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Credit Ratings and Market Information

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Accurate credit ratings are important for both investors and regulators. We demonstrate that the market for credit risk provides an important source of discipline for credit rating agencies (CRAs). We examine a model in which a CRA's rating is followed by a market for credit risk that provides a public signal – the price. More informative trading increases the CRA's incentives to be accurate by making rating errors more transparent. We show that this source of discipline is (a) robust to moral hazard, multiple CRAs, and connected primary and secondary markets and (b) specific to the market for credit risk. (*JEL* G24, G28)

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Investors rely on credit ratings to provide information about the credit risk of assets and issuers. Accurate and timely ratings facilitate the allocation of capital and have real effects (e.g., Kisgen 2006; Kisgen and Strahan 2010). Ratings also play a significant role in the risk assessments regulators and markets perform on financial institutions (see European Commission 2015). Yet credit ratings are not perfect; market events have exposed their inaccuracies. Famous examples include the East Asian Financial Crisis (1997), Enron and Worldcom (2001, 2002), and the 2007–2008 Financial Crisis, reflecting sovereign, corporate, and structured finance ratings, respectively.

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Despite the importance of ratings for the financial system, regulatory efforts have been largely ineffective. Dimitrov, Palia, and Tang (2015), for example, show how the Dodd-Frank Act did little to improve ratings accuracy.[1] In this paper, we argue that there is a source of discipline for credit rating agencies (CRAs): markets for credit risk. These markets, such as the credit default swap (CDS) market or the secondary bond market, *crowd in* rating accuracy by making rating mistakes more transparent and thus strengthening rating agencies' reputation concerns. This implies that, in some cases, markets may supplement regulators as a governance mechanism for CRAs, and policies that foster markets for credit risk may have benefits both in the markets themselves and externally.

We examine the equilibrium interaction between credit ratings and the market for credit risk. In the model, a CRA exerts effort to acquire accurate information about the quality of a risky investment project. This is costly, but the CRA may diminish its reputation (and future payoffs) if its ratings are revealed to be of poor quality. The CRA's rating is released to the public, and investors may purchase the asset. A market for credit risk then establishes a market price à la Kyle (1985): a speculator may acquire information to profit from noise traders, and a market maker clears the market. Lastly, the asset payoffs are realized, leading to monetary payoffs for investors and a reputational payoff for the CRA.

The interaction between the CRA and the market for credit risk has two contrasting effects. More informative trading increases the CRA's incentives to be accurate by providing insight about whether the rating was incorrect and, thus, augmenting the CRA's reputational concerns. The information coming from the market price disciplines the CRA; this is the crowding in effect. This demonstrates that, from the CRA's perspective, the CRA's rating accuracy and information acquisition by the speculator are strategic complements. On the other hand, ratings that are more accurate decrease the speculator's incentives to acquire information by decreasing mispricing. Rating accuracy thus crowds out market informativeness. Therefore, from the speculator's perspective, the CRA's rating accuracy and information acquisition by the speculator are strategic substitutes. This leads to a unique equilibrium.

We find that, in equilibrium, the CRA may acquire too little information or too much information from a surplus perspective. When the CRA acquires too little information, regulatory discipline, such as reputational sanctions (e.g., increased liability standards[2]), incentivize the CRA to be more accurate, but crowd out market discipline, which reduces the effectiveness of the regulation. However, policies to increase market informativeness (such as

[1] Issues with ratings have not subsided since the financial crisis; for example, Baghai and Becker (2020) discuss recent failures in the commercial mortgage-backed securities market.

[2] The 2010 Dodd-Frank bill required a higher standard of liability for CRAs. This was not enforced, as an initial attempt to do so caused CRAs to pull their ratings from asset-backed securities, freezing the market. In response, the SEC decided to delay implementation indefinitely (see Holzer 2011).

improving governance and trust in exchanges or increasing disclosure[3]) also reap informational benefits by improving CRA accuracy.[4] If, on the other hand, the CRA acquires too much information, we demonstrate that the effect of these policies may go either way. This demonstrates that policy recommendations must be market specific, not one size fits all.

These results prove robust to various settings. First, we examine the case in which investors in the primary market may be active in the secondary market due to liquidity shocks. As these investors pay dearly to trade against the informed speculator, this reduces their willingness to pay in the primary market and its viability. Second, we study competition between rating agencies. The CDS market still provides a source of discipline with multiple CRAs. Lastly, we prove that the main conclusions of the model hold when we allow the CRA to both acquire information *and* inflate ratings. In equilibrium, the CRA does not do both but may perform a mix of the two.

The result that information from the market for credit risk crowds in rating accuracy raises the question of whether this effect is specific to the market for credit risk. Many other sources of information could potentially discipline a CRA. We demonstrate that the answer may not be straightforward. First, we show that while other CRAs also provide information, their competitive interaction leads to collusive effects that may diminish information provision. Second, project outcomes (e.g., defaults) provide information, but may take time to realize, if they realize at all. Third, we discuss how biases in media may undercut that source of discipline.

Several papers look at the effect of different sources of information on credit ratings. Badoer and Demiroglu (2019), find that the introduction of TRACE, which disseminated information about OTC corporate bond transactions, makes ratings downgrades better predictors of default and more sensitive to changes in credit spreads.[5] Gopalan, Gopalan, and Koharki (2017) distinguish between listed and unlisted firms (listing makes information for market participants easier to find) and find that ratings of unlisted firms in India are higher, less sensitive to financial conditions, and contain less information about subsequent defaults than ratings of listed firms. Fong et al. (2014) shows that a plausibly exogenous decrease in information leads to less accurate ratings; when mergers reduce the number of equity analysts, ratings become less informative about

[3] One example of this is the introduction of TRACE, which provided information on over-the-counter corporate bond transactions. Badoer and Demiroglu (2019) show that this increased the explanatory power of ratings for defaults and further linked ratings to credit spread changes (though it weakened the link between ratings and equity prices).

[4] Intriguingly, this also implies that when market informativeness is low, the CRA does not fill the informational gap; that is, the CRA does not produce a lot of information either. Hence, a CRA fails to produce information at the time when it is most valuable.

[5] They also find that, in contrast, ratings downgrades are less informative in the sense of explanatory power for stock market prices. Chava, Ganduri, and Ornthanalai (2019) find that the introduction of CDS contracts lessened the impact of ratings downgrades on stock prices.

defaults and future downgrades. Bonsall, Green, and Muller (2018) find that firms with more media coverage receive more accurate ratings.[6]

A few recent papers look at the effect of different sources of information on informed speculation. Lewis and Schwert (2018) provide evidence that when more information about bond transactions is provided, prices are less informative, consistent with reducing the incentives of informed investors to trade bonds. Jayaraman and Wu (2019) show evidence consistent with mandatory disclosure leading to less informed trading.

In the text, we also provide novel empirical implications that explore the interaction between ratings and the market for credit risk. Next, we offer a summary of the related theoretical literature.

The link between ratings quality and reputation is key for our results. Mathis, McAndrews, and Rochet (2009) examine how a CRA's concern for its reputation affects its ratings quality. They present a dynamic model of reputation in which a monopolist CRA may mix between lying and truth-telling to build and/or exploit its reputation. Strausz (2005) is similar in structure to Mathis, McAndrews, and Rochet (2009), but examines information intermediaries in general.

Several papers have studied how a firm's disclosure policy affects information aggregation in the presence of a market comprising sophisticated investors and uninformed liquidity traders (e.g., Verrecchia 1982, Diamond 1985, Kim and Verrecchia 1991). Goldstein and Yang (2017) review this literature in depth. Gao and Liang (2013) focus on the real impacts of this interaction. In these models, disclosure reduces gains to information acquisition by speculators, as in our paper.[7] However, we also examine information being produced by a strategic rating agency and the two-way interaction between market information and the rating agency's information.

The interaction between market information and the rating agency's information is a type of feedback effect. A substantial literature on feedback effects of market prices examines how markets guide real decisions and the feedback loop between the two that results (for a review, see Bond, Edmans, and Goldstein 2012). Our paper examines the feedback between two sources of information, and their effects on surplus. Bond and Goldstein (2015) look at both the real and informational feedbacks between government interventions and market information. When the intervention consists of disclosing information, they also find a crowding out effect of speculators' information acquisition. Goldstein and Yang (2019) demonstrate that making an exogenous public signal more informative may reduce real efficiency. Goldstein and Huang (2020) have a feedback loop between investors, a firm undertaking an investment project, and a CRA. In their model, the CRA is the only

6 We will discuss the media and other sources of public information further in Section 7.

7 Han, Tang, and Yang (2016) show that this effect may be mitigated by the information attracting more noise/liquidity traders.

source of information and a coordination problem among investors drives the feedback loop.

1. The Model

Our model has two distinct elements, the ratings process and the market for credit risk. We first present them separately and then analyze the strategic interactions between them. The market for ratings takes place first, and is followed by the market for credit risk. All agents in the model are rational and risk neutral, and, for simplicity, we assume that there is no discounting.

1.1 The ratings process

The ratings process takes place at time $t=1$, and consists of three types of agents: an issuer, a monopoly credit rating agency (CRA), and investors.

The issuer has access to a risky investment project but needs financing from investors. The project returns 1 in case of success and 0 in case of failure, and the cost of the project is I. $y \in \{S, F, N\}$ denotes the project's outcome, where S (F) signifies *Success* (*Failure*) and N represents the case in which the project is not undertaken. The quality of the project is denoted by $\theta \in \{B, G\}$, where B (G) stands for *Bad* (*Good*) and relates to its probability of failure: a bad project fails with probability $f_B \in (0,1)$, and a good project fails with probability $f_G \in (0, f_B)$, with $f_B - f_G \equiv \Delta^f$. Both good and bad investments have an ex ante probability $\frac{1}{2}$ of occurring, and the investment quality is *a priori* unknown, even to the issuer itself.

If the issuer wishes to undertake the project, it can sell a claim to the payoff of the project to investors. $v_\theta = 1 - f_\theta - I$ denotes the net present value (NPV) of a project of quality θ. Good projects should be financed (they have positive NPV, i.e., $v_G > 0$) but, without prior knowledge about the quality of the project, no financing takes place (ex ante NPV is negative, i.e., $\frac{1}{2}v_G + \frac{1}{2}v_B < 0$). Therefore, the presence of a CRA can improve welfare by screening projects for investors.

The CRA observes a signal of project quality (more on this shortly) and privately offers a credit report, $\widetilde{r} \in \{H, L\}$, where H signifies *high* and L signifies *low*, to the issuer. The issuer either pays a rating fee φ and has the report publicized or refuses to purchase it. This allows for "rating shopping"by the issuer. The outcome of the rating process, as observed by investors, is thus $r \in \{\widetilde{r}, \varnothing\}$, where $r = \widetilde{r}$ signifies that the issuer had the credit report publicized by the CRA, and $r = \varnothing$ signifies that there is no rating. If the issuer refuses to buy the CRA's report and goes on the market as unrated, that in itself is a signal to investors.

Investors observe the rating and decide whether they wish to buy the claim. p^r denotes the price investors are willing to pay for a claim backed by the payoff of the project for a given rating r. Investors are perfectly competitive, so the market price, p^r, is equal to the expected value of the project's payoff given their posterior beliefs. We describe the expression for p^r in Section 1.3 after having introduced the CRA's reputation and its rating strategy. If the price, p^r, exceeds the investment cost, that is, if $p^r > I$, the issuer sells the claim and undertakes the project; the investment outcome $y \in \{S, F\}$ is then realized at time $t=3$ and is observed by all players. Otherwise, the project is not implemented, and all players observe $y=N$ at the end of time $t=1$.

We assume that the rating fee is an exogenously specified fraction $\phi \in (0,1]$ of the issuer's surplus from selling the claim to investors and implementing the project. When the rating is publicized ($r=\widetilde{r}$) and the issuer implements the project, the CRA receives a fraction ϕ of the issuer's surplus $p^r - I$. This captures the bargaining between the issuer and the CRA over the rating fee. The qualitative results in the paper do not depend on the value of ϕ. In Internet Appendix B.9, we show that these results continue to hold in a setting where the rating fee is endogenous.

1.2 The market price of risk

At time $t=2$, if the issuer implemented the investment project, a market for credit risk takes place. We will describe this market as the CDS market in what follows. The same setting can be used to model the secondary market for the asset.[8]

p^{cds} denotes the price of a CDS contract and x the net volume of trades.[9] A CDS contract is formalized as follows: at time $t=2$, the contract is signed, and the buyer of the swap pays an amount p^{cds} to the swap's seller. In return, the seller agrees that in the event of default at time $t=3$, the seller will pay the buyer an amount 1.[10] Trading occurs among a noise/liquidity trader, who trades an amount $x_n \in \{-n; +n\}$, with both realizations equally likely; a second trader, who may be a speculator or another noise trader (more on this shortly); and a competitive market maker. The price p^{cds} is determined in a simplified model à la Kyle (1985).

With probability $\eta \in (0,1]$, the second trader is a speculator, who acquires information to profit off the noise trader in the market. Having observed the rating r, a speculator decides how much information about the project to acquire. She chooses the precision $\iota \in \left[\frac{1}{2}, 1\right]$ of her private signal $\sigma_s \in \{b, g\}$, where

$$\iota \equiv \Pr(\sigma_s = g \mid \theta = G) = \Pr(\sigma_s = b \mid \theta = B).$$

$K(\iota) = \rho_s k(\iota)$ denotes the cost of precision, and we assume $\rho_s > 0; k' \geq 0; k'' > 0$; and $k(\frac{1}{2}) = k'(\frac{1}{2}) = 0$. To ensure that the equilibrium choice of ι is unique, we

8 The easiest way to model this as a secondary market is to endow the speculator with some of the asset. Allowing for shorting would be equally good, of course.

9 In a CDS contract, a protection buyer pays a premium to the protection seller, in exchange for a payment from the latter if a credit event (usually bankruptcy) occurs on a given reference entity within a predetermined time period. The protection buyer does not need to hold the reference entity ("naked" CDS). The amount that the protection seller has to pay in case the credit event occurs is called the notional amount. The premium is quoted in basis points per year of the contract's notional amount and is called the CDS spread.

10 We normalize the notional amount per contract to 1 and let p^{cds} represent the CDS spread.

assume that the speculator's cost function is sufficiently convex, that is, $K''(\iota) > 2n\Delta^f$ for any $\iota \in \left[\frac{1}{2}, 1\right]$. The speculator tries to use her superior information to profit from mispricing in the market. x_s denotes her demand. We use the convention $x_s < 0$ when she is selling protection and $x_s > 0$ when she is buying protection.

With probability $1-\eta$, the second trader is another noise trader, whose demand is unrelated to project quality, that is, $x_s \in \{-n; +n\}$, with both realizations equally likely. The identity of the second trader is unknown to the other traders. This ensures that the order flow and, as a consequence, the CDS price, never fully incorporate the speculator's private information.

The market maker observes the trade orders, that is, $\{x_s, x_n\}$, but not the identity of the trader submitting each order.[11] Having observed $\{x_s, x_n\}$ and r, he sets a price p^{cds} and clears the market. We assume that the market maker makes zero profits, which implies $p^{cds} = E[f_\theta \mid r, \{x_s, x_n\}]$, where the expectation takes into account equilibrium beliefs about the CRA's rating strategy and the speculator's choice of precision and trading strategy.[12]

$x = x_s + x_n$ denotes the total order flow. The informativeness of market trading is defined by the speculator's choice of precision ι. The other agents in the model do not directly observe the speculator's choice of ι; $\bar{\iota}$ denotes their conjectures about this choice.

1.3 The CRA's reputation and rating strategy

The CRA can be one of two different types: *opportunistic* or *informative*. $\tau \in \{\mathcal{O}, \mathcal{I}\}$ denotes a realized type, where $\mathcal{O}$ ($\mathcal{I}$) stands for *Opportunistic* (*Informative*). The realization of τ is the CRA's private information. Investors' prior beliefs about τ are given by $\Pr(\tau = \mathcal{I}) = q_0 \in (0,1)$ and $\Pr(\tau = \mathcal{O}) = 1 - q_0$. The probability q_0 represents the CRA's initial reputation for being an informative type. The informative CRA always learns the project quality perfectly and offers a high (low) rating for a good (bad) project. This type is behavioral;[13] it may be fully informative because of (a) a very high reputation cost, (b) a very low cost of information gathering, and/or (c) a very high private benefit from having an accurate rating. For simplicity, we assume that the informative CRA's cost of information is zero.

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[11] As in the discrete setup of Faure-Grimaud and Gromb (2004), to ensure the existence of a Perfect Bayesian equilibrium, we allow the market maker to observe trade orders (but not the identity of those trade orders). For example, suppose the speculator submits a sell order $x_s = -n$ and the noise trader submits a buy order $x_n = +n$. The market maker observes a sell order and a buy order, but does not know which trader submitted the sell order.

[12] The price equation is equivalent to the one in equation (2) in Faure-Grimaud and Gromb (2004).

[13] This follows the approach of Fulghieri, Strobl, and Xia (2014) and Mathis, McAndrews, and Rochet (2009) (who, in turn, follow the classic approach of modeling reputation of Kreps and Wilson [1982] and Milgrom and Roberts [1982]).

The opportunistic CRA, on the other hand, is not behavioral; it chooses its informativeness in response to incentives.[14] Specifically, it chooses the precision $\alpha \in \left[\frac{1}{2}, 1\right]$ of its signal $\sigma_{cra} \in \{b, g\}$ about project quality, where

$$\alpha \equiv \Pr(\sigma_{cra} = g \mid \theta = G) = \Pr(\sigma_{cra} = b \mid \theta = B).$$

$C(\alpha) = \rho_{cra} c(\alpha)$ denotes the cost of rating accuracy, and we assume $\rho_{cra} > 0$; $c' \geq 0$; $c'' > 0$; and $c(\frac{1}{2}) = c'(\frac{1}{2}) = 0$.

While there is only one CRA, the CRA's possible types vary in their informativeness. Substantial evidence points to rating informativeness varying over time (e.g., Alp 2013, Baghai, Servaes, and Tamayo 2014). The variance in rating informativeness between CRAs may be due to incentives, regulation, or both. Some explanations come from Kedia, Rajgopal, and Zhou (2017), who find that Moody's gives more favorable ratings to its main investors' other investments; Becker and Milbourn (2011), who find that Standard and Poor's and Moody's ratings increase when Fitch begins rating an industry; and Kempf and Tsoutsoura (2021), who show that ratings analysts who are not affiliated with the U.S. president's party downward-adjust corporate credit ratings more frequently. Thus, different ownership structures, market structures, and composition of their analyst pools may lead to heterogeneity in informativeness between CRAs.

Having observed the signal σ_{cra}, the opportunistic CRA offers a report $\widetilde{r}$ to the issuer. In the main model, we assume that the opportunistic CRA always offers a high (low) rating after having observed a good (bad) signal. In Section 6.3, we will let the opportunistic CRA choose the report $\widetilde{r}$ as well, allowing for the possibility that the CRA lies and offers a high rating after having observed a bad signal; we show that our qualitative results continue to hold. The other agents in the model do not observe the CRA's choice of α; $\overline{\alpha}$ denotes their conjectures about α.

An opportunistic CRA chooses the precision of its private information about project quality to maximize a weighted sum of its profits from selling the rating and its expected reputational payoff. This captures the tension between the opportunistic CRA's concern with its reputation for accuracy and the cost of acquiring information. The reputational payoff is assumed to be the CRA's reputation for being an informative type. We represent this by the posterior probability that the CRA is an informative type $q_{r,x,y}$, given the rating r, the realization of the CDS market x,[15] and the observable realizations of the investment, y. We micro-found the reputational payoff with a second rating period in Section 5 and show that our results carry through. Since good (bad)

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[14] Information acquisition is commonly used in research on the rating industry (e.g., Bar-Isaac and Shapiro [2011,2013], Kashyap and Kovrijnykh [2016] provide an overview of the benefits of this approach).

[15] As we will see, our analysis does not hinge on whether investors directly observe the number of trades or the corresponding price in the CDS market when updating their beliefs about the CRA's type, since these are observationally equivalent in equilibrium.

projects fail (succeed) with positive probability, the realization of the project's payoffs is not a perfect signal for its quality. Therefore, when a highly rated project defaults, investors cannot fully tell apart "bad luck" from "bad ratings." They use both the outcome y and the information coming from the CDS market to learn about the project's quality and, thus, about the accuracy of the rating.

γ denotes the weighting factor, which represents the relative importance of reputational payoffs to time $t=1$ profits. The weighting γ can be potentially larger than one (as in, e.g., Laffont and Tirole 1993), as future payoffs may arrive over a long time horizon after the game we have described has ended.

1.3.1 Price of the initial investment. If investors purchase the claim, they do so at the expected value of the project's payoff given their posterior beliefs; these beliefs depend on the rating, as well as the perceived accuracy of the CRA that publicized it, which is given by the initial reputation q_0 and conjectures about the opportunistic CRA's precision, $\overline{\alpha}$.

