Location and Distance in Economics

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Abstract

In this collection of essays, I explore three topics where space and distance plays a fundamental role in international economics. Not only do spatial considerations affect the pattern of trade, the frictions that arise from distance also determine where and how goods are produced and where people live.

In Chapter 1, I show that human made locational characteristics can determine the spatial allocation of economic activity. I take the standard core-periphery model and add endogenous housing to its forward-looking dynamic adjustment process. By introducing a model of adjustment with an extra state variable, which I interpret as housing, I show that the distribution of housing allows the model to converge to a unique spatial equilibrium. This explains the observed persistence and robustness of economic agglomerations in the data.

Chapter 2 is a theory of task assignment in the production of final goods and across countries. By allowing for tasks to differ in their suitability of being used in the production of multiple goods, my model endogenizes the allocation of tasks in the production of goods that use them. The resulting equilibrium task allocation defines the pattern of off-shoring. Tasks that are used in only one good concentrate in the country with a specialization in production of that good. Tasks used in many goods are allocated across countries, with the more substitutable tasks located in the country with the larger overall output. Gains from off-shoring are derived from a better mix of allocation of tasks into goods as well as larger scale of production.

Finally in Chapter 3, I study how real exchange rate fluctuations determine the size and composition of the export sector. Using the methodology set out in Dixit and Pindyck (1994) in a heterogeneous firms model, I determine the set of trigger real exchange rates for entry and exit into exporting. My primary result of this chapter is that exchange rate uncertainty coupled with sunk cost of entry causes hysteresis in the number and productivity of exporting firms. I then extend the model to allow free entry of firms. This explains the stylized fact of the existence of non-exporting firms with higher productivity than some exporting firms.

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Introduction

The study of international trade is distinctive in its focus on the consequences of distance on economic interactions. While topics in the field may be disparate, they have in common the focus on the effects rising from spatial separation and the differences of locations on firm and worker behaviour and welfare. In doing so, international trade theory isolates this important feature of the economy and uses it to explain trade patterns and informs policy.

The main motivation for the study of international trade is to understand how and why nations trade, and in doing so to see if trade is beneficial or not. Canonical models in the literature point to a number of reasons for international trade. In the early seminal works of Ricardo and Heckscher-Ohlin, countries trade because of exogenous differences among them. Endogenous advantages due to economics of scale is another motivation for trade as pointed out by Ethier (1982). Krugman (1979, 1980) explains how monopolistic competition and economics of scale can explain two way trade for the same products. All these models show net gains from trade, yet the distribution of gains and losses matter for policy. Policy makers can use this understanding of trade to shape it one way or other depending on their policy objectives.

1 Three topics of research

In this collection of essays, I dive into three topics within the international trade literature. The areas of international trade in which I explore involve the role of trade frictions in shaping trade.

Anderson and van Wincoop (2004) outline the importance of trade costs in determining the pattern of trade. Economic transactions over long distances involve many complexities, of which transportation costs are only a part of. International trade requires costs of coordinating a network of production over a large geographical area as well as trade finance to insure against any risks from exporting and importing. Moving goods through national borders further require tariff and non-tariff barriers to trade. Exchange rate uncertainly poses another trade cost when trading across international borders.

I take three topics within the literature and show how trade frictions interact with various motivations for trade. I vary the combination of which factors or goods can be moved across locations or countries and which ones are immobile in order to gain further insight into the process of economic interactions between several locations.

Dynamics of economic geography

In Chapter 1, I look at how the evolution of the housing stock selects one among multiple spatial equilibria. In my model, labour is mobile, incurring migration costs, but housing stock immobile. Spatial relocations of labour interacts with the endogenous housing sector to determine the spatial allocation of economic activity. This causes hysterisis in the location of spatial concentration and explains the observed persistence of economic concentrations.

Common tasks and the pattern of off-shoring

Chapter 2 shows how the substitutability of a task determines whether it used in the production of one good or several goods and the effects this has on the pattern of off-shoring. I thus endogenizes the allocation of tasks in the production of goods that use them. The resulting equilibrium task allocation defines the pattern of off-shoring. Tasks that are used in only one good concentrate in the country with a specialization in production of that good. Tasks used in many goods are off-shored to the country with the largest overall output. Gains from off-shoring are derived from a better mix of allocation of tasks into goods as well as larger scale of production.

Exchange rates uncertainty and exporting firms

I look at the importance of exchange rate uncertainty on the export sector in Chapter 3. I model heterogeneously productive firms with a fixed cost of entry into exporting. With exchange rate uncertainty, firms entering into exporting have to factor in the value of the option of stopping exporting in the future if exchange rates move unfavourably. This creates a band whereby exchange rates can fluctuate without inducing entry or exit into the export sector.

The main implication of this is that exchange rate uncertainty combined with sunk entry costs creates hysteresis in the export sector. The number of firms and the productivity of the sector adjusts upon large movements of the exchange rate, but remains unchanged upon smaller exchange rate fluctuations. This helps explains the stylized fact that export surges caused by changes in the real exchange rate tend to happen through the intensive rather than the extensive margin.

2 The bigger picture

The wider issue brought forward by these three topics is the all encompasing nature of spatial considerations in economics. Not only is distance important for internaitonal trade, it affects where and how goods are produced and where people live. This collection explores the widely disparate areas where space and distance plays a fundamental role.

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Chapter 1: Dynamics of New Economic Geography and a Unique Spatial Equilibrium

Abstract

I take the standard core-periphery model and add endogenous housing to its forward-looking dynamic adjustment process. I show that human made locational characteristics can determine the spatial allocation of economic activity. Existing dynamic processes provide the possibility of self-fulfilling expectations as a driving force of spatial reallocation among multiple equilibria. This does not match the observed persistence and robustness of economic agglomerations in the data. By introducing a model of adjustment with an extra state variable, which I interpret as housing, I show that the distribution of housing allows the model to converge to a unique spatial equilibrium.

1 Introduction

Economic geography is highly persistent. Cities that were centres of economic agglomeration long ago stay as such today, even after large exogenous shocks such as war and famine. Although the natural endowments of locations affect the spatial economy in a fundamental way, developments in technology and geopolitics change the advantages of locating near various natural geographic characteristics. Natural endowments shaped the past distribution of economic activity, and this distribution persists for a long time even when new circumstances make an alternative allocation Pareto superior. Krugman (1993) points to Chicago as a prime example of how a city that developed from past natural advantages stays as a centre of agglomeration long after the advantages have disappeared. The location of Chicago on the Great Lakes in not particularly remarkable especially given the decline in importance of lake transportation. Yet self-reinforcing advantages maintain the concentration of population and production in the same location. History provides an initial spatial allocation and any plausible model of economic geography therefore should be constrained by history.

The purpose of this paper is to develop a model that explains the large amounts of path dependence in the spatial distributions of economic activity. This is done in the context of a forward-looking model with multiple equilibria. I propose an alternative explanation to the existing literature, which appeals to location fundamentals or prohibitive and ad hoc migration costs for equilibrium selection. While natural endowments of locations and high migration costs are very good reasons for why economic geography is very stable, I am interested in locations where these reasons alone are not enough. My model is particularly useful in explaining why cities persist even after the natural endowments in their locations have been exhausted and there is a better location in close proximity. By explicitly modelling a dynamic process of adjustment in a model of economic geography, I reconcile the existence of multiple equilibria and the stylized fact that economic agglomerations are highly persistent.

The observed locational stability of economic agglomerations is at odds with existing models of economic geography that have forward-looking migrants. The mainstream view in the literature, initiated by Krugman (1991a) and Matsuyama (1991) and applied later to the context of economic geography by Baldwin (2001) and Ottaviano (2001) among others, is that under multiple equilibria, the locations of economic agglomerations face opposing influences of "history versus expectations". The combination of past spatial distribution of economic activity and the expectations of the mobile agents determine the trajectory of the economy towards one equilibrium or another. Baldwin (2001) shows that sufficiently high migration cost results in the current spatial distribution of economic activity (given by history) being the sole determinant spacial equilibrium, while lower migration costs allow expectations to play an increasing role in determining the equilibrium. Ottaviano (2001) shows that when forward-looking migrants face zero migration costs, history is irrelevant and only expectations matter in determining the equilibrium towards which the economy heads.

The empirical literature in economic geography is still young and far from conclusive. Redding (2011) identifies a common theme in several papers, however, namely the stylized fact that city locations are robust to large shocks and persist over long periods of time[1]. Using data from allied bombings and the subsequent recovery of Japanese cities in World War II, Davis and Weinstein (2002, 2008) show that the locations of cities are highly persistent. Even cities that suffered heavy bombings quickly recovered to the population and production levels they had before. The authors argue that the robustness of city locations to large temporary shocks affecting the population distribution provides evidence against jumps from one spatial equilibrium to another. They conclude that natural geography is a key determinant of city location and see no evidence of spatial fluctuations among multiple equilibria.

A small industry of papers using war as a natural experiment to test for multiple equilibria have sprouted since Davis and Weinstein (2002). Brakman, Garretson and Schramm (2004) reproduce the Davis and Weinstein (2002) results using data from post-war Germany. They find that large and temporary shocks only have a transitory impact on German city growth in the post-war period. Looking at Vietnam after U.S. bombings, Miguel and Roland (2006) find that the distribution of population across Vietnam was only affected temporarily by the bombings.

While spatial concentrations are very persistent, they can be shifted by policy. Becker, Heblich and Sturm (2013) look at the natural experiment of the selection of Bonn as a place for the post-war federal government. They show that the introduc-

[1]While city populations are found to be robust to large shocks, there is evidence that individual industries can be susceptible to fluctuating among multiple steady-states (Redding, Sturm, and Wolf (2011), Dumais, Ellison, and Glaeser (2002)). See Redding (2011) for a thorough exposition on the empirical literature of economic geography.

tion of public sector jobs to Bonn increases the population growth rate of the city conpared to other control cities. This suggests that a concerted effort by government can shift spatial allocations between equilibria after some time.

The history versus expectations literature does not bode well in the light of the empirical evidence. The observation that cities resume to their original size after a shock that displaces the population suggests that history as embodied in population size does not determine where agglomerations will locate. The robustness of city location also rules out the possibility of expectations driven fluctuations. Neither history nor expectations is able to consistently explain the robustness of cities to large shocks!

The results of the empirical studies suggest there are qualities about the locations of cities that make them attractive to economic activity beyond the size of population. While Davis and Weinstein (2002) point to the natural geography of locations, Redding (2011) notes that there are many physical and institutional structures that may survive even the most severe of bombings. He suggests that infrastructure, buildings, property rights, and legal institutions may persist even when the population has been displaced by war. The data is therefore consistent with having both natural and human made locational characteristics in explaining the robustness of city locations to large shocks.

The urban economics literature supports the view that human made characteristics of a location can make economic agglomerations persist. Henderson and Venables (2009) create a model of city formation where there are substantial sunk capital costs of housing and infrastructure. Rauch (1993) shows how the existence of sunk costs can prevent city relocation to a more efficient site. Glaeser and Gyourko (2005) and Glaeser, Gyourko, and Saks (2006) examine declining cities and find evidence that durable housing slows down urban decline. Durable housing implies the housing stock in cities will only decline gradually in response to a negative shock. When a negative shock moves the population away from the city, housing prices decrease as housing supply can only depreciate and is unable to adjust immediately.

This makes the city more attractive to live in and lures back the population into the city.

1.1 Summary of findings

In this paper, I develop a dynamic model of economic geography that allows housing to favour an equilibrium to explain the spatial persistence of economic agglomerations. What I refer to as housing for the remainder of this paper can also be a range of human made locational characteristics as suggested by Redding (2011). The mechanism is based on adding a state variable, which I interpret as housing, to a model with multiple equilibria. The evolution of the state variable is explicitly modelled and needs to satisfy certain assumptions in order for the model to converge to a unique equilibrium. Under multiple equilibria, the endogenous housing reacts to the migration choices of workers and is able to select a unique equilibrium.

I then apply my model to the canonical Core-Periphery model to eliminate problems of indeterminacy when the mobile workers are forward-looking. This allows me to adapt the history versus expectations literature to include the housing. I first show how the evolutionary process of housing is able to eliminate multiple equilibria under low Krugman (1991a) style migration costs. Later I dispense with such migration costs entirely and show that my model can select an equilibrium when migration costs are fixed or non-existent.

The main result of this paper is that spatial hysteresis can be caused by the existence of a housing sector that adjusts slowly to housing demand in each region. I show analytically that a housing sector that satisfies certain behavioural assumptions can cause a multiple equilibria model to converge to a single equilibrium. Numerically, I show that such a housing sector is able to select an equilibrium in a Core-Periphery model with forward-looking mobile workers. While history and past expectations determine the initial stock of housing in each region, once an equilibrium is favoured by the distribution of housing the model locks-in to that equilibrium. History, in the form of the housing stocks in each region, thus selects the equilibrium.

1.2 Relationship to the literature

The dynamics of the Core-Periphery model have been studied ever since Krugman's initial formalization of the model in Krugman (1991b). A typical new economic geography model such as the Core-Periphery model of Fujita, Krugman, and Venables (2001) explains why concentrations of economic activity exist. They give no prediction where the concentrations will be of the multiple possible locations. Fujita and Thisse (2009) point out that an equilibrium cannot be chosen without specifying a dynamic adjustment process.

Krugman (1991b) uses an ad hoc adjustment process where myopic mobile workers move to the region with the highest real wages. Indeed more recent versions of the Core-Periphery model, such as Forslid and Ottaviano (2003), still assume myopic migrants for analytical convenience. Myopic migration produces a Marshallian tatonnement process that leads the model toward the nearest stable equilibrium. This is what Baldwin et al. (2003) call the hysteresis property of economic geography; historical accidents that determine where workers are located also determine the equilibrium at which the economy will converge to and stay indefinitely. Fujita, Krugman, and Venables (2001) justify the use of the myopic adjustment process by appealing to replicator dynamics of evolutionary game theory. This is, however, just another way of imposing the assumption of myopia. While myopic migration provides tractability and allows researchers to concentrate on other features of the economic geography models, it has little economic and empirical basis.

The history versus expectations literature attempts to provide a micro-founded dynamic adjustment process of the Core-Periphery model. Baldwin (2001) points out that this gives rise to a major source of intractability, namely the existence of multiple stable equilibria. Directly adding forward-looking expectations to the migration process would allow for self-fulfilling reallocations. A richer description of the dynamic process is thus required for the model to gain some explanatory power of why agglomerations locate where they do.

There are two main models of migration dynamics in this line of research, both of which impose a friction to the migration process. The first approach follows Krugman (1991a) and Fukao and Benabou (1993) (KFB) where the migration cost is convex in migrant flow. Under KFB dynamics, the level of migration costs determine the balance between whether history, in the form of current allocation of workers, or expectations determine the spatial organization. The second approach introduced by Matsuyama (1991) uses a probabilistic mechanism to control the rate of flow of migrants. At a given period, only a fixed proportion of the migrants, chosen at random, is allowed to relocate costlessly. To what extent history or expectations determine equilibrium selection depends on the size of this proportion relative to the discount rate of migrants.

This class of models rely on a specific type of migration costs that needs to be high enough to remove indeterminacy. However, a unique solution path is really a special case of these models. Having less extreme parameter values such as lower migration costs or abandoning the migration friction completely leads to the possibility of expectations driven spatial fluctuations. When a forward-looking migrant believes other migrants will move from one location to another, and every other migrant has the same belief, it is optimal behaviour for all migrants to move and realize that belief. This means self-fulfilling expectations can cause spatial reallocation. It also implies equilibria are fragile and there is the possibility of sudden takeoffs and reversals as expectations change. Gali (1995) illustrates how expectations driven by sunspot revisions can cause random fluctuations in a forward-looking spatial model. This instability of spatial allocation is not observed in the stickiness of the growth and decline of cities.

Despite a lack of empirical evidence, history versus expectations models are still the main tools used to describe the global stability properties of new economic geography models. Neary (2001) points to Baldwin's (2001) analysis to justify the use of an ad hoc migration equation based on myopic migrants. He notes that under high migration costs, KFB dynamics and myopic migration are identical. There is an agreement within the literature that "myopia is truly an assumption of convenience", as Baldwin (2001) puts it.

Subsequent research on the dynamics of the Core-Periphery model have elaborated on properties of KFB or Matsuyama (1991) dynamics. Matsui and Matsuyama (1995) show for a general game with multiple equilibria and a small degree Matsuyama (1991) friction, the equilibrium that is risk dominant is more likely to be played[2]. This equilibrium is more stable as it is both "uniquely absorbing" and "globally accessible". In other words, the risk dominant equilibrium has a single equilibrium path in its neighborhood and it is globally stable once the equilibrium has been reached. Self-fulfilling expectations can still drive the economy when it is farther away from the risk dominant equilibrium. Using asymmetric regions to generate a risk-dominant equilibrium, Oyama (2009a) applies Matsui and Matsuyama (1995)'s technique to a forward-looking Core-Periphery model with Matsuyama (1991) dynamics. Oyama (2009b) does the same for KFB dynamics. In both of Oyama's papers he shows that asymmetric regions can cause one of the regions to be uniquely absorbing and globally accessible.

While having asymmetric regions is a step in the right direction towards equilibrium selection in a forward-looking Core-Periphery model, there are still unresolved issues inherent to the history versus expectations literature. There is little predictive power of which equilibria the economy will converge to. The risk dominant equilibrium is more likely to occur, but there is nothing that rules out the economy from staying at the risk dominated equilibrium forever. On a methodological note, this result also relies on extreme parameter values, as migration costs cannot be too low or too high relative to the discount rate of migrants. Furthermore, this method still cannot explain the empirical stylized fact that the equilibrium is robust to large

[2] The definition of risk dominance is the sense of Harsanyi and Selten (1988)

shocks to the allocation of economic activity. Once a shock moves the economy away from the neighbourhood of the risk dominant equilibrium, self-fulfilling expectations will be able to alter the outcome of the economy.

This paper is a continuation of the line of research that seeks to better define the global stability properties of the Core-Periphery model with forward-looking migrants. It provides two main contributions. First, I introduce a dynamic adjustment process that provides a mechanism for equilibrium selection. It resolves the theoretical possibility of multiple equilibria with the observed stability of actual economies. This provides a framework that solves the intractability of any model with multiple equilibria. The existence of multiple equilibria is not unique to economic geography and with a few adjustments my model can be applied to a wider set economic applications.

The second contribution is that I apply my model to a forward-looking Core-Periphery model to explain an empirical stylized fact. It provides a story for why the locations of economic agglomerations are robust to large shocks and immune to spatial fluctuations. My model is an alternative dynamic adjustment process to the history versus expectations literature with KFB or Matsuyama (1991) dynamics. In contrast to the existing literature where self-fulfilling expectations can determine the spatial equilibrium, this paper shows that the human made locational charateristics locks in over time the economy to a unique equilibrium. The results of my model conform better to the empirical literature than other existing models without the need to deviate from the insights of the Core-Periphery model (i.e. without appealing to natural geographic endowments of locations). In particular, my model is able to achieve a stable unique equilibrium in the presence of forward-looking mobile workers that is robust to shocks affecting the distribution of the workers.

The remainder of the paper is organized in the following way. In section 2, I present a general model with multiple equilibria with a dynamic adjustment process. Although the assumptions used in this section are ad hoc, they covey the underlying economic intuition of how an extra state variable can be a mechanism for equilibrium selection. Section 3 provides micro-foundations to the framework presented in section 2 by applying an endogenous housing sector to the canonical Core-Periphery model. In this section, I also set up the KFB dynamic optimization problem applied to my model. I then solve for the global solutions to my model in section 4. This is done first under KFB migration costs, and then under zero and constant migration costs. Section 5 outlines the policy implications of my model and I conclude in section 6.

2 A Simple Stylized Model

I begin by developing a generalized version of a model of equilibrium selection that distils the essential elements of my later analysis. This section sketches the economic intuition behind the mechanism of the model. I add further structure and complexity when I provide proper micro-foundations in section 3.

My model is concerned with the spatial distribution of workers among two regions when there are increasing returns and self-fulfilling multiple equilibria. I first look at what I refer to as the "Static" model, which is characterized by the following assumptions:

Assumption 1 *There are two regions, A and B.*

The regions are initially symmetric.

Assumption 2 *There is a continuum of workers and* $\lambda \epsilon [0, 1]$ *is the proportion of workers in region A.*

I model the location decisions of workers who are perfectly mobile between regions. Workers choose to locate in either region and since they are identical, and any pure strategy Nash equilibrium will either be at $\lambda = 0$ or $\lambda = 1$.

Assumption 3 *There is a stock of housing in each region and* $H\ \epsilon [0, 1]$ *is the proportion of housing in region A*

At the moment I take H as exogenously given. I later model the behaviour of H when I add a dynamic dimension to the model. Although I interpret H as housing, it can also represent other human made locational characteristics such as infrastructure, acquiring local knowledge, and the state of institutions. Since I only need to look at the relative attractiveness to workers of the two regions, I only need the relative size of the housing stocks as the units of housing can be normalized.

Assumption 4 *Each worker receives the payoff of* $\omega^A = f(\lambda, H)$ *and* $\omega^B = f(1-\lambda, 1-H)$ *in region A and B respectively, where* f *is monotonically increasing in both arguments.*

Assumption 4 implies increasing returns to labour so that $\frac{\partial \omega^A}{\partial \lambda} > 0$ and $\frac{\partial \omega^B}{\partial \lambda} < 0$; workers would like to locate in the same region where all other workers are located.

The workers play a one-shot game that is repeated. The two pure strategies are to locate in region A and in region B. A pure strategy Nash equilibrium will emerge if all workers locating in the same region results in a bigger payoff in that region:

Proposition 1 *For a given* H*,*

(i) *all workers locating in region A* ($\lambda = 1$) *is a Nash equilibrium if* $\omega^A(1, H) - \omega^B(1, H) > 0$*, and*

(ii) *all workers locating in region B* ($\lambda = 0$) *is a Nash equilibrium if* $\omega^A(0, H) - \omega^B(0, H) < 0$.

Proof. *When all workers are at region A and* $\omega^A(1, H) - \omega^B(1, H) > 0$*, an infinitesimal deviation of the continuum of workers to region B does not affect the payoffs. So any defection is not profitable and* $\lambda = 1$ *is therefore a Nash equilibrium. The proof for* $\lambda = 0$ *is opposite but analogous. q.e.d.*

(i) and (ii) can hold independently of each other in which case there will be a unique Nash equilibrium. When both (i) and (ii) hold, the game exhibits both positive spillovers and strategic complementarities, which as shown in Cooper and John (1988) implies the existence of multiple Nash equilibria.

The proportion of housing in region A, H, is a state variable that also determines the payoff differential between the two regions. Assumption 4 implies that $\frac{\partial \omega^A}{\partial H} > 0$ and $\frac{\partial \omega^B}{\partial H} < 0$. As workers live in houses, having more housing makes a region more attractive. Higher values of H would make region A more attractive and vice versa.

Now I define threshold values of H that indicates the concentration of housing in one region or another. Define $\overline{A}$ as the threshold value of H such that when H is above it the payoff in region A is always greater than that of region B. Likewise, define $\overline{B}$ to be the threshold that when H is below it the region B payoff is always greater than that of region A. This is formalised in the following assumption:

Assumption 5 $\overline{B} < 0.5 < \overline{A}$ *are threshold values of* H *and* $\lambda^* (H)$ *a threshold value of* λ *such that:*

$\omega^A - \omega^B < 0,\ \forall\ \lambda,$ *when* $H \epsilon [0, \overline{B})$

$\omega^A - \omega^B > 0$ *for* $\lambda > \lambda^* (H)$, $\omega^A - \omega^B = 0$ *for* $\lambda = \lambda^* (H)$, *and* $\omega^A - \omega^B < 0$ *for* $\lambda < \lambda^* (H)$, *when* $H \epsilon \left[\overline{B}, \overline{A}\right]$

$\omega^A - \omega^B > 0, \forall\ \lambda,$ *when* $H \epsilon (\overline{A}, 1]$.

I thus allow the value of H to determine whether there are one or multiple equilibria. Small or large values of H removes the multiple equilibria in the game by taking away the strategic complementarities. When $H < \overline{B}$, most housing is in region B such that location B provides a higher payoff no matter the distribution of workers. In this case, Proposition 1(ii) will hold but not 1(i) so there is a unique Nash equilibrium. The equivalent but opposite argument will show that there is also a unique Nash equilibrium when $H > \overline{A}$.

The model has 2 symmetric pure strategy Nash equilibria only when H is in the multiple equilibria interval $\left[\overline{B}, \overline{A}\right]$. Both Proposition 1($i$) and ($ii$) will hold, and choosing to locate in region A with probability $\lambda^* (H)$ is a symmetric mixed strategy Nash equilibrium. There is a multiplicity of equilibria since workers choose simultaneously in the presence of strategic complementarities. Every worker choosing to locate in A or in B are both symmetric pure strategy Nash equilibria and the model gives no prediction over which one will occur in the economy. When each worker believes that every other worker will choose to locate in region A, all workers will choose A and realize those initial beliefs. Self-fulfilling expectations can therefore determine the spatial distribution of workers when there are multiple equilibria.

The Static model is defined by assumptions 1 to 5. The relative proportions of housing stock determines whether there are multiple equilibria or a unique equilibrium. It is by defining the behaviour of H, therefore, that my model selects an equilibrium.

Now I add a time dimension to form the "Dynamic" model. The Static model game is repeated infinitely as the housing stock H evolves. The housing sector adjusts based on the distribution of workers and the housing stock endowed from the previous period. When workers are concentrated in one region, the proportion of housing in that region will increase gradually. The following three assumptions captures the evolution of the housing stock.

I let the proportion of housing in region A, $H(t)$, be a state variable in period t that is endogenously determined:

Assumption 6 *$H(t)$ is the solution to $\dot{H}(t) \equiv \frac{\partial H(t)}{\partial t} = g(\lambda(t), H(t))$*

That is, the first order differential equation $\dot{H}(t)$ is a function of the current distribution of workers as well as H endowed from the previous period. The housing sector responds the the distribution of workers, but is constrained by the existing distribution of housing.

In each period, workers observe the value of H endowed from the previous period and then choose their actions. The state variable H reacts to the aggregate of workers' actions. Workers' payoffs are thus determined by the combination of their actions and the endogenous state variable. As there are no adjustment costs for moving from one region to another, a one shot game is repeated in every period with H evolving endogenously.

The evolution of H is important in determining the characteristics of the global game. In order for my model to converge to a unique equilibrium, two key assumptions regarding the equation of motion for H are required:

Assumption 7 $\left|\frac{\partial \dot{H}(t)}{\partial \lambda(t)}\right| < \varepsilon$

This means the instantaneous response of $\dot{H}(t)$ to $\lambda(t)$ is small and less than some constant ε. The state variable adjusts slowly to the choices of the workers, as it takes time to build houses. Otherwise, a housing sector that reacts quickly to the choices of workers would adjust instantaneously to the distribution of workers and become redundant as a locational anchor for economic activity.

Assumption 8 $\dot{H}(t) \geq 0$ *when* $\lambda = 1$ *and* $\dot{H}(t) \leq 0$ *when* $\lambda = 0$

Assumption 8 means the state variable has to react positively to the choice variable to reinforce the equilibrium chosen. This implies that if all workers are in one region, the housing sector will grow more in that region relative to the other as there is less demand in the other region. Intuitively, the housing sector sector follows the distribution of workers as the workers demand housing where they are located

The Dynamic model is defined by assumptions 6, 7, and 8. It characterizes the reaction of the state variable H to the aggregate of workers' choices in the Static model. I argue that the Dynamic model presents a reasonably realistic behaviour of housing sectors. I combine it with the Static model to form a single larger model that characterizes the global stability properties of a model with multiple equilibria. With assumptions 1 to 8, I have the following result:

Proposition 2 *The economy will converge to a single Nash equilibrium once* H *leaves the "multiple equilibria interval"* $\left[\overline{B} - \varepsilon, \overline{A} + \varepsilon\right]$ *.*

Proof. When $H(t) \epsilon (\overline{A} + \varepsilon, 1]$*, assumption 7 means that the slow response of the* H *prevents it from re-entering* $\left[\overline{B} - \varepsilon, \overline{A} + \varepsilon\right]$ *regardless of* $\lambda(t)$*, so the single equilibrium of* $\lambda = 1$ *will be chosen. From assumption 8, the equation of motion of*

H is weakly increasing so that H will stay above $\overline{A}+\varepsilon$ and the single equilibrium is maintained. The proof for $H(t)\,\epsilon[0,\overline{B})$ is opposite but analogous. q.e.d.

Under this model, the economy experiences hysteresis; the evolution of H chooses an equilibrium, and once it is chosen it is the only equilibrium that will be played indefinitely. $\dot{H}(t)$ needs to be explicitly modelled in order to specify under what circumstances the economy leaves $\left[\overline{B}-\varepsilon,\overline{A}+\varepsilon\right]$. However it follows from assumption 6 that the cumulation of present and past λ (the latter being summarized by $H(t-1)$) determines the evolution of H.

This result shows that the housing sector has the potential to "lock in" one equilibrium over another. When the proportion of housing is spread relatively evenly across regions, the standard Baldwin (2001) history versus expectations effect holds and there are multiple equilibria. However, on the occasion that one of these equilibria occurs for a long enough period for the housing sector to adjust, the housing sector makes it so that it is the only equilibria.

So "history" in the form of the past distribution of workers for many periods, rather than the current distribution of workers as in the history versus expectations literature, determines which equilibrium the economy converges to. In a sense the mechanism in the model side-steps the multiple equilibria problem with the addition of the state variable by having the economy change into one with a unique equilibrium. Multiple equilibria will exist without the existence and the particular behaviour of this state variable.

