The effect of blockage on tidal turbine rotor design and performance

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Abstract

The performance-enhancing effects of closely packing tidal turbines in single row arrays (tidal fences) are evaluated in this computational study. Infinitely long tidal fences are simulated with a range of lateral rotor spacings using a blade element momentum method embedded in a Reynolds averaged Navier-Stokes solver (RANS-BEM).

First, a rotor design tool is applied to determine a hydrodynamically optimal rotor design for each lateral spacing. In the RANS-BEM method, the effect of blockage (the ratio of rotor swept area to channel cross-sectional area) on rotor optimization is accounted for. Increased blockage is found to result in increased optimal solidity and decreased optimal pitch. Next, each rotor design is simulated in its design spacing as well as several off-design spacings. The resulting power coefficient is largest when the rotor optimized for the highest blockage case operates in the array with the closest lateral spacing. Further, although a rotor's performance is improved through operation at a blockage higher than its design point, it still exhibits inferior performance relative to a rotor designed for that higher blockage. The results indicate that blockage must be considered in the rotor design process if the optimal rotor efficiency for a given spacing is to be achieved.

Keywords: tidal turbine, tidal fence, rotor design, blockage, RANS-BEM

1. Introduction

Unlike most wind turbines, tidal turbines will operate in partially blocked conditions, for instance in a tidal channel, in which the seabed and sea surface are close to the turbine, and/or a closely spaced array, in which the flow is confined by neighboring turbines. While actuator disc theory predicts the maximum power coefficient for a rotor operating in unbounded flow, also known as the Lanchester-Betz limit, to be $C_{p,Betz} = 0.593$ [1, 2], it is widely understood that the Lanchester-Betz limit may be exceeded by operating a rotor in blocked conditions [3]. Despite the prospective performance improvements, however, the effects of blockage on optimal rotor design are not well understood. Although blockage corrections have been introduced for tidal turbine modelling (for instance in analytical blade

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element momentum (BEM) models [4]), rotors are not generally designed with regard to the blockage conditions the rotor is expected to face in operation.

BEM methods embedded in Reynolds-averaged Navier-Stokes computational fluid dynamics (CFD) solvers, also known as RANS-BEM methods, are particularly well-suited for designing rotors for operation in specified blockage conditions, as well as for the investigation of performance comparisons of specific rotor designs. First, the steady-state solution allows for a relatively fast design iteration and gives BEM methods a computational advantage over unsteady methods of rotor analysis such as actuator line and 3D blade-resolved methods. Second, unlike actuator disc methods, which model the rotor as an idealised disc, BEM methods include the influence of the blade geometry. Finally, the advantage of this method over simple analytic BEM methods is that the flow constraints (such as blockage and boundary conditions) and the presence of supporting structures can be accounted for through the CFD model. However, an assumption of fixed mass flow rate through the domain is maintained.

In this computational study, a RANS-BEM method [5] is employed to evaluate the effects of blockage on rotor design and to investigate the improvement in the rotor power coefficient when tidal turbines are closely spaced in tidal turbine fences. First, the RANS-BEM rotor optimisation tool is utilised to determine hydrodynamically optimal rotor designs for infinitely long tidal fences with lateral intra-rotor spacings ranging from $0.25d$ to $4d$ (d is the rotor diameter), as well as a virtually unblocked case, which is included for comparison purposes. Next, the results of RANS-BEM simulations in which the rotor designs are tested in both design and off-design blockage conditions are presented.

2. Methods

2.1. Reynolds-averaged Navier-Stokes equations

The Reynolds-averaged Navier-Stokes (RANS) equations are time-averaged representations of the continuity and momentum equations which govern fluid flow (i.e. Navier-Stokes equations) and are

$$\nabla \cdot u_i = 0 \quad (1)$$

and

$$\rho \frac{\partial}{\partial x_j} (u_i u_j) = \frac{\partial}{\partial x_j} \left[-p \delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + \rho \frac{\partial}{\partial x_j} \left(-\overline{u'_i u'_j} \right), \quad (2)$$

respectively, for steady, incompressible flow. Here, ρ is the fluid density, u_i is the velocity component ($i = 1, 2, 3$), μ is the molecular viscosity of the fluid, p is the pressure, and δ_{ij} is the Kronecker delta. A turbulence closure must be employed to model the $-\overline{u'_i u'_j}$ term, which represents the Reynolds stresses, in order to close the RANS equations. The $k - \omega$ shear stress transport (SST) turbulence closure, introduced by Menter [6], is used to close the steady-state, incompressible RANS equations for all simulations in the current work.

The SIMPLE algorithm for pressure-velocity coupling, with an algebraic multigrid method, is employed in the current work. The convective and diffusion terms of the momentum equations are discretised using a second-order upwind scheme and a second-order accurate central differencing scheme, respectively.

2.2. RANS-BEM method and design tool

Analytical blade element momentum methods are based on concepts introduced in the 1920's [7, 8, 9] and provide a well-established means of analyzing and designing wind turbines [10]. However, traditional analytical BEM methods do not adequately account for the blockage effects that must be considered in the case of tidal turbines.

