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**ESTIMATING EULER EQUATIONS WITH NOISY DATA:  
TWO EXACT GMM ESTIMATORS**

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# Estimating Euler Equations with Noisy Data: Two Exact GMM Estimators.<sup>1</sup>

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## **Abstract**

In this paper we exploit the specific structure of the Euler equation and develop two alternative GMM estimators that deal explicitly with measurement error. The first estimator assumes that the measurement error is lognormally distributed. The second estimator drops the distributional assumption and solves out for the unknown, but constant, conditional mean. Our monte carlo results suggest that both proposed estimators perform much better than conventional alternatives based on the exact Euler equation or its log-linear approximation, especially with short panels. The empirical application of the proposed estimators yields plausible estimates of the coefficient of relative risk aversion and discount rate.

*Keywords:* Nonlinear models; Measurement error; Euler equation

*JEL Classification:* C13, E21

# 1 Introduction

Since Hall's (1978) seminal paper, the Euler equation for consumption derived from the intertemporal problem faced by a generic consumer has played an important role in applied consumption research <sup>1</sup>. The reason for this is because it allows one to estimate structural preference parameters and possibly test the specification of the model without having to specify fully the stochastic environment in which the consumer operates. Unfortunately, the relationships that are derived from all plausible preferences are highly non-linear. This creates a number of econometric problems, ranging from the performance of non-linear GMM estimators in small samples to the effect of measurement error (see Alan and Browning (2003) and Attanasio and Low (2004)) . The latter problem cannot be credibly ignored when working with microeconomic data, (see, for example, Shapiro (1984), Altonji and Siow (1987) and Runkle (1991) ).

Standard non-linear GMM estimation yields inconsistent estimates in the presence of (neglected) measurement error (see, for instance, Amemiya (1985)). One solution that has been followed in the literature has been to work with a log linearized version of the Euler equation, a solution first used by Hansen and Singleton (1982). Such a procedure has several advantages, ranging from the robustness to the presence of classical measurement error, to the fact that it yields a specification linear in parameters and is therefore amenable to estimation using pseudo panel methods such as those proposed by Browning, Deaton and Irish (1985). These advantages are, however, outweighed by several problems. First and foremost, there are many situations in which log linearizing the Euler equation does not give a specification linear in parameters and therefore one is again subject to the same problems mentioned above. Situations of this type include a variety of interesting models, such as, for instance, models with habit formation. Second, the 'constant' of a log-linearized Euler equation includes conditional higher moments of consumption growth and interest rate, so that one loses identification of the discount factor;

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<sup>1</sup>See Browning and Lusardi (1996) and Attanasio (1999).

an intertemporal allocation parameter that is of great importance especially in macro-finance literature. Moreover, if such conditional moments are not constant, the error term will include innovations to these moments that might be correlated with the instruments that one typically use in estimating the Euler equation. Attanasio and Low (2004) show that with long panels we can consistently estimate the elasticity of intertemporal substitution using the log-linearized Euler equation, even when the income process is heteroscedastic but long panels (Attanasio and Low suggest about 40 periods) are rare. Consequently one would like to work with an estimator that is robust to these potential problems<sup>2</sup>.

In the wider measurement error literature, resolutions of the problem for nonlinear estimators have only been possible in particular circumstances (see Hausman et al (1991), Hausman et al (1995), Schennach (2000), Hausman (2001), Wansbeek (2001), Hong and Tamer (2003)). In this paper we exploit the specific structure of the Euler equation to propose two alternative GMM estimators that deal explicitly with measurement error. Both assume that the measurement error is ‘classical’ in the sense of being stationary and multiplicative (but not necessarily with a unit mean). For the first estimator we additionally assume that the measurement error is lognormally distributed. This leads to a GMM in which the conditional expectation of the discounted change in the marginal utility is equal to a constant which is not equal to unity but which depends on the preference parameters and the variance of the measurement error<sup>3</sup>. The second estimator drops the distributional assumption and solves out for the unknown, but constant, conditional mean<sup>4</sup>. The trade-off between the two estimators is the conventional one: the first estimator is more efficient if the errors are indeed lognormally distributed but

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<sup>2</sup>Alan and Browning (2003) propose an alternative non Euler equation procedure which avoids the need to model any driving processes. This requires parameterising and simulating the martingale that expectations errors follow. Attanasio and Browning (1995) propose a different approach to the problem which consists in starting with a flexible but log-linear in parameters specification for the marginal utility of consumption. One then has to integrate such a specification to obtain the corresponding utility function.

