THE *LIBER THEOREUMACIE* (1214) AND THE EARLY HISTORY OF THE *QUADRANS VETUS*

Abstract

The *Liber theoreumacie* is a neglected work of practical geometry, written in Strasbourg in 1214, which sheds valuable light on the study and practice of astronomy in early thirteenth-century Europe. In this article, I focus on the first two chapters of Book IV, which both deal with the construction of horary instruments. The first of these chapters contains the earliest preserved account of the type of universal horary quadrant known as *quadrans vetus*, which is here given a biblical pedigree by labelling it the 'sundial of Ahaz'. The second chapter describes a graphical method of inscribing hour-markings on the surface of an astrolabe's alidade, which appears to have been introduced into Latin Europe by the twelfth-century translator John of Seville. Critical editions of the relevant passages will conclude the article.

Keywords

astronomical instruments; quadrant; astrolabe; practical geometry; precession; John of Seville; thirteenth-century astronomy

Introduction

If the availability of written descriptions is any indicator, the universal horary quadrant must be ranked behind the astrolabe as the second-most popular astronomico-mathematical instrument of medieval Europe. Although descriptions of an early form of the instrument, known to modern scholars as *quadrans vetustissimus,* already crop up in Latin manuscripts of the eleventh century,[1] the more developed and apparently much more widespread *quadrans vetus* is first attested in texts dating from the thirteenth century.[2] In its canonical form, the *quadrans vetus* is characterized by the presence of three components on its face: (i) a movable cursor showing both the signs of the zodiac and the months of the Julian calendar; (ii) curved lines for finding unequal (seasonal) hours;[3] (iii) a shadow square.[4] The inclusion of (i) allowed the quadrant's time-keeping function to work at any latitude, without recourse to solar tables, while the addition of (iii) meant that it could be employed not just for telling time, but also for planimetry and altimetry.[5]

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[1] J. Millàs Vallicrosa, "La introducción del cuadrante con cursor en Europa," *Isis*, 17, 1932, pp. 218–58; J. Samsó, "Cultura científica àrab i cultura científica llatina a la Catalunya altmedieval: el monestir de Ripoll i el naixement de la ciència catalana," *Symposium Internacional sobre els orígens de Catalunya (segles VIII–IX),* 2 vols. (Barcelona: Real Academia de Buenas Letras, 1991), I, pp. 253–69, at pp. 262–3.

[2] The earliest preserved specimen of a *quadrans vetus*, claimed to date from *c.*1300, is kept at the History of Science Museum in Oxford: http://www.mhs.ox.ac.uk/object/inv/52020 (accessed 27 Sept. 2019). On the Islamic background to this type of instrument, see D. A. King, "A *Vetustissimus* Arabic Treatise on the *Quadrans Vetus*," *Journal for the History of Astronomy*, 33, 2002, pp. 237–255; King, *In Synchrony with the Heavens*, vol. 2, *Instruments of Mass Calculation* (Leiden: Brill, 2005), pp. 111–258.

[3] On these lines, see M. Archinard, "The Diagram of Unequal Hours," *Annals of Science*, 47 (1990), pp. 173–90.

[4] On shadow squares, see King, *In Synchrony with the Heavens* (ref. 2), II, pp. 247–52.

[5] On this use of the quadrant, see A. Simi, "Celerimensura e strumenti in manoscritti dei secoli XIII–XIV," in R. Franci, P. Pagli and L. Toti Rigatelli (eds.), *Itinera mathematica: studi in onore di Gino Arrighi per il suo 90º*

Of the various medieval Latin texts devoted to this type of quadrant, the one that enjoyed the greatest diffusion begins with the words *Geometrie due sunt partes*. The over 100 surviving manuscripts of this text do not agree on the name of its author, which was either Johannes Anglicus or Robertus Anglicus, although it seems clear that he was based in Montpellier.[6] According to a source analysis published in 1997 by Wilbur R. Knorr, he probably wrote in the mid-1260s, with a definite *terminus ad quem* of 1271.[7] His identifiable sources include a *Tractatus de quadrante* attributed to John of Sacrobosco, which is extant in at least 18 manuscripts. Sacrobosco's treatise distinguishes between two forms of the instrument, *simplex* and *compositus*, whose only difference is the absence or presence of a cursor. For users of the *quadrans simplex*, which came without the cursor, it was necessary to refer to tables for the solar longitude and declination. Knorr has shown that Sacrobosco's

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compleanno (Siena: Centro studi sulla matematica medioevale, Università di Siena, 1996), pp. 71–121, at pp. 82–91.

[6] Editions of this text were published by P. Tannery, "Le Traité du quadrant de Maître Robert Anglès (Montpellier, XIII[e] siècle): texte latin et ancienne traduction grecque," *Notices et extraits des manuscrits de la Bibliothèque Nationale et autres bibliothèques*, 35.2, 1897, pp. 561–640, at pp. 593–632, and N. L. Hahn, *Medieval Mensuration:* Quadrans Vetus *and* Geometrie Due Sunt Partes Principales = Transactions of the American Philosophical Society 72.8 (Philadelphia: American Philosophical Society, 1982), pp. 6–113. See also the discussions in L. Thorndike, "Who Wrote Quadrans Vetus?", *Isis*, 37, 1947, pp. 150–53; E. Poulle, "Two Medieval Texts" (Review of Hahn, *Medieval Mensuration*), *Journal for the History of Astronomy*, 15, 1984, pp. 135–8.

[7] W. R. Knorr, "The Latin Sources of *Quadrans Vetus*, and What They Imply for Its Authorship and Date," in E. Sylla and M. McVaugh (eds.), *Texts and Contexts in Ancient and Medieval Science* (Leiden: Brill, 1997), pp. 23–67. For an extended version of this article, see Knorr, "The Latin Sources of 'Quadrans vetus': 'Humeniz', Sacrobosco, Campanus et al.," unpublished typescript in Stanford University Libraries, Department of Special Collections, Wilbur Knorr Papers, SC 933, Box 1.

description of these tables was derived from a Latin version of the *Almanach* composed in the eleventh century by al-Zarqālī (Azarquiel). Since this Latin version is known to have been redacted by a certain John of Pavia in 1239, one is left with the conclusion that Sacrobosco's treatise on the *Quadrans* was written no earlier than that year.[8]

In their attempts to establish a *terminus ad* or *ante quem* for the introduction of the *quadrans vetus,* some scholars have drawn attention to a treatise on the *saphea* by William the Englishman (*Guillelmus Anglicus*), citizen of Marseilles. This text written in 1231 describes and depicts a universal astrolabe featuring the familiar combination of curved hour-lines and shadow square in one of the four corners of its backplate. William treats the hour-lines as something well known to his readers and explicitly mentions the *quadrans* as the model for these markings.[9] An argument according to which these lines had been known in the Latin West since the first half of the twelfth century was put forward in 1972 by Emmanuel Poulle, who pointed to a brief passage in Raymond of Marseille's treatise on the astrolabe, written not long before 1141.[10] According to Raymond, some practitioners in his day made the additional step of endowing the backs of their astrolabes with hour-lines, "which to us seems completely superfluous."[11] Even though Raymond says nothing more

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[8] W. R. Knorr, "Sacrobosco's *Quadrans:* Date and Sources," *Journal for the History of Astronomy*, 28, 1997, pp. 187–222.

[9] MS Paris, Bibliothèque nationale de France, lat. 7195, fol. 74v. See Tannery, "Le Traité," pp. 585–6, 633–4; Millás Vallicrosa, "La introducción" (ref. 1), pp. 251–3; Hahn, *Medieval Mensuration* (ref. 6), pp. xxx–xxxii; Knorr, "Sacrobosco's *Quadrans*" (ref. 8), p. 213.

[10] E. Poulle, "Les instruments astronomiques de l'Occident latin aux XIe et XIIe siècle," *Cahiers de civilisation médiévale*, 15, 1972, pp. 27–40, at p. 39.

[11] Raymond of Marseille, *Tractatus astrolabii* (11c), in M.-T. d'Alverny, C. Burnett and E. Poulle (eds.), *Raymond de Marseille: Opera omnia*, vol. 1 (Paris: CNRS Éditions, 2009), p. 84: "Ecce sufficienter qualiter

about the shape and position of these lines, Poulle was probably right to argue that they were lines representing unequal hours of the type found on many later specimens.

The question remains when these lines first made their transition onto the face of Latin quadrants. Thus far, the earliest treatise to describe the standard diagram of unequal hours as a part of the *quadrans vetus* is that attributed to John of Sacrobosco, which, as already mentioned, originated no earlier than 1239. The purpose of this article is to draw attention to a neglected source from the year 1214, the so-called *Liber theoreumacie*, which documents an early variant of the *quadrans vetus*. In this version of the instrument, the three key components mentioned above are already in place, but the shadow square is found on the reverse side of the quadrant rather than its front. To my knowledge, the only scholar to have ever used this source in a discussion of the history of the universal horary quadrant is Ernst Zinner, whose condensed remarks on the *Liber theoreumacie*, published in 1943 and 1956, appear to have been ignored by everyone who has worked on the subject since.[12] Besides fleshing out our timeline of the development of the *quadrans vetus*, the *Liber theoreumacie* is also noteworthy for some of its other astronomical or astronomy-related content. Examples include a unique account of how to turn the alidade on the back of an astrolabe into an instrument for measuring seasonal hours. The construction methods described in this source appear to have been introduced into Latin Europe as part of a treatise translated by John of

astrolabium fieri debeat expositum est, nunc ad ea que per illud fiant accedamus. Solent tamen quidam in dorso ejus horas designare, quod nobis supervacaneum per omnia fore videtur, presertim cum intrinsecus necessario fieri debeant."

[12] E. Zinner, *Entstehung und Ausbreitung der Copernicanischen Lehre*, 2nd ed. (Munich: Beck, 1988 [1st ed. Erlangen, 1943]), pp. 76–77, 499; Zinner, *Deutsche und niederländische astronomische Instrumente des 11.–18. Jahrhunderts* (Munich: Beck, 1956), p. 156.

Seville in the first half of the twelfth century and have so far received little attention from historians of the astrolabe.