The present study was therefore completed using an in-house RANS-BEM code and design tool embedded in ANSYS Fluent. The code has been verified and used in the design and simulation of unducted and ducted tidal turbines [5, 11]. In addition, Belloni [12] verified the current RANS-BEM code by comparing power and thrust coefficients computed for a bare rotor with those computed using analytical BEM methods for the same rotor. Similarly, Schluntz [13] found good agreement in power coefficient and radial variation of angle of attack for RANS-BEM and actuator line simulations of a rotor operating in blocked flow.

In the RANS-BEM methods, the RANS solver passes the local velocity at the rotor blade elements to the coupled BEM code, which then computes the relative velocity and blade attack angle, and hence the force vector, for each blade element, thereby allowing for the influence of the array effects on the flow to be included in the simulations. An iteration proceeds in which the blade forces, as calculated in the BEM code, and resultant flow field are alternately updated until a converged solution is obtained. RANS-embedded BEM methods have been applied to the study of tidal turbine flow in a number of studies, for instance in Masters et al. [14], Bai et al. [15], and Malki et al. [16].

In RANS-BEM simulations, the rotor is represented as a thin disc. The effects of the forces applied by the rotor on the flow are replicated in the computational domain in the present work by the implementation of a static pressure jump, Δp , across each rotor disc element and the specification of a swirl velocity, u_θ , on the downstream side of each rotor disc element:

$$\delta p = \frac{1}{2} \rho U^2 \sigma(r) (C_l \cos \phi + C_d \sin \phi) \quad (3)$$

$$u_\theta = \frac{U^2 \sigma(r) (C_l \sin \phi - C_d \cos \phi)}{4u_x}. \quad (4)$$

u_x is the streamwise velocity component and ϕ is the flow angle (the angle between the rotor plane and the relative velocity vector, $\mathbf{U}$, at the blade segment). C_l and C_d are the sectional lift and drag coefficients, respectively, for the blade segment and are determined from experimental data tabulated as a function of the angle of attack. The solidity, $\sigma(r)$, at local radius r is dependent on the number of blades, N , and the local chord, $c(r)$:

$$\sigma(r) = \frac{Nc(r)}{2\pi r}. \quad (5)$$

The magnitude of the relative velocity at each blade section is

$$U = [(\Omega r - u_\theta)^2 + u_x^2]^{1/2}, \quad (6)$$

where Ω is the angular velocity of the rotor and is determined from the tip speed ratio (which is an input of the model).

Prandtl's tip correction [17, 9] is included to account for the influence of the unsteady wake structures that form as a result of discrete blades and can not be directly modelled in BEM methods. The tip correction factor, F , is applied to the axial induction factor and is given by

$$F = \frac{2}{\pi} \cos^{-1} \left[\exp \frac{N \left(1 - \frac{R}{r} \right)}{2 \sin \phi} \right], \quad (7)$$

where R is the outer radius of the rotor.

The RANS-embedded BEM design tool allows for a tidal rotor to be designed for optimal performance in specified operating blockage conditions. The method iteratively adjusts the sectional chord and twist for each annulus of the blade in order to achieve the maximum power coefficient, C_p , for an assigned tip speed ratio, λ , and target local thrust coefficient, c_x :

$$c_x = \frac{\delta F_x}{\frac{1}{2} \rho \delta A u_x^2}, \quad (8)$$

where δF_x is the magnitude of the streamwise force on a corresponding rotor disc element of area δA . The optimum twist for each blade section is the twist required to maintain the angle of attack, α , that maximises the lift to drag ratio for the given blade section. This value of α is determined from tabulated aerodynamic data and is set as a design angle of attack by the user. While the blade twist is primarily associated with the angle of attack, the blade solidity is primarily associated with the local thrust. Therefore, in order to define the rotor geometry, a design local thrust is also set by the user.

Thus, the in-house RANS-BEM code and design tool used in this study requires the tip speed ratio, λ , target constant local thrust coefficient, c_x , and angle of attack corresponding to the maximum lift to drag ratio for the aerofoil section to be defined by the user. In the design optimisation process, the solidity and blade twist are iteratively adjusted for constant, user-defined values of c_x , λ , and target α in order to determine a maximum rotor power coefficient, C_p , and corresponding blade geometry for the given set of inputs. This process is repeated for a range of user-defined input values for λ and c_x , with each simulation providing a different value for maximum C_p as well as different optimised geometry. The combination of λ and c_x values which leads to the greatest maximum C_p is considered to be the optimal operating point for the rotor. The design optimisation process is discussed further in Belloni [12] and Schluntz [13].

An upper limit of $\sigma = 0.95$ is set for the local solidity in accordance with practical blade design constraints.

2.3. Computational setup

This study uses RANS-BEM methods to examine the performance of two-bladed rotors of diameter $d = 10$ m operating in infinitely long tidal fences with a range of blockage ratios. With the rotor disc area constant, 4 computational domains of varying cross-sectional area are used to create the desired blockage conditions. The cross-sectional area of each domain is dependent on the specified lateral rotor tip-to-tip spacing, s , and domain height, h , as seen in Figure 1.

Figure 1: Cross-section of the computational domain at the rotor plane.