<sup>3</sup>Colera (1994) follows the same parametric approach and assumes a lognormally distributed serially independent measurement error process. While her GMM estimator identifies the coefficient of relative risk aversion, discount factor and measurement error variance are not separately identified.

<sup>4</sup>Chioda (2003) follows a somewhat similar route, however, the GMM estimator she proposes fails to identify the discount factor.

the second estimator is more robust. We present the small sample behavior of our two GMM estimators as compared with the usual exact Euler equation GMM estimator and conventional log-linearized GMM.

Our Monte Carlo results suggest that both proposed estimators perform much better than conventional alternatives based on the exact Euler equation or its log-linear approximation, especially with short panels. As the sampling period increases, the bias in the coefficient of relative risk aversion parameter becomes smaller for both exact and log-linear Euler equation estimation whereas the bias in the discount factor does not respond to the panel length in the case of the exact Euler equation. The novel feature of our estimators is that they outperform the other two estimators in estimating both the coefficient of relative risk aversion and the discount factor especially when the available panel length is short. Both proposed estimators capture the true values of the coefficient of relative risk aversion and the discount factor remarkably well even in small samples and the resulting sampling distributions have reasonable dispersions.

In the empirical section we present an application using the US Panel Study of Income Dynamics (PSID). Although it contains only food expenditure, the survey suits our purposes since its panel length is reasonably long and it gives us the opportunity to place our results in the literature<sup>5</sup>. Our empirical study using the two proposed estimators yields statistically significant estimates of the coefficient of relative risk aversion of about 1.5. This value lies between the estimate obtained by the log-linearized GMM (2.25) and that obtained by the exact GMM (1.15). Furthermore, our first estimator yields a high and precisely estimated discount factor estimate (0.99) while the second estimator provides a lower but less precise estimate of this parameter.

## 2 The model and the proposed estimators.

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<sup>5</sup>There is a substantial body of literature that explores consumer behavior by estimating Euler equations using the PSID food expenditures and assuming that utility is separable in food and other expenditures. Examples include Hall and Mishkin(1982), Shapiro (1984), Zeldes (1989), Runkle (1991) and Dynan (2000).

In this section we discuss the estimation by GMM of a non-linear Euler equation for consumption where the consumption data are contaminated by measurement error. We will assume that we have enough time series observations and variation in intertemporal prices to be able to estimate the parameters of interest, which include the elasticity of intertemporal substitution and the discount factor. For expositional purposes, we consider the simplest version of a standard life cycle model with uncertainty. We could easily consider more complicated settings. A consumer maximizes expected utility. Utility is intertemporally additive and the instantaneous utility function is of the iso-elastic form. Future utility is discounted at a rate  $\beta < 1$ . The individual consumer is not subject to liquidity constraints and has rational expectations. She can move resources over time by saving and borrowing in  $N$  assets with variable and stochastic returns. Her problem will then imply the following Euler equation for a generic asset  $i$ :

$$E_t \left[ \left( \frac{c_{t+1}^*}{c_t^*} \right)^{-\gamma} (1 + r_{t+1}^i) \beta \right] = 1 \quad (1)$$

where  $c_t^*$  is consumption at time  $t$ ,  $r_{t+1}^i$  the interest rate,  $\beta$  the discount factor and  $\gamma$  the coefficient of relative risk aversion. Such an equation can be used to estimate the preference parameters  $\beta$  and  $\gamma$ , using nonlinear GMM. In order to implement GMM, we need a vector of instruments assumed to be included in the time- $t$  information set of the consumer. In this particular example, in the absence of measurement error a constant and a single instrument (such as the lagged real rate or lagged consumption growth) would deliver identification. For our discussion it is useful to express 1 as follows:

$$\left( \frac{c_{t+1}^*}{c_t^*} \right)^{-\gamma} (1 + r_{t+1}^i) \beta = \varepsilon_{t+1} \quad (2)$$

where  $\varepsilon_{t+1}$  is an expectational error uncorrelated with the information available at time  $t$  and, by definition, has unit conditional mean:  $E_t(\varepsilon_{t+1}) = 1$ .

