Network clearing algorithms and statistical methods for risk assessment

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Abstract

This thesis presents methodological contributions for the quantification of systemic risk in financial systems. Clearing algorithms that can be used for computing financial contagion effects are presented and their relation to network centrality measures is discussed. An algorithm that allows computing clearing solutions for networks of financial contracts including debt of multiple seniority classes, equity participations as well as contingent convertible debt and bail-in-able debt is presented. We present a valuation function that allows computing network-based valuations of contingent convertible and bail-in-able debt instruments. An application to systemic risk assessment for bail-in decisions using real-world data is presented, showing that bail-ins have the potential to reduce systemic risk in some crisis situations. An alternative approach to systemic risk assessment based on a metamodel of clearing algorithms is prestented. We use quantile panel regression for learning the metamodel and present statitistical tests for deciding between different quantile panel estimators as well as goodness-of-fit measures. An application with real-world-data shows how regulators could improve their identification of systemically important banks.

Contents

Acknowledgements

Parts of this thesis are contained in the following papers:

- ‘**Clearing Algorithms and Network Centrality**’
  - Status: Published. *Proceedings of the VIIth International Conference on Complex Networks and their Applications* (pp. 499-507). Springer
  - Authors: Christoph Siebenbrunner
- ‘**Network Valuation under Equity Conversions**’
  - Status: Under Review. *Annals of Operations Research*
  - Authors: Matias Puig (Bluecap Management Consulting), Christoph Siebenbrunner
- ‘**Assessing the systemic risk impact of Bank Bail-Ins**’
  - Status: Working Paper
  - Authors: Christoph Siebenbrunner, Martin Guth (Oesterreichische Nationalbank, OeNB), Ralph Spitzer (OeNB), Stefan Trappl (OeNB)
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  - Authors: Christoph Siebenbrunner, Michael Sigmund (OeNB)

Parts of the work in this thesis have been presented at and benefited from participants' feedback at the following conferences:

- November 2016, Rome, Italy: Financial Distortions and Macroeconomic Performance Policy Clinic. Stress Testing Systems
- March 2017, London, UK: STEM for Britain. Poster Presentation: Quantifying the Systemic Impact of Bank Failures
- June 2017, Helsinki, Finland: RiskLab/BoF/ESRB Conference on Systemic Risk Analytics. Poster Presentation: Clearing Algorithms and Network Centrality
- September 2017, Mexico City, Mexico: Network models and stress testing for financial stability. Presentation: What drives pricing in interbank markets?
- September 2017, Palermo, Italy: International Rome Conference on Money Banking and Finance. Presentation (held by co-author): What drives pricing in interbank markets?
- May 2018, Helsinki, Finland: RiskLab/BoF/ESRB Conference on Systemic Risk Analytics. Presentation: Do Interbank Markets Price Systemic Risk?
- June 2018, Rome, Italy: Financial Engineering and Banking Society International Conference. Presentation (held by co-author): Do interbank markets price systemic risk?

- June 2018, Seoul, South Korea: International Panel Data conference. Presentation (invited, could not attend due to last-minute funding issues): An application of quantile panel estimation to systemic risk

- September 2018, Lucca: Workshop on Networks in Economics and Finance.

  – Presentation (held myself): Does network centrality determine pricing on interbank markets?

  – Poster presentation (held by co-author): Multilayer network valuation under bail-in and fire sales

- October 2018, Oxford, UK: Political Economy of Finance. Presentation: Do Interbank Markets Price Systemic Risk?

- December 2018, Cambridge: International Conference on complex networks and their applications. Poster presentation (invited, unable to attend): Clearing Algorithms and Network Centrality

- April 2019, Paris, OECD Headquarters: New Analytical Tools and Techniques for Economic Policymaking.

  – Presentation (held myself): Money creation and liquid funding needs.

  – Presentation (held by co-author): If stress-tests are predictable, are they still useful?

- June 2020, Chania, Greece: Financial Engineering and Banking Society (cancelled due to Covid-19).

- Paper accepted for presentation: Pricing of Risks on the Interbank Market.
- Paper accepted for presentation: Assessing the systemic risk impact of bank bail-ins

Symbols used

Table 1: Commonly used symbols

Symbol	Description
n	Size of the system
e	Vector of external assets
L	Matrix (or tensor) of bilateral liabilities, where $L_{i,j,(s)}$ represents liabilities from i to j (in seniority class s)
i, j	Generally used to designate vector, matrix or tensor entries corresponding to individual nodes
s	Generally used to designate vector entries corresponding to seniority classes
S	Most junior seniority class
Σ	Summation operator
k	Generally used to designate iteration steps
Θ	Matrix of ownership shares, where $\Theta_{i,j}$ is the share of node i owned by node j
$\bar{p}$	Vector of total liabilites (marginal sum of L)
$\bar{P}$	Matrix of total liabilities by seniority class

p, (P)	Vector (matrix) of actual payments (less than or equal to liabilities)
$\cdot^*$	Generally denotes solutions to fixed point problems, e.g. p^*
Π	Matrix (or tensor) of relative liabilities, where $\Pi_{i,j,(s)}$ corresponds to the share that loan $L_{i,j,(s)}$ makes up of node i's total liabilities (in seniority class s)
Π', $(\Pi_{\cdot,\cdot,s})$	Transpose of Π (for seniority class s). $\cdot'$ more generally denotes the transpose of a matrix.
r, (r_e)	Recovery value for (external) assets
f	Map returning the minimum of total liabilities and total assets
$\mathbf{I}_n$	n-dimensional identity matrix
$\mathbb{R}$	Field of real numbers
o	Original value of external assets
ν	Shock to value of external assets
m	Interpolation value used to scale shocks to external assets
λ, v	Eigenvalues and eigenvectors
$\omega(\cdot)$	Maximum eigenvalue of a matrix
ζ	Systemic risk measure based on Katz-centrality
$\mathbf{1}^n$	Vector of ones of dimension n
η	Vector of initial node weights for Katz-centrality
$\forall$	For all
max, min	Maximum, minimum
argmin	Argumentum minimae
s.t.	subject to

$(\cdot)^+$	Element-wise maximum of zero and element of a vector
V	Vector of equity values (used in clearing algorithms)
ψ	Equity map
Φ	Payments map
$\mathcal{A}$	Net interbank assets
H	Vector keeping track of which seniority class at which bank may potentially face losses in the current iteration
φ	Fraction of CoCo liabilities that are converted
λ^C, λ^B	Conversion trigger thresholds for CoCo's and Bail-Ins, respectively
λ^R	Recapitalization target for Bail-In
C	Conversion shares (equity shares gained when CoCo's or Bail-Ins are triggered)
Λ	Diagonal matrix keeping track of which nodes are currently in default or below a conversion threshold
$\mathbb{I}$	Indicator function
Γ	Diagonal matrix keeping track of conversions in previous iterations
γ	Fraction of equity gained in a fair conversion (ensures that a technical condition is met)
$(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in\mathbb{T}}, \mathbb{P})$	Filtered probability space
t, T	Time index and maximum time index
μ, σ	Drift and diffusion parameters for asset price process
$\mathbb{E}$	Expectation operator
$\mathbb{Q}$	Risk-neutral probability measure
Ψ	Payoff function for external assets (used for valuation)

L	Systemic losses
ρ	Check function (used to define loss function in regression
y	Dependent (target) variable in regression
X	Independent (feature) data in regression
K	Number of independent variables
α, β	Intercept and slope parameters in regression
u	Residual in regression
$\otimes$	Kronecker product
τ	Regression quantile
$\text{rank}(\cdot)$	Rank of a matrix
Q, q	Set of quantiles and generic element
W, w	Set of weights for quantiles and generic element
DV	Dummy variable estimator
QID	Quantile-independent dummy variable estimator
QIS	Quantile-independent slope estimator
QI	Quantile-independent estimator
B, $BQID$	Benchmark estimators
H	Hypothesis
$Var(\cdot)$	Variance of an estimator
$(\cdot)^{\dagger}$	Moore-Penrose generalized inverse of a matrix
S	Test statistics
0^K	K-dimensional vector of zeros

R^1	Goodness of fit for quantile models
E	Sum of errors

Table 2: Glossary of acronyms

Acronym	**Description**
GFC	Global financial crisis
US	United States
CoCo	Contingent convertible debt security
BRRD	Bank Recovery and Resolution Directive
NCWO	No creditor worse off
CDS	Credit Default Swap
ECB	European Central Bank
CCR	Central Credit Register
LDR	Liability Data Report
EUR	Euros
bn	billion
NRA	National Resolution Authority
SRB	Single Resolution Boad
EBA	European Banking Authority
ITS	Implementing Technical Standard
OeNB	Oesterreichische Nationalbank
AT1	Additional Tier 1 capital

T2	Tier 2 capital
CET1	Common Equity Tier 1
FSAP	Financial Stability Assessment Program
(O)SII	(Other) Systemically Important Institution

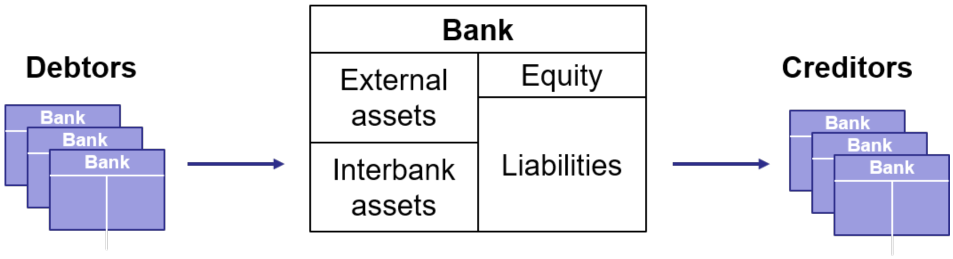
1. Introduction

Clearing algorithms for financial systems are an important building block of modern theories of financial crises. They can help explain the spread of losses through a financial system and the emergence of default cascades. Work on such algorithms predates the global financial crisis (GFC) of 2008, which has exemplified how the interconnectedness of financial institutions combined with a low loss-absorption capacity of banks can propagate distress in the financial system (Laeven and Valencia, 2013). Regulators, previously worried about banks that might have been deemed '*too big to fail*' are now equally worried about banks that are '*too interconnected to fail*'. The terminology hints at how the structure of the network formed by the various contracts between financial institutions may play an important role in shaping the risk profile of the financial system, a topic that will be explored in greater detail in this thesis.

Low loss-absorption capacity led to capital shortfalls at many banks when the crisis hit, forcing policy makers to step in with public bail-outs in order to prevent the default of major banks. The unintended consequence of these emergency measures was an increase in sovereign credit risk, creating a vicious circle between the creditworthiness of banks and governments (Fratzscher and Rieth, 2019). Bail-outs further create a moral hazard problem by rewarding less prudent banks with an implicit insurance (Dam and Koetter, 2012). Regulators have since responded with a host of new regulations aimed at safeguarding financial stability and at breaking the sovereign-bank nexus, where the creditworthiness of banks and governments decline in tandem, due to the implicit guarantee of public bail-outs for failing banks. New regulations have introduced contingent convertible debt and bail-ins, two recapitalization instruments aimed at providing a private-sector substitute for publicly funded bailouts. These new instruments can change the structure of the interbank network dynamically, and part of the contribution of this thesis is to show how such contracts can be incorporated into clearing algorithms. Bail-ins also require decisions by policymakers whenever they are faced with a failing bank, and in this thesis an operational framework that can provide quantitative guidance for such decisions is presented and applied to real-world data.

The GFC has also brought about a shift in focus from *microprudential* to *macroprudential* regulation, which analyzes financial stability at the level of the financial system rather than looking at the health of individual institutions in isolation. One prerequisite to an effective macroprudential supervision regime is the ability to identify "*key players*" in the financial network. These are *systemically important institutions*, which could trigger systemic crises in the event that they were to face an initially idiosyncratic distress event. In this thesis we will present a statistically inspired framework that allows using clearing algorithms to evaluate the effectiveness of regulators' key player identification tools.

Figure 1: Stylized balance sheet.

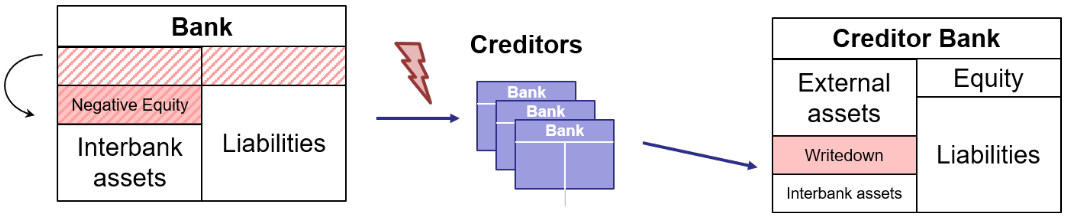


1.1. An informal presentation of the clearing problem for financial systems

We are concerned with a system of nodes that are connected through a network of bilateral loan obligations. Nodes in the system could be any economic agent, but we are primarily concerned with banks in this thesis, hence we will usually refer to nodes as banks henceforth. We represent each bank by a stylized balance sheet, consisting of interbank loans and other assets on the asset side, and interbank liabilities, other liabilities as well as equity on the other side. Other assets comprise retail and corporate loans, securities not issued by banks, holdings in non-banks, and more (infrastructure,...). Other liabilities consist of customer deposits and wholesale funding that owned by investors outside the system. Figure 1 provides an overview of such a stylized balance sheet.

One crucial feature of the clearing problem is the fact that every interbank asset is an interbank liability of another bank, and vice versa. These links provide a possible channel of contagion for financial shocks: if one bank incurs sufficiently large losses (e.g. because some of its retail borrowers default on their loans), then its creditors will be affected and will have to bear some of the losses. The basic logic of this mechanism is that the value of an interbank asset cannot be more than

Figure 2: Shock propagation. An external shock renders the first bank insolvent, implying losses for all its creditors. On the right side we show how this leads to a shock on the asset side of another bank that is a creditor of the first bank.



the corresponding share of assets of that bank. If a bank is insolvent, it means that the total assets are worth less than the total liabilities, and hence all creditors will need to write down the values of their interbank assets accordingly. Hence, while the value of interbank liabilities can be read directly from the balance sheets, the value of interbank assets following a shock has to be computed.

Whenever the creditors affected by such a loan writedown are other banks, this constitutes another shock to the assets of a bank in the system, which may potentially trigger further losses to interbank creditors. If viewed as a network, the interbank loans are the edges along which shocks are propagated. Figure 2 provides a schematic overview of such a shock propagation from one bank to another.

1.2. Overview of the thesis

Chapter 2 starts with a formal presentation of the clearing problem discussed informally in section 1.1. Section 2.1 introduces the formal notation that will be used throughout the thesis, and section 2.2 presents the model of Eisenberg and Noe (2001), which is used for solving the basic clearing problem, as demonstrated in section 2.3. Section 2.4 presents the first original contribution (published in Siebenbrunner (2019)), showing the connection between this clearing algorithm (more specifically, the extension by Rogers and Veraart (2013) of the original algorithm) and standard network centrality measures. Section 2.5 presents the extension by Elsinger (2009) of the basic clearing algorithm by Eisenberg and Noe (2001) which introduces different seniority classes for debt, as well as equity cross-holdings. This model will form the basis both for the extensions discussed in the subsequent chapter, as well as in the application to systemic risk assessment for bail-ins.

Chapter 3 presents two extensions to the previously mentioned algorithm by Elsinger (2009), both of them original contributions. Section 3.1 shows how the framework can be extended to include contingent convertible debt, a special class of structured products that automatically convert from debt to equity at pre-defined events. Section 3.2 presents a related extension that allows for bail-in-able debt, a mechanism for recapitalizing failing banks without recourse to public funds that has been introduced by recent regulations. Section 3 suggests an approach for linking the clearing algorithms presented in sections 2.5, 3.1 and 3.2 to asset pricing models based on the seminal model by Merton (1974), an approach that has been pioneered by Fischer (2014) and Barucca et al. (2020).

Chapter 4 presents an application of the models presented in the previous chapters to a policy-relevant real-world problem. Bail-ins were introduced by financial regulation, which foresees that a resolution authority should decide on a case-by-case basis whether bail-ins should be performed, or whether the bank should be allowed to enter into a regular insolvency procedure instead. Regulators should consider both the immediate impact to creditors, as well as systemic feedback effects in their decision. Section 4.2.1 presents a framework for quantifying systemic contagion losses based on the algorithms presented in the preceding sections. Section 4.2.2 presents an extension to these algorithms that allows incorporating liquidation losses for the assets of insolvent firms, an important consideration when comparing bail-ins against insolvencies. Section 4.3 presents the real-world data set used in the analysis which is presented in section 4.4. Section 4.5 discusses the results. Chapter 4 represents an original contribution.

Chapter 5 presents an alternative approach to systemic risk assessment. The approaches presented in the previous sections rely on the full knowledge of the granular network of bilateral interbank contracts. These data are not always available, and in practice regulators use a set of bank-level indicators in order to estimate the systemic importance of individual institutions. We present a metamodeling approach that allows for assessing how well these bank-level indicators can approximate systemic losses, and whether regulators are using the right weighting system to this end. Our learning strategy is based on quantile panel regression, and we review two common methods for estimating such regressions in section 5.2. In section 5.3 we present a novel test statistic to decide between these two estimators, and in section 5.4 we present how we measure goodness of fit for these models. 5.5 presents an application of our methodology to a real-world data set, and section 5.6 discusses the policy implications, representing an original contribution (published in Siebenbrunner and Sigmund (2019)).

Section 1.3 presents the relevant literature and discusses the contributions to this literature developed in this thesis, chapter 6 concludes and presents an outlook for future research directions.

1.3. Literature review

The importance of the network of interbank loans for the dynamics of financial crises and systemic risk has been discovered long before the financial crisis of 2008. Allen and Gale (2000) extended the seminal model of bank runs by Diamond and Dybvig (1983) to multiple regions, showing the potential for financial contagion from insolvent to solvent banks. Other early contributions on the role of the network formed by interbank lending relationships include Rochet and Tirole (1996), Angelini et al. (1996), Rochet and Tirole (1996), Freixas et al. (2000) and Kiyotaki and Moore (2002). The basic clearing problem presented in section 1.1 was first treated formally in the seminal works by Eisenberg and Noe (2001) and – independently – Suzuki (2002). They describe a financial network mathematically and provide an algorithm to compute a clearing payment vector in such a model. The clearing payment vector consists of the payments made by each bank to other banks in the network due to claims, taking into account the possibility of default of a bank. Elsinger (2009) extends the model of Eisenberg and Noe (2001) to include the possibility of cross-ownership of equity in the network, and to allow for multiple seniority classes of liabilities. This extension is necessary to incorporate equity conversions. Rogers and Veraart (2013) introduce bankruptcy costs in the Eisenberg and Noe (2001) model, and argue that with bankruptcy costs, solvent banks may have incentives to rescue failing banks in the network. Battiston et al. (2012) and Furfine (2003) have introduced alternative models for computing contagion effects that do not rely on the fixed-point argument of the clearing model. Furfine (2003) considers a model in which the recovery rate after the default of a bank is predetermined and equal for all banks, instead of being determined endogenously as in Eisenberg and Noe (2001)'s model. Battiston et al. (2012) present a model that is targeted at identifying "key players", or systemically important banks, by using a model that differs from that of Eisenberg and Noe (2001) by treating debt holdings like equity participations, and by limiting the number of contagion rounds that are considered. Variations and extensions of this model have been presented by Bardoscia et al. (2015) and Bardoscia et al. (2016). Fischer (2014) notes that these models can be used to extend the classical asset pricing framework of Merton (1974), where the clearing algorithms serve to compute payoff functions. This approach was pioneered by Suzuki (2002) and Fischer (2014) extended it to a model that includes multiple seniority classes of debt and equity, thereby extending the model of Elsinger (2009) to a valuation setting. Barucca et al. (2020) add to this model the concept of "local ex-ante valuation", in which banks only have information about their own counterparties. Barucca et al. (2020) further provide a general framework that encapsulates many of the clearing algorithms mentioned previously as special cases. Bardoscia et al. (2019) present an alternative to the algorithm of Eisenberg and Noe (2001) that assumes that liabilities are only repaid if they can be fully repaid. Clearing-based models of systemic risk have also been extended to include other channels of financial contagion such as overlapping portfolios (Caccioli et al., 2014, 2015), fire sales and liquidity effects (Cont and Schaaning, 2017; Cont et al., 2019) and effects of mark-to-market accounting (Cifuentes et al., 2005; Siebenbrunner et al., 2017) and methodologies to study the interplay between these channels have been developed (Siebenbrunner, 2020; Farmer et al., 2020). Other alternative approaches include approaches inspired by the literature on complex networks (Gai and Kapadia, 2010; Teteryatnikova, 2014) and agent-based modelling (Paulin et al., 2019). Caccioli et al. (2018) provide a recent review of the literature on financial contagion on systemic risk analysis, earlier reviews include Upper (2011), Glasserman and Young (2016) and Neveu (2018).

