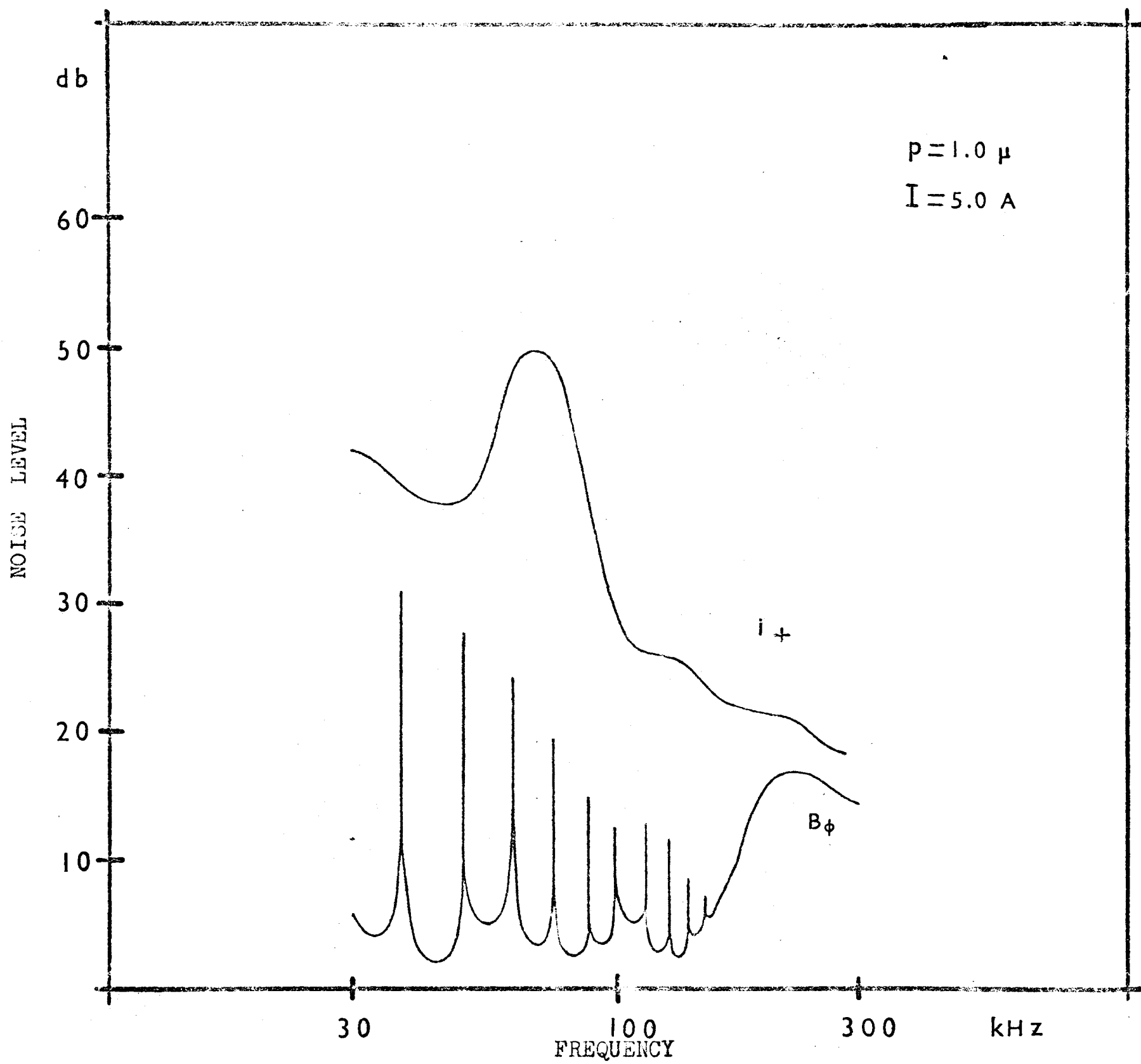


Fig. 8c. Spectra of fluctuations ; $p = 1.0 \mu$ $I = 5.0 A$.

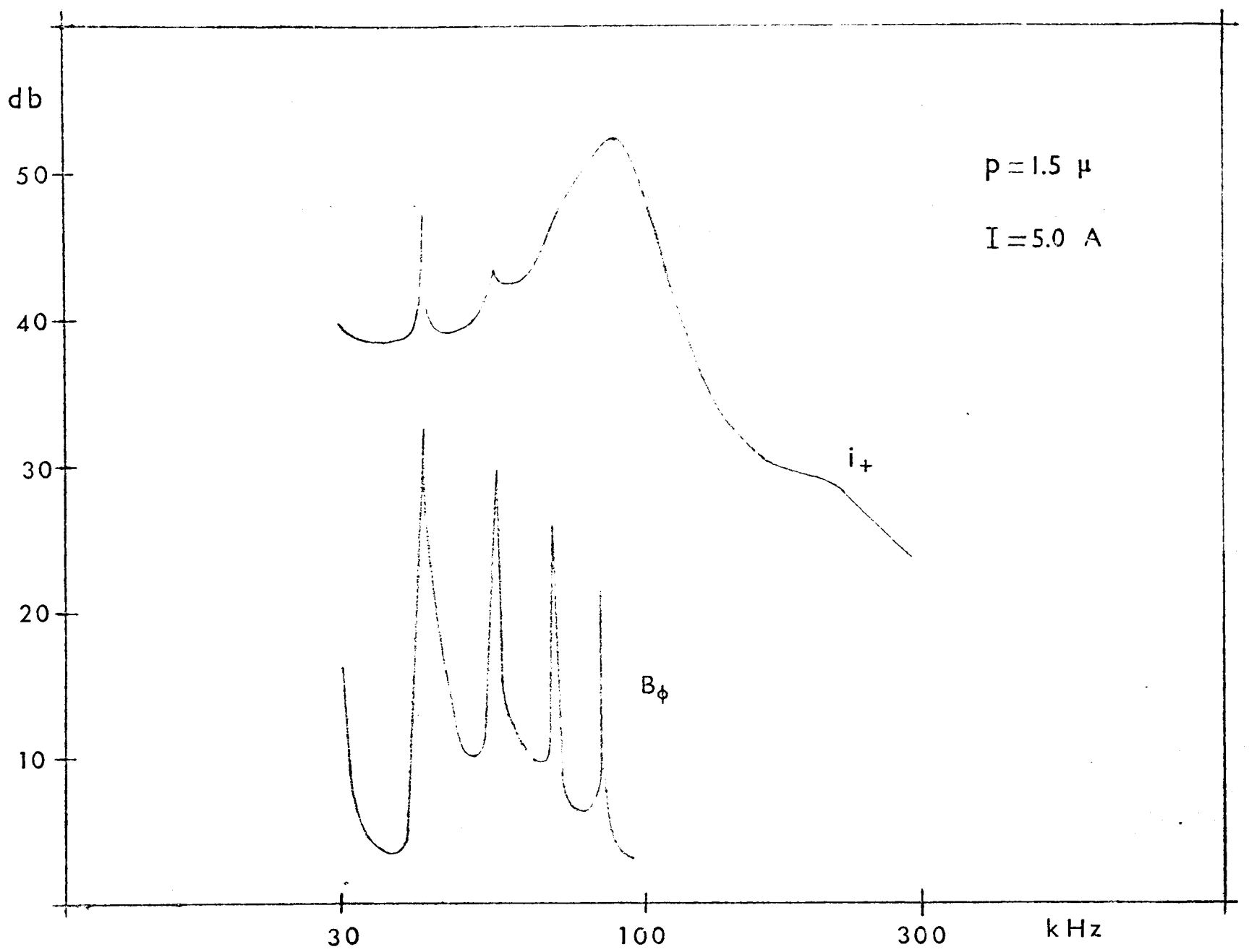


Fig. 9. Spectra of fluctuations ; $p = 1.5 \frac{\text{mTorr}}{\text{A}}$ $I = 5.0 A$.

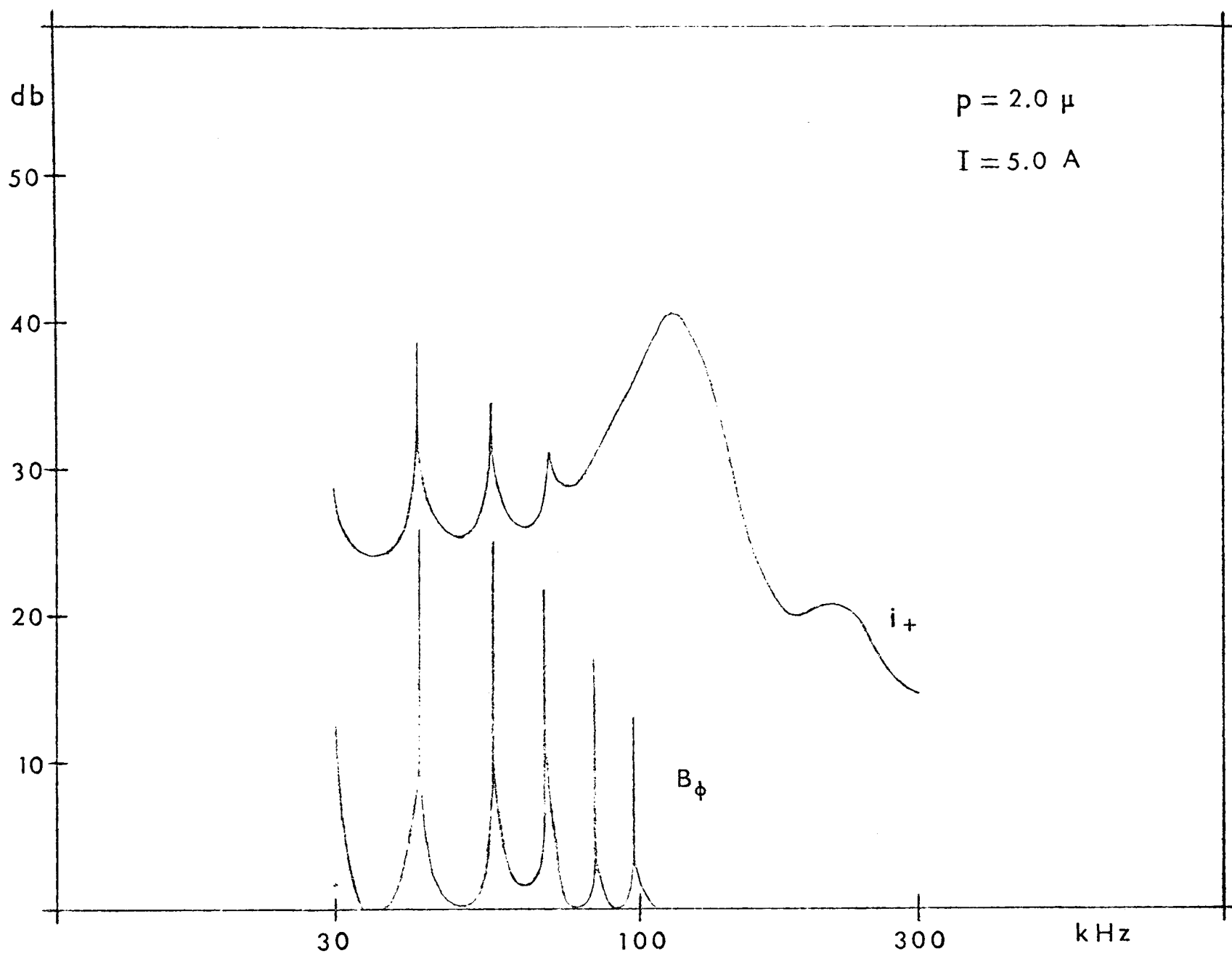


Fig. 10. Spectra of fluctuations ; $p = 2.0 \mu$ $\wedge$ $I = 5.0 A$.

3.2. SPECTRA OF SPONTANEOUS FLUCTUATIONS.

In order to investigate the low-frequency spontaneous waves and oscillations occurring in the positive column it is convenient to start by an extensive study of the spectra obtained by different probing techniques and for a number of discharge regimes. For the spectral analysis we have employed a Airmec wave-analyser to cover the range from 30 up to 300 kHz and a Wayne Kerr wave-analyser to cover the range from 0 to 20 kHz .

Figs. 6 to 10 show the spectra of the azimuthal magnetic field (B_ϕ) and of the ion saturation current to a Langmuir probe (i_+) obtained in the 30 - 300 kHz range and under different discharge regimes.

3.3. GENERAL PROPERTIES AND IDENTIFICATION OF THE SPONTANEOUS WAVES AND OSCILLATIONS

3.3.1. Now, we shall proceed to a detailed analysis of the observed spectra and to the study of the properties and identification of the waves and oscillations present in the positive column.

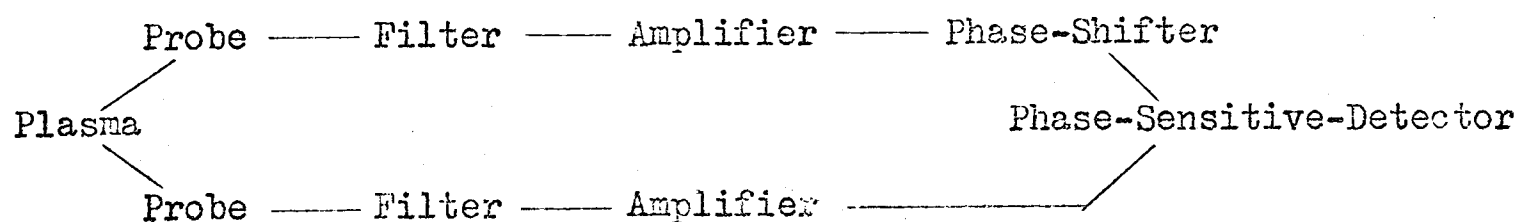
To start with, and in order to facilitate our task, we shall consider in particular the case of $p = 0.50$ mTorr controlled vapour pressure.

Up to $I = 3$ A discharge current, the magnetic and the Langmuir probes spectra are similar and the same happens with the light, capacitive probe and anode voltage spectra. At higher discharge current, the two spectra are substantially different. The B_ϕ spectrum reveals the presence of a strongly non-linear low-frequency oscillation (12 - 14 kHz) , its harmonics being regularly spaced. The non-linearity is not instrumental, because the shape of the spectrum does not change

when moving the probe away from the discharge tube, therefore decreasing the amplitude of the signal. Besides, similar spectra are obtained with the anode voltage and current. However, one should recall that the magnetic probe differentiates the magnetic signal and its harmonics are consequently enhanced. The Langmuir probe spectra reveal the presence of a pronounced band (at 55 - 60 kHz). The capacitive probe and light spectra show a combination of the features of the magnetic and Langmuir probes spectra.

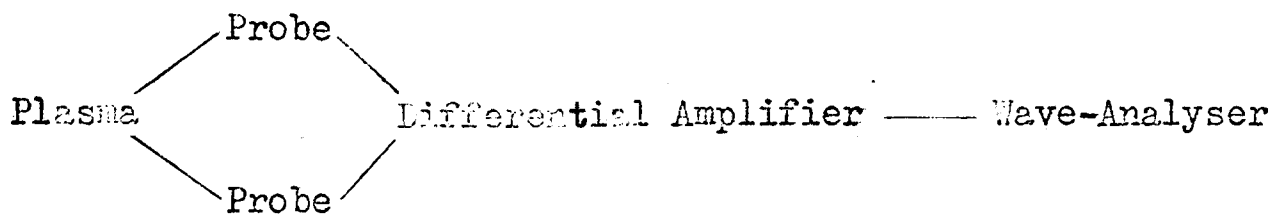
3.3.2. The observed fluctuations were afterwards analysed for their space variation. First we shall consider the discharge regimes up to $I = 3$ A anode current. To begin with, the uniformity of the positive column was examined; it was found that the level of the magnetic field spectrum decreases (about 10 db) along the initial 30 cm from the cathode end of the horizontal section and then it is substantially constant from there on. In order to carry out the phase correlation and amplitude variation study of the observed fluctuations, two experimental arrangements were employed:

(i) Using two capacitive or two radial Langmuir probes, and the following circuit:



One obtains a substantial phase correlation; however, when keeping one probe fixed and moving the other one the output is nearly constant or shows a small monotonic variation. Very little phase or amplitude variation can be detected throughout the column (both radially and longitudinally).

(ii) Using two capacitive or two magnetic probes and the following circuit:



It has been found that the differential spectrum has a level rather lower than the single probe spectrum, and that it does not change appreciably when moving one probe relatively to the other one. These facts show that the whole spectrum represents fluctuations of constant phase and amplitude (both longitudinally and azimuthally).

One arrives at the conclusion that in the 30 - 300 kHz range the plasma column oscillations are uniform in space.

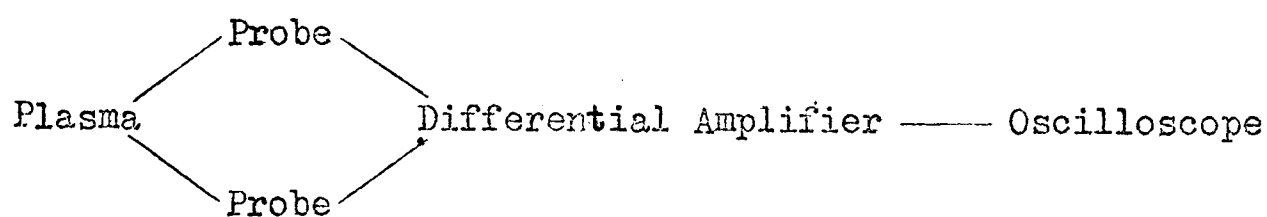
Among the continuous spectrum of oscillations there are large amplitude components which are obvious on the spectra. With $I = 1.0$ A discharge current, a large component is found at 41 kHz. Its uniformity in space can be easily checked by observation on the oscilloscope of the wave-forms picked up by moving probes; it was also found that the different probing devices yield signals which are apparently in phase. This fact can be appreciated on the oscillogram shown in fig. 11. This same oscillation can still be observed when the discharge current is lowered down to 0.2 A. At $I = 1.0$ A a smaller oscillation can be detected with the low-frequency wave-analyser at 12.3 kHz. This same oscillation is found again at $I = 3.0$ A, but slightly displaced to 12.8 kHz; by then, its amplitude is larger and its third and fifth harmonics can be seen in the spectra shown in fig. 7. At $I = 3.0$ A one can already notice a small bump corresponding to the band of fluctuations which will become important at larger discharge currents. (the 55 - 60 kHz band) . .

Now, we shall consider the discharge regimes above $I = 3$ A anode current. The main component of the magnetic field spectra is

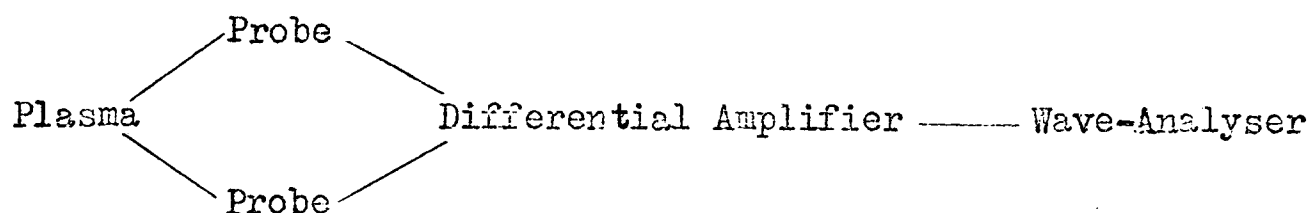
around 12 - 14 kHz ; it seems possible that this is the same oscillation already observed at lower discharge current but at a rather lower level then. The spectra of the Langmuir probe shows a band of large amplitude fluctuations (at 55 - 60 kHz) . On the oscilloscope one observes a rather noisy wave-form, but a nearly sinusoidal oscillation at 55 - 60 kHz is easily seen, and the low frequency oscillation picked up by the magnetic field can be seen there as well. On fig. 12 one can see an oscillogram showing the Langmuir and magnetic probes wave-forms.

In order to investigate the 12 - 14 kHz oscillation, two experimental arrangements were employed:

(i) Using two similar magnetic probes and the following circuit:



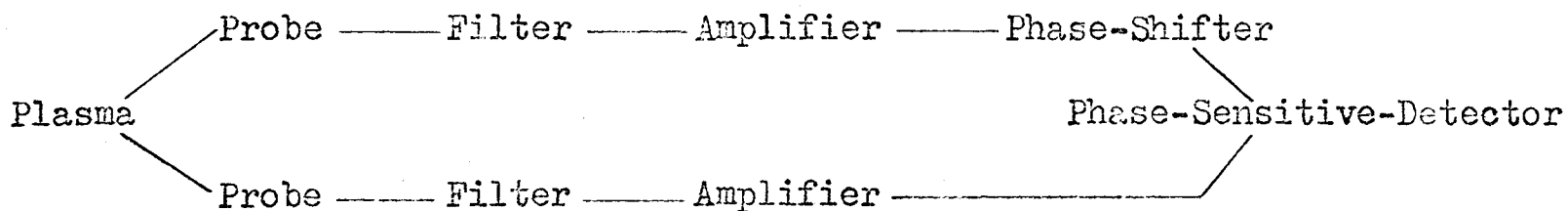
(ii) the same probes, but this other circuit instead:



One finds that the two signals always cancel each other out (the oscillogram loses its wave-form and the spectrum loses its peaks) and that this result does not change when moving the probes one relatively to the other (longitudinally and azimuthally). Next, with the help of a radial and mobile Langmuir probe, and a magnetic probe as reference, it was checked that the phase and amplitude of this oscillation does not vary substantially across the radius either (this was done by observation on the oscilloscope). One is accordingly led to conclude that the oscillation in consideration is uniform throughout the plasma column. Besides, this oscillation as observed in the Langmuir and magnetic probes is in phase.

The magnitude of the large amplitude uniform oscillations was appreciated through the magnitude of the relative perturbations of the magnetic field and of the emitted light (at both $4080 \text{ \AA}$ and $5461 \text{ \AA}$). Both the 41 kHz oscillation, observed at $I = 1.0 \text{ A}$ discharge current, and the 13 kHz oscillation, observed at $I = 7.0 \text{ A}$, show in all cases an amplitude of the order of 1% .

3.3.3. The investigation of the $55 - 60 \text{ kHz}$ band was done, to begin with, by observation on the oscilloscope. Using two Langmuir probes, radial and facing each other along the same diameter, one sees that the amplitude decreases monotonically from near the axis to the wall and that the phase is also delayed monotonically. On the other hand, two Langmuir probes spaced along the column axis show that the phase correlation exists but that it is lost with the axial distance within a few centimeters. It is, therefore, required to use the phase-sensitive-detection technique for a detailed study of this type of wave. The experimental set up is as follows:



where Langmuir probes (in the ion saturation current regime) have normally been employed, although the capacitive probe or the photomultiplier could also have been used (but apparently with less success).

From the phase correlation data collected in this way one can conclude that the $55 - 60 \text{ kHz}$ band corresponds to waves propagating axially (the phase varies linearly along the axis) towards the anode, and attenuating (the amplitude roughly decays exponentially with distance). The output of the phase-sensitive-detector oscillates with the distance along the axis yielding at once the wave-length and the attenuation. An oscillatory output can in general be due to either a periodic

variation of the amplitude or to a linear variation of the phase. The two cases can be distinguished by varying the time delay produced by the phase shifter and searching for maximum output; one can in this way measure the actual phase and amplitude at every point. It turns out that the observed oscillatory output is due to a linear phase variation, and one can conclude that the wave is propagating. A typical output of the phase-sensitive-detector is shown in fig. 13. The phase velocity of these waves falls in a range close to the ion-acoustic speed, and the waves are therefore identified as ion-acoustic waves.

The transverse structure of the observed waves is non-azimuthal (otherwise the amplitude on the axis ought to be zero) and almost axisymmetric. The phase-sensitive-detection measurements reveal that the wave amplitude has a maximum along the column diameter which generally lies off center although close to the axis, and that the wave phase has a similar behaviour. The wave mode is accordingly supposed to be the first radial non-azimuthal mode.

Away from the 55 - 60 kHz band, say, below 45 and above 70 kHz, phase correlation is still obtained but no space variation can be observed. Moreover, the ion-acoustic waves are in general detected with a simultaneous constant-phase and constant-amplitude oscillation. It is therefore obvious that the ion-acoustic band is superimposed upon a wide and continuous spectrum of uniform oscillations.

When reversing the discharge direction but maintaining the role of the two probes (the same reference probe and the same moving one), phase correlation is still obtained and the uniform component of the fluctuation is still there. However, the amplitude of the ion-acoustic waves becomes many times smaller and its decay many times faster (or the wave cannot be observed at all if one is not right at the peak of the wave band). This experiment proves clearly that the spontaneous ion-

-acoustic waves can propagate along the electron drift direction only; or, being more precise, that they are severely attenuated along the opposite direction.

The variation of the level of the ion-acoustic wave band along the column was examined as well. Using either two Langmuir probes or two capacitive probes one finds that the level rises very slowly from the cathode to the anode side of the column, between 0.05 and 0.10 db/cm. This result is not affected when reversing the discharge direction and, accordingly, it is a genuine result. The magnitude of the ion-acoustic waves can be appreciated by the magnitude of the perturbation of the ion saturation current. At $I = 3.5 \text{ A}$, when the ion-acoustic waves are about to set in, the relative perturbation of the ion saturation current is only around 0.01 %. However, at $I = 7.0 \text{ A}$ anode current, that quantity is already about 0.20 % and almost entirely due to the ion-acoustic waves.

3.3.4. In the study of the properties of the positive column it is convenient to make sure that end effects are not important and, in the case of the study of waves and oscillations, that these are not being generated in the cathode or anode regions. A few tests were therefore carried out to make clear that the observed phenomena are genuine phenomena of the positive column.

(i) A discharge was produced between the cathode (E 1) and the near by umbrella (E 2) in order to have a short discharge with a negligible positive column. Its voltage and current spectra were drawn at a number of discharge currents, and it turned out that no obvious similarity was found between these spectra and those characteristic of the long positive column.

(ii) Having the discharge running from cathode (E 1) to anode (E 6) the umbrella (E 2) was switched on in order to draw about 4 A

extra current through the cathode region. The band of ion-acoustic waves observed at large anode current ($I > 3 \text{ A}$) is not affected when switching E 2 on. On the other hand, at low current ($I < 3 \text{ A}$) when the ion-acoustic waves are not observed, they are not excited by switching E 2 on. Regarding the large uniform oscillations as detected by the magnetic probe or the photomultiplier, at large anode current no significant effect is produced; and at low current the amplitude rises and the wave-form changes a bit, but the fundamental frequency is still the same.

(iii) Connecting the cathode E 1 with the near by umbrella E 2 ; or the anode E 6 with the auxiliary electrode on the same side E 4 ; or the two auxiliary electrodes at both ends of the horizontal section of the column E 3 and E 4 , through a large capacitor ($0.5 \mu\text{F}$) , does not give rise to any detectable effect upon the signals picked up by the magnetic and Langmuir probes.

3.3.5 Although the large uniform oscillations are associated with a current flow in the discharge circuit, a substantial change of the external resistance (up to 5 times) does not affect these oscillations significantly. However, connecting a large inductive coil (nearly 1 H) in series with the discharge produces a general decrease of the spectra level (10 to 20 db) although their shapes are essentially maintained (see figs. 14a and 14b). On the oscilloscope the ion wave is seen to have the same amplitude and frequency but its wave-form is less noisy now.

When connecting a large capacitor ($0.5 \mu\text{F}$) between the cathode and the anode, the magnetic field spectra show a general level rise and the Langmuir probe spectra show a general decrease. On the oscilloscope the amplitude of the magnetic probe wave-form increases 5 to 15 times and the Langmuir probe one increases 2 to 3

times, and both wave-forms become better defined. At low anode current, ($I = 1$ A) the 41 kHz oscillation nearly disappears while the 12 - 14 kHz oscillation becomes more pronounced instead. At large anode current ($I > 3$ A) this same oscillation (12 - 14 kHz) is found to be the most important component of the signals detected by all the probing devices; even the ion-acoustic wave band becomes comparatively small in the Langmuir probe signal. The constant phase and constant amplitude characteristic of this oscillation can be easily checked now; at this amplitude this oscillation is clearly uniform throughout the whole plasma column and its magnitude is estimated to be of the order of 2 to 4 %. Spectra and oscillograms obtained when a large capacitor was connected in parallel with the discharge tube are shown in figs. 15 and 16 .

3.4. GENERAL DATA ON THE SPONTANEOUS WAVES AND OSCILLATIONS

The magnetic field and the Langmuir probe spectra (in the range from 30 to 300 kHz) obtained at different values of the controlled vapour-pressure and of the discharge current are shown in figs. 6 to 10 .

One observes a few general trends in these spectra. The ion-acoustic waves and the large amplitude non-linear uniform oscillations (10 - 15 kHz) tend to appear at the same time when the discharge current becomes large enough. The current at which this happens goes down when the pressure goes up. The peak of the ion-acoustic wave band is displaced to a higher frequency by increasing the pressure and, at the same time, its band-width widens and its amplitude is attenuated. On the other hand, the large uniform oscillations (10 - 15 kHz) exhibit a remarkably constant wave-form and frequency when varying either the vapour pressure

or the discharge current. At low discharge current the spectra reveal a continuous band of uniform oscillations of the whole column among which a few larger components emerge as well as some of their harmonics.

In table 4 one finds a record of the fundamental frequency of the larger uniform oscillations observed at different discharge regimes.

Data on the propagating ion-acoustic waves, is shown in table 5. The following dimensionless quantities are used here:

$$\sigma = \omega R/a_0 \quad h = R\beta \quad c = \omega/\beta a_0$$

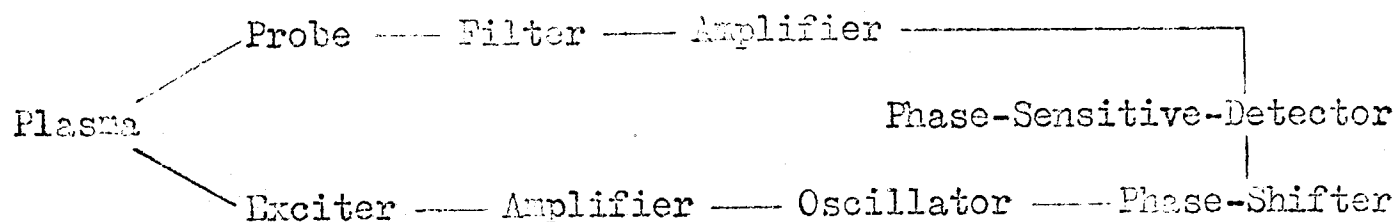
where ω and β are the frequency and the wave-number, R is the tube radius and a_0 is the ion-acoustic speed given by $a_0^2 = kT_e/n_i$. On this table one has a record of the frequency and amplitude of the peak of the ion-acoustic wave band observed on the Langmuir probe spectra, in different discharge regimes. On the same table one also finds the frequency, wave number and phase velocity of the ion-acoustic wave observed at the peak of the band. Data on a number of waves in the ion-acoustic wave band observed at $p = 0.50$ mTorr and $I = 7.0$ A is shown in table 6. The radial structure of the wave, as observed near the peak of the wave band, can be appreciated in figs. 17 to 20. One sees that both the phase and the amplitude exhibit a maximum near the axis which usually lies off-center.

3.5. EXTERNALLY EXCITED WAVES

In order to study the properties of externally excited waves it is convenient to have a positive column with a noise level as low as possible. The discharge regime which seems to fit this requirement better is the one at $p = 0.50$ mTorr and $I = 3.0$ A. To lower the noise level still further a large inductance coil was connected in series in the discharge circuit.

Two types of exciter were employed, a single grid and a coil

(see chapter II) which could be moved axially. The experimental set-up was as follows:



The probe was a transverse Langmuir probe in the ion saturation current regime which was kept fixed while the exciter would move along the column axis.

A well defined wave-form could be obtained only when the applied frequency would lie not very far from 60 kHz , that is to say, in a frequency range around which an ion-acoustic wave band would be spontaneously excited by increasing the discharge current. Below 40 kHz or above 80 kHz either a poorly defined wave-form or no wave-form at all could be observed. A nearly uniform oscillation was usually detected together with the wave. When the wave is of much smaller amplitude than the uniform oscillation, the output of the phase-sensitive-detector looks like a small amplitude oscillation with nearly constant phase. When the wave is of much larger amplitude than the uniform oscillation, the output reveals a clear linear phase variation and a slowly varying amplitude. When the wave and the uniform oscillation are of comparable magnitude, the output of the phase-sensitive-detector can be resolved into an oscillating phase and an oscillating amplitude. In fig. 21 one can see an example of the obtained outputs analysed into its phase and amplitude.

The detected wave exhibits the same general features as the spontaneous ion-acoustic waves regarding phase velocity and radial structure. The wave nature and its propagating mode are accordingly identified as being the same.

When the discharge direction is reversed, the wave-form

disappears, what means that the plasma response can be observed along the direction of the electron drift only.

Although there is some evidence that the wave is slowly attenuated within, say, 30 cm from the source, its phase and amplitude features are unchanged from there up to at least 80 cm from the source. This fact shows that the externally excited ion-acoustic waves propagate with a negligible attenuation-rate.

The data collected on the externally excited waves is summarized in table 7 .

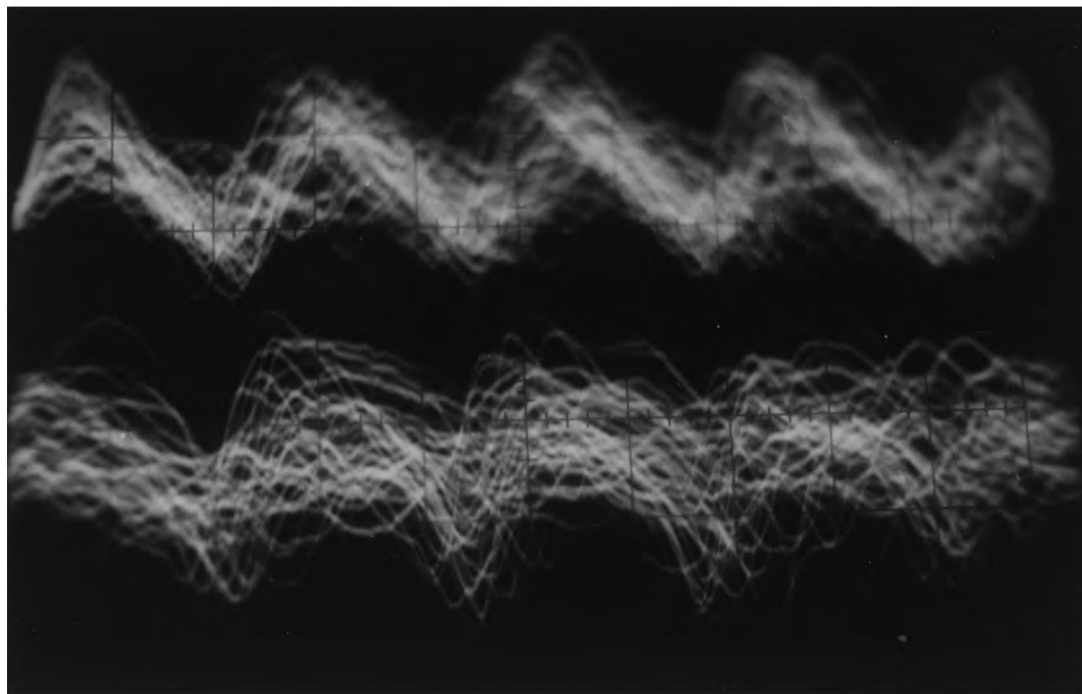


Fig. 11. Upper beam : light output (4080 Å) ; lower beam :
Langmuir probe signal (ion saturation current) .
Discharge regime : $p = 0.50 \text{ mTorr}$ $I = 1.0 \text{ A}$.

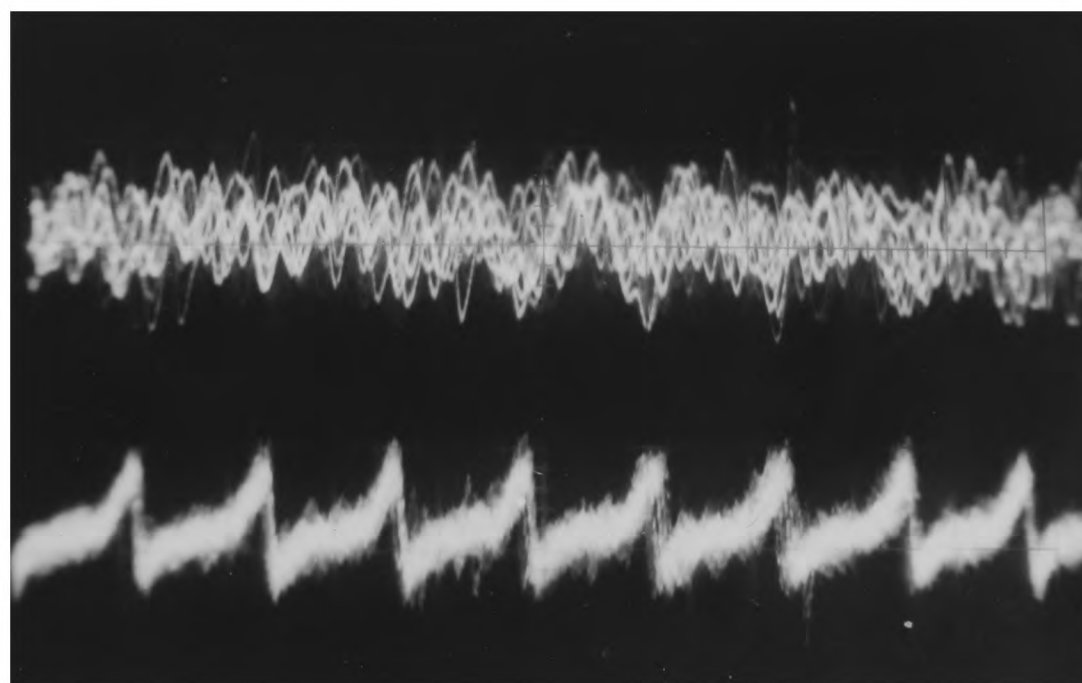


Fig. 12. Upper beam : Langmuir probe signal (ion saturation
current) ; lower beam : magnetic probe signal .
Discharge regime : $p = 0.50 \text{ mTorr}$ $I = 7.0 \text{ A}$.

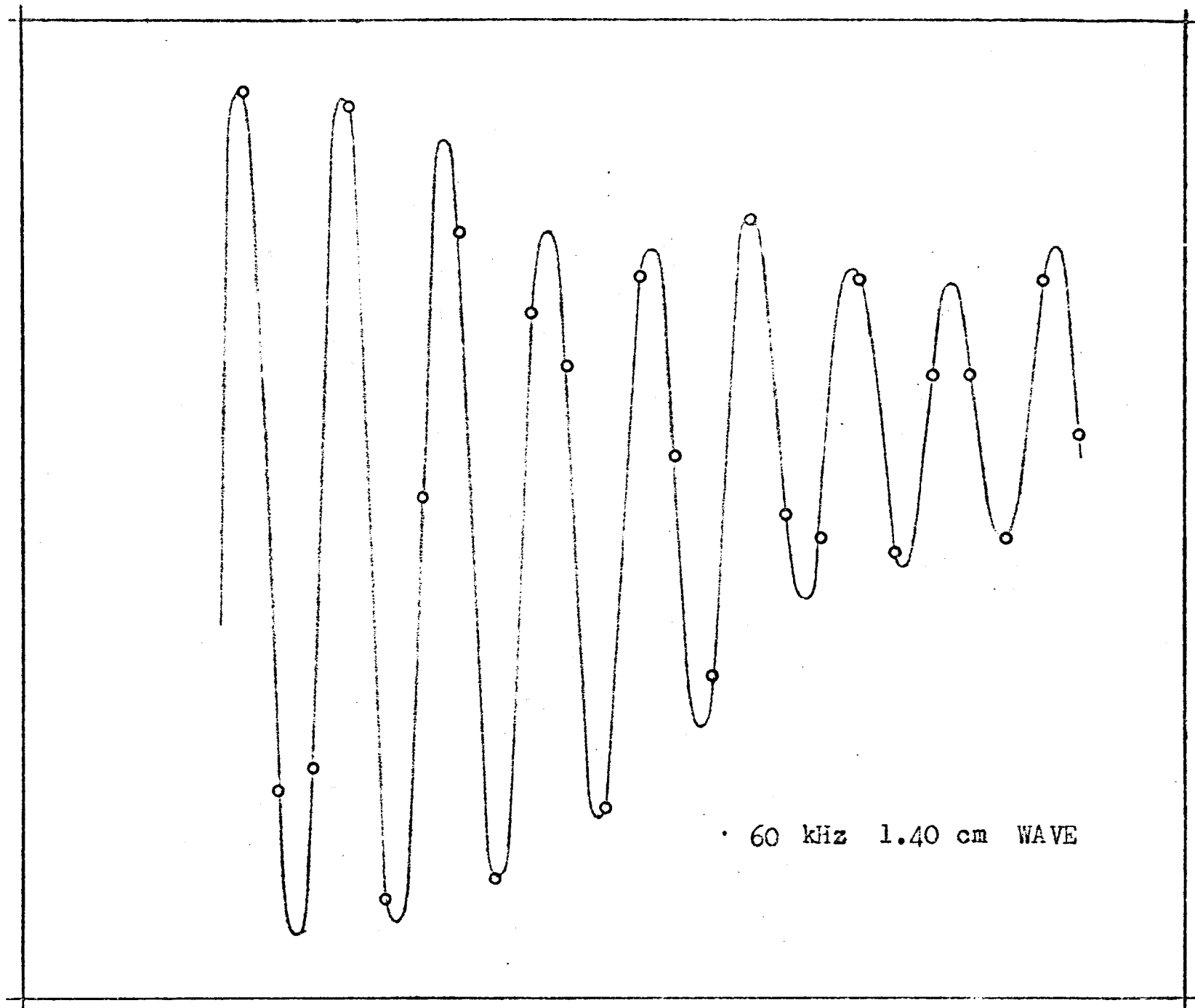


Fig. 13. The axially propagating ion-acoustic waves ; a typical example of the output of the phase-sensitive-detector .

