

Modified Minimum Spanning Tree for Optimised DC Microgrid Cabling Design

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Abstract—The construction of low-cost nanogrids could help speed up the electrification rate in remote communities in sub-Saharan Africa. One of the challenges in nanogrid design for these communities is the uncertainty in future load demand profiles impacting the cabling topology. We propose two new wiring design approaches for radial direct current power distribution systems. The first one is a modified minimum spanning tree (MST) algorithm and the second one is an adaptive shuffled frog leaping algorithm (SFLA). Their objective is to identify the optimal cable path and the lowest cost wiring characteristics to electrify rural areas with poor infrastructural development. A comparative study of computation burdens has shown the applicability limits relative to the SFLA based approach and encouraged the implementation of the MST as it is faster and does not imply any limitation regarding the number of dwellings to electrify, while providing low-cost wiring design option. The latter is applied to a village located in Kenya and demonstrated more than 25% savings on the entire system cabling cost compared to a classical wiring design based on shortest-path calculation.

Index Terms—Design optimisation, Euclidean distance, load flow, rural areas, shuffled frog leaping algorithm, shortest path problem.

NOMENCLATURE

Variable	Description
Λ	Number of trench segments.
Q	Number of homes to connect.
D	Number of cable section diameters.
Υ	Set of possible trench segments.
T_i	Trench segment number i .
l_i	Length of the trench segment number i (m).
τ_i	Trench segment i diameter (mm^2).
δ_j	Diameter of the cable j (mm^2).
C_{2k,δ_j}^{MH}	Man-hours cost per one meter of trench dug.
C_{2k,δ_j}^{JB}	Junction box cost for 2k-cores cable diameter δ_j .
C_{2k,δ_j}^W	Cost of 2k-cores cable diameter δ_j .
$\lambda_{i,j,k}$	Indicator for the presence of 2k-cores cable, section δ_j in the trench segment i .

$\Delta_{i,j,k}$	Indicator for redundancy of junction boxes in a home cluster.
$\mathcal{F}$	Feasible solutions space.
$\mathbb{E}$	Metric space.
v	Vertice.
$e_{i,j}$	Edge between vertice v_i and vertice v_j .
$c_{i,j}$	Cost per unit length of the edge $e_{i,j}$.
$\omega_{i,j}$	Weight of the edge $e_{i,j}$.
$d_{i,j}$	Length of the edge $e_{i,j}$ (m).
Ω	Weight of the tree in LeMST.

I. INTRODUCTION

THIS paper presents a methodology to optimise the cost of microgrids wiring design. To do this, we propose an algorithm that selects shortest paths, minimises dug trenches length, and defines the optimal number of cores per cable in each trench segment considering junction box cost. Nanogrids are an emerging energy access solution suitable for populations with low energy demand in isolated communities [1]. They are easy to deploy either in a standalone mode or as part of another power grid. Widespread implementation of microgrids incorporating nanogrids could rapidly increase the social impact of electricity, reaching up to 900 million people [2], with two thirds of them in sub-Saharan Africa [3]. Designing economic systems and embedding smart solutions is the best combination for an extra robust low-cost energy distribution grid.

One of the main challenges in building such energy systems is the distribution network design. In this design, three aspects are considered: the nature of the current waveform (alternating current (AC) or direct current (DC)), the cable position (under-ground or over-ground), and the grid configuration (radial or non-radial). In this context, DC wiring implementation has three major benefits: (i) it reduces the number of converters and inverters [4], (ii) it is safer than AC in residential sector [5], and (iii) it is enabled by the increasing use of DC energy sources, energy storage systems, and emerging DC loads. For reliability purposes, trenches will be dug in the ground to lay the cables and protect them from harsh weather and theft, which will ensure security and durability of the electricity supply. There are various DC microgrid wiring topologies in the literature for radial

Project funded through Engineering and Physical Sciences Research Council's Global Challenges Research Fund, UK (Grant No: EP/R030111/1). Project received ethical clearance in UK (R53727/RE001) and Kenya (SU-IERC0281/18).

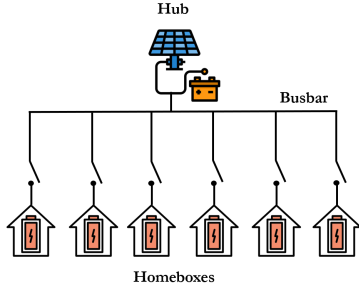


Fig. 1. *Hub-to-homebox* connections in an off-grid star distribution system.

[6–8], meshed [9] and parallel [10] configurations. However, the radial configuration is one of the most cost-efficient and appropriate designs for rural electrification [11].

Therefore, we propose in this paper a form of radial configuration called the star radial configuration. This is a *hub-to-homebox* [12] cabling design, that has the advantage of directly connecting active customers to the main source “*hub*” through the “*homeboxes*”, as shown in Fig. 1. This configuration has three main benefits: 1) minimises the copper quantity by selecting minimal cable section to each single *homebox* corresponding to its installed power, 2) allows the detection of any electricity theft, and 3) helps to mitigate the impact of abstaining customers and makes it easier to accommodate customers seeking to have access to the grid during the implementation phase. The optimisation of the proposed microgrid’s underground star radial distribution network is the focus of the present paper.

In this paper, we not only look at computing the shortest path for each *hub-to-homebox* connection, which is the straightforward design option for this problem, we also look at optimising the whole cost of the system wiring. Since multi-core cables are more economical than using several single-core cables, we opted to use them to supply the different homes to economise the electrification cost per customer per meter. The incorporation of multi-core cables, where suitable, means that we need to identify the home clusters that could be supplied by the same main cable while distributing energy to customers through diverse multi-wire cables through junction boxes. Furthermore, the definition of each cluster is related to the identification of the optimal length of each cable type and its appropriate position on the set of possible locations where cable trenches could be dug. Finally, we need to identify the trenches that will host more than one cable (i.e. with cable segments from different clusters). This gives rise to a constrained clustering problem.