Following the theoretical works that showed the connection between financial networks and stability, several authors began studying the statistical properties of empirical networks (Boss et al., 2004) and the relation of the network structure to contagion effects (Iori et al., 2006; Nier et al., 2007). A more recent strand of literature studies the empirical relation between network centrality measures contagiousness. Kuzubaş et al. (2014) study characteristics a financial institution that was key to the Turkish financial crisis of 2000 and find increasing trends in several centrality measures prior to the outbreak of the crisis. Puhr et al. (2014) and Alter et al. (2015) find that the Katz centrality and its close cousin (see Newman (2010)), the Eigenvector centrality, have the best explanatory power for contagion losses from a Eisenberg and Noe (2001)-type clearing model for the Austrian and the German banking system, respectively. Kobayashi (2013) and Gauthier et al. (2013) obtain similar results in simulations. Bardoscia et al. (2017) show that the formation of cycles in networks leads to instability. Fukker (2017) shows that contagion losses can be approximated with a special form of a harmonic distance measure and, assuming equal liabilities of all banks and a symmetric network structure, by an Eigenvector centrality. In this thesis we demonstrate that these results were indeed to be expected, as the solution of the clearing model converges to a generalized Katz centrality measure as a crisis tends to affect the entire financial system. At first sight, this result may appear at odds with the reasoning of Acemoglu et al. (2015) or Tahbaz-Salehi (2015), who argue that - notwithstanding their empirical performance - "*off-the-shelf*" centrality measures such as the Katz centrality are a poor proxy for the results of clearing models from a theoretical perspective, as the latter exhibit non-linearities that typically cannot be captured by those indicators. The work in section 2.4 contributes to this literature by highlighting the very rigorous assumptions that have to be made in order to be able equate the solution of a clearing model to a Katz centrality measure. This allows a critical appreciation of these assumptions, leading us to agree with the aforementioned authors as a conclusion.

The work in sections 3.1 and 3.2 directly contribute to the literature by extending the model of Elsinger (2009) to include two new classes of debt, contingent convertible and bail-in-able debt. Related work concerns the inclusion of credit defaults swaps in financial networks (Schuldenzucker et al., 2017, 2020). Banerjee and Feinstein (2018), published after the paper presented in this thesis, study contingent payments in networks, a related goal to this study but without the focus on the application to bail-ins. Section 3.3 proposes an approach for ex-ante-valuation in this context, thereby extending the model of Fischer (2014) to include these new classes of debt.

The introduction of the bail-in tool and its implications on financial stability has been met with interest by academic researchers. Bail-ins are considered both as a crisis prevention tool as well as an alternative to the highly undesirable practice of bail-outs, where the sovereign provides the necessary funds for recapitalizing failing banks. One of the first papers analyzing the potential benefits of bail-ins over bail-outs was Conlon and Cotter (2014). The authors used sovereign bail-outs as a proxy for the need for bail-in and computed estimates for impairment charges due to bail-ins during the GFC, thus showing that such a bail-in mechanism can stabilize the system by limiting the danger of a 'flight-to-safety' and associated contagion. Klimek et al. (2015) utilized an agent-based macroeconomic model with a financial sector and showed that bail-ins are indeed welfare-improving with respect to taxpayer-funded bailouts. Such bailouts have led to 'vicious circles' between bank and public finances during the GFC, where the credit ratings of sovereigns decline in tandem with the financial health of the banks in their country, due to or in anticipation of publicly funded bailouts. Galliani and Zedda (2015) found that bail-ins can indeed provide an efficient tool for breaking these vicious circles, if the crisis is small enough to be absorbed within the financial system. Moreover, if such a bank-sovereign nexus can be broken, the public financing costs can be reduced significantly (Benczur et al., 2017; Boccuzzi and Lisa, 2017; Bodellini, 2018; Havemann, 2019). Lintner and Lincoln (2016) provide a collection of international case studies of bail-ins in different European jurisdictions. Sufficiently large crises can still require the injection of public funds, and in such situations the bail-in regime can lead to additional risks both due to higher contagion losses as well as litigation and other legal risks. Avgouleas and Goodhart (2015) provide an extensive analysis of the institutional challenges and potential benefits of a bail-in regime. The previously mentioned case study of Bodellini (2018) also features a thorough discussion of the legal resolution framework. Bernard et al. (2017) use a game-theoretic model to show that if the threat of no bailout is credible, bail-ins can also emerge voluntarily in equilibrium, in the absence of a legal framework that enforces participation. Fiordelisi et al. (2020) use stock market data on the reactions to the announcements of bail-in regulations to show that the threat of no bailout established by European regulations on bank resolution was perceived as credible by market participants.

The work in chapter 4 contributes to the above-mentioned literature by providing an assessment of the systemic risk impact of bail-ins using real-world data. Huser et al. (2017) and Souza et al. (2019) are two of the studies most closely related to our work. Huser et al. (2017) use data from the European Central Banks's Securities Holdings Statistics to quantify the potential impairment costs to financial institutions from conducting bail-ins in the Euro Area. Souza et al. (2019) use data from Brazil to study whether bail-ins reduce the financial system's ability to provide credit to the real economy. Our study differs from these in various ways: methodologically, our analysis is based on the rich literature on contagion models mentioned above, building on the extensions to the seminal model by Eisenberg and Noe (2001). We further consider the principle that no creditor should be worse off under a bail-in than under insolvency in our analysis, thus providing a quantitative toolset that allows resolution authorities to include such considerations in their decision making. In terms of data, we make use of the data collected under new reporting requirements, thereby providing, by the best of our knowledge, the first application of this data set to studying the systemic risk impacts of bank bail-ins.

Contagion models are not the only means of systemic risk assessement that have been developed in the literature. A prominent alternative approach is the CoVaR metric by Adrian and Brunnermeier (2016), which uses market data measures the systemic risk contribution as the change in the value-at-risk of the system if it becomes distressed, estimating this contribution using quantile regression. Publicly traded institutions are also the focus of other popular systemic risk measures such as SRISK (Brownlees and Engle, 2016) and the Marginal Expected Shortfall (Brownlees and Engle, 2012).

The goal of the work in chapter 5, however, is to study the systemic importance also of non-listed institutions and to provide insights to regulators using data sets that are available for all institutions. We thus use a metamodeling approach, where we learn a surrogate model of a classic contagion model using regulatory indicators as features. Our work is related in spirit to that of Siebenbrunner et al. (2017), who use a least-squares-based regression approach and conclude that the regulatory indicators are an efficient selection of indicators. In chapter 5 we will instead focus on evaluating the weights used by the regulators. In addition, we employ a different statistical method that is better able to capture the highly skewed nature of the distribution of systemic risk losses. Our tool of choice is quantile regression (Koenker, 2004), which has been popularized by Adrian and Brunnermeier (2016) for the study of systemic risk. Our work differs from these approaches by combining it with a contagion model in the spirit of Eisenberg and Noe (2001), which imposes a causal model of how losses are transmitted through the system. Furthermore, the model does not require market data and can thus also be used for non-listed banks, an important consideration given that around 90% of banks in Europe and the US are not publicly traded (Siebenbrunner et al., 2017). The advantage of the approach taken by Adrian and Brunnermeier (2016) is that it puts the focus on high quantiles of the loss distribution, which are of particular interest in the case of systemic risk, which deals with the potential downfall of entire financial systems. We argue that our approach combines the advantages of these two approaches by allowing for counterfactual analysis of tail risks in high quantiles. In a similar vein, Klomp and De Haan (2012) use quantile regression to estimate the impact of banking regulation on banking risk. Their measures for bank risk combine multiple aspects of risk using factor analysis, their data set, however, is restricted to bank-specific variables and does not include the network data necessary for computing systemic contagion losses in the spirit of Eisenberg and Noe (2001). Covas et al. (2014) take a similar approach, using a dynamic panel quantile estimator to estimate capital shortfalls in US banks. Greenwood et al. (2015) take a similar approach of learning a metamodel of the contribution of bank-level indicators to systemic risk, our work differs in that we present a novel statistical methodology and that we emphasize the evaluation of regulatory indicators in the application.

2. A formal presentation of the clearing problem, its relation to network theory and extensions to include multiple seniority classes of debt and equity cross-holdings

This chapter introduces the notation that will be used throughout the thesis and presents the basic building blocks of and some novel results about the clearing algorithms that are discussed in this thesis.

2.1. Definition of a financial system

We consider a system of $n-1$ financial entities, and a "sink node" with label n corresponding to external creditors towards which the financial entities have liabilities. In the context of financial systems, the financial entities may be thought of as banks or other financial institutions, and the external creditors could be other banks not included in the system of consideration, corporations or individual depositors. It is worth pointing out, however, that the formal framework per se is not limited to this interpretation. Extensions to include corporations or even households are a matter of data availability and the aim of the analysis, as the model framework is agnostic to these differences. For simplicity, we will usually refer to the entities in the system as banks here. They are represented via a stylized balance sheet, which on the asset side consists of external assets as well as assets that represent either claims on or equity participations in other entities in the system. On the liability side, we distinguish several seniority classes of liabilities, going to entities within the system and outside, and equity, which is the residual quantity. The set of n financial entities and the different types of connections between them constitute a multilayer network. This corresponds to the model of Elsinger (2009), which we extend in the subsequent sections.

Definition 1 (External assets)**.** *Let* $e_i \geq 0$ *be the price of all external assets of bank* i*. The vector* $e \in \mathbb{R}^n$ *is the vector of external assets.*

Definition 2 (Liabilities)**.** *Let* $L_{i,j,s}$ *denote the liabilities from bank* i *towards bank* j *within seniority class* $s \in \{1, \ldots, S\}$*,* S *being the most junior seniority class. We assume that the "sink node" has no liabilities, i.e.* $L_{n,j,s} = 0$ *for all* j, s*. The corresponding tensor of liabilities is* $L \in \mathbb{R}^{n \times n \times S}$.

Definition 3 (Ownership)**.** *It is possible that bank* i *owns a share* $\Theta_{j,i}$ *of bank* j*. The ownership matrix* Θ *is required to be a holding matrix, as defined in Elsinger (2009), i.e it must satisfy that* $\sum_i \Theta_{j,i} \leq 1$ *for all* j*, and, for any* $x \subset \{1, \ldots, n\}$*,* $\sum_{i \in x} \Theta_{j,i} < 1$.

Hence, the financial system is fully described by the vector of assets $e \in \mathbb{R}^n$, the liabilities tensor $L \in \mathbb{R}^{n \times n \times S}$, and the ownership matrix $\Theta \in [0, 1]^{n \times n}$. It is not trivial to determine the payments between banks if some bank is in default. The clearing payment matrix is defined as follows:

Definition 4 (Clearing payment matrix)**.** *A clearing payment matrix is a matrix* $P \in \mathbb{R}^{n \times S}$ *of total payments made by each bank, so that the payments respect the following criteria:*

- *limited liability: the total payments of each bank must not exceed the total assets of the bank.*
- *priority of debt claims: the bank's stockholders receive no value unless all liabilities are repaid fully.*
- *seniority hierarchy of debt claims: lower seniority classes receive no payoff unless all liabilities of higher seniority are repaid fully.*
- *proportionality: in case of default, all creditors of the same seniority class are paid proportionally to the liabilities against them.*

2.2. The basic clearing problem for simple debt networks

We will start by considering the basic problem of clearing in a system with only one seniority class of debt ($S = 1$) and no equity participations ($\Theta = \mathbf{0}$). Let $\bar{p} \in \mathbb{R}^n$ be the vector of total liabilities of each bank, $\bar{p}_i = \sum_j L_{i,j}$. Define the matrix $\Pi \in \mathbb{R}^{n \times n}$ of relative liabilities by

$$\Pi_{i,j} = \begin{cases} \frac{L_{i,j}}{\bar{p}_i}, & \text{if } \bar{p}_i > 0 \\ 0, & \text{if } \bar{p}_i = 0 \end{cases} \tag{2.1}$$

Note that each entry in L is at the same time a liability of one bank and an asset of another. While $\bar{p}$ captures the liability values, we can compute the vector of asset values of interbank claims as $\Pi' \bar{p}$. Note that we are assuming here that every node fully repays its liabilities, which is not the case for insolvent nodes. It is assumed that insolvent nodes split the remaining value of their assets proportionally among their creditors, while solvent nodes repay their liabilities in full. The actual repayment value p of a loan thus depends on the repayment values of the other liabilities, as formalized by the following map:

$$f(p)_i = \begin{cases} \bar{p}_i & \text{if } e_i + (\Pi' p)_i \geq \bar{p}_i \\ e_i + (\Pi' p) & \text{otherwise} \end{cases} \tag{2.2}$$

Eisenberg and Noe (2001), in their seminal contribution, call every fixed point p^* of f a **clearing payment vector** (the special case of the clearing payment matrix defined in section 2.1 for only one seniority class $S = 1$). They show that it is unique under very mild conditions, requiring only the presence of positive external assets in specific subsets of nodes, and provide a polynomial-time algorithm to compute this fixed point. It is in fact highly realistic to assume that every node has some external assets $e > \mathbf{0}$, a sufficient but not necessary condition for uniqueness.

2.3. Solving a basic clearing problem

For reasons that will become apparent below we will demonstrate how to solve a basic clearing problem in a setting that incorporates the extension Rogers and Veraart (2013) of the basic clearing model by Eisenberg and Noe (2001). They introduce recovery rates $r, r_e \in [0, 1]$ for interbank and external assets of insolvent banks, respectively. An economic argument for such an extension is provided in section 4.2.2. The clearing payment vector now satisfies:

$$p^* = \begin{cases} \bar{p}_i & \text{if } e_i + (\Pi' p^*)_i \geq \bar{p}_i \\ r_e e_i + r(\Pi' p^*) & \text{otherwise} \end{cases} . \tag{2.3}$$

Introducing a diagonal matrix $\Lambda \in \{0,1\}^{n \times n}$ capturing the nodes that are insolvent under a given payment vector p:

$$\Lambda_{i,j}(p) = \begin{cases} 1 & \text{if } i = j \wedge e_i + (\Pi' x)_i < \bar{p}_i \\ 1 & \text{if } i = j = N \\ 0 & \text{otherwise} \end{cases} . \tag{2.4}$$

Note that the sink node is set to be in default by convention, which will be used in the subsequent analyis in section 2.4, and shown to be consistent with the original model formulation there. We can now write the map given by Eq. (2.3) as:

$$f(p) = \Lambda(p) \left(r\Pi' \left(\Lambda\left(p\right) p + \left(I - \Lambda\left(p\right)\right) \bar{p} \right) + r_e e \right) + \left(I - \Lambda(p)\right) \bar{p} \tag{2.5}$$

In order to solve the model, first fix $\Lambda(p) = \Lambda$ and solve for the fixed point:

$$f = r\Lambda\Pi' + r\Lambda\Pi'\bar{p} - r\Lambda\Pi'\Lambda\bar{p} + r_e\Lambda e + \bar{p} - \Lambda\bar{p}$$

$$(\mathbf{I}_n - r\Lambda\Pi'\Lambda)(f - \bar{p}) = \Lambda(r_e e + r\Pi'\bar{p} - \bar{p})$$

$$f = (\mathbf{I}_n - r\Lambda\Pi'\Lambda)^{-1}\Lambda(r_e e + r\Pi'\bar{p} - \bar{p}) + \bar{p} \tag{2.6}$$

where $\mathbf{I}_n$ is an n-dimensional identity matrix. The existence of a solution to equations essentially similar to 2.6 has been shown by various authors in the literature (Eisenberg and Noe (2001); Rogers and Veraart (2013)), albeit for a slightly different definition for Λ (without setting the sink node to defaulted). It follows as a corollary (1) of theorem 1 that this small change does not affect the existence of a solution. As Elsinger et al. (2012) show, this computation has the advantage that the matrix inversion only has to be applied for the subset of rows and columns which correspond to already defaulted nodes, which is an advantage in real-world applications where the number of banks can be high, but the number of defaults often is low. Eisenberg and Noe (2001) show that the clearing payment vector is unique under mild conditions[1] and can be obtained from the following iteration called the **fictitious default sequence** initiated with $f_0 = l$, which converges after at most n steps (Eisenberg and Noe, 2001):

$$f_{k+1} = (\mathbf{I}_n - r\Lambda(f_k)\Pi'\Lambda(f_k))^{-1}\Lambda(f_k)(r_e e + r\Pi'\bar{p} - \bar{p}) + \bar{p} \tag{2.7}$$

[1]The conditions define that certain subsets of the financial system need to have a positive value of external assets. We will assume $e_i > 0\ \forall i$ as a sufficient condition.

2.4. *Clearing algorithms and network centrality*

2.4.1. *Expressing the solution as a Katz centrality measure*

For the subsequent analysis we will set the recovery rate for external assets to $r_e = 1$ without loss of generality. Now consider the realization of an exogenous shock ν which reduces the original value o of the external assets:

$$e = o + \nu \tag{2.8}$$

In the following steps, we will write the conditions that hold for s_i $\forall\ i = 1 \ldots N-1$. For the sink node, we will assume $e = o > 0$.

Assume that $\forall\ i < N : (\Pi'\bar{p})_i < \bar{p}_i$ and choose ν_i from the interval $\nu_i \in (-o_i, -o_i + \bar{p}_i - (\Pi'\bar{p})_i)$. This implies that $e_i \in (0, \bar{p}_i - (\Pi'\bar{p})_i)$, hence all banks still have positive external assets, but $e_i + (\Pi'\bar{p})_i < \bar{p}_i$, hence they are in fundamental default. [2]. ν can thus be interpreted as a shock that renders all banks insolvent, while still keeping the value of their external assets positive (thus not violating the conditions of uniqueness for the clearing payment vector Eisenberg and Noe (2001)). Let $m \in (0, 1)$ and consider e.g. a linear interpolation:

[2]Note that the inclusion of a sink node is crucial here, as Eisenberg and Noe (2001) show that for $e_i > 0$ $\forall i$ not all nodes can be in fundamental default. In this setup, it is the sink node that has positive equity value, but is still considered to be in default under any $\Lambda(x)$ by construction.

$$v_i = m(\bar{p}_i - (\Pi'\bar{p})_i - o_i) - (1-m)o_i = m\bar{p}_i - m(\Pi'\bar{p})_i - o_i \tag{2.9}$$

The interpolation in Eq. 2.9 scales the shock to ensure that the external assets stay in the interval defined above, $e_i \in (0, \bar{p}_i - (\Pi'\bar{p})_i)$. In the limit $m = 1$, we obtain $v_i = -o$ and $e_i = 0$, corresponding to the highest shock level. In the limit $m = 0$, we obtain $e_i = \bar{p}_i - (\Pi'\bar{p})_i$. Note that since $e_i + (\Pi'\bar{p})_i < \bar{p}_i$ for all banks except the sink node, $\Lambda(\bar{p}) = I$ (using the fact that the sink node is in default by convention). Hence the shock effectively pushes all banks beyond the default threshold where the non-linearity in the Eisenberg and Noe (2001)-model occurs. Under these conditions the fictitious default sequence converges after the first iteration and $f(\bar{p}) = p$ is a clearing payment vector and the solution of the clearing model becomes:

$$p^* = (\mathbf{I}_n - r\Pi')^{-1}(a + r\Pi'\bar{p} - \bar{p}) + \bar{p} = (I - r\Pi')^{-1}((r-m)\Pi'\bar{p} - (1-m)\bar{p}) + \bar{p} \tag{2.10}$$

Theorem 1.

(a) $\mathbf{I}_n - r\Pi'$ *is invertible for* $r \in [0, 1)$ *if the system does not contain a sink node*

(b) $\mathbf{I}_n - r\Pi'$ *is invertible for* $r \in [0, 1]$ *if the system contains a sink node*

Proof.

Note that when λ and v are eigenvalues and -vectors of Π', we have:

$$(\lambda \mathbf{I}_n - \Pi')v = 0 \Leftrightarrow (\mathbf{I}_n - \frac{1}{\lambda}\Pi')v = 0 \Leftrightarrow \det(\lambda \mathbf{I}_n - \Pi') = 0 \tag{2.11}$$

So $(\mathbf{I}_n - r\Pi')$ is invertible for $r \neq \frac{1}{\lambda}$. Since $r \in [0, 1]$ by definition, we need to investigate whether there are eigenvalues $\lambda \geq 1 \Rightarrow \frac{1}{\lambda} \leq 1$. The maximum eigenvalue for a non-negative matrix can be obtained through the Collatz-Wielandt formula (Meyer, 2000, p. 666):

$$\omega(\Pi') = \max_{x : x_i \geq 0 \wedge \exists x_i > 0} g(x, \Pi') \tag{2.12}$$

$$g(x, \Pi') = \min_{i : x_i \neq 0} \frac{(x\Pi')_i}{x_i} \tag{2.13}$$

Without a sink node, if all institution have at least some claims, Π' would be column-stochastic, hence $g(x)_i = 1 \; \forall \; i = \omega(\Pi')$. This proves part (a) for the case where all institutions have some claims. In the case this condition does not hold, one or more columns of Π' consist of only zeros, which is equivalent to the case with a sink node discussed below.

