Varieties of Logic, by Stewart Shapiro. Oxford: Oxford University Press, 2014. Pp. iix + 226.

Shapiro's wide-ranging and thought provoking book marks a major milestone in the recent debate initiated by J. C. Beall and Greg Restall's influential *Logical Pluralism* (Beall and Restall 2006). Pluralism about a given subject (e.g. etiquette, logic) is loosely characterized as 'the view that different accounts of the subject are equally correct, or equally good, or equally legitimate, or perhaps even true' (p. 13). Shapiro's book offers us many (more precisely characterized) ways to adopt 'an eclectic orientation to logic'. But his official position, which sometimes takes some unexpected twists, is incrementally developed over the seven main chapters. Before we delve into some of the details, it's worth outlining the overarching argument, helpfully summarized in a short closing chapter.

The first chapter presents the framework Shapiro employs for discussing pluralism, and related issues. The key notion is Crispin Wright's (2008) *folk relativism*—slogan: 'There is no such thing as simply being Φ' (p. 7). If an apparently monadic predicate Φ is folk-relative, then 'in order to get a truth-value for a statement in the form "a is Φ," one must explicitly or implicitly indicate something else' (p. 7). Folk relativism about logical consequence leads to pluralism provided at least two values for the parameter provide 'different, but equally legitimate "accounts"' of logical consequence (pp. 13–4). Monism about logical consequence rejects both folk-relativism and pluralism, maintaining instead that there is 'but One True Logic'; anti-monists, on the other hand, are said to have an eclectic orientation (pp. 14–15).

Chapter two outlines several routes to folk relativism and pluralism about logical consequence. The principal one centres on nine accounts of what it is for a conclusion to be a *logical consequence* of a set of premisses (equivalently: for the argument to be *valid*). The accounts are variously couched in modal, semantic (including model-theoretic), and epistemic (including proof-theoretic) terms (pp. 21–4). Shapiro makes the case that at least some of them amount to 'different, mutually incompatible, but equally legitimate ways to sharpen or further articulate the intuitive notion(s) of logical consequence' (p. 17). Further, even once we've settled on a given account, some render logical consequence folk-relative to further parameters. Semantic accounts of logical consequence based on truth-preservation in all cases (such as Beall and Restall's account discussed below) make logical consequence relative to a specification of what counts as a case (pp. 29–38); accounts which invoke the distinction between logical and non-logical terms render logical consequence relative to where the boundary falls (pp. 49–57).

Chapter three presents an important new argument against monism about logical consequence. Shapiro's starting point is what he calls 'the Hilbertian perspective' towards pure mathematics: any consistent theory is legitimate (pp. 65–7, 83; cf. p. 206). Consistent theories may be unfruitful or uninteresting but 'there is no further metaphysical hoop the proposed theory must jump through before being legitimate mathematics' (p. 67). Shapiro presents three examples of mathematical theories, which are consistent in the intuitionistic logic in which they are cast, but whose axioms are inconsistent in classical logic. Smooth infinitesimal analysis and Shapiro's other two examples have all the hallmarks of legitimate mathematical theories: beyond their consistency, they are studied by real live mathematicians, and have, or at least may have, applications (pp. 67–75). But monists who take classical logic to be 'the One True Logic' must reject these classically inconsistent theories as incoherent (pp. 70, 72).

The argument against monism is subject to an important hedge: Shapiro maintains that whether or not we should reject monism depends on substantive issues concerning the semantics of object language logical terms (e.g. $\neg$, $\vee$, $\forall$). Their semantics, in addition to the semantics of metalogical terms (e.g. 'valid', 'consistent'), is the subject of chapters four and five. The 'Dummett-Quine-Carnap perspective' that the content of the logical terms varies in the context of classical and non-classical theories is pitted against the opposing view defended by Beall and Restall, amongst others (pp. 96–111). Shapiro comes to two conditional conclusions (pp. 124–5):

(C1) If the content of the logical terms remains invariant in the context of different mathematical theories, folk-relativism about logical consequence holds.

(C2) If the content of the logical terms varies in the context of different mathematical theories, monism about logical consequence holds.

In the first case, Shapiro further argues for a contextualist view of metalogical terms: 'logical consequence' expresses classical consequence when we're discussing classical mathematics, intuitionistic consequence in the context of smooth infinitesimal analysis, and so on (pp. 114–20). In the second case, Shapiro immediately dispels any appearance of a substantial concession, arguing that the resulting monism is 'toothless' (p. 122), and does not 'substantially change the eclectic attitude' (p. 123).