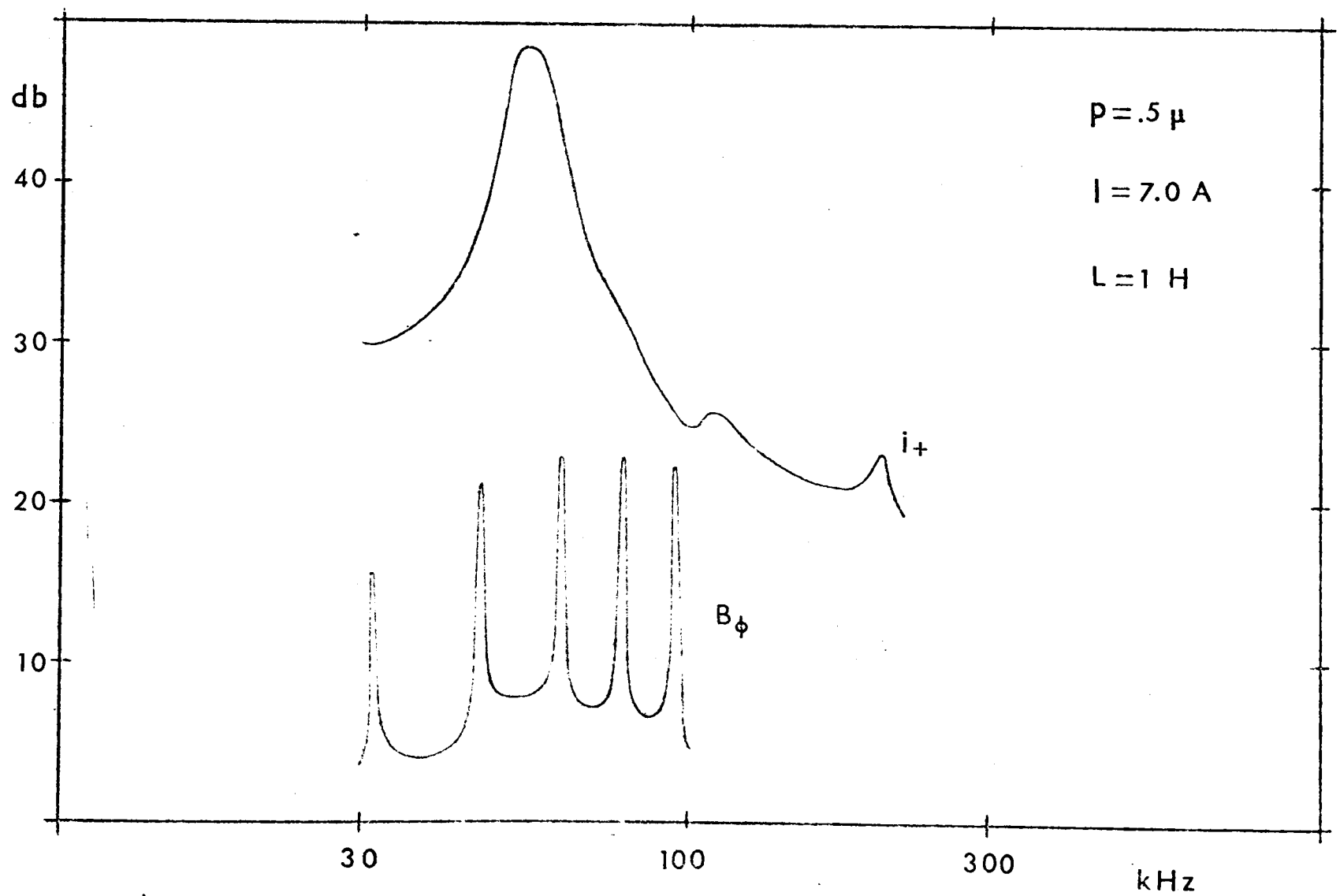
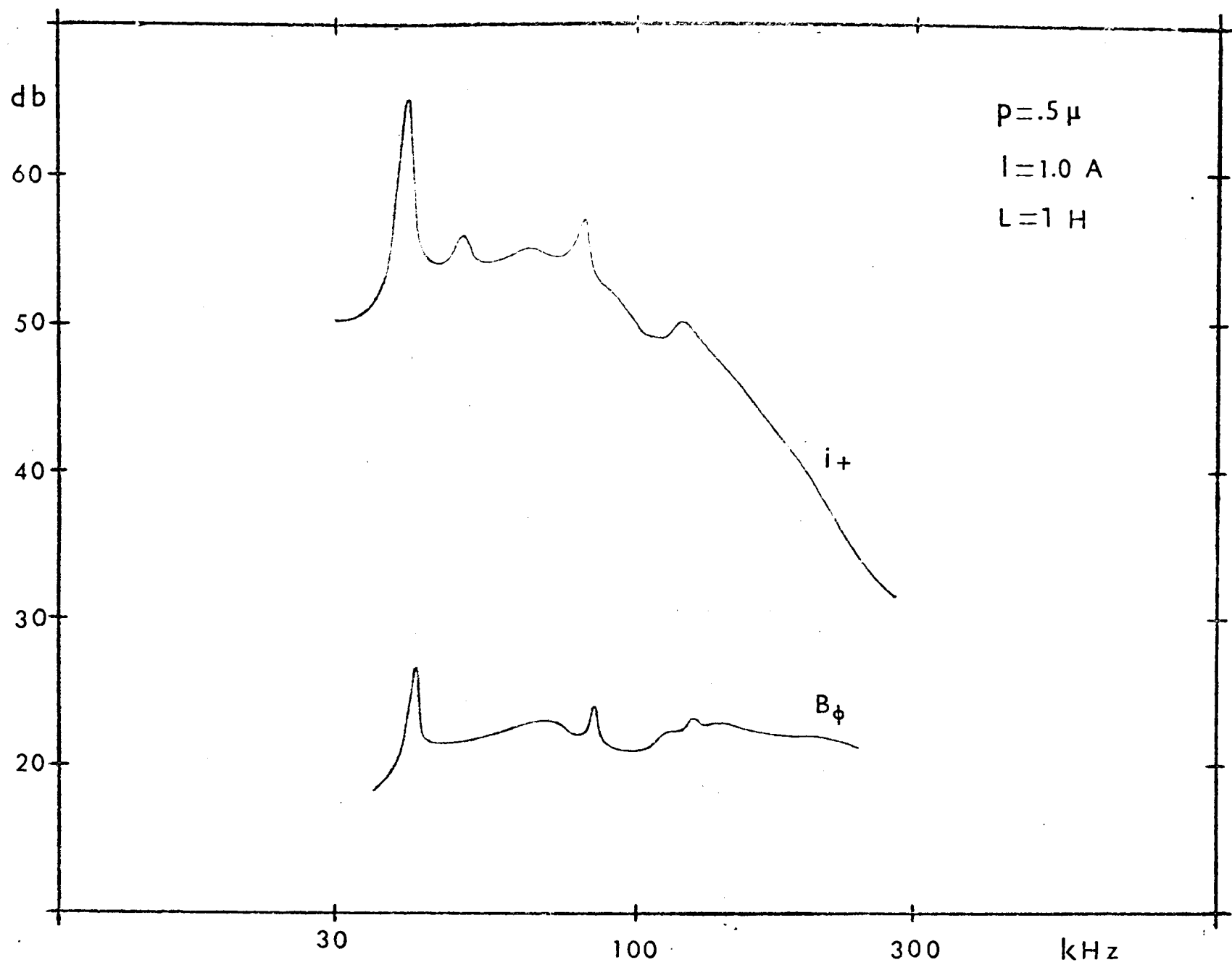


Fig. 14. The influence of the external discharge circuit impedance upon the fluctuations spectra ; large external impedance (a large inductive coil) .

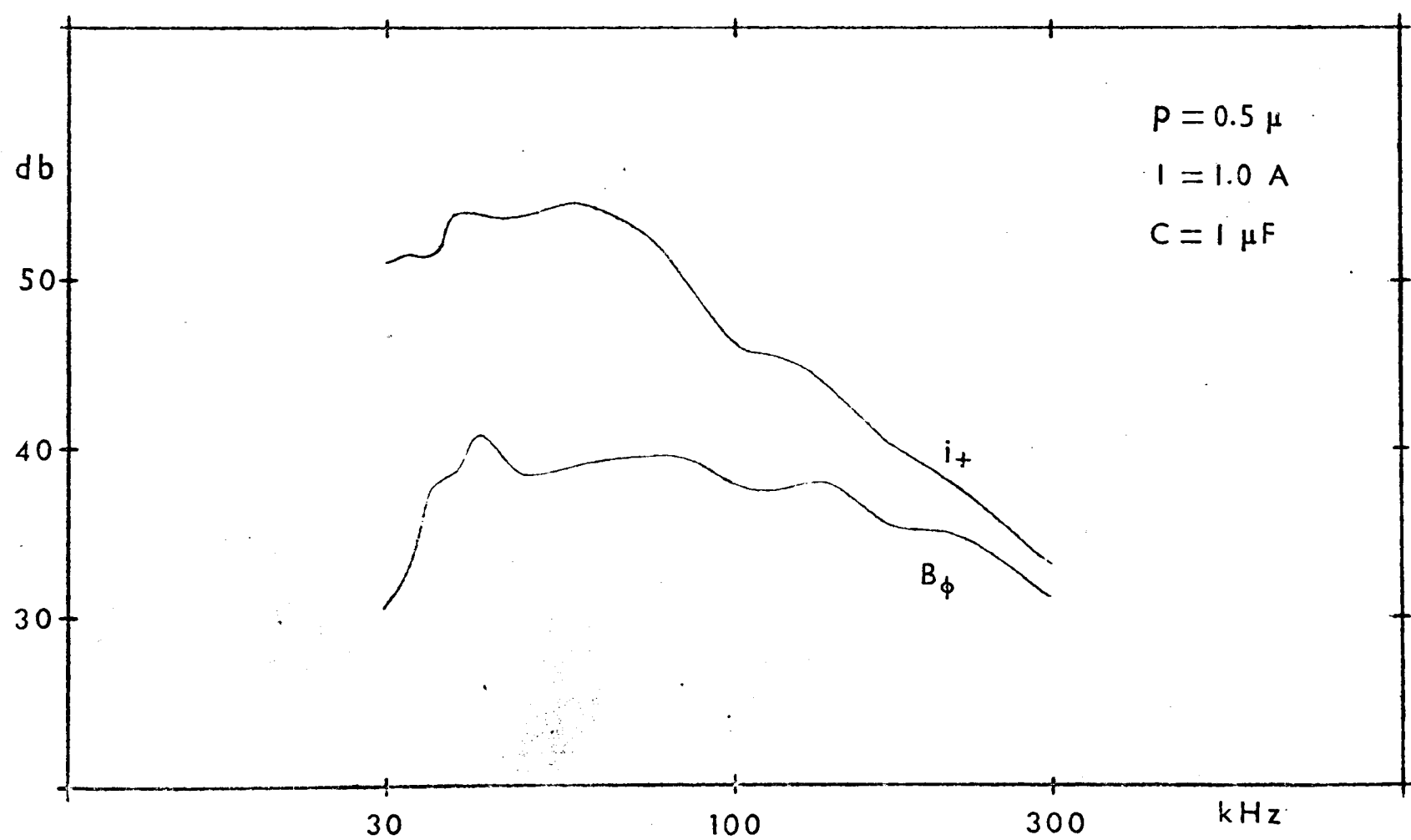
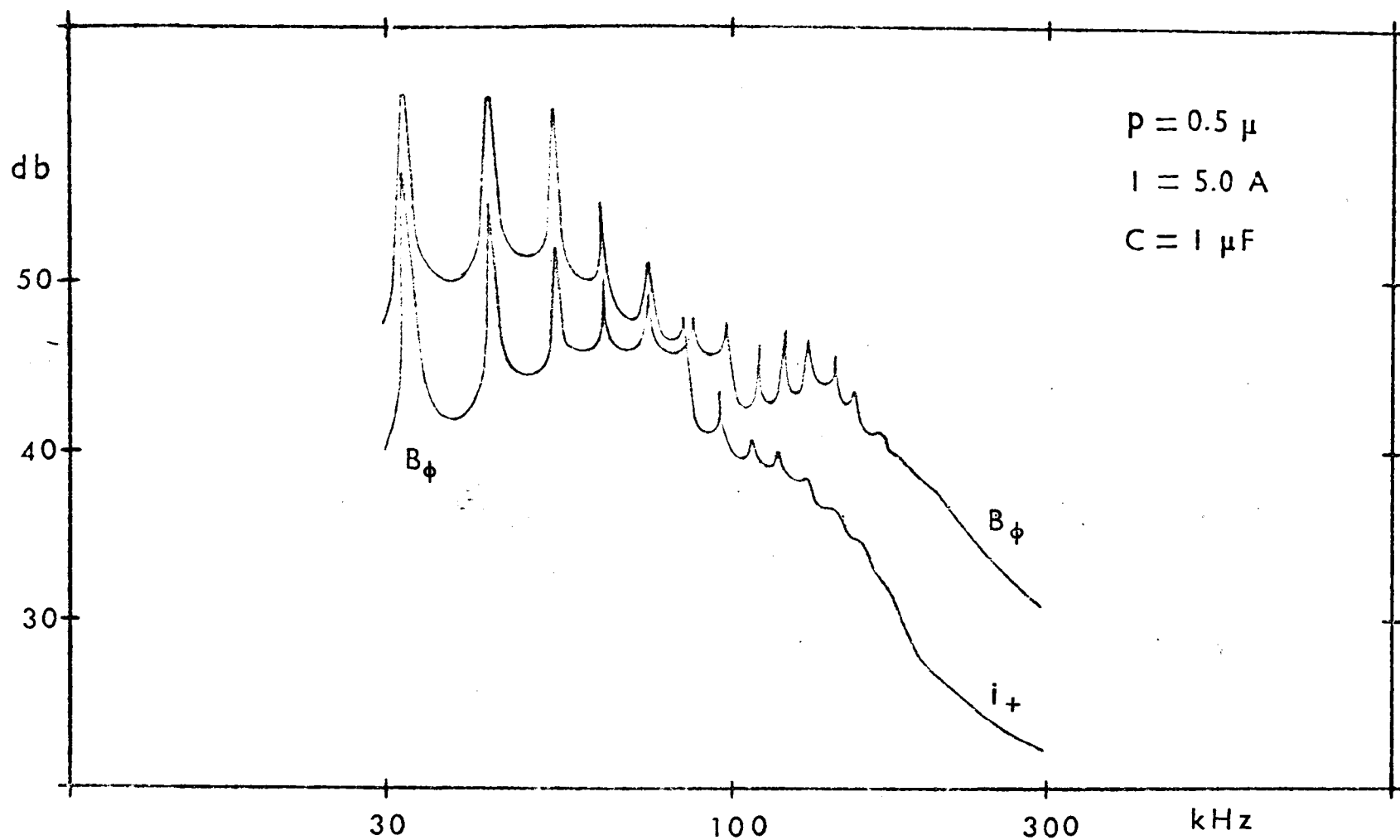


Fig. 15. The influence of the external discharge circuit impedance upon the fluctuations spectra ; small external impedance (a large capacitor) .

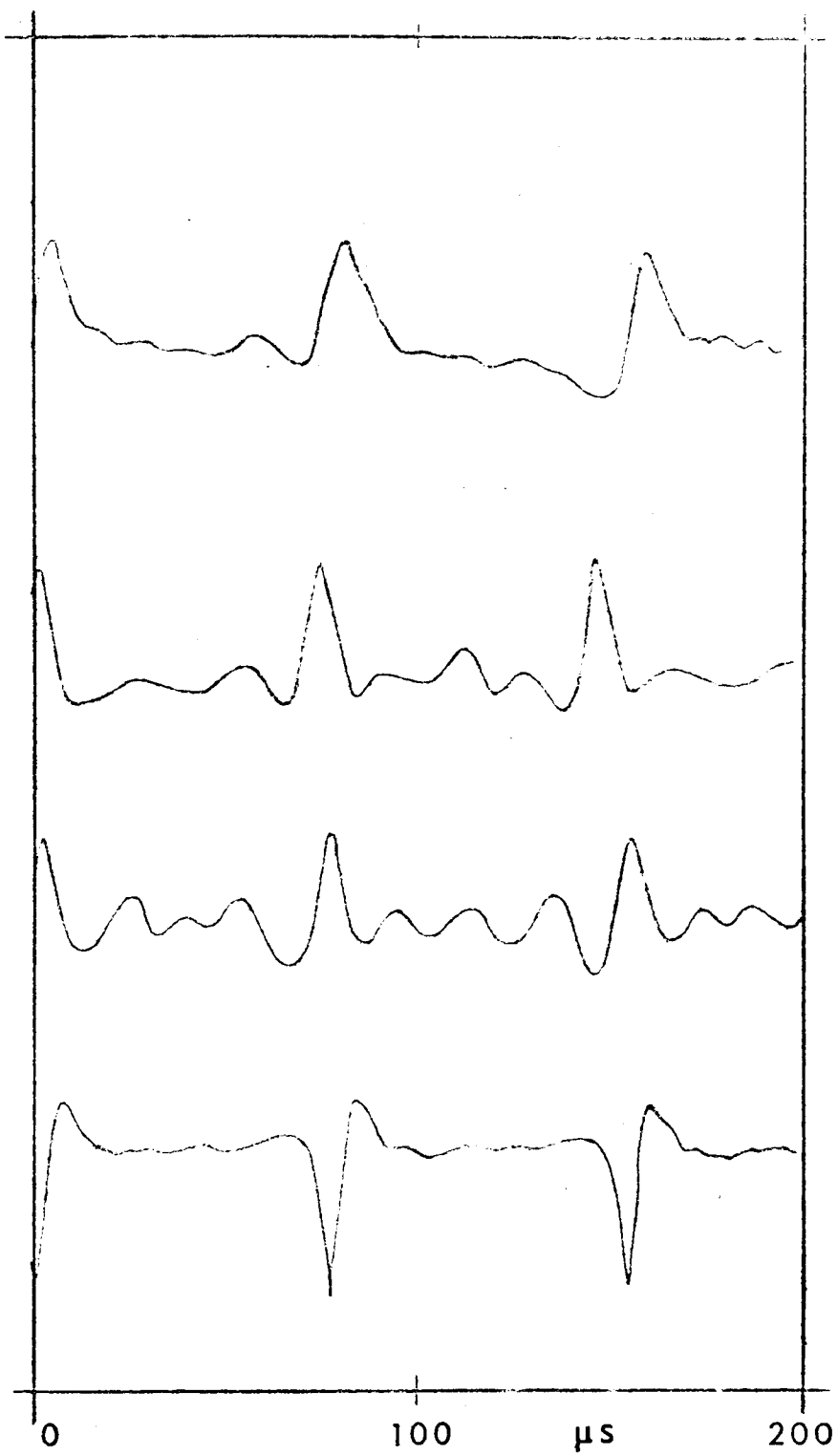


Fig. 16. Oscillograms showing the large amplitude non-linear uniform oscillations at a small discharge circuit impedance ; starting from the top : light output , capacitive probe voltage , ion saturation current and time derivative of the discharge current (as given by a magnetic probe) .

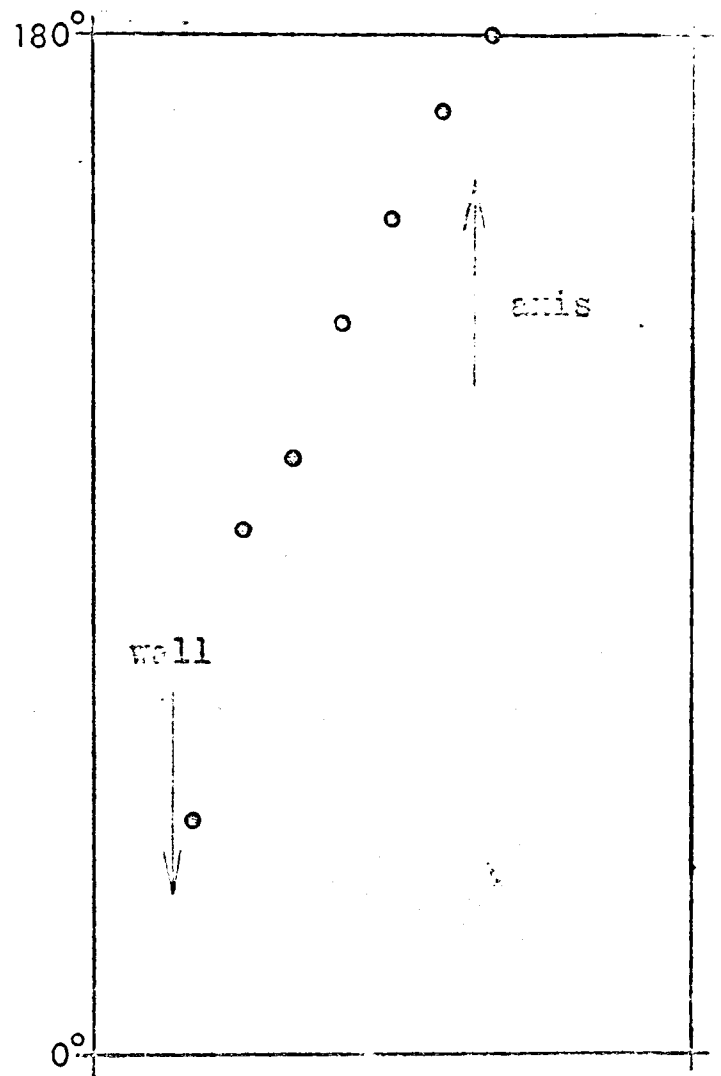
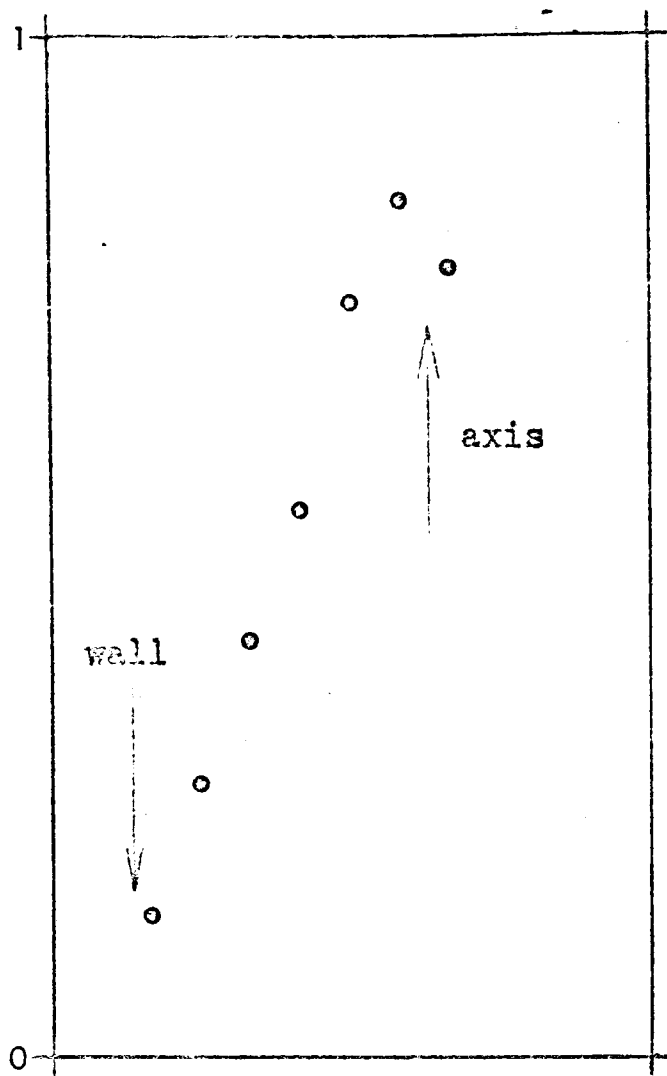


Fig. 17. Radial structure of the ion-acoustic wave : the radial dependence of amplitude and phase ; $p = 0.25 \text{ mTorr}$ $f = 53 \text{ kHz}$ $I = 10.0 \text{ A}$

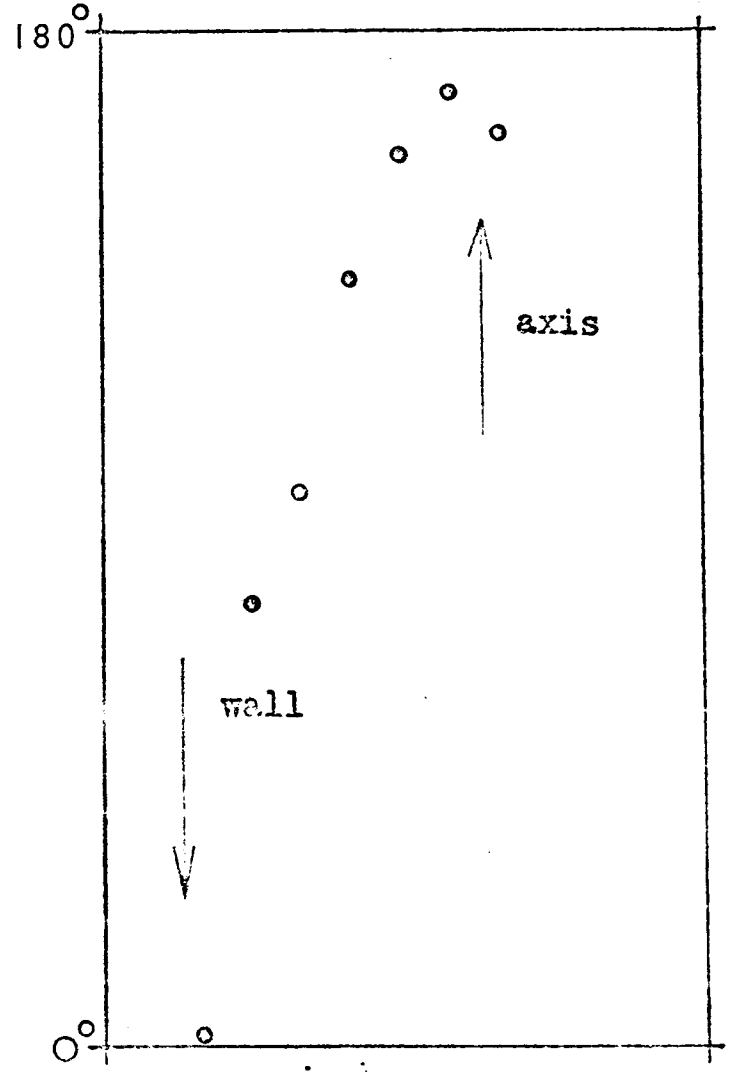
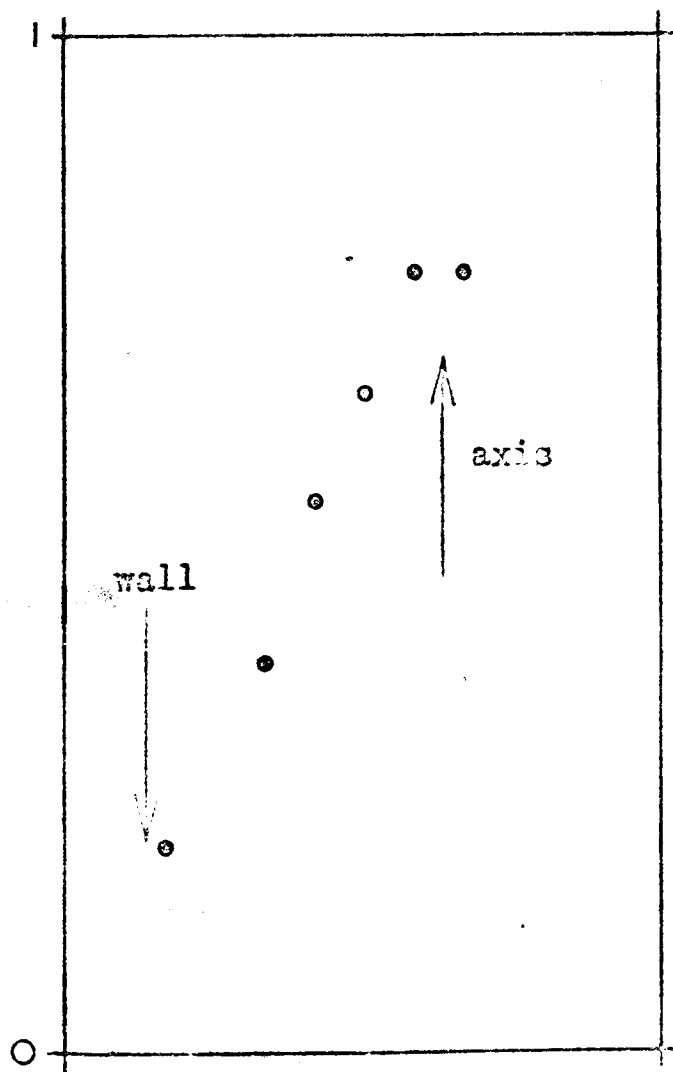


Fig. 18. Ditto ; $p = 0.50 \text{ mTorr}$ $f = 60 \text{ kHz}$ $I = 7.0 \text{ A}$

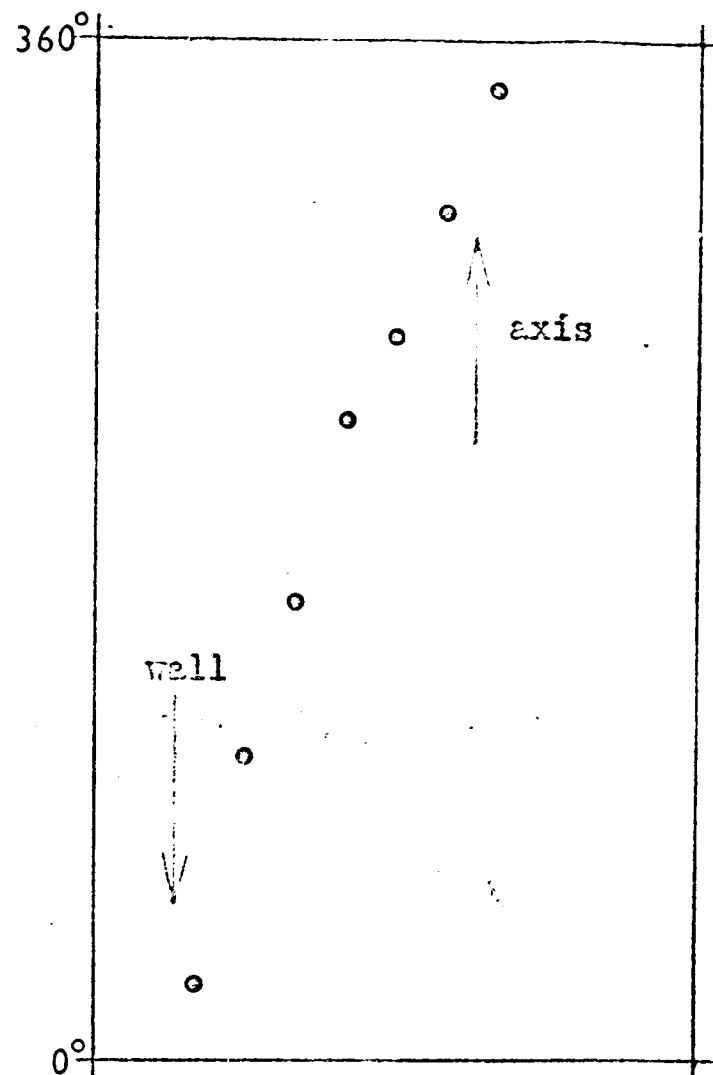
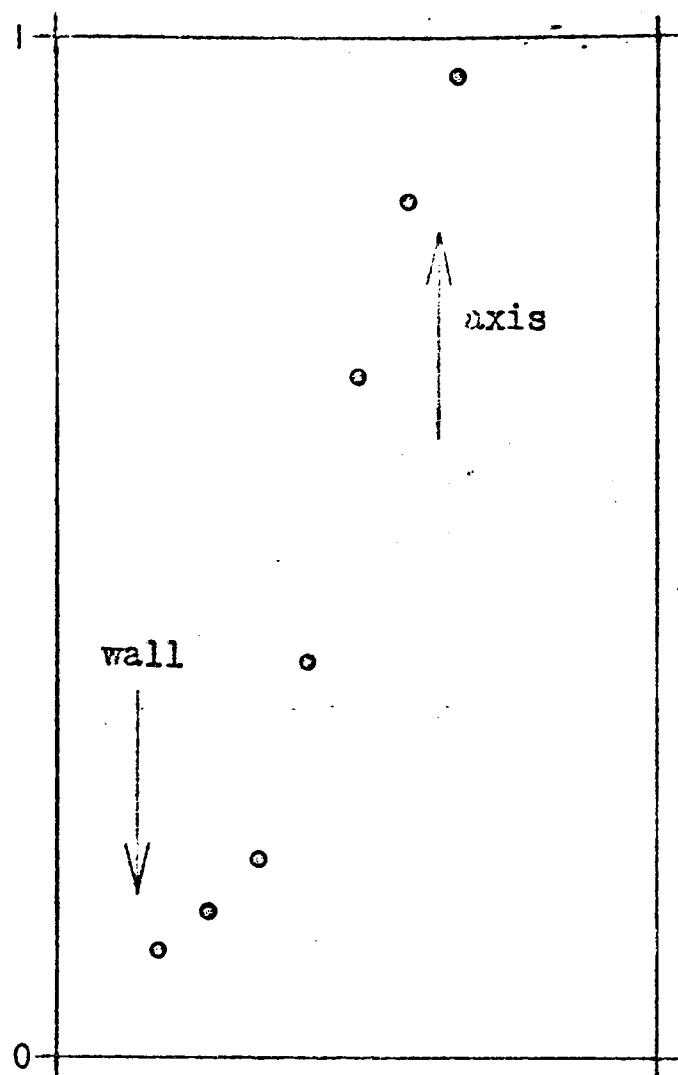


Fig. 19. Ditto ; $p = 1.0$ $\frac{\text{mTorr}}{\text{A}}$ $f = 70 \text{ kHz}$ $I = 5.0 \text{ A}$

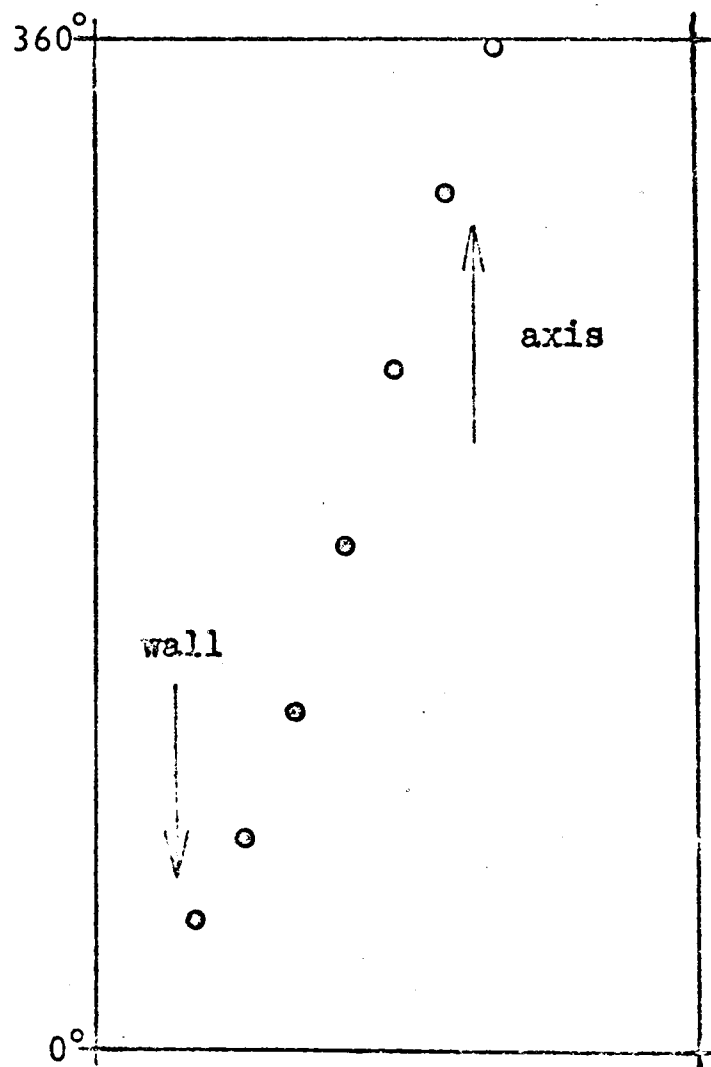
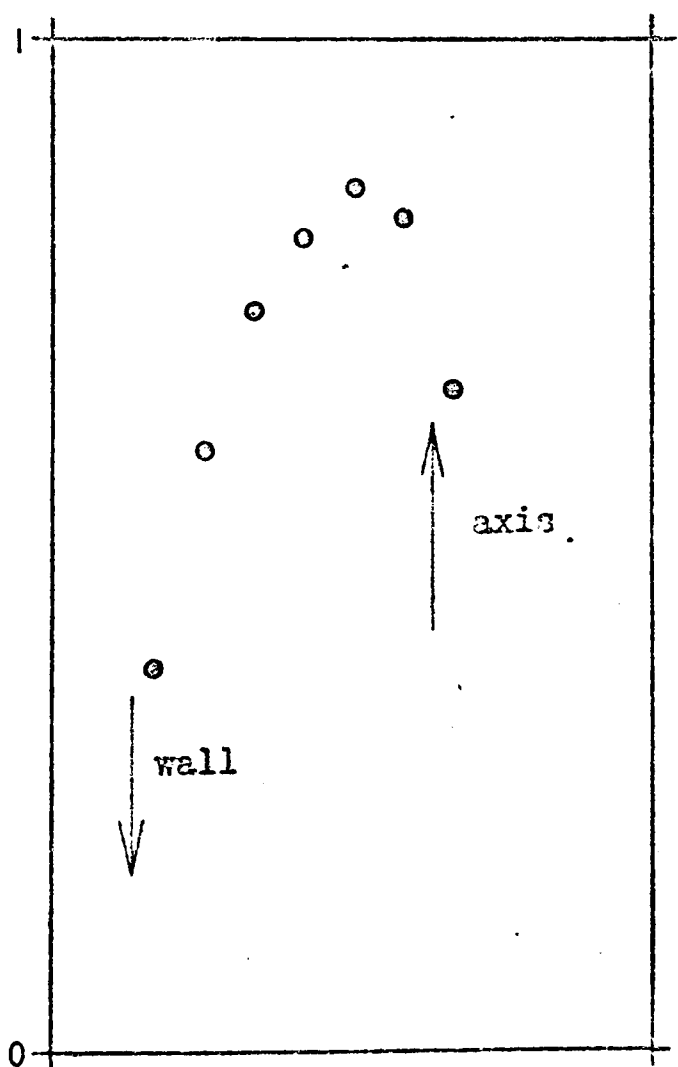


Fig. 20. Ditto ; $p = 1.5$ $\frac{\text{mTorr}}{\text{A}}$ $f = 90 \text{ kHz}$ $I = 5.0 \text{ A}$

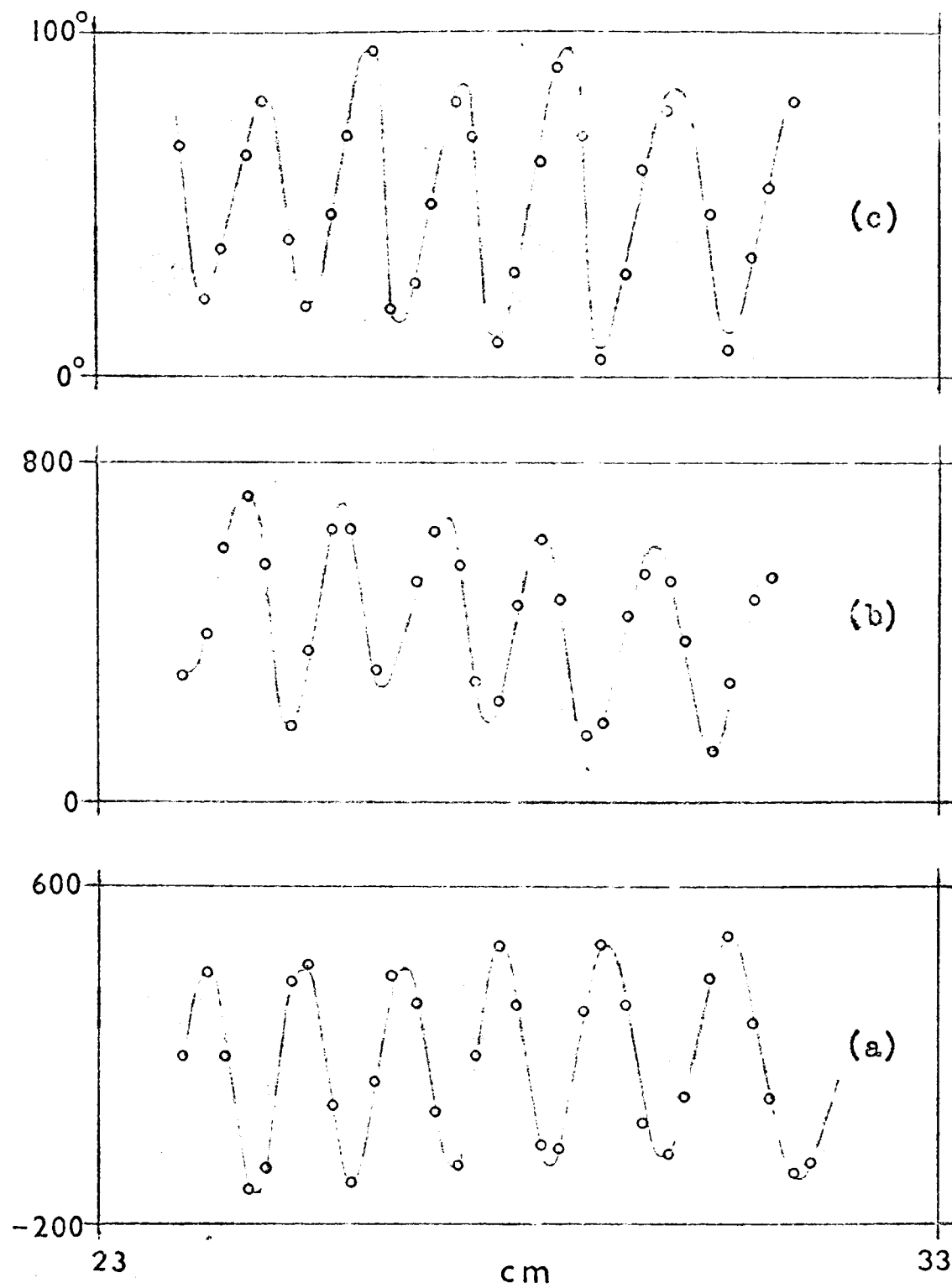


Fig. 21. Axially propagating ion-acoustic wave ; a) the output of the phase-sensitive-detector b) the variation of the wave amplitude c) the variation of the wave phase .

TABLE 4

Discharge parameters		Fundamental frequencies			
p (nTorr)	I (Amp)	(kHz)			
0.25	1.0		17		48
	3.0	10	15	30	
	5.0	10		25	
	7.0	12	17		
	9.0		16		
	10.0	13	16		
0.50	1.0	12			41
	3.0	13			
	5.0	12			
	7.0	13			
	9.0		17		
	10.0				
1.00	1.0		18		45
	3.0	5	13		
	5.0		12		
	7.0		14		
	9.0		15		
	10.0			19	
1.50	1.0				
	3.0		12		
	5.0		14	33	
	7.0			18	
3.00	1.0	3			
	3.0	3	10		
	5.0		14		
	7.0			16	

TABLE 5

Discharge parameters		Ion-acoustic wave band				
p (μTorr)	I (Amp)	f (kHz)	A (db)	σ	h	c
0.25	1.0	no	wave			
	3.0	no	wave			
	5.0	no	wave			
	7.0	no	wave			
	10.0	53	61	8.9	10.0	0.90
0.50	1.0	no	wave			
	3.0	no	wave			
	5.0	57	66			
	7.0	55	68	9.5	11.2	0.85
	10.0					
1.00	1.0	no	wave			
	3.0	68	47			
	5.0	70	50	13.3	17.6	0.75
	7.0	70	52			
	10.0	78	52			
1.50	1.0					
	3.0	90	41			
	5.0	90	52	17.0	24.0	0.72
	7.0	93	46			
	10.0					
2.00	1.0					
	3.0	110	35			
	5.0	115	41	23.2	38.0	0.69
	7.0	120	45			
	10.0					

f - frequency at the peak of the wave band
 A - amplitude at the peak of the wave band
 σ - reduced wave frequency
 h - reduced wave number
 c - reduced phase velocity

TABLE 6

Discharge parameters: $p = 0.50$ mTorr $I = 7.0$ Amp

σ	h	c
8.6	10.1	0.86
9.5	11.2	0.85
10.4	12.8	0.81
12.1	15.4	0.78

TABLE 7

Discharge parameters: $p = 0.50$ mTorr $I = 3.0$ Amp

Exciter	σ	h	c
Grid	6.7	8.3	0.81
	8.4	11.5	0.74
	10.1	14.2	0.71
	10.9	15.8	0.69
	11.8	17.1	0.69
	13.4	23.0	0.59
	16.8	26.5	0.64
Coil	6.7	8.3	0.81
	8.4	11.2	0.75
	10.1	14.4	0.70

3.6. THE LOW-FREQUENCY IMPEDANCE OF THE PLASMA SHEATH

In order to study the way in which the signal picked up by an electrostatic probe depends on the sheath impedance, we have used the probe circuit shown in fig. 22. The probe voltage is actually measured by a wave-analyser and the noise spectrum is recorded in the 30 - 300 kHz range. The discharge regime was chosen in such a way as to have a rather flat spectrum, free of ion-acoustic waves or large uniform oscillations. The discharge parameters are $p = 0.50$ mTorr $I = 3.0$ A. A large inductance coil was connected in series in the discharge circuit in an attempt to eliminate the continuous spectrum of uniform density fluctuations. In this way it is expected that the noise left is mainly space potential fluctuations.

The discharge was earthed at the cathode and, in the low-frequency range we are concerned with, the impedance of the plasma column is expected to be negligible. In these circumstances, the AC current flowing through the probe will be determined by the impedance of the plasma sheath and the impedance of the probe circuit. The AC impedance of the probe circuit is determined by the resistance r_p . When r_p is large enough (that is to say, when r_p is much larger than the sheath impedance), the noise level is independent of either the actual value of r_p or the probe bias. In fig. 23 the top curve was obtained with the probe floating and $r_p = 100$ k Ω , and it is supposed to represent the spectrum of the space potential fluctuations. When r_p is much smaller than the sheath impedance, on the other hand, the probe current spectrum practically reproduces the shape of the space potential spectrum (in a logarithmic scale). When varying the probe bias the spectrum although being displaced still keeps its shape nearly unchanged. The lower spectra shown in fig. 23 were obtained

with $r_p = 10 \Omega$ and are given with the DC bias current (in mA) as parameter. These results suggest that the sheath impedance around floating potential is nearly a pure resistance (in the 30 - 300 kHz range and for the actual parameters of the plasma).

Attempts were made to measure the sheath impedance with the help of a capacitive probe. The probe circuit shown in fig. 24 was used for this purpose. It measures both the applied voltage and the current flowing through the probe and, therefore, it would be possible to measure with it the impedance seen by the probe. The applicability of this method requires the sheath impedance to be the largest one among the dielectric wall, sheath and plasma impedances. Otherwise the measurement of the sheath impedance becomes practically impossible. We found that the impedance seen by the probe was always nearly a pure capacitance; no significant phase shift could be observed when varying the plasma parameters or the probing frequency. This result shows that the pyrex wall is too thick (about 3.5 mm), or in other words, that the wall capacitance is too small (about 1 pF/cm²) for the applicability of this experimental method. Besides, the impedance seen by the probe was also found to become dependent on the position of the earthing point of the plasma column when the probing frequency was high, say, above 5 MHz. This result points out that the plasma impedance becomes too large at high frequency for the applicability of the present method.