There is a wide range of clustering approaches in the literature, ranging from centre-based clustering algorithms [13] and genetic algorithms [14], to animal behaviour models such as insects, fishes and birds [13]. The latter algorithms are inspired by the behaviour of real-life animal colonies

resulting in either swarm-based optimisation approaches or memetic evolution algorithms [13]. In this study, we have a constrained combinatorial optimisation, incorporating design and cost constraints, which can either require an informed search or an operations research to be solved. For the informed search, we propose a meta-heuristic approach [15] referred to as the Kirchhoff’s current law based shuffled frog-leaping algorithm (KCL-SFLA) inspired by frog behaviour in obtaining food [16]. For the operations research, we develop a multi-objective evolutionary algorithm referred to as Leonardo da Vinci’s rule based minimum spanning tree (LeMST), as the MST has already been applied widely in network design [17]. The original SFLA, which is a combination of shuffled complex evolution algorithm (SCE) [15] and particle swarm optimization (PSO) [15], has been applied to other clustering and optimisation problems, e.g., PID controller optimal tuning [18], network reconfiguration [16], distributed generation placement [14], and water distribution network design [19]. Nevertheless, in our case each solution corresponds to a load flow model that should satisfy Kirchhoff’s current law (KCL) in each node before implementing it into the SFLA. For the same reason, we implement Leonardo da Vinci’s rule into the MST to generate feasible solutions which are in accordance with future roads infrastructure and are also electrically logical. The main contributions of the paper are as follows:

- 1) We present two novel variations of the SFL and MST algorithms and we adapt them to solve the wiring design problem. As this problem is distinguished by the fact that the position of each customer is unchanged while the weights corresponding to the paths leading to each customer vary constantly when searching for a global optimal solution.
- 2) We identify the computational burden of the SFLA in solving small multi-constrained system problems.

The remainder of the paper is organised as follows: Section II focuses on the problem formulation. Section III describes the proposed LeMST approach. The detailed description of the original SFLA and its application to the wiring design problem through the KCL-SFLA is presented in Section IV. Finally, the two approaches are compared and the obtained numerical results are analysed and discussed in Section V.

II. PROBLEM FORMULATION

The microgrid model proposed in this paper is composed of a ‘Virtual Power Plant’ at the central *hub* that supplies dispatchable power to each individual house separately, as illustrated in Fig. 1. This virtual power plant could be a fossil fuel generator, a renewable energy source coupled with storage, or a combination of them [20].

The microgrid distribution network design must meet three stipulations: a) high electrical efficiency, b) regulated voltage and c) low cost. This paper focuses on tackling stipulation (c).

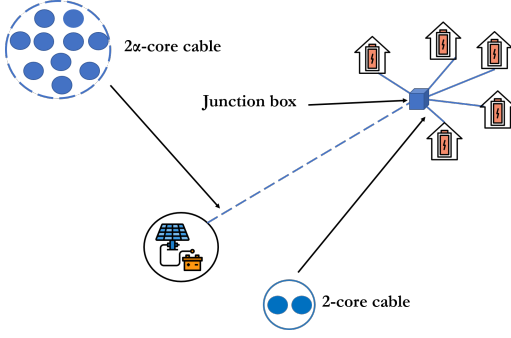


Fig. 2. Multi-core connection for a home cluster in the off-grid star distribution system. α is five in this example.

The wiring cost has two components to minimise: the assets and the man-hours. This involves the following challenges: finding the shortest path, optimal cross-sectional area, and where to locate multi-core cables and junction boxes when it is economic to do so. Besides reducing man-hours of digging trenches by minimising their length. Therefore, the problem can be formulated as minimizing the following cost function:

$$C_{\Lambda, Q, D}^{wiring} = \sum_{i=1}^{\Lambda} \sum_{k=1}^Q \sum_{j=1}^D (C_{i,j,k}^{asset} + C_i^{man-hours}) \quad (1)$$

such that

$$\begin{cases} T_i \in \Upsilon, \forall i \in [1, \Lambda] \\ \sum_{j=1}^D \delta_{i,j} \leq \tau_i, \forall i \in [1, \Lambda] \end{cases} \quad (2)$$

The trenches should only be placed in the feasible space Υ , as explained further later in this paper, and the cables in each trench should fit into the trench's diameter τ_i .

The assets cost C_{asset} is expressed by:

$$C_{i,j,k}^{asset} = \lambda_{i,j,k} \cdot (C_{2k,\delta_j}^W \cdot l_i + C_{2k,\delta_j}^{JB} \cdot \Delta_{i,j,k}) \quad (3)$$

where $\lambda_{i,j,k} = 1$ if $2k$ -cores cable is present, and $\lambda_{i,j,k} = 0$ otherwise. Likewise, $\Delta_{i,j,k} = 1$ if the $2k$ -cores cable, section δ_j is connected to a $2(k-1)$ -cores cable or less, and $\Delta_{i,j,k} = 0$ otherwise.

The man-hours cost $C_{man-hours}$ is formulated below:

$$C_i^{man-hours} = C^{MH} \cdot l_i \quad (4)$$

The multi-core cables are an image of the potential home clusters implemented in the village. If we consider α homes in one cluster, they will be supplied by a 2α -cores cable through individual 2-core connections — derived from the DC topology requiring two wires (positive and negative) — at the junction box level as illustrated in Fig.2. Therefore, the parameter $\Delta_{i,j,k}$ has been integrated into the objective function to make sure the junction boxes are properly placed in each cluster.

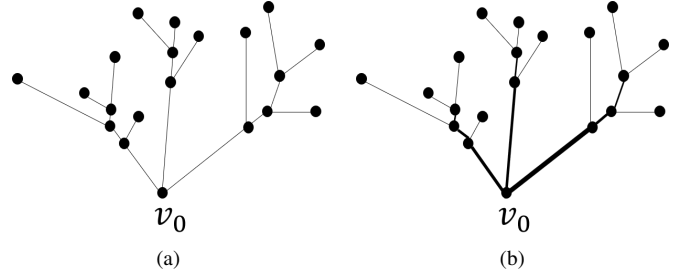


Fig. 3. (a) MST algorithm (b) LeMST algorithm.

