

# Beyond bosons and fermions: discovering anyons, Majoranas and more

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## Key advances

- Interferometric measurements in the quantum Hall regime provides the strongest experimental proof to date of exotic anionic particles.

- Individual states inside superconducting vortices, called Caroli-de Gennes-Matricon states, have been experimentally verified in low-density topological superconductors

- Access to the Caroli-de Gennes-Matricon states provides a new platform in which anyonic particles may be braided and detected in three dimensional topological superconductors

Progress in condensed matter physics can be described with an appropriate term can be borrowed from evolutionary biology: punctuated equilibrium. This term is used to describe abrupt jumps in species evolution that are separated by long periods, known as stasis, having little or no apparent change. In the early 1980s a paradigm shift occurred in condensed matter with the discovery of fractional quantum Hall effect and the near immediate theoretical prediction that such systems could, as an emergent phenomenon, harbor anyons --- particles that are neither bosons nor fermions. Thereafter the field developed slowly on both the experimental and theoretical sides for a long period. Now, after almost 40 years, these developments have finally culminated in two beautiful experiments in the quantum Hall system [1,2]. One measures correlations between particles akin to those of photons in beam splitters while the other employs interferometry to pinpoint the phase obtained by encircling a particle around another—a process known as braiding; together they provide the strongest experimental proof to date that such anyons do exist. The anyons detected in these experiments are of the simplest variety, in that they acquire a fractional phase interpolating between bosons and fermions upon braiding. However, quantum Hall systems hold promise for the realization of more exotic anyons which are sensitive to the order in which they are braided – a property which could enable certain schemes for quantum computation [5]. The search for one of these particles, the Majorana fermion, in the solid state provides us with another example of slow, but persistent, development of a field.

The Majorana fermion, a fermion which is its own antiparticle, proposed in 1937, was for a very long time considered to be mostly irrelevant to condensed matter physics. At the turn of the 21st century it was theoretically realized [3,4] that such particles, could also occur in condensed matter. In 2012 the first experimental signatures of Majorana fermions came to light in the context of topological superconducting nanowire devices [6], which offer an alternative to quantum Hall systems. Yet these signatures have so far failed to develop into definitive confirmation. After many frustrating partial results, it may now appear that the field has settled

47 into another long stasis. However, all the while, the field has been slowly advancing on several  
48 fronts, any one of which may be the key to allowing the field to make its next big jump forward.

49  
50 Here we point to two such particularly promising, interconnected developments. To understand  
51 their significance, we first visit a subtle point in the field of superconductivity that places a  
52 historic concept in modern setting: the distinction between the Bose-Einstein condensate (BEC)  
53 regime and the conventional Bardeen-Cooper-Schrieffer (BCS) superconductivity regime.

54  
55 The BEC regime is characterized by superconducting pairing that is strong enough that the pair-  
56 breaking gap ( $\Delta$ ) is at least comparable to the Fermi energy ( $E_F$ ). While this regime was not  
57 experimentally relevant to superconductors during the genesis of the celebrated BCS theory, it is  
58 the easier limit to understand. Here, extremely strong attractive interaction can, in principle, bind  
59 pairs of electrons into molecules. These molecules, being bosons, can then Bose condense to  
60 form a BEC. While this so-called molecular theory of superconductivity explained many  
61 properties of superconductors, it could not be applied to most superconductors where the pairing  
62 interaction is weak. Since the development of BCS theory, a select few systems have been found  
63 with pairing interactions strong enough to be close to the BEC regime. These include SrTiO<sub>3</sub>,  
64 high-T<sub>C</sub> superconductors, ultra-cold atomic gases, and most recently, iron-based  
65 superconductors.