The ex ante NPV of the project is negative and, thus, the market price p^r exceeds the investment cost, that is, $p^r > I$, only if the rating conveys sufficiently positive information about project quality. As a consequence, the issuer never publicizes a low rating, and the project goes unrated when the report is low, that is, $r=\varnothing$ when $\widetilde{r}=L$. μ^H denotes the probability that the project quality is good after a high rating is publicized; the price p^H is characterized as follows:

$$p^H = \mu^H (1-f_G) + \left(1-\mu^H\right)(1-f_B).$$

where $\mu^H \equiv \Pr(\theta = G \mid r = H) = q_0 + (1-q_0)\overline{\alpha}$.

If q_0 or $\overline{\alpha}$ are large enough, μ^H is close to 1 and, thus, $p^H > I$. In this case, the issuer chooses to publicize a high rating, that is, $r=\widetilde{r}$ when $\widetilde{r}=H$.

We solve the model under the conjecture that $p^H > I$ and the project is implemented when the rating is high. Therefore, investors observe either a project with a high rating ($r=H$) or an unrated project ($r=\varnothing$); in the latter case, they infer that the issuer was offered a low report that went unpurchased.[16] We then characterize conditions under which this conjecture is satisfied in equilibrium. When these conditions are not satisfied, the project is never implemented and the market for ratings fails to exist in equilibrium. Finally, given that the issuer buys the credit report only if it is high, the rating fee amounts to $\phi\left(p^H - I\right)$.

[16] The fact that investors observe when the project goes unrated simplifies the exposition but does not affect the results. We could have that, when the report is low and the project goes unrated, the issuer anticipates that $p^{\varnothing} < I$ and, hence, does not market the claim to investors. In this case, investors would infer that the issuer was offered a low report and the analysis would be essentially the same.

1.4 Timing of the model

The timing of the model is summarized below:

Time ***t=0*** **(*Quality of the investment and CRA type*):** Nature chooses the quality of the investment project $\theta \in \{B, G\}$ and the CRA's type $\tau \in \{\mathcal{I}, \mathcal{O}\}$. The CRA privately observes τ.

Time ***t=1*** **(*Rating and initial investment market*):**

- The CRA offers a report $\widetilde{r}$ to the issuer.
- The issuer either agrees to pay the rating fee and has the report publicized ($r = \widetilde{r}$) or refuses to purchase it and goes on the market as unrated ($r = \varnothing$).
- Given the rating r, the issuer sets a price p^r and markets a claim backed by the investment project to investors.
- If $p^r > I$, the issuer sells the claim and undertakes the project. If the report was publicized, the issuer pays the CRA.

Time ***t=2*** **(*CDS market*):** If the issuer undertakes the project, a noise trader, a second trader, and a market maker trade CDS contracts on the outcome of the project.

Time ***t=3*** **(*Investment outcome and reputational payoff*):** The investment outcome y is realized. Having observed (r, x, y), investors update their beliefs about the CRA's type.

We use the Perfect Bayesian equilibrium as the solution concept.

2. The CDS Market Equilibrium

We work our way backward by first characterizing the CDS market equilibrium for a given conjecture $\overline{\alpha}$ about the CRA's rating accuracy. The issuer does not undertake the project when the investment goes unrated and, thus, the CDS market does not take place in this case. Hence, we can focus on the case in which the rating is high ($r = H$) in what follows. We first solve for the equilibrium trading strategies and then, given these strategies, characterize the speculator's choice of precision.

2.1 Market equilibrium

The following two lemmas characterize the speculator's trading strategy and the market maker's inference from the order flow.

Lemma 1. Given $r = H$ and for a given $\overline{\alpha}$, in equilibrium the speculator's trading strategy is $x_s = +n$ if $\sigma_s = b$ and $x_s = -n$ if $\sigma_s = g$. Proof: See Appendix A.2.

When the speculator is trading in the CDS market, she needs to camouflage her information-based trades. She is, therefore, constrained to trade an amount $x_s \in \{+n, -n\}$, buying protection when receiving negative information about the investment, and selling it otherwise.[17,18]

The equilibrium total order flow is $x \in \{-2n, 0, +2n\}$. Given that the market maker does not observe the identity of the trader submitting each order, the total order flow conveys the market maker's information.[19] We therefore simplify the notation in the rest of the paper by using x for the market maker's pricing function rather than $\{x_s, x_n\}$.

Lemma 2. Given $r = H$ and for a given $\overline{\alpha}$, in equilibrium the market maker sets a price

$$p^{cds}(x) = \mu^{H,x} f_G + \left(1 - \mu^{H,x}\right) f_B, \text{ for } x \in \{-2n, 0, +2n\}, \tag{1}$$

where $\mu^{H,-2n} \geq \mu^{H,0} = \mu^H \geq \mu^{H,+2n}$, with strict inequalities if $\iota > \frac{1}{2}$. The appendix describes the expression for $\mu^{H,x}$ used in Equation (1). Proof: See Appendix A.2.

Given the speculator's trading strategy in Lemma 1, the market maker's inference in Lemma 2 is straightforward. When $x_s = x_n = +n$ and, thus, the total order flow is $x = +2n$, both traders submitting orders are buying. This implies that, if one of the two traders is the speculator, she must have observed a negative signal $\sigma_s = b$. As a result, the market maker interprets a large demand for CDS contracts as a negative signal of the underlying asset's quality, and revises the CDS price accordingly. The opposite logic applies when $x_s = x_n = -n$. However, for $x_s = -x_n$ (i.e., $x = 0$), the market maker is unable to infer the direction of the traders' orders and, thus, the speculator's signal if she were one of the two traders.

$\mu^{H,x}$ denotes the probability the market maker attaches to the project being good following $r = H$ and a given realization of x. If the probability the second trader is a noise trader is strictly positive, that is, if $\eta < 1$, since the market maker cannot distinguish between whether the second trader is a speculator or another noise trader, the CDS price does not fully reveal the speculator's private information, that is, p^{cds} is a noisy signal for σ_s in equilibrium. However, the CDS price fully reveals the market maker's information about the project quality. Therefore, it is not important for our analysis whether investors directly observe the total order flow, x, or the corresponding price in the CDS market when updating their beliefs about the CRA's type at time $t = 3$.

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17 If x_s and x_n were different in absolute values, the market maker could always tell them apart, and so extract the speculator's private information. Expected profits from these trades then would be zero for the speculator.

18 Since we have $x_s \in \{+n, -n\}$ on the equilibrium path, we need to specify off-the-equilibrium-path beliefs if the market maker observes a buy order different than $+n$ and a sell order different than $-n$. The belief that the market maker sets a price $p^{cds} = f_B$ in the former case and $p^{cds} = f_G$ in the latter case supports the equilibrium.

19 This is not generally the case off-the-equilibrium path, where $\{x_s, x_n\}$ provides strictly more information than x for some combinations of x_s and x_n with $x_s \notin \{-n, +n\}$.

2.2 Informativeness of market trading

We need to evaluate the speculator's expected profits in order to examine her decision on how precise a signal to obtain. Π^s denotes the speculator's expected profits when she is trading; we have

$$\Pi^s = n\mu^H \left\{\iota\left[E\left[p^{cds}\,|\,x_s=-n\right]-f_G\right]+(1-\iota)\left[f_G-E\left[p^{cds}\,|\,x_s=+n\right]\right]\right\}$$
$$+n\left(1-\mu^H\right)\left\{\iota\left[f_B-E\left[p^{cds}\,|\,x_s=+n\right]\right]+(1-\iota)\left[E\left[p^{cds}\,|\,x_s=-n\right]-f_B\right]\right\}$$
$$-K(\iota). \tag{2}$$

Given that the CRA publicized a high rating, the ex ante probability that the project is good, that is, $\theta=G$, is μ^H. When this is the case, the speculator observes a correct signal $\sigma_s=g$ with probability ι; she then sells CDS protection ($x_s=-n$) at time $t=2$, at the expected price $E\left[p^{cds}\,|\,x_s=-n\right]$. Conditional on $x_s=-n$, the exact realization of p^{cds} depends on the realization of noise trading x_n, according to the market maker's pricing function described in Lemma 2. At time $t=3$, the project fails with probability f_G, in which case she has to pay 1 to the buyer. Since $E\left[p^{cds}\,|\,x_s=-n\right]\geq f_G$, the speculator profits from the trade in this case. With probability $1-\iota$, the speculator observes the wrong signal, that is, $\sigma_s=b$; she then buys CDS protection ($x_s=+n$) at time $t=2$, at the expected price $E\left[p^{cds}\,|\,x_s=+n\right]$. At time $t=3$, the project fails with probability f_G, in which case she receives 1 from the seller. Since $E\left[p^{cds}\,|\,x_s=+n\right]\geq f_G$, the speculator makes a loss in this case.[20] The case in which the project is bad ($\theta=B$) occurs with probability $1-\mu^H$ and is the mirror image of the one for $\theta=G$.

Taking the derivative of the speculator's expected profits with respect to ι yields[21]

$$\frac{\partial \Pi^s}{\partial \iota} = n\mu^H \left\{\left[E\left[p^{cds}\,|\,x_s=-n\right]-f_G\right]-\left[f_G-E\left[p^{cds}\,|\,x_s=+n\right]\right]\right\}$$
$$+n\left(1-\mu^H\right)\left\{\left[f_B-E\left[p^{cds}\,|\,x_s=+n\right]\right]-\left[E\left[p^{cds}\,|\,x_s=-n\right]-f_B\right]\right\}$$
$$-K'(\iota). \tag{3}$$

Higher precision benefits the speculator by increasing the chances that she observes a correct signal, that is, $\sigma_s=g$ when $\theta=G$ and $\sigma_s=b$ when $\theta=B$, and profits from trading.

Notice that $\frac{\partial \Pi^s}{\partial \iota}$ depends on the conjecture $\bar{\iota}$ through the expected CDS prices $E[p^{cds}\,|\,x_s=-n]$ and $E[p^{cds}\,|\,x_s=+n]$. This is because $\bar{\iota}$ determines the weight the market maker puts on the order flow x when forming beliefs about project

[20] If $\overline{\alpha}<1$, we have $E[p^{cds}\,|\,x_s=+n]\geq E[p^{cds}\,|\,x_s=-n]>f_G$, so that the speculator strictly benefits from observing a correct signal.

[21] The speculator's gross expected payoff is positive for any level of $\iota=\bar{\iota}>\frac{1}{2}$, since σ_s gives her an informational advantage over the market maker. The speculator takes the conjectures $(\overline{\alpha},\bar{\iota})$ as fixed when choosing ι.

quality. For a given conjecture about rating accuracy $\overline{\alpha}$, the equilibrium level of precision $\iota^*(\overline{\alpha})$ sets $\frac{\partial \Pi^s}{\partial \iota}$ to zero when the conjecture about ι is consistent, that is, when $\overline{\iota}=\iota^*(\overline{\alpha})$. The existence and uniqueness of $\iota^*(\overline{\alpha})$ is guaranteed by the assumptions on the shape of the speculator's cost function. The effect of expected rating accuracy on $\iota^*(\overline{\alpha})$ depends on how $\frac{\partial \Pi^s}{\partial \iota}$ changes with $\overline{\alpha}$.

Lemma 3. Given $r=H$ and $\overline{\alpha}$, the CDS market equilibrium is unique:

1. The equilibrium trading strategies are as described in lemmas 1 and 2.
2. The equilibrium level of precision $\iota^*(\overline{\alpha})$ is unique, with $\iota^*(\overline{\alpha})\in(\frac{1}{2},1)$ if $\overline{\alpha}<1$ and $\iota^*(1)=\frac{1}{2}$.
3. The equilibrium level of precision $\iota^*(\overline{\alpha})$ decreases with expected rating accuracy $\overline{\alpha}$.

Proof: See Appendix A.3.

Interestingly, from the speculator's point of view, information acquisition and rating accuracy are strategic substitutes. That is, lower expected rating accuracy increases the incentive to acquire information. This occurs through the mispricing channel, as the speculator can take advantage of wrong valuations due to less precise ratings.[22]

3. The Rating Game

Having characterized the equilibrium trading strategies and level of precision in the CDS market, we can now characterize the equilibrium rating accuracy for a given level of expected precision $\overline{\iota}$. The properties of the strategic interaction between credit ratings and the market for credit risk then will be used in the next section to characterize the unique equilibrium of the game.

At time $t=1$, before offering a rating to the issuer, an opportunistic CRA decides how much information to acquire about the project. We can write the CRA's total payoffs as follows:

$$\Pi^{cra}=\frac{1}{2}\left\{\alpha\left[\varphi+\gamma E\left[q_{H,x,y}\,|\,\theta=G\right]\right]+(1-\alpha)\gamma q_{\varnothing}\right\}$$
$$+\frac{1}{2}\left\{(1-\alpha)\left[\varphi+\gamma E\left[q_{H,x,y}\,|\,\theta=B\right]\right]+\alpha\gamma q_{\varnothing}\right\}-C(\alpha). \qquad (4)$$

The ex ante probability that the project is good, that is, $\theta=G$, is $\frac{1}{2}$. In this case, with probability α, the CRA observes a positive signal $\sigma_{cra}=g$ and offers a high

[22] The speculator benefits more from observing a correct (negative) signal and trading on it when $\theta=B$, as the mispricing is larger in this case. First, since $\theta=B$ is relatively less likely when the rating is more accurate, an increase in $\overline{\alpha}$ has a direct negative effect on the speculator's incentives to acquire information. Second, the market maker reduces the CDS price when $\overline{\alpha}$ goes up, since the rating is a more reliable signal for $\theta=G$. Since $\theta=G$ is more likely and the speculator sells CDS protection relatively more often, the lower price also reduces the speculator's incentives to acquire information.

rating $\widetilde{r}=H$ to the issuer. The issuer chooses to publicize the rating $(r=\widetilde{r}=H)$ and the CRA collects the fee $\varphi=\phi\left(p^H-I\right)$. The continuation reputational payoff is $E\left[q_{H,x,y}\,|\,\theta=G\right]$ in this case, since the project is implemented $(p^H > I)$ and the CDS market takes place, so investors will use both the total order flow of the CDS market, x, and the investment outcome, y, to update the CRA's reputation at time $t=3$.

With probability $1-\alpha$, the CRA observes a negative signal $\sigma_{cra}=b$ and offers a low rating $\widetilde{r}=L$ to the issuer; the issuer then refuses to publicize the rating $(r=\varnothing)$ and the rating fee is not collected. This conveys bad news about project quality to investors (i.e., $p^{\varnothing}<I$). Therefore, the initial investment project is not implemented and the CDS market does not take place. As a consequence, at time $t=3$, investors update the CRA's reputation based on $r=\varnothing$ only; the continuation reputational payoff is thus $q_{\varnothing}$ in this case.

The case in which the project is bad $(\theta=B)$ occurs with probability $\frac{1}{2}$ and is the mirror image of the one for $\theta=G$.

The precision of rating accuracy α is chosen as a best response to the conjectures $(\overline{\alpha},\overline{\iota})$. Taking the derivative of Π^{cra} with respect to α yields

$$\frac{\partial\Pi^{cra}}{\partial\alpha}=\frac{\gamma}{2}\left\{E\left[q_{H,x,y}\,|\,\theta=G\right]-E\left[q_{H,x,y}\,|\,\theta=B\right]\right\}-C'(\alpha). \tag{5}$$

Higher accuracy increases the chances that the CRA observes a correct signal, that is, $\sigma_{cra}=g$ when $\theta=G$ and $\sigma_{cra}=b$ when $\theta=B$, and offers an accurate rating to the issuer, increasing the CRA's expected reputational payoff. This is because the distribution of the public signals (x,y), conditional on $\theta=G$, puts more weight on positive signals about project quality, that is, on the events where (a) there is low demand for CDS protection $(x=-2n)$ in the CDS market (b) and the investment project succeeds $(y=S)$ at time $t=3$. These positive signals suggest to investors that the project's quality was good and the H rating was accurate, increasing the CRA's posterior reputation.

Notice that $\frac{\partial\Pi^{cra}}{\partial\alpha}$ depends on the conjecture $\overline{\alpha}$, through the expected reputational payoffs. This is because $\overline{\alpha}$ pins down investors' learning about the CRA's type. If $\overline{\alpha}=1$, both an opportunistic and an informative CRA are expected to always offer accurate ratings and, thus, the investors cannot learn about the CRA's type from its rating. As $\overline{\alpha}$ decreases, an opportunistic type is expected to offer inaccurate ratings relatively more often than an informative type.

For a given conjecture about the precision of the speculator's signal $\overline{\iota}$, the equilibrium level of rating accuracy $\alpha^*(\overline{\iota})$ sets $\frac{\partial\Pi^{cra}}{\partial\alpha}$ to zero when the conjecture about $\overline{\alpha}$ is consistent, that is, when $\overline{\alpha}=\alpha^*(\overline{\iota})$. The existence and uniqueness of $\alpha^*(\overline{\iota})$ is guaranteed by the assumptions on the shape of the CRA's cost function.

Lemma 4. For a given conjecture about market informativeness $\overline{\iota}$, the equilibrium of the rating game is unique:

1. In equilibrium, an opportunistic CRA chooses a level of rating accuracy $\alpha^*(\bar{\iota}) \in (\frac{1}{2}, 1)$.
2. The equilibrium level of rating accuracy α^* increases with expected market informativeness $\bar{\iota}$.

Proof: See Appendix A.4.

This indicates that from the point of view of the CRA, rating accuracy and market trading informativeness are strategic complements. That is, more informative market trading incentivizes the CRA to be more accurate. This arises because market transparency from informative trading makes inaccurate ratings more transparent, thus making reputational incentives more important.

We now examine this effect in detail. The effect of market informativeness on $\alpha^*(\bar{\iota})$ depends on how $\frac{\partial \Pi^{cra}}{\partial \alpha}$ changes with $\bar{\iota}$. Taking the derivative of $\frac{\partial \Pi^{cra}}{\partial \alpha}$ with respect to $\bar{\iota}$ yields

$$\frac{\partial \Pi^{cra}}{\partial \alpha \partial \bar{\iota}} = \frac{\gamma}{2} \left\{ \frac{\partial E\left[q_{H,x,y} \mid \theta = G\right]}{\partial \bar{\iota}} - \frac{\partial E\left[q_{H,x,y} \mid \theta = B\right]}{\partial \bar{\iota}} \right\}. \tag{6}$$

When $\bar{\iota}$ increases, the distribution of x is expected to be more informative about the true project quality, since the speculator is more likely to observe the correct signal and sell CDS protection when $\theta = G$ and buy it when $\theta = B$. This implies that, conditional on $\theta = G$ ($\theta = B$), positive news about project quality becomes more (less) likely. As a result, $E\left[q_{H,x,y} \mid \theta = G\right]$ increases and $E\left[q_{H,x,y} \mid \theta = B\right]$ decreases, since the investors learn more about the accuracy of the CRA's rating. This effect on the CRA's expected reputation raises its incentives to offer accurate ratings.

4. Equilibrium and Comparative Statics

We have found that equilibrium rating accuracy $\alpha^*(\bar{\iota})$ is *increasing* in the expected level of the informativeness of market trading $\bar{\iota}$. On the other hand, the informativeness of market trading (the choice of precision by the speculator) $\iota^*(\overline{\alpha})$ is *decreasing* in the expected rating accuracy $\overline{\alpha}$. The fact that these effects move in opposite directions implies a unique equilibrium.

Proposition 1. There exists a unique pair of rating accuracy and market trading informativeness (α^*, ι^*) such that $\alpha^*(\bar{\iota} = \iota^*) = \alpha^*$ and $\iota^*(\overline{\alpha} = \alpha^*) = \iota^*$; it

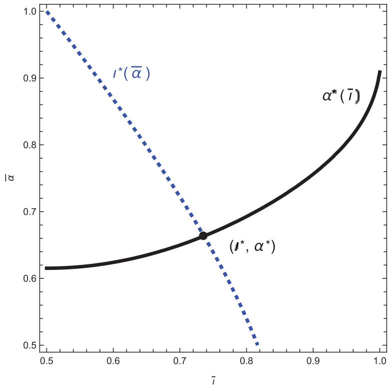

Figure 1
Equilibrium rating accuracy and trading informativeness
In this example, $C(\alpha)=(\alpha-\frac{1}{2})^2, K(\iota)=(\iota-\frac{1}{2})^2, q_0=\frac{1}{2}, \gamma=3, \eta=n=1, f_B=0.6, f_G=0.2$.

follows that the equilibrium of the game is unique, and is characterized as follows:

1. If the equilibrium level of rating accuracy is larger than a threshold, that is, if

$$\alpha^* > \underline{\alpha} \equiv \left(\frac{-v_B}{v_G - v_B} - q_0 \right)(1-q_0)^{-1},$$

then the investment project is implemented following a high rating ($p^H > I$), and both the opportunistic CRA and the speculator produce information about the project's quality, that is, $\alpha^* \in (\max\{\underline{\alpha}, \frac{1}{2}\}, 1)$ and $\iota^* \in (\frac{1}{2}, 1)$.
2. Otherwise, the project is not implemented, and the market for ratings and the CDS market fail to exist.