III. LEONARDO DA VINCI'S RULE EMBEDDED INTO THE MINIMUM SPANNING TREE ALGORITHM

In this section, we describe our strategy for transforming the wiring design problem, which is an NP-hard problem, into a connectivity problem to solve it. We propose a novel algorithm LeMST combining Dijkstra's algorithm, Euclidean distance (ED) computation, Minimum Spanning Tree (MST) and Leonardo da Vinci's rule to solve a combinatorial problem that has not been treated before in the literature. The wiring design problem constraints which have driven the conception of this new model, to improve upon the classical MST algorithm, are cited in section III-A. Then, a detailed explanation of the proposed algorithm is provided in section III-B. Finally, section III-C is dedicated to the preliminaries for the proposed algorithm implementation.

A. Problem constraints

Solving the wiring design problem has raised many issues that drove the development of a new approach. The MST is a spanning tree of a set of n points "vertices" in a metric space ($\mathbb{E}^n$), where the length of an edge is the distance between a pair of points in the point set [22] as shown in Fig.3a. Its objective is to minimize the tree's cost-radius tradeoffs [23]. However, in the way the existing MST has been constructed [23, 24], it will not work on a problem with the following constraints:

- 1) The space could be limited to a specific area $\mathcal{F}$: the problem could be geospatially constrained, the edges e and vertices v cannot be illustrated in the entire Euclidean space. $\forall (x, y) \in N, \{e_x, v_y\} \subset \mathcal{F} \subset \mathbb{E}$.
- 2) The number of final vertices could be unknown initially: we have a defined number of vertices to connect, however new parent-vertices could be generated to minimise the objective function. $\forall (x, y) \in N, \{e_x, v_y\}_{initial} \neq \{e_x, v_y\}_{final}$.
- 3) The edges' weight ω could change through the different iterations: once a vertex is connected to a subtree, the weights of the existing tree's edges change automatically. The edges' weights could be interdependant. $\forall (x, y) \in N, \omega_{e_x} = f(\omega_{e_y})$.

B. Leonardo da Vinci's MST algorithm

In the proposed strategy, we are not adopting a top-down MST construction approach and prune heavy edges [25]. Instead, we are building up edges gradually from a fixed source to form the subtrees and then the final trees, while applying the Leonardo da Vinci's rule [26] (*i.e., the sum of the cross-sectional area of all tree branches above a branching point at any height is equal to the cross-sectional area of the trunk or the branch immediately below the branching point*) on each subtree. As shown in Fig.3b, we consider that the trunk diameter of each subtree is equal to the sum of the branches diameter to verify the condition of inter-dependant edges' weights at each iteration.

There are three primary optimisation techniques adopted in the literature to compute shortest pathways: Bellman Ford [27], Dijkstra [28] and Floyd-Warshall [29] algorithms. The optimal path length obtained from the three methods is the same and the difference between the three techniques is the computation time [29]. Therefore, the Dijkstra algorithm has been selected, as unlike the others, it is classified as a search algorithm. Furthermore, the results reported by Cruz et al.[29] encourages the implementation of this algorithm in our study as the number and geography of customers to electrify can vary widely and it is needed to maintain a good performance regardless of the grid topology to design. As a result, we propose to combine Dijkstra's algorithm and ED computation for the developed approach to define the min-radius. Yet, the min-cost and edges geospatial information are also among the objectives of the LeMST. A detailed description of the algorithm steps is below.

LeMST algorithm begins with the trivial shortest-path tree, using Dijkstra's algorithm [23], consisting of the source denoted by v_0 and the N initial vertices denoted by v_j , as illustrated in Fig.4a. The edge between v_i and v_j is referred to as $e_{i,j}$, where $e_{i,j} \in \mathcal{E}$. The shortest-path tree edges have the same weight $\omega_{i,j}$ but different lengths $d_{i,j}$. As explained previously, the edges are interdependant, hence, differently from the classical MST where the edges are sorted in descending order of their weight and the heaviest are removed [23], the LeMST removes edges from the trees with the heaviest weight. The tree k 's weight Ω_k is given by:

$$\Omega_k = \frac{\sum_{i=0}^{n_k} \sum_{j=1}^{m_k} d_{i,j} \cdot c_{i,j}}{N_k \cdot \sum_{j=1}^{N_k} D_{0,j}} \quad (5)$$

where $c_{i,j}$ is the cost per unit length of the edge $e_{i,j}$, N_k the number of initial vertices $(v_i)_k$ connected to the tree k and $(D_{0,i})_k$ their length. In the shortest-path tree, all edges have the same $c_{i,j}$, i.e., the same diameter according to Leonardo da Vinci's rule. Therefore, all subtrees have the same Ω_k because each subtree only contains one vertex [23]. At that stage, the objective function $f(T)$ is set at its initial value that

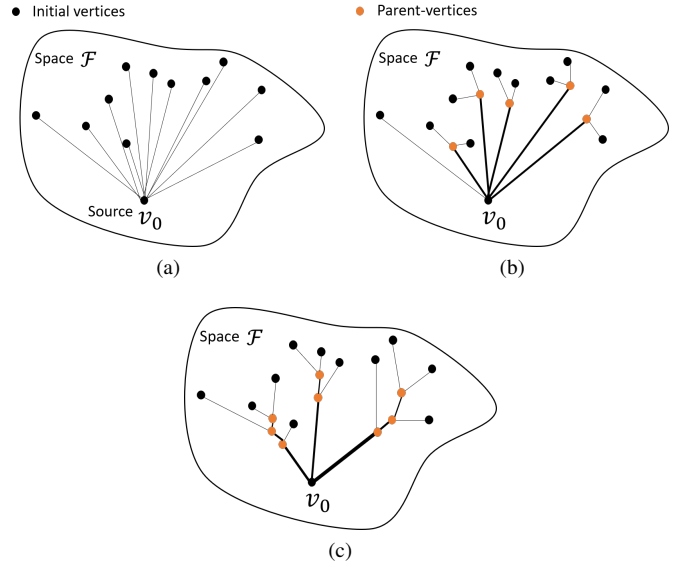


Fig. 4. (a) Dijkstra's MST (b) Pairs' MST (c) LeMST.

should be optimised to set the final number of trees T .