Now suppose that *observed* consumption in time  $t$ ,  $c_t^o$ , is observed subject to a multiplicative

error:  $c_t^o = c_t^* \eta_t$ . The multiplicative assumption is a critical assumption for identification; we also assume the following.

**Assumption** *The measurement error is stationary and independent of ‘everything’ (including lagged values of the measurement error and expectations errors, consumption levels and interest rates).*

Such an assumption is relatively weak and does *not* require that the measurement error have a unit mean. Neither does it require that measurement error has a specific distribution. If we write equation 2 in terms of observed consumption we have:

$$\left(\frac{c_{t+1}^o}{c_t^o}\right)^{-\gamma} (1 + r_{t+1})^\beta = \varepsilon_{t+1} \left(\frac{\eta_{t+1}}{\eta_t}\right)^{-\gamma} \quad (3)$$

(where we now drop the  $i$  superscript for the asset return). Assumption 1 gives the following result for conditional expectations:

$$E_t \left( \varepsilon_{t+1} \left( \frac{\eta_{t+1}}{\eta_t} \right)^{-\gamma} \right) = E_t(\varepsilon_{t+1}) E_t \left( \left( \frac{\eta_{t+1}}{\eta_t} \right)^{-\gamma} \right) = E_t \left( (\eta_{t+1})^{-\gamma} \right) E_t((\eta_t)^\gamma) = \kappa \quad (4)$$

where the expectation is taken conditional to the information available to the econometrician at time  $t$ . This is obviously a subset of the information available to the agent, which presumably includes measurement error. However, the assumption that measurement error is independent of the expectational error can be used to justify the first equality in equation 4. The stationarity of the measurement error distribution gives that the conditional expectation of the measurement error does not depend on  $t$ . If the measurement error is constant ( $\eta_t = \eta, \forall t$ ) then  $\kappa = 1$ . The exact value of  $\kappa$  depends on the distribution of the measurement error and the (unknown) value of  $\gamma$ . If, for instance, we assume that measurement error is log-normal, that is  $\ln \eta_t \sim N(\mu, \nu)$ , then we have:

$$\kappa = E_t \left( \left( \frac{\eta_{t+1}}{\eta_t} \right)^{-\gamma} \right) = \exp\{\gamma^2 \nu\} \quad (5)$$



Equation (4) implies that applying GMM to equation (3) ignoring measurement error would yield inconsistent estimates of both preference parameters. Notice that the mean of  $\ln(\eta)$  does not enter this expression so that for this and subsequent results we do *not* have to assume that the measurement error has a unit mean. In addition to the Euler equation (1) we can also consider the Euler equation which links consumption at  $t$  and at  $t+2$ . Following the same steps used to derive equation (3), we can obtain:

$$\left(\frac{c_{t+2}^o}{c_t^o}\right)^{-\gamma} (1+r_{t+1})(1+r_{t+2})\beta^2 = \varepsilon_{t+1}\varepsilon_{t+2} \left(\frac{\eta_{t+2}}{\eta_t}\right)^{-\gamma} \quad (6)$$

The assumption of a stationary measurement error implies that  $E_t\left(\left(\frac{\eta_{t+2}}{\eta_t}\right)^{-\gamma}\right) = \kappa$ . Under the assumption of lognormality this term is also equal to  $\exp\{\gamma^2\nu\}$ . Now, let's define  $u_{t+1}^1$  and  $u_{t+2}^2$  as follows:

$$\begin{aligned} u_{t+1}^1 &= \left[ \left(\frac{c_{t+1}^o}{c_t^o}\right)^{-\gamma} (1+r_{t+1})\beta - \kappa \right] \\ u_{t+2}^2 &= \left[ \left(\frac{c_{t+2}^o}{c_t^o}\right)^{-\gamma} (1+r_{t+1})(1+r_{t+2})\beta^2 - \kappa \right] \end{aligned} \quad (7)$$

With no further assumptions these equations cannot be used for estimation since the parameter  $\gamma$  enters  $\kappa$  in an unknown way. We now derive two estimators based on these equations.

