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Abstract

We study strategic interactions in markets where firms sell to consumers both directly and via a competitive channel (CC), such as a price comparison website or marketplace, where multiple sellers' offers are visible at once. We ask how a CC's size influences market outcomes. A bigger CC means more consumers compare prices, increasing *within*-channel competition. However, we show such seemingly pro-competitive developments can raise prices and harm consumers by weakening *between*-channel competition. We also use the model to study relevant active policy issues including price clauses, integrated ownership structures, and access to consumers' purchase data. (*JEL: D43, D83, L11, M3*)

Keywords: sales channels, digital markets, price competition, most-favored-nation clauses, vertical integration, non-resolicitation clauses.

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1 Introduction

Many goods and services can be purchased either through a seller’s *direct channel*, e.g., their own store or website, or through a *competitive channel* where multiple sellers’ offers can be considered simultaneously. Competitive channels (CCs) take a variety of forms: In the travel and financial services industries, price comparison websites allow consumers to view prices from many sellers at once. For durable goods, online marketplaces such as Amazon Marketplace or eBay are commonly used by sellers as an additional sales channel, and facilitate comparison for consumers. Offline, marketplaces such as shopping malls allow consumers to compare multiple sellers’ offers in one location.

Competitive channels facilitate price comparison and so strengthen competition between sellers. But the operators of competitive sales channels are economic actors too, and their incentives need not align with the interests of consumers.¹ Such concerns have led competition authorities in various jurisdictions to more closely scrutinize industries where CCs play a significant role.² This paper contributes to the ongoing academic and policy debates by studying the implications of competition between, and within, sales channels. We recognize the strategic agency of the competitive channel and examine when and how the direct sales channel serves to discipline a CC’s behavior and vice-versa.

The relative importance of direct and competitive channels has varied across markets and over time. Indeed, technology has facilitated the emergence and rapid growth of new kinds of competitive channels. Moreover, the operators of such channels often invest heavily in marketing, leading many to become household names in their own right, e.g., Expedia in the US, Compare The Market in the UK, or Amazon Marketplace globally.

Unlike existing work, we introduce a model that allows both competitive and direct channels to vary in their market power, or “size”. We then use our model to study the effects of variations in the relative size of the competitive channel and show how this is an important determinant of market prices and consumer surplus. For example, growth in the competitive channel implies that more consumers compare more prices. Although this may seem pro-competitive, we show that it can affect price competition in ways that mean consumers end up worse-off because between-channel competition is weakened.

Equipped with the model, we study practices that have attracted policy scrutiny in markets with competing sales channels including “most-favored nation” clauses (MFNs), cross-channel integration, and “non-resolicitation” clauses (NRCs). MFNs state that a seller’s direct price may not be lower than the price it lists on the CC. Cross-channel integration is when a CC and a seller are both owned by the same parent company. NRCs dictate which channel is allowed to utilize consumers’ purchase data to solicit future sales.

¹Some recent work highlights practices that reduce the benefits of CCs, such as price parity clauses (e.g., Edelman and Wright, 2015; Johnson, 2017) and high commission levels (e.g., Ronayne, 2019).

²For examples, see work by, or commissioned by, the OECD (Hviid, 2015), the EU (EU Competition Authorities, 2016), and the UK’s Competition and Markets Authority (CMA, 2017c).

We contribute a novel theory of harm of MFNs, and to our knowledge are the first to examine integration and NRCs in these markets.

To be more precise, the actors in our baseline model are sellers, a competitive sales channel, and a mass of consumers. Sellers set prices: one for consumers who buy direct and, if they wish to, another for consumers who buy via the competitive channel. The competitive channel sets a fee that is paid by a seller each time it makes a sale through the CC. We adopt a clearinghouse framework à la Varian (1980). As such, some of the consumers are savvy “shoppers” who buy at the lowest price available anywhere in the market. The remainder of the consumers are “captives” and check only one sales channel, either a seller’s direct channel or the competitive channel, and buy at the best price they see there. Captives can be thought of as being directed by brand marketing; facing high search or information frictions; or simply being relatively inactive or naive. The number of captives a seller or CC has is a measure of its market power or size. We refer collectively to the masses of captives and shoppers as the “market composition”.

We first provide a complete characterization of the equilibrium prices and fees for any given market composition. Equilibrium falls into one of two qualitatively-distinct regimes: low-fee and high-fee. In the case of a relatively small CC (i.e., a CC with few captives), the CC sets a *low fee* and hosts the lowest prices in the market. Shoppers buy through the competitive channel, while sellers set a high direct price to exploit their captive consumers. By contrast, a relatively large CC demands a *high fee*, exploiting its position as bottleneck provider of access to its captive consumers. To avoid paying the high fee, sellers compete for shoppers via direct prices, undercutting the prices on the competitive channel. In this regime it is direct prices that are the lowest in the market.

We then proceed to analyze the effects of market composition on equilibrium prices and consumer surplus. As the competitive channel becomes larger, we observe two effects. The direct effect is that more consumers are subjected to the frictionless competitive channel where prices are easily compared, which tends to increase consumer surplus. But there is a countervailing effect that arises through firms’ endogenous competitive responses. That is, when a seller has relatively few captive consumers it is more focused on attracting price-sensitive shoppers, and therefore is more willing to undercut the prices on the CC with its direct price. It eventually becomes so difficult for the CC to deter undercutting that it gives up and switches to the high-fee regime, where prices are higher, and focuses on selling to its own captive base. This means that seemingly pro-competitive changes in market composition that result in more consumers comparing more prices (such as uninformed consumers becoming informed or adopting a price comparison technology) can increase prices, harming consumers.

In the baseline model, some consumers are exogenously captive to each channel. This “reduced form” approach enables us to showcase the model’s central mechanisms. We also explore extensions that explicitly micro-found this pattern of behavior. Firstly (in

Section 5), we model market composition as resulting from consumer information frictions: some consumers are initially aware of only the CC; rather than being totally captive, they face a search cost to go on to visit sellers directly. For any level of search friction, including zero, our substantive results survive. Secondly (in Section 7.1), we model the market composition as the output of long-run costly marketing decisions by firms (both sellers and the CC) and find that although either a low or high fee regime may occur, the high fee regime results for a larger set of cost functions.

Regarding policy, we first find that MFNs weaken the undercutting incentives of sellers, leading to lower consumer surplus for any market composition. We are therefore consistent with the negative results reported in the literature on MFNs, but arrive there via a novel theory of harm: the trade-off of within vs. between channel competition. Second, we show that integration can, depending on the relative size of the CC, either reduce the competitive pressure from the direct channel (lowering consumer surplus), or increase the intensity of competition for shoppers (raising consumer surplus). Third, we find that certain types of NRCs can increase or decrease consumer surplus, depending on whether the competitive channel is relatively large or small.

The rest of the paper proceeds as follows: Section 1.1 discusses related literature and documents our contribution, Section 2 introduces the model, Section 3 characterizes equilibrium, Section 4 demonstrates the importance of market composition for equilibrium outcomes, Section 5 explicitly models search frictions, Section 6 studies policy, Section 7 contains extensions and robustness checks, and Section 8 concludes.

1.1 Related literature and contribution

Our model builds on those of the established literature on price clearinghouses. In such models, a fraction of consumers only consider one firm’s price, while the remainder use a “clearinghouse” and compare firms’ prices. Foundational papers in this literature include Rosenthal (1980), Shilony (1977) and Varian (1980); a survey can be found in Baye et al. (2006). The sustained popularity of the paradigm has led to many extensions and applications including: consumer search (e.g., Janssen and Moraga-González, 2004; Stahl, 1989); product substitutability (e.g., Inderst, 2002); competition with boundedly-rational consumers (e.g., Heidhues et al., 2018; Piccione and Spiegler, 2012); price discrimination (e.g., Armstrong and Vickers, 2019; Fabra and Reguant, 2018); and generalizations allowing for firm heterogeneity (e.g., Baye et al., 1992; Moraga-González and Wildenbeest, 2012; Shelegia and Wilson, 2017) and richer patterns of consumer comparisons (e.g., Johnen and Ronayne, 2020; Armstrong and Vickers, 2018). Unlike these papers, we consider a market with competitive channel that is a profit-maximizer in its own right. We propose a new and tractable way to model competition not only within, but also between different sales channels, enabling several contributions relative to the literature.

First, we show how inter-channel competition affects equilibrium prices for a given market composition. Second, we study the welfare effects of a change in market composition (with a particular focus on the effects of CC whose importance grows over time). Third, we endogenize consumers' decision to stick with the CC or undertake costly search, and show how equilibrium fees and prices respond to this source of discipline.

In a symmetric setup with a monopolist intermediary, Baye and Morgan (2001) allow a more active role for the clearinghouse. They model a two-sided environment in which the clearinghouse sets fixed advertising and subscription fees in order to get both sellers and consumers on board, respectively. We provide a new model with features that reflect empirical characteristics of many modern competitive channels: (i) CCs that charge per-unit commissions rather than lump-sum fees, (ii) sellers that can set different prices for direct and CC sales, and (iii) consumers that only consider prices on the competitive channel. This allows us to study a new set of strategic considerations: whereas Baye and Morgan are mostly concerned with the platform's problem of getting consumers and firms on board, the main trade-offs in our model revolve around simultaneous competition between and within the different sales channels available to firms.

Some papers study competition within platform-based sales channels, but assume these are the only sales channels (e.g., Belleflamme and Peitz, 2010; Boik and Corts, 2016; Hagiu, 2009; Karle et al., 2019).³ In contrast, we allow firms to sell through multiple channels and study the impact of market composition on equilibrium prices and consumer surplus.

Our first policy analysis concerns most-favored-nation (MFN) clauses. Others studies of MFNs include Boik and Corts (2016), Calzada et al. (2019), Edelman and Wright (2015), Foros et al. (2017), Johansen and Vergé (2016), and Johnson (2017)). The most related treatment of MFNs is Wang and Wright (2019), which like us considers the interaction of MFNs with free-riding behavior by firms (with heterogeneous products).

Although MFNs have been widely-studied, we use our framework to highlight a novel theory of harm: as well as allowing a CC to increase its fee at the margin, an MFN can trigger an anti-competitive change in equilibrium regime, reducing consumers' access to information about lower prices. We then proceed to use our framework to study other policy issues that have not been considered in the literature, including cross-channel integration and non-resolicitation clauses, and find that inter-channel competition and market composition, both novel features of our framework, play a key role.

Our work is also distinct from classical studies of vertical relations between firms. Specifically, our model involves both vertical and horizontal dimensions to competition because, as well as serving as an essential gatekeeper for access to some consumers, the

³Another recent paper that allows competition between sales channels is Shen and Wright (2019), who ask why sellers do not always undercut intermediaries to ensure sales take place direct. Their answer is that, when sellers set the fee paid to intermediaries, it is cheaper to attract consumers by raising their fee than by lowering their price. In contrast, we study settings where fees are set by the intermediary and show how undercutting incentives depend on the market composition.

CC competes with sellers' direct channels. In addition, a CC is a technology enabling comparison between sellers and so by its nature directly impacts competitive outcomes.

Our work is also related to a literature on the coexistence of retailers and direct sales by a manufacturer (e.g., Arya et al., 2007; Chiang et al., 2003; Kumar and Ruan, 2006). Unlike these papers, we focus on cases where the seller sets the price for both direct and intermediated sales, and introduce a model that allows us to study the relative size of the two channels and the implications for competitive outcomes.

2 Model

Two sellers produce a homogeneous good at a common constant marginal cost, normalized (without further loss of generality) to zero. Sales can be made both directly and through a competitive channel. Sellers are indexed $i = 1, 2$, and the competitive channel, $i = 0$. The CC chooses a commission fee, $c \geq 0$, which a seller must pay for each sale that takes place via the CC. Given c , sellers choose a direct price, $p_i^d \geq 0$, and a price to list on the CC, $p_i \geq 0$. Sellers may choose not to list on the CC, which in the baseline model is formally equivalent to listing a price above all consumers' willingness to pay.

There is a mass of consumers of measure $\mu + \lambda_0 + \lambda_1 + \lambda_2$ who wish to buy one unit of the good and have willingness to pay $v > 0$. There are two types of consumer: shoppers and captives. Shoppers are of mass $\mu > 0$. They are informed of all prices, buying at the lowest price available. Sellers each have a mass, $\lambda_i > 0$, of captive consumers who shop directly i.e., on the seller's website or at its physical store. Without loss of generality, we index the sellers such that $\lambda_1 \leq \lambda_2$. The CC also has a mass of captive consumers, $\lambda_0 > 0$, who shop exclusively via the competitive channel, buying at the lowest price listed there. If there is a tie for the lowest price where such prices are either all direct prices or all available through the CC, then shoppers (and, in the latter case, CC captives also) buy at the lowest price, from one of the tied sellers at random. If there is a tie in the lowest price where some of these prices are direct and some are via the CC, shoppers complete the purchase from one of the CC prices with probability $r_0 \in [0, 1]$, and one of the direct prices with $1 - r_0$.⁴ We refer to $\boldsymbol{\lambda} = (\lambda_0, \lambda_1, \lambda_2, \mu)$ as the market composition. The solution concept is subgame-perfect Nash equilibrium. The game's timing is as follows:

- $t = 1$ CC sets commission fee c ,
- $t = 2$ Sellers observe c and set prices p_i^d and p_i ,
- $t = 3$ Consumers shop.

⁴The "between-channel" tie-breaking rule, r_0 , does not appear in previous work where each seller has only one sales channel. Instead of fixing some value for r_0 , we solve for it as part of equilibrium (à la Simon and Zame, 1990). In some subgames, equilibrium only exists for some values of r_0 . However, the on-path equilibrium strategies of the whole game hold for any $r_0 \in [0, 1]$.

2.1 Discussion of model assumptions

One key and novel assumption is that there exist consumers who are captive to the competitive channel. This constitutes our measure of market power, or size, of the CC. Because we measure the size of sellers similarly (by the number of consumers captive to their direct channel) we have a commensurate scale by which to examine the balance of power in markets where both actors are present. In addition, the assumption that there are consumers who only visit the competitive channel is consistent with empirical experience. In the context of price comparison websites (PCWs), a survey commissioned by the UK competition authority (CMA, 2017a) found that 58% of users knew which PCW they wanted to use and went straight there. Moreover, 30% of PCW users consulted only one PCW and no other source of price quotes, and among the most commonly cited reasons for not shopping around is “brand loyalty”. Elsewhere, online marketplaces also appear to enjoy high degrees of captivity. For example, it has been reported that in some categories as much as 77% of consumers conduct their whole search experience through Amazon.^{5,6} Our baseline analysis takes a reduced form approach, assuming that such captive consumers exist. We offer two micro-foundations for this in Sections 5 and 7.1.

We allow firms to set $p_i \neq p_i^d$. Price discrimination across channels is common in many markets. For example, Hunold et al. (2020) report that over 2016-17 about 25% of hotel stays were cheaper via the hotel’s own website (the direct channel) than via Booking.com and Expedia (the competitive channel), while a European Commission investigation (EU Competition Authorities, 2016) found that approximately 40% of surveyed hotels had undercut the CC-listed price with their direct price. In Section 6.1 we analyze most-favored nation clauses, which CCs sometimes use to restrict the direct price of the seller.

We conduct our main analysis with $n = 2$ sellers because this facilitates the cleanest and simplest exposition of the forces at play in our model. However, analogous equilibria with equivalent payoffs exist in the game with $n > 2$ sellers; details are in Appendix A.⁷ The only time we require $n > 2$ is when we study most-favoured nation clauses in Section 6.1, where the assumption helps to rule-out some pathological cases.

Our assumption that the CC uses per-sale fees is consistent with modern practice in the industry (see Section 7.2 for examples and discussion). In Section 7.2 we allow the CC to use a combination of per-unit and ad valorem fees, and in Section 7.3 we allow it to set different fees for each seller. Neither modification to the model changes the substantive results of the baseline analysis.

⁵Hitwise (2017), “Amazon vs. google product searches: How do they stack up?” *Hitwise*, April 24. http://www.hitwise.com/blog/2017/04/amazon/?lang=1&bis_prd=1.

⁶The assumption that there are consumers who consider one seller is more common e.g., Varian (1980). As an example of evidence consistent with this assumption, see De los Santos et al. (2012) that finds 75% of consumers in the online market for books visit only one store during their search process.

⁷As in Baye et al. (1992), there exist a multiplicity of payoff equivalent equilibria when $n > 2$.

3 Equilibrium

3.1 Pricing subgame

We begin by studying the best-responses of sellers in stage $t = 2$ for a given choice of c by the competitive channel. In equilibrium, competition à la Bertrand in prices on the frictionless CC implies that sellers make zero profits from any sales made there.

Lemma 1 (CC pricing). *In any equilibrium, $\min_i p_i = c$.*

Proofs are in the Appendix. Lemma 1 means that every equilibrium of the subgame starting at $t = 2$ is pay-off equivalent to one with $p_1 = p_2 = c$, all else equal. We therefore continue by taking $p_1 = p_2 = c$ as given.

Now consider the sellers' choice of direct price. Suppose that the lowest price in the market, at which the shoppers buy, is on the competitive channel. This implies that sellers serve only captive consumers through their direct channel and, therefore, that $p_i^d = v$ with corresponding profit $\pi_i = v\lambda_i$. The best deviation for the seller would be to set p_i^d just low enough to induce shoppers to buy direct. But this requires the seller to undercut not only its rival's direct price, but also the prices listed on the CC, i.e., to set $p_i^d \leq c$. The best such deviation yields profit $c(\lambda_i + \mu)$ and is, therefore, not profitable if

$$c \leq \underline{c}_i \equiv \frac{v\lambda_i}{\lambda_i + \mu}. \quad (1)$$

Seller 1 finds such a deviation profitable for lower levels of c than seller 2, i.e., $\underline{c}_1 \leq \underline{c}_2$, because seller 1 has fewer captives and therefore loses less from a reduction in its direct price. If (1) is satisfied for both sellers then the unique equilibrium behavior is for sellers to set the monopoly price on their direct channel and allow the CC to serve the shoppers.

Lemma 2 (Direct pricing 1). *Suppose $0 \leq c \leq \underline{c}_1$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, $p_1^d = p_2^d = v$ and any $r_0 \in [0, 1]$. Profits are $\pi_0 = c(\lambda_0 + \mu)$, $\pi_1 = v\lambda_1$, $\pi_2 = v\lambda_2$. When $0 \leq c < \underline{c}_1$, there are no other equilibria.*

Now suppose $\underline{c}_1 < c < \underline{c}_2$. In this range, seller 1 finds it worthwhile to undercut the CC prices in order to attract shoppers: the resulting increase in demand more than compensates it for foregone monopoly rents on its captive consumers. Seller 2, on the other hand, earns more from monopoly pricing on its captives and is unwilling to undercut CC prices. This gives rise to a subgame equilibrium in which sellers pursue asymmetric strategies, with shoppers buying directly from seller 1.

Lemma 3 (Direct pricing 2). *Suppose $\underline{c}_1 \leq c \leq \underline{c}_2$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, $p_1^d = c$, $p_2^d = v$ and $r_0 = 0$. Profits are $\pi_0 = c\lambda_0$, $\pi_1 = c(\lambda_1 + \mu)$, and $\pi_2 = v\lambda_2$. When $\underline{c}_1 < c \leq \underline{c}_2$, this is the unique equilibrium.*

For $c > \underline{c}_2$, both sellers prefer to undercut the CC and fight for shoppers using direct prices. Because more than one seller now uses a single variable (direct price) to trade off competition for shoppers against sure sales to captives, equilibrium seller strategies are mixed. The strategies, which must take the CC's fee into account, are given by Lemma 4. Here shoppers buy directly from whichever seller charges the lowest direct price.

Lemma 4 (Direct pricing 3). *Suppose $\underline{c}_2 < c \leq v$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, p_1^d and p_2^d mixed over supports $[\underline{p}, c]$ and $[\underline{p}, c] \cup v$ respectively via the strategies*

$$F_1(p) = \begin{cases} \frac{\mu p - \lambda_2(v-p)}{\mu p} & \text{for } p \in [\underline{p}, c] \\ 1 & \text{for } p \geq c, \end{cases} \quad (2)$$

$$F_2(p) = \begin{cases} \frac{\mu p - \lambda_2(v-p)}{\mu p} \frac{\mu + \lambda_1}{\mu + \lambda_2} & \text{for } p \in [\underline{p}, c] \\ \frac{\mu c - \lambda_2(v-c)}{\mu c} \frac{\mu + \lambda_1}{\mu + \lambda_2} & \text{for } p \in [c, v] \\ 1 & \text{for } p \geq v, \end{cases} \quad (3)$$

where $\underline{p} = \frac{v\lambda_2}{\lambda_2 + \mu}$ and $r_0 = 0$. When $c = v$, any $r_0 \in [0, 1]$ can be supported. Profits are $\pi_0 = c\lambda_0$, $\pi_1 = \frac{v\lambda_2(\lambda_1 + \mu)}{\lambda_2 + \mu}$, and $\pi_2 = v\lambda_2$. When $\lambda_1 < \lambda_2$ these are the unique equilibrium pricing strategies.⁸

Lemmas 2–4 characterize equilibrium when $n = 2$. When $n > 2$ and $\lambda_1 \leq \dots \leq \lambda_n$, there are analogous equilibria of these subgames which leave seller 1 and 2's strategies unchanged while each $i > 2$ plays $p_i = c$, $p_i^d = v$. The profits of 0, 1, and 2 are identical to that with $n = 2$, as is consumer surplus (because the extra $\lambda_3 + \dots + \lambda_n$ consumers each have their surplus fully extracted).

3.2 Competitive channel fee-setting

Now we solve the game starting at $t = 1$. Accordingly, consider the incentives of the competitive channel when it chooses its fee level, c . For $c \in (0, v]$, the CC makes positive profit from the fees paid by sellers for purchases made by its λ_0 captives. In addition, when c is in the low-range of Lemma 2, direct prices exceed those listed on the CC so shoppers also buy through the competitive channel. For higher levels of c , at least one seller sets a direct price lower than all prices on the competitive channel, leaving the CC selling only to its captives. The CC therefore faces a choice between (i) a *low-fee regime* where it facilitates sales to shoppers and its captives; and (ii) a *high-fee regime*, defined as a situation where it facilitates sales only to its captives. By Lemmas 2–4, the highest c such that the CC sells to shoppers and its captives is \underline{c}_1 , whereas the highest c such that

⁸See the Web Appendix for the case of $\lambda_1 = \lambda_2$, where equilibrium pricing can be slightly different.

it sells only to its captives is v . Hence the CC prefers (i) to (ii) if and only if

$$\frac{v\lambda_1}{\lambda_1 + \mu}(\lambda_0 + \mu) \geq v\lambda_0 \Leftrightarrow \lambda_0 \leq \lambda_1.$$

Our model reveals that a comparison of channel size is central to a competitive channel's trade-off. If the CC is relatively large, it chooses to set a high fee and generate profit through sales to its captives. When it is relatively small, it sets a low fee in order to capture shoppers. This reflects the ways that the market composition affects the CC's profits. A higher λ_1 increases \underline{c}_1 and hence profits for the CC in the low-fee regime. On the other hand, a higher λ_0 increases profits in both regimes, but increases profit by more in the high-fee regime precisely because that is where the marginal revenue is higher. In line with this prediction, analysis in the insurance industry by the UK's competition authority finds that larger sellers tend to be associated with lower CC commissions, while commissions tend to be higher for bigger CCs (CMA, 2017b). Proposition 1 formalizes these forces and states the equilibrium.

Proposition 1 (Equilibrium). *When $\lambda_0 < \lambda_1$, the low-fee equilibrium results: the competitive channel sets $c = \underline{c}_1$ and sellers price in accordance with Lemma 2. When $\lambda_0 > \lambda_1$, the high-fee equilibrium results: the competitive channel sets $c = v$ and sellers price in accordance with Lemma 4. When $\lambda_0 = \lambda_1$, both equilibria exist.*

4 Analysis

Proposition 1 shows market composition to be an important determinant of equilibrium outcomes. In reality, composition varies both across markets and over time, with the reach of competitive channels growing in various markets. For example, UK consumers are around twice as likely to rely on a price comparison website when buying motor insurance than when buying home broadband. Over time, the sites have grown to the point that 85% of consumers have used such a site (CMA, 2017a). In this section we consider the effect on prices and consumer surplus of a change in market composition from $\boldsymbol{\lambda} = (\lambda_0, \lambda_1, \lambda_2, \mu)$ to some other $\boldsymbol{\lambda}' = (\lambda'_0, \lambda'_1, \lambda'_2, \mu') > 0$. Because much of this section is concerned with the effects of shifting consumers between groups, it is convenient to use the notation $\Delta(\alpha, \beta) \in \mathbb{R}$ to denote the size of a reduction in α and increase in β such that $\alpha + \beta$ remains constant, ceteris paribus. For example, $\Delta(\lambda_1, \mu) = 1$ denotes the exercise of informing mass 1 of firm 1's captive consumers of all prices, turning them into shoppers.