This model can easily be extended to a wider class of economic geography models to select among multiple equilibria. More than two regions can be introduced, in which case several state variables will have to be able to move the economy into and out of the multiple equilibria interval. The state variable H also need not be housing. H can be a vector of several variables that affects the payoffs of the game and follows an evolutionary path conforming to assumptions 7 and 8. Other potential candidates for state variables include physical constraints such as infrastructure, acquiring local knowledge, the state of institutions, as well as policy variables such as taxes and property rights. Furthermore, my framework can also be adopted to models where the mobile factor is something other than workers. An economic geography model with mobile firms and immobile workers as in Krugman and Venables (1995) can have factories or commercial buildings as the state variable instead of housing.

The above model can also be generalized to a range of models with multiple equilibria. Due to the assumptions required for the state variable, such a model would apply only to games with physical frictions rather than currency and equity markets where there are none. This means the model can be seen as orthogonal to that of Morris and Shin (1998) as it deals with multiple equilibria games of a different kind. Morris and Shin (1998) show that a small uncertainty in speculator knowledge can cause a model of self-fulfilling currency crisis to have a unique equilibrium based on the realized value of the economic fundamentals. Their state of economic fundamentals that shifts the economy into and out of the multiple equilibria interval is like the state variable H in my model. However, their model has imperfect information about the state variable and it is stochastic. The value of the state variable in my model is public knowledge and evolves over a predictable path based on the history of λ's. Unlike Morris and Shin's model, my model does not rely on the nature of how the game is played. Instead it works because of the particular behaviour of the state variable that is able to change the game into one with a unique equilibrium.

A possible application of this model outside of economic geography is in the "big push" literature such as Murphy, Shleifer and Vishny (1989). In fact the original history versus expectations literature of Krugman (1991a) and Matsuyama (1991) try to model the industrialization process where there are multiple equilibria. With external economies in each sector, there are two equilibria in which all people work in the low productivity sector and in the high productivity sector. Sector specific skills is a possible state variable in these models that locks-in one equilibrium or another. Additionally, the state variable can also be a policy instrument. Diamond and Dybvig (1983) show that a multiple equilibria model with and without bank runs can be resolved by a deposit insurance. The state variable, the level of deposit insurance, is too small when the economy is in the multiple equilibria interval. With a large enough insurance, this policy changes the payoffs of the game such that there is only one Nash equilibrium.

3 A dynamic model of economic geography

The main goal of this paper is to show how adding a dynamic state variable into a model of economic geography can make the model better fit an empirical stylized fact. By selecting a unique equilibrium among multiple equilibria, my model explains why economic agglomerations are robust to large shocks affecting the distribution of people and immune to expectations driven fluctuations. Section 2 is a general and intuitive presentation of my model. However, its assumptions are ad hoc and lacks proper micro-foundations of a economic geography model and of a housing sector.

I now adopt the framework of section 2 into a more specific model of economic geography to provide micro-foundations to my approach. Using the canonical Core-Periphery model (taken as Fujita, Krugman and Venables (2001)[3]), I first introduce an extension proposed by Helpman (1998) to develop a Static model. The Static model takes housing distribution as exogenous and specifies the payoffs of locating in each region as a function of the distribution of workers. There can either be one or multiple equilibria depending on the state of housing. The Static Core-Periphery model is shown to be an example of the Static model of section 2.

I then introduce a dynamic adjustment process into the Static Core-Periphery model in order to characterize its stability properties. The Dynamic Core-Periphery model has an endogenous housing sector and forward-looking workers. The evolution of housing loosely maintains the assumptions 6, 7, and 8 of section 2.

[3]Forslid and Ottaviano's (2003) version of the Core-Periphery model is often used instead for its analytical solvability, but since I will be relying on numeric simulations for my results this is not needed. I choose to use Fujita, Krugman and Venables (2001) for its more intuitive presentation.

3.1 Canonical Static Core-Periphery Model

The economy consists of two regions, A and B. There are two factors of production and two sectors: industrial workers, L_M, working in manufacturing, M, and agricultural labourers, L_Z, working in the agricultural sector, Z. I denote λ as the proportion of industrial workers in region A. Agricultural goods are assumed to be freely transported while manufactured goods are subject to iceberg transport costs τ, such that $\tau \geq 1$ units must be shipped in order for 1 unit to reach the destination.

Every consumer has the same two-tier utility function. It is Cobb-Douglas over consumption of manufacturing goods, agricultural goods, and, following Helpman (1998), housing services. There are also Constant Elasticity of Substitution (CES) preferences over the manufacturing sector. The utility function of the representative consumer is:

$$U = C_M^{\alpha} C_Z^{\beta} C_H^{1-\alpha-\beta} \text{ where } C_M = \left(\int_{i=0}^{n^A+n^B} c_i^{1-1/\sigma} di \right)^{1/(1-1/\sigma)} \tag{1}$$

α is the expenditure share on manufactured goods and β the share on agricultural goods. c_i is the consumption of variety i and n^A and n^B are the number of manufacturing varieties in regions A and B respectively. $\sigma > 1$ is the constant elasticity of substitution between varieties.

Utility maximization yields the standard CES demand function for each manufactured variety j:

$$c_j = \frac{p_j^{-\sigma}}{(P)^{1-\sigma}} \alpha E \tag{2}$$

where

$$E = w_M L_M + w_Z L_Z \tag{3}$$

E is the total expenditure and w_M and w_Z are the nominal wages of manufacturing workers and agricultural labourers respectively. With p_i as the price of the variety i manufactured good, the price index of manufactured goods, P, is defined as:

$$P \equiv \left(\int_{i=0}^{n^A+n^B} p_i^{1-\sigma} di \right)^{1/(1-\sigma)} \tag{4}$$

The agricultural sector is perfectly competitive and has constant returns to scale. Goods in this sector are homogenous and freely traded so prices are the same across regions and equal to marginal cost:

$$p_Z = w_Z^A = w_Z^B$$

The demand for housing services in region A is given by the Cobb-Douglas demand curve:

$$p_H^A = \frac{(1-\alpha-\beta)\, E^A}{H^A} \tag{5}$$

Region B has a similar demand curve for housing services. Note that it is not important what units of measure we use for housing, since H^A and $1-\alpha-\beta$ are defined under a single normalization.

The manufacturing sector faces Dixit-Stiglitz monopolistic competition. A typical firm producing variety j has the profit function:

$$\pi_j = p_j q_j - (cq_j + F)\, w$$

where q_j is the quantity of variety j produced, c is the constant marginal cost and F is the fixed cost of production.

Each firm acts as a monopolist facing demand curve (2). The constant elasticity of substitution implies that profit maximization will lead to prices being a constant mark-up over marginal costs. The prices of a typical variety produced in region A and sold in regions A and B are:

$$p^A = \frac{w_M^A c}{1-1/\sigma},\; p^B = \frac{\tau w_M^A c}{1-1/\sigma} \tag{6}$$

Given these prices, the operating profit of a firm is:

$$\pi_j = w_M \left[\frac{q_j c}{\sigma - 1} - F \right]$$

Free entry ensures firms make zero profits so the scale of a typical firm is fixed at:

$$q_j^* = \frac{F(\sigma - 1)}{c} \tag{7}$$

and the associated labour input equilibrium implies the number of firms in region A is:

$$n^A = \frac{L_M^A}{F\sigma} \tag{8}$$

Following Fujita, Krugman, and Venables (2001), I now do some simplifying normalizations. I choose the units of measure of manufacturing goods such that $c = \frac{\sigma-1}{\sigma}$. Prices (6) becomes:

$$p^A = w_M^A,\ p^B = \tau w_M^A$$

I also choose the units of firms such that $F = \frac{\alpha}{\sigma}$, (7) and (8) become:

$$q^{A*} = \alpha,\ n^A = \frac{L_M^A}{\alpha}$$

Choosing agricultural goods as the numeraire, I set $p_Z = w_Z^A = w_Z^B = 1$. I also choose the units of agricultural labourers so that the world supply is $L_Z^W = \beta$, and both regions are symmetric so each region has $\frac{\beta}{2}$ agricultural labourers. Likewise, I set the world supply of manufacturing workers to $L_M^W = \alpha$ by choosing the appropriate units. I define λ as the proportion of manufacturing workers in region A, $\lambda \equiv \frac{L_M^A}{L_M^W}$.

Market clearing ensures the value of each firm's output must equal the demand from both locations for the variety of good produced by that firm. A firm producing variety j in region A has the market clearing condition:

$$q_j^{A*} = \frac{p_j^{A-\sigma} \alpha E^A}{\left(P^A\right)^{1-\sigma}} + \frac{\tau \left(p_j^B\right)^{-\sigma} \alpha E^B}{\left(P^B\right)^{1-\sigma}}$$

which gives region A's market clearing wage as:

$$w_M^A = \left[E^A \left(P^A\right)^{\sigma-1} + \tau^{1-\sigma} E^B \left(P^B\right)^{\sigma-1}\right]^{1/\sigma} \tag{9}$$

The real wage in region A is:

$$\omega^A = \frac{w_M^A}{\left(P^A\right)^{\alpha} \left(p_H^A\right)^{1-\alpha-\beta}} \tag{10}$$

The Core-Periphery model consists of the income equation (3), price index (4), wage equation (9), real wages (10), and housing price (5) for both regions. Using the normalizations for further simplification, the complete Static Core-Periphery model is:

$$E^A = w_M^A \alpha \lambda + \frac{\beta}{2} \tag{11}$$

$$E^B = w_M^B \alpha (1-\lambda) + \frac{\beta}{2} \tag{12}$$

$$P^A = \left(\lambda w_M^{A1-\sigma} + (1-\lambda)\left(\tau w_M^B\right)^{1-\sigma}\right)^{1/(1-\sigma)} \tag{13}$$

$$P^B = \left(\lambda \left(\tau w_M^A\right)^{1-\sigma} + (1-\lambda) w_M^{B1-\sigma}\right)^{1/(1-\sigma)} \tag{14}$$

$$w_M^A = \left[E^A \left(P^A\right)^{\sigma-1} + \phi E^B \left(P^B\right)^{\sigma-1}\right]^{1/\sigma} \tag{15}$$

$$w_M^B = \left[\phi E^A \left(P^A\right)^{\sigma-1} + E^B \left(P^B\right)^{\sigma-1}\right]^{1/\sigma} \tag{16}$$

$$\omega^A = \frac{w_M^A}{(P^A)^\alpha (p_H^A)^{1-\alpha-\beta}} \tag{17}$$

$$\omega^B = \frac{w_M^B}{(P^B)^\alpha (p_H^B)^{1-\alpha-\beta}} \tag{18}$$

$$p_H^A = \frac{(1-\alpha-\beta) E^A}{H^A} \tag{19}$$

$$p_H^B = \frac{(1-\alpha-\beta) E^B}{H^B} \tag{20}$$

This Static Core-Periphery model (11) to (20) provides an instantaneous equilibrium of all endogenous variables given λ, H^A, and H^B. This includes ω^A and ω^B which are monotonically increasing with H^A and H^B respectively, as well as with λ (the latter cannot be shown analytically due to the non-linear nature of the model). This model thus satisfies assumption 4 of section 2. Note that other than the existence of housing and the price of housing, the model is identical to that of Fujita, Krugman and Venables (2001) Chapter 5.

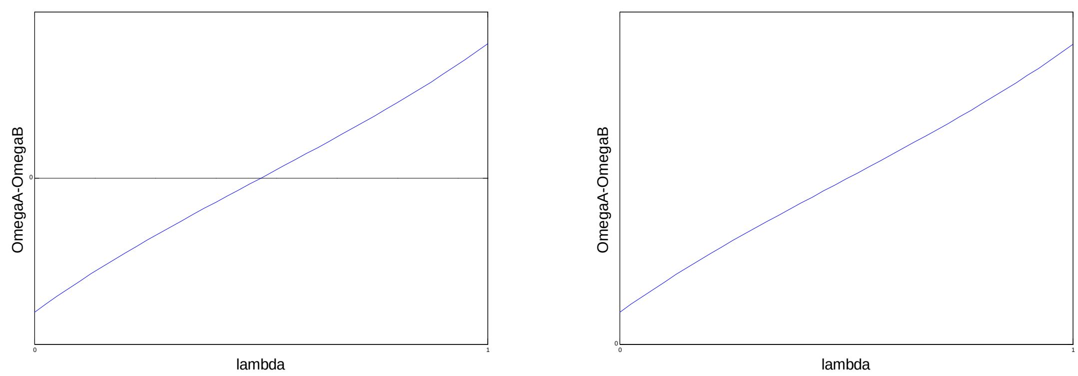
The equilibrium structure is determined by one centripetal force and two centrifugal forces in this model. The centripetal force is the increasing returns to manufactured goods production which makes workers want to locate all in the same region. The first centrifugal force is that the immobile agricultural labourers ensure demand exists in both regions so that workers would want to locate in both regions to supply them. The second centrifugal force is the need for workers to live in housing. The supply of housing services becomes more expensive as more workers move to a region, so workers would want to spread out across regions so they can pay less for housing. The balance of centripetal and centrifugal forces determines whether the model has a unique symmetric equilibrium, at which workers are located evenly across regions, or has two additional core-periphery equilibria, at which all workers concentrate in a single region.

The level of trade costs determines the balance between the centripetal and centrifugal forces, but the latter forces react in an opposite way to trade costs. Immobile agricultural labourers induce the strongest centrifugal force when trade costs are high. When it is expensive to transport goods among regions, production would tend to spread out and locate near where the consumers are. On the other hand housing generates the strongest centrifugal force when trade costs are low. Workers can take advantage of cheap housing across the regions and move their products to all other regions cheaply. Which of the centrifugal forces is stronger entirely depends on the parameters values chosen for the model.

I restrict my analysis to levels of trade costs that allow for multiple core-periphery equilibria, since there is no need for equilibrium selection when there is a unique symmetric equilibrium. If the parameter values are such that immobile agriculture is a much stronger centrifugal force than housing, then the core-periphery equilibria will emerge at low levels of trade costs. The opposite is true for when the parameter values make housing the stronger force. Either way, the problem addressed by this paper is to resolve the issue of multiple equilibria. The remaining analysis is only concerned with the cases where the set of parameter values and trade cost generate core-periphery equilibria when housing stock is distributed evenly across regions.

The Static Core-Periphery model conforms with the Static model encompassing assumptions 1 to 5 in section 2. Assumptions 1 to 3 have been directly assumed and I have already shown the model follows assumption 4. To show my model satisfies assumption 5, first define $H \equiv \frac{H^A}{H^A+H^B}$ as the proportion of housing in region A. For a set of parameter values, there exist a multiple equilibria interval $\left[\overline{B}, \overline{A}\right]$ of H such that assumption 5 of section 2 is satisfied. The main determinant of $\overline{A}$ and $\overline{B}$ is the expenditure share of housing $1-\alpha-\beta$. A larger expenditure share of housing makes the prices of housing services more important for real wages, so that the distribution of housing can skew the real wage differential across regions.

This can only be shown numerically due to the non-linear nature of the model. Figure 1 shows the real wage differential between region A and B, $\omega^A - \omega^B$, for various levels of H. When H is in the multiple equilibria interval $\left[\overline{B}, \overline{A}\right]$ as in Figure 1(a), $\omega^A - \omega^B > 0$, for $\lambda > \lambda^*(H)$, and $\omega^A - \omega^B < 0$, for $\lambda < \lambda^*(H)$. In this case there are multiple pure strategy symmetric Nash equilibria where all workers choose region A or all workers choose region B. Figure 1(b) shows the real wage differential when H is above $\overline{A}$ and thus outside the multiple equilibria interval. For all λ, $\omega^A - \omega^B > 0$, so there is a unique Nash equilibrium where all workers choose region A. The state variable H is able to transform the model into and out of a game with multiple equilibria. This Static Core-Periphery model is therefore a micro-founded version of the Static model of section 2.



(a) (b)

Fig. 1 Real wage differential $\omega^A - \omega^B$ when (a) $H \in \left[\overline{B}, \overline{A}\right]$, (b) $H > \overline{A}$

3.2 Dynamic Core-Periphery Model

I now develop the Dynamic part of my model by introducing endogenous housing. The housing sector in each region is characterized by a durable housing stock. H conforms with assumptions 6 and 7 and behaves very similarly to assumptions 8.

To relate the above model to the literature, I then adapt it into a dynamic model developed by Krugman (1991a) and Fukao and Benabou (1993) (KFB). The migration friction of KFB is convex in migrant flow so workers spread out their migration and not move at the same time. This friction is ad hoc and is mainly used for analytical convenience. I later relax the KFB migration friction in section 4 to show that it is irrelevant in obtaining my results.

3.2.1 Endogenous housing

I add a housing sector that produces a dynamic adjustment process for H. Given λ, H^A, and H^B the Static model determines the price of housing services p_H^A and p_H^B. The housing sector constructs houses based on the stock of housing and the price of housing services. Assume housing stocks are owned by a separate group of developers (i.e. foreigners) so I can abstract from the revenues of housing. I also assume that housing construction does not hire manufacturing workers or agricultural labourers, but has a separate workforce not in the model. Following Glaeser and Gyourko (2005), housing is durable and the stock of housing can only fall with depreciation. Housing depreciates at a constant rate δ over time and construction in every period is given by $\zeta^A\left(p_H^A(t), p_H^A(t+1), ...\right) \geq 0$.

I now derive the evolution of $\dot{H}$. As $H = \frac{H^A}{H^A + H^B}$, I look at the housing evolution in each region separately first. The housing stock in region A has the equation of motion:

$$\dot{H}^A = -\delta H^A + \zeta^A\left(p_H^A(t), p_H^A(t+1), ...\right) \qquad (21)$$

and the housing stock in region B behaves analogously.

However, in order for my model to be tractable, I assume ζ^A is only a function of the current price $p_H^A(t)$ and not future prices. I take a simple linear form

$$\zeta^A = a p_H^A + b$$

This is a necessary simplification for analytical tractability to reduce the number of jump variables when I combine worker migration behaviour with the evolution of housing. While it is certainly restrictive, realistic behaviour of a housing sector is not sacrificed. Under multiple equilibria the housing sector cannot distinguish whether a movement in workers is temporary or permanent, so the behaviour of a forward-looking housing sector would be similar to a myopic one. When there is a unique equilibrium, a forward-looking housing sector would build more in response to the permanent increase of housing demand in the agglomeration region than a myopic one. However, since both types of housing sectors will construct housing until construction is just enough to replace depreciation, there is a common terminal value of housing stock. The only difference between the myopic and the forward-looking housing sector is that the former spreads construction over time and the latter builds all at once. With the existence of diminishing returns and time to build in reality, the behaviour of a myopic housing sector may be a plausible description of how a housing sector evolves.

$\dot{H}$ can be derived from $\dot{H}^A$ and $\dot{H}^B$ and can be shown to *roughly* conform to the Dynamic model in section 2. Since $\dot{H}^A$ and $\dot{H}^B$ are a function of λ, p_H^A, and p_H^B, and the latter two are also a function of H^A and H^B, a continuous time version of assumption 6 is satisfied. The durability of housing slows down the evolution of H and I can further appeal to time to build and diminishing returns to make ζ^A and ζ^B small such that $\frac{\partial \dot{H}(t)}{\partial \lambda(t)}$ is also small. Like assumption 7, $\frac{\partial \dot{H}(t)}{\partial \lambda(t)}$ is smaller than some ε so that the interval of multiple equilibria of the Static Core-Periphery model will be $\left[\overline{B} - \varepsilon, \overline{A} + \varepsilon\right]$.

My housing model departs from assumption 8 somewhat more, but it still maintains the essence of the same housing sector behaviour. The housing sector of (21) would conform with assumption 8 if not for immobile agricultural workers. When there are only mobile manufacturing workers, an equilibrium with all workers concentrated in one region would have no demand for housing services in the other region as it is empty. In which case, the long run steady states are $H = 1$ when $\lambda = 1$ and $H = 0$ when $\lambda = 0$ and assumption 8 is satisfied.

The introduction of immobile agricultural labourers add a degree of complexity because the labourers need housing, too. Consider the case where $\lambda = 1$, that is, all workers in region A. If there is not enough housing for the immobile labourers in a region B, $\dot{H}$ may actually fall as more housing is constructed in region B than in region A, which contradicts assumption 8.

When the economy stays at a given λ, the housing stock will continue to evolve until $\dot{H}^A = 0$ and $\dot{H}^B = 0$ such that there will be a steady state $H^A(t) = \frac{\zeta^A\left(p_H^A(t)\right)}{\delta}$ and $H^B(t) = \frac{\zeta^B\left(p_H^B(t)\right)}{\delta}$ for $t \to \infty$. This means $H(t) = \frac{\zeta^A\left(p_H^A(t)\right)}{\zeta^A\left(p_H^A(t)\right)+\zeta^B\left(p_H^B(t)\right)}$ with $t \to \infty$ is the steady state for a given λ. I replace assumption 8 by imposing the following weaker assumption to the housing sector:

Assumption 9 *$H(t) > \overline{A}+\varepsilon$ when $\lambda = 1$ and $H(t) < \overline{B}-\varepsilon$ when $\lambda = 0$ as $t \to \infty$*

Assumption 9 is equivalent to imposing a limit on the size of parameter β, the world supply of agricultural labourers. The assumption ensures there are few enough immobile workers that the steady state of housing in a core-periphery equilibrium is outside of the multiple equilibria interval $\left[\overline{B} - \varepsilon, \overline{A} + \varepsilon\right]$. Otherwise, in a world where the distribution of housing is primarily determined by immobile agricultural labourers, the housing sector would not be able to sustain a large enough concentration of housing to eliminate equilibria. Recall also that the expenditure share of housing $1 - \alpha - \beta$ is the main determinant of $\overline{A}$ and $\overline{B}$. A larger $1 - \alpha - \beta$ narrows the interval $\left[\overline{B} - \varepsilon, \overline{A} + \varepsilon\right]$ so can also help satisfy assumption 9.

Assumption 9 is not a descriptive assumption of an economy but rather a limitation of the scope of my model. Multiple equilibria cannot be resolved by an endogenous housing sector if this assumption is not satisfied. However, since I am using my model to explain the behaviour of city location, the share of agricultural labourers to manufacturing workers tends to be small in cities. This assumption is thus applicable to my area of study. In addition, my model can accommodate asymmetric regions where the immobile agricultural labourers differ across regions as long as assumption 9 is satisfied. Once I accept this assumption, the evolution of endogenous housing is a micro-founded model very similar to the Dynamic model of assumptions 6 to 8 in section 2.

3.2.2 Forward-looking dynamics

I now characterize the migration process of mobile workers. Workers optimize their location taking real wages and housing prices as given under the dynamic adjustment process of KFB. They can move between regions at any time, but will incur moving costs that increase quadratically to the flow of migrating workers in the economy. More specifically, workers pay $\frac{\dot{\lambda}^2}{2\gamma}$ to migrate from one region to another.

Workers are homogeneous in my model. In order to allow for a division of labour in both regions, I group workers into representative households that optimize the location of labour. Hence the economy consists of multiple representatives households all optimizing the same utility function.

The representative household divides labour between regions A and B and chooses the level of migration, m, to maximize its real wages[4]. The optimal migration behaviour is the path of m and λ that maximizes the present value of utility of the representative worker, U_t, given some initial condition λ_o. The optimal control problem of a representative manufacturing workers is:

$$U_t = \max_m \int_0^\infty e^{-\rho t} \left[\lambda \omega^A - (1-\lambda)\,\omega^B - \frac{m^2}{2\gamma} \right] dt \quad (22)$$

$$s.t.$$

$$\dot{\lambda} = m \quad (23)$$

$$\lambda(0) = \lambda_o \quad (24)$$

$$\lambda \geq 0,\ \lambda \leq 1 \quad (25)$$

where γ is the KFB migration cost and ρ is the subjective discount rate.

The current value Hamiltonian of the problem $(22) - (25)$ is:

[4] Ottaviano (2001) derives the exact same results without the need for migrants dividing their labour between two regions. He has migrants being indifferent between moving and postponing on the equilibrium trajectory.

$$\mathbf{H} = \lambda\omega^A - (1-\lambda)\,\omega^B - \frac{m^2}{2\gamma} + \mu m$$

where μ is the co-state variable that captures the asset value of migration, the Lagragian is:

$$\mathcal{L} = \mathbf{H} + \boldsymbol{\theta}_1\lambda + \boldsymbol{\theta}_2\,(1-\lambda)$$

where $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ are Lagrange multipliers for the inequality constraints.

A system of 2 differential equations and 2 complementary slackness conditions define the solution to the optimal control problem:

$$\dot{\lambda} = \mu\gamma \tag{26}$$

$$\dot{\mu} = \rho\mu - \omega^A + \omega^B - \boldsymbol{\theta}_1 + \boldsymbol{\theta}_2 \tag{27}$$

$$\lambda \geq 0,\ \ \boldsymbol{\theta}_1 \geq 0,\ \ \lambda\boldsymbol{\theta}_1 = 0 \tag{28}$$

$$1-\lambda \geq 0,\ \ \boldsymbol{\theta}_2 \geq 0,\ \ (1-\lambda)\,\boldsymbol{\theta}_2 = 0 \tag{29}$$

(26) is the migration equation that decribes the law of motion of λ, and μ is the asset value of migrating now.

Adding housing dynamics (21) for each region closes the model:

$$\dot{H}^A = -\delta + \left(ap_H^A + b\right) \tag{30}$$

$$\dot{H}^B = -\delta + \left(ap_H^B + b\right) \tag{31}$$

Real wages ω^A and ω^B as well as housing stocks H^A and H^B are endogenously determined when migrants choose m.

The Dynamic Core-Periphery model consists of (30) and (31) while the KFB dynamics are represented by (26) to (29). Combined with the Static Core-Periphery model (11) to (20), I now have a full description of the economy.

4 Results

Now that I have defined the forward-looking dynamic Core-Periphery model, I next solve the model to identify its global stability properties. Since the model consists of a set of non-linear differential equations, I resort to using numerical techniques to obtain my solutions. Just as in section 2, the evolution of the state variable in my model is able to select a unique equilibrium. I then allow workers to be perfectly mobile and then with fixed migration costs. I can show that my results are not reliant on the specific form of KFB dynamics.

4.1 Global stability under KFB dynamics

4.1.1 Solution Method

The solution can only be found using numerical techniques due to the non-linear nature of both the Static and Dynamic Core-Periphery model. μ is a jump variable, m the choice variable, and λ, H^A, H^B are state variables. The Euler method is a common technique used to numerically approximate the solutions of non-linear ordinary differential equations[5]. The Euler method works by specifying a starting value for all the state variables and jump variables, and then iterating over time with the system of ordinary differential equations. However, since movement along the solution path becomes very rapid as the KFB migration friction is reduced, I use the Runge-Kutta method instead. This method improves on the Euler method by allowing for smaller iteration steps and thus providing more accurate approximations.

[5] See Butcher (2008) for a textbook presentation of the Euler and Runge-Kutta method.

The exact solution cannot be identified unless the starting values and terminal values are specified. History provides the starting values of the state variables λ_0, H_0^A, H_0^B. The appropriate transversality condition is $\lim_{t\to\infty} e^{-\rho t}\mu(t) = 0$ as pointed out by Fukao and Benabou (1993). The solution method, therefore, picks the level of μ_0 such that the transversality condition is satisfied. I implement this by doing a grid search for values of μ_0 that meets this condition.

Since my model has the state-space constraints (25), I have to consider the possibility that the co-state variable μ can jump when the state space constraints become binding. Chiang (1992) Chapter 10 provides the technical details for the required modification to the solution procedure. Additional jump conditions need to be satisfied at junction points where the constraint turns from nonbinding to binding status. As adopted from Oyama (2009b), the state-space constraints provide another condition for the solution path:

Lemma 1 $\mu(\bar{t}) = 0$, where $\bar{t}$ is the time when the λ first reaches 0 or 1 (i.e. the junction point).

Proof. I show this for $\lambda(\bar{t}) = 1$ but the proof is analogous for when $\lambda(\bar{t}) = 0$. Since (25) are state space constraints, it cannot be ruled out that the co-state variable μ may experience jumps. The conditions (26)-(29) are accompanied by the following jump condition:

$$\begin{aligned} \mu(\bar{t}+\epsilon) &= \mu(\bar{t}-\epsilon) - b_1 + b_2, \\ b_1 &\geq 0, \lambda b_1 = 0, \\ b_2 &\geq 0, (1-\lambda) b_2 = 0 \end{aligned} \tag{32}$$

where ϵ is small. For λ to reach 1 in the first place, I require $\dot{\lambda}(\bar{t}-\epsilon) \geq 0$ and from (26) this implies $\mu(\bar{t}-\epsilon) \geq 0$. Since λ cannot be greater than 1 due to the constraint in (25), I need $\dot{\lambda}(\bar{t}+\epsilon) \leq 0$ for the constraint to hold. This means $\mu(\bar{t}+\epsilon) \leq 0$. When $\lambda(\bar{t}) = 1$, from (32) I have $b_1(\bar{t}) = 0$ and $b_2(\bar{t}) \geq 0$. This implies the jump condition is $\mu(\bar{t}+\epsilon) = \mu(\bar{t}-\epsilon) + b_2$. The left hand side $\mu(\bar{t}+\epsilon) \leq 0$ and the right hand side $\mu(\bar{t}-\epsilon)+b_2 \geq 0$. Only when $\mu(\bar{t}+\epsilon)=\mu(\bar{t}-\epsilon)=b_2=0$ will the jump condition be satisfied and therefore $\mu(\bar{t})=0$. q.e.d.

Lemma 1 says that once the economy reaches $\lambda=0$ or $\lambda=1$, μ will stay at 0 even if $\dot{H}^A$ and $\dot{H}^B$ are still non-zero. Thus a solution path that reaches a core-periphery allocation will stay there until it reaches a long run steady state. Note that as μ equals 0 at $\bar{t}$, lemma 1 also satisfies the transversality condition.