Proof: See Appendix A.5.

Figure 1 depicts the result in Proposition 1. The equilibrium values for market informativeness and rating accuracy are $\iota^*(\overline{\alpha}) \in [\frac{1}{2}, 1)$, with $\iota^*(1) = \frac{1}{2}$, and $\alpha^*(\overline{\iota}) \in (\frac{1}{2}, 1)$; this implies that the two functions in Figure 1 always cross each other (and do so only once) at some interior level $(\alpha^*, \iota^*) \in (\frac{1}{2}, 1)^2$. However, the rating accuracy level α^* might not be enough to make the NPV of a highly rated project positive (recall that the ex ante NPV of the project is negative). When this is the case, the project is not implemented and, thus, the market for ratings and the CDS market on the underlying asset fail to exist in equilibrium.

We now analyze the effect of varying some of the key parameters of the model.[23] We first look at their effect on the equilibrium pair (α^*, ι^*) and then explore their implications for the total surplus in the model in Section 5. We find:

Lemma 5. The following results hold in equilibrium:

1. An increase in the volume of the noise traders' demand, n, increases both equilibrium market informativeness, ι^*, and equilibrium rating accuracy, α^*. The same results apply to a decrease in the speculator's cost of information (ρ_s).
2. An increase in the weight on reputational payoffs in the CRA's objective function (γ) decreases ι^* and increases α^*. The same results apply to a decrease in the CRA's cost of information (ρ_{cra}).
3. Assume $\rho_{cra} = \rho_s = \rho$; an increase in ρ decreases α^*, while it may increase or decrease ι^*.

Proof: See Appendix A.6.

The comparative statics on the volume of noise trading n are straightforward. When n increases, the speculator can trade more and, thus, makes more profits. Therefore, she chooses a larger $\iota^*(\overline{\alpha})$ for any level of $\overline{\alpha}$ ($\iota^*(\overline{\alpha})$ rotates counterclockwise).[24] Note that the volume of noise trading does not enter the CRA's objective function, so that a change in n affects the equilibrium only through its direct effect on $\iota(\overline{\alpha})$. This implies that the new equilibrium pair will feature a higher level of precision in the CDS market and higher rating accuracy. Similar results apply to a decrease in the cost of precision for the speculator.

This comparative static has an intriguing implication. When the volume of trading is low (e.g., during a crisis or when markets are fragmented), the informativeness of market trading is low. However, the CRA does not respond and act as a substitute for information production by the market, because of the lack of monitoring. In this sense, CRAs fail to produce information precisely when it would be most valuable.

An increase in the CRA's reputation concern γ moves only $\alpha^*(\overline{\iota})$, shifting it upward.[25] This is quite intuitive: the opportunistic CRA offers more accurate

[23] We look at the comparative statics results on the asset's information sensitivity (Δ^f) in Section 7.1.

[24] Note that $\iota^*(\overline{\alpha}=1)=\frac{1}{2}$ since, when $\overline{\alpha}=1$, ratings are expected to be perfectly accurate and, thus, the speculator cannot profit from mispricing in the CDS market. This pins down the speculator's reaction function so that, when a parameter in the speculator's objective function changes, $\iota^*(\overline{\alpha})$ rotates either clockwise or counterclockwise around the point $\left(\overline{\alpha}=1, \overline{\iota}=\frac{1}{2}\right)$ in Figure 1.

[25] The CRA's reaction function $\alpha^*(\overline{\iota})$ is not pinned down by any particular value of $\overline{\iota}$ and, thus, a change of parameters in the CRA's objective function causes both a shift of $\alpha^*(\overline{\iota})$ upward or downward and a change in its slope. The direction of the shift in $\alpha^*(\overline{\iota})$ determines the comparative results in Lemma 5.

ratings because it is more concerned about future reputation. Thus, the new equilibrium pair features a higher level of rating accuracy but lower market informativeness. Similar results apply to a decrease in the cost of rating accuracy for the CRA.

The comparative static on γ has an important ramification: policies such as reputational sanctions (e.g., increased liability standards), which are aimed at improving CRA incentives to produce higher rating quality, reduce the discipline of these incentives provided by the market. In other words, the regulation of CRAs crowds out their market discipline, which reduces the effectiveness of such policies. Moreover, since the increase in rating accuracy leads to less informed trading, the effect on the overall information produced in the economy is ambiguous.

Finally, we explore the effect of an increase in a common cost parameter for both the CRA and the speculator. We assume $\rho_{cra} = \rho_s = \rho$ and look at the effect of an increase in ρ; this moves both $\iota^*(\overline{\alpha})$, which rotates clockwise, and $\alpha^*(\overline{\iota})$, which shifts downward, since the higher cost of information reduces the incentives of both the CRA and the speculator to learn about the asset. The fact that $\alpha(\overline{\iota})$ shifts downward, however, has a positive effect on the speculator's incentives to acquire information; this may compensate for the increase in the cost of information and lead to an increase in market informativeness.

5. Real Effects of Information

In this section, we explore the real effects of the information produced in the model. The main model captures the CRA's reputational payoff in a reduced-form way and is thus not suited to explore welfare implications. Here, we endogenize the CRA's reputational payoff by adding a second market for ratings to the main model.[26] This formulation is consistent with the formulation of the main model and thus all of the results there carry through.

At time $t=3$, after investors update the CRA's type, a new issuer seeks financing for a new investment project. Since the ex ante NPV of the project is negative, the issuer needs a high rating to obtain financing from investors. The features of the rating process and the market for the investment taking place at time $t=3$ are the same as the ones that take place at $t=1$; we use a subscript t to differentiate between the two. The opportunistic CRA chooses the precision of her signal about project quality in each period, while the informative CRA offers accurate ratings in both periods. The qualities of the projects in the two periods are uncorrelated. The weight γ represents the relative importance of time $t=3$ profits in the opportunistic CRA's payoffs.

To simplify the exposition, we assume that the CRA's initial reputation q_0 is large enough ($q_0 > \overline{q}$, where $\overline{q}$ is described in Appendix A.7), so that the

26 We don't consider the payoffs of noise traders in this section, but will do so in Section 6.1, where we endogenize their decision to participate in the CDS market.

market for ratings always takes place in both periods. First, we show that the opportunistic CRA's incentives to choose α_1 directly map to the ones in the main model, so that all the results in the main model continue to hold in this extension of the model. Then we define our measure of the real effects of information and discuss its comparative statics.

5.1 Preliminaries

Since the market ends at time $t=3$ and there is no need to foster a reputation, an opportunistic CRA has no incentives to collect information in this period, that is, $\alpha_3=\frac{1}{2}$ and $C(\alpha_3)=0$. Investors anticipate this and price the claim to the project's payoff accordingly. q_3 denotes the CRA's reputation at the beginning of time $t=3$, which corresponds to the CRA's posterior reputation in the main model, that is, $q_3\in\left\{q_{H_1,x,y_1},q_{\varnothing_1}\right\}$. When $r_3=H$, the equilibrium price is $p_3^H=\mu_3^H(1-f_G)+\left(1-\mu_3^H\right)(1-f_B)$, where $\mu_3^H=q_3+(1-q_3)\frac{1}{2}$ is the probability investors attach to the project being good. At time $t=3$, the rating fee is $\varphi_3(q_3)=\phi\left(p_3^H-I\right)$, which increases with q_3 (the larger the probability the CRA is informative, the more accurate and valuable the rating is in the eye of investors). The opportunistic CRA then has an incentive to issue an accurate rating at time $t=1$ (i.e., $\alpha_1\in(\frac{1}{2},1)$), in order to preserve her future reputation and time $t=3$ profit.

The opportunistic CRA's total payoffs are as follows:

$$\Pi^{cra}=\frac{1}{2}\varphi_1+\frac{1}{2}\left\{\alpha_1\frac{\gamma}{2}E\left[\varphi_3\left(q_{H_1,x,y_1}\right)|\theta_1=G\right]+(1-\alpha_1)\frac{\gamma}{2}\varphi_3\left(q_{\varnothing_1}\right)\right\}$$
$$+\frac{1}{2}\left\{(1-\alpha_1)\frac{\gamma}{2}E\left[\varphi_3\left(q_{H_1,x,y_1}\right)|\theta_1=B\right]+\alpha_1\frac{\gamma}{2}\varphi_3\left(q_{\varnothing_1}\right)\right\}-C(\alpha_1). \quad (7)$$

The only difference with the CRA's payoff in the main model (Equation (4)) is that the reputational payoff is now the expected value of φ_3 times $\frac{1}{2}$ – the CRA sells the rating and collects the rating fee at time $t=3$ only if it observes a good signal and offers a high rating, which occurs with probability $\frac{1}{2}$. Given that φ_3 is a linear increasing function of q_3, the characterization of the equilibrium rating accuracy in the first period is qualitatively the same as in the main model.

Lemma 6. For a given conjecture about market informativeness $\bar{\iota}$, an opportunistic CRA chooses rating accuracy $\alpha_3^*=\frac{1}{2}$ and $\alpha_1^*(\bar{\iota})\in(\frac{1}{2},1)$, where $\alpha_1^*(\bar{\iota})$ is unique and such that

$$\frac{\gamma}{2}\left(\phi\frac{\Delta_f}{4}\right)\left\{E\left[q_{H_1,x,y_1}|\theta_1=G\right]-E\left[q_{H_1,x,y_1}|\theta_1=B\right]\right\}=C'(\alpha_1^*(\bar{\iota})) \quad (8)$$

at $\alpha_1=\overline{\alpha}_1=\alpha_1^*(\bar{\iota})$. Equilibrium rating accuracy $\alpha_1^*(\bar{\iota})$ increases with $\bar{\iota}$. Proof: See Appendix A.7.

Since the CDS market does not change in this extension, the equilibrium characterization in Proposition 1 and the comparative statics in Lemma 5 continue to hold as well.

Corollary 1. The equilibrium of the game with two rating markets is unique with $\alpha_3^* = \frac{1}{2}$, $\alpha_1^* \in (\frac{1}{2},1)$, $\iota^* \in (\frac{1}{2},1)$, and lemmas 1 to 5 continuing to hold in this extension of the model.

5.2 Real effects

There are two sources of information in the model. The first is the information produced by the CRA, which has direct real effects since it affects whether the investment projects are undertaken at both times $t=1$ and $t=3$. The second is the information produced by the speculator in the CDS market, which does not guide any real decision but has an indirect effect on the initial investment by disciplining the CRA's incentives to produce information at time $t=1$.

As a measure of the real effects of information, we consider the ex ante expected total surplus, which we write as S. This consists of the NPV of a project with a high rating at both times $t=1$ and $t=3$, minus the information costs sustained by both the CRA and the speculator.[27] We can write S as follows:

$$S = \sum_t \gamma_{t=3} \left\{ \frac{1}{2} \left[\widetilde{\mu}_t^H v_G + \left(1 - \widetilde{\mu}_t^H\right) v_B \right] - (1 - q_0) C(\alpha_t) \right\} - \frac{\eta}{2} K(\iota), \qquad (9)$$

where $t \in \{1,3\}$, $\widetilde{\mu}_t^H = q_0 + (1-q_0)\alpha_t$, and $\gamma_{t=3}$ takes values $\gamma_S > 0$ when $t=3$ and 1 otherwise.

A project is implemented only if it receives a high rating, which occurs with probability $\frac{1}{2}$ in each period.[28] The expected NPV of a project that received a high rating at time t is $\widetilde{\mu}_t^H v_G + (1-\widetilde{\mu}_t^H) v_B$; since good (bad) projects have positive (negative) NPV, that is, $v_G > 0$ and $v_B < 0$, this is increasing in rating accuracy α_t. The terms $C(\alpha_t)$ and $K(\iota)$ represents the cost of information production for the opportunistic CRA (which is incurred with probability $1-q_0$ at both time $t=1$ and $t=3$) and the speculator (which is incurred only at time $t=2$, if the second trader is the speculator and if $r_1 = H$, which occurs with probability $\frac{\eta}{2}$). γ_S denotes the weight on the time $t=3$ surplus in S value of γ_S.

The surplus S is maximized at $\alpha_1 = \alpha_3 = \widehat{\alpha} \in (\frac{1}{2},1]$ and $\iota = 0$, since market informativeness has no direct real effect.[29] From this perspective, the opportunistic CRA underinvests in rating accuracy at time $t=3$, since $\alpha_3^* = \frac{1}{2} < \widehat{\alpha}$. The posterior reputation q_{H_1,x,y_1} matters for the CRA's accuracy incentives at time $t=1$ (as it affects the pricing of the project and, thus, the rating fee at

[27] Since prices in the model are transfers, they do not affect the total surplus. Therefore, we don't need to consider the prices in the markets for the investment, the CDS market, or the rating fees.

[28] At time t, the project is good (bad) with probability $\frac{1}{2}$ and receives a high rating with probability $q_0 + (1-q_0)\alpha_t$ $((1-q_0)(1-\alpha_t))$. Therefore, the probability the project is rated high is $\frac{1}{2}$ in each period.

[29] We have $\widehat{\alpha} = 1$ if $\Delta^f \geq 2C'(1)$; otherwise, we have $\widehat{\alpha} \in (\frac{1}{2},1)$ such that $\Delta^f = 2C'(\widehat{\alpha})$.

time $t=3$) but does not directly affect S (as the price does not affect surplus).[30] This creates a wedge between the equilibrium level α_1^* and the optimal level $\widehat{\alpha}$, which may lead to either underinvestment or overinvestment at time $t=1$. When underinvestment arises (i.e., when $\alpha_1^* < \widehat{\alpha}$), market informativeness can increase surplus because of the discipline it provides to the CRA's incentives.

Proposition 2. $\widehat{\alpha}$ denotes the level of rating accuracy that maximizes the expected total surplus S. The following comparative statics results hold in equilibrium:

1. Equilibrium rating accuracy is smaller than $\widehat{\alpha}$ at time $t=3$ (i.e., $\alpha_3^* = \frac{1}{2} < \widehat{\alpha}$), while it may be larger or smaller than $\widehat{\alpha}$ at time $t=1$.
2. If equilibrium rating accuracy is lower than $\widehat{\alpha}$ at time $t=1$ (i.e., if $\alpha_1^* < \widehat{\alpha}$), we have:
   (a) S is increasing in the weight on reputational payoffs in the CRA's objective function (γ) and decreasing in the CRA's cost of information (ρ_{cra}).
   (b) If the speculator's cost of information ρ_s is sufficiently small, S is decreasing in ρ_s and increasing in the volume of the noise trader's demand (n).
3. If $\alpha_1^* \geq \widehat{\alpha}$, S is decreasing in n, and may increase or decrease with γ, ρ_{cra}, and ρ_s.

Proof: See Appendix A.8.

The equilibrium rating accuracy at time $t=1$ (α_1^*) is pinned down by Equation (8). The left-hand side of this equation describes the CRA's private benefit of accuracy; α_1^* is lower than its optimal level $\widehat{\alpha}$, if the CRA's weight on reputational payoffs γ or market informativeness ι^* are small. In this case, increasing α_1^* directly, via an increase in γ, increases surplus. Perhaps more surprisingly, if the marginal cost of ι is sufficiently small, increasing α_1^* indirectly, via an increase in market discipline ι^*, also increases surplus. In this case, the marginal cost of an increase in ι^* is lower than the marginal benefit of the consequent rise in α_1^*. Therefore, an increase in the volume of noise trading n or a reduction in the speculator's information cost ρ_s increase market discipline and, as a result, total surplus S.[31] This result suggests that,

[30] If we remove the assumption $q_0 > \overline{q}$, the posterior reputation has a direct effect on S, since the market for ratings fails to exist at time $t=3$ if q_3 is too small, as the rating is too inaccurate for investors. In this case, ι has an additional positive effect on S, as it increases the probability that the market for ratings exists (fails to exist) when the CRA is an informative (opportunistic) type, given that the project is more likely to be NPV positive (negative).

[31] Of course, holding fixed ι^* and α_1^*, a reduction in ρ_s or in ρ_{cra} have always also a direct positive effect on S, as they reduce $K(\iota)$ and $C(\alpha_1)$, respectively.

when $\alpha_1^* < \widehat{\alpha}$, policies to increase market informativeness (such as improving governance, transparency, and trust in exchanges) reap informational benefits beyond the market itself and increase investment efficiency. Moreover, they can act as a substitute to more stringent regulation of CRAs, which is often argued to be limited by practical and economic constraints (White 2010).

The CRA's concern with its reputation also may be larger than what is socially optimal, in which case the CRA overinvests in rating accuracy. Increasing n always reduces surplus in this case, since it pushes both α_1^* and ι^* further from their optimal levels. An increase in γ (or a reduction in ρ_{cra}) may still increase S, since it induces α_1^* to go up but ι^* to go down. This result suggests that a *one size fits all* type of approach to CRA regulation may be ill-suited, since rating accuracy may vary largely across asset classes, and policies that improve rating accuracy may increase welfare in some markets and reduce it in others.

6. Extensions

This section explores the robustness of our main results. First, we extend the main model to endogenize the noise trader's demand in the CDS market. Second, we consider an extension with two CRAs in the market for ratings. Finally, we allow for a second source of moral hazard for the CRA, namely, inflating ratings. The main qualitative results continue to hold in all three settings.

6.1 Linking the primary and secondary markets

It is natural to wonder about the identity of the noise traders in the model, and whether they participated in the primary market for investment. This section considers an extension in which investors in the primary market may receive liquidity shocks and trade as noise traders in the CDS market.[32] Investors' anticipation of losses from trading with the speculator reduces their willingness to pay for the investment project. The price in the primary market then reflects both the benefits (in terms of higher rating accuracy) and the costs (in terms of a larger discount due to the possibility of trading against the speculator) of market discipline.

We consider a mass 1 of ex ante identical investors who participate in the market for the initial investment. At time $t=1$, having observed the rating r, each investor chooses whether to acquire a unit share of the claim to the project's payoff and, if so, at what price. At time $t=2$, a randomly chosen fraction n of investors who acquired a share of the claim at time $t=1$ receive a common shock $z=(\ell,0)$ or $z=(0,\ell)$, with each realization equally likely. Investors know about the possibility of the shock, but do not know whether they will be hit with it. The shock z represents a liquidity shock and its realization is the investor's

[32] Diamond and Verrecchia (1991) and Gao and Liang (2013) model participation in both primary and secondary markets in a similar fashion.

private information. If $z=(\ell,0)$, an investor hit by the shock has a liquidity need ℓ at time $t=2$; if $z=(0,\ell)$, the investor has a liquidity need ℓ at time $t=3$. The investor incurs a cost κ if she cannot raise an amount greater or equal than her liquidity need in that period. Such illiquidity may be costly if the investor is an institution who faces withdrawals (or a reduced rate of debt rollover) from its investors, costly bankruptcy, or missing out on a lucrative investment opportunity.

Investors can hedge their liquidity shocks by trading in the CDS market. An investor with $z=(\ell,0)$ can *sell* a unit of CDS protection at time $t=2$, to raise money and satisfy her liquidity need in that period. An investor with $z=(0,\ell)$ can *buy* a unit of CDS protection at $t=2$, to avoid the risk that the project defaults and she is illiquid at $t=3$. We assume that κ is large enough that the investor chooses to sell CDS protection when $z=(\ell,0)$ and to buy it when $z=(0,\ell)$.[33] Like in the main model, the market maker cannot distinguish the identity of the traders submitting the orders. To simplify the exposition, we assume that the speculator always participates in the market, that is, $\eta=1$. The following lemma describes the equilibrium in this extension of the game.

Proposition 3. An equilibrium of the game where the primary and secondary markets are connected always exists, and is unique. In this equilibrium, we have:

1. Investors with a liquidity shock at time $t=2$ ($t=3$) sell (acquire) a unit of CDS protection; their aggregate demand is $x_n=-n$ if $z=(\ell,0)$ and $x_n=+n$ if $z=(0,\ell)$.
2. The speculator's trading strategy and the market maker's pricing function are the same as in lemmas 1 and 2, respectively.
3. The equilibrium pair of rating accuracy and market trading informativeness $\left(\alpha^{link},\iota^{link}\right)$ is characterized as in Proposition 1, except that the market for ratings and the CDS market fail to exist if $\alpha^{link}\leq\underline{\alpha}^{link}$, where $\underline{\alpha}^{link}>\underline{\alpha}$.

Proof: See Internet Appendix B.1.