The remainder of this article has three parts. In the first, I shall introduce the *Liber theoreumacie* itself, showing that this little-known source offers some precious evidence of the study of astronomy in Strasbourg in the first quarter of the thirteenth century. In the second, I shall focus on its construction plan for the quadrant, which is here labelled *horologium Achaz* ('the sundial of Ahaz'). To this I shall add, in part three, some observations on the following chapter in the treatise, which deals with the hour-markings on the alidade. The article will conclude with a critical edition of the two chapters in question.

The 'Liber theoreumacie' and its manuscripts

The *Liber theoreumacie* (henceforth: *LT*) is an anonymous compilation on all branches of the quadrivium beginning with the words *Cum Ptolomeus in Almagesti ediscerat quod bonum fuit*.[13] In its extant version, the text comprises four books dealing with arithmetic, geometry, music, and astronomy. Each part is further divided into individual *theoreumata*, or teaching units, which accounts for the somewhat strange title: to proceed *theoreumacie* is to proceed 'according to theorems'.[14] Although the four-book structure would seem to follow

[13] L. Thorndike and P. Kibre, *A Catalogue of Incipits of Mediaeval Scientific Writings in Latin*, rev. ed. (London: The Mediaeval Academy of America, 1963), p. 332. See K.-J. Sachs, "Die 'Musica' aus dem 'Liber theoreumacie', einem anonymen Quadriviums-Traktat (des 14. Jahrhunderts?)," *Archiv für Musikwissenschaft*, 52, 1995, pp. 241–78, and the brief remarks on the text in S. Günther, *Geschichte des mathematischen Unterrichts im deutschen Mittelalter bis zum Jahre 1525* (Berlin: Hofmann & Comp., 1887), pp. 128–9; M. Cantor, *Vorlesungen über Geschichte der Mathematik*, vol. 2, *Von 1200–1668*, 2nd ed. (Leipzig: Teubner, 1900), p. 237; L. Thorndike, "Unde Versus," *Traditio*, 11, 1955, pp. 163–93, at p. 175.

[14] See Sachs, "Die 'Musica'" (ref. 13), pp. 247–8, 252.

straightforwardly from the work's focus on the quadrivium, the introduction announces a work in five parts, adding *physica* to the four quadrivial disciplines.[15] From the way the introduction extols the virtues of this fifth discipline, one can infer that it was roughly equivalent to medical astrology.[16] That said, the fifth part is absent from all extant manuscripts, some of which only contain smaller parts of the text. In the following list, I add sigla to those five copies of the *LT* that will be used in the edition appended to this article.

G Munich, Bayerische Staatsbibliothek, Clm 56, fols. 123r–153v (bk. I–IV); Salzburg; a. 1436[17]

M Munich, Bayerische Staatsbibliothek, Clm 14684, fols. 52v–70r (bk. I–IV); St Emmeram; s. XIV$^{2/2}$[18]

Munich, Bayerische Staatsbibliothek, Clm 14783, fols. 466r–482r (bk. II.1–31); St Emmeram; s. XV^{med} (*c.*1450)[19]

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[15] *M*, fol. 52v: "Opus istud itaque in quinque distinctiones distinguo. In prima arismetrice subservio. In secunda geometrie rudimenta erudio. In tertia musicam tango. In quarta ad astronomiam ascendo. In quinta ad phisicam quasi ad divinam properabo."

[16] *M*, fol. 52v: "Quid excellentius quadruvii notis effectibus, sicut in libro secretorum Ypocras testatur, vitam infirmo mortemque prenunciare?"

[17] D. Juste, "MS Munich, Bayerische Staatsbibliothek, Clm 56 (update: 07.05.2019)," *Ptolemaeus Arabus et Latinus. Manuscripts*, URL = http://ptolemaeus.badw.de/ms/76.

[18] D. Juste, "MS Munich, Bayerische Staatsbibliothek, Clm 14684 (update: 16.03.2018)," *Ptolemaeus Arabus et Latinus. Manuscripts*, URL = http://ptolemaeus.badw.de/ms/85.

[19] D. Juste, "MS Munich, Bayerische Staatsbibliothek, Clm 14783' (update: 03.11.2018)," *Ptolemaeus Arabus et Latinus. Manuscripts*, URL = http://ptolemaeus.badw.de/ms/663.

Munich, Bayerische Staatsbibliothek, Clm 14908, fols. 186r–201r (bk. I); St Emmeram; a. 1457[20]

S St Florian, Stiftsbibliothek, XI.619, fols. 118r–132r (bk. I–IV); Regensburg (?); s. XV$^{\text{med}}$ (*c.*1446/47)[21]

P Vatican City, Biblioteca Apostolica Vaticana, Pal. lat. 1376, fols. 185vb–189rb (bk. IV), 308r–313v (bk. I), 329r–330v (bk. II.1–14); St Emmeram, s. XV$^{\text{med}}$ (*c.*1447)[22]

Vienna, Österreichische Nationalbibliothek, 4775, fols. 181r–193v (bk. II); s. XV$^{\text{med}}$ (*c.*1455/1457)

Vienna, Österreichische Nationalbibliothek, 5303, fols. 274r–276v (bk. IV.1–3); s. XV/XVI[23]

V Vienna, Österreichische Nationalbibliothek, 5418, fols. 209r–211v (bk. IV.1–3); s. XV$^{2/4}$ (*c.*1433/34)[24]

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[20] See K. Vogel, *Die Practica des Algorismus Ratisbonensis: Ein Rechenbuch des Benediktinerklosters St. Emmeram aus der Mitte des 15. Jahrhunderts nach den Handschriften der Münchner Staatsbibliothek und der Stiftsbibliothek St. Florian* (Munich: Beck, 1954), pp. 10–19, who notes that the text was probably copied from *M*.

[21] D. Juste, "MS St Florian, Stiftsbibliothek, XI.619 (update: 01.10.2018)," *Ptolemaeus Arabus et Latinus. Manuscripts*, URL = http://ptolemaeus.badw.de/ms/150; Vogel, *Die Practica* (ref. 20), p. 24.

[22] D. Juste, "MS Vatican, Biblioteca Apostolica Vaticana, Pal. lat. 1376 (update: 24.11.2018)," *Ptolemaeus Arabus et Latinus. Manuscripts*, URL = http://ptolemaeus.badw.de/ms/719.

[23] D. Juste, "MS Vienna, Österreichische Nationalbibliothek, 5303 (update: 07.05.2019)," *Ptolemaeus Arabus et Latinus. Manuscripts*, URL = http://ptolemaeus.badw.de/ms/648.

[24] D. Juste, "MS Vienna, Österreichische Nationalbibliothek, 5418 (update: 07.05.2019)," *Ptolemaeus Arabus et Latinus. Manuscripts*, URL = http://ptolemaeus.badw.de/ms/647.

Of the nine manuscripts I have been able to identify,[25] at least four come from the abbey of St Emmeram in Regensburg, Bavaria. A possible fifth witness sharing this background is *S*, which is closely related to the copy of book IV in *P*, in that they both attach the same brief text starting *Item si quantitatem corporis sperici vis scire*. We shall see below that book IV, and presumably also the rest of the *LT*, was written in the year 1214. Despite this relatively early date, none of the *LT*'s known witnesses predate the second half of the fourteenth century. The earliest among them is *M*, which is part of a mathematical and astronomical miscellany of 100 folios written by three different hands. None of the items contained in this miscellany are later than Jean de Lignères's *Algorismus de minutiis* (fols. 22r–29v), a text from around 1320, which is here present in a copy dated 1356. The texts in *G* were all copied in the 1430s by Reinhard Gensfelder of Nuremberg, a noted cartographer.[26] They range from the *Almagesti minor* (fols. 3r–120r, copied in 1434), a work originally written in the late-twelfth or early-thirteenth century,[27] to Johannes Schindel of Nuremberg's *Tractatus de*

[25] Zinner, *Entstehung* (ref. 12), p. 499, already mentioned *GMSV* and Clm 14908. See also E. Zinner, *Verzeichnis der astronomischen Handschriften des deutschen Kulturgebietes* (Munich: Beck, 1925), pp. 274–5 (no. 8849–8852), which lists the astronomical part of *M* and *S* as well as the Viennese MSS 5303 and 5418 (*V*). Manuscripts *GMP* are also recorded in Thorndike and Kibre, *A Catalogue of Incipits* (ref. 13), cols. 332, 640, although the folios in *P* are here mistakenly given as "308r-324r".

[26] On Gensfelder, see D. B. Durand, *The Vienna-Klosterneuburg Map Corpus of the Fifteenth Century: A Study in the Transition from Medieval to Modern Science* (Leiden: Brill, 1952), pp. 44–48; K. Pilz, *600 Jahre Astronomie in Nürnberg* (Nuremberg: Carl, 1977), pp. 50–51; E. Dekker, *Illustrating the Phaenomena: Celestial Cartography in Antiquity and the Middle Ages* (Oxford: Oxford University Press, 2013), pp. 367–70; D. Becher-Hedenus, "Das astronomische 15. Jahrhundert in Kloster Reichenbach," in *900 Jahre Kloster Reichenbach: Das Kloster und sein Dorf* (Regensburg: Morsbach 2018), pp. 48–61.

[27] See H. Zepeda, *The First Latin Treatise on Ptolemy's Astronomy: The* Almagesti minor *(c. 1200)* (Turnhout: Brepols, 2019).

quantitate trium solidorum (1420), which comments on a part of the fifth book of Ptolemy's *Almagest*.[28] The *LT* is here complete and comes with a colophon stating that Gensfelder finished the copy in Salzburg in 1436. His hand also features in *V*, which contains the first three chapters of book IV. This manuscript served as the exemplar for folios 130r–279r of MS Vienna, ÖNB, 5303, which is also the latest of the extant witnesses to the *LT*.