For the first estimator we assume that the errors are lognormally distributed so that the form of  $\kappa$  is given by (5) and we have:

$$\begin{aligned} u_{t+1}^1 &= \left[ \left(\frac{c_{t+1}^o}{c_t^o}\right)^{-\gamma} (1+r_{t+1})\beta - \exp\{\gamma^2\nu\} \right] \\ u_{t+2}^2 &= \left[ \left(\frac{c_{t+2}^o}{c_t^o}\right)^{-\gamma} (1+r_{t+1})(1+r_{t+2})\beta^2 - \exp\{\gamma^2\nu\} \right] \end{aligned}$$

A constant and a single instrument (for instance the lagged value of the interest rate,  $r_t$ ) would

therefore deliver four orthogonality conditions that would over-identify the three parameters of the model  $\{\beta, \gamma, \nu\}$ . We refer to this estimator as the GMM-LN estimator.

Our second estimator, uses the two equation in (7) to difference out  $\kappa$  and obtain orthogonality conditions that involve only  $\beta$  and  $\gamma$ . In particular, define:

$$\zeta_{t+2} = \left[ \left( \frac{c_{t+1}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})\beta \right] - \left[ \left( \frac{c_{t+2}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})(1 + r_{t+2})\beta^2 \right] \quad (8)$$

From (7) is clear that  $\zeta_{t+2}$  has zero mean and is uncorrelated with information available at  $t - 1$ . This residual term can then be used to form orthogonality conditions to identify the parameters of the model. Notice that equation (8) does not depend on the parameters of the measurement error process. We refer to this estimator as GMM-D. As equation (8) is a form of double-differencing, we expect a significant decline in precision, as compared to the GMM-LN estimator.

In this section we have developed two GMM estimators for the consumption Euler equation if consumption is contaminated by stationary multiplicative measurement error. The first estimator, GMM-LN (see (7)), makes use of the lognormality assumption. The second, GMM-D (see (8)), does not require any supplementary assumptions. We turn now to the small sample properties of four estimators: the linearized (approximate) GMM equation (AGMM), the exact GMM (EGMM, with no allowance for measurement error), GMM-LN and GMM-D.

### 3 Small Sample Results

The simulated data used in our experiments are generated by solving a standard intertemporal utility maximization problem under interest rate and labor income uncertainty. The details of the model and its solution are given in the Appendix. The parameter values for the coefficient of relative risk aversion and the discount factor are set to 4 and 0.95 respectively. The interest rate series for each agent is a stationary  $AR(1)$  process with a mean of 0.05 and an  $AR$  coefficient

of 0.6; thus the discount factor multiplied by the mean gross interest rate is very close to unity. We solve the model for 100 periods and drop the initial 19 and final 18 observations to avoid starting and ending effects. For periods 20 to 81 the optimal consumption functions are all very similar and consumption growth is effectively stationary. To construct the data used in the experiments below we first generate consumption paths using the optimal program and then add multiplicative lognormal measurement error. The measurement error has a unit mean and a variance of 0.004. The latter is chosen so that 75% of the period to period variance in consumption growth is due to noise. This is at the upper end of estimates of the amount of noise in the real world data.

The experiments are performed to obtain the sampling distribution of the estimators developed in the previous section. For comparison, we also document the sampling distributions of conventional estimators based on the exact nonlinear Euler equation (EGMM) and its log-linear approximation (AGMM)<sup>6</sup>. We experiment with different panel lengths and we set the number of households for each estimation to 100. To be as close to real data availability as possible, we experiment with panel lengths of 15 and 25. We use a single instrument, lagged real rates. Two sets of experiments are performed: The first set uses simulated data that are generated allowing for correlation of expectational errors across households. For this, we simply assume that every group of 100 households face the same interest rates and estimations are performed by pooling these 100 households together. The second set of experiments uses simulated data generated allowing for cross-sectional variation in interest rates so that every household faces different series (although the underlying interest rate process is the same for every household, ex-post series are different). Again, estimations are performed by pooling 100 households together. Notice that in both experiments individual households experience independent (across households) wage shocks. We set the number of Monte Carlo replications for both sets to 1000.

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<sup>6</sup>In the case of log-linearized Euler Equation estimation, we attempt to identify only the coefficient of relative risk aversion since the discount factor is buried in the constant after linearization.

