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Nonrenewable Resource Allocation under Intertemporally Dependent Demand

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Oxford Institute for Energy Studies

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ABSTRACT

This paper investigates the implications of an intertemporally dependent demand structure in the market for an exhaustible resource. The demand structure is derived endogenously in a partial equilibrium model where price-taking users combine the resource with capital to produce final output. It is shown that, when demand for the resource is modelled explicitly as a derived demand, the common assumption of an intertemporally independent sequence of instantaneous demand relationships is justified if there is a perfect rental market for capital equipment and there are no internal adjustment costs associated with either adding to the stock of equipment or subtracting from it. In the more realistic case where changes in capital stocks incur adjustment costs, the time-profile of demand is determined by the underlying programme of (dis)investment. In this case, changes in the resource price over time induce a lagged demand response.

The paper goes on to study the properties of an intertemporal competitive equilibrium, with emphasis on the demand structure with underlying adjustment costs. Agents are assumed to transact on perfectly functioning forward markets, so expectations about the resource price are always fulfilled <u>ex post</u>. The principal results to emerge from the analysis are that the resource utilization rate declines to zero in the long term as processes that use the resource in question are replaced by others; however, the precise time-path of substitution depends sensitively on the nature of adjustment costs. For example, if adjustment costs exhibit non-convexities, substitution away from processes that use the resource occurs in "pulses", in contrast to the smooth substitution programme under convex adjustment costs.

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1. INTRODUCTION

It is often argued informally that demand responses to the major oil price increases of 1973 and 1979 were still not complete several years after the events. The evidence, moreover, is that the long-run price elasticity of demand exceeds the short-run price elasticity, both for energy aggregates and individual energy carriers (see Kouris, 1983, for a survey). One reason is undoubtedly that the possibilities for substituting away from a particular energy resource, or energy in general, in "production" cannot be activated instantaneously, even at prohibitive cost. Existing processes have to be modified, new capital equipment ordered and installed, etc. (see Sweeney, 1983, for an overview of these arguments). The upshot of this is that the demand for energy and energy carriers has a dynamic structure.

This paper formulates a simple partial equilibrium model that endogenizes the phenomenon of non-instantaneous adjustment to changes in the price of an energy resource. To base such a result on an explicit optimization exercise is preferable to simply assuming demand for the resource to have an ad hoc lagged adjustment structure.[1] The construct that is developed is then used to study the properties of the equilibrium depletion profile for the resource, which is assumed to be non-replenishable. Briefly, the ingredients of the model are these: the resource is bought and used - along with productive capital - as an input by price-taking producers of "final output", who alter the input mix in response to changes in the resource price (anticipated or otherwise). Supply of the resource is competitive. Two cases are considered: the first where buyers are able to adjust the volume of capital costlessly and instantaneously, and the second where they face adjustment costs, so that capital is a quasi-fixed input.

Costless adjustment refers to the case where the only cost of adding a unit of capital to the firm's installation is the purchase price for that unit, assumed parametric to the firm and constant over time. Moreover, there is no cost associated with the removal of a unit of capital from the firm's installation, and capital equipment can be resold in a perfect second-hand market anytime. In this case, firms need only tailor decisions about their purchases of capital equipment to the current prices of other inputs that they use (in this case the resource). Because of this, resource demand is, for a given output price, a function only of current input prices. Adapting to whatever the future holds in store is costless, so there is no incentive to - say - forecast the resource price.[2]

Conversely, where there are adjustment-specific costs, or where second-hand markets for capital goods are imperfect, investment decisions have to be made on the basis of the entire expected future resource price trajectory. The role of price expectations is therefore a crucial one. Any change in expectations about the future price of the resource alters, in general, current investment behaviour. Among others, Nickell (1974) has emphasized - in the context of irreversible investment - that firms' current investment decisions are sensitive to anticipated future swings in demand conditions and input prices. Because expectations do play such a crucial role, it is as well to clarify at the outset that, aside from the occasional comparison, the analysis in this paper is based on the long-run perfect foresight postulate. In terms of the results, this is equivalent to the assumption that there is a complete set of forward markets for transactions in the resource at the initial date.

The assumption that resource demand has a static specification is deep-rooted in the theoretical literature on exhaustible resource depletion. Strictly speaking, however, it is justified only if the resource is used directly as a consumption good - a supposition with little appeal - or is used as an input in a productive process wherein all inputs are costlessly and instantaneously variable. But the assumption has been relaxed, implicitly or explicitly, in a handful of cases. For example, in aggregate models of growth with an exhaustible resource, where output is produced using capital and the resource, the implicit demand price of the resource (its marginal product) depends not only on the current flow of supply, but also on the current capital stock. The current capital stock is determined by past accumulation decisions; in the simplest case the savings ratio is an exogenously given constant. This mechanism establishes an intertemporal dependence in the demand relation: current demand for the resource depends partly on the rate at which it has been supplied in the past. This is precisely the case in the model considered by Samuelson and Moussavian (1984), who go on to show that a monopolist who recognizes the intertemporal dependence has an incentive to exploit it by violating the Hotelling r per cent rule.

Another example of intertemporal dependence to which attention has been drawn in the literature arises in connection with durable exhaustible resources. To analyze this case, Levhari and Pindyck (1981) define the marginal value of the resource on a flow of services derived from a <u>stock</u> in circulation. If, as they assume, there is perfect foresight, the marginal value of a unit of stock must at all dates equal unit carrying costs (forgone interest plus physical deterioration less capital appreciation). This equilibrium condition gives an expression for the rate of change of the resource price. Under competitive supply, the equilibrium condition is simply the Hotelling rule - price minus marginal cost rises at the rate of discount. Interestingly, however, if the resource is perfectly durable its price <u>falls</u> continuously so long as it is supplied. Under partial durability, price falls on an initial interval of time and later rises. Stewart (1980) observes similarly that, under competitive supply, the Hotelling rule holds independently of the durability of an exhaustible resource, though she is principally concerned with demonstrating that a monopolist fails to sustain it (in particular, the resource price declines over time under monopolistic supply if the resource is perfectly durable). It should be noted, however, that Stewart equates the marginal value of a unit of stock in circulation with the resource price (not carrying costs), and thereby blurs the distinction between stock and flow dimensions.

In this paper the dynamic resource demand structure stems from the premise that resource users - industrial, say - incur costs in the process of adjusting to any changes in the resource price. The objective is to study the properties of depletion paths under this demand structure, and interest focuses on equilibrium paths that satisfy the Hotelling rule. In other words, the PV of price (marginal profits) received by resource owners remains constant over time in equilibrium. This result follows from the assumption that resource supply is competitive. Under this assumption, suppliers do not perceive, at the initial date, that it is their entire (collective) planned sales profile that determines users' planned (dis)investment trajectories, and therefore the precise time-stream of instantaneous demand relationships for the resource.

The paper is arranged as follows: the remainder of this section consists of a review of the literature on firms' optimal investment decisions in the face of adjustment costs. Sections 4, 5 and 6 draw on, and adapt, parts of this work. Section 2 examines buyers' decisions and the resource demand structure under costless factor adjustment and discusses the interpretation of the model. Section 3 characterizes the intertemporal resource market equilibrium under the demand conditions derived in section 2, and shows that the common prediction that a higher interest rate speeds up resource utilization holds equally in this model. Section 4 parallels section 2 by looking at users' decisions and the resulting demand structure where changes in the capital stock incur adjustment costs. Both the case of (A) convex adjustment costs, and (B) non-convex adjustment costs are considered.

Section 5 elucidates the properties of the intertemporal equilibrium that emerges under the demand conditions derived in section 4A. Section 6 does likewise for the demand conditions derived in section 4B. Finally, section 7 contains some concluding remarks.

The prototype model of investment behaviour under adjustment costs (and certainty) on which is based the analysis of sections 4A and 5 has been studied by several authors. Among these is Gould (1968), who formulates the investment decision problem of a competitive firm that seeks to maximize discounted profits - market price times output less the wage and interest bill. The main point is that output is assumed to be an additively separable function of the factors of production (capital and labour) on the one hand, and the gross investment rate, on the other. That is, real adjustment costs (output lost as a direct result of adding to or subtracting from the stock of capital) are an increasing (and strictly convex) function of the gross investment rate, but are independent of the rate of use of factor services. With a linear homogeneous technology, constant (input and output) prices and a quadratic adjustment cost function, Gould shows that the capital stock adjusts towards its long-run steady state level according to a flexible accelerator mechanism - its rate of change is proportional to the discrepancy between the steady state and actual stocks. This also underpins an inverse relation between the gross rate of investment and the interest rate.

Treadway (1969) deals with a similar problem but assumes that real adjustment costs depend on <u>net</u> investment. His principal results are these: under constant returns to scale, a uniquely optimal rate of expansion exists, but an optimal long-run capital stock does not. Conversely, where returns to scale increase over an initial range but later decrease, a uniquely optimal asymptotic target stock can be found, with a unique investment path leading to it. In a subsequent paper, Treadway (1971) drops the separability assumption and demonstrates that without it the firm does not necessarily exhibit (a) a negatively sloped demand relation for the variable factor; or (b) a positively sloped output supply relation. This is so both in the long run (steady state) equilibrium, and in the "short run" (in motion according to a linearized system in the neighborhood of the steady state). Also, the short-run price effects are not unambiguously less elastic than the long run effects. Furthermore, (c) symmetry of cross-price effects in the long-run factor demand functions does not hold in general, so the long-run demand functions cannot be derived by maximizing a surrogate static profit function.

In Lucas (1967) the individual competitive firm's output is specified as a linear homogeneous (non-separable) function of the capital and labour inputs and the gross investment rate. Under this assumption - for time invariant prices - the firm accumulates (or runs down) capital at a constant proportional rate that depends on these prices and the depreciation rate. This percentage rate of accumulation (equal to the rate of growth of the firm's supply) is independent of the firm's size as measured by its capital stock. To progress from the individual firm to the industry's investment demand function, these steps are followed: an output price is determined to clear the market over time (demand growth equals supply growth). This price is then fed back into individual firms' investment demand functions, and the latter "summed horizontally" to determine the industry's equilibrium rate of capacity expansion (or contraction). This rate depends on the rate of growth of market demand, but the market price is solved out.

Because the model allows a time-invariant price to clear the ouput market over time, the initial expectation by firms that price will remain unchanged is fulfilled along the equilibrium path. In general this is not so, and Lucas (1966) contrasts the dynamic investment behaviour of a competitive industry under static product price expectations with that under rational expectations. The industry is composed of many small profit-maximizing firms, and for each of these output is proportional to its current stock of capital. To capture the notion of adjustment costs, Lucas specifies the factor of proportionality (the average product of capital) as a declining function of the ratio of the gross investment rate to the current capital stock. Where firms have static price expectations, the time-path of price and industry output can be determined by deriving individual firms' time-paths of supply (given their expectation at each date that price will thereafter remain constant), aggregating them, and combining them with the time-path of industry demand. But where firms have rational expectations, the equilibrium supply trajectories of individual firms and the industry as a whole must be determined simultaneously rather than sequentially. The main result is that the long-run equilibrium price and rate of growth of output are the same under the two expectations hypotheses, but adjustment to equilibrium is less rapid under rational expectations (but see proposition 5 in section 5 below). The rational expectations path can also be shown to maximize a discounted integral of consumer plus producer surplus.

A distinguishing feature of all the above treatments is that they assume increasing marginal adjustment costs, which means that motion towards a "target" stock of capital is "staggered" over time. As discussed in Nickell (1978, Ch. 3) and Rothschild (1971, pp. 608-9), to assume that costs associated with current additions to a firm's capital stock rise at the margin may be credible over certain ranges in some contexts, but does not stand up to empirical scrutiny in others. It is thus worthwhile tracing, at the conceptual level, the implications of non-convexities in the adjustment cost function. Sections 4B and 6 take up this task, though in some respects they circumvent the problem. Of the previous work on this issue, two papers in particular deserve mention.

Rothschild (1971) uses a discrete-time model to demonstrate the existence of an optimal investment programme for a (monopolistic) firm without restrictions on the form of the adjustment cost function. If the firm expects that factor prices and demand conditions will remain at their current levels, the optimal (planned) programme has the following features: (a) with a differentiable and strictly convex cost function, if it is optimal to sustain a positive investment rate in one period, it is optimal to do so in all periods; but (b) with a linear or strictly concave cost function, it is optimal to sustain a positive rate of investment for at most one period. In both cases, if it is optimal to invest and the investment programme is unique, it is optimal to invest in the first period. Also noteworthy is that, if marginal adjustment cost at a zero rate of investment is non-zero, "small" changes in market conditions may fail to induce any changes in the equilibrium stock of capital (see also section 4B below).

Davidson and Harris (1981) use a continuous-time framework to examine three types of non-convexity in the competitive firm's investment problem: (i) increasing returns to scale in the production technology; (ii) a non-convex (flow) adjustment cost function; and (iii) a fixed cost incurred whenever a phase of zero investment is succeeded by a phase of positive investment. Under (i), a multiplicity of long-run stationary equilibria can arise, and the value of the programme along every candidate path must be computed to locate the optimal investment policy. Under (ii), "chattering" investment programmes may have to be used to "convexify" approximately the non-convex range of the adjustment cost function (on this point, see also section 5B and the Appendix to this paper). Finally, a non-convexity of type (iii) (a "startup" cost) affects the optimal programme only if it coexists with an adjustment cost function that has a fixed (flow) component (as in Figure 8.2 below). If this is the case, it can be shown that if startup costs are "large" relative to the flow component, it pays to avoid "shutting down" the process by letting the investment rate drop to zero. Conversely, for "small" start-up costs, the optimal policy features alternate cycles of investment and no investment, so that start-up costs are borne more than once, but fixed flow costs are avoided over the intervals of zero investment.

The papers surveyed above provide many of the basic ingredients for the analytical framework constructed in sections 4, 5 and 6 of this paper. Several of them also relax one or more of the simplifying assumptions (for example, that resource users have technologies that are separable in factor services and the investment rate) underlying that framework. Those assumptions are used to concentrate attention on the objective of this paper: the analysis of investment programmes under costs of adjustment when the price of an input (the resource) is rising over time, and the derivation of the resulting time-path of demand for the resource. As will become clear shortly, the analysis of steady-state behaviour - a technique common to all the papers reviewed here - is no longer appropriate in this context.

2. RESOURCE DEMAND UNDER COSTLESS FACTOR ADJUSTMENT

This section derives the time-path of demand for the resource where all factors of production are variable. The resource-using sector is assumed to consist of a large number of identical firms, for notational convenience modelled as a single <u>price-taking</u> group ("the buyer") in the output and factor markets. The buyer is a productive entity that is profit maximizing in the usual sense, and produces a gross "final output" denoted by F(R,K) where R is the rate of supply of the exhaustible resource (equal to the rate of utilization) and K the stock of (homogeneous) productive capital. The following assumptions will be employed here and in subsequent sections:

(A.1) F(R,K) is twice-continuously differentiable in both of its arguments, displays diminishing returns to each input and to scale everywhere (to compensate for the explicit recognition of any other inputs), and satisfies the Inada conditions $F_R(0,K)=F_K(R,0)=+\infty$, $R,K>0$. Also $F_K(R,\infty)=0$ and $F_R(R,0)=F_K(0,K)=0$, $R,K>0$.

(A.2) There is no capital equipment depreciation.

(A.3) There are no adjustment costs associated with changes in either R or K.

If the price of output is assumed to stay constant at unity through time,[3] the buyer's criterion can be expressed as

$$(1) \quad \underset{\{R\}_{t=0}^{\infty},\ \{K\}_{t=0}^{\infty}}{\text{maximize}} \int_0^{\infty} e^{-rt}\{F(R,K) - pR - rK\}\, dt, \text{ subject to}$$

$$(2) \quad R \geq 0,\ K \geq 0,\ K(0) = K_0 \text{ given;}$$

where the time-dependence of the variables R, K and p (the spot real price of the resource) has been suppressed for convenience, and $r > 0$ denotes the rate of return on the numeraire asset, assumed exogenous and time-invariant. The criterion is then simply the maximization of the discounted value of net output (gross output less the resource bill and the interest bill).[4]

It is straightforward that, because of assumption (A.3), the solution is devoid of any dynamic properties. It is simply a succession - through time - of profit maximizing solutions in a static framework (a thorough treatment of the latter appears, for example, in Varian (1978, Chapter 1)) . Note that an implication of this is that the buyer's expectations about future resource prices have no bearing whatsoever on current factor employment decisions. This is simply because present decisions do not impinge on the buyer's future opportunity set.