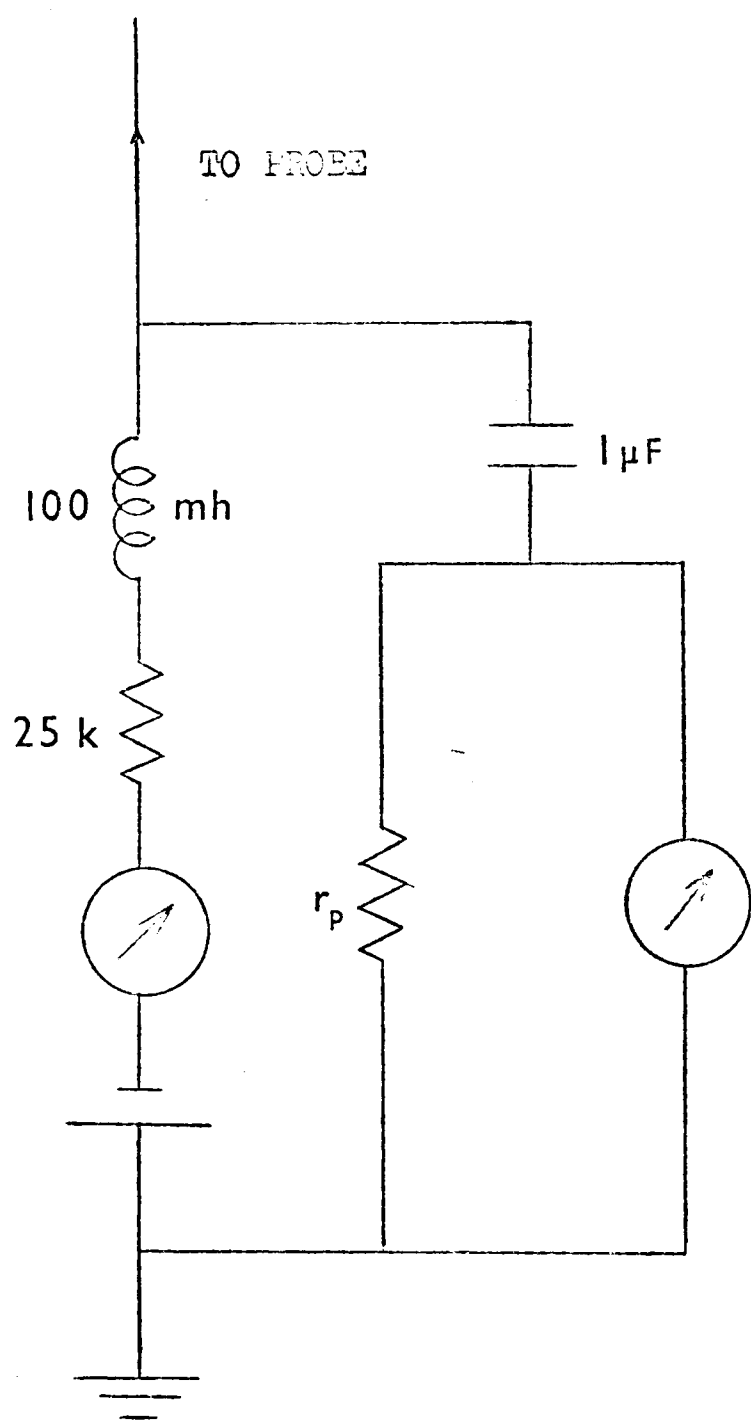


Fig. 22. Circuit employed in the study of the sheath impedance .

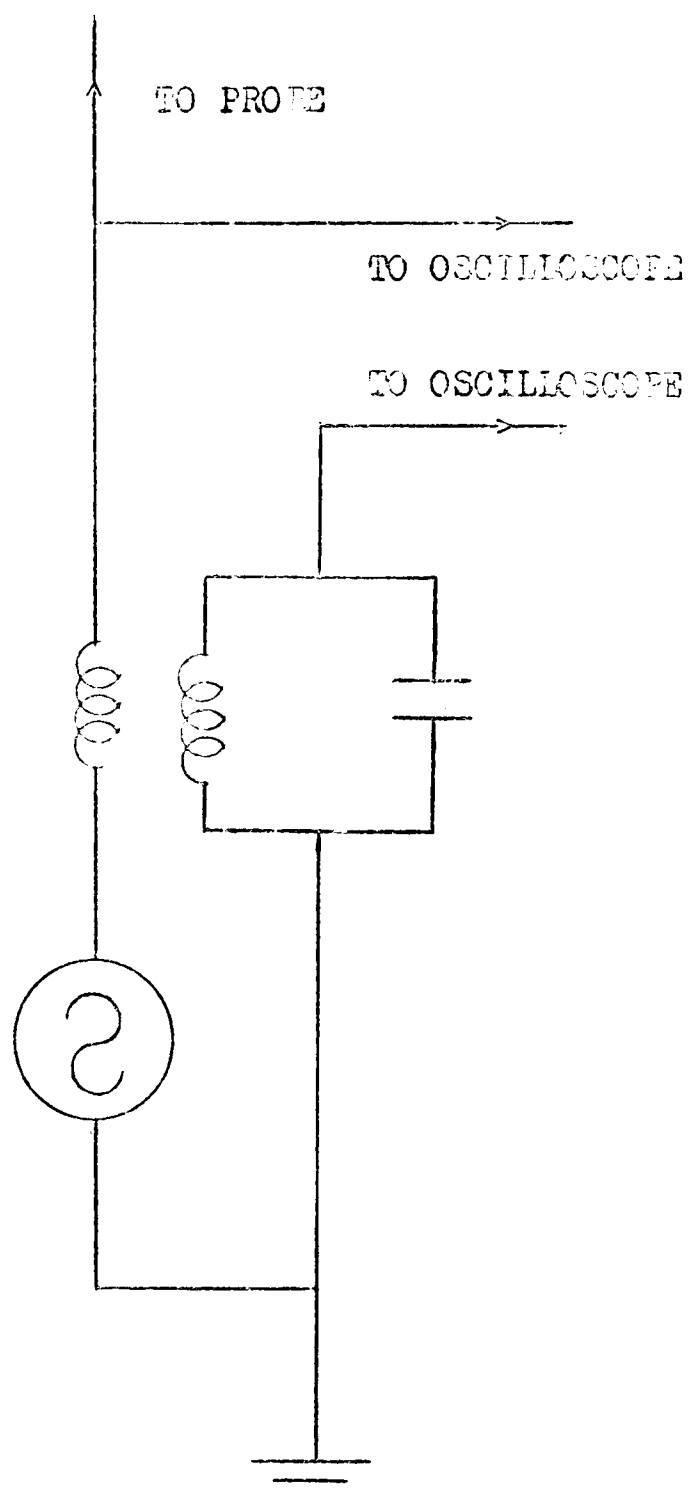


Fig. 24. Circuit employed in the study of the sheath impedance .

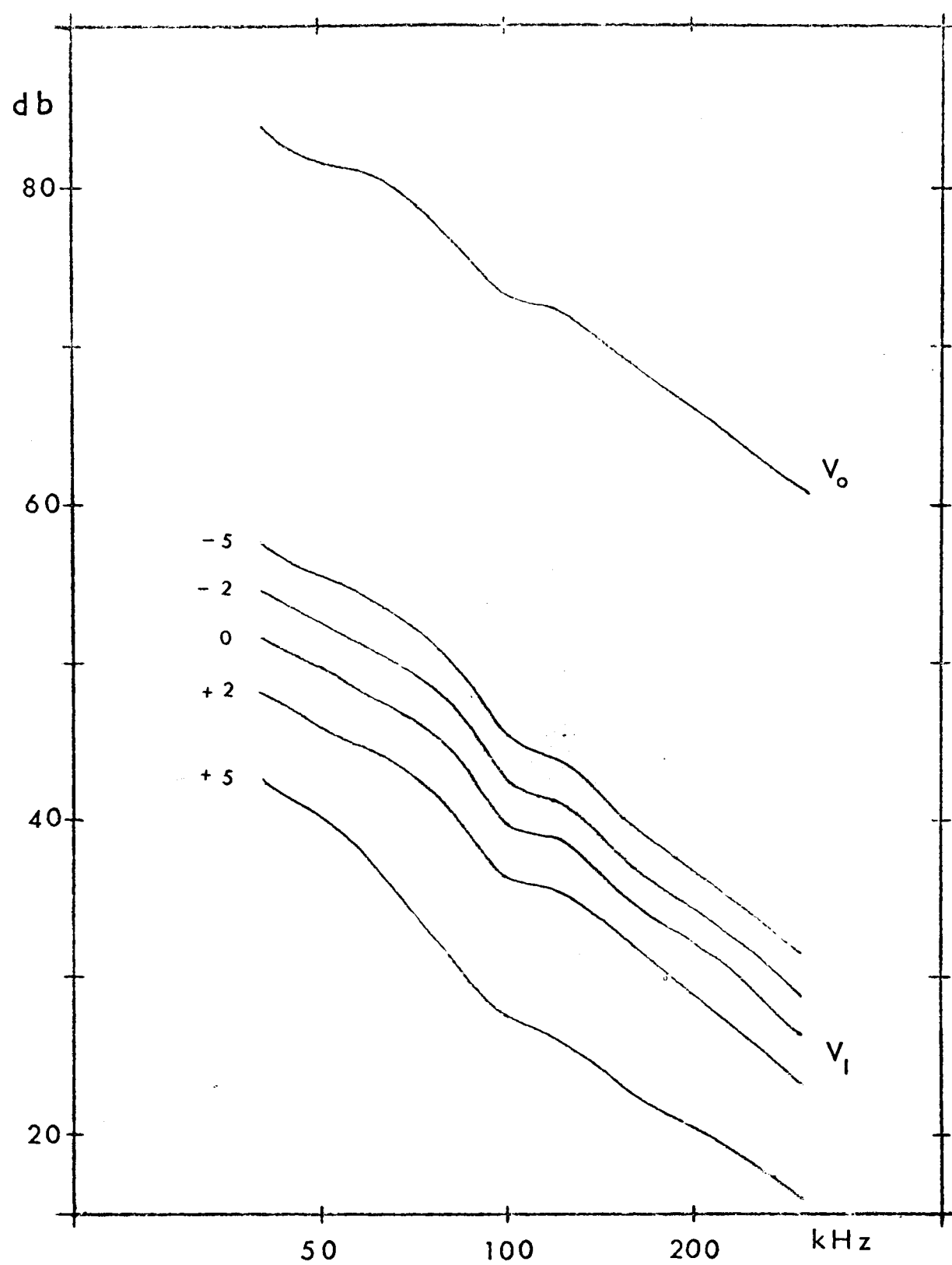


Fig. 23. The influence of the probe bias upon the spectrum level .

Chapter IV

THE POSITIVE COLUMN

The positive column of gas discharges has been the object of extensive study for many years. Nevertheless, its properties are still far from being entirely understood. A better knowledge of its structure and typical phenomena is necessary not only for the sake of this knowledge itself but also because the plasma of the positive column is very often used as a suitable medium for the study of the properties of weakly ionized gases.

In this chapter we deal with the structure of the positive column, the conservation equations which govern it and the similarity laws which can be deduced therefrom. We consider in some detail the electron dynamics and in particular the ionization and mobility mechanisms. The "average-electron" and the "distribution function" viewpoints are presented and developed. The plasma sheath which bounds the positive column is of fundamental importance in determining the boundary conditions upon which many of the plasma phenomena depend. We give the general properties of the plasma sheath in the actual conditions of a low-pressure arc. In particular we deal with the generalization of Bohn's criterion in the presence of oscillations, and with the concept of sheath impedance and techniques to measure it. Finally, pressure effects in low-pressure arc discharges, which have been little studied so far, are extensively analysed here.

4.1. THE STRUCTURE OF THE POSITIVE COLUMN

4.1.1. THE EQUILIBRIUM STRUCTURE OF THE POSITIVE COLUMN

The theory of the equilibrium structure of a low-pressure positive column was first developed by Tonks and Langmuir⁷² and later improved in details by a number of other authors. The classic theory assumes cold ions governed by continuity and momentum equations, and hot electrons governed by the Boltzmann law; the ion mean free path is considered to be indefinitely long. We shall be concerned with the particular case of cylindrical geometry and ionization proportional to electron density; we shall also assume a negligibly small Debye length.

Let R be the column radius, b the ionization rate, a the ion-acoustic speed $a = (kT/n_i)^{1/2}$, η the reduced potential $\eta = eV/kT$, $n(0)$ the plasma density on the axis. The relevant results of Tonks and Langmuir theory are summarized by a number of relations as follows:

1. The potential profile

$$\begin{aligned} br/a &= (2\eta)^{1/2} (1 - 0.200\eta - 0.02606\eta^2 - 0.006489\eta^3 \\ &\quad - 0.00198\eta^4 - 0.000679\eta^5 - 0.000253\eta^6 \dots) \end{aligned}$$

2. The plasma balance equation

$$bR/a = 1.0920$$

3. Potential at the plasma boundary

$$\eta = 1.155$$

4. Plasma density at the plasma boundary

$$n = 0.3150 n(0)$$

5. Total number density per unit length

$$N = 0.700 \pi R^2 n(0)$$

6. Ion current density at the plasma boundary

$$I = 0.3822 en(0)a$$

7. Ion speed at the plasma boundary

$$(\langle v^2 \rangle)^{1/2} = 1.215 a$$

8. Wall potential

$$\eta = \ln [1.044 (n_i/n_e)^{1/2}]$$

The classic theory has been the object of renewed calculations which confirmed the early results. New refinements have also been introduced, as for instance the effect of the finite Debye length (see Parker⁷³ and Self⁷⁴). Experimental confirmation of different features of the theory has been obtained by a number of authors in many circumstances (see Tonks and Langmuir⁷², Killian⁷⁵, Klarfeld⁷⁶, Irish and Byrant⁷⁷, Hoyaux and Gans⁷⁸).

Working on this model, we further evaluate:

9. Mean potential of ion generation

$$\eta = 0.420$$

10. Life-time of the average ion

$$\tau = 0.54 R/a$$

4.1.2. THE DISCHARGE KINETICS

4.1.2.1. Consider a discharge tube a few centimeters in diameter filled with mercury vapour at about 1 mTorr and at about room temperature.

First of all let us consider the collisional regime of the heavy particles. From cross-section values it is found that de-excitation of metastable levels and transitions between metastable levels through atomic impact with ground state atoms have mean free paths too long to be of any importance. But the mean free path for energy exchange, specially so for charge exchange, are of magnitude comparable with the discharge diameter.

The mean life-time of ground or metastable atoms for transitions induced by electronic impact can be calculated through the rate constants.

Let i and k denote two atomic levels and z_{ik} the rate constant for the transition $i-k$. Let n_1 and n_0 be the electron and ground state densities. The mean life-time of the species i for a particular transition $i-k$ is given by $\tau_{ik} = 1/n_1 z_{ik}$, and the mean life-time of the same species is $\tau_i = 1/n_1 \sum_k z_{ik}$. For an electron density of the order of 10^{11} cm^{-3} and temperature of a few electron-volts, τ_i turns out to be comparable with the time of flight of atoms across the discharge.

From the foregoing considerations it can be concluded that in discharges of the type described above, all sorts of heavy particles will be nearly, but not quite, in a free-fall regime. A particular atom, initially in its ground state when entering the discharge, will have a fairly high probability of going through one or two consecutive impacts, specially with electrons, while crossing the discharge. The effect of collisions among heavy particles is just to lengthen the time of flight across the discharge and to increase the chance of electronic impact.

The transition from the i -th level to any other level by electronic impact follows a probability law given by $p_i(t) = (1/\tau_i) \exp(-t/\tau_i)$, which law is similar to the distribution of free paths. Let t_i denote the mean time for boundary loss of the i -th species; the probability of loss of this species by electronic impact transition to any other level is $p_i(t) dt = 1 - \exp(-t_i/\tau_i)$. Consider a ground state atom entering the discharge; while crossing it, this particle may go through a one-step or a two-step process. For a one-step process $0-i$ the rate is $z_{0i} n_0 n_1$ and for a two-step process $0-i-k$ the rate is $\pi_{ik} z_{0i} (1 - \exp(-t_i/\tau_i)) n_0 n_1$, where π_{ik} is the probability of the transition $i-k$ among all the possible ones starting from the i -th level: $\pi_{ik} = z_{ik} / \sum_j z_{ij}$.

2.2. There are two types of optical transitions from the point of view of discharge kinetics. A first type involves the ground or a metastable level; a second one involves two resonant levels.

In a discharge of the kind we are concerned with, the radiation due to transitions involving the ground or a metastable level is propagated by diffusion to the boundary and lost there. As the life-time of an excited resonant atom is much longer than the time the light takes to travel along one of its mean free paths, the energy associated with the optical transition spends most of its time as excitation energy. However, in the low-pressure range we are interested in, light diffuses at a much faster rate than the atomic species. Accordingly, resonance radiation diffuses at a rate determined by the life-time of the upper level and the density of the lower one, as though the diffusing particles were immobile. Besides, resonant levels will tend to have a considerably smaller concentration than metastable levels. Finally, notice that the loss of a photon at the boundary is accompanied by the formation of a metastable or ground state atom.

Radiation from the optical transitions between resonant levels, owing to the low concentration of such levels, escapes directly from the discharge. As a consequence of this high loss rate, the density of energy associated with this type of optical transition is extremely low. Nevertheless, such transitions are important to the energy balance.

4.1.3. THE CONTINUITY EQUATIONS

In what follows, greek subscripts will denote resonant levels and latin subscripts metastable ones. Time constants for boundary loss will be represented by t_i and so on, and transition probabilities will be represented by π_{ij} and so on, according to the nature of both levels involved. The subscripts 0 and 1 will represent the

ground and ionized states.

4.1.3.1. On the basis of the foregoing considerations one can write down the net production rate of the different species in the form $z n_0 n_1$ where z is a rate constant strongly dependent on electron temperature and slowly dependent on electron and neutral densities. For instance the rate constant of excitation to a metastable level k is given by

$$z_k = (z_{ok} + \sum \pi_{ak} z_{oa} + \sum \pi_{a\beta} \pi_{\beta k} z_{oa}) \exp(-t_k/\tau_k) + \sum_j (\pi_{jk} z_{oj} + \sum \pi_{aj} \pi_{jk} z_{oa} + \sum \pi_{ja} \pi_{ak} z_{oj} + \sum \pi_{a\beta} \pi_{\beta j} \pi_{jk} z_{oa}) (1 - \exp(-t_j/\tau_j))$$

The rate constants of excitation to resonant levels and of ionization can also be expressed in more or less similar terms.

Now, it becomes possible to write down in a rather simplified form the continuity equations governing the different populations. Assume that the plasma follows a Tonks-Langmuir density profile at every moment and that the ground state density is uniform. That first condition means that the applicability of this model is limited to the frequency range below the ionization frequency. Let the number densities represent the average values over the discharge cross-section. The continuity equation read:

For the ground state:

$$\dot{n}_0 = -z_0 n_0 n_1 - n_0/t_0 + \sum n_v/t_v + \Phi$$

where v represents states in resonance with the ground level.

For the plasma:

$$\dot{n}_1 = z_1 n_0 n_1 - n_1/t_1$$

For the metastable states:

$$\dot{n}_k = z_k n_0 n_1 - n_k/t_k + \sum n_\beta/t_\beta$$

where β represents states in resonance with the k -th metastable level

For resonant states:

$$\dot{n}_a = z_a n_0 n_1 - n_a/t_a$$

Excited states in resonance with other resonant levels have negligible densities, and the rates of excitation and loss may be considered to be instantaneously equal.

The total rate of excitation and ionization is denoted by z_0 ; for the sake of mass conservation one must have:

$$z_0 = z_1 + \sum z_k + \sum z_a$$

The quantity Φ is the influx of ground state atoms through the boundary; when the time constants associated with de-excitation, recombination and re-evaporation on the wall are negligible, then:

$$\Phi = n_0/t_0 + n_1/t_1 + \sum n_k/t_k + \sum n_a/t_a$$

where a stands for all resonant states except those in resonance with the ground level.

The rate constants for boundary losses depend on the densities and temperatures of the different species. While t_0 and t_k are approximately constant in the nearly free-fall regime, the situation is very different with t_1 and t_a . As a matter of fact:

$$1/t_1 = 1.09 a / R$$

for the plasma loss; for the loss of resonance radiation:

$$1/t_a = 4/(3\tau q^2 n_k^2 R^2)$$

where q is the cross-section for photon reabsorption, τ is the mean life-time of the resonant state, n_k is the density of the metastable or ground state involved in the optical transition (see Appendix A.2.) .

4.1.4. THE ENERGY CONTENT AND THE ENERGY CONSERVATION EQUATION

4.1.4.1. In order to study the energy balance in the positive column, it is convenient to start with the evaluation of the functional dependence and magnitude of the different contributions to the total energy. In the computation of some of these contributions we have used relations and data of the equilibrium structure as given by the Tonks - Lenzmair theory.

Number densities will represent average values over the cross-section. Neutral states densities are assumed uniform. Energy contributions are evaluated per unit length of the column.

The radial drift ion kinetic energy: this was estimated on the basis of the average kinetic energy of each ion during its life-time: $m_i \bar{v}^2/2 = 0.138 kT$ hence:

$$U = 0.138 \pi R^2 kT n_i$$

which is independent of the ion mass.

The electron thermal energy is simply:

$$U = 1.50 \pi R^2 kT n_e$$

The axial drift ion kinetic energy. In a free-fall regime, the drift velocity is: $u = 0.27 eXR / m_i a$ therefore:

$$U = 0.037 \pi e^2 R^4 X^2 n_i / m_i a^2$$

Where X stands for the axial field strength.

The axial drift electron kinetic energy. The drift velocity is given by $u = eX / m_e Q n_0$ where Q stands for the cross-section for electron-neutral momentum transfer; therefore:

$$U = 0.50 \pi e^2 R^2 X^2 n_e / m_e Q^2 n_0^2$$

The ionization and excitation energy. There are contributions due to ions, metastable and resonant states, all of the same form:

$$U = \pi R^2 (\epsilon_i n_i + \sum \epsilon_k n_k + \sum \epsilon_a n_a)$$

where ϵ_i is the ionization energy, ϵ_k is the energy of the k -th

metastable level and ϵ_a is the photon energy associated with the optical transition from the a -th level. When required, transition probabilities can be introduced.

The energy of the self-consistent radial electric field is

$$U = \int X_r^2 r \, dr / 4 = 1.52 (kT/e)^2$$

The energy in the axial electric field, on the other hand, is

$$U = \int X_z^2 r \, dr / 4$$

The sheath energy. This was calculated assuming a Childs -

- Langmuir positive space-charge potential profile across the sheath and a negligible electron density, and in the limit $m_e/m_i \rightarrow 0$.

The field energy and the ion kinetic energy contribution are in the ratio 2:1, the total value being:

$$U = 0.15 \pi R \ln^{5/4}(m_i/m_e) (en_1)^{1/2} (kT/e)^{3/2}$$

It is interesting to know which energy contributions are the important ones. If one takes, say, $en_1 = 10^2$ e.s.u., $kT/e = 10^{-2}$ e.s.u. and $\sum n_k = 10^{12} \text{ cm}^{-3}$ one may estimate the magnitude of the different contributions in a typical low-pressure arc:

excitation energy	75%
ionization energy	15%
electron thermal energy	8%
radial drift ion kinetic energy	1%

all other terms being negligibly small. The total energy contained per unit length of the positive column of a low-pressure arc is, therefore, very nearly given by:

$$U = (1.64 kT + \epsilon_1)n_1 + \sum \epsilon_k n_k + \sum \epsilon_a n_a$$

4.1.4.2. To obtain the energy balance and the energy conservation equation governing the positive column, one has to know the form of the different input and output terms.

Energy loss rates associated with metastable states and radiation are proportional to the loss rates at the boundary of the species involved in the process and all of them are of the same form

$$\epsilon_k n_k / t_k \quad \text{and} \quad \epsilon_a n_a / t_a$$

Radiation due to transitions between resonant states is lost instantaneously at the rate of its production: $\epsilon_a z_a n_0 n_1$. The problem of energy loss associated with the flow of charged particles to the wall is not so obvious. Each ion carries away a mean energy given by the difference between the wall potential and the mean ion birth-potential; in the case of Hg ions this quantity is $\eta = 6.45 - 0.42 = 6.03$. Regarding the electrons, one has to assume that they are maxwellian and to apply a thermodynamic argument. According to the "equipartition principle", a maxwellian gas has $kT/2$ mean energy per quadratic term of energy; when the particle moves with three degrees of freedom under the influence of a one-dimensional parabolic potential, then the mean energy per particle is $2kT$. It is still necessary to add the ionization energy per ion-electron pair. Therefore, the energy loss rate associated with the charged particles is

$$(\epsilon_1 + 8.0 kT) n_1 / t_1$$

The energy input consists of a single term due to the electron drift in the applied field: $e u_e K n_1$, the contribution due to ion drift being several orders of magnitude smaller.

To obtain the energy conservation equation for the positive column one has to equate the rate of change of energy content to the difference between input and output energy terms. When one does this and next eliminates the terms in $\dot{n}_1$, $\dot{n}_k$ and $\dot{n}_a$ with the help of the continuity equations, and further assumes all radiation loss rates to be infinitely large, then one arrives at:

$$1.64 \, d \, kT / dt = e u_e X - 6.4 \, kT / t_1 - \left[(1.64 \, kT + \epsilon_1) z_1 + \sum \epsilon_k z_k + \sum \epsilon_a z_a + \sum \epsilon_i z_\beta \right] n_0$$

where β denotes a state in resonance with the metastable i -th level.

4.1.5. THE ELECTRON MOBILITY EQUATION

A momentum conservation equation for the electron gas is easily obtained. The electrons are accelerated by the axial field and lose momentum by electron-neutral, electron-ion and electron-wall collisions. One can therefore write down

$$d \, n_1 u / dt = e X n_1 / m_e - (Z_0 n_0 + \sum Z_k n_k) n_1 u - n_1 u / t_1 - Z_1 n_1^2 u$$

Here Z_0 and Z_k denote the rate constants for electron-ground level and electron-metastable states momentum transfer; Z_1 denotes the rate constant of the electron-ion Coulomb interaction. Neutral particles densities were assumed uniform; n_1 denotes the electron average density in the Tonks - Langmuir model; use was made of the relation $\langle n_1^2 \rangle \simeq \langle n_1 \rangle^2$. With the help of the plasma continuity equation, the equation above can be considerably simplified. Moreover, in a typical low-pressure arc the ionization rate is much smaller than the rate of momentum transfer, and the Coulomb term is negligibly small. Finally one arrives at:

$$\dot{u} = eX/m_e - (Z_0 n_0 + \sum Z_k n_k) u$$

The rate $\dot{u}/u$ is very large when out of equilibrium. Therefore, in the study of low-frequency phenomena one may assume electron momentum balance at all times, and one obtains the electron mobility equation

$$u = eX/m_e (Z_0 n_0 + \sum Z_k n_k)$$

4.1.6. THE SIMILARITY LAWS

We are now going to consider the laws governing the equilibrium

state of the positive column of a low-pressure discharge. At equilibrium, the conservation equations express mass, momentum and energy balance. With a little manipulation it is possible to cast the equations into more convenient forms. As far as equilibrium densities are concerned, one obtains:

$$n_0 R = g_0$$

$$n_k = g_k T_n^{-1/2} n_1$$

$$n_a = g_a R T_n^{-1} n_1^3$$

where T_n denotes the temperature of the neutral species; the g 's are functions of the electron temperature and little dependent on densities. Regarding field and electron drift one arrives at:

$$XR = (f_1 + f_2 R T_n^{-1/2} n_1)^{1/2}$$

$$u = (h_1 + h_2 R T_n^{-1/2} n_1)^{-1/2}$$

where the f 's and h 's are in the same conditions as the g 's

In the limit when $n_1 \rightarrow 0$, one obtains three laws relating $n_0 R$, u and XR to the electron temperature; it turns out immediately that X/n_0 is also a function of the electron temperature alone. With the help of these laws, when two of the plasma parameters are known then all the other ones will be determined as well. It is also interesting to notice that they provide information about the magnitude and functional dependence of the rate constants of electron impact induced reaction. These laws have been the object of extensive analysis (see Francis⁷⁹ for the foundations of the similarity laws) and experimental investigation. In fig. 25 we have plotted $n_0 R$ and $1/u$ to fit the extensive data of Hoyaux (78 and 80) on the low-pressure range of a mercury-vapour positive column; according to this data one has

$$n_0 R \propto T^{-3}$$

$$u \propto T^3$$

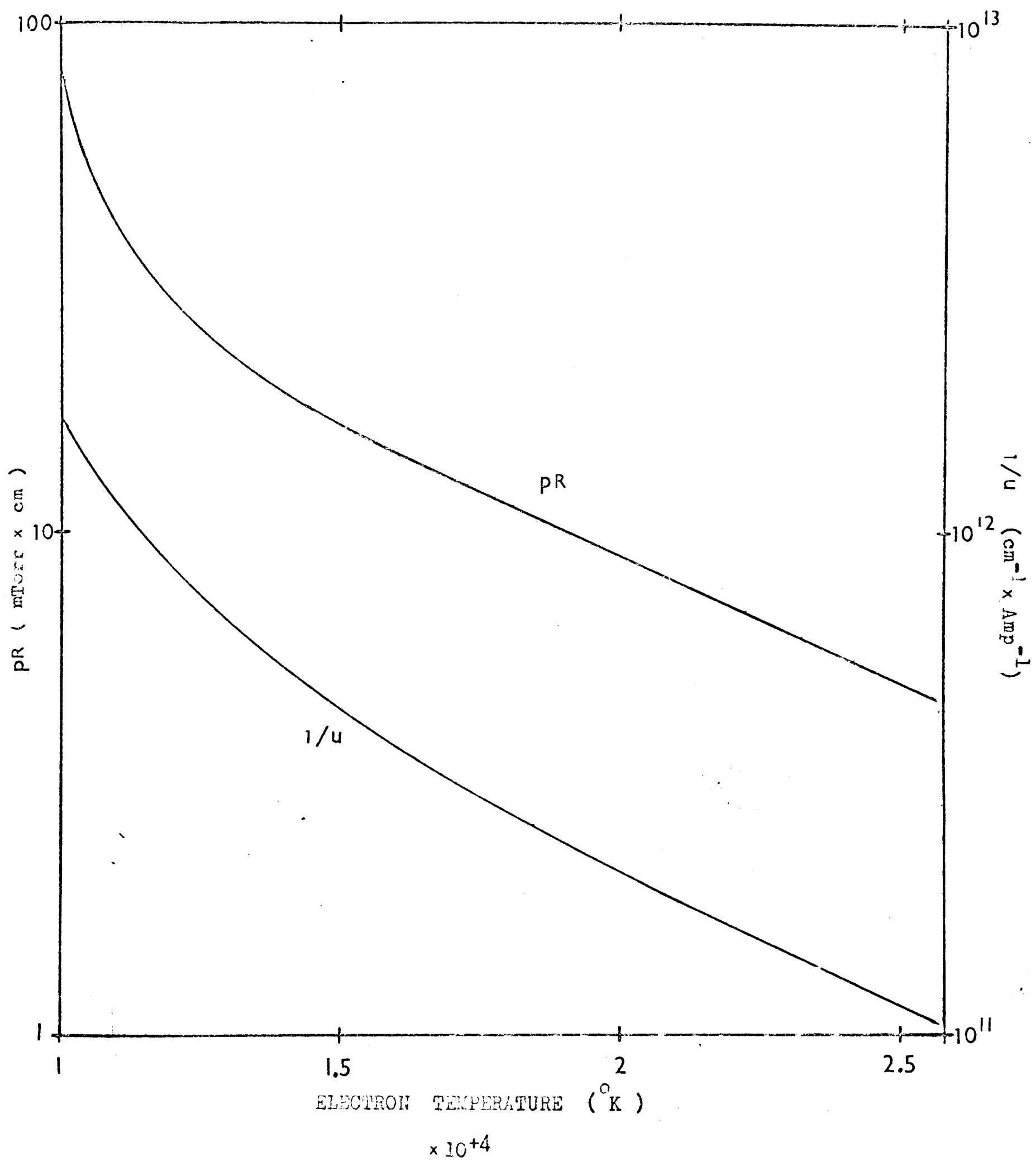


Fig. 25. The similarity laws of the low-pressure mercury-vapour positive column according to the data of Hoyaux and Gans (78) .

4.2. THE ELECTRON DYNAMICS

4.2.1. COLLISION PROCESSES IN MERCURY VAPOUR

4.2.1.1. Elastic electron-neutral collisions are very effective as a momentum randomizing process. The energy exchange per collision is extremely low, however: the average fractional energy loss in the case of collisions between electrons and neutral spherical and rigid atoms in a strong applied field is $2m_e/m_n$ only. This energy process is negligible except in lighter gases. The cross-section for momentum transfer was originally studied in a wide energy range and with high energy resolution by Brode ⁸¹ and ⁸². Later, the work of McCutcheon ⁸³ and Tan ⁸⁴, extending the experimental data to the lower energy range, showed a marked peak in the cross-section near 0.5 eV, about $130 \times 10^{-16} \text{ cm}^2$ high. Below this energy the Ramsauer-Townsend effect shows up and the cross-section decreases. According to Choudhury ⁸⁵, there is a minimum of $10.5 \times 10^{-16} \text{ cm}^2$ at zero energy. With the exception of the range below 0.5 eV, the cross section varies roughly as $1/v$, and $Q(v) = 0.60 \times 10^{-6}/v \text{ C.G.S.}$ may prove to be a convenient approximation.

4.2.1.2. Inelastic electron-neutral collisions are important in determining the electron distribution function, although they are not important as a momentum loss process because their frequency is low relatively to the elastic collision frequency. Regarding the direct atomic exciting collisions from the ground level, the best known transitions are the $6^1S_0-6^3P$ ones, but it is likely that the total cross-section is appreciably larger than that of the transitions to the 6^3P levels. At the higher electron energy range the transitions $6^1S_0-6^1P_1$ and 6^1S_0-Hg^+ are dominant, and the total inelastic cross-section for excitation and ionization from the ground level apparently has a flat maximum of about $13 \times 10^{-16} \text{ cm}^2$ around 30 eV. The total inelastic

cross-section for excitation and ionization from the ground level up to the 6^3P , 6^1P_1 , 7^3S_1 , 6^3D , Hg^+ levels is plotted in fig. 26 according to the data obtained by several authors (see appendix A.1). From this curve, and assuming a maxwellian electron distribution, the total rate constant for excitation and ionization from the ground level was computed and is shown in fig. 27 . In the same graph is shown the rate constant for direct ionization as computed by Cayless ⁸⁶ . Also on the graph is marked the ionization rate constant **at discharge currents of about $I = 5$ Amp** as obtained from our experimental data by means of the plasma balance equation of the positive column:

$$z = 1.09 a / Rn_0$$

(see paragraph 4.1.1.). The experimental ionization rate constant is from two to three times larger than the direct ionization rate constant, but still about ten times smaller than the total inelastic rate constant. These facts strongly suggest that a predominantly stepwise ionization mechanism is going on. As a matter of fact, the concentration of excited states increases rapidly with the electron concentration (see paragraph 4.1.6.), in such a way that cumulative ionization, cumulative excitation and superelastic collisions become important energy processes when the plasma density is high enough. This is the case of the low-pressure arc in opposition to the low-pressure glow discharge. Provided that the concentration of excited states is relatively large, the energy exchange between the electron gas and the excited states population becomes an important mechanism for electron energy randomization. Moreover, electrons tend to lose their energy to the excited states in small steps but tend to recover that energy in large exchanges. In fact, the larger the energy exchange in a transition the larger the rate constant for the reverse superelastic process; and the higher the threshold the smaller the rate constant for the direct inelastic process.

Therefore, a large population of excited states provides also a mechanism of repletion of the high energy tail of the distribution function.

4.2.1.3. In a weakly ionized gas, charged particle interaction is unimportant by definition. In a low pressure arc which typically has a degree of ionization of the order of 10^{-2} , although this type of interaction is of relatively little importance as an electron momentum randomizing process, it can however be important as an electron energy exchange one. There are two types of Coulomb collisions: binary and cumulative collisions, the former produced by the interaction of a single pair of near by particles the latter due to the simultaneous interaction of an electron with all the other particles in its Debye sphere. The rate constants for these two processes are respectively (Holt and Haskell⁸⁷) :

$$Z \simeq 1.0 \times 10^{-10} (kT/e)^{-3/2}$$

$$Z \simeq 4.1 \times 10^{-10} \ln \Lambda (kT/e)^{-3/2}$$

where:

$$\Lambda \simeq 8.0 \times 10^{13} n^{-1/2} (kT/e)^{3/2}$$

(all in e.s.u.) . One should notice that $\Lambda^{2/3}$ is approximately the ratio of the average kinetic energy per electron to the average potential energy between neighbour particles. In all cases of laboratory plasmas this ratio is very large, and therefore the frequency of multiple collisions will be considerably larger than the frequency of direct collisions. It is worth notice that the condition $\Lambda \gg 1$ also implies that (i) the electron gas follows the classic statistical law and behaves like a perfect gas (ii) the Debye sphere contains a large number of particles.

In a low pressure arc one has, say $kT/e \approx 10^{-2}$ e.s.u. , $n \approx 10^{11} \text{ cm}^{-3}$, and $\ln \Lambda \approx 12$. The rate constant for Coulomb collisions turns out to be of the order of $10^{-6} \text{ cm}^3/\text{sec}$. Having

in mind that the rate constant for ionization is of the order 10^{-9} , and that the degree of ionization is of the order of 10^{-2} , one finds that each electron goes through an average number of Coulomb collisions of the order of 10 during its life-time. This shows how important this type of interaction can be even in a plasma with a relatively low degree of ionization.

1.4. In a typical low-pressure mercury-vapour arc, where we have, say,

$$p \approx 0.5 \text{ mTorr}, \quad n \approx 2 \cdot 10^{11} \text{ cm}^{-3}, \quad kT \approx 2.5 \text{ eV}$$

the different collision frequencies are

elastic collisions	1×10^7	sec^{-1}
inelastic collisions	5×10^5	sec^{-1}
Coulomb collisions	1×10^6	sec^{-1}
ionization collisions	3×10^4	sec^{-1}

4.2.2. THE ELECTRON DISTRIBUTION FUNCTION

The electron distribution function in a low-pressure current-sustained discharge is generally considered to be nearly maxwellian. Early experimental data supported this view in contradiction to theoretical elementary arguments (Langmuir paradox). More recent data proves that the distribution function is not quite maxwellian indeed, although it may sometimes be nearly so with the exception of the high energy tail where only a small minority of the whole population is contained.

A maxwellian distribution is in general approached together with thermodynamic equilibrium in an isolated system of particles and after a time long compared with the momentum and energy relaxation times. A low-pressure discharge is far from being an isolated system, given that energy is being continuously supplied by the applied field and disposed of by the plasma, radiation and excited atoms losses at the boundary and by heating

up the background gas. Moreover, at low plasma density the electron mean life-time may be shorter than the relaxation times. It is therefore evident that the electron gas in a low-pressure glow will be far from being in thermodynamic equilibrium, while in the low-pressure arc it may be somewhat closer to it (due to the higher electron-electron collision rate).

The electron distribution function in a number of different idealized condition has been the object of theoretical study with the help of the Boltzmann equation. Cahn⁸⁸ and Gureyvitsh⁸⁹ studied the distribution function of an electron gas in a static electric field and subjected to electron-electron and electron-neutral elastic collisions and found that the distribution will be maxwellian for any electron density provided that the **electron-neutral** elastic collisions frequency is constant (which is roughly the **electron-neutral** case of mercury). For a constant elastic collision cross-section instead, the distribution will change from the Druyvesteyn distribution to the maxwellian one with increasing degree of ionization. The study of the distribution function in more realistic situations, that is to say including both elastic and inelastic collisions and the role of metastable states and radiation as well, has been attempted by a few authors: Haylen⁹⁰, Kagan⁹¹, Surdo⁹². It has been concluded that a nearly maxwellian distribution may in some case be expected, but this will not necessarily be the case in most circumstances. No comprehensive analysis of the low-pressure mercury-vapour discharge is known up to now.

Data obtained by means of the Langmuir probe characteristic are often found compatible with the assumption of a maxwellian distribution^{75 93}. It is helpfull to recall here that the electron current drawn by the probe is due to the whole electron population above an energy treshold determined by the probe bias. Consequently, the Langmuir probe characteristic technique tends to be reliable for

the low electron energy range, but not anymore for the higher range when the collected electron current is much smaller than the ion current.

Even so, Howe ⁹⁴ was able to observe a depletion of the high energy tail in Langmuir probe characteristics obtained in an extensive series of measurements in a low-pressure mercury-vapour arc. But this effect is often either overlooked or not observed at all. Druyvesteyn ⁹⁵ introduced an important extension to the electrostatic probe method by proving that the true energy distribution can be obtained from the second derivative of the probe characteristic. Rayment and Twiddy ⁹⁶ applied this technique to the positive column of a low-pressure mercury-vapour glow discharge, and found that the distribution was anisotropic, and depleted of high energy electrons in the direction normal to the axis. He also found that the distinct groups in the energy distribution persist along the column though shifted and attenuated, which excludes the existence of any strong energy-exchanging process which have been invoked by different authors in connection with the Langmuir paradox. It is convenient to point out that in the low-pressure arc the situation can be substantially different, owing to the role of electron-electron collisions and excited states populations, which will be able to bring about further energy randomization.

4.2.3. THE ELECTRON MOBILITY

The electron motion in an applied field and subjected to purely elastic collisions has been the object of extensive studies (see Loeb ⁹⁷). The classic Langevin's theory applies to an isotropic velocity distribution of electrons in thermal equilibrium with the scattering gas and yields the drift velocity proportional to the reduced field (the field divided by the gas number density). A recent improvement due to Davidson ⁹⁸ includes the effect of the velocity dependence of the

electron mean free path. Another extension of the Langevin's theory is due to Compton⁹⁹ who considered the case of a slightly anisotropic distribution. From the analysis of the energy balance of the drifting electrons the drift velocity results proportional to the square root of the reduced field. This theory is found to agree with experimental data at moderate fields. The electron drift when inelastic collisions are dominant was treated by Cobine¹⁰⁰. He assumed that the probability of the electron free path follows an exponential law (which means a constant cross-section) and that every free path is started from rest. In this case the drift velocity turns out proportional to the square root of the reduced field.

The problem of the electron drift under the conditions operating in the positive column of a low-pressure discharge was studied in detail by Compton and Langmuir¹⁰¹, Killian⁷⁵ and Tonks and Langmuir⁷². Recently Forrest¹⁰² studied this problem in the specific case of mercury vapour including the effects of both elastic and inelastic collisions.

4.2.4. THE ELECTRON DYNAMICS IN THE POSITIVE COLUMN.

THE AVERAGE-ELECTRON APPROACH

4.2.4.1. Each electron is produced with little energy. Afterwards it is continuously accelerated by the axial field but at the same time confined inside the discharge by the potential barrier of the wall sheath. An electron with enough energy will escape only when its velocity lies within a certain escape cone. The discharge volume is a reservoir for electrons possessing enough energy to escape but which are reflected elastically by the wall sheath because they are not suitably directed. The axial and the transverse velocity are uncoupled in the absence of scattering agents. The energy due to the applied field is not available

to overcome the wall barrier unless an elastic collision scatters the electron in such a way that this acquires a substantial radial energy component (see Crawford¹⁰³).

In what follows we shall neglect end effects upon the electron dynamics in the positive column. This will apply in experimental situations when wall current is much larger than the anode current. We shall also be dealing with a weakly ionized gas so that charged particle interaction may be neglected.

The energy balance is easily formulated. The energy input per unit time is $e u X$; the electron mean life time is $\tau = 1/z_1 n_0$; consequently, the average energy acquired from the axial field per electron is $e u X / z_1 n_0$. Now, the average electron has to produce one ionizing collision to maintain the charge balance. Eventually, it will escape to the wall, and in the process it takes away $6kT$ which is given to the radial field (because $6kT$ is the wall barrier in a mercury vapour discharge) plus $2kT$ excess energy (because this is the average energy per electron). To complete the energy balance picture one has to incorporate the mechanisms of loss of excitation energy as well; let E stand for the excitation energy loss per electron. The energy balance equation reads :

$$e u X = z_1 n_0 (\epsilon_1 + E + 8kT)$$

In a typical low-pressure mercury-vapour arc the quantity E accounts for most of the total loss : Killian⁷⁵ .

4.2.4. 2. Under the action of a steady applied field each electron will go through a sequence of cycles which begin and end with inelastic collisions. The electron starts each cycle with relatively low energy (around 1 eV as it will be seen later) and from then on it is uniformly accelerated. At each elastic collision its velocity is

randomized but its energy is maintained. When its energy becomes large enough, then there will be a finite chance for an inelastic collision to occur, and both kinds of collision become competitive. Eventually there will be an inelastic collision which will bring the energy down to a relatively small value again, and another cycle starts. Eventually it will leave the discharge when its velocity becomes large enough and gets within its escape cone.

Assume the electron scattering process to be isotropic.

Also assume $eXdt \ll nv$ where dt is the mean free time for elastic collision. In the limit $eXdt / nv \rightarrow 0$, one has a continuous process of momentum randomization. Let dv denote the mean speed increment between two collisions. It is straightforward that

$$dt = 1/Qvn$$

$$dv = \left(\frac{\pi eXdt}{4m} \right)^2 / 2v$$

The electron drift velocity is the average velocity along the field during one average cycle:

$$u = \frac{eX}{2m} \Sigma dt^2 / \Sigma dt$$

where the summations extend over all the elastic collisions in one complete cycle. These can be put in an integral form:

$$\Sigma dt = 2n \left(\frac{4m}{\pi eX} \right)^2 \int Qv^2 dv$$

$$\Sigma dt^2 = 2 \left(\frac{4m}{\pi eX} \right)^2 \int v dv$$

Let us introduce the rate constant for elastic electron-neutral collision $Z = \langle Qv \rangle$. When the cross-section varies as $1/v$, as it is nearly the case of the mercury ground level, then the rate constant is velocity independent. We shall include the contributions

due to the metastable levels and assume that the cross-section for these states also vary as $1/v$. The integrals above are now readily carried out and we obtain

$$u = eK/2m(Z_0 n_0 + \sum Z_k n_k)$$

The condition required for the applicability of the drift velocity concept is that the characteristic time one is interested in must be rather longer than the time involved in one cycle as described in this theory. This time is of the order of ϵ^*/euX where ϵ^* is a characteristic excitation energy. In a low-pressure arc this time is of the order of several μsec .

Now, having in mind that dv is the average speed increment between two collisions, one can evaluate the average number of elastic collisions produced per electron while it is being accelerated up to the characteristic excitation energy :

$$N = 4 \left(\frac{4n}{\pi eX} \right)^2 \int_0^{\epsilon^*} Q^2 \epsilon \, d\epsilon$$

In our experimental conditions this number is of the order of 10^2 . Therefore, our initial assumption that the momentum randomization is a continuous process, is justified.