Given that the sink node adds a column of all zeros to Π', the maximum eigenvalue is attained for $g(x : x_N = 0)$. Since the remainder of the columns still sum to 1, we obtain $0 < \omega(\Pi') < 1$, hence $\frac{1}{\lambda} > 1$ for all positive eigenvalues. □

Corollary 1. *Note that for* $0 < \Lambda < \mathbf{I}_n$, $0 < \omega(\Lambda\Pi'\Lambda) \leq \omega(\Pi')$, *which shows the existence of a solution to equation 2.6.*

A typical form of a systemic risk measure based on a clearing model is to consider the difference between total liabilities and the clearing payment vector (Glasserman and Young (2016)) $\zeta = \bar{p} - p$. A systemic risk measure based on equation 2.10 can thus be written as a centrality measure:

$$\zeta = (\mathbf{I}_n - r\Pi')^{-1}\eta \tag{2.14}$$

With $\eta_i = (1-m)\bar{p}_i - (r-m)(\Pi'\bar{p})_i \ \forall \ r, m \in (0,1), i < N$ (the framework would also allow for setting bank-specific recovery and interpolation values by taking r and m as diagonal matrices). The functional form of ζ is equivalent to that of a Katz centrality measure (Newman (2010); Katz (1953)). In the standard definition, Π' is an adjacency matrix and $\eta = \mathbf{1}^n$, so in the general case ζ could be seen as a generalization. In order for Π' to be an adjacency matrix, we would have to assume that each bank has at most one creditor, s.t. $\Pi'_{ij} \in \{0,1\} \ \forall \ i, j$.[3] If we further set $r = m$, η simplifies to $\eta_i = (1-r)\bar{p}_i \ \forall \ r \in (0,1), i < N$ and we could obtain $\eta = \mathbf{1}^n$ through normalization if every bank has at least some liabilities. Under these – admittedly very strict – conditions the solution of the clearing model is a specific form of a **Katz centrality measure**.

A large systemic shock that immediately renders all banks in the system insolvent is admittedly a strong assumption. We can relax this assumption by choos-

[3]Of course, Katz-type centrality measures can also be computed for weighted networks, so the assumption of at most one creditor is not required to obtain a solution.

ing $v_i \in (-o_i, -o_i + \bar{p}_i - (\Pi' p)_i)$ [4], which implies a less severe minimum shock since $\Pi' p \leq \Pi' \bar{p}$. Under this condition, $a_i + (\Pi' p)_i < \bar{p}_i \ \forall\ i < N$ and hence $\lim_{k \to n} \Lambda(f_k) \to I$. I.e. all banks are pushed into default after the algorithm converges (which happens after at most n steps as shown by Eisenberg and Noe (2001)). By choosing an analogous interpolation and inserting into $f(p)$, we again obtain a centrality measure (assuming that $I - (r - m)\Pi'$ is invertible):

$$v_i = m(\bar{p}_i - (\Pi' p)_i - o_i) - (1 - m)o_i = m\bar{p}_i - m(\Pi' p)_i - o_i \tag{2.15}$$

$$p = (\mathbf{I}_n - r\Pi')^{-1}(a + r\Pi'\bar{p} - \bar{p}) + \bar{p} = (I - r\Pi')^{-1}(m(\bar{p} - \Pi' p + r\Pi'\bar{p}) - \bar{p}) + \bar{p}$$

$$p = (I - (r - m)\Pi')^{-1} m(\bar{p} + r\Pi'\bar{p}) \tag{2.16}$$

Which for $r = m$ simplifies to $p = r\bar{p} + r^2\Pi'\bar{p}$.

2.4.2. Discussion

It is often discussed and/or assumed that standard network centrality measures can give insights about the systemic risk contribution of individual banks (Puhr et al.,

[4] In order to find suitable values for v, the model has to be solved first. Given that the aim is to have a small shock size, a reasonable approach would be to start by setting $v_i = (-o_i + \bar{p}_i - (\Pi'\bar{p})_i)) - \frac{k}{MaxSteps}(\bar{p}_i - (\Pi'\bar{p})_i)$ for a suitable $MaxSteps = 1000$, e.g., and iterating over $k = 1, 2, \ldots$ until $\Lambda(p) = I$.

2014). At the same time, sophisticated systemic stress testing tools have been designed using contagion models based on a model of interbank clearing (see e.g. Elsinger et al. (2006)). In this paper, we have shown that one common methodology used in this context, the clearing model developed by Eisenberg and Noe (2001) (and in particular, the extension by Rogers and Veraart (2013)), converges to a special form of a Katz centrality measure as a crisis becomes so severe that all banks in the system are pushed into default (the existence of a world outside the system to which there exist financial liabilities is necessary - this is a mild assumption for a banking system, where deposit-taking is a central part of the business model).

Given this result, we advocate using contagion models instead of centrality measures when analyzing systemic risk (in the case where the required data is available - Anand et al. (2015) provide a good overview of methods that can be used in the absence of granular network data). As Puhr et al. (2014) and others show, different versions of the Katz centrality measure and its cousins have the highest explanatory power for the output of a contagion model. We have shown that this result is due to a mathematical relation between those measures. Their analysis is based on a different type of shock scenario – idiosyncratic instead of systemic – but the similarity is still statistically identifiable. A key difference is, however, that clearing models allow to interpret the assumptions made and the results obtained from an economic perspective. Such an analysis shows that the assumptions that one is making when using even a highly specific, specially derived form of a Katz centrality are strong and unrealistic for typical applications (most notably that the entire banking system is in fundamental default)[5]. So even if one is looking at the best possible centrality measure - as demonstrated both empirically by Puhr et al. (2014) and formally in this study - one is making strong, potentially unfounded assumptions that could be avoided by using a clearing model instead.

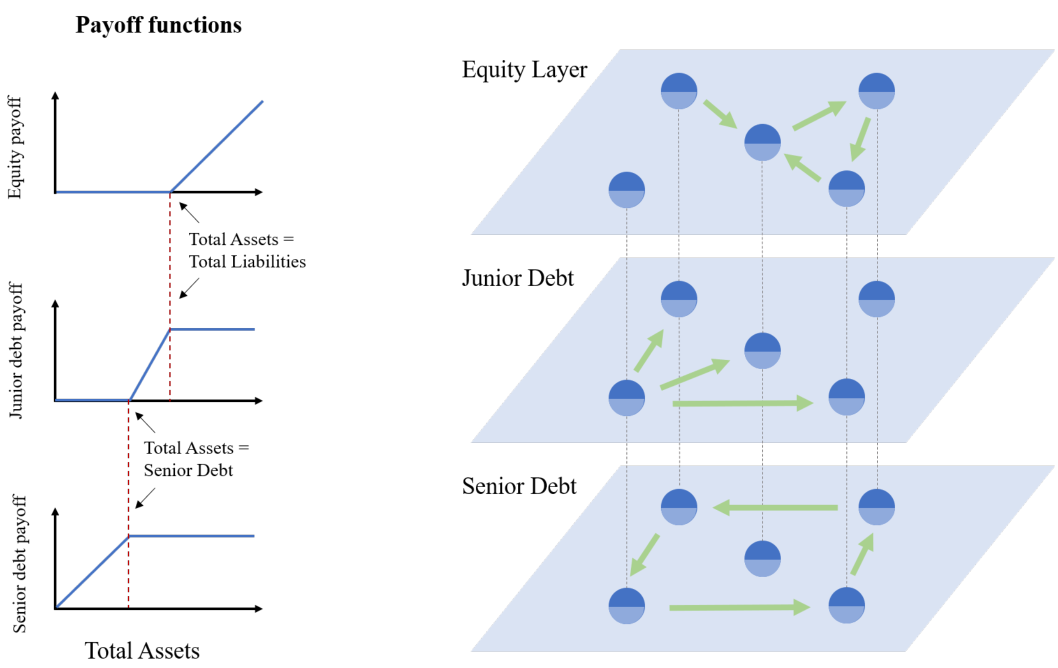
2.5. *Clearing algorithms for equity and debt contracts with multiple seniority classes*

We will now relax the assumptions that there are no equity participations and only one seniority class of debt. Before we begin the formal introduction of the general model, we will lay out its main logic qualitatively. Figure 3 shows the different layers of the network model, in this case a simple example with five banks and two seniority layers of liabilities. As can be seen, the same set of five banks have different links in each of the layers. Each of these links is valued using a different payoff function, depending on the layer. The payoff for the equity layer looks like the payoff for a long call option with strike price equal to the total liabilities. The payoff for the most junior layer can be described as long call option with strike equal to the value of senior liabilities plus a short call option with strike equal to total liabilities. Payoffs for more senior equity classes can be described analogously, with strikes equal to the combined value of the more senior liability classes (or 0 for the most senior class) in the long position and equal to this value plus the value of the current class for the short position. The reminiscence of the model of Merton

[5] Another shortcoming of the ζ-measure is that differences in initial capitalization, which are a significant driver of contagion dynamics, are canceled out through the shock. If one wanted to use the ζ measure for systemic risk analysis, one should consider including another factor like $\frac{v_i}{o_i+(\Pi'\bar{p})_i}$ to capture the initial financial health of firm i.

(1974) here is not misleading, and will be discussed in greater detail in section 3.3.

Figure 3: Payoff functions for multiple layers of cross-holdings



2.5.1. *Equity*

Consider first the case with no seniority structure ($S = 1$) but with equity participations ($\Theta \geq \mathbf{0}$). In order to determine the clearing payment vector, it is first necessary to define the concept of the equity of a bank. The equity of a bank consists of its total amount of assets (external assets plus assets from interbank lending and holdings) minus its total liabilities, $\bar{p}_i$. In the case that the holdings matrix $\Theta = 0$, the equity vector is simply expressed as:

$$V^*(p) = (Assets - Liabilities)^+ = (e + \Pi' p - \bar{p})^+ \tag{2.17}$$

This definition corresponds to the value of equity for the owner (equity payoff), which is non-negative. We will also consider the related definition of accounting value of equity (which can be negative), which we will also refer to as equity:

$$Equity = Assets - Liabilities = e + \Pi' p - \bar{p} \tag{2.18}$$

If the holdings matrix Θ is not null, the equity is defined as follows.

Definition 5 (Equity)**.** *Given a financial system with no seniority structure, and a payment vector $p \in \mathbb{R}^n$, the vector $V^*(p) \in \mathbb{R}^n$ is an equity vector if and only if it is a fixed point of the following map:*

$$\psi(V) = (e + \Pi' p - \bar{p} + \Theta' V)^+ \tag{2.19}$$

So that,

$$V^*(p) = (e + \Pi' p - \bar{p} + \Theta' V^*(p))^+ \tag{2.20}$$

Elsinger (2009) (lemma 5, Appendix) show that the map has a unique fixed point $V^*(p)$ for any $p \in \mathbb{R}^n$, provided that Θ is a holding matrix.

To be consistent with the clearing criteria, a payment vector must satisfy:

$$
p_i = \begin{cases} 0, & \text{if } (e + \Pi' p + \Theta' V^*(p))_i \leq 0 \\ (e + \Pi' p + \Theta' V^*(p))_i, & \text{if } 0 \leq (e + \Pi' p + \Theta' V^*(p))_i \leq \bar{p}_i \\ \bar{p}_i, & \text{if } \bar{p}_i \leq (e + \Pi' p + \Theta' V^*(p))_i \end{cases} \tag{2.21}
$$

Elsinger (2009) prove the existence of a clearing payment vector under this framework and discuss conditions for its uniqueness. Of more interest from a practical perspective is to calculate the (largest) clearing vector. First, it is necessary to define a new variable $W^*(p)$ given a payment vector $p \in [0, \bar{p}]$, such that $W^*(p)$ is a fixed point of the map $\psi^W(W) = e + \Pi' p - \bar{p} + \Theta'(W \vee 0)$.

Lemma 1. *[Elsinger (2009)] The sequence W^k defined by $W^0(p) = e + \Pi' p - \bar{p}$ and $W^{k+1}(p) = e + \Pi' p - \bar{p} + \Theta' \Lambda^k W^{k+1}$, where $\Lambda^k = diag(W^k > \mathbf{0})$ converges to the largest fixed point of ψ^W.*

Theorem 2. *[Elsinger (2009)] If Θ is a holding matrix, the sequence $p^{i+1} = (W^*(p^i) + \bar{p})^+ \wedge \bar{p}$, with $p^0 = \bar{p}$, converges to the largest clearing payment vector.*

2.5.2. Seniority structure

The previous results show how to calculate a clearing payment vector for a financial system in the particular case that there is no seniority structure. Consider now the case of a financial system with seniority structure ($S > 1$). In this case, a payment matrix P is determined by its value in each seniority class: $P \in \mathbb{R}^{n \times S}$, and the total liabilities are $\bar{P} \in \mathbb{R}^{n \times S}$ where $\bar{P}_{i,s} = \sum_j L_{i,j,s}$. Similarly, the tensor of relative liabilities is defined for each seniority class:

$$\Pi_{i,j,s} = \begin{cases} \frac{L_{i,j,s}}{\bar{P}_{i,s}}, & \text{if } \bar{P}_{i,s} > 0 \\ 0, & \text{if } \bar{P}_{i,s} = 0 \end{cases}$$

Definition 6 (Equity Vector)**.** *The equity vector under a given payment matrix is defined by the equation:*

$$V^*(P) = \left(e + \sum_{s=1}^{S} \Pi'_{\cdot,\cdot,s} P_{\cdot s} - \sum_{s=1}^{S} \bar{P}_{\cdot s} + \Theta' V^*(P)\right)^+ \tag{2.22}$$

It is a fixed point of the map

$$\psi^S(V) = \left(e + \sum_{s=1}^{S} \Pi'_{\cdot,\cdot,s} P_{\cdot,s} - \sum_{s=1}^{S} \bar{P}_{\cdot,s} + \Theta' V\right)^+ \tag{2.23}$$

A clearing payment matrix must respect the seniority structure. Elsinger (2009) provide a characterization for a clearing payment matrix in this case:

Definition 7. *[Clearing payment matrix] A matrix P^* satisfying $P^*_{i,j} \in [0, \bar{P}_{i,j}]$, $\forall\, i, j$ is a clearing payment matrix if and only if it is a fixed point of the map*

$$\Phi^S(P)_{i,T} = \begin{cases} \bar{P}_{i,T}, & \textit{if } e_i + \mathcal{A}_{i,T} \geq \bar{P}_{i,T} \\ (e_i + \mathcal{A}_{i,T})^+, & \textit{otherwise} \end{cases} \tag{2.24}$$

Where $\mathcal{A}_{\cdot,T} = \sum_{s=1}^{S} \Pi'_{\cdot,\cdot,s} P_{\cdot,s} - \sum_{s=1}^{T-1} \bar{P}_{\cdot,s} + \Theta' V^*(P)$.

Hence, $\forall\ T \in \{1, \dots, S\}$,

$$P^*_{\cdot,T} = \left(e + \sum_{s=1}^{S} \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s} - \sum_{s=1}^{T-1} \bar{P}_{\cdot,s} + \Theta' V^*(P^*)\right)^+ \wedge\ \bar{P}_{\cdot,T} \tag{2.25}$$

Intuitively, a bank will only pay liabilities for a certain seniority class T *if all seniority classes* $s < T$ *have been fully paid.*

Elsinger (2009) define a sequential clearing procedure for calculating the clearing payment matrix, starting at the most junior seniority class assuming that all senior seniority classes have been fully paid, and proceeding iteratively to more senior classes if the bank is insolvent. Formally, following the notation in Elsinger (2009), define $H^k = (h_1^k, \dots, h_n^k)$ to be the vector of seniority classes at iteration k, with elements $h_i^k \in \{1, \dots, S\}$. Define

$$e_i^{H^k} = e_i + \sum_{j=1}^{n} \sum_{s=1}^{h_j^k - 1} \Pi_{j,i,s} \bar{P}_{j,s} - \sum_{s=1}^{h_i^k - 1} \bar{P}_{i,s} \tag{2.26}$$

Which corresponds to the assets of bank i in the current iteration k. Similarly, define

$$\Pi_{i,j}^{H^k} = \Pi_{i,j,H_i^k}, \quad (p_H)_i = P_{i,H_i^k} \tag{2.27}$$

Start the iteration with $H^0 = (S, \dots, S)$, and let $p^*_{H^0}$ be the clearing vector for the financial system with no seniority structure $(e^{H^0}, \Pi^{H^0}, \bar{p}_{H^0}, \Theta)$. If the equity of all banks in this financial system is non-negative, or the only banks with negative equity have reached the most senior seniority structure in the iteration, the procedure is completed. Otherwise, let $\Lambda = diag(e^{H^0} + \Pi^{H^0\prime} p^*_{H^0} - \bar{p}_{H^0} + \Theta' V^*(p^*_{H^0}) < 0)$, and set $H^1 = H^0 - 1'\Lambda$. Iterating this procedure leads to a clearing matrix in finite steps (Elsinger (2009)).

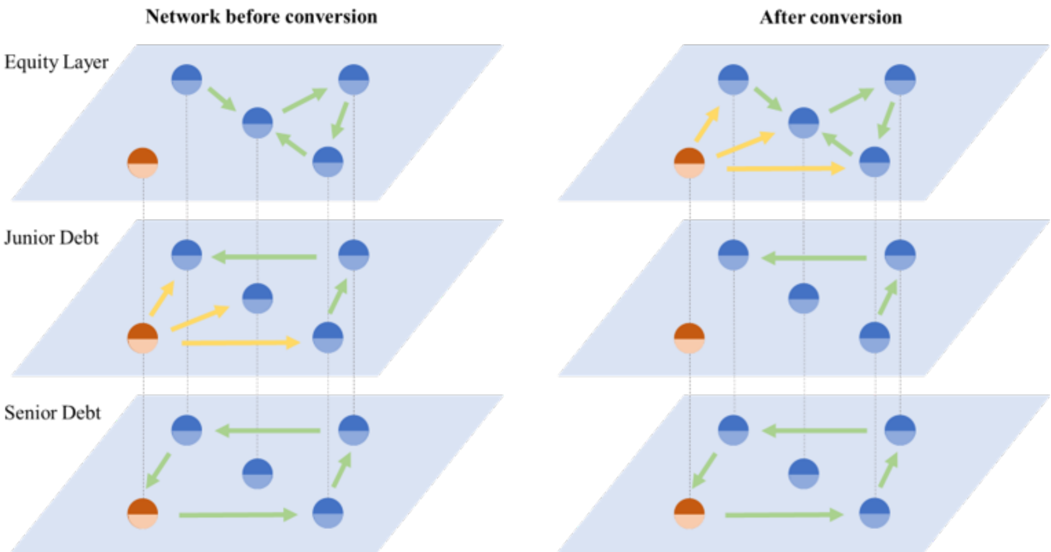
3. Extensions to include contingent convertible debt and the link to asset pricing models

The contribution of the work presented in this chapter is to introduce bail-in mechanisms and Contingent convertible debt securities (CoCo's) into formal models of financial contagion. Both are new mechanisms introduced by recent banking regulations aimed at eliminating the need for recapitalizing failing banks with public funds ('bail-outs'). The mechanics of these mechanisms will be discussed in the next sections. Here we note that our model extends the model of Fischer (2014) and the related model of Elsinger (2009) by including the possibility of a bail-in mechanism and CoCo's, and ex-ante valuation as introduced by Barucca et al. (2020). It extends the model of Hüser et al. (2017) by including higher-order contagion effects from cascades and the aforementioned extensions, and generalizes it to allow for different allocations of equity between bailed-in creditors and old equity owners. The potential scope of applications of the model includes central banks, where it can serve as an add-on to stress test exercises, as well as regulators who have to decide on capital buffers for systemically important institutions and for systemic risk (CRD-IV (2013)).

3.1. Clearing algorithms for contingent convertible securities

CoCo's are bonds which are converted to equity after a predefined trigger event occurs. In this section we will describe how to include CoCo's in the Elsinger (2009) framework. CoCo triggers can be based on the share price, an accounting

Figure 4: *Network reconfiguration effect of a debt-to-equity conversion*: A conversion is performed at the highlighted node. After the Bail-In, all junior liabilities of this bank have been converted to equity participations. No liabilities by other banks are affected.



capital ratio as defined in Basel III, or a regulatory decision. We will model the accounting trigger, which is a fairly general case, as the main examples of adoption of CoCo's are based on these types of triggers (De Spiegeleer and Schoutens, 2011).

In the trigger event, CoCo's may be written down or partially or completely converted to equity. This is pre-specified in the contract. Let φ be the conversion fraction, so that $\varphi = 1$ corresponds to a complete conversion and $\varphi = 0$ corresponds to no conversion (i.e, no change in liabilities).

The triggering of a CoCo implies a reconfiguration of both the liability as well as the ownership network. Figure 4 shows a graphical representation of the network reconfiguration effects implied by a conversion in the simple case of only two seniority classes of debt (CoCo's and senior debt): as can be seen, all links are deleted from the CoCo layer and added to the equity holdings layer. Note that the payoff functions, as illustrated in figure 3, do not change under this operation – the converted liabilities will be subject to the equity payoff function instead of the payoff function of their original seniority class. The conversion by one or several banks in the network may lead to losses in equity for other banks in the network possibly causing further rounds of CoCo conversions. In the following section we will present a formal model of this process.

With the previous specifications of the contract, CoCo's can be incorporated into the network model described in section 2.5. We assume that each bank has no more than one type of CoCo, which is consistent with past adoptions of CoCo's (De Spiegeleer and Schoutens, 2011), although this assumption could be easily extended.

It makes sense to place CoCo's into an own seniority class in the model. Let s_c be the seniority class of CoCo's, λ^C the vector of bank-specific CoCo conversion thresholds, φ be the vector of bank-specific conversion fractions, and C be the conversion matrix, which determines the amount of shares received by bank j if the CoCo issued by bank i is triggered in position $C_{i,j}$. The conversion matrix can depend on the value of equity of the issuing bank, $C = C(V^*(P^*))$ but we will not make this relationship explicit generally here.