In equilibrium, all investors acquire a share of the claim if the project is implemented. As a consequence, the aggregate demand of CDS protection from the investors who face a liquidity shock (*liquidity traders)* is $x_n\in\{-n,+n\}$, with both realizations equally likely (according to the distribution of z). Since the distribution of x_n is the same as in the main model, the speculator's and

[33] Formally, we assume (i) $\kappa\geq f_B[\mu^H f_G+\left(1-\mu^H\right)f_B]^{-1}-1$; and (ii) $\ell\in\left(0,f_G\right)$, so that both the notional amount 1 and the CDS price p^{cds} satisfy the investor's liquidity need. Investors are not financially constrained in periods in which they are not hit by the liquidity shock, but they cannot transfer liquidity across periods.

the market maker's equilibrium strategies are the same as well. The investor's willingness to pay for the claim to the payoff of a highly rated project is:

$$p^H_{link} = \mu^H(1-f_G) + \left(1-\mu^H\right)(1-f_B) - \underbrace{n\Delta^f(2\iota^{link}-1)\mu^H\left(1-\mu^H\right)}_{\text{Liquidity discount}}. \quad (10)$$

The price p^H_{link} is the same as in the baseline model minus a liquidity discount. Since trading in the CDS market is a zero-sum game, this is equal to the speculator's expected trading profits excluding the cost of precision, that is, $\Pi^s + K(\iota) = n\Delta^f(2\iota^{link}-1)\mu^H\left(1-\mu^H\right)$.

The liquidity discount does not affect the CRA's marginal benefit of rating accuracy in Equation (5). If the market for the initial investment takes place, the equilibrium rating accuracy α^{link} is then the same as in the main model—characterized in Lemma 4—and the information produced in the CDS market continues to discipline the CRA's accuracy incentives. However, by reducing investors' willingness to pay for the project, the liquidity discount increases the threshold level of rating accuracy $\underline{\alpha}^{link}$ (which is determined by $p^H_{link} = I$) below which the markets for the investment and ratings fail to exist.

If the market for the initial investment takes place, the effects on total surplus of the parameters are the same as in the main model (see Proposition 2). However, through the liquidity discount, these parameters have now a new effect on the price in the primary market p^H_{link}. The new effects on p^H_{link} are interesting for two reasons. First, as discussed above, p^H_{link} now incorporates both the benefits and costs of market discipline, as it reflects the expected payoff of the uninformed traders in the model. Second, when p^H_{link} decreases, the condition $p^H_{link} \geq I$ becomes more difficult to satisfy, so that a market failure is more likely. The following lemma explores these effects.

Lemma 7. The equilibrium price in the primary market p^H_{link} is increasing in the weight on reputational payoffs in the CRA's objective function (γ) and decreasing in the CRA's cost of information (ρ_{cra}), and it may increase or decrease with the speculator's cost of precision ρ_s and the volume of liquidity trading n. Proof: See Internet Appendix B.2.

The results in Lemma 7 draw a distinction between our paper's model and other models of market monitoring (e.g., Holmström and Tirole 1993). In their paper, more informative stock prices improve economic efficiency by making it easier to incentivize managers. This efficiency gain, however, comes at the cost of lower payoffs for uninformed traders, who suffer from trading against more informed speculators. In our model, an increase in information acquisition by the speculator improves rating accuracy, which leads to more public information about the security. This can reduce asymmetric information, ultimately benefiting uninformed traders.

6.2 Multiple rating agencies

In this section, we explore an extension of the baseline model in which the issuer can seek ratings from two different CRAs ($j \in \{\mathcal{A}, \mathcal{B}\}$). Since the modeling of the CDS market does not change, we now elaborate on the interaction between the issuer and the CRAs.

We make the following assumptions.[34] The issuer can seek ratings from both CRAs. Each of the two CRAs is identical to the CRA in the baseline model. The realizations of the CRAs' types and of their signals are independent across CRAs. α_j denotes an opportunistic CRA j's level of rating accuracy. Each CRA j observes a signal about project quality and privately offers a rating $\widetilde{r}_j \in \{H, L\}$ to the issuer. The issuer either pays a fee to the CRA and has the rating publicized or refuses to purchase it. If a rating is publicized, it becomes public knowledge. The issuer can observe both offers $\widetilde{r}_{\mathcal{A}}$ and $\widetilde{r}_{\mathcal{B}}$ at no cost and then decide which ratings to publicize. Investors only observe the publicized ratings $r = (r_{\mathcal{A}}, r_{\mathcal{B}})$, namely, one of four possible cases: a project with two ratings; a project only rated by $\mathcal{A}$; a project only rated by $\mathcal{B}$; or an unrated project. Therefore, we have $r \in \{(\widetilde{r}_{\mathcal{A}}, \widetilde{r}_{\mathcal{B}}), (\widetilde{r}_{\mathcal{A}}, \varnothing), (\varnothing, \widetilde{r}_{\mathcal{B}}), (\varnothing, \varnothing)\}$, where $\varnothing$ signifies the project is *unrated*. Since the CRAs are ex ante identical, here we restrict our attention to symmetric equilibria, that is, equilibria in which the two CRAs choose the same level of rating accuracy. In Internet Appendix B.10, we show that our main qualitative results continue to hold in the case of asymmetric equilibria. Like in the main model, we assume that the rating fee is a fraction of the issuer's surplus from implementing the project: $\varphi_{\mathcal{A}} = \varphi_{\mathcal{B}} = \frac{\phi}{2}\left(p^{\widetilde{r}_{\mathcal{A}}, \widetilde{r}_{\mathcal{B}}} - I\right)$ if the issuer buys ratings from both CRAs; and $\varphi_j = \phi\left(p^{\widetilde{r}_j, \varnothing} - I\right)$ if the issuer only buys the rating from CRA j.

The following lemma describes the issuer's disclosure strategy and the rating fees.

Lemma 8. In a symmetric equilibrium of the rating game with two CRAs, the issuer publicizes the ratings, sells the claim, and undertakes the project only when both ratings are high ($r = (H, H)$); otherwise, the claim is unrated ($r = (\varnothing, \varnothing)$) and the project is not undertaken. Proof: See Internet Appendix B.3.

Since the NPV of the asset is ex ante negative, the issuer needs two high ratings to sell the claim and implement the project. Split ratings (one high and one low) would cancel each other out and leave investors' valuation of the project negative. As a consequence, split ratings are never publicized by the issuer and, thus, never observed by investors. This result is consistent with stylized facts in the rating industry, where disagreement among CRAs is rarely observed, mainly because it rarely occurs for rated issues.[35]

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[34] The analysis in this section adapts the Piccolo (2021) setup, which is a model of competition between CRAs, where the CRAs do not acquire information about the asset quality but can choose to offer inflated ratings.

[35] Kronlund (2020) documents that 76% of U.S. corporate bonds are rated by both S&P and Moody's (the two largest CRAs), and their ratings differ by two or more notches in only 13.7% of the cases.

The following proposition characterizes the effect of market discipline on rating accuracy and the strategic interaction between the CRAs.

Proposition 4. For a given conjecture about market informativeness $\bar{\iota}$, a symmetric equilibrium of the rating game with two CRAs always exists, and there may be more than one; moreover:

1. There always exists a symmetric equilibrium in which both opportunistic CRAs' level of rating accuracy weakly increases with the conjecture about the speculator's signal precision $\bar{\iota}$.
2. The equilibrium accuracy α_j^{mult} may increase or decrease with j's conjecture about an opportunistic CRA $-j$'s level of rating accuracy ($\overline{\alpha}_{-j}$), and may be larger or smaller than the equilibrium rating accuracy in the baseline model (α^*).

Proof: See Internet Appendix B.4.

Proposition 4 offers three main insights. First, the speculator continues to discipline the CRAs' accuracy incentives in this extension of the model.[36] Second, the CRAs' investments in rating accuracy may be strategic substitutes or strategic complements; we discuss the intuition below. Third, the implication of the strategic interaction between the CRAs is that their incentives to invest in accuracy may be weakened, leading to lower accuracy choices than in a setting with a single CRA. This draws a contrast between the role of the CDS market and that of competition in disciplining a CRA's incentives to be accurate. Section 7.2 explores this contrast further by comparing the surplus in both cases.

We now examine the strategic interaction between the two CRAs further. Since the issuer only publicizes two high ratings, both CRAs must offer a high rating for each one to sell its rating. The more accurate the ratings, the higher the chance they match. This relation can be explained by the fact that precise ratings are more correlated. If the CRAs' payoffs from selling ratings at time $t=1$ are high, this need for correlation leads to a strategic complementarity between the CRAs' accuracy choices.[37] If the CRAs have higher payoffs from maintaining their reputation, one way to maintain reputation is to reduce the likelihood that the issuer buys the ratings at $t=1$ by reducing

36 The equilibria in which α_j^{mult} decreases with $\bar{\iota}$ are all unstable, such that small perturbations of the equilibrium value do not converge back to the equilibrium. When an equilibrium in which α_j^{mult} decreases with $\bar{\iota}$ exists, there always exists also another equilibrium that is stable and has larger α_j^{mult}, in which α_j^{mult} increases weakly with $\bar{\iota}$.

37 It is worth emphasizing that the strategic complementarity between the two CRAs' accuracy levels derives from a *reward* channel, which operates through the probability of selling the rating. In contrast, from the point of view of the CRA, the strategic complementarity between its accuracy level and the speculator's precision derives instead from a *reputation* channel.

their correlation. In this case, the CRAs' choices of accuracy are strategic substitutes.

It is worth emphasizing that, even when the CRAs' accuracy choices are strategic complements, each CRA may choose a lower accuracy than in the setting with only one CRA. Since the issuer does not disclose split ratings, investors may learn less about each CRA's type than in the setting with a single CRA. This diminishes the CRAs' reputational concerns, potentially leading to lower investments in accuracy.

6.3 Inflated ratings

In the main model, we assume that the opportunistic CRA is truthful after it acquires information; it always offers a high (low) rating after having observed a good (bad) signal. Nevertheless, given the issuer pays the rating fee only if the rating is high, the CRA may have an incentive to inflate ratings. In this section, we allow for this possibility; the opportunistic CRA may offer a high rating to the issuer after a bad signal. We show that our main qualitative results continue to hold.

To make this extension tractable, we assume that the opportunistic CRA cannot deflate ratings; that is, it cannot offer a low rating to the issuer after having observed a good signal. Moreover, we restrict our attention to equilibria where the opportunistic CRA always offers a H rating when it chooses not to acquire information about the project.

The following proposition describes the equilibrium rating strategy. For presentational purposes, we let the CRA's cost of accuracy be such that $C'(1) > \gamma/2$ and simplify the main model slightly by assuming the CRA's reputational payoff is only conditional on the realization of market trading x, but not on the realization of the project outcome, y.[38,39] The modeling of the CDS market does not change here, so we focus on the CRA's incentives in this section.

Proposition 5. For a given conjecture about market informativeness $\bar{\iota}$, an equilibrium of the rating game where the CRA inflates its rating always exists, and there may be more than one. In equilibrium, an opportunistic CRA mixes between acquiring information (i.e., $\alpha^{infl} \in (\frac{1}{2}, 1)$) and truthfully revealing it to investors (with probability $\lambda^{infl} \in [0,1]$), and staying uninformed and offering a high rating to the issuer (with probability $1-\lambda^{infl}$), where α^{infl} is such that

$$\frac{\gamma}{2}\left\{E\left[q_{H,x} \,|\, \theta=G\right]-E\left[q_{H,x} \,|\, \theta=B\right]\right\}|_{\bar{\alpha}=\alpha^{infl},\bar{\lambda}=\lambda^{infl}}=C'(\alpha^{infl}). \tag{11}$$

38 This would be the case if investors cannot observe the realization, y, before the game we have described has ended.

39 The assumption $C'(1) > \gamma/2$ ensures that $\alpha^{infl} < 1$, which simplifies the exposition.

1. If $\{\gamma q_{\varnothing} - (\varphi^{infl} + \gamma E[q_{H,x}])\}|_{\overline{\alpha}=\alpha^{infl}, \overline{\lambda}=0} > 0$ (**Condition 1**), an opportunistic CRA is informed with strictly positive probability in equilibrium (i.e., $\lambda^{infl} > 0$). Otherwise, the CRA always stays uninformed and offers a high rating to the issuer (i.e., $\lambda^{infl} = 0$).
2. If $\{\gamma q_{\varnothing} - (\varphi^{infl} + \gamma E[q_{H,x}])\}|_{\overline{\alpha}=\alpha^{infl}, \overline{\lambda}=1} > 0$ (**Condition 2**), there exists an equilibrium where the CRA is always informed (i.e., $\lambda^{infl} = 1$).
3. If Condition 1 holds but Condition 2 does not hold, the equilibrium features $\lambda^{infl} \in (0,1)$, where λ^{infl} is such that

$$\{\gamma q_{\varnothing} - (\varphi^{infl} + \gamma E[q_{H,x}])\}|_{\overline{\alpha}=\alpha^{infl}, \overline{\lambda}=\lambda^{infl}} = 0. \quad (12)$$

The expressions for $q_{H,x}$, $q_{\varnothing}$, and φ^{infl} are described in the Internet Appendix. If α^{infl} and λ^{infl} are such that $p^H > I$, the investment project is implemented following a high rating. Otherwise, the project is not implemented and the market for ratings and the CDS market fail to exist. Proof: See Internet Appendix B.5.

We offer several remarks on Proposition 5. First, the CRA never acquires information to then misreport the signal in equilibrium. If the CRA intends to ignore the signal when rating the asset, then it is better off not acquiring the signal in the first place. Second, the issuer pays the rating fee only if the rating is high, which creates a conflict of interest for the CRA. Holding fixed investors' conjecture about their rating strategy, the CRA makes a higher profit from offering a high rating to the issuer. However, *inflated* ratings come at the cost of a lower expected reputation, since the CRA has no information about the quality of the project and is thus more exposed to reputational losses.

Condition 1 describes the trade-off discussed above. It compares the CRA's payoff from offering a low rating (i.e., $\gamma q_{\varnothing}$) versus a high rating (i.e., $\varphi^{infl} + \gamma E[q_{H,x}]$) to the issuer when (a) the CRA is uninformed and (b) the investors' conjecture is that an opportunistic CRA is always uninformed, that is, $\overline{\lambda} = 0$, where $\overline{\lambda}$ denotes investors' conjecture about λ. The CRA's reputation is larger if the rating is low (since $q_{\varnothing} > E[q_{H,x}]$), but the issuer buys the rating and pays the fee only if the rating is high. Condition 2 describes the same trade-off, but it is evaluated at investors' conjecture $\overline{\lambda} = 1$. This condition is more stringent than Condition 1, since the reputational loss of inaccurate ratings is less severe when $\overline{\lambda} = 1$. When Condition 1 holds but Condition 2 does not, the equilibrium is in mixed strategies, that is, $\lambda^{infl} \in (0,1)$, where the opportunistic CRA is indifferent between inflating and being truthful (and acquiring information). Finally, since the system of inequalities in Proposition 5 may admit multiple solutions, there may be multiple equilibria with information acquisition.

The following lemma further characterizes the equilibrium rating strategy and describes how this changes with the conjecture about the speculator's signal precision $\overline{\iota}$.

Lemma 9. The following results hold in equilibrium:

1. The set of parameters for which the opportunistic CRA acquires information with positive probability ($\lambda^{infl} > 0$; Condition 1 is satisfied) increases with $\bar{\iota}$.
2. There always exists a value $\overline{\gamma}$ of the weight on the reputational payoff in the CRA's objective function such that the opportunistic CRA acquires information with probability 1 ($\lambda^{infl} = 1$; Condition 2 holds) for any $\gamma \geq \overline{\gamma}$. Similarly, there always exists a value $\underline{\phi}$ of the fraction of the issuer's surplus in the rating fee such that $\lambda^{infl} = 1$ for any $\phi \leq \underline{\phi}$.
3. Holding $\overline{\lambda}$ fixed, the value of α^{infl} that solves Equation (11) increases with $\bar{\iota}$. Holding $\overline{\alpha}$ fixed, the value of λ^{infl} that solves Equation (12) increases with $\bar{\iota}$.

Proof: See Internet Appendix B.6.

When market informativeness $\bar{\iota}$ increases, inaccurate ratings are more likely to be revealed to investors via the realization of x. This makes inflating ratings less attractive for the CRA and, thus, an equilibrium with information acquisition more likely to exist. If γ is sufficiently large (or ϕ is sufficiently small), the opportunistic CRA's concern with its reputation is sufficient to fully discipline its incentives to inflate ratings: the opportunistic CRA always acquires information in equilibrium.[40] In this case, the equilibrium rating strategy is the same as in the baseline model and, thus, the same comparative statics results apply.

When γ and ϕ have intermediate values, the equilibrium is in mixed strategies, that is, $\lambda^{infl} \in (0, 1)$. An increase in $\bar{\iota}$ has two types of effects on the equilibrium pair in this case. First, a direct effect, for example, holding λ^{infl} (α^{infl}) fixed, the value of α^{infl} (λ^{infl}) that solves Equation (11) (Equation (12)) changes with $\bar{\iota}$. Second, an indirect effect, since the change in α^{infl} (λ^{infl}) affects Equation (12) (Equation (11)) and, as a result, the equilibrium values of λ^{infl} (α^{infl}). Lemma 9 shows that all of the direct effects lead to an increase in rating accuracy. That is, the direct effect of an increase in $\bar{\iota}$ leads to an increase in α^{infl} and an increase in λ^{infl}. The characterization of mixed-strategy equilibria is complex and we cannot derive analytical results for the overall effect on the equilibrium, that is, the sum of direct plus indirect effects. In principle, the indirect effects could lead to a perverse effect on rating accuracy. For example, an increase in $\bar{\iota}$ may lead the CRA to acquire less precise information but more often, that is, $\alpha^{infl} \downarrow$ and $\lambda^{infl} \uparrow$, since the CRA's incentives to acquire precise information diminish when $\overline{\lambda}$ increases. In this case, the overall effect on rating accuracy may be negative. However, in Internet Appendix B.6, we perform a numerical simulation of the model and show the direct effects dominate for the set of parameters examined.

40 Nevertheless, given information acquisition is costly, the opportunistic CRA does not acquire full information.

7. Other Sources of Discipline

We have shown that the information produced by the speculator in the CDS market disciplines the CRA by providing investors with an informative signal about asset quality that they can use to assess the CRA's performance in rating the asset. In this section, we describe some other signals of asset quality that might be available to investors, and discuss the differences and similarities in their effects on the CRA's incentives.

7.1 Project outcomes

In the model, investors learn about the quality of the rating from two signals of asset quality: the volume of trades in the CDS market (x) and the realization of the project outcome (y). The informativeness of y depends on the relative likelihood of default across project qualities, which is measured by the asset's information sensitivity $\Delta^f = f_B - f_G$.

Lemma 10. An increase in the asset's information sensitivity Δ^f always increases equilibrium rating accuracy α^*, while it may increase or decrease equilibrium market informativeness ι^*. Proof: See Internet Appendix B.7.

When Δ^f goes up, y becomes a more informative signal of project quality, since bad assets fail relatively more often and good assets relatively less often. This makes inaccurate ratings ex post more transparent and, thus, disciplines the CRA. As a result, the increase in Δ^f has two contrasting effects on the speculator's incentives to acquire information. On the one hand, the value of the CDS contract is more sensitive to the quality of the underlying asset, which makes learning about the asset more profitable for the speculator. On the other hand, the increase in rating accuracy reduces mispricing and, thus, profits from informed trading. As a consequence, α^* always increases with Δ^f, while the effect of Δ^f on ι^* is ambiguous.

In the limit as f_B approaches 1 and f_G approaches zero, a project's outcome becomes a perfect signal of its quality and, thus, investors only use y to assess the accuracy of the rating. However, there are two caveats to the disciplining role of y. First, defaults are unlikely to be observed in the short term, while trading in the CDS market is continuously providing information.[41,42] Second, although y is eventually realized and observed in the long term, the CDS market still plays a role in providing *interim* discipline in the short term. This is easier to see when we specify the CRA's objective function to allow for both an interim

[41] According to Standard & Poor's, the global corporate default rate in 2019 was 1.3% for investment-grade and 2.5% for speculative-grade issuances. The average time to default—the time between first rating and date of default—was 19.7 years for investment-grade issuances, with an associated standard deviation of 11.9 years, and 5.2 years for speculative grade issuances, with an associated standard deviation of 5.5 years. Source: https://www.spglobal.com/ratings/en/research/articles/200429-default-transition-and-recovery-2019-annual-global-corporate-default-and-rating-transition-study-11444862#ID7890

[42] Notice that a small Δ^f also decreases information acquisition in the CDS market; however, CDS trading may still be active if the volume of noise trading is large and information costs are low for speculators.

reputational payoff at time $t=2$, which depends on x only, and a final reputation payoff at time $t=T>2$, which depends on both x and y. We can then write the CRA's total payoff as follows:

$$\Pi^{cra}=\Pr(g)\left\{\varphi+\gamma E\left[q_{H,x}\mid g\right]+\gamma^{T}E\left[q_{H,x,y}\mid g\right]\right\}$$
$$+\Pr(b)\left(\gamma+\gamma^{T}\right)q_{\varnothing}-C(\alpha).$$

Notice that, if $\gamma<1$ and T is large, the weight on $E\left[q_{H,x,y}\mid g\right]$ approaches zero, while the one on $E\left[q_{H,x}\mid g\right]$ does not. Therefore, the discipline provided by the CDS market still can be important even when the project outcome is a perfect signal about the asset's quality.[43]

7.2 Multiple rating agencies

An alternate source of information about a CRA's rating quality could be the rating of another rating agency. In this section, we show that the presence of an additional CRA is not sufficient to align the CRAs' incentives with their social optimal level. As a result, market discipline can still improve welfare. We also show that improving market discipline may be more effective at increasing surplus than allowing another CRA to enter the market.