A nacelle of diameter $d_{nacelle} = 0.15d$ (similar to many prototype devices) is explicitly included in each simulation via a no-slip boundary. It is necessary to include the nacelle because it blocks flow through the rotor centre and thus increases blockage. However, no other support structures are included in the simulations. The rotor disc area and nacelle geometry are fixed throughout the study. In the design process, the rotor discs are discretised into 80 concentric annuli of uniform radial width, δr . Symmetry boundary conditions are used at the side boundaries of each domain in order to simulate an infinite fence of turbines, each positioned at a tip-to-tip distance s from its neighbours. The lateral intra-rotor spacing, height, and blockage ratio, B , for each of the 4 domains considered are listed in Table 1. The blockage is the ratio of the rotor disc area to the cross-sectional area of the computational domain at the rotor plane.

Table 1: Computational domain parameters.

Domain	s	h	B	Cell Count
1	$d/4$	$2d$	0.314	1.22×10^6
2	d	$2d$	0.196	1.38×10^6
3	$4d$	$2d$	0.079	1.56×10^6
4	$100d$	$100d$	0.0001	1.86×10^6

The height of the first 3 domains is held constant at $h = 2d$. The range of blockage ratios for these domains is achieved through variation in the lateral rotor spacing alone. Domain 4 approximates unblocked conditions, with the height increased to $h = 100d$. All domains extend $5d$ upstream and $10d$ downstream of the rotor.

The surface of the channel is approximated with a rigid lid and the floor and lid of each domain are slip wall boundaries (free surface effects are not considered in this work, and have been shown to only be significant at very high levels of flow blockage [18]). A simple uniform velocity profile with water ($\rho = 1025 \text{ kg/m}^3$) flowing at $U_\infty = 2 \text{ m/s}$ is used as the inlet boundary condition for each of the simulations in this study. This allows the effects of blockage on rotor design to be studied in isolation from the effects of shear, yaw, and gravity waves, which are not considered at present. The gauge pressure is prescribed to be constant

at the outflow boundary.

Following the results of a grid convergence study, a mesh with element length $e = 0.25$ m is utilised in the rotor plane region, and a growth rate of 1.1 for the mesh elements is used in the wake region. The last column in Table 1 indicates the the number of cells in each domain. The low cell count, enabled by the BEM representation of the rotor, and the implementation of steady solution techniques result in low computational cost for the simulations.

The Risø-A1-24 blade section is used for all rotors from $0.15d/2 < r < d/2$, the entire span of the blades. The selected aerofoil has a realistic thickness for hydrokinetic applications (24%) as opposed to the much thinner aerofoils used in most wind turbines, which would face structural difficulties in the high-load conditions experienced by tidal turbines. In addition, experimental aerodynamic data for this blade is available for a wide range of attack angles, which allows for the blade forces to be calculated at the non-optimal attack angles that are expected to occur when the rotors are operated in off-design blockage conditions. The aerodynamic data used in the current study was experimentally determined for $Re = 1.6 \times 10^6$ using an aerofoil with leading edge roughness [19].

3. Results and discussion

3.1. Rotor designs

The blade optimisation tool is run for each of the 4 domains detailed in Table 1 using increments of $\Delta c_x = 0.1$, and the optimum design for each domain is taken to be the design that results in the highest rotor power coefficient. The tip speed ratio is $\lambda = 5$ for all cases. The maximum values of C_p obtained for each of the 4 domains are listed in Table 2. Also included in Table 2 is the corresponding global thrust coefficient, C_t , required to achieve the maximum C_p for each domain.

Table 2: Maximum power coefficient and corresponding thrust coefficient for each domain.

Domain	$C_{p,max}$	C_t at $C_{p,max}$
1	1.14	1.97
2	0.83	1.32
3	0.66	1.04
4	0.58	0.92

As expected, the power coefficient of the optimum rotor designs is highest for the domain with the lowest lateral spacing and highest blockage. Conversely, the unblocked rotor yields the lowest rotor efficiency. The thrust coefficient required to achieve $C_{p,max}$ increases as the rotor spacing decreases. The reasons for this are explained in Section 3.3, which includes a discussion of the mechanisms causing performance improvement in blocked flow.

The simulation for Domain 2 at its peak operating point was repeated with a longer domain (but with the same height and width) to ensure that the streamwise extent of the domains used was sufficient. In the long domain, the inlet was $10d$ (rather than $5d$) upstream

of the rotor plane and the outlet was $20d$ (rather than $10d$) downstream of the rotor plane. Results for C_p and C_t for this simulation were within 0.1% of the values obtained using the original domain; hence, we consider the streamwise extent of the original domain to be sufficient.

It is notable that $C_{p,max}$ for the Domain 1 design is greater than 1. Garrett and Cummins [3] showed that the maximum power coefficient for a single rotor operating in a bounded flow passage is $C_{p,max} = C_{p,Betz}(1 - B)^{-2}$, where $C_{p,Betz} = 16/27$ is the Lanchester-Betz limit. With this definition, the theoretical $C_{p,max}$ is greater than 1 if $B > 0.23$. The blockage for Domain 1 is $B = 0.314$, which leads to a theoretical maximum power coefficient of $C_{p,max} = 1.259$. Thus, the computed value of $C_{p,max} = 1.10$ is consistent with the theoretical results, falling below the theoretical limit due to viscous losses in the RANS-BEM simulations.