4.2.4.3. When the collision cross-section is velocity independent, the free path obeys a simple exponential distribution law. When the cross-section is velocity dependent, as it is generally the case, the distribution law is more complex. Let Q and q denote the elastic and inelastic cross-sections respectively. Let $P(v)$ be the probability of an electron starting from rest to produce an inelastic collision when its velocity attains the value v . One can imagine the electron motion divided in a large number n of small steps such that the probability for inelastic collision is the

same in every individual step. Obviously one can write:

$$dP = (1 - qndx)^m qndx$$

From here one easily comes to

$$dP = \exp(-X) qndx$$

where

$$X = \int qndx$$

Now, $dx = vdt$, and dt/dv is known in terms of the elastic cross-section as it was found when dealing with the electron mobility mechanism. Therefore,

$$P = - \frac{d}{dv} \exp(-X)$$

$$X = 4 \left(\frac{4n}{\pi eX} \right)^2 \int_{\epsilon_k}^{\epsilon} Qq\epsilon d\epsilon$$

Where ϵ_k denotes the energy threshold for excitation. It is readily seen that the value $\bar{\epsilon}$ where $X(\bar{\epsilon}) = 0.70$ divides the energy axis in two ranges such that

$$\int_0^{\bar{\epsilon}} P d\epsilon = \int_{\bar{\epsilon}}^{\infty} P d\epsilon = 1/2$$

Also, the maximum of P lies near $\bar{\epsilon}$. In fact, P is the product of a decaying exponential of a certain power of $\epsilon - \epsilon_k$ by another power of $\epsilon - \epsilon_k$, and therefore its maximum lies in the neighbourhood of the point $\bar{\epsilon}$ where the exponential is $1/2$. Accordingly, $\bar{\epsilon} - \epsilon_k$ is a suitable estimate for the excess energy left to the electron when it produces an inelastic collision.

The integral in the formula defining X was estimated from the known data concerning the mercury ground level cross-sections. In our experiments, $4n/\pi eX \simeq 2 \times 10^{26}$ and we obtain $\bar{\epsilon} - \epsilon_k \simeq 0.7$ eV. It turns out that it is improbable for an electron to have such a low energy as to be subjected to the Ramsauer - Townsend effect, and that is

why the assumption that the elastic cross-section varies as $1/v$ can be acceptable. Also, the magnitude of $\bar{\epsilon} - \epsilon_k$ clearly shows that most excitation transitions are to the 6P levels.

4.2.5. THE ELECTRON DYNAMICS IN THE POSITIVE COLUMN.

THE DISTRIBUTION FUNCTION METHOD.

4.2.5.1. Consider an electron gas in interaction with a cold neutral gas background and under influence of an applied electric field. Assume that the distribution function is nearly maxwellian, in which case it will be enough to express the actual distribution as the sum of the two first terms of a spherical harmonics expansion:

$$f = f^0 + \cos\theta f^1$$

where θ stands for the angle between the vector f^1 and the velocity vector, and f^0 is the maxwellian distribution. With the help of Boltzmann's equation, one obtains a set of two differential equations governing the isotropic f^0 and anisotropic f^1 terms of the distribution (see Holt and Haskell⁸⁷):

$$df^1/dt + w df^0/dx - \frac{eX}{m} df^0/dw + \nu f^1 = 0$$

$$df^0/dt + \frac{w}{3} df^1/dx - \frac{eX}{3mw^2} d(w^2 f^1)/dw - \frac{g}{2w^2} d(\nu w^3 f^0)/dw = 0$$

where g is the average fractional energy loss per collision, ν is the collision frequency and w is the velocity magnitude. Further on we shall denote by n^0 and n^1 the number densities associated with the isotropic and anisotropic terms of the distribution.

Introduce a small isothermal perturbation of the form $\exp j(\omega t - \beta x)$; the equilibrium and perturbed quantities will bear the lower subscripts 0 and 1 respectively.

Throughout the following calculation it will be assumed that the collision frequency is velocity independent. This condition is

strictly satisfied in the case of the maxwellian-type of interaction, that is to say when the potential varies as the power -4 of the distance. This condition is roughly satisfied in the case of the interaction between the electron and the mercury ground level, because the largest cross-section, the one for elastic collision, varies roughly as $1/v$ over most of the energy range of interest.

It will also be assumed that $\omega \ll v$ which in practical terms means applicability up to the MHz range.

Take the first differential equation, linearize and Fourier-transform. One then obtains f_0^1 and f_1^1 . From these it is possible to evaluate the zero-th and first order drift velocities:

$$u_0 = -eX_0/mv$$

$$u_1 = (2\pi kT/m)^{1/2} n_1^1 / n_0^0$$

The steady state drift velocity formula is identical to the one obtained by Kihara¹⁰⁴ with a similar method.

Now, linearize and Fourier-transform the two differential equations and next integrate in velocity space. The zero-th order solutions yield:

$$X^2 = \frac{3}{2} \frac{mv^2 kTg}{e^2}$$

$$n_0^1 = \left(\frac{3g}{4\pi} \right)^{1/2} n_0^0$$

The equation for the steady-state field is analogous to the one obtained by Von Engel¹⁰⁵ for the glow discharge. On the other hand, the first order solutions yield:

$$-\frac{u_1}{u_0} + (1 - j \frac{\beta kT}{eX_0}) \frac{n_1^0}{n_0^0} + \frac{X_1}{X_0} = 0$$

$$(2 + j \frac{\beta kT}{eX_0}) \frac{u_1}{u_0} + 3(gv - j\omega) \frac{kT}{eu_0 X_0} \frac{n_1^0}{n_0^0} + 2 \frac{X_1}{X_0} = 0$$

which relates the density, drift and field perturbations at constant

electron temperature.

The applicability of the foregoing model to the actual bounded discharge still has to be justified. In fact, the boundary affects the idealized situation which has been analysed in two ways. First, in the presence of the boundary there is loss of high energy electrons with velocity vectors within the escape cone and the consequent disturbance of the distribution function. Second, the loss of particles to the wall introduces an extra energy loss mechanism (an average $8kT$ per every pair ion-electron lost to the wall in the case of a mercury-vapour plasma). The first of these effects has already been discussed in paragraph 4.2.2. and we shall not discuss it further. The second effect is not a serious one from the theoretical point of view because it can be accounted for in g . However, it is only a small effect provided that the excitation frequency is much larger than the ionization frequency, which is always the case at low electron temperature.

4.2.5.2. Combining the equations in U_0 and X_0 one arrives at:

$$u_0^2 = \frac{3}{2} \frac{kT}{m} g$$

which defines the electron drift velocity in terms of the electron temperature. The average fractional energy loss per collision g can be computed in terms of the electron-neutral rate constants and of the fractional energy loss in the different sorts of collisions. The elastic rate constant depends little on the temperature; it will be assumed constant and taken to be $Z = 0.60 \times 10^{-6}$ C.G.S. (see paragraph 4.2.1.1). The inelastic rate constant z has already been computed earlier (see paragraph 4.2.1.2). The fraction of energy lost per collision is negligibly small in the case of elastic collisions and is nearly 1 in the case of inelastic collisions. Given that the actual temperature dependence of the fraction of energy lost per inelastic collision is

rather small and hard to estimate, the effect of this dependence is not included in our calculation. We can therefore write:

$$g = z/(Z + z)$$

In this way we obtain the following table:

kT	z	g
(eV)	($\times 10^{-9}$ C.G.S.)	($\times 10^{-2}$)
1.0	0.75	0.124
1.5	5.2	0.88
2.0	13.8	2.30
3.0	39.0	6.2
6.0	126	17.1

From here the drift velocity was computed and is shown in fig. 28 . For comparison, the curve computed by Forrest¹⁰² is also plotted. Experimental data obtained by us is shown on the same graph. Our experimental drift velocity points were obtained in two ways: first, by measuring the axial field strength and applying the mobility equation; second, from the plasma density measured on the axis and assuming a Tonks-Langmuir density profile. In both cases data was obtained at moderate plasma density (near $1 \times 10^{11} \text{ cm}^{-3}$) to avoid the effects which otherwise would be produced by a larger metastable states population. Experimental points due to Allen and Magistrelli¹⁰⁶ are also marked. Having in mind that the computation of our theoretical curve involves a few small simplifications, and given the way how theory and experimental data compare, one can say that the foregoing theoretical model seems to give an acceptable description of the physical situation.

4.2.5.3. The presence of drift affects the distribution function in such a way as to produce significant changes in the rate constants. The influence of drift upon the ionization rate and electron temperature in a mercury-vapour discharge were studied by Forrest¹⁰². Now, we shall simply deduce an analytic expression for the direct ionization rate including the effect of the drift.

It was shown by Killian⁷⁵ and by Von Engel¹⁰⁵ that the direct ionization rate in mercury can be computed by assuming a linear approximation of the ionization cross-section. The error introduced is not significant because the cross-section has a rather slow slope and reaches its maximum at an energy considerably larger than the threshold. We therefore take the cross-section

$$q(\epsilon) = \sigma(\epsilon - \epsilon_1) \quad (\epsilon > \epsilon_1) \quad \quad q(\epsilon) = 0 \quad (\epsilon < \epsilon_1)$$

and a drift-maxwellian distribution:

$$f(\epsilon) = (1/\pi(2mkT)^{1/2}u) \exp(-(\epsilon + mu^2/2)/kT) \\ \left[\exp((2n\epsilon)^{1/2}u/kT) - \exp(-(2m\epsilon)^{1/2}u/kT) \right]$$

Performing the integration of qfv over the velocity space, one obtains:

$$z = [4\sigma(kT)^{3/2}/(2\pi m)^{1/2}] \exp(-\epsilon_1/kT) \\ \left[(2 + \epsilon_1/kT) + (2 + 5\epsilon_1/3kT + (2/3)(\epsilon_1/kT)^2) \frac{mu^2}{2kT} \right]$$

where terms in $(\frac{mu^2}{2kT})^2$ have been neglected. At the limit when $(\frac{mu^2}{2kT}) \rightarrow 0$ one obtains the same formula as obtained by Killian and Von Engel.

In our experiments in a low-pressure mercury-vapour arc we have:

$$mu^2/2kT \simeq 3 \times 10^{-2} \quad \epsilon_1/kT \simeq 4$$

and we estimate an increment in direct ionization rate of about +10% due to the presence of electron drift. Once the temperature dependence of the ionization rate is known, it is possible to estimate next the

correspondent increment in electron temperature necessary to maintain the charge balance. In the example we are considering, this electron temperature increment is about -2% which is small. The electron drift effect, however, becomes significant when the drift velocity is comparable with the thermal speed.

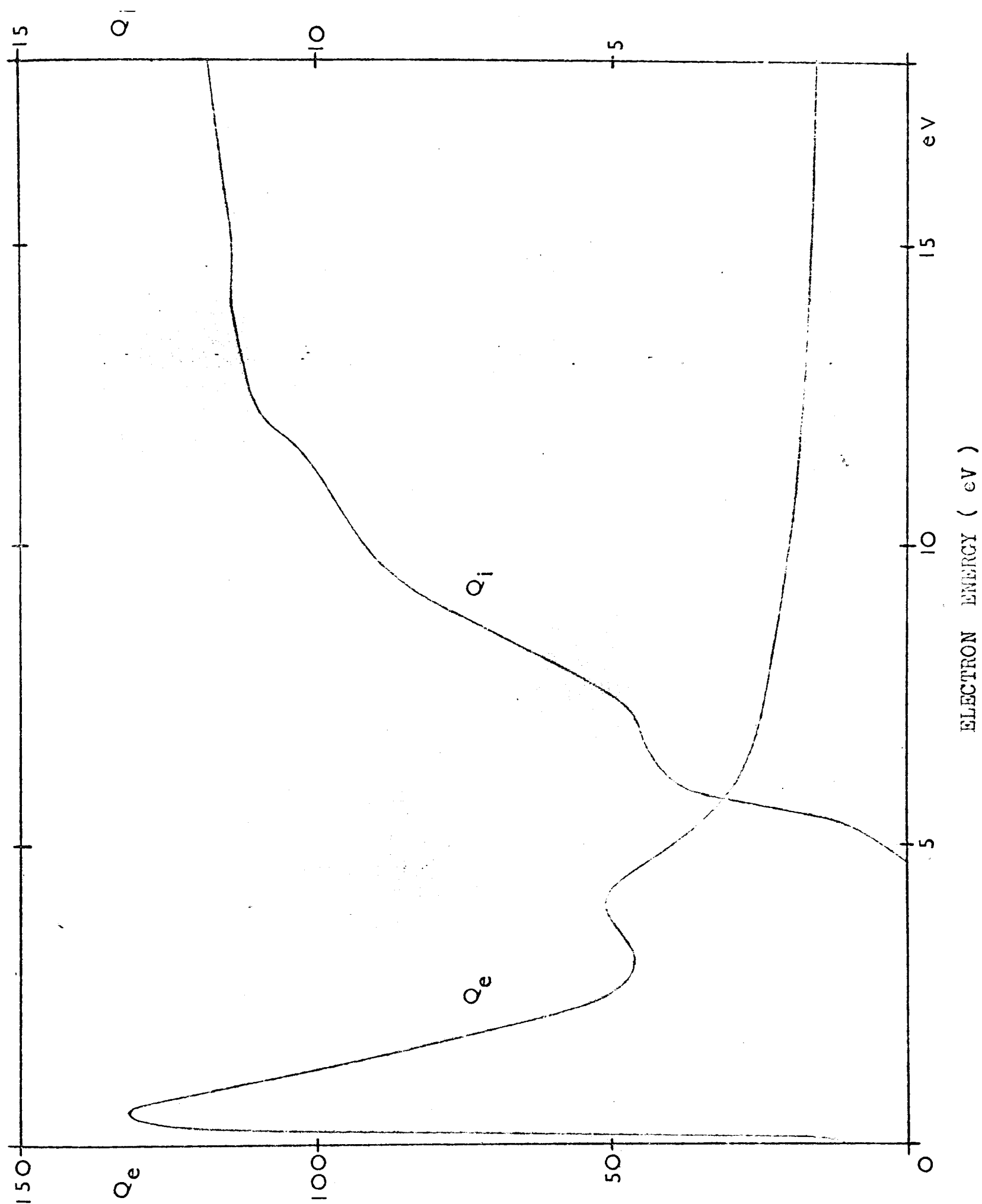


Fig. 26. Elastic and inelastic collision cross-sections in mercury-vapour (10^{-16} cm^2) .

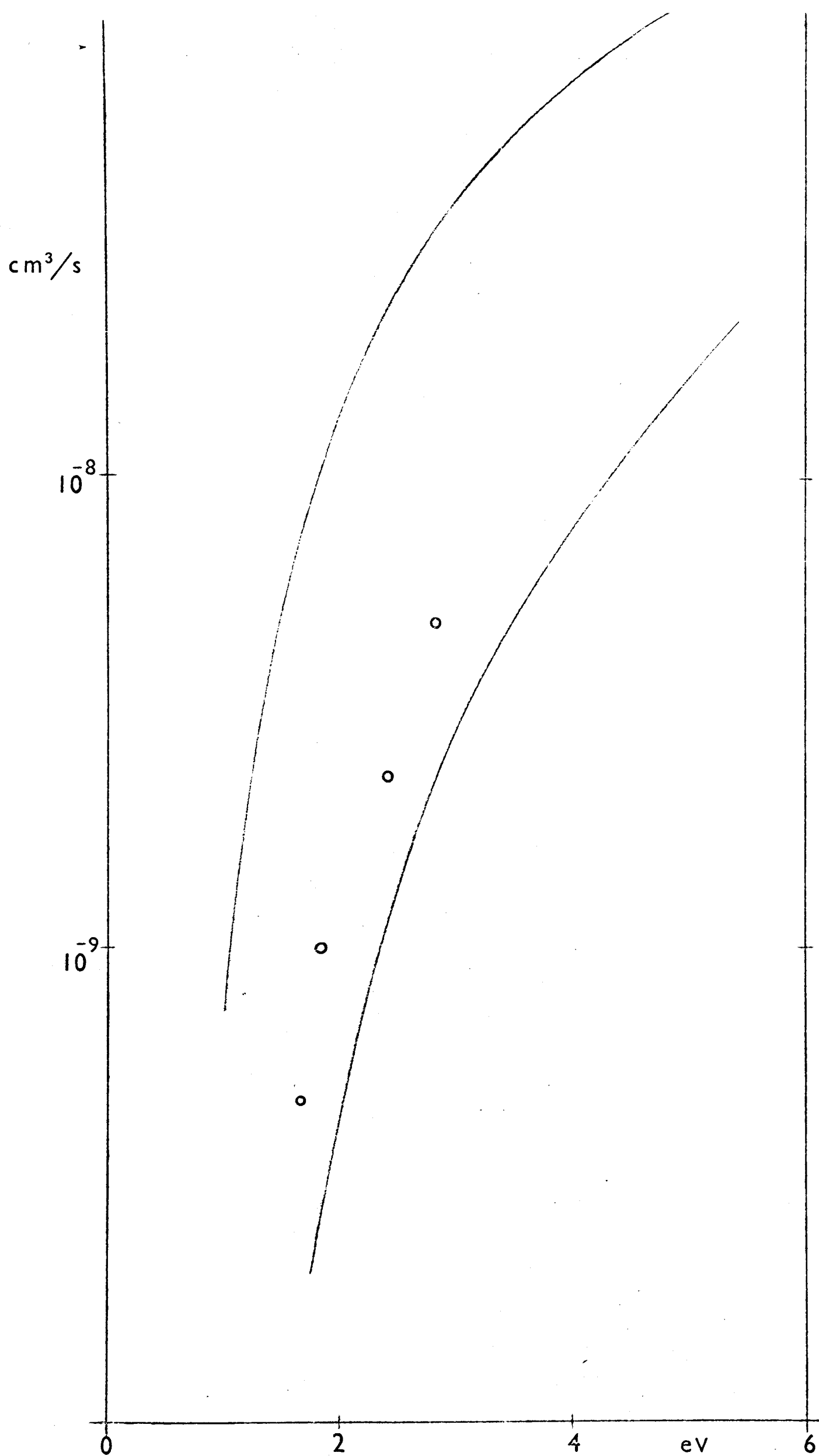


Fig. 27. Total inelastic rate constant and direct ionization rate constant for the ground level of mercury . Experimental points : data on the actual ionization rate constant .

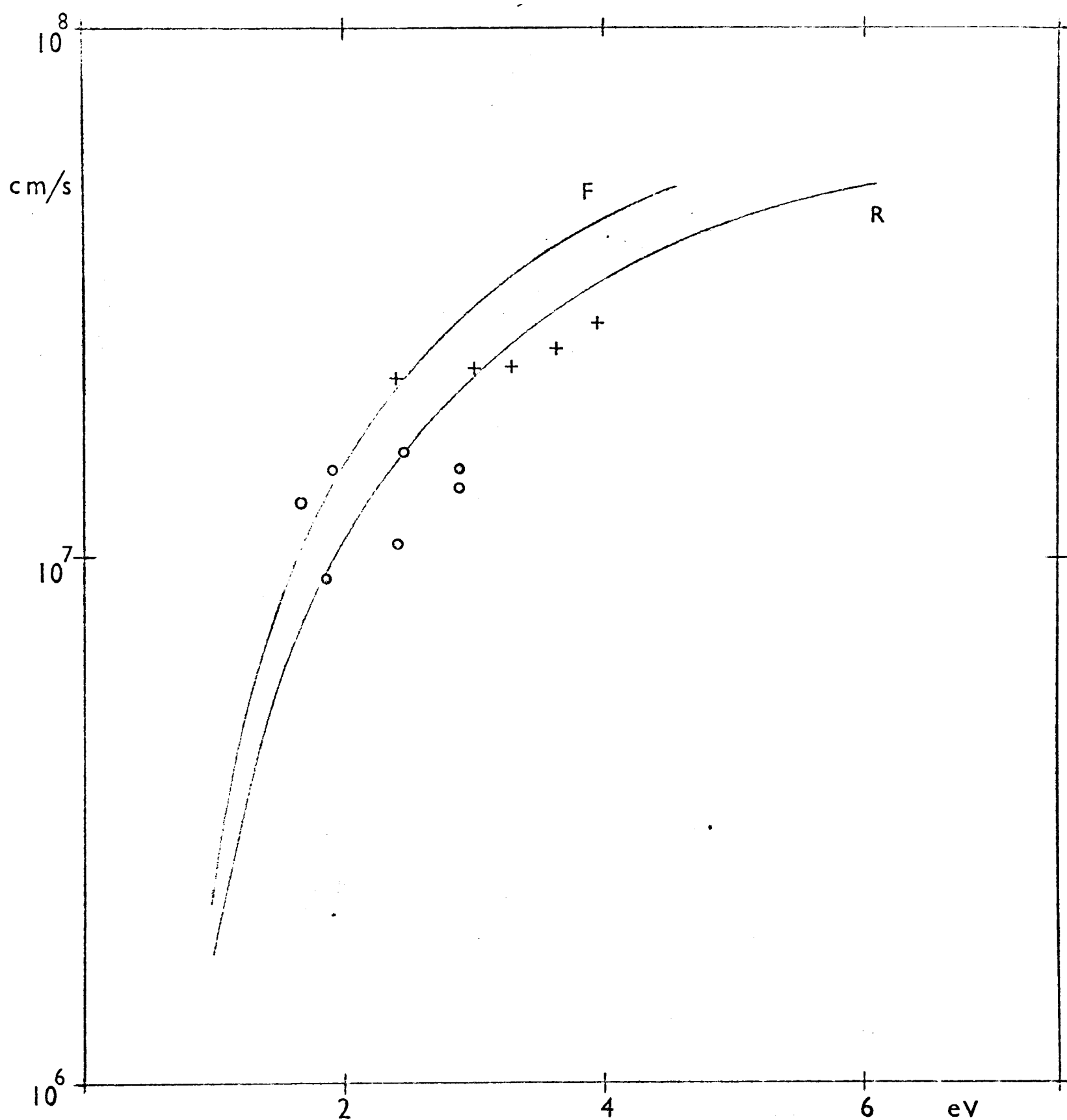


Fig. 28. Electron drift speed as computed by Forrest ¹⁹⁰² (F) and by ourselves (R) . Experimental points : according to Allen and Magistrelli ¹⁹⁰⁶ (+) and our own data (o) .

4.3. THE PLASMA SHEATH

4.3.1. SHEATH FORMATION.

4.3.1.1. A bounded plasma requires in general the presence of a charged double-layer to control the net current flow across the boundary. A floating wall is charged up by the more mobile species and a sheath containing the cold species develops on it, in such a way as to balance the electron and ion fluxes reaching the wall and therefore maintaining the wall potential and the plasma neutrality. The flow of the cold species is governed by a space-charge limited regime and the flow of the hot species by the potential barrier. Beyond the sheath, however, there is an almost complete charge neutrality and very weak fields only can be observed. The sheath thickness is of the order of the electrostatic shielding characteristic length, the Debye length.

In a plasma of hot maxwellian electrons and cold ions, small fields exist which accelerate the ions and eventually bring them to the boundary. Fast moving electrons form a negative layer on the wall and an ion-rich sheath develops between the plasma and the wall. When entering the sheath the average ion speed has a critical value determined by the electron temperature and the ion mass (the Bohm speed). The criterion for the plasma-sheath transition was originally studied by Langmuir¹⁰⁷ and Bohm¹⁰⁸ and later by Allen and Thonemann¹⁰⁹. It was established that the critical ion speed is determined by the requirements of a smooth transition from the plasma into the sheath and of the existence of a monotonic potential profile across this.

4.3.1.2. Consider a hot maxwellian electron gas of temperature T and a cold ion beam of velocity v in a one-dimensional space. Assume the low-pressure collisionless limit. Let us define a characteristic speed c and a characteristic distance h (the Debye length):

$$c = (kT/n_i)^{1/2}$$

$$h = (kT/4\pi n_o e^2)^{1/2}$$

where n_o is the density at the plasma edge. Let us also define the following dimension-less quantities:

$$u = v/c \quad \eta = eV/kT \quad \xi = x/h$$

where V is the electrostatic potential relative to the plasma edge.

The ion beam is completely governed by the momentum and continuity equations. Ionization will be neglected within the sheath so that no generation terms need be included in these equations. The electron gas density follows a Boltzmann's law. Consequently one simply has

$$n_i = n_o (1 + 2\eta u_o^{-2})^{-1/2}$$

$$n_e = n_o \exp(-\eta)$$

where the subscript o refers to the plasma edge. Poisson's equation governs the field and reads

$$n_o d^2\eta/d\xi^2 = n_i - n_e$$

The critical ion speed is determined by the following boundary conditions:

$$n_i = n_e = n_o$$

$$dn_i/d\eta = dn_e/d\eta = 0$$

at the plasma edge. These imply $u_o = 1$ or $v_o = c$ which is Bohm's criterion for sheath formation.

The rigorous boundary-layer analysis of the plasma-sheath transition was given by Caruso and Cavaliere¹¹⁰ and by Bertotti¹¹¹ in the low-pressure collisionless limit.

4.3.2. THE GENERALIZATION OF BOHM'S CRITERION

It will be shown now that Bohm's criterion can be generalised to the case when oscillations are present in the plasma. This question was first studied by Bertotti and Cavaliere³⁶ and¹¹² who proved that the ion speed at the plasma edge, in the presence of ion-acoustic

oscillations and in the collisionless limit, is still the Bohm speed c .

Now, both density and temperature oscillations will be allowed. It is required that frequency ω lies in the range $\omega \ll \Omega_i$, where Ω_i denotes the ion plasma frequency

$$\Omega_i = (4\pi n_0 e^2 / m_i)^{1/2}.$$

When an ion crosses the plasma edge into the sheath, it is subjected to a field impulse whose characteristic frequency is of the order of $c/h = \Omega_i$. Consequently, if $\omega \ll \Omega_i$, the ion crossing the boundary moves as though in a static field, and time derivatives can be dropped in the equations governing the ion beam. It is also required that the ionization frequency is much smaller than Ω_i as well. Under this assumption ionizing collisions can be neglected and the generation terms can be dropped. Finally, in the frequency range of interest the electron is inertia free, that is to say, it follows in equilibrium with the oscillations. In the light of all these considerations, it becomes evident that the steady-state analysis of the sheath formation can be carried out to every instantaneous state in the course of an oscillation, and a generalized Bohm's criterion is obtained. This means that the velocity of the ion beam entering the sheath equals the instantaneous Bohm speed.

One arrives at the same conclusion when, under the same assumptions, one considers the plasma motion approaching the plasma edge. Now, the space charge is negligible because one is within the plasma region and oscillations are limited to the range $\omega \ll \Omega_i$. Consequently, Poisson's equation need not be introduced and one can simply use the one-fluid description of the plasma. In the conservation equations only those terms which involve space derivatives have to be kept because in the plasma solution these terms tend to infinity when approaching the plasma edge and all other terms become relatively unimportant. Therefore, in the neighbourhood of the plasma edge one simply has:

$$u du/d\xi = d\eta/d\xi$$

$$dn/d\xi = 0$$

$$n = n_0 \exp(-\eta)$$

From this set of equations one easily arrives at $u_0 = 1$ at the plasma edge, and the generalized Bohm's criterion is obtained once again. This same type of argument has been applied in paragraphs 5.4. and 6.3.1. in connection with the problem of the boundary conditions satisfied by the radial ion-acoustic oscillation and by the radial plasma pulsation of a cylindrical plasma column.

4.3.3. DC STRUCTURE AND PROPERTIES

In order to study the structure of the ion sheath it will be assumed that the electron density in the sheath is negligibly small. This situation is actually approached when either the wall is biased increasingly negative or the ion mass is increasingly large. In this case the electron density throughout the sheath is very low and the ion motion closely follows a space charge limited regime. Once again we are considering a plane sheath, a maxwellian electron gas of temperature T , a cold ion beam, and the low-pressure collisionless limit. At the plasma edge the ion beam has the Bohm speed and the field is supposed vanishingly small there.

The total current through the sheath is

$$j = env + \frac{1}{4\pi} dE/dt$$

Taking in consideration Poisson's equation one obtains

$$j = \frac{1}{4\pi} dE/dt$$

and introducing the ion continuity equation one arrives at

$$dj/dx = 0$$

First of all it is convenient to analyse the equilibrium structure of the sheath, in which case $dj/dt = 0$ and $dj/dt = 0$.

Take the equation of motion of an individual ion

$$m d^2 x / dt^2 = e E$$

Its first total derivative yields

$$m d^3 x / dt^3 = 4 \pi e j$$

which is a constant. The complete integration of the equation of motion becomes quite easy now. The boundary conditions to be satisfied are

$$t = 0 \quad x = 0 \quad \dot{x} = c \quad \ddot{x} = 0$$

which mean that every ion enters the sheath with the Bohm speed and that the field is vanishingly small at the plasma edge. One readily obtains the ion motion and the potential profile:

$$x = (\Omega_i^2 t^2 / 6 + 1) c t$$

$$\eta = (\Omega_i^2 t^2 / 4 + 1) c^2 t^2 / 2 h^2$$

where $h^2 = (kT / 4 \pi e^2 n_0)$ is the Debye length at the plasma edge. At this point it is convenient to determine the potential drop across the sheath (on a floating wall). This is obtained when equating the ion and electron fluxes to the wall:

$$\eta_s = \ln \left[0.40 (m_i / m_e)^{1/2} \right]$$

Making $t = \tau_i$ the ion transit time, $x = X$ the sheath thickness and $\eta = \eta_s$ one obtains

$$X = 0.47 (2 + \sqrt{1 + 2 \eta_s}) (-1 + \sqrt{1 + 2 \eta_s})^{1/2} h$$

$$\Omega_i^2 \tau_i^2 = 2 (-1 + \sqrt{1 + 2 \eta_s})$$

In the particular case of mercury-vapour one has

$$\eta_s = 5.5 \quad X = 4.0 h \quad \Omega_i \tau_i = 2.2$$

Using the same sheath model, the ion kinetic energy and the field energy contained in the sheath per unit area were estimated. These two contributions are nearly in the ratio 1 : 2. This ratio corresponds to a perfect balance between the kinetic stress of the ions $m n v^2$ and the field stress $E^2 / 8 \pi$ throughout the sheath. In the limit when $m_i / m_e \rightarrow \infty$

this ratio is approached and the total sheath energy per unit area can be put in the form

$$U = 0.42 \left(1 + \frac{1}{3} \Omega_i^2 \tau_i^2 + \frac{1}{20} \Omega_i^4 \tau_i^4 \right) (en_0)^{1/2} (kT/e)^{3/2} \quad (\text{e. s. u.})$$

4.3.4. AC PROPERTIES AND SHOT NOISE

4.3.4.1. Let us consider a stationary plasma and inquire about the properties of the response of its sheath to an applied R.F. field. If a voltage oscillation is applied across the sheath, its thickness will oscillate as well. The motion of the plasma edge then gives rise to a displacement current:

$$j = en_0 dx/dt$$

and an equivalent sheath capacitance can be defined

$$j = C_s dV/dt$$

two limit cases are particularly easy to consider: when either $\omega \ll \Omega_i$ or $\Omega_i \ll \omega \ll \Omega_e$.

(i) When $\omega \ll \Omega_i$, then the sheath structure follows the static space-charge-limited regime at every moment (see Bertotti¹¹² and Butler and Kino¹¹³). One can, therefore, apply the analysis of the foregoing paragraph to obtain dx/dt and dV/dt . The equivalent sheath capacitance turns out to be

$$C_s = (1 + \frac{1}{6} \Omega_i^2 \tau_i^2) / 4\pi X$$

In the case of mercury-vapour this gives

$$C_s = 1.80 / 4\pi X$$

(ii) When $\Omega_i \ll \omega \ll \Omega_e$ the ions can be regarded as being immobile.

Poisson's equation gives, to first order,

$$dV/dX = 4\pi en_0 X$$

and consequently

$$C_s = 1 / 4\pi X$$

(all in e. s. u.).

While $\omega < \Omega_e$ there is a conduction current across the sheath associated with the electron flow. An equivalent resistance can hence be defined by the ratio of the voltage and electron current increments:

$$r_s = 16 \pi \exp(\eta_s) h^2 / c_e \quad (\text{e. s. u.})$$

where c_e is the electron random speed. Now consider the ion contribution to the conduction current. Having in mind that, first, the R.F. field is unable to penetrate into the plasma while $\omega \ll \Omega_e$ (Pavlovich and Kino¹¹⁴ and Montgomery and Holmes¹¹⁵) and that, second, the plasma flows at its edge at the Bohm speed while $\omega \ll \Omega_i$, one can prove that the ion current entering the sheath is not modulated by the applied field in the range $\omega \ll \Omega_i$. When $\Omega_i \ll \omega \ll \Omega_e$ the ion current is not modulated either, as can be concluded with a similar argument.

In the two frequency ranges considered here, the sheath has both equivalent resistance and capacitance, and an equivalent time constant $r_s C_s$ can be defined. This quantity has different values in the two different ranges, but both values are of the order of $1/\Omega_i$. Therefore, the sheath behaves mainly like a resistor when $\omega < \Omega_i$ and mainly like a capacitor when $\Omega_i < \omega < \Omega_e$. For a mercury-vapour plasma whose parameters are, say,

$$n_0 = 4 \times 10^{10} \text{ cm}^{-3} \quad kT = 2.5 \text{ eV}$$

one finds

$$r_s \simeq 4 \text{ k}\Omega/\text{cm}^2 \quad C_s \simeq 4 \text{ pF}/\text{cm}^2$$

4.3.4.2. Shot noise is generated in the sheath by both electron and ion currents. We shall consider the case of the ion-rich sheath. In this case electrons give full shot noise and ions give a space-charge suppressed shot noise because their flow is space-charge limited. Secondary electrons can make the picture more complex. Secondary electrons can be extracted from the wall by the ion bombardment and the extra

charge transferred per ion in this process produces an enhancement of the ion contribution to the shot noise.

The total shot noise generated in the ion-rich sheath is given by

$$\overline{i^2} = 2eI_e \Delta f + 2eI_i F_i^2 \Delta f$$

where F_i embodies the space-charge suppression and the secondary emission enhancement mechanisms. A statistical analysis shows that (see Galbraith and Van der Ziel¹¹⁶ and Burgess¹¹⁷)

$$F_i^2 = (1 + p)\Gamma^2 + p(1 - p)/(1 + p)$$

where p denotes the probability of secondary emission per impinging ion, and Γ is the space-charge suppression factor (see Robinson¹¹⁸).

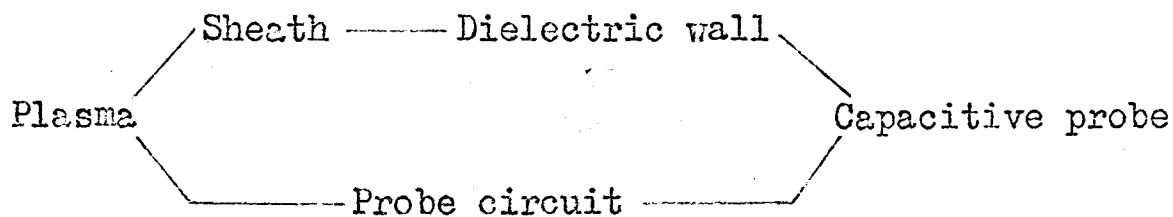
Shot noise is important in electrodes of large area because $\overline{i^2}$ due to shot noise increases with the collected current, while the contribution to $\overline{i^2}$ due to fluctuations in the plasma is fairly independent of the electrode area. In small electrostatic probes shot noise is found to be negligibly small.

When varying the bias of an electrode, a minimum of shot noise is to be expected somewhere near the point where the ion saturation regime starts. When moving from this point towards the floating bias, the total particle current increases and so does the shot noise as well. When moving into the saturation region, on the other hand, the secondary emission enhancement mechanism starts to operate and the ion saturation current tends to grow, both mechanisms leading to increasing shot noise too. These phenomena were observed and analysed by Galbraith and Van der Ziel.

4.3.5. THE MEASUREMENT OF THE SHEATH IMPEDANCE

The impedance of the plasma sheath is determined by the plasma parameters as discussed in paragraph 4.3.4.1. An experimental device for the measurement of the sheath impedance can accordingly be employed as a probing tool.

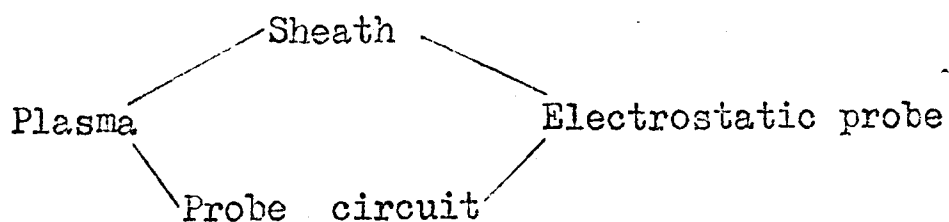
Consider the following experimental arrangement:



It will be assumed that the plasma has negligible impedance and that the wall capacitance is known. Assume also that the probe circuit includes a wave generator and instruments to measure the applied voltage and the current flowing to the probe. Such an arrangement can be employed to measure the sheath impedance and justifies the use of the capacitive probe for the plasma diagnosis. This possibility had already been pointed out by Butler and Kino¹¹³.

The applicability of this method requires the sheath impedance to be the largest of all the impedances in the whole circuit, otherwise the measurement would become difficult or even impossible to carry out. In the specific case of our discharge tube the wall impedance was found to be too large and this method could not be employed (see paragraph 3.6.).

Consider now an electrode directly immersed in the plasma:



In this case the difficulty arising from the presence of a dielectric wall does not exist anymore, and it becomes easier to measure the sheath

impedance.

It will be considered here a diagnosis method based on the measurement of the sheath resistance. In the frequency range far below the ion plasma frequency, the sheath impedance is purely resistive and the ^{AC} sheath current is the electron conduction current alone. Assume that an oscillator applies an alternate voltage V_0 to the sheath and that the AC current is measured by the voltage drop V_1 in a small resistor r_1 connected in series with the probe. Let r_s denote the sheath resistance. One has, provided that $r_1 \ll r_s$,

$$V_0/V_1 = r_s/r_1$$

Now recall that the sheath resistance is just the ratio of the voltage to the electron current increments, that is to say,

$$r_s = kT/ej_e$$

where j_e is the electron ^{DC} current. Hence:

$$V_0/V_1 = kT/ej_e r_1$$

Here, V_0 and r_1 have known values, and j_e and V_1 are easily measured. Plotting $\log(V_0/V_1)$ versus $\log(j_e)$ one expects to obtain a linear plot of slope -1 and whose position determines the electron temperature.

The present method is based on two assumptions: (i) that the sheath impedance is the dominant one in determining the AC current flowing through the probe, and (ii) that the conduction is due to the electron current alone. The range of applicability of this technique is around floating potential. When $j_e \rightarrow 0$, the sheath conductivity decreases but does not vanish. Instead, it tends to a minimum and starts increasing again on account of secondary mechanisms which begin to be important then (secondary emission by ion bombardment and enlargement of the sheath area). This minimum is expected to lie close to the onset of the ion saturation regime, and this fact would provide a way to

determine the ion saturation current and hence to separate the ion from the electron contributions to the probe current. However, experimentally one finds that the whole spectrum does not go through its minimum level uniformly. The largest bias current at which there is still a point in the spectrum passing its minimum was found to give the best estimate of the ion saturation current as given by the Langmuir characteristic of the probe. When $j_e \rightarrow \infty$, the sheath impedance vanishes with the increasing electron current. The impedance of the probe circuit or of the plasma itself will eventually become comparable with the sheath impedance, and a saturation level will eventually be attained.

The data collected in the experiment described in paragraph 3.6. will be used now to test the foregoing method. In that experiment the alternate voltage source was provided by the plasma noise. The source voltage V_0 was easily obtained by replacing the small resistance r_1 by a large one, having in mind that when $r_1 \gg r_s$, the $V_1 \simeq V_0$. Fig. 29 is a plot of $\log(V_0/V_1)$ in db versus $\log(j_e)$. The experimental points are marked with the spread observed over the measured frequency range. The straight line has been drawn according to the theoretical slope (-1) and the electron temperature as given by the double probe method. The agreement between the two methods is good.

An analogous method based on the measurement of the sheath capacitance could also be outlined. It would require a higher probing frequency, and the plasma impedance would more likely interfere in the technique.

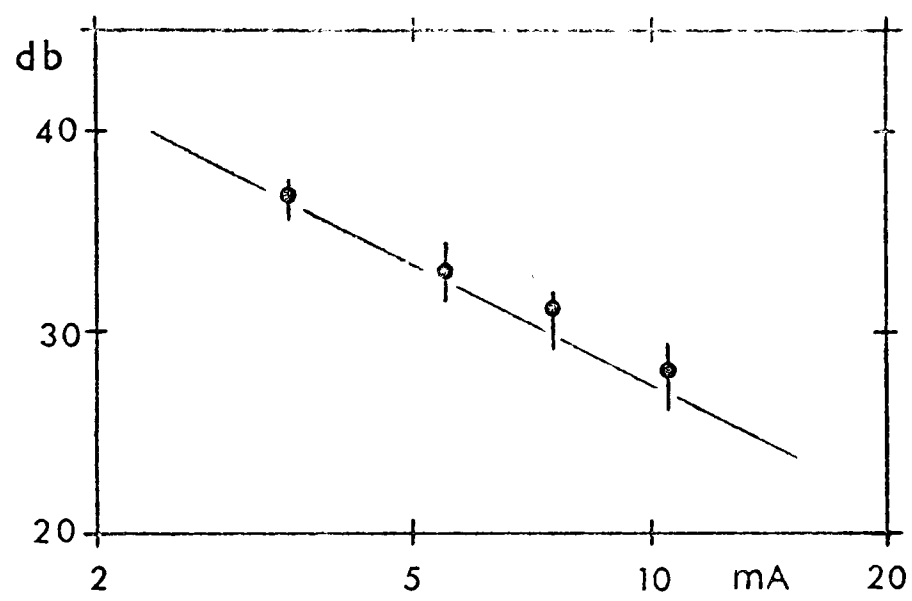


Fig. 29.

4.4. PRESSURE EFFECTS

4.4.1. INTRODUCTION

In a current sustained discharge it is usual to observe a build up of pressure on the anode side or a gas flow from cathode to anode. This effect was the object of a theory due to Langmuir¹¹⁹. According to this theory, the net momentum transferred to the wall of a discharge by the ion and electron currents is directed to the cathode, so that there is a net anode-directed momentum transferred to the gas within one ion mean-free-path from the wall. Here arises the problem of the nature of the elastic collision of an electron with the wall sheath. This collision can be either specular or diffusive: in the first case the axial momentum of the electron is kept and in the second case it is transferred to the wall. The probability of either event should be included in any complete theory of electron mobility and gas pumping in a wall-confined discharge. However, this effect is generally overlooked and the collision assumed to be purely specular. We shall also take this view in the present work.

The original theory of Langmuir was later developed and applied to the electrophoresis phenomena in moderate pressure discharges (in the Torr range) by Druyvesteyn¹²⁰. More recently, Leiby¹²¹ suggested that a volume force which does not require the presence of the wall also exists, and Chester¹²² has pointed out to the existence of another effect, caused by the radial ion motion, leading to gas pumping as well.

Pressure effects in low-pressure discharges (in the mTorr range) are also known, but they have been little studied and are not entirely understood so far. Kenty¹²³ and Klarfeld and Poletaek¹²⁴ reported some experimental observations and advanced explanations in line with Langmuir's theory, but these proved to be insufficient to account for the observed effects.

In the following sections we consider the momentum balance among the ion, electron and neutral fluxes under the action of the applied field and subjected to wall losses. Next, the similarity laws of the positive column are introduced. An equation is deduced which relates the electric current I and the net heavy particle flux Φ to the neutral gas temperature and density only.

Two effects operating in low-pressure discharges are then predicted.

These ideas are then applied in detail to the present experiment. Our theoretical model gives an answer with the correct qualitative features and of the same magnitude as our experimental observations.

4.4.2. THEORY OF GAS DYNAMICS EFFECTS IN A LOW PRESSURE ARC

4.4.2.1. To solve the problem of the equilibrium fluxes of ions electrons and neutrals in a uniform positive column, one has to start with the macroscopic momentum equations for the different species and drop time and space derivatives. One is left with the applied force, the momentum losses to the wall and the momentum exchange among species. The momentum exchange terms will always be assumed proportional to the relative drift velocity, which is strictly true in the case of constant collision frequency only (see Holt and Haskell⁸⁷), but which is commonly used as a good first approximation for any type of collision frequency (see Van Kampen and Felderhof¹²⁵). Therefore one is left with the following set of algebraic equations:

$$v_{ni}(u_i - u_n) + v_{ne}(u_e - u_n) - v_{nw}u_n = 0$$

$$v_{in}(u_n - u_i) + v_{ie}(u_e - u_i) - v_{iw}u_i = \frac{eX}{m_i}$$

$$v_{en}(u_n - u_e) + v_{ei}(u_i - u_e) - v_{ew}u_e = -\frac{eX}{m_e}$$

where ν_{pq} denotes the collision frequency for momentum transfer from particles of type p to particles of type q . According to total momentum conservation

$$n_p m_p \nu_{pq} = n_q m_q \nu_{qp}$$

Also notice that $\nu_{pq} = Z_{pq} n_q$ where Z_{pq} is the corresponding rate constant, a function of the two particles temperatures. In mercury-vapour the frequencies for both electron-neutral and ion-neutral collisions are roughly independent of the particles temperatures (for data on the cross-sections see Brown¹²⁶). Moreover, one has:

$$\nu_{nw} \simeq \left(\frac{kT_n}{m_n} \right)^{1/2} / 2R$$

$$\nu_{iw} = \nu_{ew} \simeq \left(\frac{kT_e}{m_i} \right)^{1/2} / R$$

In a typical low-pressure arc in mercury-vapour one has, say,

$n_e = 4 \cdot 10^{11} \text{ cm}^{-3}$, $kT_e = 2.5 \text{ eV}$, $R = 3 \text{ cm}$, and the estimated magnitude of the different collision frequencies are as follows (sec^{-1}):

$$\begin{array}{lll} \nu_{en} \approx 1 \cdot 10^7 & \nu_{in} \approx 2 \cdot 10^4 & \nu_{nw} \approx 3 \cdot 10^3 \\ \nu_{ne} \approx 1 & \nu_{ni} \approx 3 \cdot 10^2 & \nu_{iw} = \nu_{ew} \approx 3 \cdot 10^4 \end{array}$$

We shall limit ourselves to the case $\nu_{ni} \gg \nu_{ne}$ and $\nu_{en} \gg \nu_{in}$, which conditions are always fully satisfied in a mercury-vapour discharge.