Table 1 presents the results for the first set of experiments in which the pooled households for a given estimation face the same interest rate series. The table presents the means, medians and standard deviations of the sampling distributions for all four estimators. As can be seen, all estimators improve in estimating the coefficient of relative risk aversion ( $\gamma$ ) as the sampling period ( $T$ ) increases. Both *AGMM* and *EGMM* display significant bias in the estimates of  $\gamma$  but our two new estimators have negligible bias. The same holds for the estimates of the discount factor. As expected, the distribution of  $\gamma$  is more dispersed for *GMM-D* as compared to *GMM-LN*.

Table 2 presents the results for the second set of experiments in which the pooled households face different interest rate series. Both of our estimators, *GMM-LN* and *GMM-D*, still perform better than the other two estimators and capture the true value of  $\gamma$  and  $\beta$  fairly well. The difference between our estimators and *AGMM* and *EGMM* is again substantial for small  $T$ . Both *EGMM* and *AGMM* display upward biases (in means) although both estimators greatly improve as  $T$  becomes large. Moreover, medians of the sampling distributions of  $\gamma$  for *EGMM* and *AGMM* are very close to the true value and this is not true for our proposed estimators. As was the case for the common interest rate experiments, both our new estimators do an excellent job in capturing the true value of  $\beta$ , while *EGMM* seems hopeless.

Overall, the results suggest that the type of measurement error assumed in the literature causes a serious downward bias in the discount factor estimates if it is ignored and this bias does not go away as the panel length increases. The bias in the coefficient of relative risk aversion is not as severe and it seems to become smaller as the panel length increases (particularly in the common interest rate case). It is quite interesting that with a short panel of 15 years, the log-linearized Euler equation yields almost the same result as the exact one, whether interest rate series are common or there is a cross section variation. Recall that the main reason for using the log-linearized version of the Euler equation is that linearizing the exact Euler equation removes the measurement error by shifting it to the expectation errors. The log-linear version

seems to be superior to the exact version only as the panel length becomes (unrealistically) long.

Turning to our estimators, they both perform very well in estimating the parameters of interest. The most striking feature of our estimators is that they outperform the other two estimators especially when the panel length is short. This is clearly an important feature since panel data on consumption are rare, let alone long panels. These estimators' advantage over traditional approaches is more pronounced in the case where households face the same interest rate series (which is perhaps a more realistic assumption than cross sectional variation in interest rates).

## 4 An Empirical Application

In this section we present an empirical application of our GMM estimators using the Panel Study of Income Dynamics (PSID). The survey contains annual information on food at home and food at restaurants. Despite its shortcomings (no expenditure variable other than food, large measurement error, representativeness issue due to attrition) we chose to work with the PSID because it is the longest available panel survey on consumption and it has been used extensively for Euler equation estimation previously. Our sample covers the periods between 1974 and 1987. Although the actual panel length is much longer, some of the food variables are hard to interpret prior to 1974 and food related questions were suspended for several years after 1987.

Our sample excludes households that did not report food expenditure for at least five consecutive years. We treat split-ups as separate household units. We also exclude any household that may be liquidity constrained in the sample period; we do this by keeping only the households that reported some savings<sup>7</sup>. We exclude single headed households, households whose marital

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<sup>7</sup>Note that the Euler equation estimation is based on the implicit assumption that the equation holds every period for a given household. If a household carries forward some liquid assets from one period to the next then the relevant Euler equation for that household should hold. Even though this is not a perfect way of handling

status changed over the sample period and those whose head is less than 22 in 1974 and more than 22 in 1987. Our final unbalanced panel (a minimum of 5 years and a maximum of 14 years) has a total of 1,906 households (19,317 observations). We assume that all households face a common interest rate series calculated based on the U.S. three-month treasury bill rates and the consumer price index. This amounts to using only the "time variation" in the intertemporal price.

Following Attanasio et al (1999), we specify the within period utility function in the following way:

$$U(C_t, Z_t) = \frac{1}{1-\gamma} C_t^{1-\gamma} \exp(\theta' Z_t) \quad (9)$$

where  $\gamma$  is the coefficient of relative risk aversion,  $Z$  variables are demographics such as family composition, the number of children and labor supply variables. It is important to incorporate them into empirical specifications since they are likely to affect the level of utility for a given level of consumption. The resulting Euler equation is

$$E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 + r_{t+1}^i) \beta \exp(\theta \Delta Z_{t+1}) \right] = 1 \quad (10)$$