To explore surplus with multiple CRAs, we extend the model with two CRAs from Section 6.2 to the setting where the reputational payoff is replaced with a second ratings market as described in Section 5. Similar to the main model, in this setting the time $t=3$ rating fee depends on the CRAs' reputation in that period, which contributes to their accuracy incentives at time $t=1$. Unlike the main model, however, the CRAs have an incentive to produce information also at time $t=3$, to increase the likelihood the issuer receives and acquires two high ratings. To make this extension tractable, we modify the model slightly and assume that rating accuracy is sufficiently costly that the CRAs choose not to produce information at time $t=3$ (i.e., $\alpha_{\mathcal{A},3}=\alpha_{\mathcal{B},3}=\frac{1}{2}$), but may choose to do so at $t=1$, due to their reputational incentives in that period.[44]

The formulation described above is consistent with the reduced-form approach to reputational payoffs, so that the qualitative results in Lemma 8 and Proposition 4 carry through. To simplify the exposition, we assume that the project outcome y_1 is not observed at time $t=3$, so the CRAs' payoffs are not contingent on its realization.

We can write the expression for total surplus as follows:

$$S_{mult}=\sum\nolimits_{t}\gamma_{t=3}\left\{\frac{1}{2}\left[\mu_t^{H_{\mathcal{A}}}\mu_t^{H_{\mathcal{B}}}v_G+(1-\mu_t^{H_{\mathcal{A}}})(1-\mu_t^{H_{\mathcal{B}}})v_B\right]\right.$$
$$\left.-(1-q_0)\sum\nolimits_{j}C\left(\alpha_{j,t}\right)\right\}-\eta\frac{\Upsilon}{2}K(\iota), \qquad (13)$$

[43] Of course, y is a noisy signal and could be even noisier in reality than in our model. An example of this is y depending in part on an unpredictable systemic risk factor.

[44] Formally, we assume $C'(\frac{1}{2})>\frac{\phi}{4}v_G$ and $C''>0$, so that we always have $\alpha_{\mathcal{A},3}=\alpha_{\mathcal{B},3}=\frac{1}{2}$ in equilibrium.

where $j \in \{\mathcal{A}, \mathcal{B}\}$, $t \in \{1,3\}$, $\mu_t^{H_j} = q_0 + (1-q_0)\alpha_{j,t}$, $\Upsilon = \mu_1^{H_\mathcal{A}} \mu_1^{H_\mathcal{B}} + (1-\mu_1^{H_\mathcal{A}})(1-\mu_1^{H_\mathcal{B}})$, and $\gamma_{t=3}$ takes values γ_S when $t=3$ and 1 otherwise.

The expression for S_{mult} is similar to the one in the main model (S in Equation (9)), with the difference that a project is now implemented when *both* ratings are high. The unconditional probability that the project is implemented in the first period is $\frac{\Upsilon}{2}$. Therefore, the cost of the speculator's precision is incurred with probability $\eta\frac{\Upsilon}{2}$. The surplus S_{mult} is maximized at $\iota = 0$, and $\alpha_{\mathcal{A},t} = \alpha_{\mathcal{B},t} = \widehat{\alpha}^{mult}$ for $t \in \{1,3\}$, where $\widehat{\alpha}^{mult} \in [\frac{1}{2}, 1]$.[45] The optimal level, $\widehat{\alpha}^{mult}$, is smaller than that in the main model, that is, $\widehat{\alpha}^{mult} \leq \widehat{\alpha}$. This difference in the optimal level occurs because two CRAs screen the projects here, and this screening improves the average quality of the projects that are implemented for a given rating accuracy.

The next proposition describes comparative statics results on S_{mult}.[46]

Proposition 6. $\widehat{\alpha}^{mult}$ denotes the level of rating accuracy that maximizes the expected total surplus, S_{mult}. The following comparative statics results hold in equilibrium:

1. Equilibrium rating accuracy at time $t=1$ (α_1^{mult}) may be larger or smaller than $\widehat{\alpha}^{mult}$. If $\alpha_1^{mult} < \widehat{\alpha}^{mult}$ and the speculator's cost of information ρ_s is sufficiently small, S_{mult} is decreasing in ρ_s and increasing in the volume of the noise trader's demand (n).
2. Assume that the CRAs and the speculator have the same cost function, that is, $C(\alpha) = K(\iota)$ for any $\alpha = \iota$. Surplus when there are two CRAs and no information provided by the CDS market (i.e., S_{mult} evaluated at $\eta = 0$) may be larger or smaller than surplus in the model with one CRA with information provided by the CDS market (i.e., S evaluated at $\eta > 0$).

Proof: See Internet Appendix B.8.

The CRAs may still underinvest or overinvest in information production in the setting with two CRAs, depending on the parameters of the model. The CDS market can still increase surplus from its disciplining effect; if $\alpha_1^{mult} < \widehat{\alpha}^{mult}$ and the speculator's cost of information is sufficiently small, an increase in market discipline ι—spurred by a decrease in ρ_s or an increase in n—leads to higher surplus, by pushing the equilibrium accuracy α_1^{mult} closer to its optimal level.

The second part of the proposition compares surplus across two different scenarios: one with two CRAs and no speculator in the CDS market; the other

45 We have $\widehat{\alpha}^{mult} = 1$ if $v_G > 2C'(1)$; $\widehat{\alpha}^{mult} = \frac{1}{2}$ if $\frac{1}{2}(1+q_0)(v_G+v_B) - v_B < 2C'(\frac{1}{2})$; otherwise, we have $\widehat{\alpha}^{mult} \in (\frac{1}{2}, 1)$ such that $(q_0 + (1-q_0)\widehat{\alpha}^{mult})(v_G+v_B) - v_B = 2C'(\widehat{\alpha}^{mult})$.

46 For brevity, Proposition 6 focuses on the parameters that drive market discipline ρ_s and n, and the case $\alpha_1^{mult} < \widehat{\alpha}^{mult}$. The qualitative results in Proposition 2 hold also in this setting for the other parameters γ, ρ_{cra}, and the case $\alpha_1^{mult} > \widehat{\alpha}^{mult}$.

with only one CRA but the speculator trading in the CDS market. We show that total surplus may be larger in the latter scenario. This result is driven by a combination of two factors. First, the speculator always disciplines the CRAs' accuracy incentives. As we discussed in Section 6.2, the strategic substitutability in the CRAs' choices of accuracy may instead weaken their incentives. Second, disagreement among CRAs is not observed in equilibrium, which hinders investors' ability to learn about the quality of a rating from the rating of another CRA. This is different from the CDS market, where both confirmatory (i.e., low demand for CDS following a high rating) and contradictory (i.e., high demand for CDS following a high rating) signals are observable and informative.

7.3 Media

Another important source of information is the media. Gentzkow and Shapiro (2006) point to media building reputation from confirming reader bias rather than providing information. There may be a selection effect to what media reports; they may select surprising news or positive news (Goldman, Martel, and Schneemeier forthcoming, Tetlock 2014) and be biased toward large, well-known firms (Ahern and Sosyura 2015). Nevertheless, Bonsall, Green, and Muller (2018) find that firms who are more widely covered by the business press have more accurate ratings.

8. Empirical Implications

In the introduction, we discussed evidence in the literature that is consistent with our model's implications. We now examine some new empirical implications of the paper that are not present in the literature (i.e., papers that only examine CRA *or* market behavior rather than their interactions).

First, many papers in the literature on credit ratings use a shock to rating agencies to examine how ratings accuracy is affected (e.g., Dimitrov, Palia, and Tang (2015) look at the passage of the Dodd-Frank bill; Becker and Milbourn (2011) study the entry of Fitch; and Badoer and Demiroglu (2019) analyze the introduction of TRACE). To measure ratings accuracy, these papers use the responsiveness of market prices (bond yields, stock market prices) to credit ratings. Our paper demonstrates that the feedback effect between market prices and ratings may muddle this measure. Any shock that affects ratings will feed through to market prices in two ways. First, the information from ratings will be incorporated into the price. Second, the change in ratings accuracy will change the incentives of the speculator to acquire information, affecting the informativeness of the market price. Presumably these analyses are trying to only measure the first effect, that is, whether ratings themselves have become more accurate (measuring μ_H). One way to get around this is to look at how ratings match up with subsequent defaults.[47] Another possibility is to use an expected default measure, such as distance to default.

Second, our model predicts that exogenous shocks to the speculator's cost of producing information or to the volume of trades in the CDS or secondary market will increase the CRA's rating accuracy. These shocks change the speculator's level of information acquisition, affecting the informativeness of the market price and therefore the discipline imposed on the CRA. These effects have, to our knowledge, not been examined. A couple of exogenous shocks to the cost of producing information have been recently used in the finance and accounting literatures. Gao and Huang (2020), Goldstein, Yang, and Zuo (2020), and others study the implementation of the SEC's EDGAR system to provide access to corporate filings online. Dong et al. (2016), Bhattacharya, Cho, and Kim (2018), and others examine the SEC's subsequent requirement that all filings be tagged in a standardized machine readable (extensible Business Reporting Language (XBRL)) format. A recent literature also exploits exogenous shocks to the volume of trades (rule changes for corporate bond ETFs in Dannhauser [2017], mechanical corporate bond index rules in Dick-Nielsen and Rossi [2019], and the implementation of the Volcker Rule in Bao, O'Hara, and Zhou [2018]).

Third, many papers in the literature on the real effects of financial markets identify a positive correlation between price informativeness and corporate policies (e.g., Chen, Goldstein, and Jiang 2007, Bakke and Whited 2010, Ferreira, Ferreira, and Raposo 2011). The standard explanation is that managers' learning from stock prices links price informativeness to corporate policies. Our paper suggests an additional channel that could be contributing to this correlation: more informative prices crowd-in more rating accuracy, which provides more information to managers and potentially spurs firms' investments. Thus, an exogenous shock to market information also affects outside sources of information (e.g., CRAs), so this effect should be taken into account.

9. Conclusion

Investors and regulators monitor credit ratings, which are critical for well-functioning debt markets. We focus on a different source of discipline for ratings: the market for credit risk. By providing timely information, this market makes rating errors transparent and increases the reputational concerns of rating agencies, leading to increases in ratings accuracy. We demonstrate that this effect is robust to competition between rating agencies, allowing for rating inflation, and connected primary and secondary markets. Market information is special in this role; discipline from other rating agencies and project outcomes

[47] Note that both Becker and Milbourn (2011) and Badoer and Demiroglu (2019) look at defaults in robustness tests.

may be weaker. Incorporating dynamics into market trading to create a richer environment for study would be an interesting way forward for future research.

A. Appendix

A.1 Preliminaries

A.1.1 Reputation. If the CRA offers a high rating to the issuer ($r = H$), the issuer implements the project, and the CDS market takes place. At time $t = 3$, investors observe the realization of the order flow x and the project outcome y. For a given realization of x and y, the CRA's posterior reputation is

$$q_{H,x,y} = \Pr(\tau = \mathcal{I} \mid H,x,y) = \frac{q_0 \Pr(H,x,y|\mathcal{I})}{q_0 \Pr(H,x,y|\mathcal{I}) + (1-q_0)\Pr(H,x,y \mid \mathcal{O})}$$
$$= \left\{1 + \frac{1-q_0}{q_0}\frac{\Pr(H,x,y \mid \mathcal{O})}{\Pr(H,x,y \mid \mathcal{I})}\right\}^{-1},$$

where

$$\frac{\Pr(H,x,y \mid \mathcal{O})}{\Pr(H,x,y \mid \mathcal{I})} = \frac{\Pr(H \mid \mathcal{O},G)\Pr(x,y \mid G) + \Pr(H \mid \mathcal{O},B)\Pr(x,y \mid B)}{\Pr(H \mid \mathcal{I},G)\Pr(x,y \mid G) + \Pr(H \mid \mathcal{I},B)\Pr(x,y \mid B)}$$
$$= \Pr(H \mid \mathcal{O},G) + \Pr(H \mid \mathcal{O},B)\frac{\Pr(x,y \mid B)}{\Pr(x,y \mid G)},$$

since an informative CRA always produces accurate ratings, that is, $\Pr(H \mid \mathcal{I},G) = 1$ and $\Pr(H \mid \mathcal{I},B) = 0$, and the distributions of (x,y) and $r = H$ are independent, conditional on the project's quality.

If investors only observe x, the CRA's posterior reputation is

$$q_{H,x} = \left\{1 + \frac{1-q_0}{q_0}\left[\Pr(H \mid \mathcal{O},G) + \Pr(H \mid \mathcal{O},B)\frac{\Pr(x \mid B)}{\Pr(x \mid G)}\right]\right\}^{-1}.$$

If the CRA offers a low rating to the issuer, the issuer does not disclose the rating ($r = \varnothing$), the investment project is not implemented, and the CDS market does not take place. In this case, the CRA's posterior reputation is

$$q_{\varnothing} = \Pr(\mathcal{I} \mid \varnothing) = \left\{1 + \frac{1-q_0}{q_0}\frac{\Pr(\varnothing \mid \mathcal{O})}{\Pr(\varnothing \mid \mathcal{I})}\right\}^{-1},$$

where $\Pr(\varnothing \mid \mathcal{I},G) = 0$ and $\Pr(\varnothing \mid \mathcal{I},B) = 1$.

The expressions for $\Pr(H \mid \mathcal{O},G)$, $\Pr(H \mid \mathcal{O},B)$, and $\Pr(\varnothing \mid \mathcal{O})$ depend on the opportunistic CRA's rating strategy. In the main model, we assume that the opportunistic CRA always offers a high (low) rating after having observed a good (bad) signal, and focus on its choice of signal precision. In Section 6.3, we extend the model and let the opportunistic CRA choose both the precision of its signal and the report $\tilde{r}$ it offers to the issuer, allowing for the possibility that the CRA lies and offers a high rating after having observed a bad signal. In either case, notice that $q_{H,x,y}$ decreases with the ratio $\frac{\Pr(x,y|B)}{\Pr(x,y|G)}$.

A.1.2 NPV of the initial investment. Depending on the rating r, the probability investors attach to the project being good are:

$$\mu^H = \Pr(\theta = G|H) = \left\{1 + \frac{\Pr(H|B)}{\Pr(H|G)}\right\}^{-1}; \qquad \mu^{\varnothing} = \Pr(\theta = G|\varnothing) = \left\{1 + \frac{\Pr(\varnothing|B)}{\Pr(\varnothing|G)}\right\}^{-1}.$$

The issuer is willing to sell the claim and undertake the project as long as $p^r > I$. For a given rating r, this means:

$$\mu^r(1-f_G)+\left(1-\mu^r\right)(1-f_B)>I \Leftrightarrow \mu^r > \frac{-v_B}{v_G-v_B}. \tag{A1}$$

Notice that $\frac{-v_B}{v_G-v_B} > \frac{1}{2}$, since the ex ante NPV of the project is assumed to be negative, that is, $\frac{1}{2}(v_G+v_B)<0$, which implies $v_G < -v_B$. Therefore, whenever the project is implemented, the issuer and investors believe this is more likely to be good, that is, $\mu^r > \frac{1}{2}$. Notice also that the larger the ratio $\frac{-v_B}{v_G}$, the closer μ^r must be to one for inequality (A1) to hold.

In the baseline model, we have $\mu^H = q_0+(1-q_0)\overline{\alpha} > \mu^{\varnothing} = (1-q_0)(1-\overline{\alpha}) = 1-\mu^H$, where $\overline{\alpha}$ is the investors' conjecture about the opportunistic CRA's rating accuracy. Since $\overline{\alpha}\in[\frac{1}{2},1]$, we have $\mu^{\varnothing} < \frac{1}{2}$ and, thus, the project never takes place when the asset goes unrated. When the CRA's initial reputation q_0 and $\overline{\alpha}$ are small compared to $\frac{|v_B|}{v_G}$, the inequality in (A1) does not hold for $r=H$ either. This has two implications for the equilibrium. First, for a given initial reputation q_0, equilibrium rating accuracy α^* must be large enough for the project to take place, that is,

$$q_0+(1-q_0)\alpha^* > \frac{-v_B}{v_G-v_B} \Leftrightarrow \alpha^* > \left(\frac{-v_B}{v_G-v_B}-q_0\right)(1-q_0)^{-1} \equiv \underline{\alpha}.$$

Second, if q_0 is sufficiently small, there exist conjectures α' such that $\mu^H < \frac{-v_B}{v_G-v_B}$ and the issuer chooses to not publicize a high rating. In this case, the opportunistic CRA is indifferent between any rating strategy and, hence, is fine with choosing the strategy α': this implies that the conjecture α' represents indeed an equilibrium of the game. In the literature on Communication Games, this type of equilibrium is referred to as a *babbling equilibrium*. However, since ratings play no role in this type of equilibria, they are not of interest for our analysis.

The only possible equilibria of the game are the ones we characterize in the text and the babbling equilibria described above. However, when the CRA's initial reputation is sufficiently large, that is, if

$$q_0+(1-q_0)\frac{1}{2} > \frac{-v_B}{v_G-v_B} \Leftrightarrow q_0 > \frac{-2v_B}{v_G-v_B}-1 \equiv \underline{q},$$

a high rating is always credible enough that investors always pay $p^H > I$ for a highly rated project and, thus, any babbling equilibrium fails to exist. In this case (i.e., $q_0 > \underline{q}$), the equilibrium we characterize in the analysis is the unique equilibrium of the game.

A.2 Proofs of Lemmas 1 and 2

We combine the proofs of Lemmas 1 and 2. First, we prove that the speculator's strategy in Lemma 1 is the unique possible one in equilibrium. We then verify that, given this strategy, the market maker's inference in Lemma 2 is consistent with Bayes' rule.

For a given realization of x, the CDS price is given by $p^{cds}(x) = \mu^{H,x} f_G + \left(1-\mu^{H,x}\right) f_B$, where

$$\mu^{H,x} = \Pr(G\,|\,r=H,x) = \left\{1+\Gamma(x)\frac{1-\mu^H}{\mu^H}\right\}^{-1}, \tag{A2}$$

is the probability the market maker attaches to the project being good after having observed $r=H$ and x, and $\Gamma(x) = \frac{\Pr(x|B)}{\Pr(x|G)}$ describes the likelihood ratio for each realization of x:

$$\Gamma(-2n) = \frac{\eta\frac{1-\bar{\iota}}{2}+(1-\eta)\frac{1}{4}}{\eta\frac{\bar{\iota}}{2}+(1-\eta)\frac{1}{4}}; \qquad \Gamma(+2n) = \frac{\eta\frac{\bar{\iota}}{2}+(1-\eta)\frac{1}{4}}{\eta\frac{1-\bar{\iota}}{2}+(1-\eta)\frac{1}{4}}; \qquad \Gamma(0)=1. \tag{A3}$$

Consider a speculator who has observed a positive signal $\sigma_s = g$. If she sells CDS protection, her expected payoff is

$$E[p^{cds} \mid x_s = -n] - \left[\mu^{H,g} f_G + \left(1-\mu^{H,g}\right) f_B\right], \tag{A4}$$

where $\mu^{H,g} = \left\{1 + \frac{1-\iota}{\iota} \frac{1-\mu^H}{\mu^H}\right\}^{-1}$ is the probability the speculator attaches to the project being good after having observed both $r = H$ and $\sigma_s = g$.

The expected CDS price conditional on submitting a sell order is $E\left[p^{cds} \mid x_s = -n\right]$ and the speculator pays the buyer an amount 1 in case of default, which occurs with probability $\mu^{H,g} f_G + \left(1-\mu^{H,g}\right) f_B$ conditional on $\sigma_s = g$. When $x_s = -n$, we have $x \in \{-2n, 0\}$, with both realizations being equally likely. The speculator's payoff from selling in Equation (A4) thus simplifies to

$$\frac{1}{2}\left[\mu^H f_G + \left(1-\mu^H\right) f_B\right] + \frac{1}{2}\left[\mu^{H,-2n} f_G + \left(1-\mu^{H,-2n}\right) f_B\right] - \left[\mu^{H,g} f_G + \left(1-\mu^{H,g}\right) f_B\right]. \tag{A5}$$

In equilibrium, the market maker's conjecture about the speculator's signal precision is correct, that is, $\bar{\iota} = \iota$. This implies $1 \geq \Gamma(-2n) \geq \frac{1-\iota}{\iota}$ and, thus, $\mu^{H,g} \geq \mu^{H,-2n} \geq \mu^H$, with $\mu^{H,g} > \mu^H$ if $\iota \in (\frac{1}{2}, 1]$ and $\overline{\alpha} < 1$. Given that $f_G < f_B$, the expression in Equation (A5) is then always positive in equilibrium (strictly positive if $\iota \in (\frac{1}{2}, 1]$ and $\overline{\alpha} < 1$). The speculator thus profits from selling CDS protection when $\sigma_s = g$.