To obtain the time-path of demand for the resource, define the current value profit function at date t as

$$\pi(p(t),r) = \max_{R(t),K(t)>0} \{F(R(t),K(t)) - rK(t) - p(t)R(t)\}.$$

Then, by Hotelling's lemma,

$$(3) \quad R(p(t),r) = -\pi_p(p(t),r), \text{ and}$$

$$(4) \quad K(p(t),r) = -\pi_r(p(t),r)$$

give the instantaneous factor demand functions at t, where a subscript variable denotes the derivative with respect to that variable. Again omitting time dependence, the following results are standard:

(5.1) $R_p(p,r) = -F_{KK}(F_{KR}^2 - F_{RR}F_{KK})^{-1} < 0,$

(5.2) $K_r(p,r) = -F_{RR}(F_{KR}^2 - F_{RR}F_{KK})^{-1} < 0,$ and

(5.3) $K_p(p,r) = R_r(p,r) = F_{KR}(F_{KR}^2 - F_{RR}F_{KK})^{-1}$

$> (=) (<) 0$ according as $F_{KR} < (=) (>) 0,$

where the second order derivatives are evaluated at the optimal choice. (5) uses (3) and (4), the symmetry of cross-derivatives and the convexity of the profit function (Legendre condition for a maximum of (1)), which ensures that $(F_{KR}^2 - F_{RR}F_{KK}) < 0$.

Now (3) and (4) yield the entire time profiles of inputs into the production of final output by the resource-using sector. Both are continuous functions of time (with the exception, in general, of K at the initial date) if p(t) is a continuous function of time. In the case of a resource price that is monotonically increasing over time, by (5.1) demand for the resource dwindles steadily over time. The buyer's holdings of productive capital also decline over time if (and only if) $F_{KR}>0$ (which implies that $K_p<0$). This is henceforth assumed to be the case. Under assumption (A.1) (Inada conditions and diminishing returns to scale), the desired capital stock is always positive provided the resource price is finite.

Given its implications here, is the assumption that $F_{KR}>0$

defensible? Certainly many "common" technologies have this property of "cooperancy".[5] The implication that the capital stock is run down alongside demand for the resource as its price increases over time might appear at first blush to be rather an unwanted one: the interesting case is typically that where other inputs replace an exhaustible resource in the productive process, not where they contract as the supply of the resource contracts. But, suitably reinterpreted, this assumption does allow the model to capture "substitution" in a specific, albeit indirect, sense. As the resource price rises over time, the marginal productivity of capital employed in the resource-using industry - where the "industry" is defined very narrowly as the collection of existing processes that transform the resource into output - declines. Since capital is perfectly malleable, it is progressively shifted out of the latter industry to earn the normal rate of return r in "other industries" (the "rest of the economy"). These others presumably employ different resources or backstop technologies, engage in research and development, etc. This is the precise sense in which substitution occurs here: emphasis shifts to modes of production that do not depend upon the resource.[6]

3. MARKET EQUILIBRIUM UNDER PERFECTLY MALLEABLE CAPITAL

This section characterizes the temporal allocation of the resource under the demand conditions traced in the previous section. The following assumptions are used:

(A.4) Resource owners are price takers. The total stock of the resource (the sum of individual firms' stocks) available initially (S_0) is homogeneous, non-augmentable, of known size, and costless to extract.

(A.5) There is a complete set of forward markets for transactions in the resource at the initial date.

Under (A.4) and (A.5), the resource price trajectory is determined by the equations

(6) $p(t) = p(0)e^{rt}$, all t, and

(7) $$\int_0^\infty R(p(t),r)\, dt = S_0$$

(6) is the basic no-arbitrage condition under price-taking resource supply. In its absence, every resource owner has a preference for supplying the resource at some dates over other dates. Also, because of (A.5), resource owners are able to peer into the indefinite future and avoid paths along which some of the resource remains unextracted. Condition (7) (complete exhaustion) captures this.

The use of (6) in (7) uniquely determines the initial condition p(0), hence $\{p(t)\}^{\infty}_{t=0}$. Figure 1 shows the optimal path in (p,S) space. The unique stable arm emanates at p_0*. Paths that start at lower values of p(0) cross the vertical axis (that is, they exhaust the resource in finite time). Paths that begin at values of p(0) above p_0* do not meet the vertical axis, even asymptotically, and are therefore also not optimal. Once the optimal resource price trajectory is determined, (3) and (4) give the entire time-stream of input values, hence the time-path of output supplied by the resource-using industry. Because the resource price rises throughout, the industry experiences a secular contraction. Otherwise put, there is substitution away from processes (suitably narrowly defined) that use the resource in production. It is obvious that the larger is the stock of the resource available initially, the lower must be its price at the initial and subsequent dates. So the larger is S_0, the greater the viability of processes that use the resource.

Because in this construct the interest rate enters the resource demand function, it is worth enquiring about the sensitivity of the optimal path to interest rate changes. In fact it transpires that, under the present assumptions, the usual result that a higher rate entails quicker depletion continues to hold. Time-differentiation of (6) and (7) gives the slope of the phase path in Figure 1 as

$$dp/dS = \dot{p}/\dot{S} = - rp/R(p,r) < 0 \quad , \text{ whence}$$

$$d/dr(dp/dS) = - p \{R(p,r) - rR_r(p,r)\}/R(p,r)^2 < 0$$

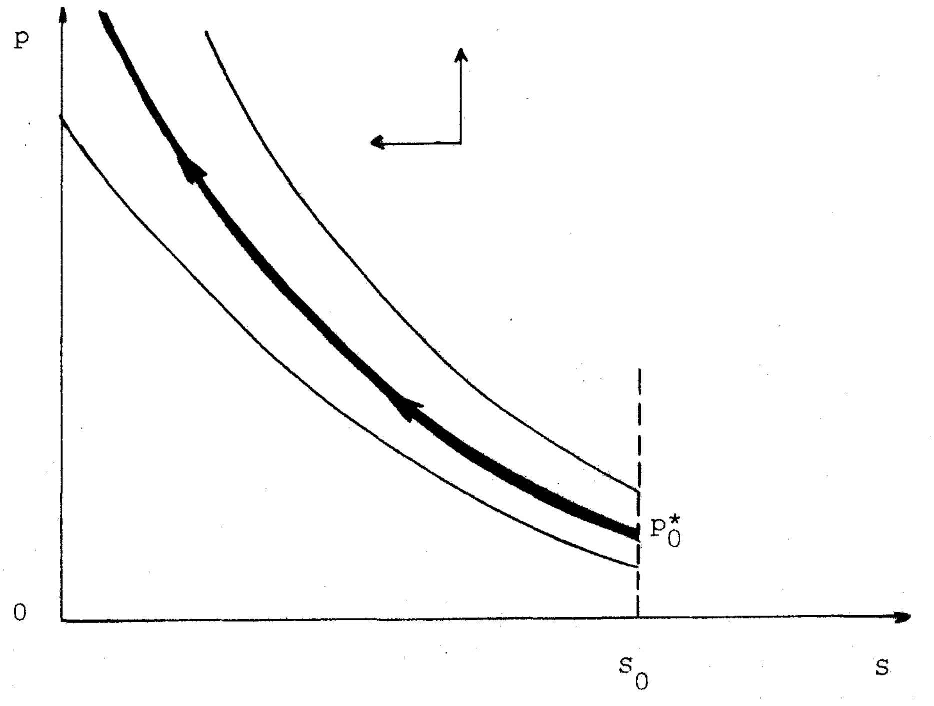

FIGURE 1

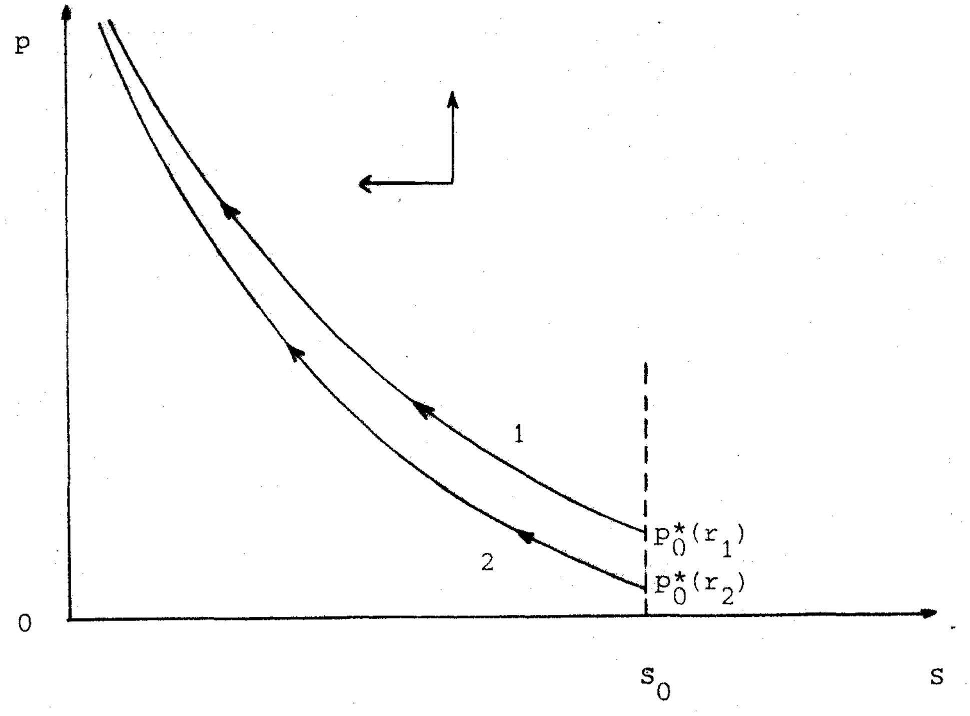

FIGURE 2

provided that $R_r < 0$, as assumed. So for any given p, the phase path is steeper the larger is r. Figure 2 illustrates this. The main feature is that the path corresponding to the higher interest rate r_2 (path 2) begins at a lower initial price and has the larger slope (in absolute value), but cannot intersect the other path.

To see this, suppose instead that $p_0^*(r_2)>p_0^*(r_1)$. Path 2 then lies entirely above path 1. For any given state S, path 2 implies both a larger price and a larger interest rate, and therefore a smaller extraction rate. One of the paths thus violates the asymptotic exhaustion condition, which contradicts optimality. Consequently $p_0^*(r_2)<p_0^*(r_1)$. Suppose now that the paths intersect - they can do so only once - at the point S=S'. Then, for any state S<S', path 2 implies a smaller rate of extraction than path 1. Starting from a given state, therefore, cumulative depletion along path 2 is less, which again raises a contradiction.

Alternatively, substituting (6) into (7) gives, by implicit differentiation [7]

$$(8) \quad \frac{\partial p(0)}{\partial r} = - \frac{\int_0^\infty \{R_p(p(t),r)\, p(0)e^{rt}t + R_r(p(t),r)\}\, dt}{\int_0^\infty R_p(p(t),r)\, e^{rt}\, dt},$$

Since $R_P<0$ and by assumption $R_r < 0$, (8) is negative, indicating that a higher interest rate will tilt the equilibrium price trajectory in the direction of the arrows (see Figure 3). It is then easy to show that the remaining resource stock is, at any

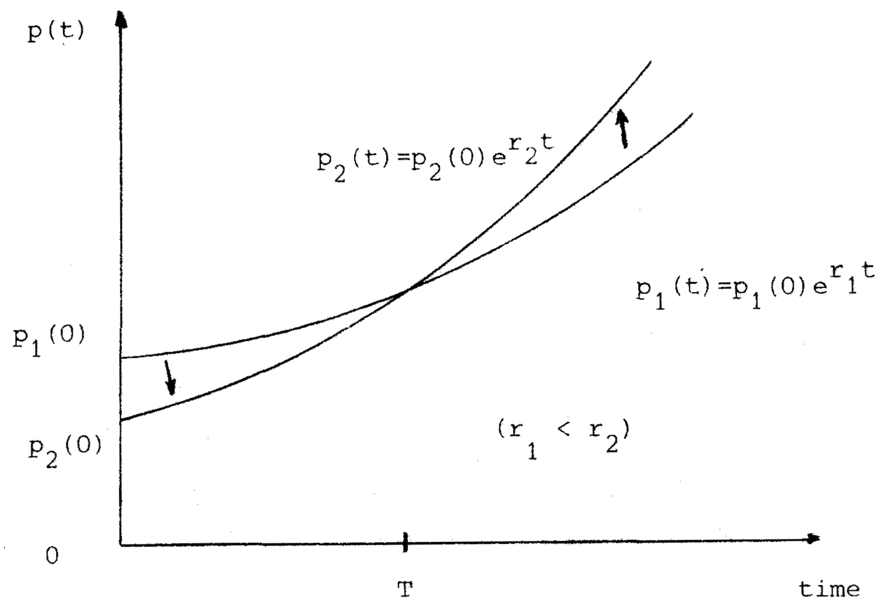

FIGURE 3

finite date, larger along profile 1 than along profile 2.[8] When this is the case, profile 1 will be said to be strongly more conservationist than profile 2. Profile 1 will be said to be weakly more conservationist than profile 2 if, for an initial interval of time, profile 1 has a larger remaining stock than profile 2 (see also section 5 below).

Note that the term Rr in (8) accentuates the required reduction in p(0) consequent on a higher r, compared with the "usual" case where r does not enter directly into the resource demand function. The reason is simply that a higher interest rate entails, in the present case, a direct reduction in demand that is absent in the usual case. This comes from the fall in the demand for capital services that accompanies a larger interest rate. To ensure that the stock exhaustion condition (7) is met, price must be lowered a little to offset this indirect effect, in addition to the price reduction dictated by the increased "impatience" of the resource sellers if r rises.

Example

Suppose that $F(R,K) = R^{\alpha} K^{\beta}$ Then

(6') $R(p,r) = (r/\beta)^{-\beta/(1-\alpha-\beta)} \quad (p/\alpha)^{-(1-\beta)/(1-\alpha-\beta)}$, and

(4') $K(p,r) = (r/\beta)^{-(1-\alpha)/(1-\alpha-\beta)} \quad (p/\alpha)^{-\alpha/(1-\alpha-\beta)}$.

Using (6) and (3') in (7), a solution for p(0) in terms of the parameters can be obtained:

$$p(0) = \frac{(1-\beta)}{(1-\alpha-\beta)} \alpha \{ S_0^{-(1-\alpha-\beta)} \ r^{-(1-\alpha)} \ \beta^{\beta} \}^{1/(1-\beta)} .$$

so that $\partial p(0)/\partial r<0$. Note also that p(0) is independent of K_0, a result that depends strongly on assumption (A.3).[9] Figures 3 and 4 show the time paths of R and K. The output of the resource-using industry likewise declines monotonically with time.

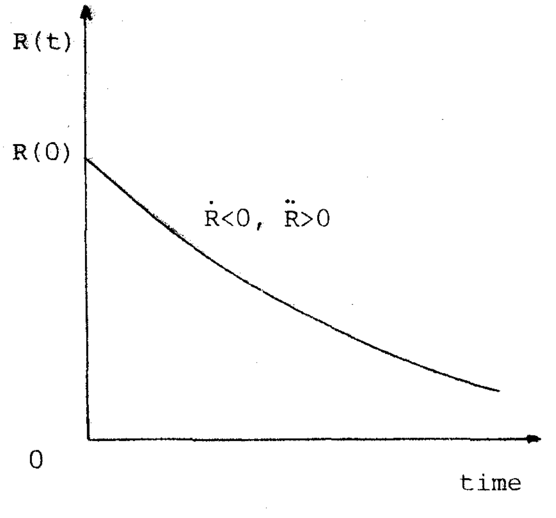

FIGURE 4

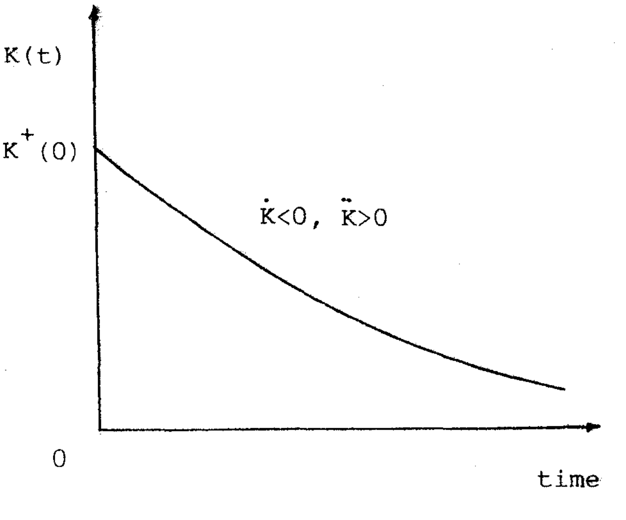

FIGURE 5

4. RESOURCE DEMAND UNDER COSTLY FACTOR ADJUSTMENT

Section 2 indicated that, if all factors of production are costlessly and instantaneously variable, decision rules about factor service purchases for the buyer are given in terms of current prices only. This is not in general the case where adjustments in the quantity of the capital input involve a cost. It is to this case that the analysis now turns. Two different assumptions about the general shape of the adjustment cost function are considered in turn.