The power coefficient is calculated by normalising the power by the kinetic energy flux upstream of the rotor. However, in addition to extracting kinetic energy, the ability of a blocked flow to develop a pressure gradient from far upstream to far downstream enables pressure head to be extracted as well. This is most notable for highly blocked conditions, and is the mechanism by which the power coefficient, as currently defined, is able to exceed unity.

In addition, the theoretical limit for the maximum power coefficient, determined using the Garrett and Cummins [3] result discussed above, is $C_{p,max} = 0.917$ for Domain 2, $C_{p,max} = 0.699$ for Domain 3, and $C_{p,max} = 0.593$ for Domain 4. Comparison of these values with the computational results included in Table 2 confirm that the current BEM results are consistent with analytical predictions.

The results in Table 2 suggest that the optimum lateral spacing between rotors for a tidal fence spanning a channel is less than $1d$ and possibly less than $0.25d$. Other works, see Bai et al. [15], have determined that in the case of multiple rows of staggered turbines wider spacings produce maximum power but these results are not directly comparable to the current investigation of a single tidal array.

The local solidity, $\sigma(r)$, and blade twist, $\beta(r)$, for the optimal rotor design for each lateral spacing are shown in Figures 2 and 3, respectively. In addition, sketches of the rotor blades corresponding to each design (for a 2-bladed rotor) are included in Figure 4.

It is seen in Figure 2 that the local solidity required for optimal C_p increases as the spacing between rotors decreases. The upper limit for the local solidity ($\sigma = 0.95$) is reached for the design with the highest blockage conditions ($B = 0.314$; $s/d = 0.25$). It is expected that if this limit is relaxed, $C_{p,max}$ for this domain would exceed the value computed with the present solidity limit.

The optimal solidity for the closest rotor spacing is approximately double the solidity for the unblocked rotor design through most of the blade span. The effects of this increased solidity on blade design can be seen in the rotor blade planform sketches included in Figure 4. The results indicate that local blockage conditions have a large effect on optimal solidity and suggest that it is important to adapt rotor solidity for high blockage operating conditions in order to exert greater thrust and extract power with maximum rotor efficiency. This is further discussed in Section 3.3.

Figure 2: Comparison of the blade solidity, σ , for the 4 rotor designs.

The blade twist is iteratively modified during the blade optimisation tool computation. The optimum twist for each blade section is the twist required to maintain the angle of attack that maximises the lift to drag ratio for the given blade section, which, for the current aerofoil, is 7° .

The effect of local blockage on the local blade twist is not as pronounced as its effect on the local solidity. Still, the results in Figure 3 show that β generally decreases as the rotor spacing is reduced. The optimal twist near the blade root for the closest intra-rotor spacing, $s/d = 0.25$ ($B = 0.314$), differs from the pattern seen for the other rotors. This is due to the imposed limit on solidity in the vicinity of the blade root for this rotor.

Contours of the streamwise velocity, u_x , on a horizontal centre plane for each of Domains 1-4 are shown in Figures 5a-5d, respectively. The contours depict the streamwise velocity when the optimised rotor is operated in its design blockage conditions at its design tip speed ratio ($\lambda = 5$ for all cases). The entire cross-stream width of Domains 1-3 is shown in Figures 5a-5c, whilst Figure 5d only shows part of Domain 4 due to its large width. The disc area in each of the figures is kept constant to allow for direct visual comparison of the varying levels of disc spacing.

The effects of the lateral rotor spacing on the streamwise velocity are quite evident. The speed of the flow bypassing the rotor becomes faster with each reduction in lateral rotor spacing. In addition, u_x downstream of the rotor is much lower for low lateral rotor spacing. The observed increase in momentum reduction across the rotor is consistent with the higher thrust applied by high blockage rotors to achieve maximum power.

Figure 3: Comparison of the blade twist, β , for the 4 rotor designs.

3.2. Rotor performance in off-design blockage conditions

Next, a series of rotor simulations is completed in which each of the optimised rotors discussed in Section 3.1 is operated in the domain for which the rotor was designed as well as in each of the other domains used in the present study. For each combination of rotor design and domain, RANS-BEM simulations are performed over a range of tip speed ratios, allowing for a peak C_p to be determined. The blade optimisation component of the RANS-BEM code is deactivated for this portion of the study.

The power coefficient curves resulting from this series of simulations are shown in Figure 6. Figure 6a shows the resultant C_p curve for each of the rotor designs when simulated in Domain 1, the domain with the lowest lateral spacing and the highest blockage ratio. Similarly, the results for each rotor design when simulated in Domains 2 - 4 are presented in Figures 6b - 6d, respectively.

All of the rotors exhibit improved performance with each reduction in lateral rotor spacing, regardless of whether the rotor is operating in design or off-design blockage conditions. Therefore, operating a rotor under highly blocked conditions is seen to improve performance irrespective of the rotor blade design. It is also seen in Figure 6 that for low blockage rotor designs, a higher tip speed ratio is required in order to achieve maximum C_p as the lateral spacing is reduced from the design spacing. Operating at increased tip speed ratio (i.e. spinning faster) in high blockage conditions allows rotors designed for lower blockage to achieve greater thrust and thus improved power at the new blockage condition.