Chapter five argues that statements such as 'the connectives mean the same in classical and intuitionistic theories' contain vague terms. On Shapiro's preferred, contextualist account of vagueness, this makes the statement context sensitive: whether or not it is true in a context depends on which similarities and differences between the theories are salient (pp. 127–131, 136–7). Combining this with (C1) and (C2), Shapiro tentatively concludes that 'the question of whether to adopt a monism or a contextualism for 'valid' [is itself] context-sensitive' (pp. 154–55).

The final two main chapters concern metatheoretic issues. Chapter six argues that no difficulty is posed by using a metatheory with one logic to shed light on an object theory using another, and suggests set theory and category theory as 'two good candidates for this sort of study' (p. 177). Chapter seven considers 'explicitly relational propositions' such as 'the law of excluded middle is valid *in classical logic*'. Shapiro argues that such statements are often, but not always, insensitive to the metalogic (pp. 194–99).

Shapiro's eclectic orientation differs in some important ways from Beall and Restall's pluralism. The latter centres on the Generalized Tarski Thesis (GTT), which they take to be the key constraint on the concept of logical consequence:

> An argument is valid$_x$ if and only if, in every case$_x$ in which the premises are true, so is the conclusion. (2006, p. 29)

Their pluralism consists in there being two or more admissible instances of GTT whose valid arguments additionally conform to what they identify as three further core features of logical consequence (p. 35): (i) necessity (i.e. it's necessary that, if the premisses of a valid argument are true, so is the conclusion), (ii) normativity (in that there is something wrong in accepting the premisses of a valid argument while rejecting the conclusion), and (iii) formality (in various respects Beall and Restall elucidate) (pp. 14–23). Admissible instances of GTT are argued to give rise, *inter alia*, to classical, intuitionist and a species of relevant logic (pp. 35–74).

This makes for a more constrained view than Shapiro's eclectic orientation. Shapiro observes that GTT-based accounts fall within the 'modal-cum-semantic-cum-model-theoretic conception of logical consequence', and build in certain structural features to the consequence relation (e.g. reflexivity and transitivity) (p. 32). On the other hand, the eclectic perspective is free to embrace accounts outside this tradition, or lacking these features, most notably proof-theoretic accounts (e.g. Neil Tennant's (1987) non-transitive core logic) (p. 33). But not completely free: in taking logical consequence to be a relation between a set of premisses and a conclusion, Shapiro excludes accounts deploying different relata (e.g. linear logic) which he acknowledges might have provided 'more grist for the present, eclectic mill' (p. 20).

Shapiro's widening of the potential plurality is welcome, and opens up new avenues in the pluralism debate. But does further freedom bolster the philosophical case for pluralism? Let's start with the argument from chapter two. There's a question as to how many *distinct* accounts of *logical* consequence he presents us with (as Shapiro acknowledges, p. 24). But assuming that we have more than one, does chapter two establish that there is a plurality of *legitimate* accounts, as per Shapiro's more liberal pluralism?

The answer depends on what we mean by 'legitimate'. One option is to take a legitimate account of logical consequence to be something like a correct conceptual analysis, or at least, an admissible precisification, of a pre-theoretic concept. This is Beall and Restall's approach. Even if it's hazy at the edges, their view is that the logical consequence relation has a settled core (analysed in terms of GTT, as above), and they argue that their favoured instances amount to precisifications of the concept in virtue of conforming to its core features. Shapiro, too, speaks of 'articulations or sharpenings of the intuitive notion or notions of logical consequence' (p. 25). But he doesn't engage in a Beall and Restall style programme of analysing the core of the notion(s), or arguing that the various accounts conform to the settled core(s). Instead Shapiro observes that texts in logic, and its philosophy, often deploy different accounts of logical consequence, and claims that 'it does seem to be at least prima facie correct that at least *some* of the seemingly different notions listed here are more or less on target, concerning what should be rightly called *logical* consequence' (p. 24). Taken as something like conceptual analyses, however, it's not clear why a mere plurality of accounts provides prima facie grounds for pluralism. The plurality of post-Gettier conceptual analyses of knowledge, for instance, does not seem to provide a prima facie case for pluralism about knowledge.