The only part of the *LT* to have received close attention in the literature is Book III (on *musica*), which was analysed and edited by Klaus-Jürgen Sachs on the basis of *M* and *G*.[29] For the three remaining books, Sachs was able to expand on a finding by Marshall Clagett, who noticed their dependence on an anonymous treatise on practical geometry known from its incipit as *Artis cuiuslibet consummatio* (henceforth: *ACC*).[30] This work survives in at least 14 manuscripts of the thirteenth to sixteenth centuries. From a dating clause contained in one of its astronomical chapters, it appears that the original time of writing was 1193.[31] Sachs did not name a precise date for the *LT*, which he suspected was

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[28] D. Juste, "Johannes Andree Schindel, *Tractatus de quantitate trium solidorum* (update: 19.06.2019)," *Ptolemaeus Arabus et Latinus. Works*, URL = http://ptolemaeus.badw.de/work/78.

[29] Sachs, "Die 'Musica'" (ref. 13), 253–78. Sachs (ibid., pp. 243–47) also printed a full list of chapter headings, the prologue, and the transitional passages between the four books.

[30] M. Clagett, *Archimedes in the Middle Ages*, vol. 3, *The Fate of the Medieval Archimedes 1300 to 1565* (Philadelphia: American Philosophical Society, 1978), pp. 220–2; Sachs, "Die 'Musica'" (ref. 13), pp. 249–51. For an edition of the text, see S. K. Victor, *Practical Geometry in the High Middle Ages:* Artis cuiuslibet consummatio *and the* Pratike de geometrie (Philadelphia: American Philosophical Society, 1979).

[31] *Artis cuiuslibet consummatio*, II.26, ed. and trans. Victor (ref. 30), pp. 282–3. The dating clause was dismissed by Wilbur Knorr, who attributed *ACC* to the thirteenth-century geometer Gerard of Brussels, whose period of activity he dated to the 1230s and 1240s. See W. R. Knorr, "John of Tynemouth *alias* John of London: Emerging Portrait of a Singular Medieval Mathematician," *British Journal of the History of Science*, 23, 1990,

only written much later, in the early fourteenth century.[32] A closer look at the chapters of book IV reveals, however, that one of them (IV.28) identifies the 'present time' (*hodierno tempore*) as August 1214.[33] Before I go on to comment on the content of book IV, it will be expedient to provide a full listing of its chapters, in Latin and English, with parallel chapters in *ACC* added in square brackets:[34]

1. *Horologium Achaz collocare* (to assemble the sundial of Ahaz)
2. *Horas naturales in regula locare* (to locate the natural hours on the alidade)
3. *Lineam meridionalem invenire* (to find the meridian line) [II.3]
4. *Solis, terre et lune magnitudines eorumque proportiones invenire* (to find the sizes of the Sun, Earth, and Moon and their proportions) [III.5bis]
5. *Altitudinem solis per umbram metire* (to measure the altitude of the Sun by means of a shadow) [II.1]
6. *Stelle altitudinem per triangulum ortogonium invenire* (to find the altitude of a star by means of a right-angled triangle) [II.2]
7. *Meridionalem altitudinem invenire* (to find the [Sun's] noon altitude) [II.4]
8. *Equinoctialem solis altitudinem invenire* (to find the Sun's equinoctial altitude) [II.5]
9. *Solstitialem solis altitudinem meridianam habere* (to obtain the Sun's noon altitude at the solstices) [II.6]

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pp. 293–330, at pp. 315–22. Since the *LT* posits a *terminus ad quem* for the composition of *ACC*, this conclusion can now be regarded as invalid.

[32] Sachs, "Die 'Musica'" (ref. 13), p. 267.

[33] See n. 41 below.

[34] Most of these correspondences were already noted by Sachs, "Die 'Musica'" (ref. 13), pp. 244–5.

10. *Per equinoctialem et altitudinem solis meridianam in quovis die remotionem eius scire* (to know the declination of the Sun by means of its equinoctial [altitude] and noon altitude in any day) [II.20]
11. *Per altitudinem equinoctialem latitudinem regionis invenire* (to find the latitude of a region by means of the [Sun's] equinoctial altitude) [II.7]
12. *Poli altitudinem invenire* (to find the altitude of the pole) [II.8]
13. *Per poli altitudinem summam imperfectorum almucantarat in quolibet climate reperire* (to ascertain the sum of incomplete almucantars in any climate by means of the altitude of the pole) [II.9]
14. *Per poli altitudinem distantiam cenit ab ipso et paralellis habere* (by means of the altitude of the pole to obtain the distance of the zenith from it and the parallels) [II.10]
15. *Per solis altitudinem horam diurnam investigare* (to investigate the hour of the day from the Sun's altitude) [II.11]
16. *Per altitudinem stelle horam noctis invenire* (to find the hour of the night from the altitude of a star) [II.13]
17. *Per solis altitudinem umbram invenire* (to find the [length of the] shadow from the Sun's altitude) [II.14]
18. *Per umbram altitudinis 29 graduum ascensum Arietis in spera recta invenire* (to find the right ascension of Aries using the shadow of an altitude of 29 degrees) [II.15]
19. *Per umbram altitudinis 27 graduum ascensum Tauri in spera recta invenire* (to find the right ascension of Taurus using the shadow of an altitude of 27 degrees) [II.16]

20. *Per umbram altitudinis 26 graduum ascensum Geminorum in spera recta investigare* (to investigate the right ascension of Gemini using the shadow of an altitude of 26 degrees) [II.17]
21. *Per equinoctialis altitudinis umbram ascensus signorum in spera declivi ubique invenire* (to find everywhere the oblique ascensions of the signs using the shadow of the [Sun's] equinoctial altitude) [II.18]
22. *Solis gradum in zodiaco reperire* (to ascertain the Sun's degree in the zodiac)
23. *Per maximam stelle altitudinem gradum eius scire* (to know the degree of a star by means of its greatest altitude) [II.21]
24. *Per altitudinem stelle note et altitudinem planete locum eius invenire* (to find the place of [a planet] by means of the altitude of a known star and the altitude of the planet) [II.22]
25. *Per altitudinem stelle et planete directionem eius investigare* (to investigate the direction of [a planet] by means of the altitude of a star and the planet) [II.23]
26. *Per altitudinem planete et gradum in quo est latitudinem planete invenire* (to find the latitude of a planet by means of the altitude of the planet and the degree in which it is) [II.24]
27. *Per distantiam planete ab alto loco excentrici equationem eius habere* (to obtain the equation of a planet from its distance from the apogee of the eccentric) [II.25]
28. *Auges 6 planetarum reperire secundum motum fixarum* (to ascertain the apogees of the six planets according to the motion of the fixed [stars]) [II.26]
29. *Secundum reditum solis ad altum locum annum computari* (to assess the year according to the Sun's return to the apogee) [II.27]

30. *Per alta loca planetarum 5 geuzahar eorum invenire* (to find the five nodes of the planets by means of their apogees) [II.28]
31. *Per altitudinem poli orizontis depressionem et dierum variationem designare* (to describe the depression of the horizon and the variation of days by means of the altitude of the pole) [II.29]
32. *Locum lune sic invenire* (to find the place of the Moon in the following manner)
33. *Ex curva planam elicere speram* (to turn a curved object into a plane sphere)

Of the 33 chapters that make up this book, only four (IV.1–2, 32–33) lack an identifiable counterpart in *ACC*, while a fifth (IV.22) shares with the corresponding chapter in *ACC* (II.19) no more than its overall subject matter. In fact, while the chapter in *ACC* explains how an astrolabe can be used to infer the Sun's degree in the zodiac from its noon altitude, the *LT* gives a much briefer and simpler description of how to obtain the same information by means of the quadrant, or *horologium Achaz*, whose construction is described in chapter IV.1 (see below).[35] The degree to which the remaining chapters depend on *ACC* varies a great deal, including both cases of outright repetition (e.g., IV.12, IV.16, IV.29–31) and efforts to modify the source text. Chapter IV.3, on the construction of a meridian line, resembles chapter II.3 in *ACC* for its first half, but then adds an entirely new description of how to determine empirically whether a planet or star is crossing the meridian. In chapter IV.5, the author supplemented a reference to Ptolemy's *Almagest* already found in *ACC* by explicitly stating the relevant book and chapter (I.9). In chapter IV.29, a reference to the *Almagest* is

[35] *M*, fol. 68r–v: "Perpendiculum ponatur super diem mensis in linea curva horologii Achaz et quem gradum cuius signi tetigerit in illo gradu illius signi necessario sol erit, quod per primam huius libri probatur. Quomodo etiam per astrolabium inveniatur patebit."

newly inserted into a text otherwise copied from *ACC*, which would seem to indicate that the author had independent access to Ptolemy's work.

Chapters in *ACC* that deal with geographic latitude or correlated values always refer to the climate or region of Paris, which is assigned a latitude of 48°.[36] The author of the *LT* is consistent in replacing Paris with Strasbourg, presumably because this was his place of writing.[37] Already in chapter IV.1 he informs us that the Sun's equinoctial altitude at the latitude of Strasbourg will be 41;40°, while the corresponding solstitial altitudes are of 17;50° and 65;30° (see ll. 23–25 in the edition below). By implication, the obliquity of the ecliptic is here assumed to be 65;30° – 41;40° = 23;50° (rounded from Ptolemy's 23;51°) and the latitude of Strasbourg is taken as 90° – 41;40° = 48;20°, just 0;14° below the correct value. Not only is this the earliest evidence of a medieval latitude measurement for the city of Strasbourg, but the value in the *LT* is independent of those recorded in geographical coordinate tables of the late Middle Ages, where Strasbourg's latitude appears, less accurately, as 47° or 47;50°.[38]

An explicit statement as to the latitude of Strasbourg can be found in chapter IV.11, where manuscript *G* (written by Reinhard Gensfelder) has the expected 48;20°, whereas the three other witnesses for this chapter omit the degree value, leaving only *20 minuta*. In

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[36] *Artis cuiuslibet consummatio*, II.5–7, 18, ed. and trans. Victor (ref. 30), pp. 230–5, 258–9.

[37] The references to Strasbourg led Zinner to identify the author with Master Conrad of Strasbourg, author of a *computus* treatise approximately datable to the beginning of the thirteenth century. This, however, is as baseless as Zinner's speculation that Conrad or one of his students installed the sundial at the facade of Strasbourg Cathedral. See Zinner, *Entstehung* (ref. 12), p. 76.