$$f(T) = \frac{\sum_{k=1}^T \Omega_k}{N} \quad (6)$$

To start the optimisation process, we generate all possible pairs of edges $\{e_{o,p}, e_{q,r}\}$ that could be merged to form one subtree ζ_i by creating a new parent-vertex for each pair. We sort the subtrees' weight in ascending order without repeating any vertex twice $\{\zeta_1, \zeta_2, \dots, \zeta_k\}$. The obtained subtrees obey Leonardo da Vinci's rule. We refer to this process results as *Pairs MST* as shown in Fig.4b. If the initial number of vertices is odd, one vertex will be rearranged later in one of the subtrees. After that, we have the choice between two iterative search approaches: unconditioned and conditioned. The first one consists of splitting the vertices $\{v_x, v_y\}$ of the subtree with the heaviest weight ζ_k , connecting the first vertex v_x to the subtree with the lightest weight ζ_1 , and calculating its new weight $\Omega_{1,new}$. If $\Omega_{1,new} < \Omega_1$ the vertex v_x will join the subtree; if not, the same process will be done for the subtree ζ_2 as illustrated in Fig.5a and then for v_y and so on. The conditioned iterative search is adopted when the final number of trees should not exceed a certain limit χ due to a certain constraint depending on the problem, or when the trees with the lightest weight could be merged if geospatially possible. This is illustrated by doing the unconditioned iterative search at the ψ trees with the smallest weights as shown in Fig.5b. The connecting process in each of the iterative search approaches is the same, and it is based on two main steps:

Step 1: calculation of the minimal ED between the vertex v_x to rearrange and the edge e_{0,m_1} representing the edge between the source and the parent-vertex v_{m_1} of the subtree

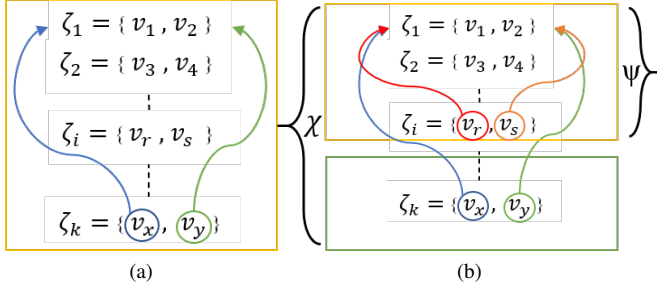


Fig. 5. (a) Unconditioned iterative search (b) Conditioned iterative search.

I to define the connection point. If this point is different from v_{m_1} , a new parent-vertex v_{n_1} will be created and e_{0,m_1} will be split into two edges e_{0,n_1} and e_{n_1,m_1} .

Step 2: calculation of the shortest-path between v_x and v_{m_1} or v_{n_1} in the space Γ to measure d_{x,m_1} or d_{x,n_1} and update the different weights accordingly to calculate the new value of $\Omega_{1,new}$.

The new subtrees' weight will be again sorted in ascending order until the final set of trees or clusters that optimises the objective function is obtained and do not violate Leonardo da Vinci's rule. We can have some vertices that will not fit in any of the trees, which will be directly connected to the source.

C. LeMST algorithm applied to wiring design

In this section, we introduce the preliminaries to solve the wiring design problem using LeMST. Before implementing the aforementioned algorithm, maps and data are prepared to serve as inputs to the algorithm through the identification of:

- 1) Road infrastructure: all possible potential road infrastructure from the map [30] by considering the customers that will be electrified in the village. The map of a village showing the road infrastructure is shown in Fig.6a.
- 2) Trench pathways: processing of the road map to identify possible trench routes, and connect customers to those trenches via small trenches (considering an underground topology) as shown in Fig.6b.

The *hub* is the source in our problem as shown by a small array connected to a battery symbol in the bottom of Fig.6a, the initial vertices are the *homeboxes* illustrated by green (shop) and red (home) dots in the same figure. The edges are the sections of cable. The parent vertices are the junction boxes needed to electrify the microgrid. The space $\mathcal{F}$ of the optimization problem is limited to the available trenches where the cables and junction boxes will be placed. An illustration of Leonardo da Vinci's rule application is when we have in our problem a junction box connecting a 2α -cores cable to two $2\beta_1$ -cores and $2\beta_2$ -cores cables, the condition $\alpha=\beta_1+\beta_2$ should be satisfied. As a result, we can have more

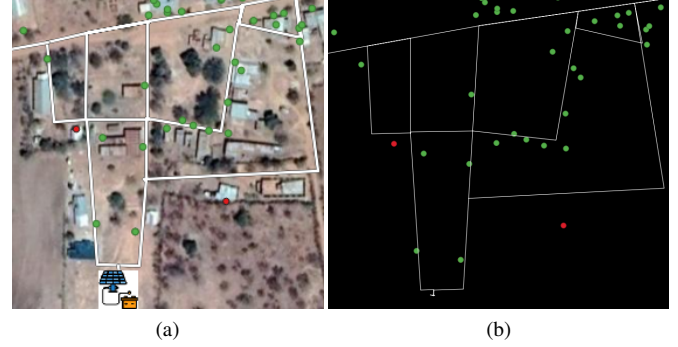


Fig. 6. (a) Village map with road infrastructure (b) Corresponding one road side trenches. Green and red dots refer to the location of *homeboxes*.

than one multi-core cable type per cluster or tree. Regarding the iterative search, it could be conditioned or unconditioned, this depends on the population spread and density on the village map and on the grid planner constraints such as limits regarding the number of cables per trench or the maximum number of cores to be implemented per cable. In the case of conditioned search, we set different values for ψ and χ to find the one that optimises the objective function which is the cost of electrification per customer per meter for the entire system. The weight of each tree is expressed as follows for the wiring design problem:

$$\Omega_k = \frac{\sum_{i=0}^{n_k} \sum_{j=1}^{m_k} (d_{i,j} \cdot c_{i,j}) + J_{i,j}}{N_k \cdot \sum_{j=1}^{N_k} D_{0,j}} \quad (7)$$

where $d_{i,j}$ is the length of the trench segment, $c_{i,j}$ is the cost of cable per meter, and $J_{i,j}$ the junction box cost if present in the aforementioned trench segment $e_{i,j}$. N_k is the number of dwellings connected to the tree k and $D_{0,j}$ is the length of the shortest path between the source and the dwelling j .