4.1.2 Simulation Results

Recalling that I restrict my analysis to the cases where there are multiple core-periphery equilibria when housing stock is symmetric, my results are analogous to proposition 2 that show the possibility of the model converging to a unique equilibrium. The phase portraits for the four dimensional system of ordinary differential equations (26)-(31) are shown in Figure 2.

Figures 2 (a), (b), and (c) plot the zero contour surfaces $\dot{\lambda}=0$, $\dot{\mu}=0$, and $\dot{H}=0$ respectively, where I use the definition $H \equiv \frac{H^A}{H^A+H^B}$ so that I can plot my solution in a three dimensional graph. The arrows indicate the direction of motion in relation to the surfaces.

Figure 2 (a) depicts the law of motion of λ from (26): $\dot{\lambda}=\mu\gamma$. It is apparent as that $\dot{\lambda}=0$ when $\mu=0$. Then the asset value of migrating, μ, is zero, there is no migration. When μ is positive, workers migrate towards region A ($\lambda=1$) and when it is negative workers migrate towards region B ($\lambda=0$).

Figure 2 (b) shows how the asset value of migrating now, μ, changes depending on the levels of λ and H. This is the co-state variable. Note that in addition to the surface in Figure 2 (b), μ and therefore $\dot{\mu}$ are also equal to zero along the $\lambda=0$ and $\lambda=1$ lines according to Lemma 1. The shape of the surface is explained by (27). In order for $\dot{\mu}=0$, the size of μ has to equate $\rho\mu=\omega^A-\omega^B+\boldsymbol{\theta}_1-\boldsymbol{\theta}_2$. For example, when both housing and workers are concentrated in region A (H and λ equal to 1), the payoff in region A, ω^A, is high so the require μ for $\dot{\mu}=0$ is also high.

The evolution of the housing sectors of both regions is shown in Figure 2 (c).

$\dot{H} = 0$ is increasing with λ showing that the housing sector will adjust so that a larger share of housing will locate where the workers are. Above the zero contour surface, there is too much housing in region A relative to region B than justifiable by workers' demand, so H falls. Below the surface there is too little housing and H rises. Note H is not at 0 and 1 when λ is at 0 and 1 because of the existence of immobile agricultural labourers. Even when all workers are in one region, housing is not all concentrated in that region as the other region also has demand for housing from the labourers.

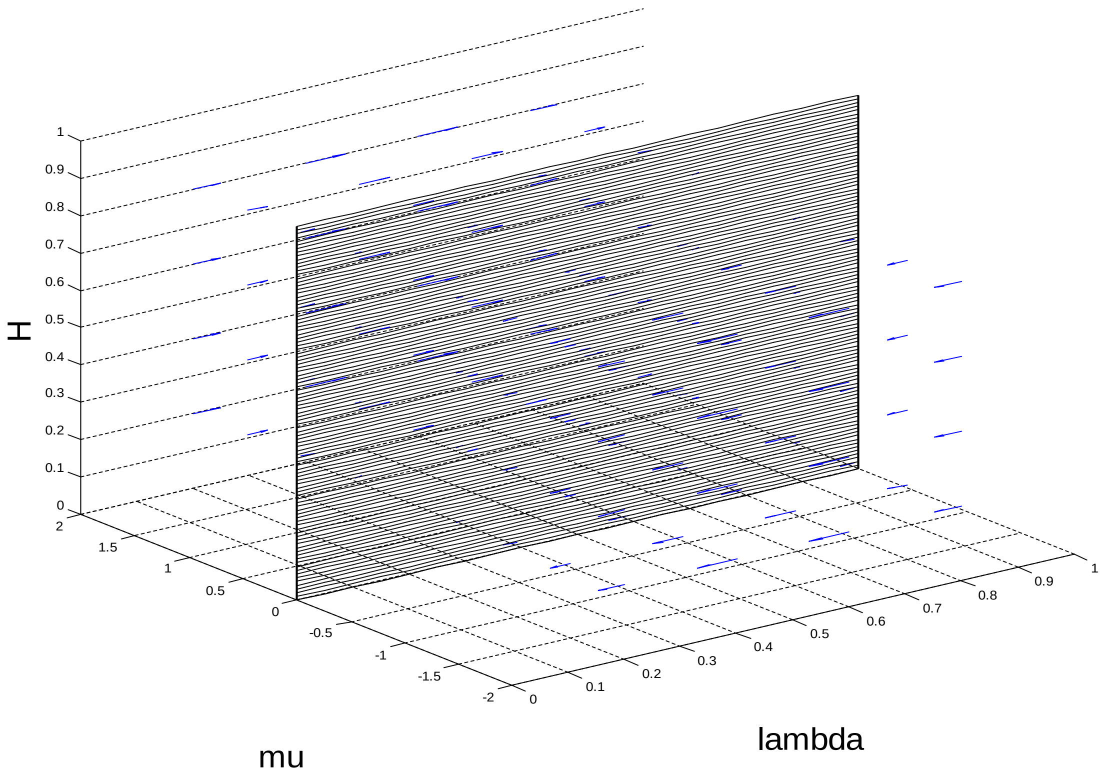


Figure 2 (a). Phase portraits of $\dot{\lambda} = 0$

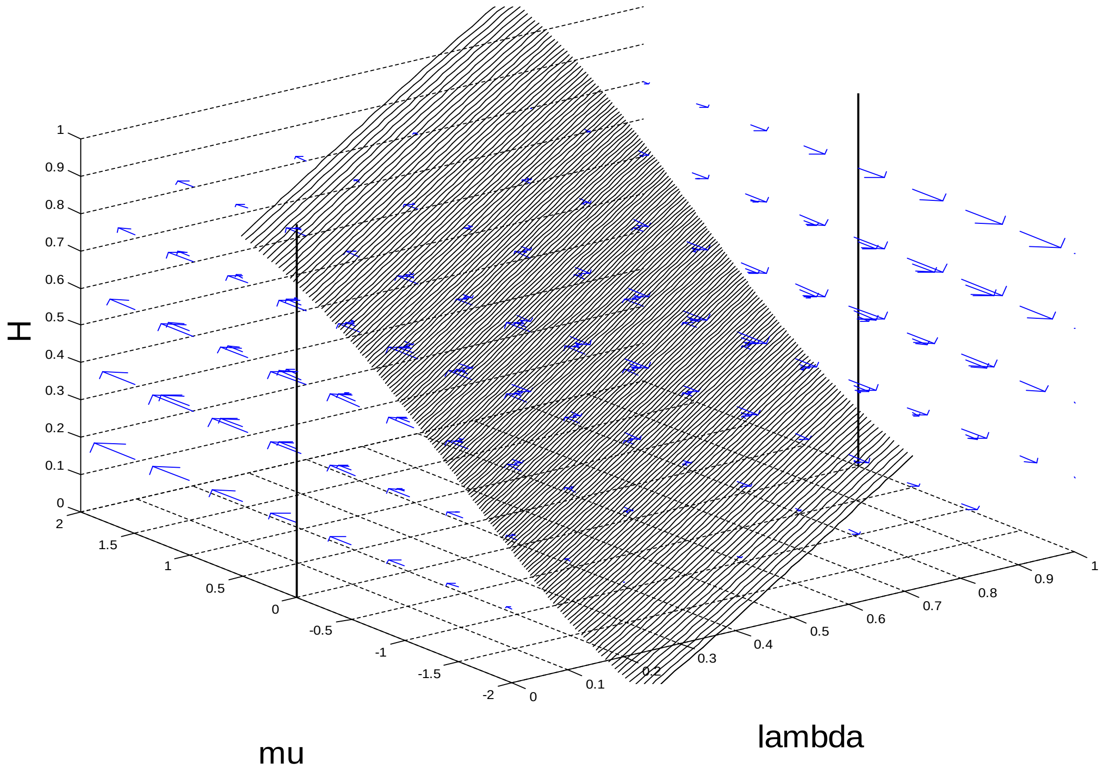


Figure 2 (b). Phase portraits of$\dot{\mu} = 0$

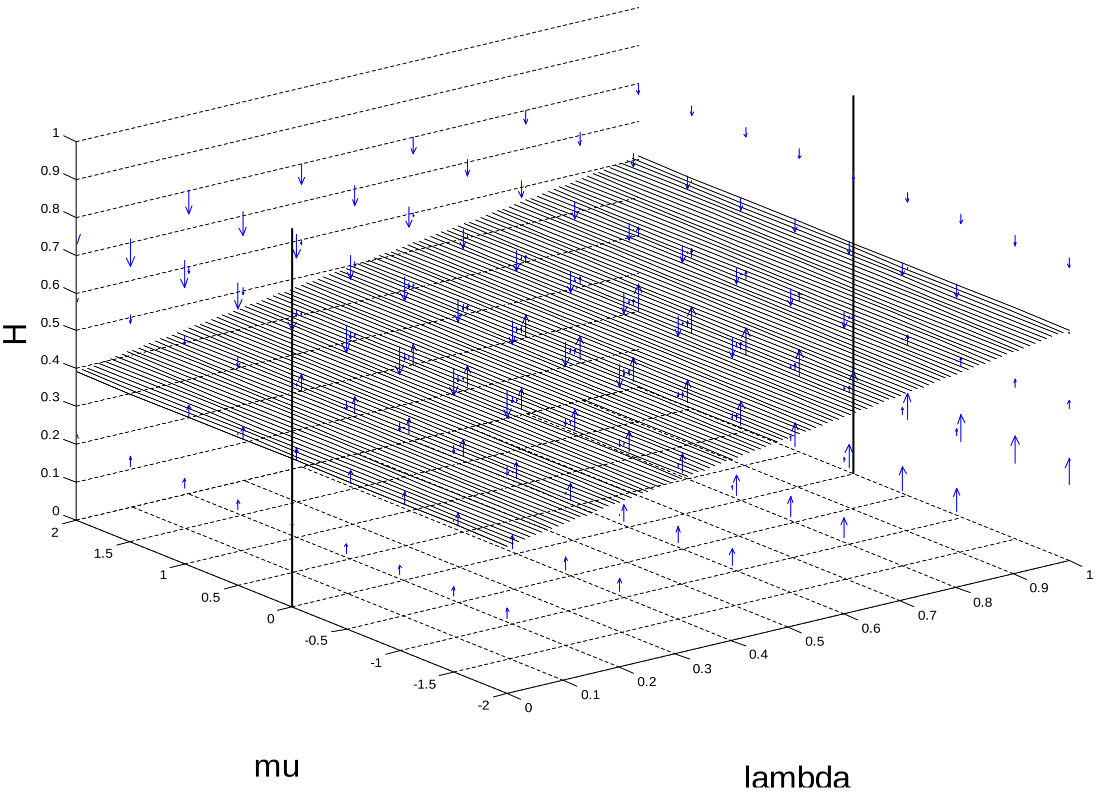


Figure 2 (c). Phase portraits of $\dot{H} = 0$

Superimposing the $\dot{\lambda} = 0$, $\dot{\mu} = 0$, and $\dot{H} = 0$ surfaces gives Figure 3. The model has three steady states where the surfaces overlap (recall that $\dot{\mu} = 0$ when λ is zero and one), two of which are labelled CP and one Sym. Since the set of differential equations (26)-(31) describe the workers' optimized behaviour, these are also the long run equilibria of the model.

Note that the Sym equilibrium is locally unstable, as the direction of motion will move away from it after a small perturbation, while the CP equilibria are locally stable. The evolution of housing prevents the stability of Sym. This is because when λ is slightly above 0.5, the housing in region A will increase relative to region B, thus making region A even more attractive and increasing λ more. This is in stark contrast to the results of Baldwin (2001) where the Core-Periphery model without housing allows for the symmetric equilibrium to be stable.

Assumption 9 ensures the CP equilibria are outside of the interval $\left[\overline{B}-\varepsilon,\overline{A}+\varepsilon\right]$ along the H axis (recall $\left[\overline{B}-\varepsilon,\overline{A}+\varepsilon\right]$ is the multiple equilibria interval of the Static Core-Periphery model). This is necessary for the model to converge to a unique equilibrium. Otherwise, the model will converge to the multiple equilibria interval and only self-fulfilling expectations will determine spatial distribution.

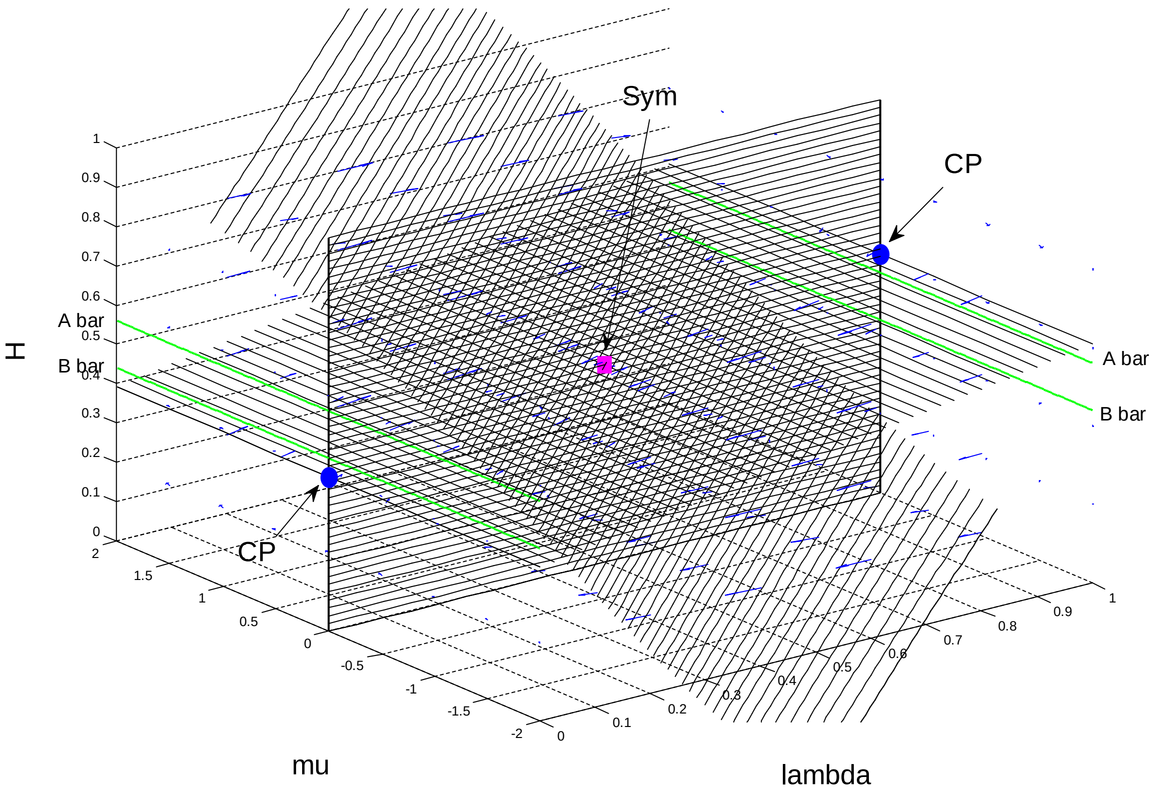


Figure3. Phase portrait of the Core-Periphery model with KFB dynamics and endogenous housing

I now turn to numerical simulations of the saddle path solution. Using a grid search to find the starting values of μ that statisfies the transversality condition, Figure 4 plots three examples of saddle paths for low KFB migration costs[6]. The

[6]When migration costs are high, migration will happen very slowly so that the housing sector

behaviour of the model depends of the initial value of H. For H_0 that is above $\overline{A}+\varepsilon$ in solution path i, the real wage differential $\omega^A - \omega^B$ is greater than zero for all λ. All workers move to region A and the solution path leads to the CP equilibrium with $\lambda = 1$. When housing is all concentrated in one region, that region becomes so attractive that all workers would choose that region no matter how workers are allocated before hand.

On the other hand, for H_0 that is within the interval $\left[\overline{B} - \varepsilon, \overline{A} + \varepsilon\right]$ as in saddle paths ii and iii, there is a value λ_0 where two paths originate each leading towards a separate CP equilibrium. As within this interval of H the distribution of workers can affect the relative payoffs among the two regions, it is rational for forward-looking workers to follow either solution path and the model provides no prediction over which one is taken.

These paths form the characteristic spiral of the KFB dynamics as seen in Krugman (1991b), Baldwin (2001) and Oyama (2009b). The "overlap" in the saddle paths shows the indeterminacy in the model. In Figure 4 the originating point of paths ii and iii illustrates this overlap. At any given point in time, the state variables of the economy λ and H are derived from the migration choice and their own values from the previous period. There is an overlap over the solution paths when there are two paths for a given set $[\lambda, H]$. All solution paths within the overlap are rational adjustment paths of forward-looking optimizing workers. Individual workers will migrate based on which solution path they think other workers are following. The aggregate of workers' actions determines the path of the economy. Thus expectations will determine which solution path is taken when there is an overlap.

would be able to adjust to the distribution of workers. Thus housing will not alter the model and the results revert back to the same as Krugman (1991b) where only history determines the equilibrium chosen.

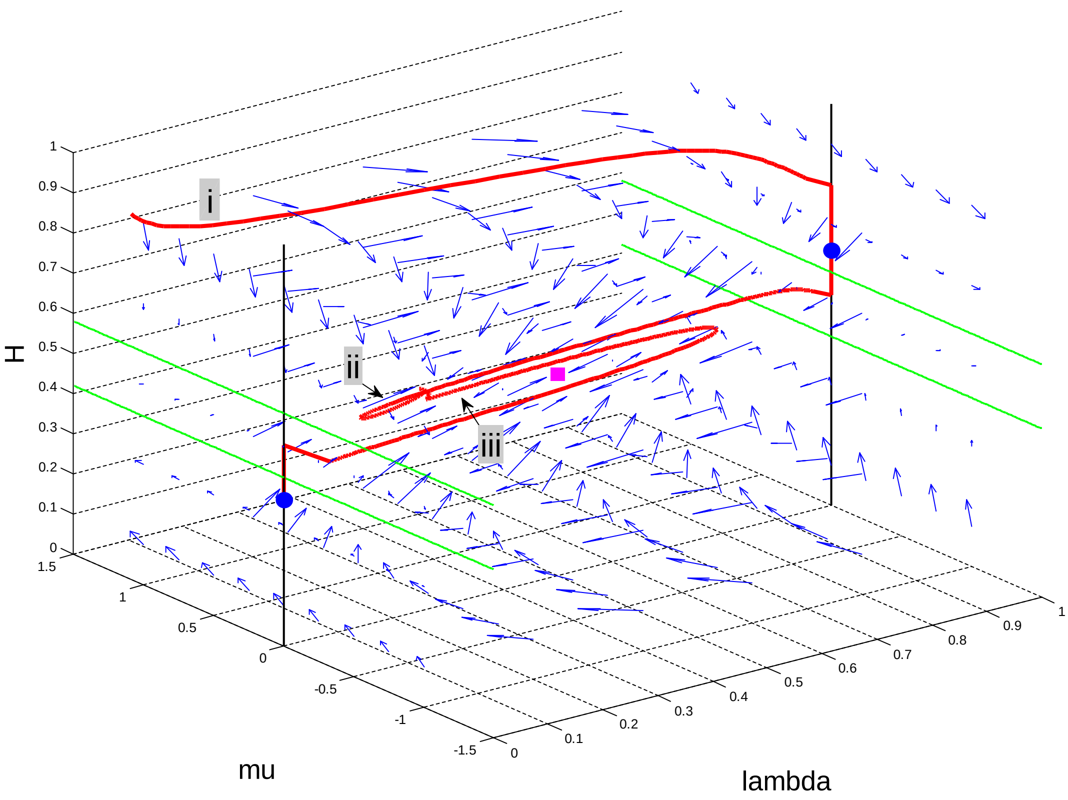


Figure 4. Example of three solution paths

The existence of overlap in my model is more apparent when I plot more solution paths. Figure 5 plots the family of saddle paths of my model. Not the entire set of solutions is plotted as there is a continuum of infinitely many paths. For H above $\overline{A}+\varepsilon$ or below $\overline{B}-\varepsilon$ the solution paths line up above and below each other so there can be no overlap. The model converges uniquely to one of the CP equilibria. In the case where H is within the interval $\left[\overline{B}-\varepsilon, \overline{A}+\varepsilon\right]$, the solution paths overlap one another. Expectations can lead the economy to jump from one path to another and there is no prediction of the CP equilibrium towards which the economy is heading.

This result is analogous to proposition 2 of section 2. When housing is concentrated in one region, the economy evolves to move the centre of agglomeration to that region. When housing is evenly divided, expectations determine which solution path the economy is on. $\dot{H}(t)$ depends on which solution path is chosen and the economy will converge to a single Nash equilibrium if H leaves the multiple equilibria interval $\left[\overline{B}-\varepsilon, \overline{A}+\varepsilon\right]$.

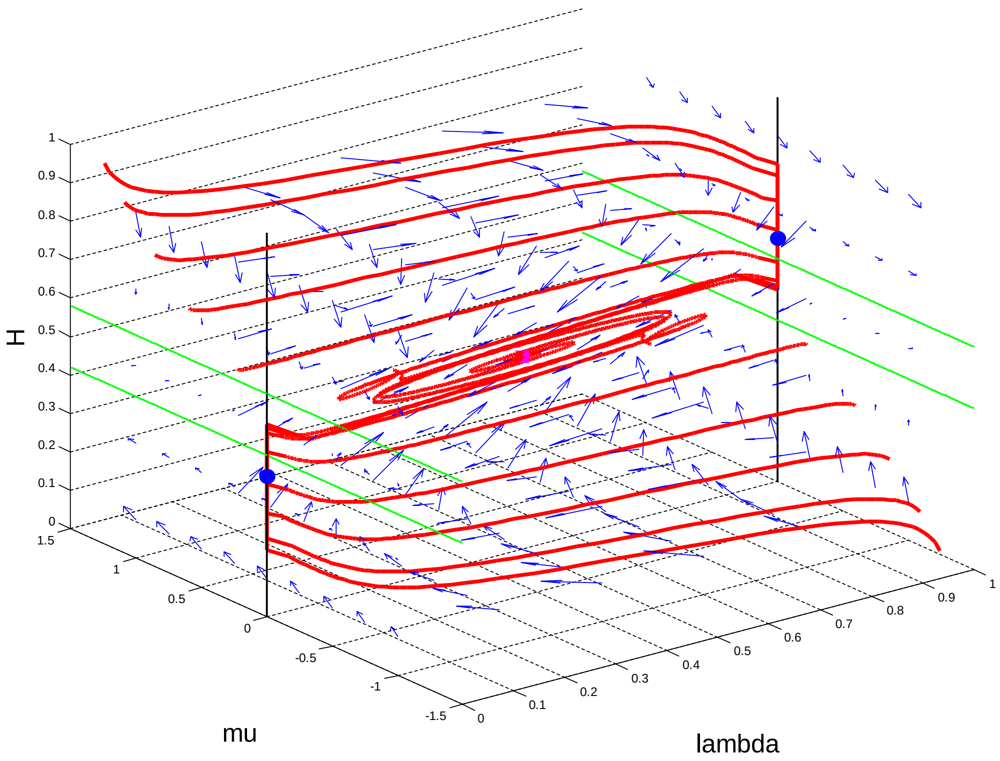


Figure 5. Global stability of the dynamic Core-Periphery model

My result adds an extra dimension to the standard KFB models since the overlap only occurs only for certain values of H. Unlike, Krugman (1991b) and Baldwin (2001) where the migration cost is the sole determinant of whether history or expectations determines the equilibrium, my model allows the housing sector to select the equilibrium.

In the Core-Periphery model with multiple equilibria and low migration costs, the housing sector locks the economy into one unique equilibrium. If expectations cause a concentration of workers in one region for a long enough time, H will eventually be large or small enough to leave $\left[\overline{B}-\varepsilon,\overline{A}+\varepsilon\right]$ and make that region the only equilibrium. The past distribution of workers for many periods determines the value of H and thus is able to select the equilibrium to which the economy converges.

4.2 Beyond KFB dynamics

Given that I have shown how H can select a unique equilibria even when KFB migration costs are low, I now dispense with KFB migration costs altogether. The KFB migration costs are ad hoc and have little economic motivation. They are usually introduced for analytical tractability and to eliminate multiple equilibrium in a forward-looking Core-Periphery model. Now that I have introduced endogenous housing, it is no longer necessary to rely on high KFB migration costs for this.

One way to escape from KFB dynamics in (22) is to simulate (26)-(31) with γ going to ∞. Figure 5 already shows the model with very low migration costs and even lower migration costs would result in faster migration relative to the housing sector adjustment. The solution paths with a very large γ more or less correspond to the ones in Figure 5, but will have straighter paths moving up and down λ.

Perhaps a better way to leave KFB dynamics is to use a game theoretic approach as in section 2. If there are no costs to migration, workers can choose which region to locate costlessly in each period as in a repeated game. The Static Core-Periphery model in section 3.1 micro-founds assumptions 1 to 5 and the Dynamic Core-Periphery model section 3.2.1 provides a basis for assumptions 6, 7 and 9. So except for the use of assumption 9 instead of 8, there is no difference between the structures of my models in section 2 and 3.

With assumptions 1 to 7 and 9, the result is identical to proposition 2. While $H\epsilon\left[\overline{B}-\varepsilon,\overline{A}+\varepsilon\right]$, the economy has multiple equilibria and the model provides no prediction of which equilibrium will occur in the economy. However if the history of worker distribution is such that there are more workers in one region for an extended period of time, the housing sector will adjust so that $H\notin\left[\overline{B}-\varepsilon,\overline{A}+\varepsilon\right]$. This causes the economy to converge to a single Nash equilibrium, at which H converges to its steady state $H(t)$ as $t \to \infty$ for the equilibrium value of λ. Assumption 9 ensures the steady state of H is outside the multiple equilibria interval so the model will be locked-in to a unique equilibrium. The economy exhibits hysteresis based on H, shaped by the history of λ.

Similar analysis can be extended to the case where migration costs are positive, but fixed. Let x be the cost to a worker of moving from one region to another. A worker in one region will only move to the other if the present value of future gains from the move is larger than x. Let $V^A \equiv \int_0^\infty e^{-\rho t}\omega^A dt$ and $V^B \equiv \int_0^\infty e^{-\rho t}\omega^B dt$ be the present value of all future payoffs from locating in region A and B respectively. All workers in region A would move to region B if $V^A - \left(V^B - x\right) < 0$ and would move from region B to A if $\left(V^A - x\right) - V^B > 0$.

Once again, I rely on the ability of H to alter the payoffs such that equilibria can be eliminated when housing is skewed in favour of one region or another. Assume that H is exogenous for now. Let $\overline{B}' < \overline{B} < \overline{A} < \overline{A}'$ be threshold values of H, where $\overline{A}$ and $\overline{B}$ are as defined in assumption 5. We now assume there exist two more intervals of H in addition to those in assumption 5 that determine the characteristics of the economy:

Assumption 10 $\overline{B}' < \overline{A}'$ *are threshold values of* H *such that:*

$V^A - \left(V^B - x\right) < 0$, $\forall\lambda$, *when* $H\epsilon[0, \overline{B}')$

$V^A - x - V^B > 0$, $\forall\lambda$, *when* $H\epsilon(\overline{A}', 1]$

Note that a larger x implies a larger $\overline{A}'$ and a smaller $\overline{B}'$. Assumption 10 can be micro-founded by the Static Core-Periphery model of Section 3.1. We have already established that H can alter the payoffs ω^A and ω^B. Assumption 10 requires the expenditure share of housing, $1 - \alpha - \beta$, be large enough, or the value of x small enough, to allow H to be able to skew the payoffs for the assumption to hold.

When $H\epsilon[0, \overline{B}')$, all housing is concentrated in region B such that $V^A - \left(V^B - x\right) < 0$ for all λ. Since all workers will move to region B, the payoffs ω^A and ω^B will take their values when λ equals zero. V^A and V^B can therefore be easily calculated with ω^A and ω^B constant over time. All workers will move to region A if $H\epsilon(\overline{A}', 1]$ by a similar argument.

When $H\epsilon[\overline{B}', \overline{B})$, there are some values of λ where all workers move from region A to B just as in the previous case, but also some values where the $V^A - \left(V^B - x\right) > 0$ so workers would not move from region A to B on the margin. In the latter case, expectations can still cause everyone to move from A to B although it is possible for workers to stay in region A indefinitely. Note that since H is still less than $\overline{B}$, no worker in region B would move to A. So once they move to region B they will not move back. Likewise, when $H\epsilon(\overline{A}, \overline{A}']$ workers may move from region B to A, but not vice versa.

Not surprisingly, when H is in the interval $\left[\overline{B}, \overline{A}\right]$, both being in region A and in region B are pure Nash equilibrium strategies. There are multiple equilibria and the model cannot predict which equilibrium will occur.

I now add the same dynamic process for $\dot{H}$ as presented in section 3.2.1 to make H endogenously determined. Since I allow for $\frac{\partial \dot{H}(t)}{\partial \lambda(t)} \neq 0$, I need to account for the instantaneous movement of H (ε from assumption 7) in my analysis. Provided that the steady states of housing, $H(t)$ as $t \to \infty$ when $\lambda = 1$ and $\lambda = 0$, are greater than $\overline{A}' + \varepsilon$ and less than $\overline{B}' - \varepsilon$ at their associated equilibrium, the economy converges to a unique equilibrium once H leaves the interval $\left[\overline{B}' - \varepsilon, \overline{A}' + \varepsilon\right]$. In such cases, all workers move to one region and the housing sector moves H towards its equilibrium steady state. Whereas when H is within the interval $[\overline{B}' - \varepsilon, \overline{B} - \varepsilon)$ or $(\overline{A} + \varepsilon, \overline{A}' + \varepsilon]$, the value of λ is able to determine whether the economy converges to a unique equilibrium or whether expectations determine the equilibrium path. When H is in $\left[\overline{B} - \varepsilon, \overline{A} + \varepsilon\right]$, expectations will drive spatial distribution.