[38] See, e.g., MSS Berlin, Staatsbibliothek, lat. fol. 246, fol. 155v (47°; s. XV^{med}); Bern, Burgerbibliothek, 483, fol. 198v (47;50°; s. XIV); D. B. Durand, *The Vienna-Klosterneuburg Map Corpus of the Fifteenth Century: A Study in the Transition from Medieval to Modern Science* (Leiden, 1952), pp. 106 (n. 2), 336.

chapter IV.8, by contrast, the equinoctial noon altitude of Strasbourg is given as 41;10°, which would imply a latitude of 48;50°.[39] Another such inconsistency occurs in chapter IV.9, where the noon altitude at the summer solstice is written as 65;36° (65;306° in *M* and *S*), while that at the winter solstice is alleged to be 17;10° in all manuscripts. Scribal corruption appears to be the likely cause behind these divergences.

Yet another numerical puzzle arises from chapter IV.28, which deals with apogeal longitudes and the rate of precession, as does *ACC*'s chapter II.26. In *ACC*, the apogeal longitudes are claimed to be valid for 1193, but the text contradicts itself insofar as it begins by citing Ptolemy's precession rate of 1°/100y, yet subsequently instructs readers to add or subtract 0;0,51° per year, which would instead point to a rate of 1°/70y.[40] The *LT* removes this inconsistency by sticking to the Ptolemaic rate, although it is unclear why the author initially equates 1°/100y with 0;0,36,14°/y rather than simply 0;0,36°/y, as he does later. He also offers a completely different set of apogeal longitudes, giving their reference date as August 1214 while claiming that these values were confirmed both on the basis of the *Almagest* and "by means of eclipses" (*per eclipses*).[41] If the value for the Sun is emended

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[39] Gensfelder, the scribe of *G*, corrected this number after the fact to 41;40°.

[40] *Artis cuiuslibet consummatio*, II.26, ed. and trans. Victor (ref. 30), pp. 282–5.

[41] *M*, fol. 69r: "Probatur in Almagesti libri septimi, capitulo tertio, quod quelibet stella fixa in omnibus 100 annis uno gradu movetur ab occidente in orientem, hoc est quolibet anno 36 secundis et 14 tertiis, et eodem modo altus locus sive aux cuiusvis planete preter quam lune. Est autem hodierno tempore, scilicet anno Domini M° CC° XIIII° in aureo numero 18, in mense Augusto aux Solis 2 signa, 17 gradus, 59 minuta, 36 secunda; aux Saturni 8 signa, 6 gradus, 39 minuta, 36 secunda; aux Iovis 5 signa, 24 gradus, 38 minuta, 36 secunda; aux Martis 4 signa, 6 gradus, 9 minuta, 36 secunda; aux Veneris 2 signa, 8 gradus, 39 minuta, 36 secunda; aux Mercurii 6 signa, 23 gradus, 39 minuta, 36 secunda. Hec omnimodo, tam in Almagesti, quam per eclypses, probavi. Si vis scire ubi erunt post multos annos, pro singulis annis addas 36 secunda. Si vis scire ubi fuerint ante multos annos, pro singulis annis 36 secunda subtrahas. Fuit autem aux solis primo anno Nabuchodonosor

from 77;59,36° to 87;59,36° and that for Mars is emended from 126;9,36° to 129;9,36°, the six apogees listed are all 22;29,36° ahead of those given in Ptolemy's *Almagest* (III.4, III.7, IX.4) for the Sun and the five planets.[42]

Sun	[8]7;59,36° = 65;30° + 22;29,36°
Saturn	246;39,36° = 224;10° + 22;29,36°
Jupiter	174;38,36° = 152;9° + 22;29,36°
Mars	12[9];9,36° = 106;40° + 22;29,36°
Venus	68;39,36° = 46;10° + 22;29,36°
Mercury	203;39,36° = 181;10° + 22;29,36°

Whether he was aware of it or not, the author here effectively altered Ptolemy's original doctrine by assuming that the solar apogee will be subject to precession and that the value of 65;30° given in the *Almagest*, rather than being permanent, applies to the epoch of the corresponding mean motion tables, which was 1 Thoth in the first year of Nabonassar = 26 February 747 BC. Yet this does not suffice to explain his decision to increase Ptolemy's apogeal values by +22;29,36°. The interval between February 747 BC and August 1214 is approximately 1960;40y. On an annual precession rate of 0;0,36°, a period of 1960;40y would produce a precessional increment of no more than 19;36,24°. The solar apogee should hence have been at 65;30° + 19;36,24° = 85;6,24° rather than 87;59,36°. Conversely, the

quinto gradu, 3<0> minuto Geminorum. Et hoc erat propositum probare." The date mentioned here is repeated in chapter IV.32. See ibid., fol. 69v: "Verbi gratia, ut si annus domini fuerit 1214 imperfectus, mense Augusto, in aureo 18".

[42] See Ptolemy, *Almagest*, trans. G. J. Toomer, rev. ed. (Princeton, NJ: Princeton University Press, 1998), pp. 153, 168, 427, 430, 433, 436, 439.

precession rate implied by an increase of 22;29,36° over 1960;40y is 1° in 86;15,45... years, or 0;0,41,18...°/y, which is too far removed from the values normally transmitted in medieval sources (e.g., 1°/66y, 1°/70y, 1°/72y, 1°/100y) to provide a good explanation.[43] Presumably, then, the numbers in this chapter must be ascribed to some arithmetical blunder on the part of the author.

The 'horologium Achaz'

One of the most striking oddities about the *LT* is its use of the label *horologium Achaz* to refer to the universal horary quadrant, the construction of which is the main subject of its chapter IV.1.[44] This unconventional name harbours an allusion to a story recounted in 2 Kings (20:8–11) and Isaiah (38:7–8), where God gives King Hezekiah a sign by making the shadow on the *ma'alot* of his father Ahaz (אָחָז בְּמַעֲלוֹת) go back by ten steps or degrees. Although some modern scholars have suggested that the *ma'alot* were intended to be a reference to the stairs outside Ahaz's house,[45] the text of the Latin Vulgate passed down a firm tradition according to which the miracle involved a *horologium* or sundial, the earliest in recorded history. Inspired by the biblical story, instrument makers and writers of late

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43 See C. P. E. Nothaft, "Criticism of Trepidation Models and Advocacy of Uniform Precession in Medieval Latin Astronomy," *Archive for History of Exact Sciences*, 71, 2017, pp. 211–244.

44 Besides chapter IV.1, the name *horologium Achaz* is also used in IV.22, which refers back to IV.1, whereas in IV.5 the variant *quadrans Achaz* is used.

45 S. Iwry, "The Qumrân Isaiah and the End of the Dial of Ahaz," *Bulletin of the American Schools of Oriental Research*, 147, 1957, pp. 27–33; cf. J. van Dorp, "The Prayer of Isaiah and the Sundial of Ahaz (2 Kgs 20:11)," in B. Becking and E. Peels (eds.), *Psalms and Prayers: Papers Read at the Joint Meeting of the Society of Old Testament Study and Het Oudtestamentische Werkgezelschap in Nederland en België, Apeldoorn August 2006* (Leiden: Brill, 2007), pp. 253–65.

medieval and early modern Europe occasionally used the moniker '*horologium* of Ahaz' for certain sundials.[46] In the case of the famous refractive sundials made in the sixteenth century by Georg Hartmann (1489–1564) and Christoph Schissler (*c.*1531–1608), the purpose of building an instrument named *horologium Achaz* was to recreate the biblical miracle in question, by having the Sun's rays refracted by water poured into a hemispheric bowl.[47] With the quadrant, attaching this label could be justified by the fact that the altitude of the Sun, as measured by the instrument's plumb line, will ascend between sunrise and noon, but then go back in the other direction between noon and sunset. This was a different sort of reversal than the one described in the Bible, but a reversal it was nonetheless. In fact, the construction sketch in chapter IV.1 reinforces the instrument's biblical grounding by citing a passage in the New Testament, John 11:9 (l. 3 in the edition below), which was a way of signalling that the twelve hours of the Sun's daily ascent and descent measured by the quadrant had already been acknowledged by Jesus Christ.

The following construction plan is relatively straightforward in that it tells us to draw a circle and divide the circumference of one of its quarters (the bottom right quarter, to be precise) into 90 degree-units [ll. 3–5]. To the graduated border of the quadrant, here designated as line A, one is instructed to add three more concentric arcs B, C, and D [ll. 6–8], as indicated in Fig. 1 below. The gap between D and C is meant to serve as a track for the cursor [ll. 8–10] and is hence considerably wider than that between C and B. Although the text lacks clarity on this point, one way of reading the following instructions [ll. 11–14] is

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[46] Zinner, *Deutsche und niederländische astronomische Instrumente* (ref. 12), pp. 51–52, 54.