The clearing payment matrix is calculated with the following iteration, in which successively a bank triggers its CoCo if it has breached its capital ratio trigger:

1. Start the iteration with $\bar{P}^1 = \bar{P}$, $\Theta^1 = \Theta$.

2. Compute P^* according to the Elsinger algorithm using $\bar{P}^k$ and Θ^k from the current iteration.

3. Check if any CoCo trigger threshold has been breached.

   Λ^k is the diagonal matrix which indicates whether a bank has breached its CoCo threshold in the current iteration:

$$\Lambda^k = diag\left(\frac{e + (\Theta^k)' V^*(P^*) + \sum_s \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s} - \sum_s \bar{P}^k_{\cdot,s}}{e + (\Theta^k)' V^*(P^*) + \sum_s \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s}} < \lambda_C\right) \tag{3.1}$$

   Γ^k keeps track of the banks who's CoCo's have triggered in some previous iteration:

$$\Gamma^k = diag\left(\max\left(\Lambda^1, \ldots, \Lambda^{k-1}\right)\right) \tag{3.2}$$

4. Update the liabilities vector and the holding matrix. Intuition: no changes are made if the CoCo threshold has not been breached in the current iteration (first term). If the CoCo threshold has been breached, the liabilities are reduced by the contractually defined amount φ (second term).

$$\bar{P}^{k+1}_{\cdot,s_c} = \left(\mathbf{I} - \Lambda^k(\mathbf{I} - \Gamma^k)\right)\bar{P}^k_{\cdot,s_c} + (1-\varphi) \cdot \Lambda^k(\mathbf{I} - \Gamma^k)\bar{P}^k_{\cdot,s_c} \tag{3.3}$$

   The updated holding matrix is defined as follows. The intuition is similar to the update of the liabilities: only make a change if a conversion has been triggered. If a conversion happens (second term), the converted creditors gain a contractually defined equity share, and the shares of old equity owners are adjusted to reflect this dilution. Let $\bar{c}_i = \sum_j C_{i,j}$. Then,

$$\Theta^{k+1} = \Big(\mathbf{I} - \Lambda^k(\mathbf{I} - \Gamma^k)\Big)\Theta^k + \Lambda^k(\mathbf{I} - \Gamma^k)\left(diag\Big(1 - \bar{c}\Big) * \Theta^k + C\right) \tag{3.4}$$

5. If $\Lambda^k(\mathbf{I} - \Gamma^k) = \mathbf{0}$, stop the iteration. Otherwise, continue to step 2.

It is easy to see that the above procedure converges after at most n iterations by noting that Γ^k is monotonously increasing by construction, i.e. $\Gamma^{k+1}_{i,j} \geq \Gamma^{k+1}_{i,j}$ for all i, j. Since the triggering of a CoCo is irreversible, the iteration ends after each bank that fell below its trigger threshold at some point has triggered its conversion. The resulting equilibrium value $P^{*,n}$ gives the **clearing payments in the presence of CoCo**'s, which we denote $P^{*,C}$.

3.2. Clearing algorithms for bail-in-able debt

Bail-in-able debt is similar to CoCo's in the sense that it may be used for recapitalizing failing banks by converting debt to equity. The difference is that bail-ins affect a much wider proportion of the liabilities of a bank, and is applied to existing asset classes instead of creating a new one. The legal differences between bail-ins and CoCo's are important (Avgouleas and Goodhart, 2015), from an economic perspective the main differences that we will be concerned with here is that bail-ins can concern debt of different seniority classes and do not necessarily convert all debt even within the same seniority class to equity, but rather seeks to recapitalize the bank up to a given recapitalization target. Chapter 4 presents more on the legal and regulatory background on bail-ins.

3.2.1. Conversion shares

To model bail-in, we assume that the K most junior seniority classes are bail-in-able. If a certain amount $\text{Bail-In}_{i,j,s}$ of the liabilities $L_{i,j,s}$ are to be converted to equity, bank j will gain a share $C_{i,j,s}$ of bank i. This share can be dependent on the bail-in amount $\text{Bail-In}_{i,j,s}$ and the equity values of the banks, $\text{Equity}^n_j = \left(e + (\Theta^n)' V^*(P^*) + \sum_s \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s} - \sum_s \bar{P}^n_{\cdot,s}\right)_j$, so we model the conversion share as a function:

$$C : \mathbb{R}^{n \times n \times S} \times \mathbb{R}^n \to \mathbb{R}^{n \times n \times S} \tag{3.5}$$

$$(\text{Bail-In}, \text{Equity}) \mapsto C\,(\text{Bail-In}, \text{Equity}) \tag{3.6}$$

Clearly, $C_{i,j,s} = 0$ must hold for $s \leq S - K$. We also impose that $C(0, \text{Equity}) = 0$ for all values of Equity. For convenience of notation, we will usually refer to $C\,(\text{Bail-In}, \text{Equity})$ as simply the conversion matrix C. In order for the updated holdings matrix to still be consistent with the definition of a holdings matrix (definition 3), we have to make one additional assumption on the conversion matrix:

Assumption 1 (Conversion matrix)**.** *The shares gained in each bank sum to less* *than one:*

$$\sum_s \sum_j C_{i,j,s} < 1 \ \forall\, i \tag{3.7}$$

Note that while the bail-in is overall neutral for the investors of the bailed in bank, it could imply mutually offsetting gains and losses for either the old equity owners or the bailed in creditors, depending on the conversion factors C. A too low conversion factor (or a complete write-down of the liabilities, corresponding to a conversion factor of 0) would mean a transfer of wealth from the bailed in creditors to the old equity owners of the bank, and vice versa for too high conversion factors.

In practice, conversion factors would have to be determined from applicable regulations or covenants. In the absence of such data, we present here what we consider a canonical choice for the conversion factor, the *fair share*, which ensures that the bail-in is wealth-neutral for all affected investors:

Definition 8 (Fair conversion matrix)**.** *A conversion matrix is said to be fair if the share gained by the bailed in creditors is equal to the share of the bailed in liabilities in the new equity, consisting of the bailed in liabilities plus the old equity, in the case the latter is positive. Let Bail-In*$^n_{j,s} = \bar{P}^n_{j,s} - \bar{P}^{n+1}_{j,s}$ *and Bail-In*$^n_j = \sum_{s=1}^{S} \bar{P}^n_{j,s} - \bar{P}^{n+1}_{j,s}$ *denote the amount of liabilities currently bailed in at bank j over one seniority class and all seniority classes, respectively, and let Equity*$^n_j = \left(e + (\Theta^n)' V^*(P^*) + \sum_s \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s} - \sum_s \bar{P}^n_{\cdot,s}\right)_j$ *denote the current equity of bank j. Then the fair conversion factor for creditor i in seniority class s is given by:*

$$C^{fair}_{j,i,s} = \begin{cases} y_{j,i,s} & \textit{if } Equity^n_j \leq 0 \\ \\ \frac{\Pi_{j,i,s} Bail\text{-}In^n_{j,s}}{\left(Equity^n_j + Bail\text{-}In^n_j\right)} & \textit{otherwise} \end{cases} \tag{3.8}$$

where $y_{j,i,s}$, $\sum_{i,s} y_{j,i,s} \leq 1$, *is a share that remains to be determined otherwise, in practice likely by computing a ratio similar to the above using paid-in capital instead of current book value equity. While it would seem consistent with the idea of the fair share that the bailed in creditors would gain full control of the bailed in bank, we have to assume that the* $y_{j,i,s}$ *are such that* C^{fair} *is still consistent with assumption 1 (i.e.* $\sum_{i,s} y_{j,i,s} < 1$*). We thus propose the fair choice of* y *as:*

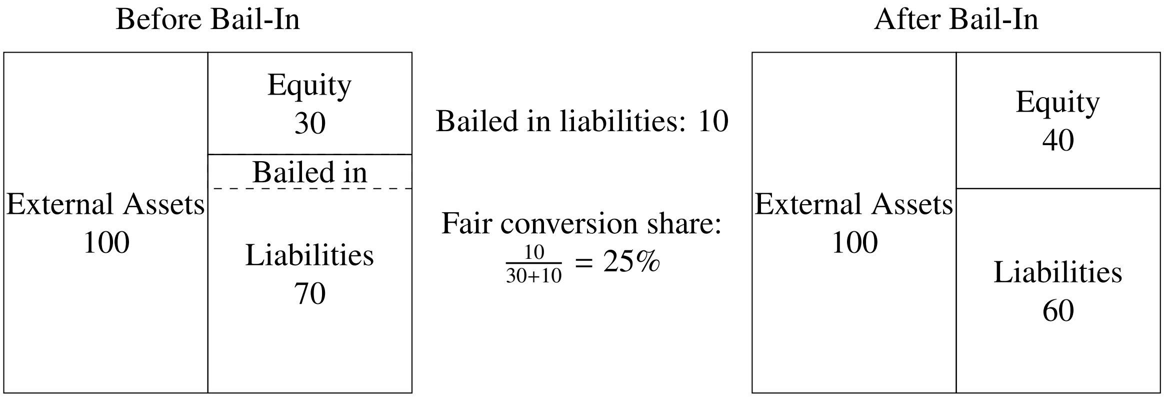
$$y^{fair}_{j,i,s} = \begin{cases} 0 & \textit{if } Bail\text{-}In^n_{j,s} = 0 \\ \\ \gamma \frac{\Pi_{j,i,s} Bail\text{-}In^n_{j,s}}{Bail\text{-}In^n_j} & \textit{otherwise} \end{cases} \tag{3.9}$$

where $\gamma \in [0, 1)$ *ensures* $\sum_{i,s} y_{j,i,s} < 1$. *In order to be able to ensure that the non-positive wealth impact of the fair conversion matrix also holds for the old equity owners, we further define* $C^{fair,\, y=1}$*, which explicitly violates assumption 1 (i.e.* $\sum_{i,s} y_{j,i,s} = 1$*), and will be used later:*

$$C^{fair,\ y=1}_{j,i,s} = \begin{cases} 0 & if\ Bail\text{-}In^n_{j,s} = 0 \\ \\ \frac{\Pi_{j,i,s} Bail\text{-}In^n_{j,s}}{\left(\max\left(0, Equity^n_j\right) + Bail\text{-}In^n_j\right)} & otherwise \end{cases} \tag{3.10}$$

We will see later that if $C^{\text{fair}}_{j,i,s} = C^{\text{fair, } y=1}_{j,i,s}$, the bail-in is neutral for all affected creditors, and deal with the general case there as well. For now we present a simple example to demonstrate the mechanics of a bail-in. Consider a bank with only external assets of 100 monetary units, only one creditor in one seniority class of liabilities totalling 70 monetary units, of which 10 are bailed in (figure 5). The fair share in this example is given by 25%, leaving the old equity owners with an unchanged equity stake of $(1 - 25\%) * 40 = 30$ and the bailed in creditor with a $25\% * 40 = 10$ equity stake, equal to the amount of bailed in liabilities. A higher conversion share, e.g. 50% would imply equal equity stakes of 20 for both investors, thus transferring 10 monetary units from the old shareholder to the creditor. A lower share, e.g. 0%, corresponding to a complete write-down for the creditor, would imply a transfer of 10 units from the creditor to the old shareholder. Note that the structure of the balance sheet, as shown in the right part of figure 5, would look the same under all conversion shares, as these only affect the investors.

Figure 5: Fair conversion share example



3.2.2. Computing payoffs

Note that a bail-in implies a similar network reconfiguration as the triggering of a CoCo, albeit the effects are somewhat more difficult to track, as the amount of bailed-in liabilities has to be determined as a function of the current balance sheet composition. A bail-in by another bank in the network may lead to equity losses at a bank that has already conducted a bail-in, necessitating a further bail-in of some share of liabilities. We refer to this process as the 'bail-in iteration'. We provide a framework in which one step of this iteration is modeled, and we show the convergence of the iteration.

The iteration starts with the initial configuration of liabilities and equity holdings, $\bar{P}$ and Θ. We model a step of the iteration by $(P^{\bar{k}+1}, \Theta^{k+1}) = \Phi(\bar{P}^k, \Theta^n)$, where Φ is calculated according to the below algorithm.

1. Start the iteration with $\bar{P}^1 = \bar{P}$, $\Theta^1 = \Theta$ and Bail-In$^0 = \mathbf{0}$.

2. Compute P^* according to the Elsinger algorithm using $\bar{P}^k$ and Θ^k.

3. Check which banks have breached the bail-in threshold by computing the corresponding capital ratio,

$$\Lambda^k = diag\left(\frac{e + (\Theta^k)' V^*(P^*) + \sum_s \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s} - \sum_s \bar{P}^k_{\cdot,s}}{e + (\Theta^k)' V^*(P^*) + \sum_s \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s}} < \lambda_B\right) \tag{3.11}$$

4. Compute the amount of required bail-in under P^*, $\bar{P}^k$ and Θ^k for each bank. The bail-in amount is capped at the amount of bail-in-able liabilities, and we further do not allow negative bail-ins (lowering of the capital ratio for banks that are above the threshold). We thus obtain the vector Bail-In^k of bail-in amounts at each bank:

$$\text{Bail-In}^k = \Lambda^k \min\left(\sum_{s=S-K+1}^{S} \bar{P}^k_{\cdot,s}, \left(\sum_s \bar{P}^k_{\cdot,s} - \left(1 - \lambda^R\right)\left(e + (\Theta^k)' V^*(P^*) + \sum_s \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s}\right)\right)^+\right) \tag{3.12}$$

5. Update the liabilities and holdings matrices:

$$\bar{P}^{k+1}_{\cdot,s} = \begin{cases} \bar{P}^k_{\cdot,s} & \text{if Bail-In}^k \leq \sum_{\kappa>s} \bar{P}^k_{\cdot,\kappa} \\ \bar{P}^k_{\cdot,s} - \left(\text{Bail-In}^k - \sum_{\kappa>s} \bar{P}^k_{\cdot,\kappa}\right) & \text{if } \sum_{\kappa>s} \bar{P}^k_{\cdot,\kappa} \leq \text{Bail-In}^k < \sum_{\kappa\geq s} \bar{P}^k_{\cdot,\kappa} \\ 0 & \text{otherwise} \end{cases}$$

(3.13)

The updated holding matrix follows from $\bar{P}^{k+1}$. Define $\text{Bail-In}^k_{i,j,s}$ to be the liabilities of bank i bailed in in seniority class s, $\varphi^k_{i,j,s}$ to be the share bank j gains of bank i due to the bail-in of the liabilities $L_{i,j,s}$, and $\bar{\varphi}^k$ to be the vector of total shares gained over each bank during the bail-in in iteration k,

$$\text{Bail-In}^k_{i,j,s} = \Pi_{i,j,s}\left(\bar{P}^k_{i,s} - \bar{P}^{k+1}_{i,s}\right) \tag{3.14}$$

$$\varphi^k_{i,j,s} = C_{i,j,s}\left(\text{Bail-In}^k, \text{Equity}^k\right) \tag{3.15}$$

$$\bar{\varphi}^k_i = \sum_{j,s} \varphi^k_{i,j,s} \tag{3.16}$$

As alluded to in definition 8, even when a fair conversion matrix is used, the conversion may lead to a gain for the old equity owners if the equity was negative prior to the bail-in and positive afterwards. In order to make the impact of the bail-in non-positive for all investors we define a penalty $\varphi^p_i = \sum_{j,s}\left(C^{\text{fair, } y=1}_{i,j,s} - C_{i,j,s}\right)$, by which the equity shares of the old owners will be decreased. The updated conversion matrix is then given by:

$$\Theta_{j,i}^{k+1} = \left(1 - \bar{\varphi}_j^k - \mathbb{I}_{\bar{\varphi}_j^k > 0} f_j^p\right)\Theta_{j,i}^k + \sum_s \varphi_{j,i,s}^k, \quad \forall\ i, j \tag{3.17}$$

where $\mathbb{I}$ represents an indicator function. Note that the penalty is optional and φ_j^p may be set to zero if a non-positive bail-in (see definition 9 below) is not required.

6. If Bail-In^k = Bail-In^{k-1}, stop the iteration. Otherwise, continue to step 2.

We now establish a useful characterization of bail-ins:

Definition 9 (Neutral and non-positive bail-ins)**.** *We say that a bail-in is neutral if it does not change the payoff value of all investments (debt plus equity) for all investors:*

$$\forall\ i, j :\ \Theta_{j,i}^{k+1} \max\left(0, Equity_j^k + Bail\text{-}In_j^k\right) - \Theta_{j,i}^k \max\left(0, Equity_j^k\right) = \sum_s \Pi_{j,i,s} Bail\text{-}In_{j,s}^k \tag{3.18}$$

Note that this relates to both bailed-in creditors as well as diluted equity investors.

Non-positive bail-ins are defined analogously:

$$\forall\ i, j :\ \Theta_{j,i}^{k+1} \max\left(0, Equity_j^k + Bail\text{-}In_j^k\right) - \Theta_{j,i}^k \max\left(0, Equity_j^k\right) \leq \sum_s \Pi_{j,i,s} Bail\text{-}In_{j,s}^k \tag{3.19}$$

Lemma 2.

(i) *When* $Equity_j^k + Bail\text{-}In_j^k \leq 0$*, any bail-in is non-positive.*

(ii) *When* $Equity_j^k \leq 0, Equity_j^k + Bail\text{-}In_j^k \geq 0$ *and* $\varphi_i^p = \sum_{j,s}\left(C_{i,j,s}^{fair,\, y=1} - C_{i,j,s}\right)$*,* $\forall\ i$*, a fair conversion matrix implies a non-positive bail-in.*

(iii) *When* $Equity_j^k \geq 0$ *and* $\varphi_i^p = \sum_{j,s}\left(C_{i,j,s}^{fair,\, y=1} - C_{i,j,s}\right)$*,* $\forall\ i$ *, a fair conversion matrix implies a neutral bail-in.*

Proof. See Appendix B. □

Lemma 3.

If the bail-in is neutral and $Equity^k + Bail\text{-}In^k \geq 0$*, then*

$$V^*(\bar{P}^{k+1}, \Theta^{k+1}) = Equity^k + Bail\text{-}In^k \tag{3.20}$$

Proof. Consider the equity map $\psi^S(V, P) = \left(e + \sum_{s=1}^S \Pi'_{\cdot,\cdot,s} P_{\cdot,s} - \sum_{s=1}^S \bar{P}_{\cdot,s} + \Theta' V\right)^+$ defined in section 2.5.2. Assume the clearing payment matrix under $\bar{P}^{k+1}$ and Θ^{k+1} is $\bar{P}^{k+1}$, so that all banks are solvent. If, under this assumption, $\text{Equity}^{k+1} \geq 0$, then the assumption is validated, due to the definition of a clearing payment matrix 7. Applying definition 9, we obtain for all j,

$$\psi^S(\text{Equity}^k + \text{Bail-In}^k, \bar{P}^{k+1})_j = \tag{3.21}$$

$$= \left(e_j + \sum_s \sum_i \Pi_{i,j,s} \bar{P}^{k+1}_{i,s} - \sum_s \bar{P}^{k+1}_{j,s} + \sum_i \Theta^{k+1}_{i,j} (\text{Equity}^k_i + \textit{Bail-In}^k_i) \right)^+ \tag{3.22}$$

$$= \left(e_j + \sum_s \sum_i \Pi_{i,j,s} \bar{P}^{k+1}_{i,s} - \sum_s \bar{P}^{k+1}_{j,s} + \sum_i \left(\Theta^k_{i,j} V^*(\bar{P}^k, \Theta^k)_i + \sum_s \Pi_{i,j,s} \textit{Bail-In}^k_{i,s} \right) \right)^+ \tag{3.23}$$

$$= \left(e_j + \sum_s \sum_i \Pi_{i,j,s} (\bar{P}^{k+1}_{i,s} + \textit{Bail-In}^k_{i,s}) - \sum_s \bar{P}^k_{j,s} + \textit{Bail-In}^k_j + \sum_i \Theta^k_{i,j} V^*(\bar{P}^k, \Theta^k)_i \right)^+ \tag{3.24}$$

$$= \text{Equity}^k_j + \textit{Bail-In}^k_j \geq 0 \tag{3.25}$$

This proves that $\text{Equity}^k + \text{Bail-In}^k$ is a fixed point of $\psi^S(V)$. Elsinger (2009) prove that such a solution is unique (see Lemma 5). Hence, $V^*(\bar{P}^{k+1}, \Theta^{k+1}) = \text{Equity}^{k+1} = \text{Equity}^k + \text{Bail-In}^k \geq 0$, and the clearing payment matrix is $\bar{P}^{k+1}$.

□

Theorem 3.

(i) The sequence $\bar{P}^k$ converges to a limit $\bar{P}^B$ for any conversion matrix.

(ii) If the conversion is neutral, under $\bar{P}^B$ every bailed-in bank either reaches its recapitalization ratio or fully bails in all bail-in-able liability classes.

Proof.

(i) Note that by definition, $\bar{P}^{k+1} \leq \bar{P}^k$ and further note that $\bar{P}^k \geq \mathbf{0}$ for all k, where the inequalities are understood to be component-wise. Hence there exists a monotone limit $\lim_{k\to\infty} \bar{P}^k = \bar{P}^B$, in the norm $\|A\|_\infty = \sup_{i,j} |A_{i,j}|$.