Eliminating the field strength from the momentum equations one obtains the flux-balance equations:

$$(\nu_{nw} + \nu_{ni} + \nu_{in} \nu_{nw} / \nu_{iw}) \phi_n = \frac{m_e}{m_i} \nu_{en} \phi_e$$

$$\phi_i = (\nu_{nw} / \nu_{iw}) \phi_n$$

where $\phi_n = n_n u_n$ is the neutral gas flux, and so on. These two flux-balance equations relate the three particle fluxes, and once one is known the other two will be determined.

4.4.2.2. As is well known, the different plasma parameters are

inter-related by the similarity laws (see paragraph 4.1.6.). We shall introduce now the two similarity laws which determine the neutral density and the electron drift as functions of the electron density:

$$Rn_n = \text{function}(T_e)$$

$$u_e = \text{function}(T_e)$$

Here we shall neglect any contributions due to metastable states populations. To be precise we shall take the actual similarity laws governing a low-pressure mercury-vapour discharge. According to the data collected by Hoyauss and Gans ⁷⁸, one can write:

$$Rn_n = AT_e^{-3}, \quad u_e = BT_e^3$$

Now, let us also introduce the equations of definition of I , the discharge current, and Φ , the total mass flux:

$$I = \phi_e + \phi_i, \quad \Phi = \phi_n - \phi_i$$

In what follows, the ion contribution to I will be neglected.

Taking the two flux balance equations, the two similarity laws and the two equations of definition of I and Φ , and proceeding to the elimination of u_n , u_e , u_i , n_e and T_e one arrives at:

$$\begin{aligned} \left[(k/m_n)^{1/2}/R + (Z_{in} + Z_{ne} \frac{I}{\Phi}) A^{-1/6} R^{1/6} n_n^{7/6} \right] T_n^{1/2} = \\ = 2(Z_{ne} \frac{I}{\Phi} - Z_{ni} A^{-1} B^{-1} RI) n_n \end{aligned}$$

This equation relates the neutral gas density and temperature to the discharge current I and mass flux Φ . From its analysis one can discover two pressure-effects operating in low-pressure discharges. This is what we shall do in the next paragraphs.

4.4.2.3. First, assume that there is a perfect control of neutral gas temperature and density in a particular section of the discharge. This condition is satisfied, for instance, in a temperature-controlled cooling-section of a mercury-vapour tube. Under such circumstances Φ

turns out to be a function of I alone; in fact, their dependence can be expressed in the form:

$$I/\Phi = a_1 + a_2 I$$

This equation governs the pumping effect observed in low-pressure gas discharges. One sees that $d\Phi/dI$ is always positive. Φ is proportional to I , except at considerably large currents ($I \simeq a_1/a_2$) when the pumping effect is somewhat slowed down, and will eventually attain a saturation regime where $\Phi = 1/a_2$ (when $I \gg a_1/a_2$).

This effect can be observed in a number of ways: as a flow of matter when this can be condensed or evacuated at the anode end or re-circulated to the cathode end; otherwise, as a pressure gradient which will build up in such a way as to produce an opposite flow and balance the pumping effect; or as differential pressure between two pressure gauges facing anode and cathode respectively.

In a mercury-vapour discharge such that $p \simeq 0.5$ mTorr and $R \simeq 3$ cm, one estimates

$$a_1 \simeq 2 \times 10^2, \quad a_2 \simeq 0.5 \times 10^{-17} \quad (\text{c.g.s.})$$

so that $\Phi \simeq 5 \times 10^{-3} I$ at low current and $\Phi \simeq 2 \times 10^{17}$ in saturation regime. In the conditions of our experiments we are well within the former of these regimes.

The physical mechanism behind the pumping-effect is the mechanism of Langmuir's classic theory, although this is not very apparent in our formalism of this phenomenon.

It should be emphasized that the pumping effect is not necessarily associated with a pressure effect. Provided that the anode end is a perfect sink for the pumped gas, no pressure gradient will be observed. The gas flow does not produce a pressure gradient because the two forces acting upon the gas (the driving force due to the electron flow and the frictional force due to the wall) act uniformly all along the discharge and balance

each other everywhere.

4.4.2.4. Now, assume that both I and Φ are conserved along the discharge. This second condition is not satisfied in every particular situation, but it surely is in many; it will not be satisfied when there is condensation or evacuation of matter somewhere, but it will otherwise be satisfied. When I and Φ are kept constants along the discharge then n_n is a sole function of T_n , and all other quantities are automatically determined accordingly. The general equation obtained in paragraph 4.4.2.2 can now be cast in the more convenient form:

$$T_n^{1/2} = b_1 n_n / (1 + b_2 n_n^{7/6})$$

This equation shows that, provided that the gas density is not too large ($b_2 n_n^{7/6} < 6$), n_n will be an increasing function of T_n ($dn_n/dT_n > 0$). It should be noticed that this effect opposes the gas density equilibrium distribution in the absence of the discharge. Therefore, it will be observed only when the transport of gas by the pumping-effect is of considerably larger magnitude than the transport of gas due to the temperature and density gradients. This new effect being considered here will be referred to as the temperature-density effect. The physical mechanism behind this effect can be understood as follows: when the neutral gas flux comes to a higher temperature region, the neutral-wall collision frequency increases, producing a decrease of the neutral drift velocity and, for the sake of mass conservation, a density rise.

In a mercury-vapour discharge such that $p \simeq 0.5$ mTorr and $R \simeq 3$ cm, we find

$$b_2 n_n^{7/6} \simeq 0.55$$

$$dn_n/n_n \simeq 0.85 dT_n/T_n$$

As in our experiments the gas temperature may rise as much as 30 % of its value at the temperature-controlled cooling-jackets, we have to conclude that the foregoing temperature-density effect can be of sizeable

magnitude. It should also be noticed that the saturation pressure of mercury-vapour increases with temperature faster than the pressure rise due to the effect we are considering, and therefore no vapour condensation will occur. This is why the condition of mass flux conservation is satisfied in our experimental conditions.

4.4.2.5. The gas dynamics effects we have been analysing here will be seriously affected at high current when the arc-starvation effect operates. This effect sets in when the plasma flux to the boundary becomes comparable with the neutral random flux, and it produces a depletion of gas around the discharge axis (see Allen¹⁰⁹ and¹²⁷). It is readily seen that, in our experimental conditions, we are still far from the conditions in which this effect operates.

4.4.3. EXPERIMENTAL DATA AND THEORETICAL ESTIMATES

4.4.3.1 In a current-sustained discharge the gas kinetic stress tensor will in general be anisotropic due to the directed gas flow. In a volume external to the plasma the pressure will build up until a balance of mass flux into and out of the volume will be attained, unless the gas is being condensed or evacuated. This pressure will in general be different from the gas pressure inside the discharge itself.

The relative magnitude of the anisotropic component of the stress tensor is given by the ratio of the directed mass flux to the gas random flux:

$$4\Phi/n_n c_n$$

where c_n is the atomic mean speed. The relative magnitude of the isotropic pressure increment due to the density-effect in the presence of the discharge is given by the relative increment of the gas random flux:

$$dn_n/n_n + 0.5 dT_n/T_n$$

Another contribution to the isotropic pressure increment is due to the

flow of plasma to its boundary, and its relative magnitude is given by the ratio of the plasma flux to the random gas flux. When a Vonk-Langmuir density profile is assumed, one has:

$$1.53 \, n_e c_s / n_n c_n$$

where c_s is Bohm's speed and n_e is the plasma density on the axis.

When two pressure gauges are connected to a uniform discharge, one facing the cathode and the other facing the anode, two different readings are obtained in the presence of the discharge. The anisotropic and isotropic components of the stress tensor are given by ~~half~~ the difference and half the sum of the two readings respectively.

4.4.3.2. Consider a discharge tube of uniform diameter and comprising three parts: two temperature-controlled end sections and one middle section whose temperature depends on the power released by the discharge. At the two joints gradients will set up but we shall not be interested in their structure. On the other hand, the discharge will be uniform in the different sections, provided that they all are rather longer than the tube diameter and the electron mean-free-path for energy and momentum transfer. In this case one can speak of plasma parameters and of neutral gas density and temperature in each one of those sections.

The arc discharge of our experiments comprises three nearly uniform sections, the two extreme ones being provided with temperature-controlled cooling-jackets. Two ionization gauges are connected to the middle section, one facing the anode and the other one facing the cathode, providing the means to measure both the isotropic and the anisotropic stress tensor components in the middle section of the discharge.

All along the discharge there is conservation of mass flux and current. The mass flux is certainly conserved due to the symmetry of the arrangement and because no vapour condensation is observed on the middle section.

The gas can flow freely from the cathode to the anode side because there are cooling-jackets and mercury pumps operating at both ends, and the middle section of the discharge is therefore expected to be uniform.

In view of these conditions, the foregoing theory can be applied to the actual experimental situation. In the following table (see TABLE 1) experimental data and estimates based on our theory are condensed. They refer to discharges at $p = 0.50$ mTorr controlled vapour pressure.

1. The anisotropic stress component:

Discharge current (Amp):	5.0	10.0	15.0
Theoretical estimates:	0.11	0.22	0.33
Experimental data:	0.08	0.31	0.64

2. The isotropic stress component:

Experimental conditions

(i) current (Amp)	5.0	10.0	15.0
(ii) dT_n/T_n	0.15	0.25	0.34
Theoretical estimate of dp/p			
(i) temperature-density effect	0.20	0.34	0.46
(ii) plasma pressure	0.10	0.19	0.28
(iii) total	0.30	0.53	0.74
Experimental data	0.15	0.42	0.75

When comparing experimental and theoretical data one must recall that the estimated magnitude of the density effect is to be attained asymptotically with an increasingly large discharge current, that our estimates are based on little known collision frequencies, and also that the similarity laws which have been used do not involve the role of the metastable populations. From the experimental point of view, a differential

pressure sensing device ought to be used in order to obtain reliable measurements of the directed gas flow, and it must be noticed that the actual discharge tube is still not quite the idealized model with three perfectly uniform sections. Even so, the theory provides an explanation with the right features and of the right magnitude to the observed gas dynamics phenomena.

Chapter V

ION-ACOUSTIC WAVES

The study of waves in plasmas, given the complexity of this medium, has been usually approached from either of two simplified and opposite points of view. On one side, the fluid point of view, in which the plasma is defined by the density, mean velocity and sometimes also other mean quantities of its particles, but which overlooks the detailed structure of the actual distribution function. However, boundary effects and collisions (elastic and inelastic) can easily be included and dealt with. On the other side, the kinetic point of view, but in which collisions are neglected, although giving a full microscopic picture of the plasma, can strictly be applied to short time scale phenomena only. The actual plasma dynamics is in general more complex than that which either of the above mentioned approaches can describe. A full description of plasma dynamics requires, in fact, a kinetic model in which all collisions and boundary effects are included, but such a model brings in a much more elaborate mathematical analysis. Nevertheless, the fluid model and the collisionless kinetic model are remarkably complementary to each other and, together, they seem to be enough to understand most of the properties of the ion and ion-acoustic waves in plasmas. In this chapter we have attempted to review those features of ion-acoustic waves which are more relevant in the physics of gas discharge plasmas.

In the last paragraph of this chapter we consider in detail the purely radial ion-acoustic oscillation of a low-pressure plasma column. The equilibrium structure of the column as well as its first radial ion-acoustic oscillatory mode were computed and are reported.

5.1. THE FLUID THEORY OF ION-ACOUSTIC WAVES

5.1.1. The theoretical foundations of ion and ion-acoustic waves have been analysed by a number of authors. This question will be briefly reviewed now, in a rather simplified situation, just for the sake of obtaining the most fundamental features of these types of wave.

Consider the two-fluid plasma model and take continuity and momentum equations. No collisions, no drifts and no boundaries are included here, but a uniform magnetic field may be allowed. Take Maxwell's electrodynamic equations as well. Proceed to the linear analysis of the plane wave motion in the plasma (see Holt and Haskell⁸⁷ for instance). The particular case of a purely longitudinal wave propagating along the magnetic field will be considered now.

Let the subscripts e and i refer to electrons and ions respectively. Define the ion plasma frequency Ω_i and the ion thermal speed a_i by

$$\Omega_i^2 = 4\pi n e^2 / m_i, \quad a_i^2 = \gamma_i k T_i / m_i,$$

where γ_i is the compression coefficient of the ion gas. Finally, define the corresponding quantities of the electron gas similarly. The particular case of a longitudinal wave propagating along the magnetic field is governed by the following dispersion relation (see Holt and Haskell⁸⁷ and Stringer¹²⁸)

$$(\omega^2 - \beta^2 a_i^2 - \Omega_i^2) (\omega^2 - \beta^2 a_e^2 - \Omega_e^2) = \Omega_i^2 \Omega_e^2$$

Notice that this kind of wave motion is not affected by the magnetic field at all and that it can equally be observed in a non-magnetized plasma. This fact is reflected in the absence of the field in the dispersion relation. This embodies two branches of which the low-frequency one concerns the ion and ion-acoustic waves:

$$\omega^2 - \beta^2 a_i^2 - \Omega_i^2 = -\Omega_i^2 \Omega_e^2 / (\beta^2 a_e^2 + \Omega_e^2)$$

Here the wave-number has to be restricted by the requirement that the distance travelled by an average ion during one wave period must be much smaller than the wave-length, otherwise the wave field would have extremely little efficiency in driving the ions into the wave motion.

This requirement yields an upper bound to β : $\beta \ll \Omega_i/a_i$

For the long wave-length range ($\beta a_e \ll \Omega_e$) one has

$$\omega^2 = a_o^2 \beta^2$$

$$a_o^2 = (\gamma_i kT_i + \gamma_e kT_e)/m_i$$

which represents a non-dispersive wave named the ion-acoustic wave, whose phase velocity a_o is the ion-acoustic speed.

For the short wave-length limit ($\beta a_e \gg \Omega_e$), on the other hand, one has

$$\omega^2 = \Omega_i^2 + a_i^2 \beta^2$$

which represents a wave of frequency close to the ion plasma frequency called the ion wave. It is worth noticing that the existence of this type of wave requires that both the conditions $\beta a_i \ll \Omega_i$ and $\beta a_e \gg \Omega_e$ be satisfied; this is possible only when $T_e \gg T_i$.

It is interesting to examine the nature and importance of the different contributions to the energy of the wave. It is assumed that $a_i < \omega/\beta < a_e$; this implies that the electron gas is too hot to be efficiently driven by the wave field; it also implies that the ion gas can be regarded as completely cold. Starting with the equation of motion of the individual ion in the field of the wave, one easily arrives at the following conclusion: the density of field energy and the density of ion kinetic energy are in the ratio ω^2/Ω_i^2 . This result shows that when $\omega \ll \Omega_i$, which is the case of the ion-acoustic wave, then the field energy of the wave is comparatively small, which also means that ions and electrons do move nearly together. Having in mind the expression of the wave phase velocity, and in analogy with the case of the sound

wave, it becomes apparent that the ion inertia, on one hand, and the kinetic pressure of the electron and ion gases, on the other hand, provide the driving and the restoring forces involved in the ion-acoustic motion. The mechanic kinetic energy and the thermal kinetic energy of the plasma give the two largest contributions to the wave energy; but the field energy, although relatively small plays an essential role in this type of wave motion.

5.1.2. Consider a plasma of cold ions and hot maxwellian electrons. The ion speed at the plasma edge has a critical value, the Bohm speed. This, in the case of plane geometry and mono-kinetic ions is given by $c = (kT_e/m_i)^{1/2}$. This critical value is satisfied even when ion-acoustic waves are present in the plasma (see chapter 4.3.2.). In the same plasma the ion-acoustic phase velocity is $a_o = (\gamma_e kT_e/m_i)^{1/2}$. It will be assumed now that the electron gas is isothermal, in which case the wave speed and the Bohm speed turn out to be equal. It is therefore clear that the plasma flow becomes transonic approaching its edge, and that an ion-acoustic wave propagating towards the boundary is simply swept out with the plasma flow. In the light of this argument it is apparent that an ion-acoustic wave in a plasma of cold ions and hot maxwellian and isothermic electrons will be absorbed at the plasma boundary. (That the actual physical circumstances are not exactly the ideal ones as stated here, does not invalidate the essence of this argument and its conclusion.) From another point of view, according to the generalized Bohm criterion the perturbation of the plasma drift to its edge has a node there (and the corresponding density perturbation has an anti-node); the sheath behaves like a rigid wall and the principal mode can be observed. In the opposite case, when the perturbation of the plasma drift would show an anti-node at the boundary, the sheath would behave like a free wall and the principal mode could not be

observed.

The detailed analysis of the properties of the plasma boundary to ion-acoustic waves has been carried out recently by Bertotti et al¹¹² and¹²⁹ and Cavaliere et al¹³⁰. It was established that the ion-rich and the electron-rich sheaths behave quite differently from one another. The former is highly absorvent but the latter is highly reflective to ion-acoustic waves. Wong¹³¹ also proved on theoretical grounds that transverse boundary effects do not operate in a magnetically supported plasma provided that the ion Larmor radius is much smaller than the plasma diameter.

Experimental evidence concerning the boundary effects of the sheath has been the object of some controversy. Wong et al¹³² did experiments in a magnetically supported plasma column which agreed with a plane wave analysis even when the wave-length was comparable with the plasma diameter. Experiments with plasma produced in gas discharge tubes produced results which are of less obvious interpretation than those obtained in strong magnetic fields. Crawford²⁸, Crawford and Self²¹ and Little and Jones¹³³ obtained data which seemed to suggest the density node type of boundary condition. However, Crawford and Kuhler¹³⁴ and Alexeff and Neidigh⁵⁴, in analogous experimental circumstances, obtained results which suggested the density anti-node type of boundary condition instead. A more direct investigation of the behaviour of the ion-rich sheath is due to Jones and Alexeff¹³⁵; they tried to observe the reflection of an ion-acoustic wave beam by the plasma sheath formed on either glass or metal walls, but without success, and concluded that the beam was highly absorbed by the sheath. This result fully supports the theoretical expectation.

5.1.3. The properties of ion-acoustic waves in a weakly ionized gas may be affected by collisions between the charged particles and the

background neutral gas.

This problem was first analysed by Hatta and Sato³³ and Liperovskii³⁴. In the former paper the authors used a two-fluid plasma model, in which collision terms were included but drifts and boundaries were discarded, in order to study the properties of a plane ion-acoustic wave. Define an effective collision frequency ν by

$$\nu = \nu_{in} + (m_e/m_i) \nu_{en}$$

where ν_{in} and ν_{en} denote the ion-neutral and the electron-neutral collision frequencies for momentum transfer. When ν becomes comparable to the frequency of the wave, then the wave exhibits dispersion and attenuation which are governed by

$$(\omega/\beta_R)^2 = 2a_0^2/[1 + (1 + \nu^2/\omega^2)^{1/2}]$$

$$\beta_I^2 = \omega^2[(1 + \nu^2/\omega^2)^{1/2} - 1]/2a_0^2$$

where β_R and β_I are the real and the imaginary parts of the wave-number.

The influence of collisions between the charged particles and the background gas upon the properties of ion-acoustic waves in a bounded plasma has already been studied by a few authors. Alexeff and Weidigh

⁵⁴ considered the case of standing waves in a drift-free plasma. Given an oscillating system, a quality factor Q can always be defined as 2π times the ratio of the energy stored in the wave to the power loss per cycle. Recalling that half the energy of the wave is the mechanic kinetic energy of the oscillating mass, one easily arrives at

$$Q = 2\omega/\nu$$

where ν is the effective collision frequency. When $Q = 1/2$ the oscillation is critically damped. Therefore, only those modes which satisfy the condition $\omega/\nu > 1/4$ can be observed otherwise they are damped away. The same authors checked these facts experimentally.

In a weakly ionized gas whose neutral density is substantially

high, not only the collisions between charged particles and neutrals but also the collisions among neutrals themselves can cause new wave effects. At moderate pressures, around 1 Torr, when the neutral-neutral mean free path is already much smaller than the dimensions of the container, gases are dense enough to support sound waves. In such circumstances the motion of the neutral gas itself can give rise to new wave effects, and the full picture of the plasma wave phenomena requires the consideration of a three-fluid model of the ionized gas.

The three-fluid model has been analysed by a number of authors (Sessler¹³⁶, Ingard and Gentle¹³⁷, Parker¹³⁸, Tanenbaum and Meskan¹³⁹ and others). The deduction of a dispersion relation from such a model is rather involved, and only highly idealized cases have been solved up to now (adiabatic fluids, no drifts and no boundaries). The dispersion relation is complex and some confusion has arisen from its interpretation. Nevertheless, a few facts are well established. There are three branches which, over a wide range of physical conditions, represent: (i) a sound wave propagating at the sound speed of the neutral gas, in which the three fluid components move in phase and with the same relative amplitude, without transfer of wave momentum among components and without damping; (ii) an ion wave propagating at near the ion-acoustic speed, in which electrons and ions move together and neutrals can be regarded as being immobile, whose properties are in agreement with the conclusions of the theory of Hatta and Sato; (iii) a degenerate (overdamped) plasma wave, in which electrons and ions move in opposite directions and whose wave-length is near the ion Debye length.

Sound waves were observed in gas discharges by a few authors (see Alexeff and Neidigh⁵⁴, Nygaard¹⁴⁰ and Siche and Subertova¹⁴¹). Sound waves modulate the plasma density giving rise to further

complexity of the electrical noise spectrum and providing a mechanism for the modulation of the plasma density.

In favourable conditions, that is to say, when the ion-acoustic and the sound wave modes are coupled (the same ω and the same β) and the former is less damped than the latter, it will be possible to excite ion-acoustic waves with a sound transmitter, but this method has never been attempted yet.

5.1.4. The combined influence of collisions and drifts on ion-acoustic waves propagating in a weakly ionized gas was studied by Liperovskii³⁴. This author made use of a plasma model similar to the one studied by Hatta and Sato³³ but he further allowed for an applied DC electric field along the direction of wave propagating and for the corresponding ion and electron drifts. He proved that this medium will in general show anisotropic propagation and attenuation properties, and also that in some particular circumstances the wave may be amplified (although he did not go into the analysis of this last effect). When dropping the drifts in Liperovsku theory, the results of Hatta and Sato are recovered. When dropping the collisions instead, one obtains:

$$\omega/\beta_R = B/[-A \pm (A^2 - B)^{1/2}]$$

$$\beta_i = 0$$

where the quantities A and B are defined by

$$A = -u_i + (m_e/m_i) u_e$$

$$B = u_i^2 - a_i^2 + (m_e/m_i)(u_e^2 - a_e^2)$$

and where u_i and u_e denote the ion and the electron drift velocities. The dispersion relation shows clearly how the presence of the drifts introduces the anisotropy of the wave propagation. When $m_e/m_i \ll 1$ and $u_i \ll u_e$, as one has in most cases, the anisotropy amounts to the usual Doppler effect due to the motion of the center of mass of the plasma relatively to the observer.

5.1.5. The compression coefficient of the electron gas is an important quantity involved in the phase velocity of ion-acoustic waves in plasma with a large ratio of electron to ion temperatures:

$$a_0 = (\gamma_e k T_e / m_i)^{1/2}$$

The equation of state of a fluid involves three quantities, two of which are independent variables the other one being a function of state (as the pressure and the internal energy are usually regarded to be). Taking the first law of thermodynamics in consideration and assuming that the fluid behaves adiabatically (no heat produced irreversibly as a result of heat conduction, viscosity, diffusion, electrical conduction or chemical reactions) it is possible to eliminate one of the independent variables in the equation of state. In the case of the ideal gas one arrives at the following adiabatic law

$$p \propto n^\gamma$$

where γ is the ratio of the specific heats. Two cases are of interest to the electron gas: first, the adiabatic collisionless one-dimensional compression, when the wave period is much shorter than the relaxation time for momentum randomization, which corresponds to $\gamma = 3$; second, the adiabatic collision-dominated three-dimensional compression, when the wave period is much longer than the same relaxation time, and which corresponds to the value $\gamma = 5/3$.

When the compression-expansion cycle is not adiabatic, it can in general be defined a relaxation time τ for the mechanism responsible for the irreversible generation of heat. A compression coefficient can always be defined and, in this case, it will be a complex quantity and a function of the frequency. Accordingly, there will be attenuation and dispersion of the wave in a frequency range around $1/\tau$. This is what happens in intermediate frequency ranges where the transitions between different adiabatic regimes take place. (see Rayleigh¹⁴² and Hirschfelder

et al ¹⁴³).

Another situation of very special interest is that of the ideal isothermal electron gas, in which the temperature is uniform and constant throughout the whole plasma volume. In this case $\gamma = 1$. The electron gas is expected to behave isothermally when its mean free path is much longer than the wave-length and its thermal speed is much larger than the wave phase speed and provided that the electron-electron collision rate is higher than the wave frequency. This is clearly the case of the ion-acoustic wave in a low-pressure arc. Owing to the strong temperature dependence of the inelastic electron-neutral collision frequency, the presence of the neutral gas has further stabilizing influence upon the electron temperature, provided that the wave frequency is much smaller than the inelastic collision frequency.

The compression coefficient of the electron gas was investigated experimentally in a number of quiescent drift-free plasmas and in the low pressure collisionless limit (the electron mean free path longer than the dimensions of the container) by Alexeff and Jones ¹⁴⁴ . The experimental method was based on the measurement of the phase velocity of ion-acoustic waves far away from the influence of the boundaries. Their data suggests that the electron gas is isothermal under the physical circumstances quoted above. Sessler ¹⁴⁵ observed an increase of the phase velocity and attenuation constant of ion waves in a frequency range close to the electron-neutral collision frequency; he interpreted that fact as being due to the transition from the isothermal to the collisionless uni-dimensional adiabatic regimes.

5.1.6. The presence of a foreign ion in a plasma influences its ion-acoustic speed. In some circumstances even a minute percentage of a contaminant can produce a very considerable change of the phase velocity. It will be deduced here the general way in which the ion-

-acoustic speed is related to the composition of the plasma. (Sessler¹⁴⁶ and Allen¹⁴⁷).

We assume that the ions are perfectly cold and the electron gas is maxwellian and isothermic. We start by recalling that in an ion-acoustic wave charge neutrality must be satisfied; accordingly, one has:

$$\sum z \, dn_i = dn_e$$

where z is the number of elemental charges in a particular ionic species. If the electron gas is isothermic, then

$$dn_e = n_e e \phi / kT_e$$

where ϕ is the perturbed potential. Besides, for the mass continuity of each ion species one has

$$dn_i = -n_i d\xi / dx$$

where ξ is an infinitesimal displacement from the equilibrium which is governed by Newton's law:

$$m_i d^2 \xi / dt^2 = - e z d\phi / dx$$

Incidentally, it should be noticed that $m_i \xi / z$ is the same for all the species. Now, notice that the condition of charge neutrality can be cast in the form

$$z n_i d\xi / dx + n_e e \phi / kT_e = 0$$

Taking its derivative in order to x and combining it with the equations of motion of the different ionic species one arrives at the following wave equation:

$$d^2 \xi / dt^2 = a^2 d^2 \xi / dx^2$$

whose phase velocity a is given by:

$$a^2 = kT_e \sum (f z^2 / m_i)$$

where f denotes the relative amount of each species, that is to say, the ratio of its density to the electron density. This equation shows how the ion-acoustic speed is affected by the plasma composition. It

is obvious now that traces of a light ion in a plasma of heavy ions can increase the phase velocity very considerably. Also, that small amounts of multi-ionized atoms can produce a substantial effect too. Finally, in gases in which both the mono-atomic and di-atomic ions can coexist in comparable amounts, the phase speed is rather dependent on the exact ion composition.

Barrett ¹⁴⁸ observed ion-acoustic waves in a low-pressure mercury-vapour discharge whose relatively low phase velocity he attributed to the presence of the Hg^{++} ion. Jones ²³, in comparable experimental conditions, claimed to have observed two separate branches of ion-acoustic waves, one associated with the Hg^+ and the other one with the Hg^{++} ions. Alexeff ⁴² carried out experiments on ion-acoustic waves propagating in a xenon plasma with traces of helium; he observed that the addition of about 2% of helium was enough to give rise to a detectable increase of phase velocity.

5.2. WAVE PROPAGATION IN PLASMA COLUMNS

5.2.1. The propagation of ion-acoustic waves along a plasma column was formerly interpreted in analogy with the case of sound waves in gases. This point of view overlooks the ionization and recombination processes which go on in most experimental plasmas, but it enables one to carry out a straight forward wave analysis. In fact, consider a cylindrical plasma column of cold ions and hot maxwellian electrons where plasma generation and drifts are neglected, and assume perturbations of the form

$$n(r) \exp j(\omega t - \beta z - m\phi)$$

One arrives at a conventional wave equation for $n(r)$ whose solutions are given in terms of Bessel functions (see Crawford and Self ²¹ and Davies ³⁸). This model, besides giving an incomplete picture of the

plasma reality, has the further disadvantage of requiring a somehow arbitrary choice of boundary condition to be completed. This is one reason (besides the uncertainty about the electron compression coefficient) why the experimental investigation of the behaviour of the plasma boundary in the presence of the ion-acoustic waves, led in the light of the present model, gave rise to much confusion and proved to be little successful.

The wave propagation in a plasma is substantially influenced by the ionization and recombination processes. In a bounded plasma, maintained by volume ionization and surface recombination, the wave energy is lost at a rate determined by the plasma generation and loss, and the wave consequently exhibits dispersion and attenuation. Woods

³⁷ studied this effect first, using a plasma model of a hot isothermic electron gas and cold ions in which volume ionization was included but axial drifts were not. The theory was developed for the case of propagation of ion-acoustic waves along a cylindrical plasma column in the low-pressure collisionless limit. The correct boundary conditions were discussed and introduced, and the wave dispersion and attenuation, as well as the radial dependence of the different wave modes, were computed. The same plasma model was next analysed by Davies³⁸ who further introduced elastic collisions to cover the intermediate pressure range as well.

5.2.2. Let us define the dimensionless quantities

$$\sigma = \omega R / a_0$$

$$h = R \beta$$

$$\mathbf{x} = \mathbf{r} / R$$

where R is the tube radius, and $a_0 = (kT_e / m_i)^{1/2}$ is the ion-acoustic speed. The subscripts r and i of the reduced wave-number h will denote its real and imaginary parts. We shall be concerned here with the non-azimuthal modes; these are represented by the pair $(0, N)$ where N is the number of order of the radial

mode.

We consider the collisionless limit first. In the zeroth or principal mode (0,0) (see Woods' paper) the amplitude of the density perturbation has a radial variation which is proportional to the density equilibrium profile and its phase does not show any radial dependence. It exhibits little dispersion and attenuation; for $\sigma \gg 1$ one has

$$h_R = \sigma \quad h_1 = 0.554 \quad .$$

For the first mode (0,1) (see Davies' thesis) the dispersion (σ versus h_R) and attenuation (h_1 versus σ) are shown in fig. 30. The radial dependence of this mode is shown in figs. 31 and 32 for several values of the reduced frequency σ . The plot of h_1/h_R versus σ (see fig. 33) is of special interest because it gives information about the relative attenuation over the frequency spectrum; a minimum is found at $\sigma = 8.25$, $h_R = 8.35$.

We consider now the influence of collisions; for this purpose it is convenient to define the quantity $\delta = 1 + \nu/b$ where ν is the effective collision frequency and b is the ionization frequency. When collisions are taken into account, the (0,1) mode exhibits a decrease in phase velocity at low frequency and an increase at high frequency, and in such a way that the frequency at which the phase velocity equals a_0 increases fast with δ :

$\delta = 1$		$\sigma = 7.5$
2		10.5
4		15

The plot of h_1/h_R versus σ shows that the attenuation increases with δ while σ is below the point where h_1/h_R is minimum; besides, the minimum h_1/h_R is displaced towards higher frequency when δ goes up:

$\delta = 1$		$\sigma = 8.2$		$h_i/h_R = 0.203$
2		10.0		0.209
4		>15		< 0.156

On the experimental side the information available is still not enough for a full understanding of the propagation of ion-acoustic waves along the positive column of low-pressure discharges. In papers by Barrett and Little ²² and Crawford and Kuhler ¹³⁴ there is however clear evidence that both the principal and the first symmetric modes can be excited and propagate. Experiments carried out by Little and Jones ¹³³ on spontaneous and excited waves yielded data consistent with the first symmetric mode of propagation too. Two features of the experimental results are worth reporting here. On one hand the phase velocity of the principal mode is generally somewhat lower than the expected ion acoustic speed. On the other hand, data on the first mode suggest the existence of a cut-off frequency which was originally interpreted in terms of the properties of the Bessel functions. Regarding the radial structure of the waves, there is still little information available; a few measurements are reported by Little ³² and Little and Jones ¹³³ and also in Woods' paper, but more work is required along these lines.

We come to an end with a short reference to asymmetric (azimuthal) modes. These were also analysed in Davies' thesis. Here only two facts will be mentioned: that the relative attenuation h_i/h_R does show a minimum for the (1,2) mode but not for the (1,1) mode; and that the radial structure of both modes shows a density perturbation node on the axis at all frequencies. Asymmetric modes have never been reported to occur spontaneously in non-magnetized plasma columns. An attempt to excite them proved unsuccessful (Crawford and Kuhler ¹³⁴).

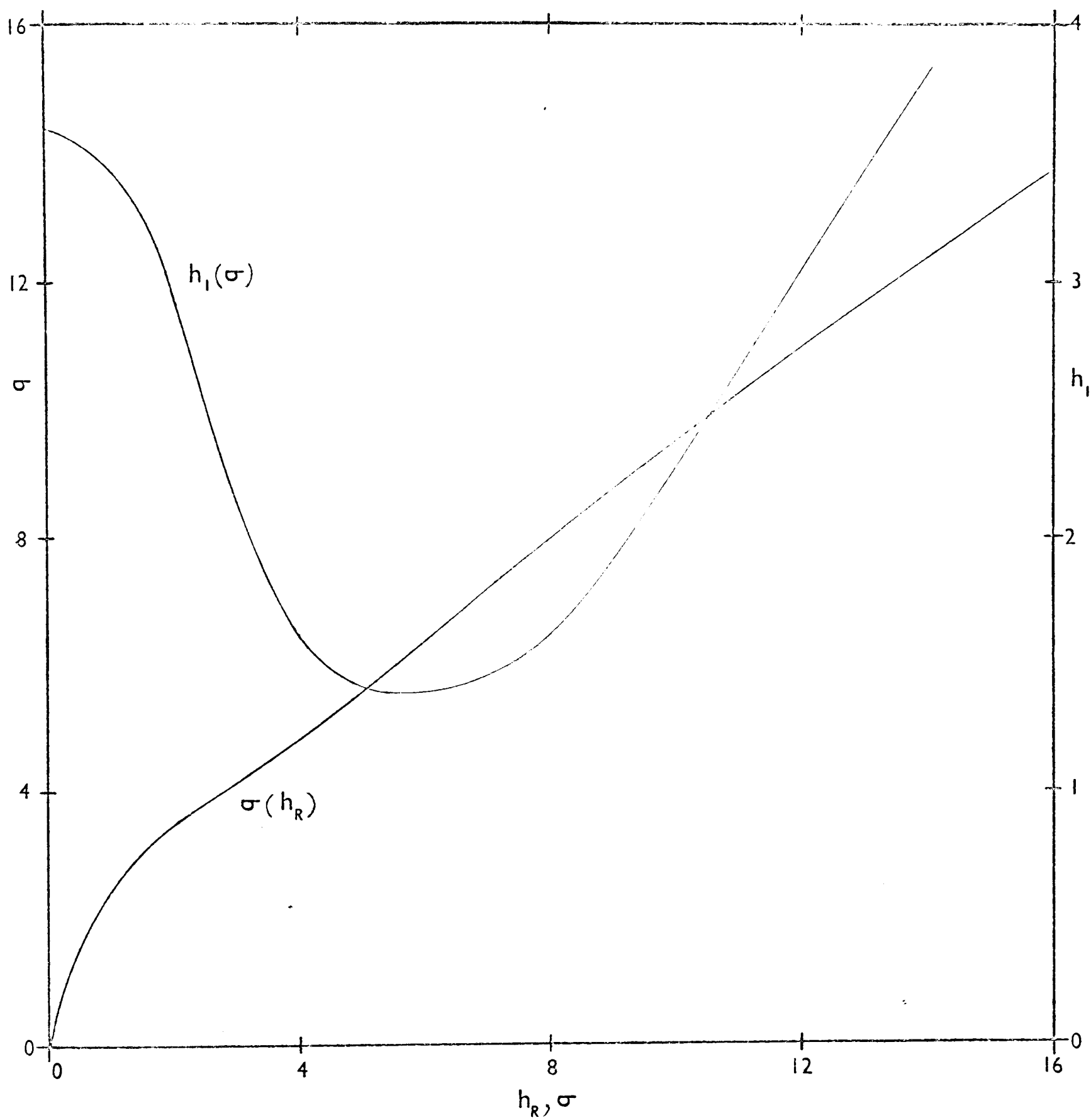


Fig. 30. Dispersion and attenuation of the (0,1) mode according to Davies 38 .

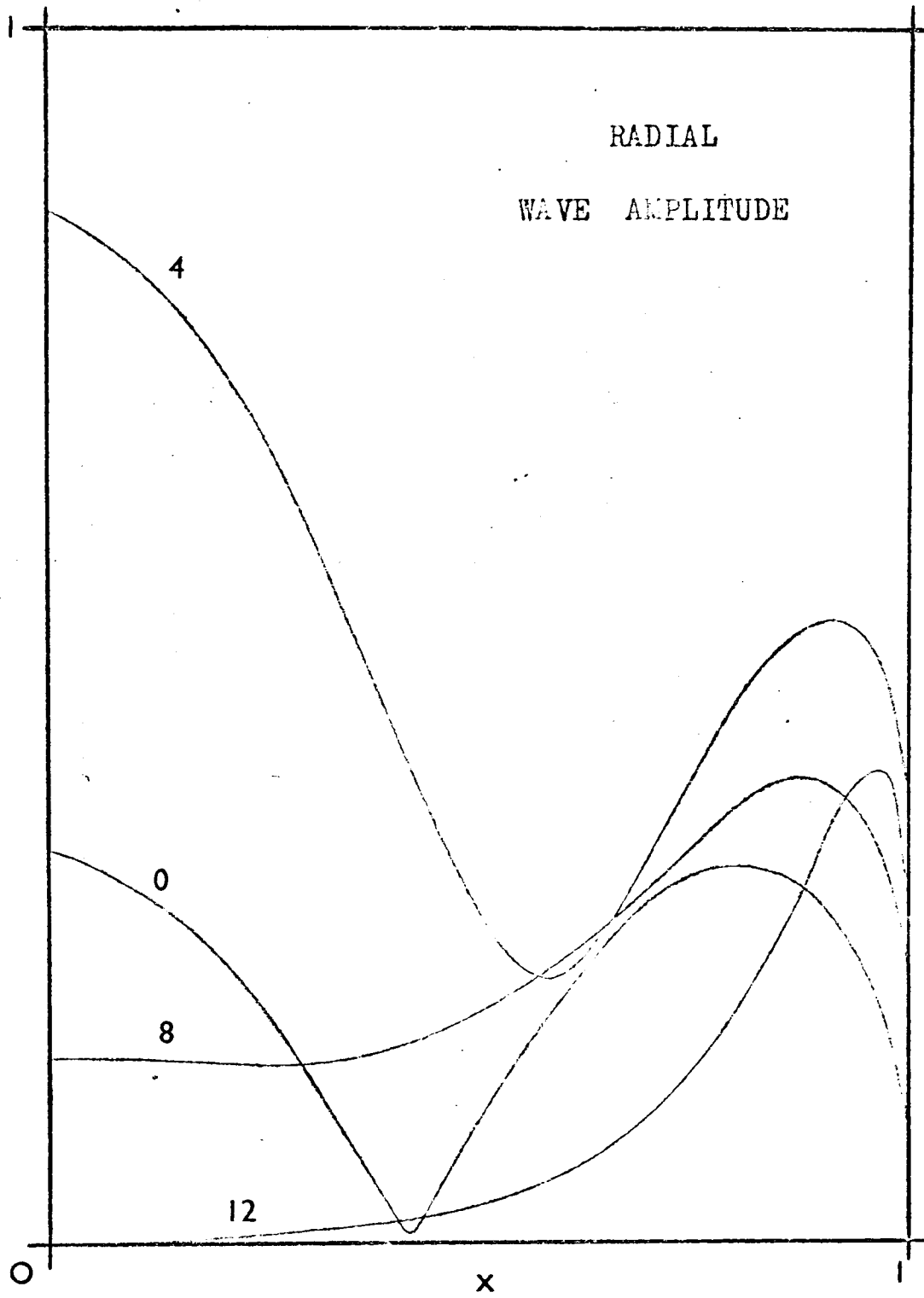


Fig. 31. Radial structure of the (0,1) mode : wave amplitude (Davies) .

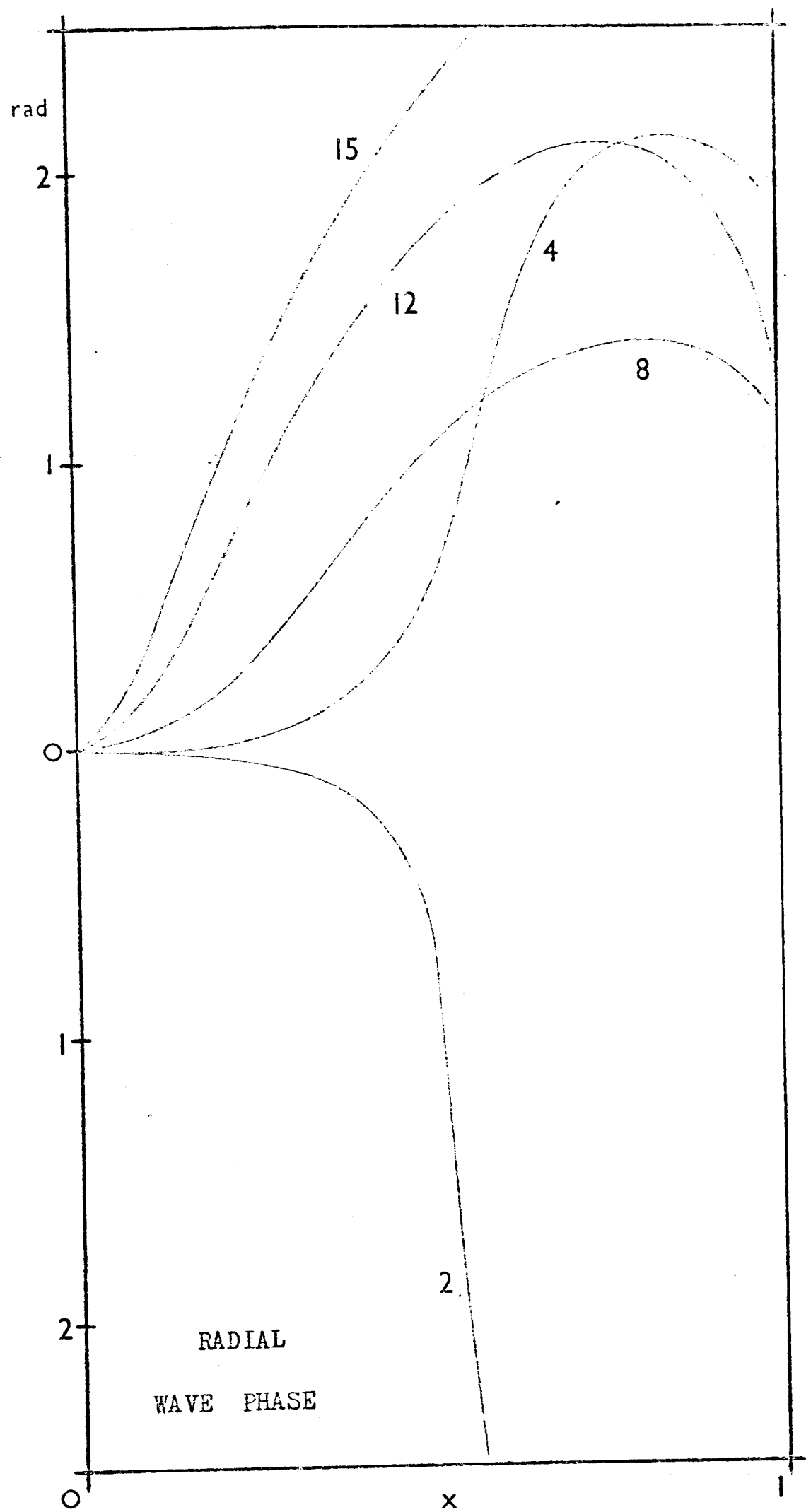


Fig. 32. Radial structure of the (0,1) mode : wave phase (Davies) .