[47] Zinner, *Deutsche und niederländische astronomische Instrumente* (ref. 12), pp. 108–9; J. Mersmann, "Moving Shadows, Moving Sun: Early Modern Sundials Restaging Miracles," *Nuncius*, 30, 2015, pp. 96–123, at pp. 111–22. See also P. M. Sadler, "An Ancient Time Machine: The Dial of Ahaz," *American Journal of Physics*, 63, 1995, pp. 211–16.

that the space between C and B must be divided into eighteen segments representing 5° each, while the space between B and A breaks these segments down further to display a total of 90 degree-units. If this was the intention, the design of the limb of the *horologium Achaz* would effectively have been the same as that described in the most popular thirteenth-century text on the *quadrans vetus,* which post-dates the *LT* by half a century. The eighteen segments between C and B are here reserved for figures from 5 to 90 showing the number of degrees on the limb.[48]

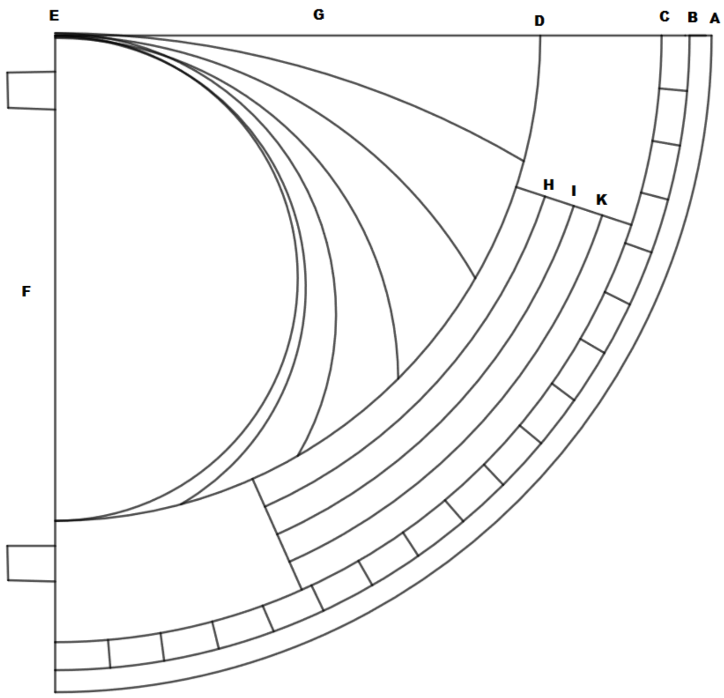

Fig. 1: the basic design of the face of the *horologium Achaz*

The cursor sliding between arcs D and C is referred to as *regula curva* [ll. 9, 20, 25–27, 46–47], which is a term not found in any other Latin treatise on this subject. In thirteenth-century

[48] *Geometrie due sunt partes*, c. 6–7, ed. Hahn (ref. 6), pp. xlii–xliii, 16–18.

descriptions of the *quadrans vetus*, the standard label is *cursor*,[49] whereas early accounts of the *quadrans vetustissimus* may have *curriculum* instead.[50] In sources of the latter category, the construction plan for the cursor includes markings for the months of the Julian calendar, but no zodiacal degrees. With one recently discovered exception,[51] these texts also fail to factor in the non-linear change of the solar declination in the course of the year, which makes it necessary to vary the width of the cursor-segment allotted to each month. This is also a flaw of the *LT*, which it shares with the treatises by John of Sacrobosco and Campanus of Novara.[52] At the same time, however, the *LT* is the earliest known Latin text to equip the cursor with parallel rows of data, taking into account the Sun's degree in the zodiac in addition to the calendrical date [ll. 29–41].[53] The correlation of dates and degrees is here based on the simple assumption that the summer solstice occurs on 15 June [ll. 42–47], which

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[49] John of Sacrobosco, *Tractatus de quadrante*, MS Paris, Bibliothèque nationale de France, lat. 7196, fols. 25rb–27va, at fol. 26r; Campanus of Novara, *Practica quadrantis*, MS Vatican City, Biblioteca Apostolica Vaticana, Pal. lat. 1414, fols. 208ra–211ra, at fol. 209ra; *Geometrie due sunt partes*, c. 11–29, ed. Hahn (ref. 6), pp. 23–39; *De compositione quadrantis*, MS Paris, Bibliothèque nationale de France, lat. 7475, fols. 60v–63v, at fol. 61v (s. XIII/XIV); *Cum quadrantem componere volueris*, MS Paris, Bibliothèque nationale de France, lat. 7416B, fols. 62ra–63va (s. XIV$^{\text{in}}$).

[50] Millás Vallicrosa, "La introducción" (ref. 1), pp. 226, 229, 236, 238

[51] See C. P. E. Nothaft, "An Overlooked Construction Manual for the *Quadrans Vetustissimus*," *Nuncius* (forthcoming).

[52] See on this point Knorr, "The Latin Sources" (ref. 7), pp. 49–50; Knorr, "Sacrobosco's *Quadrans*" (ref. 8), pp. 212–13.

[53] John of Sacrobosco, *Tractatus de quadrante*, MS Paris, Bibliothèque nationale de France, lat. 7196, fols. 25rb–27va, at fol. 26rb–va; Campanus of Novara, *Practica quadrantis*, MS Vatican City, Biblioteca Apostolica Vaticana, Pal. lat. 1414, fols. 208ra–211ra, at fol. 209ra–va.

is correct for Strasbourg and 1214, although other years in this period (first quarter of the thirteenth century) would have had the solstice on 14 June.

Having mentioned the two small sighting vanes that are affixed to the instrument's left flank (line F in Fig. 1) [ll. 48–50], the chapter concludes by describing as an additional option the construction of a shadow square [ll. 51–69], which was already familiar from the backplates of certain astrolabes. A likely model for some of the pertinent instructions in IV.1 is chapter II.37 in *ACC*, which explains how to draw a shadow square on the surface a quadrant in vaguely similar terms, using some of the same wording.[54] It is interesting to observe, however, that the *ACC* has nothing to say regarding the quadrant's horological function, which may tell us something about the instrument's evolution in medieval Europe. Perhaps the *horologium Achaz* described in the *LT* arose as a conscious effort to combine two types of quadrant that had previously existed separately from each other: (i) a horary quadrant with a cursor and curved lines; and (ii) a quadrant for the purposes of altimetry and planimetry, which only featured the shadow square. The *horologium Achaz* added the latter function to the horary quadrant by drawing the shadow square on its reverse side. In a further step, both functions were integrated more neatly by representing them on a single plate. This

[54] *Artis cuiuslibet consummatio*, II.37, ed. and trans. Victor (ref. 30), pp. 318–321. That this chapter was a source behind *LT*'s chapter IV.1 is suggested by the following passage (loc. cit., ll. 11–14): "Et in termino linee protense a c usque ad mediam sectionem, fiat ibi quadratum orthogonium. Dataque latitudine ad libitum, dividantur duo latera in sena et in 12 spatia." Compare the verbal overlap in chapter IV.1 [ll. 59–62 in the edition below]: "*Et in termino linee protense usque* utrumque latus ducantur linee recte orthogonaliter. *Fiat*que ibi *quadratum orthogonium* cuius unum latus vocetur C, reliquum B. *Dataque* ei *latitudine* per duas lineas equidistantes linea C in sex equas partes dividatur, similiter linea B, quarum quelibet in duo."

gave rise to the *quadrans vetus*, whose standard construction plan places the shadow square above or beneath the curved hour-lines.[55]

A graduated alidade

When Ernst Zinner wrote a highly condensed account of the quadrant described in the *LT* for his 1956 survey of astronomical instruments, he assumed that the text's construction plan extended to the second *theoreuma* of book IV, which is headed *horas naturales in regula locare*. What this brief chapter explains is how to turn the rotatable rule or alidade found on the reverse side of a typical astrolabe into a portable gnomon by having one of its sighting vanes (also known as pinnules) cast a shadow on some markings for the seasonal hours inscribed on its surface.[56] Since Zinner read chapter IV.2 in the *LT* as a continuation of IV.1, he assumed that such an alidade was supposed to be included on the back of the *horologium Achaz* as a second feature besides the shadow square. According to his understanding, this version of the quadrant offered two ways of telling time based on the Sun's altitude: the curved hour-lines on the front and the alidade mounted on the back.[57] It is far from obvious, however, that this is what the author of these chapters intended. Rather than following on from the previous chapter, the topic of chapter IV.2 appears to be intended as a new one, with

[55] John of Sacrobosco, *Tractatus de quadrante*, MS Paris, Bibliothèque nationale de France, lat. 7196, fol. 25v; Campanus of Novara, *Practica quadrantis*, MS Vatican City, Biblioteca Apostolica Vaticana, Pal. lat. 1414, fol. 209va; *Geometrie due sunt partes*, c. 30–31, ed. Hahn (ref. 6), pp. 40–41; *De compositione quadrantis*, MS Paris, Bibliothèque nationale de France, lat. 7475, fol. 61v; *Cum quadrantem componere volueris*, MS Paris, Bibliothèque nationale de France, lat. 7416B, fol. 62vb.

[56] Such graduated alidades can indeed be found on a handful of European astrolabes, although none of them appear to pre-date the fifteenth century. See King, *In Synchrony with the Heavens* (ref. 2), II, pp. 253–5.

[57] Zinner, *Deutsche und niederländische astronomische Instrumente* (ref. 12), p. 156.

no indication that it relates to the *horologium Achaz* in any direct way. It is true that the chapter in question begins very abruptly, as it does not specify what the *regula* under discussion is or where it is located. This may simply mean, however, that the description of the graduated alidade was wrenched from its original context, the possibility being that it was taken from an unknown construction manual on the astrolabe.

Our text describes a simple graphical method for placing hour-marking on the alidade, which is summarized in Fig. 2.[58] Its begins by informing us that the distance between the tip of the alidade and the line that marks the end of the fifth and beginning of the sixth hour (or, what is the same, the end of the seventh and beginning of the eighth hour) will be four times the height of the shadow-casting pinnule, which puts a constraint on the minimum length of the alidade in relation to the pinnule [ll. 72–73]. These proportions are subsequently used for an auxiliary drawing, in which the tip of the pinnule is used as the vertex of a quadrant consisting of sides AB and AC and arc BC. In a next step, the arc is divided into six equal segments whose divisions are marked by letters DEFGH [ll. 73–77]. Each of these points is then used to draw a line passing through the vertex of the quadrant (point A), which will meet the long line representing the alidade at a position that marks the end of one of the seasonal hours as well as the beginning of the next one (points IKLMN) [ll. 78–90].

[58] On the presence of this method in (earlier) Arabic and (later) Hebrew works, see J. Rodríguez-Arribas, "A Treatise on the Construction of Astrolabes by Jacob ben Abi Abraham Isaac al-Corsuno," *Journal for the History of Astronomy*, 49, 2018, pp. 27–82, at pp. 33, 63–65, 76–82.