A. Convex Adjustment Costs

The construct is precisely that of Section 2, except that assumption (A.3) is replaced by the following:

(A.3') The adjustment cost function $C(I)$ is twice-continuously differentiable and displays $C(I)>0$ for $I\neq 0$, $C(0)=0$, $C'(I)>(=)<0$ as $I>(=)(<)0$, and $C''(I)>0$, all I.

Thus $C(I)$ is assumed to be a strictly convex function passing through the origin. In spite of mixed evidence concerning its likely validity, the implications of this type of adjustment cost function have been widely analyzed in the literature.

Now the rate of resource utilization, R, can be varied at will. Therefore define the current value restricted profit function at t:

$$\pi^0(p(t),r,\bar{K}(t),I(t)) = \max_{R(t)>0} \{F(R(t),\bar{K}(t)) - r\bar{K}(t) - p(t)R(t) - C(I(t))\}$$

(recall that the output price is assumed to be fixed at unity). Then

$$(9)\ R(p(t),\bar{K}(t)) = -\pi^0_p(p(t),r,\bar{K}(t),I(t))$$

gives the instantaneous demand relationship for the resource at date t. The bar (omitted from now on) indicates that the buyer's capital stock is fixed at t. Also

$$(10)\ -\pi^0_K(p(t),r,K(t),I(t)) = r - F_K(R(p(t),K(t)),K(t))$$
$$= r - f(p(t),K(t)) \quad \text{say.}$$

The right-hand side of (10) is the difference between the actual rental rate for capital and its "virtual" rental rate. The latter refers to that rate which, if actually prevailing, would induce an unconstrained buyer - one able to adjust the capital stock costlessly - to choose precisely $\bar{K}$ (Neary and Stiglitz, 1983, Appendix). Alternatively, it is the increment in output gained from a small increase in the capital stock, if the rate of resource input is allowed to adjust optimally to the change. The following assumption will be employed in the next section:

$$(A.6)\ \lim_{K\to 0} f(p,K)=+\infty \text{ and } \lim_{K\to\infty} f(p,K)=0.$$

These are stronger requirements than the Inada conditions of assumption (A.1). The first states that a unit of capital should be infinitely valuable at the margin if none of it is currently available, even if the rate of resource utilization has been adjusted optimally to reflect this. The second states that, if there is an infinitely large stock of capital equipment in place, then an incremental unit is worthless, even when resource use can be increased alongside.[10]

For a given resource price path $\{p(t)\}_{t=0}^{\infty}$, continuous in time, the buyer's problem is now simply to choose an associated investment programme $\{I(t)\}_{t=0}^{\infty}$ to maximize

(11) $$\int_0^{\infty} e^{-rt}\, \pi^0(p(t),r,K(t),I(t))\, dt$$, where

(12) $$\dot{K}(t) = I(t) \text{ with } K(0) = K_0 \text{ given.}$$

(Note that a non-negativity restriction on K is redundant if the production function satisfies the Inada conditions). Apart from the explicit appearance of a time-trend through the resource price, this is the well-known problem of finding the optimal path of investment demand under costs of adjustment (see, for example, Treadway, 1969). A maximizing solution satisfies the first-order differential equation system given by (12) and [11]

(13) $$\dot{I} = \frac{r(1+C'(I)) - f(p,K)}{C''(I)}$$, with

(14) $$\lim_{t\to\infty} e^{-rt}\, C'(I)K = 0.$$

Under the present assumptions, this system has an absolutely continuous solution $\{K(t),I(t)\}$ on $[0,\infty)$. Furthermore, because under (A.1) and (A.3'), the Hamiltonian corresponding to (11) and (12) is strictly concave in (K,I), a programme satisfying (12)-

(14) is uniquely optimal (see, for example, Long and Vousden, 1977, Theorems 7 and 8).

It will prove useful for the analysis that follows to examine a hypothetical steady state solution, defined by $I=\dot{I}=0$. Fix p at a given level $\bar{p}$. (12) and (13) is then an autonomous system, and, from (13), $\dot{I}=0$ defines, for $p=\bar{p}$, a locus in (K,I) space with slope $dI/dK = f_K(\bar{p},K)/rC''(I)$. The slope is negative since $f_K(p,K)<0$ everywhere under the assumption of diminishing returns to scale. Moreover, the characteristic equation of the linear approximation to (12) and (13), that is

$$\phi^2 - r\phi + \frac{f_K(\bar{p},K^*)}{C''(0)} = 0, \text{ has solution}$$

$$\phi = \frac{r}{2} \pm \left\{ \left(\frac{r}{2}\right)^2 - \frac{f_K(\bar{p},K^*)}{C''(0)} \right\}^{1/2};$$

so the roots are real and of opposite sign. The steady state solution corresponding to a given level of p is, as is well known, saddlepoint stable. Figure 6 illustrates this. If p remains fixed at $\bar{p}$, the buyer adjusts its holdings of productive capital asymptotically to the level K^*. Whatever the initial condition for K, the solution is characterized by a movement along the stable arm towards the long-run steady state. For $K_0=K_0^1$ ($K_0=K_0^2$) the initial stock falls short of (exceeds) the desired stock, and the solid path emanating at the point (K_0^1,I_0^1) (the point (K_0^2,I_0^2)) shows the path of investment (disinvestment) that is pursued. In the neighborhood of the steady state solution the accumulation (decumulation) path of the buyer approximates a flexible accelerator rule.[12]

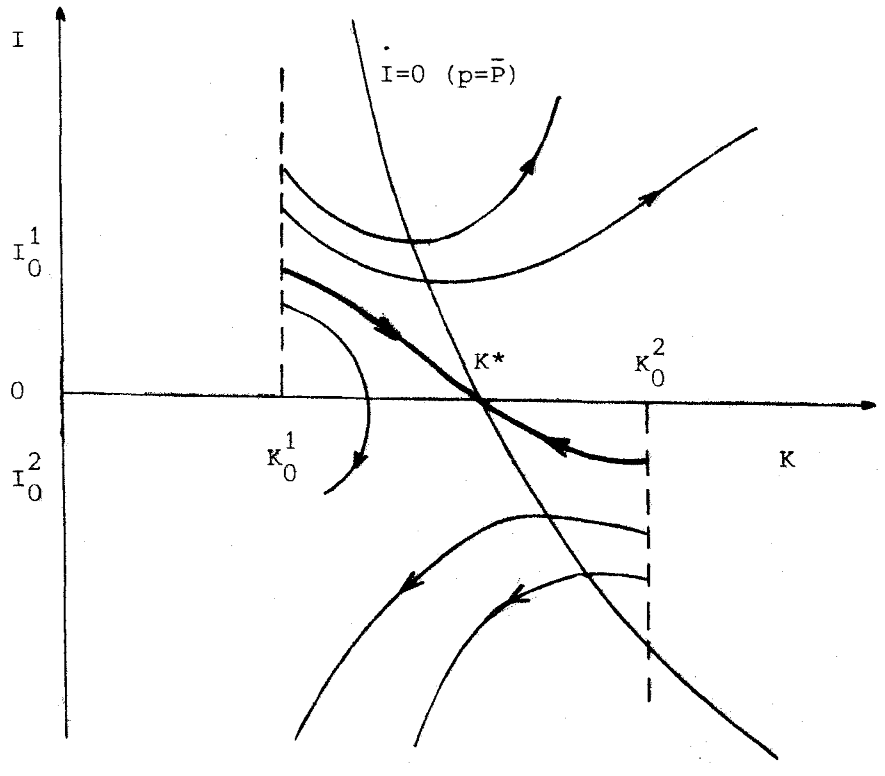

FIGURE 6

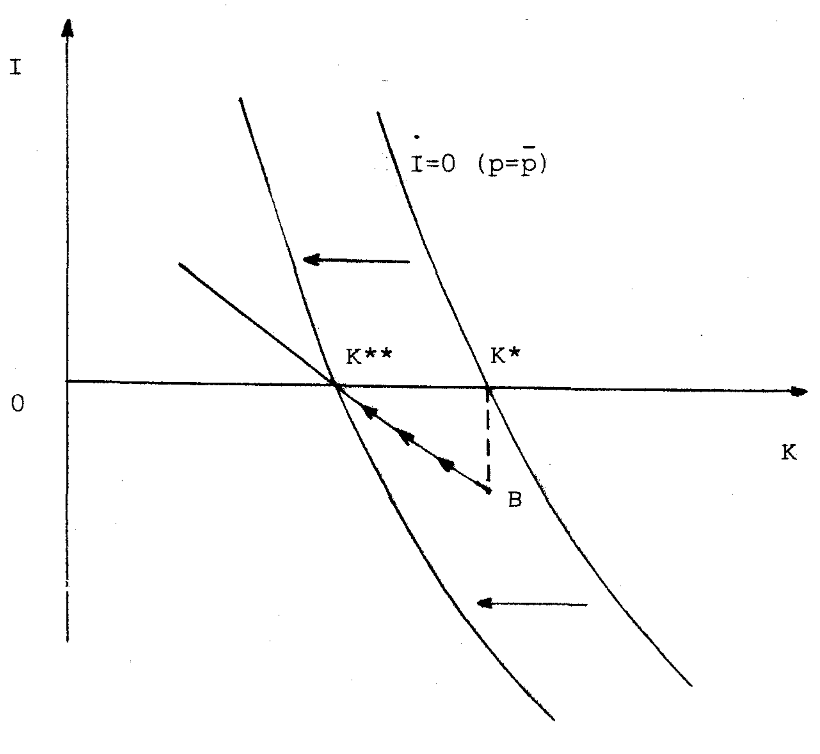

FIGURE 7

By way of illustration, suppose that the buyer is in long-run steady-state equilibrium at K*, and consider the effects of an unanticipated discontinuous increase in the resource price. In Figure 7, this displaces the $\dot{I}=0$ isocline in a leftward direction. The rate of investment jumps down to point B, and thereafter moves along the stable arm towards the new steady-state equilibrium K**. The upshot is that the full adjustment to the new equilibrium occurs over an extended period of time, during which demand for the resource is larger than it would be in the case of costless adjustment.

The above is useful as a reference point, and would be applicable if, say, the resource were a conventionally produced and competitively supplied commodity with a constant unit production cost of $\bar{p}$. But if the resource stock is of finite size and nonreplenishable, this precludes a constant price; instead, as indicated in the next section, price rises monotonically over time. Under (A.1) (Inada conditions and diminishing returns to scale), demand does not choke off, and there is no upper bound on the resource price. A stationary solution cannot, therefore, be characterized in the above manner. Section 5 turns to the task of describing the features of solution paths, but first the analysis turns to the case of non-convex adjustment costs.

B. Non-Convex Adjustment Costs

Two examples of non-convexities in the adjustment cost function are shown in figure 8. Figure 8.1 demonstrates the case where marginal adjustment costs diminish over an initial range but later increase. Figure 8.2 illustrates the presence of fixed

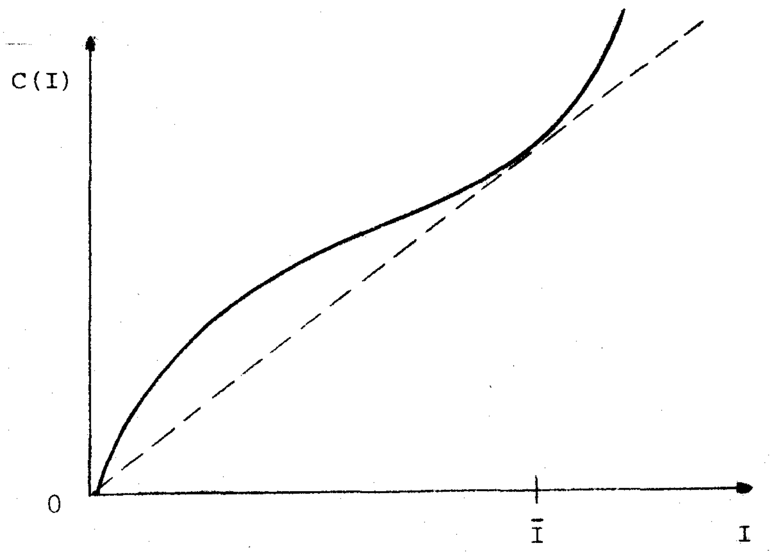

FIGURE 8.1

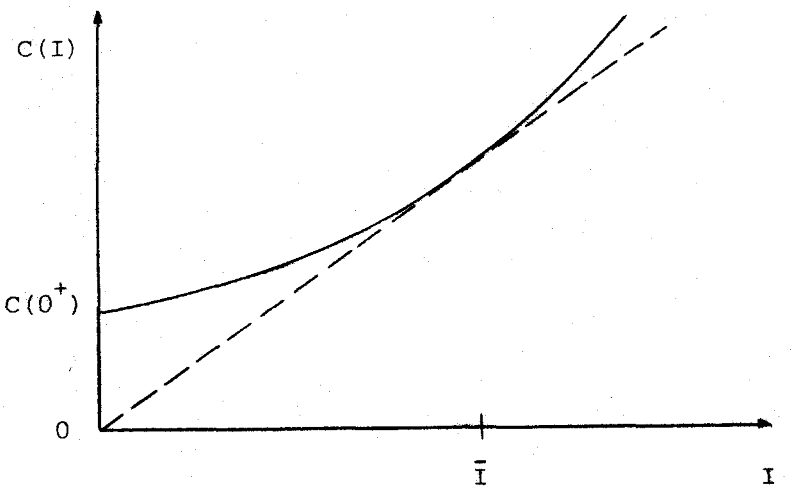

FIGURE 8.2

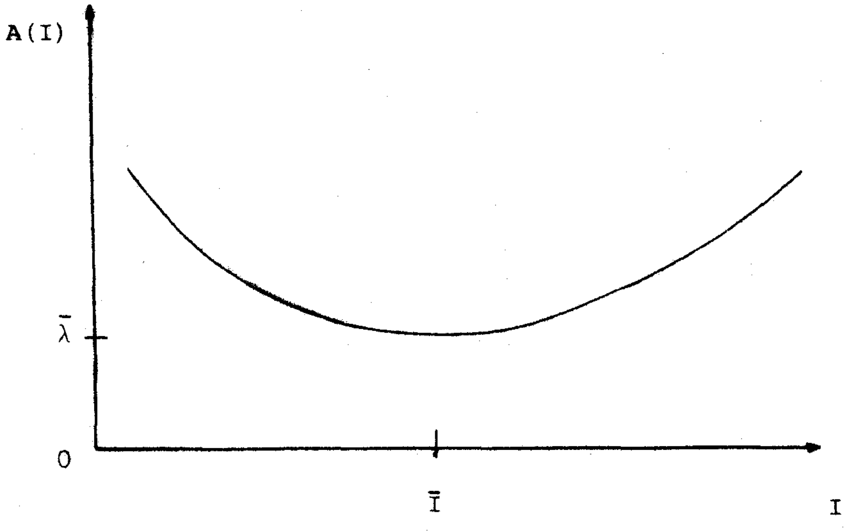

FIGURE 9

(flow) costs of adjustment.[13] The point is that both types of non-convexity imply that the unit adjustment cost is declining over an initial range, and later increasing, so that the function has the common U-shape (Figure 9). Figures 8 and 9 show only the positive quadrant, but the picture is assumed roughly symmetric on the disinvestment side. Clearly various hybrids of the two cases can occur, but for expositional convenience assumption (A.3') is here replaced by the following straightforward assumption:

(A.3") (Non-convexity) The unit adjustment cost function $A(I) \equiv C(I)/|I|$ displays $0<A(0)\leq+\infty$, $A'(I)<0$ for $-\bar{I}<I<\bar{I}$, $I\neq0$ and $A'(I)>0$, $I<-\bar{I}$ and $I>\bar{I}$.