IV. KIRCHHOFF'S LAW BASED SHUFFLED FROG LEAPING ALGORITHM

In this section, we will first introduce briefly the original concept of the SFLA. Then, we will identify the frog in the wiring design problem and how we encoded it for the optimisation algorithm through the edge-set encoding technique. Finally, we will describe the performance evaluation in the proposed algorithm conditioning the SFLA's local search.

A. Brief overview of the conventional SFLA

In the classic SFLA, we have a group of frogs leaping in a swamp containing several stones that the frogs use to move from a point to another. The frogs' goal is to search for the best stone with the maximum food by improving their memes, which is the unit of cultural information spread by imitation [19], using bilateral communication. Therefore, improving those memes induces an immediate change in each frog's position by adjusting its leaping step size so

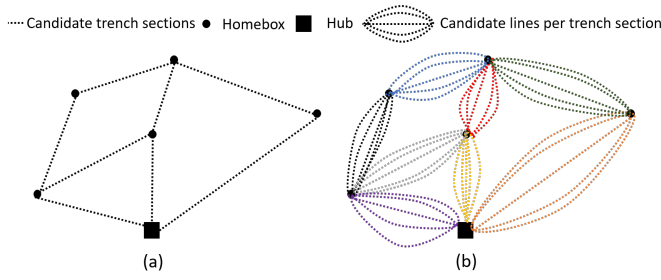


Fig. 7. (a) Edge-set encoding and (b) 2-level edge-set encoding for connecting five *homeboxes* to the main *hub*.

that an individual frog's position converges to the best stone [19]. The SFLA involves a population of possible solutions defined by a set of virtual frogs that are split into subsets referred to as memplexes [15]. Parallel local searches are then performed in each memplex using PSO to ensure a memetic evolution [31]. After a certain number of local search iterations, the virtual frogs are shuffled and reorganised into new memplexes. Shuffling enhances the meme quality after being infected by frogs from different memplexes. After shuffling, the searches for the optimal solutions are pursued. This local search and shuffling process continues until predefined convergence criteria are met. A detailed description of the SFLA is done by Eusuff et al. [19].

B. Two-level edge-set encoding technique for wiring design problems' modelling

To encode each possible solution of the problem (frog), an edge-set encoding technique is suggested. This technique has already been used for substations' location definition in a power distribution network [32]. However, in our case the modeling is much more complex. Each solution cannot be only encoded with an array containing the lines of the solutions, but each line can have several possible solutions. Therefore, the number of bits is not proportional to the number of branches in the network as stated by Rivas-Davalos et al. [32]. This leads us to a two-level edge-set encoding as our solution needs to define which trench segments will be used to connect the different households, and how many cables will be laid in each trench and what are their characteristics. Characteristics here refers to the number of cores per cable and length that are used to calculate the objective function. Therefore, if we want to solve this optimisation problem through the edge-set modelling of the network topology using SFLA, section IV-A, the frog will be defined as follows:

A frog contains a meme comprised of N memotypes, representing the trench segments in our problem:

The memotype representation is a string that encodes how many cables we have in each trench segment per type:

M_1	M_2	M_3	...	M_N
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m_{α_1}	m_{α_2}	m_{α_3}	...	m_{α_n}
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where $m_{\alpha_i}, i \in \{1, 2, \dots, n\}$, is the number of cables type α_i and n the number of cable types. If the latter is equal to zero this means that this type of cable doesn't exist in the corresponding trench segment: a candidate line per trench section is omitted as shown in Fig.7b. Likewise, if $M_j = 0$ where $M_j = \sum_{q=1}^n m_{\alpha_q}, j \in \{1, 2, \dots, N\}$, this means that the trench segment M_j doesn't contain any cable: a candidate trench section is omitted Fig.7a. If we consider the graph illustrated in Fig.7b as an example, we have $n=5$ cable types to be used to design the wiring to connect the five *homeboxes* to the *hub*: 2-core, 4-cores, 6-cores, 8-cores and 10-cores. The possible combinations in one trench segment could be $\{2\text{-cores} \times 5, 0, 0, 0, 0\}$, $\{2\text{-cores} \times 3, 4\text{-cores} \times 1, 0, 0, 0\}$, $\{2\text{-cores} \times 3, 4\text{-cores} \times 2, 0, 0, 0\}$, $\{2\text{-cores} \times 2, 0, 6\text{-cores} \times 1, 0, 0\}$, $\{0, 4\text{-cores} \times 1, 6\text{-cores} \times 1, 0, 0\}$, $\{2\text{-cores} \times 1, 0, 0, 8\text{-cores} \times 1, 0\}$ or $\{0, 0, 0, 0, 10\text{-cores} \times 1\}$. The number of possible trench segments is $N=8$ as shown in Fig.7a.

SFLA uses the value generated by each memotype for the evolution process as will be explained in the next subsection. The frog's position or meme represents the selected trench segments, their number and length as well as the number of cables laid in each segment and their type. Therefore, a change in the frog's position, means a change in the number of cables and their type in each single trench segment as well as the number and reference of the trenches used.

C. KCL-SFLA for performance evaluation in wiring design problems

The above mentioned SFLA's evolution process cannot be directly applied to the wiring design problem. This is because some infeasible and wrong grid configuration could be generated. For example, if we consider the example illustrated in Fig.7b, the algorithm could generate two solutions as shown in Fig.8a and Fig.8b. If we consider that all cables are 2-core and each *homebox* requires two amperes, the first solution is infeasible according to Kirchhoff's law. We have only 2A flowing from the *hub* through trench segment T_2 while 6A on T_5 and 10A on T_3 . However, according to this configuration the current going through T_2 should be equal to the sum of T_3 and T_5 's amperes and so on for all junction boxes located at each *homebox* position considered as a node in the circuit. The second solution is feasible as it obeys the Kirchhoff's law following the same reasoning. Therefore, a performance evaluation algorithm to suppress infeasible solutions, before the local search and shuffling process, will be inserted into the SFLA to make the process quicker and increase the quality of the obtained results. In fact, the presented algorithm KCL-SFLA combines SFLA and a load flow simulation model to