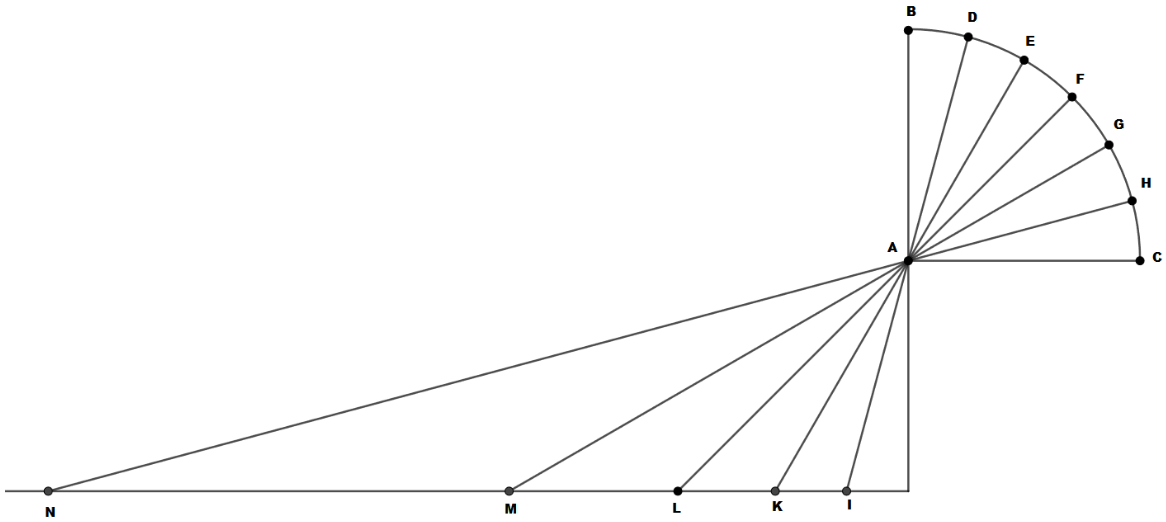

Fig. 2: graphical method of placing hour-markings on the surface of an alidade

The method just described appears to have been introduced into Latin Europe by John of Seville (*Johannes Hispalensis*), a prolific translator of Arabic texts active in the first half of the twelfth century.[59] His Christian name features at the beginning of a treatise on astrolabe construction (*Dixit Johannes: Cum volueris facere astrolabium accipe auricalcum optimum*), which survives in at least six manuscripts. In two of these, the text is expressly identified as a translation from Arabic made by "John of Spain," while another comes with an attribution to *Hispalensis*.[60] The final paragraphs of this text propose two different ways of marking hours

[59] See C. Burnett, "Introduction: John of Seville and Limia," in C. Burnett and P. Mantas-España (eds.), *"Ex Oriente Lux": Translating Words, Scripts and Styles in Medieval Mediterranean Society* (Córdoba: UCOPress, 2016), pp. 11–17; Burnett, "John of Seville and John of Spain: A *Mise au point*," *Bulletin de philosophie médiévale*, 44, 2002, pp. 59–78.

[60] MS Florence, Biblioteca Riccardiana, 866, fols. 1r–6va (s. XIV): "Incipit liber astrolabii editus a magistro Iohanne Yspano interprete quem translavit de Arabico in Latino." MS Vienna, Österreichische Nationalbibliothek, 2452 (s. XIII), fols. 1r–5v, at fol. 1r: "Incipit liber astrolabii editus a magistro Iohanne Yspano interprete quem translatavit de Arabico in Latino." The name *Hispalensis* appears at the start of MS

on the astrolabe. One is based on the actual shadow lengths for different angles of solar altitude, as they can be derived from numerical tables or from the shadow square on the back of the astrolabe. The other is the graphical method involving the drawing of a quarter-circle. The description of this method was originally accompanied by a diagram, as seen from the fact that the copies of John's translation all finish with the words "as you can see in this *figura*" (*quemadmodum vides in hac figura*).[61]

From *Dixit Johannes*, the account of how to equip the alidade with hour-markings passed into at least two other Latin texts on astrolabe construction. One begins *Astrologi(c)e speculationis exercitium habere volentibus* and ends with a much abbreviated account of the two ways of inscribing the hour-markings.[62] In two of the known manuscripts, this account is

Oxford, Bodleian Library, Canon. Misc. 340, fols. 49r–54r (s. XIV). Other copies I am aware of are MSS Edinburgh, Royal Observatory, Crawford Collection Cr. 3.28, fols. 31r–40r (s. XV; not seen); Paris, Bibliothèque nationale de France, lat. 7293A, fols. 1r–14v (s. XIII/XIV); Toledo, Archivo y Biblioteca Capitulares, 98-27, fols. 35r–48v (s. XV; not seen); Vatican Library, Biblioteca Apostolic Vaticana, lat. 4087, fols. 74v–80v (s. XIV). See Thorndike and Kibre, *A Catalogue of Incipits* (ref. 13), col. 454. Larger extracts from the manuscript in Toledo were published in J. M. Millás Vallicrosa, *Las traducciones orientales en los manuscritos de la Biblioteca Catedral de Toledo* (Madrid: CSIC, 1942), pp. 322–7. See also P. Kunitzsch, *Typen von Sternverzeichnissen in astronomischen Handschriften des zehnten bis vierzehnten Jahrhunderst* (Wiesbaden: Harrassowitz, 1966), p. 31; Kunitzsch, "Glossar der arabischen Fachausdrücke in der mittelalterlichen europäischen Astrolabliteratur," *Nachrichten der Akademie der Wissenschaften in Göttingen*, phil. hist. Kl., Jg. 1982.1, pp. 459–571, at p. 488. It should be noted that the text of *Dixit Johannes* often precedes John of Seville's translation of a treatise on astrolabe use by Ibn aṣ-Ṣaffār (ibid., pp. 486–487).

[61] A defective diagram is preserved in MS Oxford, Bodleian Library, Canon. misc. 340, fol. 53v, but its authenticity is uncertain.

[62] A transcription of this text, based on MS Madrid, Biblioteca nacional de España, 10009, fols. 18rb–20rb (s. XIII$^{2/2}$), was published in Millás Vallicrosa, *Las traducciones* (ref. 60), pp. 316–21, who merged the text with the preceding item in the same manuscript. See also Kunitzsch, "Glossar" (ref. 60), pp. 496–7. Other copies

supplemented with a schematic drawing similar to Fig. 2 above.[63] The earliest copy of *Astrologi(c)e speculationis*, which is now in Pommersfelden, dates from the first half of the thirteenth century.[64] It starts with a note in the top margin identifying the text as a translation by John of Seville (*Io. Yspolensis transtulit*),[65] as did the author of the *Speculum astronomiae* (mid-thirteenth century).[66] Given the close similarities between *Astrologi(c)e speculationis* and *Dixit Johannes*, however, it seems more likely that we are dealing with an adaptation of John's original translation rather than an additional translation from his hands. Further

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include MSS Madrid, Biblioteca nacional de España, 10063, fols. 12ra–13ra (s. XIII); Munich, Bayerische Staatsbibliothek, Clm 28229, fols. 67r–69v (s. XIV); Oxford, Merton College, 259, fols. 89v–90v (s. XIII); Paris, Bibiliothèque de l'Arsenal, 879, fols. 48v–51v (s. XIII); Pommersfelden, Gräflich Schönbornsche Bibliothek, 66 (2640), fols. 106ra–109rb (s. XIII$^{1/2}$); Vatican City, Biblioteca Apostolica Vaticana, lat. 3114, fols. 75r–77v (s. XIII/XIV); Vatican City, Biblioteca Apostolica Vaticana, Reg. lat. 1285, fols. 36ra–38rb (s. XIII). I am grateful to David Juste for helping me compile this list. The treatise on astrolabe construction starting *Astrologi(c)e speculationis exercitium habere volentibus* must not be confused with the identical incipit of the prologue accompanying a treatise on astrolabe use in manuscripts such as Bergamo, Biblioteca Civica Angelo Mai, MA 388, fol. 76r (s. XV$^{2/2}$); Erfurt, Universitäts- und Forschungsbibliothek, Dep. Erf. CA 4º 386, fol. 110v (s. XIV); Paris, Bibliothèque nationale de France, lat. 7292, fol. 292v (s. XV).

[63] MSS Madrid, Biblioteca nacional de España, 10009, fol. 20rb; Vatican City, Biblioteca Apostolica Vaticana, Reg. lat. 1285, fol. 38rb.

[64] D. Juste, "MS Pommersfelden, Gräflich Schönbornsche Bibliothek, 66 (2640) (update: 05.08.2019)," *Ptolemaeus Arabus et Latinus. Manuscripts*, URL = http://ptolemaeus.badw.de/ms/542.

[65] MS Pommersfelden, Gräflich Schönbornsche Bibliothek, 66 (2640), fol. 106ra. On the attribution to John of Seville, see Burnett, "John of Seville" (ref. 59), p. 60n10.

[66] *Speculum astronomiae*, ed. in P. Zambelli, *The* Speculum Astronomiae *and Its Enigma: Astrology, Theology and Science in Albertus Magnus and his Contemporaries* (Dordrecht: Kluwer, 1992), p. 218: "Sunt praeterea libri necessarii de hac parte scientiae, qui per viam narrationis demonstrationem planisphaerii imitantur, ut est ille quem transtulit Ioannes Hispalensis, qui sic incipit: *Astrologicae speculationis etc.*"

research on the transmission and sources of *Astrologi(c)e speculationis* would be needed, however, to settle this question.

The other work on the astrolabe to incorporate the instructions in *Dixit Johannes* is the one traditionally, albeit spuriously, attributed to Māshā'allāh ibn Athari. This widely diffused text often comes in two parts, on the construction (inc.: *Scito quod astrolabium est nomen grecum*) and on the use of the astrolabe (inc.: *Nomina instrumentorum sunt hec*), which may have originated independently from each other.[67] The construction part is itself the result of multiple accretions of material over time, as chapters 1–6, 7–16, and 17–22 each form a separate unit of undetermined origin.[68] It reached its final form no earlier than 1246, the year of a star table included in the final chapter.[69] The two methods of inscribing hour-markings are here presented in chapter 5, which is only a lightly modified version of the text at the end of *Dixit Johannes*.[70]

Whoever wrote the chapter in the *LT* was clearly aware of both approaches mentioned in John's translation, although he addressed one of them only in a very cursory fashion in the penultimate sentence of chapter IV.2 [ll. 91–93]. What is described there differs significantly from the account of the first method in *Dixit Johannes* and the other two texts just mentioned insofar as the author takes no recourse to shadow tables or squares. Instead, he suggests

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[67] P. Kunitzsch, "On the Authenticity of the Treatise on the Composition and Use of the Astrolabe Ascribed to Messahalla," *Archives internationales d'histoire des sciences*, 31, 1981, pp. 42–62; Kunitzsch, "Glossar" (ref. 60), pp. 499–502.