The results in Figure 6a indicate that although the rotor efficiency for the virtually unblocked design improves more than 50%, from $C_p = 0.58$ to $C_p = 0.90$, when moved from unblocked to highly blocked flow (Domain 4 to 1), the efficiency remains significantly lower than the efficiency of the rotor specifically designed for operation in high blockage flow,

Figure 4: Sketch of the rotor designs. The leading edge (top) and trailing edge (bottom) of the rotor blade for a 2-bladed rotor are shown for each rotor design.

$C_p = 1.14$.

Conversely, the results presented in Figure 6d indicate that rotors designed for high blockage conditions are far inferior to the rotors designed for low blockage and unblocked flow when all are operated in minimal blockage conditions.

The relative performance for off-design rotor operation is shown in Figure 7. In this figure, the maximum power coefficients achieved for each of the 4 rotor designs operating at a given lateral spacing are normalised by the maximum power coefficient achieved by any rotor design at that spacing. For every spacing, the rotor with the highest power coefficient is found to be the rotor designed for the given spacing (as intended).

The rotors designed for low blockage see the greatest relative performance decrement when operated in the domain with the highest blockage, Domain 1 ($s/d = 0.25$; $B = 0.314$). The unblocked rotor design has the worst performance in this domain and has a decrement of 21% when compared to the rotor designed for Domain 1. Conversely, the rotor designed for high blockage conditions has the greatest power coefficient decrement when operated in Domain 4 ($s/d = 100$; $B = 0.0001$), which is the virtually unblocked domain. The maximum power coefficient for the rotor designed for Domain 1 is 19% lower than the maximum power coefficient for the Domain 4 rotor design when both are operated in Domain 4. The results indicate that all devices perform within 21% of the optimum achievable for a given spacing. Notably, the device designed for $s/d = 1$ ($B = 0.196$) does not perform more than 9% below the optimum achievable at any spacing.

Operating a rotor in off-design blockage conditions has significant effects on the bypass and wake flow velocities. These effects are seen in Figure 8. In this figure, streamwise velocity contours are shown for rotors with design spacing $s/d = 0.25$ (Figure 8a), $s/d = 1$

(a) Domain 1 ($s/d = 0.25$; $B = 0.314$)

(b) Domain 2 ($s/d = 1$; $B = 0.196$)

(c) Domain 3 ($s/d = 4$; $B = 0.079$)

(d) Domain 4 ($s/d = 100$; $B = 0.0001$)

Figure 5: Contours of streamwise velocity, u_x , on the horizontal centre plane for the design rotor in each of the 4 domains. The flow is from left to right.

(a) Domain 1 ($s/d = 0.25$; $B = 0.314$)

(b) Domain 2 ($s/d = 1$; $B = 0.196$)

(c) Domain 3 ($s/d = 4$; $B = 0.079$)

(d) Domain 4 ($s/d = 100$; $B = 0.0001$)

Figure 6: Power coefficient curves for each of the 4 rotor designs when operated in (a) Domain 1, (b) Domain 2, (c) Domain 3, and (d) Domain 4.

(Figure 8b), $s/d = 4$ (Figure 8c), and $s/d = 100$ (Figure 8d) when operated at actual spacing $s/d = 1$ (Domain 2; $B = 0.196$) at the optimal tip speed ratio for each rotor in Domain 2. The maximum power coefficient and corresponding thrust coefficient and tip speed ratio for each of the cases shown in Figure 8 are included in Table 3.

The rotor in Figure 8a is operating in lower blockage than it was designed for. The rotor has high solidity and has to apply greater than optimal thrust on the flow to produce maximum power. There is therefore greater resistance on the upstream flow than there is for the design rotor for Domain 2 (Figure 8b), resulting in increased streamwise velocity in the bypass flow and decreased streamwise velocity in the wake. This, in turn, creates greater shear and an increase in mixing losses between the bypass and core flows in the wake. Although not investigated in detail in the current study, the magnitude of mixing losses may be computed as the difference in total energy flux between the inlet of the domain and the fully remixed downstream condition (beyond the domain outflow but can be computed analytically [12]), less the energy extracted by the tidal rotor.

Figure 7: Maximum power coefficient for each rotor design for lateral rotor spacings $0.25 \leq s/d \leq 4$. Power coefficients are normalised with respect to the maximum power coefficient achieved by any rotor for that spacing.

Table 3: Maximum power coefficient and corresponding thrust coefficient and tip speed ratio for each domain.

s_{design}	$s_{operation}$	$C_{p,max}$	C_t at $C_{p,max}$	λ at $C_{p,max}$
$0.25d$	$1d$	0.84	1.76	5
$1d$	$1d$	0.85	1.46	6
$4d$	$1d$	0.80	1.26	6
$100d$	$1d$	0.77	1.24	7

The rotors in Figures 8c and 8d are operating in higher blockage than they were designed for. These rotors have lower than optimal solidity and are unable to apply a high enough thrust to achieve the optimum power generation possible for this domain. As these rotors impose less resistance on the flow than the rotor designed for the domain, the bypass flow speed is lower than that for the design rotor and the streamwise velocity of the flow in the wake is higher. Therefore, there is less shear between the bypass and core flows in the wake and hence reduced mixing losses.