(ii) Recall that the bail-in amount Bail-In^k is set as

$$\text{Bail-In}^k = \min\left(\sum_{s=S-K+1}^{S} \bar{P}^k_{\cdot,s}, \left(\sum_s \bar{P}^k_{\cdot,s} - (1-\lambda^R)\Big(e + (\Theta^k)' V^*(\bar{P}^k, \Theta^k) + \sum_s \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s}\Big)\right)^+ \right) \tag{3.26}$$

If $\text{Bail-In}^k_j = \left(\sum_{s=S-K+1}^{S} \bar{P}^k_{\cdot,s}\right)_j$, the bank fully bails in all bail-in-able liability classes. If $\text{Bail-In}^k_j = 0$, there is no bail-in. Otherwise,

$$\text{Bail-In}^k_j = \left(\sum_s \bar{P}^k_{\cdot,s} - (1-\lambda^R)\Big(e + (\Theta^k)' V^*(\bar{P}^k, \Theta^k) + \sum_s \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s}\Big)\right)_j \tag{3.27}$$

By Lemma 3,

$$V^*(\bar{P}^{k+1}, \Theta^{k+1})_j = \text{Equity}^k_j + \text{Bail-In}^k_j \tag{3.28}$$

$$= \lambda^R_j \left(e + (\Theta^k)' V^*(\bar{P}^k, \Theta^k) + \sum_s \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s} \right)_j \geq 0 \tag{3.29}$$

(3.30)

The new capital ratio is:

$$\lambda_j = \frac{V^*(\bar{P}^{k+1}, \Theta^{k+1})_j}{V^*(\bar{P}^{k+1}, \Theta^{k+1})_j + \sum_s \bar{P}^{k+1}_{j,s}} \tag{3.31}$$

$$= \frac{\lambda^R_j \left(e + (\Theta^k)' V^*(\bar{P}^k, \Theta^k) + \sum_s \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s}\right)_j}{\text{Equity}^k_j + \text{Bail-In}^k_j + \sum_s \bar{P}^k_{j,s} - \text{Bail-In}^k_j} \tag{3.32}$$

$$= \lambda^R_j \tag{3.33}$$

Hence, the recapitalization ratio has been reached. In fact, this argument shows that the iteration ends after one step, after which every bailed-in bank either reaches its recapitalization ratio or fully bails in all bail-in-able liability classes, and every non bailed-in bank maintains its original capital structure, having a capital ratio higher than the bail-in ratio.

□

The matrix Θ^B is defined from $\bar{P}^B$ analogously to how Θ^{k+1} is defined from $\bar{P}^{k+1}$.

Combining the Bail-in mechanism described above with the CoCo's procedure described in section 2.5.2 would require checking whether CoCo's or bail-in are triggered first ($\lambda^B < \lambda^C$ or $\lambda^C < \lambda^B$) and then ensuring that in the first iteration where at least one of the two is breached, only the corresponding update operations will be effected. If $\lambda^C = \lambda^B$ then the update operations need to be performed in the same iteration, unless one assumes that e.g. the CoCo's will be converted first in this case as well.

3.3. Valuation of financial contracts in the network model

The algorithms of the previous sections corresponds to debt valuation at maturity (i.e, a calculation of payoffs). We now consider the problem of valuing debt before maturity in the previous framework, thus contributing to the research which began with Merton's structural model for pricing debt (Merton (1974)), and was expanded by Suzuki (2002) and Fischer (2014), who adapted Merton's model to the interbank network framework. The model allows to price all of the seniority classes (equity, junior debt, etc.), and also can be extended to the case of equity conversions which will be introduced in the next sections.

Consider a time frame $\mathbb{T} = [t_0, T]$, where t_0 is the time at which the pricing is made, a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in\mathbb{T}}, \mathbb{P})$, and a n-dimensional Brownian motion W, such that the filtration $(\mathcal{F}_t)$ is generated by W. Assume that the external assets $e_t \in \mathbb{R}^{n\times\mathbb{T}}$ of each bank follow a diffusion process of the form:

$$\mathrm{d}e_t = \mu(t, e_t)\mathrm{d}t + \sigma(t, e_t)\mathrm{d}W_t \tag{3.34}$$

Where $\mu(t, e_t) \in \mathbb{R}^n$ is the vector of drifts, and $\sigma(t, e_t) \in \mathbb{R}^n$ is the vector of volatilities. For simplicity, we will restrict the model to the case where e_t is a multi-dimensional geometric Brownian motion, with $\mu(t, e_t) = \mu e_t$ and $\sigma(t, e_t) = \sigma e_t$ - however, more general models such as 3.34 could be considered without adding much more complexity. We also assume the interest rate is $r = 0$. The external assets can be correlated, so assume W is a n-dimensional Brownian motion, with correlation matrix Σ. Under certain assumptions, this framework corresponds to a multidimensional Black-Scholes model (see Merton (1974)).

In order to compute the valuation at time t, we first define a payoff function that returns the payoffs for all liabilities in the network as a function of the value of external assets at time T:

Definition 10 (Payoff function). *The payoff for a given value of external assets is*

$$\Psi \colon \mathbb{R}^n \to \mathbb{R}^{n \times n \times S} \tag{3.35}$$

$$e_T \mapsto \left(\Pi_{i,j,s} * P^*(e_T)_{i,s}\right)_{i,j,s} \tag{3.36}$$

The valuation of those payoffs for a liability $L_{i,j,s}$ at time t is then given by the expectation $\mathbb{E}^{\mathbb{Q}}[\Psi(e_T)_{i,j,s}|\mathcal{F}_t]$, where $\mathbb{Q}$ is the risk-neutral measure. It follows from the measurability of the payoff function (Theorem 5) that this expectation is well-defined.

Theorem 4. *[Measurability of the payoff function] The function Ψ is measurable.*

Proof. See Appendix A □

In order to define a valuation for CoCo's we first need to define a payoff function at maturity. The payoff of a CoCo is given by the total amount of liabilities if the trigger event has not occurred at maturity, and the value of the equity conversion if else. Let $P^{*,C}$ be the clearing payment matrix with CoCo's obtained through the previous iteration. The payoff function is thus defined as:

Definition 11 (Payoff function with CoCo's)**.** *The CoCo payoff for a given value of external assets is*

$$\Psi^C : \mathbb{R}^n \to \mathbb{R}^{n\times n} \tag{3.37}$$

$$e_T \mapsto \Big(\Pi_{i,j,s_c} * P^{*,C}(e_T)_{i,s_c} + (\Theta^{*,C}_{i,j} - \Theta_{i,j})V^*_i(e_T, \bar{P}^{*,C}, \Theta^{*,C})\Big)_{i,j=1,\ldots,n} \tag{3.38}$$

The measurability of Ψ^C follows from the same argument as Theorem 5. Hence, the ex-ante valuation of the CoCo issued by bank i to bank j at time t is given by:

$$\mathbb{E}^{\mathbb{Q}}[\Psi^C(e_T)_{i,j}|\mathcal{F}_t] \tag{3.39}$$

In order to compute valuations for Bail-Ins, we again need to define a payoff function for bail-in-able securities at maturity. The payoff of such a security is given by the total payoff of those liabilities that have not been bailed in, plus the equity value of the shares gained in each bail-in iteration. The payoff function is thus given by:

Definition 12 (Payoff function with Bail-In)**.**

$$\Psi^B : \mathbb{R}^n \to \mathbb{R}^{n \times n \times S} \tag{3.40}$$

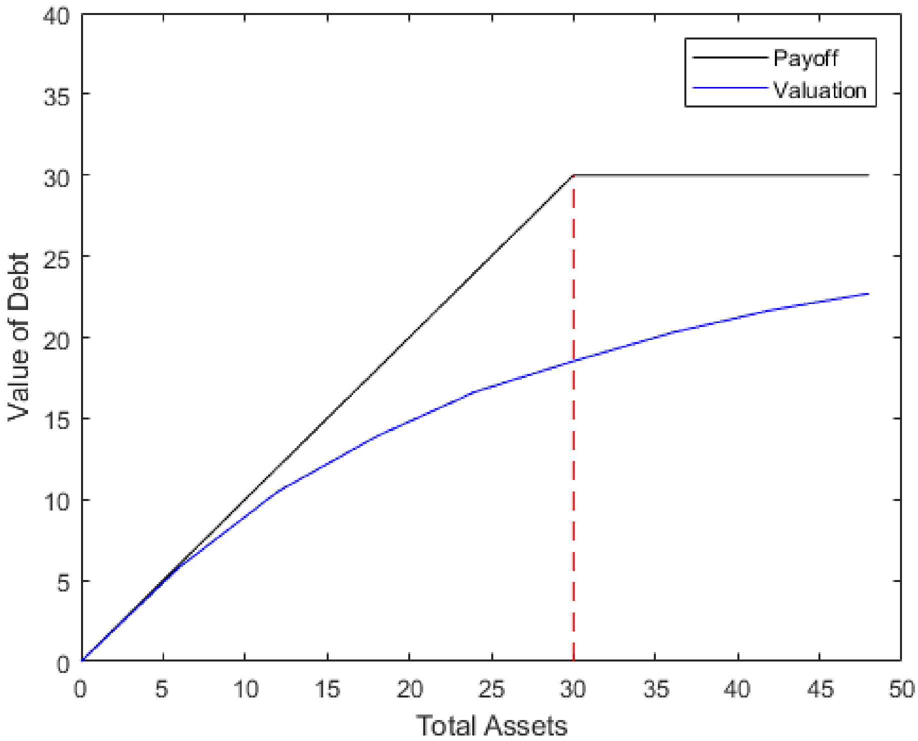
$$e_T \mapsto \left(\Pi_{i,j,s} * P^{*,B}(e_T)_{i,s} + (\Theta^{*,B}_{i,j} - \Theta_{i,j})V^*_i(e_T, \bar{P}^{*,B}, \Theta^{*,B})\right)_{i,j,s} \tag{3.41}$$

The measurability of Ψ^B again follows from the same argument as Theorem 5. Hence, the ex-ante valuation of a bail-in-able security issued by bank i to bank j at time t is thus given by:

$$\mathbb{E}^{\mathbb{Q}}[\Psi^B(e_T)_{i,j}|\mathcal{F}_t] \tag{3.42}$$

It should be noted here that the addition of valuation functions to the model is not primarily intended to be used for asset pricing or for trading of CoCo debt securities. The main point of interest from the perspective of systemic risk assessment is how it changes how the system reacts to small shocks to the solvency of some banks. In the model of Eisenberg and Noe (2001), no contagion effects at all are observed as long as there are no defaults. This restrictive assumption has motivated alternative models of contagion such as that of Battiston et al. (2012), which also capture '*pre-default contagion*', i.e. valuation losses of interbank holdings that occur before insolvency. Figure 6 illustrates how pre-default-contagion enters the model, by plotting the valuation function (for a neutral bail-in) against the corresponding payoff function. As can be seen, the valuation function is also decreasing

Figure 6: Payoff vs. valuation function



when the solvency position of a solvent creditor deteriorates, leading its creditors to record a (small) loss on their interbank holdings. See Barucca et al. (2020) for a more in-depth discussion of how different contagion models such as Furfine (2003) and Battiston et al. (2012) can be unified in a common framework by considering more general valuation functions than the piecewise linear payoff function of the model of Eisenberg and Noe (2001).

3.4. Discussion

In this chapter we have presented a model of network valuation for different financial contracts, namely equity participations, debt liabilities of different seniority levels and contingent convertible debt instruments. The starting point of our model is a combination of the work of Elsinger (2009), who model the valuation of equity and debt of different seniority levels, with the work of Barucca et al. (2016) and Fischer (2014), who discuss the network valuation of debt at time points before maturity under stochastic prices for external assets. The combined model can be seen as an extension of the model of Merton (1974) to the multidimensional case with cross-holdings of different instruments. We extend the combined model to include contingent convertible debt instruments as well as the bail-in of entire seniority classes of financial instruments to reach capitalization targets.

4. An application to systemic risk assessment for Bail-Ins

In this chapter we will make use of the methodologies developed in the previous chapters to show they can be used for assisting with real-world policy decisions.

4.1. Background on bail-ins and financial regulation

As a consequence of the GFC policy-makers devised a framework with the aim to resolve banks quickly while minimising risks to financial stability, the real economy and taxpayers. In 2011 the Financial Stability Board published its "Key attributes for effective resolution regimes" (FSB, 2011). The European Commission created the Bank Recovery and Resolution Directive (BRRD), which became effective in 2014 (BRRD, 2014).

Resolution is therefore a cornerstone of the post-crisis regulatory framework. Its **objectives** are to ensure the continuity of critical functions banks provide, to avoid significant negative effects on the financial system, to protect insured depositors, public funds, and client assets and funds. However, resolution is not intended to be applied to banks automatically – it has to be in the public interest and private sector measures have to be exhausted. This can be the case for banks which provide critical functions or for banks, which, due to their size or complexity, would threaten financial stability at large if they were liquidated.

The legal framework addresses banks, supervisory authorities and resolution authorities and foresee a set of measures for different stages of financial distress:

- Under **normal operations**, banks have to prepare recovery plans, including specific measures they will implement if in distress. Together with higher capital requirements (which were also implemented after the GFC) this aims at increasing the financial and operational resilience of banks and reducing the probability of bank failures.

- Should conditions deteriorate the bank goes into **recovery mode** and activates its recovery plan. Supervisory authorities were given powers to intervene early in case the bank's measures prove to be insufficient and breaches of legal provisions are imminent.

- Should the bank reach a point where it is failing or likely to fail, resolution authorities take over responsibility from supervisors and the bank goes into either **resolution or liquidation**. This fundamental decision is based on the criteria outlined above. If resolution is possible, the resolution authority activates the resolution plan it has prepares in advance.

While the legal framework does not exclude **government stabilisation**, the use of public funds should be limited to exceptional situations of a systemic crisis. Instead, resolution authorities have a number of tools at their disposal to resolve a bank without the consent of existing shareholders. The "**bail-in tool**" allows recapitalising a bank by writing down and/or converting to equity the claims of unsecured creditors. Three "transfer tools" can be used to transfer (parts) of a bank's business to other entities. The "**sale of business tool**" allows selling the bank or part of its business to other private buyers. With the help of the "**bridge bank tool**" parts of the business can be transferred to a publicly owned bridge bank ("good bank") which upholds critical functions until a private sector solution can be found, while the "**asset separation tool**" allows transferring parts of the business to a publicly owned asset management vehicle ("bad bank"), which would be gradually wound-down, avoiding fire-sale losses.

The "**bail-in tool**", which we examine in this study, is a means for recapitalising a bank which is failing or likely to fail. It allows recapitalising a bank by writing down and/or converting to equity all liabilities which are not explicitly excluded (BRRD, 2014, Art. 44).[6]

Our goal in this paper is to study whether bail-ins can contribute to the objectives of improving stability described above using real-world data. We do this by comparing hypothetical crises scenarios under different regimes, one where bail-ins are always performed if possible, and one where they are never performed, i.e. banks are always sent into liquidation. We then compare the systemic impact under both regimes. In this analysis, we account for another important difference between resolution and liquidation: the former maintains at least parts of the bank's business and therefore its franchise value, which has important implications for the valuations of its assets. Our model also accounts for general resolution principles defined in the

[6]Excluded are deposits to the extent covered by a deposit guarantee scheme, secured liabilities (up the to the collateral value), certain liabilities with an original or residual maturity of less than 7 days, liabilities in relation to employee salary and pension benefits, tax and social security authorities and providers of critical services to the bank in resolution (e.g. IT operations, rental of premises). A detailed description of the seniority classes is presented in section 4.3.

BRRD, namely that current shareholders shall bear losses first, that creditors shall bear losses next and in the order of priority their claims would have under normal insolvency proceedings, that creditors of the same class shall be treated equitably and that no creditor shall incur greater losses following resolution than would have been incurred if the bank had been wound up under normal insolvency proceedings. The latter is referred to as the "no creditor worse off (NCWO) principle" (BRRD, 2014, Art. 34), which we will analyze separately in this study.

4.2. Methodology

In order to provide quantitative answers to our stated goal of analysing the financial stability impact, we concretize this by assessing whether creditors will be worse off when a Bail-In is performed versus when the bank is allowed to enter into insolvency. We want to consider the full systemic implications of performing a Bail-In, i.e. take into account second- and further-round effects of potential losses that creditors face under a Bail-In or an insolvency. Such losses may lead to defaults of creditors of bailed-in banks, leading to further Bail-Ins or insolvencies.

Default cascades have been studied extensively in the literature on financial contagion. However, in this analysis we want to take into account for an effect that is often overlooked in such studies: typically, contagion models often assume that all creditors are either equally or proportionally affected by a default, thereby ignoring the seniority order of creditors. This is because prior to the introduction of the BRRD, data on bilateral exposures with seniority information was typically not collected. The seniority structure has significant implications for contagion, however, as it concentrates contagion losses at the junior creditors. These creditors will thus have to bear higher losses than what would be implied if losses were distributed proportionally among all creditors. In order to allow for a fair comparison of Bail-Ins and Insolvencies, we thus consider the effects of the seniority structure in both regimes. In the following chapters we present our approach to measuring systemic risk in these two regimes.

4.2.1. Computing systemic losses

As discussed, we wish to quantify systemic losses both in the presence and in the absence of bail-ins. We assume that either all banks that have bail-in-able capital perform bail-ins, or that none of them do. We will refer to these two different situations as the **bail-in regime** and the **insolvency regime**, respectively.

Our analysis of the **insolvency regime** is based on the model by Elsinger (2009), i.e. we do account for multiple seniority classes. We recall that the **clearing payment matrix** captures the actual payoffs of each node in the system in each seniority class. We recall that the clearing payment matrix satisfies

$$P^*_{\cdot,T} = \min\left(\bar{P}_{\cdot,T}, \left(e + \sum_{s=1}^{S} \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s} + \Theta' \max\left(\mathbf{0}, V(P^*)\right) - \sum_{s=1}^{T-1} \bar{P}_{\cdot,s}\right)^+\right) \tag{4.1}$$

for each seniority class $T \in 1, \ldots, S$, where $\mathbf{0}$ is a vector of 0's, the max, min operations are applied element-wise. $V(P)$ captures the equity of each bank under a payment matrix P and satisfies

$$V(P) = e + \sum_{s=1}^{S} \Pi'_{\cdot,\cdot,s} \, P_{\cdot,s} + \Theta' \max\left(\mathbf{0}, V(P)\right) - \sum_{s=1}^{S} \bar{P}_{\cdot s}. \tag{4.2}$$

The proposed model is equivalent to the one presented in section 2.5 and allows for a straightforward quantification of the losses faced by each bank due to contagion, by considering the differences between the initial equity values $\bar{E} = V(\bar{P})$, and the equity values after accounting for contagion, $V^* = V(P^*)$. Bank i's losses are then given by:

$$L_i^* = \bar{V}_i - V_i^*. \tag{4.3}$$

We use an analogous setup to quantify systemic losses in the **insolvency regime**. Using the new model described in chapter 3 with fair conversion shares, we obtain a new interbank holdings matrix Θ^B, a total and relative liabilities matrices $\bar{P}^B$ and Π^B, as well as a clearing payment matrix P^B. From these we compute the equity values after contagion and Bail-Ins satisfying

$$V^B = e + \sum_{s=1}^{S} (\Pi^B_{\cdot,\cdot,s})' \, P^B_{\cdot,s} + \Theta' \max\left(\mathbf{0}, V^B\left(P^B\right)\right) - \sum_{s=1}^{S} \bar{P}^B_{\cdot s} \tag{4.4}$$

The equity V^B is increased mechanically through a bail-in, an effect that we have to correct for when computing contagion losses under the bail-in regime:

$$L_i^B = \bar{V}_i - V_i^B - \sum_{s=1}^{S} \bar{P}_{i,s}^B - \bar{P}_{i,s} \tag{4.5}$$

Comparing L^* and L^B then allows comparing contagion losses under the Insolvency and Bail-In Regimes. Since the results are computed on a bank-by-bank basis, the model also allows assessing whether any creditors are worse off under a Bail-In than under Insolvency, an important consideration for regulators. We will return to the discussion about comparing the two regimes in Section 4.4.

4.2.2. An extension to include gone-concern valuation

The moment when a bank is allowed to enter into insolvency has crucial implications for the valuation process. Normally, the going-concern principle is state of the art in valuation of firms. The value of a firm is therefore the sum of all expected future cash flows available for distribution to shareholders, discounted to the present value. However, in case of financial distress and imminent insolvency, the expected future cash flows in common sense are zero (or negative) and therefore these valuation methods are not useful. In this case, the future cash flows are taken as the liquidation price of the firms' assets to be achieved on the market (gone-concern principle).

Gone-concern valuations may be lower than going-concern due to various reasons: the assets may require company-specific knowledge in order to extract full profitability (e.g. production equipment) or asset markets may react with negative price movements to large amounts being sold (e.g. for financial assets). Assets like goodwill represent an inherent belief in the value of the business that may be void in the case of an insolvency.

The BRRD expressly foresees that the resolution authority carry out a "*fair, prudent and realistic valuation of the assets and liabilities of the institution*" (BRRD, 2014, Art. 36(1)). The Valuation Handbook by the European Banking Authority recommends various valuation methods and distinguishes between the "*hold value*" and the "*disposal value*" (EBA, 2019, p. 19). The hold value is described in Art. 1(e) of the technical supplement to the BRRD as the "*present value, discounted at an appropriate rate, of cash flows that the entity can reasonably expect under fair, prudent and realistic assumptions from retaining particular assets and liabilities*", i.e. as a going-concern valuation. The disposal value on the other hand is described as the "*valuation on the basis of the cash flows, net of disposal costs and net of the expected value of any guarantees given, that the entity can reasonably expect in the currently prevailing market conditions through an orderly sale or transfer of assets or liabilities*" (Art. 12(5) of the Technical Supplement to the BRRD), i.e. a gone-concern valuation. It is expressly pointed out that the liquidation of the company's assets may result in discounts in the valuation.