If the speculator buys CDS protection after having observed $\sigma_s = g$, her expected payoff is

$$\left[\mu^{H,g} f_G + \left(1-\mu^{H,g}\right) f_B\right] - E[p^{cds} \mid x_s = +n]. \tag{A6}$$

The expected CDS price conditional on submitting a buy order is $E\left[p^{cds} \mid x_s = +n\right]$ and the speculator is paid by the seller an amount 1 in case of default. When $x_s = +n$, we have $x \in \{0, +2n\}$, with both realizations being equally likely. The speculator's payoff from buying in Equation (A6) simplifies to

$$\left[\mu^{H,g} f_G + \left(1-\mu^{H,g}\right) f_B\right] - \frac{1}{2}\left[\mu^H f_G + \left(1-\mu^H\right) f_B\right] - \frac{1}{2}\left[\mu^{H,+2n} f_G + \left(1-\mu^{H,+2n}\right) f_B\right]. \tag{A7}$$

When $\bar{\iota} = \iota$, we have $\Gamma(+2n) \geq 1 \geq \frac{1-\iota}{\iota}$ and, thus, $\mu^{H,g} \geq \mu^H \geq \mu^{H,+2n}$. It follows that the expression in Equation (A7) is always negative in equilibrium (strictly negative if $\iota \in (\frac{1}{2}, 1]$ and $\overline{\alpha} < 1$). The speculator thus chooses to *sell* CDS protection when she observes $\sigma_s = g$.

The case in which the speculator observes $\sigma_s = b$ is the mirror image of the one in which she observes $\sigma_s = g$. That is, the speculator profits (loses) from buying (selling) CDS protection when $\sigma_s = b$.

This proves that the strategies described in Lemma 1 represent an equilibrium of the game. It is straightforward to show that any other possible market maker's inference from the submitted orders admit profitable deviations by the speculator. Therefore, this is also the unique equilibrium.

When $x \neq 0$, the market maker's inference trivially satisfies Bayes' rule.

A.3 Proof of Lemma 3

We prove Lemma 3 in two steps. In step 1, we derive a condition under which, for a given conjecture about rating accuracy $\overline{\alpha}$, the equilibrium value of precision $\iota^*(\overline{\alpha})$ is unique. In step 2, we show that the equilibrium value of precision decreases with $\overline{\alpha}$.

Step 1. The first-order condition for the speculator choice of precision is:

$$\frac{\partial \Pi^s}{\partial \iota} = 0 \Leftrightarrow \begin{array}{c} n\mu^H \left\{\left[E\left[p^{cds} \mid x_s = -n\right] - f_G\right] - \left[f_G - E\left[p^{cds} \mid x_s = +n\right]\right]\right\} \\ +n\left(1-\mu^H\right)\left\{\left[f_B - E\left[p^{cds} \mid x_s = +n\right]\right] - \left[E\left[p^{cds} \mid x_s = -n\right] - f_B\right]\right\} \end{array} = K'(\iota)$$

$$\Leftrightarrow \underbrace{\begin{array}{c} n\mu^H\left\{E\left[p^{cds}\,|\,x_s=-n\right]+E\left[p^{cds}\,|\,x_s=+n\right]-2f_G\right\} \\ +n\left(1-\mu^H\right)\left\{2f_B-E\left[p^{cds}\,|\,x_s=+n\right]-E\left[p^{cds}\,|\,x_s=-n\right]\right\}\end{array}}_{\equiv\frac{\partial R^s}{\partial\iota}}=K'(\iota). \tag{A8}$$

The left-hand side of Equation (A8) represents the speculator's marginal benefit of precision. R^s denotes the speculator's gross profit from trading and $\frac{\partial R^s}{\partial\iota}$ the marginal benefit of precision. The speculator's problem is strictly concave, since we have $\frac{\partial^2\Pi^s}{\partial\iota^2}=-K''(\iota)<0$. It follows that the optimal value of ι for given conjectures $\overline{\alpha}$ and $\overline{\iota}$ is unique. For a given conjecture $\overline{\alpha}$, the equilibrium value $\iota^*(\overline{\alpha})$ solves Equation (A8) evaluated at $\iota=\overline{\iota}$, that is, when the market maker's conjecture is consistent. The CDS price and, thus, $\frac{\partial R^s}{\partial\iota}$ only depend on the precision of the speculator's signal through the market maker's conjecture $\overline{\iota}$. The equilibrium level of precision is then $\iota^*(\overline{\alpha})=\frac{1}{2}$ if $\frac{\partial R^s}{\partial\iota}\leq 0$ at $\overline{\iota}=\frac{1}{2}$, $\iota^*(\overline{\alpha})=1$ if $\frac{\partial R^s}{\partial\iota}\geq K'(1)$ at $\overline{\iota}=1$, and $\iota^*(\overline{\alpha})\in(\frac{1}{2},1)$ if $\frac{\partial R^s}{\partial\iota}=K'(\overline{\iota})$ for some interior value $\overline{\iota}=\iota^*(\overline{\alpha})$.

Using the expression for $\mu^{H,x}$ in Equations (A2) and (A3), and given that $E\left[p^{cds}\,|\,x_s=-n\right]=\frac{1}{2}[p^{cds}(-2n)+p^{cds}(0)]$ and $E\left[p^{cds}\,|\,x_s=+n\right]=\frac{1}{2}[p^{cds}(+2n)+p^{cds}(0)]$, we can rewrite $\frac{\partial R^s}{\partial\iota}$ as

$$\frac{\partial R^s}{\partial\iota}=2n\Delta^f(1-\mu^H)\mu^H\frac{\left[\eta(1-2\overline{\iota})(1-2\mu^H)\right]^2-2}{\left[\eta(1-2\overline{\iota})(1-2\mu^H)\right]^2-1}. \tag{A9}$$

If $\overline{\alpha}=1$, we have $\mu^H=1$ and, thus, $\frac{\partial R^s}{\partial\iota}=0$; the unique CDS market equilibrium is $\iota^*(1)=\frac{1}{2}$ in this case. If $\overline{\alpha}<1$, we have $\frac{\partial R^s}{\partial\iota}>0$ at $\overline{\iota}=\frac{1}{2}$, which implies that either $\frac{\partial R^s}{\partial\iota}$ is always above $K'(\overline{\iota})$, in which case $\iota^*(\overline{\alpha})=1$, or $\frac{\partial R^s}{\partial\iota}$ meets $K'(\overline{\iota})$ at least one time. If $\frac{\partial R^s}{\partial\iota}$ meets $K'(\overline{\iota})$ more than once, there are multiple CDS market equilibria for a given value of $\overline{\alpha}$. In what follows, we describe a condition on the cost of precision such that $\frac{\partial R^s}{\partial\iota}$ and $K'(\overline{\iota})$ meet only once at some interior level $\overline{\iota}\in(\frac{1}{2},1)$ for any $\overline{\alpha}\in[\frac{1}{2},1)$, so that the equilibrium level of precision $\iota^*(\overline{\alpha})$ is unique.[48]

In what follows, we consider the case $\overline{\alpha}<1$, which implies $\mu^H<1$. One can easily verify that $\frac{\partial R^s}{\partial\iota}$ is increasing and convex in $\overline{\iota}$, and satisfies $\frac{\partial R^s}{\partial\iota}\in(0,n\Delta^f)$. Let $\underline{K}'(\overline{\iota})\equiv n\Delta^f(2\overline{\iota}-1)$; we have $\underline{K}'(\frac{1}{2})=0<\frac{\partial R^s}{\partial\iota}$ at $\overline{\iota}=\frac{1}{2}$ and $\underline{K}'(1)=n\Delta^f>\frac{\partial R^s}{\partial\iota}$ at $\overline{\iota}=1$. We prove next that $\frac{\partial R^s}{\partial\iota}$ and $\underline{K}'(\overline{\iota})$ meet only once in the interval $\overline{\iota}\in[\frac{1}{2},1]$, and $\frac{\partial R^s}{\partial\iota}$ meets $\underline{K}'(\overline{\iota})$ from above at some interior value $\overline{\iota}\in(\frac{1}{2},1)$.

For $\frac{\partial R^s}{\partial\iota}$ to meet $\underline{K}'(\overline{\iota})$ from below, we would need that $\frac{\partial R^s}{\partial\iota}$ increases with $\overline{\iota}$ at a faster rate than $\underline{K}'(\overline{\iota})$ around the intersection point, that is, $\partial\frac{\partial R^s}{\partial\iota}/\partial\overline{\iota}>\partial\underline{K}'(\overline{\iota})/\partial\overline{\iota}$. Notice that (i) $\frac{\partial R^s}{\partial\iota}$ is increasing and convex in $\overline{\iota}$, so $\partial\frac{\partial R^s}{\partial\iota}/\partial\overline{\iota}$ increases with $\overline{\iota}$; (ii) $\partial\underline{K}'(\overline{\iota})/\partial\overline{\iota}=2n\Delta^f$, so that $\partial\underline{K}'(\overline{\iota})/\partial\overline{\iota}$ does not depend on $\overline{\iota}$. It follows that $\frac{\partial R^s}{\partial\iota}$ cannot intersect $\underline{K}'(\overline{\iota})$ from below, since then $\frac{\partial R^s}{\partial\iota}$ would stay above $\underline{K}'(\overline{\iota})$ for any $\overline{\iota}$ larger than the intersection point, which violates $\underline{K}'(1)=n\Delta^f>\frac{\partial R^s}{\partial\iota}$ at $\overline{\iota}=1$. Finally, since $\frac{\partial R^s}{\partial\iota}>\underline{K}'(\overline{\iota})$ at $\overline{\iota}=\frac{1}{2}$ and $\frac{\partial R^s}{\partial\iota}<\underline{K}'(\overline{\iota})$ at $\overline{\iota}=1$, $\frac{\partial R^s}{\partial\iota}$ crosses $\underline{K}'(\overline{\iota})$ once from above at some interior value $\overline{\iota}\in(\frac{1}{2},1)$.[49]

If $K''(\iota)\geq 2n\Delta^f$ for any $\iota\in[\frac{1}{2},1]$, we have that $K'(\frac{1}{2})=\underline{K}'(\frac{1}{2})=0$ and $K'(\overline{\iota})$ increases with $\overline{\iota}$ at a faster rate than $\underline{K}'(\overline{\iota})$. Under this condition, $K'(\overline{\iota})$ is then above $\underline{K}'(\overline{\iota})$ and steeper than $\underline{K}'(\overline{\iota})$ for any $\iota\in[\frac{1}{2},1]$. It follows that $\frac{\partial R^s}{\partial\iota}$ meets $K'(\overline{\iota})$ from above at a lower value of $\overline{\iota}$ than the one at which $\frac{\partial R^s}{\partial\iota}$ meets $\underline{K}'(\overline{\iota})$. It also follows that $\frac{\partial R^s}{\partial\iota}$ and $K'(\overline{\iota})$ meet only once; otherwise, we would need either that $\frac{\partial R^s}{\partial\iota}$ and $\underline{K}'(\overline{\iota})$ never meet or that that $\frac{\partial R^s}{\partial\iota}$ and $\underline{K}'(\overline{\iota})$ meet more than once, both of

48 It is worth emphasizing that all the comparative static results in the paper hold for all the stable equilibria of the CDS market, that is, the ones where $\frac{\partial R^s}{\partial\iota}$ meets $K'(\iota)$ from above or where $\iota^*(\overline{\alpha})=1$. In general, a stable equilibrium always exists and there may be more than one.

49 The two functions cannot meet more than once, since that would require $\frac{\partial R^s}{\partial\iota}$ crossing $\underline{K}'(\overline{\iota})$ from below at least once, which as shown above is not possible.

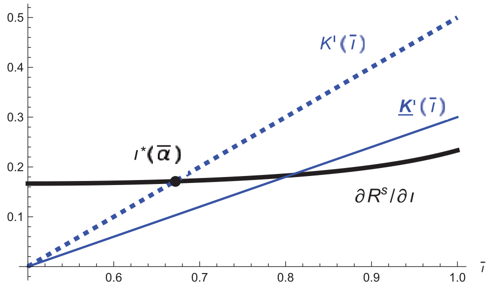

Figure A1
Equilibrium market informativeness for a given level of rating accuracy
In this example, $K(\iota)=(\iota-\frac{1}{2})^2, q_0=\frac{1}{2}, \overline{\alpha}=\frac{2}{3}, \eta=1, n=\frac{3}{4}, f_B=0.6, f_G=0.2$.

which lead to a contradiction. Notice also that $\frac{\partial R^S}{\partial \iota}$ crosses $K'(\bar{\iota})$ at some interior level $\bar{\iota}\in(\frac{1}{2},1)$, since $\frac{\partial R^S}{\partial \iota} > K'(\bar{\iota})=0$ at $\bar{\iota}=\frac{1}{2}$.

Therefore, under the condition $K''(\iota)\geq 2n\Delta^f$ for any $\iota\in[\frac{1}{2},1]$, $\iota^*(\overline{\alpha})$ is unique and satisfies $\iota^*(\overline{\alpha})\in(\frac{1}{2},1)$ if $\overline{\alpha}<1$ and $\iota^*(1)=\frac{1}{2}$, and we have $K''(\iota)>\frac{\partial^2 R^S}{\partial \iota^2}$ in equilibrium (since $\frac{\partial R^S}{\partial \iota}$ meets $K'(\iota)$ from above). This is described in Figure A1.

Step 2. The conjecture $\overline{\alpha}$ about rating accuracy affects the speculator's and market maker's ex ante beliefs about project quality and, thus, the speculator's marginal benefit of precision. Here, we show that $\frac{\partial R^S}{\partial \iota}$ decreases with $\overline{\alpha}$, which implies that the equilibrium level of precision decreases with $\overline{\alpha}$.[50] Taking the derivative of $\frac{\partial R^S}{\partial \iota}$ (Equation (A8)) with respect to $\overline{\alpha}$ yields

$$\frac{\partial R^s}{\partial \iota \partial \overline{\alpha}} = 2n \underbrace{\frac{\partial \mu^H}{\partial \overline{\alpha}}}_{>0} \left\{ E[p^{cds} \mid x_s=-n] + E[p^{cds} \mid x_s=+n] - f_G - f_B \right\}$$

$$+ n\left(2\mu^H - 1\right) \frac{\partial \mu^H}{\partial \overline{\alpha}} \underbrace{\left\{ \frac{\partial E[p^{cds} \mid x_s=+n]}{\partial \mu^H} + \frac{\partial E[p^{cds} \mid x_s=+n]}{\partial \mu^H} \right\}}_{<0}.$$

Therefore, it suffices that

$$f_G + f_B \geq E[p^{cds} \mid x_s=-n] + E[p^{cds} \mid x_s=+n] \tag{A10}$$

for $\frac{\partial R^S}{\partial \iota \partial \overline{\alpha}}$ to be negative.

Using the expressions for $E[p^{cds} \mid x_s=-n]$ and $E[p^{cds} \mid x_s=+n]$, the right-hand side of inequality (A10) simplifies to

$$\mu^H f_G + \left(1-\mu^H\right) f_B + \frac{1}{2}\left[\mu^{H,-2n} f_G + \left(1-\mu^{H,-2n}\right) f_B\right] + \frac{1}{2}\left[\mu^{H,+2n} f_G + \left(1-\mu^{H,+2n}\right) f_B\right]$$

Therefore, inequality (A10) can be rearranged as

$$\left(1-\mu^H\right) f_G + \mu^H f_B \geq \frac{1}{2}\left[\mu^{H,-2n} f_G + \left(1-\mu^{H,-2n}\right) f_B\right] + \frac{1}{2}\left[\mu^{H,+2n} f_G + \left(1-\mu^{H,+2n}\right) f_B\right]$$

[50] Since $\frac{\partial R^S}{\partial \iota}$ meets $K'(\iota)$ from above, the two functions cross at a lower value of ι when $\frac{\partial R^S}{\partial \iota}$ shifts downward.

$$\Leftrightarrow \left[\mu^H + \frac{1}{2}\mu^{H,-2n} + \frac{1}{2}\mu^{H,+2n} - 1\right]\Delta^f \geq 0$$

$$\Leftrightarrow \mu^H + \frac{1}{2}\mu^{H,-2n} + \frac{1}{2}\mu^{H,+2n} \geq 1. \tag{A11}$$

The inequality above is always satisfied. This implies that $\frac{\partial R^s}{\partial\iota\partial\overline{\alpha}} < 0$ and, thus, that $\iota^*(\overline{\alpha})$ decreases with $\overline{\alpha}$.

A.4 Proof of Lemma 4

We prove Lemma 4 in two steps. In step 1, we show that, for a given conjecture $\overline{\iota}$, the equilibrium level of rating accuracy $\alpha^*(\overline{\iota})$ is unique. In step 2, we show that $\alpha^*(\overline{\iota})$ increases with $\overline{\iota}$.

Step 1. The first-order condition for the CRA's choice of accuracy is:

$$\frac{\gamma}{2}\left\{E\left[q_{H,x,y} \mid G\right] - E\left[q_{H,x,y} \mid B\right]\right\} = C'(\alpha), \tag{A12}$$

where $q_{H,x,y} = \left\{1 + \frac{1-q_0}{q_0}\left[\overline{\alpha} + (1-\overline{\alpha})\frac{\Pr(x,y|B)}{\Pr(x,y|G)}\right]\right\}^{-1}$.

Notice that the CRA's problem is strictly concave, since we have $\frac{\partial^2 \Pi^{cra}}{\partial\alpha^2} = -C''(\alpha) < 0$. Therefore, the optimal value of α for given conjectures $\overline{\alpha}$ and $\overline{\iota}$ is unique.

The left-hand side of Equation (A12) represents the CRA's marginal benefit of accuracy. This depends on the investor's conjecture $\overline{\alpha}$. The weight investors put on the realizations of x and y when updating their beliefs about the CRA's type depends on $\overline{\alpha}$. If $\overline{\alpha} = 1$, both an opportunistic and an informative CRA are expected to issue perfect ratings. In this case, investors do not update their beliefs about the CRA's type; that is, $q_{H,x,y} = q_0$ for any realizations of x and y. As $\overline{\alpha}$ becomes smaller, investors put more weight on x and y in learning about the accuracy of the rating and the CRA's type.[51]

For a given conjecture $\overline{\iota}$, the equilibrium rating accuracy $\alpha^*(\overline{\iota})$ is characterized by Equation (A12) evaluated at $\overline{\alpha} = \alpha$. The distribution of the likelihood ratio $\frac{\Pr(x,y|B)}{\Pr(x,y|G)}$ conditional on $\theta = B$ represents a First-Order Stochastic Dominance shift of the distribution of $\frac{\Pr(x,y|B)}{\Pr(x,y|G)}$ conditional on $\theta = G$. Since $q_{H,x,y}$ is decreasing in $\frac{\Pr(x,y|B)}{\Pr(x,y|G)}$, it follows that $E\left[q_{H,x,y} \mid G\right] \geq E\left[q_{H,x,y} \mid B\right]$ (with strict inequality if $\overline{\alpha} < 1$). Given that $C'(\frac{1}{2}) = 0$, Equation (A12) then cannot be satisfied at $\alpha = \frac{1}{2}$ and, thus, $\alpha^*(\overline{\iota}) > \frac{1}{2}$ for any $\overline{\iota}$. As we discussed above, we have $q_{H,x,y} = q_0$ for any realizations of x and y at $\overline{\alpha} = 1$, but $C'(1) > 0$. Therefore, Equation (A12) cannot be satisfied at $\alpha = 1$ either. This means that we have $\alpha^*(\overline{\iota}) \in (\frac{1}{2}, 1)$ in equilibrium.

In what follows, we show that $E\left[q_{H,x,y} \mid G\right] - E\left[q_{H,x,y} \mid B\right]$ is decreasing in $\overline{\alpha}$; given that $C''(\alpha) \geq 0$, this implies that the equilibrium level of accuracy is unique, since there exists a unique value of $\overline{\alpha} = \alpha$ for which Equation (A12) holds ($\frac{\gamma}{2}\left\{E\left[q_{H,x,y} \mid G\right] - E\left[q_{H,x,y} \mid B\right]\right\}$ and $C'(\alpha)$ cross only once). This is described in Figure A2. We have

$$\frac{\partial q_{H,x,y}}{\partial\overline{\alpha}} = -\left(q_{H,x,y}\right)^2 \frac{1-q_0}{q_0}\left[1 - \frac{\Pr(x,y \mid B)}{\Pr(x,y \mid G)}\right].$$

It follows that we can write $\frac{\partial\left(E\left[q_{H,x,y}|G\right] - E\left[q_{H,x,y}|B\right]\right)}{\partial\overline{\alpha}}$ as

$$-\frac{1-q_0}{q_0}\sum_{(x,y)}\left(q_{H,x,y}\right)^2\left[1 - \frac{\Pr(x,y \mid B)}{\Pr(x,y \mid G)}\right]\left[\Pr(x,y \mid G) - \Pr(x,y \mid B)\right]$$

51 Notice that, when $\eta = \overline{\iota} = \overline{\alpha} = 1$, $q_{H,x,y}$ does not satisfy Bayes' rule, since investors believe that the project is good (as their conjecture is that the CRA is perfectly informative), but $x = +2n$ reveals that the project is bad (as $\eta = \overline{\iota} = 1$). The off-equilibrium-path belief that investors don't update the CRA's reputation in this case supports the equilibrium rating strategy described in Lemma 4. It is worth noticing that this case never arises when the equilibrium value of ι is considered, since we always have $\iota = \frac{1}{2}$ when $\overline{\alpha} = 1$.