As mentioned in Section 2.3, free surface effects have not been included in the current simulations. Rather, a rigid lid is used to approximate the surface of the channel. It is known that free surface deformations occur downstream of tidal turbines. For example, Myers and Bahaj [20] measured a drop in free surface height downstream of a rotor operating in a flume. Further, it has been demonstrated in experimental and computational studies, for instance in Birjandi et al. [21] and Bai et al. [22], that drops in free surface elevation downstream of tidal rotors can lead to improvements in C_p .

3.3. Coupled effects of blockage and design on rotor performance

Curves showing power coefficients as a function of coefficient of mass flow rate, $C_{\dot{m}}$, for rotors operating in Domains 1 ($B = 0.314$) and 4 ($B \approx 0$) are included in Figure 9. We

(a) $s/d = 0.25$ design operating in $s/d = 1$ ($B = 0.314$) domain

(b) $s/d = 1$ design operating in $s/d = 1$ ($B = 0.314$) domain

(c) $s/d = 4$ design operating in $s/d = 1$ ($B = 0.314$) domain

(d) $s/d = 100$ design operating in $s/d = 1$ ($B = 0.314$) domain

Figure 8: Contours of streamwise velocity, u_x , on the horizontal centre plane. Figures 8a-8d show contours for the $s/d = 0.25$, $s/d = 1$, $s/d = 4$, and $s/d = 100$ rotor designs, respectively, operating at their optimal tip speed ratio in the $s/d = 1$ domain (Domain 2; $B = 0.196$).

define $C_{\dot{m}}$ as the ratio of mass flow rate through the rotor disc to the mass flow rate through the projected disc area at the domain inlet ($\dot{m}_{inlet} = \rho U_{\infty} A_{disc}$). Because ρ and the rotor disc area remain constant in all simulations, $C_{\dot{m}}$ is effectively the ratio of the mean velocity through each disc to the upstream flow speed.

Figure 9: C_p variation with C_m for rotors operating in Domains 1 (blue) and 4 (red).

In Figure 9, the power curves for rotors operating in Domain 4 (virtually unblocked conditions) are shown in red and those operating in Domain 1 (high blockage conditions) are blue. The solid black line connects the peak operating points achieved by any rotor design for each blockage ratio. As previously discussed, the peak operating point for each blockage ratio is achieved by the rotor designed for that configuration. It may be shown using the Garrett and Cummins result [3] that when blockage is increased, the velocity through the actuator disc at the peak power point for the new blockage will be lower than for the peak power point for a lower blockage configuration. The current results are in agreement with the Garrett and Cummins theory, as the solid black line shows.

The dashed lines in Figure 9 connect the peak operating points achieved by each individual rotor in the two blockage conditions shown. It is seen from these dashed lines that, for each specific rotor design, C_m (and therefore the mean velocity) through the rotor at the peak operating point increases as blockage increases. It is notable that peak performance for a specific rotor design occurs with higher C_m through the rotor at high blockage than at low blockage, whereas overall peak performance achieved by any rotor occurs with lower C_m at high blockage.

To understand these results, we must first understand the fundamental mechanism through which blockage can improve rotor performance. Nishino and Willden [23] provide the following explanation: increased blockage results in increased bypass flow acceleration and a corresponding increase in pressure head drop in the bypass flow passages. The pressure head in the bypass and core flow regions must be in equilibrium both far upstream and far downstream of the rotor plane. Hence, the increased pressure head drop in the bypass

leads to an increased pressure head drop in the core flow and thus also across each rotor disc, which in turn results in increased thrust and power coefficients. Thus, the principal drivers for rotor performance in blocked conditions are the magnitude of the flow acceleration and corresponding pressure head drop in the bypass flow region. For further explanation of this mechanism, see Nishino and Willden [23].

For a given rotor design, an increase in blockage results not only in greater acceleration in the bypass regions, but also in greater resistance to the flow (due to the rotor wake expansion/bypass flow streamtube constriction) in the bypass regions. This resistance results in greater flow being pushed through the swept rotor area; hence the increase in $C_{\dot{m}}$ and therefore mean velocity through the rotor when blockage is increased and the rotor design is unchanged.

However, while operating a rotor designed for low blockage conditions in higher blockage conditions will improve $C_{p,max}$ for that rotor, a further adjustment must be made to achieve the overall $C_{p,max}$ for the higher blockage condition. The thrust the rotor applies to the flow must be increased in order to counter the increased resistance to the flow in the bypass region. This will reduce flow through the rotor and further increase the acceleration and pressure head drop in the bypass region, thereby improving C_p . A simple way to increase the thrust applied by a rotor is to increase its solidity. This helps explain why, in the rotor design process discussed in Section 3.1, the solidity for the rotor designed for the highest blockage conditions was significantly larger than that for the rotor designed for unblocked conditions.

3.4. Radial variation of c_x

The discussion thus far has been limited to global rotor properties, including C_p and C_t . In this section, the radial variation of the local thrust coefficient will be discussed for rotors operating in both design and off-design blockage conditions.