66  
67 One particular feature of interest – though present in the BCS regime, is only practically  
68 accessible in the BEC regime – are the so-called Caroli de Gennes Matricon (CdGM) states in  
69 the core of a superconducting vortex, first predicted in 1964. The high current density in the core  
70 of vortices is sufficient to locally destroy superconductivity. In the BCS regime, the high current  
71 density in the core of vortices is sufficient to locally destroy superconducting pairing. The  
72 almost metallic excitations in the vortex core then display particle-in-a-box physics, confined by  
73 the surrounding superconductor, with discrete energy levels having energy spacing of  $\frac{\Delta^2}{E_F}$  (Fig. 1a)  
74 which is far too small to be measured for conventional BCS superconductors. In contrast, the  
75 "molecular" pairing deep in the BEC limit remains robust in the vortex core, which has almost  
76 no low-lying excitations in this case. The discrete energy levels in the vortex core reappear in the  
77 cross-over regime between the BEC and the BCS regime where the energy spacing is  
78 comparable to the superconducting gap which is measurably large. Unfortunately for attempts to  
79 measure CdGM states, amongst the few exotic superconductors near the BEC regime, each  
80 system has had its own difficulties. For example, high-T<sub>C</sub> superconductors, while having very  
81 strong pairing, also have gapless or nodal superconductivity, which implies that they fail to have  
82 proper CdGM states since the particles-in-the-box can leak into the bulk of the superconductor.  
83 Thus, the recent observations of CdGM states in some iron superconductors [7,8] has been a  
84 major triumph. These superconductors may be the "Goldilocks" materials which have strong  
85 pairing and no other show-stopping problems, providing a flexible family of materials that are  
86 fully gapped in the bulk and can be tuned close to the BEC regime.

87  
88 The observation of CdGM states holds particular significance in the context of topological  
89 superconductivity. In any superconductor, the set of CdGM states are particle-hole symmetric,  
90 meaning that if there is a state for electron addition at energy  $E$  there is also a state for electron  
91 removal at energy  $-E$ . This symmetry is assured by the fact that adding an electron is  
92 essentially the same as removing an electron and adding a pair of electrons (a boson) from the

93 condensate. What is special about the case of the topological superconductor is that one of the  
94 CdGM states sits at zero energy (Fig. 1b). The zero-energy state is unique in that it must be its  
95 own partner under particle-hole symmetry i.e. it is a Majorana. Such a Majorana mode cannot  
96 move away from zero energy because any continuous deformation of the spectrum that moves  
97 the Majorana away to either a positive or negative energy would break the required particle-hole  
98 symmetry inherent to superconductivity. It is also this symmetry which underlies the topological  
99 protection of qubits which might be built from Majoranas, should they be found in experiment.  
100 Crucially, it is suspected that at least some of the iron-based superconductors are topological  
101 superconductors. This suggests that the observed zero-energy state in a vortex core is the long-  
102 sought Majorana.

103  
104 Conclusive identification of excitations in vortices may prove to be more than a scientific  
105 footnote: vortices on the surface of a superconductors can be manipulated, which may provide a  
106 key technological advance needed for topological quantum computation [9]. Braiding two  
107 vortices and then fusing them together produce quantum states that depend on how the braiding  
108 was performed. This process is a key requirement needed to determine the anyonic properties of  
109 Majorana and essential in the creation of a topological qubit. This type of manipulation could  
110 potentially be assimilated into a scalable device architecture. This is due to the fact that, in  
111 addition to bulk superconductors, vortices are also possible in Josephson junctions. These  
112 junctions are superconducting devices where two superconductors are coupled via a weak link.  
113 Unlike bulk vortices, around which supercurrents flow full circle, these Josephson vortices are  
114 located at points along the extended junction where the current switches direction. These points  
115 can be controlled with external knobs like current pulses and magnetic fields. Multiple candidate  
116 topological materials can be used as the weak link in Josephson junctions, offering prospects for  
117 creating entire networks in which vortices can be manipulated via local fields and current pulses  
118 [10]. These manipulations would enable the Majoranas-based devices to take steps towards  
119 quantum information processing. And as with its original nanowire cousins, the Josephson  
120 junction system is a scalable platform, allowing for the coupling of many Majorana-based qubits  
121 needed for topological quantum computation.

122  
123 As developments in theory, experiment, and materials research have worked hand in hand over  
124 the last decades, a period of stasis in physics seems to have culminated. Heartened by the recent  
125 discovery of anyons, we may very well be at a cusp for establishing the existence of Majorana  
126 fermions and other more exotic anyons. Optimistically, we might hope that the field is now  
127 poised to make its next abrupt jump forward.

128  
129 **Fig. 1. Comparison of states localized inside a vortex. a.** In conventional superconductors,  
130 states localized inside a vortex are spaced by  $\frac{\Delta^2}{E_F}$ , where  $\Delta$  is a metric of the strength of the  
131 superconducting pairing strength and  $E_F$  is the Fermi energy. These states are situated at equal  
132 but opposite energies about  $E = 0$ , a result of the particle-hole symmetry provided by the  
133 superconductor. The lowest energy state is displaced from  $E = 0$ . **b.** States inside a vortex in a  
134 topological superconductor are also spaced by  $\frac{\Delta^2}{E_F}$  but include a state pinned to  $E = 0$ . This state  
135 is a Majorana fermion.

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