Notice that the point at which average cost "turns up" has been assumed symmetric as between investment and disinvestment. This will help save on notation subsequently but is easily relaxed.

How does replacing (A.3') with (A.3") affect the solution to the problem of optimizing (11)? As demonstrated by Davidson and Harris (1981), one gains insight into the problem initially by looking at its solution subject to a "convexified" adjustment cost function. In other words, (A.3") is altered to:

(A.3*) ("Convexification" of A(I)) The unit adjustment cost function $A^*(I)$ is given by $A^*(I)=A(-\bar{I})$, $-\bar{I}<I<0$; $A^*(I)=A(\bar{I})$, $0<I<\bar{I}$, and $A^*(I) = A(I)$, $I<-\bar{I}$ and $I>\bar{I}$.

Under (A.2) and (A.3*), necessary and sufficient conditions for an optimal accumulation programme are

(15) $C'(I) = \lambda$ for $\lambda>\bar{\lambda}$, $\lambda<-\bar{\lambda}$;

$I\in[0,\bar{I}]$ for $\lambda=\bar{\lambda}$;

$I\in[-\bar{I},0]$ for $\lambda=-\bar{\lambda}$;

$I=0$, for $-\bar{\lambda}<\lambda<\bar{\lambda}$;

where $\bar{\lambda}=C'(\bar{I})$, $-\bar{\lambda}=C'(-\bar{I})$, and

(16) $\dot{\lambda} = r\lambda - f(p,K) + r$,

and (12) and (14), where λ (the current value co-state of K) is a continuously differentiable function of time. Now define two "target" capital stocks by

(17.1) $\underline{K}>0$: $r(1+\bar{\lambda}) = f(p,\underline{K})$; and

(17.2) $\bar{K}>0$: $r(1-\bar{\lambda}) = f(p,\bar{K})$.

In particular, if $1-\bar{\lambda}<0$ (the "disturbance" cost of removing any equipment exceeds that equipment's resale value), $\bar{K}$ does not exist (see also below). In general it is important to note that there is no longer a unique "target" stock of capital to which the buyer adjusts in the long run. Figure 10 illustrates the solution in (λ,K) space. It is characterized as follows:

(i) For $K_0\in[\underline{K},\bar{K}]$ (or $K_0\in[\underline{K},\infty)$ if $\bar{K}$ does not exist), $K(t)=K_0$ for $t\geq0$. In this case the cost of any adjustment in the capital stock exceeds the discounted stream of returns from it.

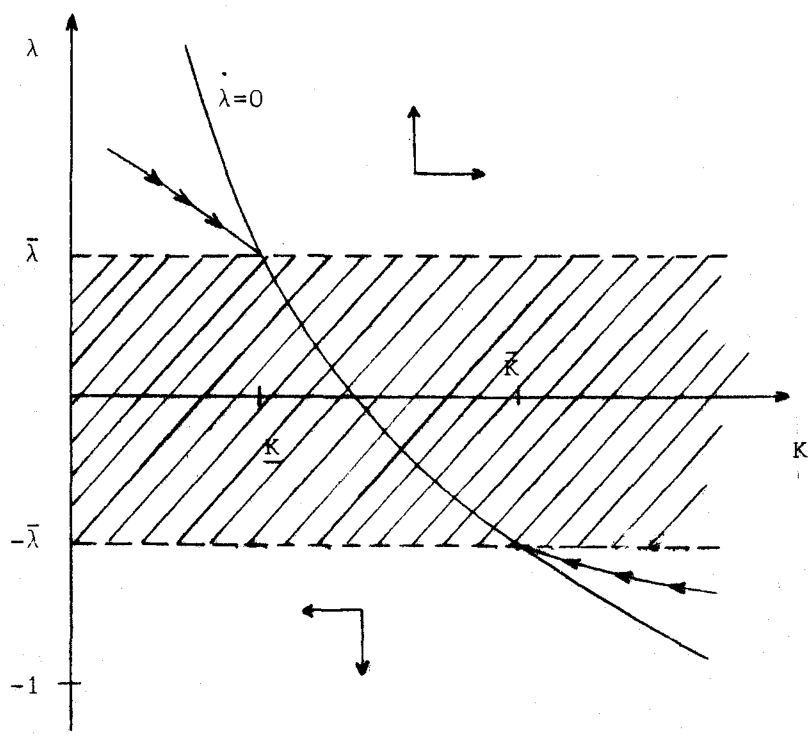

FIGURE 10

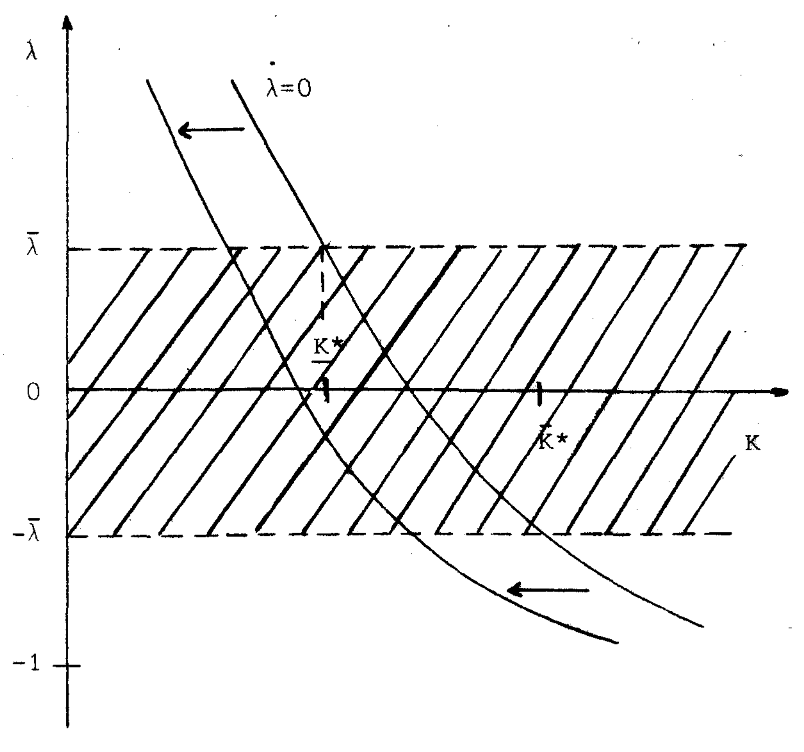

FIGURE 11

(ii) For $K_0<\underline{K}$, accumulation takes place along the unique stable arm leading to $\underline{K}$. For each K_0 there is a unique corresponding value of λ_0 (and, by (15), of I_0) so that the accumulation path leads to $\underline{K}$. Since $I\geq\bar{I}$ along the entire path, the target $\underline{K}$ is reached in finite time.

(iii) For $K_0>\bar{K}$, decumulation occurs along the unique stable arm, and the capital stock reaches the target $\bar{K}$ in finite time.

An interesting implication of this analysis is that discontinuous changes in p may have no effect on the steady-state equilibrium scale of the buyer industry. Thus, for example, if the buyer is currently in long-run equilibrium at $\underline{K}^*$ and there is a discontinuous increase in p, there is no adjustment as long as this shock is not "too large". Figure 11 illustrates this. The price shock shifts the $\dot{\lambda}=0$ isocline towards the vertical axis, but the previous equilibrium point remains an equilibrium point. Given the magnitude of the shift, the gains to decumulation are insufficient to offset the costs.

Finally, how does the solution under (A.3*) relate to the solution under (A.3")? The answer is that, since along the optimal investment path corresponding to (A.3*) $I\notin\mathcal{I}=\{I:-\bar{I}<I<\bar{I},I\neq0\}$ at any date, then a fortiori $I\notin\mathcal{I}$ under (A.3"), because total adjustment costs are greater over that range under the latter assumption. However, even if the solution under assumption (A.3*) does exhibit $I\in\mathcal{I}$ over a period of time (say because the capital stock evaporates and a gross rate of investment $0<I<\bar{I}$ is needed to sustain the steady-state capital stock), provided certain restrictions on the form of the adjustment cost function are met, it is still true that the value of the optimal programme under (A.3*) can be approximated to any desired degree of accuracy under (A.3") by adopting a "chattering" programme. This point is taken up in the Appendix.

5. MARKET EQUILIBRIUM UNDER COSTLY FACTOR ADJUSTMENT: CONVEX ADJUSTMENT COSTS

This section builds on the previous one by providing a detailed analysis of the equilibrium accumulation profile that takes explicit account of the constraint on the total supply of the resource. As argued in section 3, the resource price must, under assumptions (A.4) and (A.5), satisfy the Hotelling r per cent rule, with the initial price determined to ensure that the stock of the resource is just exhausted asymptotically. The analyses of the previous section that dealt with autonomous systems (because the resource price was assumed to remain constant over time) then need to be modified. This section does this for the case of strictly convex adjustment costs (assumption (A.3')).

As outlined in the previous section, for a continuous resource price trajectory $\{p(t)\}^{\infty}_{t=0}$ there is, under (A.1) and (A.3') a unique (and absolutely continuous) investment trajectory that optimizes (11). For completeness, the equations describing the optimal path are written out here:

(12) $\dot{K} = I$

(13) $\dot{I} = \{r(1+C'(I))-f(p,K)\}/C''(I)$

(14) $\lim_{t\to\infty} e^{-rt}C'(I)K = 0$; and

(18) $\dot{p}/p = r$,

with initial conditions $K(0)=K_0>0$ and $p(0)=p_0>0$ given. (18) is the Hotelling rule that now specifies the temporal behaviour of the resource price. The remainder of this section is concerned with characterizing the solution to this system and deriving some comparative dynamic results. Propositions 1 - 4 below summarize the principal attributes of the optimal programme. The capital stock and the rate of investment at date t along the optimal programme are occasionally denoted by $K(K_0,t)$ and $I(K_0,t)$, respectively, to emphasize that the solution depends on the stock of capital that the buyer industry has in place at the initial date.

Proposition 1 Along the optimal programme (i) $\lim_{t\to\infty} K(t)=0$; and (ii) $K(t)>0$ for all finite t.

Proof (i) Suppose to the contrary that $0 < \lim_{t\to\infty} K(t) = \hat{K} \leq +\infty$,[14] and pick $\Delta>0$ such that $\hat{K}-\Delta>0$. Then for large enough t, $K>\hat{K}-\Delta$ so $f(p,K)<f(p,\hat{K}-\Delta)$. But since $\lim_{t\to\infty} p(t)=+\infty$, p is arbitrarily large for arbitrarily large t. Thus $f(p,\hat{K}-\Delta)\equiv F_K(R(p,\hat{K}-\Delta),\hat{K}-\Delta))$ is by assumption (A.1) close to zero for sufficiently large t, and optimal motion is approximately described by

$$\dot{I} = r(1+C'(I))/C''(I).$$

Integration yields that

$$(19)\quad C'(I) = be^{rt} - 1$$

where b is a constant of integration. From (19) it is clear that the programme either violates the transversality condition (14) (if b>0), or contradicts the original supposition that $\lim_{t\to\infty} K(t)=\hat{K}>0$ (if $b\leq 0$).

(ii) Let $K(t_1)=0$ for finite $t_1>0$ with $K(t)>0$ $0\leq t_0<t<t_1$. It is easily shown that this contradicts the continuity requirement on the optimal investment programme. Since $K(t)>0$, $t_0<t<t_1$ and $\lim_{t\to t_1^-} K(t)=0$, assuming I(t) is well-behaved there exists $\varepsilon'>0$ such that $I(t-\varepsilon)<0$, $0<\varepsilon<\varepsilon'$. Now $\lim_{K\to 0} f(p,K)=+\infty$ for finite $p>0$, so

$$\lim_{t\to t_1^-} \dot{I}(t) = \lim_{t\to t_1^-} \frac{r(1+C'(I(t))) - f(p(t),K(t))}{C''(I)} = -\infty.$$

Thus

$$\lim_{t\to t_1^-} I(t) < 0 \leq \lim_{t\to t_1^+} I(t)$$

which establishes the required discontinuity. □

Corollary $\lim_{t\to\infty} I(t)=0$.

Next, proposition 2 states that an interval of zero or negative investment cannot be followed by an interval of positive investment.

Proposition 2 If $I(t')\leq 0$ for some $t'\geq 0$, $I(t)\leq 0$ $\forall t>t'$.

Proof Suppose to the contrary that $I(t'')>0$ for some $t''>t'$. By the continuity of the optimal investment programme, it is possible to choose t" such that $\dot{I}(t'')>0$. But then (13) implies that $I(t)>0$ for <u>all</u> $t>t''$, because the numerator on the right-hand side of (13) grows over time and the denominator remains positive. The asymptotic behaviour of the investment rate is thus given by (19) with $b>0$, and the programme violates (14), contradicting optimality. □

Propositions 3 and 4 now outline the relationship between the optimal path and the initial condition on the capital stock.

Proposition 3 For given $p_0>0$ $I(K_0,0)$ (the starting value of investment) is a continuous and everywhere declining function of K_0.

Proof Assumptions (A.1) and (A.3') ensure that the integrand in (11) is twice continuously differentiable and strictly concave in (K,I), $K>0$. This implies that the value function $V(K_0,0)$ (the maximum attainable value of (11) subject to (12)) is continuously differentiable and strictly concave in K_0 with

$$(20)\quad V_K(K_0,0) = C'(I(K_0,0)).$$

(see Benveniste and Scheinkman, 1979, section 3). That $I(K_0,0)$ is continuous in K_0 then follows from the continuity of V_K and C', and differentiating both sides of (20) with respect to K_0 establishes, using the strict concavity of V, that $I(K_0,0)$ is everywhere declining in K_0. □

Proposition 4 shows, moreover, that if the initial capital stock is "small" ("large"), the initial rate of investment is positive (negative).

Proposition 4 For given $p_0>0$, (i) $\lim_{K\to 0} I(K_0,0) > 0$; and (ii) $\lim_{K\to\infty} I(K_0,0) < 0$.

Proof Recall that $\lim_{K\to 0} f(p,K) = +\infty$ and $\lim_{K\to+\infty} f(p,K) = 0$ (assumption (A.6)).

(i) Integrating (13) and recalling that $\lim_{t\to\infty} I(t)=0$, the optimal programme $\{I(K_0,t),K(K_0,t)\}^{\infty}_{t=0}$ satisfies

$$(21)\quad 1 + C'(I(K_0,0)) = \int_0^{\infty} e^{-rt} f(p(t),K(K_0,t))\, dt.$$

Suppose then for a contradiction that $\lim_{K_0\to 0} I(K_0,0)\leq 0$. This implies that $\lim_{K_0\to 0} C'(I(K_0,0)) \leq 0$. Thus

$$1 \geq \lim_{K_0\to 0} \int_0^{\infty} e^{-rt}\, f(p(t),K(K_0,t))\, dt$$

$$> \lim_{K_0\to 0} \int_0^{T} e^{-rt}\, f(p(t),K(K_0,t))\, dt \quad \text{for arbitrary (finite) } T>0$$

$$> \lim_{K_0\to 0} f(p(T),K_0)(1-e^{-rT})/r = +\infty \text{ , a contradiction,}$$

where the last inequality follows from the fact that K is a nonincreasing function of time on $[0,\infty)$ ($I(K_0,0)\leq 0$ implies $I(K_0,t)\leq 0$ $\forall t>0$; see proposition 2) and p a strictly increasing function of time. This establishes that $I(K_0,0)>0$ if K_0 is sufficiently small.