3.4.1. Rotors operating in design blockage conditions

Radial variation in c_x is plotted for each the 4 rotors operating in design blockage conditions in Figure 10. Data for 5 different operating tip speed ratios are included in the plots, and the design c_x is denoted by the dashed lines. The region from $0 < r/R < 0.15$ does not contain data as this is the nacelle region.

For most rotors, c_x for the $\lambda = 5$ cases is approximately equal to the design c_x , as expected, and is nearly constant, with a slight aberration for the blade element closest to the blade tip. The exception for $\lambda = 5$ is the rotor designed for and operating in Domain 1 ($s/d = 0.25$; $B = 0.314$). For this rotor, there is non-constant c_x in the vicinity of the blade root for $\lambda = 5$ (see Figure 10a). This, and deviation from the patterns seen in results for the other rotors near the blade root for all tip speed ratios, is a result of the solidity limit imposed in the root region of the $s/d = 0.25$ rotor design.

c_x for the $\lambda = 4$ simulations is also nearly constant for most of the rotors. However, the magnitude is much less than that of the design tip speed ratio for this lower than optimal rotational speed. This is because this rotor is unable to impart sufficient thrust on the flow, which results in $C_p < C_{p,max}$ at this tip speed ratio.

(a) Domain 1 ($s/d = 0.25$; $B = 0.314$)

(b) Domain 2 ($s/d = 1$; $B = 0.196$)

(c) Domain 3 ($s/d = 4$; $B = 0.079$)

(d) Domain 4 ($s/d = 100$; $B = 0.0001$)

Figure 10: Spanwise variation of c_x for rotors operating in their respective design domains.

Conversely, the rotors operating above the design tip speed ratio (the $\lambda = 6, 7$, and 8 cases) all experience increases in c_x as r/R increases, with the magnitude of c_x in the tip region increasing with λ for each of these cases. For the $\lambda = 6$ case, this non-constant variation in local thrust coefficient enables the rotor to achieve a power coefficient slightly higher than that for the design tip speed ratio, $\lambda = 5$ for the $s/d = 0.25, 1$, and 4 rotors, as seen in Figures 6a-6c. For even higher tip speed ratios, however, the power coefficient drops, despite the extreme radial variations in c_x seen for $\lambda = 7$ and 8 . For these high tip speed ratios, the rotor applies greater than optimal thrust to the flow.

Although the $\lambda = 5$ and $\lambda = 6$ cases achieve similar power, the increased c_x as r/R increases for the $\lambda = 6$ case is not desirable from a structural standpoint, and it is thus preferable to operate at the lower of the two tip speed ratios.

3.4.2. Rotors operating in off-design blockage conditions

It is seen in Figures 6 and 7 that, for some blockage conditions, rotors operating in off-design conditions are capable of achieving maximum power coefficients that are very near

the $C_{p,max}$ achieved by the rotor designed for the given domain. For example, the $s/d = 0.25$ rotor design, when operated in Domain 2 ($s/d = 1$; $B = 0.196$) achieves a $C_{p,max}$ that is only 1% lower than the maximum achieved by the rotor designed for Domain 2.

Although the rotor power coefficients for these two cases are similar, the local thrust coefficients are quite different. The radial variation in local thrust coefficient is included in Figure 11.

Figure 11: Radial loading experienced at the peak operating tip speed ratio for the $s/d = 0.25$ and $s/d = 1$ rotor designs operating in Domain 2 ($s/d = 1$; $B = 0.196$).

c_x for the rotor designed for operation in Domain 1 is nearly twice that for the rotor designed for Domain 2 when each is operated at their respective $C_{p,max}$ in Domain 2. These results agree with the conclusions stemming from an analysis of Figures 8a and 8b; the higher solidity of the $s/d = 0.25$ design causes this rotor to apply greater than optimal thrust on the flow in order to achieve maximum power. This results in a larger than optimal reduction in momentum downstream of the rotor and, in turn, greater shear between the core and bypass flows in the wake, causing increased mixing losses.

4. Conclusions

An investigation into the impact of local blockage conditions on turbine performance in tidal fences spanning infinitely long channels has been carried out. The effects of local blockage are of particular concern for tidal power device design as tidal devices are far more likely to operate in blocked conditions than wind turbines due to the geometry of tidal channels.

A series of tidal rotors are designed for maximum efficiency in a range of local blockage conditions using an in-house RANS-BEM optimisation tool. The aerofoil section and rotor radius remain unchanged throughout the study. Results confirm that the highest power coefficient is achieved by the rotor designed for the highest blockage. Further, it is shown that in order to achieve maximum efficiency, rotors designed to operate in high local blockage

require greater solidity and lower blade twist than rotors designed for operation in unblocked flow.

An assumption of fixed mass flow rate through the domain is employed in all simulations in this study. If the flow through the channel was head driven, enabling a variable mass flow rate, then the thrust applied by the rotor would act to resist the upstream flow, which would in turn, would result in reduced rotor thrust and power, the degree of reduction being dependent on the channel characteristics (driving head, dimensions and bed friction).