[68] Kunitzsch, "On the Authenticity" (ref. 67), pp. 45–48.

[69] Kunitzsch, *Typen von Sternverzeichnissen* (ref. 60), pp. 39–50; Kunitzsch, "On the Authenticity" (ref. 67), p. 47.

[70] pseudo-Māshā'allāh, *De compositione astrolabii*, c. 5, ed. and trans. Ron B. Thomson (Toronto, 2018), pt. 2, pp. 78–107. This edition is available at https://shareok.org/handle/11244/14221 (accessed 27 Sept. 2019).

measuring shadow lengths with the aid of circles that are somehow divided into twelve digits of different colour. This cryptic passage must be interpreted in the light of chapter II.38 in *ACC*, which describes the construction of a gnomon surrounded by a circular measuring scale that converts the length of the shadow cast by the gnomon into digits.[71] If this additional echo of *ACC* is taken to one side, the remainder of chapter IV.2 could well have been fashioned out of the account in *Dixit Johannes*. What the author of *LT* contributed is a more geometrical way of phrasing things, as manifested by his way of assigning letters to all important points in the graphic and his use of the term *katheta* to refer to the vertical line representing the pinnule [ll. 74–75, 91, 93]. This, of course, should come as no surprise given the text's rootedness in the tradition of practical geometry embodied by *ACC*, its principal source.

Edition of the text

The following edition of *LT*, chapters IV.1–2, is based on five witnesses, which I shall indicate by the same sigla as above:

G Munich, Bayerische Staatsbibliothek, Clm 56, fols. 145r–146v

M Munich, Bayerische Staatsbibliothek, Clm 14684, fols. 65r–66v

P Vatican City, Biblioteca Apostolica Vaticana, Pal. lat. 1376, fols. 185vb–186va

S St Florian, Stiftsbibliothek, XI.619, fols. 127v–128v

V Vienna, Österreichische Nationalbibliothek, 5418, fols. 209r–211r

I shall disregard a sixth copy of the text, MS Vienna, Österreichische Nationalbibliothek, 5303, fols. 274r–276r, for being a straightforward copy of *V*. The main text of the edition is oriented towards *M* with significant variants taken from *G*. Their orthography has been

[71] *Artis cuiuslibet consummatio*, II.38, ed. and trans. Victor (ref. 30), pp. 322–5.

normalized throughout. To aid an understanding of the Latin text, I also attach free English translations of both chapters, which are oriented towards the sense rather than the letter of the original.

Acknowledgements

I am deeply grateful to Charles Burnett and David Juste for their helpful comments on an earlier draft of this article.

Primum theoreuma: horologium Achaz[1] *collocare*

[…] Horologium itaque Achaz[2] sic componens,[3] secundum quod dicit dominus in evangelio ‘Nonne XII[4] sunt hore diei’ [Ioh. 11:9], primo itaque circulum in aliquo plano describes, quem in quatuor equa dividas per duos diametros. Quartam illam que est inferior et tibi[5] dextra dividas in tres partes equales, quarum quelibet 30 partes habebit, et sic[6] tota quarta 90. Postea dividas quamlibet trium in sex equas partes et quamlibet sex partium in quinque. Et linea curva iam dictas divisiones continens vocetur A. Postea dabis ei latitudinem per duas lineas B, C. Deinde adhuc dabis ei maiorem latitudinem per tertiam lineam curvam que vocetur D, ita ut regula curva signa cum mensibus continens in concavitate facta inter D et C normaliter valeat moveri. Centrum autem vocetur E, semidiameter sinister F, dexter vero G vocetur. Inde sic pone unum caput regule recte super E punctum et aliud super primas tres intersectiones: duc lineas rectas usque ad lineam C, ita ut[7] a divisionibus per sex factas [*sic*] alie recte linee ducantur ad B inter singulas factis lineis quinquennis, qui[8] sunt gradus ascensionum.

Orisonta[9] vero sic compone: diameter F, qui est ab E puncto ad D lineam, in duo equa partiatur et posito pede circini in puncto invento semicirculum ab E in D describe et hic erit

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[1] Achaz] acham *PS* achab *V*

[2] Achaz] achab *V*

[3] componens] compones *V*

[4] XII] 12 *GV*

[5] tibi] ubi *PS*

[6] sic] fit *P* sit *S*

[7] ut] *om. PS*

[8] qui] que *V*

[9] orizonta] orisontem *G(a.c.) V* orizanta *P*

orison. Postea totam lineam D in sex equales partes[10] partire et inventis centris in F linea singulas lineas ab E ad singulas sectiones ducas et ante meridiem ascendendo habebis sex horas, alias vero descendendo post meridiem, que sunt hore totius anni artificiales.

Signa autem cum suis mensibus in regula curva sic describe: vide que sit altitudo solstitialis[11] hiemalis per quadrantem iam constitutum, sive per astrolabium civitatis tue, quam invenire docebimus,[12] et illam nota[13] inter gradus ascensionis versus dextram, que est super Argentinam 17[14] graduum et 50 minutorum. Iterum sume altitudinem solstitialem estivalem et eam notes eodem modo versus sinistram, que est Argentine 65 graduum et 30 minutorum. Et ita erit altitudo equinoctialis 41 gradus et 40 minuta.[15] Deinde posita regula super E et super iam dictum solstitium estivale lineam ducas in regula curva. Simili modo facias de solstitio hiemali. Et quod ex utraque parte de regula curva superfluum fuerit recidatur. Residuum vero latitudinem regionis representat.

Signa autem cum suis mensibus in regula sic describes:[16] latitudinem regule in quatuor equas partes dividas per lineas HIK. Post hec lineam H in sex equas partes[17] divide. Similiter linea K. Postea posita regula recta super E et super singulas sectiones duc lineas 12 que linee spatia[18] 12 signorum presentabunt. Postea quodlibet signum in 30 equas partes, ut

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[10] equales partes] partes equales *GV*

[11] solstitialis] solstitii *GV*

[12] *Cf.* caput IV.9.

[13] nota] notam *PV*

[14] 17] *a.c.* 47 *V*

[15] minuta] *add.* etc. *V*   minutorum *GMPS*

[16] describes] describas *V*

[17] partes] portiones *V*

[18] spatia] spatium *PS*

dictum est de ascensionibus, partiaris, et sic gradus signorum habebis. Inscribes itaque Cancrum primo signo linee H incipiens a sinistra et sic procedas[19] usque ad Geminos linee K versus sinistram; et sic habebis 12 signa.

Menses sic compones: lineam I dividas in 183 partes et lineam K in 182, que simul faciunt 365 dies. Quod sic facies: pone cum circino tres dies in linea I secundum graduum quantitatem quos ante invenisti et similiter duos dies in linea K incipiendo a sinistra. Postea utramque linearum IK in tres equas partes[20] metire quarum quamlibet in duo, utramque duarum in tres, quarum quamlibet in duas,[21] duarum utramque[22] in quinque; et habebis 365 dies.

Sed quia 15[a] die Iunii sol intrat Cancrum, 15 dies numera in linea I et hoc a sinistra et ibi[23] fac distinctionis[24] lineam, et habes dimidium mensem in I linea et dimidium in K linea. Hoc facto superscribe 'Iunius' incipiendo in K linea et sic, singulis mensibus sequentibus in ordine numerum suorum dierum sibi tribuendo et distinctionis lineas protrahendo, singulis[25] mensibus nomen proprium superscribe; et sic usque ad finem Maii procede; et sic[26] hec curva regula astrolabii posticam tibi representat.

[19] procedas] procede *G* procedes *V*

[20] equas partes] partes equas *G*

[21] duas] duo *V*

[22] utramque] quamlibet *G(p.c.)V*

[23] et ibi] ubi *V*

[24] distinctionis] distinctiones *V*

[25] Singulis] singulas *V*

[26] sic] *om. V*

Duas quidem pennulas super semidiametrum F pone, quarum utraque foramen proportionale ad aliam habebit. Tandem perpendiculum E puncto infigatur, perla[27] perforata transcurrente in ipso. Et sic habemus[28] propositum.

In dorso, si placeat,[29] ad aliquem[30] locum[31] profundum metiendum quadratum constitues,[32] cuius compositionem[33] in astrolabii compositione invenies. Tene itaque quod habes. Secretum enim est. In dorso iam dicti quadrantis Achaz[34] unus pes circini E puncto infigatur et alter pes quartam partem circuli describat extra eiusdem quadrantis extremitatem et vocetur A dividaturque A in tres partes equales et vocentur LMN.[35] Unaquamque vero[36] illarum in sex quarum quelibet in quinque,[37] ut supradictum est, partiatur, et habebis partes 90. Ducaturque alia[38] linea recta ab E puncto, scilicet[39] a centro, ad medietatem linee A, scilicet[40] ad 45^{am}[41] sectionem.