This case for pluralism is stronger, however, if we take a legitimate account to be an acceptable definition of 'logical consequence'. This reading is suggested by Shapiro's later discussion of the semantics of metalogical terms. While part of English, the terms 'are, primarily, terms of art' and as such, 'the "artist" who introduces such terms of art gets to stipulate their meaning, hoping that the introduced terms help shed light on whatever is in focus, and hoping that the stipulated notions line up, in part, with ordinary usage, at least sometimes' (p. 91). This freer, primarily stipulative view fits much better with the way 'accounts' of logical consequence are typically presented in logic textbooks (often prefixed with labels like 'Definition 2.9'). Moreover, there is no doubt that many of these, often fruitful, stipulations are perfectly acceptable definitions. But even if this provides an easy route to pluralism, it's less clear why a plurality of acceptable definitions for a term of art amounts to an interesting version of pluralism. The plurality of acceptable definitions for the astronomical term of art 'planet' proposed around the time Pluto was downgraded, for instance, does not seem to sustain an interesting pluralism about planethood.

Whether or not the arguments from chapter two (not all of which have been examined here) establish an interesting species of pluralism, Shapiro's anti-monist argument in chapter three provides an important, independent reason to depart from Beall and Restall's more constrained view. The necessity of classical consequence, on their view, means that classical contradictions are necessarily false. Consequently, Shapiro argues, 'Beall and Restall cannot accept, as legitimate, those branches of intuitionism that are in outright conflict with classical logic', as they acknowledge (2006, p. 120). 'The reason is that those theories have, as theorems, what Beall and Restall take to be necessary falsehoods' (p. 37). Shapiro takes the opposite view (cf. n. 6, p. 69). 'One of the main motivations for the present, eclectic orientation to logic . . . is to show how a wide variety of theories, studied by mathematicians whose credentials can hardly be challenged, are legitimate' (p. 38).

This brings us to the heart of Shapiro's case against monism. To set up the argument, imagine a monist tempted by the Hilbertian perspective who pursues mathematical research both in classical real analysis and in smooth infinitesimal analysis. (Let's identify each of these theories simply with a suitable set of axioms—CRA and SIA—so we may follow Shapiro in speaking of a single theory being inconsistent in one logic and consistent in another.) We may then regiment what seems to me the core of Shapiro's argument as an inconsistent triad:

(1) The theories CRA and SIA are formulated in the same language, using the same logical terms.

(2) The consequence relation of the One True Logic (OTL) extends the classical consequence relation employed in the context of CRA (i.e. whenever a CRA-formula is a classical consequence of a set of CRA-formulas, it is also a consequence of this set in OTL).

(3) The axioms of SIA are consistent in OTL.

The three theses cannot be jointly accepted by the monist. Consider the classically valid CRA-formula $\forall x(x = 0 \vee \neg x = 0)$. The CRA-formula is also an SIA-formula, by (1), and a consequence of the SIA-axioms in OTL, by (2). But, as Shapiro observes, the SIA-formula leads to a contradiction in intuitionistic logic in conjunction with the SIA-axioms (p. 74). Consequently SIA is inconsistent in OTL, contrary to (3).

The argument invites monist responses falling into two broad categories. The first is for the monist to accept that SIA is inconsistent in OTL. The cost, Shapiro suggests, is that the monist must reject a legitimate branch of mathematics as incoherent (pp. 70, 72, 75). But the monist may challenge this assessment. For one thing, it's not clear that the Hilbertian perspective mandates the legitimacy of SIA. For the Hilbertian monist who takes OTL to have a classical consequence relation, we should expect classical logic to be the ultimate arbiter of consistency. Even if a theory's consistency suffices for its legitimacy, by her lights, SIA fails to meet this sufficient condition. Shapiro, as noted above, has other reasons for thinking this branch of mathematics legitimate. But some monists (and indeed some pluralists) may be inclined to stand firm on this point, and maintain that classical consistency is a red line that legitimate theories never cross.

Monists who take this view may sugar the pill by allowing that inconsistent theories may still have mathematical interest. Shapiro acknowledges that work on intuitionistic theories may adopt a 'more hypothetical spirit, exploring the consequences of the theory, rather than adopting it directly' (p. 69). (The point is attributed to Alexander Paseau, p. 72, n. 9.) However, Shapiro adds, 'one cannot even do that if one is serious about classical logic being the One True Logic. For this monist, *every* sentence is a consequence of this theory' (p. 69). But what's to stop the classical monist exploring the *intuitionistic* consequences of SIA in a hypothetical spirit? And who's to say in advance whether such flights of mathematical fancy may prove fruitful (or publication worthy) in ways that don't rely on their being consistent in OTL.