It is possible for x to be high enough such that the values of $\overline{A}'$ and $\overline{B}'$ expand the interval $\left[\overline{B}' - \varepsilon, \overline{A}' + \varepsilon\right]$ to include the steady states of H at each equilibria. In this case, migration costs are too high to allow H to guarantee a unique equilibrium. Additionally, in the case where x is larger than $V^A - V^B$ when both λ and H equal

1 (the largest possible gain from moving), the migration costs become prohibitive. No worker will move between regions no matter the distribution of housing or population. Then only the history of λ determines the spatial allocation of economic activities.

This insight is not unlike that of the history versus expectations literature. For a small enough fixed migration cost, the distribution of housing is able to select a unique spatial equilibrium as decribed in proposition 2. However when migration costs become higher, λ becomes consequential in determining whether the H is able to influence the solution path of the economy. So "history", in the form of λ, becomes more important with higher migration costs.

5 Policy implications

The policy insight of my model is the ability of housing (or other human made locational characteristics) to determine which equilibrium path taken by the economy. Policies can take advantage of the influence of H by either directly affecting its value or adding a policy dimension to H. Encouraging physical construction such as housing subsidies, the building of infrastructure, and zoning policy to control housing stock can shift H in favour of a region. Policy can also move H away from a region by not investing enough in H to keep up with depreciation. Governments can also shift the population for some time, thus directly adjusting λ, and wait until the housing stock adjusts to lock in the new equilibrium.

Policies that directly affect the payoffs ω's can also be added to make H a vector. Local tax rates, defined property rights, law and order and other such location specify policies can skew payoffs in favour of one region over another and can eliminate equilibrium by moving the economy out of the multiple equilibria interval $\left[\overline{B}-\varepsilon, \overline{A}+\varepsilon\right]$. The wide range of possible interpretations of what H consists of allows for policy makers to be creative in their methods for guiding the spatial economy.

Indeed Becker, Heblich and Sturm (2013) provides an example of such policies. By shifting the federal government into Bonn, the relocation of government employees pinns down λ. Although rents increased, the housing sector adjusted over time. The government also built housing of its own. The population of Bonn grew much more rapidly than other control cities and the government sucessfully shifted the spatial equilibrium.

So far my model is ex-ante symmetric so there is no Pareto dominant equilibrium. Now suppose the regions differ in the shares of agricultural labourers so that there is a Pareto dominant equilibrium, but the skew in the distribution of immobile labourers is not enough as to eliminate multiple equilibria altogether. Policy makers would like to shift the agglomeration to the Pareto dominant region if it is not already there. There is thus a role for policy to affect H to bring the economy to a Pareto superior equilibrium.

It should be noted that my model does not imply economic agglomerations do not relocate or disappear without policy intervention. I also need to account for natural geography; H is just another locational characteristic in the same way as natural geography. Even when housing and infrastructure is concentrated in one region, changes technology or the exhaustion of naturally endowed resources may make another region even more attractive. In this case, natural geography can override the human made characteristics to determine spatial allocation. Furthermore, agglomeration economics may also reduce the number of agglomerations sustainable in the economy such that at least one centre of economic concentration will disappear. Richer new economic geography models such as those presented in Fujita, Krugman, and Venables (2001) show that a change in population size can determine the equilibrium number of cities. The H cannot sustain an extra city when the underlying agglomeration economics call for one less city, although it may determine which location will looses its city.

When I expand the interpretation of my model to other multiple equilibria scenarios as in section 2, I can apply my insights to assist in other policy problems.

For all the applications of the model, the relevant policy is something that can affect H to delete one equilibrium in favour of another. Take the Murphy, Shleifer and Vishny (1989) "big push" industrialization literature for example. A candidate state variable is the sector specific skills of workers who can only acquire them over time. The economy may be locked-in to a Pareto inferior equilibrium where all workers work in agriculture because they only have the skills to do so. My model would suggest that the policy maker should either temporarily subsidize working in manufacturing or directly provide training to workers in manufacturing skills. As workers move to the manufacturing industry, they slowly acquire the skills required for their new jobs. The state of industry specific skills will become skewed towards an economy where the only equilibrium is that everyone works in manufacturing. This application of my model provides an insight into how industrial policy can shift an economy from one equilibrium to another.

6 Conclusions

Existing models of dynamics of economic geography ignore the role of human made locational characteristics in determining the attractiveness of a region to workers and firms. My model shows that slowly adjusting endogenous housing play a key role in determining spatial allocation and is able to select a unique equilibrium in a forward-looking model. Spatial hysteresis occurs when the distribution of housing stock, determined by many periods of past distribution of workers, locks the economy into a single equilibrium. The resulting spatial allocation of economic activities is consistent with the stylized fact that economic agglomerations are robust to shocks to the distribution of workers and to expectations driven fluctuations.

The primary testable implication of my model is whether housing stock can indeed determine spatial allocation. Since housing stock is strongly correlated with the population size of a city or region, it is an empirical challenge to disentangle any causal relationships. One possible direction of analysis can be to look for natural experiments where there is a large shock to the housing stock. Davis and Weinstein (2002) style regressions can be conducted using wars as an exogenous shock to the housing stock. With the decline in housing as an additional independent variable, the implication of my model suggests that cities with a larger decline in housing stock would be less likely to revert to their pre-war size.

There are certainly limitations to my model that can be improved on in future work. Firstly, the housing sector is not modelled very realistically. This is necessary to simplify the problem so that I can solve the system of non-linear first order differential equations. However if I completely abandon the use of KFB dynamics, the dynamic adjustment process may be simple enough to allow the introduction of more jump variables. It would be an improvement to allow the housing sector to build according to expected future prices. Also requiring land and labour inputs for housing construction may also lead to a more realistic housing sector. Secondly, once I abandon KFB dynamics the model will have two fewer non-linear differential equations. Perhaps introducing an analytically solvable version of the Core-Periphery model such as Forslid and Ottaviano (2003) may allow for analytic results that are more precise.

Yet despite its short comings, this paper provides an explanation for an empirical stylized fact not addressed in previous models. My model is an alternative to the ones found in the history versus expectations literature. This paper is an attempt to continue the line of research that seeks to better characterize the dynamics of economic geography.

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Chapter 2: Common Tasks and the Pattern of Off-shoring

Abstract

This is a theory of task assignment in the production of final goods and across countries. By allowing for tasks to differ in their suitability of being used in the production of multiple goods, my model endogenizes the allocation of tasks in the production of goods that use them. Tasks are distinguished between those that are only used in one good and those that are used in multiple goods. The resulting equilibrium task allocation defines the pattern of off-shoring. Tasks that are used in only one good concentrate in the country with a specialization in production of that good. Tasks used in many goods are allocated across countries, with the more substitutable tasks located in the country with the larger overall output. Gains from off-shoring are derived from a better mix of allocation of tasks into goods as well as larger scale of production.

1 Introduction

The increasing fragmentation of the production process has allowed for the specialization of a single component of production to take on a global scale. This is due to advances in transportation and communication technology. Consider the tire manufacturer Bridgestone that makes tires for vehicles of all shapes and sizes manufactured across the world. Or examine the computer chip maker Qualcomm that makes chips for smart phones and tablet computers, used by a range of companies including Samsung, Sony, Toshiba, LG, HTC, and Lenovo. These are a few examples of how firms and countries have taken up to specializing not only in the production of particular goods, but are also focusing their efforts on a narrow range of components used in a wide range of products.

Since the advent of mass production, interchangeable parts have played a major role in modern manufacturing. The use of a common part or task allow producers to take advantage of increasing economies of scale. Yet not all tasks are suitable for multiple goods. Some tasks are better performed as specific tasks for one particular good. This is a decision made by producers as to which tasks are *common* and which are *specific* in the production process of each good. The selection of tasks into the production of final goods across countries influences the location of tasks and the production process of goods.

I isolate this feature of common tasks used in several goods and look at its implications on the pattern of off-shoring. With the existence of off-shoring cost, tasks used for a single good will tend to co-locate to minimize on off-shoring cost. Therefore each specific task will locate in the country with the largest output of the set of goods that uses it. Common tasks are allocated amongst counties, with more locating near the largest output of goods using them. Hence countries specialize in final goods also specializes in specific tasks, and countries with larger output will attract more common tasks. This differential motive for the location of common and specific tasks helps determine the pattern of off-shoring.

This paper's approach to the trading of tasks is markedly different from that of the existing literature. To my knowledge this is the first paper to allow for the endogenous determination how a product is produced under off-shoring. It provides a description of the pattern of off-shoring that is more nuanced than models that rely on exogenous heterogeneous off-shoring cost, and factor or technological heterogeneity among countries.

The contributions of this paper are twofold. First, I identify a new determinant of the pattern of off-shoring based on the equilibrium allocation of common and specific tasks. This feature of my model give a testable prediction of the international allocation of tasks. The resulting pattern of off-shoring from my model is a more nuanced description of the global supply chain that emphasizes the effect off-shoring has on how goods are produced. Furthermore, by providing an explanation of this process I can identify a source of gains from off-shoring through a better balance of common and specific tasks. I am not aware of other efforts in the literature that identify this dimension of gains from off-shoring.

My second contribution is a methodological innovation in the organizational trade literature. My model provides an example where the technology of production has a level of endogeneity caused by the unbundling of production. The set of tasks used in the production of each good is endogenous and I model the factors that determine which tasks enter into the production of each product. The trade-off between increased scale of common tasks and better suitability for goods of a specific tasks means that it makes a difference which tasks are used, and not all tasks are used in the final equilibrium. This flexibility of how goods are produced in the global supply chain is a increasing reality as the unbundling of production becomes more common, and my efforts are an attempt to capture this.

1.1 Methodology

The distinctive feature of my approach is the endogenous determination of which tasks are used to produce each final good. Following Grossman and Rossi-Hansberg (2012), each final good is produced by a continuum of tasks. These tasks can be performed in the country of the final good producers or off-shored. There are increasing returns at the task level by location, so that the same tasks benefit from being produced in one location.

Instead of requiring the same set of tasks for the production of every good, the key innovation of this paper is that not all goods use the same set of tasks. There are sets of mutually exclusive tasks, each best suited for the production of one type of good. Although each type of good has a set of tasks best suited for their production, they can choose to substitute tasks with those suited for other goods.

I first sketch out a model of task matching where task producers compete to serve final good firms. Tasks vary in their suitability for substitution into goods they are not originally designed for through a heterogeneous adaptation cost. Tasks that are easily substitutable among goods are used as common tasks in the production of several goods. Whereas tasks that require hefty adaptation costs when being used for another good remain as specific tasks used for only one good.

The equilibrium actions of task producers distinguishes which tasks will be common and which will be specific . Task producers compete through price, and the lowest priced producer sells to the entire market. This enables them to take full advantage of economies of scale. The resulting allocation of tasks in equilibrium is the result of competing task producers either selling to firms of only one good or selling to firms of several goods.

In the next step, I locate the production of each task given the outputs of goods in each countries. The scale of each market matters for location of tasks. Task producers face a trade-off when moving from one country to another. It distances itself from final goods firms in the original country, incurring more off-shoring costs, while it makes it easier to sell its task to firms in the destination country, thus saving in off-shoring costs. Factor price variations compound the cost differences of the two countries. Price competition again drives producers of each task to locate where they can provide their tasks at the lowest costs to the market they choose to serve. Identifying the location of each tasks allows me to characterize the off-shoring equilibrium of tasks.

I finally examine the efficiency of task allocation equilibrium and explore a few comparative statics.

1.2 Findings

In this paper, I show that the existence of common tasks helps determine where each part of the production process will be located. Producers of specific tasks locate in countries that specialize in their good. Common tasks producers are split amongst countries, but have a tendency to locate in the country with the larger overall output. This allocation of tasks also determines which good producers do the most off-shoring. With countries specializing in specific tasks, producers of goods not of the country's specialization must off-shore all their specific tasks as well as any common tasks not available in their country.

My model shows how heterogeneous adaptation costs can determine the mix of common and specific tasks that are produced. When tasks vary by adaptation costs, this mix of common and specific tasks depends on the size of the incremental gain from larger economies of scale of common tasks versus higher adaptation costs. Poorly substitutable tasks with high adaptation costs are sold to firms of one good (specific tasks) and easily substitutable tasks to firms of several goods (common tasks). So the distinction between common and specific tasks shapes the pattern of off-shoring.

Furthermore, I newly identify a benefit of off-shoring derived from better mix of common and specific tasks. Previous attempts to model off-shoring in the literature do not take into account the change in availability of tasks when tasks can be traded. The access to more suitable specific task is a potential gain from off-shoring. For example, firms making a good that is produced at a small scale in their own country can off-shore tasks to another country where more of that good's tasks are produced. Also, goods firms can also gain access to larger scale production in the case where off-shoring allows for more common tasks.

I also find the off-shoring equilibrium to be inefficient. This is due to the existence of externalities in the production of tasks. By producing specific tasks, task producers remove the economies of scale accrued by common task producers. The disregard of such externalities results in an inefficient task allocation equilibrium.

This paper proceeds with a overview of the literature in Section 2. A description of my model of task allocation is in Section 3. Here I also apply my model for an economy in autarky and I develop the idea of common and specific tasks. I then look at the model in a two country setting in Section 4. The task allocation equilibrium is different for complete and incomplete goods specialization and I look at each case in turn. In Section 5, I look at two comparative statics. I make my concluding remarks in Section 6.

2 Relationship to Literature

I am not aware of other attempts in the literature to endogenize the choice of tasks involved in the production process to explain the pattern of off-shoring. There is a growing literature focusing on what happens inside the global supply chain. As surveyed by Antràs and Rossi-Hansberg (2009), this line of research seeks to answer the question of "how to produce" in an international setting.

The literature points to several main determinations of the organization of international production. Each group of papers differ in how they organize the supply chain across different countries. Heterogeneity comes in parts of the production process that interact with the differences across countries.

The first group of papers look at how Hechscher-Ohlin type differences in factor endowments across countries motivates some tasks to be produced in one country and not another. Since countries are endowed with different varieties of factors, the productive technology is suboptimal when using factors from only one country. This creates a motivation for off-shoring. In an early attempt to explain the pattern of off-shoring, Dixit and Grossman (1982) use a model with a vertical production process where each stage of production differs in factor intensities. They show that the determination of the location of production of each stage is endogenously determined by the comparative advantage of each country.

Heterogeneity of factor endowments also comes in the form of differing skills of the labour force. Grossman and Maggi (2000) take two industries with technologies that differ in how they combine workers of different skills. They reach the conclusion that the country with a larger variance in skill of workers specializes in the industry that works best with workers of opposing types. Expanding on the knowledge economy of Garciano (2000), Antràs et al. (2006, 2008) use heterogeneously skilled workers that match into cross-country teams with one manager and several workers. Under a technology that leads to positive assortative matching, allowing teams to be formed internationally results in fewer managers in the lower skilled country. Other examples using Hechscher-Ohlin forces to determine the pattern of off-shoring include Jones and Kierzkowski (1990, 2001), Egger (2002), Nocke and Yeaple (2008), and Baldwin and Robert-Nicoud (2007, 2010).

The next group of papers appeal to Ricardian difference in technologies across countries to explain the pattern of off-shoring. Yi (2003) develops a model with production done sequentially in three stages, in which only the first two can be off-shored. With technological differences for each stage of production across each country, he looks at the patterns of off-shoring as tariffs fall. Costinot, Vogel and Wang (2011) consider a continuum of intermediate goods used sequentially in the production of the final good. Countries differ in their productivity and as a result countries with higher productivity specialize in later stages of the supply chain. Deardoff (2001) and Rodriguez-Clare (2010) also based their models of off-shoring in a Ricardian world.

A third line of research in the determining the pattern of off-shoring appeals to heterogeneous off-shoring costs to sort out which tasks are off-shored. Methodologically speaking, these papers are closest to my approach. The trade in tasks literature such as Grossman and Rossi-Hansberg (2008) model production as a continuum of tasks all with different levels of off-shoring costs. Tasks with low off-shoring costs are off-shored and they use their model to look at off-shoring's effects on factor prices. In a later work, Grossman and Rossi-Hansberg (2012) use a heterogeneous off-shoring cost model with increasing returns to scale to determine the pattern of off-shoring. They show that countries specialize in easily off-shorable tasks as determined by their off-shoring costs and tasks with high off-shoring costs are not traded. They relate the resulting pattern of task trade to wages and levels of output.

Then finally in the off-shoring literature, there are firm based theories of international decisions of multinational enterprises and how they guide the allocation of tasks. These models include McLaren (2000) and Grossman and Helpman (2002) who use a transactions-cost approach to motivate vertical integration of final good producers and their suppliers. Antràs and Helpman (2004, 2008) look at a world with incomplete contracts where suppliers and final goods producers engage in relationship specific investments to determine which industries will outsource production. Other works in this line include Grossman and Hart (1986) and Antràs (2003).

Though not a model of off-shoring, the productive technology of my model mirrors that of Hausmann and Hidalgo (2011). They use a Ricardian approach to argue that countries have a range of ”capabilities” that determine the mix of products they produce. Each good requires a different mix of capabilities and countries have an incentive to increase their range of capabilities to produce a wider range of products. Some capabilities are unique to one good while others are used in several goods.

This is the first paper to consider the components of the supply chain as endogenous. Many of the papers outlined above have a fixed method of production in which a predetermined set of tasks must be performed. Some models such as Antràs et al. (2006, 2008) combine heterogeneous factors in their production process in which the matching of factors into production is endogenous. However all factors will be employed and can only be employed once. In contrast, my model allows the tasks involved in production to be only a subset of the universe of task. Which tasks are used in production is endogenously determined and some tasks can be used for many different goods. The heterogeneity in tasks in my model arises in how well tasks are suited in being used to produce several goods, and this drives the equilibrium allocation of tasks among countries.

3 A Model of Task Allocation

I model the international allocation of tasks by allowing for common tasks in the production of different goods, and look at the consequences on the pattern of off-shoring. This model his three key features i) tasks can be off-shored to other countries, ii) production has the ability to substitute tasks from one good to another, incurring a heterogeneous adaptation cost, iii) and increasing returns to scale in task production. These features capture how tasks from each country will be combined in the production of goods.

Let there be only two types of goods, C and T (cars and trucks, or computers and televisions, chairs and tables, or any two different goods). Manufacturing of these final goods happens in two stages. First, task producers make the intermediate tasks. These tasks are then sold to the final goods firms to be assembled into C or T goods.

Production is done by combining many tasks to produce a good. Tasks are divided into two mutually exclusive sets: a continuum of C tasks and a continuum of T tasks. However, each C task has a T counterpart and vice versa, so that there is a one-to-one matching from C tasks to T tasks. The combination of C and T tasks used in the production of each good determines the costs of production.

A unit measure of C (T) tasks is performed exactly once to produce a C (T) good. In addition, goods can substitute a task from the other kind of good.

Tasks vary in how suited they are for substituting into the good they are not originally designed for. Some tasks may be practically identical for two different types of goods, while some tasks are extremely good-specific such that they are of no use for the production of any other type of good. A C (T) good firm using a T (C) task i incurs an extra adaptation cost $\gamma(i) \geq 1$. Index tasks $i\epsilon[0,1]$ such that $\gamma'(i) > 0$. $\gamma(i)$ measures the good-specificity of task i. The different $\gamma(i)$ captures the heterogeneous substitutability of tasks among the production of different types of goods. For example, headlights may have a low $\gamma(i)$ as they can easily be adopted for both cars and trucks, while the engine would have a high $\gamma(i)$.

I refer to tasks used in the production of both C and T goods as *common tasks*. These can be C tasks used in both goods or T tasks used in both goods. For the C tasks used only for C goods and T tasks only for T goods, I refer to as *specific tasks*. Which of the continuum of tasks are produced as common tasks and as specific tasks is endogenously determined.

External economies of scale provide a motivation for substituting to a less suitable tasks. The cost of producing each task falls as the total scale of production of that task across all task producers increases. The labour cost of performing task i for any producer is $A(Y_i)$, where Y_i is the total number of times task i is performed. $A(.)$ is continuously differentiable, decreasing and convex. If several producers of different final goods use the exact same task, increasing returns to scale results in lower cost of production for that task. The external economies of scale at the task level follow closely in line with Grossman and Rossi-Hansberg (2010, 2012) in representing the positive spillovers of production.

Task producers compete in the market for tasks to sell to good firms. Following Grossman and Rossi-Hansberg (2012), I assume task producers choose their location and engage in Bertrand competition. Task producers compete by price, and only the producers selling at the lowest price serve the entire market of the task. With identical technology across task producers, this leads to pricing at expected cost for tasks. The market structure of task production endogenously determines the scale of task production and therefore the price of tasks in each country to be sold to goods firms.

Goods firms buy tasks from task producers. Each goods firm has to decide the mix of common tasks and specific tasks that will enter their final product based on the price of tasks and the adaptation cost.

Task and good production takes place over two countries, East and West. The relative wage among the two countries determines the cost of employing workers to produce tasks. Workers are paid a wage of 1 in the East and w in the West. For the purpose of my analysis, I take w as exogenously determined by clearing of the international labour markets. I will not model this process explicitly, and will instead analyze the range of possible w's and determine the implications on the task allocation equilibrium.

3.1 Autarky Equilibrium

I first look at an Autarky economy in order to gain an insight in the endogenous determination how tasks are produced for each good. *Common tasks* are tasks with low enough adaptation costs such that the benefits from larger scale of production for both goods outweigh the adaptation costs that this incurs. *Specific tasks* have such high adaptation costs that they are not worth using to substitute into the production of other goods.

Suppose C goods firms produce an output of Y_C and T goods firms an output of Y_T. The wage rate clears the labour market and is normalize to 1. Tasks can be produced as a common task, where a single type of task (C or T) is used in the production of both C and T goods. Otherwise tasks are produced as a specific task, where a C task is used for C goods and T task for T goods.

I now determine whether a task is common or specific. Consider some task, task i, and how it is produced. Let task i be initially a common task of the C variety. This C task is made to be sold to the entire market $Y_C + Y_T$. Bertrand (price) competition leads to pricing at expected cost. T goods firms will incorporate in their price of the task the adaptation cost $\gamma(i)$. The C task producer thus quotes p_{Ci} to C firms and p_{Ti} T firms:

$$p_{Ci} = A(Y_C + Y_T) \qquad p_{Ti} = A(Y_C + Y_T)\gamma(i)$$

A T task producer could announce a price a bit below p_{Ti} and attract sales of Y_T, and achieve labour productivity of $A(Y_T)$. This strategy would be profitable if its per-unit cost, $A(Y_T)$, is less than the price quoted by the common task producer. Therefore for task i to remain a common task, no T task producer can profitably undercut the current common task provider, which requires

$$A(Y_C + Y_T)\gamma(i) < A(Y_T)$$

I generalize this result to other tasks in the continuum. Define $\gamma(I_T)$ as the level of adaptation costs that equates the above inequality while keeping I_T between zero and one:

$$I_T \equiv min\left\{max\left\{0, \gamma^{-1}\left(\frac{A(Y_T)}{A(Y_C + Y_T)}\right)\right\}, 1\right\} \tag{1}$$

I_T is the marginal task to be produced as a common C task. So $i < I_T$ is a necessary condition for task i to be performed as a common C task. Otherwise, T task producers would enter the market and produce T tasks for T firms as a specific task.

Now consider the case where task i is a T task, though still initially a common task that is produced for both C and T goods. Now that the common task is of the T variety, the C goods firms need to pay the adaptation costs $\gamma(i)$. This implies now that $p_{Ci} = A(Y_C + Y_T)\gamma(i)$ and $p_{Ti} = A(Y_C + Y_T)$. A C task producer could announce a price a bit below p_{Ci} to capture the sales of Y_C. This is not profitable when

$$A(Y_C + Y_T)\gamma(i) < A(Y_C)$$

So a necessary condition for task i to be performed as a common task, this time a T task, is $i < I_C$, where I^C is defined by

$$I_C \equiv min\left\{max\left\{0, \gamma^{-1}\left(\frac{A(Y_C)}{A(Y_C + Y_T)}\right)\right\}, 1\right\} \tag{2}$$

Therefore, in order for task i to be a common task (as either C or T), a necessary condition is for $i < max\{I_C, I_T\}$.

To identify the sufficient condition for a common task, I now consider the case where task i is performed as a specific task. This means C task producers serve only C goods firms with output Y_C and price at expected cost of $A_C(Y_C)$. T task producers sell to Y_T and price at $A_T(Y_T)$. A C task producer can capture the entire market of $Y_C + Y_T$ by pricing a little below the price of T tasks. This is only profitable if it's per-unit cost, $A(Y_C + Y_T)\gamma(i)$, is less than $A_T(Y_T)$. Therefore, the level of adaptation cost that makes task producers indifferent between producing specific tasks or as C common tasks is given by (1). Likewise (2) gives the level of adaptation cost at which task producers are indifferent between producing specific tasks or to produce as a common T task. Hence $i > max\{I_C, I_T\}$ is a necessary condition for task i to be performed as a specific task.

Combining the two parts, I can now determine whether tasks are common or specific. A necessary condition for task i to be a common task is $i < max\{I_C, I_T\}$ and a necessary condition for task i to be a specific task is $i > max\{I_C, I_T\}$. Since I have examined all possible switches from common to specific and vice versa, I can conclude that the necessary and sufficient condition for task i to be a common task is:

$$i < max\{I_C, I_T\} \tag{3}$$

Task $i = max\{I_C, I_T\}$ is the marginal task that is performed as a common task, so it is a measure of the share of common and specific tasks in the economy. By examination of (1) and (2), the relative size of I_T and I_C depends on the relative size of Y_C and Y_T. For example, if $Y_T < Y_C$, then the marginal common tasks is just I_T.

Now that I have determined which is the marginal common task, I will examine whether the common tasks are of C or T variety. If a common task i is initially a C task, a T task producer can undercut it's prices to both C and T goods firms and capture the entire market. That is, the T tasks must be priced below $p_{Ci} = A(Y_C+Y_T)$

and $p_{Ti} = A(Y_C + Y_T)\gamma(i)$ while incurring a cost of $A(Y_C + Y_T)$ for each task produced. This is not profitable for the T task producer if:

$$[A(Y_C + Y_T) - A(Y_C + Y_T)\gamma(i)]Y_C$$
$$+[A(Y_C + Y_T)\gamma(i) - A(Y_C + Y_T)]Y_T < 0$$

The left hand side of the equality is the profits that a T task firm entering to replace the incumbent common C task producer can make. The first square bracketed term represents the loss of a T task producer serving the C goods firms at below cost. The price of the T task has to match the incumbent at p_{Ci}, while the cost to sell to C firms incurs the additional adaptation cost $\gamma(i)$ which is internalized by the task firm. The second square bracketed term is the gain in profits from capturing the T goods firms.

This simplifies to:

$$Y_C > Y_T$$

Hence the common task will remain a C task if the output of C is greater than that of T. Repeating same exercise for common T tasks will show that $Y_T > Y_C$ is required for the common task to be a T task. Therefore, the common task will be the same as the good that has the larger output.

The task allocation in equilibrium under autarky is depicted in Figure 1. Tasks with low adaptation costs are produced as common task of either C or T type while tasks with high adaptation costs are performed as specific tasks for both goods. Figure 1 a) shows the equilibrium when more C goods are produced than T goods. In this case, I_T is the marginal common task and the common tasks are C tasks. In Figure 1b), there are more T produced than C. Tasks with high adaptation costs are still produced specifically for their respective goods, but common tasks are now T tasks. I_C is the marginal common task when the output of T is larger than that of C.

There are two observations that can be made from this equilibrium. The first is that the split between common and specific tasks varies with the relative size of C

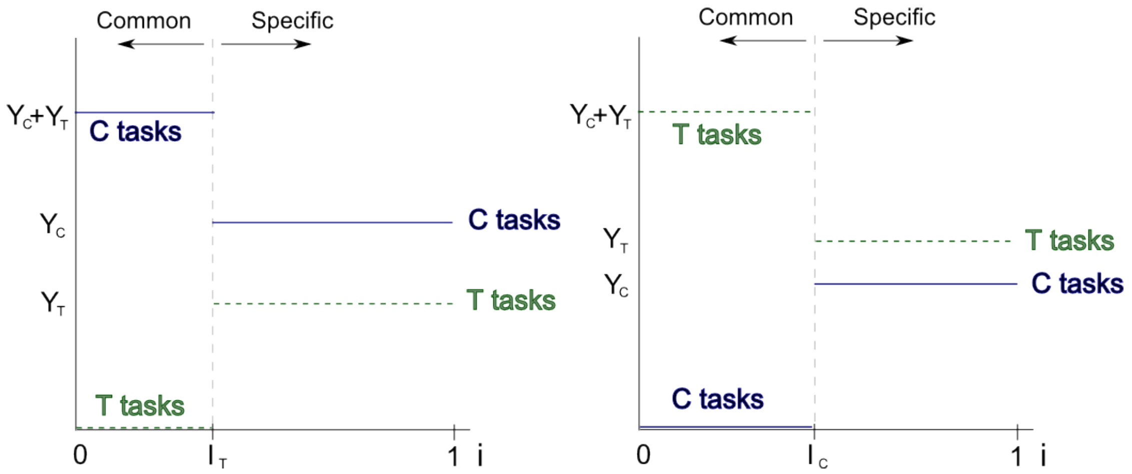

Figure 1: Task allocation equilibrium under autarky with a) $Y_T < Y_C$ b) $Y_T > Y_C$

and T output. When the economy is evenly split between C and T goods production, $Y_T = Y_C$, the share of common tasks is at its smallest since $I_T = I_C$. This result is fairly intuitive as economies of scale are the same for both types of tasks so there is less incentive to substitute with tasks from another good. And obviously, the share of common tasks is at its largest when only one good is produced. The scale of the good not produced is so small that it is better to pay the adaptation cost than to have specific tasks. This is summarized in the following proposition:

Proposition 1. *The share of common tasks to specific tasks increases with the difference in output levels of the two goods.*

A second observation is to note that the type of task used as common tasks is endogenous to the relative size of outputs of each good. Since common tasks are tasks used in the production of both C and T goods, it is quite intuitive that they are tasks suited to the good produced at larger quantity. If fewer T goods are produced than C goods, T firms will use C tasks and not the other way around.