We are now interested in estimating  $\gamma$  and  $\beta$  controlling for changes in demographics. The first estimator,  $GMM - LN$ , can be re-written as follows

$$\begin{aligned} u_{t+1}^1 &= \left[ \left( \frac{C_{t+1}^o}{C_t^o} \right)^{-\gamma} (1 + r_{t+1}) \beta \exp(\theta \Delta Z_{t+1}) - \exp\{\gamma^2 \nu\} \right] \\ u_{t+2}^2 &= \left[ \left( \frac{C_{t+2}^o}{C_t^o} \right)^{-\gamma} (1 + r_{t+1})(1 + r_{t+2}) \beta^2 \exp(\theta \Delta Z'_{t+2}) - \exp\{\gamma^2 \nu\} \right] \end{aligned} \quad (11)$$

where  $\Delta Z'_{t+2} = Z_{t+2} - Z_t$

The second estimator is based on:

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binding and/or expected to bind liquidity constraints, it seems to be a widely accepted one.

$$\zeta_{t+2} = \left[ \left( \frac{c_{t+1}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1}) \beta \exp(\theta \Delta Z_{t+1}) \right] - \left[ \left( \frac{c_{t+2}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})(1 + r_{t+2}) \exp(\theta \Delta Z'_{t+2}) \beta^2 \right] \quad (12)$$

Note that we now have an additional parameter vector ( $\theta$ ) to estimate. In practice, we assume that the only relevant demographic variable is 'family size'. We assume that changes in this are perfectly foreseen so that we use  $\Delta Z_{t+1}$  as an instrument.

Table ?? presents the estimates from the PSID. As in the simulations, we also present the results based on the approximate GMM (AGMM) and the exact GMM (EGMM) for comparison. For all estimators, the parameters are estimated by allowing for conditional heteroskedasticity of the disturbances but not the serial correlation. Moreover, the weighting matrices used do not allow for cross section correlation of the disturbances. In the case of GMM-LN, we allow for correlation across equations. The values for the elasticity of intertemporal substitution ( $\gamma$ ) given in Table ?? are all within a familiar range. The AGMM estimation results in the highest estimate while EGMM yields the lowest. Although the estimates of the coefficient of relative risk aversion we obtain from GMM-LN and GMM-D are similar (around 1.5), the discount factor estimates are quite different. GMM-LN yields a reasonably precisely estimated value of 0.99 (corresponding to a discount rate of 1%) whereas GMM-D yields an estimate of 0.93 (a discount rate of 7.5%) albeit less precisely. In comparison, the discount factor estimate obtained from EGMM is implausibly low (0.87 corresponding to a discount rate of 15%). The estimated measurement error variance using GMM-LN is 0.085. This value corresponds to 62% of the variation in consumption growth being noise. In all cases the parameter for the effect of family size ( $\theta$ ) is positive but imprecisely estimated for all models. The fact that the estimates for GMM-LN and GMM\_D are so similar statistically suggests that the lognormal assumption is a reasonable one. If this is the case then the conclusion is that our best estimate of the mean eis is 1.45 (with a standard error of 0.3) and the mean discount factor is 0.99 (with a standard

error of 0.1).

## 5 Conclusion

In this paper we have proposed two new GMM estimators for the Euler equation that take account of the measurement error in consumption. A feature of dealing with measurement error in nonlinear models is that the solutions are specific to the context and that is the case here. In particular, we have exploited the iso-elastic assumption which gives a ratio of consumptions in the Euler equation. If some other functional form used then different assumptions will be needed to achieve identification. Monte Carlo simulations suggest that when the panel length is short both of our estimators perform significantly better than the linearized Euler equation or the exact GMM estimator that ignores measurement error. The estimator that makes a specific lognormal assumption, GMM-LN, performs better when that assumption is correct but is likely to be less robust than the estimator that simply assumes stationarity, GMM-D. The empirical application of the proposed estimators yields plausible estimates of the coefficient of relative risk aversion and discount rate. In particular, the estimate of the elasticity of intertemporal substitution is 1.45 with a standard error of 0.3.