(ii) Now suppose that $\lim_{K_0\to\infty} I(K_0,0)\geq 0$. This implies that $\lim_{K_0\to\infty} C'(I(K_0,0))\geq 0$. Since then for finite $p_0>0$

$$\lim_{K_0 \to \infty} \frac{\{r[1+C'(I(K_0,0))]-f(p_0,K_0)\}}{C''(I(K_0,0))} \geq \lim_{K_0 \to \infty} r/C''(I(K_0,0) > 0,$$

from (13) the rate of increase of investment at the initial date is positive for arbitrarily large K_0. I and $\dot{I}$ therefore increase without bound over time, which contradicts optimality. □

To sum up: the uniquely optimal programme that satisfies (12)-(14) and (18) for given p_0 and K_0 features, in general, an initial phase of accumulation followed by a phase of decumulation that runs the capital stock down to zero asymptotically (as the resource becomes prohibitively expensive). The initial phase exists if and only if K_0 is sufficiently "small" relative to the initial price of the resource p_0.

Figures 12.1 - 12.6 show, in general terms, the evolution of the solution path in (K,I) space for "small" K_0. Each Figure is to be viewed as a snapshot of the temporal path already completed by the variables at the date in question.[15] Thus Figure 12.1, which pertains to the initial date t=0 (at which date $p=p_0$) shows only the point A from which the phase path emanates. The capital stock $K_0{*}$ is given implicitly by $r=f(p_0,K_0{*})$ and represents an instantaneous "target" in the sense that it is the stock to which the buyer would wish to adjust instantaneously if there were no adjustment costs whatsoever associated with either investment or disinvestment. Alternatively, if the buyer believed that p were going to remain at p_0 in perpetuity (that is, if, contrary to the implications of assumption (A.5), the buyer had static expectations about the resource price), it would

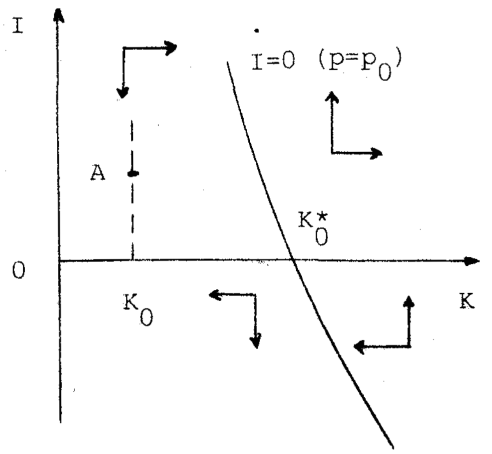

FIGURE 12.1

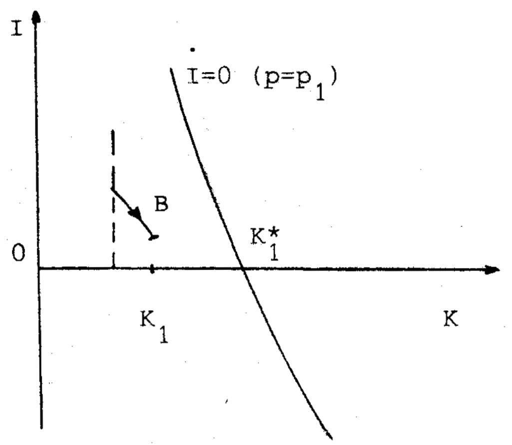

FIGURE 12.2

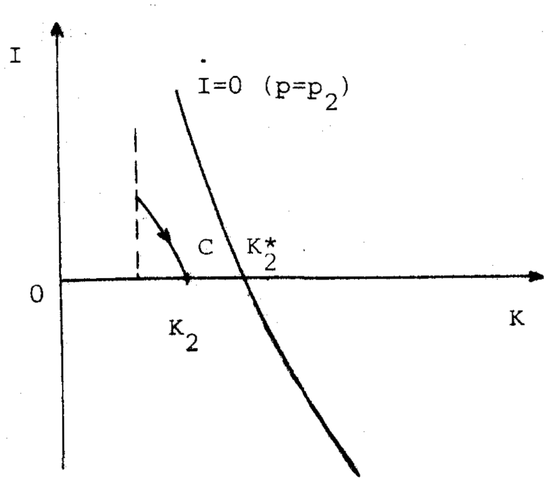

FIGURE 12.3

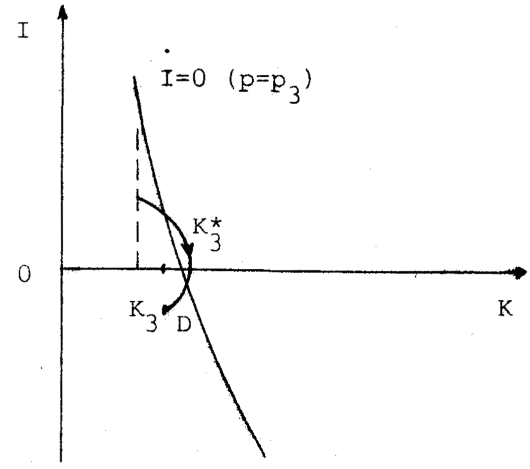

FIGURE 12.4

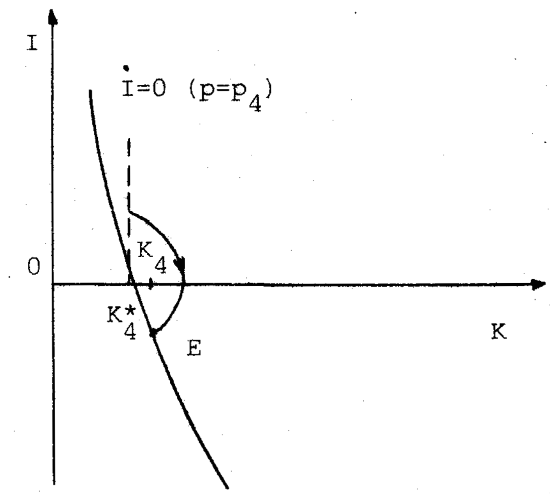

FIGURE 12.5

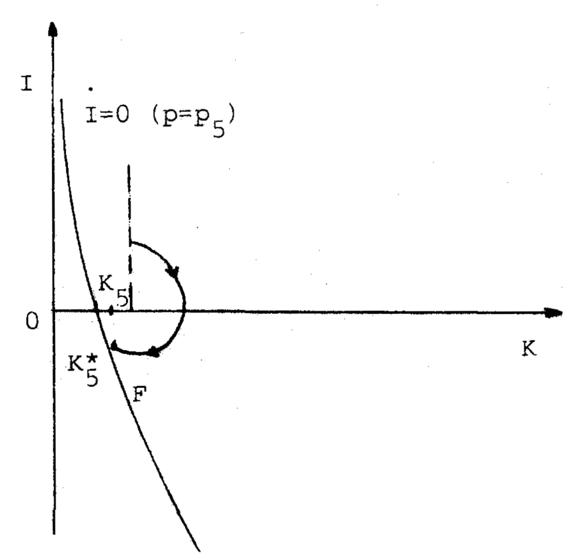

FIGURE 12.6

momentarily set itself on the unique path which, were p indeed to remain constant over time, would represent the stable arm leading to K_0*. Note that the initial rate of investment associated with this latter path is strictly larger than the initial investment rate associated with point A. This is perhaps intuitively appealing; that it is so is demonstrated below.

Figure 12.2 provides a picture at date $t=t_1>0$ (and corresponding resource price $p_1=p_0\exp(rt_1)>p_0$). The system is by then at point B, and the $\dot{I}=0$ locus has shifted leftward. Note again that the path on which the buyer would momentarily set itself if p were expected to be maintained at p_1 always (i.e., the path that, for p remaining fixed at p_1, is targeted to K_1*) emanates at a point that lies vertically <u>above</u> point B.

Figures 12.3 - 12.6 show the state of the system at dates $t=t_2$, t_3, t_4 and t_5 and corresponding resource prices p_2, p_3, p_4 and p_5. At date t_2, I changes sign. Both the path and the $\dot{I}=0$ locus then move leftward. The latter overtakes the former at date t_4 (point E in Figure 12.5) at which date $\dot{I}$ changes sign. After this date the path "chases" the $\dot{I}=0$ isocline, approaching the origin and reaching it asymptotically. Figure 13 shows the paths, as explicit functions of time, of the stock of capital and the rate of investment demand. It is noteworthy that one may observe a time-increasing rate of resource input on part of the initial phase $[0,t_2)$; although the resource price is increasing, the capital stock is also expanding.

As demonstrated in proposition 4, if (for given p_0) K_0 is "large" the phase during which accumulation takes place does not occur. This case also can be illustrated by referring to Figure

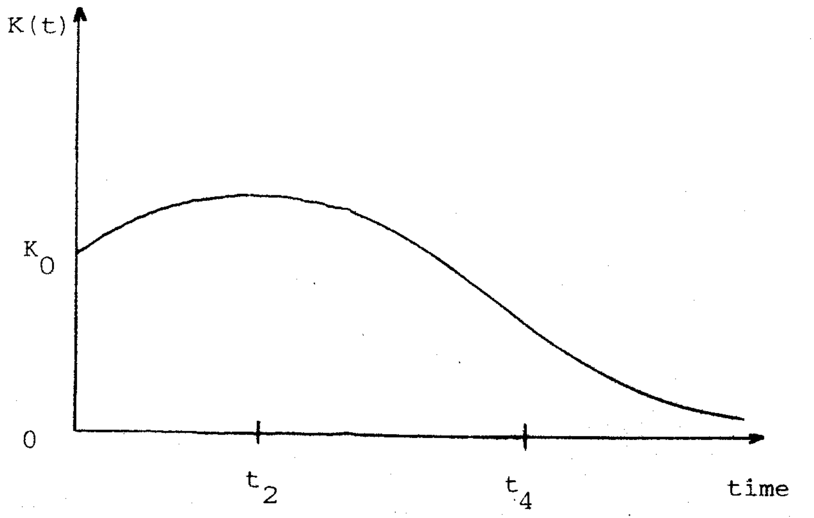

FIGURE 13.1

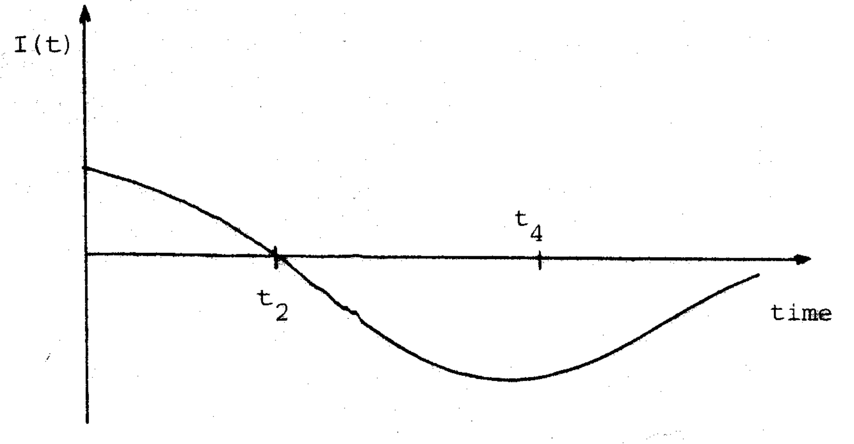

FIGURE 13.2

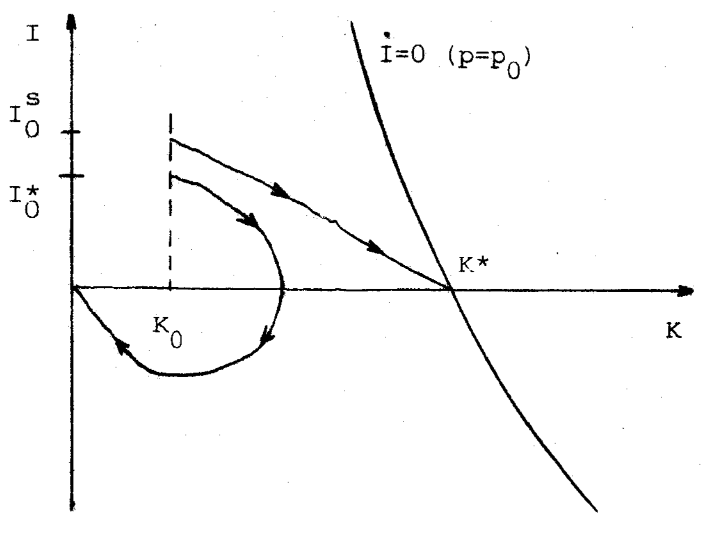

FIGURE 14

12. Because the path traced there is intertemporally consistent, it is clear that for an "initial" state such as ($p=p_3, K=K_3$) (Figure 12.4) the rate of investment along the optimal programme is negative and declining on an initial interval. Similarly, if the initial state were given by ($p=p_5, K=K_5$)(Figure 12.6), the investment rate could be negative but increasing over the entire programme.

Return now briefly to the earlier assertion that the expectation of a stable resource price in the future induces a larger current rate of investment demand than the expectation of a secularly increasing resource price. Suppose that the current ("initial") state is given by (K_0, p_0). In Figure 14, let $I_0^s = I^s(K_0, p_0)$ denote the current rate of investment chosen if the resource price is expected to remain at p_0. This is depicted as a point on the stable arm leading to K_0^*. Also label $I_0^* = I^*(K_0, p_0)$ the rate that is chosen if the resource price is expected to evolve according to (18). Suppose first, contrary to what is depicted, that $I^*(K_0, p_0) > I^s(K_0, p_0)$. Since the path starting at $I^*(K_0, p_0)$ must cross the horizontal axis at a value of K that is less than K_0^*, it must cross the phase path originating at I_0^s from above in the positive quadrant at least once. It must therefore have the lower (larger negative) slope of the two paths at that point of intersection. Now the slope of a phase path is given generically by

$$dI/dK = \dot{I}/\dot{K} = \{r(1+C'(I)) - f(p,K)\}/C''(I)I,$$

but because p is by assumption greater along the path starting at I_0^* (except in the initial state), this path must have the larger slope at any point of intersection in the positive quadrant. So the paths cannot cross in the manner required. The remaining possibility is that they coincide in the initial state. But then the slope of the I^* path exceeds the slope of the other path at the initial point, which again prevents it crossing the horizontal axis at a value of K that is less than K_0^* and thereby raises a contradiction. Thus $I^*(K_0,p_0)<I^s(K_0,p_0)$, and a symmetric argument shows that this holds equally if initial conditions are such that $I^s(K_0,p_0)<0$.

Since the "initial" state (K_0,p_0) was arbitrary, we have that $I^s(K,p)>I^*(K,p)$ for all states (K,p), $K,p>0$. In fact it is possible to deduce from this the following proposition:

Proposition 5 Let $K^s(t)$ denote the capital stock maintained at date $t\geq0$ by a buyer who expects $p(t)$ to prevail for $s>t$, (static expectations), and $K^*(t)$ the stock maintained by a buyer with perfect foresight (who expects $p(t)$ to evolve according to (18) for $s>t$). Then $K^s(t)>K^*(t)$, all (finite) $t>0$.

Proof Since $K^s(0)=K^*(0)=K_0$, $I^s(K^s(t),p(t))>I^*(K^*(t),p(t))$ for $0\leq t<\Delta$, some $\Delta>0$ and $K^s(t)>K^*(t)$ on an initial interval. Now let $t_1>0$ be the first (interior) date at which $K^s(t)=K^*(t)$. Since K^* and K^s are continuous functions of time, this requires that

$$I^s(K^s(t_1),p(t_1)) \leq I^*(K^*(t_1),p(t_1))$$

which contradicts the earlier result that $I^s(K,p)>I^*(K,p)$, $K,p>0$. □

The next line of enquiry is the nature of the relationship between the resource price trajectory and the optimal accumulation programme. Towards this, consider two alternative scenarios (call them 1 and 2). Along 1, the resource is always at least as expensive as along 2, and at some dates strictly more expensive. Proposition 6 asserts that the capital stock held by the buyer along 1 is never larger than the stock held along 2, and at some dates is strictly smaller.

Proposition 6 Consider two price profiles $\{p^1(t)\}^{\infty}_{t=0}$ and $\{p^2(t)\}^{\infty}_{t=0}$ satisfying $p^1(t) \geq p^2(t)$ for all t and $p^1(t)>p^2(t)$ for some t. If $K^1(0)=K^2(0)$, the associated solution paths for the capital stock, $\{K^1(t)\}^{\infty}_{t=0}$ and $\{K^2(t)\}^{\infty}_{t=0}$, satisfy $K^1(t) \leq K^2(t)$ for all t, and $K^1(t) < K^2(t)$ for some t (and, by continuity, over some interval of time).