4 Off-shoring in the Two Country Equilibrium

I now allow for off-shoring in a two country setting to find out the implications of common tasks on the international allocation of production.

The choice to off-shore imposes a trade-off between economies of scale and off-shoring costs. While concentrating tasks in one country allows the full extent of economies of scale, selling the tasks to firms in other country incurs off-shoring costs. On the other hand, having tasks dispersed among the two countries would not incur off-shoring costs, but does not allow for the same level of economies of scale.

Task producers now can decide to locate in East or West and sell to C and T goods firms in either country. Firms off-shoring a task to another country requires an additional $\tau > 1$ units of labour input to pay for the cost of off-shoring. To focus my analysis to the effect of common goods on off-shoring, I assume a low value of τ such that the production of a particular task is ”concentrated” in one location as described by Grossman and Rossi-Hansberg (2012). So in my model I take $\tau = 1 + \epsilon$, where ϵ is arbitrarily small.

From the perspective of task producers, levels of output in each country and of each product are exogenous, and for the purpose of my analysis I will treat them as such. Eastern firms produce Y_C^E and Y_T^E of C and T goods respectively. Firms in the West produce Y_C^W and Y_T^W. World output is given by $Y^\Omega \equiv Y_C^E + Y_T^E + Y_C^W + Y_T^W$.

I set Eastern wages as the numaraire so that w is the relative wage of the West to the East. I also take the relative wage w as exogenous. This w is determined by the labour market clearing in each country and is outside the scope of my analysis of only two goods. While there is value in taking a general equilibrium approach, I maintain my model as a partial equilibrium model to highlight the forces that shape the pattern of off-shoring.

As tasks only vary in their adaptation costs $\gamma(i)$, which has no direct locational consequences, the location of production of a task is determined by the size output in each country and the relative wage.

4.1 A Simple Model with One Good per Country

Before solving for the task allocation equilibrium more generally, I first illustrate the mechanisms at work by solving a simplified model. Here I derive the equilibrium task allocation in a world where each country produces only one good.

Let Y_C^E and Y_T^W both be positive and $Y_C^E > Y_T^W$. As only one good is produced in each country, $Y_T^E = Y_C^W = 0$. That is, East has complete specialization in C goods and West in T goods, and East has more output than the West. Furthermore, I will assume $w = 1$ so that relative wages are not a determinant of the task allocation equilibrium.

4.1.1 Common C Task

First, I determine whether the common task is C or T in equilibrium. Consider some task i initially produced in the East as a C task, and sold to C goods firms in the East and T goods firms in the West. This common task prices at expected cost in Bertrand competition so that the price of C and T tasks are respectively $p_{Ci}^E = A(Y_C^E + Y_T^E)$ and $p_{T\,i}^W = \tau A(Y_C^E + Y_T^E)\gamma(i)$.

A T task producer can enter as a common task only if it prices below the C task for both goods producer. This is not profitable if:

$$[A(Y_C^E + Y_T^E) - A(Y_C^E + Y_T^E)\gamma(i)]Y_C^E + [\tau A(Y_C^E + Y_T^E)\gamma(i) - \tau A(Y_C^E + Y_T^E)]Y_T^W < 0$$

$$\Leftrightarrow Y_C^E > \tau Y_T^W$$

That is, the common task cannot be a T task in the East if $Y_C^E > \tau Y_T^W$. A similar exercise shows that a common task in the West cannot be a T task if $\tau Y_C^E > Y_T^W$.

As $\tau = 1 + \epsilon$ is close to one, $Y_C^E > Y_T^W$ implies that the common tasks are C tasks in equilibrium. With more C goods being produced than T goods, it is intuitive that the common task is a C task.

4.1.2 Marginal Common Task I

Second, I determine the marginal common task. With $w = 1$, there is no reason for a T specific task producer to locate in the East since all of T goods are produced in the West. So I only need to focus on the deviations of T specific tasks located in the West.

When there is a C common task produced in the East, it prices at expected cost $p_{T\ i}^{W} = \tau A(Y_C^E + Y_T^W)\gamma(i)$ for the Western T goods producers. It is not profitable for a T task producer to locate in the West and only sell to T goods firms if:

$$\tau A(Y_C^E + Y_T^E)\gamma(i) < A(Y_T^W)$$

And when the common C task is made in the West, T cannot profitably enter when:

$$A(Y_C^E + Y_T^E)\gamma(i) < A(Y_T^W)$$

Defining $\gamma(I)$ as the adaptation cost of a task that equates the above inequalities (and with $\tau = 1 + \epsilon$) gives $I \equiv min\left\{max\left\{0, \gamma^{-1}\left[\frac{A(Y_T^W)}{A(Y_C^E+Y_T^E)}\right]\right\}, 1\right\}$.

The marginal common task is given by I such that tasks $i < I$ are common and tasks $i > I$ are specific. Tasks that are specific tasks have too high adaptation costs to gain from the economies of scale of a common task. Common tasks have low enough adaptation costs so as to take advantage of production at a larger scale for both C and T goods.

4.1.3 The Division of Common Tasks

Third, I determine the location of common tasks. A common C task in the East will remain if it is not profitable for a C task producer to enter in the West and produce the common task instead. A Western C task producer cannot undercut the existing

C producer in the East when:

$$[A(Y_C^E + Y_T^W) - w\tau A(Y_C^E + Y_T^W)]Y_C^E + [\tau A(Y_C^E + Y_T^W) - wA(Y_C^E + Y_T^W)]\gamma(i)Y_T^W < 0$$

$$\Leftrightarrow Y_C^E > \gamma(i)Y_T^W$$

Defining J as the task with adaptation costs that equates the above inequality gives $J \equiv \gamma^{-1}\left(\frac{Y_C^E}{Y_T^W}\right)$. Among common tasks, J is the marginal task produced in the East. Tasks $i < J$ are common tasks in the East and $i > J$ are produced in the West.

The reason why common tasks are split between two countries, despite relative wages equal to one, is due to the assumption of multiplicative off-shoring cost τ with the adaptation cost $\gamma(i)$. When a common task has relatively low adaptation costs, it locates near the bigger market, hence in the East. However, common tasks with higher adaptation costs also have higher off-shoring costs for T goods producers and not for C goods producers (since τ is multiplied with $\gamma(i)$ and the common tasks are C tasks). Therefore, common task with relatively high adaptation cost avoid the higher off-shoring costs by locating near the T goods producers in the West.

4.1.4 Specific C Tasks in the East and Specific T Tasks in the West

Forth, I determine the location of specific tasks. It is intuitive for a specific C task to be produced in the East and T tasks in the West since there are no C goods made in the West and no T goods in the East.

This intuition can easily be verified. When a specific C task i is produced in the East, C producers in the East pay $p_{Ci}^E = A(Y_C^E)$. For the same task, another C firm cannot produce in the West so long as $A(Y_C^E) - \tau w A(Y_C^E) < 0$. This inequality holds even with $w = 1$ as $\tau > 1$. Therefore specific tasks made as C tasks are located in the East in equilibrium. Similarly, specific T tasks can be shown to locate in the West.

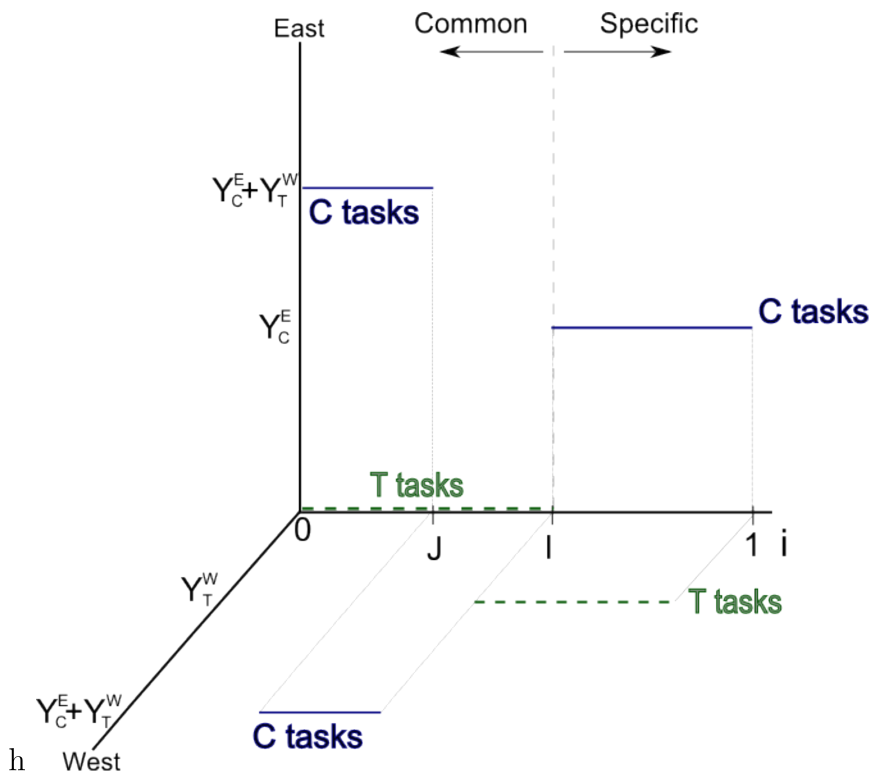

Figure 2: Task allocation equilibrium in the two country model with one good per country

4.1.5 Task Allocation Equilibrium

Now I have fully described the task allocation equilibrium for the simplified model with one good per country. Common tasks are produced in both countries while specific tasks production is located in the country that makes the tasks' goods. This is characterized by the following proposition:

Proposition 2. *Under complete specialization of goods production amongst countries, goods producers off-shore common tasks but procure specific tasks locally.*

This result is depicted in Figure 2. Tasks $i > I$, ones with relatively higher adaptation costs are produced in the same country as the goods production at the same amounts as the goods outputs. These tasks are not off-shored. Tasks $J < i < I$ are common task, produced in the West. They are produced as C tasks in the West for both C and T goods firms worldwide. Tasks $i < J$ are tasks that are common tasks produced in the East.

This result illustrates the importance of common tasks in the off-shoring equilibrium. Even without heterogeneous off-shoring costs as in Grossman and Rossi-Hansberg (2012), off-shoring of tasks is determined by how many goods the task is used for. Common tasks can take advantage of the full economies of scale of multiple goods and locate in either country, while specific tasks locate in the proximity of where their good is produced.

4.2 The General Two-Country Model

I now allow for both countries to produce both C and T goods. However, for concreteness I assume incomplete specialization of goods such that $Y_C^E > Y_C^W$ and $Y_T^E < Y_T^W$. This means that East specializes in producing C goods while the West specializes in T goods. The exact same analysis can be applied to the opposite specialization pattern to reach equivalent results.

I also assume $Y_C^E + Y_C^W > Y_T^E + Y_T^W$, that is more C goods are produced than T goods globally. This allows for defining the type of common tasks.

4.2.1 Common Tasks

I first establish whether the common task is C or T variety in the Bertrand competitive task equilibrium.

As in the previous section, the common tasks in the two country equilibrium is the same as the good that is produced in larger quantities. Intuitively, when the scale of C goods production is larger than that of T goods, choosing to perform common tasks as C task will impose less adaptation costs overall since there are relatively fewer T goods. If a common task is produced as a T task, a C task producer can profitably enter the market and undercut the incumbent common task producer to capture the entire market.

This is outlined in the following lemma:

Lemma 1. *Given $Y_C^E + Y_C^W > Y_T^E + Y_T^W$, common tasks are performed as C tasks.*

Proof. For some task i, let it initially be produced as a C task in the East and is sold to both C and T goods producers worldwide. Pricing at expected cost in Bertrand competition, task i's producer charges Eastern C goods firms $p^E_{Ci} = A(Y^\Omega)$ and Western C goods firms $p^W_{C\,i} = \tau A(Y^\Omega)$. It also charges T goods firms $p^E_{Ti} = A(Y^\Omega)\gamma(i)$ and $p^W_{T\,i} = \tau A(Y^\Omega)\gamma(i)$ for T goods firms in the East and West respectively.

A T task producer in the East can enter the market by pricing slightly below the C task prices and sell to the entire market of Y^Ω. This is not profitably when:

$$[A(Y^\Omega) - A(Y^\Omega)\gamma(i)]Y^E_C + [\tau A(Y^\Omega) - \tau A(Y^\Omega)\gamma(i)]Y^W_C$$
$$+[A(Y^\Omega)\gamma(i) - A(Y^\Omega)]Y^E_T + [\tau A(Y^\Omega)\gamma(i) - \tau A(Y^\Omega)]Y^W_T < 0$$

Or simply

$$Y^E_C + \tau Y^W_C > Y^E_T + \tau Y^W_T$$

With $\tau = 1+\epsilon$ and $Y^E_C + Y^W_C > Y^E_T + Y^W_T$, this means it is never profitable for T task producers in the East enter this market. A similar argument will show that when a task is a C task produced in the West, a T task producer will not find it profitable to enter.

Therefore, whether located in the East or in the West, common tasks will be performed as C tasks when output of C is larger than that of T. □

Lemma 1 extends to the two country setting the result from section 3.1 that the common task in equilibrium is the same type as the good that is produced more. By assuming more C goods are produced than T goods, I have thus restricted common tasks to be C tasks. This simplifies my analysis in the following sections.

4.2.2 The Marginal Common Task

Next, I identify the equilibrium split of common versus specific tasks given Y^E_C, Y^E_T, Y^W_C, and Y^W_T. I do this by looking at the interaction of competing task producers.

Consider some task i, initially produced as a common C task in the East. This means a C task producer sells to the entire world market, Y^{Ω}.

A prospective T task producer has to decide which T market to capture and where it locates its production. Recall that the off-shoring cost $\tau = 1+\epsilon$ means that production of any task will be "concentrated" in one location. This means T task producer will not produce in both East and West at the same time. It also rules out the possibility of it being produced for goods firms in only one country, since selling to T firms in both countries allow for greater economies of scale at minimal off-shoring costs.

There are therefore only two possible ways for a T task firm to enter the market. The T task producer may i) locate in the East or ii) in the West and sell to T goods firms in both East and West.

I now find the necessary conditions for task i to be a common task by looking at when T task producers can enter profitably. The incumbent C producer in the East sets prices $p^E_{T\,i} = A(Y^{\Omega})\gamma(i)$ and $p^W_{T\,i} = \tau A(Y^{\Omega})\gamma(i)$ for Eastern and Western T firms respectively. A T task producer can undercut these prices and capture the market $Y^E_T + Y^W_T$ of the world's T firms. This is not profitable for a T task producer to enter the market and produce in East when:

$$\left[A(Y^{\Omega})\gamma(i) - A(Y^E_T + Y^W_T)\right] Y^E_T + \left[\tau A(Y^{\Omega})\gamma(i) - \tau A(Y^E_T + Y^W_T)\right] Y^W_T < 0$$

The left hand side of the above inequality is the profits the T task producer can earn from entering and locating in the East.

Likewise, t is not profitable to enter and produce in the West when the profits of doing so are negative:

$$\left[A(Y^{\Omega})\gamma(i) - w\tau A(Y^E_T + Y^W_T)\right] Y^E_T + \left[\tau A(Y^{\Omega})\gamma(i) - wA(Y^E_T + Y^W_T)\right] Y^W_T < 0$$

I apply the two above inequalities for the entire continuum of tasks $i\epsilon[0,1]$ by identifying the threshold tasks. The $\gamma(i)$ that equates the first above inequality gives the threshold adaptation cost $\gamma(I)$, where I is:

$$I \equiv min\left\{max\left\{0, \gamma^{-1}\left[\frac{A(Y_T^E + Y_T^W)}{A(Y^\Omega)}\right]\right\}, 1\right\} \tag{4}$$

and $\gamma(I^E)$ is the equivalent for the second inequality:

$$I^E \equiv min\left\{max\left\{0, \gamma^{-1}\left[\frac{wA(Y_T^E + Y_T^W)}{A(Y^\Omega)}\left(\frac{\tau Y_T^E + Y_T^W}{Y_T^E + \tau Y_T^W}\right)\right]\right\}, 1\right\} \tag{5}$$

Therefore, necessary conditions for a common task to be produced in the East are $i < I$ and $i < I^E$.

The two necessary conditions $i < I$ and $i < I^E$ are required for i to be a common task in the East. That is, a necessary condition for task i to be a common task in the East in equilibrium is:

$$i < \min\{I, I^E\} \tag{6}$$

Otherwise, a T task producer may enter the market profitably and rendering it no longer a common task. This happens when the adaptation costs of the task is too high relative to the economies of scale of producing the task as a common task.

With the same reasoning, I derive the necessary conditions for a common task to be produced in the West as:

$$i < \min\{I, I^W\} \tag{7}$$

where I is as defined by (4) and I^W is defined as:

$$I^W \equiv min\left\{max\left\{0, \gamma^{-1}\left[\frac{A(Y_T^E + Y_T^W)}{wA(Y^\Omega)}\left(\frac{Y_T^E + \tau Y_T^W}{\tau Y_T^E + Y_T^W}\right)\right]\right\}, 1\right\} \tag{8}$$

So if either one of (6) and (7) is satisfied, that is $i < \max\{\min\{I, I^W\}, \min\{I, I^E\}\}$, the adaptation cost is low enough for the task to be produced as a common task.

To find the sufficient conditions for a common task, I consider the case where a task i is initially performed as a specific task. C task producers can undercut the T tasks and sell to the entire market of C and T goods in both countries (i.e. a common task). I find that the threshold tasks for maintaining specific tasks in each country are also I, I^E, and I^W. In this case $i > max\{I, I^W\}$ is the necessary condition for a task to be a specific task in the East, and $i > max\{I, I^E\}$ necessary condition for it to be in the West. Task i is therefore a specific task if $i > \min\{\max\{I, I^W\}, \max\{I, I^E\}\}$.

Combining the conditions for common tasks and specific tasks, I conclude that the necessary an sufficient condition for task i to be a common task in equilibrium is $i < \max\{\min\{I, I^W\}, \min\{I, I^E\}\}$. For a task to be a specific task, the necessary and sufficient condition is $i > \min\{\max\{I, I^W\}, \max\{I, I^E\}\}$.

In fact, the marginal common task is simply given by I:

Lemma 2. *I, as given by* (4)*, is the marginal common task in the two country competitive equilibrium of task producers:*

(i) Highly substitutable tasks ($i < I$) are performed as common tasks.

(ii) Poorly substitutable tasks ($i > I$) are performed as specific tasks.

Proof. See Appendix 1 □

This equilibrium task allocation is therefore the outcome of task producers' competition, which determines the mix of tasks used for production of final goods in both countries. Under low off-shoring costs with all task having concentrated production, C and T goods firms in both countries produce their goods with the same mix of common and specific tasks. Whether it is located in the East or West, all final goods firms use $[0, I)$ common tasks and $[I, 1]$ specific tasks.

Just as in the Autarky case, Proposition 1 applies to the two-country equilibrium. That is, the share of common tasks depends on the relative size of C and T goods production. When world production is evenly split between C and T, the share of common tasks is at its smallest. On the other hand, when the world output of C largely out number that of T, the share of common tasks becomes very large as T goods firms use more C tasks.

4.2.3 The Location of Common Tasks

I continue by examining the location of the common tasks in equilibrium. Consider task i with $i < I$. This task is a common task in equilibrium. When there is an incumbent common task producer in the East, it sets prices $p^E_{Ci} = A(Y^\Omega)$ and $p^E_{Ti} = A(Y^\Omega)\gamma(i)$ for C goods firms and T goods firms in the East respectively. It also charges to C goods firms and T goods firms in the West $p^W_{C\,i} = \tau A(Y^\Omega)$ and $p^W_{T\,i} = \tau A(Y^\Omega)\gamma(i)$ respectively.

An entrant common task firm in the West can undercut these prices to capture the entire market for common tasks. Doing so will yield negative profits if

$$[A(Y^\Omega) - w\tau A(Y^\Omega)](Y^E_C + \gamma(i)Y^E_T) + [\tau A(Y^\Omega) - wA(Y^\Omega)](Y^W_C + \gamma(i)Y^W_T) \leq 0$$

When the inequality holds, it ensures no common task producer can enter the market profitably in the West.

Now consider the case where the common task is produced by an incumbent producer in the West. The expression for the profits of a common task producer entrant into the East is equal to the left hand side of the above inequality multiplied by -1. So if the above inequality fails to hold (i.e. is greater than 0), then no common task producer can enter the market profitably in the East.

Thus I can identify the condition that determines the location of common tasks. Since the relative wage w is crucial in determining the location of task production, I define the relative wage that equates the above inequality for some task i as $\hat{w}(i)$:

$$\hat{w}(i) \equiv \frac{Y^E_C + \gamma(i)Y^E_T + \tau(Y^W_C + \gamma(i)Y^W_T)}{\tau(Y^E_C + \gamma(i)Y^E_T) + Y^W_C + \gamma(i)Y^W_T} \tag{9}$$

The location of a common task is determined by the market relative wage w and its relation to the $\hat{w}(i)$ threshold for that task i.

Common task i will locate in the East when $w > \hat{w}(i)$, that is, when the relative wage of the West is higher than the threshold. These same tasks will locate in the West $w < \hat{w}(i)$. When the relative wage is exactly at $\hat{w}(i)$, the location of common tasks is indeterminate.

I note two important of properties of $\hat{w}(i)$. First, $\hat{w}(i)$ is increasing in Western output and decreasing in Eastern output (since τ is greater than 1). So as the output of the East increases, while holding Western output constant, it becomes more attractive for common task to locate in the East as this would save in off-shoring costs. Second, note that $\hat{w}(i)$ is a function of i, meaning that the common tasks with different adaptation costs may locate differently from one another. It is increasing in i if Y_T^W is greater than Y_T^E and vice versa.

I can now determine the location of common tasks with the use of $\hat{w}(i)$. I define task J as the task with $\hat{w}(i)$ equal the actual (exogenous) relative wage w, so that:

$$J \equiv \hat{w}^{-1}(w) \tag{10}$$

I can deduce the following lemma from J:

Lemma 3. *The location of common tasks:*

(i) When $Y_T^W > Y_T^E$, common tasks $i > J$ will locate in the West and $i < J$ in the East in equilibrium.

(ii) When $Y_T^W < Y_T^E$, common tasks $i > J$ will locate in the East and $i < J$ in the West in equilibrium.

Proof. With $Y_T^W > Y_T^E$, $\hat{w}(.)$ is an increasing function in i. So for any task $i > J$, $\hat{w}(i)$ is greater than w and the common task locates in the West. Likewise, tasks $i < J$ have $\hat{w}(i)$ less than w so that the common tasks locate in the East in equilibrium.

For the case of $Y_T^W < Y_T^E$, $\hat{w}(.)$ is decreasing in i. Thus tasks $i < J$ have larger $\hat{w}(i)$'s and tasks $i > J$ have smaller $\hat{w}(i)$'s than w. □

Therefore, common tasks are split among both countries and the ones with higher adaptation cost $\gamma(i)$ locate near where there are more T goods being produced.

This result is due to only T goods firms incurring adaptation costs when common tasks are C tasks and tasks with higher adaptation costs also have higher off-shoring costs. This is an artifact of the off-shoring cost τ, which magnifies the adaptation costs of tasks as it is a multiplicative cost. For instance, with $Y_T^W > Y_T^E$ common tasks with higher adaptation costs would prefer to be closer to the Y_T^W output to incur less off-shoring cost.

4.2.4 The Location of Specific Tasks

Finally, I turn my focus to the location of specific tasks. Tasks $i > I$ are performed as specific tasks. For such tasks, T goods firms use T tasks, while C goods firms still use C tasks. The location of C and T tasks can be analyzed independently as they are produced by two different group of task producers.

I look at the threshold wage that determines the equilibrium location of specific C tasks. Comparing the profitability of locating the production of such a task, it is more profitable to perform the C tasks in East if:

$$[A(Y_C^E + Y_C^W) - w\tau A(Y_C^E + Y_C^W)]Y_C^E + [\tau A(Y_C^E + Y_C^W) - wA(Y_C^E + Y_C^W)]Y_C^W \leq 0$$

Just as with common tasks, I define the relative wage threshold for specific C tasks as w^C:

$$w^C \equiv \frac{Y_C^E + \tau Y_C^W}{\tau Y_C^E + Y_C^W} \tag{11}$$

In the Bertrand equilibrium, specific C tasks will locate in the East when $w > w^C$ and in the West when $w < w^C$.

Likewise, T tasks will locate in the East when $w > w^T$ and in the West when

$w < w^T$, where w^T is defined as:

$$w^T \equiv \frac{Y_T^E + \tau Y_T^W}{\tau Y_T^E + Y_T^W} \tag{12}$$

So the relative outputs of C and T in each country determines w^C and w^T. The actual w in relation to these thresholds determines the location of both specific tasks for C and specific tasks for T.

This result is summarized in the following lemma

Lemma 4. *The location of specific tasks:*

(i) Specific C tasks will locate in the East when $w > w^C$ and in the West when $w < w^C$

(ii) Specific T tasks will locate in the East when $w > w^T$ and in the West when $w < w^T$

Note that w^C decreases with Y_C^E and increases with Y_C^W. When the share of C goods output is higher in the West, the relative wage of the West does not need to be as low for the specific C tasks to be produced in the West. This is because the proximity to the output C goods is another reason other than the w to locate in one country or another.

4.3 The General Task Allocation Equilibrium

In this section, I identify the equilibrium of the general two country model. The resulting off-shoring equilibrium in the two country model is characterized by Lemmas 1 to 4.

The resulting equilibrium task allocation has each country gaining a complete specialization in specific tasks of the same product as their final goods specialization. Common tasks are split among countries, but more will locate in the country with greater total output since proximity to more output reduces off-shoring costs incurred.

As mentioned previously, for concreteness I assume $Y_C^E > Y_C^W$, $Y_T^E < Y_T^W$, and $Y_C^E + Y_C^W > Y_T^E + Y_T^W$. I maintain the assumption that $w = 1$ so that the relative wage is neutral in the task allocation equilibrium. This lets me focus on the effect of the adaptation costs on how and where each task is produced.

From Lemmas 1 and 2 I can deduce that the task equilibrium has C common tasks with tasks $i < I$ as common tasks and $i > I$ as specific tasks.

From Lemma 3, J determines the distribution of common tasks. As $Y_T^W > Y_T^E$, common tasks $i > J$ will locate in the West and tasks $i < J$ in the East in the equilibrium. Since $\hat{w}(i)$ is increasing with Western output and decreasing with Eastern output, J as (defined by (10)) increases with $\frac{Y_C^E + Y_T^E}{Y_C^W + Y_T^W}$. The intuition behind this is that as the global share of East final goods output increases relative to the West, more common goods are produced in the East to minimize on off-shoring costs.

From Lemma 4, the location of specific tasks are determined by the relation of w^C and w^T to $w = 1$. $w^C > 1$ when $Y_C^E < Y_C^W$ and $w^T > 1$ when $Y_T^E < Y_T^W$. C task producers will locate in the country with higher output of C goods and T task producers in the country with more T output.

I have now characterized the task allocation equilibrium of off-shoring in the two country model. The following proposition summarizes the result:

Proposition 3. *In the two country task allocation equilibrium with $w = 1$ and low off-shoring costs $\tau = 1 + \epsilon$:*

(i) Common tasks are produced in both countries, with more common tasks locating in the country that has a larger overall output

(ii) Specific tasks are produced only in the country with a higher output of their good.

This equilibrium is shown in Figure 3. Common and specific tasks are determined jointly in both countries by I. Unlike the task allocation equilibrium with one good per country, here each task is produced in one location for goods firms in both countries. For tasks with high adaptation costs, tasks with $i > I$, the East produces C

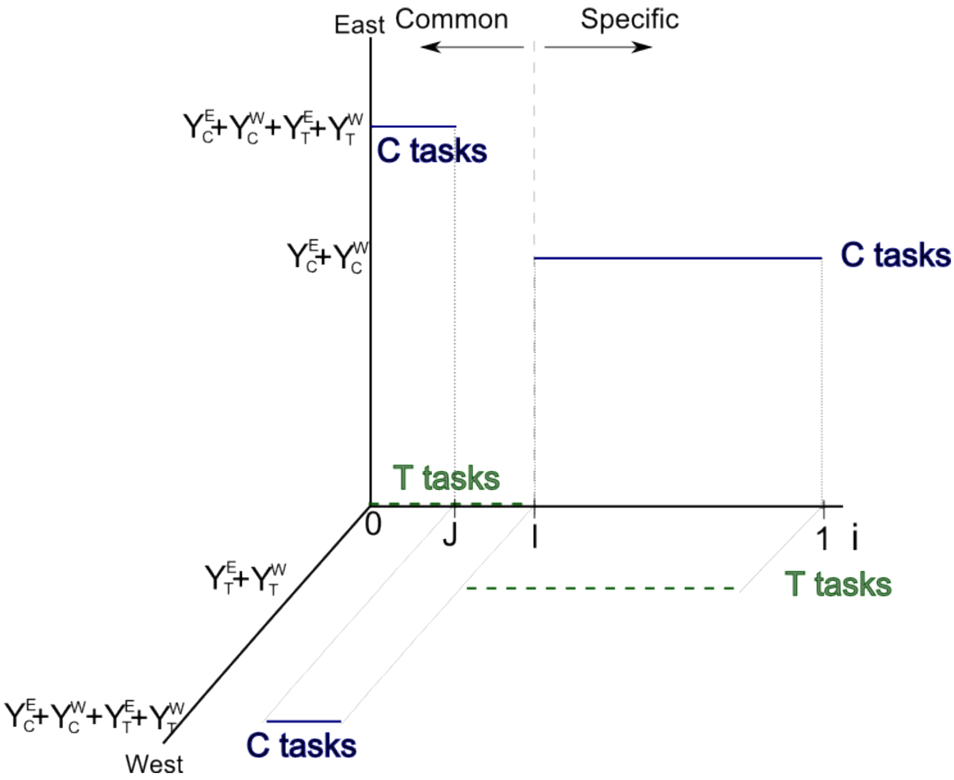

Figure 3: Task allocation equilibrium in the general two country model

specific tasks and the West T specific tasks for all C goods firms and all T goods firms respectively.