The second broad strategy open to the monist is to attempt to render SIA consistent in OTL. Shapiro anticipates two ways to implement this response. One is simply for the monist to take OTL to be no stronger than intuitionistic logic (p. 70). Rejecting (2) in this way, renders SIA consistent in OTL (assuming it is consistent in its original intuitionistic logic). The intuitionistic monist can also retain classical reasoning in the context of classical analysis by adding the relevant instances of the law of excluded middle to the non-logical axioms of CRA (p. 82). Of course, intuitionistic monists find themselves in the same predicament as their classical counterparts if there are legitimate mathematical theories whose axioms are trivial over intuitionistic logic (i.e. every formula is a consequence of the axioms). Shapiro moots inconsistent (but non-trivial) mathematical theories based on paraconsistent logic as a potential example (pp. 82–7). He leaves open the question of whether or not there are inconsistent mathematical theories that are interesting or fruitful, but questions 'what sort of argument one might give to dismiss them out of hand, in advance of seeing what sort of fruit they may bear' (p. 84).

Another response, which Shapiro anticipates and criticizes (p. 75–81), retains the full strength of classical logic. This response seeks to render smooth infinitesimal analysis consistent in an extension of classical logic by taking the theory to have tacit modal content. The monist can exploit one of the well-known interpretations of intuitionistic logic in a suitable modal extension of classical logic. An SIA-formula A may be 'modalized' to obtain a modal formula A^* by inserting modal operators in appropriate places. The translation is defined so that A is an intuitionistic consequence from some premisses (in the language of SIA) if and only if its modalization A^* follows from the modalizations of the premisses in the modal extension of classical logic. Assuming SIA is consistent in intuitionistic logic, the theory SIA* which results from modalizing its axioms is consistent in the modal logic. Consequently, the monist can coherently accept the legitimacy of SIA* while taking the consequence relation of OTL to extend classical consequence.

Shapiro objects that 'the language of a modalized smooth infinitesimal analysis does not have a natural reading' (p. 79). On the usual sort of epistemic interpretation of the modal operators given in the context of intuitionistic logic, he argues, the axioms are hard to motivate, and may lead us close to Moore's paradox (p. 78–81). A monist who adopts Shapiro's Hilbertian perspective, however, should be unimpressed by this style of objection. She maintains, on the contrary, that the axioms of SIA* serve to implicitly define its (logical and non-logical) primitives and the theory's consistency ensures its legitimacy (cf. p. 166). The demand for a well-motivated antecedent interpretation which renders the axioms true should be rejected as exactly the sort of extra metaphysical hoop legitimacy doesn't require. All the same, some monists may agree with Shapiro that, at least prima facie, 'smooth infinitesimal analysis is not a modalized enterprise, epistemic or otherwise' (p. 79), and that 'it is much more natural for those trained in classical mathematics to "go native," as Quine might put it, and to read the axioms and theorems of this theory at face value, learning the ins and outs of intuitionistic logic along the way' (p. 81).

This suggests a third way for the monist to attempt to render SIA consistent in OTL, without weakening OTL or inserting unpronounced modal operators in SIA-formulas. The monist may claim that although SIA and CRA are both standardly formulated using the same symbols for logical expressions ($\neg$, $\vee$, $\dots$), these are ambiguous between the classical connectives employed in CRA ($\neg_c, \vee_c, \dots$), and the intuitionistic connectives employed in SIA ($\neg_i, \vee_i, \dots$). On this view—call it liberal monism—both sets of connectives are governed by OTL which respectively sustains classical and intuitionistic reasoning in either case, in that (2), above, holds, together with its intuitionistic analogue:

(4) The consequence relation of OTL extends the intuitionistic consequence relation employed by SIA (i.e. whenever an SIA-formula is an intuitionistic consequence of a set of SIA-formulas, it is also a consequence of this set in OTL).

The liberal monist may hope that rejecting (1) in this way, permits her to maintain (2)–(4). For although the classically valid formula $\forall_c x(x = 0 \vee_c \neg_c x = 0)$ is a consequence of SIA in OTL, as above, it is a different, intuitionistic formula, $\forall_i x(x = 0 \vee_i \neg_i x = 0)$ which leads to a contradiction in intuitionistic logic in conjunction with the axioms of SIA.

This monist manoeuvre comes up several times in the course of the book (pp. 123–4, 161, 175). Shapiro responds that 'it is well-known that if the various connectives and quantifiers are just thrown together in a single language, they will collapse into each other, at least if the natural deduction rules for all of the terms are applied in full generality' (p. 161). Liberal monism risks classical-intuitionistic collapse:

Classical-intuitionistic collapse: formulas that differ only in their i- or c-subscripts are equivalent in the combined classical–intuitionistic logic.

This kind of collapse would fatally undermine the liberal monist response. For assuming $\forall_i x(x = 0 \vee_i \neg_i x = 0)$ and its classical counterpart are equivalent in OTL reinstates the inconsistency in SIA. The liberal monist response, however, is not yet sunk. Shapiro's qualification about applying the natural deduction rules 'in full generality' points to a substantive caveat. Before we come to that, it's important to note the wider role classical-intuitionistic collapse plays in his overarching argument.