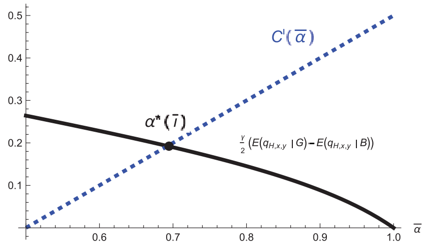

Figure A2
Equilibrium rating accuracy for a given level of market informativeness
In this example, $C(\alpha)=(\alpha-\frac{1}{2})^2, \bar{\iota}=\frac{4}{5}, q_0=\frac{1}{2}, \gamma=3, f_B=0.6, f_G=0.2$.

$$=-\frac{1-q_0}{q_0}\sum_{(x,y)}\frac{\left(q_{H,x,y}\right)^2}{\Pr(x,y\mid G)}\left[\Pr(x,y\mid G)-\Pr(x,y\mid B)\right]^2<0.$$

Therefore, $E\left[q_{H,x,y}\mid G\right]-E\left[q_{H,x,y}\mid B\right]$ decreases with $\overline{\alpha}$ and the equilibrium level of rating accuracy is unique.

Step 2. The conjecture $\bar{\iota}$ about market informativeness affects the CRA's marginal benefit of rating accuracy in Equation (A12) in two ways: first, through the probability distribution over realizations of x, and, second, for a given realization of x and y via learning about project quality, which affects the CRA's posterior reputation. In the following, we will show that the difference $E\left[q_{H,x,y}\mid G\right]-E\left[q_{H,x,y}\mid B\right]$ and, thus, the equilibrium level of accuracy increases with $\bar{\iota}$.

Since the conditional distributions of x and y are independent, we have

$$E\left[q_{H,x,y}\mid\theta\right]=E_y\left[E_x\left[q_{H,x,y}\mid y,\theta\right]\right].$$

We show that $E_x\left[q_{H,x,y}\mid y,G\right]$ increases and $E_x\left[q_{H,x,y}\mid y,B\right]$ decreases with $\bar{\iota}$ for any realization of y. Since the distribution of y does not depend on $\bar{\iota}$, this implies that $E\left[q_{H,x,y}\mid G\right]-E\left[q_{H,x,y}\mid B\right]$ increases with $\bar{\iota}$.

We can rearrange the expression for $q_{H,x,y}$ as

$$q_{H,x,y}=\left\{1+\frac{1-q_0}{q_0}\left[\overline{\alpha}+(1-\overline{\alpha})\frac{\Pr(x\mid B)}{\Pr(x\mid G)}\frac{\Pr(y\mid B)}{\Pr(y\mid G)}\right]\right\}^{-1}=\left\{k_1+k_2\frac{\Pr(x\mid B)}{\Pr(x\mid G)}\right\}^{-1}, \quad \text{(A13)}$$

where $k_1=1+\frac{1-q_0}{q_0}\overline{\alpha}>0$ and $k_2=\frac{1-q_0}{q_0}(1-\overline{\alpha})\frac{\Pr(y|B)}{\Pr(y|G)}>0$.

Notice that $x\in\{-2n,0,+2n\}$ and $\Pr(x\mid B)$ and $\Pr(x\mid G)$ are characterized as follows:

$$\Pr(-2n\mid G)=\eta\frac{\bar{\iota}}{2}+(1-\eta)\frac{1}{4};\qquad \Pr(-2n\mid B)=\eta\frac{1-\bar{\iota}}{2}+(1-\eta)\frac{1}{4}, \quad \text{(A14)}$$

$\Pr(0\mid B)=\Pr(0\mid G)=\frac{1}{2}$, $\Pr(+2n\mid B)=\Pr(-2n\mid G)$, $\Pr(+2n\mid G)=\Pr(-2n\mid B)$, and $\Pr(x\mid\theta)\in[0,\frac{1}{2}]$ for any realizations of x and θ.

Let us first focus on $E_x\left[q_{H,x,y}\mid G,y\right]$; we have

$$E_x\left[q_{H,x,y}\mid G,y\right]=\Pr(-2n\mid G)q_{H,-2n,y}+\Pr(+2n\mid G)q_{H,+2n,y}+\frac{1}{2}q_{H,0,y},$$

and, thus,

$$\frac{\partial E_x\left[q_{H,x,y}\mid G,y\right]}{\partial\bar{\iota}}=\frac{\eta}{2}\underbrace{\left[q_{H,-2n,y}-q_{H,+2n,y}\right]}_{>0}+\Pr(-2n\mid G)\underbrace{\frac{\partial q_{H,-2n,y}}{\partial\bar{\iota}}}_{>0}$$

$$+\Pr(+2n\mid G)\underbrace{\frac{\partial q_{H,+2n,y}}{\partial\bar{\iota}}}_{<0},$$

where

$$\frac{\partial q_{H,-2n,y}}{\partial\bar{\iota}}=\left(q_{H,-2n,y}\right)^2\frac{\eta\frac{1}{4}k_2}{\Pr(-2n\mid G)^2};\qquad \frac{\partial q_{H,+2n,y}}{\partial\bar{\iota}}=-\left(q_{H,+2n,y}\right)^2\frac{\eta\frac{1}{4}k_2}{\Pr(+2n\mid G)^2}.$$

$\mathbf{1}_x$ denotes an indicator function that takes values 1 if $x=-2n$ and -1 if $x=+2n$; we can rearrange $\frac{\partial E_x\left[q_{H,x,y}\mid G,y\right]}{\partial\bar{\iota}}$ as follows:

$$\begin{aligned}\frac{\partial E\left[q_{H,x,y}\mid y,G\right]}{\partial\bar{\iota}}&=\sum_{x\in\{-2n,+2n\}}\mathbf{1}_x\left[\frac{\eta}{2}q_{H,x,y}+\frac{\eta\frac{1}{4}k_2}{\Pr(x\mid G)}\left(q_{H,x,y}\right)^2\right]\\&=\frac{\eta}{2}\sum_{x\in\{-2n,+2n\}}\mathbf{1}_x q_{H,x,y}\left[1+\frac{\frac{1}{2}k_2}{\Pr(x\mid G)}q_{H,x,y}\right]\\&=\frac{\eta}{2}\sum_{x\in\{-2n,+2n\}}\mathbf{1}_x\frac{1}{k_1+k_2\frac{\Pr(x|B)}{\Pr(x|G)}}\left[1+\frac{\frac{1}{2}k_2}{\Pr(x\mid G)}\frac{1}{k_1+k_2\frac{\Pr(x|B)}{\Pr(x|G)}}\right]\\&=\frac{\eta}{2}\sum_{x\in\{-2n,+2n\}}\mathbf{1}_x\Pr(x\mid G)\frac{k_1\Pr(x\mid G)+k_2\Pr(x\mid B)+\frac{1}{2}k_2}{[k_1\Pr(x\mid G)+k_2\Pr(x\mid B)]^2}\\&=\frac{\eta}{2}(\Psi_G(-2n,y)-\Psi_G(+2n,y)),\end{aligned}\tag{A15}$$

where, using the fact that $\Pr(x\mid B)=\frac{1}{2}-\Pr(x\mid G)$ for $x\in\{-2n,+2n\}$, we can write

$$\Psi_G(x,y)=\frac{(k_1-k_2)\Pr(x\mid G)^2+k_2\Pr(x\mid G)}{\left[(k_1-k_2)\Pr(x\mid G)+\frac{1}{2}k_2\right]^2}.\tag{A16}$$

Notice that $\Psi_G(x,y)$ is an increasing function of $\Pr(x\mid G)$. Since $\Pr(-2n\mid G)>\Pr(+2n\mid G)$, then $\Psi_G(-2n,y)>\Psi_G(+2n,y)$ for any realization of y. It follows that $\frac{\partial E_x\left[q_{H,x,y}\mid G,y\right]}{\partial\bar{\iota}}>0$ and, as a consequence, that $E\left[q_{H,x,y}\mid G\right]$ increases with $\bar{\iota}$.

We can now focus on $E_x\left[q_{H,x,y}\mid B,y\right]$; we have

$$E_x\left[q_{H,x,y}\mid B,y\right]=\Pr(-2n\mid B)q_{H,-2n,y}+\Pr(+2n\mid B)q_{H,+2n,y}+\frac{1}{2}q_{H,0,y},$$

and, thus,

$$\frac{\partial E_x\left[q_{H,x,y}\mid B,y\right]}{\partial\bar{\iota}}=\frac{\eta}{2}\underbrace{\left[q_{H,+2n,y}-q_{H,-2n,y}\right]}_{<0}+\Pr(-2n\mid B)\underbrace{\frac{\partial q_{H,-2n,y}}{\partial\bar{\iota}}}_{>0}$$

$$+\Pr(+2n\mid B)\underbrace{\frac{\partial q_{H,+2n,y}}{\partial\bar{\iota}}}_{<0}.$$

We can rearrange $\frac{\partial E_x\left[q_{H,x,y}|B,y\right]}{\partial \bar{\iota}}$ as follows:

$$\frac{\partial E\left[q_{H,x,y}\mid y,B\right]}{\partial \bar{\iota}}=\sum_{x\in\{-2n,+2n\}}\mathbf{1}_x\left[-\frac{\eta}{2}q_{H,x,y}+\frac{\Pr(x\mid B)}{\Pr(x\mid G)^2}\frac{\eta}{4}k_2\left(q_{H,x,y}\right)^2\right] \tag{A17}$$

$$=\frac{\eta}{2}\sum_{x\in\{-2n,+2n\}}\mathbf{1}_x q_{H,x,y}\left[-1+\frac{\Pr(x\mid B)}{\Pr(x\mid G)^2}\frac{1}{2}k_2 q_{H,x,y}\right]$$

$$=\frac{\eta}{2}\sum_{x\in\{-2n,+2n\}}\mathbf{1}_x\frac{1}{k_1+k_2\frac{\Pr(x|B)}{\Pr(x|G)}}\left[-1+\frac{\Pr(x\mid B)\frac{1}{2}k_2}{\Pr(x\mid G)^2}\frac{1}{k_1+k_2\frac{\Pr(x|B)}{\Pr(x|G)}}\right]$$

$$=\frac{\eta}{2}\sum_{x\in\{-2n,+2n\}}\mathbf{1}_x\frac{-k_1\Pr(x\mid G)^2-k_2\Pr(x\mid B)\left[\Pr(x\mid G)-\frac{1}{2}\right]}{[k_1\Pr(x\mid G)+k_2\Pr(x\mid B)]^2}$$

$$=\frac{\eta}{2}\left(\Psi_B(-2n,y)-\Psi_B(+2n,y)\right)$$

where

$$\Psi_B(x,y)=\frac{-k_1\Pr(x\mid G)^2+k_2\left[\frac{1}{2}-\Pr(x\mid G)\right]^2}{\left[(k_1-k_2)\Pr(x\mid G)+\frac{1}{2}k_2\right]^2}. \tag{A18}$$

Notice that $\Psi_B(x,y)$ is a decreasing function of $\Pr(x\mid G)$. Since $\Pr(-2n\mid G)>\Pr(+2n\mid G)$ and, thus, $\Psi_B(-2n,y)<\Psi_B(+2n,y)$, we have $\frac{\partial E\left[q_{H,x,y}|y,B\right]}{\partial \bar{\iota}}<0$ for any realization of y. It follows that $\frac{\partial E_x\left[q_{H,x,y}|y,B\right]}{\partial \bar{\iota}}<0$ and, as a consequence, that $E\left[q_{H,x,y}\mid B\right]$ decreases with $\bar{\iota}$. Therefore, the difference $E_x\left[q_{H,x,y}\mid y,G\right]-E_x\left[q_{H,x,y}\mid y,B\right]$ increases with $\bar{\iota}$ for any realization of y.

A.5 Proof of Proposition 1

Putting together the results in Lemma 3 and Lemma 4, the proof for Proposition 1 is straightforward. For the speculator, we have $\iota^*(\overline{\alpha})\in(\frac{1}{2},1)$ for any $\overline{\alpha}\in[\frac{1}{2},1)$, with $\iota^*(1)=\frac{1}{2}$. For the CRA, we have $\alpha^*(\bar{\iota})\in(\frac{1}{2},1)$ for any $\bar{\iota}\in[\frac{1}{2},1]$. This implies that the two functions in Figure 1 always cross (and do so only once) at some interior level $(\alpha^*,\iota^*)\in\left(\frac{1}{2},1\right)^2$.

A.6 Proof of Lemma 5

The equilibrium pair (α^*,ι^*) satisfies the following system of equations:

$$\frac{\gamma}{2}\left\{E\left[q_{H,x,y}\mid G\right]-E\left[q_{H,x,y}\mid B\right]\right\}-C'(\alpha)=0; \tag{A19}$$

$$\frac{\partial R^s}{\partial \iota}-K'(\iota)=0. \tag{A20}$$

Equation (A19) pins down the equilibrium α for a fixed value of ι, say $\alpha^*(\iota)$. $\frac{\partial\alpha}{\partial\iota}$ denotes the derivative of $\alpha^*(\iota)$ with respect to ι. Applying the implicit function theorem on Equation (A19), we have

$$\frac{\partial\alpha}{\partial\iota}=\frac{\frac{\gamma}{2}\frac{\partial\left(E\left[q_{H,x,y}|G\right]-E\left[q_{H,x,y}|B\right]\right)}{\partial\iota}}{C''(\alpha)-\frac{\gamma}{2}\frac{\partial\left(E\left[q_{H,x,y}|G\right]-E\left[q_{H,x,y}|B\right]\right)}{\partial\alpha}}>0. \tag{A21}$$

Similarly, we can find the derivative of $\alpha^*(\iota)$ with respect to the parameters in the CRA's objective function, that is, γ and ρ_{cra}, where $C(\alpha)=\rho_{cra}c(\alpha)$. We have

$$\frac{\partial\alpha}{\partial\gamma}=\frac{\frac{1}{2}\left(E\left[q_{H,x,y}\mid G\right]-E\left[q_{H,x,y}\mid B\right]\right)}{C''(\alpha)-\frac{\gamma}{2}\frac{\partial\left(E\left[q_{H,x,y}|G\right]-E\left[q_{H,x,y}|B\right]\right)}{\partial\alpha}}>0;$$

$$\frac{\partial\alpha}{\partial\rho_{cra}}=\frac{-c'(\alpha)}{C''(\alpha)-\frac{\gamma}{2}\frac{\partial\left(E\left[q_{H,x,y}|G\right]-E\left[q_{H,x,y}|B\right]\right)}{\partial\alpha}}<0.$$

Equation (A20) pins down the equilibrium value of ι for a fixed value of α, say $\iota^*(\alpha)$. $\frac{\partial\iota}{\partial\alpha}$ denotes the derivative of $\iota^*(\alpha)$ with respect to α. Applying the implicit function theorem on Equation (A20), we have

$$\frac{\partial\iota}{\partial\alpha}=\frac{\frac{\partial R^S}{\partial\iota\partial\alpha}}{K''(\iota)-\frac{\partial R^S}{\partial\iota\partial\iota}}<0. \tag{A22}$$

Similarly, we can find the derivative of $\iota^*(\alpha)$ with respect to the parameters in the speculator's objective function, that is, n and ρ_s, where $K(\iota)=\rho_s k(\iota)$. We have

$$\frac{\partial\iota}{\partial n}=\frac{\frac{\partial R^S}{\partial\iota\partial n}}{K''(\iota)-\frac{\partial R^S}{\partial\iota\partial\iota}}>0; \qquad \frac{\partial\iota}{\partial\rho_s}=\frac{-k'(\iota)}{K''(\iota)-\frac{\partial R^S}{\partial\iota\partial\iota}}<0,$$

since $\frac{\partial R^S}{\partial\iota\partial n}>0$ ($\frac{\partial R^S}{\partial\iota}$ is directly proportional to n).

The equilibrium interaction between the CRA and the speculator is such that when we vary a parameter in the CRA's objective function, say γ in Equation (A19), we have a direct effect on rating accuracy $\frac{d\alpha^*}{d\gamma}$ and an indirect effect on market informativeness $\frac{\partial\iota}{\partial\alpha}\frac{d\alpha^*}{d\gamma}$. The effect of an increase in γ on the equilibrium pair (α^*,ι^*) is then characterized by the following system of equations:

$$\frac{d\alpha^*}{d\gamma}=\frac{\partial\alpha}{\partial\gamma}+\frac{\partial\alpha}{\partial\iota}\frac{d\iota^*}{d\gamma}; \qquad \frac{d\iota^*}{d\gamma}=\frac{\partial\iota}{\partial\alpha}\frac{d\alpha^*}{d\gamma},$$

whose solution is

$$\frac{d\alpha^*}{d\gamma}=\frac{\frac{\partial\alpha}{\partial\gamma}}{1-\frac{\partial\alpha}{\partial\iota}\frac{\partial\iota}{\partial\alpha}}>0; \qquad \frac{d\iota^*}{d\gamma}=\frac{\partial\iota}{\partial\alpha}\frac{d\alpha^*}{d\gamma}<0. \tag{A23}$$

Similarly, the effect of an increase in ρ_{cra} on (α^*,ι^*) is described by

$$\frac{d\alpha^*}{d\rho_{cra}}=\frac{\frac{\partial\alpha}{\partial\rho_{cra}}}{1-\frac{\partial\alpha}{\partial\iota}\frac{\partial\iota}{\partial\alpha}}<0; \qquad \frac{d\iota^*}{d\rho_{cra}}=\frac{\partial\iota}{\partial\alpha}\frac{d\alpha^*}{d\rho_{cra}}>0. \tag{A24}$$

The effect of parameters in the speculator's objective function is similar. The effect of an increase in n on the equilibrium pair (α^*,ι^*) is characterized by the following system of equations:

$$\frac{d\iota^*}{dn}=\frac{\partial\iota}{\partial n}+\frac{d\alpha^*}{dn}\frac{\partial\iota}{\partial\alpha}; \qquad \frac{d\alpha^*}{dn}=\frac{\partial\alpha}{\partial\iota}\frac{d\iota^*}{dn},$$

whose solution is

$$\frac{d\iota^*}{dn}=\frac{\frac{\partial\iota}{\partial n}}{1-\frac{\partial\alpha}{\partial\iota}\frac{\partial\iota}{\partial\alpha}}>0; \qquad \frac{d\alpha^*}{dn}=\frac{\partial\alpha}{\partial\iota}\frac{d\iota^*}{dn}>0. \tag{A25}$$

Similarly, the effect of an increase in ρ_s on (α^*,ι^*) is described by

$$\frac{d\iota^*}{d\rho_s}=\frac{\frac{\partial\iota}{\partial\rho_s}}{1-\frac{\partial\alpha}{\partial\iota}\frac{\partial\iota}{\partial\alpha}}<0; \qquad \frac{d\alpha^*}{d\rho_s}=\frac{\partial\alpha}{\partial\iota}\frac{d\iota^*}{d\rho_s}<0. \tag{A26}$$

Finally, we consider the effect of parameters that are common to both Equations (A19) and (A20). Here, we consider a common cost parameter; that is, we assume $\rho_s=\rho_{cra}=\rho$, and consider the effect of varying ρ on the equilibrium outcomes. In Lemma 10, we consider instead the information sensitivity of the asset Δ^f. The effect of an increase in ρ on (α^*, ι^*) is characterized by the following system of equations:

$$\frac{d\iota^*}{d\rho} = \frac{\partial \iota}{\partial \rho} + \frac{\partial \iota}{\partial \alpha}\frac{d\alpha^*}{d\rho}; \qquad \frac{d\alpha^*}{d\rho} = \frac{\partial \alpha}{\partial \rho} + \frac{\partial \alpha}{\partial \iota}\frac{d\iota^*}{d\rho},$$

whose solution is

$$\frac{d\alpha^*}{d\rho} = \frac{\frac{\partial \alpha}{\partial \rho} + \frac{\partial \iota}{\partial \rho}\frac{\partial \alpha}{\partial \iota}}{1 - \frac{\partial \alpha}{\partial \iota}\frac{\partial \iota}{\partial \alpha}} < 0; \qquad \frac{d\iota^*}{d\rho} = \frac{\frac{\partial \iota}{\partial \rho} + \frac{\partial \alpha}{\partial \rho}\frac{\partial \iota}{\partial \alpha}}{1 - \frac{\partial \alpha}{\partial \iota}\frac{\partial \iota}{\partial \alpha}}. \tag{A27}$$

Notice that while $\frac{d\alpha^*}{d\rho}$ is unambiguously negative, $\frac{d\iota^*}{d\rho}$ may be positive or negative, since $\frac{\partial \iota}{\partial \rho} < 0$ but $\frac{\partial \alpha}{\partial \rho}\frac{\partial \iota}{\partial \alpha} > 0$. This is because the reduction in rating accuracy may compensate for the increased cost of information and incentivize the speculator to acquire more information.