Tasks with low adaptation costs, with $i < I$, are produced as common task and no T versions of these tasks are produced at all. Common tasks are split between the two countries by J, which is increasing in the size of Eastern output relative to Western output. So when East produces a lot more goods in total than the West, J will be higher so that most if not all of the common tasks locate in the East in Equilibrium.

The common tasks with lower adaptation costs in Figure 3 are located in the East and ones with higher adaptation costs are in the West. This is because in this case the West produces more T goods than the East. Common tasks with higher adaptation costs locate in the country with higher output of T because off-shoring costs magnifies the adaptation costs. This is a result of how I have assumed off-shoring costs to interact with adaptation costs.

One final feature of this result I highlight is the specialization in goods output translates into the specialization of specific tasks in equilibrium. $Y_C^E > Y_C^W$ and the East produces all C specific tasks in equilibrium. $Y_T^W > Y_T^E$ and the West produces all T specific tasks. This result shows how the substitutability of tasks in production shapes the pattern of off-shoring. Tasks used in the production of one type of good will locate near where that good is produced in larger quantities. Adaptation costs matter for the equilibrium allocation of tasks.

The results from this section highlight the importance of task substitutability on the pattern of off-shoring. Even with homogeneous off-shoring costs and equal wages, the allocation of tasks is defined by whether tasks are used in one good or in several goods. More common tasks locate in the country with larger total output and specific tasks locate in countries with larger output of the task's good.

4.4 Efficiency

Intuitively, one would expect the task allocation equilibrium to be inefficient due the existence of externalities. In this section, I confirm this by comparing the Bertrand equilibrium with the efficient allocation of tasks. I show that the equilibrium allocation has fewer common tasks than there would in the efficient allocation.

Firstly there are aspects of the allocation equilibrium that are efficient. The choice of whether the common task is C or T task is efficient (Lemma 1). Competing task firms supply the common task with the type that minimizes on the adaptation costs by producing the type of good produced at a larger scale.

Also, the location choice of task production (Lemmas 3 and 4), conditional on a task being produced as a common or specific task, is efficient. This is because the scale of production is the same no matter where the common or specific tasks are produced, and the only consideration for location is relative wages. Competition among task producers allows common task producers to concentrate in the East if it's overall costs are lower in the East and likewise in the West. Specific task producers also locate in the lower cost country.

Inefficiency arises from the determination of common versus specific tasks (Lemma 2). Consider task i. I compare the cost of producing task i as a common task in the East to it being produced as specific tasks in the East. Producing it as a common task has a lower cost if:

$$\begin{aligned} &A(Y_C^E + Y_C^W)(Y_C^E + \tau Y_C^W) + A(Y_T^E + Y_T^W)(Y_T^E + \tau Y_T^W) \\ &> A(Y^\Omega)(Y_C^E + \gamma(i) Y_T^E + \tau(Y_C^W + \gamma(i) Y_T^W)) \end{aligned}$$

The task with adaptation cost that equates this inequality is

$$I^* = \min\left\{\max\left\{0, \gamma^{-1}\left[\frac{A(Y_T^E + Y_T^W)}{A(Y^\Omega)} + \frac{Y_C^E + \tau Y_C^W}{Y_T^E + \tau Y_T^W}\left(\frac{A(Y_C^E + Y_C^W)}{A(Y^\Omega)} - 1\right)\right]\right\}, 1\right\} \tag{13}$$

The cost of producing task I^* as a common task in the East is the same as producing it as specific task with both C and T tasks in the East.

The efficient international task allocation is the allocation of tasks that provides the lowest aggregate costs. Tasks with $i > I^*$ are optimally produced as specific tasks. Tasks $i < I^*$ are performed as common tasks in the efficient allocation.

Comparing (4) to (13), it can readily be seen that $I^* \geq I$ as the second term in the squared bracket is positive. Recall that tasks $i > I$ are specific tasks according to Lemma 2 in the Bertrand task competition equilibrium. This means that there are tasks $I \leq i \leq I^*$ that cannot be sustained as common tasks in the equilibrium that are cheaper to perform if they remained as common tasks. This is because the T task producers that enter the market do not take into account their negative externalities of decreasing the scale of production of the C common task producers.

The equilibrium has specific tasks that would be more efficiently be performed as common tasks. That is, the task allocation equilibrium provides too many specific tasks and too few common tasks. This result is summarized in the following proposition:

Proposition 4. *The task allocation equilibrium is inefficient. Compared to the efficient outcome, the international task allocation from Bertrand competition results in too few common tasks and too many specific tasks.*

The reason behind the inefficient level of common tasks is that the Bertrand competitive task producers do not take into account the negative externalities of their actions. While common tasks take full advantage of scale economies, specific task producers by entering into the market take away some of the scale economies of the common tasks. Their disregard for the effects of their actions mean that the overall loss, due the the higher cost to the firms using the common task, outweighs the gain from saving on adaptation costs by providing a specific task.

5 Comparative Statics

Now that I have characterized the equilibrium of my model, I continue by looking into some results drawn from comparative statics exercises. I first show how a fall in adaptation costs increases the number of common tasks. Then I look at how the change in off-shoring costs affects the equilibrium task allocation. In particular, the comparison between the autarkic task equilibrium and the international task equilibrium will show how the supply chain is influenced by off-shoring.

5.1 Size of Adaptation Costs

I look at how a fall in adaptation costs affect the task allocation equilibrium. Before the invention of interchangeable parts, products were made in their entirety of tasks specifically intended for the good. Advancements in technology allow for standardization tasks, which made it easier to outsource and interchange tasks among other products. The standardization of technology across industries reduces the adaptation costs and will affect the share of common and specific tasks.

Suppose there is an advancement in technology or design that reduced the adaptation costs for at least some tasks in the production of C and T goods. Let $\gamma'(i)$ be less than or equal to $\gamma(i)$ for all task i in $[0, 1]$.

The level of adaptation cost of the marginal common task is unchanged, so $\gamma(I)$ is equal to $\gamma'(I')$, where I' is the new marginal common task in equilibrium. As $\gamma(i) \geq \gamma'(i)$ and tasks are indexed so that i increases with adaptation costs, I' needs to be larger or equal than I in order to keep the original adaptation costs.

With the index of the marginal task weakly increasing, this represents a larger share of tasks being used as common tasks and fewer as specific tasks. It is unsurprising that a fall in adaptation costs allows more tasks to take advantage of larger economies of scale by being common tasks. Thus I have the following proposition:

Proposition 5. *A fall in adaptation costs $\gamma(i)$ increases the share of common tasks and reduces the share of specific tasks*

As technology improves over time, the fall in adaptation costs allows for a improvements in productivity. The increase in the share of common tasks lets task producers take greater advantage of economies of scale, while letting goods firms pay less adaptation costs for common tasks. This result introduces to the literature the gains from off-shoring, derived from the rearrangement of how goods are produced as technology improves.

The reality of the global supply chain is that there are common tasks and they will only be more prevalent with time. This means it will be increasingly important for us to understand the implications of common tasks in international economics.

5.2 Gains from off-shoring

Off-shoring with common tasks allows for two types of gains in productivity. The first is through larger economies of scale for each task which lowers to cost of production. Both common and specific task producers can sell to firms both at home and away when off-shoring is introduced. The second source of gain comes from the change in the mix of common and specific tasks. Better task allocation reduces the adaptation costs incurred in the final product by using more specific tasks. In this section, I focus my attention on this second effect and compare the marginal common task of the autarky economy with that of the off-shoring equilibrium.

How do common tasks change when we move from no off-shoring to off-shoring? I revisit Section 3.1 for the autarky equilibrium. From (3), the marginal task under autarky depends on the relative sizes of I_T and I_C, as given by (1) and (2). I maintain the assumption that the East produces more C goods and the West produces more T goods: $Y_C^E > Y_C^W$ and $Y_T^E < Y_T^W$. From this, the autarky equilibrium of the two countries is I_T in the East and I_C in the West as the marginal common task in the respective countries.

Whether the common task is C or T in autarky depends on which good's output is larger. Thus the common tasks in the East are C tasks, and the common task in the West are T tasks.

Contrast this with the task allocation in the two country equilibrium from Section 4, where I as given by (4) is the marginal common task used by all firms in both countries. With the output of C goods greater than that of T goods, the common task is a C task.

I am now ready to compare I_T and I_C with I. I reproduce these terms below from (1), (2), and (4) to aid my comparison:

The marginal common task of the East under autarky

$$I_T = min\left\{max\left\{0, \gamma^{-1}\left(\frac{A(Y_T^E)}{A(Y_C^E + Y_T^E)}\right)\right\}, 1\right\}$$

The marginal common task of the West under autarky

$$I_C = min\left\{max\left\{0, \gamma^{-1}\left(\frac{A(Y_C^W)}{A(Y_C^W + Y_T^W)}\right)\right\}, 1\right\}$$

The marginal common task of both East and West with off-shoring

$$I = min\left\{max\left\{0, \gamma^{-1}\left[\frac{A(Y_T^E + Y_T^W)}{A(Y^\Omega)}\right]\right\}, 1\right\}$$

While it is unclear what the relative magnitudes of I_T, I_C, and I are, as these depend on the relative sizes of outputs and the convexity of $A(.)$, I can see that the marginal common task need not be the same under autarky and under off-shoring.

In the case where off-shoring brings more common tasks, i.e. I is greater than I_T or I_C, tasks that were specific in autarky become common tasks with off-shoring. This means for the tasks that have changed, T goods firms must incur the adaptation cost for there tasks. However, these tasks will still be cheaper overall for the T goods producers as they are able to produce at larger scales. This is guaranteed by the fact that the common task producer must undercut the price of specific task producers when they enter the market.

In the alternative case where off-shoring brings fewer common tasks, i.e. I is less than I_T and I_C, this gives goods firms greater access to specific tasks. Overall, goods firms will have to incur less adaptation costs.

In both cases, C goods firms in the country where the autarky common tasks are T tasks (the West) gain from having access to C common tasks under off-shoring. T goods firms in that country incur the adaptation costs as the common T tasks that will disappear under off-shoring, but the gains from the larger scale of production more than compensates the extra costs.

So, a part of the gains from off-shoring comes from the different mix of common and specific tasks. This result is summarized in the following proposition:

Proposition 6. *A source of gain for goods firms from off-shoring is through a superior mix of common and specific tasks:*

(i) Off-shoring that results in more common tasks allows for firms to access even larger scale of production by shifting some specific tasks to common tasks.

(ii) *Off-shoring that results in fewer common tasks allows firms access to specific tasks, thereby reducing the need to incur adaptation costs of using less suitable common tasks.*

This result comes intuitively when considering how the marginal common task under off-shoring, I, is derived. In the Bertrand competition equilibrium, task producers compete to provide goods firms with the lowest price. The winners of price competition are the task firms that can provide the lowest price, by selecting the scale and location of production. As a result, goods firms are better off under off-shoring than in Autarky as task producers gain new ways to reduce prices.

This newly identified source of the gain from off-shoring is not present in other models in the literature, where the set of tasks used in each good is fixed. The ability to access greater scales with more common tasks and/or the ability to access more suitable specific tasks provide an important motivation for firms to off-shore.

6 Conclusion

This paper is an attempt to capture an important feature of the unbundling of the production process in the context of the global supply chain. The existence of common tasks shapes the international allocation of task production. Off-shoring allows for a reorganization of production on an international scale and as task firms compete as common and specific tasks.

The resulting task allocation equilibrium from off-shoring divides tasks into common and specific tasks. Countries specialize in specific tasks while common tasks are spread across the world, but with more located in the country with the larger output. Gains from off-shoring will is derived from a better mix of common and specific tasks in addition to the gains from larger scale production.

The implication of these findings allows for useful policy lessons for countries operating in the globalized supply chain. The main lesson that can be drawn from my model is that the process of off-shoring generates a structural change to a country's productive capacity, and perhaps more importantly, a change to the process of how each good in produced. As the cost of off-shoring falls with the advancement of technology, countries must be prepared for these changes in their economic makeup. No longer will countries produce goods in their entirety, instead tasks of each good will be dispersed across different locations. Countries will specialize in specific tasks and firms will have access to more kinds of tasks. Any industrial policy must take into account the new mix of tasks used resulting from the international allocation of tasks.

Knowing about the different roles of common and specific tasks gives guidance for policy makers seeking to alter their country's production. In a world with off-shoring, industrial policy is no longer about championing specific goods but rather aiding specific tasks. There are gains to be had from not producing the entire supply chain and only focusing on a few tasks.

This research initiates a further line of inquest towards a better understanding of the pattern of off-shoring. A natural extension of my model would be to introduce more than two goods and allow tasks to be shared by any combination of such goods. For example, while cars and trucks may use the same headlights, trucks and buses may have the same engines. The intricate web of tasks that go into the production of many goods is in part determined by technology, but in part determined by organizational choice. Knowing how the determination of which tasks matches into which good will help us understand in more detail how off-shoring works.

Further work can also be done empirically to investigate the availability of tasks in each country. Although countries specialize in tasks, my model predicts that they specialize in specific rather than common tasks. Common tasks will tend to locate in the country with the larger overall output, though they will be spread throughout the countries. More generally, a study on how common and specific tasks are allocated across countries provides a better understanding of the pattern of off-shoring.

The advancement of technology has made the global unbundling of production an increasing reality. The only way to understand this process is to recognize the endogeneity of the production process as goods can be produced in a number of different ways. This brings in another dimension of gains from off-shoring that we are only beginning to understand.

Appendix

Appendix 1: The Marginal Common Task

Lemma 2

Proof. First note that because $\tau = 1 + \epsilon$, for any w and C and T goods outputs, we have either $I^E \leq I \leq I^W$ or $I^W \leq I \leq I^E$. That is, I is always between I^E and I^W.

In the case of $I^E \leq I \leq I^W$, tasks $i < I^E$ are obviously common tasks in equilibrium. As a common task, T firms cannot enter profitably as dictated by (6) and (7). And if they were specific tasks, a C task producer will be able to profitably enter and take over the world market in equilibrium.

Tasks $I^E < i < I$ cannot be common tasks in the East, as according to (6) a T firm can profitably enter in that case. However if the task were produced in the West as a common task, it will remain so since it is still below I^W. Were the task a specific task, a C task producer will be able to produce in the West as a common task profitable as a common task. Therefore these tasks are also common task in the Bertrand equilibrium.

Following the similar reasoning, I find that tasks $i > I^W$ are specific tasks in equilibrium. Also, tasks $I < i < I^W$ are specific task as a T producer of these task can enter profitably in the East were it a common task.

The same arguments applied to the case when $I^W \leq I \leq I^E$ will show that I is the marginal common task in that case as well.

Therefore, tasks $i < I$ are common tasks and tasks $i > I$ are specific tasks in the two country Bertrand equilibrium. □

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Chapter 3: Exporting decisions of heterogeneous firms under exchange rate uncertainty

Abstract

In this paper, I study how real exchange rate fluctuations determine the size and composition of the export sector. My model puts investment under uncertainty into modern trade theory. Using the methodology set out in Dixit and Pindyck (1994) in a heterogeneous firms model, I model firms as call options for changing their export status. I then identify the marginal exporting firm at a given point of time and then determine the set of trigger real exchange rates for entry and exit for this firm. There is a middle band of real exchange rates where firms neither enter or exit and is the source of hysteresis. My primary result is that exchange rate uncertainty coupled with sunk cost of entry causes hysteresis in the number and productivity of exporting firms. This hysteresis provides an explanation for the stylized fact that export surges caused by changes in the real exchange rate tend to happen through the intensive rather than the extensive margin. I then extend the model to allow free entry of firms. This explains a second stylized fact of the existence of non-exporting firms with higher productivity than some exporting firms.

1 Introduction

In this paper, I aim to uncover the effects of exchange rate fluctuations on international trade. Exchange rates move around in a highly unpredictable manner and they play an important part in determining export profits. Even with the use of hedging, exporting firms can only mitigate some of the exchange rate risks. Furthermore, non-exporting potential exporters would not be engaged in hedging currencies. For exporters selling to countries with the same currency, such as countries within the European Union, purchasing power parity almost never holds and real exchange rates fluctuate due to changes in risks and expectations. Exchange rates therefore play an important role in firm exporting decisions.

This paper studies how exchange rate fluctuations affect endogenous firm entry and exit decisions, and how this determines the composition of trading firms and the subsequent pattern of trade. Under exchange rate uncertainty, my model describes the entry and exit behaviour of heterogeneous exporting firms when entry is costly. The investment under uncertainty decision of exporting firms characterizes the circumstances in which a change in exporting opportunities can cause intra-industry reallocations in the process of trade.

I focus on the firm level responses to uncertainty in the real exchange rate when firms differ in their level of productivity. The real exchange rate is modeled as an exogenous stochastic process, justifiable by the number of largely differing factors (e.g. from changes in country risk premium, in productivity, in monetary policy, in consumer preferences etc.) outside of the focus of our paper. This generates uncertainty in the profits from exporting for all firms, no matter their productivity level.

Uncertain exporting profits when coupled with the need for a sunk entry cost into the export market generates hysteresis in the number of the firms exporting. This is due to what some in the literature have called the "beachhead" effect (Baldwin (1988)). When entry costs are sunk, a sufficiently large exchange rate change can induce a persistent shift to the market structure of exporters. This effect is exacerbated with more uncertainty in the exchange rate.

I recasts the beachhead effect in a richer model with heterogeneous firms. When firms differ by their productivity level, the effect also influences intra-industry allocations.

This means that large shocks in the real exchange rate can have implications to the composition of trading firms. Firms that exit from exporting upon an adverse shock have lower productivity, so the exporting sector experiences a "cleansing" effect. Thus the productivity of the export sector is subject to hysteresis, which has implications on exchange rate pass-through and can explain trade sector adjustments along the intensive and extensive margins.

Part of why exchange rate uncertainty matters to exporting firms is because the decision to enter into exporting incurs irreversible sunk costs. The uncertainty in exchange rates coupled with sunk investment makes this a classic investment under uncertainty problem as explored by Dixit and Pindyck (1994). This combination of factors means that the fluctuation of the exchange rate triggers entry and exit of firms into exporting asymmetrically. This creates hysteresis in the export sector that has real economic implications.

When applied to heterogeneous firms, the export sector hysteresis suggests that the productivity level of firms exporting is path dependent. Firms with lower productivity may not be able to afford the sunk entry cost if not for a temporary depreciation of the exchange rate. Once they have entered they will continue exporting even if the exchange rate returns to its original level. This provides an explanation of why otherwise similar countries with similar domestic productivities have different profiles of exporting firms.

I also explain in this paper several stylized facts identified in the literature. The first, as identified by Bernard and Jensen (2004a) and Bernard, Jensen, Redding, Schott (2009) among others, is that export surges caused by changes in the exchange rate happen through the intensive margin rather than the extensive margin. This effect is exhibited by the hysteresis of exporting firms; while existing exporters adjust on the intensive margin to exchange rate changes, there are no new exporters due to the prohibitive sunk entry costs for potential entrants.

A second stylized fact I explain is the existence of non-exporting firms with higher productivity than some exporting firms. This phenomenon, as pointed out in Bernard, Eaton, Jensen and Kortum (2003) and Castro, Li, Maskus, and Xie (2013) among others, is explained in my model by combining exporter hysteresis with new successive cohorts of firms. While the firms exporting currently are determined by the path of past exchange rates, the cohort of new entrants all start off as non-exporters. This creates the possibility that an incumbent firm is exporting while a new firm with the same productivity is not.

1.1 Methodology

This paper puts investment under uncertainty into modern trade theory. My model uses a Melitz (2003) heterogeneous firms setting of trade with monopolistic competition to formulate the export entry decision. There is fixed mass of firms all of which produce for the domestic market and they differ in their productivity. These firms decide whether to enter the export market by incurring a sunk entry cost. Exporting also requires fixed costs and transport costs. Firms face uncertainty in the real exchange rate in the form of a Brownian motion process. Forward-looking firms make their entry and exit decisions by evaluating the value of the option they hold.

By modeling these firms in a small open economy, I focus on the the economic mechanisms within one country. I also abstract from labour market feedback by introducing a homogeneous sector that pins down wages.

Using the methodology set out in Dixit and Pindyck (1994), I determine the set of trigger real exchange rates for entry and exit. Having characterized the export profits as a function of the real exchange rate, I model a non-exporting firm as an asset that is a call option to become an exporting firm. Likewise, I model an exporting firm as an asset that is a call option to become a non-exporting firm. The entry trigger requires the operating profits from exporting to exceed the interest on the cost of entry, while the exit trigger requires operating profits to be less than zero. Greater exchange rate variability makes these options more valuable and less likely to be exercised. I identify a middle band of real exchange rates where firms neither enter or exit and is the source of hysteresis. This means that only significant changes to the real exchange rate can have persistent effects on the export sector.

I then expand this to a heterogeneous firms setting. As firms only differ in their productivity, the same microeconomic decision of entry and exit applies to all firms. A firm having decided to enter the export market suggests that all firms with higher productivity will also do so. Similarly, if a firm decides to stop exporting, all firms with lower productivity would also stop exporting. Therefore I can identify the marginal exporting firm at a given point of time. This allows for the trade sector to be endogenously determined. Thus the same hysteresis effect in entry and exit applies to all firms.

My model is then extended to allow free entry of firms. This is achieved by treating the above analysis as that of an incumbent cohort. I introduce new entrants as a separate cohort, all of which enter with the asset of a non-exporting firm. Potential entrants face uncertainty over their productivity, which they only realize once they pay a sunk entry cost to start operating. Firms that do not enter today do not have the option to enter at a later period. This is therefore a one off decision and forward-looking firms will only enter if their expected lifetime profits exceeds the cost of entry.

As this new cohort of firms all start off as non-exporters, the entry trigger real exchange rate will be activated for the more productive firms which will start exporting. I determine the marginal exporting firm for the entrant cohort. The hysteresis in entry and exit among the incumbent firms means that the marginal exporting firm can differ among the cohorts. I show that it is possible for two firms of the same productivity level to have made different exporting decisions.

1.2 Findings

My primary result is that exchange rate uncertainty causes hysteresis in the number of firms exporting and the productivity of those firms. This is useful in explaining the composition of exporters as well as the market structure of the export sector. For a given number of existing exporters, there is a range of exchange rates that is an area of inaction, within which the exchange rate can fluctuate without triggering entry and exit of exporters. Only when the exchange rate moves past the entry or exit triggers will the number of firms in the export sector change, after which there is a new area of inaction. Therefore I show that the number of exporters and their productivities is path dependent.

Another result of my paper is that I provide an explanation for the stylized fact that export surges due to exchange rate changes are mainly through the intensive margin rather than the extensive margin. My model provides a framework to understand trade adjustments to macroeconomic shocks. My final result of this paper is that I explain how two firms with the same productivity can make a different decision on whether to export or not. This, like the first result, provides a better understanding on which firms are involved in exporting.

In the next section, I explore the relationship between this paper and others in the literature. I outline the model in section 3. In section 4, I derive analytical and numerical results to show the implications of my model. I then expand the model by allowing for entrants and show additional results from this extension in section 5. In section 6, I close with some concluding remarks.

2 Relationship to the literature

My research in this paper follows the literature that looks into the relationship between exchange rates and international trade. This is surveyed in Auboin and Ruta (2011), although they point out that the main research questions in this area are on exchange rate volatility and exchange rate misalignments effects on trade.

The first papers to look into the entry and exit decisions of exporters when facing a sunk cost of entry include Baldwin (1988), Dixit (1989a, b), and Baldwin and Krugman (1989). These models show how uncertainty in export prices or the exchange rate generates hysteresis in trade. However, these papers abstract from firm heterogeneity. Continuing this line of inquiry, Baldwin and Lyons (1994) incorporates a trade hysteresis model in a simple model of international finance. They show that trade hysteresis can cause the exchange rate to be path dependent. This general class of models incorporates sunk cost investment with uncertainty in profits. This topic is thoroughly addressed in Dixit and Pindyck (1994).

Countering the claim of trade hysteresis, Alessandria and Choi (2007) uses an open economy business cycle model to analyze the effects of sunk costs on export dynamics. They analyze the effects of exchange rate changes, caused by technological shocks, on net exports and find that introducing sunk costs makes very little difference. They also allow for firm heterogeneity with each firm having a firm-specific technology level that is random over time and has no persistence. A positive innovation to technology in one country causes its real exchange rate to depreciate. So their model is more about how sunk costs have little effect on trade dynamics in response to technological shocks, rather than solely exogenous exchange rate shocks. Furthermore, the authors focus on net exports and therefore abstract from the pattern of trade.

In this paper, I also follow the line of research that seeks to understand the Melitz (2003) heterogeneous firms model in a dynamic setting. Ghironi and Melitz (2005) models firm heterogeneity in a dynamic stochastic general equilibrium two country setting. They recognize that sunk cost may be important but do not model them for simplicity. Indeed in a survey article, Melitz and Redding (2013) concede that modeling sunk entry costs in a dynamic general equilibrium model with heterogeneous firms adds substantial complexity. Ottaviano (2012) looks at how technological shocks have a compositional effect on heterogeneous firms, highlighting how extensive margin adjustments are important to aggregate productivity.

The idea that sunk costs investments are wiped out upon a negative shock is a very similar concept to the ideas explored in the creative destruction literature (first outlined in Schumpeter (1942)). Caballero and Hammour (1994) have a model where firms optimize between buying new production units and destroying old ones that are less productive. Upon a recession, old production units are "cleansed" as more of them are destroyed. Mortensen and Pissarides (1994) apply similar concepts to job creation and destruction. The abandonment (or destruction) of sunk cost investments of exporters upon an unfavorable exchange rate appreciation of this paper fits in line with this literature.

Empirically, there are papers looking at the entry into exporting decision with firm heterogeneity. Among these, including Roberts and Tybout (1997), Das, Roberts, and Tybout (2007), and Clerides, Lach, and Tybout (1998), the papers estimate the size of the sunk export costs, which they generally find to be significant. Bernard and Jensen (2004b) found that exporting today increases the probability of exporting tormorrow by 36% and that plant heterogeneity is important for export decisions.

There is also empirical work seeking to understand the dynamic adjustment process of trade. In works including Bernard and Jensen (2004a), Iacovone, Rauch, and Winters (2013), and Freund and Pierola (2012), the adjustment of trade along the intensive and extensive margins is described. Bernard, Jensen, Redding, and Schott (2009) find that variation in trade in the short term is dominated by the intensive margin, but the role of the extensive margin increases in longer time horizons.

To this literature, my paper makes several contributions. First, I highlight a mechanism in which exchange rates can affect international trade and illustrates the consequences of exchange rate fluctuations. This expands on the trade hysteresis models by showing that hysteresis also occurs in exporter productivity. The productivity of exporters is shown to be path dependent. Second, I expand the methodology developed in Dixit and Pindyck (1994) into a different class of models. By using a Melitz (2003) heterogeneous firms model, my paper updates the investment under uncertainty literature to modern trade theory.

Finally, I provide an explanation to several stylized facts identified in the empirical literature. I explain how firms adjust along the intensive and extensive margins. This matches findings in papers such as Bernard and Jensen (2004a) and Bernard, Jensen, Redding, and Schott (2009) that export surges are primarily along the intensive margin rather than the extensive margin. Another stylized fact that my model provides a story for is the observation that there are some non-exporting firms with higher productivity than exporters. Trade hysteresis working on successive cohorts of firms allows for the possibility of firms in different cohorts with the same productivity to have made different export decisions.

3 Model

In this section, I develop a model where there is no entry. There is a fixed mass of N firms in the home country, a subset of which export.

3.1 Setup

3.1.1 Demand

In my model, the world consists of a home country that is a small open economy and a foreign rest of the world. I assume that consumers in the home country maximize their preferences over goods produced in a homogeneous sector and a differentiated sector:

$$U = c_0^{\beta} \left[(\int_{\omega \in \Omega} c(\omega)^{(\theta-1)/\theta} d\omega)^{\theta/(\theta-1)} \right]^{(1-\beta)}$$

The above utility function is a Cobb-Douglas upper tier utility with lower tier Dixit and Stiglitz (1977) Constant Elasticity of Substitution (CES) preferences for one sector. c_0 is the amount of the homogeneous good consumed. This good is produced with a unit input, is costlessly traded, and is the numeraire good. In the other sector there is a continuum Ω of horizontally differentiated varieties that are consumed. $\theta > 1$ is the elasticity of substitution across the differentiated goods.

With Y as the aggregate income, the upper tier Cobb-Douglas utility means that $C = (1 - \beta)Y$ is the consumer spending on the differentiated sector. I define $p(\omega)$ as the home price of a good $\omega \in \Omega$. The demand for each differentiated variety ω in this sector is given by:

$$c(\omega) = \left(\frac{p(\omega)}{P}\right)^{-\theta} C$$

where $P \equiv (\int_{\omega \in \Omega)} p_t(\omega)^{(1-\theta)} d\omega)^{1/(1-\theta)}$ is the price index for consumed goods.

Demand in the rest of the world, $c^*(\omega)$, is defined analogously, and so is the foreign price index P^*. The only difference is that in a small open economy, P^* is treated as a constant.