We first show how the presence of competing sales channels changes the standard intuition from clearinghouse models of price competition, before analyzing the effects of a growing competitive channel.

Key to these analyses is the surplus derived by consumers across the different fee regimes. Given the optimal commission levels reported in Proposition 1, consumer surplus in the low and high fee regimes (CS_{LFR} and CS_{HFR} , respectively) is found by subtracting profits from total welfare, $CS = v(\lambda_0 + \lambda_1 + \lambda_2 + \mu) - \pi_0 - \pi_1 - \pi_2$:

$$CS = \begin{cases} CS_{LFR} \equiv v\mu \frac{\lambda_0 + \mu}{\lambda_1 + \mu} & \text{for } \lambda_0 \leq \lambda_1 \\ CS_{HFR} \equiv v\mu \frac{\lambda_1 + \mu}{\lambda_2 + \mu} & \text{for } \lambda_0 \geq \lambda_1. \end{cases} \quad (4)$$

4.1 Competition within versus between channels

It is important, both for policy and more generally, to understand the effects of consumers' information on market outcomes. While an individual consumer's own welfare of course improves if they become aware of more of the alternatives available on the market *ceteris paribus*, it is not obvious (i) how firms respond to broader shifts in consumer awareness; (ii) how the information of some consumers affects others; and (iii) how the presence of a competitive channel alters any effects. In this section we shed light on these issues.

In standard clearinghouse models (e.g., Narasimhan, 1988), consumer surplus increases as consumers become more informed (in the sense that some captive consumers are turned into shoppers). Conditional on remaining within the same regime, the same is true here. Indeed, from (4) consumer surplus in each regime has the form $v\mu(\lambda_i + \mu)/(\lambda_j + \mu)$, which increases in $\Delta(\lambda_i, \mu)$ and $\Delta(\lambda_j, \mu)$. This happens for different reasons in each regime. In the high-fee regime, more shoppers leads firms to compete more fiercely in price within the direct channel, benefiting seller-captives and existing shoppers (a standard clearinghouse result).¹⁰ In the low-fee regime, the CC lowers its fee in response to an increase in shoppers to fend off the increased temptation of sellers to undercut CC prices and win the shoppers for themselves. This benefits CC-captives and existing shoppers. In either case, more information for some consumers benefits others.

The discussion so far is based on within-regime forces. However, a change in market composition can change the regime, with consequences summarized by Proposition 2.

Proposition 2 (Between-regime effects). *$CS_{HFR} \leq \lim_{\lambda_0 \uparrow \lambda_1} CS_{LFR}$, with a strict inequality when $\lambda_1 < \lambda_2$. Therefore, for generic λ and λ' such that $\lambda_1 - \lambda_0 > 0 > \lambda'_1 - \lambda'_0$, there exists an $\epsilon > 0$ such that λ yields higher consumer surplus than λ' if $\|\lambda - \lambda'\| < \epsilon$.*

Proposition 2 says that, for generic parameters, consumer surplus discontinuously decreases as the market tips from the low- to the high-fee regime. Between-channel

⁹Proposition 1 reports two equilibria at $\lambda_0 = \lambda_1$ (but uniqueness otherwise) hence the two values in (4).

¹⁰Armstrong (2015) terms this phenomenon a "search externality".

competition in the low-fee regime forces the CC to keep prices (and hence c) low to prevent undercutting. When it ceases to be worthwhile for the CC to compete for shoppers, the disciplining effect of between-channel competition disappears and the CC's fee and prices increase. Instead, only within-channel competition remains to discipline prices in the high-fee regime. But because of the information frictions present in the direct channel, the within-channel competition that takes place there is insufficient to offset the loss of competition between channels. The result shows that the interplay of within- and between-channel competition can overturn the standard intuition for the effects of informing consumers. To highlight this phenomenon, we obtain the following result as an immediate corollary to Proposition 2.

Corollary 1. *Consumer surplus is non-monotonic in $\Delta(\lambda_1, \mu)$.*¹¹

In standard models a higher shopper-to-captive ratio would unambiguously reduce prices (by intensifying within-channel competition). We find that the same is not true once competition between channels is also considered. Informing a seller's captives so that there are more shoppers (i) increases the seller's payoff from undercutting prices on the CC to serve shoppers directly, and (ii) reduces the benefit of charging high direct prices that serve only captives. Thus, the CC finds it more difficult to deter undercutting, forcing it to set a lower fee. Eventually, it is so costly to deter undercutting that the CC switches to a higher fee and focuses on extracting surplus from its own captive base. As we saw in Proposition 2, the result is a fall in consumer surplus. Thus, while the standard pro-competitive effect of consumer information is active in our model, technologies or interventions that promote transparency or awareness can also have anti-competitive effects by changing the balance of power between competing sales channels.

4.2 A growing competitive channel

The rise in the presence and market power of competitive channels around the world stresses the importance and urgency to study the impact on competitive outcomes. Towards that end, we now build on Proposition 2 to examine the effects of a growing CC on consumer surplus. To do so, it is useful to express the relationship between any λ and λ' as

$$\lambda'_1 = \lambda_1 - \Delta(\lambda_1, \lambda_0); \quad \lambda'_2 = \lambda_2 - \Delta(\lambda_2, \lambda_0); \quad \mu' = \mu - \Delta(\mu, \lambda_0); \quad \lambda'_0 = \lambda_0 + \sum_{x \in \{\lambda_1, \lambda_2, \mu, \emptyset\}} \Delta(x, \lambda_0).$$

Expressing λ' in this way recognizes that a competitive channel's captive audience can grow through market expansion (denoted $\Delta(\emptyset, \lambda_0)$), through its adoption by consumers who would otherwise be captive to seller i (higher $\Delta(\lambda_i, \lambda_0)$), or through adoption by

¹¹When $\lambda_0 < \lambda_1$, consumer surplus is also non-monotonic in $\Delta(\lambda_2, \mu)$.

shoppers (higher $\Delta(\mu, \lambda_0)$). The following proposition describes how the different sources of CC-growth influence consumer surplus.

Proposition 3 (Consumer surplus and CC growth). *Consumer surplus is non-monotonic in the growth of the competitive channel whenever that growth reduces the power of the direct channel or expands the market; and is decreasing if it reduces the number of shoppers. Specifically, consumer surplus is:*

1. *non-monotonic in $\Delta(\lambda_i, \lambda_0)$;*
2. *non-monotonic in $\Delta(\emptyset, \lambda_0)$ (if $\lambda_1 < \lambda_2$; else non-decreasing);*
3. *decreasing in $\Delta(\mu, \lambda_0)$.*

A larger $\Delta(\lambda_i, \lambda_0)$ or $\Delta(\emptyset, \lambda_0)$ reduces the share of consumers captive to a single seller, increases the share of consumers exposed to frictionless competition on the CC, and increases the average number of prices consumers see. Nevertheless, the result can be lower overall consumer surplus. These types of CC-growth have three effects: (i) consumers compare more prices; (ii) in the case of $\Delta(\lambda_i, \lambda_0) > 0$, sellers have fewer captives and are more willing to fight for shoppers; and (iii) the CC has more captive consumers to exploit. The first and second effects, in isolation, tend to make the market more competitive. But the second and third effects together leave the CC less interested in between-channel competition, and more inclined to set high fees. Proposition 3 establishes that these latter, anti-competitive effects can dominate, to the detriment of consumers. An example of point 1 of Proposition 3 is illustrated in Figure 1 (an illustration of point 2 would look similar). Point 3 follows because although CC captives cause fierce pricing *within* the CC channel, shoppers also exert competitive pressure *between* channels, making shoppers a more effective competitive discipline.

5 Imperfectly captive CC users

In the preceding analysis, consumers captive to the CC have their surplus fully-extracted in the high-fee regime. In practice, one might expect these consumers to look elsewhere if they routinely encounter such high prices on the CC. This would be a source of discipline for the CC, which would have to keep its hosted prices (and thus its fee) low to avoid driving away consumers to alternative sales channels. In this section we show that the insights we developed are robust to allowing the CC’s “captive” consumers to be more active in response to CC prices, and we show there exists an equilibrium analogous to that in the baseline model with the same high- and low- fee regime structure.

One interpretation of our baseline model is that the CC benefits from two kinds of friction. Firstly, some consumers visit the CC first (e.g., because it is initially the only channel they are aware of). Once on the CC, consumers see a list of (and so become aware of) other firms. Thus, the second friction benefiting the CC is a search friction that

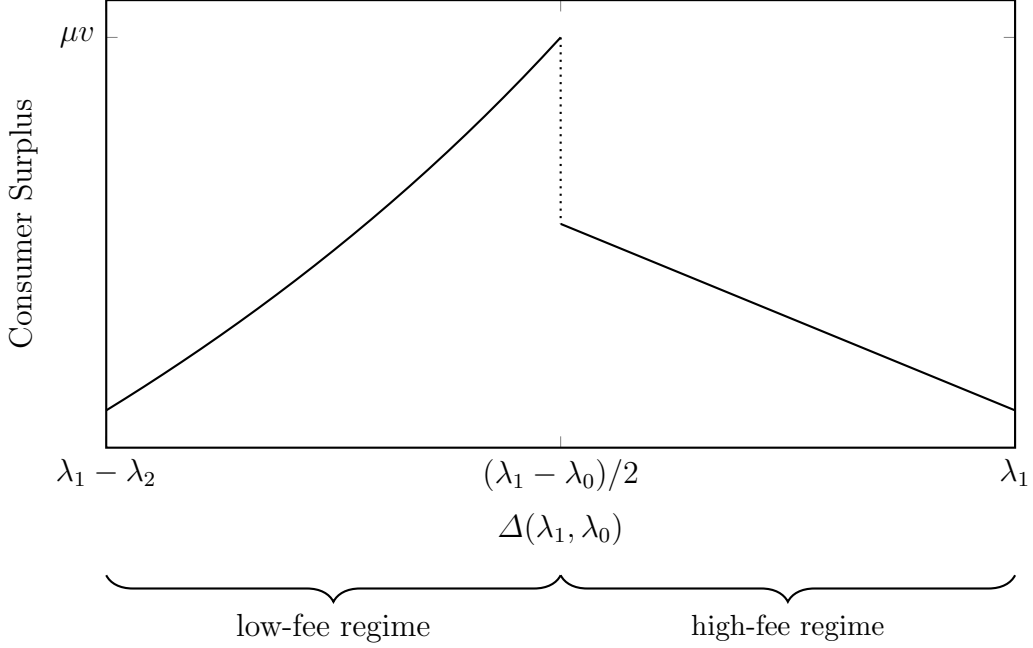


Figure 1: Effect of $\Delta(\lambda_1, \lambda_0)$ on consumer surplus, ceteris paribus.

discourages consumers from checking those firms' direct prices. We now explicitly model this search friction. As conjectured above, reducing the search friction forces the CC to decrease its fee to lower the prices it hosts and deter consumers from searching. However, our main results are robust, even when the friction disappears completely. In particular, we show that low- and high-fee regime equilibria continue to exist, such that an increase in λ_0 has a non-monotonic effect on consumer welfare by causing a regime change.

Formally, the model is modified as follows: the μ , λ_1 , and λ_2 consumers behave as before.¹² The λ_0 consumers visit the CC first. Once there, they can choose to buy immediately or to visit firms' direct channels to check prices there. The first and second such visits are made simultaneously and cost σ_1 and σ_2 , respectively, where $0 \leq \sigma_1 \leq \sigma_2$. Finally, we assume players coordinate on equilibria without search, where those exist.

As a first step in the analysis: if CC-captive consumers expect $\min_i p_i^d \geq \min_i p_i$ then they prefer not to check any direct prices. Moreover, we saw in Section 3 that sellers indeed set prices such that $\min_i p_i^d \geq \min_i p_i$ if $c \leq \underline{c}_2$ and the λ_0 consumers do not check direct prices. Thus, for any σ_1, σ_2 , the equilibrium in the baseline model survives unchanged in sub-games following $c \leq \underline{c}_2$.

Lemma 5. *If $c \leq \underline{c}_2$ then there is an equilibrium with no consumer search, and the prices of Lemmas 2 and 3.*

We now study how search costs may alter the analysis in the case of higher fees, where consumers have a strict incentive to check direct prices. Examining several such

¹²One interpretation is that λ_i consumers are initially aware only of i 's existence. If i is a seller the consumer does not learn about other firms by visiting i directly, and so remains functionally captive to i .

configurations, we find that our earlier results emerge qualitatively in all cases.

5.1 Prohibitive search costs

When search costs are high enough, our baseline equilibria are, of course, unaffected. Specifically, if σ_1 is sufficiently large, CC-captives will never search.

Suppose consumers are not expected to search. The pricing incentives are unchanged from those in Section 3. Given such prices, consumers do not search if $\sigma_1 > v - E_{F_1}[p]$, where the expectation is calculated using (2). The left-hand side is the cost of search, and the right-hand side is the expected gain from searching the firm with the lowest prices in the high-fee regime.¹³

Proposition 4. *If $\sigma_1 > v - E_{F_1}[p]$ then there is an equilibrium with no consumer search, and the prices of Proposition 1.*

Because the pricing sub-game is unchanged when search costs are prohibitively high, the CC faces exactly the same incentives in setting its fee and the consumer surplus results are also unchanged from the baseline analysis.

5.2 Zero search costs

We now consider the extreme case of $\sigma_1 = \sigma_2 = 0$. Following the reasoning in Section 3, any $c > \underline{c}_2$ must leave both sellers wanting to undercut the CC ($\min_i p_i^d < \min_i p_i$). Moreover, because both firms mix over a common interval, $\Pr(p_2^d < p_1^d) \in (0, 1)$. The CC-captives therefore find it strictly worthwhile to search both sellers and, from sellers' perspective, are functionally equivalent to shoppers. Equilibrium in the pricing sub-game is thus found by taking the $c > \underline{c}_2$ pricing strategies (2) and (3) but replacing λ_0 by 0, and μ by $\mu + \lambda_0$. It is immediate that this cannot be part of an equilibrium of the overall game. Indeed, because the lowest prices are in the direct channel and there are no search frictions, no consumers buy through the CC when $c > \underline{c}_2$. The CC would therefore earn zero profit and prefer to deviate to a lower fee.¹⁴

Thus, returning to Lemma 5, when $\sigma_1 = \sigma_2 = 0$ the CC faces a choice between: (i) a low-fee regime in which $c \leq \underline{c}_1$, shoppers buy via the CC, and $\pi_0 = c(\lambda_0 + \mu)$; or (ii) a high-fee regime with $c \in [\underline{c}_1, \underline{c}_2]$, shoppers buying via the direct channel, and $\pi_0 = c\lambda_0$. Comparing maximum profits in the two regimes yields Proposition 5.

Proposition 5. *Suppose $\sigma_1 = \sigma_2 = 0$. There is an equilibrium where: the CC prefers to implement the low-fee regime with $c = \underline{c}_1$ and the prices of Lemma 2 if $\lambda_0 \leq \tilde{\lambda}_1 \equiv$*

¹³Consumers would choose to search firm 1 first because $F_1(p) \geq F_2(p)$ for all p . It is never optimal to search both firms, given that a consumer does not gain by searching the first.

¹⁴The best a seller could do by undercutting the CC is to attract the shoppers and CC captives for profit $c(\lambda_i + \lambda_0 + \mu)$. Thus, if $c < v\lambda_1/(\lambda_1 + \lambda_0 + \mu)$ no firm can profitably undercut the CC. Such a c is a way for the CC to guarantee itself positive profit.

$\lambda_1(\lambda_2 + \mu)/(\lambda_2 - \lambda_1)$. The CC prefers the high-fee regime with $c = \underline{c}_2$ and the prices of Lemma 3 otherwise.

Therefore, for generic parameter values, equilibrium switches from a low-fee regime (where the lowest price in the market is on the CC) to a high-fee regime (where the lowest price is a direct price) as the CC becomes larger, just as in the baseline model. The main difference to the baseline is that the extent to which the CC can increase its fee in the high-fee regime is limited by consumers' willingness to shop around for better prices. In the baseline, the CC optimally charged v in the high-fee regime and hosted only the monopoly price, but with zero search costs it can only achieve \underline{c}_2 , hosting lower prices. This makes the high-fee regime less attractive to the CC so the threshold for entering that regime in equilibrium is higher i.e., $\tilde{\lambda}_1 > \lambda_1$. Consumer surplus is

$$\text{CS} = \begin{cases} v\mu \frac{\lambda_0 + \mu}{\lambda_1 + \mu} & \text{for } \lambda_0 \leq \tilde{\lambda}_1 \text{ (low-fee regime)} \\ v\mu \frac{\lambda_0 + \lambda_1 + \mu}{\lambda_2 + \mu} & \text{for } \lambda_0 \geq \tilde{\lambda}_1 \text{ (high-fee regime),} \end{cases}$$

which has a downward discontinuity at $\lambda_0 = \tilde{\lambda}_1$ (the point at which the regime switches). Then, as in the baseline model, consumer surplus is non-monotonic in the CC's size:

Proposition 6. *Suppose $\sigma_1 = \sigma_2 = 0$. For generic parameter values and under the equilibrium of Proposition 5, consumer surplus is non-monotonic in $\Delta(\lambda_1, \lambda_0)$, $\Delta(\lambda_2, \lambda_0)$, and $\Delta(\emptyset, \lambda_0)$ (where these quantities are defined in Section 4).*

5.3 Intermediate search costs

We now summarize the analysis for the case in which σ_1 is positive but not prohibitive.¹⁵ Consumers' gain to search is increasing in the difference between CC and direct prices, which, in turn, is affected by c . In equilibrium, if c is only slightly above \underline{c}_2 , direct prices are slightly below c and consumers do not search. Thus, in equilibrium, the CC can increase its fee a little above \underline{c}_2 without prompting search, but does not want to raise it so high that it loses all consumers to the direct channel. The CC's optimal high-fee regime commission level is therefore an intermediate $c \in (\underline{c}_2, v)$, chosen to resolve this trade-off.¹⁶ The less costly is search, the less the CC can exploit consumers, lowering the optimal c . As search costs approach zero, equilibrium converges smoothly to the case discussed in Section 5.2, and as they become large, it converges to the case discussed in Section 5.1, which is the case considered in the baseline analysis. The equilibrium trade-off faced by the CC is analogous to the baseline model: it can implement a low-fee regime with $c \leq \underline{c}_1$

¹⁵Details are lengthy and parallel to the preceding analysis, so are relegated to the Web Appendix.

¹⁶Depending on parameters, the optimal fee may prevent all direct-channel search, or accommodate some.

and serve shoppers, or it can opt for a higher fee and focus on exploiting those consumers who face search costs. The latter regime is more attractive when λ_0 is large and growth of the CC therefore eventually triggers a change in regime.

6 Policy Analysis

We now use our framework to examine a number of practices that have attracted regulator scrutiny. These include most-favored-nation clauses (where a seller’s direct price cannot be lower than their competitive channel price); cross-channel integration (when a CC and a seller have the same owner); and non-re-solicitation clauses (which restrict firms’ ability to poach others’ past consumers). These issues share the feature that they affect competition both within and between channels, which is the core focus of our paper.

6.1 Most-favored-nation clauses

To capture shoppers’ business without paying the competitive channel’s fee, sellers want to undercut prices on the CC. Deterring such undercutting constrains the value of c and thus limits CC profits in the low-fee regime. This naturally raises the question: are there ways for the CC to relax this constraint? Much policy activity and scholarship has focused on most-favored nation (MFN) clauses, which explicitly forbid undercutting. We now use our model to examine the structure of equilibrium under MFNs (which impose $p_i \leq p_i^d$) and the implications of the clauses for equilibrium outcomes.

The second respect in which we modify the baseline model is to introduce “showrooming” consumers who use the CC to learn of sellers’ existence, but then go elsewhere to check for lower prices.¹⁷ Showrooming has played an important role in the MFN debate. We show that it gives MFNs bite in our model through its effects on intra- and inter-channel price competition. We let the mass of shoppers be decomposed into a fraction s of “showroomers” and fraction $1 - s$ “true shoppers”. Showroomers see all CC prices, plus the direct prices of firms that list on the CC. True shoppers observe all prices in the market, as in the baseline (where $s = 0$). In the baseline model without MFNs, allowing $s > 0$ leaves the equilibrium and results unchanged.¹⁸ We compare equilibrium with MFNs to our baseline case without. To eliminate implausible equilibria, we focus on the case where the number of sellers, n , is not too small, with $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$.¹⁹

¹⁷In a related analysis, Wang and Wright (2019) also study the relationship between showrooming and MFNs. Also related, Janssen and Ke (2020) study the free-riding problem related to service provision (Telser, 1960) and show that the problem can be mitigated in the presence of search frictions.

¹⁸Since firms never have a reason to delist from the CC in the baseline model, showroomers are aware of both firms’ prices and behave exactly like true shoppers.

¹⁹Specifically, we take $n > 3$, which allows us to focus on the empirically plausible case in which there are always multiple sellers on the CC in equilibrium.

When MFNs are in use, a firm faces a choice between listing on the CC with $p_i \leq p_i^d$, or delisting from the CC and setting only a direct price. We define

$$\underline{c}'_i \equiv \frac{v\lambda_i}{(1-s)\mu + \lambda_i} > \underline{c}_i,$$

and describe the effects of an MFN in our framework in the following result.

Proposition 7 (Most-favored-nation clauses). *With MFNs, equilibrium falls into one of two regimes.*

1. *If $\lambda_0 \leq \lambda_1 - s\mu$, a low-fee equilibrium exists in which the CC sets $c = \underline{c}'_1$, and all sellers list, setting $p_i = \underline{c}'_1$ and $p_i^d = v$.*
2. *If $\lambda_0 \geq \lambda_1 - s\mu$, a high-fee equilibrium exists in which the CC sets $c = v$, firms 1 and 2 do not list on the CC and mix over prices $p_i^d \leq c = v$, while $i > 2$ list and set $p_i = p_i^d = c = v$.*

For any market composition, MFNs are profitable for the CC and reduce consumer surplus.

Full details of the equilibrium pricing strategies can be found in Appendix C.1. Proposition 7 reports that the basic structure of equilibrium in the baseline model is preserved under MFNs i.e., equilibrium is divided into low-fee and high-fee regimes. Differences include a lower cut-off between regimes in terms of λ_0 , and slightly different prices.

A firm that delists loses access to the showroomers, whereas a firm that undercuts while listing can continue to serve them. MFNs force undercutting firms to delist, degrading sellers' outside option and allowing the CC to increase its commission in the low-fee regime. The higher fee generates higher prices, which harm consumers.

In the high-fee regime, MFNs also benefit the competitive channel by allowing it to retain the business of showroomers despite the presence of lower prices in the market. Because showroomers do not learn about the lower prices (because those firms cannot list a price on the CC), consumer surplus is again reduced.

Lastly, the lower threshold for switching between regimes means that as λ_0 grows, equilibrium tips into a high-fee regime earlier than in the setting without MFNs. This occurs because the CC experiences a larger increase in profit when adopting MFNs within the high-fee regime (because that is where its marginal revenue is higher). Together, these factors leave consumers unambiguously worse-off under MFNs.

Discussion. A key argument advanced in defence of MFNs is that competitive channels exist to help consumers find low prices and MFNs have a legitimate business purpose in making this proposition viable. Otherwise, goes the argument, CCs would be vulnerable to free-riding by showroomers. Our analysis suggests another perspective: A CC, like any other firm, should be expected to compete for consumers through its choice of c . It is already within a CC's power to prevent free-riding by merely lowering its fee and

hosting lower prices. The fee (and CC profits) necessary to deter undercutting might be relatively low, but this is cause for concern only if CCs face particularly high fixed costs or other significant barriers to entry.

6.2 Cross-channel integration

It is not uncommon for a seller and a competitive channel to be jointly owned by the same parent company. For example, in the UK both Confused.com (a price comparison website specialized in financial services) and Admiral Insurance (an automobile insurer) are part of the Admiral Group. To study this kind of arrangement, we suppose that the CC and seller $i \in \{1, 2\}$ are integrated in a joint entity called $0i$ that controls both c , and i 's prices. We compare the resulting equilibrium to the baseline, using the same timing: $0i$ first sets c before firms simultaneously set prices. Note that, when i sells through the CC, the integrated firm essentially pays c to itself (or, equivalently, pays no fee).