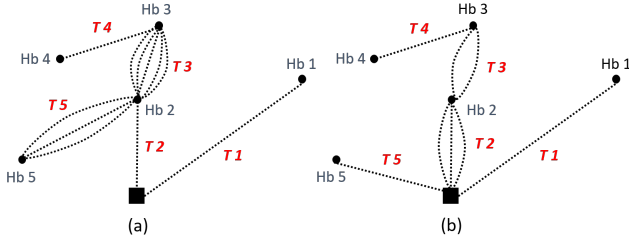


Fig. 8. (a) Infeasible and (b) feasible wiring design solutions.

delete unfeasible solutions. A similar process applied to water distribution network design has been initiated by Eusuff et al. [19], however the performance process in that work has not ignored unfeasible solutions, instead assigned a penalty to those solutions as the problem is less constrained than the one exposed in this paper. As a result, the performance function g for each frog f_o to be maximised in the proposed KCL-SFLA is expressed below:

$$g(f_o) = \frac{1}{\sum_{i=0}^N \sum_{j=1}^n M_{i,j,f_o} \cdot (D_j \cdot C_j + J_j)} \quad (8)$$

where C is the cost of the cable depending on its type, D the different trench segments' diameters and J the junction boxes' cost. N is the total number of trench segments and n the total number of dwellings to electrify. We have modelled $M_{i,j}$ as a matrix of dimensions $n \times N$. The detailed steps for computing the algorithm are described as follows:

Step 0: Transform the map of possible trenches into a graph, define the length of all trench segments, insert the prices corresponding to the possible cable's type to implement (this is equal to the *homeboxes* number).

Step 1: Choose a set of trench segments and a set of cable numbers and types per trench segment to represent the frog i i.e define the memotypes for the trench segments $(T_1, T_2, \dots, T_N)$ representing the meme.

Step 2: Apply the Kirchhoff's law on all nodes of the first design network. **if the law is verified in all of the circuit's nodes then**

keep this as a feasible solution (frog) and;
go to next design network.

else

delete frog from the solution set and;
check the next design network.

end

Step 3: Calculate performance function g on all selected frogs j from *step 2*.

After those steps that represent the particularity of the KCL-SFLA, the values issued from *step 3* will be ranked to partition the frogs into memplexes and go through the local search and shuffling processes of the classical SFLA as shown in Fig.9 [19].

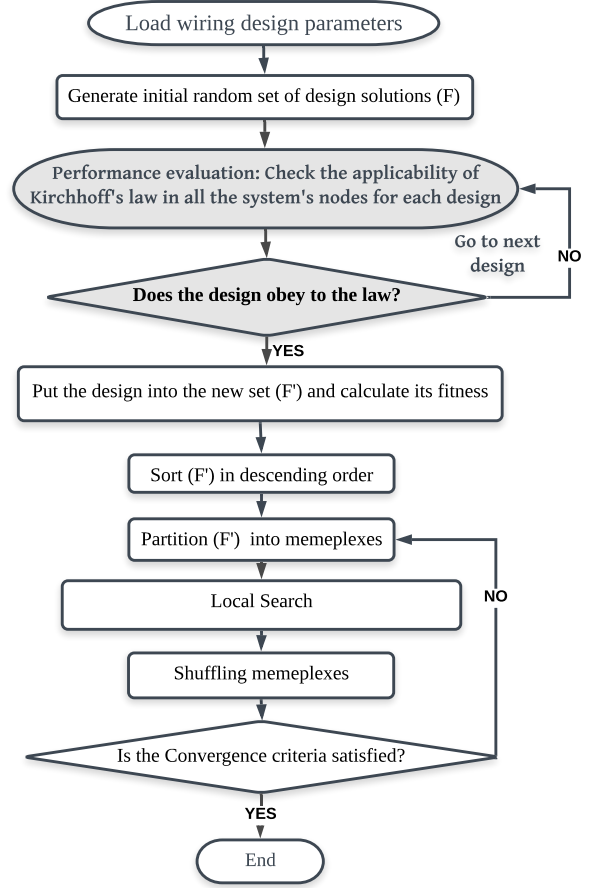


Fig. 9. Flowchart for the KCL-SFLA.

V. SIMULATION RESULTS AND DISCUSSION

The proposed LeMST and KCL-SFL algorithms have been implemented in Python on a computer using an Intel® core™ i7-8850U processor base frequency 1.80 GHz. The performance of the proposed algorithms is tested on an unelectrified village southeast of Nairobi, Kenya. For this village, the microgrid's central *hub* is composed of a solar system and lead acid battery banks. The latter is sized to provide sufficient power to support the main electric demand of the system. Small lithium-ion batteries have been placed inside the *homeboxes*, to provide energy during short-term power outages and faults in the microgrid [12].

The 18 premises that are to be electrified are illustrated on the map shown in Fig. 6a. Each connection between the *hub* and the *homebox* requires two wires rated at +60 V and -60 V with respect to ground at the *hub* [12]. Regrouping these two wires in one single-core cable is the first approach in reducing the cost and improving the reliability of the connections. The cross-section of each wire is 1.5 mm^2 to support the 2A maximum current to each home [12], as all

the *homeboxes* have the same installed power. Steel Wire Armour PVC is used to protect the cables from the impact of weather and increase the durability of the system in an underground design configuration. Cable and junction box costs are illustrated in Table I according to current market prices and the cost per customer per meter corresponding to each cable type is also presented. Those costs are adopted in the different upcoming simulations.