Proof (i) Suppose first that $K^1(t) \geq K^2(t)$ for all t. This is easily contradicted to demonstrate that $K^1(t)<K^2(t)$ for some t>0. Recall that the accumulation programme satisfies, for $t \geq 0$,

$$(22)\quad 1 + C'(I(t)) = \int_t^{\infty} e^{-r(s-t)} f(p(s),K(s))\, ds$$

Since $f_p(p,K)<0$ and $f_K(p,K)<0$, $p^1(t) \geq p^2(t)$ for all t and $p^1(t)>p^2(t)$ for some t, it is clear that $f(p^1,K^1) \leq f(p^2,K^2)$ for all t, and $f(p^1,K^1) < f(p^2,K^2)$ for some t. From (22), and using $C''(I) > 0$, it follows that $I^1(t) \leq I^2(t)$ for all t, with $I^1(t) < I^2(t)$ for some t, which contradicts the initial supposition that $K^1(t) \geq K^2(t)$ for all t.

(ii) Suppose now, again for a contradiction, that $K^1(t)>K^2(t)$ for some t. By continuity, $K^1(t)>K^2(t)$ for $a>t>b$, some $0\leq a<b\leq+\infty$. By part (i) we cannot have $K^1(t)\geq K^2(t)$ for all t, so there are two possible cases:

<u>Case A</u> $a\geq 0$, $b<+\infty$. In this case it is possible to find t' and t", $a<t''<t'<b$, such that $I^1(t')=I^2(t')$ and $I^1(t'')>I^2(t'')$. Using this in (22) and taking differences,

$$(23)\quad \int_{t'}^{\infty} e^{-r(s-t')}\{f(p^1,K^1)-f(p^2,K^2)\}ds = 0.$$

But $\int_{t''}^{\infty} e^{-r(s-t'')}\{f(p^1,K^1)-f(p^2,K^2)\}ds > 0$, so that

$$(24)\quad e^{r(t'-t'')}\int_{t''}^{\infty} e^{-r(s-t'')}\{f(p^1,K^1)-f(p^2,K^2)\}ds > 0.$$

Subtracting (23) from (24) yields that

$$\int_{t''}^{t'} e^{-r(s-t')}\{f(p^1,K^1)-f(p^2,K^2)\}ds > 0,$$

a contradiction, because $p^1\geq p^2$ and $K^1>K^2$ on (t',t'').

<u>Case B</u> $a>0$, $b=+\infty$. Here it must be possible to find $t'>a$ such that $I^1(t')>I^2(t')$. This implies that

$$\int_{t'}^{\infty} e^{-r(s-t')}\{f(p^1,K^1)-f(p^2,K^2)\}ds > 0,$$

which cannot hold, since $p^1\geq p^2$ and $K^1>K^2$ on (t',∞). □

Corollary If $p^1(t)>p^2(t)$ for $t>0$ and $K_0^1=K_0^2$, then $K^1(t)<K^2(t)$ for all finite $t>0$.

Proof By Proposition 5, $K^1(t) \leq K^2(t)\ \forall t>0$. Suppose then for a contradiction that $K^1(t)=K^2(t)$, some $a<t<b$. It follows that $I^1(t)=I^2(t)$, $a<t<b$. Thus for t' and t'', $a<t'<t''<b$,

$$(25) \quad \int_{t'}^{\infty} e^{-r(s-t')}\{f(p^1,K^1)-f(p^2,K^2)\}ds = 0, \text{ and also}$$

$$(26) \quad \int_{t''}^{\infty} e^{-r(s-t'')}\{f(p^1,K^1)-f(p^2,K^2)\}ds = 0.$$

Multiplying (26) by $e^{r(t'-t'')}$ and subtracting (25) from it, one obtains

$$\int_{t''}^{t'} e^{-r(s-t')}\{f(p^1,K^1)-f(p^2,K^2)\}ds = 0,$$

which is impossible, since $p^1>p^2$ and $K^1=K^2$ on (t',t''). The one remaining possibility is that $K^1(t) \leq K^2(t)$, all $t>0$, with $K^1(t)=K^2(t)$ for a set of single points (of tangency). Let t' be one such point. Because we require $K^1(t)<K^2(t)$ in an ε-neighborhood on either side of t', it must be that $\dot{I}^1(t')<\dot{I}^2(t')$. From (13), one verifies that, because $K^1(t')=K^2(t')$, $I^1(t')=I^2(t')$ and $p^1(t')>p^2(t')$, this cannot hold. □

In particular, proposition 6 and its corollary imply that if $p^i(t)=p_{0}^{i}e^{rt}$ $t \geq 0$ (see (18)), then for two profiles such that $p_0^1>p_0^2$ and with the same initial stock of capital, the equilibrium capital stock is larger for all (finite) subsequent dates along profile 2.

The analysis thus far has taken $p(0)=p_0$ as given. How is the resource price at the initial date determined? The answer is that, under assumption (A.5), p_0 is set at that level which ensures that the stock of the resource is just exhausted asymptotically. That is, p_0 satisfies

$$(27) \quad \int_0^{\infty} R(p_0 e^{rt}, K(p_0, K_0, r, t))\, dt = S_0, \quad S_0>0 \text{ given,}$$

where, as previously, S_0 denotes the stock of the resource available at the start of the planning horizon, and the fact that $p(t)=p_0 e^{rt}$, $t \geq 0$, follows by integrating (18). The notation $K(p_0, K_0, r, t)$ now recognizes the dependence of the optimal path for the capital stock on the initial resource price and the interest rate, in addition to K_0 and the date in question. From the corollary to proposition 6, K is everywhere decreasing in p_0 for given r, K_0 and t.

It is intuitive from constraint (27) that p_0 is a continuous and monotonically declining function of S_0. To verify this, note that the solution to (12)-(14), (18) and (27) (with K_0 and S_0 given) is equally the (unique, under assumption (A.1)) solution to the problem of choosing, for $K(0)=K_0$ and $S(0)=S_0$, $\{R(t), I(t)\}_{t=0}^{\infty}$ to maximize

$$(28) \quad \int_0^{\infty} e^{-rt}\{F(R,K) - rK - C(I)\}\, dt,$$

subject to (12), and

$$(29) \quad \dot{S} = -R, \quad \lim_{t\to\infty} S(t) \geq 0,$$

with the current value multiplier function associated with (29) taken to be p. Express the maximum attainable value of (28) as $V(S_0,K_0)$. Under assumptions (A.1) and (A.3'), $V(S,K)$ is continuously differentiable and concave in (S,K) for $S,K>0$ (Benveniste and Scheinkman, 1979, section 3). Thus

$$V_S(S_0,K_0) = p_0(S_0,K_0)$$

is continuous in (S_0,K_0). (27) then confirms that p_0 is a strictly declining function of S_0 for given K_0, and also that $K(p_0,K_0,r,t)$ is continuous and strictly declining in p_0 for given K_0, r and t.

Finally, how is the optimal programme altered if the rate of interest during the planning horizon is different? Towards this, let programme 1 correspond to interest rate r_1, and programme 2 to interest rate r_2, $r_1>r_2$. Also let $S^i(t)$ denote the stock of the resource that remains at date t along programme i, i=1,2. It is assumed that $S^1(0)=S^2(0)$ and $K^1(0)=K^2(0)$. Although there appears to be no straightforward way of demonstrating that profile 2 is strongly more conservationist than profile 1 (that is, $S^1(t)<S^2(t)$ for <u>all</u> $t>0$ in this case, it is easy to show that $S^1(t)<S^2(t)$, $0<t<t_1$, for some t_1. That is, a larger interest rate induces greater profligacy "early on" in the programme; viewed from the initial date, therefore, programme 1 might be labelled (weakly) the less conservationist of the two.

The starting point is to show that, if $S^1(0)=S_2(0)$, then $p^2(0)>p^1(0)$, and so $p^2(t)=p^2(0)\exp(r_2t)>p^1(t)=p^1(0)\exp(r_1t)$ on an initial interval of time. Assume instead that $p^2(0)\leq p^1(0)$. Since

$r_2 < r_1$, this means that $p^2(t)=p^2(0)\exp(r_2 t)<p^1(t)=p^1(0)\exp(r_1 t)$ for $t>0$. If $K^1(0)=K^2(0)=K_0$, one infers by the corollary to proposition 6 that $K^2(t)>K^1(t)$, all $t>0$. But then

$$R(p^1(t),K^1(t)) < R(p^2(t),K^2(t)) \quad \text{all } t>0, \text{ so}$$

$$\int_0^\infty R(p^1(t),K^1(t))dt < \int_0^\infty R(p^2(t),K^2(t))dt,$$

and at least one of the depletion programmes is either inefficient or infeasible. This establishes that $p^2(0)>p^1(0)$. Thus $R(p^2(0),K_0)<R(p^1(0),K_0)$. The initial rate of resource use is therefore larger along the profile with the larger interest rate. Now because p and K are continuous on $[0,\infty)$ along either programme, and R is continuous in (p,K), it follows that R is continuous on $[0,\infty)$ along either programme. This implies that if $R^2(t)$ starts off below $R^1(t)$ at $t=0$, it remains below it for some time, say up to t_1. Thus for $0\leq t<t_1$

$$S^1(0)-S^1(t) = \int_0^t R(p^1(s),K(s))ds$$

$$> \int_0^t R(p^2(s),K(s))ds \equiv S^2(0)-S^2(t),$$

or $S^1(t)<S^2(t)$, as claimed. Note that the result is independent of the buyer's initial stock of capital; whatever value K_0 takes, a higher interest rate discourages conservation, at least in the weak sense.

In sum, this section has shown that under smooth and convex adjustment costs, the long-run tendency is for technologies (or "sub-technologies") that employ the resource in question to be gradually phased out. In a properly functioning competitive market, the transmission mechanism whereby this occurs is a steadily increasing real price for the resource. Underlying this are discounted rent-maximizing resource owners who ensure that in equilibrium marginal profit is the same at every date. However, if resource-using technologies are not in widespread use initially but the resource is sufficiently plentiful - as evidenced by a "sufficiently small" capital-resource stock ratio - a secular increase in the use of these technologies occurs for an initial period. Over part of this initial period, the rate of resource utilization may increase over time if the rate of expansion in the resource-using sector is sufficiently rapid.

6. MARKET EQUILIBRIUM UNDER COSTLY FACTOR ADJUSTMENT: NON-CONVEX ADJUSTMENT COSTS

It was argued in section 4B that the case of non-convex adjustment costs can be treated, under suitable assumptions, by focusing on the solution under a "convexified" adjustment cost function. Recall that the buyer's problem is one of choosing a time-profile of (dis)investment in order to optimize the objective function (11), subject to (12) and (18) (with $p(0)=p_0$ given), under assumption (A.3"). But suppose it is the case that the buyer is able to implement (costlessly) a policy of "chattering" between a zero rate of investment and that rate of investment (or disinvestment, as appropriate) at which unit adjustment costs are minimized. Then an insight into the solution of the optimization problem under assumption (A.3") is gained by looking at the solution to the optimization problem under (A.3*). This is because the latter can be approximated arbitrarily closely by the former, using a chattering policy that "averages out" the rate of investment over time. A formal justification of this statement is given in the Appendix.

What, then, are the general features of a solution to the optimization problem under (A.3*) when the resource price evolves according to the Hotelling rule (18)? Recall that a solution must satisfy the optimality condition

(15) $C'(I) = \lambda$ for $\lambda>\bar{\lambda}$, $\lambda<-\bar{\lambda}$;

$I\in[0,\bar{I}]$ for $\lambda=\bar{\lambda}$;

$I\in[-\bar{I},0]$ for $\lambda=-\bar{\lambda}$; and

$I=0$ for $-\bar{\lambda}<\lambda<\bar{\lambda}$

where $\bar{\lambda}=C'(\bar{I})$, $-\bar{\lambda}=C'(-\bar{I})$, as well as

$$(16)\quad \dot{\lambda} = r\lambda - f(p,K) + r \ ,$$

$$(18)\quad \dot{p}/p = r \ , \text{ and}$$

$$(12)\quad \dot{K} = I$$

with K_0 and p_0 given.

To deal with the problem piecewise, suppose first that $C(I) = \bar{\lambda}|I|$. That is, adjustment costs are linear throughout. Suppose also that $\bar{\lambda}<1$ (if $\bar{\lambda}>1$, disinvestment is never worthwhile, so any investment decision is effectively irreversible). Then the (current value) Hamiltonian function

$$H(p,K,I,\bar{\lambda},\lambda) = F(R(p,K),K) - rK - pR(p,K) - \bar{\lambda}|I| + \lambda I$$

is linear in the control variable I and condition (15) instead reads

$$(15')\quad \begin{aligned} &I=+\infty \quad \text{whenever } \lambda>\bar{\lambda}, \\ &I=-\infty \quad \text{whenever } \lambda<-\bar{\lambda}, \\ &I\in[0,\infty) \text{ if } \lambda=\bar{\lambda}, \\ &I\in(-\infty,0] \text{ if } \lambda=-\bar{\lambda} \\ &I=0 \quad \text{if } -\bar{\lambda}<\lambda<\bar{\lambda}. \end{aligned}$$

Consider the attributes of a programme that satisfies (12), (15'), (16) and (18). Points (i) - (iv) below go through these, and Proposition 7 provides a summary of them. To begin with, recall the "target" stocks $\underline{K}(p)$ and $\bar{K}(p)$ defined implicitly by

(17.1) $\underline{K}(p)$: $r(1+\bar{\lambda}) = f(p,\underline{K})$; and

(17.2) $\bar{K}(p)$: $r(1-\bar{\lambda}) = f(p,\bar{K})$.

(i) A positive (negative) jump in the capital stock arising from a momentarily infinite accumulation (decumulation) rate cannot be optimal except perhaps at the initial date $t=0$. This follows from the strict concavity of $\max_{I} H(p,K,I,\bar{\lambda},\lambda)$ in K for given p, $\bar{\lambda}$ and λ, and is demonstrated, for example, by Arrow and Kurz (1970, pp. 51-57). The strict concavity of the maximized Hamiltonian in K also ensures that the optimal programme is unique.

(ii) Along the optimal programme, any increase in the capital stock is effected by means of a jump (and can therefore occur only at $t=0$). For suppose instead that the rate of investment is positive (but finite) on some interval of time (t_1,t_2). Then $\lambda(t)=\bar{\lambda}$, or $\dot{\lambda}(t)=0$, $t_1<t<t_2$. Using (16), this requires that $f_p(p(t),K(t))\dot{p}(t)+f_K(p(t),K(t))I(t)=0$, $t_1<t<t_2$. Because $f_p<0$, $f_K<0$ and $\dot{p}>0$, the suppostion that $I(t)>0$, $t_1<t<t_2$, raises a contradiction. For $t>0$, the optimal programme thus features possible phases of zero investment (phase 2) and phases along which disinvestment takes place at a finite rate (phase 3).

(iii) Phase 2, if it exists, must always precede phase 3. The basic reason for this is that the resource price is a strictly increasing function of time. Suppose that this ordering is violated, and instead

$I(t)<0$, $t_0<t<t_1$; and

$I(t)=0$, $t_1\leq t<t_2$,

Then $\lambda(t)=-\bar{\lambda}$, or $\dot{\lambda}(t)=0$, and thus $K(t)=\bar{K}(p(t))$, $t_0<t<t_1$. Also, $-\bar{\lambda}\leq\lambda(t)\leq\bar{\lambda}$, $t_1\leq t<t_2$. In particular, because λ and K are continuous, $\lambda(t_1)=-\bar{\lambda}$ and $K(t_1)=\bar{K}(p(t_1))$. Now assume - this is verified shortly - that t_2 is finite. Then $\lambda(t_2)=-\bar{\lambda}$, and $\dot{\lambda}(t'')\leq 0$ for some $t_1<t''<t_2$ by the continuity of $\lambda(t)$. It is clear also that $\lim_{t\to t_1^+}\lambda(t)\geq 0$. Then, from (16),

$$\lim_{t\to t_1^+}\dot{\lambda}(t') = -r\bar{\lambda} - f(p(t_1),K(t_1)) + r \geq 0, \text{ and}$$

$$\dot{\lambda}(t'') = r\lambda(t'') - f(p(t''),K(t'')) + r \leq 0.$$

Subtracting the first of these from the second yields that

$$r\{\lambda(t'')+\bar{\lambda}\} - \{f(p(t''),K(t''))-f(p(t_1),K(t_1))\} \leq 0.$$

The first term in curly brackets is non-negative, and the second term in curly brackets negative (because $p(t'')>p(t_1)$ and $K(t'')=K(t_1)$). This contradicts the proposed inequality. It remains to confirm that t_2 is indeed finite. Suppose otherwise; then, integrating (16)

$$\lambda(t_1) = \int_{t_1}^{\infty} e^{-r(t-t_1)}\{f(p(t),K(p(t_1)) - r\}dt$$

$$< \frac{\{f(p(t_1),K(p(t_1))) - r\}}{r} = -\bar{\lambda}$$

again a contradiction.