In both 01 and 02 cases, the equilibrium falls into either a low-fee or high-fee regime, as in the baseline model. We begin with the case where the CC is integrated with firm 1.

Proposition 8 (01 Integration). *Suppose the CC and seller 1 are integrated. When $\lambda_0 \leq \lambda_1$, consumer surplus falls relative to no integration; when $\lambda_0 \geq \lambda_1$, consumer surplus is unaffected by integration.*

In the baseline low-fee regime, competition between channels imposed a constraint on how high CC fees could be without triggering undercutting. When 0 and 1 are jointly owned, seller 1 no longer wishes to undercut the CC (because it would be undercutting itself). Moreover, because seller 1's "no undercutting" constraint was binding in the baseline model, relaxation of the constraint allows c to increase.²⁰ This increases the 01's profit, but harms consumers (because higher fees translate into higher prices).

For higher fee levels, undercutting takes place as firms fight to serve shoppers. Because it is integrated and does not have to pay the fee, firm 01 can set $p_1 < c$ without incurring a loss. Therefore, 01 decides whether to fight for shoppers with its CC price or its direct price based solely on a comparison of how many captive consumers each channel has. It chooses the latter in a high-fee equilibrium because such an equilibrium emerges when λ_0 is large relative to λ_1 which is precisely when setting a low CC price would be most costly. We thus have a game in which sellers fight for shoppers in direct prices, just as in the baseline. The upshot is that equilibrium strategies and pay-offs in the high-fee regime are the same as in the baseline. Comparison of the integrated firm's profits between regimes finds that the cut-off between regimes is also unchanged.

Now consider a merger between firms 0 and 2, creating firm 02.

²⁰Specifically, c can be increased from c_1 to c_2 .

Proposition 9 (O2 Integration). *Suppose the CC and seller 2 are integrated. When $\lambda_0 \in [\lambda_1, \lambda_2]$, consumer surplus rises relative to no integration; when $\lambda_0 \notin [\lambda_1, \lambda_2]$, consumer surplus is unaffected by integration.*

As under O1 integration, joint ownership of 0 and 2 relaxes 2’s “no undercut” constraint in the low-fee regime. However, because this constraint is not binding in equilibrium, strategies and payoffs are unchanged. Equilibrium strategies and payoffs are also unchanged in the high-fee regime for $\lambda_0 > \lambda_2$: O2 prefers to exploit them with a high CC price and fight for shoppers with its direct price, as in the baseline.

When $\lambda_0 \in [\lambda_1, \lambda_2]$, the CC is large enough that it does not want to deter undercutting by firm 1, but small enough that it would rather fight for shoppers with its CC price than its direct price. Moreover, because $\lambda_0 < \lambda_2$, the integrated firm fights more fiercely for shoppers than would an independent firm 2 (the latter being reluctant to cut its direct price when it has so many captives to exploit). Prices are thereby lower under integration. These low prices come at the cost of firm 1’s profit, but cause consumer surplus to increase.

Discussion. A number of papers have considered the conflicts of interest that arise when an intermediary has an ownership or other financial stake in the firms it recommends.²¹ We have shown that, even putting such concerns aside, integration with an intermediary can distort inter-channel competition in two respects. Firstly, although competitive channels and sellers may not appear to be direct competitors with each other, integration between them can soften competition. This is because the direct channel is an important source of competitive discipline for the CC’s fee setting, which in turn affects consumer prices. Secondly, integration is a way for a firm to commit to be a tough competitor, which can benefit consumers but also reduces rivals’ profits and might therefore trigger concerns about anticompetitive foreclosure in the spirit of Whinston (1990).

6.3 Non-resolicitation clauses

Consumers often make repeat purchases, e.g., insurance policies are often renewed annually. When a sale is conducted through the CC, both the chosen seller and the CC are party to the transaction and the issue of which firm is allowed to use consumers’ purchase data to solicit future sales becomes relevant. A non-re-solicitation clause (NRC) is an agreement between a CC and a seller that governs who is allowed to send targeted communications to consumers that previously bought through the CC. Competition regulators have raised concerns about NRCs, worrying that a ban on CCs contacting past consumers, which we refer to as a “CC-binding” NRC, may lead inactive, older, or more vulnerable consumers to automatically renew directly with their existing provider at inflated prices.²² In contrast, Amazon and eBay use conditions of sale preventing third-party

²¹E.g., Armstrong and Zhou (2011), De Cornière and Taylor (2019), and Inderst and Ottaviani (2012).

²²See, for example, paragraphs 65–69 of CMA (2017d).

sellers from contacting buyers, which we classify as “DC-binding” NRCs.²³ The worry that this arrangement, too, may interfere with inter-channel competition has also arisen.²⁴

To study non-resolicitation clauses, we consider a stationary overlapping generations extension of our model in which consumers each live for two periods. Consumers are “young” in the first period of their life and “old” in the second; a mass of $\lambda_0 + \lambda_1 + \lambda_2 + \mu$ young consumers enters the population in each period. To speak to regulatory concerns, we study how different NRCs operate in our model with vulnerable consumers. Specifically, we make two simplifying assumptions of limited rationality that reflect plausible biases in the relevant markets: (i) the young generation are first-time buyers and act as in the one-period baseline model according to the market composition λ i.e., without regard to how their actions may affect their future price offers; (ii) the old generation (regardless of their type when young) are all inert or inactive and only respond to direct solicitations, buying at the lowest price offered to them.

Old consumers who bought direct when young receive a renewal price, \hat{p}_i^r , from their previous supplier, i , and see no other prices. Old consumers who bought via the competitive channel receive a set of offers determined by the NRC (if any) in place. We consider three cases. Firstly, under a DC-binding NRC only the CC is able to contact them, so they are offered prices p_1 and p_2 . Under a CC-binding NRC, these old consumers receive only a personalized renewal quote (p_i^r) from their existing supplier. Lastly, if no NRCs are in place (e.g., because they are banned) then both the CC and existing supplier are able to contact the consumer who therefore considers all three prices (p_1 , p_2 , and p_i^r).²⁵

We look for stationary equilibria in which strategies are constant across periods. Appendix C.3 details the equilibrium strategies. As in the baseline, equilibrium falls into either a low-fee regime where shoppers buy through the CC, or a high-fee regime where they buy direct. The high-fee regime arises when λ_0 is sufficiently large. But NRCs induce changes in pricing and fee-setting behavior. In particular, sellers offer introductory discounts when they anticipate serving consumers at a profit in future periods, and the competitive channel takes lifetime competition with sellers’ direct channels into account when setting c . The following result states the effects of NRCs on consumer surplus.

Proposition 10 (Non-resolicitation clauses). *There exists $\tilde{\lambda} \in [\frac{\lambda_1 - \mu}{2}, \lambda_1]$ such that, relative to a policy intervention banning the use of NRCs,*

²³For example, Amazon outlaws such practices as can be seen in their General Guidelines: <https://www.amazon.com/gp/help/customer/display.html?nodeId=200414320>, as does Ebay in their Policy Overview: <http://pages.ebay.com/help/policies/rfe-spam-ov.html>, links accessed 14 May 2018.

²⁴E.g., Rankin, Jennifer (2015) “Third-party sellers and Amazon - a double-edged sword in e-commerce.” *Guardian*, June 23. <https://www.theguardian.com/technology/2015/jun/23/amazon-marketplace-third-party-seller-faustian-pact>.

²⁵We have assumed that old consumers receive personalized renewal quotes from sellers, but see the same CC prices as young consumers. This assumption is motivated by current practice in some price comparison markets where the use of NRCs has been subject to policy scrutiny.

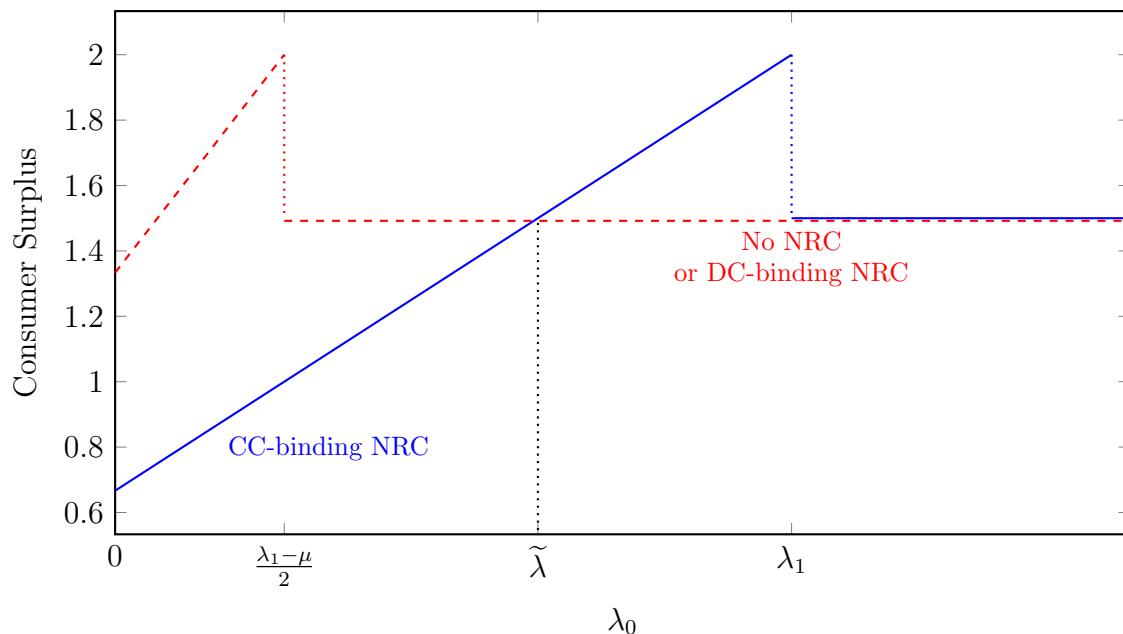


Figure 2: Consumer surplus under various NRCs where $v = \mu = 1$, $\lambda_1 = 2$, $\lambda_2 = 3$.

1. a CC-binding NRC decreases consumer surplus if $\lambda_0 < \tilde{\lambda}$ and (weakly) increases consumer surplus if $\lambda_0 \geq \tilde{\lambda}$.
2. a DC-binding NRC has no effect on consumer surplus;

Figure 2 illustrates proposition 10. Under a CC-binding non-resolicitation clause, prices on the competitive channel are especially low because sellers compete to attract young consumers who will become captive when old. Since undercutting these low prices is costly, the CC can increase its fee without fear of being undercut. The CC-binding NRC therefore makes the low-fee regime more attractive to the competitive channel. We thus observe two effects of this type of NRC: (i) within the low-fee regime, it increases CC fees and hence prices, but (ii) it makes a low-fee regime result for a wider range of market compositions. A small CC (low λ_0) implements a low-fee regime regardless of whether NRCs are permitted or not, so only the first effect survives and an NRC ban benefits consumers. If λ_0 is larger, a ban on NRCs may cause a switch from the low-fee to the high-fee regime, which can harm consumers.

In contrast, a DC-binding NRC reduces the lifetime value to sellers of young consumers served via the competitive channel and thus the c that they can be charged. But it also guarantees that the CC will serve consumers in both periods of their life instead of only the first. These two effects (lower fees, paid more often) exactly offset each other so that a ban on NRCs leaves both the CC's incentive to induce a low-fee regime, and consumers' lifetime expenditure, unchanged.

7 Extensions and Robustness

7.1 Advertising and Market Composition

In Section 4, we conducted comparative statics on the market composition. This allowed us both to make comparisons across markets with different compositions, and to consider the implications of shifts in power within a particular market. We found consumer surplus can fall both when consumers make more price comparisons and when the competitive channel is larger. Both of these effects are due to the presence of the high-fee regime. In this section, we ask which regime we may expect to see in the long-run by studying the endogenization of market composition via the costly marketing efforts of firms.²⁶ We add a pre-stage to the game where all firms (sellers and the competitive channel) compete for market power using a costly technology. This can be interpreted as determination of the brand loyalty through advertising and marketing. In reality, there is fierce competition between channels as reflected by high ad spends, not only by sellers themselves, but also by competitive channels. Online giant Amazon’s total global marketing spend was \$7.2bn in 2016 and it appears to have accumulated a position as the go-to source for shopping searches. In the UK, price comparison websites spent £123m in 2014-15²⁷, with the biggest four entering the top 100 companies in the country by ad spend.²⁸

Formally, we model a new stage, $t = 0$, where firms $i = 0, 1, 2$ simultaneously choose their level of market power i.e., number of captive, or “loyal” consumers, λ_i .²⁹ The cost to i of choosing λ_i given the others’ marketing choices is determined by the function ψ_i . We assume all firms have access to the same technology, hence ψ is symmetric across players. Our other notable assumptions are that ψ is: increasing and convex in own λ , and inter-dependent (it is more costly to generate the same loyal share when others advertise more).

We consider equilibria with pure strategies at $t = 0$. Let Ψ_L (Ψ_H) denote the set of all functions ψ for which a low-fee (high-fee) regime equilibrium exists. In general, for a given cost function ψ there may be many equilibria of the game. But, without the need to be more specific about the cost function in question, we report the following result.

²⁶This exercise builds on the informative advertising literature such as Baye and Morgan (2009); Chioveanu (2008); Ireland (1993). Unlike those papers, we allow sellers to set a direct price and choose to advertise a price on the CC; for the CC to be an explicit and profit-maximizing player; and for any firm, seller or CC, to engage in marketing activities to determine its market power.

²⁷Ralph, Oliver (2016). “Going gets tough for GoCompare the SuperMeerkat.” *Financial Times*, 13 June. <https://www.ft.com/content/8997ec7a-2f01-11e6-bf8d-26294ad519fc>.

²⁸Straus, Rachel Rickard (2015). “We can’t act surprised that comparison sites make a mint from us switching - Snoop Dogg doesn’t work for free.” *This is Money*, 3 February. <http://www.thisismoney.co.uk/money/bills/article-2933401/Energy-price-comparison-sites-spend-110m-annoying-adverts.html>.

²⁹We hold μ fixed for this exercise. The results presented hold for any $\mu > 0$.

Proposition 11 (High-fee regime predominance). *There are more cost functions that lead to a high-fee regime than a low-fee regime in equilibrium, i.e., $|\Psi_H| \geq |\Psi_L|$.*

The proof, in the Web Appendix, shows that for any given cost function ψ , if there is a low-fee regime equilibrium, then there is also a high-fee regime equilibrium. This follows because sellers are more tempted to deviate from a market composition giving the low-fee regime than one giving the high-fee regime. Given a composition yielding the low-fee regime (i.e., when the CC is small), deviating to a level of advertising just low enough to prompt the high-fee regime causes a jump up in profit for a seller. The jump arises from the ensuing subgame in which sellers fight for shoppers with their direct price where the deviating seller fights the hardest and is rewarded with a high chance of selling to the shoppers. In contrast, for a composition yielding a high-fee regime, no such jump exists because when the CC is large, sellers already serve the shoppers in equilibrium.

The result shows that in our framework, the high-fee regime is not an abstract possibility. In fact, the model predicts more situations where the competitive channel is large and exploits its market power through high commissions.

7.2 Percentage fees and positive marginal cost

In practice, competitive channels typically use per-sale fees.³⁰ Sometimes these are in the form of flat commission rates, independent of the sale price; sometimes they are ad valorem, usually a percentage of the sale price; and sometimes there is a combination of the two.³¹ Our baseline model employs a flat fee, c , rather than an ad valorem commission. In this subsection we instead allow for arbitrary combinations of flat and percentage fees. To do so, denote the competitive channel’s choice of flat and a percentage commission, $c \geq 0$ and $\tau \in [0, 1]$, respectively. We also allow for a positive marginal cost of production, $m \in [0, v]$, which sellers incur for each unit sold.³²

³⁰In a clearinghouse context, Baye et al. (2011) allow intermediaries to choose between any combination of fixed or per-sale payment and find that they optimally set the fixed component to zero.

³¹Amazon (https://sellercentral.amazon.com/gp/help/external/200336920/ref=asus_soa_p_fees?ld=NSGoogle) and eBay (<http://pages.ebay.com/help/sell/storefees.html#fvf>) charge both percentage and flat per-sale fees to sellers, making the majority of their revenue through the latter. Travel reservation websites such as Booking.com and Expedia are understood to charge their sellers 15-25% of the sale price; see, e.g., Poulter, Sean (2015). “Hotel guests ‘are being fleeced by online agents’: Hoteliers say they are forced to hike prices to cover 25% commission taken by websites.” *Daily Mail*, 18 April. <http://www.dailymail.co.uk/news/article-3044298/Hotel-guests-fleeced-online-agents-Hoteliers-say-forced-hike-prices-cover-25-commission-taken-websites.html>.

Price comparison sites in the UK are reported to charge flat per-sale fees; see, e.g., BBC (2013). “Price comparison sites to be investigated over deals.” *BBC News*, 23 November. <http://www.bbc.co.uk/news/business-25068588>. Shopping Malls often collect rent from the stores they host as a percentage of their sales (Gould et al., 2005).

³²Note that it can be shown that $m + c > 0$ must hold in equilibrium: Briefly, if $m + c = 0$ then sellers’ net marginal cost is zero and Bertrand competition on the CC drives prices there to zero, implying the CC earns zero profit. The CC will therefore always choose a strictly positive c if $m = 0$.

In our framework, the price set by sellers on the competitive channel is a sufficient statistic to characterize equilibrium. This price, in turn, is determined by c , τ , and m . In other words, it does not matter what the exact level of c and τ are, what matters is the price on the CC, p , that they generate. Indeed, one p corresponds to many different (c, τ) pairs. Furthermore, in equilibrium the competitive channel's profits are not affected by (c, τ) other than through p , and hence the CC is indifferent between any of the (c, τ) pairs corresponding to that p . For a given p , sellers also have the same equilibrium profit, and consumers have the same surplus. Therefore, the analysis of the baseline model is unchanged under the alternate assumption that commissions are a percentage of the sale price. For completeness, the equilibrium is stated in terms of p in Appendix W.4.

Here, we make this equivalence between variables explicit. Bertrand competition between sellers on the CC ensures sellers make zero profit through sales there: $p(1 - \tau) - c - m = 0 \Leftrightarrow p = \frac{c+m}{1-\tau}$. This identity shows how the baseline analysis of $\tau = 0$ is without loss: For some m , consider arbitrary flat and percentage commissions (c, τ) . The induced CC prices are the same as those with alternative fee structure $(c', \tau') = (\frac{c+m\tau}{1-\tau}, 0)$. Moreover, because p is a sufficient statistic for equilibrium characterization, equilibrium under the two fee structures is equivalent and there is no loss from assuming $\tau = 0$.

7.3 Commission discrimination

Competitive channels often charge different sellers the same commission levels. For example, one can easily check the standard seller fees charged by eBay and Amazon on their respective websites.³³ In other markets, e.g., the hotel reservation industry, although the fees are set in private it is understood there is a common base commission.³⁴ However, our framework allows sellers to have different levels of market power. This suggests that our model should not constrain the competitive channel to charge the same commission level to all sellers. Here we show that this assumption is without loss of generality.

Formally, we allow the competitive channel to set two fee levels in $t = 1$, c_1 and c_2 , which are the commissions that sellers 1 and 2, respectively, pay upon a sale through the CC. In the Web Appendix, we prove that the CC nevertheless sets $c_1 = c_2$. For the intuition, suppose $c_i < c_j$. Prices via the CC are competed à la Bertrand down to c_j , but i can keep undercutting as $c_i < c_j$ and so secures all the sales through the CC at a cost of c_i . This means the CC's income is solely determined by c_i but prices on the CC are set by c_j . Therefore, the CC wants either to increase c_i to receive more per sale over the same number of sales, or to decrease c_j to lower CC prices and make undercutting more costly. Thus, when the CC can set discriminatory fee levels, it chooses not to.

³³Amazon: https://sellercentral.amazon.com/gp/help/external/200336920/ref=asus_soa_p_fees?ld=NSGoogle. eBay: <http://pages.ebay.com/help/sell/storefees.html#fvf>, 14 May 2018.

³⁴E.g., 15% on booking.com: <http://blog.directpay.online/booking-com>, accessed 14 May 2018.

8 Conclusion

We studied an environment in which selling occurs both directly and via a competitive sales channel (CC). A CC is a platform (such as a price comparison website, online marketplace, or shopping mall) that lists price offers from multiple sellers. In our model the CC is a strategic actor in its own right, just as sellers are. Furthermore, our flexible framework features a measure of each actor’s market power, or size, which is commensurate across types of actor, both sellers and competitive channel operators.

A key question in our model is whether the most price-sensitive consumers buy directly from the seller or via a competitive sales channel; a tension that causes equilibrium to fall into one of two regimes. In the first regime, the competitive channel attracts the most sensitive consumers by demanding a low fee which results in it hosting the lowest prices on the market. Sellers, meanwhile, focus on serving their captive consumers at high prices through their direct channel. In the second regime, the situation is reversed in the sense that fees (and hence prices) on the CC are high, while sellers undercut it and compete with each other for price-sensitive consumers in their direct prices. Which regime occurs in equilibrium depends on how much power the CC holds over consumers, relative to the power sellers hold in their direct channels. The more consumers rely on the CC, the more tempted it is to increase its commission and exploit its position as a bottleneck gatekeeper. Moreover, as sellers’ captive audience shrinks, price-sensitive shoppers account for a growing share of their potential demand, which increases the temptation to undercut prices on the CC and serve these consumers directly.

These equilibrium dynamics have implications for consumer welfare. Suppose consumers become less captive to individual firms and start comparing prices across firms more often (either by using the competitive channel, or by independently becoming more informed shoppers). The direct and immediate effect of more widespread price comparisons is to increase the intensity of competition between sellers and cause them to lower their prices, i.e., competition *within* channels is strengthened. However, there is a countervailing effect, namely that the equilibrium regime can change in such a way that prices on the CC increase, making it a less effective discipline on prices more generally, i.e., competition *between* channels is weakened. In contrast to models that neglect the role of inter-channel competition, this countervailing effect means that consumer surplus can fall, even as consumers become informed of a greater number of price offers.

Within this framework, we analyze the effect of most-favored nation clauses, integrated ownership structures, and non-resolicitation clauses; all features of markets with both direct and competitive channels that have received attention from regulators and competition authorities. Through these analyses, we report a common theme in that the size, or market power, of the competitive channel matters for the implications of these practices, and hence also for interventions that weaken or prevent them.

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Appendix

Appendices A, B and C contain the main proofs for the results of Sections 3, 4 and 6. For the remaining proofs and details we refer the reader to our Web Appendix.

Allow there to be any number of sellers, n , indexed $i = 1, \dots, n$ without loss of generality such that $\lambda_i \leq \lambda_j \Leftrightarrow i \leq j$. After Appendix A we set $n = 2$ unless otherwise specified. We now define the tie-break parameter, r_0 . Suppose the lowest price on the market is p , set by the set of sellers $T \neq \emptyset$ on the CC and $D \neq \emptyset$ via direct prices. There are then $|T| + |D|$ sources of the lowest price on the market. Let $r_0 \in [0, 1]$ such that if:

- (1) $i \notin T, i \notin D$: the probability that i sells to a shopper is 0;
- (2) $i \in T, i \notin D$: the probability that i sells to a shopper is $\frac{r_0}{|T|}$;
- (3) $i \notin T, i \in D$: the probability that i sells to a shopper is $\frac{1-r_0}{|D|}$;
- (4) $i \in T, i \in D$: the probability that i sells to a shopper is $\frac{1-r_0}{|D|} + \frac{r_0}{|T|}$.

In words, when at least one direct price and at least one competitive channel price are tied and are the lowest on the market, r_0 is the probability with which shoppers buy at one of the lowest prices listed on the competitive channel. For all the results that follow, if equilibrium restricts the value of r_0 , the restriction is reported.

A Baseline Model

We first prove Lemma 1 via some constructive intermediate results. Denote the (possibly degenerate) distribution of the *lowest* price listed on the CC as G_{\min} . Denote the minimum and maximum of the support of G_{\min} by \underline{s} and \bar{s} respectively.

Lemma A1. $\underline{s} \geq c$.

Proof. Suppose $\underline{s} < c$. Then some seller, i , has a positive probability of serving a positive mass of consumers via the CC at price $p_i < c$. A deviation to $p_i > v$ (or equivalently to not listing a price on the CC), holding p_i^d fixed, cannot reduce i 's demand via its direct channel, but reduces demand via the CC to zero. Since each consumer served via the CC at $p_i < c$ incurred a loss, the deviation is profitable. \square

Lemma A2. When $c < v$, $\underline{s} = c$.