TABLE I
COST OF WIRING PER CUSTOMER ACCORDING TO CABLE DESCRIPTION

SWA/PVC cables description	Number of cores	Cost in GBP per meter	Junction box cost in GBP
$2 \times 1.5\text{mm}^2$	2	0.460	-
$4 \times 1.5\text{mm}^2$	4	0.685	3.28
$6 \times 1.5\text{mm}^2$	6	0.834	4.01
$8 \times 1.5\text{mm}^2$	8	1.052	4.33
$10 \times 1.5\text{mm}^2$	10	1.480	4.40
$12 \times 1.5\text{mm}^2$	12	1.934	4.50
$14 \times 1.5\text{mm}^2$	14	2.230	5.09
$16 \times 1.5\text{mm}^2$	16	2.520	5.52
$18 \times 1.5\text{mm}^2$	18	2.911	6.01
$20 \times 1.5\text{mm}^2$	20	3.800	6.42
$22 \times 1.5\text{mm}^2$	22	3.850	7.23
$24 \times 1.5\text{mm}^2$	24	3.930	8.09

While considering an unconditioned iterative search for the LeMST, we compare the cost and computation time for electrifying two to 18 premises using LeMST as the multi-core wiring design options, and the classical Dijkstra's algorithm as the 2-core wiring design option. From Fig.10a, we can notice that LeMST is always cheaper than Dijkstra's algorithm, with a cost benefit up to 20% for 18 customers. Therefore, we conclude that the more the population increases, the more economically advantageous it is to put dwellings into clusters. Yet, the computation time of the LeMST is slightly higher than the Dijkstra's algorithm as shown in Fig.10b, especially when the number of customers to electrify is odd as the algorithm takes longer to place the last customer in the appropriate tree. From the map shown in Fig. 6a, there are some zones with important population density, so we could have several cables going through the same trench's segment that could be part of one main cable (one edge). For this reason, we test the conditioned iterative search in the LeMST, to see to what extent our wiring design could be optimised. As a result of varying χ and ψ (defined in section III-B), the optimal configuration is obtained for $\psi = 60\%$ and $\chi = 90\%$. The cost of electrification dropped by 7% from £733.8 to £680.53 for the same set of 18 premises. Therefore, we conclude that LeMST reduces the cost of electrification by 25% compared to the classical Dijkstra's algorithm that would cost £906.02. The final results obtained from implementing the LeMST are illustrated in Table II.

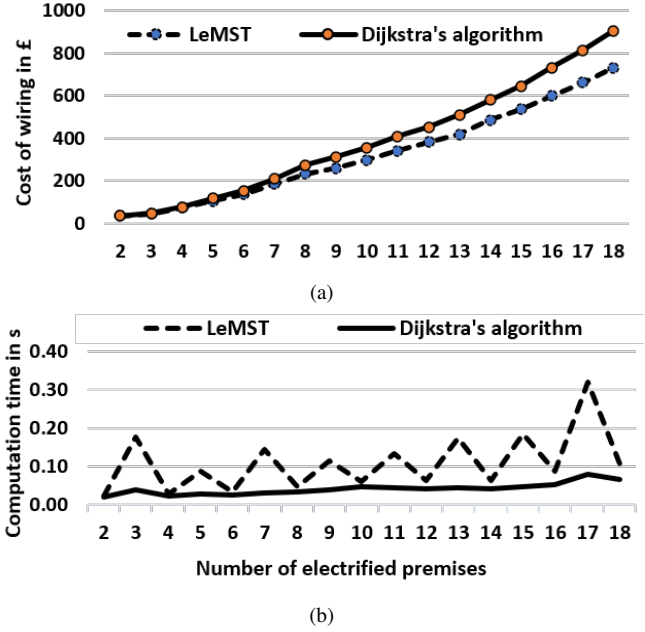


Fig. 10. Comparison between the cost (a) and computation time (b) of LeMST and Dijkstra's algorithm.

TABLE II
WIRING DESIGN CONFIGURATION USING LEMST FOR 18 PREMISES

Tree ref.	Edge ref.	Number of cores per edge	Length in m	Customers ref.
1	$e_{0,1}$	10-cores	23.4	19,21,20, 15,18
	$e_{1,2}$	4-cores	157.2	
	$e_{2,3}, e_{2,4}, e_{1,5}, e_{1,6}, e_{1,7}$	2-cores	79.2	
2	$e_{0,1}$	10-cores	18.4	24,26,28, 25,23
	$e_{1,2}$	4-cores	99.2	
	$e_{2,3}, e_{2,4}, e_{1,5}, e_{1,6}, e_{1,7}$	2-cores	125.2	
3	$e_{0,1}$	6-cores	79.6	11,12,16
	$e_{1,2}$	4-cores	33.4	
	$e_{2,3}, e_{2,4}, e_{1,5}$	2-cores	27	
4	$e_{0,1}$	4-cores	153.2	27,22
	$e_{1,2}, e_{1,3}$	2-cores	36.4	
5	$e_{0,1}$	6-cores	54.8	17,14,13
	$e_{1,2}$	4-cores	28.4	
	$e_{2,3}, e_{2,4}, e_{1,5}$	2-cores	53.6	

For the same range of customers, the heuristic KCL-SFLA approach is implemented on the graph shown in Fig.11 that has been based on the trenches' map as explained in section III-C. The 18 green circles refer to the premises coordinates projection on the trench and the 11 orange circles refer to the crossroads, where the cable can take more than one distinct path. The maximum number of possible trench segments is 34. To find the best wiring design option, we examined the effect of changing the input parameters on the convergence toward the optimum. We varied the initial set of feasible

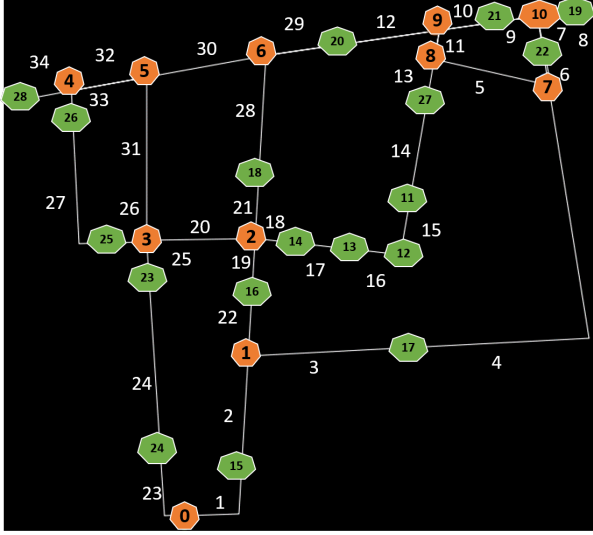


Fig. 11. Trench segments graph for the KCL-SFLA. Green circles refer to customers connection and orange circles to crossroads

solutions (not violating KCL in any of the nodes) from 15 to 100 and the number of memplexes from 3 to 20. The number of iterations per memplex has been set to 25. Results show that the more we increase the set of initial solutions the cheapest our solution become, but more time is required to run the simulations. This is an algorithm limitation that has already been raised by Asrari et al. [16] for applying SFLA on large distribution networks reconfiguration. Despite the fact that it has been encouraged previously by its rapidness compared to genetic algorithms or simulated annealing [15].