A.7 Proof of Lemma 6

We prove Lemma 6 in two steps. We first prove the lemma under the conjecture that the market for ratings always takes place at time $t=3$. We then derive a condition on the CRA's initial reputation q_0 under which this conjecture is always satisfied in equilibrium.

Taking the derivative of Π^{cra} in Equation (7) with respect to α_1 yields

$$\frac{\partial \Pi^{cra}}{\partial \alpha} = \frac{\gamma}{4}\left\{E\left[\varphi_3\left(q_{H_1,x,y_1}\right)|\theta_1 = G\right] - E\left[\varphi_3\left(q_{H_1,x,y_1}\right)|\theta_1 = B\right]\right\} - C'(\alpha_1). \tag{A28}$$

The equilibrium value α_1^* sets $\frac{\partial \Pi^{cra}}{\partial \alpha}$ in Equation (A28) to zero when evaluated at $\overline{\alpha} = \alpha_1^*$. Using $\mu_3^H = q_3 + (1-q_3)\frac{1}{2}$, we can write $p_3^H = \frac{\Delta_f}{2}q_3 + 1 - \frac{1}{2}(f_G + f_B)$. Given that the time $t=3$ rating fee is $\varphi_3(q_3) = \phi\left(p_3^H - I\right)$, Equation (A28) simplifies to:

$$\frac{\partial \Pi^{cra}}{\partial \alpha} = \frac{\gamma}{2}\left(\phi\frac{\Delta_f}{4}\right)\left\{E\left[q_{H_1,x,y_1}|\theta_1 = G\right] - E\left[q_{H_1,x,y_1}|\theta_1 = B\right]\right\} - C'(\alpha_1). \tag{A29}$$

The derivative of the CRA's profits in Equation (A29) is identical to $\frac{\partial \Pi^{cra}}{\partial \alpha}$ in the main model (Equation (5)), except that in Equation (A29) the difference $E\left[q_{H_1,x,y_1}|\theta_1 = G\right] - E\left[q_{H_1,x,y_1}|\theta_1 = B\right]$ is multiplied by a constant $\phi\frac{\Delta_f}{4}$. Under the conjecture that the market for ratings always takes place at $t=3$, $E\left[\varphi_3\left(q_{H_1,x,y_1}\right)|\theta_1\right]$ is differentiable in both $\bar{\iota}$ and $\overline{\alpha}$. Therefore, the proof of Lemma 4 in Appendix A.4 holds.[52] It follows that the equilibrium value α_1^* has the same properties as the equilibrium rating accuracy α^* of the main model, even though their exact values are different.

We now derive a condition on q_0 such that the market for ratings takes place at time $t=3$ for any realization of the CRA's posterior reputation. The issuer discloses a high rating when $p_3^H - I > 0$, where $p_3^H = \mu_3^H(1-f_G) + \left(1-\mu_3^H\right)(1-f_B)$ and $\mu_3^H = q_3 + (1-q_3)\overline{\alpha}_3$. Since the NPV of a project of quality θ is $v_\theta = 1 - f_\theta - I$, we can write

$$p_3^H - I = [q_3 + (1-q_3)\overline{\alpha}_3]v_G + (1-q_3)(1-\overline{\alpha}_3)v_B.$$

The difference $p_3^H - I$ increases with both q_3 and $\overline{\alpha}_3$. The lowest possible reputation is $q_{H,+2n,F}$, since both $x = +2n$ and $y_1 = F$ convey negative information about the accuracy of the time $t=1$ rating. We can write $q_{H,+2n,F}$ as

$$q_{H,+2n,F} = \left\{1 + \frac{1-q_0}{q_0}\left[\overline{\alpha}_1 + (1-\overline{\alpha}_1)\frac{\eta\bar{\iota} + \frac{1-\eta}{2}}{\eta(1-\bar{\iota}) + \frac{1-\eta}{2}}\frac{f_B}{f_G}\right]\right\}^{-1}. \tag{A30}$$

The reputation $q_{H,+2n,F}$ is smallest when evaluated at $\overline{\alpha}_1 = \frac{1}{2}$, since investors learn the most about the CRA's type when $\overline{\alpha}_1 = \frac{1}{2}$. It follows that $p_3^H - I$ is smallest when evaluated at $\overline{\alpha}_3 = \overline{\alpha}_1 = \frac{1}{2}$ and $q_3 = q_{H,+2n,F}$.

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52 Notice that the proof of Lemma 4 can be easily extended to the case in which $E\left[\varphi_3\left(q_{H_1,x,y_1}\right)|\theta_1\right]$ is nondifferentiable.

Notice that we have $\frac{\partial q_{H,+2n,F}}{\partial q_0}>0$, $\lim_{q_0\to 1} q_{H,+2n,F}=1$ and, thus, $\lim_{q_0\to 1} p_3^H - I = v_G > 0$ for any values of $\overline{\alpha}_3, \overline{\alpha}_1$, and either $\overline{\iota}<1$ and $\eta\in[0,1]$ or $\overline{\iota}=1$ and $\eta<1$. Since in equilibrium we always have $\overline{\iota}<1$, there always exists a value $\overline{q}$ sufficiently close to 1 such that $p_3^H - I>0$ for any $q_0>\overline{q}$. The exact value of $\overline{q}$ is pinned down by the equation $\frac{1}{2}\left[\left(q_{H,+2n,F}+1\right)v_G+\left(1-q_{H,+2n,F}\right)v_B\right]=0$, where $q_{H,+2n,F}$ is evaluated at $\overline{\alpha}_1=\frac{1}{2}$ and $\overline{\iota}=1$ if $\eta<1$ or at $\overline{\alpha}_1=\frac{1}{2}$ and $\overline{\iota}$ such that $\frac{\partial R^S}{\partial \iota}|_{\overline{\alpha}_1=\frac{1}{2}}=K(\overline{\iota})$ if $\eta=1$, where $\frac{\partial R^S}{\partial \iota}$ is the speculator's marginal benefit of precision in Equation (A8).

Finally, notice that, since $q_0>q_{H,+2n,F}$, when $q_0>\overline{q}$ the market for ratings takes place at time $t=1$ for any conjecture $\overline{\alpha}_1$ about rating accuracy in that period.

A.8 Proof of Proposition 2

The expected total surplus in equilibrium is

$$S=\sum_t \gamma_{t=3}\left\{\frac{1}{2}\left[\widetilde{\mu}_t^H v_G+\left(1-\widetilde{\mu}_t^H\right)v_B\right]-(1-q_0)C(\alpha_t)\right\}-\frac{\eta}{2}K(\iota), \tag{A31}$$

where $t\in\{1,3\}$, $\widetilde{\mu}_t^H=q_0+(1-q_0)\alpha_t$, and $\gamma_{t=3}$ takes values γ_S when $t=3$ and 1 otherwise.

Holding α_1 constant, the speculator's precision has only a direct negative effect on S. Therefore, surplus is largest at $\iota=0$. Taking the derivative of S with respect to α_t yields

$$\frac{\partial S}{\partial \alpha_t}=\gamma_{t=3}(1-q_0)\left\{\frac{\Delta^f}{2}-C'(\alpha_t)\right\}. \tag{A32}$$

Since the sign of $\frac{\partial S}{\partial \alpha_t}$ does not depend on $\gamma_{t=3}$, the level of accuracy that maximizes S is the same in both periods, that is, $\alpha_1=\alpha_3=\widehat{\alpha}$. Moreover, since S is a concave function of α_t, $\widehat{\alpha}$ is unique. We have $\widehat{\alpha}=1$ if $\Delta^f\geq 2C'(1)$; otherwise, we have $\widehat{\alpha}\in(\frac{1}{2},1)$ such that $\Delta^f=2C'(\widehat{\alpha})$. All else equal, S increases (decreases) with α_t^* when $\alpha_t^*<\widehat{\alpha}$ ($\alpha_t^*>\widehat{\alpha}$) and always decreases with ι^*. Since $C'(\frac{1}{2})=0$ and $\Delta^f>0$, we have $\widehat{\alpha}>\frac{1}{2}$. As a result, the opportunistic CRA always underinvests in rating accuracy at time $t=3$, since $\alpha_3^*=\frac{1}{2}<\widehat{\alpha}$. The equilibrium value α_1^* solves Equation (A29). Notice that α_1^* approaches $\frac{1}{2}$ when γ approaches 0, so we can always find values of γ and Δ_f such that $\alpha_1^*<\widehat{\alpha}$. Similarly, $\widehat{\alpha}$ approaches $\frac{1}{2}$ when Δ_f approaches 0; so we can find values of γ and Δ_f such that Δ_f approaches 0 but $\Delta_f\gamma$ does not, so that $\alpha_1^*>\widehat{\alpha}$.

Now consider the effect on S of a parameter e in the CRA's or the speculator's objective functions, when S is evaluated at the equilibrium choices α_1^*, α_3^*, and ι^*. Notice that e only affects α_1^*, since we always have $\alpha_3^*=\frac{1}{2}$ in equilibrium. The parameter e may have a direct effect on S, as is the case for the cost parameters ρ_{cra} and ρ_s. There is always an indirect effect on S via the effect of e on α_1^* and ι^*. Taking the derivative of S with respect to e yields

$$\frac{dS}{de}=(1-q_0)\left(\frac{\Delta^f}{2}-C'\left(\alpha_1^*\right)\right)\frac{d\alpha_1^*}{de}-\frac{\eta}{2}K'\left(\iota^*\right)\frac{d\iota^*}{de}+\frac{\partial S}{\partial e},$$

where $\frac{\partial S}{\partial e}$ describes the direct effect of e on S.

Lemma 5 describes the effect of various parameters on $\left(\alpha_1^*,\iota^*\right)$. Let us first consider the parameters of the CRA's objective function, that is, γ and ρ_{cra}. We have $\frac{d\alpha_1^*}{d\gamma}>0$ and $\frac{d\iota^*}{d\gamma}<0$ for γ; $\frac{d\alpha^*}{d\rho_{cra}}<0$, $\frac{d\iota^*}{d\rho_{cra}}>0$, and $\frac{\partial S}{\partial \rho_{cra}}=-(1-q_0)c\left(\alpha_1^*\right)<0$ for ρ_{cra}.[53] Therefore, the effect of an increase in γ on S is always positive when $\alpha_1^*<\widehat{\alpha}$ (since we have $\frac{\Delta^f}{2}-C'\left(\alpha_1^*\right)>0$ in this case). When $\alpha_1^*>\widehat{\alpha}$, the effect on S is ambiguous, since α_1^* moves away from its optimal level $\widehat{\alpha}$ but ι^* moves closer to its optimal level $\iota^*=0$. The same holds for a decrease in ρ_{cra}.

[53] It is worth mentioning that, since $\alpha_3^*=\frac{1}{2}$ and $c(\frac{1}{2})=0$, a change in ρ_{cra} has no effect on the time $t=3$ component of S. The same holds for ρ_s, given that the CDS market only occurs at time $t=1$.

We can now consider the parameters of the speculator's objective function, that is, n and ρ_s. We have $\frac{d\iota^*}{dn} > 0$ and $\frac{d\alpha_1^*}{dn} > 0$ for n; $\frac{d\iota^*}{d\rho_s} < 0$, $\frac{d\alpha_1^*}{d\rho_s} < 0$, and $\frac{\partial S}{\partial \rho_s} = -\frac{\eta}{2} k(\iota^*) < 0$ for ρ_s. Since $K'(\iota) = \rho_s k'(\iota)$, when ρ_s is sufficiently small, the direct effect on S of an increase in ι is negligible. Therefore, when $\alpha_1^* < \widehat{\alpha}$ and ρ_s is sufficiently small, an increase in n or a decrease in ρ_s have a positive effect on S. Otherwise, an increase in n decreases S, while the effect of ρ_s is ambiguous, since the direct effect of a decrease in ρ_s is positive.

References

Ahern, K. R., and D. Sosyura. 2015. Rumor has it: Sensationalism in financial media. *Review of Financial Studies* 28:2050–93.

Alp, A. 2013. Structural shifts in credit rating standards. *Journal of Finance* 68:2435–70.

Badoer, D. C., and C. Demiroglu. 2019. The relevance of credit ratings in transparent bond markets. *Review of Financial Studies* 32:42–74.

Baghai, R. P., and B. Becker. 2020. Reputations and credit ratings: evidence from commercial mortgage-backed securities. *Journal of Financial Economics* 135:425–44.

Baghai, R. P., H. Servaes, and A. Tamayo. 2014. Have rating agencies become more conservative? Implications for capital structure and debt pricing. *Journal of Finance* 69:1961–2005.

Bakke, T.-E., and T. M. Whited. 2010. Which firms follow the market? An analysis of corporate investment decisions. *Review of Financial Studies* 23:1941–80.

Bao, J., M. O'Hara, and X. A. Zhou. 2018. The Volcker Rule and corporate bond market making in times of stress. *Journal of Financial Economics* 130:95–113.

Bar-Isaac, H., and J. Shapiro. 2011. Credit ratings accuracy and analyst incentives. *American Economic Review* 101:120–24.

———. 2013. Ratings quality over the business cycle. *Journal of Financial Economics* 108:62–78.

Becker, B., and T. Milbourn. 2011. How did increased competition affect credit ratings? *Journal of Financial Economics* 101:493–514.

Bhattacharya, N., Y. J. Cho, and J. B. Kim. 2018. Leveling the playing field between large and small institutions: evidence from the SEC's XBRL mandate. *Accounting Review* 93:51–71.

Bond, P., A. Edmans, and I. Goldstein. 2012. The real effects of financial markets. *Annual Review of Financial Economics* 4:339–60.

Bond, P., and I. Goldstein. 2015. Government intervention and information aggregation by prices. *Journal of Finance* 70:2777–2812.

Bonsall, S. B., J. R. Green, and K. A. Muller. 2018. Are credit ratings more rigorous for widely covered firms? *Accounting Review* 93:61–94.

Chava, S., R. Ganduri, and C. Ornthanalai. 2019. Do credit default swaps mitigate the impact of credit rating downgrades? *Review of Finance* 23:471–511.

Chen, Q., I. Goldstein, and W. Jiang. 2007. Price informativeness and investment sensitivity to stock price. *Review of Financial Studies* 20:619–50.

Dannhauser, C. D. 2017. The impact of innovation: Evidence from corporate bond exchange-traded funds (ETFs). *Journal of Financial Economics* 125:537–60.

Diamond, D. W. 1985. Optimal release of information by firms. *Journal of Finance* 40:1071–94.

Diamond, D. W., and R. E. Verrecchia. 1991. Disclosure, liquidity, and the cost of capital. *Journal of Finance* 46:1325–59.

Dick-Nielsen, J., and M. Rossi. 2019. The cost of immediacy for corporate bonds. *Review of Financial Studies* 32:1–41.

Dimitrov, V., D. Palia, and L. Tang. 2015. Impact of the Dodd-Frank act on credit ratings. *Journal of Financial Economics* 115:505–20.

Dong, Y., O. Z. Li, Y. Lin, and C. Ni. 2016. Does information-processing cost affect firm-specific information acquisition? Evidence from XBRL adoption. *Journal of Financial and Quantitative Analysis* 51:435–62.

European Commission. 2015. Study on the feasibility of alternatives to credit ratings. Available at https://ec.europa.eu/info/system/files/alternatives-to-credit-rating-study-01122015_en.pdf.

Faure-Grimaud, A., and D. Gromb. 2004. Public trading and private incentives. *Review of Financial Studies* 17:985–1014.

Ferreira, D., M. A. Ferreira, and C. C. Raposo. 2011. Board structure and price informativeness. *Journal of Financial Economics* 99:523–45.

Fong, K. Y., H. G. Hong, M. T. Kacperczyk, and J. D. Kubik. 2014. Do security analysts discipline credit rating agencies? In *AFA 2013 San Diego Meetings Paper*.

Fulghieri, P., G. Strobl, and H. Xia. 2014. The economics of solicited and unsolicited credit ratings. *Review of Financial Studies* 27:484–518.

Gao, M., and J. Huang. 2020. Informing the market: The effect of modern information technologies on information production. *Review of Financial Studies* 33:1367–1411.

Gao, P., and P. J. Liang. 2013. Informational feedback, adverse selection, and optimal disclosure policy. *Journal of Accounting Research* 51:1133–58.

Gentzkow, M., and J. M. Shapiro. 2006. Media bias and reputation. *Journal of Political Economy* 114:280–316.

Goldman, E., J. Martel, and J. Schneemeier. Forthcoming. A theory of financial media. *Journal of Financial Economics*.

Goldstein, I., and C. Huang. 2020. Credit rating inflation and firms' investments. *Journal of Finance* 75:2929–72.

Goldstein, I., and L. Yang. 2017. Information disclosure in financial markets. *Annual Review of Financial Economics* 9:101–25.

———. 2019. Good disclosure, bad disclosure. *Journal of Financial Economics* 131:118–38.

Goldstein, I., S. Yang, and L. Zuo. 2020. The real effects of modern information technologies. Technical report, National Bureau of Economic Research.

Gopalan, R., Y. Gopalan, and K. Koharki. 2017. Market information and rating agency catering. Working Paper, Washington University.

Han, B., Y. Tang, and L. Yang. 2016. Public information and uninformed trading: Implications for market liquidity and price efficiency. *Journal of Economic Theory* 163:604–43.

Holmström, B., and J. Tirole. 1993. Market liquidity and performance monitoring. *Journal of Political Economy* 101:678–709.

Holzer, J. 2011. House panel votes to free raters from ABS liability. *Wall Street Journal*, July 20. https://www.wsj.com/articles/SB10001424053111903461104576458393767069116.

Jayaraman, S., and J. S. Wu. 2019. Is silence golden? Real effects of mandatory disclosure. *Review of Financial Studies* 32:2225–59.

Kashyap, A. K., and N. Kovrijnykh. 2016. Who should pay for credit ratings and how? *Review of Financial Studies* 29:420–56.

Kedia, S., S. Rajgopal, and X. A. Zhou. 2017. Large shareholders and credit ratings. *Journal of Financial Economics* 124:632–53.

Kempf, E., and M. Tsoutsoura. 2021. Partisan professionals: Evidence from credit rating analysts. *Journal of Finance* 76:2805–56.

Kim, O., and R. E. Verrecchia. 1991. Market reaction to anticipated announcements. *Journal of Financial Economics* 30:273–309.

Kisgen, D. J. 2006. Credit ratings and capital structure. *Journal of Finance* 61:1035–72.

Kisgen, D. J., and P. E. Strahan. 2010. Do regulations based on credit ratings affect a firm's cost of capital? *Review of Financial Studies* 23:4324–47.

Kreps, D. M., and R. Wilson. 1982. Reputation and imperfect information. *Journal of Economic Theory* 27:253–79.

Kronlund, M. 2020. Do bond issuers shop for favorable credit ratings? *Management Science* 66:5944–68.

Kyle, A. S. 1985. Continuous auctions and insider trading. *Econometrica: Journal of the Econometric Society* 53:1315–35.

Laffont, J.-J., and J. Tirole. 1993. *A theory of incentives in procurement and regulation*. Cambridge: MIT Press.

Lewis, R., and M. Schwert. 2018. The effects of transparency on trading profits and price informativeness: Evidence from corporate bonds. Working Paper, University of Colorado, Boulder.

Mathis, J., J. McAndrews, and J.-C. Rochet. 2009. Rating the raters: Are reputation concerns powerful enough to discipline rating agencies? *Journal of Monetary Economics* 56:657–74.

Milgrom, P., and J. Roberts. 1982. Predation, reputation, and entry deterrence. *Journal of Economic Theory* 27:280–312.

Piccolo, A. 2021. Credit ratings and competition. Working Paper, Indiana University.

Strausz, R. 2005. Honest certification and the threat of capture. *International Journal of Industrial Organization* 23:45–62.

Tetlock, P. C. 2014. Information transmission in finance. *Annual Review of Financial Economics* 6:365–84.

Verrecchia, R. E. 1982. The use of mathematical models in financial accounting. *Journal of Accounting Research* 20:1–42.

White, L. J. 2010. Markets: The credit rating agencies. *Journal of Economic Perspectives* 24:211–26.