Proof. By Lemma A1, $\underline{s} \geq c$. Suppose $\underline{s} > c$. Then all sellers must make positive profits via the CC: if not the i with zero profit could deviate to $p_i = \underline{s} - \epsilon$, ϵ small (in the case where $p_i^d = \underline{s}$ and there is a positive probability this is the cheapest direct price, then in addition consider i deviating to $p_i^d = \bar{s} - 2\epsilon$). As sellers make positive profits through the CC and must be indifferent between all prices they play, all prices listed are no greater than v . Consider the highest price in the union of all the supports of all sellers, \bar{p} , where $\bar{s} \leq \bar{p} \leq v$. If there is a positive probability of a tie at \bar{p} , any seller i would

shift the associated probability mass to $\bar{p} - \epsilon$ (and wherever $p_i^d = \bar{p}$ and there is a positive probability this is the cheapest price, then in addition i deviates to $p_i^d = \bar{p} - 2\epsilon$). If there is a zero probability of a tie, any seller playing \bar{p} makes zero profit, a contradiction. \square

Lemma A3. *When $c < v$, $\bar{s} = c$.*

Proof. From Lemma A2, $\underline{s} = c$. Therefore some seller i has c in its support and makes zero profit via the CC when it plays $p_i = c$ (or vanishing profit as $p_i \downarrow c$). If $\bar{s} > c$ then, when i is called upon to play $p_i = c$ (or $p_i \downarrow c$), it can instead play $\bar{s} - \epsilon$ for $\epsilon > 0$ small and net a strictly higher profit. Such an increase in p_i cannot reduce demand through i 's direct sales channel, but would strictly increase expected profit through the CC. \square

Lemma A4. *When $c < v$, at least two sellers list $p = c$ with probability one.*

Proof. Lemmas A2 and A3 show $\bar{s} = \underline{s} = c$. This implies that there is at least one seller, say i , which lists c on the CC with probability one. Suppose i was the only firm to do so. If so, i makes zero profit through the CC, yet could profitably deviate to some slightly higher p_i at which there is a positive probability of having the lowest listed price. Such an increase in p_i cannot reduce demand through i 's direct sales channel, but would strictly increase expected profit through the CC. \square

Lemma A5. *When $c = v$, at least one seller lists $p = v$ with probability one.*

Proof. By Lemma A1, every seller sets $p_i \geq c$. When $c = v$, sellers are indifferent between any $p \geq v$. However, suppose that there is no seller that plays $p = v$ with probability one. This implies CC profit is lower than $v\lambda_0$. This can't be an equilibrium because the CC could slightly reduce c to induce at least two sellers to list $p = c$ with probability one (by Lemma A4), which increases CC profit. \square

Proof of Lemma 1. Follows from Lemmas A1-A5. \square

Proof of Lemma 2. Sellers earn zero profit from consumers served through the CC in any equilibrium (Lemma 1). If a seller charges $p_i^d = v$ it makes total profit of $\pi_i = v\lambda_i$. Prices $p_i^d \in (c, v)$ are dominated by v . The highest profit from setting any other value of p_i^d is from $p_i^d = c - \epsilon$ as $\epsilon \downarrow 0$ (conditional on all other direct prices being the same or higher), which in the limit generates $\pi_i' = c(\lambda_i + \mu)$. Because $\pi_i' - \pi_i$ is decreasing in i , it suffices to check that seller 1 does not find this deviation profitable, $\pi_1 \geq \pi_1' \Leftrightarrow c \leq v\lambda_1/(\lambda_1 + \mu)$. For $c < \underline{c}_1$, given that profit $v\lambda_i$ is the best a seller can ever do, uniqueness follows because a seller can always obtain such profit (only) by $p_i^d = v$.³⁵ Let $n = 2$ for the lemma. \square

³⁵When $c = \underline{c}_1$, i : $\lambda_i = \lambda_1$ is indifferent between $p_i^d = v$ and $p_i^d = c$. Therefore, there are equilibria in which $Pr(p_i^d = c) \equiv \alpha \in [0, 1]$ and $Pr(p_i^d = v) = 1 - \alpha$, while $p_j^d = v$ for $j \neq i$. (Note $\alpha > 0$ cannot be part of the equilibrium of the whole game: If so, 0 would slightly lower c to get discretely higher sales).

Proof of Lemma 3. Sellers can guarantee themselves profit of $v\lambda_i$ by setting $p_i^d = v$; no greater profit can be earned by serving only captives via the direct channel (by Lemma 1). Attracting shoppers to the direct channel would require $p_i^d \leq c$, which is not profitable by definition when $c \leq \underline{c}_i$. Thus, in any equilibrium with $c < \underline{c}_2$, we must have $p_i^d = v$ for all $i \geq 2$. Therefore, when $c < \underline{c}_2$, 1 chooses a direct price of either c or v (all other direct prices are dominated). When $\underline{c}_1 < c < \underline{c}_2$, 1 strictly prefers c because $c > \underline{c}_1$. Note that $r_0 = 0$: if $r_0 > 0$, 1 strictly prefers direct prices arbitrarily below c to c , which are not well-defined. This leaves us with the unique equilibrium. Let $n = 2$ for the lemma.

When $c = \underline{c}_2$, any firm i such that $\lambda_i = \lambda_2$ is indifferent between $p_i^d = v$, and $p_i^d = c$, so long as (i) no other firm plays a direct price of c with positive probability and (ii) $r_0 = 0$, $p_i^d = c$ (all other direct prices are strictly dominated by v). When $\underline{c}_1 < c = \underline{c}_2$, playing $p_i^d = c$ for $i \geq 2$ would therefore require 1 to play a price $p_1^d \leq c$ with probability zero, but this is not possible in equilibrium because $c > \underline{c}_1$, hence $p_i^d = v$ for $i \geq 2$. This again leaves us with the unique equilibrium. Let $n = 2$ for the lemma. Footnote 35 explains how uniqueness fails when $\underline{c}_1 = c \leq \underline{c}_2$. \square

Proof of Lemma 4. We show the strategies of Lemma 4 constitute an equilibrium when $\lambda_1 < \lambda_2$. For the uniqueness proof and the special case $\lambda_1 = \lambda_2$, see the Web Appendix.

Suppose $n = 2$. When seller 1 sets $p_1^d \in [\underline{p}, c]$, $\pi_1(p_1^d) = p_1^d \{ \lambda_1 + \mu [1 - F_2(p_1^d)] \}$. Firm 1's profits when $p_1^d = \underline{p} \equiv \frac{v\lambda_2}{\lambda_2 + \mu}$ are $\pi_1(\underline{p}) = \frac{v\lambda_2}{\lambda_2 + \mu} (\lambda_1 + \mu)$. Setting $\pi_1(p_1^d) = \pi_1(\underline{p})$:

$$F_2(p) = \frac{\mu p - \lambda_2(v - p)}{\mu p} \frac{\mu + \lambda_1}{\mu + \lambda_2}.$$

Similarly, when seller 2 sets $p_2^d \in [\underline{p}, c]$, its profit is $\pi_2(p_2^d) = p_2^d \{ \lambda_2 + \mu [1 - F_1(p_2^d)] \}$. Seller 2's profits when $p_2^d = v$ are $\pi_2(v) = v\lambda_2$. Setting $\pi_2(p_2^d) = \pi_2(v)$:

$$F_1(p) = \frac{\mu p - \lambda_2(v - p)}{\mu p}.$$

By construction, F_2 ensures 1 is indifferent over every $p_1^d \in [\underline{p}, c]$ and F_1 makes 2 indifferent over every $p_2^d \in [\underline{p}, c) \cup v$. Notice seller 1's strategy includes an atom at c when $c < v$. If $r_0 > 0$, F_1 cannot be part of an equilibrium because seller 1 could profitably deviate downwards slightly in its direct price. Hence, $r_0 = 0$. When $c = v$, there is no atom, CC prices are never the lowest on the market and so any $r_0 \in [0, 1]$ can be supported.

We now need to check that no seller can profit from a deviation outside of its support. Deviations to $p_i^d > v$ yield zero profit and are not profitable. Neither seller can profit from a deviation to $p_i^d < \underline{p}$: this would result in the same demand as $p_i^d = \underline{p}$ but at a lower price. Any $p_i^d \in (c, v)$ is greater than the lowest price on the CC and therefore only attracts captives. Such prices are dominated by $p_i^d = v$. Since 1 has a mass point at c , $p_2^d = c$ induces a tie and yields strictly lower expected profit than does $p_2^d = c - \epsilon$ (for

$\epsilon > 0$ small). Lastly, we check that seller 1 cannot profit from a deviation to $p_1^d = v$. We have $\pi_1(v) > \pi_1(\underline{p}) \Leftrightarrow v\lambda_1 > \frac{v\lambda_2}{\lambda_2 + \mu}(\lambda_1 + \mu)$, which fails because $\lambda_1 < \lambda_2$. This gives the strategies stated in Lemma 4.

When $n > 2$, there is an equilibrium where, in addition to the strategies derived above, sellers $i > 2$ set $p_i = c$, $p_i^d = v$ and earn profit $v\lambda_i$. We now show that these sellers have no profitable deviations. A deviation to $p_i^d \in (c, v)$ fails to attract any shoppers and is not profitable. A deviation to $p_i^d = c$ induces ties with seller 1's mass point, and are therefore dominated by deviations to $p_i^d = c - \epsilon$ for some small ϵ . Deviations to $p_i^d \in [\underline{p}, c)$ yield profit $p_i^d \{\lambda_i + \mu [1 - F_1(p_i^d)] [1 - F_2(p_i^d)]\}$. We observe that (i) seller i earns lower profit from $p_i^d \in [\underline{p}, c)$ than does seller 2; and (ii) $v\lambda_i \geq v\lambda_2$. Since seller 2 is indifferent between $p_2^d \in [\underline{p}, c)$ and $p_2^d = v$, seller $i > 2$ must strictly prefer the latter. \square

Proof of Proposition 1. When $c \in [0, \underline{c}_1)$, $\pi_0 = c(\lambda_0 + \mu)$ by Lemma 2. The equilibrium fee, c , cannot fall in this range: if it did, the competitive channel would have a strictly profitable deviation to $c + \epsilon < \underline{c}_1$ for $\epsilon > 0$. When $c \in (\underline{c}_1, v)$, $\pi_0 = c\lambda_0$ by Lemmas 3 and 4. The equilibrium fee, c , cannot fall in this range: if it did, the competitive channel would have a strictly profitable deviation to $c + \epsilon < v$ for $\epsilon > 0$. Therefore, the only values of c possible in equilibrium are \underline{c}_1 and v .

When $c = \underline{c}_1$, either $\pi_0 = \underline{c}_1(\lambda_0 + \mu)$ by Lemma 2, or $\pi_0 = \underline{c}_1\lambda_0$ by Lemma 3. However, the prices of Lemma 3 cannot follow $c = \underline{c}_1$ in equilibrium: if it did, the CC would have a strictly profitable deviation to $c - \epsilon$ for some small $\epsilon > 0$. When $c = v$, Lemma 4 applies and $\pi_0 = v\lambda_0$. We are left with two equilibrium candidates: (i) $c = \underline{c}_1$ and firms price in accordance with Lemma 2; or (ii) $c = v$ and firms price in accordance with Lemma 4. Case (i) constitutes an equilibrium if the CC does not profit from a deviation to v , i.e., if $\underline{c}_1(\lambda_0 + \mu) \geq v\lambda_0 \Leftrightarrow \lambda_1 \geq \lambda_0$. Case (ii) constitutes an equilibrium if the CC does not profit from a deviation to any $c \in [0, \underline{c}_1]$. This is ensured when $v\lambda_0 \geq \underline{c}_1(\lambda_0 + \mu) \Leftrightarrow \lambda_0 \geq \lambda_1$. \square

B Analysis

Proof of Proposition 2. The first statement follows by inspection of (4). For the second, consider the change in consumer surplus from $\boldsymbol{\lambda}$ to $\boldsymbol{\lambda}'$ such that $\lambda'_1 < \lambda'_2$:

$$\frac{v(\mu - \Delta(\lambda_\mu, \lambda_0))(\lambda_1 - \Delta(\lambda_1, \lambda_0) + \mu - \Delta(\lambda_\mu, \lambda_0))}{\lambda_2 - \Delta(\lambda_2, \lambda_0) + \mu - \Delta(\lambda_\mu, \lambda_0)} - \frac{(\lambda_0 + \mu)v\mu}{\lambda_1 + \mu}, \quad (5)$$

where, as defined in the main text prior to Corollary 1, when we move from $\boldsymbol{\lambda}$ to $\boldsymbol{\lambda}'$, $\Delta(\alpha, \beta) \in \mathbb{R}$ denotes the size of a reduction in α and increase in β such that $\alpha + \beta$ remains constant. We have $\|\boldsymbol{\lambda} - \boldsymbol{\lambda}'\| = \sqrt{(\lambda_0 - \lambda'_0)^2 + \Delta(\lambda_1, \lambda_0)^2 + \Delta(\lambda_2, \lambda_0)^2 + \Delta(\lambda_\mu, \lambda_0)^2} < \epsilon$, which implies $|\lambda_0 - \lambda'_0|$, $|\Delta(\lambda_1, \lambda_0)|$, $|\Delta(\lambda_2, \lambda_0)|$, and $|\Delta(\lambda_\mu, \lambda_0)|$ must all be small for ϵ small. Moreover, this observation along with $\lambda_1 - \lambda_0 > 0 > \lambda'_1 - \lambda'_0$ implies that $\lambda_1 - \lambda_0$

must approach zero with ϵ . Letting $\lambda_0 \rightarrow \lambda_1$ and $\Delta(\lambda_1, \lambda_0), \Delta(\lambda_2, \lambda_0), \Delta(\lambda_\mu, \lambda_0) \rightarrow 0$, (5) becomes $[(\lambda'_1 + \mu)/(\lambda'_2 + \mu) - 1] v\mu < 0$. \square

Proof of Proposition 3. Part 1. For the purposes of this proof, index the two sellers i and j rather than 1 and 2 to distinguish between their identities and their relative sizes. For seller i and any λ there exists some Δ_i^* such that $\lambda'_0 = \lambda'_i$.

- If $\lambda'_i < \lambda_j$: then consider values $\Delta(\lambda_i, \lambda_0)$ close to Δ_i^* ,

$$CS = \begin{cases} v\mu \frac{\lambda_0 + \Delta(\lambda_i, \lambda_0) + \mu}{\lambda_i - \Delta(\lambda_i, \lambda_0) + \mu} & \text{for } \Delta(\lambda_i, \lambda_0) < \Delta_i^*; \text{ increasing in } \Delta(\lambda_i, \lambda_0) \\ v\mu \frac{\lambda_i - \Delta(\lambda_i, \lambda_0) + \mu}{\lambda_j + \mu} & \text{for } \Delta(\lambda_i, \lambda_0) > \Delta_i^*; \text{ decreasing in } \Delta(\lambda_i, \lambda_0) \end{cases}$$

hence consumer surplus is non-monotonic in $\Delta(\lambda_i, \lambda_0)$.

- If $\lambda'_i = \lambda_j$: then consider values $\Delta(\lambda_i, \lambda_0)$ close to Δ_i^* ,

$$CS = \begin{cases} v\mu \frac{\lambda_0 + \Delta(\lambda_i, \lambda_0) + \mu}{\lambda_j + \mu} & \text{for } \Delta(\lambda_i, \lambda_0) < \Delta_i^*; \text{ increasing in } \Delta(\lambda_i, \lambda_0) \\ v\mu \frac{\lambda_i - \Delta(\lambda_i, \lambda_0) + \mu}{\lambda_j + \mu} & \text{for } \Delta(\lambda_i, \lambda_0) > \Delta_i^*; \text{ decreasing in } \Delta(\lambda_i, \lambda_0) \end{cases}$$

hence consumer surplus is non-monotonic in $\Delta(\lambda_i, \lambda_0)$.

- If $\lambda'_i > \lambda_j$: then there is Δ_i^+ such that $\lambda'_0 = \lambda_j < \lambda'_i$. Consider values $\Delta(\lambda_i, \lambda_0)$ close to Δ_i^+ ,

$$CS = \begin{cases} v\mu \frac{\lambda_0 + \Delta_i^+ + \mu}{\lambda_j + \mu} = v\mu & \text{for } \Delta(\lambda_i, \lambda_0) \uparrow \Delta_i^+ \\ v\mu \frac{\lambda_j + \mu}{\lambda_i - \Delta_i^+ + \mu} < v\mu & \text{for } \Delta(\lambda_i, \lambda_0) \downarrow \Delta_i^+ \end{cases}$$

hence consumer surplus is non-monotonic in $\Delta(\lambda_i, \lambda_0)$.

Part 2. Denote Δ_\emptyset^* the value of $\Delta(\emptyset, \lambda_0)$ such that $\lambda'_0 = \lambda_1$ and consider value of $\Delta(\emptyset, \lambda_0)$ close to Δ_\emptyset^* . The jump discontinuity in consumer surplus is of size

$$\lim_{\Delta(\emptyset, \lambda_0) \downarrow \Delta_\emptyset^*} CS - \lim_{\Delta(\emptyset, \lambda_0) \uparrow \Delta_\emptyset^*} CS = \frac{(\lambda_1 - \lambda_2)v\mu}{\lambda_2 + \mu}$$

which is strictly negative for $\lambda_1 < \lambda_2$. When $\lambda_1 = \lambda_2$, consumer surplus is increasing in $\Delta(\emptyset, \lambda_0)$ up to Δ_\emptyset^* , then remains constant thereafter.

Part 3. For $\Delta(\lambda_\mu, \lambda_0)$ such that $\lambda'_0 < \lambda_1$: from (4), consumer surplus is decreasing in $\Delta(\lambda_\mu, \lambda_0)$. For $\Delta(\lambda_\mu, \lambda_0)$ such that $\lambda'_0 > \lambda_1$: from (4), consumer surplus is decreasing in $\Delta(\lambda_\mu, \lambda_0)$. Denote the value of $\Delta(\lambda_\mu, \lambda_0)$ such that $\lambda'_0 = \lambda_1$ as Δ_μ^* and observe that the

jump discontinuity is weakly negative:

$$\lim_{\Delta(\lambda_\mu, \lambda_0) \downarrow \Delta_\mu^*} CS - \lim_{\Delta(\lambda_\mu, \lambda_0) \uparrow \Delta_\mu^*} CS = \frac{(\lambda_1 - \lambda_2)v\mu}{\lambda_2 + \mu},$$

hence consumer surplus is everywhere decreasing in $\Delta(\lambda_\mu, \lambda_0)$. \square

C Policy

C.1 Equilibrium with MFNs

Here we first solve the game from $t = 2$, assuming MFNs are imposed by the CC; i.e., if seller i lists, $p_i^d \geq p_i$. The equilibria, which are again separated by their level of c similarly to Lemmas 2 to 4 in the baseline save the differences we describe here.

Firstly, sellers who undercut the prices on the CC do not list on the CC. We write $p_i = \emptyset$ to denote a firm not listing on the CC. Secondly, there are now $(1 - s)\mu$ shoppers and $s\mu$ showroomers; in the baseline, $s > 0$ is equivalent to $s = 0$. In addition, we let $n > 3$ to rule-out implausible outcomes with only one seller listed on the CC. Also, because there are now two types of consumers who are active across channels, we need two tie-breaking rules for when there is a tie for the lowest price across channels. Accordingly, we let the probability that true shoppers and showroomers buy through the CC in the case of a such a tie be denoted r_0^t and r_0^s . For brevity, we let $r_0^t = 0$ and $r_0^s = 1$. The former ensures equilibrium exists in all subgames starting at $t = 2$ as in the baseline's Lemmas 2 to 4 (where there are only true shoppers).³⁶ The latter best gels with the intuition for MFNs and showrooming: under $r_0^s = 1$, there is no way for a firm to get showroomers to buy direct with MFNs in place, i.e., MFNs completely prevent showrooming.³⁷ Finally, it is also useful to denote \underline{c}'_i as the c which solves $v\lambda_i = c(\lambda_i + (1 - s)\mu)$, which is analogous to \underline{c}_i in the baseline without MFNs i.e, where $s = 0$.

Lemma C1 (Direct pricing 1 under MFNs). *Suppose $0 \leq c \leq \underline{c}'_1$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = \dots = p_n = c$, $p_1^d = \dots = p_n^d = v$. The resulting equilibrium profits are $\pi_0 = c(\lambda_0 + \mu)$, and $\pi_i = v\lambda_i$ for $i \geq 1$.*

³⁶If $r_0^t > 0$, then as in the baseline, there are subgames in which equilibrium fails to exist because the sellers' profit is downwards-discontinuous at $p_i^d = c$. This leaves the firm wanting to play $\max\{p_i^d : p_i^d < c\}$; not well-defined. When $r_0^t = 0$, equilibrium direct price can be c . Therefore, the tie-break rule of $r_0^t = 0$ is simply a tool which allows firms to undercut the CC in a technically well-defined way in equilibrium.

³⁷An equilibrium with MFNs exists for any $r_0^s \in [0, 1]$. However, it is not meaningful to study MFNs under $r_0^s = 0$. Such a tie-break rule along with $r_0^t = 0$ would allow a seller to "undercut" with $p_i^d = c$ and attract both true shoppers and showroomers, just as in the baseline. In other words, an MFN only bites if $r_0^s > 0$. For any $r_0^s > 0$, undercutting the CC in direct price is less attractive for a firm because it wins fewer showroomers (who instead buy through the CC). As a result, the low-fee regime results for a smaller range of λ_0 and CC profit in both regimes increases. Therefore, for any $r_0^s > 0$, CC profit is higher and consumer surplus is lower than in the baseline model without MFNs (which is nested at $r_0^s = 0$). For simplicity, we set $r_0^s = 1$, but the substantive results are unchanged for any $r_0^s > 0$.

Proof. The proposed equilibrium leaves each seller with profit $v\lambda_i$. No profitable deviation involving de-listing exists: A deviation $\hat{p}_i^d \leq c$ yields profit no greater than $c(\lambda_i + (1-s)\mu)$, which is less than $v\lambda_i$ because $c \leq \underline{c}'_i$; any $\hat{p}_i^d \in (c, v)$ yields less than $\hat{p}_i^d = v$; and $\hat{p}_i^d = v$ gives the same profit as in equilibrium.

No profitable deviation involving listing exists for a seller. To see this, first note that, due to the MFN, $\hat{p}_i^d \geq \hat{p}_i$. If $\hat{p}_i \leq \hat{p}_i^d \leq c$ then i 's profit is at most $c(\lambda_i + (1-s)\mu)$, which is no greater than equilibrium profit because $c \leq \underline{c}_i$. If $\hat{p}_i^d > c$ then any such deviation leaves the firm serving only captives with its direct price, hence $\hat{p}_i^d = v$ is part of the optimal such deviation. However, for $p_i^d = v$: deviations with $\hat{p}_i < c$ yield negative profit from shoppers and hence are dominated by $p_i = c$; deviations with $\hat{p}_i^d > c$ do not serve shoppers and thus leave profit unchanged. \square

Lemma C2 (Direct pricing 2 under MFNs). *Suppose $\underline{c}'_1 \leq c \leq \underline{c}'_2$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = \emptyset$, $p_2 = \dots = p_n = c$, $p_1^d = c$, and $p_2^d = \dots = p_n^d = v$. The resulting equilibrium profits are $\pi_0 = c(\lambda_0 + s\mu)$, $\pi_1 = c(\lambda_1 + \mu(1-s))$, and $\pi_i = v\lambda_i$ for $i \geq 2$.*

Proof. Seller 1 has no profitable deviation involving delisting: For any $\hat{p}_1^d \leq c$, its profit is $\hat{p}_1^d(\lambda_1 + \mu(1-s)) \leq c(\lambda_1 + \mu(1-s))$; and, for any $\hat{p}_1^d \in (c, v]$, its profit is $\hat{p}_1^d\lambda_1 \leq v\lambda_1$. Given $\underline{c}'_1 \leq c$, $v\lambda_1 \leq c(\lambda_1 + \mu(1-s))$.

Seller 1 has no profitable deviation involving listing: If $\hat{p}_1 \geq 0$ then the MFN implies $\hat{p}_1 \leq \hat{p}_1^d$. If $\hat{p}_1 \leq \hat{p}_1^d \leq c$ then 1's profit is at most $c(\lambda_1 + (1-s)\mu)$. Suppose, instead, that $\hat{p}_1^d > c$. Then, firm 1 serves only captives via its direct channel and the best such deviation is $\hat{p}_1^d = v$. Given $\hat{p}_1^d = v$ and $p_2 = c$, there is no way to serve shoppers with \hat{p}_1 at a profit. Thus, firm 1's deviation profit if it lists is at most $v\lambda_1$, which is less than its equilibrium profit because $\underline{c}'_1 \leq c$.