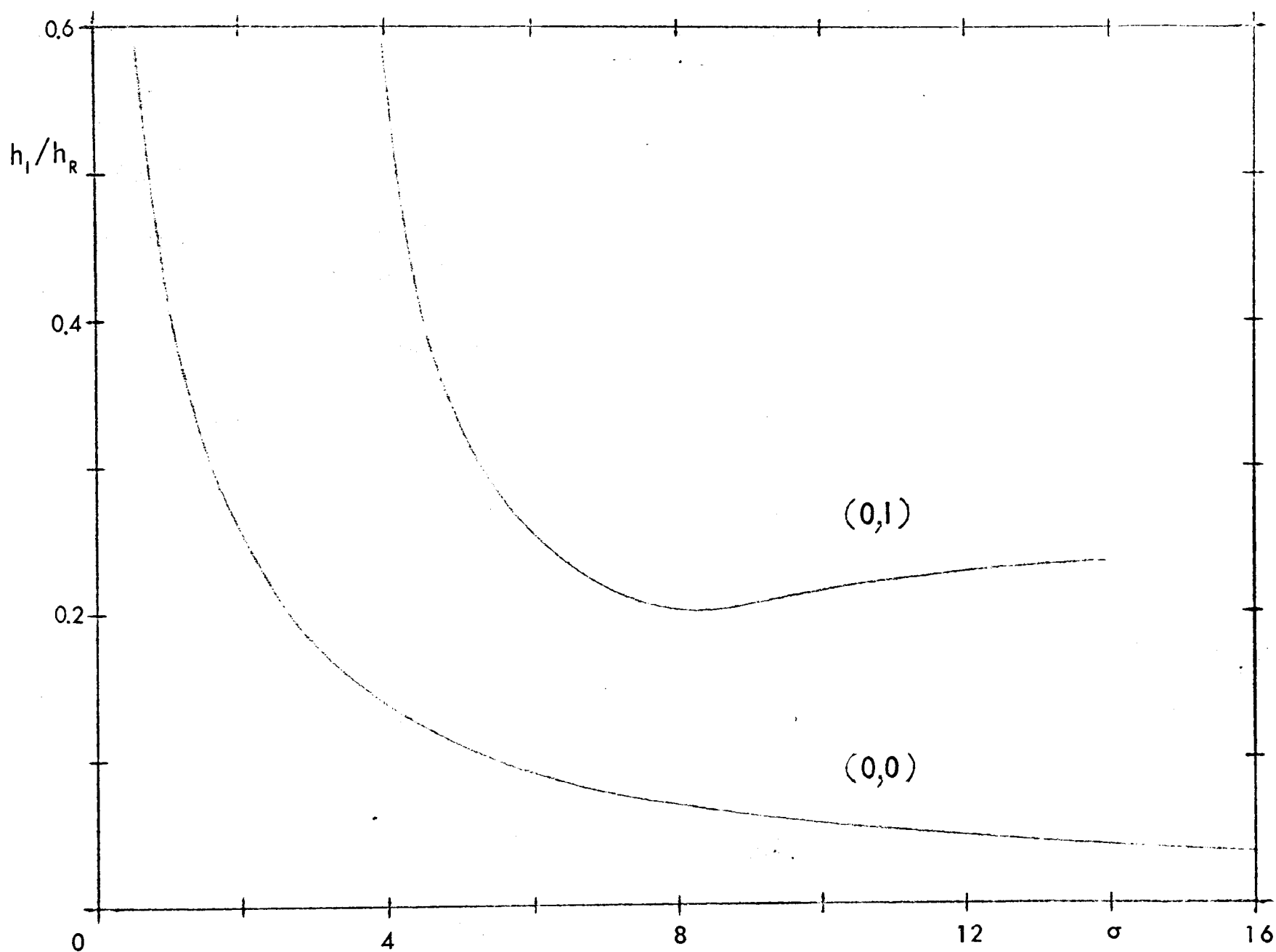


Fig. 33. Attenuation rate of the $(0,0)$ and $(0,1)$ modes (Davies) .

5.3. THE KINETIC THEORY OF ION ACOUSTIC WAVES.

THE TWO-STREAM INSTABILITY.

5.3.1. The longitudinal component of the dielectric tensor can be expressed in terms of the electric field and the charged particles densities with the help of Poisson's equation. When the medium is a fluid which can be described by one or more Boltzmann equations then the longitudinal dielectric constant can be expressed in terms of the distribution functions alone. On the other hand, the longitudinal dielectric constant when equated to zero yields the dispersion relation of the medium to electrostatic oscillations (for a full discussion of these matters, see Akhiezer et al ¹⁴⁹).

The investigation of plasma dynamics problems in the light of the kinetic theory was pioneered by Vlasov ¹⁵⁰ and Landau ¹⁵¹ and ³⁹ . Vlasov introduced the self-consistent field and the collisionless Boltzmann equation in order to study the plasma evolution in time intervals shorter than the reciprocal of the collision frequency. Landau's contributions comprise the study of the Boltzmann collision integral for Coulomb interaction, and the analysis of the integrals over the velocity space which play an important role in the kinetic formalism. His interpretation generalizes to the complex plane the Dirac's interpretation of the integral of a function having a pole on the real axis. Landau's analysis of the initial value problem opened the way to the prediction of a new damping mechanism of the electrostatic waves in collisionless plasmas. Bohm and Gross ¹⁵² discussed the problem of energy exchange between a wave in a plasma and a beam of particles and the relation between the structure of the distribution function and the wave instability, at the same time uncovering the physical mechanism behind Landau's damping phenomenon. Landau damping has been the object of experimental investigation and this has recently brought some clear

evidence to its support: Wong et al ¹³² and Alexeff et al ⁴²

The kinetic theory of plasmas as originally laid down by Vlasov and Landau has been followed by many other authors. Jackson ¹⁵³ analysed the conditions necessary for the growth or decay of longitudinal oscillations and the physical mechanisms involved. The same author and also Jackson ¹⁵⁴ treated the problem of the stability of a plasma of two maxwellian components in relative drift motion. This problem is related to the two-stream instability already known earlier and analysed in hydrodynamic terms by Buneman ¹⁵⁵. Fried and Gould ³⁵ analysed the stability of ion-acoustic waves and emphasized that their damping may be reduced by either increasing the ratio of electron to ion temperatures or increasing the relative drift speed; this last factor can eventually bring about the instability of the wave.

5.3.2. Consider a plasma of two maxwellian components both governed by the collisionless Boltzmann equation (Vlasov) and coupled by Poisson's equation. The electron component is assumed to be drifting at speed u relatively to the ion component and this one is taken to be at rest in the observer's frame. Assume one-dimensional electrostatic perturbations of the form $\exp j(\beta x - \omega t)$, Fourier analysed in space and time and linearize all the equations. Let h denote the Debye length ($h^2 = kT/4\pi ne^2$), a thermal speed ($a^2 = kT/m$) and

Θ the ratio of electron to ion temperatures. Define the dimensionless quantities

$$\zeta_e = (\omega/\beta - u) / 2^{1/2} a_e$$

$$\zeta_i = \omega / 2^{1/2} \beta a_i$$

One obtains a dispersion relation which is usually cast into the form (see Fried and Gould ³⁵):

$$2\beta^2 h^2 = Z'(\zeta_e) + \Theta Z'(\zeta_i)$$

Here Z is a complex function, called the plasma dispersion function,

defined by

$$Z(\zeta) = \pi^{-1/2} \int_{-\infty}^{+\infty} dx \exp(-x^2)/(x - \zeta)$$

when $\text{Im}(\zeta) > 0$ and by its analytic continuation when $\text{Im}(\zeta) < 0$

(see Fried and Conte¹⁵⁶ and Faddeeva and Terentev¹⁵⁷ for numerical tables and general properties and also Fried et al¹⁵⁸ for a two-pole approximation of this function).

We are concerned with the study of the two-stream instability of ion waves in a plasma of hot electrons and cold ions drifting in an applied electric field. This problem was analysed by Gordeyev⁴³ and afterwards by Jackson¹⁵⁴ and Fried and Gould³⁵. Jackson investigated the boundary between the stable and unstable regions in the β versus u plane. This boundary was experimentally investigated recently by Tanaka et al⁴⁶ whose results are in good agreement with the theoretical expectations. Fried and Gould studied the influence of Θ upon the wave-number and drift speed at which the onset of the instability occurs.

We shall consider now in detail the two-stream instability mechanism under those physical conditions which operate in low-pressure current-sustained discharges and are of interest to us.

It will be assumed here that, besides $\Theta \gg 1$, one has $a_i \ll |\omega/\beta| \ll a_e$ and $u \ll a_e$, which implies $|\zeta_e| \ll 1$ and $|\zeta_i| \gg 1$.

First we shall investigate the stable-unstable boundary in the β versus u plane (in the case of large Θ). It should be noticed that both ζ 's are real quantities on the boundary. In the case of real ζ $Z(\zeta)$ can be put into the following form:

$$Z(x) = j\pi^{1/2} \exp(-x^2) - 2xY(x)$$

where

$$Y(x) = (\exp(-x^2)/x) \int_0^x \exp t^2 dt$$

$$Y(x) = 1 - 2x^2/3 + 4x^4/15 \dots \quad (x \ll 1)$$

$$Y(x) = (1/2x^2)(1 + 1/2x^2 + 3/4x^4 \dots) \quad (x \gg 1)$$

with the help of the power series and the asymptotic expansion of Y the dispersion relation was solved for $\Theta = 20, 50$ and 100 in the range $0 < u/2^{1/2}a_e < 0.20$ and the solutions are shown in fig. 34. The intersection of the stability boundary with the u -axis is determined by taking $\beta = 0$ in the dispersion relation and is given by

$$u/a_e = \Theta^{3/2} \exp(-\Theta/2)$$

which means that when $\Theta \gg 1$ the drift required for the onset of the instability is vanishingly small. The frequency along the stability boundary is found to be related to the wave-number by

$$\omega/\beta = a_e/(1 + h^2\beta^2)^{1/2}$$

In order to study the unstable region one should notice that both $\text{Im}(\zeta)$ are positive there (either in the case of real positive β and complex ω or in the case of real positive ω and complex β). When its argument lies on the upper half plane, $Z(\zeta)$ admits the following approximations:

$$Z(\zeta) = j\pi^{1/2} \exp(-\zeta^2) - 2\zeta(1 - 2\zeta^2/3 + 4\zeta^4/15 \dots) \quad (|\zeta| \ll 1)$$

$$Z(\zeta) = -(1/\zeta)(1 + 1/2\zeta^2 + 3/4\zeta^4 \dots) \quad (|\zeta| \gg 1)$$

With the help of the power series and the asymptotic expansion of Z the dispersion function was solved for the case when

$$a_i \ll |\omega/\beta| \ll u \ll a_e$$

which corresponds to the physical situation of a low-pressure current-sustained gas discharge. A small growth-rate is assumed and is justified a posteriori.

(i) Assume β to be real and solve for complex ω . One arrives at

$$\omega_i/\omega_R = \frac{\pi^{1/2} u}{2^{3/2} a_e} (1 - \omega_R^2/\Omega_i^2)$$

$$h\beta = 1/(\Omega_i^2/\omega_R^2 - 1)^{1/2}$$

Recall now that the wave number is assumed to be real and positive; on the other hand, it is obvious that the instability ($\omega_i > 0$) requires $\omega_R u$ to be positive. Therefore, only the wave propagating along the electron drift grows unstable. In fig. 35 the relative growth rate ω_i/ω_R versus ω_R/Ω_i has been plotted; one sees that the growth rate decreases with increasing frequency and eventually vanishes when the frequency approaches the ion plasma frequency. From there on the wave is increasingly damped. At the low frequency limit one simply has

$$\omega_i/\omega_R = \pi^{1/2}u/2^{3/2}a_e$$

$$\omega_R/\beta = a_e$$

The low-frequency growth rate obtained here is identical to the value given by Stringer¹⁵⁹.

(ii) Now assume that ω is real and solve for complex β . This time one has

$$\beta_i/\beta_R = -\pi^{1/2}u/2^{3/2}a_e$$

$$\omega/\Omega_i = h\beta_R/(1 + h^2\beta_R^2)^{1/2}$$

Once again it becomes apparent that the instability appears along the direction of the electron drift only. It also turns out that the relative amplification rate β_i/β_R is independent of the wave frequency. It is worth while emphasizing that both the absolute and convective instability analysis yield the same phase velocity.

5.3.3. In the experimental investigation of the properties of ion and ion-acoustic waves these are often impressed on the plasma by means of a localized source. A coil which produces a magnetic compression, a double grid which applies an electric field, or a single grid which modulates the plasma density have been commonly used as excitation devices. A detailed analysis of the excitation mechanism and of the plasma response to it is an important requirement for the adequate interpretation of the

experimental data. This study has been initiated recently in a paper by Gould ¹⁶⁰ who investigated the process of the excitation of ion-acoustic waves by the electric field produced by a pair of closely spaced grids. He assumed that both ions and electrons are Maxwellian and governed by the collisionless Boltzmann equation. From his analysis it emerges that the character of the wave changes with the distance from the exciter. In the very neighbourhood of the source there is first an electrostatic shielding region whose characteristic length is h . There is next a region which extends over a distance of the order of a few times a_i/ω in which the wave is typical ion wave with exponential amplitude decay and linear phase shift. Far from the source the phase becomes stationary and the wave amplitude oscillates with a periodicity of the order of the ion-acoustic wave-length. Regarding the ion-acoustic wave, its amplitude is proportional to ω^2 while $\omega \ll \Omega_i$, and from there on, when $\omega \rightarrow \Omega_i$, its amplitude is progressively reduced. When Θ rises, the ion regime extends over an increasing distance (due to the reduced ion Landau damping). Finally, it should be noticed that, besides the excited wave, a non-propagating space potential oscillation is also excited. Sessler and Pearson ¹⁶¹ carried out some more computations along the lines of Gould's theory, but for $\Theta \gg 1$ and $\omega > \Omega_i$, and did an experimental study of the ion-wave properties around and above the ion plasma frequency. However, theory and data show little agreement, and it has been suggested that the wave phenomenon which was observed is of another nature different from the ion-wave (see Lonngren et al ¹⁶²). Nevertheless, the change of wave character with the distance from the source has been found experimentally and Gould's theoretical predictions were essentially confirmed by Wong ⁴⁹ (in a thermally ionized Cs plasma). The experimental conditions were substantially different from the theoretical ideal ones: the actual electron-ion and

electron-electron collision rates were larger than the wave frequency, and the exciter and detector were single grids instead of double ones. These differences give rise to a considerable discrepancy between theory and experiment, but the essential features of the plasma response are still the same.

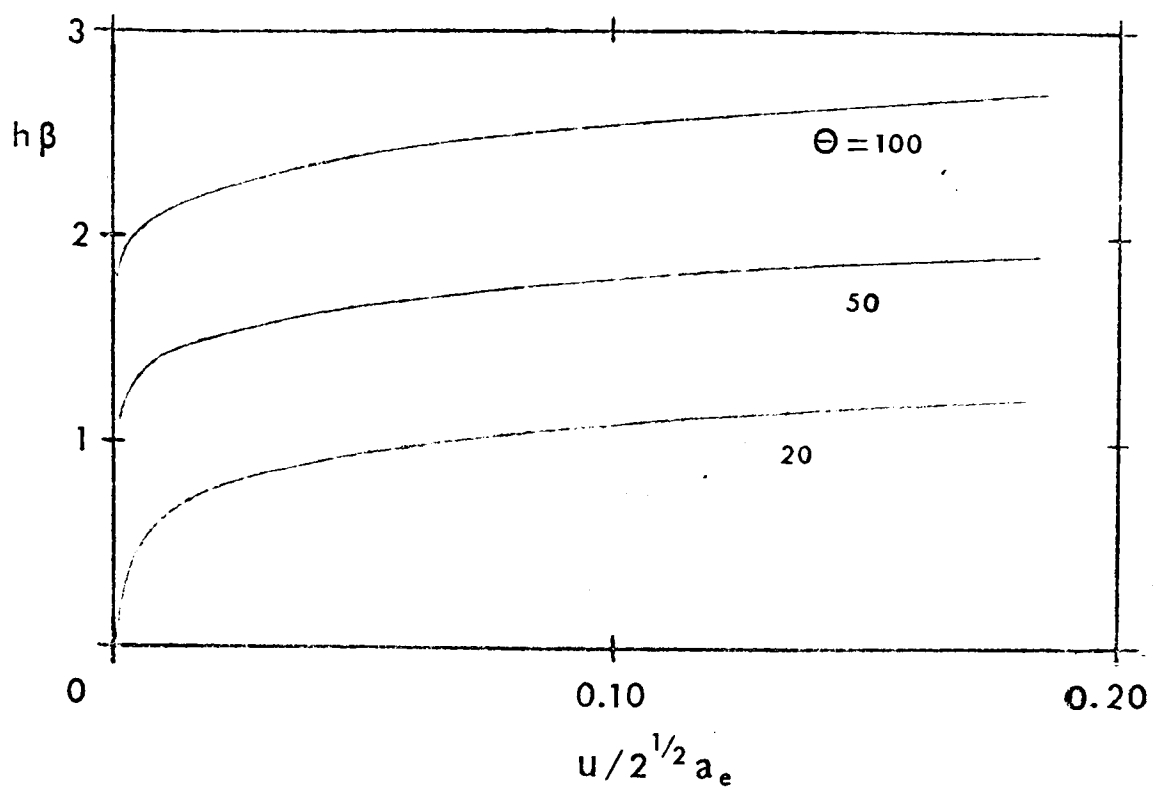


Fig. 34. The two-stream instability . The stability boundary at different values of Θ .

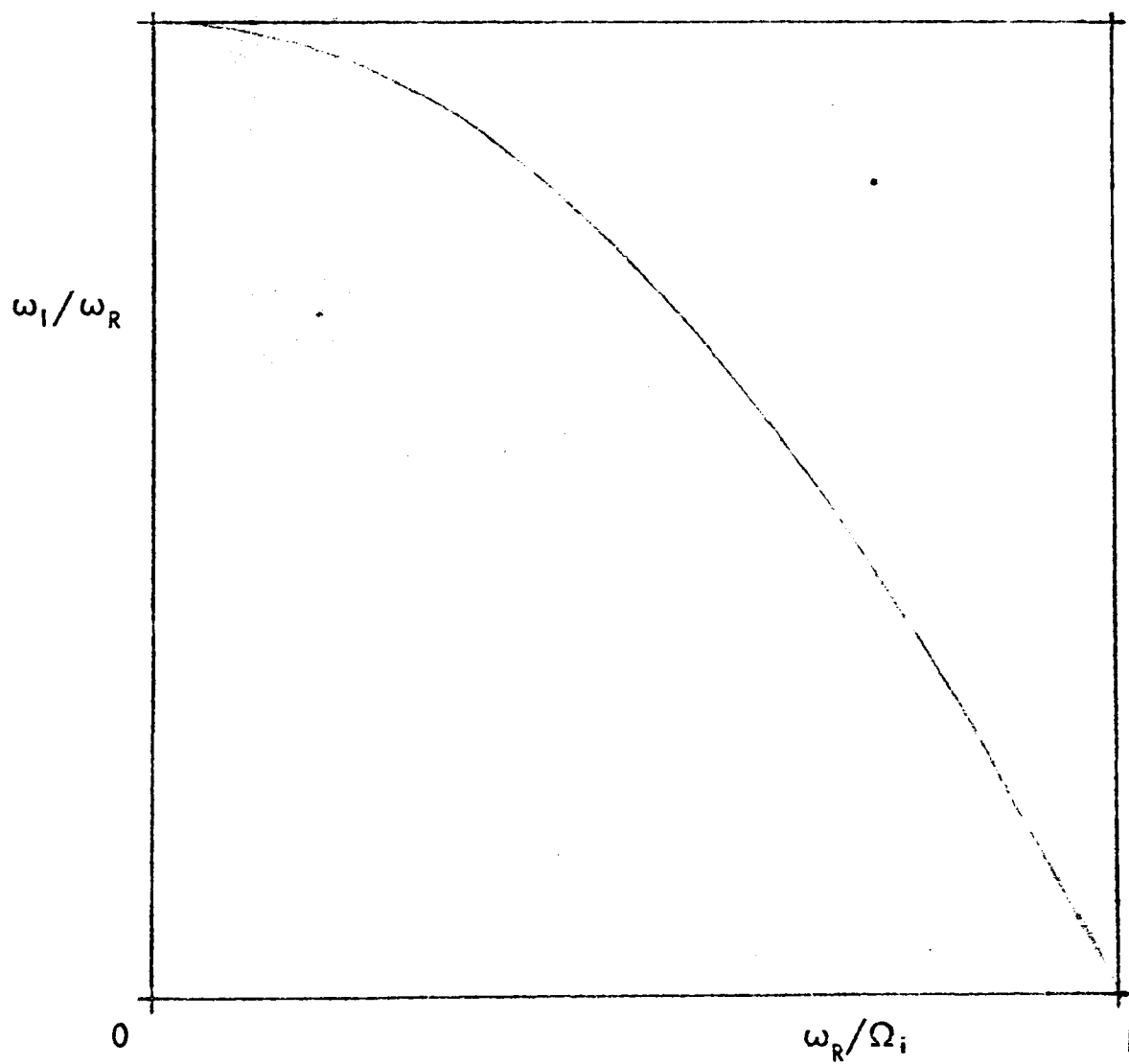


Fig. 35. The two-stream instability . Growth-rate versus frequency .

5.4. ION-ACOUSTIC RADIAL OSCILLATIONS

5.4.1. Consider the bounded plasma of a low-pressure positive column, and let us inquire about the ion-acoustic oscillatory modes which may be observed in it. Such a type of oscillation can usually be described by a plasma model involving purely electrostatic forces but in which space-charge is neglected. Assume an isothermic and inertia-free electron gas, and cold ions generated at rest. The equations governing this situation are:

$$dn/dt + \nabla \cdot n\mathbf{v} = bn$$

$$d\mathbf{v}/dt + \mathbf{v} \cdot \nabla \mathbf{v} = -b\mathbf{v} - a_0^2 \nabla n/n$$

where $a_0^2 = kT_e/n_i$ and b is the ionization rate (see Davies³⁸ and Woods³⁷). Woods has shown that the equilibrium state defined by this set of differential equations is closely equivalent to the equilibrium solution defined by the rather more complex integral formulation in the theory of Tonks and Langmuir⁷². Woods also used this set of differential equations to investigate the dispersion and attenuation of ion-acoustic waves propagating down a cylindrical plasma column.

First of all let us consider the problem of the steady state structure of the cylindrical column. Let R be the discharge radius, r the radial distance and v the radial drift velocity. Define the dimensionless quantities

$$u = v/a_0, \quad a = Rb/a_0, \quad x = r/R,$$

and denote the radial derivative d/dx by a dash, the equilibrium quantities by the subscript zero. The equations governing the steady state can be put in the form:

$$u'_0 = [a(1 + u_0^2) - u_0/x]/(1 - u_0^2)$$

$$n'_0 = -n_0 u_0 (2a - u_0/x)/(1 - u_0^2)$$

Here a is a parameter to be determined by the boundary conditions.

These are of two sorts: on the axis and on the edge. On the axis one

has to satisfy regularity; and at the plasma edge the plasma solution has to break up: therefore:

$$x = 0 \quad \text{---} \quad u_0 = 0$$

$$x = 1 \quad \text{---} \quad u_0 = 1$$

this problem has been solved numerically and the solution is shown in fig. 36b. The value of the parameter a which fits the boundary conditions is:

$$a = 1.109$$

in close agreement with the values given by Tonks and Langmuir (1.092) and by Woods (1.1088) .

5.4.2. Let us now consider the perturbed equations. We shall be interested in purely radial ion-acoustic modes of a cylindrical plasma column. For simplicity, we shall linearize the perturbed equations. Take perturbations of the form

$$n = n_0 + n_1(x) \exp(j\omega t)$$

Note that $n_1(x)$ will in general be complex, and will represent the radial phase and amplitude dependence. Define the two dimensionless quantities:

$$\sigma = \omega_R R/a_0 \quad \gamma = -\omega_I R/a_0$$

where ω_R and ω_I are the real and imaginary parts of the frequency.

The equations governing the perturbed quantities n_1 and u_1 are:

$$(1 - u_0^2)u_1' = \left\{ (j\sigma + \gamma)u_0 + 2u_0(2a - u_0/x)/(1 - u_0^2) - 1/x \right\} u_1 - (j\sigma + \gamma)n_1/n_0$$

$$(1 - u_0^2)n_1' = \left\{ j\sigma + \gamma - 2a + u_0/x \right\} u_0 n_1 - \left\{ j\sigma + \gamma + 2(a(1 + u_0^2) - u_0/x)/(1 - u_0^2) \right\} n_0 u_1$$

As u_1 and n_1 are complex, each one of these two equations can be split into two real ones. The two parameters σ and γ are to be determined by the boundary conditions. The choice of the boundary conditions for the perturbed quantities is a major problem. On the axis

one must have regularity, but the condition at the edge is not so obvious.

Bertotti has shown that a "generalized Bohm condition" exists for the sheath formation in the presence of ion oscillation, both in the case of a magnetic field-free plasma and also when a magnetic field exists normal to the plasma edge ³⁶ 111. This generalized condition means that the ion drift speed at the plasma edge is still the Bohm speed even in the presence of oscillations. The same condition has to be satisfied in the present problem as a result of the structure of our equations. As a matter of fact, the coefficient of u_1 in the equations of u_1' and n_1' go to infinity at $x = 1$, which suggests that u_1 should vanish there. Consideration of the behaviour of u_1 and n_1 in the neighbourhood of $x = 1$ shows that it must indeed be so. One can therefore write the boundary conditions

$$x = 0 \quad u_1 = 0$$

$$x = 1 \quad u_1 = 0$$

For convenience, the two equations in u_0 and n_0 and the four equations in n_1 and u_1 are solved as a set of 6 ordinary differential equations. At this point one already knows that $a = 1.109$ and the boundary conditions for u_0 are automatically satisfied. Only the two parameters σ and γ are to be varied in order to fit the boundary conditions imposed on u_1 .

The lowest mode is the one for which u_1 does not vanish inside the interval $0 < x < 1$ but at its extremes only. The computation yields for the characteristic values associated with the lowest mode

$$\sigma = 3.170, \quad \gamma = -0.913$$

or:

$$\omega_R = 2.858 b, \quad \omega_i = 0.823 b$$

This mode is always damped, e-fold in about 3.48 periods. The characteristic frequency as predicted by this formula agrees with the

observed frequency of the lower ion-acoustic radial mode reported by Crawford²⁸ in a low-pressure mercury-vapour discharge; according to Crawford's data the value of σ is estimated to lie between 3.1 and 3.4 which compares well with our theoretical value. The radial variation of phase and amplitude of both density and drift perturbations are shown in figs. 37a and 37b. It is interesting to notice that the radial variation of this mode is rather similar to the radial variation exhibited by the (0,1) mode of an ion-acoustic wave of the same frequency propagating along the plasma column, as computed by Davis³⁸ and Woods³⁷. However, the two cases are essentially different; those authors considered a real frequency and a complex axial wave number, while we have considered a zero axial wave number and complex frequency instead.

5.4.3. A similar calculation was carried out in plane geometry for a plasma slab. In this case we find $\alpha = 0.5708$ which compares well with the figure 0.5722 obtained by Harrison and Thompson¹⁶³ for identical physical conditions. The characteristic values of the first oscillatory mode are $\sigma = 2.711$, $\gamma = -0.857$. Equilibrium and mode structures are shown in figs. 36a, 38a and 38b.

5.4.4. The radial ion-acoustic modes presented in this paragraph have apparently been observed in the past (Crawford²⁸ and Alexeff and Neidigh⁵⁴). They were identified then as standing ion-acoustic waves but such a viewpoint is in conflict with the absorbing properties of the ion rich sheath (see paragraph 5.1.2.) and the present viewpoint seems more acceptable.

It should be noticed now that the frequency of the first radial ion-acoustic mode (σ) is comparable with the ionization frequency (α). Moreover, this type of oscillation drives the whole plasma boundary in phase and produces a net modulation of the plasma loss rate. In these circumstances one can expect the electron temperature to enter in oscillation too and the ion-acoustic mode will accordingly change its character. The new type of oscillation which thus arises will be studied in paragraph 6.2.

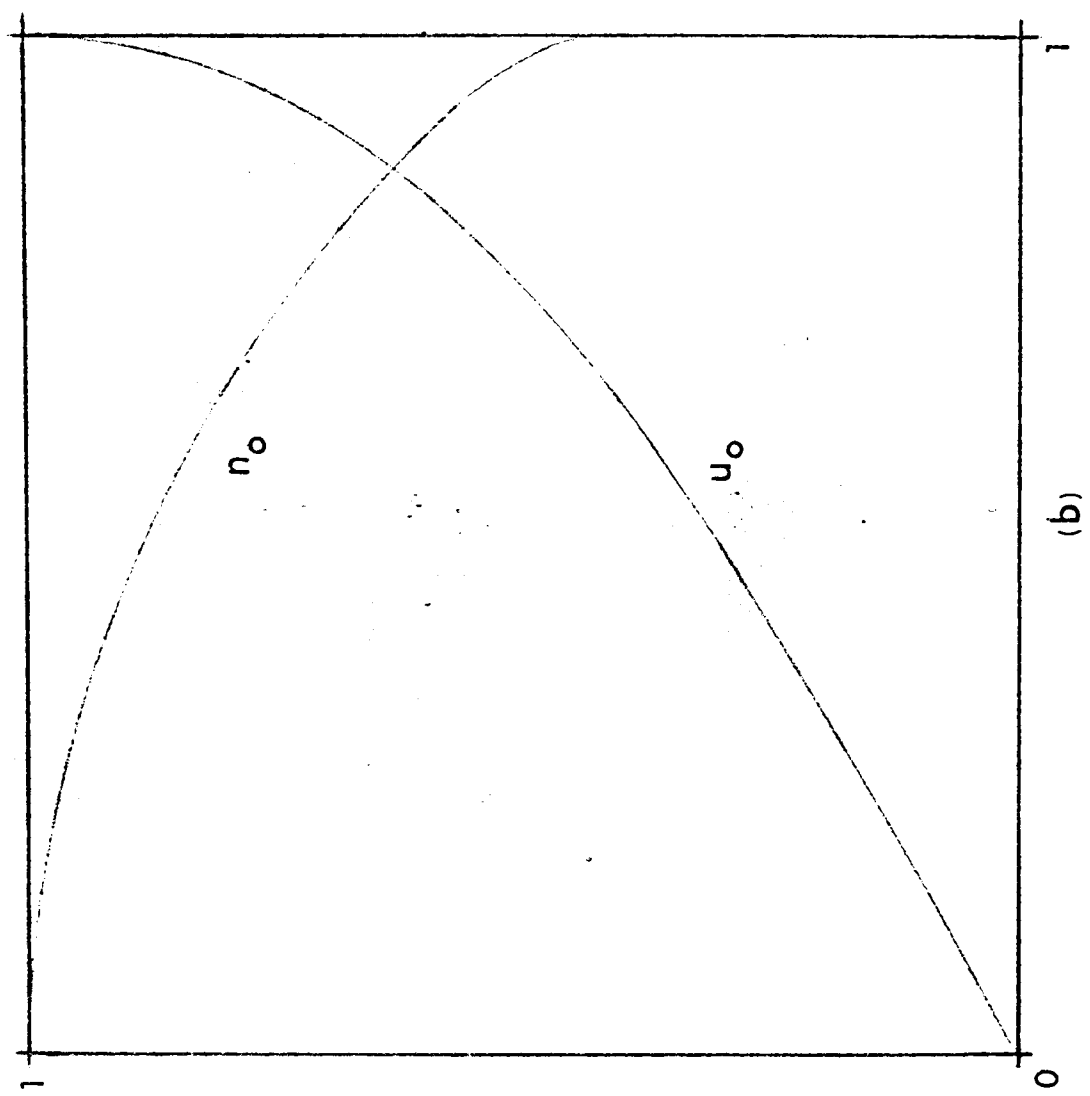
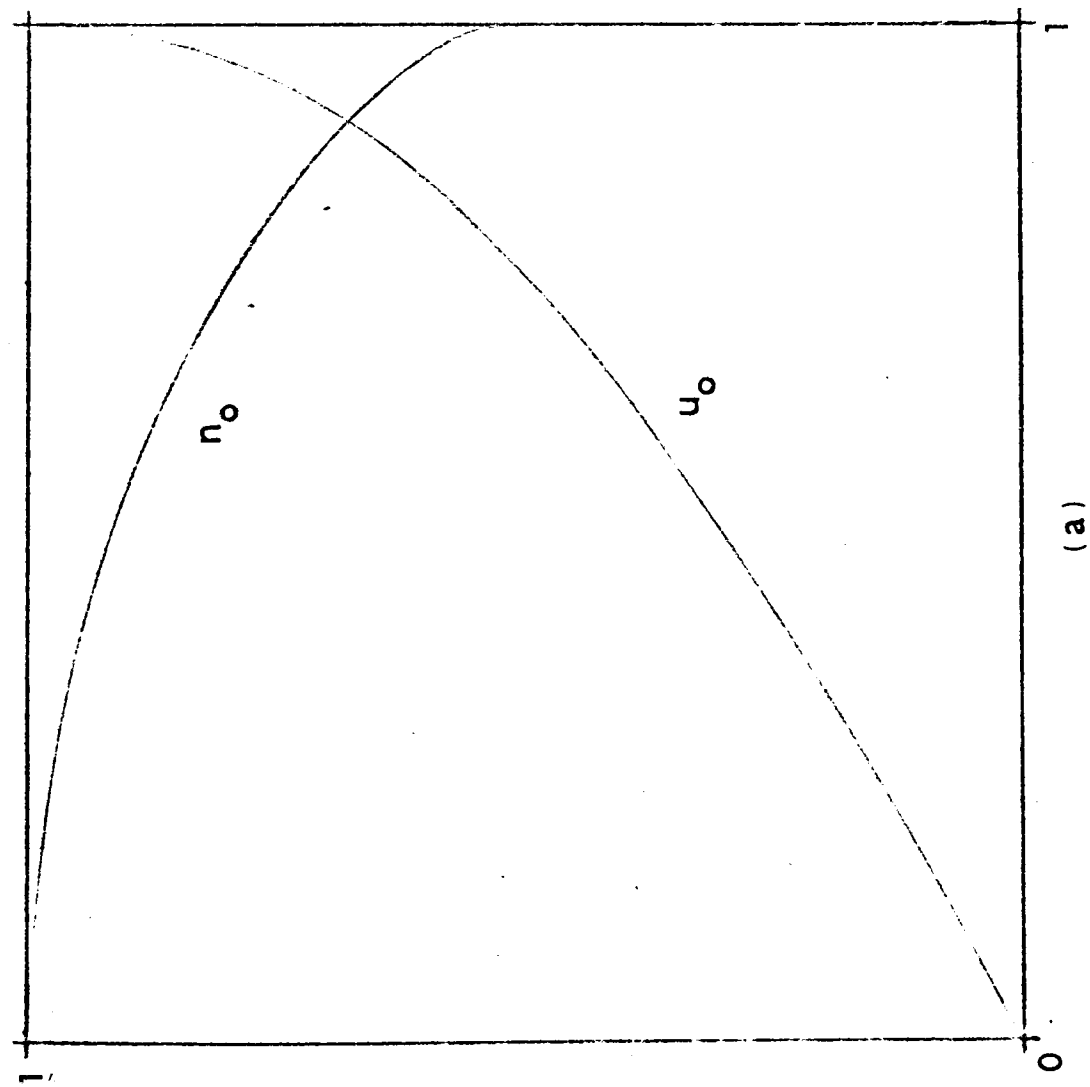


Fig. 36. Equilibrium structure of a plasma slab (a) and of a plasma cylinder (b) .

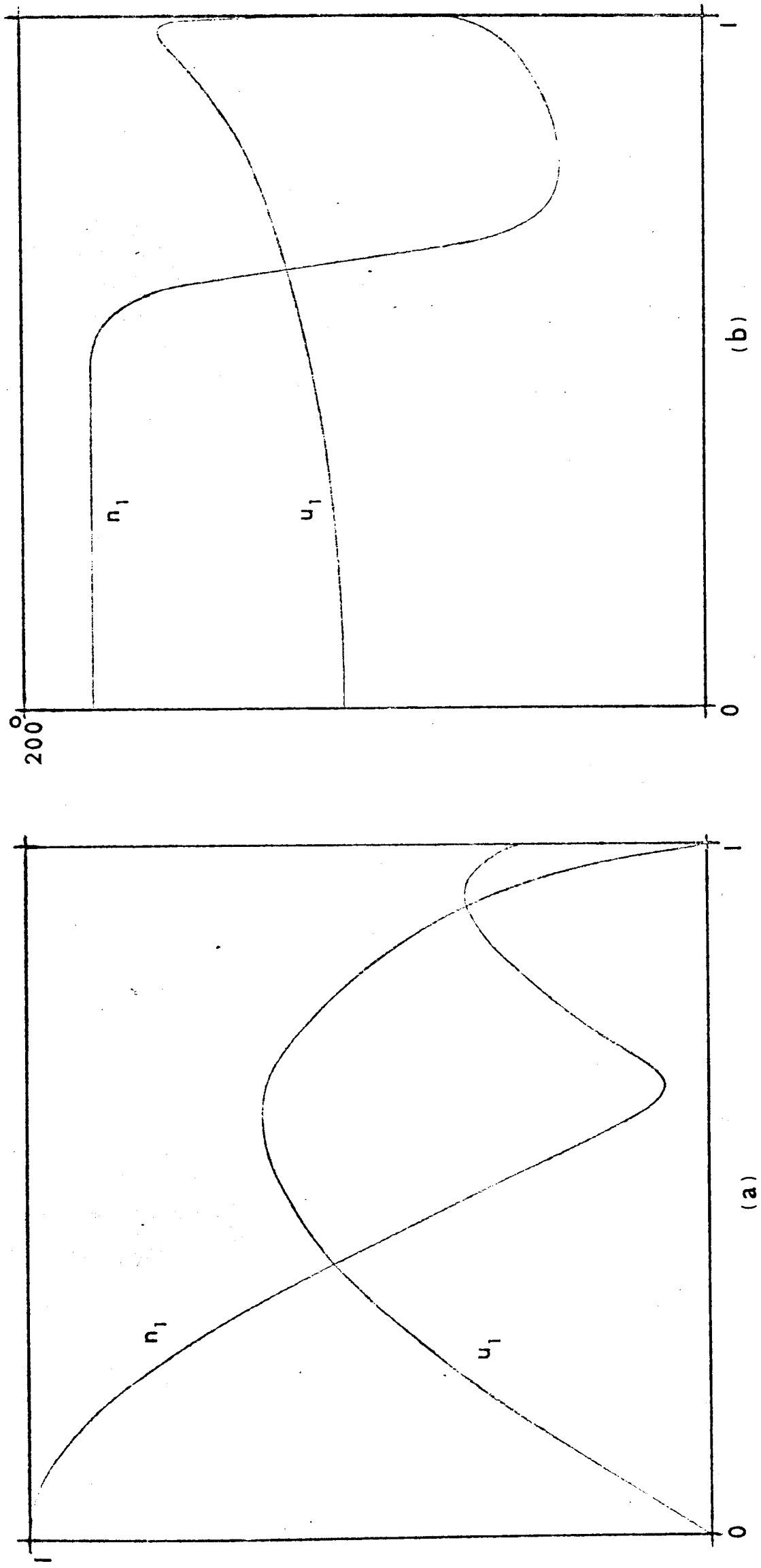


Fig. 37. Structure of a purely radial ion-acoustic oscillation of a plasma cylinder ; amplitude (a) and phase (b) radial dependence .

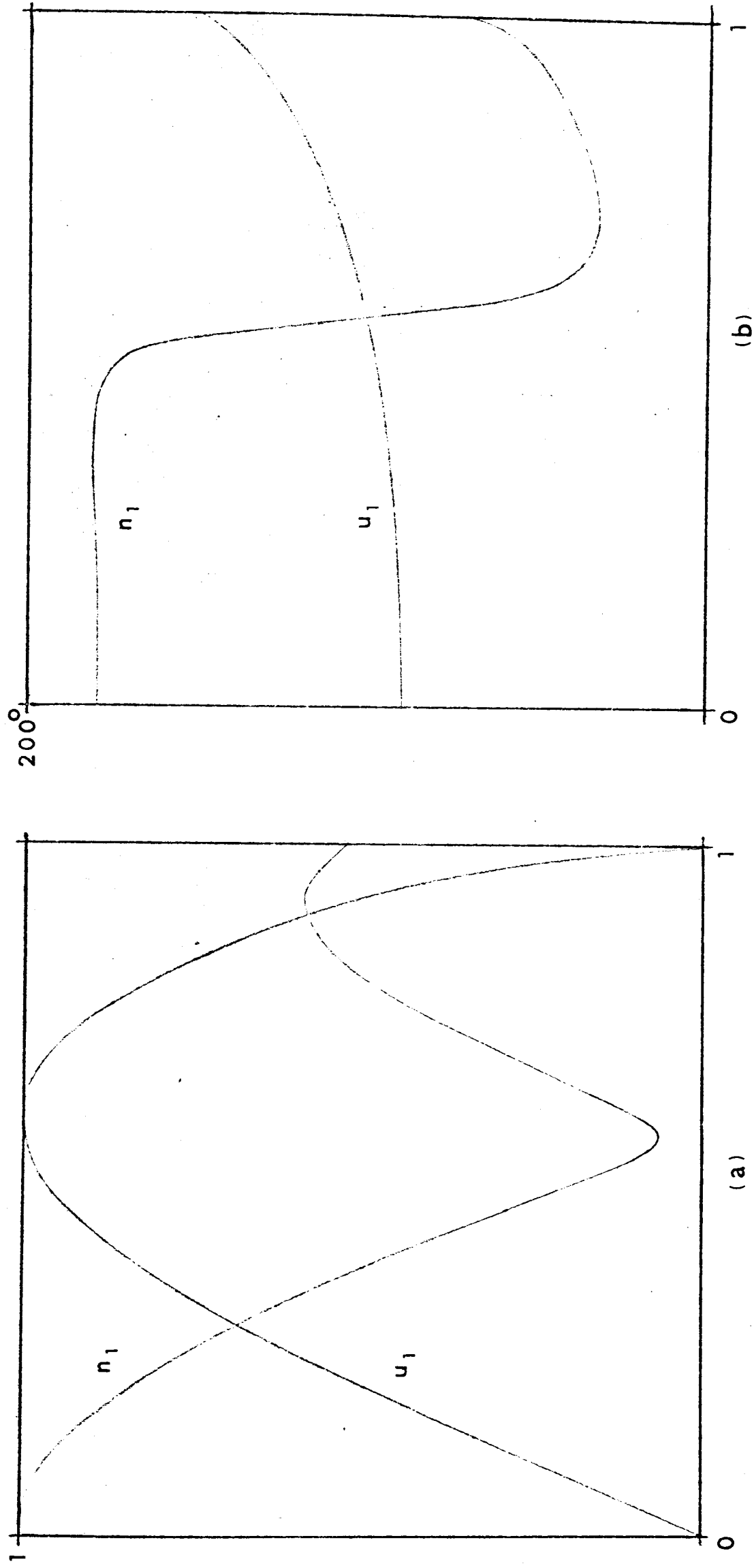


Fig. 38. Structure of a purely transverse ion-acoustic oscillation of a plasma slab ; amplitude (a) and phase (b) transverse dependence .

Chapter VI

IN-PHASE OSCILLATIONS

In-phase oscillations (or oscillations which do not exhibit phase variation in space) and uniform oscillations (which do not exhibit either phase or amplitude variation in space) have been sometimes reported to occur in plasmas. However, very little work exists in this particular field so far. The origin and nature of these oscillatory phenomena are examined and discussed in this chapter. A few uniform oscillatory mechanisms and their stability are reviewed.

The specific case of a low-pressure positive column is considered, and the theory of the radial pulsation of a plasma column involving both electron density and temperature perturbations is developed here.

It is shown that, in the light of the present theory, in-phase and even uniform oscillations of a plasma column can be predicted. Moreover, when electron density and temperature perturbations coexist, there is a net perturbation of the boundary losses. In this circumstance, the power loss experienced by the plasma column will be changed by the presence of this type of oscillation. The modulation of the boundary losses might therefore give way to the instability of the uniform oscillation of the plasma column.

6.1. IN-PHASE AND UNIFORM OSCILLATIONS

6.1.1. ON THE NATURE OF IN-PHASE OSCILLATIONS

6.1.1.1. In-phase oscillations have sometimes been observed in plasmas (see chapter I). Their nature is in general little understood, although an explanation has been advanced by Roth⁵⁸ concerning a particular class of such oscillations⁵⁹. In order to shed some light on the nature of in-phase oscillations let us consider the conservation equations governing the mechanical and thermal properties of a body. Assume a vanishingly small perturbation of its equilibrium state. For the time being, one is ignoring the question of the existence of non-linear oscillations whose features and stability depend on their amplitude, and one is also ignoring the possible existence of "hard oscillations" which can occur at finite amplitude only. Proceed to the linearization of the conservation equations and assume that all the Eulerian perturbed quantities vary as

$$q(x) \exp j\omega t$$

In this way one obtains a set of linear differential equations governing the space variation of the perturbed quantities where the complex frequency ω plays the role of a parameter. This set of equations has to be considered in connection with the boundary conditions imposed upon the perturbed quantities. The normal-mode type of analysis can be carried out now. When the problem has no trivial solutions, one obtains a number of eigen-frequencies and the corresponding eigen-modes. The question of the mode stability is answered by the sign of the imaginary part of ω .

In general, this set of differential equations has complex coefficients owing to the time derivatives which bring $j\omega$ in.

Therefore, the perturbed quantities $q(x)$ will in general be complex functions of the space coordinates, that is to say, they will exhibit phase variation in space. But it is important to stress that this is not necessarily always the case. In particular problems, some quantities may show a slow phase variation. This will happen, for instance, when second order time derivatives only are involved in the conservation equations and ω is nearly real. This situation is illustrated by the classic example of certain variable stars of the Cepheid and RR Lyrae type, which show a radial vibrational instability which can be described by a model involving the mass, momentum and energy conservation equations. Most of the volume of these stars is an adiabatic core, and in the adiabatic regime, the phase of the radial vibration is constant in space. It turns out that, with the exception of a thin surface layer, the whole star vibrates in-phase (on this subject, see Ledoux 1964).