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[27] perla] particula *V*

[28] habemus] habebis *PV*

[29] placeat] placet *V*

[30] aliquem] alium *GMPSV*

[31] locum] lacum *GM*

[32] constitues] constituens *PS*

[33] compositionem] compositio *V*

[34] Achaz] achab *V* acham *PS*

[35] LMN] L in *P*

[36] vero] *om. V*

[37] quinque] 6 *P*

[38] alia] A *MPSV*

[39] scilicet] id est *V*

[40] scilicet] scibilis *V*

[41] 45^{am}] 40 *GMPSV*

Et in termino linee protense usque utrumque latus ducantur linee recte orthogonaliter. Fiatque ibi quadratum orthogonium cuius unum latus vocetur C, reliquum B. Dataque ei latitudine per duas lineas equidistantes linea C in sex[42] equas partes[43] dividatur, similiter linea[44] B,[45] quarum[46] quelibet in duo. Positaque regula super E punctum et super singulas sectiones per sex[47] divisas ducantur singule linee ab[48] linea ad lineam intimam latitudinis. Item posita linea super idem punctum E et super singulas sectiones factas per duo linee B ducantur singule linee[49] ad proximam sue latitudinis lineam et habebis unum latus quadrati in 12 partes equas divisum, qui vocantur digiti. Simili modo de C linea[50] procedas per omnia et habebis aliud latus quadrati. Infixoque E[51] puncto perpendiculo omnem longitudinem, latitudinem, altitudinem, profunditatem per iam dictum quadratum, ut infra dicetur, mensurabis.[52]

SECUNDUM THEOREUMA: HORAS NATURALES IN REGULA LOCARE

Erigatur tabula orthogonaliter in regule capite, ad cuius tabule longitudinem ipsam regulam quater vel amplius divide, quia in quadruplo numero est finis hore quinte. Postea ad

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[42] sex] G *S*

[43] equas partes] partes equas *V*

[44] linea] *om. S*

[45] dataque ... B] *om. P*

[46] quarum] deinde *GV*

[47] sex] G *PS*

[48] ab] *add.* E *V*

[49] linee] *add.* linee B *GMPS*   *add.* B *V*

[50] C linea] linea C *V*

[51] E] *s.l. S*

[52] mensurabis] *add.* Et exemplum predictorum subsequens declarat figura *G*

similitudinem huius regule atque katheti in plano lineam cum suo katheto scribe, in cuius katheti summitate quartam partem circuli describe,[53] cuius centro A, angulo superiori B, inferiori vero C appone. Lineam itaque a B in C in[54] sex equas partes[55] metire et eas per litteras DEFGH memento notare.

Postea capite regule posito in D, ita ut regula per A punctum transiens contingat lineam divisam, ibi fac[56] punctum I,[57] cui superscribe finis hore prime. Postea pone regulam super E et super A et ubi intersecat lineam divisam fac punctum K, cui superscribe finis secunde. Ita posita regula super F et super A, ubi intersecat lineam divisam, fac punctum L, cui supernotabis finis tertie. Tandem regulam pone[58] super G et super A et ubi tangit lineam divisam fac punctum M, cui superpone finis quarte hore. Postea regula constituta super H et punctum A, ubi ceciderit super lineam divisam fac punctum N, cui supernotabis finis quinte hore. Sexta vero finem non habet, sicut nec principium prima.

Sic ergo sex horas habes ab ortu solis ascendendo ad meridiem, a meridie vero alias sex descendendo ad occasum. Et ita finis prime hore erit initium duodecime, finis secunde initium undecime, finis tertie initium decime, finis quarte initium none, finis quinte initium octave, sexte similiter septime, que omnia normaliter in regula divisa transferre memento et habebis instrumentum ad horas capiendas necessarium.

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[53] describe] inscribe *V*

[54] in] *om. V*

[55] equas partes] partes equas *GV*

[56] fac] *om. P*

[57] I] *add.* et *V*

[58] regulam pone] pone regulam *V*

Hoc idem sit per circulos per duodecim digitos ad quantitatem katheti diversi coloris divisas [*sic*], ita ut umbra plana, quam invenire docebimus, proportione digitorum ad suum kathetum coaptetur. Et hec ad presens de horis capiendis sufficiant explanata.

First theorem: to assemble the sundial of Ahaz

[...] And so, when you build the sundial of Ahaz, according to what the Lord says in the Gospel (‘Are there not twelve hours of the day?’ [John 11:9]), you must first draw a circle on some level surface, which you must divide into four equal parts by means of two diameters. Divide the lower quarter to your right into three equal parts, of which each will have 30 parts, such that the whole quarter [will have] 90. Next divide each of the three into six equal parts and each of the six parts into five. And the curved line containing these divisions shall be called A. Next you will give [A] a width by [adding] two lines, B [and] C. Afterwards you will give it a greater width by [adding] a third curved line, which shall be called D, such that the cursor containing the signs with the months can move normally within the cavity made between D and C. The centre shall be called E, the semidiameter to the left F, whereas the one to the right shall be called G. Next place one end of the ruler above point E and the other above the three divisions made at the beginning. Draw straight lines down to line C, such that you make divisions by six and draw further lines down to B, making five lines between each [pair], which are the degrees of [the Sun’s] ascensions.

Construct the horizon as follows: diameter F, which runs from point E to line D, must be divided into two equal [parts] and by placing the foot of the compass on the point thus found you must draw a semicircle from E to D; and this will be the horizon. Next divide the whole of line D into six equal parts and with centres found on line F you must draw lines from E to these divisions and you will have six hours ascending before noon, whereas after the noon they are descending. These are the artificial hours of the entire year.

Inscribe the signs with their months on the cursor as follows: see what the [Sun’s noon] altitude at the winter solstice is, using the quadrant already constructed or an astrolabe for your city, which [i.e., the solstitial altitude] we shall teach to find [later],[130] and make a note of it among the degrees of the [Sun's] ascensions towards the right. It is 17 degrees and 50 minutes at [the latitude of] Strasbourg. Likewise, take the [Sun's noon] altitude at the summer solstice and make a note of it towards the left. It is 65 degrees and 30 minutes in Strasbourg. And the equinoctial altitude will accordingly be 41 degrees and 40 minutes. Next place the ruler over E and the already mentioned summer solstice and draw a line towards the cursor. Do the same for the winter solstice. And whatever superfluous space is left on the cursor on either side shall be cut away. The remainder, however, represents the latitude of the region in question.

This is how you will inscribe the signs with their months on the cursor: divide the width of the cursor into four equal parts by [drawing] lines H, I, and K. Next divide line H into six equal parts. Do the same for line K. Then place the ruler above E and above these divisions and draw 12 lines. The spaces between these lines will represent the 12 signs. Next divide each sign into 30 equal parts, in the same way as was said about the [Sun's] ascensions, and this way you will have the degrees of the signs. Inscribe the name Cancer into the first sign of line H, starting from the left, and proceed in this way until [you reach] Gemini on line K towards the left. And this way you will have 12 signs.

This is how you will construct the months: divide line I into 183 parts and line K into 182, which together make up 365 days. Do this as follows: use the compass to put three days on line I according to the size of the degree you found earlier and do likewise with two days on line K, starting from the left. Then divide both lines I and K into three equal parts and each of these into two, each of the two into three, each of the [three] into two, and each of the two into five. And you will have 365 days.

[130] In Chapter IV.9.

Yet since the Sun enters Cancer on 15 June, you must count 15 days on line I, and do this from the left, and there draw a line of separation, and what you have is one half of the month on line I and one half on line K. Having done this, write ‘June’ above it, starting on line K. And this is how you must write above each month its name, giving each of the following months in their sequence the appropriate number of days and drawing lines of separation [between them]. Proceed in this manner until the end of March. And this way the cursor will show you [the same as what is found on] the reverse of an astrolabe.

Place two little wings on semidiameter F, of which each will have an aperture proportionate to the other. At last, one ought to attach to point E a plumb line on which a perforated pearl can run up and down. And this way we have [reached] the proposed [result].

If you want, you can construct on the reverse side a quadrant to measure some deep place, the composition of which you will find in [the rules of] composing an astrolabe. Keep, then, what you have [learned], for it is a secret. On the back of the aforementioned sundial of Ahaz one must attach one foot of the compass to point E while the other foot describes a quarter-circle outside of the perimeter of the same quadrant. And [this quarter-circle] shall be called A and must be divided into three equal parts, which shall be called L, M, and N. Each of these, however, must be divided into six [parts] and each of them into five, as has been said above, and you will have 90 parts. Draw another straight line from point E, that is, from the centre, towards the middle of line A, that is, towards the 45th division.

And at the end-point of a line extended to either side one must draw lines orthogonal to it. And this results in a square whose one side shall be called C, the other B. Give C a width by drawing two lines at the same distance and divide it into six equal parts, each of these into two, doing the same with line B. Placing the ruler over point E and each of the six divisions, one must draw lines for each of them that cross the width from one line to the other. Likewise, placing a line above the same point E and the each of the subdivisions into two on line B, draw lines that for each of them go to the neighbouring line of its width, and you will have one side of the square divided into twelve equal parts, which shall be called ‘digits’. Proceed in exactly the same manner with line C and you will have the other side of the square. If you affix to point E a plumb line you will be able to use the said square to measure any length, width, height, or depth, as shall be shown below.[131]

SECOND THEOREM: TO LOCATE THE NATURAL HOURS ON THE ALIDADE

Erect a plate that is orthogonal to the tip of the alidade and divide the alidade four times or more according to the length of this plate, because the end of the fifth hour is [located] at four times [this length]. Next draw a line with its vertical that is as long as this alidade with its vertical. Draw a quarter-circle at the tip of the vertical, assigning the letter A to its centre, B to its higher corner, C to its lower one. Divide the line from B to C into six equal parts and remember to mark them with the letters D, E, F, G, and H.

Next place one end of the ruler at D, such that by passing through point A the ruler touches the line divided [before]. Mark this spot with point I, above which you must write the end of the first hour. Next place the ruler over E and A and where it intersects with the divided line make point K, above which you must write the end of the second [hour]. Having placed in this way the ruler above F and A, make point L where it intersects the divided line, and write above it the end of the third [hour]. At last, place the ruler above G and A and where it touches the divided line make point M, above which you must write the end of the fourth hour. Afterwards position the ruler above H and point A, make point N where it falls on the divided line, above which you will write the end of the fifth [hour]. The sixth [hour], however, has no end, just as the first has no beginning.

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[131] Note that no such instructions appear in the preserved text of the *Liber theoreumacie*.

This way you have six hours ascending from sunrise towards noon, while another six descend from noon to sunset. And the end of the first hour will accordingly be the beginning of the twelfth, the end of the second will be the beginning of the eleventh, the end of the third will be the beginning of the tenth, the end of the fourth will be the beginning of the ninth, the end of the fifth will be the beginning of the eight, while the sixth is like this in relation to the seventh. Remember to transfer all of these in the usual way onto the divided alidade and you will have an instrument needed for measuring the hours.

The same can be had by means of circles divided into twelve digits of different colour according to the size of the vertical, such that the plane shadow, which we shall teach to find [later],[132] is measured by the ratio of the digits to its vertical. And this shall suffice for the moment as an explanation of how to measure the hours.

[132] In Chapter IV.5.