Proof that seller $i > 1$ has no profitable deviation follows that of Lemma C1. \square

Lemma C3 (Direct pricing 3 under MFNs). *Suppose $\underline{c}'_2 \leq c \leq v$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = \emptyset$, $p_i = c$ and $p_i^d = v$ for $i > 2$, p_1^d and p_2^d mixed over supports $[\underline{p}, c]$ and $[\underline{p}, c] \cup v$ respectively via the strategies*

$$F_1(p) = \begin{cases} \frac{(1-s)\mu p - \lambda_2(v-p)}{(1-s)\mu p} & \text{for } p \in [\underline{p}, c) \\ 1 & \text{for } p \geq c, \end{cases}$$

$$F_2(p) = \begin{cases} \frac{(1-s)\mu p - \lambda_2(v-p)}{(1-s)\mu p} \frac{(1-s)\mu + \lambda_1}{(1-s)\mu + \lambda_2} & \text{for } p \in [\underline{p}, c) \\ \frac{(1-s)\mu c - \lambda_2(v-c)}{(1-s)\mu c} \frac{(1-s)\mu + \lambda_1}{(1-s)\mu + \lambda_2} & \text{for } p \in [c, v) \\ 1 & \text{for } p \geq v, \end{cases}$$

where $\underline{p} = \frac{v\lambda_2}{\lambda_2 + (1-s)\mu}$; $\pi_0 = c(\lambda_0 + s\mu)$, $\pi_1 = \frac{v\lambda_2(\lambda_1 + (1-s)\mu)}{\lambda_2 + (1-s)\mu}$, and $\pi_i = v\lambda_i$ for $i \geq 2$.

Proof. Sellers 1 and 2 have no profitable deviation. To see this, and to derive F_1, F_2 , the steps in the proof of Lemma 4 can be followed. The only new deviations to check involve 1 or 2 choosing to list a $\hat{p}_i \leq \hat{p}_i^d$. If $\hat{p}_1 \leq \hat{p}_1^d \leq c$ then 1's profit is at most $\hat{p}_1^d[\lambda_1 + (1-s)\mu(1 - F_2(\hat{p}_1^d))]$ (i.e., at most equal to equilibrium profit). Suppose, instead, that $\hat{p}_1^d > c$. Then, firm 1 serves only captives via its direct channel and the best such deviation is $\hat{p}_1^d = v$. Given $\hat{p}_1^d = v$ and $p_3 = c$, there is no way to serve shoppers with \hat{p}_1 at a profit. Thus, firm 1's deviation profit if it lists is at most $v\lambda_1$, which is not more than its equilibrium profit because $\underline{c}'_1 \leq c$. The same argument rules out a deviation by firm 2.

For sellers $i > 2$, there is no profitable deviation involving listing. Start with deviations involving delisting. Any $\hat{p}_i^d \in (c, v)$ yields less than $\hat{p}_i^d = v$, and $\hat{p}_i^d = v$ gives the same profit as equilibrium. To see that no $\hat{p}_i^d \leq c$ is profitable, note that for such a deviation with $\hat{p}_i^d \in [\underline{p}, c)$ yields profit equal to:

$$\hat{\pi}(\hat{p}_i^d) = \hat{p}_i^d \lambda_i + \hat{p}_i^d \mu (1-s) (1 - F_1(\hat{p}_i^d)) (1 - F_2(\hat{p}_i^d))$$

which is convex in \hat{p}_i^d . Therefore, if (i) $\hat{\pi}(\underline{p}) \leq v\lambda_i$ and (ii) $\hat{\pi}(c) \leq v\lambda_i$ there is no profitable such deviation. Computing: (i) $\hat{\pi}(\underline{p}) = v\lambda_2 \frac{\lambda_i + (1-s)\mu}{\lambda_2 + (1-s)\mu} \leq v\lambda_i$ because $\lambda_2 \leq \lambda_i$; (ii) $\hat{\pi}(c) \leq v\lambda_i \Leftrightarrow c \geq \bar{c}$,

$$\bar{c} = v \frac{\lambda_2^2 (\lambda_1 + (1-s)\mu)}{(\lambda_2 + (1-s)\mu)(\lambda_1 \lambda_2 + \lambda_i (1-s)\mu)},$$

which is always satisfied because $\bar{c} \leq \underline{c}'_2$. For deviations involving listing: the same argument as above implies the best a deviating firm can do is $p_i \leq p_i^d \leq c$ with $p_i^d \in \{\underline{p}, c\}$ (which is worse than delisting and setting the same direct price); or $p_i^d \in (c, v]$ (in which case we must have $p_i^d = v$ and profits are no greater than in equilibrium). \square

Proof of Proposition 7. When $c \leq \underline{c}'_1$, $\pi_0 = c(\lambda_0 + \mu)$ by Lemma C1, maximized at $c = \underline{c}'_1$. For $\underline{c}'_1 < c \leq v$, $\pi_0 = c(\lambda_0 + s\mu)$ by Lemma C2 and Lemma C3, maximized at $c = v$. Note that for $c = \underline{c}'_1$, the pricing of Lemma C2 cannot occur in equilibrium: if it did, the CC would prefer to slightly reduce its fee to induce the pricing of Lemma C1. Rearranging terms shows that the CC prefers $c = \underline{c}'_1$ to $c = v$ when $\lambda_0 \geq \lambda_1 - s\mu$.

To show that the CC wants to impose MFNs for any market composition: When $\lambda_0 \leq \lambda_1 - s\mu$, the low fee regime results with and without MFNs. The CC prefers MFNs because $\underline{c}'_1(\lambda_0 + \mu) \geq \underline{c}_1(\lambda_0 + \mu)$. When $\lambda_0 \in [\lambda_1 - s\mu, \lambda_1]$, the high (low) fee regime results with (without) MFNs. The CC prefers MFNs because $v(\lambda_0 + s\mu) \geq \underline{c}_1(\lambda_0 + \mu) \Leftrightarrow \lambda_0 \geq \lambda_1 - s\mu - s\lambda_1$. When $\lambda_0 \geq \lambda_1$ the high fee regime results with and without MFNs. The CC prefers MFNs because $v(\lambda_0 + s\mu) \geq v\lambda_0$.

Now we show that MFNs reduce consumer surplus for any market composition. Under

MFNs, consumer surplus is given by:

$$\text{CS} = \begin{cases} v\mu(1-s)\frac{\lambda_0 + \mu}{\lambda_1 + (1-s)\mu} & \text{for } \lambda_0 \leq \lambda_1 - s\mu \\ v\mu(1-s)\frac{\lambda_1 + (1-s)\mu}{\lambda_2 + (1-s)\mu} & \text{for } \lambda_0 \geq \lambda_1 - s\mu. \end{cases} \quad (6)$$

Equation (4) provides the analogous expressions without MFNs. Comparing expressions piece-wise for the cases $\lambda_0 \leq \lambda_1 - s\mu$, $\lambda_0 \in [\lambda_1 - s\mu, \lambda_1]$, and $\lambda_0 \geq \lambda_1$ shows that surplus is lower when MFNs are imposed. \square

C.2 Equilibrium with cross-channel integration

We consider integration across channels, firstly considering an entity formed of 0 and 1, then 0 and 2. We denote the integrated entities 01 and 02 respectively. We first present equilibrium pricing strategies for the firms for a given c . We then consider the optimal c set in $t = 1$, which completes the equilibrium characterization.

C.2.1 01 Integration

Lemma C4 (01 low-fee regime). *Suppose $0 \leq c \leq \underline{c}_2$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, $p_1^d = p_2^d = v$ and any $r_0 \in [0, 1]$. The resulting equilibrium profits are $\pi_{01} = c(\lambda_0 + \mu) + v\lambda_1$, $\pi_2 = v\lambda_2$.*

Proof. 01's profit is $c(\mu + \lambda_0) + v\lambda_1$. It cannot earn more than v from λ_1 's captives. It also cannot earn more than c from shoppers or CC captives because an increase in p_1 would result in those consumers buying from seller 2. For seller 2 the deviation checks are the same as in Lemma 2. \square

Lemma C5 (01 high-fee regime). *Suppose $\underline{c}_2 < c \leq v$ and $\lambda_0 \leq \lambda_1$; or $\underline{c}_2 < c \leq v\frac{\lambda_0\lambda_2 + \mu\lambda_1}{\lambda_0\lambda_2 + \mu\lambda_0}$ and $\lambda_0 \geq \lambda_1$. An equilibrium of the subgame starting at $t = 2$ has $p_2 = c$, $p_1^d = v$; p_1 and p_2^d are mixed over supports $[\underline{c}_2, c]$ and $[\underline{c}_2, c] \cup v$ respectively via the strategies*

$$F_{01}(p) = \begin{cases} 1 - \frac{\lambda_2(v-p)}{\mu p} & \text{for } p \in [\underline{c}_2, c) \\ 1 & \text{for } p \geq c, \end{cases}$$

$$F_2(p) = \begin{cases} 1 - \frac{\lambda_0(\underline{c}_2 - p) + \mu \underline{c}_2}{\mu p} & \text{for } p \in [\underline{c}_2, c) \\ 1 - \frac{\lambda_0(\underline{c}_2 - c) + \mu \underline{c}_2}{\mu c} & \text{for } p \in [c, v) \\ 1 & \text{for } p \geq v, \end{cases}$$

and any $r_0 \in [0, 1]$. Profits are $\pi_{01} = \underline{c}_2(\lambda_0 + \mu) + v\lambda_1$ and $\pi_2 = v\lambda_2$.

Proof. In order for 2 to be indifferent over all direct prices in its support, $p \in [\underline{c}_2, c] \cup v$, $v\lambda_2 = p\lambda_2 + p\mu(1 - F_{01}(p))$, which is rearranged to give F_{01} . Residual mass is allocated to c . In order for 01 to be indifferent over all prices in its support, we equate profit when it charges the CC price \underline{c}_2 (and the direct price $p_1^d = v$) to profit when it charges some CC price $p \in [\underline{c}_2, c]$ (and the direct price $p_1^d = v$): $\underline{c}_2(\lambda_0 + \mu) + v\lambda_1 = p\lambda_0 + p\mu(1 - F_2(p)) + v\lambda_1$, which is rearranged to give F_2 . Residual mass is allocated to v . In order for F_2 to be well-defined over $[\underline{c}_2, c]$, we need that $1 - F_2(c) \geq 0$ which is satisfied either when both $\lambda_0 \leq \lambda_1$ and $c \leq v$ or when both $\lambda_0 \geq \lambda_1$ and $c \leq v \frac{\lambda_0\lambda_2 + \mu\lambda_1}{\lambda_0\lambda_2 + \mu\lambda_0}$.

Most deviation checks are similar to those in Lemma 4 and not repeated here. However, new deviations to consider include those by the integrated entity, which may deviate in both its prices (p_1 or p_1^d). To see that 01 has no such profitable deviation, first note that there are two cases to consider. Case (i): if $p_1 \leq p_1^d$ then p_1^d could be increased to v to yield extra profit on captives without sacrificing profit from other types of consumer; so such a deviation must have $p_1 \leq p_1^d = v$. Case (ii): if $p_1^d < p_1$ then p_1 serves only captive consumers from whom 01 earns $\min\{p_1, p_2\}\lambda_0 = \min\{p_1, c\}\lambda_0$; thus, any such deviation must have $p_1^d < p_1 = c$. Firm 01 prefers to set a CC price of p (and a direct price of v) rather than a direct price of p (and a CC price of c) if:

$$v\lambda_1 + p\lambda_0 + p\mu(1 - F_2(p)) \geq p\lambda_1 + c\lambda_0 + p\mu(1 - F_2(p)) \Leftrightarrow \lambda_1(v - p) \geq \lambda_0(c - p), \quad (7)$$

satisfied if $\lambda_0 \leq \lambda_1$, or if both $\lambda_0 \geq \lambda_1$ and $p \geq \frac{c\lambda_0 - v\lambda_1}{\lambda_0 - \lambda_1}$. When $\lambda_0 \geq \lambda_1$, note $\underline{c}_2 \geq \frac{c\lambda_0 - v\lambda_1}{\lambda_0 - \lambda_1}$ rearranges to $c \leq v \frac{\lambda_0\lambda_2 + \mu\lambda_1}{\lambda_0\lambda_2 + \mu\lambda_0}$, so 01 has no such profitable deviation. There is no equilibrium restriction on r_0 because no firm charges c in direct price with positive probability. \square

Lemma C6 (01 high-fee regime). *Suppose $v \frac{\lambda_0\lambda_2 + \mu\lambda_1}{\lambda_0\lambda_2 + \mu\lambda_0} < c \leq v$ and $\lambda_0 \geq \lambda_1$. An equilibrium of the subgame starting at $t = 2$ has $p_2 = c$. Denote $p_{min} = \min\{p_1, p_1^d\}$, $p_{max} = \max\{p_1, p_1^d\}$ and $\underline{p} = \frac{c\lambda_0 - v\lambda_1}{\lambda_0 - \lambda_1}$. Prices p_{min} , p_{max} and p_2^d are mixed over supports $[\underline{c}_2, c]$, $\{c, v\}$ and $[\underline{c}_2, c] \cup v$ respectively. Price p_{min} is distributed according to*

$$F_{min}(p) = \begin{cases} 1 - \frac{\lambda_2(v-p)}{\mu p} & \text{for } p \in [\underline{c}_2, c) \\ 1 & \text{for } p \geq c. \end{cases}$$

When $p_{min} \in [\underline{c}_2, \underline{p}]$, $p_1^d = p_{min}$ and $p_1 = p_{max} = c$; when $p_{min} \in [\underline{p}, c]$, $p_1 = p_{min}$ and $p_1^d = p_{max} = v$. Price p_2^d is distributed according to

$$F_2(p) = \begin{cases} 1 - \frac{\lambda_1(\underline{c}_2 - p) + \mu \underline{c}_2}{p\mu} & \text{for } p \in [\underline{c}_2, \underline{p}) \\ 1 - \frac{\lambda_1(\underline{c}_2 - v) + \mu \underline{c}_2 + \lambda_0(c - p)}{p\mu} & \text{for } p \in [\underline{p}, c) \\ 1 - \frac{\lambda_1(\underline{c}_2 - v) + \mu \underline{c}_2}{c\mu} & \text{for } p \in [c, v) \\ 1 & \text{for } p \geq v. \end{cases}$$

Any $r_0 \in [0, 1]$ is supported. Profits are $\pi_{01} = \underline{c}_2(\lambda_1 + \mu) + c\lambda_0$ and $\pi_2 = v\lambda_2$.

Proof. F_{min} is derived in the same way as F_{01} in Lemma C5. In order for 01 to be indifferent over all prices in its support where it fights for shoppers with its direct price, we equate profit when it charges the direct price \underline{c}_2 (and the CC price $p_1 = c$) to profit when it charges some direct price p (and the CC price $p_1 = c$): $\underline{c}_2(\lambda_1 + \mu) + c\lambda_0 = p\lambda_1 + p\mu(1 - F_2(p)) + c\lambda_0$, which is rearranged to give the first expression defining F_2 . In order for 01 to be indifferent over all prices in its support where it fights for shoppers with its CC price, we equate profit when it charges the direct price \underline{c}_2 (and the CC price $p_1 = c$) to profit when it charges some CC price p (and the direct price $p_1^d = v$): $p\lambda_0 + p\mu(1 - F_2(p)) + v\lambda_1 = \underline{c}_2(\lambda_1 + \mu) + c\lambda_0$, which is rearranged to give the second expression defining F_2 . Residual mass is allocated to v .

Other deviation checks mirror those in Lemma C5 and not repeated here. At \underline{p} , firm 01 is indifferent between fighting for shoppers with direct rather than its CC price, and is that which equates the expression in (7), i.e., $\underline{p} = \frac{c\lambda_0 - v\lambda_1}{\lambda_0 - \lambda_1}$. (Firm 01 prefers fighting with its direct price rather than its CC price when $p < \underline{p}$.) Therefore, both strategies are employed in equilibrium when $\underline{p} \in [\underline{c}_2, v]$, which occurs when $c \geq v \frac{\lambda_0\lambda_2 + \mu\lambda_1}{\lambda_0\lambda_2 + \mu\lambda_0}$ and $\lambda_0 \geq \lambda_1$. \square

Proof of Proposition 8. Firstly, we solve for the optimal choice of c by 01. That provides the equilibrium strategies and payoffs, allowing consumer surplus to be computed.

Suppose $\lambda_0 \leq \lambda_1$. From Lemmas C4 and C5:

$$\pi_{01} = \begin{cases} c(\lambda_0 + \mu) + v\lambda_1 & \text{if } c \in [0, \underline{c}_2] \\ \underline{c}_2(\lambda_0 + \mu) + v\lambda_1 & \text{if } c \in (\underline{c}_2, v] \end{cases}$$

which is maximized at $\underline{c}_2(\lambda_0 + \mu) + v\lambda_1$ by any $c \in [\underline{c}_2, v]$. Equilibrium strategies are described by Lemma C4 or C5 accordingly. Suppose $\lambda_0 \geq \lambda_1$. From Lemmas C4 to C6:

$$\pi_{01} = \begin{cases} c(\lambda_0 + \mu) + v\lambda_1 & \text{if } c \in [0, \underline{c}_2] \\ \underline{c}_2(\lambda_0 + \mu) + v\lambda_1 & \text{if } c \in \left(\underline{c}_2, v \frac{\lambda_0\lambda_2 + \lambda_1\mu}{\lambda_0\lambda_2 + \lambda_0\mu} \right] \\ \underline{c}_2(\lambda_1 + \mu) + c\lambda_0 & \text{if } c \in \left(v \frac{\lambda_0\lambda_2 + \lambda_1\mu}{\lambda_0\lambda_2 + \lambda_0\mu}, v \right] \end{cases}$$

maximized at $\underline{c}_2(\lambda_1 + \mu) + v\lambda_0$ by $c = v$. Equilibrium strategies are described by Lemma C6.

Consumer surplus is $v(\lambda_0 + \lambda_1 + \lambda_2 + \mu)$ minus firms' equilibrium profits. Suppose $\lambda_0 \leq \lambda_1$. In equilibrium, $\pi_{01} = \underline{c}_2(\lambda_0 + \mu) + v\lambda_1$ and $\pi_2 = v\lambda_2$. Suppose $\lambda_0 \geq \lambda_1$. In equilibrium, $\pi_{01} = \underline{c}_2(\lambda_1 + \mu) + v\lambda_0$ and $\pi_2 = v\lambda_2$. Computing yields

$$CS = \begin{cases} v\mu \frac{\lambda_0 + \mu}{\lambda_2 + \mu} & \text{for } \lambda_0 \leq \lambda_1 \\ v\mu \frac{\lambda_1 + \mu}{\lambda_2 + \mu} & \text{for } \lambda_0 \geq \lambda_1. \end{cases} \quad (8)$$

Comparison of (8) to (4) shows that the integration of 0 & 1 causes consumer surplus to fall when $\lambda_0 \leq \lambda_1$, and leaves it unchanged otherwise. \square

C.2.2 02 Integration

Lemma C7 (02 low-fee regime). *Suppose $0 \leq c \leq \underline{c}_1$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, $p_1^d = p_2^d = v$ and any $r_0 \in [0, 1]$. The resulting equilibrium profits are $\pi_{02} = c(\lambda_0 + \mu) + v\lambda_2$, $\pi_1 = v\lambda_1$.*

Proof. Identical to that of Lemma C4, with the roles of sellers 1 and 2 switched. \square

Lemma C8 (02 high-fee regime). *Suppose $\underline{c}_1 < c \leq v$ and $\lambda_0 \leq \lambda_1$; or $\underline{c}_1 < c \leq v \frac{\lambda_1}{\lambda_1 + \mu} \frac{\lambda_0 + \mu}{\lambda_0}$ and $\lambda_0 \geq \lambda_1$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = c$, $p_2^d = v$; p_1^d and p_2 are mixed over supports $[\underline{c}_1, c] \cup v$ and $[\underline{c}_1, c]$ respectively via the strategies*

$$F_1(p) = \begin{cases} 1 - \frac{\lambda_0(\underline{c}_1 - p) + \mu \underline{c}_1}{\mu p} & \text{for } p \in [\underline{c}_1, c) \\ 1 - \frac{\lambda_0(\underline{c}_1 - c) + \mu \underline{c}_1}{\mu c} & \text{for } p \in [c, v) \\ 1 & \text{for } p \geq v, \end{cases}$$

$$F_{02}(p) = \begin{cases} 1 - \frac{\lambda_1(v - p)}{\mu p} & \text{for } p \in [\underline{c}_1, c) \\ 1 & \text{for } p \geq c, \end{cases}$$

and any $r_0 \in [0, 1]$. Profits are $\pi_{02} = \underline{c}_1(\lambda_0 + \mu) + v\lambda_2$ and $\pi_1 = v\lambda_1$.

Proof. In order for 1 to be indifferent over all direct prices in its support, $p \in [\underline{c}_1, c] \cup v$, $v\lambda_1 = p\lambda_1 + p\mu(1 - F_{02})$, which gives F_{02} . Residual mass is allocated to c . In order for 02 to be indifferent over all prices in its support, we equate profit when it charges the CC price \underline{c}_1 (and the direct price $p_2^d = v$) to profit when it charges some CC price $p \in [\underline{c}_1, c]$ (and the direct price $p_2^d = v$): $\underline{c}_1(\lambda_0 + \mu) + v\lambda_2 = p\lambda_0 + p\mu(1 - F_1) + v\lambda_2$, which gives F_1 . Residual mass is allocated to v . In order for F_1 to be well-defined over $[\underline{c}_1, c]$, we need that $1 - F_1(c) \geq 0 \Leftrightarrow c \leq \frac{\lambda_1}{\lambda_1 + \mu} \frac{\lambda_0 + \mu}{\lambda_0}$ which is satisfied when $\lambda_0 \leq \lambda_1$ and $c \leq v$.

Most deviation checks are similar to those in Lemma C5, but with the integrated firm being 02 rather than 01, and are not repeated here. The analog of (7) is found by comparing where firm 02 prefers to set a CC price of p (and a direct price of v) rather than a direct price of p (and a CC price of c) if:

$$v\lambda_2 + p\lambda_0 + p\mu(1 - F_1(p)) \geq p\lambda_2 + c\lambda_0 + p\mu(1 - F_1(p)) \Leftrightarrow \lambda_1(v - p) \geq \lambda_0(c - p) \quad (9)$$

which is satisfied when $\lambda_0 \leq \lambda_2$ (and hence when $\lambda_0 \leq \lambda_1$). Expression (9) is satisfied with equality when $p = \frac{c\lambda_0 - v\lambda_2}{\lambda_0 - \lambda_2}$. When $\lambda_0 \geq \lambda_1$, note that $\underline{c}_1 \geq \frac{c\lambda_0 - v\lambda_2}{\lambda_0 - \lambda_2}$ is rearranged to $c \leq v \frac{\lambda_0 \lambda_1 + \mu \lambda_2}{\lambda_0 \lambda_1 + \mu \lambda_0}$. Because $v \frac{\lambda_1}{\lambda_1 + \mu} \frac{\lambda_0 + \mu}{\lambda_0} \leq v \frac{\lambda_0 \lambda_1 + \mu \lambda_2}{\lambda_0 \lambda_1 + \mu \lambda_0}$, 02 does not have such a profitable deviation when $c \leq v \frac{\lambda_1}{\lambda_1 + \mu} \frac{\lambda_0 + \mu}{\lambda_0}$. \square

Lemma C9 (02 high-fee regime). Suppose $v \frac{\lambda_1}{\lambda_1 + \mu} \frac{\lambda_0 + \mu}{\lambda_0} < c \leq v$ and $\lambda_1 \leq \lambda_0 \leq \lambda_2$; or $v \frac{\lambda_1}{\lambda_1 + \mu} \frac{\lambda_0 + \mu}{\lambda_0} < c \leq v \frac{\lambda_2}{\lambda_2 + \mu} \frac{\lambda_0 + \mu}{\lambda_0}$ and $\lambda_0 \geq \lambda_2$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = c$, $p_2^d = v$; p_1^d and p_2 are mixed over supports $\left[\frac{c\lambda_0}{\lambda_0 + \mu}, c \right]$ via the strategies

$$F_1(p) = 1 - \frac{\lambda_0(c-p)}{\mu p} \quad \text{for } p \in \left[\frac{c\lambda_0}{\lambda_0 + \mu}, c \right]$$

$$F_{02}(p) = \begin{cases} 1 - \frac{\lambda_1 \left(\frac{c\lambda_0}{\lambda_0 + \mu} - p \right) + \mu \frac{c\lambda_0}{\lambda_0 + \mu}}{\mu p} & \text{for } p \in \left[\frac{c\lambda_0}{\lambda_0 + \mu}, c \right] \\ 1 & \text{for } p \geq c \end{cases}$$

and any $r_0 \in [0, 1]$. Profits are $\pi_{02} = c\lambda_0 + v\lambda_2$ and $\pi_1 = \frac{c\lambda_0}{\lambda_0 + \mu}(\lambda_1 + \mu)$.