6.1.1.2. When the situation is such that all perturbations show negligible phase and amplitude variations throughout most of the volume of the body, the system becomes one with a finite number of degrees of freedom. In this case one can proceed to the space integration or averaging of the conservation equations over most of the volume, with the exception of any outer layer where uniformity of amplitude and phase may fail appreciably. One can then obtain a set of first order ordinary differential equations which can be cast in the canonical form:

$$d\psi/dt = F(\psi)$$

The different terms in these equations represent either volume or surface integrals. In this context, the properties of any outer layer of material where uniformity does not prevail, as the plasma sheath, are accounted for by surface integral terms.

In this particular case in which the system has a finite number of dimensions, the dependent variables are just the integrated or average quantities. Furthermore, the problem has been simplified sufficiently to allow the investigation, not only of the stability of the equilibrium state, but also of the existence of closed orbits in the phase space and their stability.

The analysis of uniform oscillations, that is to say, oscillations which show negligible phase and amplitude variation in space, can in some particular cases be brought into the frame of classic mathematical examples. The study of the pulsation of Cepheid stars can be reduced to the study of an equation of the Van der Pol type. The Van der Pol equation is well known for the limit cycles of its phase plane, which correspond to stable periodic solutions, and it has been employed in the analysis of the oscillating triode. The limit cycles in the phase plane of the equation governing the evolution of Cepheid stars were investigated by Krogdahl ¹⁶⁵, and were found to give an acceptably good description of the observed pulsations. Roth ⁵⁸ formulated the problem of the "unison" ionization oscillation of a weakly ionized gas in analogy with the classic problem of Volterra of two conflicting populations (the neutral gas and the plasma itself). This problem is known to admit periodic solutions which can be expressed in terms of elliptic functions (see Davies ¹⁶⁶). The wave-form of the predicted oscillation as well as the dependence of its frequency with the neutral gas and plasma densities were in general agreement with the observed data.

6.1.2. ON THE NATURE OF UNIFORM OSCILLATORY MECHANISMS AND THEIR INSTABILITY

6.1.2.1. Consider a continuous body in an equilibrium state and

let's enquire into its stability. The state of the system is defined by a point in its configuration space. Take as definition of stability the following one: given an arbitrary small neighbourhood R_1 of the equilibrium configuration there is always another neighbourhood R_2 such that a disturbance initially contained in R_2 will always be confined to R_1 in the course of time. Two general methods exist to investigate the stability of a steady equilibrium configuration. A first method consists in assuming a vanishingly small perturbation and in linearizing the equations governing the system. After some manipulation, the problem can in principle be reduced to the analysis of an equation of the form

$$d^2\xi/dt^2 = F(\xi)$$

ξ being an arbitrary fluid displacement and F being a linear operator. Stability is then investigated by evaluating the eigen-values of this equation considered in connection with the actual boundary conditions. This is the "normal-mode" method. A second method consists in defining a generalized potential and in searching for its absolute minima. The necessary and sufficient condition for stability is the existence of an absolute minimum of the generalized potential at the equilibrium configuration. This theorem is called the "energy principle". Incidentally, the sufficiency of this condition is a consequence of Liapunov's direct method of stability analysis. An early report on the use of the "energy principle" is due to Tolman¹⁶⁷ and it has been widely employed in plasma physics lately (see Kruskal and Oberman¹⁶⁸ and Rose and Clark¹⁶⁹). Both the "normal mode" and the "energy principle" methods are currently used in the investigation of the stability of steady equilibrium configurations of plasmas, and the two of them are known to be equivalent at least when F is a self-adjoint operator (Kulsrud¹⁷⁰). However, the "normal mode" method further

provides the characteristic frequencies at which the system may oscillate besides the information concerning its stability.

Finally, it should be emphasized that the above cited definition of stability is incomplete in that it leaves out some types of instability such as the sort of instability that gives rise to hard self-excited oscillations which can manifest themselves at finite amplitude only.

6.1.2.2. The question arises as to whether any known mechanisms, uniform in space, might be involved in the uniform plasma oscillations which have been observed. For a mechanism to be uniform it is a necessary condition the existence of a physical agent (particle, field or wave) able of travelling across the whole body in a time much shorter than the characteristic time of the oscillation. This condition may be satisfied, for instance, when there is some kind of particle whose time of flight or diffusion across the system is smaller than the period of the oscillation and provided that the relaxation lengths are longer than the dimensions of the body.

In a series of papers Crawford and others (⁶³ , ⁶⁴ and ¹⁷¹) attempted to locate and identify the sources of instability responsible for the low-frequency noise in a low-pressure glow discharge. They analysed the noise spectrum in different discharge regimes and measured its variation along the positive column. They found noise components which could possibly be ascribed to ion plasma oscillations and to potential minimum oscillations, but other components could not be explained. They also found that some components are generated in the cathode region and propagate down the column, while other components exhibit remarkable constant phase and amplitude along the whole column. Moreover, when the discharge circuit impedance is made very large and the noise current is accordingly vanishingly small, the shape of the spectrum does not change much and its level

is still finite. These facts suggest the existence of a uniform and unstable oscillatory mechanism operating throughout the column. The same authors also analysed the impedance of a low-pressure glow discharge. This was found to be uniform and have characteristic frequencies which could be related to the different regions and physical processes in the discharge. They emphasized the apparent correlation between the uniformity of both the impedance and the noise generation, as well as the apparent correlation between the characteristic frequencies of the impedance and those of the noise spectrum. These facts suggest that the same physical processes are involved in both the impedance and the noise generation mechanisms in a low-pressure glow discharge (Crawford ⁶³). However, these authors were not able to identify the instability.

Roth's mechanism of uniform oscillation ⁵⁸ is based in a model of conflicting populations (in analogy with Volterra's classic problem); it can operate in a weakly ionized gas giving rise to stable oscillations. He assumed a time delay associated with the recirculation of the particles lost from the plasma. In this way, a detailed balance between the loss of plasma and the influx of neutrals cannot take place, and a relaxation oscillation of the plasma density becomes possible.

Another mechanism of uniform oscillation, which may operate in a thermonuclear reactor and is apparently operating in some variable stars, is based on the sensitivity of the fusion reaction rates upon the plasma temperature. In this case there may exist an exchange between thermal kinetic energy and expansion work, in such a way that the fusion reactions provide a thermal energy source in the right phase of the cycle to act as a driving agent of instability (see Eddington ¹⁷²).

Another mechanism which may operate in weakly ionized gases

involves an energy exchange between a gas of heavy excited particles and a hot electron gas. By first kind collisions energy is transferred from the hot electron gas to the heavy particle gas at a rate which increases with the electron temperature. By superelastic collisions the energy is given back to the electron gas at a rate which decreases with the electron temperature. If the energy input to the system occurs at an appropriate phase of this cycle, then energy will be periodically interchanged in a self-supported way.

However, a uniform mechanism operating throughout the whole plasma volume is not an essential requirement to have uniform oscillations. Some mechanisms can be operating at the boundary only and nevertheless be responsible for the instability driving a uniform oscillation of the whole volume. This situation can be illustrated by the "valve mechanism" which acts upon the radiation escaping from pulsating stars of the Cepheid type. This mechanism is due to a surface layer of material of larger specific heat and whose opacity increases quickly with temperature in such a way as to reduce substantially the loss of radiation associated with the pulsation. This mechanism, which operates solely at the boundary, makes possible the adiabatic oscillation of the core of the star which would otherwise be overdamped (Eddington 172).

A similar mechanism can also operate in a plasma column. The energy loss associated with the outflow of plasma is determined by the electron temperature and density at the boundary. When both density and temperature oscillations coexist, the average rate of energy loss is changed. When the phase relation between the two oscillating quantities is appropriate, the oscillation may become unstable.

6.2. THEORY OF THE RADIAL IN-PHASE PULSATION OF A PLASMA COLUMN

6.2.1. THE CONTINUITY AND MOMENTUM EQUATIONS

When a purely radial oscillation exists in a plasma column, the whole boundary oscillates in-phase. In such a circumstance there is a net modulation of the boundary losses. Moreover, when one is concerned with in-phase oscillations, one must have in mind that in this circumstance there is a net modulation of all the volume mechanisms as well. It is therefore quite clear that when one has a purely radial or an in-phase oscillation of a plasma column both the energy input and loss mechanisms will in general be modulated. One therefore ought to take the energy equation into account.

However, and in order to achieve a considerable simplification of this problem, we shall assume that the electron temperature, although oscillating, is nevertheless uniform throughout the plasma at every instant. This hypothesis is acceptable when one is dealing with low-frequency oscillations, so that the average distance travelled by an electron during one period is much larger than the plasma dimensions and so that the relaxation times of the energy processes involving the electrons are negligible small.

We shall take a plasma model in which the ions, assumed to be cold and generated at rest, are governed by continuity and momentum equations. The electron gas is assumed to be Maxwellian, isothermal and inertia free so that Boltzmann's law can be employed. The two fluids are coupled through purely electrostatic forces but space-charge is neglected. This theoretical model is similar to the one used in paragraph 5.4. to study ion-acoustic radial oscillations, but with

the difference that the electron temperature is now allowed to oscillate. The continuity and momentum equations governing the plasma are

$$dn/dt + \nabla \cdot n\mathbf{v} = 0$$

$$d\mathbf{v}/dt + \mathbf{v} \cdot \nabla \mathbf{v} = -b\mathbf{v} - a^2 \nabla n/n$$

where $a^2 = kT_e/n_i$ and b is the ionization frequency. Let r and v denote the radial coordinate and radial plasma drift, and let R be the column radius. Let a_0 denote the ion-acoustic speed at the equilibrium electron temperature and define the following dimensionless quantities

$$u = v/a_0 \quad a = Rb/a_0 \quad x = r/R$$

Assume small perturbations of the form:

$$n = n_0(x) + n_1(x) \exp j\omega t$$

and so on, and define the dimensionless frequency and damping rate by:

$$\sigma = \omega_R R/a_0 \quad \gamma = -\omega_I R/a_0$$

where ω_R and ω_I are the real and imaginary parts of the angular frequency. According to the equilibrium analysis of this model of the plasma column, already discussed in paragraph 5.4.1., it is known that a is a constant, $a = 1.109$, which expresses the plasma balance equation.

Proceed to the linearization of the two conservation equations. One arrives at a set of ordinary differential equations governing u_1 and n_1 which read as follows:

$$\begin{aligned} (1 - u_0^2)u_1' &= \left[(j\sigma + \gamma)u_0 + 2u_0(2a - u_0/x)/(1 - u_0^2) - 1/x \right] u_1 \\ &\quad - (j\sigma + \gamma) n_1/n_0 \\ &\quad + \left[aA(1 + u_0^2) - u_0^2(2a - u_0/x)/(1 - u_0^2) \right] T_1/T_0 \\ (1 - u_0^2)n_1' &= (j\sigma + \gamma - 2a + u_0/x) u_0 n_1 \\ &\quad - \left[j\sigma + \gamma + 2(a(1 + u_0^2) - u_0/x)/(1 - u_0^2) \right] n_0 u_1 \\ &\quad - \left[2aA - (2a - u_0/x)/(1 - u_0^2) \right] n_0 u_0 T_1/T_0 \end{aligned}$$

The quantity A is an atom parameter given by $A = d \ln b / d \ln T$.

It depends on the ionization cross-section of the particular atomic species and on the equilibrium temperature.

Once again one faces the question of which boundary conditions are to be imposed upon u_1 . On the axis, regularity must be satisfied. On the other hand, the analysis of the behaviour of the two equations near $x = 1$, similar to the analysis referred to in paragraph 5.4.2., shows that a further generalization of Bohm's criterion is required at the plasma edge. This is $u_1 = T_1/2T_0$, which means that the ion drift speed at the plasma edge, in the presence of density and temperature oscillations, equals the instantaneous Bohm speed. This generalization had already been discussed in paragraph 4.3.2. in connection with the properties of the plasma sheath.

The energy equation has to be taken into account now in order to obtain a full picture of the oscillatory phenomenon. However, it should be noticed, the continuity and momentum equations form a closed set of equations which can be solved separately from the energy equation. The two parameters σ and γ can be adjusted in order to fit the boundary conditions, but three arbitrary parameters are still left. These are the phase and amplitude of the density perturbation relatively to the temperature perturbation somewhere in the column, say, on its axis, plus the magnitude of the temperature perturbation itself. The actual mode of oscillation will be determined by the phase and amplitude of the quantity $T_0 n_1 / n_0 T_1$ considered on the axis. The amplitude of the oscillation is determined by the magnitude of T_1/T_0 but, as the equations are linearized, this magnitude does not influence the actual mode in any way.

6.2.2. THE ENERGY EQUATION

Let us consider the energy equation now. In the positive column energy is supplied throughout the volume at a rate I and lost at the boundary at a rate L while an amount U is stored in the column. Assume that the gas pressure is very low so that the energy loss will be entirely due to the wall loss of charged particles (Killian ⁷⁵ and Klarfeld ⁷⁶). It can be shown that, under the same assumption, the energy content can be ascribed to the ionization and kinetic energy of charged particles alone.

The energy content U , including the electron thermal kinetic energy, the radial drift kinetic energy and the ionization energy (ϵ), is given by:

$$U = 2\pi R^2 k T_0 n_0(0) \int_0^1 \left(\frac{3T}{2T_0} + \epsilon/kT_0 + u^2/2 \right) \frac{n}{n_0(0)} x dx$$

The power loss due to the outflow of plasma, in the case of a mercury-vapour plasma column (see paragraph 4.1.4.2) is given by:

$$L = 2\pi R k T_0 a_0 (8T/T_0 + \epsilon/kT_0) n(1) u(1)$$

Regarding the power input one has:

$$I = 2\pi R^2 k T_0 n_0(0) \int_0^1 \frac{i}{kT_0} \frac{n}{n_0(0)} x dx$$

where i denotes the power input per electron due to its drift in the axial electric field.

The energy-balance equation at equilibrium is simply

$$I_0 = L_0 \quad \text{which yields:}$$

$$R i_0 / a_0 k T_0 = 1.138 (8 + \epsilon/kT_0)$$

Let us take the first order perturbations now:

$$U_1 = 2\pi R^2 k T_0 n_0(0) \int_0^1 \left\{ \frac{3}{2} \frac{T_1}{T_0} + u_0^2 \frac{u_1}{u_0} + \left(\frac{3}{2} + \epsilon/kT_0 + u_0^2/2 \right) \frac{n_1}{n_0} \right\} \frac{n_0}{n_0(0)} x dx$$

$$L_1 = 2\pi R k T_0 a_0 n_0(1) \left\{ (12 + \epsilon/2kT_0) \frac{T_1}{T_0} + (8 + \epsilon/kT_0) \frac{n_1(1)}{n_0(1)} \right\}$$

$$I_1 = 2\pi R^2 k T_0 n_0(0) \frac{i_0}{kT_0} \int_0^1 \left(\frac{n_1}{n_0} + \frac{i_1}{i_0} \right) \frac{n_0}{n_0(0)} x dx$$

At this point it is convenient to define another atomic parameter by

$B = d \ln i / d \ln T$. This quantity can be defined because the power

input per electron i is a sole function of the temperature, as that is also the case of both the electron axial drift and the axial field, provided that the electron momentum and energy relaxation times are much shorter than the period of the oscillation. This quantity depends on the elastic and inelastic electron-neutral cross-sections and the equilibrium temperature T_0 . A last atomic parameter will be defined by $C = \epsilon/kT_0$ whose meaning is quite obvious.

The energy conservation equation of the plasma column in the presence of oscillations is, to first order,

$$dU_1/dt = I_1 - L_1.$$

This energy equation, considered together with the first order continuity and momentum equations introduced in the preceding paragraph form a closed set of equations which fully governs and determines the oscillation of the plasma column. In this set of equations there is one arbitrary parameter only, the magnitude of T_1/T_0 . But, as the equations are linearized, the actual magnitude of T_1/T_0 solely determines the amplitude of the oscillation and it has no influence at all upon the oscillatory mode itself.

We shall now solve the first order energy equation under the assumption that the oscillatory mode is perfectly uniform (neither amplitude nor phase space dependence of the relative perturbations n_1/n_0 , and so on). In this case the space integrations which appear in the energy equation can be easily carried out. It should also be noticed that we can take $u_1/u_0 = T_1/2T_0$ because that is so at the boundary, in the light of the generalized Bohm criterion, and it ought to be so throughout the column because the oscillation is assumed to be uniform. Finally, the time derivative dU_1/dt should be replaced by $j\omega U_1$. Solving for $T_0 n_1/n_0 T_1$ one eventually arrives at:

$$T_0 n_1 / n_0 T_1 = \left[-0.61 (j\sigma + \gamma) + 0.43 ((\varepsilon + C)B - (12 + C/2)) \right] / \left[(0.61 + 0.38 C)(j\sigma + \gamma) \right]$$

6.2.3. THE INSTABILITY MECHANISM

Purely radial ion-acoustic oscillations give rise to mass, momentum and energy loss perturbations which average zero in time. Therefore, when this type of oscillations are present, the plasma column does not experience any net excess of losses. But once electron density and temperature oscillations coexist the situation is quite different. Due to the geometric symmetry of the system the momentum loss perturbation is still zero, however, the average mass and energy loss perturbations are already finite now. Of course, this effect is negligible when the amplitude of the oscillation is vanishingly small. But, when its amplitude increases, and one leaves the context of a strictly linear theory, this effect can become important. This loss mechanism will naturally lead to either damping or anti-damping of the oscillation. The magnitude of this effect can be appreciated by its contribution $\delta\omega_i$ to the imaginary part of the frequency of the oscillation. In order to calculate this contribution, we shall assume that the ratio $\delta\omega_i/\omega_R$ must be equated to the ratio of the average energy loss perturbation per period to the total energy stored in the oscillation. The average of the mass loss perturbation need not be considered separately, because mass and energy losses actually are the same loss mechanism. Mass loss is already accounted for in the ionization energy loss term.

The energy contained per unit length of the plasma column, U , as well as its first order perturbation, whose magnitude represents the energy stored in the oscillation, U_1 , have already been given in the preceding paragraph. The energy loss per unit length, L ,

has also been given there. Its first order perturbation averages zero in time, so that one has to proceed to the evaluation of the time average of its second order perturbation, L_2 . With this step the theory leaves the frame of a strictly linear theory in order to obtain non-linear effects. One arrives at the time average of L_2 which reads:

$$\overline{L}_2 = 2\pi R k T_0 a_0 n_0(1) \left[(6 + C/4) \left| \frac{n_1(1)}{n_0(1)} \right| \cos \theta + \frac{1}{4} (6 - C/4) \frac{T_1}{T_0} \right] \frac{T_1}{T_0}$$

where θ denotes the phase angle of $n_1(1)$ relatively to T_1 .

Now, according to the definition of $\delta\omega_1/\omega_R$ which has been put forward, one has

$$\delta\omega_1 = 2\pi\overline{L}_2/|U_1|$$

It is now obvious that when $\overline{L}_2$ is negative the modulation of the boundary loss mechanisms produces an anti-damping effect. This will be the case provided that:

$$\text{Re} (T_0 n_1(1)/n_0(1) T_1) < - (6 - C/4)/4(6 + C/4)$$

Moreover, as $\delta\omega_1$ is proportional to the amplitude of the oscillation, the anti-damping effect may become dominant and off set the damping effect due to the continuous ionization and plasma loss, provided that the amplitude of the oscillation is sufficiently large. The oscillation can therefore become unstable, but at a finite amplitude only, that is to say, it would be a typical hard oscillation. Of course the linear theory may break up even before the amplitude of the oscillation is large enough for the system to be unstable. Besides, once the system becomes unstable, non-linear effects will eventually have to determine the actual mode of the oscillation, its amplitude and waveform. All that we have shown is that, owing to a second order effect, the system will tend to become unstable.

6.2.4. NUMERICAL RESULTS

In order to investigate whether in-phase oscillations of a plasma column can be predicted by the foregoing theory, we have solved numerically the continuity and momentum equations governing the oscillating plasma column (see paragraph 6.2.1.). As it was pointed out when these equations were discussed, these equations form a closed set which can be solved independently from the energy equation. In this case, the oscillatory mode will be dependent on the choice of the arbitrary parameter $T_0 n_1(1)/n_0(1)T_1$, which we shall denote by Y . To take the energy equation into account is equivalent to introducing two more boundary conditions. If these are to be satisfied, then Y cannot be arbitrary any longer, and its actual value will accordingly be determined. Nevertheless, Y is considered to be an arbitrary parameter for the time being.

We are particularly concerned with unstable in-phase oscillations. Since, in order to have instability it is necessary that $\text{Re}(T_0 n_1(1)/n_0(1)T_1) < 0$ (see paragraph 6.2.3.), we need to care only about the left-hand side of the complex Y -plane. A lattice was drawn on the half-plane $\text{Re}(Y) < 0$ and the oscillatory modes corresponding to every node of this lattice were investigated. To start with, the continuity and momentum equations were split into their real and imaginary parts. The continuity and momentum equations governing the equilibrium structure of the column were incorporated in the same set of equations for convenience of calculation. The numerical program solved this set of six ordinary differential equations, step by step, starting at the axis until the boundary was reached. Once an arbitrary magnitude is ascribed to the oscillation and a point was chosen on the Y -plane all quantities are determined on the column axis and the numerical process can start. In order to satisfy the boundary condition concerning the radial drift

at the plasma edge, the reduced frequency and damping constant have to be varied and adjusted. This adjustment is performed automatically with the help of a function minimizing procedure. In this way one can establish a correspondence between any point of the Y-plane, on the one hand, and a pair (σ, γ) and the associated oscillatory mode, on the other hand. Once the Y-plane had been extensively explored, the most interesting features of the modes were examined and singled out.

The first order continuity and momentum equations involve the atomic parameter A . The calculations we are reporting next were carried out for the particular value $A = 5$. In-phase oscillations were actually found. In fig. 39 one can see two distinct lines along which the corresponding oscillatory modes exhibit no phase variation across the column. Lines corresponding to modes which exhibit 10° and 20° phase variation along the column radius are shown as well. It is quite clear that over most of the left-hand side of the Y-plane modes show a high degree of phase uniformity. In fig. 40 one finds the ratio of the amplitude of the oscillation at the boundary to its amplitude on the axis of the column. As the modes in general show a gradual variation of amplitude across the column, the plotted ratio is an adequate measure of the uniformity of the amplitude of the oscillation. It is now also clear that the modes exhibit a substantial amplitude uniformity over most of the left-hand side of the Y-plane. Finally, on fig. 41 one can appreciate how the reduced frequency σ and damping constant γ vary in the Y-plane. As two general features notice that both frequency and damping decrease with $|Y|$. One should also notice that when $\text{Im}(Y)$ changes sign then σ changes sign too but γ is still the same.

The computing programme can be found in Appendix A.6.

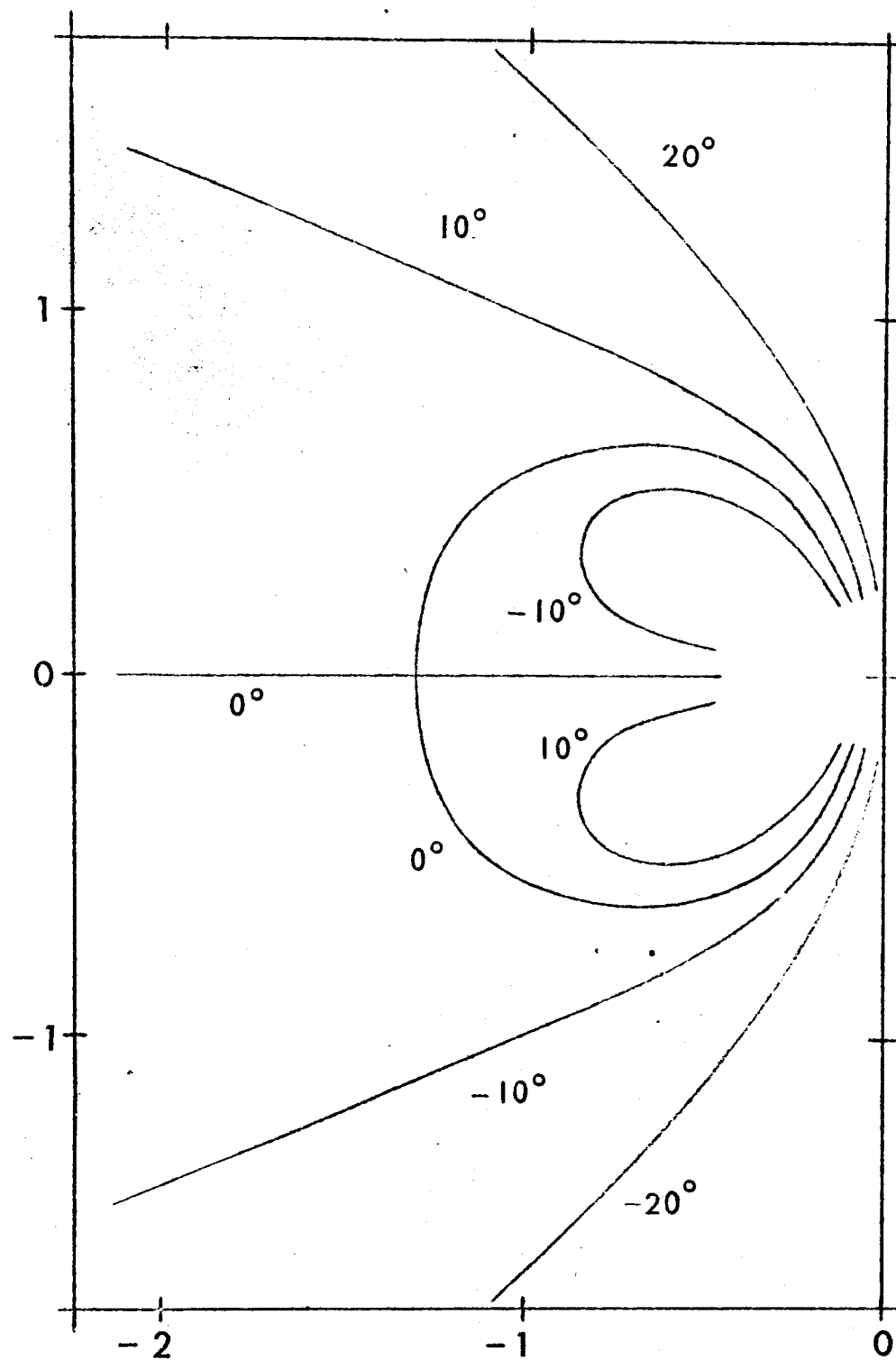


Fig. 39. The degree of phase uniformity of the oscillatory modes in the Y-plane .

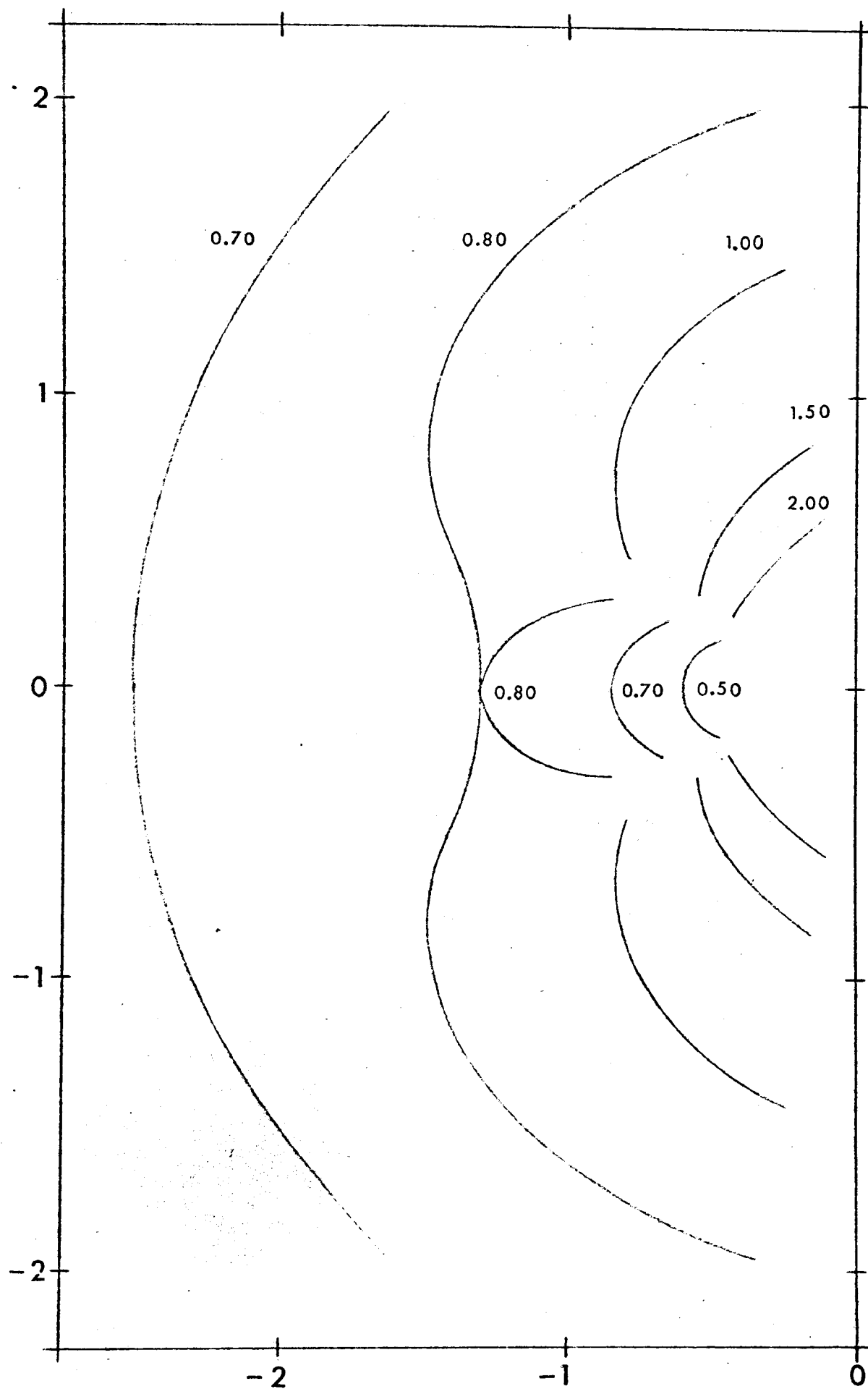


Fig. 40. The degree of amplitude uniformity of the oscillatory modes in the Y-plane .

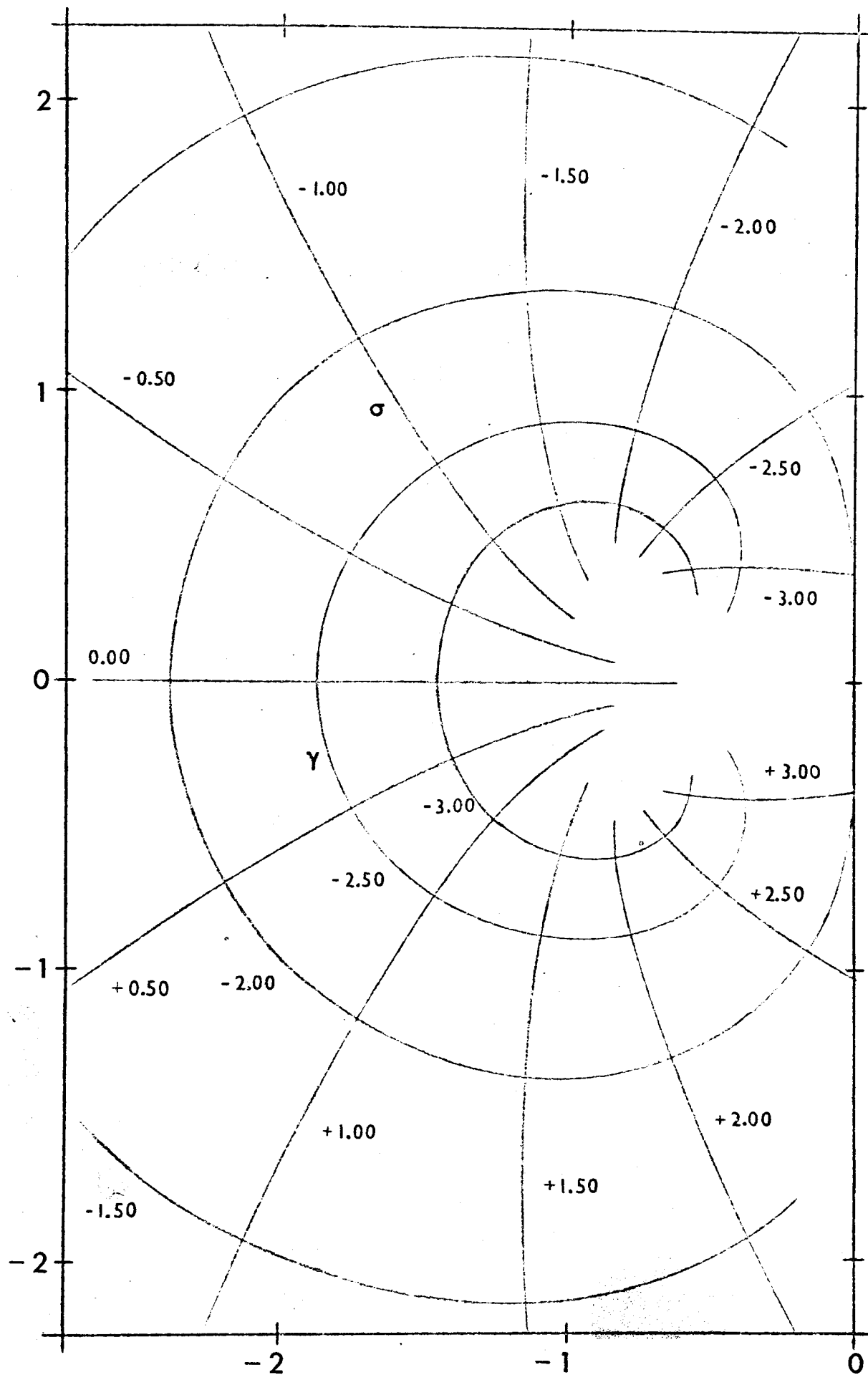


Fig. 41. The reduced frequency (the radial lines) and the reduced damping constant (the concentric lines) of the oscillatory modes in the Y-plane .

Chapter VII

DISCUSSION OF THE EXPERIMENTAL RESULTS

7.1. THE SPONTANEOUS ION-ACOUSTIC WAVES

7.1.1. In order to interpret and discuss the experimental results obtained on the spontaneous ion-acoustic waves, it is convenient to start by recalling a few experimental facts which seem most worthy of attention.

One observes the excitation of ion-acoustic waves when raising the discharge current. The current interval in which the wave amplitude rises from zero to a significant and nearly stationary level is a relatively narrow interval. Therefore, it seems acceptable to speak of a critical current for the onset of the ion-acoustic waves. The higher the vapour pressure, the lower the discharge current required to excite the wave. The ion-acoustic waves are observed in a narrow frequency band whose width and frequency increases with the vapour-pressure. The level of the wave band is nearly constant along the plasma column which could be due to either negligible attenuation or continuous generation throughout the column. The phase correlation of the wave is obtained along the electron drift direction only, and is quickly lost with the axial distance. Similar findings were reported by Tanaca et al.⁴⁶. Both the amplitude and the phase of the wave exhibit a maximum close to the axis and from there they both vary monotonically towards the wall. No amplitude node was observed over the column radius. The wave mode was accordingly supposed to be the first radial non-azimuthal mode, although it is not quite axisymmetric. The asymmetry of this mode has been observed by other authors as well (see Woods et al.¹⁷³ and Little³²). The observed phase velocity was consistently below the ion-

-acoustic speed and even below the expected phase velocity of the first radial non-azimuthal mode. Similar results were obtained by Crawford and Kuhler ¹³⁴ and Barrett and Little ²² .

7.1.2. In a low-pressure arc discharge the conditions required for the two-stream instability to operate are entirely satisfied. At the same time one also expects that a number of damping factors will stabilize the instability. These factors are the continuous generation and loss of plasma, the ion-neutral collisions and the presence of an absorbing boundary. Once the system is unstable, the wave amplitude will eventually be limited by non-linear effects which involve a feedback of energy from the wave to the plasma distribution function. The non-linearity is apparent in the Langmuir probe spectra where the ion-acoustic wave band is in general accompanied by smaller bumps at frequencies corresponding to its harmonics. The feedback from the wave to the plasma is apparently reflected in the experimental fact that, although the electron drift velocity normally increases with the discharge current while the ion waves are absent, it turns stationary or even starts decreasing with the discharge current once the ionwaves are present (see tables 3 and 5).

The generation of ion-acoustic waves by two-stream instability, in a plasma of hot electrons and cold ions drifting relatively to each other with a velocity small by comparison to the electron thermal speed, occurs within a cone the angle of whose apex is the Cherenkov angle $\cos^{-1}(a_0/u)$ where a_0 denotes the ion-acoustic speed and u the electron drift speed (see Akhiezer et al. ¹⁴⁹). In a low-pressure arc this angle is very nearly $\pi/2$ rad. and practically all wave-vectors with a positive projection on the electron drift direction are excited. Nevertheless, radial waves are quickly lost at the boundary and are accordingly the most severely damped, while waves in a narrow cone

centered on the discharge axis are the least damped of all.

The generation of ion-acoustic waves by the two-stream instability under the physical conditions operating in a low-pressure arc was studied in detail in paragraph 5.3. . The most elementary feature of the instability which was predicted is that it operates along the direction of electron drift only. It is thought that this property is reflected in the experimental fact that the wave phase correlation is only observed along the same direction as well.

The relative growth-rate ω_i/ω_R was found to be a function of ω_R/Ω_i alone (in the physical conditions we are concerned with) in such a way that the growth-rate increases with the plasma density (see fig. 35). This theoretical fact offers an explanation to the observed existence of a critical discharge current for the onset of the waves. At low frequency ($\omega_R \ll \Omega_i$) the relative growth-rate and amplification-rate are frequency independent and given by

$$\omega_i/\omega_R = -\beta_i/\beta_R = \pi^{1/2}u/2^{3/2}a_e$$

This quantity has been calculated from the experimental data at different vapour-pressures and at discharge currents just above the onset of the observed waves:

$p = 0.25$ mTorr	$I = 10.0$ Amp	$-\beta_i/\beta_R = 0.155$
0.50	5.0	0.158
1.00	3.0	0.160

7.1.3. Woods' ³⁷ and Davies' ³⁸ theory provides the best available theoretical basis to discuss the propagation of ion-acoustic waves in a positive column, although it neglects the axial drift (see paragraph 5.2.). The principal (0,0) mode is the least attenuated of all modes (smaller β_i/β_R) and it is therefore not obvious why it has not been observed. According to the theory, the first radial of the non-azimuthal modes (0;1) shows a few relevant features which we shall recall now. (i) The attenuation-rate exhibits a minimum (see

fig. 33) (ii) which is displaced to higher frequency and (iii) broadened with increasing collision frequency. At the same time (iv) the attenuation-rate at its minimum shows a general trend to decrease. These properties suggest that waves propagating in this particular mode should be observed in a limited wave band whose frequency and width would increase with the vapour-pressure. Besides, the critical current required for the onset of the waves should exhibit a general trend to decrease when the vapour pressure goes up. As a matter of fact, all these four predicted properties are actually found in the experimental results. This point comes as an important argument in favour of the identification of the observed wave mode as the non-azimuthal first radial mode (see paragraph 3.3.). We shall now seek a more quantitative comparison of theory with experiment. For this purpose one has to introduce the parameter $\delta = 1 + \nu/b$ where ν is the equivalent collision frequency for momentum loss and b is the ionization frequency. Owing to the very small electron to ion mass ratio, only the ion- neutral collisions are to be taken into account in practice; this collision frequency was calculated assuming the collision rate to be constant and the cross section to be $0.56 \times 10^{-13} \text{ cm}^2$ (see appendix A.4.). The ionization frequency was calculated by means of the equation $b = 1.09 a_0/R$ (see paragraph 4.1.1.). The gas density increment produced by the discharge (discussed in paragraph 4.4.2.) was taken in consideration as well. In the next table we have recorded the frequency of the peak of the wave band observed at a number of discharge regimes together with the corresponding estimated values of δ :

p (mTorr)	I (Amp)	δ	σ
0.25	10.0	1.4	8.9
0.50	7.0	1.6	9.5
1.00	5.0	2.1	13.3
1.50	5.0	2.6	17.0
2.00	5.0	3.3	23.2

This correspondence between σ and δ should now be compared with the theoretical prediction:

δ	σ	β_i/β_R
1	8.2	0.202
2	10.0	0.209
4	>15.0	<0.156

Unfortunately, the theoretical information available on this matter is rather limited and an extensive quantitative appreciation cannot be done. Notwithstanding one can see that both the magnitude of σ and its variation trend with δ are similar in the two sets of values.

7.1.4 The observed phase velocities are consistently lower than the expected values. This fact seems to be fully explainable in terms of the Doppler shift. We shall consider in detail the case of the discharge at $p = 0.50$ mTorr controlled vapour-pressure and $I = 7.0$ A anode current. The axial ion drift speed, assuming a free fall regime, would be $u_i = 0.25 \times 10^4$ cm/s ; assuming a collision dominated regime instead, it would be $u_i = 2.0 \times 10^4$ cm/s (see appendix A.4). The actual value lies somewhere in between because the actual discharge is in the transition region. This can be calculated with the help of the flux balance equations of the positive column which were discussed in paragraph 4.4.2. . In this way, we arrive at the correct drift speed which turns out to be, for the particular discharge regime,

$$u_i = 0.8 \times 10^4 \text{ cm/s}$$

Here, the increment of vapour-density due to the presence of the discharge was taken into account. Once the drift speed of the ions is known, the wave propagation data can be easily corrected for the Doppler shift effect (having in mind that $m_e \ll m_i$) in the following way :

$$\sigma \rightarrow \sigma + h u_i / a_0$$

$$c \rightarrow c + u_i / a_0$$

After the introduction of this correction, the experimental points practically fall on the theoretical dispersion curve. In fig. 42 the experimental data obtained at $p = 0.50$ mTorr $I = 7.0$ A, corrected for the Doppler effect, is shown together with the theoretical dispersion curve of the $(0,1)$ mode, corresponding to the collisionless low-pressure limit ($\delta = 1$). The actual regime is not quite collisionless, therefore the appropriate dispersion relation corresponding to the actual value of δ ($\delta = 1.6$) should have been used, but this is not available. Nevertheless, the influence of collisions upon the dispersion relation is very small indeed for such a small δ (see Davies' thesis).

7.1.5. One could expect other wave modes to be present in the positive column besides the $(0,1)$ one. In fact, the $(0,0)$ and the $(1,1)$ modes are less attenuated and the $(1,2)$ mode is about as much attenuated as the $(0,1)$ one. Moreover, around $\sigma = 8$ all the mentioned modes have very close wave numbers, therefore they can be hardly distinguished by axial wave correlation measurements in a narrow frequency range around $\sigma = 8$. Outside this range one ought to observe beats resulting from the superposition of distinct modes. However, these have not been observed although our measurements have extended far away from $\sigma = 8$, and over distances long enough to observe the beats if they were present.

An argument was advanced by Bertotti et al.¹¹² which suggests that the principal wave mode can be eliminated by the presence of a substantial longitudinal drift, as it is the case of a low-pressure arc. It should also be noticed that the principal mode of propagation along a positive column, although often observed as an externally excited wave, seems to be uncommon as a spontaneous wave.

The observed radial structure of the wave (see figs. 17 to 20) is rather different from the theoretical structure of the $(0,1)$

mode. In particular, the wave was found to be slightly axi-asymmetric. This fact was not due to a geometric error because the maxima of amplitude or of phase do not appear consistently on any particular side of the axis. It would therefore be apparent that an azimuthal mode should be present. However, for $\sigma > 8$ the amplitude maxima of the two lower azimuthal modes lie close to the wall, while the observed maximum lies near the axis. Another important difference between theory and experiment concerns the amplitude on the axis. The theoretical $(0;1)$ mode has an amplitude minimum on the axis for $\sigma > 8$ which vanishes for $\sigma > 12$, while the observed structure exhibits a maximum close to the axis and still shows a very high amplitude on it. This difference would suggest the presence of the $(0,0)$ mode (since all the azimuthal modes have an amplitude node on the axis). However, the combination of the $(0,0)$ and $(0,1)$ modes should exhibit, for $\sigma > 12$, a rather broad constant-phase plateau on the center, which has not been observed.

One is accordingly led to rule out the possibility of the presence of modes other than the $(0,1)$ one. The remarkable disagreement between theory and experiment over the radial wave structure remains an open question.

7.1.6. The ion-acoustic wave level was found to be very nearly uniform along the positive column. At the same time, the wave phase correlation is lost with the axial distance rather quickly. One can think that either the wave loses its phase coherence while keeping its amplitude, or the wave is attenuated while being continuously generated throughout the column.

In order to distinguish among these different phenomena, we shall start by comparing their magnitudes. We shall consider in particular a discharge at around $p = 0.50$ mTorr vapour-pressure. (i) The

amplification-rate associated with the two stream instability is estimated to be about

$$-\beta_i/\beta_R = 0.16$$

(ii) The attenuation-rate due to the continuous generation and loss of plasma and to the collision of charged particles with the neutral gas is, in the light of Davies' theory, of the order of

$$\beta_i/\beta_R = 0.20$$

(iii) Finally, the observed loss of phase correlation yields

$$\beta_i/\beta_R = 0.20$$

From points (i) and (ii) one expects a net attenuation-rate given by

$$\beta_i/\beta_R = 0.04 \quad . \text{ Comparing this figure with the observed figure (iii)}$$

one is led to conclude that the wave attenuation is not mainly due to its amplitude attenuation but rather to loss of its phase coherence.