Thus the phase of decumulation must be the last. Note that during this phase $K(t)=\bar{K}(p(t))$. Note also that $\lim_{t\to\infty} K(t) = \lim_{t\to\infty} \bar{K}(p(t))=0$.

(iv) Under what circumstances is it optimal to effect a discontinuous increase or decrease in the capital stock at the initial date? If K_0 denotes the buyer's initial stock of capital (before any adjustment is made), let $K^+(0)$ denote the capital stock at the initial date _after_ a jump - if desired - has been effected. Now suppose first that $K_0 > \bar{K}(p_0)$ (the "upper target"). In this case, the buyer does best by disposing of a block of capital at the initial date. It sets $K^+(0)=\bar{K}(p_0)$ and disinvests "smoothly" along the upper target stock $\bar{K}(p(t))$ at subsequent dates. Basically, it pays to jump down to the target stock along which disinvestment occurs because the target is going to decline monotonically in any case. The story is not quite symmetric, as one might expect, where adding a block of capital at the initial date is concerned.

Suppose next that $K_0 \leq \bar{K}(p_0)$. Recalling that phase 2 - if it exists - precedes phase 3, define K^* and T^* such that

$$\bar{\lambda} = \int_0^{T^*} e^{-rt}\{f(p(t),K^*) - r\}dt + \int_{T^*}^{\infty} e^{-rt}\{f(p(t),\bar{K}(p(t))) - r\}dt$$

and

$$K^* = \bar{K}(p(T^*)) = \bar{K}(p_0 \exp(rT^*)).$$

Now $T^*>0$ because in fact $K^* < \underline{K}(p_0) < \bar{K}(p_0)$. To see this, note that

$$\bar{\lambda} = \int_0^{T^*} e^{-rt}\{f(p(t),K^*) - r\}dt - \bar{\lambda}\exp(-rT^*)$$

(because $f(p(t),\bar{K}(p(t))) - r = -r\bar{\lambda}$). Now since $f_p<0$ everywhere and p is increasing on $(0,T^*)$,

$$\bar{\lambda} < \{f(p_0,K^*) - r\}\frac{(1 - \exp(-rT^*))}{r} - \bar{\lambda}\exp(-rT^*); \text{ or}$$

$$r\bar{\lambda} < \{f(p_0,K^*) - r\}\frac{(1 - \exp(-rT^*))}{(1 + \exp(-rT^*))} < f(p_0,K^*) - r.$$

But $r\bar{\lambda} = f(p_0,\underline{K}(p_0)) - r$, so that

$$f(p_0,\underline{K}(p_0)) < f(p_0,K^*),$$

which implies that $K^*<\underline{K}(p_0)$, as asserted.

Using the above definition, we can now state the following: if $K^*\leq K_0\leq\bar{K}(p_0)$, then optimally $K^+(0)=K_0$. That is, no jump in the capital stock takes place. The buyer just waits until the resource price has risen sufficiently so that the "upper target", $\bar{K}(p)$, coincides with K_0, then enters phase 3. However, if $0<K_0<K^*$, it pays to add a block of capital at the initial date by setting $K^+(0)=K^*$. Thereafter the buyer again sets investment equal to zero on $(0,T^*)$. $\bar{K}(p)$ reaches K^* at T^* and disinvestment takes place on (T^*,∞).

To verify these assertions, let $0<K\leq\bar{K}(p_0)$, and define

$$\lambda(K) = \int_0^{T(K)} e^{-rt}\{f(p(t),K) - r\}dt + \int_{T(K)}^{\infty} e^{-rt}\{f(p(t),\bar{K}(p(t))) - r\}dt$$

where $T=T(K)$ solves $r(1-\bar{\lambda})=f(p_0e^{rT},K)$. It is straightforward to confirm that $T(K)$ is everywhere decreasing and that

$$\lim_{K\to 0} T(K) = +\infty \quad \text{and} \quad \lim_{K\to K(p_0)} T(K) = 0.$$

Consequently $\lambda(K)$ is everywhere decreasing with

$$\lim_{K\to 0} \lambda(K) = +\infty \quad \text{and} \quad \lim_{K\to K(p_0)} \lambda(K) = -\bar{\lambda}$$

with $\lambda(K^*)=\bar{\lambda}$. Thus for $K^* \leq K_0 \leq \bar{K}(p_0)$, $-\bar{\lambda} \leq \lambda(K_0) \leq \bar{\lambda}$, and no jump takes place. Conversely, for $0<K_0<K^*$, $\lambda(K_0)>\bar{\lambda}$, and optimality requires a momentarily infinite rate of accumulation.

The results in (i) - (iv) above are summarized in Proposition 7:

Proposition 7 The unique solution to (12), (15'), (16) and (18) is characterized as follows. There exists a stock K^*, $0<K^*<\underline{K}(p_0)<\bar{K}(p_0)$, such that:

(a) If $K_0<K^*$, the buyer effects an upward jump in the capital stock to $K^+(0)=K^*$ at the initial date. On an initial interval $(0,T^*)$, where T^* is defined above, the rate of investment is set equal to zero. For $t>T^*$, disinvestment occurs on the target path $\bar{K}(p(t))$.

(b) If $K^* \leq K_0 \leq \bar{K}(p_0)$, there is no jump in the capital stock. On $(0,T)$, where $0 \leq T \leq T^*$ is defined so that $K_0=\bar{K}(p_0 e^{rT})$, the rate of investment is zero. After T the buyer disinvests to stay on the path $\bar{K}(p(t))$.

(c) If $K_0>\bar{K}(p_0)$, there is a downward jump in the capital stock to $K^+(0)=\bar{K}(p_0)$. On $(0,\infty)$ the capital stock is run down along the path $\bar{K}(p(t))$.

Irrespective of the initial conditions K_0 and p_0, the capital stock dwindles to zero asymptotically.

Return now to the case where the adjustment cost function "turns up" (assumption (A.3*), so that the optimality condition is given by (15) and not (15'). Points (i) - (iii) below clarify the main properties of the optimal programme in this case, and these are summarized in proposition 8.

(i) "Bang-bang" adjustment does not occur at any time. This is because by assumption it is infinitely costly to effect an infinite rate of accumulation (or decumulation), even momentarily.

(ii) Let phase 1 refer to an interval of time on which accumulation occurs, phase 2 an interval of zero investment, and phase 3 an interval on which decumulation occurs. Then along the optimal programme (again unique by the strict concavity of the relevant maximized Hamiltonian) phases must occur in the order (1,2,3), with phase 3 as the final phase. The argument that was used earlier to rule out a transition from phase 3 to phase 2 applies equally here. Furthermore, since on phase 3 $\lambda \leq -\bar{\lambda}$, while on phase 1 $\lambda \geq \bar{\lambda}$, a transition from the former to the latter is ruled out by the continuity of $\lambda(t)$. Phase 2 must therefore intercede. But a transition from phase 3 to phase 2 has already been ruled out.

It remains then to rule out a transition from phase 2 to phase 1. Suppose instead that

$I(t)=0, \quad t_0<t<t_1 \quad$ (so $-\bar{\lambda} \leq \lambda \leq \bar{\lambda}$ on this interval), and

$I(t)>0, \quad t_1<t<t_2 \quad$ (so $\lambda \geq \bar{\lambda}$ on this interval).

Since λ is continuous and piecewise continuously differentiable on $[0,\infty)$, we can find $t' \in (t_0, t_1)$ and $t'' \in (t_1, t_2)$ such that $\dot{\lambda}(t') \geq 0$ and $\dot{\lambda}(t'') \leq 0$ (the latter follows because t_2 is finite, so λ must be nonincreasing at some time in the interval (t_1, t_2)). Substituting for $\dot{\lambda}(t')$ and $\dot{\lambda}(t'')$ from (16) and subtracting the latter from the former yields that

$$\dot{\lambda}(t')-\dot{\lambda}(t'') = r\{\lambda(t')-\lambda(t'')\} - \{f(p(t'),K(t'))-f(p(t''),K(t''))\}\geq 0.$$

But the first term in curly brackets is negative, and the second positive, so this inequality cannot hold. Therefore phase 1 - if it exists - precedes phase 2 in an optimal programme, and phase 2 in turn leads phase 3, as asserted originally.

(iii) Phase 1 exists if (and only if) for given p_0, K_0 is "sufficiently small". Broadly speaking, given that the "target" stock of capital declines monotonically to zero in the long run in any case, it does not pay to add any capital unless the marginal value product stream is large enough to offset the disinvestment costs that will have to be incurred later. Further, if it does pay to expand the capital stock, it pays to do so promptly, and it is intuitive that the rate of investment will exceed the value $\bar{I}$, the rate at which unit adjustment costs begin to rise.

To verify these assertions, suppose first that, even though K_0 is arbitrarily small, phase 1 is not part of the optimal programme. Let $\tau\geq 0$ be the transition date from phase 2 to phase 3. Integrating (16) and using the result, we have

$$\lambda(K_0) = \int_0^{\tau} e^{-rt}\{f(p(t),K_0) - r\}dt + \int_{\tau}^{\infty} e^{-rt}\{f(p(t),K(t)) - r\}dt$$

$$= \int_0^{\tau} e^{-rt}f(p(t),K_0)dt + \int_{\tau}^{\infty} e^{-rt}f(p(t),K(t))dt - 1.$$

Now because $K(t)<K_0$, $t>T$, and $f_K<0$ everywhere

$$\lambda(K_0) > \int_0^{\infty} e^{-rt} f(p(t),K_0)dt - 1$$

$$> \int_0^{T} e^{-rt} f(p(t),K_0)dt - 1 \quad \text{for any (finite) } T>0$$

$$> f(p(T),K_0)\frac{(1 - \exp(-rT))}{r} - 1.$$

But the right-hand side is arbitrarily large for arbitrarily small K_0, which confirms that $\lambda(K_0)>\bar{\lambda}$ for sufficiently small K_0. Phase 1 thus exists provided K_0 is small enough.

Conversely, if $K_0>K^*$, where K^* has been defined in Proposition 7, phase 1 must be absent. To see this, recall that, for $K_0>K^*$, it did not pay to add to the capital stock when the unit adjustment cost was given by $\bar{\lambda}$ for any non-zero value of I. A fortiori, then, it does not pay to do so here (cet. par.) because the unit adjustment cost is at least as large as $\bar{\lambda}$ for all non-zero values of I, and strictly larger than $\bar{\lambda}$ for $I<-\bar{I}$ and $I>\bar{I}$.

It remains to establish that the rate of investment is larger than $\bar{I}$ during phase 1. This is done by showing that λ cannot equal $\bar{\lambda}$ for longer than an isolated instant of time. For if instead $\lambda(t)=\bar{\lambda}$, $t_0<t<t_1$, then according to (16) $f(p,K)$ must be constant on that interval. But clearly $f_p\dot{p} + f_K\dot{K} < 0$, because $\dot{p}>0$ $f_p<0$, $f_K<0$, and by assumption $\dot{K}>0$, so this cannot hold.

Finally, if the initial stock of capital is, for given p_0, sufficiently large, the optimal programme features only phase 3.

For example, $K_0 > \bar{K}(p_0)$ is a sufficient condition for the absence of phases 1 and 2. If unit adjustment costs were equal to $\bar{\lambda}$ for all non zero values of I, the buyer would jump to $\bar{K}(p_0)$ immediately. Here this is prohibitively costly, but it clearly pays to push the disinvestment rate initially at least up to the point $(-\bar{I})$ at which unit adjustment costs begin to rise. Thus the initial and only phase is one of disinvestment.

From these points, the basic features of the solution to (12), (15), (16) and (18) are summarized in Propostion 8:

Proposition 8 The unique solution to (12), (15), (16) and (18) exhibits, in general, three phases, in the following chronological order. During phase 1 the investment rate is positive and exceeds $\bar{I}$. In phase 2, investment is zero, and in the final phase (3), disinvestment takes place. Phase 1 exists if the capital stock is "small enough", but $K_0 > K^*$ is a sufficient condition for its absence. $K_0 > \bar{K}(p_0)$ is a sufficient condition for the absence of both phases 1 and 2.

Return finally to the original assumption (A.3") that the adjustment cost function is non-convex on $\mathbb{I} = \{I: -\bar{I} \leq I \leq \bar{I},\ I \neq 0\}$. Recall that the solution under the convexified cost function assumption (A.3*) requires that $I(t) \in \mathbb{I}$ only during phase 3, when the capital stock is run down along the target $\bar{K}(p(t))$. As the capital stock approaches zero asymptotically, it must be the case that $-\bar{I} < I(t) < 0$ on a final interval of time.

The Appendix shows that this part of the convexified solution can still be approximated arbitrarily closely - at arbitrarily close cost - under assumption (A.3"). This is done by alternating rapidly between a disinvestment rate of zero and a disinvestment rate of $-\bar{I}$. Thus, provided this policy of rapid switching does not of itself entail any costs, proposition 8 applies equally to the solution under assumption (A.3"), except that disinvestment in "pulses" rather than a smooth disinvestment profile would be observed.

It may be, however, that the buyer must bear a non-negligible "startup" cost every time a disinvestment phase begins after the disinvestment rate has been reduced to zero. It can be noted immediately that this is of no consequence provided that $\lim_{I \to 0^-} C(I)=0$ (as in a mirror image of Figure 8.1). In this case the buyer can still support a "chattering" disinvestment profile by alternating with arbitrary rapididty between a disinvestment rate of $-\bar{I}$ and a disinvestment rate of $-\varepsilon$, where ε is an arbitrarily small positive number. By so doing the buyer avoids "shutting down" the disinvestment operation and hence avoids incurring the start-up costs (except once, at the start of the disinvestment phase).[16]

If, on the other hand, there are flow fixed costs of (dis)investment (that is, $\lim_{I \to 0^-} C(I)>0$, as in a mirror image of Figure 8.2), the policy of ε-chatter (which avoids the startup cost) does incur the flow fixed cost. As is intuitive, whether the solution under assumption (A.3") features a policy of ε-chatter or shutdown-startup depends on the relative magnitudes of the flow fixed costs and the startup costs (see Davidson and Harris, 1981, section 4, especially Theorem 3).[17] Of course it is no longer possible to approximate the convexified solution at arbitrarily close cost, and in general the <u>entire</u> programme of decumulation and depletion under assumption (A.3") will differ from the convexified solution. In particular, it is not clear that the buyer's optimal disinvestment policy involves reducing the capital stock to zero in the long run.

Thus, although it is of potential importance, this added difficulty introduces a number of complications into the analysis and is not taken up here. That is, it is supposed that either startup costs are negligible, or that there are no flow fixed adjustment costs. Under these circumstances, the solution under assumption (A.3*) also gives the optimal programme under assumption (A.3"). The results under non-convex adjustment costs are then broadly similar to those that emerge under strictly convex adjustment costs, and it can be shown similarly for this case that the initial price of the resource is inversely related to the stock available initially, that the equilibrium capital stock at any date is non-increasing in p_0, and that a larger interest rate results in a quicker depletion rate on an initial interval of time.

There are three basic differences between the results in this section and those under strictly convex adjustment costs (assumption (A.3')): (i) on phase 1, the observed investment rate is "large" throughout in the present case (assumption (A.3")); (ii) phase 2 (the interval of zero investment) occurs in the present case but not under assumption (A.3'); and (iii) the (dis)investment programme under (A.3") is "lumpy", at least on a final interval of time, whereas it is smooth under assumption (A.3'). Roughly speaking, the capital stock expands more rapidly (or falls less rapidly) initially in the non-convex case. For a given resource price path, demand for the resource declines less quickly initially.