Proof. In order for 1 to be indifferent over all direct prices in its support, $p \in \left[\frac{c\lambda_0}{\lambda_0 + \mu}, c \right]$, $\frac{c\lambda_0}{\lambda_0 + \mu}(\lambda_1 + \mu) = p\lambda_1 + p\mu(1 - F_{02})$, which is rearranged to give F_{02} . Residual mass is allocated to c . In order for F_{02} to be well-defined over $[\underline{c}_1, c]$, we need that $1 - F_{02}(c) \geq 0 \Leftrightarrow \lambda_0 \geq \lambda_1$. In order for 02 to be indifferent over all prices in its support, we equate profit when it charges the CC price c (and the direct price $p_2^d = v$) to profit when it charges some CC price $p \in \left[\frac{c\lambda_0}{\lambda_0 + \mu}, c \right]$ (and the direct price $p_2^d = v$): $c\lambda_0 + v\lambda_2 = p\lambda_0 + p\mu(1 - F_1) + v\lambda_2$, which is rearranged to give F_1 . There is no residual mass.

Most deviation checks mirror those in Lemma C8. In addition we check 1 does not deviate to $p_1^d = v$: $\frac{c\lambda_0}{\lambda_0 + \mu}(\lambda_1 + \mu) \geq v\lambda_1 \Leftrightarrow c \geq v \frac{\lambda_1}{\lambda_1 + \mu} \frac{\lambda_0 + \mu}{\lambda_0}$. Finally, we know that for $\lambda_0 \leq \lambda_2$, (9) holds. When $\lambda_0 \geq \lambda_2$, note $v \frac{\lambda_1}{\lambda_1 + \mu} \frac{\lambda_0 + \mu}{\lambda_0} \geq \frac{c\lambda_0 - v\lambda_2}{\lambda_0 - \lambda_2}$ rearranges to $c \leq v \frac{\lambda_2}{\lambda_2 + \mu} \frac{\lambda_0 + \mu}{\lambda_0}$. \square

Lemma C10 (02 high-fee regime). Suppose $v \frac{\lambda_2}{\lambda_2 + \mu} \frac{\lambda_0 + \mu}{\lambda_0} < c \leq v$ and $\lambda_0 \geq \lambda_2$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = c$. Denote $p_{min} = \min\{p_2, p_2^d\}$, $p_{max} = \max\{p_2, p_2^d\}$ and $\underline{p} = \frac{c\lambda_0 - v\lambda_2}{\lambda_0 - \lambda_2}$. Prices p_{min} , p_{max} and p_2^d are mixed over supports $[\underline{c}_2, c]$, $\{c, v\}$ and $[\underline{c}_2, c] \cup v$ respectively. Price p_1^d is distributed according to

$$F_1(p) = \begin{cases} 1 - \frac{\lambda_2(v-p)}{p\mu} & \text{for } p \in [\underline{c}_2, \underline{p}] \\ 1 - \frac{\lambda_0(c-p)}{p\mu} & \text{for } p \in [\underline{p}, c] \end{cases}$$

Price p_{min} is distributed according to

$$F_{min}(p) = \begin{cases} 1 - \frac{\lambda_1(\underline{c}_2 - p) + \mu \underline{c}_2}{\mu p} & \text{for } p \in [\underline{c}_2, c) \\ 1 & \text{for } p \geq c. \end{cases}$$

When $p_{min} \in [\underline{c}_2, \underline{p}]$, $p_2^d = p_{min}$ and $p_2 = p_{max} = c$; when $p_{min} \in [\underline{p}, c]$, $p_2 = p_{min}$ and $p_2^d = p_{max} = v$. Any $r_0 \in [0, 1]$ is supported; $\pi_{02} = c\lambda_0 + v\lambda_2$ and $\pi_1 = \underline{c}_2(\lambda_1 + \mu)$.

Proof. F_{min} is derived in the same way as F_{02} in Lemma C9. In order for 02 to be indifferent over all prices in its support where it fights for shoppers with its direct price, we equate profit when it charges the CC price c (and the direct price $p_2^d = v$) to profit when

it charges some direct price p (and the CC price $p_2 = c$): $c\lambda_0 + v\lambda_2 = p\lambda_2 + p\mu(1 - F_1) + c\lambda_0$, which is rearranged to give the first expression defining F_1 . In order for 01 to be indifferent over all prices in its support where it fights for shoppers with its CC price, we equate profit when it charges the CC price c (and the direct price $p_2^d = v$) to profit when it charges some CC price p (and the direct price $p_2^d = v$): $c\lambda_0 + v\lambda_2 = p\lambda_0 + p\mu(1 - F_1) + v\lambda_2$, which is rearranged to give the second expression defining F_1 . There is no residual mass.

The price \underline{p} is that where firm 02 is indifferent between fighting for shoppers with direct rather than its CC price, and is that which equates the expression in (9), i.e., $\underline{p} = \frac{c\lambda_0 - v\lambda_2}{\lambda_0 - \lambda_2}$. (Firm 02 prefers fighting with its direct price rather than its CC price when $p < \underline{p}$.) To check the equilibrium strategies are well-defined, we check: $\underline{p} \leq v \Leftrightarrow c \leq v$ and $\underline{p} \geq \underline{c}_2 \Leftrightarrow c \geq v \frac{\lambda_2}{\lambda_2 + \mu} \frac{\lambda_0 + \mu}{\lambda_0}$. Other deviation checks are similar to preceding Lemmas. \square

Proof of Proposition 9. Firstly, we solve for the optimal choice of c by 02. That provides the equilibrium strategies and payoffs, allowing consumer surplus to be computed.

Suppose $\lambda_0 \leq \lambda_1$. From Lemmas C7 and C8:

$$\pi_{02} = \begin{cases} c(\lambda_0 + \mu) + v\lambda_2 & \text{if } c \in [0, \underline{c}_1] \\ \underline{c}_1(\lambda_0 + \mu) + v\lambda_2 & \text{if } c \in (\underline{c}_1, v] \end{cases}$$

maximized at $\underline{c}_1(\lambda_0 + \mu) + v\lambda_2$ by any $c \in [\underline{c}_1, v]$. Equilibrium strategies are described by Lemma C7 or C8 accordingly. Suppose $\lambda_1 \leq \lambda_0 \leq \lambda_2$. From Lemmas C7 to C9:

$$\pi_{02} = \begin{cases} c(\lambda_0 + \mu) + v\lambda_2 & \text{if } c \in [0, \underline{c}_1] \\ \underline{c}_1(\lambda_0 + \mu) + v\lambda_2 & \text{if } c \in \left(\underline{c}_1, \frac{v\lambda_1}{\lambda_1 + \mu} \frac{\lambda_0 + \mu}{\lambda_0} \right] \\ c\lambda_0 + v\lambda_2 & \text{if } c \in \left(\frac{v\lambda_1}{\lambda_1 + \mu} \frac{\lambda_0 + \mu}{\lambda_0}, v \right] \end{cases}$$

which is maximized at $v\lambda_0 + v\lambda_2$ by $c = v$. Equilibrium strategies are described by Lemma C9. Suppose $\lambda_0 \geq \lambda_2$. From Lemmas C7 to C10:

$$\pi_{02} = \begin{cases} c(\lambda_0 + \mu) + v\lambda_2 & \text{if } c \in [0, \underline{c}_1] \\ \underline{c}_1(\lambda_0 + \mu) + v\lambda_2 & \text{if } c \in \left(\underline{c}_1, \frac{v\lambda_1}{\lambda_1 + \mu} \frac{\lambda_0 + \mu}{\lambda_0} \right] \\ c\lambda_0 + v\lambda_2 & \text{if } c \in \left(\frac{v\lambda_1}{\lambda_1 + \mu} \frac{\lambda_0 + \mu}{\lambda_0}, \frac{v\lambda_2}{\lambda_2 + \mu} \frac{\lambda_0 + \mu}{\lambda_0} \right] \\ c\lambda_0 + v\lambda_2 & \text{if } c \in \left(\frac{v\lambda_2}{\lambda_2 + \mu} \frac{\lambda_0 + \mu}{\lambda_0}, v \right] \end{cases}$$

maximized at $v\lambda_0 + v\lambda_2$ by $c = v$. Equilibrium strategies are described by Lemma C10.

Consumer surplus is $v(\lambda_0 + \lambda_1 + \lambda_2 + \mu)$ minus firms' equilibrium profits. Suppose $\lambda_0 \leq \lambda_1$. In equilibrium, $\pi_{02} = \underline{c}_1(\lambda_0 + \mu) + v\lambda_2$ and $\pi_1 = v\lambda_1$. Suppose $\lambda_1 \leq \lambda_0 \leq \lambda_2$. In equilibrium, $\pi_{02} = v\lambda_0 + v\lambda_2$ and $\pi_1 = \underline{c}_0(\lambda_1 + \mu)$. Suppose $\lambda_0 \geq \lambda_2$. In equilibrium,

$\pi_{02} = v\lambda_0 + v\lambda_2$ and $\pi_1 = c_2(\lambda_1 + \mu)$. Computing yields

$$\text{CS} = \begin{cases} v\mu \frac{\lambda_0 + \mu}{\lambda_1 + \mu} & \text{for } \lambda_0 \leq \lambda_1 \\ v\mu \frac{\lambda_1 + \mu}{\lambda_0 + \mu} & \text{for } \lambda_1 \leq \lambda_0 \leq \lambda_2 \\ v\mu \frac{\lambda_1 + \mu}{\lambda_2 + \mu} & \text{for } \lambda_0 \geq \lambda_2. \end{cases} \quad (10)$$

Comparison of (10) to (4) shows that the integration of 0 & 2 causes consumer surplus to rise when $\lambda_1 \leq \lambda_0 \leq \lambda_2$, and leaves it unchanged otherwise. \square

C.3 Equilibrium under NRCs

Lemma C11. *In each regime (CC-binding, DC-binding, no NRCs) there exists an equilibrium with a similar structure to the baseline case. In particular:*

1. *equilibrium is either a high-fee regime with firms mixing their direct prices and shoppers buying direct or a low-fee regime with high direct prices and shoppers buying via the CC;*
2. *there exists a cut-off value of λ_0 below which the low fee regime emerges and above which the high fee regime emerges.*

We prove Lemma C11 in Sections C.3.1–C.3.3, and Proposition 10 in Section C.3.4. We treat ties between the direct and competitive channel in the same way as in the baseline model: the probability with which consumers break such ties in the CC's favour, r_0 is irrelevant for equilibrium except in some sub-games where that probability must equal 0 (otherwise a firm would have to put positive mass at $p_i^d = \max\{p : p < c\}$, which is not well-defined). We explicitly note where a particular tie-break probability is needed to support sub-game equilibrium.

Begin with the observation that, because old consumers who bought direct when young are exposed only to \hat{p}_i^r , we must have $\hat{p}_i^r = v$ in any equilibrium.

C.3.1 Equilibrium under CC-binding NRCs

If a CC-binding NRC is in place then old consumers receive solicitation messages only from their previous supplier, which can therefore price as a monopolist: $p_i^r = v$. A young consumer attracted via the CC is worth $p_i + p_i^r - c = p_i + v - c$ to firm i . In equilibrium, we must have $p_i = c - v$ (otherwise, a seller would undercut its rival to capture the CC consumers). To analyze pricing for young consumers, we proceed by looking for an equilibrium with a similar structure to Lemmas 2 to 4 of the baseline case.

Direct pricing with a low fee is such that shoppers buy through the CC and sellers' direct prices serve only captive young consumers. We must therefore have $p_i^d = v$. To check when this is equilibrium behavior, note that firm i 's, $i = 1, 2$, per-period profit is $\pi_i = [v\lambda_i + (c - v - c)(\lambda_0 + \mu)/2] + [v\lambda_i + v(\lambda_0 + \mu)/2] = 2vLi$, where the first square bracket is profit from young consumers and the second is profit from the old. The best deviation for i is to $p_i^d = p_i = c - v$ (or slightly below if $r_0 > 0$) and $p_i^r = \hat{p}_i^r = v$, which results in young shoppers buying from it direct rather than through the CC. This yields profit $\pi_i = [(c - v)(\lambda_i + \mu) + (c - v - c)\lambda_0/2] + [v(\lambda_i + \mu + \lambda_0/2)] = c(\lambda_i + \mu)$. We therefore find that the deviation is not profitable if $2v\lambda_i \geq c(\lambda_i + \mu)$, i.e., if $c \leq 2c_2$.

Direct pricing with an intermediate fee is such that firm 1 finds it worthwhile to undercut prices on the CC and serve shoppers directly, while firm 2 does not. The preceding analysis established that this will be the case if $2c_1 \leq c \leq 2c_2$. Then $p_1^d = c - v$, $p_2^d = v$, $p_1 = p_2 = c - v$, $p_1^r = p_2^r = \hat{p}_1^r = \hat{p}_2^r = v$, $r_0 = 0$.

Direct pricing with a high fee is such that both sellers undercut CC prices, which they wish to do if $c \geq 2c_2$. The same Bertrand logic as above implies $p_i = c - v$ if $c \leq 2v$ (if $c > 2v$ there is no way for firms to recoup the cost of sales through the CC so they will opt not to participate; this yields zero CC profit and cannot be an equilibrium).

We start with the conjecture that, as in the baseline case, the equilibrium is such that seller 1 mixes over $[p, p_i]$ (with a mass point at p_i), while 2 mixes over $[p, p_i] \cup v$ (with a mass point at v), and verify that this is indeed consistent with equilibrium.

When playing $p_2^d = v$, firm 2's profit is $2v\lambda_2$ (its direct prices attracts only captives who buy in both periods of their life). An arbitrary price in $[p, p_i]$ yields profit $(p_2^d + v)\lambda_2 + [1 - F_1(p_2^d)](p_2^d + v)\mu$, where F_1 is the distribution of 1's direct prices. Setting these two expressions for 2's profits equal and solving yields: $F_1(p) = 1 - (v - p)\lambda_2/((v + p)\mu)$. Additionally, $F_1(p) = 0$ implies $p = v(\lambda_2 - \mu)/(\lambda_2 + \mu)$.

If seller 1 sets $p_1^d = p$ then its profit is $(p + v)(\lambda_1 + \mu)$. This must be compared to profit from some $p_1^d \equiv p \in [p, p_1]$, which is $(p + v)\lambda_1 + [1 - F_2(p)](p + v)\mu$. Solving for indifference yields $F_2(p) = 1 - ((p + v)(\mu + \lambda_1) - (p + v)\lambda_1)/((p + v)\mu)$. These strategies along with $p_1 = p_2 = c - v$, $p_1^r = p_2^r = v$, and $r_0 = 0$ form a well-defined equilibrium. In particular, $p \leq c - v$ (as required) whenever $c \geq 2c_2$ and the distributions are increasing.

CC's decision and equilibrium payoffs. If $c \geq 2c_1$, CC profit is $c\lambda_0$ so the optimal c is $c = 2v$. If $c \leq 2c_1$, CC profit is $c(\lambda_0 + \mu)$; the optimal c is therefore $2c_1$. Thus, the CC implements a low-fee regime with $c = 2c_1$ if $2c_1(\mu + \lambda_0) \geq 2v\lambda_0 \Leftrightarrow \lambda_1 \geq \lambda_0$, and implements a high-fee regime with $c = 2v$ otherwise.

In the low-fee regime, the profits are $\pi_0 = 2v\lambda_1(\lambda_0 + \mu)/(\lambda_1 + \mu)$, $\pi_1 = 2v\lambda_1$, and $\pi_2 = 2v\lambda_2$. Consumer surplus is $2v(\lambda_0 + \lambda_1 + \lambda_2 + \mu) - \pi_0 - \pi_1 - \pi_2 = 2v\mu(\lambda_0 + \mu)/(\lambda_1 + \mu)$.

In the high-fee regime, the profits are $\pi_0 = 2v\lambda_0$, $\pi_1 = 2v\lambda_2(\lambda_1 + \mu)/(\lambda_2 + \mu)$, and $\pi_2 = 2v\lambda_2$. Consumer surplus is $2v(\lambda_0 + \lambda_1 + \lambda_2 + \mu) - \pi_0 - \pi_1 - \pi_2 = 2v\mu(\lambda_1 + \mu)/(\lambda_2 + \mu)$.

C.3.2 Equilibrium under DC-binding NRCs

If a DC-binding NRC is in place, sellers can resolicit only old consumers who bought direct when young. There is no competition for these consumers, so sellers set $p_i^r = v$. Because sellers have no possibility of serving old consumers who bought through the CC, the lifetime profit associated with a young consumer buying through the CC is $p_i - c$ and competition à la Bertrand implies $p_i = c$. We proceed by looking for an equilibrium with a similar structure to Lemmas 2 to 4 of the baseline case.

Direct pricing with a low fee is such that shoppers buy through the CC and sellers' direct prices serve only captive young consumers. We must therefore have $p_i^d = v$. To check when this is equilibrium behavior, note that firm i 's per-period profit is $\pi_i = 2v\lambda_i$. The best deviation for i is to $p_i^d = p_i = c$ (or slightly below if $r_0 > 0$) and $p_i^r = \hat{p}_i^r = v$, which results in young shoppers buying from it direct rather than through the CC. This yields profit $\pi_i = c(\lambda_i + \mu) + v(\lambda_i + \mu)$. We therefore find that the deviation is not profitable if $c \leq \underline{c}_i'' \equiv [v(\lambda_i - \mu)]/[\lambda_i + \mu]$.

Direct pricing with an intermediate fee is such that firm 1 finds it worthwhile to undercut prices on the CC and serve shoppers directly, while firm 2 does not. The preceding analysis established that this will be the case if $\underline{c}_1'' \leq c \leq \underline{c}_2''$. We then have an equilibrium in which $p_i^d = c$, $p_2^d = v$, $p_i = c$, $p_i^r = \hat{p}_i^r = v$, and $r_0 = 0$, $i = 1, 2$.

Direct pricing with a high fee can be found in the same way as in the CC-binding NRC case (the sole difference being that now the maximum admissible c is v —because firms are prevented from directly selling to firms that bought through the CC when young). Indeed, one can verify that the expressions for equilibrium profit functions, and hence the equilibrium F_1 , F_2 , and \underline{p} are identical in the two cases.

CC's decision and equilibrium payoffs. Notice that if $c \geq \underline{c}_1''$ CC profit is $2c\lambda_0$ hence $c = v$ is optimal. If $c \leq \underline{c}_1''$, CC profit is $2c(\lambda_0 + \mu)$; the optimal c is therefore \underline{c}_1'' . Thus, the CC implements a low-fee regime with $c = \underline{c}_1''$ if $2\underline{c}_1''(\mu + \lambda_0) \geq 2v\lambda_0 \Leftrightarrow (\lambda_1 - \mu)/2 \geq \lambda_0$, and implements a high-fee regime with $c = v$ otherwise.

In the low-fee regime, the profits are $\pi_0 = 2v(\lambda_0 + \mu)(\lambda_1 - \mu)/(\lambda_1 + \mu)$, $\pi_1 = 2v\lambda_1$, and $\pi_2 = 2v\lambda_2$. Consumer surplus is $2(\lambda_0 + \lambda_1 + \lambda_2 + \mu)v - \pi_0 - \pi_1 - \pi_2 = 4v\mu(\lambda_0 + \mu)/(\lambda_1 + \mu)$.

In the high-fee regime, the profits are $\pi_0 = 2v\lambda_0$, $\pi_1 = [1 + (\lambda_2 - \mu)/(\lambda_2 + \mu)](\lambda_1 + \mu)v$, and $\pi_2 = 2v\lambda_2$. Consumer surplus is $2(\lambda_0 + \lambda_1 + \lambda_2 + \mu)v - \pi_0 - \pi_1 - \pi_2 = 2v\mu(\lambda_1 + \mu)/(\lambda_2 + \mu)$.

C.3.3 Equilibrium under no NRCs

In the absence of an NRC, old consumers who bought through the CC are exposed to offers from their previous supplier and the CC. If they buy through the CC again then their previous supplier earns at most $p_i - c$. But if they are offered $p_i^r \leq p_i$ they will buy direct yielding profit p_i^r .³⁸ Thus, consumers buying through the CC are offered $p_i^r = p_i$

³⁸Assuming $r=0$. If $r_0 > 0$ then the best p_i^r is $\max\{p : p < p_i\}$, which is not well-defined.

when old so that their lifetime value to a seller is $2p_i - c$. Bertrand competition on the CC is such that all profits are dissipated and $p_i = c/2$. We proceed by looking for an equilibrium with a similar structure to Lemmas 2 to 4 of the baseline case.

Direct pricing with a low fee is such that shoppers buy through the CC and sellers' direct prices serve only captive young consumers. We must therefore have $p_i^d = v$. To check when this is equilibrium behavior, note that firm i 's per-period profit is $\pi_i = 2v\lambda_i$. The best deviation for i is to $p_i^d = p_i = c/2$ (or slightly below if $r_0 > 0$) and $p_i^r = \hat{p}_i^r = v$, which results in young shoppers buying from it direct rather than through the CC. This yields profit $\pi_i = (v + c/2)(\lambda_i + \mu)$. We therefore find that the deviation is not profitable if $c \leq \underline{c}_i \equiv [2v(\lambda_i - \mu)]/[\lambda_i + \mu]$.

Direct pricing with an intermediate fee is such that firm 1 finds it worthwhile to undercut prices on the CC and serve young shoppers directly, while firm 2 does not. The preceding analysis established that this will be the case if $\underline{c}_1 \leq c \leq \underline{c}_2$, yielding an equilibrium with $p_1^d = p_1 = p_2 = c/2$, $p_2^d = v$, $p_i^r = c/2$, $\hat{p}_i^r = v$, and $r_0 = 0$.

Direct pricing with a high fee can be found in the same way as in the CC-binding NRC case. Indeed, one can verify that the expressions for equilibrium profit functions, and hence the equilibrium F_1 , F_2 , and \underline{p} are identical in the two cases.

CC's decision and equilibrium payoffs. Notice that if $c \geq \underline{c}_1$ CC profit is $c\lambda_0$ so the optimal c is $c = 2v$. If $c \leq \underline{c}_1$, CC profit is $c(\lambda_0 + \mu)$; the optimal c is therefore \underline{c}_1 . Thus, the CC implements a low-fee regime with $c = \underline{c}_1$ if $\underline{c}_1(\mu + \lambda_0) \geq 2v\lambda_0 \Leftrightarrow (\lambda_1 - \mu)/2 \geq \lambda_0$, and implements a high-fee regime with $c = 2v$ otherwise.

In the low-fee regime, the profits are $\pi_0 = 2v(\lambda_0 + \mu)(\lambda_1 - \mu)/(\lambda_1 + \mu)$, $\pi_1 = 2v\lambda_1$, and $\pi_2 = 2v\lambda_2$. Consumer surplus is $2(\lambda_0 + \lambda_1 + \lambda_2 + \mu)v - \pi_0 - \pi_1 - \pi_2 = 4v\mu(\lambda_0 + \mu)/(\lambda_1 + \mu)$.

In the high-fee regime, the profits are $\pi_0 = 2v\lambda_0$, $\pi_1 = [1 + (\lambda_2 - \mu)/(\lambda_2 + \mu)](\lambda_1 + \mu)v$, and $\pi_2 = 2v\lambda_2$. Consumer surplus is $2(\lambda_0 + \lambda_1 + \lambda_2 + \mu)v - \pi_0 - \pi_1 - \pi_2 = 2v\mu(\lambda_1 + \mu)/(\lambda_2 + \mu)$.

C.3.4 Proof of Proposition 10

Part 2 of the proposition follows immediately from the above equilibrium characterization. To see part 1: First, if $\lambda_0 \leq (\lambda_1 - \mu)/2$ there is a low-fee equilibrium under both CC-binding NRCs and no NRCs, but the latter implies twice the consumer surplus of the former. Second, if $(\lambda_1 - \mu)/2 < \lambda_0 \leq \lambda_1$ then a CC-binding NRC supports a low-fee regime with consumer surplus equal to $2v\mu(\lambda_0 + \mu)/(\lambda_1 + \mu)$, while a ban on NRCs results in a high fee regime with consumer surplus of $2v\mu(\lambda_1 + \mu)/(\lambda_2 + \mu)$. Consumer surplus under a ban is constant in λ_0 , while that under a CC-binding NRC is increasing. Moreover, as $\lambda_0 \rightarrow \lambda_1$, consumer surplus is (weakly) higher under a CC-binding NRC (strictly so if $\lambda_1 < \lambda_2$). Third, if $\lambda_0 > \lambda_1$ then both regimes yield the same consumer surplus. \square

W Web Appendix

The Web Appendix houses the lengthier proofs not central to the main results.