This fact can be understood if one assumes the waves to be generated

by a continuous distribution of correlated sources along the plasma

column. According to this model, one ^{might} expect a substantial phase mixing to take place, leading to the observed loss of phase coherence. As

the positive column is found to be rather uniform, one should admit

that the wave generation process is not only continuous but also uniform

throughout the column. The existence of uncorrelated noise sources

distributed in the positive column had already been suggested by

Little and Jones ¹³³ although by a different argument.

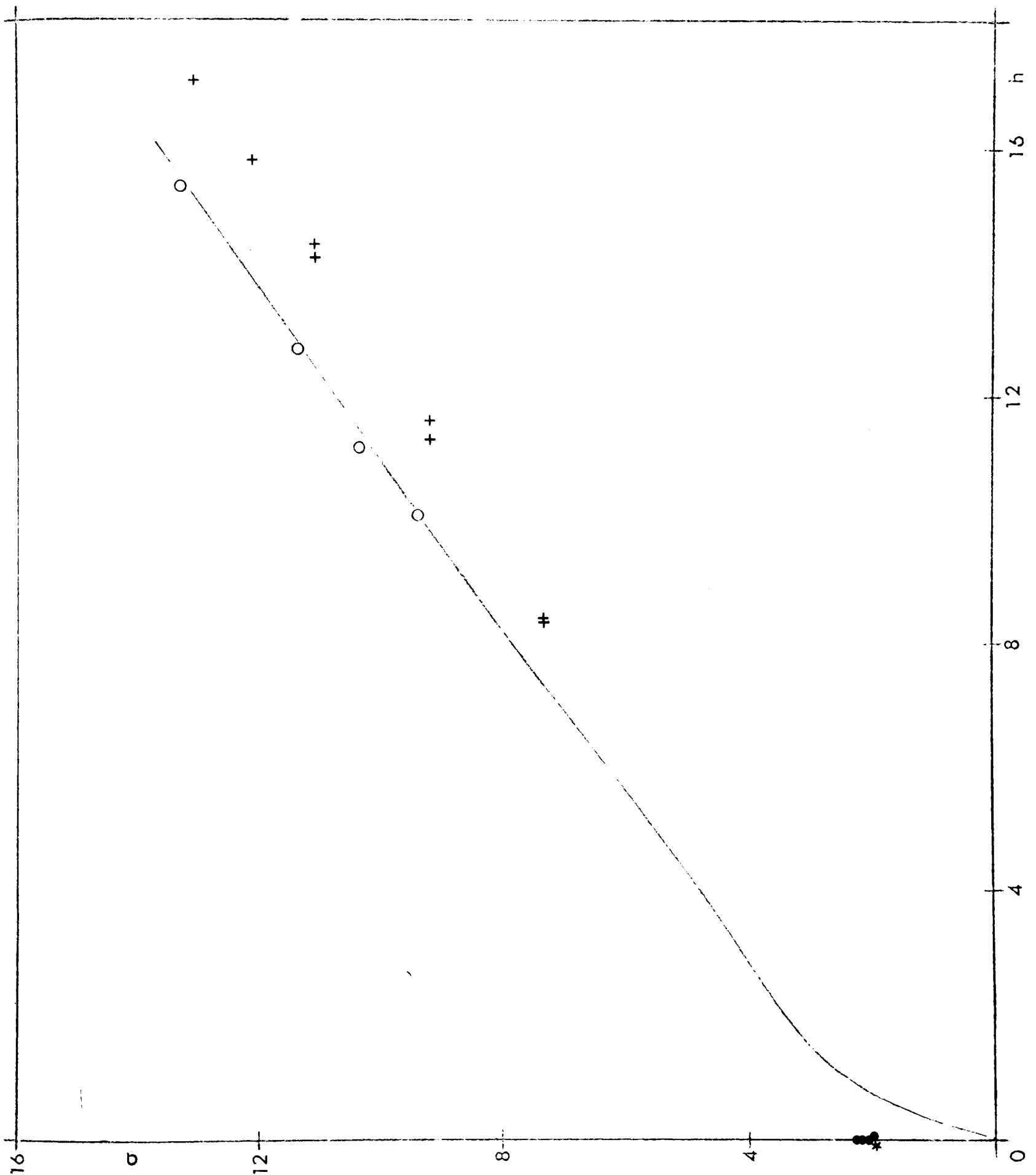


Fig. 42 . Dispersion relation of the (0,1) mode of the propagating ion-acoustic wave according to Woods ³⁷ . Experimental points obtained with spontaneous waves (o) and with externally excited waves (+), at $p = 0.50$ mTorr vapour pressure, both corrected for the Doppler shift .

Non-linear uniform oscillations : experimental data obtained at $p = 0.50$ mTorr (●) and the theoretical estimate (*) .

7.2. THE EXTERNALLY EXCITED ION-ACOUSTIC WAVES

The results obtained with the externally excited waves are summarized in table 7. One immediately sees that the two sorts of exciters, the grid and the coil, produce very similar results in spite of the difference between the excitation mechanisms. However, it is apparent that the coil is a more efficient exciter at low frequency, while the grid is more efficient at high frequency.

It should be recalled that the externally excited waves can be successfully observed in a frequency range around the ion-acoustic wave band which would be spontaneously excited by raising the current above a certain critical value. This result is to be expected since both waves occur in the same mode and therefore should exhibit minimum attenuation at the same frequency.

Another feature of the experimental results is that the frequency band over which the waves can be excited is much more sharply defined on the low frequency side than on the high frequency side. At high frequency, the excited wave shows a gradually decreasing amplitude with increasing frequency. At low frequency, however, the wave shows a very irregular wave-form or seems to disappear all together below 30 kHz. This fact is probably related to the general behaviour of the theoretical attenuation-rate of the $(0,1)$ mode (see fig. 33) which grows sharply at low frequency while it increases slowly at high frequency.

The externally excited waves consistently show a phase velocity substantially smaller than the theoretical one, and even smaller than the phase velocity of the corresponding spontaneous waves. This result cannot be entirely explained by the Doppler shift effect and it remains to be explained. It can possibly be related to the increase of the wave-number far from the wave source as observed by

Little ³² in analogous experimental conditions. Nevertheless, in the present case the effect (if it is the same) is operating nearer to the source, certainly within 5 cm of it.

According to a few authors (²² , ¹³⁴) who did similar experiments in low-pressure mercury-vapour arcs, a long wave-length backward wave may be present. In our experiments we cannot completely rule out the presence of such waves because we have not carried continuous phase correlation measurements over more than 20 cm at one time. But we can say, if present, such waves would have comparatively small amplitude. Moreover, it is known that, in the present physical conditions, backward waves are heavily damped (²¹ and ²²). However , they could account for the observed small and slow amplitude variation of the uniform oscillations which are normally detected together with the ion-acoustic waves.

A remarkable aspect of the externally excited ion-acoustic waves is that both the phase velocity and the attenuation-rate are substantially constant over long distances. Changes of the wave propagation characteristics, regarding either the attenuation-rate or the phase velocity, have been reported in analogous experimental conditions by other authors (³² , ¹³⁴ and ¹³³). It is also known that at large distance from the source (much farther than the distance travelled by an ion during one wave cycle at its thermal speed) the attenuation is reduced, the amplitude oscillates and the phase becomes nearly stationary (Wong ⁴⁹); but in a plasma of large electron to ion temperature ratio this may only be expected at very large distances (Gould ¹⁶⁰). None of the above mentioned effects is apparently operating in the present experiments.

Another remarkable aspect of our results is the very small attenuation-rate which has been observed. We shall now estimate the

amplification-rate due to the two stream instability and the attenuation-rate due to ionization and boundary loss in order to obtain an estimate of the net attenuation-rate (in the same way as we did in paragraph 7.1.6. for the spontaneous waves). For the discharge regime in which the externally excited waves were studied one obtains a net attenuation-rate of about $\beta_i/\beta_R = 0.05$ near 60 kHz. On the other hand, the observed value, far from the source, is smaller than $\beta_i/\beta_R = 0.01$. One sees that the wave is less attenuated than predicted. The difference between the highly attenuated spontaneous wave and the slightly attenuated externally excited wave can be understood if one recalls that in the first case there is a continuous distribution of correlated sources which brings about a destructive phase mixing, while in the second case there is one single source.

It is known that non-linear effects can influence the propagating characteristics of ion-acoustic waves (⁴⁸) by producing a modulation of the wave amplitude with distance. In the case of grid-excited waves we have varied the applied voltage from 3.5 to 15 V r.m.s. without noticing any change of the wave character. In the case of coil-excited waves we have employed a magnetic field which is about one order of magnitude smaller than the magnitude of the field employed by other authors. In no case have we obtained any clear evidence of non-linear effects. However, such effects as the change of wave properties with distance (observed by Little ³², Crowford and Kuhler ¹³⁴ and Little and Jones ¹³³) are probably due to non-linear effects.

Experimental data concerning the ion-acoustic waves externally excited in the positive column, at $p = 0.50$ mTorr and $I = 3.0$ Amp, corrected for the Doppler shift, is shown in fig.42 together with the spontaneous wave data. Notice the rather lower phase velocity of the externally excited waves.

7.3. THE SPONTANEOUS UNIFORM OSCILLATIONS

7.3.1. In the study of low-frequency spontaneous waves and oscillations in a low-pressure positive column we have invariably found a continuous background of uniform oscillations. Superimposed on this, large amplitude components would in general appear as well. Among these, there is a particular class of uniform oscillations which is remarkable for a number of features: (i) they are strongly non-linear oscillations with a peculiar waveform, (ii) their frequency lies in the range from 10 to 15 kHz but is little dependent on the discharge parameters and (iii) they tend to be excited, together with the axially propagating ion-acoustic waves, when increasing the anode current.

As far as the continuous background of oscillations is concerned, their nature is not definitely understood, but they can possibly be accounted for as "drift waves" as predicted by the theory of Ewald et al.⁵⁰. According to this theory "drift waves" are backward waves whose phase velocity is much larger than the ion-acoustic speed. These can propagate along a plasma column, in a fundamental mode which does not exhibit either phase or amplitude radial dependence, at any frequency. The fundamental mode of such a wave was successfully excited and observed by Self et al.⁵¹ in a low-pressure discharge in mercury-vapour. These authors found that the wave can be externally excited at all pressures and propagates without damping with a very high phase velocity. Their amplitude was found to be dependent on the external impedance of the discharge circuit. In our experimental conditions an instability mechanism, with a broad low-frequency spectrum and operating in the cathode region, could excite "drift waves" which, propagating along the positive column, would appear as a continuous spectrum of oscillations

of the column as a whole. The observed dependence of the level of the spectral background on the discharge circuit impedance would fit in this picture.

The "conflicting populations" type of oscillation (Roth 59) could possibly account for the very low frequency (3 - 5 kHz) large amplitude components which are sometimes detected at the higher side of the explored pressure range (from 1 to 3 mTorr). Roth's oscillatory mechanism predicts frequencies which fall in a rather low frequency range (1 - 5 kHz) for the discharge regimes covered in our experimental work. It also predicts a substantial variation of the frequency with both the neutral and electron densities. This dependence was not found, but a more extensive survey at the high pressure side would be required before a conclusive answer could be reached.

3.2. Regarding the large amplitude non-linear oscillations which appear in the 10 - 15 kHz frequency range, we wish to start by drawing the attention to some of their most meaningful features. The frequency and wave form of the oscillation are little sensitive to substantial changes in the conditions of the cathode region or in the external discharge impedance. These facts do suggest that such oscillations correspond to oscillatory modes of the plasma column itself and that the source of instability must be found there as well. However, the amplitude of the oscillation depends on the external discharge impedance, which means that the discharge current must play some role in the oscillatory mechanism. Another significant feature of this class of oscillations is that they invariably appear, when increasing the anode current, at the same time as the axially propagating ion-acoustic waves. This fact suggests a common origin for these two, otherwise quite distinct, phenomena.

It is well known that the two-stream instability mechanism

excites all the ion-acoustic waves with wave vectors contained within a cone whose axis is directed along the electron drift and the angle of whose apex is given by the Cherenkov angle $\cos^{-1}(a_0/u)$ (where a_0 is the ion-acoustic speed and u is the relative electron-ion drift) (see Gordeyev ⁴³). As in a low-pressure arc discharge one has $u \gg a_0$, one expects that the waves whose wave vectors lie at the edge of the cone of excitation will practically be radial waves. A purely radial ion-acoustic oscillation of the plasma column, as discussed in paragraph 5.4., can accordingly be excited. It was noticed at that time that this oscillatory mode is subjected to a substantial damping, e-fold in 3.48 periods. Moreover, this oscillation will be subjected to further damping due to the destructive interference of the radial waves generated by the uncorrelated sources distributed along the column. But as the generation is continuous, one can admit that the radial ion-acoustic oscillation of the plasma column sets in. Once this oscillation occurs, it will drive the whole boundary into an in-phase oscillation which will in turn produce a modulation of the boundary loss mechanisms. This effect will lead to a modulation of the energy mechanisms and will consequently drive the electron temperature into oscillation. At this point, an in-phase radial oscillation of the plasma column, as discussed in paragraph 6.2., may be expected to appear.

We shall now choose reasonable values for the atomic parameters **A, B and C (which are defined in paragraphs 6.2.1. and 6.2.2.)** in the actual physical conditions of a mercury-vapour low-pressure arc. We chose the values $A = 5$ (see paragraph 4.2.1.), $B = 3$ (see paragraph 4.2.5.) and $C = 4$. From here one can now predict whether, in the light of the theory developed in paragraph 6.2., in-phase radial oscillations of the positive column can be expected, and which frequency and amplitude may they have. Without going into

the exact solution of this problem one can however conclude that, for the given set of atomic parameters, nearly uniform oscillations can indeed be predicted whose frequency and damping are of the order of

$\sigma \approx \pm 2$ $\gamma \approx -2$. The oscillatory mode falls in the instability region and this gives rise to an anti-damping increment of the order of $\delta\gamma \approx 10 T_1/T_0$. This means that the oscillation would become unstable at an amplitude of about $T_1/T_0 \approx 0.20$. However, the oscillation has been observed at levels down to $T_1/T_0 \approx 0.01$. Therefore, if the oscillatory mechanism is the same one, other non-linear effects ought to be operating at very small amplitude, well below $T_1/T_0 \approx 0.01$, and ought to enhance substantially the instability mechanism. In this circumstance, the oscillation is expected to be strongly non-linear even at the lowest level at which they have been detected.

The observed uniform oscillation is a strongly non-linear one and its frequency lies in the interval $1.6 < \sigma < 2.6$. These two facts fit the theoretical expectations in an acceptable way.

Attention should be drawn to the uncertainty about the atomic parameters upon which all the theoretical predictions are based. Although the values of the atomic parameters can easily be estimated from equilibrium data or for equilibrium conditions, [as it was done here , the problem is more involved as far as the actual values in the presence of oscillations are concerned. This situation is due to the important role that the metastable states play in the ionization and momentum and energy relaxation processes. In a change between equilibrium states the population of metastables changes very substantially. But, in a fast change in the course of an oscillation (whose period is smaller or comparable with the relaxation time of the metastable states density) the population of metastables has not enough time to change much. According to this argument, the quantities A and B , in the

presence of oscillations, are expected to be smaller than at equilibrium.

7.4. FINAL REMARKS

In view of the evidence concerning the time variation of the electron temperature in association with the large amplitude non-linear uniform oscillations of a plasma column, and allowing for the possibility of a continuum of electron temperature fluctuations, it would be convenient to proceed to the time-resolution of the electron temperature in the positive column. As the variation of the electron temperature leads to the variation of the floating potential as well, it would be advisable, as a technical detail, to employ screened Langmuir probes in spite of the fact that one is dealing with low-frequency phenomena. In this way it would be possible (i) to check the actual role of the electron temperature in the mechanism of the uniform oscillations and (ii) to establish whether the variation of the electron temperature is influencing the propagating ion-acoustic waves in any way.

It would be interesting to investigate theoretically how the variation of the electron temperature can affect the radial structure and propagating properties of the axial ion-acoustic waves. In this way one might account for a few discrepancies found between the observed wave properties and the predictions of Woods' ³⁷ and Davies' ³⁸ theory. Another question worthy of theoretical investigation is the influence of the variation of the electron temperature upon the two-stream instability mechanism itself.

APPENDIXESA.1. ELECTRON IMPACT EXCITATION AND IONIZATION OF MERCURY
ATOMS

Data on the excitation and ionization of mercury atoms by electronic impact is given in table 8 . It should be noticed that the actual magnitude of many of these cross-sections are uncertain. The cross-sections listed here are collected from a number of sources: Kenty ¹⁷⁴ , Massey and Burhop ¹⁷⁵ , Cayless ⁸⁶ and Vriens ¹⁷⁶ .

The rate-constants (the average of the product of the cross-section by the electron speed over the velocity space) of the direct reactions were computed by Cayless on the assumption of a maxwellian distribution of 2.5 eV temperature. The rate-constants of the reverse reactions were estimated from the direct reaction-rates with the help of the "detailed balance principle". This principle states that the direct and reverse transitions to and from any state or between two any states by any particular mechanism are equal at thermodynamic equilibrium.

The three-body recombination is the reverse reaction of ionization by electron impact. Its rate-constant can be estimated with the help of the principle of detailed balance and Saha's equation (¹⁷⁷), but it turns out to be negligibly small in the conditions of a low-pressure gas discharge.

A recent survey of the electron impact excitation of atoms is due to Moiseiwitsch and Smith ¹⁷⁸ .

A.2. OPTICAL TRANSITIONS

The optical spectrum of mercury is of the two-electrons type.

The orbital momenta of the outermost electrons combine into

a resultant L which determines the term character (S, P, D, F). On the other hand, the spin momenta of those electrons also combine into a resultant S . The multiplicity of a term is defined as the number of all possible combinations of L with S and it is given by $2S + 1$. In a two-electrons atom there are singlet ($S = 0$) and triplet ($S = 1$) terms. In the majority of atoms electron coupling is in general of the LS coupling type and the total atomic momenta J is the sum of L and S .

There are three selection rules for optical transitions:

(i) the change in L may only be $+1$ or -1 , (ii) the change in J may only be $+1$, -1 or 0 , but the transition $0 \rightarrow 0$ is excluded, (iii) the intercombination of two different **term systems** is forbidden.

This last rule means that the reversal of an electron spin is improbable. This property is also reflected in the fact that the cross-section for electron impact excitation from a singlet (like the ground state) to a triplet state is much sharper than the cross-section of excitation to another singlet state.

It should be noticed that a perturbation of the atomic structure can make an improbable transition to become less improbable. In the case of the mercury atom, the disturbance of the two external electrons due to the presence of the atomic core is such that some singlet-triplet transitions occur with rather high probability. That is the case of the following lines:

Transition	Wave-length ($\text{\AA}$)
$^1P_1 - ^3D_{1,2}$	5790 , 5770
$^1P_1 - ^3D_{1,2}$	4344 , 4339
$^3P_1 - ^1S_0$	4078
$^1S_0 - ^3P_1$	2536

The line intensity in multiplets obeys the following rules:

(i) the sum of the intensities of all the lines in a multiplet which start from a common level is proportional to the quantum weight $(2J + 1)$ of that level and (ii) the sum of the intensities of all the lines in a multiplet which end on a common level is proportional to the quantum weight of this level.

The line spectrum of a plasma depends very much on the plasma parameters. A very rich line spectrum of a "vacuum arc" in mercury vapour is shown in fig.43 (Harnwell and Livingood ¹⁷⁹). A number of mercury lines were observed in a moderate-pressure electric discharge by Kenty ¹⁷⁴ .

Recombination by radiative capture is a process with a rather small cross-section. The cross-section varies roughly as the reciprocal of the kinetic energy of the impinging electron. The probability of capture to the ground state is about $1/3$ only, so that a rich line spectrum appears superimposed on the continuum due the radiative capture process itself.

At 1 eV electron kinetic energy the cross-section is theoretically expected to be of the order of 10^{-20} cm^2 (Massey and Burhop ¹⁷⁵). In the case of mercury ions experimental evidence shows that the rate-constant of this process at 2000 °K is about $2.3 \times 10^{-10} \text{ cm}^3/\text{s}$ (Mohler ¹⁸⁰). Such a process is negligible in gas discharges.

Transport of resonance radiation is an important process in gas discharges. This process was first theoretically investigated by Milne ¹⁸⁰ in plane geometry. One can arrive at similar predictions about the transport of resonance radiation with the help of a simple model of photon diffusion as in the kinetic theory of gases. In cylindrical geometry one

obtains : $t = 3\pi \tau q^2 n^2 R^2 / 8$ where τ is the mean life-time for spontaneous emission, q is the absorption cross-section, n is the density of normal atoms and R is the radius of the cylinder . For data on the life-times and cross-sections of resonant atoms and resonance radiation see Mitchel and Zemansky ¹⁸² .

Milne's formula proves acceptable provided that the optical thickness , qnR , is low enough. When the number of absorption and re-emission events is large, however, and when the line shape is determined by Doppler or collision broadning, the resonance radiation behaves as if it were incoherently scattered, that is to say, its spectral distribution is the same as the frequency distribution of the absorption cross-section. In this case the transport process cannot be described on the basis of the average absorption coefficient and the photon mean free path, and the appropriate equation of transport of the resonance radiation has to be employed instead (Holstein ¹⁹⁵ and Biberman ¹⁹⁶) . This theory was shown to agree well with laboratory measurements of the decay constants for the mercury 2537 Å resonance line in mercury vapour at 340°K by Phelps and McCoubrey ¹⁹⁷ .

A.3. THE METASTABLE STATES OF MERCURY

Metastable states are excited states whose optical transition, either direct or indirect, to the ground state is improbable. While resonant states spontaneously decay in, say, 10^{-8} s , metastable states can stay there for much longer times, say, up to 1 s . They normally decay through mechanisms other than optical transition. Only at extreme conditions of low density is it possible to observe the optical decay of metastables. While resonant levels can only exist in small concentrations, metastable levels can exist in larger concentrations because they have longer life-times. They play an important role in the physics of weakly

ionized gases because they have low ionization potential, carry considerable amounts of energy and are not influenced by electro-magnetic fields.

Among the excited levels of mercury, the most important metastable levels are the 6^3P_0 and 6^3P_2 . The 6^3D_3 level was recently observed (¹⁸³) and its life-time for spontaneous decay was shown to be longer than 10^{-2} s.

Metastables can be de-excited by collision with ground state atoms. Biondi ¹⁸⁴ gives the value $Q = 8 \times 10^{-17} \text{ cm}^2$ for the de-excitation cross-section of the 6^3P_2 level in mercury-vapour at 350°K .

Metastables can also be de-excited by collision with a wall. Coulliette ¹⁸⁵ gives the figure of 100 % of probability of de-excitation of the 6^3P_0 level on a glass wall.

Collisions between metastables and ground state atoms can also lead to a transition to another excited state with either loss or gain of kinetic energy. The transition $6^3P_1 \rightarrow 6^3P_0$ was studied by Coulliette ¹⁸⁵ who gave the cross-section $Q = 5 \times 10^{-18} \text{ cm}^2$ in mercury-vapour at 338°K .

The collision of a metastable with a ground state atom will more probably lead to excitation transfer. Such a process has a cross-section which is given by:

$$Q = 2\pi^3\mu^2/hv \quad (\text{cm}^2)$$

where μ is the dipole moment of the optical transition (which is of the order of 10^{-18} e.s.u.), h is Planck's constant and v is the relative velocity (see Massey and Burhop ¹⁷⁵). The cross-section for diffusion of metastables is the sum of the cross-sections due to direct scattering and to excitation transfer. The last one is the largest of the two. The cross-section for diffusion of the 6^3P_0 and 6^3P_2 levels were studied by Coulliette ¹⁸⁵ and Biondi ¹⁸⁴ who give the figures $Q = 4.8 \times 10^{-15}$ and $Q = 6.0 \times 10^{-15} \text{ cm}^2$, respectively, in mercury-

-vapour at 338°K .

As already pointed out earlier, metastable states play an important role in weakly ionized gases. Fabrikant et al ¹⁸⁶ measured the concentration of the 6^3P levels in a low-pressure arc in mercury-vapour. They found that they represent a few per cent of the global neutral population and that their concentration increases with the vapour-pressure.

A.4. CHARGE TRANSFER , IONIC MOBILITY AND DIFFUSION

The motion of an ion in its parent gas will lead to symmetrical resonance charge transfer and to elastic collisions. The cross-section of the former process, which is in general larger than the cross-section of the latter one, falls with increasing impact velocity v according to the equation:

$$Q^{1/2} = a - b \ln v$$

with a , b constants (Sena ¹⁸⁷). Information on the charge transfer cross-section for mercury ions in mercury vapour can be found in the works of Palyukh and Sena ¹⁸⁸ (for large kinetic energy) and of Kovar ¹⁸⁹ (for low kinetic energy). According to this last author,

$$Q = \frac{558}{445} \times 10^{-16} \text{ cm}^2 \quad \text{at} \quad \frac{X/p_0}{1000} = 200 \text{ V/cm x Torr}$$

where $p_0 = 273 \text{ p/T}$ is the reduced vapour pressure and X is the applied electric field. Dalgarno ¹⁹⁰ offers the following theoretical figures:

$$Q = \frac{400}{356} \times 10^{-16} \text{ cm}^2 \quad \text{at} \quad \frac{mv^2/2}{10} = 1 \text{ eV}$$

The mobility of ions in their own gas can be interpreted in

the light of two different theories. Langevin's theory ¹⁹¹ is based in a model in which ions and atoms are in thermodynamic equilibrium, neglects drift as well as velocity and free paths distributions. It is applicable when the energy gain between collisions is smaller than the thermal energy, and yields the drift speed proportional to the applied field and the reciprocal of the collision cross-section. Wannier's theory ¹⁹² applies when the energy gain between two collisions is larger than the thermal energy or, in practical terms, $X/p_0 > 2 \text{ V/cm} \times \text{Torr}$. This theoretical model assumes an active charge exchange leading to isotropic scattering and constant cross-section. It yields the following drift speed:

$$u = 1.147 (eX/mQn)^{1/2}$$

The diffusion of a swarm of particles in chaotic motion follows the statistical law

$$n = n_0 \exp(-x^2/4Dt) / (4\pi Dt)^{1/2}$$

(see Loeb ⁹⁷). The first and second moments of x are:

(i) one-dimensional case

$$\bar{x} = (4Dt/\pi)^{1/2} \quad \overline{x^2} = 2Dt$$

(ii) two-dimensional case

$$\bar{x} = (8Dt/\pi)^{1/2} \quad \overline{x^2} = 4Dt$$

(iii) three-dimensional case

$$\bar{x} = (12Dt/\pi)^{1/2} \quad \overline{x^2} = 6Dt$$

The diffusion coefficient and the mobility are related to each other. When the distribution function of the charged particles is maxwellian then

$$D/\mu = kT/e$$

which is known as Einstein's relation.

As far as mercury ions in their own gas are concerned, Biondi¹⁸⁴ gives the figure $D = 1.8 \times 10^{17}/n \text{ cm}^2/\text{s}$ at 350°K and Kovar¹⁸⁹ gives the figure $\mu = 0.24 \text{ cm}^2/\text{V} \times \text{s}$ at zero field and about 500°K . Dalgarno¹⁹⁰ calculated the mobility of the mercury ions at zero field and its dependence with the vapour temperature:

$T = 200^\circ\text{K}$	$\mu = 0.26 \text{ cm}^2/\text{V} \times \text{s}$
300	0.23
400	0.21
500	0.19

In a typical low-pressure arc in mercury-vapour one finds that the mean free path for charge exchange is comparable with the column diameter. If one assumes a free-fall regime, the axial ion drift is the average axial speed of an ion during its life-time, and one obtains:

$$u = 0.272 eRX/ma_0$$

where a_0 is the ion-acoustic speed and R is the column radius. If one assumes a collision-dominated regime instead, then Wannier's theory applies. In typical conditions one has, say, $p = 0.5 \text{ mTorr}$, $X = 0.10 \text{ V/cm}$, $R = 3 \text{ cm}$, $a_0 = 1 \times 10^5 \text{ cm/s}$ and one can estimate:

(i) in free-fall regime $u \approx 2.5 \times 10^3 \text{ cm/s}$

(ii) in diffusion regime $u \approx 2.0 \times 10^4 \text{ cm/s}$

It is now apparent that the actual ion drift in the transition regime of an arc discharge cannot be estimated by extrapolation from the free-fall or the diffusion regimes, and that the question has to be attacked from a point of view more suitable for the present situation.

A.5. DIATOMIC MERCURY

Diatomic mercury molecules and ions do exist (see Herzberg

¹⁹³). The metastable level ^3O_u of the diatomic molecular mercury can be produced in a three-body process involving two ground state atoms and one 6^3P_0 metastable (McCoubrey ¹⁹⁴). This molecule, although metastable, is subjected to an internal perturbation by which the molecule is allowed to radiate spontaneously at $3350 \text{ \AA}$. Its diffusion coefficient in mercury-vapour is $D = 0.68 \times 10^{18}/n \text{ cm}^2/\text{s}$ at 473°K . The probability of its dissociation by collision with a glass wall is 0.11 only.

The diatomic mercury ion can be produced by associative ionization of one 6^3P_0 and one 6^3P_1 metastable (Tan ⁸⁴). The mean cross-section for this process is $\langle Q \rangle = 650 \times 10^{-16} \text{ cm}^2$ at around 400°K . The cross-section for charge exchange of Hg_2^+ in mercury-vapour is:

$$Q = \frac{137}{118} \times 10^{-16} \quad \text{at} \quad \frac{X/p_0}{1000} = 200 \text{ V/cm} \times \text{Torr}$$

and its mobility at zero field is $\mu = 0.45 \text{ cm}^2/\text{V} \times \text{s}$ at about 500°K (Kovar ¹⁸⁹).

TABLE 8

Electron-impact excitation and ionization of mercury atoms

Transition	Threshold (eV)	$V_{\max}$ (eV)	$Q_{\max}$ (10^{-16} cm^2)	Z_{dir} ($10^{-9} \text{ cm}^3/\text{s}$)	Z_{rev}
$6^1S_0 - 6^3P_0$	4.66	5.5	0.58	0.7	5
- 6^3P_1	4.86	5.6	1.74	3	7
- 6^3P_2	5.43	6.8	3.2	6	11
- 6^1P_1	6.7	17.5	7.4	6	30
- 7^3S_1	7.7	9.0	0.45	0.4	3
- 7^1S_0	7.9				
- 6^3D	8.8			0.3	2.2
- Hg^+	10.4	34	5.2	0.7	
$6^3P_0 - 6^3P_1$	0.20		15		
- 7^3S_1	3.04	13	3.9	15	17
- 6^3D	4.2			10	11
- Hg^+	5.7	48	9.3	5	
$6^3P_1 - 6^3P_2$	0.57	1.0	19.3	150	70
- 7^3S_1	2.83	12	0.35	15	46
- 7^1S_0	3.0	25	0.07		
- 6^3D	4.0			10	33
- Hg^+	5.5			5	
$6^3P_2 - 7^3S_1$	2.26	12	0.28	15	
- 6^3D	3.4			10	
- Hg^+	5.0	48	9.5	5	

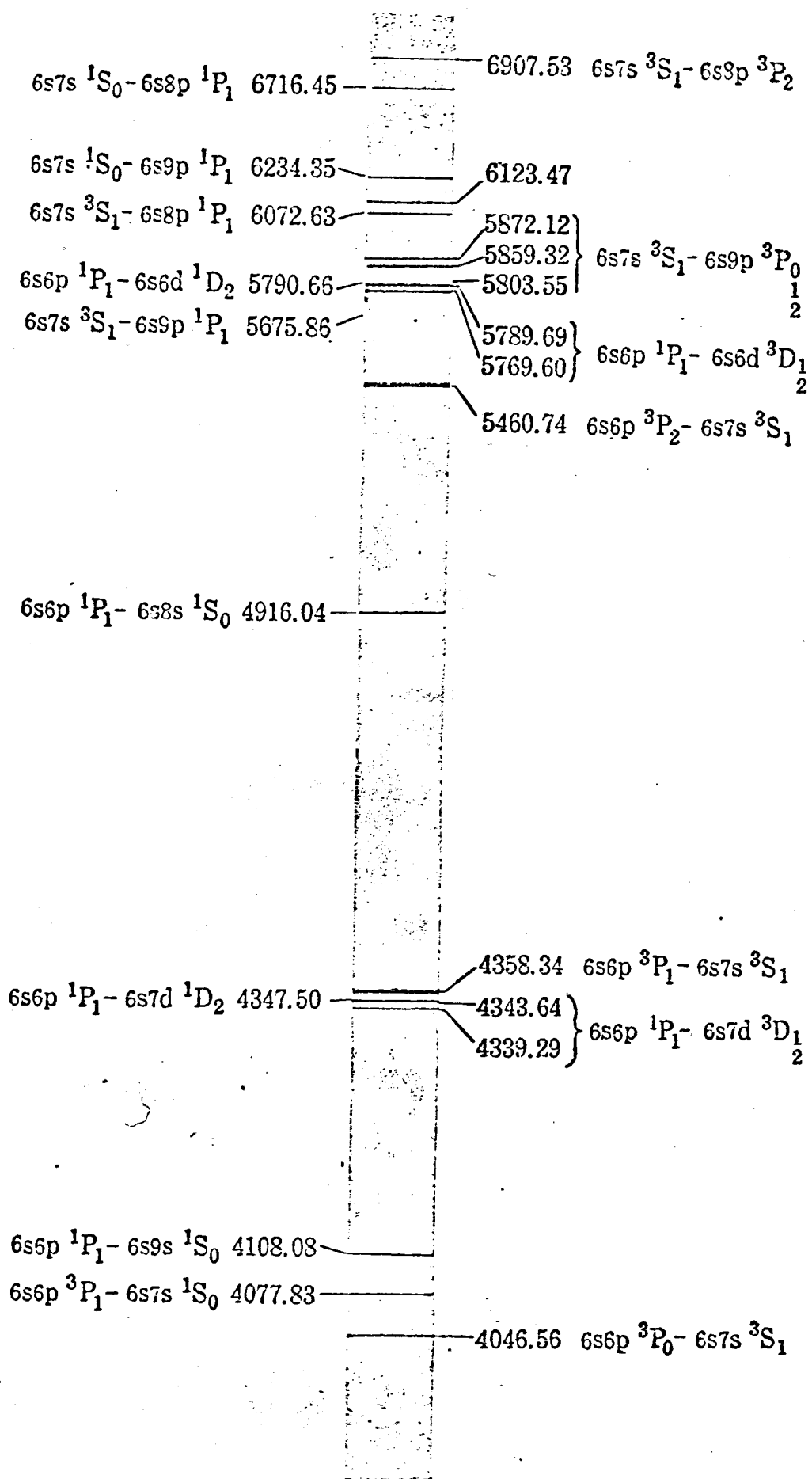


Fig. 43. Emission spectrum of a "vacuum arc" in mercury-vapour
(Harnwell and Livingood 179) .

A.6. COMPUTING PROGRAMME

Radial In-Phase Pulsation of a Plasma Column

```
FAEDVRT00APU;
SAVE;TL 99999;O/PL;->
```

```
begin library A6,A30;
      real FMIN,E,h1,mx1,alpha,A,delta,x0;
      integer f1,f2,j,n;
      array X0,VO,XMIN[1:2],y0[1:6];
```

```
real procedure func(a); value a; array a;
begin real x1; array y1,e1[1:6];
```

```
      procedure gf(q,r,s); value q; real q; array r,s;
begin
s[n-5]:=(alpha+alpha*xr[n-5]^2-r[n-5]/q)/(1-r[n-5]^2) ;
s[n-4]:=-r[n-4]*xr[n-5]*(2*alpha-r[n-5]/q)/(1-r[n-5]^2) ;

s[n-3]:=((2*xr[n-5]*(2*alpha-r[n-5]/q)/(1-r[n-5]^2)-1/q+a[2]*xr[n-5])
xr[n-3]-a[1]*xr[n-5]*xr[n-2]+(a[1]*xr[n]-a[2]*xr[n-1])/r[n-4]
+0.50*delta*(alpha*2*Ax(1+r[n-5]^2)+2*xr[n-5]*x
(-2*alpha*xr[n-5]+r[n-5]^2/q)/(1-r[n-5]^2)))/(1-r[n-5]^2) ;
s[n-2]:=((2*xr[n-5]*(2*alpha-r[n-5]/q)/(1-r[n-5]^2)-1/q
+a[2]*xr[n-5])*xr[n-2]+a[1]*xr[n-5]*xr[n-3]-
(a[1]*xr[n-1]+a[2]*xr[n])/r[n-4])/(1-r[n-5]^2) ;

s[n-1]:=(-r[n-5]*(2*alpha-r[n-5]/q-a[2])*xr[n-1]-a[1]*xr[n-5]*xr[n]
-2*xr[n-4]*(alpha+alpha*xr[n-5]^2-r[n-5]/q)*xr[n-3]/(1-r[n-5]^2)
+a[1]*xr[n-4]*xr[n-2]-a[2]*xr[n-4]*xr[n-3]-delta*xr[n-4]*x
(alpha*2*Axr[n-5]+(-2*alpha*xr[n-5]+r[n-5]^2/q)/(1-r[n-5]^2)))
/(1-r[n-5]^2) ;
s[n]:=(-r[n-5]*(2*alpha-r[n-5]/q-a[2])*xr[n]+a[1]*xr[n-5]*xr[n-1]
-2*xr[n-4]*(alpha+alpha*xr[n-5]^2-r[n-5]/q)*xr[n-2]/(1-r[n-5]^2)
-a[1]*xr[n-4]*xr[n-3]-a[2]*xr[n-4]*xr[n-2])/(1-r[n-5]^2)
end;
```

```
procedure KUTMER(N,h,x,y) differential:(dy) error:(e);
value N,h; real x,h; integer N; array y,e; procedure dy;
begin integer i; array yo,dya,dyb,dyc[1:N];
dy(x,y,dya);
for i:=1 step 1 until N do begin yo[i]:=y[i];
y[i]:=yo[i]+hxdya[i]/3.0 end; x:=x+h/3.0; dy(x,y,dyb);
for i:=1 step 1 until N do y[i]:=yo[i]+hxdya[i]/6.0+hxdyb[i]/6.0;
dy(x,y,dyb);
for i:=1 step 1 until N do y[i]:=yo[i]+0.125*hxdya[i]+0.375*hxdyb[i];
x:=x+h/6.0; dy(x,y,dyc);
for i:=1 step 1 until N do y[i]:=yo[i]+0.5*hxdya[i]-1.5*hxdyb[i]
+2*hxdyc[i]; x:=x+0.5*h; dy(x,y,dyb);
for i:=1 step 1 until N do begin e[i]:=y[i]*0.2;
y[i]:=yo[i]+hxdya[i]/6.0+2*hxdyc[i]/3.0+hxdyb[i]/6.0;
e[i]:=e[i]-0.2*y[i] end;
end;
```

```

for j:=1 step 1 until n do y1[j]:=y0[j]; x1:=x0;
for j:=1 step 1 until 2 do write(30,f2,a[j]);

L1:
if x1>mx1 then goto L2;
  KUTMER(n,h1,x1,y1,gf,e1);
  write(30,f1,x1);
  for j:=1 step 1 until n do write(30,f1,y1[j]);
  for j:=1 step 1 until n do
    if j≠n then write(30,f1,e1[j]) else write(30,f2,e1[j]);
  goto L1;

L2:
func:=(y1[3]-delta)x(y1[3]-delta)+y1[4]xy1[4];
write(30,f2,(y1[3]-delta)x(y1[3]-delta)+y1[4]xy1[4])
end;

```

```

procedure minwogr2(f,n,xo)result:(fmin,xmin)initial step:(vo)
relative precision:(eps)monitor:(dev)timelimit in seconds:(TL);
value n,eps,dev,TL;
real fmin,eps,TL; integer n,dev; array xo,xmin,vo;
real procedure f;

```

```

begin
  array x1[1:n,1:n],a,b,v,ao[1:n]; real max,fao,fa,fb,TO;
  integer i,j,imax;

```

```

real procedure time;
KDF9 2/0/0/0;
SET3; OUT; SET23; FLOAT; EXIT;
ALGOL;

```

```

procedure minoline(a,v,fa)result:(b,vb,fb); value a,v,fa;
array a,v,b,vb; real fa,fb;

```

```

begin

```

```

real procedure val(x); value x; real x;
begin integer i; array y[1:n];
  if (if evals÷10×10≠evals then false else time>TL)
  then goto runout1;
  evals:=evals+1;
  for i:=1 step 1 until n do y[i]:=a[i]+xxv[i];
  val:=f(y);
end val;

```

```

real d,c1,c2,c3,c4,f1,f2,f3,f4,z;
integer i,evals;

```

```

evals:=0; c1:=0; f1:=fa; d:=1;
for z:=c1+d,c2+d while abs(f2-f1)≤10-10xabs(f2) do
  begin c2:=z; f2:=val(c2) end;
if f2>f1 then
begin z:=c1; c1:=c2; c2:=z; z:=f1; f1:=f2; f2:=z end;
d:=c2-c1;

```

L1:

```

for z:=c2+d,c3+d while abs(f3-f2)<10-10×abs(f3) do
  begin c3:=z; f3:=val(c3) end;
if f2>f3 then
  begin c1:=c2; c2:=c3; f1:=f2; f2:=f3; d:=2×d;
    go to L1
  end; comment Now f1>f2<f3;

```

L2:

```

c4:=c2+
((c1-c2)2×(f3-f2)-(c3-c2)2×(f1-f2))/
((c1-c2)×(f3-f2)-(c3-c2)×(f1-f2))/2;
if abs(c4-c2)<eps×abs(c2) then goto exit;
f4:=val(c4);
if abs(f4-f2)<10-10×abs(f2) then go to exit;

if c3>c1 eqv c4>c2 then
begin if f4>f2 then
  begin c3:=c4; f3:=f4 end
  else begin c1:=c2; c2:=c4; f1:=f2; f2:=f4 end
end else
begin if f4>f2 then
  begin c1:=c4; f1:=f4 end
  else begin c3:=c2; c2:=c4; f3:=f2; f2:=f4 end
end;
go to L2; comment Again f1>f2<f3;

```

runout1:

```

if dev>30 and dev<40 then
  writetext(dev,
  [[c]minwogr2*exceeds*timelimit,*stopped*at*last
  *point*printed[c]]) else writetext(0,
  [Warn*User*that*minwogr2*exceeded*time*limit]);
  goto runout2;

```

exit:

```

for i:=1 step 1 until n do
  if c2≠0 then begin vb[i]:=c2×v[i]; b[i]:=a[i]+vb[i] end
  else begin vb[i]:=v[i]; b[i]:=a[i] end;
  fb:=f2; comment Always fb<fa;

```

```

if dev>30 and dev<40 then
begin integer f1,f2; real a,ab;
  f1:=format([s-nddd.ddddd]);
  f2:=format([s-d.ddddd10-nd]);
  writetext(dev,[[c]Function*=]);
  for i:=0 step 1 until n do
    begin a:=if i=0 then fb else b[i]; ab:=abs(a);
      write(dev,if ab<10-4 and ab>10-1 then f1 else f2,a);
      if i=n or i÷10×10=i then writetext(dev,[[c]])
    end
  end
end

```

end minoline;

```

if TL<0 then TL:=3600; TC:=time; TL:=TL+T0;
if eps<n-10 then eps:=n-10;
for i:=1 step 1 until n do a[i]:=xo[i];
fa:=f(a);
for i:=1 step 1 until n do
begin
  for j:=1 step 1 until n do
    xi[i,j]:=0;
    xi[i,i]:=vo[i]
end;

L1:
  for i:=1 step 1 until n do ao[i]:=a[i];
fao:=fa;
max:=-1;
for i:=1 step 1 until n do
begin
  for j:=1 step 1 until n do v[j]:=xi[i,j];
  minoline(a,v,fa,a,v,fb);
  if fa-fb>max then begin max:=fa-fb; imax:=i end;
  for j:=1 step 1 until n do xi[i,j]:=v[j];
  fa:=fb
end;
for i:=1 step 1 until n do b[i]:=2xa[i]-ao[i];
fb:=f(b);

if if fb>fao then false else
  (fao-2xfa+fb)x(fao-fa-max)12<maxx(fao-fb)12/2
then
begin for j:=1 step 1 until n do
  begin for i:=imax+1 step 1 until n do xi[i-1,j]:=xi[i,j];
    v[j]:=xi[n,j]:=a[j]-ao[j]
  end;
minoline(a,v,fa,a,v,fa)
end;

if abs(fao-fa)>n-10xabs(fao) then
begin
  for i:=1 step 1 until n do
    if abs(ao[i]-a[i])>epsxabs(ao[i]) then goto L1
end;

if dev>30 and dev<40 then
begin writetext(dev,[lc]minwogr2*run*time*in*seconds*=);
write(dev,format([nddddc]),time-T0)
end;

runout2:
  for i:=1 step 1 until n do xmin[i]:=a[i];
  fmin:=fa
end minwogr2;

```

```

open(20);    open(30);
f1:=format([4s+d.dddwb+nd]);  f2:=format([4s+d.dddwb+ndcc]);

alpha:=read(20);  A:=read(20);  n:=read(20);
delta:=read(20);
h1:=read(20);    x0:=read(20);    mx1:=read(20);
E:=read(20);
for j:=1 step 1 until n do y0[j]:=read(20);
for j:=1 step 1 until 2 do x0[j]:=read(20);
for j:=1 step 1 until 2 do v0[j]:=read(20);

close(20);

```

```

minwogr2(func,2,x0,FMIN,XMIN,v0,E,30,3600);

```

```

write(30,f2,FMIN);
for j:=1 step 1 until 2 do write(30,f2,XMIN[j]);
close(30)

```

end→

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