Additional RANS-BEM computations are completed in which each of the rotor designs are simulated in each of the 4 domains over a range of tip speed ratios. The maximum C_p achieved for each rotor design is improved with each reduction in rotor spacing. Though there is significant improvement in performance when a rotor designed for unblocked flow is operated in highly blocked conditions, C_p is further increased if the rotor is specifically designed for the operational blockage conditions. The best performing rotor for every domain is found to be the rotor designed for the respective domain. Although this result is intuitive, current rotor design practices do not take blockage conditions into account. Therefore, it is suggested that the anticipated operational blockage conditions be considered during the rotor design process if true optimal rotor efficiency is to be achieved.

Examination of the radial variation in local thrust coefficient reveals that the power output from a rotor with uniform thrust loading may be matched, or slightly exceeded, by a rotor operating at a higher tip speed ratio, for which there is increased loading at the blade tip as compared to the blade root. Still, a rotor with uniform c_x may be preferable due to structural implications.

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References

- [1] F. Lanchester, A contribution to the theory of propulsion and the screw propeller, Transactions of the Institution of Naval Architects 57 (1915) 98–116.
- [2] A. Betz, Das maximum der theoretisch möglichen ausnützung des windes durch windmotoren, Zeitschrift für das gesamte Turbinenwesen 26 (1920) 307–309.
- [3] C. Garrett, P. Cummins, The efficiency of a turbine in a tidal channel, J. Fluid Mech. 588 (2007) 243–251.
- [4] J. I. Whelan, J. M. R. Graham, J. Peiró, A free-surface and blockage correction for tidal turbines, J. Fluid Mech. 624 (2009) 281–291.
- [5] S. C. McIntosh, C. F. Fleming, R. H. J. Willden, Embedded RANS-BEM tidal turbine design, in: Proc. 9th European Wave and Tidal Energy Conf., 2011.
- [6] F. Menter, Two-equation eddy-viscosity turbulence models for engineering applications, AIAA Journal 32 (1994) 1598–1605.
- [7] H. Glauert, The analysis of experimental results in the windmill brake and vortex ring states of an airscrew, Reports and Memoranda, No. 1026, Her Majesty’s Stationery Office.

- [8] C. Lock, H. Bateman, H. Tiwsend, An extension of the vortex theory of airscrews with applications to airscrews of small pitch, including experimental results, Reports and Memoranda, No. 1014, Her Majesty's Stationery Office.
- [9] H. Glauert, Airplane Propellers. In *Aerodynamic Theory*, 4th Edition, Julius Springer, Berlin, 1935, pp. 169–360.
- [10] T. Burton, N. Jenkins, D. Sharpe, E. Bossanyi, *Wind Energy Handbook*, John Wiley & Sons Ltd., West Sussex, UK, 2011.
- [11] C. F. Fleming, S. C. McIntosh, R. H. J. Willden, Design and analysis of a bi-directional ducted tidal turbine, in: *Proc. 9th European Wave and Tidal Energy Conf.*, 2011.
- [12] C. Belloni, Hydrodynamics of ducted and open-centre tidal turbines, Ph.D. thesis, University of Oxford (2013).
- [13] J. Schluntz, Tidal turbine array modelling, Ph.D. thesis, University of Oxford (2014).
- [14] I. Masters, J. C. Chapman, M. R. Willis, J. Orme, A robust blade element momentum theory model for tidal stream turbines including tip and hub loss, *Journal of Marine Engineering and Technology* 10 (2011) 25–35.
- [15] G. Bai, J. Li, P. Fan, G. Li, Numerical investigations of the effects of different arrays on power extractions of horizontal axis tidal current turbines, *Renewable Energy* 53 (2013) 180–186.
- [16] R. Malki, A. J. Williams, T. N. Croft, M. Togneri, I. Masters, A coupled blade element momentum - computational fluid dynamics model for evaluating tidal stream turbine performance, *Applied Mathematical Modelling* 37 (2013) 3006–3020.
- [17] L. Prandtl, A. Betz, Vier abhandlungen zur hydrodynamik und aerodynamic, Tech. rep., Göttingen (1927).
- [18] C. Consul, R. Willden, S. McIntosh, Blockage effects on the hydrodynamic performance of a marine cross-flow turbine, *Royal Society Philosophical Transactions A* 371 (2013) 1–16.
- [19] P. Fuglsang, K. S. Dahl, I. Antoniou, Wind tunnel tests of the Risø-A1-18, Risø-A1-21 and Risø-A1-24 airfoils, Tech. Rep. Risø-R-1112(EN), Risø National Laboratory, Roskilde, Denmark (1999).
- [20] L. Myers, A. S. Bahaj, Wake studies of a 1/30th scale horizontal axis marine current turbine, *Ocean Engineering* 34 (2007) 758–762.
- [21] A. H. Birjandi, E. L. Bibeau, V. Chatoorgoon, A. Kumar, Power measurement of hydrokinetic turbines with free-surface and blockage effect, *Ocean Engineering* 69 (2013) 9–17.
- [22] X. Bai, E. J. Avital, A. Munjiza, J. J. R. Williams, Numerical simulation of a marine current turbine in free surface flow, *Renewable Energy* 63 (2014) 715–723.
- [23] T. Nishino, R. H. J. Willden, Effects of 3-D channel blockage and turbulent wake mixing on the limit of power extraction by tidal turbines, *International Journal of Heat and Fluid Flow* 37 (2012) 123–135.