Table III summarises the simulation results in terms of cost and time for the three methods discussed in this paper. For the KCL-SFLA, we simulated two scenarios considering the input parameters providing the cheapest solution: 1) multi-cables per trench segment and 2) single-cable per trench segment which is similar to a radial design. For up to six customers, the KCL-SFLA (scenario 2) gives the cheapest wiring design option, however, its computation time could be up to 8167 times higher than the LeMST, which limits its application to wider networks. As a result, we can conclude that the SLFA is not suitable for multi-dimensional combinatorial problems where the number of constraints is two-levels or more. In summary, if we consider the algorithm performance in terms of cost and time, we find that LeMST is the best approach for wiring design in DC microgrids, as it is quick and gives high quality results. Contrarily to the KCL-SFLA, LeMST is not limited to six customers but has been tested on up to 50 customers, and this is for two reasons: (1) the average number of households in rural unelectrified villages subject to microgrids implementation varies between 30 and 50 [33, 34] and (2) larger villages are more likely to benefit from grid connection as the profit of utility companies

increases over larger settings.

However, this doesn't mean that the proposed method cannot be implemented on larger systems in other contexts. To explore its scalability and robustness, we added a set of 650 possible houses to our map and ran the algorithm considering an unconditioned iterative search. As reported in Table III, results show the wiring cost of the 700 premises all together was reduced by 22.5% (compared the Dijkstra's algorithm) with the algorithm's execution time not exceeding 49s. The simulation time of large systems is either impacted by an odd number of customers to connect, as mentioned earlier, or by how scattered is the customers' distribution around the village trading centre, or by both of them. Lastly, the time taken to run the LeMST algorithm grew at a rate of approximately $O(N^2)$; where N is the number of houses.

VI. CONCLUSION

In this paper, we introduce and compare two optimisation techniques that we customised for the multi-objective wiring design problem. The first approach, which is a variation of the shuffled frog leaping algorithm (KCL-SFLA), generally produces better solutions, but its run time becomes prohibitively large when the number of customers to electrify becomes larger than six on the rural village studied. In contrast, the second algorithm, which is a Leonardo Da Vinci's rule based minimum spanning tree (LeMST) algorithm, has shown promising results in terms of computation time and solution cost after its implementation on 18 premises in the same village. The LeMST algorithm was further tested for up to 700 houses and ran in 49s. The time taken to run the LeMST algorithm grow approximately with the square the number of houses. The LeMST algorithm has been developed into a tool for designing low-cost distribution network for star configurations by defining optimal cable paths, appropriate number of cores per cable, junction boxes position and the trenches length. It is suitable for designing network expansion as well in the cheapest way. Likewise, it gives a wiring design coherent with the village future road infrastructure.

Furthermore, the use of the present approach could be extended to cover interconnecting various neighbouring DC microgrids to enable energy trading at the lowest cost. However, for AC microgrids, substantial modifications should be brought to address additional constraints relative to the three phases balancing at the inverter level, limited implementation of multicore wires due to pre-fixed number of trees (one per phase), and cables' cross-sectional area optimisation challenge, especially for radial configurations. Other important parameters to consider are the microgrid design and the specificity of the targeted settings as those could implicate economic limitations in terms of infrastructure investment and potential technical constraints

TABLE III
COMPARISON BETWEEN THE COMPUTATION TIME AND COST OF WIRING USING LEMST, DIJKSTRA'S ALGORITHM AND KCL-SFLA

Number of homes	Dijkstra's algorithm		LeMST		KCL-SFLA			
					Multi-cables per trench		Single-cable per trench	
	Cost in \$	Time in s	Cost in \$	Time in s	Cost in \$	Time in s	Cost in \$	Time in s
3 homes	48.1	0.04 ± 0.03	47.1	0.09 ± 0.02	47.1	0.5	47.1	0.4
4 homes	77.9	0.02	74.7	0.03	74.7	1.2	74.7	1.1
5 homes	118.5	0.03	105.5	0.09 ± 0.01	105.4	14.7 ± 0.6	105.4	9.9 ± 0.4
6 homes	155.6	0.03	138.2	0.03	130.6	501.4 ± 38	130.3	229.4 ± 15.6
7 homes	209.7	0.03 ± 0.01	185.2	0.14 ± 0.02	Long execution time			
18 homes	906.0	0.07 ± 0.02	733.8	0.11 ± 0.03				
30 homes	1,717.0	0.08 ± 0.01	1,381.5	0.15 ± 0.02				
40 homes	2,238.3	0.10 ± 0.01	1,786.7	0.22 ± 0.04				
50 homes	2,731.4	0.11 ± 0.01	2,170.6	0.29 ± 0.08				
150 homes	6,679.6	0.19 ± 0.01	5,239.1	1.31 ± 0.14				
250 homes	12,627.2	0.36 ± 0.02	9,866.6	3.46 ± 0.22				
350 homes	18,088.5	0.45 ± 0.02	14,081.9	7.45 ± 0.53				
700 homes	38,720.4	1.14 ± 0.03	30,033.6	48.9 ± 1.01				

such as voltage drops and losses. Therefore, this should be addressed carefully through another research paper for a more appropriate replicability of the proposed model on AC distribution systems.

Finally, regarding the potential applicability of the findings of this paper in the transport sector, the proposed approach could be used to optimise the civil engineering cost of roads design. Through the optimisation of roads width and length, and subsequent material (e.g. soil, stone aggregate, cement concrete, etc) depending on traffic, population size and distribution in rural areas. Applying this approach on data transmission optimisation or for solving shipping problems are also potential topics to explore in future researches.

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