W.1 Uniqueness of the high-fee regime subgame equilibrium

Here we show that the uniqueness claims in the equilibrium of the subgame denoted in Lemma 4 i.e., the sellers' equilibrium responses following a fee level $c \in (\underline{c}_2, v]$.³⁹ Proof that the equilibrium given is an equilibrium in the subgame is given in Appendix A. Given Lemma 1, we need to determine the equilibrium distribution of direct prices p_i^d for $i = 1, 2$, F_1, F_2 to complete the subgame's equilibrium characterization. We denote \underline{p}_i and \bar{p}_i as the min and max of the support of F_i . Firstly, we derive seller responses to $c \in (\underline{c}_2, v]$.

Lemma W1. $\bar{p}_1, \bar{p}_2 \leq v$.

Proof. Any $p_i^d > v$ results in no direct sales and $\pi_i = 0$, whereas $p_i^d = v$ results in direct sales to λ_i , giving $\pi_i = v\lambda_i$. \square

Lemma W2. $p_1^d, p_2^d \notin (c, v)$.

Proof. Any $p_i^d \in (c, v)$ results in sales to λ_i only, such a price is therefore strictly dominated by v . \square

Lemma W3. $\underline{p}_1, \underline{p}_2 \geq \underline{c}_2$.

Proof. The best $p_i^d < \underline{c}_2$ can do for i is to sell to shoppers with probability one, netting $\pi_i = p_i^d(\mu + \lambda_i)$, less than the $v\lambda_i$ made from setting $p_i^d = v$. \square

Lemma W4. $\min\{\underline{p}_1, \underline{p}_2\} \leq c$.

Proof. Suppose not. Then by Lemmas W1-W2, both sellers play pure strategies $p_i^d = v$. However, as $c > \underline{c}_2$, both sellers have a profitable deviation slightly below c (selling to shoppers and λ_i at c netting arbitrarily close to $c(\mu + \lambda_i)$, whereas selling only to λ_i at v nets $v\lambda_i$). \square

Lemma W5. $\underline{p}_1 = \underline{p}_2 \equiv \underline{p}$.

Proof. By Lemma W4, $\min\{\underline{p}_1, \underline{p}_2\} \leq c$. Suppose (without loss of generality) that $\underline{p}_1 < \underline{p}_2$. Case 1: $\underline{p}_1 < c$. Prices $p \in [\underline{p}_1, \underline{p}_2)$ are strictly worse than $p \nearrow \underline{p}_2$ for 1. Case 2: $\underline{p}_1 = c$. By Lemmas W1-W2, $p_2^d = v$, giving $\pi_2 = v\lambda_2$. However, as $c > \underline{c}_2$, deviations to $p \nearrow \underline{p}_1$ are profitable for 2. \square

Lemma W6. $\underline{p} < c$.

³⁹When $c = \underline{c}_2$, the equilibrium of Lemma 4 collapses to that of Lemma 3.

Proof. By Lemmas W4-W5, $\underline{p} \leq c$. Now we rule out $\underline{p} = c$. Case 1: $\underline{p} = c$ and there is a zero probability of a tie at c . Then $c < v$ and both sellers place all their mass on v by Lemmas W1-W2, $\pi_i = v\lambda_i$ but deviating to $p_i^d \in (\underline{c}_2, c)$ nets $\pi_i = p_i^d(\mu + \lambda_i) > v\lambda_i$. Case 2: $\underline{p} = c$ and there is a positive probability of a tie at c . Both sellers have an incentive to shift the mass placed on c to slightly below c (the arbitrarily small loss in price is compensated by the discrete gain in the probability of selling to shoppers). \square

Lemma W7. *Direct prices $[\underline{p}, c]$ are in the support of both sellers.*

Proof. Suppose not. By Lemma W5 there is a common element to sellers' supports. Denote $p < c$ highest common element to the supports such that $[p, p]$ is in both supports. Suppose without loss of generality that i then has a "hole" in their support of p_i^d , denoted by the interval $(p_l, p_h) \subseteq [p, c]$. This cannot be true in equilibrium because for prices close to p_l , j has a profitable deviation of $p_j^d \nearrow p_h$. \square

Lemma W8. *No seller has a mass point in $[\underline{p}, c)$.*

Proof. Suppose i had a mass point at $\dot{p} \in [\underline{p}, c)$. By Lemma W7, j 's support includes prices in small neighborhoods around \dot{p} , where j has a profitable deviation to relocate those p_j^d slightly above \dot{p} , to prices slightly below \dot{p} (the arbitrary loss in price is compensated by the discrete gain in the probability of selling to shoppers). \square

Lemma W9. *At most one seller has a mass point at c .*

Proof. If both did, then there is a positive probability of a tie at c . Hence, it would be profitable for either to shift the mass they place on c to slightly below c , (the arbitrarily small loss in price is compensated by the discrete gain in the probability of selling to shoppers). [Note that the point the mass is shifted to will not coincide with another mass point: if $\underline{p} = c$, because no prices below \underline{p} are charged; if $\underline{p} < c$, by Lemma W8.] \square

Lemma W10. *When $\lambda_1 < \lambda_2$, not both sellers have mass points at v .*

Proof. Suppose so. Case 1: $c < v$. Then $\pi_1 = \underline{p}(\mu + \lambda_1)$ (from Lemmas W7-W8) $= v\lambda_1$ so $\underline{p} = \frac{v\lambda_1}{\mu + \lambda_1} \equiv \underline{c}_1 < \underline{c}_2$, ruled out by Lemma W3. Case 2: $c = v$. Lemma W9 applies. \square

Lemma W11. *When $c < v$, at least one seller has a mass point at v .*

Proof. Suppose neither seller has a mass point at v . By Lemma W9, at least one seller does not have a mass point at c , call this i . Note that for $j \neq i$, c is in their support by Lemma W7 and $\pi_j = c\lambda_j$, but then j has a profitable deviation $p_j^d = v$, netting $v\lambda_j$. \square

Lemma W12. *When $\lambda_1 < \lambda_2$ and $c = v$, exactly one seller has a mass point at v .*

Proof. By Lemma W10 it is not true that both have mass points at v . Now suppose neither seller has a mass point at v . Then seller $i = 1, 2$ makes $\pi_i = v\lambda_i$, but we know $\pi_i = \underline{p}(\mu + \lambda_i)$ (from Lemmas W7-W8) which gives $\underline{p} = \frac{v\lambda_i}{\mu + \lambda_i} \equiv \underline{c}_i$, which cannot be satisfied for both $i = 1, 2$ because $\lambda_1 < \lambda_2$. \square

Lemma W13. *When $\lambda_1 < \lambda_2$, seller 1 has no mass point at v , seller 2 does.*

Proof. When $c < v$, Lemmas W10 and W11 imply that exactly one seller has a mass at v . When $c = v$, Lemma W12 says the same. Next, we show that this seller is seller 2. Suppose instead it was 1, then $\pi_1 = v\lambda_1$. However, 1 has a profitable deviation to $p_1^d = \underline{c}_2$ which generates $\pi_1 = \underline{c}_2(\mu + \lambda_1)$ (the deviation wins shoppers with probability one by Lemmas W3 and W8), which is greater because $\lambda_1 < \lambda_2$. \square

Lemma W14. *When $\lambda_1 < \lambda_2$, the unique equilibrium pricing strategies of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, p_1^d and p_2^d mixed over supports $[\underline{c}_2, c]$ and $[\underline{c}_2, c] \cup v$ respectively via the strategies*

$$F_1(p) = \begin{cases} \frac{\mu p - \lambda_2(v-p)}{\mu p} & \text{for } p \in [\underline{c}_2, c) \\ 1 & \text{for } p \geq c, \end{cases}$$

$$F_2(p) = \begin{cases} \frac{\mu p - \lambda_2(v-p)}{\mu p} \frac{\mu + \lambda_1}{\mu + \lambda_2} & \text{for } p \in [\underline{c}_2, c) \\ \frac{\mu c - \lambda_2(v-c)}{\mu c} \frac{\mu + \lambda_1}{\mu + \lambda_2} & \text{for } p \in [c, v) \\ 1 & \text{for } p \geq v. \end{cases}$$

Profits are $\pi_0 = c\lambda_0$, $\pi_1 = \frac{v\lambda_2(\lambda_1 + \mu)}{\lambda_2 + \mu}$, and $\pi_2 = v\lambda_2$. If $c < v$, $r_0 = 0$; if $c = v$, $r_0 \in [0, 1]$.

Proof. In equilibrium, F_1 must be such that seller 2 is indifferent over all prices in their support. By Lemma W13, 2 has a mass at v , hence $\pi_2 = v\lambda_2$. Therefore, F_1 must satisfy $\pi_2 = p_2^d\lambda_2 + (1 - F_1(p_2^d))\mu p_2^d = v\lambda_2$ for $p_2^d \in [\underline{p}, c]$. Solving gives F_1 over $[\underline{p}, c]$ as stated. By Lemmas W1, W2, W3, W8 and W13, the residual mass of $1 - F_1(c)$ must be located at c . In order for there to be no profitable deviation for 1 to shift this mass slightly below c , $r_0 = 0$. When $c = v$, there is no mass, so any $r_0 \in [0, 1]$ can be supported. Solving $F_1(\underline{p}) = 0$ gives $\underline{p} = \underline{c}_2$. Similarly, F_2 must keep 1 indifferent over all prices in their support. Solving $\pi_1 = \underline{p}(\mu + \lambda_1) = p_1^d\lambda_1 + (1 - F_2(p_1^d))\mu p_1^d$ gives F_2 over $[\underline{p}, c]$ as stated. With the addition of Lemma W9 (to W1, W2, W3, W8 and W13) the residual mass of $1 - F_2(c)$ must be located at v . Checks that the strategies described indeed constitute an equilibrium can be found in the proof of Lemma 4. \square

The following Lemmas correspond to the special case of $\lambda_1 = \lambda_2$, which as shown in Lemma W26, does not occur when market composition is endogenously determined. Therefore, we include these results mostly for the sake of completeness.

Lemma W15. *When $\lambda_1 = \lambda_2 \equiv \lambda$ and $c = v$, the unique equilibrium pricing strategies of the subgame starting at $t = 2$ is as given in Lemma W14.*

Proof. By Lemma W9, not both sellers have mass points at v . This means there is at least one seller, i , who makes $\pi_i = v\lambda$ (if neither have a mass at v , $\pi_i = v\lambda$, $i = 1, 2$; if i does, then $\pi_i = v\lambda$). In equilibrium, F_j , $j \neq i$, must be such that seller i is indifferent over all prices in their support: $\pi_i = v\lambda = p_i^d\lambda + (1 - F_j(p_i^d))\mu p_i^d$ for $p_i^d \in [\underline{p}, v]$. Solving for F_j gives the strategy stated in Lemma W14 when $\lambda_1 = \lambda_2 \equiv \lambda$ and $c = v$. There is no residual mass, hence j 's strategy features no mass points. Given F_j , we know $\pi_j = \underline{p}(\lambda + \mu) = v\lambda$ and hence i 's strategy is dictated by $\pi_j = v\lambda = p_j^d\lambda + (1 - F_i(p_j^d))\mu p_j^d$ for $p_j^d \in [\underline{p}, v]$, which is the same as we had for j . Therefore, the unique equilibrium strategies are symmetric, and are given in Lemma W14 when $\lambda_1 = \lambda_2 \equiv \lambda$ and $c = v$. \square

Lemma W16. *When $\lambda_1 = \lambda_2 \equiv \lambda$ (so that $c_1 = c_2 \equiv c$) and $c < v$, equilibria of the subgame starting at $t = 2$ have $p_i = p_j = c$, p_i^d and p_j^d mixed over supports $[c, c] \cup v$ and $[c, c] \cup v$ respectively for $i, j = 1, 2$ and $i \neq j$ via the strategies*

$$F_i(p) = \begin{cases} \frac{\mu p - \lambda(v-p)}{\mu p} & \text{for } p \in [c, c) \\ \alpha & \text{for } p \in [c, v) \\ 1 & \text{for } p \geq v, \end{cases}$$

$$F_j(p) = \begin{cases} \frac{\mu p - \lambda(v-p)}{\mu p} & \text{for } p \in [c, c) \\ \frac{\mu c - \lambda(v-c)}{\mu c} & \text{for } p \in [c, v) \\ 1 & \text{for } p \geq v, \end{cases}$$

where $\alpha \in \left[\frac{\mu c - \lambda(v-c)}{\mu c}, 1 \right]$. [Note if $\alpha = 1$, v is not in the support of seller i .] Profits are $\pi_0 = c\lambda_0$, $\pi_1 = \pi_2 = v\lambda$. If $\alpha \in \left(\frac{\mu c - \lambda(v-c)}{\mu c}, 1 \right]$, $r_0 = 0$; if $\alpha = \frac{\mu c - \lambda(v-c)}{\mu c}$, $r_0 \in [0, 1]$.

Proof. By Lemma W11, at least one seller puts mass on v . This means that at least one makes $\pi_i = v\lambda$ which gives F_j (and \underline{p}), over $[c, c)$ as stated, in the same way as in proof of Lemma W14. Similarly, $\pi_j = \underline{p}(\mu + \lambda)$ and again the proof of Lemma W14 can be followed to find F_i over $[c, c)$ as stated. Both firms have residual mass $1 - F_i(c) = 1 - F_j(c)$ to be allocated. First, and again similar to Lemma W14, by Lemma W9 there can be only one seller with mass on c , hence by Lemmas W1, W2, W3 and W8, there must be one seller (denoted j here) with all this residual mass located at v . However, unlike Lemma W14, there is nothing dictate exactly how firm i should allocate their residual mass $(1 - \alpha)$ between c and v in equilibrium which leaves us with the strategies as stated above. Note that for there to be mass on c , $r_0 = 0$ to a shift of that mass to slightly below c , but if there is none, then any $r_0 \in [0, 1]$ can be supported. Checks similar to those of Lemma 4 confirm these are equilibria. \square

W.2 Imperfectly captive CC users

For this appendix we find it helpful to draw a distinction (in terms of notation) between a firm's mass of captive consumers (λ_i) and the mass of consumers who are initially aware of that firm (which we denote $I_i > 0$). Consumers only aware of firm 1 or 2 (I_1, I_2 such that $I_2 \geq I_1 > 0$) act as captives in the baseline. Consumers aware of the CC, firm 0 ($I_0 > 0$), become aware of firm 1 and 2 when they see them listed on the CC, and then make a rational decision of whether to check those firms' direct prices, subject to marginal search costs (they do not otherwise observe firms direct prices, but they have correct expectations in equilibrium). Consumers' first search is free, hence consumers I_1 and I_2 incur no search cost, and I_0 incur no search cost for their visit to the CC, but any subsequent visit to firms is costly. The first and second such visits cost σ_1 and σ_2 , respectively, where $\sigma_1 \leq \sigma_2$. It is costless to return to any previously-seen price. Shoppers are aware of all prices (they incur no information or search costs). The I_0 consumers make their search decisions simultaneously, based on firms' equilibrium price distributions. Finally, we assume consumers coordinate around not searching when it is equilibrium behavior for them to do so. The game is otherwise unchanged. We proceed by considering equilibria in the subgames following the CC's choice of commission, c . Our first observation is that when $c \in [0, vI_2/(I_2 + \mu)]$ the strategies of Lemmas 2 and 3, are also equilibrium strategies in our search model, for any $0 \leq \sigma_1 \leq \sigma_2$. The details are given in the Lemmas below. In this section, all the equilibria we report feature $r_0 = 0$ so we do not report it when stating the results.

Lemma W17 (Direct pricing 1 with search). *Suppose $0 \leq c \leq vI_1/(I_1 + \mu)$ and $0 \leq \sigma_1 \leq \sigma_2$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, $p_1^d = p_2^d = v$, and consumers informed of the CC search only the CC. Profits are $\pi_0 = c(I_0 + \mu)$, $\pi_1 = vI_1$, $\pi_2 = vI_2$.*

Lemma W18 (Direct pricing 2 with search). *Suppose $vI_1/(I_1 + \mu) \leq c \leq vI_2/(I_2 + \mu)$ and $0 \leq \sigma_1 \leq \sigma_2$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, $p_1^d = c$, $p_2^d = v$, and consumers informed of the CC search only the CC. Profits are $\pi_0 = cI_0$, $\pi_1 = c(I_1 + \mu)$, and $\pi_2 = vI_2$.*

In Lemmas W17 and W18, the lowest prices are hosted on the CC. Therefore, I_0 consumers prefer not to search (weakly if $\sigma_1 = 0$). Given they do not search, firms have no incentive to deviate from their strategies, and the deviation checks in the baseline model apply. For higher levels of c , the baseline's high-fee regime strategies continue to be equilibrium strategies when search is sufficiently costly. To express our results we define the following distribution functions.

$$\begin{aligned}
 A_1(p; s_2) &= \begin{cases} \frac{\mu p - (I_2 + s_2 I_0)(v - p)}{\mu p} & \text{for } p \in \left[\frac{v(I_2 + s_2 I_0)}{\mu + I_2 + s_2 I_0}, c \right) \\ 1 & \text{for } p \geq c, \end{cases} \\
 A_2(p; s_1, s_2) &= \begin{cases} \frac{\mu p - (I_2 + s_2 I_0)(v - p)}{\mu p} \frac{\mu + I_1 + s_1 I_0}{\mu + I_2 + s_2 I_0} & \text{for } p \in \left[\frac{v(I_2 + s_2 I_0)}{\mu + I_2 + s_2 I_0}, c \right) \\ \frac{\mu c - (I_2 + s_2 I_0)(v - c)}{\mu c} \frac{\mu + I_1 + s_1 I_0}{\mu + I_2 + s_2 I_0} & \text{for } p \in [c, v) \\ 1 & \text{for } p \geq v \end{cases} \\
 A_S(p; s_2) &= \begin{cases} \frac{(\mu + s_2 I_0)p - I_2(v - p)}{\mu p} & \text{for } p \in \left[\frac{v I_2}{\mu + I_2 + s_2 I_0}, c \right) \\ \frac{(\mu + s_2 I_0)c - I_2(v - p)}{\mu c} & \text{for } p \in [c, v) \\ 1 & \text{for } p \geq v. \end{cases} \\
 B_S(p; s_2) &= \begin{cases} \frac{\mu p - (I_2 + s_2 I_0)(c - p)}{\mu p} & \text{for } p \in \left[\frac{c(I_2 + s_2 I_0)}{\mu + I_2 + s_2 I_0}, c \right) \\ 1 & \text{for } p \geq c. \end{cases}
 \end{aligned}$$

Let $\mathbb{E}_X[\mathbf{z}]$ denote the expected price from distribution X at parameter values \mathbf{z} , and $\mathbb{E}_{X,Y}[\mathbf{z}]$ denote the expected minimum of one draw from each of X and Y , both evaluated at parameter values \mathbf{z} .

Lemma W19 (Direct pricing 3 with high search costs). *Suppose $vI_2/(I_2 + \mu) < c \leq \min\{\sigma_1 + \mathbb{E}_{A_1}[0], v\}$ and $0 \leq \sigma_1 \leq \sigma_2$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, $p_1^d \sim A_1(\cdot; 0)$, $p_2^d \sim A_2(\cdot; 0, 0)$, and consumers informed of the CC search only the CC. Profits are $\pi_0 = cI_0$, $\pi_1 = vI_2(I_1 + \mu)/(I_2 + \mu)$, and $\pi_2 = vI_2$.*

The equilibrium benefit from searching the smaller firm when there is no search is $c - \mathbb{E}_{A_1}[0]$. Where consumers search one firm, they search firm 1 because $\mathbb{E}_{A_1}[0] < \mathbb{E}_{A_2}[0, 0]$. Note that if it is unprofitable to search firm 1, then it is unprofitable to search both 1 and 2 because the marginal search cost is weakly increasing, but the marginal expected benefit of the second search is less than the first: $c - \mathbb{E}_{A_1}[0] > \mathbb{E}_{A_1}[0] - \mathbb{E}_{A_1, A_2}[0, 0]$.

For lower levels of σ_1 , high-fee regime equilibria of the subgame starting in $t = 2$ can feature search. Below we characterize an equilibrium when $\sigma_1 = \sigma_2 = 0$ and I_0 consumers choose to search the CC and both firms.

Lemma W20 (Direct pricing 3 with zero search costs). *Suppose $vI_2/(I_2 + \mu) < c \leq v$ and $\sigma_1 = \sigma_2 = 0$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, $p_1^d \sim A_1(\cdot; 0)$, $p_2^d \sim A_2(\cdot; 0, 0)$ where μ is replaced by $\mu + I_0$. Consumers informed of the CC also visit both firms. Profits are $\pi_0 = 0$, $\pi_1 = vI_2(I_0 + I_1 + \mu)/(I_2 + \mu)$, and $\pi_2 = vI_2$.*

Lemma W21 (Market equilibrium with zero search costs). *When $I_0 < I_1(I_2 + \mu)/(I_2 - I_1)$ the CC sets $c = vI_1/(I_1 + \mu)$ while sellers price and consumers search in accordance with Lemma W17. When $I_0 > I_1(I_2 + \mu)/(I_2 - I_1)$ the CC sets $c = v$ while*

sellers price and consumers search in accordance with Lemma W18. When $I_0 = I_1(I_2 + \mu)/(I_2 - I_1)$, both equilibria exist.

We have derived equilibria with no search and full search. We now derive equilibria with partial search. We assume that $\sigma_1 \geq 0$ while searching twice is prohibitively costly e.g., $\sigma_2 = \infty$. Let s_1 and s_2 denote the share of the I_0 consumers who search firm 1 and 2, respectively. Firstly, note that when $vI_2/(I_2 + \mu) < c \leq \min\{\sigma_1 + \mathbb{E}_{A_1}[0], v\}$, Lemma W19 applies and $s_1 = s_2 = 0$. However, for higher levels of c , there is partial search in equilibrium:

Lemma W22 (Direct pricing 3 with intermediate search costs). *Suppose $\sigma_1 + \mathbb{E}_{A_1}[0] < c \leq v$, $\sigma_1 \geq 0$ and $\sigma_2 = \infty$. Define $R = I_0 - I_2 + I_1$. There exist the following equilibria in the subgames starting at $t = 2$:*

1. $R \leq 0$. $p_1 = p_2 = c$, $p_1^d \sim A_1(\cdot; 0)$, $p_2^d \sim A_2(\cdot; 1, 0)$. In $t = 3$, $s_1 = 1$ and $s_2 = 0$. Profits are $\pi_0 = 0$, $\pi_1 = vI_2(I_0 + I_1 + \mu)/(I_2 + \mu)$, and $\pi_2 = vI_2$.
2. $R \in (0, 2\mu)$.
 - $c \in \left[\frac{vI_2}{\mu + I_2}, \frac{vI_2}{\mu + R/2} \right]$. $p_1 = p_2 = c$, $p_1^d, p_2^d \sim A_S(\cdot; s_2)$, $s_1 = (2I_0 - R)/(2I_0)$ and $s_2 = R/(2I_0)$. The resulting profits are $\pi_0 = 0$, $\pi_1 = vI_2$, and $\pi_2 = vI_2$.
 - $c \in \left[\frac{vI_2}{\mu + R/2}, v \right]$. $p_1 = p_2 = c$, $p_1^d, p_2^d \sim B_S(\cdot; s_2)$, $s_1 = (I_2 - I_1)/I_0 + s_2$, where $s_2 = \min\{\tilde{s}, R/(2I_0)\}$ and \tilde{s} is the unique solution to

$$c - \mathbb{E}_{B_S}[\tilde{s}] = \sigma_1. \quad (11)$$

Profits are $\pi_0 = c(1 - s_1 - s_2)$, $\pi_1 = c(I_2 + s_2I_0)$, and $\pi_2 = c(I_2 + s_2I_0)$.

3. $R \geq 2\mu$. Same as $R \in (0, 2\mu)$ in the case of $c \in \left[\frac{vI_2}{\mu + R/2}, v \right]$.

Proof. Start in the scenario of Lemma W19 where $s_1 = s_2 = 0$, and consider exogenously increasing search of one firm. At some point, $\sigma_1 + \mathbb{E}_{A_1}[0] < c$ i.e., consumers strictly prefer to search firm 1 than not to search (because $\mathbb{E}_{A_1}[0] < \mathbb{E}_{A_2}[0, 0]$, consumers search firm 1 not firm 2). Therefore, consider s_1 increasing.