We incorporate liquidation losses into our model by introducing a recovery rate $r \in [0, 1]$ for the assets of insolvent banks and we replace Eq. (4.1) by

$$P^*_{\cdot,T} = \begin{cases} \bar{P}_{\cdot,T} \text{ if } e + \sum_{s=1}^{S} \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s} + \Theta' \max\left(\mathbf{0}, E(P^*)\right) - \sum_{s=1}^{T-1} \bar{P}_{\cdot,s} \geq \bar{P}_{\cdot,T} \\ r\left(e + \sum_{s=1}^{S} \Pi'_{\cdot,\cdot,s} P^*_{\cdot,s} + \Theta' \max\left(\mathbf{0}, E(P^*)\right) - \sum_{s=1}^{T-1} \bar{P}_{\cdot,s}\right)^+ \text{ otherwise} \end{cases} \tag{4.6}$$

for the insolvency regime, and we make an equivalent change in the bail-in regime.

The extent of valuation losses – the value for r – should acknowledge the additional losses imposed on creditors through the liquidation of an insolvent banks. Diamond and Dybvig (1983) made such valuation losses for short-term liquidation the foundation of their seminal model on bank runs. The liquidity model by Shleifer and Vishny (1992) stated that industry condition plays an important role in determining the price a company is able to achieve. Concrete figures for liquidation losses are, however, difficult to obtain. Pulvino (1998) studied a different sector compared to this chapter – commercial aircraft transactions – and found that asset prices realized by distressed firms were between 14% and 30% below market prices. For large firms the settlement process of credit default swaps (CDS) provides a potential data source. It involves an auction process aimed at determining the losses to creditors. These losses can be above 40% for banks (Risk.net, 2008). However, the resulting valuation losses are a market-estimate, highly specific to the nature of CDS, and conceptually different from the valuation loss due to gone concern valuation discussed above. Fleming and Sarkar (2014) performed a case study on the resolution of Lehman brothers which provides the best approximation to the situation that we wish to study. They found that estimates of these losses can go as high as 79%, although realized values may be lower. As a takeaway for our analysis, we note that there is considerable uncertainty regarding gone-concern valuation losses. We will thus consider the full range of possible values for r and keep in mind that high values of r may still be considered realistic.

4.3. Data

We combine different regulatory data sources on the Austrian banking system in order to assess the impact of bail-ins in a granular fashion. Specifically, the asset-side of the interbank network is based on the central credit register (CCR) and the liability-side is based on the newly introduced Liability Data Report (LDR). The analysis uses a snapshot as of December 2018, which marks the last reporting date for the CCR and the first for the LDR.

The CCR contains detailed exposure information of on and off-balance sheet exposures and risk information (e.g. credit ratings) from credit and financial institutions and insurance companies based in Austria of domestic and foreign borrowers. The exposures have to be reported by the institutions if the claims exceed EUR 350.000 per borrower (Bachmann et al., 2016).[7] The central credit register contains unconsolidated information on 599 bank creditors and 537 bank debtors, with a total of

[7] From autumn 2018 onward, the ECB project "Analytical Credit Dataset" (AnaCredit) extends and harmonises the credit risk reporting in Europe. At the OeNB, the Granular Credit Data survey officially replaced the Central Credit Register (CCR) as of January 2019. The new collection of credit risk data now combines the contents of the CCR report (under Article 75 Banking Act) and the AnaCredit report (under the Regulation (EU) 2016/867 of the European Central Bank of 18 May 2016) (OeNB, 2018a).

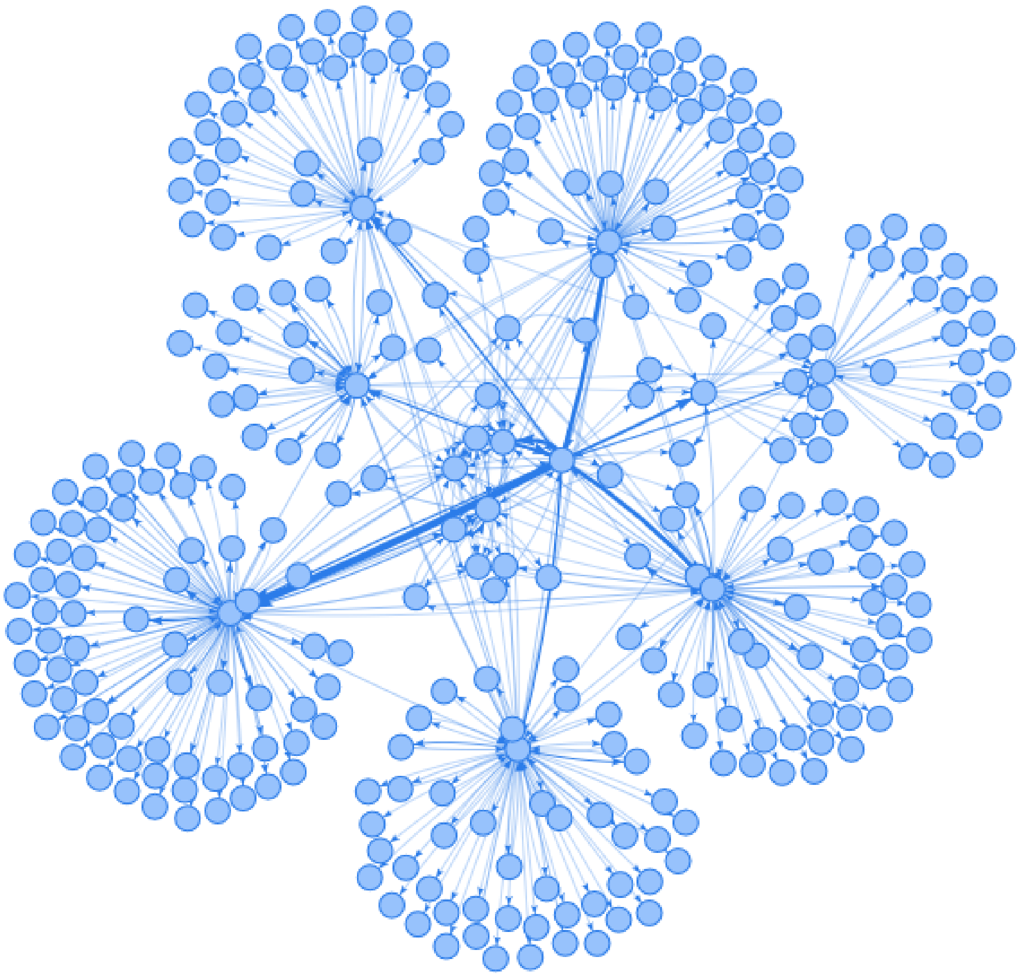
2886 links (or edges) between them amounting to EUR 158bn of exposure.

Figure 7 depicts the consolidated Austrian interbank network based on the central credit register. The Austrian banking system encompasses, as of year end 2018, 440 consolidated banks with total assets amounting to EUR986 billion. There are six significant institutions under ECB supervision representing 55 percent of total assets.[8] The banking system is characterised by its diverse business models, high concentration and multi-tiered structure. Especially the three-tier sector banks, including the Raiffeisen sector with its inverse consolidation scheme, can be clearly seen in figure 7. The most central bank in the graph is the Raiffeisen Bank International with its connections to the eight Raiffeisen clusters representing all Austrian federal states, besides Vienna.

The LDR was introduced by the Single Resolution Board (SRB) for the purpose of drafting resolution plans in cooperation with the National Resolution Authorities (NRAs). The aim is to collect data that goes beyond EBA's Implementing Technical Standards (ITS) on resolution planning. The main differences between the CCR and LDR surveys is that CCR data was collected monthly, whereas LDR data is only collected on an annual basis and that the sample of reporting banks is very different. Specifically, the LDR sample is much smaller compared to the CCR sample, as the reporting group for the LDR is based on banks that are under the responsi-

[8] As of December 2018, these banks are BAWAG Group AG, Erste Group Bank AG, Raiffeisen Bank International AG, Raiffeisenlandesbank Oberoesterreich Aktiengesellschaft, Volksbank Wien AG and Sberbank Europe AG. The third largest bank, UniCredit Bank Austria AG, is a systemic subsidiary of UniCredit. Starting with October 7 2020, Addiko Bank AG will become the seventh significant insitution.

Figure 7: The Austrian interbank network. The network is based on the Central Credit Register and thus shows the interbank claims as edges for each bank (i.e. the direction of the arrow represents the exposure). For illustration purposes, the edges of the network are only shown from a EUR 25 million upwards.



bility of the SRB and, according to the Single Resolution Mechanism Regulation (Regulation (EU) 2014/806), institutions and groups of institutions that are subject to supervision by the ECB and cross-border groups will be subject to the SRB's jurisdiction (OeNB, 2018b). To be precise, the Liability Data Report contains information on 51 Austrian unconsolidated banks and their balance sheets covering EUR 595bn or 76% of total assets of the Austrian banking system. Furthermore, for a total of 16 unique debtors the survey provides information on their respective

279 unique creditors, which makes up 5856 links with a total of EUR 10.5bn of liabilities. The outstanding feature of this database are the provided seniority classes for the liabilities. With these eight ranks - equity, Additional Tier 1 capital, Tier 2 capital, junior unsecured, senior unsecured, uncovered deposits, covered deposits and secured financing - we are able to add another valuable layer of information to our interbank network.

To the best of our knowledge, the combination of these two data sources yields the first example of leveraging assets and liabilities into one interbank network in order to assess the systemic risk impact of bail-ins. However, the combination of the central credit register and Liability Data Report cannot be achieved in a straightforward fashion. Even though, in theory, the two regulatory data sources should give the vice versa image of assets and liabilities, there a certain mismatches between the two that need to be addressed.

4.3.1. Combining data from different sources

In order to interweave the two interbank networks, we need to come-up with an algorithm to match the data between the central credit register and the Liability Data Report. Not only do we see a gap between the reported data in two data sources, but also within the Liability Data Report. The LDR provides information of each debtor-creditor relationship and also an aggregated balance-sheet for each reporting debtor. However, for certain banks, the debtor-creditor relationships don't add up to the aggregated balance-sheet data. In the following steps, we will take care of both gaps.

The CCR does not contain any information on the seniority of the provided exposures, except for the reported collateral values, which we can treat the same way as the highest seniority class provided in the LDR templates (i.e. secured financing). In order to calculate the *true* secured financing liabilities, we take the secured financing liabilities from the LDR and add the surplus value between the collateralized CCR data and the LDR. If the collateral value should be smaller than the secured liabilities, we take the liabilities from the LDR as is.

The unsecured part of the central credit register will distributed across the remaining six seniority classes based on a share between the respective liabilities in each seniority class to the aggregated balance-sheet liabilities as reported in the LDR. This marks also the step in which the above-mentioned LDR mismatch will be treated. Specifically, we calculate the surplus between the total balance-sheet liabilities per seniority class and the summation across the creditor per seniority class. We then calculate the share between the LDR surplus to the unsecured CCR exposures and multiply the exposure with this share. On the one hand, this leads to a distribution of the CCR data among the seniority classes and, on the other hand, closes the gap in the LDR data.

Additionally, we perform sanity checks through out this process in order to capture reporting or rounding errors, severe mismatches between the two data sources or make sure that the final database respects the balance sheet identity for each bank.

Specifically, the total exposure may not exceed the total assets, the total liabilities may not be larger than the total assets minus equity and define the external assets as the difference between the total interbank claims and total assets.

4.3.2. *Mapping seniority classes*

Table 3 presents an overview of the liability structure as reported in the Liability Data Reports. The Single Resolution Board (SRB, 2020) provides guidance to banks on how to report the data and is a thus a source of information on the proper mapping between the granular liabilities to the eight seniority classes and their *bail-in-ability*.

Table 3: Mapping of seniority classes

LDR Variables	Seniority Class (#rank)	bail-in-able?
Liabilities excluded from Bail-In		
Secured liabilities - collateralized part	Secured Financing (8)	No
Covered Deposits and Critical Liabilities	Covered Deposits (7)	No
Liabilities not excluded from Bail-In		
Deposits, not covered but preferential	Uncovered Deposits (6)	Yes
Deposits, not covered and not preferential	Senior Unsecured (5)	Yes
Balance sheet liabilities arising from derivatives	Senior Unsecured (5)	Yes
Uncollateralized secured liabilities	Senior Unsecured (5)	Yes
Structured notes	Senior Unsecured (5)	Yes
Senior unsecured liabilities	Senior Unsecured (5)	Yes
Senior non-preferred liabilities	Senior Unsecured (5)	Yes
Other MREL eligible liabilities	Senior Unsecured (5)	Yes
Subordinated liabilities (not recognised as own funds)	Junior Unsecured (4)	Yes
Non-financial liabilities	Junior Unsecured (4)	Yes
Residual liabilities	Junior Unsecured (4)	Yes
Own Funds		
Tier 2 Capital	T2 (3)	Yes
Additional Tier 1 capital	AT1 (2)	Yes
Common Equity Tier 1 Capital	Equity (1)	/

We start the description of the seniority classes in the same way the Bail-In mechanism works - from the most vulnerable to the most secure liabilities. The first group represents the own funds which make up the first three seniority classes. Common Equity Tier 1 capital (”Equity (1)”) is strictly speaking not a Bail-In-able class, however, it consists of shareholders equity and retained earnings and is used by banks to cover unexpected losses. Hence, it is directly linked to the Bail-In mechanism. The second and third seniority classes are the Additional Tier 1 (”AT1 (2)”) and Tier 2 capital (”T2 (3)”) respectively. The former includes preferred shares and high contingent convertible securities (CoCos), the latter represents supplementary capital such as various reserves, hybrid capital instruments and subordinated debt.

The second part of the Bail-In-able classes cover the actual liabilities that will be converted into equity in the case of losses. Specifically, the fourth seniority class (”Junior Unsecured (4)”) consists out of subordinated liabilities which will only be repaid after all classes of ordinary creditors have been repaid in full. We exclude the non-financial liabilities from the model as we focus on the interbank network and we do not utilize the residual liabilities as these are covered implicitly by the matching algorithm described above. The next seniority class (”Senior Unsecured (5)”) comprises the most unsecured debt instruments, ranging from deposits that do not qualify for exclusion from Bail-In or preferential treatment over structured notes (i.e. debt obligations that contain an embedded derivative component) to unsecured or non-preferred liabilities. The sixth category and thus last Bail-In-able seniority class (”Uncovered Deposits (6)”) are based on deposits for which a preferential treatment is foreseen.

The last group of seniority classes are both not Bail-In-able and can thus not be used to cover emerging losses. The first secure class ("Covered Deposits (7)") are different forms of covered deposits and critical liabilities (e.g. client, employee, fiduciary or institution liabilities). The highest ranking class ("Secured Financing (8)") is based on the collateralized part of secured liabilities (e.g. covered bonds and central bank liabilities which are covered by a collateral pool such as MRO, LTRO, TLTRO, etc.).

Table 4: Summary statistics for the seniority classes

Seniority Class (#rank)	Min	Q25%	Mean	Median	Q75%	Max	Std.Dev.
Equity (1)	2.6%	5.8%	10.6%	7.3%	10.0%	76.6%	12.9%
AT1 (2)	0.0%	0.0%	0.2%	0.0%	0.0%	1.8%	0.4%
T2 (3)	0.0%	0.3%	1.3%	1.0%	1.7%	9.1%	1.5%
Junior Unsecured (4)	0.0%	0.0%	0.3%	0.1%	0.5%	2.0%	0.4%
Senior Unsecured (5)	0.1%	13.5%	21.5%	20.5%	27.4%	54.5%	11.8%
Uncovered Deposits (6)	0.0%	11.4%	18.5%	18.1%	25.2%	44.2%	10.1%
Covered Deposits (7)	0.1%	21.9%	37.9%	36.3%	47.9%	91.4%	21.1%
Secured Financing (8)	0.0%	0.1%	9.8%	5.2%	16.2%	46.7%	12.1%

Note: the summary statistics are based on the reported balance sheets by the 51 debtors in the Liability Data Reports. All metrics are expressed as shares relative to total assets.

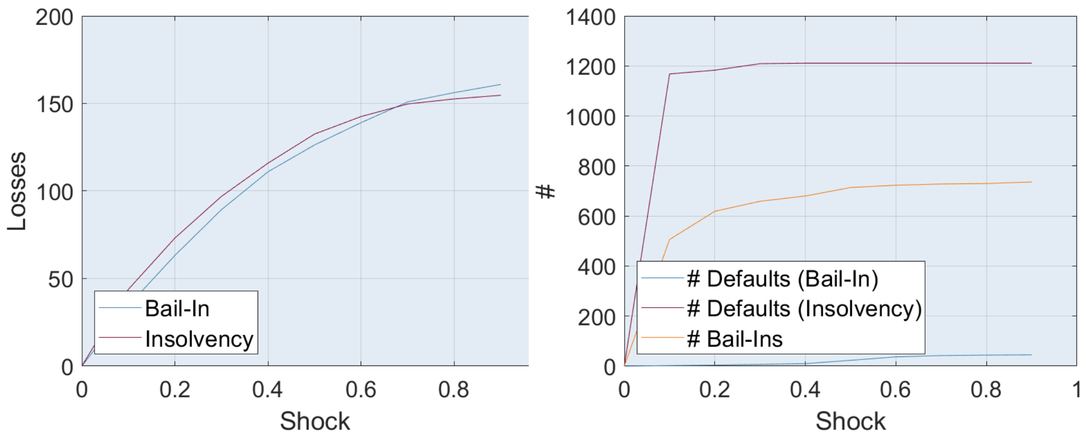
Table 4 gives a compact overview of the summary statistics across the seniority classes. The metrics are based on the 51 reported balance sheets for which we have calculated the relative shares of the respective seniority classes against the total balance sheet. Thus the mean column represents an average bank in our sample. The reported own funds - equity, AT1 and T2 - make up roughly 12.1% of the average balance sheet. The lion's share of own funds is held as equity and to a lesser extent as Tier 2 (T2) and additional Tier 1 (AT1) capital. The distribution across these capital classes is inline with the Basel III accord, which requires banks to hold the majority of their capital in Tier 1 instruments (BCBS, 2017). Within the Bail-in-able classes, we see a slight overhang of senior unsecured debt making up 21.5% of the average balance sheet, followed by uncovered deposits with 18.5% and, by a considerable margin, junior unsecured debt with only 0.3%. On average, the largest part of the liability side of the balance sheet is taken up by covered deposits with 37.9%. Furthermore, the ongoing monetary policy operations can also be observed with the fairly large share of the secured financing class making up 9.8%.

4.4. Analysis

Our goal is to assess how the introduction of Bail-Ins affects systemic risk in the financial system. In Section 4.2 we presented our approach computing contagion losses in the case that no Bail-Ins whatsoever are performed (**insolvency regime**), and when all banks that have bail-in-able liabilities perform a Bail-In once they reach a predefined Bail-In threshold (**bail-in regime**).

We start by quantifying systemic risk in the case of idiosyncratic crises: we assume an idiosyncratic shock for one bank that wipes out part of its external assets and turns its capital ratio negative. We the compute the vectors of bank-by-bank losses under the Insolvency and the Bail-In regime L^* and L^B as discussed in Section 4.2. In the Bail-In regime, we assume the same Bail-In and recapitalization thresholds of 4.5% and 12% CET1, respectively. These choices are based on discussions with regulators from a resolution authority. We compute the systemic losses by summing L^* and L^B, excluding the initial effects and losses at the sink node. We then repeat

Figure 8: Systemic impact as a function of the idiosyncratic shock size

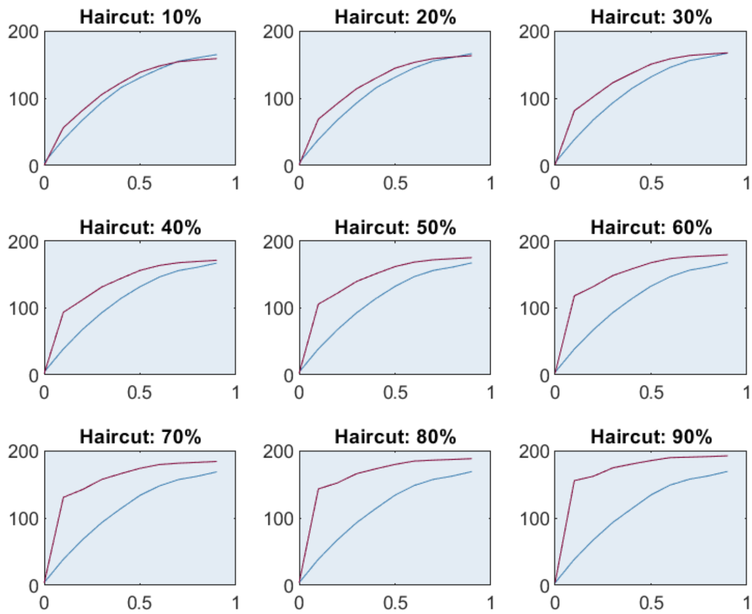


(a) Systemic risk losses (b) Number of defaults and Bail-Ins

the above procedure for all banks. Figure 8a shows the aggregate losses in the two regimes as a function of the size of the initial capital shock (as a percentage of total assets). Tables C.12-C.15 in the appendix provide numeric results. Figure 8b shows the number of contagious defaults happening in both regimes, as well as the number of bail-ins that are performed. As can be seen, the overall losses are similar in both regimes. Losses in the bail-in regime are slightly lower for moderate shock sizes below 60% of total assets, and slightly larger for higher shock sizes. Figure 8b shows a much clearer distinction between the two regimes, as the number of contagious defaults is dramatically reduced in the bail-in regime. These bail-ins, while still incurring losses for the creditors, manage to avoid bank defaults, thereby showing that the bail-in tool is effective at reducing the number of bank failures. We therefore note that, in terms of the number of institutions affected, the bail-in tool clearly manages to fulfill its objective of reducing systemic risk.