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Estimator	Instrument	$\gamma$		$\beta$		$\sigma^2$	
		$T = 15$	$T = 25$	$T = 15$	$T = 25$	$T = 15$	$T = 25$
AGMM	$(1 + r_t)$	2.43	3.76	—	—	—	—
		[2.54]	[3.29]	—	—	—	—
		(25.1)	(3.10)	—	—	—	—
EGMM	$(1 + r_t)$	2.92	3.59	0.90	0.89	—	—
		[2.52]	[3.22]	[0.92]	[0.91]	—	—
		(2.89)	(2.31)	(0.09)	(0.08)	—	—
GMM-LN	$(1 + r_t)$	3.68	4.00	0.95	0.95	0.006	0.003
		[3.56]	[3.88]	[0.95]	[0.95]	[0.004]	[0.004]
		(2.43)	(2.29)	(0.01)	(0.01)	(0.03)	(0.03)
GMM-D	$(1 + r_t)$	4.05	4.04	0.95	0.95	—	—
		[2.82]	[3.43]	[0.95]	[0.95]	—	—
		(3.93)	(2.77)	(0.02)	(0.01)	—	—

Note: Values are means, medians (in square brackets) and standard deviations (in brackets) of sampling distributions.  $N = 100$ . True values:  $\gamma = 4$ ,  $\beta = 0.95$ ,  $\sigma^2 = 0.004$

Table 1: Monte Carlo Results: Common Interest Rates

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Estimator	Instrument	$\gamma$		$\beta$		$\sigma^2$	
		$T = 15$	$T = 25$	$T = 15$	$T = 25$	$T = 15$	$T = 25$
AGMM	$\ln(1 + r_t)$	4.65	4.38	—	—	—	—
		[4.01]	[3.97]	—	—	—	—
		(9.72)	(1.66)	—	—	—	—
EGMM	$(1 + r_t)$	4.60	4.38	0.87	0.88	—	—
		[4.07]	[3.94]	[0.89]	[0.89]	—	—
		(2.04)	(1.68)	(0.08)	(0.07)	—	—
GMM-LN	$(1 + r_t)$	3.83	3.88	0.95	0.95	0.004	0.004
		[3.60]	[3.71]	[0.95]	0.95	[0.004]	0.004
		(1.18)	(0.98)	(0.004)	(0.01)	(0.001)	(0.001)
GMM-D	$(1 + r_t)$	4.32	3.75	0.95	0.95	—	—
		[3.86]	[3.53]	[0.95]	[0.95]	—	—
		(1.83)	(1.05)	(0.003)	(0.002)	—	—

Note: Values are means, medians (in square brackets) and standard deviations (in brackets) of sampling distributions.  $N = 100$ . True values:  $\gamma = 4$ ,  $\beta = 0.95$ ,  $\sigma^2 = 0.004$

Table 2: Monte Carlo Results: Cross Section Interest Rate Variation

Estimator	$\gamma$	$\beta$	$\sigma^2$	$\theta$
AGMM	2.33	—	—	0.045
	(0.77)			(0.03)
EGMM	1.15	0.87		0.186
	(0.23)	(0.05)		(0.47)
GMM-LN	1.45	0.99	0.085	0.894
	(0.27)	(0.10)	(0.05)	(0.51)
GMM-D	1.53	0.93		0.729
	(0.66)	(0.71)		(0.51)

Table 3: Estimates from the PSID (1974-1987). Standard errors in parantheses.

## A The Simulation Model.

We assume that the utility function is intertemporally additive and the sub-utilities are iso-elastic. The problem of the generic household is:

$$\max_{c_t} E_t \sum_{t=0}^T \beta^t u(C_t)$$

$$s.t \ X_{t+1} = (1 + r_{t+1})(X_t - C_t) + Y_{t+1}$$

where  $C_t$  is nondurable consumption in period  $t$ ,  $X_t$  is cash-on-hand (total financial and non-financial wealth) and  $Y_t$  is current labor income. We assume that durable consumption and leisure are separable from the nondurable consumption. The income process is assumed as follows:

$$Y_{t+1} = P_{t+1}U_{t+1}$$

$$P_{t+1} = P_t N_{t+1}$$

$P_t$  is permanent income which is subject to lognormally distributed shocks  $N_t$  with mean unity and variance  $\exp(\sigma_n^2) - 1$ , current income  $Y_t$  equals permanent income multiplied by a transitory shock,  $U_{t+1}$ , which is distributed lognormally with mean one and variance  $\exp(\sigma_u^2) - 1$ . The interest rate series is assumed to be generated by a stationary first order autoregressive process with long-run mean  $\mu$  and autoregressive coefficient  $\rho$ . Interest rates shocks  $\epsilon_{t+1}$  are assumed to be white noise with variance  $\sigma_\epsilon^2$ . The process is