Shapiro's argument that 'toothless monism' lacks teeth plays a crucial role in reconciling his eclectic orientation towards logic with the ostensibly non-eclectic conclusion from chapter five that monism is true in some contexts (pp. 122–5). The monism described as 'toothless' is closely related but different to the view we've labelled liberal monism. Shapiro's monism takes a contextualist view towards the logical terms, replacing the liberal monist's two (or more) context invariant negation symbols, for instance, with a single context-sensitive connective, expressing intuitionistic or classical (or other kinds of) negation according to context (pp. 121–2). This, Shapiro suggests, permits the 'toothless monist' to avoid collapse at the expense of giving up on some of the distinctive benefits of monism.

One traditional motivation for monism is the thought that logic should be topic neutral or universal. Shapiro identifies two aspects to this monist thought:

Universal applicability: 'logic is universal, applying to all legitimate discourses' (p. 93).

Universal expressibility: 'one can use a logical term in any situation whatsoever, no matter what one is talking about' (p. 123).

The reason his monism is 'toothless', Shapiro argues, is that it is unable to sustain a non-trivial version of either 'venerable slogan' (pp. 122-3). And the reason for this, once again, is collapse. Contrary to universal expressibility, 'it is simply not possible to coherently add a classical negation to the language of smooth infinitesimal analysis' (p. 124). The resulting view, while strictly monist, is substantially eclectic in spirit. 'The substantive thesis is that the principles and inferences in question do not apply everywhere, and cannot be imposed everywhere, which is just what the other formulations of logic relativism or pluralism say, in different words' (p. 124).

As mentioned above, Shapiro's arguments from collapse are subject to a qualification, which may yet rescue liberal monism. His qualification concerns how the proof systems for classical and intuitionistic connectives are combined. This matters, since recent work of Joshua Schechter (2011) (cited by Shapiro n. 9, p. 123; n. 16, p. 161; n. 4, p. 175) shows that classical–intuitionistic collapse is sensitive to features of the combined proof system used to axiomatize the classical–intuitionistic logic. As Schechter observes (in the propositional case), if we pool together the usual classical and intuitionistic rules in a single natural deduction system, then classical–intuitionistic collapse results, unless we apply additional side-conditions to certain rules, restricting their use in the context of 'mixed' proofs, which deploy both sets of rules. On the other hand, there is no collapse, if we suitably restrict the natural deduction rules in mixed proofs, or pool together the usual classical and intuitionistic axioms and rules of inference in a single Hilbert-style axiomatic system. Indeed Schechter shows that the result of 'juxtaposing' classical and intuitionistic consequence relations in this way conservatively extends both intuitionistic and classical consequence: an intuitionistic formula (a formula with only i-subscripts) is an intuitionistic consequence of a set of the same if and only if it is a consequence of the set in the combined classical–intuitionistic system; similarly, *mutatis mutandis*, in the classical case (2011, p. 595).

Shapiro's anti-monist argument consequently raises an interesting technical question (which, to the best of my knowledge, is unsettled): do Schechter's non-collapse results in the propositional case extend to the first-order languages used to formulate CRA and SIA? If not, then, as above, the game is up for the liberal monist's response to Shapiro's argument. But a positive answer permits the liberal monist to accept the legitimacy of SIA while retaining a non-toothless version of monism. To explore this hypothesis, let's suppose that, in parallel with the propositional case, the consequence relation of the liberal monist's OTL, conservatively extends both classical and intuitionistic first-order consequence. In contrast to the eclectic orientation (or Shapiro's 'toothless monism') the liberal monist can give non-trivial content to universal expressibility and applicability. Since SIA is formulated using the intuitionistic connectives and quantifiers, its mathematical study typically focuses on proofs which only contain intuitionistic formulas (for which OTL sustains intuitionistic reasoning, and Schechter's natural deduction restrictions are vacuously met). But, so long as we're careful with mixed proofs, this is no bar to using classical vocabulary in the context of discussing this theory: $\forall_c x(x = 0 \vee_c \neg_c x = 0)$ is valid in OTL (whether the salient theory is SIA or any other); likewise $\forall_i x(x = 0 \vee_i \neg_i x = 0)$ is not valid in OTL (regardless of the context). Despite this, because the consequence relation of OTL conservatively extends intuitionistic logic, assuming it is consistent in intuitionistic logic, SIA is consistent in OTL.

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