1. Can we have an equilibrium with firms mixing via A_1, A_2 in direct prices? Distribution A_1 is independent of s_1 , and hence if consumers strictly prefer to search, they all search. The question is whether A_1 is valid when $s_1 = 1$. A_1 is only valid if firm 1 remains weakly smaller than firm 2 after search i.e., $I_1 + s_1I_0 \leq I_2 + s_2I_0$, which is $I_1 + s_1I_0 \leq I_2$ because $s_2 = 0$ when A_1 applies. If $R = I_0 - I_2 + I_1 \leq 0$ then we can have an equilibrium where $s_1 = 1$ and A_1 applies because $I_1 + I_0 \leq I_2$. This covers the first bullet.

2. What if $R \geq 0$? The equilibrium of the previous point cannot continue: consumers cannot all search firm 1 because that would make firm 1 larger than firm 2, which would make the expected price of firm 2 more attractive which would contradict consumers

searching firm 1. Therefore, once there is sufficient search to make firms of equal size, any further search must be equally split across firms 1 and 2 such that they are of equal size i.e., $I_1 + s_1 I_0 = I_2 + s_2 I_0$, or $s_1 = (I_2 + s_2 I_0 - I_1)/I_0$. For this to be consistent with equilibrium search behavior, firms must price via identical distributions, which is achieved by A_S , which is also a symmetric mutual best response for firms. A_S is only valid if the mass point exists $A_S(c; s_2) \geq 0 \Leftrightarrow c \leq vI_2/(s_2 I_0 + I_2)$. A_S has the feature that $\partial E_{A_S}[s_2]/\partial s_2 < 0$, and therefore all consumers search in any equilibrium with A_S i.e., $s_1 + s_2 = 1$, and $s_2 = R/(2I_0)$. A_S is valid if $c \leq vI_2/(R/2 + I_2)$. Because $c > vI_2/(\mu + I_2)$, this equilibrium applies only if additionally, $R < 2\mu$.

3. What if we follow the logic in the case of $R \geq 0$ above, but $R \geq 2\mu$? A_S is no longer valid, but B_S is, and is a mutual best response for firms. In contrast to A_S , $\partial E_{B_S}[s_2]/\partial s_2 < 0$ so that equilibria with partial search are possible. Whether or not there is full or partial search depends on how many consumers there are left to search beyond the number required to make the firms equally sized. If this number is small, there is full search; if large, partial search. The relevant threshold is the level of search required beyond that to make the firms equally sized such that consumers are indifferent to search, \tilde{s} : $c - \mathbb{E}_{B_S}[\tilde{s}] = \sigma_1$, which has a unique solution. If $\tilde{s} \geq R/2$, the level of search required to make consumers indifferent exceeds the number of consumers available and so there is full search, $s_1 + s_2 = 1$. If $\tilde{s} < R/2$, there is partial search, $s_1 + s_2 < 1$ where $s_1 = (I_2 - I_1)/I_0 + \tilde{s}$ and $s_2 = \tilde{s}$. \square

Unlike the case of zero search costs, the CC optimally chooses a fee beyond $vI_2/(I_2 + \mu)$ in the high-fee regime because consumers face search costs which deter search at high price levels. This makes the high-fee regime more profitable. Naturally, the CC does (weakly) better when search costs are higher. Below we report an equilibrium of the game as a whole which features an interior CC-commission level and partial consumer search.

Lemma W23 (Market equilibrium with intermediate search costs). *Suppose $0 < \sigma_1 < v - \mathbb{E}_{A_1}[0]$, $\sigma_2 = \infty$ and $I_0 < I_2 - I_1$. Define \tilde{c} as the unique solution to $\sigma_1 = c - \mathbb{E}_{A_1}[0]$.⁴⁰ When $I_0 < v\mu I_1/(\tilde{c}(I_1 + \mu) - vI_1)$, a low-fee equilibrium results: the competitive channel sets $c = vI_1/(I_1 + \mu)$, sellers price and consumers search in accordance with Lemma W17. When $I_0 > v\mu I_1/(\tilde{c}(I_1 + \mu) - vI_1)$, a high-fee equilibrium results: the competitive channel sets $c = \tilde{c} \in (vI_2/(I_2 + \mu), v)$, sellers price and consumers search in accordance with Lemma W22. When $I_0 = v\mu I_1/(\tilde{c}(I_1 + \mu) - vI_1)$, both equilibria exist.*

W.3 Advertising and Market Composition

This section provides the results allowing for an endogenous market composition λ by allowing each firm to determine their stock of loyal consumers. We hold μ constant

⁴⁰Note that \tilde{c} is independent of I_0 and that as $\sigma_1 \downarrow 0$, $\tilde{c} \downarrow vI_2/(I_2 + \mu)$.

throughout where the proofs hold for any $\mu > 0$. Note that in the previous sections where $\boldsymbol{\lambda}$ was exogenous, we adopted the convention of naming the direct channels such that $\lambda_1 \leq \lambda_2$, but now that order of the labels should be discarded: There are simply three firms, of any size, indexed $i = 0, 1, 2$ where 0 is a competitive channel and 1, 2 are sellers. Formally, we add a stage, $t = 0$ to the game where each player $i = 0, 1, 2$ simultaneously chooses λ_i subject to a C^2 cost function $\psi_i : \mathbb{R}_+^3 \rightarrow \mathbb{R}_+$. The cost to player i of choosing λ_i given the other players choose λ_{-i} is written $\psi_i(\lambda_i|\lambda_{-i})$ where $\lambda_{-i} = \boldsymbol{\lambda} \setminus \{\lambda_i, \mu\}$. We make the following additional assumptions on the cost function for $i, j = 0, 1, 2$:

1. Symmetry: $\psi_i = \psi_j \equiv \psi$
2. Costs nothing to do nothing: $\psi(0|\cdot) = 0$
3. Increasing: $\frac{\partial \psi(\lambda_i|\lambda_{-i})}{\partial \lambda_i} = 0$ for $\lambda_i = 0$; $\frac{\partial \psi}{\partial \lambda_i} > 0$ for $\lambda_i > 0$
4. Convex: $\frac{\partial^2 \psi(\lambda_i|\lambda_{-i})}{\partial \lambda_i^2} > 0$
5. Inter-dependency: $\frac{\partial \psi(\lambda_i|\lambda_{-i})}{\partial \lambda_j} = 0$ for $\lambda_i = 0$; $\frac{\partial \psi(\lambda_i|\lambda_{-i})}{\partial \lambda_j} > 0$ for $\lambda_i > 0$

Players are assumed to have access to the same marketing tools, hence ψ is common across players (assumption 1). Assumptions 3 and 4 state respectively that determining market power is costly, and increasingly so. Assumption 5 states that it becomes more costly to generate the same market power when other players are marketing more.

Denote low-fee regime, LFR, and high-fee regime, HFR. We view this marketing stage of the game as reflecting long-run decisions and therefore restrict attention to pure-strategy equilibria. Denote a cost function satisfying assumptions 1-5 as ψ , and the set of all these functions Ψ . Let Ψ_L (Ψ_H) denote the set of all functions ψ under which a low-fee regime (high-fee regime) equilibrium exists.

Lemma W24. *In any equilibrium, $\lambda_i^* > 0$ for all i .*

Proof. Suppose $\lambda_i^* = 0$ for some i . Equilibrium profit for i is weakly positive and the marginal cost of λ_i at $\lambda_i = 0$ is zero. Therefore, if the marginal benefit of λ_i at $\lambda_i = 0$ is strictly positive, there is some strictly profitable deviation away from $\lambda_i^* = 0$.

If $i = 0$, the marginal benefit at $\lambda_i = 0$ is $\frac{d(v\lambda_i)}{d\lambda_i} = v > 0$ if the HFR results; and $v \frac{\lambda_1^*}{\lambda_1^* + \mu} > 0$ if the LFR results (note the LFR implies $\lambda_1^* > 0$). If $i > 0$, then $\hat{\lambda}_i > 0$ small produces the LFR with a marginal benefit of $v \frac{\lambda_2^*}{\lambda_2^* + \mu} > 0$ if $\hat{\lambda}_i \leq \lambda_j^*$ for all $j > 0$ or $v > 0$ otherwise. \square

Lemma W25. *No symmetric equilibrium exists.*

Proof. Suppose instead that $\lambda_i = \lambda^*$ for all i . By Lemma W24, $\lambda^* > 0$. A deviation of any player to $\hat{\lambda}_i$ yields profit:

$$\hat{\pi}(\hat{\lambda}_i) = \begin{cases} v \frac{\lambda^*}{\lambda^* + \mu} (\hat{\lambda}_i + \mu) - \psi(\hat{\lambda}_i|\boldsymbol{\lambda}_{-i}^*) & \text{for } \hat{\lambda}_i \leq \lambda^* \\ v \hat{\lambda}_i - \psi(\hat{\lambda}_i|\boldsymbol{\lambda}_{-i}^*) & \text{for } \hat{\lambda}_i \geq \lambda^*. \end{cases}$$

[Notice this is regardless of whether the LFR or HFR results in equilibrium.] The function $\hat{\pi}$ is continuous, but non-differentiable at $\hat{\lambda}_i = \lambda^*$. To see this, note that the left and right derivatives at $\hat{\lambda}_i = \lambda^*$ are $\partial_- \hat{\pi}(\lambda^*) = v \frac{\lambda^*}{\lambda^* + \mu} - \frac{\partial \psi(\lambda_i | \lambda_{-i})}{\partial \lambda_i} \Big|_{\lambda = \lambda^*}$ and $\partial_+ \hat{\pi}(\lambda^*) = v - \frac{\partial \psi(\lambda_i | \lambda_{-i})}{\partial \lambda_i} \Big|_{\lambda = \lambda^*}$ respectively, hence $\partial_- < \partial_+$. However, for there to be no strictly profitable deviation slightly below or above λ^* we need $\partial_- \geq 0$ and $\partial_+ \leq 0$ which implies $\partial_- \geq \partial_+$, a contradiction. \square

Lemma W26. *If the LFR obtains in equilibrium, $\lambda_0^* < \lambda_1^*$. If the HFR obtains in equilibrium, $\lambda_1^* < \lambda_2^*$.*

Proof. If instead $\lambda_0^* = \lambda_1^*$, the profit from deviations of the CC, $\hat{\lambda}_0$ is given by:

$$\hat{\pi}(\hat{\lambda}_0) = \begin{cases} v \frac{\lambda_1^*}{\lambda_1^* + \mu} (\hat{\lambda}_0 + \mu) - \psi(\hat{\lambda}_0 | \lambda_{-0}^*) & \text{for } \hat{\lambda}_0 \leq \lambda_0^* \\ v \hat{\lambda}_0 - \psi(\hat{\lambda}_0 | \lambda_{-0}^*) & \text{for } \hat{\lambda}_0 \geq \lambda_0^*. \end{cases}$$

Following the proof of Lemma W25 shows that such an equilibrium implies a contradiction. Similarly, if $\lambda_1^* = \lambda_2^* \equiv \lambda^* < \lambda_0^*$, the profit from deviations by $i = 1, 2$, $\hat{\lambda}_i$ is given by:

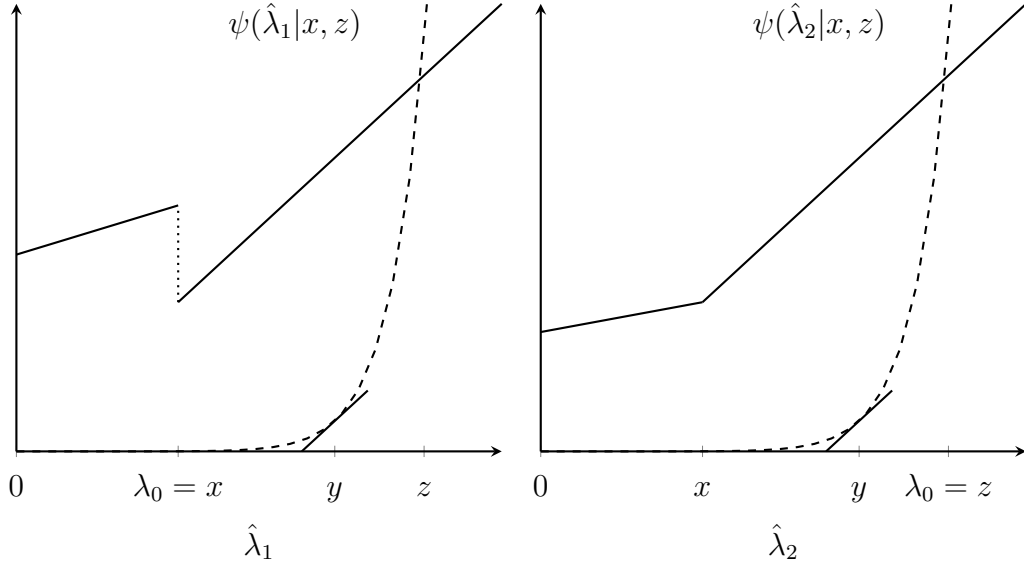
$$\hat{\pi}(\hat{\lambda}_i) = \begin{cases} v \frac{\lambda^*}{\lambda^* + \mu} (\hat{\lambda}_i + \mu) - \psi(\hat{\lambda}_i | \lambda_{-i}^*) & \text{for } \hat{\lambda}_i \leq \lambda^* \\ v \hat{\lambda}_i - \psi(\hat{\lambda}_i | \lambda_{-i}^*) & \text{for } \hat{\lambda}_i \geq \lambda^*. \end{cases}$$

Following the proof of Lemma W25 shows that such an equilibrium implies a contradiction. \square

Proposition 11 (High-fee regime predominance). *There are more cost functions that lead to a high-fee regime than a low-fee regime in equilibrium, i.e., $|\Psi_H| \geq |\Psi_L|$.*

Proof. We show that $\psi \in \Psi_L \implies \psi \in \Psi_H$ i.e., for any ψ that supports an equilibrium which gives the low-fee regime, then under the same cost function there is also an equilibrium which gives the high-fee regime.

Suppose $\psi \in \Psi_L$ and denote an equilibrium which gives the low-fee regime as $(\lambda_0, \lambda_1, \lambda_2) = (x, y, z)$ where $x \leq y \leq z$. We show that this implies that there also exists an equilibrium where firms choose (z, x, y) which gives the high-fee regime. By Lemmas W26 and W24, $0 < x < y$ for (x, y, z) to be an equilibrium. Below on the left is an illustration of such a low-fee equilibrium (x, y, z) . On the right is an illustration of the corresponding high-fee equilibrium strategies $(\lambda_0, \lambda_1, \lambda_2) = (z, x, y)$. The x-axis represents unilateral deviations of the player choosing y (seller in the left-hand panel and seller 2 in the right-hand panel). [To prevent clutter, the panels show only the revenue and cost curves of the players choosing y , but the procedure is unchanged if done instead for the players choosing x or z .]



Left panel: Deviation incentives for the smaller seller from the low-fee regime equilibrium (x, y, z) .
 Right panel: Deviation incentives for the larger seller from the high-fee regime equilibrium (z, x, y) .

As (x, y, z) is an equilibrium, there are no profitable deviations for the players either (i) locally: the slope of the cost curve is equal to the slope of the revenue curve; or (ii) globally: there is no profitable deviation to, or slightly below the choice of the smallest player. It can now be seen that (z, x, y) is also an equilibrium. Because the same three values x, y, z are chosen (albeit by different players) in (x, y, z) and (z, x, y) , we show that (z, x, y) satisfies (i) and (ii), which implies it is an equilibrium. That it is a high-fee regime equilibrium is immediate because in (z, x, y) , $\lambda_0 > \lambda_1$. To see this for the player choosing y , notice that the figures in both panels are identical except for the revenue curve which is the same for deviations above x , but strictly higher in the left-hand panel for deviations below x . [Although only the deviations for the player at y are shown, it is also true that the revenue curve in the left panel is weakly above the revenue curve in the right panel for the players at x and z , hence the argument can be repeated for all players.] This implies immediately that there are no local deviations. It also shows that if a deviation to at or slightly below x is not profitable in the left panel, it is not profitable for the player in the right panel, which shows there are no global deviations. Hence if (x, y, z) is an equilibrium, then (z, x, y) is too. \square

W.4 Commission types and a marginal cost of production

Through its selection of fees (flat or ad valorem), the CC determines the equilibrium price available through it, p . As such, in this section it is convenient to write as if the CC is directly determining p . We state the equilibria of the subgames starting at $t = 2$ in terms of p in the same order as Lemmas 2 to 4. In the baseline model, \underline{c}_i was the threshold fee level such that seller i would prefer to sell to shoppers and their captives direct, but at the

CC price, and sell only to their captives direct at the monopoly price. The corresponding expressions in terms of p , while allowing for a marginal cost of production, $m \in [0, v]$, are given below for $i = 1, 2$.

$$p_i = \frac{v\lambda_i + m\mu}{\lambda_i + \mu}$$

Lemma W27 (Direct pricing 1’). *Suppose $0 \leq p \leq p_1$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = p$, $p_1^d = p_2^d = v$ and any $r_0 \in [0, 1]$. The resulting equilibrium profits are $\pi_0 = (p\tau + c)(\lambda_0 + \mu) = (p - m)(\lambda_0 + \mu)$,⁴¹ $\pi_1 = (v - m)\lambda_1$, $\pi_2 = (v - m)\lambda_2$. When $0 \leq p < p_1$, this equilibrium is unique.*

Lemma W28 (Direct pricing 2’). *Suppose $p_1 \leq p \leq p_2$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = p$, $p_1^d = p$, $p_2^d = v$ and $r_0 = 0$. The resulting equilibrium profits are $\pi_0 = (p - m)\lambda_0$, $\pi_1 = (p - m)(\lambda_1 + \mu)$, and $\pi_2 = (v - m)\lambda_2$. When $p_1 < p < p_2$, this equilibrium is unique.*

Lemma W29 (Direct pricing 3’). *Suppose $p_2 \leq p \leq v$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = p$, p_1^d and p_2^d mixed over supports $[\underline{p}, p]$ and $[\underline{p}, p] \cup v$ respectively via the strategies*

$$F_1(p^d) = \begin{cases} 1 - \frac{(v-p^d)\lambda_2}{(p^d-m)\mu} & \text{for } p^d \in [\underline{p}, p) \\ 1 & \text{for } p^d \geq p, \end{cases}$$

$$F_2(p^d) = \begin{cases} 1 - \frac{(p-m)\mu - (p^d-\underline{p})\lambda_1}{(p^d-m)\mu} & \text{for } p^d \in [\underline{p}, p) \\ \frac{(p-m)\mu - (p-\underline{p})\lambda_1}{(p-m)\mu} & \text{for } p^d \in [p, v) \\ 1 & \text{for } p^d \geq v, \end{cases}$$

where $\underline{p} = p_2$ and $r_0 = 0$. Profits are $\pi_0 = (p - m)\lambda_0$, $\pi_1 = \frac{(v-m)\lambda_2(\lambda_1+\mu)}{\lambda_2+\mu}$, and $\pi_2 = (v - m)\lambda_2$. When $p = v$, any $r_0 \in [0, 1]$ can be supported. When $p_2 < p \leq p$, this equilibrium is unique.

Proposition W1. *When $\lambda_0 \leq \lambda_1$, there is a low-fee equilibrium: the competitive channel sets $p = p_1$ and sellers price in accordance with Lemma 2. When $\lambda_0 \geq \lambda_1$, there is a high-fee equilibrium: the competitive channel sets $p = v$ and sellers price in accordance with Lemma 4.*

W.5 Commission discrimination

Proposition W2. *Suppose the CC can charge fee c_1 to seller 1 and c_2 to seller 2. It will choose $c_1 = c_2$ in any equilibrium.*

⁴¹Here and below we use that fact that $p(1 - \tau) - c - m = 0$ because firms compete away all profit on the CC.

Proof. If the CC offers fees $c_i < c_j \leq v$ Bertrand competition on the CC implies that all sales via the CC go to firm i at a price of $p_i = c_j$.

Part (i): Suppose that $\min\{p_1^d, p_2^d\} \leq \min\{p_1, p_2\}$ with probability 1 in the pricing sub-game and $c_i < c_j \leq v$. Then the CC's profit is $\pi_0 = c_i \lambda_0$.⁴² There is a profitable deviation to $c_1 = c_2 = v$.

Part (ii): Suppose that $\min\{p_1^d, p_2^d\} \leq \min\{p_1, p_2\}$ with probability 0 and $c_i < c_j \leq v$. We show that the CC can serve the same mass of consumers at a higher fee by reducing c_j . Indeed, firm i earns profit of $v\lambda_i + (c_j - c_i)(\lambda_0 + \mu)$ (assuming it sets $p_i^d = v$, which is optimal conditional on not undercutting the CC). The best deviation would be to $p_i^d = c_j$ to undercut the CC, yielding profit $c_j(\lambda_i + \mu) + (c_j - c_i)\lambda_0$. The deviation is not profitable if $c_i \leq \tilde{c}_i \equiv \frac{\lambda_i(v - c_j)}{\mu}$. Similarly, firm j earns $v\lambda_j$. A deviation to $p_j^d = c_j$ yields profit $c_j(\lambda_j + \mu)$ and is not profitable if $c_j \leq \underline{c}_j$. Thus, the best that the CC can do, conditional on deterring undercutting, is to solve $\max_{c_i, c_j} c_i(\mu + \lambda_0)$ s.t. $c_i \leq c_j, c_i \leq \tilde{c}_i, c_j \leq \underline{c}_j$. At least one of $c_i \leq \tilde{c}_i$ and $c_j \leq \underline{c}_j$ must bind and both slacked by a reduction in c_j .

Part (iii) Lastly, if $c_i < c_j$ we show that it can't be the case that $\min\{p_1^d, p_2^d\} \leq \min\{p_1, p_2\}$ with probability in $(0, 1)$. Indeed, this implies that both firms put positive mass on direct prices below c_j and positive mass on prices above. Direct prices in (c_j, v) never serve shoppers and are dominated by v . Standard arguments then imply that both firms must share a support, $[\underline{p}, c_j] \cup \{v\}$. To be indifferent between $p_j^d = v$ and $p_j^d = \underline{p}$, j must have $\underline{p}(\lambda_j + \mu) = v\lambda_j \Leftrightarrow \underline{p} = \underline{c}_j$, which does not depend on c_i or c_j . Similarly, i is indifferent if $\underline{p} = \frac{\lambda_i v + \mu(c_j - c_i)}{\lambda_i + \mu}$. For the two values of \underline{p} to coincide, we require $(\lambda_j - \lambda_i)v = (\lambda_j + \mu)(c_j - c_i)$, which implies that $\lambda_j > \lambda_i$ (i.e., $j = 2$ and $i = 1$).

To be indifferent between \underline{p} and a $p_2^d < c_2$, 2 must have $\underline{p}(\lambda_2 + \mu) + \lambda_0(c_2 - c_1) = p_2^d \lambda_2 + \lambda_0(c_2 - c_1) + \mu p_2^d(1 - F_1(p_2^d))$, and the case for 1 follows similarly, i.e.,

$$F_1(p_2^d) = \frac{(\lambda_2 + \mu)(p_2^d - \underline{p})}{p_2^d \mu} = \frac{(\lambda_2 + \mu)(p_2^d - \underline{c}_2)}{p_2^d \mu},$$

$$F_2(p_1^d) = \frac{(\lambda_1 + \mu)(p_1^d - \underline{p})}{p_1^d \mu} = \frac{(\lambda_1 + \mu)(p_1^d - \underline{c}_2)}{p_1^d \mu}.$$

The CC's profit is $\pi_0 = c_1[1 - F_1(c_2)][1 - F_2(c_2)]\mu + c_1\lambda_0$.⁴³ Observe that F_1 and F_2 do not depend on c_1 , so π_0 is linear in c_1 and maximized when $c_1 \uparrow c_2$. Letting $c_1 = c_2 = c$, it is easily verified that π_0 is convex and maximized at either $c = v$ (inducing firms to undercut with probability 1) or at $c = \underline{c}_1$ (inducing firms to undercut with probability zero). Profit when $c_1 \neq c_2$ must be strictly less so that the CC has a profitable deviation to the case considered in either part (i) or part (ii). \square

⁴²If $\min\{p_1^d, p_2^d\} = \min\{p_1, p_2\}$ then the CC's profit is $c_i(\lambda_0 + r_0\mu)$. But we must have $r_0 = 0$ in such an equilibrium or else a firm would want to reduce p_i^d to break the tie.

⁴³A tie between CC and direct prices must be broken in the direct channel's favor, otherwise a seller would wish to undercut the tie with its direct price.