$$r_{t+1} = (1 - \rho)\mu + \rho r_t + \epsilon_{t+1}$$

The intertemporal model described above does not have an analytical solution due to the assumed income uncertainty. Therefore we utilized the standard numerical dynamic programming methods to obtain a solution. Since the utility function is additive over the life cycle we solved the model recursively starting from the last period of life. We assume away any bequest motive so that consumption in period  $T$  is:

$$c_T(x_T) = x_T$$

The problem is solved via policy function iteration using the terminal value condition. Having a nonstationary income process makes the problem harder to solve since the range of possible income values is too large. Instead, we redefine all relevant variables in terms of their ratios to permanent income and solve for the consumption to income ratio. By doing this we reduced the number of state variables to two, namely the cash on hand to income ratio and the interest rate. Moreover, we obtain an iid income process which can be approximated by standard Quadrature methods. Given the redefinition of the variables, the Euler equation can be written as

$$\theta_t(w_t, r_t)^{-\gamma} - \frac{1}{(1 + \delta)} E_t \left[ (1 + r_{t+1}) \theta_{t+1} (w_{t+1}, r_{t+1})^{-\gamma} n_{t+1}^{-\gamma} \right] = 0$$

where  $\theta_t = \frac{C_t}{P_t}$ ,  $w_t = \frac{X_t}{P_t}$ . At the terminal date  $T$ , consumption to income ratio is a function of only the cash on hand to income ratio and since the bequest motive is assumed away it follows that  $\theta_T = w_T$ .

For the income process, we use 10 point Gaussian Quadrature and we approximate the interest rate process by forming a 10 point first order discrete Markov process. We use a cubic spline to approximate the consumption function at each iteration. The agent is allowed to borrow the amount he can pay back with certainty. In practice this constraint will never bind because the functional form of the utility function implies that zero consumption results in infinite marginal utility. Since we do not assume an explicit borrowing limit, the consumption

Parameter	Value
Coefficient of risk aversion ( $\gamma$ )	4
Discount factor ( $\beta$ )	.95
Discount rate ( $\delta = \frac{1}{\beta} - 1$ )	.053
Standard deviation of permanent income shocks ( $\sigma_n$ )	.02
Standard deviation of transitory income shocks ( $\sigma_u$ )	.1
Unconditional mean of interest rate process ( $\mu$ )	.05
AR(1) coefficient of interest rate process ( $\rho$ )	.6
Standard deviation of interest rate process ( $\sigma_\epsilon$ )	.025
Variance of log measurement error ( $\sigma_\eta$ )	.004 (75% noise)

Table 4: Parameter Values

functions are continuously differentiable. In fact, in our case where agents have iso-elastic preferences and income uncertainty, consumption functions are strictly concave.

In order to solve the problem, we define an exogenous grid for the cash on hand to income ratio:  $\{x_j\}_{j=1}^J$ . It is important to adjust the grid as the solution goes back in time. The algorithm finds the consumption level that makes the standard Euler equation hold for each value of  $x$  and  $r$ . In practice, we took 100 points for  $x$  and 10 points for  $r$ . The grid for  $x$  is finer at lower levels in order to capture the curvature of the consumption function. After solving for the consumption function of a generic household for 100 periods, we simulate consumption paths for 100,000 ex-ante identical households facing different interest rate realizations. Then, using the same consumption function, we simulate paths for 100,000 ex-ante identical households where groups of 100 households face the same realized interest rate series. We use only 15 and 25 periods for the estimations and these observations are taken from the middle of each path. Table 4 presents the assumed parameter values for our experiments. The discount rate and the mean interest rate are chosen to be close in order to prevent consumers to quickly go towards the borrowing constraint<sup>8</sup>.

<sup>8</sup>When the discount rate is large relative to the interest rate, consumers borrow close to the maximum possible amount. Then the movement of consumption is largely driven by income and the identification of interest rate impact on consumption growth becomes very difficult. See Attanasio and Low (2004) and Alan and Browning (2003) for a detailed discussion on the problems of identification and impatience.