3.1.2 Production

By assuming the home country to be a small open economy, I focus my model on the behavior of firms in the home country exporting to the rest of the world. The home country has a continuum of firms. Each producer supplies a distinct horizontally differentiated variety. They produce their good with factor of production L, which has unit cost w. Firms vary by their level of productivity, indexed by φ, and they have a constant marginal cost that is inversely proportionate to their productivity. A firm with productivity φ produces φ units of output for each unit of labour input. The unit cost of production is therefore w/φ. The rest of the world firms are exogenous to the model of a small open economy.

Firms at home can sell to both the domestic market and the export market. As my model seeks to learn more about exporting decisions, I abstract from the domestic market by assuming there are no fixed costs to sell at home. All firms will produce for the local market. Exporting incurs additional costs in the form of an ice-berg trade cost $\tau \geq 1$ as well as a **fixed cost** f_x **of exporting**. In addition, there is a cost of entry into exporting, a **sunk entry cost of** f_{ex}. The values of both fixed costs and entry costs are measured in units of effective labour.

Let $p_D(\varphi)$ denote the nominal domestic price and $p_X(\varphi)$ the export price of a home firm with productivity φ. Prices are denominated in the currency of the destination market. Each firm maximizes its profits by choosing its price subject to a residual demand curve with constant elasticity θ. Monopolistic pricing implies the equilibrium price to be a constant mark-up over marginal cost for each variety, which gives the home price and export price respectively as:

$$p_D(\varphi) = \frac{\theta}{(\theta-1)}\frac{w}{\varphi}, \qquad p_X(\varphi, Q) = Q^{-1}\frac{\theta}{(\theta-1)}\frac{w\tau}{\varphi}$$

where Q is the exchange rate in units of home currency per unit of foreign currency (where a depreciate is marked by a rise in Q).

As not all firms choose to export, a firm's profits can be divided into profits earned from domestic sales, $\pi_D(\varphi)$, and profits from exporting, $\pi_X(\varphi, Q)$. Total period profits of a home firm with productivity φ is given by $\pi(\varphi, Q) = \pi_D(\varphi) + \pi_X(\varphi, Q)$, where

$$\pi_D(\varphi) = \frac{1}{\theta}[p_D(\varphi)]^{1-\theta} P_t^{\theta} C_t \tag{1}$$

$$\pi_X(\varphi, Q) = \begin{cases} \frac{Q}{\theta}[p_X(\varphi, Q)]^{1-\theta} P^{*\theta} C^* - f_x & \text{if firm } \varphi \text{ exports} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The small open economy assumption means that foreign price index P^* is taken as exogenous even though the domestic price index P is endogenously determined.

3.1.3 Aggregation and market clearing

The distribution of firm productivity in the home country is given by $G(\varphi)$, with support on $[\varphi_{min}, \infty)$. Since there are no fixed costs for selling domestically, all firms will operate in the domestic market. Whereas only a subset of firms will export. I denote the productivity cutoff φ_X be the productivity level of the marginal exporting firm in the home country. More productive firms with $\varphi \geq \varphi_X$ all export and less productive firms with $\varphi < \varphi_X$ do not.

The productivity cutoff is to be determined endogenously in a later section. This section outlines the market equilibrium for a given value of φ_X.

While the total mass of active firms N in the home country is fixed, the mass of exporting firms n is an the equilibrium value determined by φ_X. Using the definition of φ_X as the productivity cutoff of exporters, there are $n(\varphi_X) = (1 - G(\varphi_X))N$ firms exporting in the home country.

With the productivity distribution of firms and the exporting productivity cutoff, I define the aggregate productivity measures for all active firms and for all exporters in the home country. The measure of aggregate productivity of each group for a given level of φ_X is respectively:

$$\tilde{\varphi}_D \equiv \left[\int_{\varphi_{min}}^{\infty} \varphi^{\theta-1} g(\varphi) d\varphi \right]^{\frac{1}{\theta-1}}$$

$$\tilde{\varphi}_X \equiv \left[\frac{1}{1 - G(\varphi_X)} \int_{\varphi_X}^{\infty} \varphi^{\theta-1} g(\varphi) d\varphi \right]^{\frac{1}{\theta-1}}$$

These measures of aggregate productivity makes the heterogeneous firm model isomorphic to one with N identical firms with productivity $\tilde{\varphi}_D$ selling domestically and n firms with productivity $\tilde{\varphi}_X$ exporting. Therefore the average profit of a home firm from selling domestically can be expressed as $\pi_D(\tilde{\varphi}_D)$ and the average profit of a home firm form exporting is $\pi_X(\tilde{\varphi}_X, Q)$.

As $(1 - G(\varphi_X)$ is the proportion of home firms exporting, the average profits per period home firms is given by:

$$\bar{\pi} = \pi_D(\tilde{\varphi}_D) + (1 - G(\varphi_X))\pi_X(\tilde{\varphi}_X, Q) \qquad (3)$$

To close the model, I impose market clearing in both the factor market and the goods market. Recall the homogeneous good sector in the model. Assuming labour is perfectly mobile across the homogeneous sector and the differentiated sector, wages must be the same for both sectors. Hence the homogeneous good sector pins down the wage. Set the homogeneous good as the numeraire to give $w = 1$.

With the small open economy, I abstract from the international macroeconomic feedback from the rest of the world. Rest of the world variables P^* and C^* are fixed for the purpose of this paper so as to isolate the mechanisms in the home country. So net exports of the home country is just the difference between income and spending. The goods market equilibrium in the home country is given by: $Y = wL + \bar{\pi}N - \max\{0, \Delta n\}f_{ex} + NX$. The goods market clears when aggregate income/consumption equals labour income, total profits net of entry costs of new firms, and net exports NX. $\Delta n \equiv n_t - n_{t-1}$ is the change is the number of exporting firms. As export entry cost is only paid upon entry of firms into exporting, the total export entry cost is $\max\{0, \Delta n\}f_{ex}$.

3.2 Export entry and exit under uncertainty

Having characterized the model for a given value of φ_X, in this section I derive the entry and exit decisions when there is exchange rate uncertainty. This is an application of Dixit (1989a, 1989b) and Dixit and Pindyck (1994). By modeling firms' entry and exit choices as options, I can obtain the value of each decision and compare them to characterize each firm's optimal decision.

3.2.1 Exchange rate uncertainty

Let the exchange rate evolve exogenously following Brownian motion:

$$\frac{dQ}{Q} = \mu dt + \sigma dz$$

where dz is an increment of a standard Wiener process that is uncorrelated across time and satisfies:

$$E(dz) = 0, \quad E(dz^2) = dt$$

Starting at time $t = 0$, the initial exchange rate is $Q_0 = Q$. The random exchange rate Q_t under Brownian motion implies that lnQ_t follows a normal distribution with mean $lnQ_0 + (\mu - 1/2\sigma^2)t$ and variance $\sigma^2 t$. So μ is the trend growth rate of the exchange rate.

The equilibrium value of φ_X depends on the level of Q. The exogenously determined Q induces entry or exit into exporting, which in turn determines φ_X.

Q determines exporting profits. All home firms produce to sell to the domestic market, but they make a choice as to whether to export. The relevant profit function for the entry and exit decision is the exporting profits $\pi_X(\varphi, Q)$, given by (2), which is reproduced below with the value of $p_X(\varphi, Q)$ substituted in:

$$\begin{aligned} \pi_X(\varphi, Q) &= \frac{Q}{\theta}\left[Q^{-1}\frac{\theta}{(\theta-1)}\frac{w\tau}{\varphi}\right]^{1-\theta} P^{*\theta}C^* - f_x \\ &= Q^{\theta}\Pi(\varphi) - f_x \end{aligned} \tag{4}$$

where $\Pi(\varphi) \equiv \frac{1}{\theta}\left[\frac{\theta}{(\theta-1)}\frac{w\tau}{\varphi}\right]^{1-\theta} P^{*\theta}C^*$. As outlined in the previous section, $w = 1$ from the labour market equilibrium and P^* and C^* are constants for the small open economy. Therefore the only endogenous variable that affects the value of $\Pi(\varphi)$ is the productivity of the firm φ.

The indexing of productivity by φ makes this problem computationally convenient since the profits from exporting are increasing in firm productivity. If a firm with some productivity φ is exporting and does not choose to exit, then all firms with productivity greater than φ are more profitable and thus will also be exporting and not exit. Likewise, if a firm that is not exporting decides not to enter into exporting, then all firms that are less productive will not enter either.

3.2.2 Establishing value functions

I will now model the value of a firm based on its exporting status in order to determine the optimal switching between non-exporting and exporting.

The decision of a firm with productivity φ is based on two state variables: the current exchange rate Q, and an indicator for it's exporting status. The firm's exporting status is shown by a indicator variable that is 1 if the firm is exporting and 0 if it is not. So in state $(Q, 1)$, the firm decides whether to continue exporting or to exit when the exchange rate is at Q. Whereas in state $(Q, 0)$ the firm decides whether to continue not exporting or to enter into exporting.

Let δ be the discount rate for all firms and let Q be the starting exchange rate. I define $V_0(\varphi, Q)$ as the expected net present value of a firm with productivity φ that is not exporting and following optimal policies. Likewise, $V_1(\varphi, Q)$ is similarly defined for an active exporter.

The dynamic programming problem posed here is to find the optimal switching conditions from one value function to the other for firms at each possible productivity cutoff level $\varphi_X \in [\varphi_{min}, \infty)$.

Suppose the initial exporting productivity cutoff is φ_X. Over an interval of exchange rates Q where firms neither enter or exit exporting (so the equilibrium φ_X unchanged), each asset must be willingly held so that in expectation the return from the asset is equal to the normal return set by the discount rate δ. As Q follows a Brownian motion process, the evolution of $V_0(\varphi_X, Q)$ is given by Itô's lemma:

$$\begin{aligned} dV_0(\varphi_X, Q) &= V_0^{'}(\varphi_X, Q)dQ + \frac{1}{2}V_0^{''}(\varphi_X, Q)(\sigma Q dz)^2 \\ &= [\mu Q V_0^{'}(\varphi_X, Q) + \frac{1}{2}\sigma^2 Q^2 V_0^{''}(\varphi_X, Q)]dt + \sigma Q V_0^{'}(\varphi_X, Q)dz \end{aligned}$$

Therefore the capital gain of holding asset $V_0(\varphi_X, Q)$ is:

$$E[V_0(\varphi_X, Q)] = [\mu Q V_0^{'}(\varphi_X, Q) + \frac{1}{2}\sigma^2 Q^2 V_0^{''}(\varphi_X, Q)]dt$$

At the asset equilibrium condition, the asset has to be willingly held. The sum of the capital gain and period profit would equal the normal return $\delta V_0(\varphi_X, Q)$. This is represented in the differential equation:

$$\frac{1}{2}\sigma^2 Q^2 V_0^{''}(\varphi_X, Q) + \mu Q V_0^{'}(\varphi_X, Q) - \delta V_0(\varphi_X, Q) = 0 \quad (5)$$

I obtain the return on the asset of an active exporter in a similar way. In addition to the normal capital gain, an exporting firm is also receiving a dividend that is the operating profits from exporting. Therefore, for the values of Q where φ_X is unchanged, the asset equilibrium for the value of $\delta V_1(\varphi_X, Q)$ is given by differential equation:

$$\frac{1}{2}\sigma^2 Q^2 V_1^{''}(\varphi_X, Q) + \mu Q V_1^{'}(\varphi_X, Q) - \delta V_1(\varphi_X, Q) = f_x - Q^\theta \Pi(\varphi_X) \quad (6)$$

The general solution of (5) is given by:

$$V_0(\varphi_X, Q) = A_{0,\varphi_X} Q^{-\alpha} + B_{0,\varphi_X} Q^\beta$$

For (6), using a linear form for $V_1(\varphi_X, Q)$ and solving for the coefficients gives $\left(\frac{Q^\theta \Pi(\varphi_X)}{\delta - \mu\theta - \frac{1}{2}\theta(\theta-1)\sigma^2} - \frac{f_x}{\delta}\right)$. So the general solution is:

$$V_1(\varphi_X, Q) = A_{1,\varphi_X} Q^{-\alpha} + B_{1,\varphi_X} Q^\beta + \left(\frac{Q^\theta \Pi(\varphi_X)}{\delta - \mu\theta - \frac{1}{2}\theta(\theta-1)\sigma^2} - \frac{f_x}{\delta}\right)$$

A_{0,φ_X}, B_{0,φ_X}, A_{1,φ_X}, and B_{1,φ_X} are constants to be determined, and $-\alpha$ and β are roots to the quadratic:

$$f(\xi) \equiv \frac{1}{2}\sigma^2\xi(\xi-1) + \mu\xi - \delta = 0$$

The final term in parentheses in the expression for $V_1(\varphi_X, Q)$ has a meaningful economic interpretation. It is in fact the expected present value for staying an exporter forever, with an initial exchange rate Q:

$$E\left[\int_0^\infty (Q^\theta \Pi(\varphi_X) - f_x)e^{-\delta t}dt\right] = \left(\frac{Q^\theta \Pi(\varphi_X)}{\delta - \mu\theta - \frac{1}{2}\theta(\theta-1)\sigma^2} - \frac{f_x}{\delta}\right)$$

Hence the remaining parts of $V_1(\varphi_X, Q)$ can be interpreted as the value of the option to stop exporting optimally. And since the asset $V_0(\varphi_X, Q)$ does not in itself provide a flow of operating profits, the entire term is the value of the option of a non-exporting firm to begin exporting optimally.

By using some natural endpoint conditions, I now narrow down the solutions for $V_0(\varphi_X, Q)$ and $V_1(\varphi_X, Q)$. First consider the case when Q is extremely small, such that the chance of the exchange rate depreciating enough for a firm with productivity φ_X to warrant entry into exporting has a very low probability. In this case the value of the option to enter is near worthless, which requires $A_{0,\varphi_X} = 0$. Likewise, considering a high value for Q shows that $B_{1,\varphi_X} = 0$. Now it is possible to omit these two terms and rewrite the candidate solutions as:

$$V_0(\varphi_X, Q) = B_{\varphi_X} Q^\beta \tag{7}$$

$$V_1(\varphi_X, Q) = A_{\varphi_X} Q^{-\alpha} + \left(\frac{Q^\theta \Pi(\varphi_X)}{\delta - \mu\theta - \frac{1}{2}\theta(\theta-1)\sigma^2} - \frac{f_x}{\delta}\right) \tag{8}$$

3.2.3 Optimal switching between exporting and non-exporting status

I now link the two asset values through optimal switching from non-exporting to exporting and from exporting to non-exporting. Optimal switching is characterized by trigger values of the exchange rate that causes entry or exit when there are n firms exporting initially.

Arbitrage ensures that the Value-matching condition and the High-order Contact condition are satisfied when the options are exercised. The Value-matching condition requires that at the exchange rate at which it is optimal to exercise the option, the value of the option must equal the value of the asset acquired minus the exercise price. The High-order Contact condition requires the slopes of the two value functions to equal. Otherwise, the kink from the differing slopes would allow supposedly optimal trigger exchange rates to be improved upon, and therefore are not optimal policies to begin with.

Let I_{φ_X} be the exchange rate where it is optimal for the marginal non-exporting firm with productivity φ_X to begin exporting. The non-exporting firm pays a fixed export entry cost of f_{ex} to exercise it's option and gets $V_1(\varphi_X, I_{\varphi_X})$. So I_{φ_X} must satisfy the Value-matching condition:

$$V_0(\varphi_X, I_{\varphi_X}) = V_1(\varphi_X, I_{\varphi_X}) - f_{ex} \tag{9}$$

and the High-order Contact condition:

$$V_0'(\varphi_X, I_{\varphi_X}) = V_1'(\varphi_X, I_{\varphi_X}) \tag{10}$$

Likewise, let O_{φ_X} be the exchange rate where it is optimal for the marginal exporting firm with productivity φ_X to stop exporting. O_{φ_X} satisfies the Value-matching condition:

$$V_1(\varphi_X, O_{\varphi_X}) = V_0(\varphi_X, O_{\varphi_X}) \tag{11}$$

and the High-order Contact condition:

$$V_1'(\varphi_X, O_{\varphi_X}) = V_0'(\varphi_X, O_{\varphi_X}) \tag{12}$$

The economic interpretation for I_{φ_X} and O_{φ_X} is that these are trigger values of the real exchange rate Q that cause entry or exit into exporting, given an initial exporting cutoff of φ_X. When the marginal exporting firm has productivity φ_X, there are $n(\varphi_X)$ firms exporting. When $Q \geq I_{\varphi_X}$, the marginal non-exporting firm will enter into exporting. The new equilibrium exporting cutoff φ'_X will be below the original φ_X and $n(\varphi'_X) > n(\varphi_X)$. Whereas when $Q \leq O_{\varphi_X}$, the marginal exporting firm will optimally stop exporting.

I note by inspection that $O_{\varphi_X} < I_{\varphi_X}$. Therefore when $Q\epsilon(O_{\varphi_X}, I_{\varphi_X})$, the equilibrium value of φ_X will remain constant. The marginal exporting firm with productivity φ_X will neither switch from exporting to non-exporting and vice versa.

I now solve for the values of I_{φ_X} and O_{φ_X} by substituting in the functional forms of the candidate solutions from (7) and (8). The Value-matching conditions can be rewritten as

$$A_{\varphi_X} I_{\varphi_X}^{-\alpha} + \frac{I_{\varphi_X}^{\theta} \Pi(\varphi_X)}{\delta - \mu\theta - \frac{1}{2}\theta(\theta-1)\sigma^2} - \frac{f_x}{\delta} = B_{\varphi_X} I_{\varphi_X}^{\beta} + f_{ex} \tag{13}$$

$$A_{\varphi_X} O_{\varphi_X}^{-\alpha} + \frac{O_{\varphi_X}^{\theta} \Pi(\varphi_X)}{\delta - \mu\theta - \frac{1}{2}\theta(\theta-1)\sigma^2} - \frac{f_x}{\delta} = B_{\varphi_X} O_{\varphi_X}^{\beta} \tag{14}$$

And the High-order Contact conditions rewritten as

$$- A_{\varphi_X} \alpha I_{\varphi_X}^{-\alpha-1} + \frac{\theta I_{\varphi_X}^{\theta-1} \Pi(\varphi_X)}{\delta - \mu\theta - \frac{1}{2}\theta(\theta-1)\sigma^2} = B_{\varphi_X} \beta I_{\varphi_X}^{\beta-1} \tag{15}$$

$$- A_{\varphi_X} \alpha O_{\varphi_X}^{-\alpha-1} + \frac{\theta O_{\varphi_X}^{\theta-1} \Pi(\varphi_X)}{\delta - \mu\theta - \frac{1}{2}\theta(\theta-1)\sigma^2} = B_{\varphi_X} \beta O_{\varphi_X}^{\beta-1} \tag{16}$$

From (13)-(16), I can obtain the optimal policy by solving for I_{φ_X}, O_{φ_X}, A_{φ_X}, and B_{φ_X} for $\varphi_X \in [\varphi_{min}, \infty)$. I will conduct both analytical analysis as well as numerical calculations in the next section, which will shine light on the insights provided by the optimal export entry and exit decisions.

4 Results

In this section, I show the implications of the model through analytical and numerical analyses. I look at the consequences of optimal exporting/non-exporting switching policies in determining the number of firms exporting and their productivities.

4.1 Analytical results

The optimal policies described above in section 3.2 refer to the optimal switching decisions made by the marginal exporting firm with productivity φ_X, for each value of $\varphi_X \in [\varphi_{min}, \infty)$. I now extrapolate the implications of optimal switching for all other exporters with different productivity levels than the marginal firm.

Consider an economy with the equilibrium cutoff of φ_X and only firms with greater productivity are exporting. The number of firms exporting is thus $n(\varphi_X)$. The exit trigger for marginal exporting firm is O_{φ_X}. Recall that firm productivity is indexed by φ, and from (4) I know that π_X is increasing in φ. So all firms with productivity greater than φ_X make greater profits from exporting than the marginal exporting firm. Therefore, if the firm with productivity φ_X continues exporting (i.e. $Q > O_{\varphi_X}$), all firms that are more productive will also continue to export. Similarly, as all firms with productivity less than the firm with productivity φ_X make less profit from exporting. So long as this firm does not begin exporting (i.e. $Q < I_{\varphi_X}$), all firms with lower productivity will not begin exporting.

For a given equilibrium with marginal exporter productivity φ_X, $Q \epsilon (O_{\varphi_X}, I_{\varphi_X})$ is the area of inaction for the entire economy. Firms will neither enter into or exit from exporting if the exchange rate Q stays within this interval. By extension, firms will only switch into or out of exporting when Q is outside this interval. This is my first result

Proposition 1. *Upon small fluctuations within the exchange rate (i.e.* $Q\epsilon(O_{\varphi_X}, I_{\varphi_X})$ *for some equilibrium* φ_X*), firms do not enter into or exit from exporting.*

Proof. When $Q > O_{\varphi_X}$, no firms exit from exporting. When $Q < I_{\varphi_X}$, no firms start exporting. Since $O_{\varphi_X} \leq I_{\varphi_X}$, firms neither begin exporting or stop exporting when $O_{\varphi_X} < Q < I_{\varphi_X}$. □

While the effect in the above proposition is due to the existence of the sunk cost of entry into exporting f_{ex}, the uncertainty in the exchange rate contributes to the size of the range $(O_{\varphi_X}, I_{\varphi_X})$. I will now show the role of uncertainty by comparing the exporting profits with f_{ex}.

For an economy at the equilibrium with exporting productivity cutoff of φ_X, I define the incumbency premium $J_{\varphi_X}(Q)$ as:

$$J_{\varphi_X}(Q) = V_1(\varphi_X, Q) - V_0(\varphi_X, Q) \tag{17}$$

By substituting the candidate solutions for $V_0(\varphi_X, Q)$ and $V_1(\varphi_X, Q)$ from (7) and (8), I can write $J_{\varphi_X}(Q)$ as:

$$J_{\varphi_X}(Q) = A_{\varphi_X} Q^{-\alpha} - B_{\varphi_X} Q^{\beta} + \left(\frac{Q^{\theta}\Pi(\varphi_X)}{\delta - \mu\theta - \frac{1}{2}\theta(\theta-1)\sigma^2} - \frac{f_x}{\delta} \right)$$

This allows the value-matching conditions be written in terms of $J_{\varphi_X}(Q)$

$$J_{\varphi_X}(I_{\varphi_X}) = f_{ex} \qquad J_{\varphi_X}(O_{\varphi_X}) = 0 \tag{18}$$

And the higher-order contact conditions as

$$J'_{\varphi_X}(I_{\varphi_X}) = 0 \qquad J'_{\varphi_X}(O_{\varphi_X}) = 0 \tag{19}$$

Note also that $J''_{\varphi_X}(I_{\varphi_X}) < 0$ and $J''_{\varphi_X}(O_{\varphi_X}) > 0$.

I then subtract the asset equilibrium conditions (5) from (6) to show that $J_{\varphi_X}(Q)$ satisfies the differential equation:

$$\frac{1}{2}\sigma^2 Q^2 J''_{\varphi_X}(Q) + \mu Q J'_{\varphi_X}(Q) - \delta J_{\varphi_X}(Q) = f_x - Q^{\theta}\Pi(\varphi_X)$$

Evaluating this at I_{φ_X} gives:

$$\begin{aligned} f_x - I^{\theta}_{\varphi_X}\Pi(\varphi_X) &= \frac{1}{2}\sigma^2 I_{\varphi_X} Q^2 J''_{\varphi_X}(I_{\varphi_X}) + \mu Q J'_{\varphi_X}(I_{\varphi_X}) - \delta J_{\varphi_X}(I_{\varphi_X}) \\ &< -\delta f_{ex} \end{aligned}$$

where the latter inequality is obtained by using (18) and (19). In other words:

$$\pi_X(\varphi_X, I_{\varphi_X}) > \delta f_{ex} \tag{20}$$

Similarly, evaluating at O_{φ_X} gives:

$$\pi_X(\varphi_X, O_{\varphi_X}) < 0 \tag{21}$$

Therefore in order for a firm to enter, the exchange rate has to allow exporting profits to cover more than the interest on the cost of entry. Likewise, firms need to make worse than zero profit to optimally exit from exporting.

This is because exchange rate uncertainty means that firms will only enter when their export profits are high enough to cover the cost of entry and the value of the option to enter into exporting. It also means that firms will exit only when they make a bigger loss than the value of option to exit from exporting. Were there no uncertainty in the exchange rate (20) and (21) would simply be equalities. Uncertainty widens the range $(O_{\varphi_X}, I_{\varphi_X})$ in which firms neither enter or exit exporting.

4.2 Numerical results

In order to get a better understanding of what the export entry and exit exchange rate triggers are, I turn to numerical calculations in this section. I show how the number of firms currently exporting changes the levels of O_{φ_X} and I_{φ_X}. I then show how the exchange rate affects the number of firms exporting and the productivities of those firms.

First, I specify the distribution of productivity $G(\varphi)$. I assume the Pareto distribution for $G(\varphi)$, so that $G(\varphi) = 1-(\varphi_{min}/\varphi)^{\gamma}$, where φ_{min} is the minimum value for φ and γ is a positive parameter. Recall that φ_X is the productivity of the marginal exporting firm when there are n firms exporting, such that $n(\varphi_X) = (1 - G(\varphi_X))N$. The level of n for each φ_X can be obtained.

I now obtain the numerical solutions for I_{φ_X} and O_{φ_X} by solving simultaneously (13)-(16) for each level of $\varphi_X \in [\varphi_{min}, \infty)$. Let the total number of firms N be 100 and φ_{min} be 1. For $\mu = 0$ and $\sigma = 0.2$, the results of the numerical calculation is shown in Figure 1.

The area between the two lines O_{φ_X} and I_{φ_X} show the region of inaction. For an initial equilibrium of φ_X, firms will neither start or stop exporting when exchange rates are within this interval. This illustrates the range of Q for each equilibrium in which Proposition 1 applies. When the exchange rate goes above I_{φ_X} some firms will enter exporting and when it goes below O_{φ_X} some firms will stop exporting.

Note that both exchange rate triggers are decreasing with the productivity of the marginal exporter φ_X. This can be understood by considering the relationship of φ_X with the profits from exporting. When there are only a few firms exporting, the only firms that do so are high productivity firms and the marginal exporting firm also has high productivity. This means that firms that are exporting have relatively high exporting profit, π_X. These profitable firms will require a lower exit trigger O_{φ_X} to deter them from exporting. Whereas when the marginal exporting firm has a lower productivity, and more firms are exporting, export profits of the marginal exporter

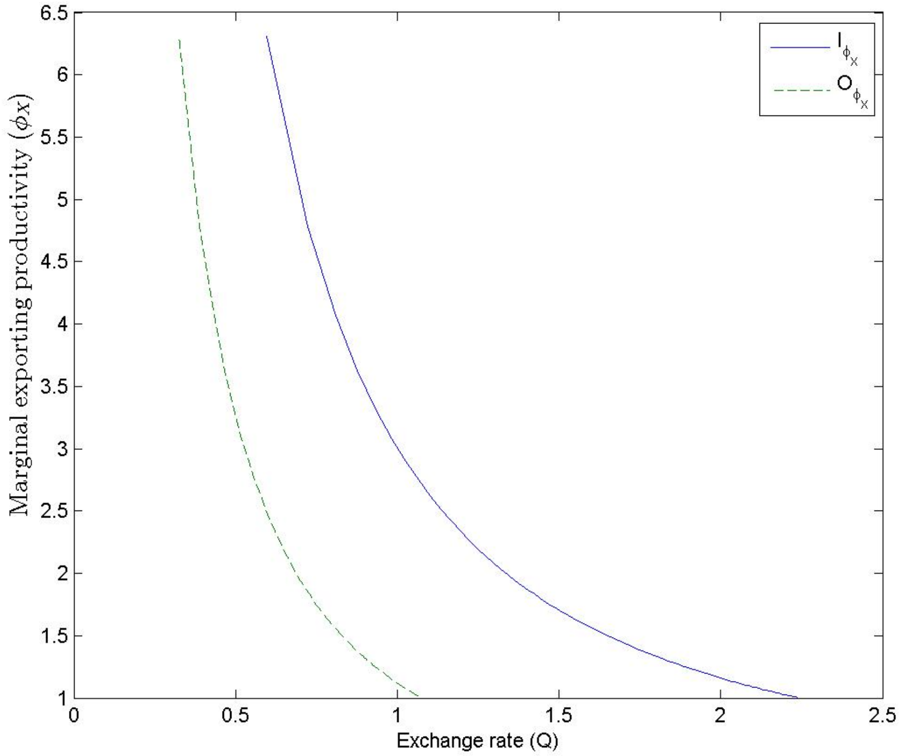

Figure 1: Exchange rate triggers for levels of φ_X

are lower and there is higher exit trigger exchange rate. The relationship between I_{φ_X} and φ_X can similarly be explained by the productivity of firms deciding to enter into exporting at each φ_X.

Another feature of Figure 1 is the the narrowing of the interval $(O_{\varphi_X}, I_{\varphi_X})$ as φ_X increases, which suggests the area of inaction decreases with the productivity of firms exporting. The intuition behind this can be seen when considering the relative size of the sunk entry cost f_{ex} to the productivity levels of exporting firms. When there are only a few highly productive firms exporting, firms at both sides of the margin of exporting have relatively high profits from exporting, π_X, due to their high productivity. Whether to pay f_{ex} to enter exporting is a minor consideration for these firms. On the other hand, when there are more firms exporting the marginal exporting firm has lower productivity. The firms on either side of the marginal exporter have comparatively lower π_X, and so the need to incur f_{ex} is more of a determining factor of their exporting status.