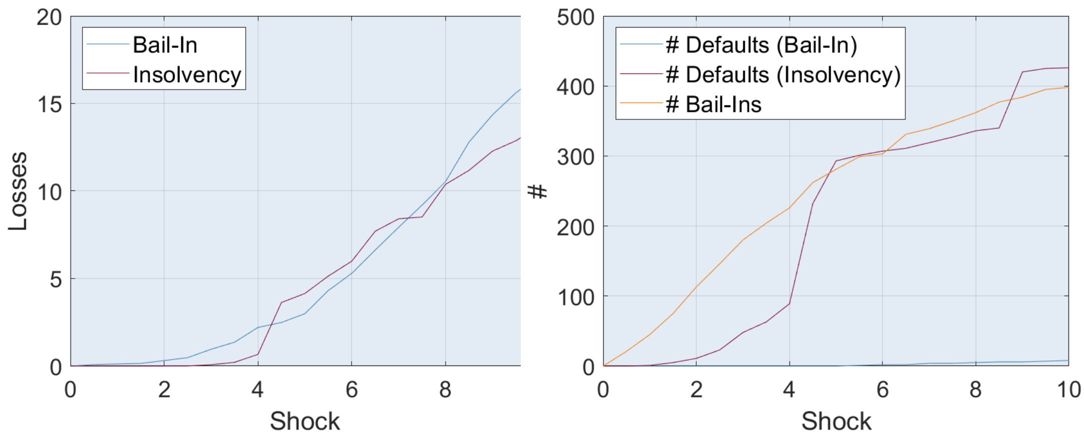
In terms of systemic losses, we recall that assuming a 100% recovery rate – or con-

Figure 9: Systemic losses for different liquidation haircuts (idiosyncratic shocks)



versely a 0% haircut through switching to gone-concern valuation – for the assets of insolvent banks, as we did in the calculations in Figure 8a, should be seen as unrealistically optimistic (see Section 4.2.2). Figure 9 shows the resulting systemic losses for different levels of haircuts. It should be noted that the charts show aggregate losses, which are largely driven by the defaults a small set of systemically important institutions. Chapter 5 discusses the skewed nature of the loss distribution and presents methods to account for this in statistical models. As can be seen, higher haircuts increase the losses in the insolvency regime far more than in the bail-in regime. This is as expected, considering that the bail-in tool manages to avoid insolvencies for most banks (as was noted in Fig. 8b), thus avoiding the losses from discontinued operations. We note that even for relatively mild haircuts around 20%-30%, losses in the bail-in regime are strictly lower than in the insolvency regime for

Figure 10: Systemic impact as a function of the systemic shock size



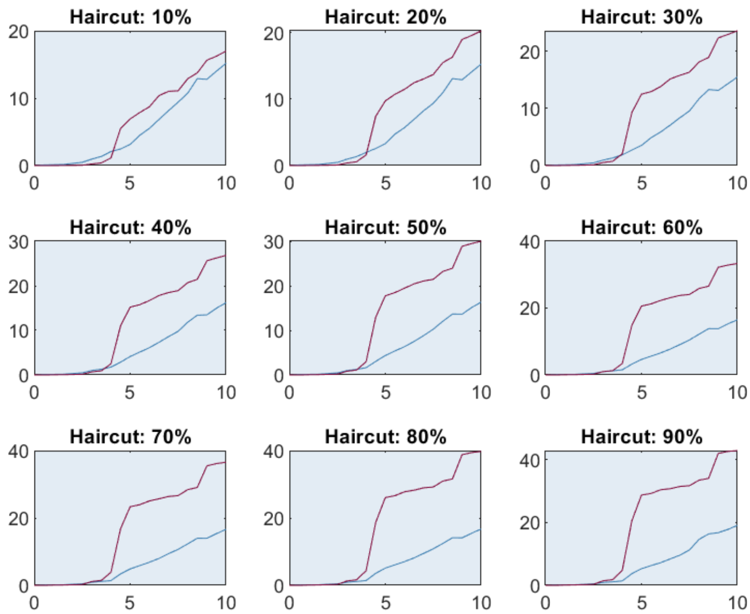
(a) Systemic risk losses

(b) Number of defaults and Bail-Ins

all shock levels. We note that haircuts around 30% are a more realistic assumption in the case of bank failures than assuming zero liquidation losses, as we did in Figure 8a. We also note that even for equal amounts of systemic losses, an outcome with significantly fewer defaults or insolvencies would still be preferable, in line with the overall goals of the BRRD which aims to preserve the functioning of financial institutions, if it can be done without recourse to public funds or negative effects on other market participants. Overall, we conclude that in the case of idiosyncratic crises, bail-ins manage to fulfill the regulatory objective of reducing systemic risk, both in terms of the number of institutions that become insolvent, as well as in terms of systemic losses, once reasonably realistic assumptions for liquidation haircuts are made.

We now turn our attention to analyzing the systemic risk contribution in the case of a systemic crisis. Figure 10a and 10b show the contagion losses and the number of defaults and bail-ins as a function of an increasingly severe systemic crisis, not yet

Figure 11: Systemic losses for different liquidation haircuts (systemic shock)



accounting for haircuts. The systemic crisis is based on the adverse scenario of the stress test conducted by the International Montery Fund in the context of its latest Financial Stability Assessment Program for Austria (FSAP, 2020). The increasing severity is produced by scaling the capital impact of the stress scenario in the range from 0 to 10 (a shock of 1 thus corresponds to the adverse scenario impact, a shock of 10 scales up this impact by a factor of 10).

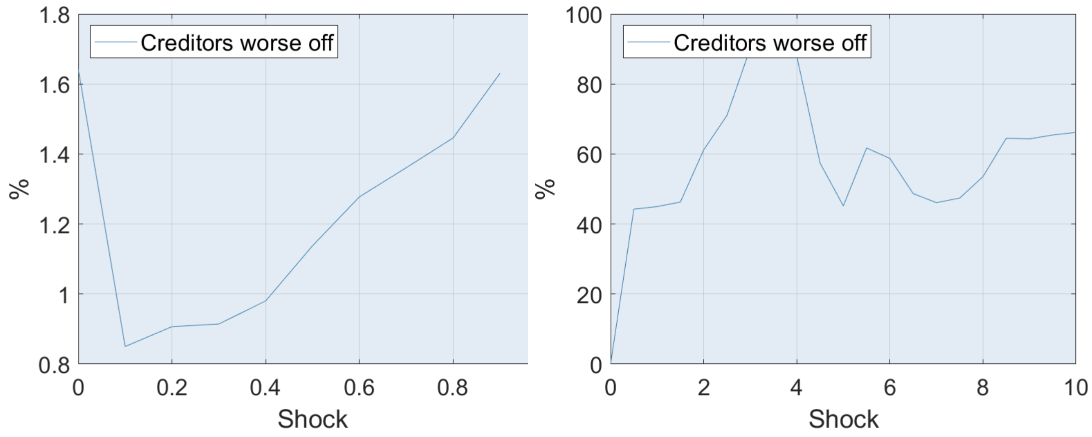
The results are less clear in the case of a systemic crisis than for an idiosyncratic crisis. The systemic losses are similar in both regimes, and generally higher in the bail-in regime than in the insolvency regime, especially for very high shock levels. This finding is in line with the literature (Galliani and Zedda, 2015). The bail-in regime still manages to drastically reduce the number of insolvencies, but the number of bail-ins is now higher than the number of insolvencies for most shock levels.

As pointed out by Avgouleas and Goodhart (2015), bail-ins incur litigation risks and a large number of simultaneous bail-ins, while arguably being preferable to insolvencies, could thus still be seen as a negative impact financial stability. Therefore, we extend the scope of our analysis again to incorporate liquidation losses for insolvent institutions. As can be seen in Figure 11, losses in the bail-in regime are significantly lower than in the insolvency regime even for relatively low levels of haircuts of 30% once the systemic shock reaches a significant size (about four times the stress test impact). For lower shock levels up to 3-4, the insolvency regime is still preferable, albeit by a much smaller margin. It should be noted that a shock level of 1, corresponding to the FSAP stress test scenario, is already calibrated to represent a severe adverse shock to the overall economy. We thus argue that a multiple of 3-4 of this shock is already an extremely adverse shock, and we thus focus our analysis on these shock levels. The charts show higher shock levels for completeness' sake, to show which shock levels would be required to affect the majority of the system. We conclude that in the case of a systemic crisis, it is less clear whether the bail-in tool can fulfill its goal of reducing systemic risk. It is still able to reduce the number of insolvencies dramatically, but in the more realistic situations at a higher cost than allowing for banks to go into insolvency without a prior bail-in. Given the significant litigation risks involved with performing a bail-in (Avgouleas and Goodhart, 2015), a resolution authority might thus prefer to abstain from a bail-in in such a situation.

As a final point in our analysis, we now turn to the consideration the NCWO principle. Even if a bail-in may be beneficial at the systemic level, there might still be a significant number of individual creditors that could be worse off in the bail-in regime than in the insolvency regime. We interpret the term ‘creditor’ here in a wide sense, including all firms that are potentially affected by the bail-in decision. The reason for this choice is that litigation can be expected from any party that is negatively affected by a bail-in decision, even if only indirectly exposed to the bailed-in bank (Avgouleas and Goodhart, 2015). Figure 12a shows that in the case of an idiosyncratic crisis, the share of institutions that are worse off under the bail-in regime than under the insolvency regime is very low, between 1% and 2% for most shock levels (figures including haircuts are similar, even lower). The situation is markedly different in the case of a systemic crisis (Figure 12b). For most shock levels over 50% of creditors are worse off under the bail-in regime than under the insolvency regime. This should hardly be surprising given that the overall losses are also higher in this scenario.

This leads us to conclude that the NCWO principle provides a highly useful tool for resolution authorities, if applied with some degree of common sense: in situations where the bail-in is clearly preferable from a social perspective, it points regulators towards performing a bail-in, by showing that only a tiny fraction of creditors would be worse off after a bail-in than after an insolvency in this situation. In situations where it is far less clear whether the bail-in is socially preferable, such as a systemic crisis event, it shows that the majority of creditors would left be worse off by performing a bail-in, thus indicating to regulators that other resolution tools

Figure 12: Share of creditors worse off under insolvency than under bail-in



(a) Idiosyncratic shocks

(b) Systemic shock

than a bail-in should be considered in the resolution process. A strict application of the NCWO principle could, however, still hinder bail-ins even in situations like an idiosyncratic shock, where it would be socially preferable. Given the clear systemic benefits of the application of the bail-in tool in such situations, we thus suggest considering the NCWO principle in conjunction with the systemic impact.

4.5. Discussion

In this chapter we presented an operational framework to assess the systemic risk implications of bank bail-ins. Our analysis is based on established models of systemic risk in financial systems. We use a contagion model that includes bail-in-able debt and extend it to account for possible valuation losses that occur during an insolvency, when switching from going to gone concern valuation. We use this model to study systemic contagion losses in two different regimes, a 'bail-in regime' where every bank that has bail-in-able debt performs a bail-in when it reaches a bail-in threshold (before insolvency), and an 'insolvency regime', where no bail-ins at all are performed. We quantify the systemic risk impact of bail-ins by comparing systemic losses, the number of defaults and bail-ins across different shock scenarios, and the number of creditors that are worse off in the bail-in regime than in the insolvency regime across different shock scenarios.

We use a real-world data set for our analysis, making using of the Liability Data Reports that are collected under the Bank Recovery and Resolution Directive (BRRD, 2014) to provide resolution authorities with data to base their bail-in decisions on. To the best of our knowledge, our analysis represents the first application of this novel data set to the assessment of the systemic risk impact of bail-ins. We map asset classes in the LDR reports to seniority classes using guidance from the European Single Resolution Authority. We obtain LDR reports for 51 Austrian banks, and we combine them with existing regulatory reports, most notably the central credit register, to study the impact of bail-ins in the full Austrian banking system of 599 bank creditors.

In our analysis, we study systemic risk impact across two different types of scenarios: in the idiosyncratic shock scenarios, we assume an idiosyncratic default of each bank in the system and then add up contagion losses across all idiosyncratic scenarios. We further consider a systemic shock scenario, which is based on the stress test results of the macroeconomic stress test performed by the International Monetary Fund in the Austrian Financial Stability Assessment. We extrapolate the stress test impact to study contagion impact across multiple shock sizes.

Our results show that bail-ins dramatically reduce the number of defaults both in the case of idiosyncratic and systemic shock scenarios. There is thus sufficient bail-in-able capital available in the system to prevent defaults. In terms of systemic losses, the results are more varied. In the case of idiosyncratic shocks, we find that bail-ins generally reduce systemic risk, especially once liquidation losses from switching to gone concern valuation are taken into account. In the case of systemic shocks, we find that in the absence of such liquidation losses, bail-ins tend to increase systemic losses for most shock levels. This problem is mitigated, especially for high shock levels, once liquidation losses are taken into account. But combined with the very high number of bail-ins, which would put strain on the resolution authorities and create litigation risks, we argue that bail-ins may not be an effective tool for reducing systemic risk during systemic crises. Finally, we find that the principle of looking at whether any creditors are worse off under a bail-in than under an insolvency (NCWO) is a useful tool for guiding regulatory decisions about whether to perform bail-ins. The share of creditors that are worse off under bail-in than under insolvency is very low in the case of idiosyncratic shocks, where a bail-in seems socially preferable, and very high in the case of systemic shocks, where bail-ins do not seem socially desirable. As a guidance for policy makers, we argue that the NCWO-principle should be treated not as a binary criterion but rather be considered in conjunction with the systemic welfare gains from performing a bail-in.

5. A metamodeling approach to assessing systemic risk and systemic risk regulations

5.1. Motivation

A key topic in financial regulation is the identification of "*systemically important*" institutions that would cause high losses for the financial system upon failure. As the recent financial crisis has shown, there are many lessons to be learned for policymakers and researchers. For example, many standard macroeconomic models lacked a banking sector and contagion effects between economic sectors, in particular banks were not adequately addressed. On the policy side, most countries did not have a clear regulate for bank recovery and resolution to prevent bank crises and ensure the orderly resolution of failing banks while minimizing their impact on the real economy and the financial sector. The devastating cascading effects observed during the financial crisis of 2008 have shown that even crises in moderately sized institutions and markets can put global financial stability at risk (Fratzscher and Rieth, 2015; Haldane and May, 2011).

The ability to identify systemically important institutions for the purpose of regulatory scrutiny is thus key to addressing the problem of moral hazard inherent in any institution that cannot be allowed to fail. This insight has – in part – triggered a paradigm shift in financial regulation, which instead of trying to identify institutions that are "*too big to fail*" now focuses on institutions that are "*too interconnected to fail*" (Haldane and May, 2011). This new approach is most visible in the capital add-ons for globally and domestically systemically important institutions (see BIS (2012, 2013); FED (2015); EBA (2014) – henceforth referred to as SII regulations). These regulations define weighted scorecard metrics aimed at quantifying not only an institution's size, but also its interconnectedness and systemic importance within the global and domestic financial system.

Studies of systemic interconnectedness are, however, plagued by scarce availability of data bilateral interbank relations, which are both highly confidential and rarely comprehensively collected. International projects of significant scale have been undertaken to try to close some of those data gaps (Anand et al., 2018). In this study we make use of a unique data set that combines both time series of the complete network of interbank relations of the entire banking system of a country, as well as of the indicators used in the SII regulations. This allows us to both assess the weighting scheme used by current regulations, as well as provide regulators who do not have access to such data with additional guidance. In our approach we aim to capture some peculiar features of interconnected financial systems, which are said to exhibit "*robust-yet-fragile*" tendencies (Gai and Kapadia, 2010): most of the times, small shocks are easily absorbed by the system and do not cause significant systemic losses. In very few instances, however, even small, isolated shocks have the capacity to affect huge parts of the system. This leads to highly skewed loss distributions, which we wish to explain in this paper, focussing in particular on the high importance of tail events in the context of systemic risk.

Our tool of choice for this approach is quantile regression. The origins of quantile regression date back to some of the very first works on regression in the mid 18-th century. Its application in modern econonometrics can be attributed to Koenker and Bassett (1978). Quantile regression differs from standard, least-squares regression in that it provides conditional forecasts of the quantiles rather than the mean of the distribution of a dependent variable. Its application is thus particularly warranted in the case of highly skewed distributions, such as those describing the systemic losses in *robust-yet-fragile* systems (Koenker, 2005).

5.2. Quantile panel estimators

Following Koenker and Bassett (1978), we define the piecewise linear check function:

$$\rho_\tau(y) = y(\tau - \mathbb{I}_{y<0}), \tag{5.1}$$

where $\tau \in [0, 1]$ is the quantile in consideration and $\mathbb{I}$ is the indicator function. We wish to estimate a classic, linear fixed effects model

$$y_{it} = \alpha_i + X_{it}\beta + u_{it}, i = 1 \ldots n, t = 1 \ldots T, \tag{5.2}$$

for a dependent variable $y \in \mathbb{R}^{nT}$ capturing information for $n \in \ltimes$ individuals over $T \in \ltimes$ observation periods. $\alpha \in \mathbb{R}^n$ is a vector of time-constant, individual-specific effects, $X \in \mathbb{R}^{nT\times K}$ is a matrix of $K \in \ltimes$ independent variables with coefficients

$\beta \in \mathbb{R}^K$ and $u \in \mathbb{R}^{nT}$ is an error term.[9] We will denote by $\mathbf{1}^n$ an n-dimensional vector of ones and by $A \otimes B$ the Kronecker product of A and B and write the model in matrix notation:

$$y = \alpha \otimes \mathbf{1}' + X\beta + u, \tag{5.3}$$

We will denote by $\begin{pmatrix} \alpha^\tau \\ \beta^\tau \end{pmatrix}$ the coefficients for a given quantile τ. The conditional quantile of y for a given set of coefficients and data is given by $\alpha^\tau \otimes \mathbf{1}' + X\beta^\tau$. The unconstrained estimator for these coefficients, which we denote the *dummy-variable*[10] estimator $DV \in \mathbb{R}^{n+K}$, is given by Kato et al. (2012):

$$DV^\tau = \begin{pmatrix} \hat{\alpha}^\tau_{DV} \\ \hat{\beta}^\tau_{DV} \end{pmatrix} = \min_{\alpha,\beta} \operatorname*{argmin}_{\alpha,\beta} f_\tau(\alpha, \beta) \tag{5.4}$$

where

$$f_\tau(\alpha, \beta) = \sum_{i=1}^{n} \sum_{t=1}^{T} \rho_\tau \left(y_{it} - \alpha_i - X_{it}\beta\right) \tag{5.5}$$

[9] X_{it} refers to the K-dimensional row vector representing the row of X corresponding to individual i at time t. For consistency with the matrix notation, X will be assumed to contain all T entries for the i^{th} individual in rows $(i-1)T + 1 \dots iT$. Other than this exception, vectors here in general are column vectors and A' denotes the transpose of A.

[10] The reason for choosing the name *dummy variable* rather than the – more commonly used – term *fixed effect* is the equivalence between dummy variable and fixed effect regression, and the fact that the within transformation associated with fixed effect estimation is generally not feasible for quantile estimators, as these are not linear operators, unlike least-squares estimators.

While the problem as posed is not linear, due to the non-linearity of ρ, it can be solved using methods of linear programming by exploiting its properties. The objective function $f : \mathbb{R}^{K+1} \rightarrow \mathbb{R}$ is an addition of $(n * T)$ ρ-functions. Since ρ is a convex, continuous and piece-wise linear function that is bounded below by zero, f itself has these properties. The feasible region of the minimization problem is thus a convex polytope that is bounded below and thus at least one (though not necessarily exactly one, as we will discuss shortly) of its corners defines a minimum of f. The corners are given by the points of non-differentiability of the objective function, which are the roots of the $(n * T)$ ρ-functions. The basic solutions of the optimization problem are thus given by coefficient combinations that fit a regression line exactly through one or more observations (setting their corresponding ρ-functions to 0). Note, however, that the uniqueness of the global solution here is not guaranteed by the usual condition that the number of linearly independent observations be no less than the number of coefficients: $\text{rank}(XX') \geq K + 1$. When $\tau * n * T$ is an integer (e.g. when there is an even number of observations in the case of a median estimator), the slopes of the $t_1 = \tau * n * T$ and $t_2 = (1 - \tau) * n * T$ ρ-functions cancel each other out in the interval between the ${t_1}^{\text{th}}$- and $(t_1 + 1)^{\text{th}}$-largest observations (unless the two fall together), creating a compact affine subspace of $\mathbb{R}^{K+1}$ with slope $t_1 * (\tau - 1) + t_2 * \tau = 0$. Due to the convexity of f_τ each point on the affine subspace is a minimum of the objective function and it thus defines a continuum of estimators. It is conventional in the literature to use the lowest coefficient values to obtain a uniquely defined estimator DV^τ in this case, hence the second operator

$\min_{\alpha,\beta}$ in Eq. 5.4. The solution to the optimization problem can be found by solving the following linear program: (Koenker, 2005)

$$\begin{aligned}
&\min_{u^+,u^- \in \mathbb{R}^{n*T}} (\tau u^+ + (1-\tau)u^-)' \mathbf{1}^{n*T} \\
&\text{s.t.} \\
&\forall\ i \in \{1 \ldots n\}, \forall\ t \in \{1 \ldots T\}\colon y_{it} = \alpha_i + X_{it}\beta + u^+_{it} - u^-_{it} \\
&u^+, u^- \geq 0
\end{aligned} \tag{5.6}$$

where the vectors $u^+ = \max(0, u)$ and $u^- = -\min(0, u)$ split the errors into their positive and negative components.