7. CONCLUDING COMMENTS

The cornerstone of the analysis of nonrenewable resource supply is the intertemporal dependence in supply decisions. This paper has drawn on factor demand analysis under adjustment costs to rationalize an intertemporal dependence in resource demand decisions. Thus in the present framework resource users as well as resource sellers peer at the entire future path of extraction and prices when making plans. The expectations of both sets of agents now determine the equilibrium depletion profile in a market economy, although the role of expectations has been downplayed here by the assumption of universal long-run perfect foresight. Nonetheless, there is a two-way interdependence between the entire profile of demand and the entire profile of supply for the resource.

The long-run properties of an equilibrium depletion profile in this model are broadly similar to those under static resource demand conditions. Under the price-taking assumption, the Hotelling r per cent price growth rule remains valid, and resource use falls to zero in the long run as the resource becomes prohibitively expensive (although resource use may rise over time early on in the programme as resource-using industries expand). This broad tendency is invariant to the shape of the adjustment cost function. The effect of the type of demand structure considered here - underlying which are adjustment costs associated with accumulation or decumulation - is to "flatten out" the path of resource use in comparison with the case where capital adjustment is costless. It also endogenizes sluggish or limited responses to unanticipated price shocks, although these are difficult to account for convincingly in the simple model developed here.

There are two broad issues in the model developed in this paper that merit attention in future research. The first of these concerns the assumption that the stock of capital equipment used by the buyer is homogeneous. A better approach would be to identify different vintages of equipment by the amount of the resource needed to produce a unit of output using a unit of capital, and to specify a functional relationship between the resource-efficiency of the current vintage and - say - cumulative R&D expenditure. However, because of the large number of control variables involved (the investment rate, the "cutoff" vintage in use at any date, and the rate of expenditure on R&D) this approach is cumbersome, and except in special cases would not appear to yield many conceptual insights.

The other issue that is worth exploring further concerns the implications of dropping the price-taking assumption, particularly on the resource supply side.[18] To take a polar example, suppose that the resource stock is controlled by a monopolist - who behaves as a Stackelberg leader - and that resource users are price takers with the type of structure considered in this chapter. It is intuitive that the monopolist will in general have an incentive to deviate from the price profile that it announces initially (once it has fooled buyers into building up their capital stock). If buyers perceive this, the appropriate equilibrium concept depends on whether or not the monopolist is able to make a binding commitment at the start of the programme. Again, however, it would appear that the prospects for getting general analytical results are severely circumscribed by the complexity of the problem.

APPENDIX

Denote by V^* the value of the optimal investment programme under assumption (A.3*) ("convexified" adjustment costs). Let $K^*(t)$, $t \geq 0$ denote the optimal capital stock along this programme, and $I^*(t)$ its time-derivative (if it exists) at t. Also let V^c denote the value of the optimal investment programme under assumption (A.3") (non-convex adjustment costs), and $K^c(t)$ and $I^c(t)$ the associated capital stock and investment rate, respectively, at t. This Appendix provides a justification for the claim in sections 4B and 6 that V^c can be made arbitrarily close to V^* through the use of a "chatter" policy if $I^*(t)$ enters the range of decreasing unit adjustment costs.

For $I < -\bar{I}$ and $I > \bar{I}$, the adjustment costs under (A.3*) and (A.3") coincide by construction. Thus for $t \geq 0$ such that $I^*(t) < -\bar{I}$ or $I^*(t) > \bar{I}$ $I^c(t)$ can be set equal to $I^*(t)$. The problem arises if $-\bar{I} < I < \bar{I}$. Over this range adjustment costs under (A.3") everywhere exceed those under (A.3*). Suppose then that $-\bar{I} < I^*(t) < \bar{I}$ on a given interval of time. The objective is to show that on this interval $K^c(t)$ can be made arbitrarily close to $K^*(t)$, and the PV of costs associated with the investment programme $I^c(t)$ arbitrarily close to the PV of costs associated with $I^*(t)$.

Choose t_0 and t_1 sufficiently close together such that $K^*(t)$ is strictly increasing with $0 < I^*(t) < \bar{I}$ on (t_0, t_1). The argument for $K^*(t)$ decreasing on (t_0, t_1) is symmetric. Without loss of generality, let (t_0, t_1) be an interval of unit length (see Figure a.1). Define

$$D^* = \int_0^1 e^{-rt}\bar{\lambda}\, I^*(t)dt = \bar{\lambda}\int_0^1 e^{-rt}dK^*(t)$$

to be the PV of adjustment costs over the interval of sustaining the path $K^*(t)$. Now partition the unit interval into $n>1$ subintervals of equal length, and consider the alternative path for the capital stock given by

$$K^A(t) = K^*(i-1/n),\ i-1/n \leq t < i,\quad i = 1/n,\ 2/n\ ,..,\ n-1/n,\ 1.$$

The PV of adjustment costs over the interval associated with this path is

$$D^A(1/n) = \bar{\lambda}\sum_{i=1/n}^{1} e^{-ri}\ \{K^*(i) - K^*(i-1/n)\}\quad .$$

Finally, suppose that $I^c(t)$ is chosen as follows over the interval:

$I^c(t)=0,\quad i-1/n \leq t < i-(1/n)(1-\alpha(i))$, and

$I^c(t)=\bar{I},\quad i-(1/n)(1-\alpha(i)) \leq t < i,$

where $0\leq\alpha(i)\leq1$ satisfies

$$\text{(a.1)}\ K^*(i)-K^*(i-1/n) = \int_{i-1/n}^{i-(1/n)(1-\alpha(i))} 0\ dt + \int_{i-(1/n)(1-\alpha(i))}^{i} \bar{I}\ dt = \bar{I}\{1-\alpha(i)\}\frac{1}{n},$$

$i=1/n,2/n,..,1$. The PV of adjustment costs over the interval associated with the latter path is

$$D^c(1/n) = \sum_{i=1/n}^{1}\int_{i-(1/n)(1-\alpha(i))}^{i} e^{-rt}\ \bar{\lambda}\ \bar{I}\ dt$$

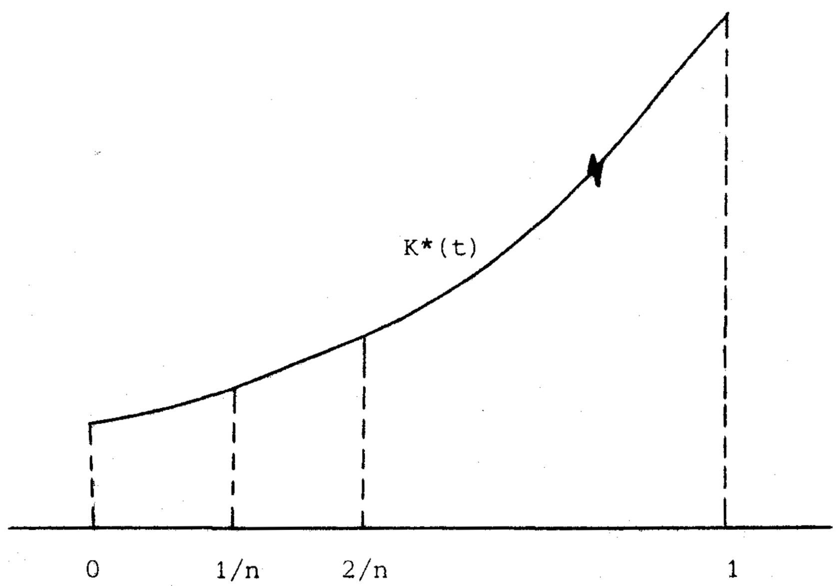

FIGURE a.1

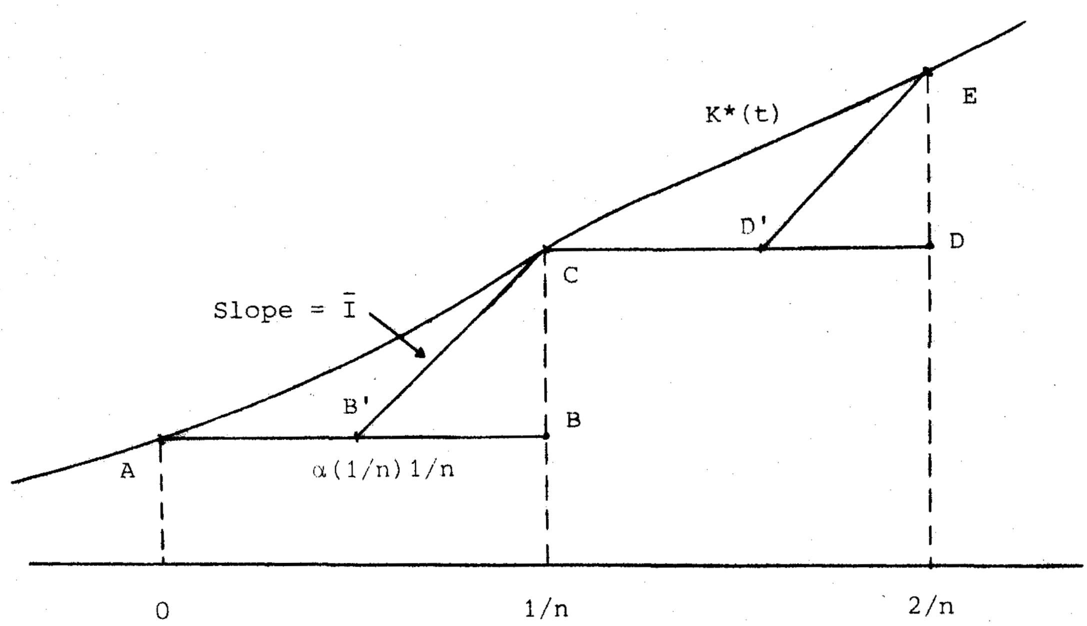

FIGURE a.2

$$= \bar{\lambda} \sum_{i=1/n}^{1} \int_{i-(1/n)(1-\alpha(i))}^{i} e^{-rt} \, dt \, \frac{\{K^*(i) - K^*(i-1/n)\}}{\{1-\alpha(i)\}(1/n)} .$$

Figure a.2 compares the paths $K^A(t)$ and $K^c(t)$ with $K^*(t)$ over a magnified section of the interval. $K^A(t)$ is the step function traced by the points ABCDE, and $K^c(t)$ the "slanted" step function traced by AB'CD'E. Now it is clear that for $n \geq 1$, no matter how large,

$$D^* > D^c(1/n) > D^A(1/n).$$

But, according to the definition of a Riemann-Stieltjes integral

$$\lim_{n \to \infty} D^A(1/n) = D^*,$$

whence it follows that

$$\lim_{n \to \infty} D^c(1/n) = D^*$$

provided the limits exist. In other words, a policy of "chattering" sufficiently (in the limit, infinitely) frequently between $I^c = 0$ and $I^c = \bar{I}$ ensures not only that K^c approximates K^* to any desired degree of accuracy but also that the associated discounted adjustment cost streams are arbitrarily close to one another.

NOTES

1. Berndt et al. (1981) provide a comprehensive survey of the development of the theoretical underpinnings for empirical work on energy demand. Early ("first generation") econometric work essentially superimposed arbitrary short-term adjustment structures on long-run static models and looked at the demand for energy largely in isolation from the demand for other inputs. Later ("second generation") studies recognized and took account of the interdependencies in input demands. More recent ("third generation") work has been based explicitly on input demand functions derived from intertemporal cost minimization exercises and has yielded considerable improvements in the results.

2. Except of course insofar as storage of the resource is feasible and future price movements are likely to make hoarding for capital gains worthwhile.

3. This assumption is made for convenience. It implies that the resource-using "industry" (appropriately defined) contributes only an incremental amount to the economy-wide flow of final output.

4. "Productive capital" and "final output" are therefore one and the same commodity.

5. For example, it is readily verified that this is so for any CES technology homogeneous of degree $v<1$ provided that the elasticity of substitution between capital and the resource does not exceed unity by too much. Arguing along similar lines to those of Berndt and Wood (1979), $F(R,K)$ might be interpreted here as a production "sub-function" that represents processes that use the resource ("utilized oil-fired boilers", for example). Using this type of approach, Berndt and Wood show that the econometric evidence points largely to a negative cross-price elasticity between energy and capital.

6. Naturally if the resource-using sector is at all "large", one would expect a secular decline in the rate of return outside it as it contracts.

7. Reversing the operations of differentiation and integration is legitimate provided that $R(p,r)$ is jointly continuous in its arguments and p is a continuous function of time.

8. Clearly for $t \geq T$, $R(p_1(t),r_1) > R(p_2(t),r_2)$ (recall $r_2 > r_1$). Thus

$$S^1(t) = \int_t^{\infty} R(p^1(s),r_1)ds > \int_t^{\infty} R(p^2(s),r_2)ds = S^2(t),$$

implying that profile 1 has larger remaining reserves. Because

$S_1(0)=S_2(0)$, it follows that

$$\int_0^T R(p_1(s),r_1)ds < \int_0^T R(p_2(s),r_2)ds,$$

and thus profile 1 depletes a smaller total on (0,T), implying greater conservation "on average" during this time interval. However, R_1 and R_2 cannot be unambiguously ranked on (0,T), so the possibility that $S_1(t)<S_2(t)$ for some $0<t<T$ cannot be ruled out in general, though it can be ruled out where the buyer's technology is Cobb-Douglas.

9. Note, however, that the factor demand equations are degenerate in the case of constant returns to scale (i.e., $\alpha+\beta = 1$). In this case the marginal product of each factor depends only on the ratio of factors. The fixity of r, the marginal product of capital, then implies a constant input ratio. This implies that p, the marginal product of the resource, remains constant over time, which contradicts the Hotelling rule. The solution is rather unappealing: the entire stock of the resource is traded and used in the productive process at the initial date.

10. (A.6) is met, for example, by a diminishing returns Cobb-Douglas technology. Many of the results derived below using (A.6) hold under much weaker conditions; (A.6) is retained for clarity. Note that, of course, a sufficiently large capital stock places a strain on the partial equilibrium nature of the model.

11. Benveniste and Scheinkman (1982) derive the conditions under which infinite horizon counterparts of transversality conditions in finite horizon problems are <u>necessary</u> for optimality. Briefly, these conditions are that the integrand in the objective function is concave and continuously differentiable, and that the optimal programme is "interior" and yields a finite payoff at any given date.

12. A proof can be found in Nickell (1978, pp. 43-4).

13. Flow fixed costs are borne at each instant in time if the rate of investment is non-zero. They differ from (stock dimension) startup costs that are incurred before a phase of non-zero investment can commence.

14. Limit cycles are ruled out because the investment rate cannot change sign more than once; see proposition 2 below.

15. This diagrammatic technique is used by Kamien and Schwartz (1978).

16. This is accurate provided the buyer's initial capital stock is sufficiently large, so that only phases 2 and 3 feature along the programme. In this case, the startup costs are borne once, at the beginning of phase 3, but otherwise leave the programme

unaffected (provided they are not so large that they entail the buyer dropping the disinvestment programme altogether).

However, suppose the original programme (without startup costs) features phase 1 as well as phases 2 and 3. In this case, retaining the original plan would incur startup costs twice; once at the beginning of phase 1, and the second time at the beginning of phase 3. Depending on the magnitude of the startup costs, it may be preferable to modify the original programme (even if ε-chatter is permissible). For example, it may now pay to exclude phase 1 if the startup costs saved thereby exceed the gains forgone by not adjusting to the "desired" profile of investment as closely.

17. Davidson and Harris are concerned with the firm's steady-state replacement investment policy. For that relatively simpler problem they are able to show that, if startup costs exceed a certain magnitude, the firm has a preference for an ε-chatter policy; otherwise, the optimal policy features regular cycles of investment and no investment.

In the present case there is no time-autonomous steady state, and it is the effect of the joint presence of startup costs and flow fixed adjustment costs on the time-dependent optimal trajectory that is of interest. It is clear that in general the equilibrium (dis)investment profile is going to differ from the case of no startup costs.

18. An example of a step in this direction appears in Dasgupta, Gilbert and Stiglitz (1982). In their model, a resource-consuming country (the buyer) decides on how much to allocate to R&D for a substitute technology on the basis of the resource price trajectory announced by a monopolistic seller. The authors pay particular attention to the equilibrium that results when the buyer acts as a Stackelberg leader and the seller as a passive follower, and derive (among other results) the proposition that the buyer may wish to delay development of the substitute even if earlier development entails no extra cost.

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