I reinterpret the results in terms of $n(\varphi_X)$ in Figure 2 for a more intuitive representation. For an economy with n firms exporting, the exchange rate can fluctuate between $(O_{\varphi_X}, I_{\varphi_X})$ for the corresponding φ_X to maintain the equilibrium with n exporting firms. When $Q > I_{\varphi_X}$, firms will enter exporting resulting in more exporters. When $Q < O_{\varphi_X}$, firms will stop exporting and n will fall.

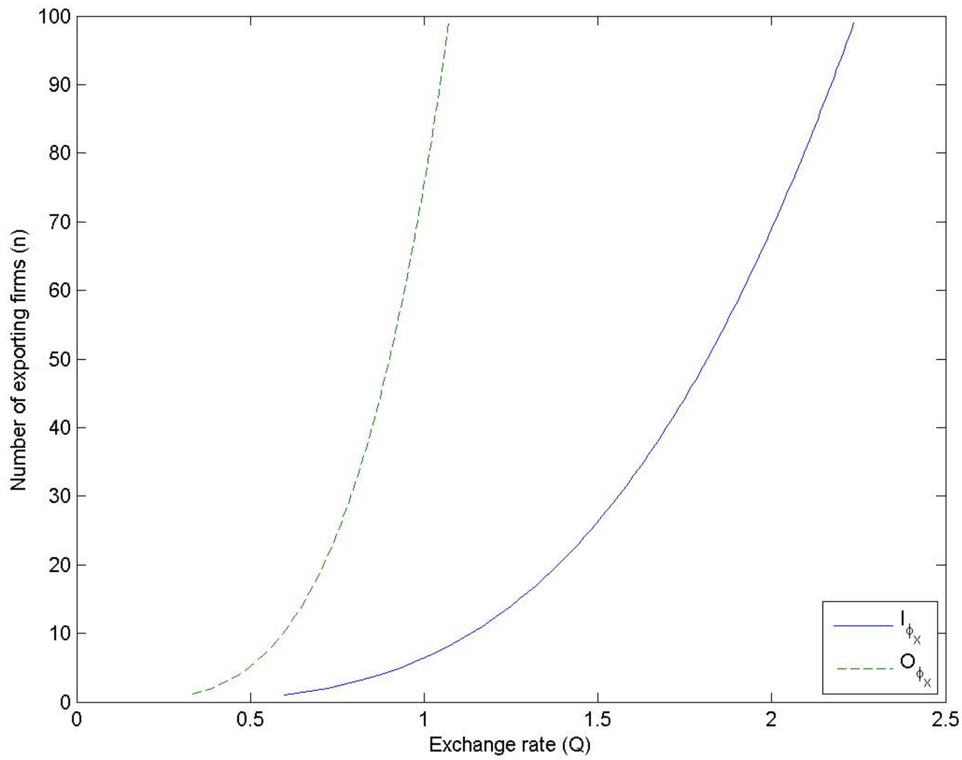

Figure 2: Exchange rate triggers for levels of $n(\varphi_X)$

4.2.1 Intensive and extensive margins

I now characterize how exporting firms adjust to fluctuations of different sizes in the exchange rate. From Proposition 1 and Figures 1 and 2, I have established that the exchange rate can fluctuate between O_{φ_X} and I_{φ_X} without trigger entry or exit into exporting, at some initial level of φ_X. In this section, I will show when firms adjust through the intensive margin, by changing the scale of their exporting activities, and when the adjustment is through the extensive margin.

Pricing at monopolistic levels, a home exporting firm with productivity φ sells to the rest of the world the demand for for their good:

$$c^*(\varphi) = \left(\frac{p_X(\varphi)}{P^*}\right)^{-\theta} C^* = \left(\frac{Q^{-1}\frac{\theta}{(\theta-1)}\frac{w\tau}{\varphi}}{P^*}\right)^{-\theta} C^*$$

As these firms operate in a small open economy, the only variable that is not a constant in the above expression is Q. The size of each exporting firm changes with Q. When Q fluctuates within $(O_{\varphi_X}, I_{\varphi_X})$, the number of firms does not change. Hence the economy adjusts purely on the intensive margin while the exchange rate fluctuates within the area of inaction.

The following proposition summarizes this finding:

Proposition 2. *Exporting firms adjust only on the intensive margin to fluctuations of Q within $(O_{\varphi_X}, I_{\varphi_X})$.*

This result is illustrated in Figures 3 to 6 by the solid lines. In Figure 3, the marginal exporter has productivity level $\varphi_X = 1.9$ and there are initially 20 exporting firms (i.e. n=20). The solid line shows a one time depreciation of the real exchange rate Q to a level still within $(O_{1.9}, I_{1.9})$. The number of firms exporting, n, and the aggregate productivity of those firms, $\bar{\varphi}_X$, are unchanged. All the adjustment is through the increase in average exporting profits, $\bar{\pi}_X$ and in total exports. Figure 4 shows that if the depreciation is reversed soon after, exporters adjust back to their initial sizes through the intensive margin.

I repeat this exercise with an appreciation of the exchange rate in Figures 5 and 6. The solid lines show an appreciation of the real exchange rate within $(O_{1.9}, I_{1.9})$. Again, all the adjustment is only through the intensive margin.

This result is useful in explaining the stylized fact that export surges caused by a depreciation in the exchange rate are due to adjustments along the intensive rather than the extensive margin. As long as Q fluctuates within $(O_{\varphi_X}, I_{\varphi_X})$, all of the adjust is made along the intensive margin.

I now analyze what happens to the exporting firms when Q moves outside of the area of inaction $(O_{\varphi_X}, I_{\varphi_X})$. When Q moves above I_{φ_X}, lower productivity firms previously not exporting will begin to find it profitable to enter. φ_X will fall until the new level of φ_X^*, where φ_X^* equates $I_{\varphi_X^*}$ with the higher level of Q. The number of exporting firms increases by $n(\varphi_X^*) - n(\varphi_X)$ and therefore the adjustment is along the extensive margin in addition the the intensive margin.

Similarly, when Q falls below O_{φ_X} some firms with lower productivity currently exporting will no longer find it attractive to export. φ_X will increase until φ_X^{**}, which equates $O_{\varphi_X^{**}}$ with the lower Q. The number of exporting firms fall by $n(\varphi_X) - n(\varphi_X^{**})$ and again the adjustment is in the extensive margin.

I state this result in the proposition:

Proposition 3. *Exporting firms adjust on both the intensive and extensive margins when Q fluctuates above or below $(O_{\varphi_X}, I_{\varphi_X})$.*

This adjustment is shown in Figures 3 to 6 by the dotted lines. The dotted lines in Figure 3 show the effect of Q depreciating above $I_{1.9}$. More firms begin exporting and n rises from 20 to 30. Hence the adjustment is also on the extensive margin. The exporting productivity cutoff φ_X falls and so does the aggregate productivity of exporters, $\bar{\varphi}_X$. Profits from exporting increases as do total exports. The dotted lines in Figure 5 shows the same effects in the other direction for an appreciation of Q below $O_{1.9}$.

In Figures 4 and 6, a hysteresis effect of exporting firms is apparent; larger exchange rate fluctuations have persistent effects, whereas smaller fluctuations within the area of inaction do not have such effects.

The dotted lines in Figure 4 show the effects of a depreciation of the exchange rate above $I_{1.9}$ that is reversed soon after. This temporary change in the exchange rate causes the number of firms to increase permanently even as the exchange rate reverts back to its original level. Exporters become on average less productive, with lower

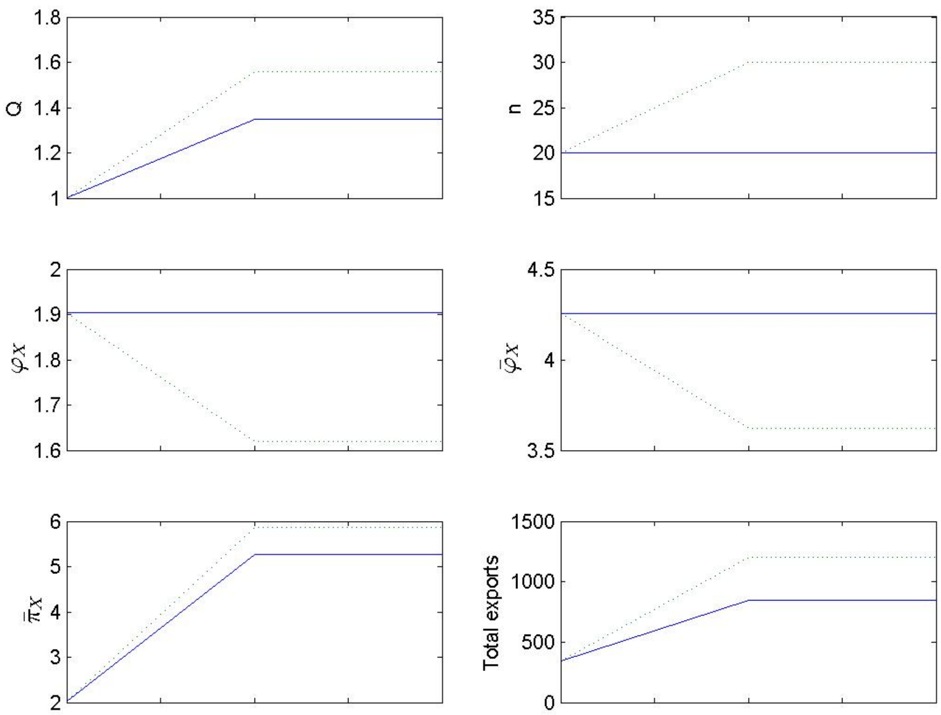

Figure 3: Response to a one off depreciation

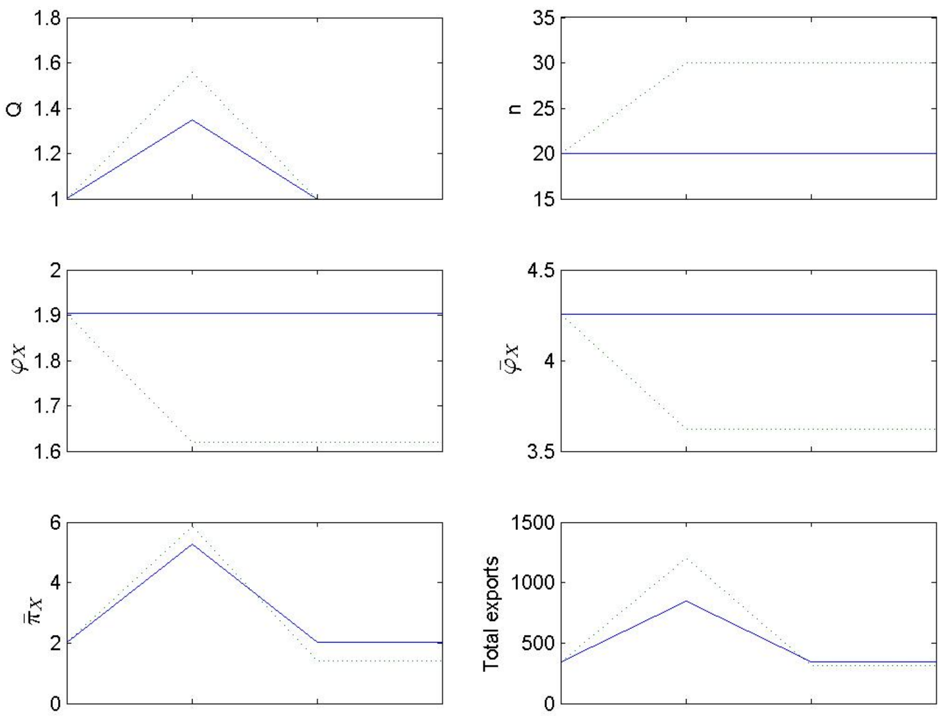

Figure 4: Response to an depreciation and reversal

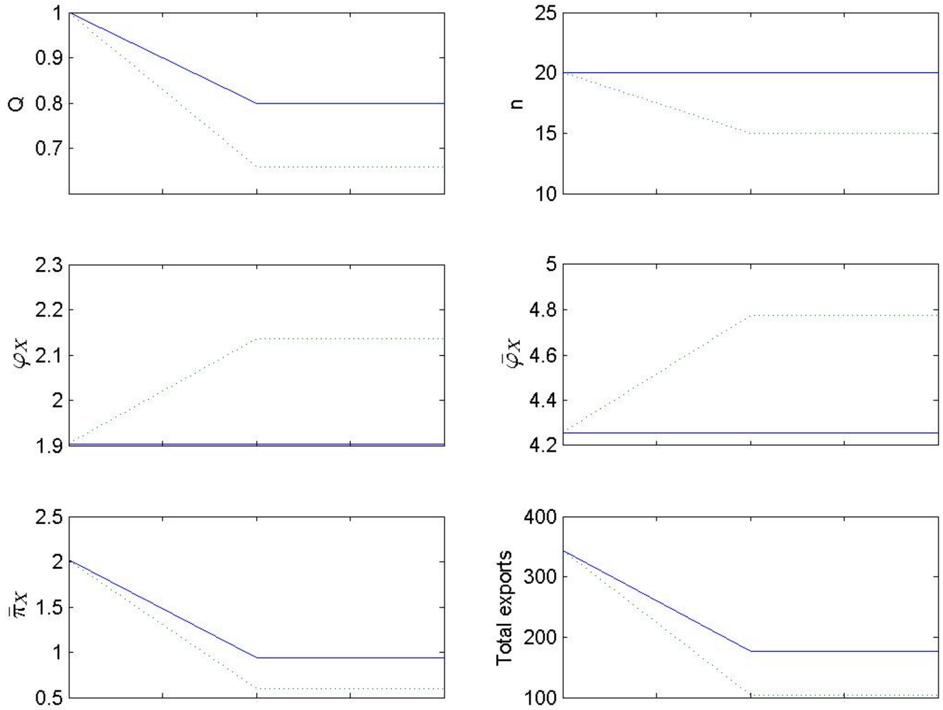

Figure 5: Response to a one off appreciation

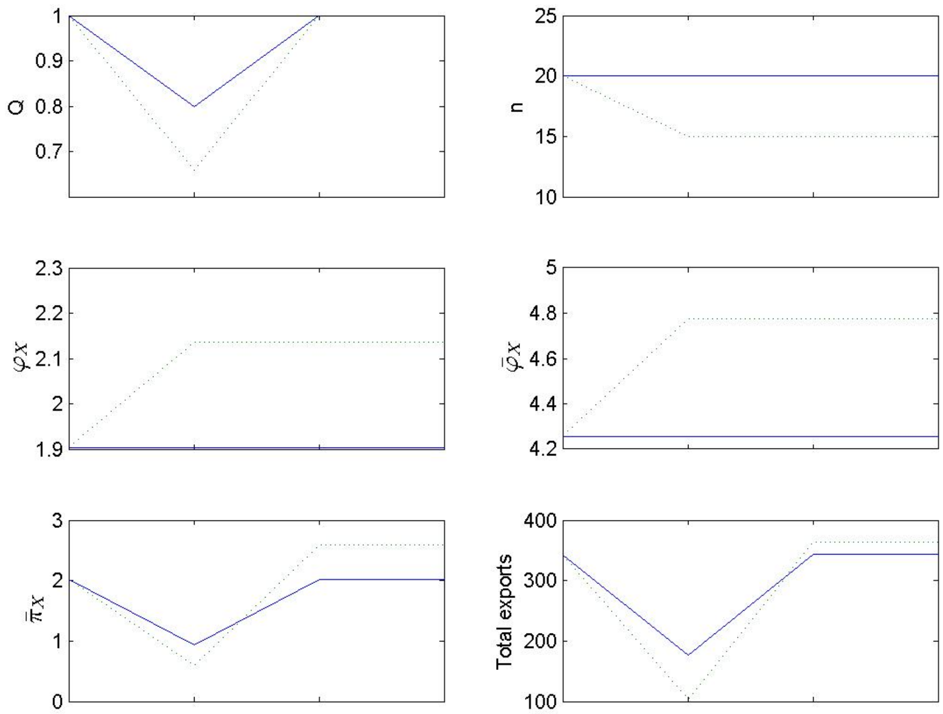

Figure 6: Response to an appreciation and reversal

$\bar{\varphi}_X$, and subsequently the average profits from exporting is lower. With more firms exporting, but the average exporters being less productive, total exports returns to only slightly below its original level. Therefore a temporary change in the exchange rate can have a persistent effect of the number of firms exporting and the productivities of those exporters.

Figure 6 shows the hysteresis upon an appreciation of the exchange rate. As shown by the dotted lines, a large, temporary appreciation of the exchange rate causes the number of firms exporting to permanently decrease and the productivity of those exporting firms to increase. The average profits of exporters rises in the long run and the total exports increase slightly. In contrast, a smaller temporary appreciation as shown by the solid lines have no such effect.

I now summarize this result in the following proposition:

Proposition 4. *The number exporting firms and the productivity of those exporters experience hysteresis due to fluctuations in the real exchange rate*

In terms of the hysteresis in the number of firms in Proposition 4, this finding is similar to the one reached in Dixit (1989b). Exchange rate fluctuations can have persistent effects on the number of firms exporting. What is different in my model is the implications of the hysteresis on firm productivity. Exporters with lower productivity that entered upon a large favourable exchange rate shock do not abandon their sunk cost investments when the exchange rate resumes its previous level. And a large appreciation in the exchange rate can persistently remove higher productivity exporters from exporting. Hence the hysteresis of exporting firms is also one of productivity. Exchange rate fluctuations can have a persistent effect on the aggregate productivity level of exporting firms.

By extension, I can extend the implications of exporter hysteresis onto results on exchange rate pass-through. Recall that export prices are set depending on the productivity of each firm with $p_X(\varphi, Q) = Q^{-1}\frac{\theta}{(\theta-1)}\frac{w\tau}{\varphi}$. With small exchange rate fluctuations within the area of inaction, the exchange rate Q has a one-to-one inverse relationship with the export price for all exported goods. This effect is more nuanced as soon as the extensive margin changes. When the exchange rate depreciates above I_{φ_X}, firms entering into exporting have relatively lower productivity. Their prices are higher than the original set of exporters which pulls up the average price of exports. With a fall in the exchange rate below O_{φ_X} the remaining firms after firms have exited have higher productivity and therefore lower export prices.

While the exchange rate pass-through in this model is a direct result of the pricing assumptions of exports, the insight I gain from the productivity hysteresis is that the exchange rate pass-through is smaller when the entry and exit triggers are crossed and adjustments are made along the extensive margin.

4.2.2 Different levels of firm heterogeneity

I now show how the distribution of productivity among firms, $G(\varphi)$, affects the nature of the trade and productivity hysteresis. In a Pareto distribution, the shape parameter γ determines the distribution of firm productivities. The higher the value of γ, the more low productivity firms and fewer high productivity firms (less heterogeneous). As γ approaches infinity, every firm will be have productivity φ_{min} (homogeneous firms).

Since the mean of a Pareto distribution is $E(\varphi) = \frac{\gamma\varphi_{min}}{\gamma-1}$ if $\gamma > 1$, I compare the effects of a change in the distribution by keeping the same mean as I change the level of γ. Thus the value of φ_{min} is adjusted accordingly to maintain a constant mean. Changing γ in this way will enable the isolation of the effects of changing the shape of distribution $G(\varphi)$.

As I am changing the distribution of firm productivity, comparing exporting cutoff productivity φ_X's is not meaningful. Instead, I focus on the effects on equilibrium n. Figure 7 shows how the shape of distribution $G(\varphi)$ affects the entry and exit triggers for each equilibrium value of n. The solid line set of entry and exit triggers are of parameter values $\gamma = 2.5$ and $\varphi_{min} = 1$. The set of dotted lines correspond to the parameter values $\gamma = 5$ and $\varphi_{min} = 4/3$. Economically speaking, the former

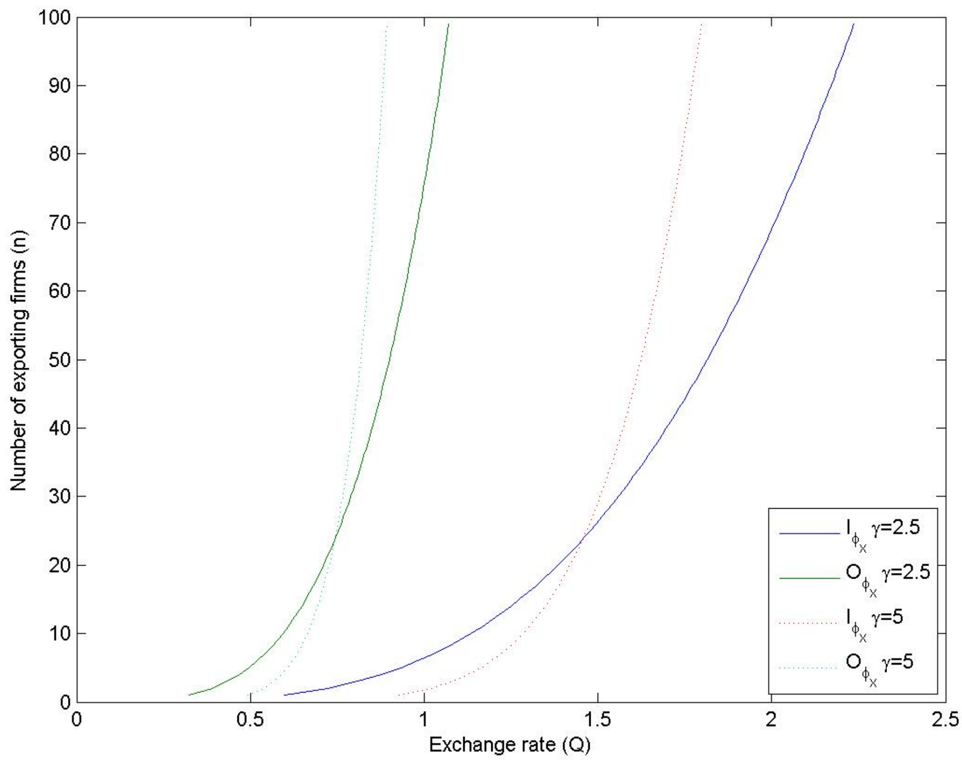

Figure 7: The effects of changing the shape parameter γ of distribution $G(\varphi)$

case has greater firm heterogeneity than the latter. In both cases, the mean is set to $E(\varphi) = 5/3$.

This figure shows that higher firm heterogeneity decreases the size of extensive margin adjustments. The lines corresponding to $\gamma = 2.5$ are steeper than the lines for $\gamma = 5$. This implies that were the exchange rate to go above $I_{\varphi X}$ or below $O_{\varphi X}$, the change in n would be smaller. So upon large exchange rate realizations, the extensive margin would adjust less the more firm heterogeneity there is. Whereas in the extreme case when there is no heterogeneity and firms are homogeneous, the entry and exit triggers will induce all firms to enter or exit at once.

Another observation I make from Figure 7 is that increased firm heterogeneity makes the area of inaction larger when many lower productivity firms are exporting, but smaller when only a few high productivity firms do so. This suggests that heterogeneity reduces hysteresis effect for when there is a small number of firms exporting and increases the effect for when there are more firms.

This exercise demonstrates how the extent of firm heterogeneity affects the export sector responses through the intensive and extensive margins. More heterogeneity implies a smaller extensive margin adjustment overall, and a larger hysteresis effect when there are many firms exporting.

5 Free entry

In this section I extend the model by allowing free entry of firms into the domestic market. By doing so, new cohorts of firms may have a different split between firms exporting and not exporting as compared to incumbent firms. I show how two firms with the same productivity level may have a different decision on their exporting status.

Suppose there is a mass of firms looking to enter into the domestic, and potentially export, sales. Prior to entry, they face uncertainty over their level of productivity. These firms upon paying a fixed cost of entry, F, draw their productivity from distribution $G(\varphi)$ (which is the same as the distribution of incumbent firms).

Profits of existing firms, the incumbent cohort, are altered by firm entry. However, since wages remain constant due to the homogeneous goods sector, only the profits of domestic sales change via the price index P. The profits from exporting $\pi_X(\varphi, Q)$ are not affected by firm entry. With no fixed cost to selling domestically, the N firms from the incumbent cohort would never exit. Hence the results from the previous sections regarding the entry and exit decisions among the incumbent firms are unaltered.

I denote all variables that pertain to the cohort of entrants with a superscript E. A firm from the entry cohort, having decided to enter already and realizing productivity φ^E, will export if the value of doing so, $V_1(\varphi^E, Q) - f_{ex}$, exceeds the value of not exporting, $V_0(\varphi^E, Q)$, at a given exchange rate Q. The marginal exporting firm among entrants will be indifferent between exporting and not exporting. Hence the productivity cutoff for the marginal exporting firm among the entrants, φ_X^E, is given by:

$$\varphi_X^E : V_0(\varphi_X^E, Q) = V_1(\varphi_X^E, Q) - f_{ex} \tag{22}$$

So only entrants with productivity greater than φ_X^E will export.

The mass of firms entering can then be determined by free entry. Free entry means that firms will enter until the expected profits from entering would just equal the fixed cost of entry F. Recall that $\tilde{\varphi}_D$ is defined as the aggregate productivity of all active firms given their distribution of $G(\varphi)$. Further define $\tilde{\varphi}_X^E$ in the same way as $\tilde{\varphi}_X$ so that it is the aggregate measure of productivity of exporters in the entrant cohort. The expected profits from (3) of an entrant would thus be:

$$\bar{\pi}^E = \pi_D(\tilde{\varphi}_D) + (1 - G(\varphi_X^E))\pi_X(\tilde{\varphi}_X^E, Q) \tag{23}$$

Free entry would therefore imply that the flow of expected profits in present value would equal the cost of entry F:

$$\frac{\bar{\pi}^E}{\delta} = F \tag{24}$$

The above condition holds because the mass of entrants, N^E, directly contributes into lowering domestic profits π_D through the price index. Firms will enter until $\bar{\pi}^E$ is low enough for (24) to hold, so N^E can be solved explicitly. The number of entrant firms exporting, n^E, can then be obtained from $n^E = (1 - G(\varphi_X^E))N^E$.

With free entry, I can draw some results by comparing the productivity cutoff of new entrants, φ_X^E, with the corresponding value of the incumbents, φ_X. I do this by referring back to Figure 1.

For the entrant cohort, firms are only entering. This means the value of φ_X^E will be at the point where Q meets the I_{φ_X} line in Figure 1. This can be verified by comparing (9) with (22). The exporter productivity cutoff among entrants φ_X^E will be equal to φ_X when $Q = I_{\varphi_X}$. Hence the level of φ_X^E is equal to the level of φ_X where the current exchange rate Q meets the I_{φ_X} line. That is, the marginal entrant exporter is determined by the lowest productivity at which a firm will begin exporting.

Whereas, for that same level of Q, the incumbents could have their marginal exporter φ_X anywhere between the I_{φ_X} and O_{φ_X} lines. Lower productivity incumbents have already paid the sunk entry cost f_{ex}. They may still be exporting even though they would not have entered at the current Q. As I_{φ_X} is weakly decreasing with φ_X, it is clear that $\varphi_X^E \geq \varphi_X$.

I illustrate the relative productivities of entrants and incumbents by examining three simple cases. For each case, the exchange rate changes differently between when the incumbents entered and when the entrants enter:

1. Q is constant. Both the incumbent cohort and the entrants entered exporting based on the same Q. Clearly, both cohorts have the same exporter productivity cutoff, so entrants and incumbent exporters have the same productivity.

2. Q appreciates (falls). Incumbent exporters had entered under a more favourable exchange rate than entrant exporters. Having paid the export entry cost, less productive incumbent firms can continue exporting. While new entrant firms must pay the export entry cost to export, and therefore must be more productive than incumbents exporters.

3. Q depreciates (rises). At the higher exchange rate, non-exporting incumbent firms can pay the export entry cost and begin exporting. New entrants must pay export entry costs to export. Since the marginal exporter in both cohorts have to pay the export entry cost, exporters in both cohorts have the same productivity.

From this I reach the following result on the relative productivities of exporters among cohorts:

Proposition 5. *New cohorts of exporters are more productive or at least as productive as incumbent exporters.*

Proof. Recall the definition for aggregate exporter productivity:
$\tilde{\varphi}_X \equiv \left[\frac{1}{1-G(\varphi_X)}\int_{\varphi_X}^{\infty}\varphi^{\theta-1}g(\varphi)d\varphi\right]^{\frac{1}{\theta-1}}$, which is increasing in the exporting productivity cutoff φ_X. As $\varphi_X^E \geq \varphi_X$, the aggregate productivities of both cohorts can be calculated to show that $\tilde{\varphi}_X^E \geq \tilde{\varphi}_X$. □

The result that $\varphi_X^E \geq \varphi_X$ explains the stylized fact that two firms with the same productivity can have a different exporting status. The marginal exporter among the incumbent firms with productivity φ_X has already paid the sunk export entry cost, f_{ex}. As the marginal exporter among the entrant firms must incur this entry cost in the current period, an entrant with the same productivity may not be able to afford the entry cost. The marginal exporter among the entrants must therefore have a higher productivity level, φ_X^E.

My model is easily expandable into more cohorts. This period's entrants will in the next period be a cohort of incumbents. As more periods are added, each cohort is defined by the total number of firms in the cohort (N and N^E for the two cohorts used in this paper) and the number of these firms that are exporting (n and n^E). The model will thus consist of one entrant cohort and many incumbent cohorts, all adjusting or not adjusting to the current exchange rate Q.

6 Conclusion

In this paper I have outlined a model that shows how exchange rates can cause hysteresis in an export sector of heterogeneous firms. My model shows how temporary exchange rate changes can have permanent effects on the number of exporters and the productivity of those exporters. I also explain several stylized facts. Trade hysteresis provides an explanation to when and how trade adjusts along the intensive and extensive margins. Also by expanding the model to free entry, I provide a story for why there may be non-exporting firms with higher productivity than some exporters.

These results are useful in explaining the composition and the market structure of the export sector. My model also provides a framework for understanding trade adjustments to macroeconomic shocks.

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