One potentially important problem with the DV^τ estimator is the incidental parameter problem (Neyman and Scott, 1948), as the number of individual-specifc intercepts grows with n. It is important to note that, in contrast to mean regression, to our knowledge, there is no general transformation that can suitably eliminate the individual effects in the DV^τ model. Asymptotically, if $n \to \infty$ and T fixed, there is a potentially important bias in the DV^τ estimator. Fixing this incidental parameter problem has drawn a lot of attention in the recent literature (Galvao and Kato, 2016). However, for empirical applications no uniquely accepted method to remove the bias has been established yet. Our argument for the consistency of the DV^τ estimator in our setting thus relies on two results. Firstly, the asymptotic result of Fernández-Val (2005) shows that the DV^τ estimator is consistent for $n \to \infty$, $T \to \infty$, and $log(n)/T \to 0$. Secondly, Kato et al. (2012) perform a simulation exercise, and in their setting ($n = 200$, $T = 50$) closest to our setting ($n = 716$, $T = 32$), they show that the bias is extremely small.

Koenker (2004) suggests fixing the individual-specific effects α for all quantiles and letting only the slopes β depend on the quantile. Under this assumption, which we denote the *quantile-independence hypothesis*, the location shift effect of the individual-specific effects on the distribution of the response is the same for all quantiles. Estimating the coefficients under this restriction requires choosing a set of quantiles $Q = \{q \mid q \in [0,1]\}$ of finite cardinality $|Q|$ and a vector of corresponding weights $W \in [0,1]^{|Q|}$ with the property that $W'\mathbf{1}^{|Q|} = 1$. The *quantile-independent dummy* estimator $QID \in \mathbb{R}^{n+K}$ of the coefficients for a given quantile $\tau \in Q$ under a given quantile set Q and weighting scheme W is then given by:

$$
QID^{\tau}_{Q,W} = \begin{pmatrix} \hat{\alpha}^{q=\tau}_{QID_{Q,W}} \\ \hat{\beta}^{q=\tau}_{QID_{Q,W}} \end{pmatrix} = \min_{\alpha^{q_1},\ldots,\alpha^{q_{|Q|}},\beta^{q_1},\ldots,\beta^{q_{|Q|}}} \quad \underset{\alpha^{q_1},\ldots,\alpha^{q_{|Q|}},\beta^{q_1},\ldots,\beta^{q_{|Q|}}}{\operatorname{argmin}} \quad \sum_{q\in Q}\sum_{i=1}^{n}\sum_{t=1}^{T} w(q)\rho_q\left(y_{it} - \alpha_i^q - X_{it}\beta^q\right)
$$
$$
\text{s.t.}
$$
$$
\forall\, q_1, q_2 \in Q,\ i \in \{1\ldots n\} :\ \alpha_i^{q_1} = \alpha_i^{q_2} \tag{5.7}
$$

where the map $w\colon Q \to [0,1]$ returns the element of W corresponding the respective quantile of Q. Note that QID can be seen as a generalization of the DV estimator, which can be recovered by setting $Q = \{\tau\}$ and $W = 1$. In the following we will sometimes assume – without loss of generality – a uniform weighting scheme $W_i =$

$\frac{1}{|Q|}$ $\forall\, i \in \{1 \ldots |Q|\}$ and drop the subscript W for notational convenience.

Note that every quantile set Q uniquely determines a set of estimators $QID_Q = \{QID_Q^\tau \mid \tau \in Q\}$, where the estimators QID_Q^τ and LS_Q^τ for the same quantile τ under different quantile sets $Q \neq \mathcal{P}$ can differ.[11] Hence, while $\hat{\alpha}^\tau_{QID_Q}$ by construction does not depend on which quantile $\tau \in Q$ is considered, both $\hat{\alpha}^\tau_{QID_Q}$ and $\hat{\beta}^\tau_{QID_Q}$ do depend on the set of sample quantiles Q, and thus $\hat{\beta}^\tau_{QID_Q}$ can differ from the unconstrained estimate $\hat{\beta}^\tau_{DV}$.[11] This raises questions about the consistency of the QID estimator in the case that the quantile-independence hypothesis does not hold. Testing for the validity of this assumption is of particular concern to us, as the individual-specific effect carries an important interpretation in our application (see Section 5.5). We will discuss the question of how to test this hypothesis in greater detail in Section 5.3.

Analyzing the incidental parameter problem in the $QID^\tau_{Q,W}$ estimator, Koenker (2004) has shown that the estimator is consistent and asymptotically normally distributed when $n^a/T \to \infty$ for $a > 0$.

The idea of fixing parameters across quantiles can also be applied differently, e.g. by fixing for example the slopes across quantiles and letting only the individual-specific effects depend on the quantile, or fixing both of them across quantiles. Versions of these estimators can sometimes be encountered in the literature, typi-

[11]Examples proving these claims are easily constructed. They also hold true in the data set which we use.

cally for non-panel estimators (e.g. the *composite quantile estimator* by Zou and Yuan (2008)). We denote the panel-versions of such estimators as the *quantile-independent slopes* and *quantile-independent* estimators, respectively defined as:

$$QIS^{\tau}_{Q,W} = \begin{pmatrix} \hat{\alpha}^{q=\tau}_{QIS_{Q,W}} \\ \hat{\beta}^{q=\tau}_{QIS_{Q,W}} \end{pmatrix} = \min_{\alpha^{q_1},\dots,\alpha^{q_{|Q|}},\beta^{q_1},\dots,\beta^{q_{|Q|}}} \operatorname*{argmin}_{\alpha^{q_1},\dots,\alpha^{q_{|Q|}},\beta^{q_1},\dots,\beta^{q_{|Q|}}} \sum_{q\in Q}\sum_{i=1}^{n}\sum_{t=1}^{T} w(q)\rho_q\left(y_{it} - \alpha_i^q - X_{it}\beta^q\right)$$

$$\text{s.t.}$$

$$\forall\ q_1, q_2 \in Q:\ \beta^{q_1} = \beta^{q_2} \tag{5.8}$$

$$QI_{Q,W} = \begin{pmatrix} \hat{\alpha}_{QI_{Q,W}} \\ \hat{\beta}_{QI_{Q,W}} \end{pmatrix} = \min_{\alpha,\beta}\ \operatorname*{argmin}_{\alpha,\beta}, \quad \sum_{q\in Q}\sum_{i=1}^{n}\sum_{t=1}^{T} w(q)\rho_q\left(y_{it} - \alpha_i - X_{it}\beta\right) \tag{5.9}$$

Note that QIS and QI are again generalizations of the DV estimator. Every quantile set Q again determines a set QIS_Q of quantile-independent slope estimators, while the QI estimator is unique for every quantile set and weighting scheme. One may still choose to interpret the QI estimator for a given quantile, e.g. by centering the quantile set Q symmetrically around this quantile (e.g. by considering $QI_{\{0.25,0.5,0.75\}}$ an augmented median estimator).

5.3. Statistical tests

There exist tests in the literature to test whether individual coefficients or all coefficients jointly vary between two or more quantiles. The null hypothesis:

$$H_0 : DV^{q_i} = DV^{q_j} \; \forall \; q_i \neq q_j \in Q \tag{5.10}$$

can be tested by considering the following test statistic (Koenker and Bassett, 1982) for a subset $\underline{\text{Q}} \subseteq Q$ of quantiles that are compared jointly:

$$S_n = (n + K)(|\underline{\text{Q}}| - 1)(H * \underline{\text{DV}})'(H * Var(\underline{\text{DV}}) * H')^{-1}(H * \underline{\text{DV}}) \tag{5.11}$$

where the matrix H expresses the linear hypothesis H_0 for the $|\underline{\text{Q}}|$ quantiles that are compared jointly and $Var(\cdot)$ denotes the variance-covariance matrix of the composite estimator $\underline{\text{DV}} = \left(DV^1, \ldots, DV^{|\underline{\text{Q}}|}\right)'$ (Koenker and Bassett, 1982). We choose to make pair-wise comparisons of two quantiles, hence $H = \left(-\mathbf{I}^{n+K}, \mathbf{I}^{n+K}\right)$, where $\mathbf{I}^{n+K}$ is an $n + K$-dimensional identity matrix. Koenker and Bassett (1982) show that Tn asymptotically follows a noncentral χ^2 distribution with rank$(H) = n + K$ degrees of freedom under common regularity assumptions.

We wish to test, however, whether the quantile-independence hypothesis causes the coefficients $\hat{\alpha}^{\tau}_{QID_{Q,W}}$ and $\hat{\beta}^{\tau}_{QID_{Q,W}}$ to be inconsistent for a given quantile. Our rationale for performing such a test is based on the argument that the DV estimator is (at least weakly) consistent and asymptotically normal regardless of whether the hypothesis of quantile-independent individual-specific effects holds or not. Weak consistency under fairly standard assumptions and asymptotic normality under slightly stricter assumptions, which we will assume to hold here, of the DV estimator has been shown by Kato et al. (2012). The QID estimator would only be consistent in the case where this assumption holds, but it would be more efficient in this case, as the restrictions placed on the coefficients should reduce the variance of the estimator. We are thus comparing a potentially inconsistent but more efficient estimator with a consistent one. This situation resembles the case of the classical Hausmann test for fixed versus random effects in least-squares regression. We thus propose the following test statistic to test the null hypothesis that both the QID and DV estimator are consistent against the alternative hypothesis that only the DV estimator is consistent:

$$S_Q^\tau\left(QID_Q^\tau\right) = \left(DV^\tau - QID_Q^\tau\right)' \left(Var\left(DV^\tau\right) - Var\left(QID_Q^\tau\right)\right)^\dagger \left(DV^\tau - QID_Q^\tau\right) \quad (5.12)$$

where $A^\dagger$ denotes the Moore-Penrose inverse of the matrix A [12]. Note that every estimator set QID_Q determines a set of test statistics $S_Q = \{S_Q^\tau \mid \tau \in Q\}$, hence con-

[12] Refer to Kato et al. (2012); Lamarche (2010) for the variance-covariance matrix of the DV and QID estimators, respectively. In the case of estimators of multiple quantiles we refer to the $(n + K) \times (n + K)$ submatrix corresponding to the tested quantile.

sistency has to be established separately for each quantile in Q.[13] We note that both QID and DV are asymptotically normally distributed – Kato et al. (2012) show both weak consistency and asymptotic normality for the DV estimator and Lamarche (2010) shows asymptotic normality for the QID estimator, under slightly different but compatible assumptions. Hence, every element in S_Q follows a χ^2 distribution with degrees of freedom equal to the rank of $Var(DV^\tau) - Var(QID^\tau)$. This allows applying a Wald test (Wald, 1945) with the null hypothesis that the quantile estimate QID^τ_Q is consistent and the alternative hypothesis that it is inconsistent.

Test statistics $S^\tau_Q\left(QIS^\tau_Q\right)$ and $S^\tau_Q\left(QI_Q\right)$ for the alternatively restricted estimators QIS and QI can be computed analogously to $S^\tau_Q\left(QID^\tau_Q\right)$. Note that while QI technically does not depend on the quantile, consistency still has to be established against a reference quantile of DV^τ, as the restricted coefficients $\hat{\alpha}_{QI_{Q,W}}$ and $\hat{\beta}_{QI_{Q,W}}$ may be consistent with regards to some quantiles and inconsistent when compared to others. One thus has to make a choice which quantile one wants to compare the QI estimator against, e.g. by interpreting it as an augmented estimator for the central quantile in Q, as discussed in Section 5.2.

5.4. *Goodness of fit measures for quantile panel estimators*

Extending the ideas from Koenker and Machado (1999) to the panel setting, we define a benchmark estimator B containing only individual-specific effects, which

[13]This can be interpreted in the sense that, even for a fixed quantile set Q, it is possible that the estimate $\hat{\beta}^\tau_{QID_Q}$ for one quantile $\tau \in Q$ is consistent while the estimate $\hat{\beta}^{\mathcal{T}}_{QID_Q}$ for a different quantile $\mathcal{T} \in Q \setminus \tau$ may be inconsistent.

can be seen as the panel analogue of an intercept:

$$B^{\tau} = \begin{pmatrix} \hat{\alpha}_B^{\tau} \\ \mathbf{0}^K \end{pmatrix} = \min_{\alpha} \operatorname{argmin}_{\alpha} \sum_{i=1}^{n} \sum_{t=1}^{T} \rho_{\tau} \left(y_{it} - \alpha_i \right) \tag{5.13}$$

We then define the sum of prediction errors $E_B = \rho_{\tau}^{nT} \left(y - Q \left(B^{\tau}, X \right) \right)' \mathbf{1}^{nT}$, where $\rho_{\tau}^{nT} : \mathbb{R}^{nT} \to \mathbb{R}^{nT}$ describes the element-wise application of the ρ_{τ}-map to the elements of a vector. We further define the sum of errors for the DV estimator $E_{DV} = \rho_{\tau}^{nT} \left(y - Q \left(DV^{\tau}, X \right) \right)' \mathbf{1}^{nT}$ to define an R^1-measure as suggested by Koenker and Machado (1999):

$$\mathrm{R}_{DV}^{1} = 1 - \frac{E_{DV}}{E_B} \tag{5.14}$$

We apply the same idea of the R^1 measure by Koenker and Machado (1999) to the QID estimator by replicating the restrictions placed on the individual-specific coefficients in the benchmark estimator $BQID \in \mathbb{R}^n$:

$$\begin{gathered} BQID_{Q,W}^{\tau} = \begin{pmatrix} \hat{\alpha}_{BQID_{Q,W}}^{q=\tau} \\ \mathbf{0}^K \end{pmatrix} = \min_{\alpha^{q_1}, \ldots, \alpha^{q_{|Q|}}} \operatorname{argmin}_{\alpha^{q_1}, \ldots, \alpha^{q_{|Q|}}} \sum_{q \in Q} \sum_{i=1}^{n} \sum_{t=1}^{T} w(q) \rho_q \left(y_{it} - \alpha_i^q \right) \\ \text{s.t.} \\ \forall\, q_1, q_2 \in Q,\ i \in \{1 \ldots n\} :\ \alpha_i^{q_1} = \alpha_i^{q_2} \end{gathered} \tag{5.15}$$

We then define the error sum and R^1_{DV}-measure for the restricted benchmark model:

$$E_{BQID} = \rho^{nT}_{\tau}\left(y - \hat{\alpha}^{\tau}_{BQID_{Q,W}} \otimes \mathbf{1}^{T}\right)' \mathbf{1}^{nT} \tag{5.16}$$

$$\mathrm{R}^1_{QID} = 1 - \frac{E_{QID}}{E_{BQID}} \tag{5.17}$$

where E_{QID} is defined in analogy to E_{DV}. The same considerations can be applied to the QI estimator. For the QIS estimator, a direct analogue of R^1_{DV} can be employed, as the restrictions in this case do not affect the benchmark model. One should be cautious, however, when comparing the goodness of fit R^1_{DV} and R^1_{QID} of the different estimators, as these relate to different benchmark models.

5.5. *Application to systemic risk measurement*

The main idea behind capital-based systemic risk regulations such as the SII regulations is that the extent of regulatory scrutiny or pressure that a financial institution receives should be proportional to the risk posed to the financial system by this particular institution.[14] The indicator set of the SII regulations aims to proxy this systemic relevance using a set of bank-specific indicators. These indicators have the advantage of ready availability to regulators. However, they do not explicitly incor-

[14]For simplicity, we will refer to all financial institutions covered by the appropriate regulations such as EBA (2014) as banks.

porate aspects of interconnectedness, which is a main driver of systemic contagion effects and robust-yet-fragile dynamics (Gai and Kapadia, 2010).

Our work builds on that of Siebenbrunner et al. (2017) who are the first to investigate whether bank-specific indicators such as those used in the SII regulations can provide a suitable proxy for network-wide contagion effects. Using fixed-effects and LASSO regression they conclude that the SII indicator set is a good choice of bank-specific indicators when compared to a large universe of possible indicator sets. They also find, however, that a significant portion of the explanatory power of these indicator sets comes from the contribution of the individual-specific (fixed) effect. They conclude that these individual-specific effects can be interpreted as the average network position of a bank, which the bank-specific SII indicators are not able to take into account. In this study we leverage the methodological toolkit introduced in section 5.2 to take a closer look at this question and study the contribution of these individual-specific effects across the quantiles of the systemic loss distribution.

The remainder of this section is structured as follows: in Section 5.5.1, we briefly outline our approach to measuring systemic risk and describe the resulting contagion losses, and in Section 5.5.2 we present the application of the methodological toolkit presented in sections 5.2 – 5.4 to this data set. In Section 5.5.3, we discuss our results and compare them to the regulatory regime defined in EBA (2014).

5.5.1. Systemic risk measurement

It is common in the literature to measure the systemic risk contribution of a bank as the sum of losses by all system participants that are caused by the idiosyncratic default of a particular bank (Glasserman and Young, 2016). Such losses can arise through multiple channels and there exists a rich literature on how to model and measure these effects. In this study we will be using the framework developed by Siebenbrunner et al. (2017), which has the advantage of being able to both incorporate multiple contagion channels, and to consistently separate the contribution of individual channels.

We use the basic clearing model of Eisenberg and Noe (2001) (section 2.2), that has widely been adopted in the literature (Elsinger et al., 2006; Upper, 2011; Barucca et al., 2020), to compute systemic contagion effects. We arrive at the **systemic contagion losses** caused by the default of institution s by summing up the losses received by each bank at each point in time t, $y_{s,t} = \sum_{i \neq s}^{n} \Pi'(\bar{p}_t - p_t^*)_i$, where p^* is the clearing payment vector discussed in section 2.2. We repeat the computation for each time point $t \in \{1, \ldots, T\}$, yielding the dependent variable $y_{i,t}$ for our estimations. We will estimate this dependent variable using the indicators from the SII regulations presented in Table 5. Note that this data set includes 9 out of the 10 indicators used in the current SII regulations, which we will discuss in greater detail in Section 5.5.3. The one missing variable, value of payment transactions, could not be reconstructed retroactively for the time series we consider here.

Table 5: Description of variables

Variable name	Description	Unit
Total assets	Total assets	EUR
Private sector deposits	Deposits taken from domestic and foreign nonbanks (no public sector), all currencies	
Private sector loans	Loans to foreign and domestic nonbanks (no public sector)	EUR
Face value of derivatives	Notional value of OTC derivatives	EUR
Cross border loans	Loans to foreign domiciled nonbanks and banks	EUR
Cross border deposits	Deposits from foreign domiciled nonbanks and banks	EUR
Bank deposits	Deposits taken from domestic and foreign banks, all currencies	EUR
Bank loans	Loans to domestic and foreign banks, all currencies	EUR
Securitized debt	Liabilities in the form of securitized debt obligations and transferable certificates	EUR

Source: Oesterreichische Nationalbank.
All indicators, except for value of private domestic payment transactions are taken from the regulatory reporting data, which are available on a quarterly basis. Due to data restrictions, we cannot include value of domestic payment transactions in our analysis on a a quarterly basis and before 2014. We therefore decided to exclude this variable from our analysis.
The data set covers the periods 2008Q1–2016Q1, yielding a panel of $n = 716$ banks and $T = 32$ points in time.

We use data for the Austrian banking system, taken from the regulatory reporting system and the central credit registry. The time series of quarterly observations ranges from the second quarter of 2008 to the first quarter of 2016, yielding a panel of $n = 716$ banks and $T = 32$ points in time. Table E.17 in the appendix provides summary statistics for all variables.

5.5.2. *Estimation and Results*

We estimate a linear model with individual-specific effects, as introduced in Section 5.5.1 to estimate the contagion losses as described in Section 5.5.1:

$$y_{it} = \alpha_i + X_{it}\beta + u_{it}, i = 1 \dots n, t = 1 \dots T, \tag{5.18}$$

The dependent variable y_{it} is the sum of losses incurred by all other banks in the system following an idiosyncratic default of bank i at time t. The explanatory variables X_{it} are the contemporaneous SII indicators for bank i, as presented in Table 5. We consider the quantiles $Q = \{0.25, 0.50, 0.75, 0.9, 0.95, 0.99\}$, which we also use as the quantile set for the constrained estimators.

As described in the introduction to Section 5.5, we are particularly interested in studying the individual-specific effects as these can be interpreted as the contribution of network effects that cannot be captured by the SII indicators. We thus first estimate the system described above using the *DV* estimator with Koenker (2018). Next, we employ the Anova-test described by Koenker and Bassett (1982) to test whether the coefficients differ between each pair of quantiles in our quantile set Q. Table D.16 in the appendix reports the results of this test applied to each combination of two or more quantiles in the set Q. As can be seen from the results, the null hypothesis is rejected for all quantile combinations, showing that the parameters (including the individual-specific effects) jointly differ between quantiles.

We then proceed to estimating the system using the *QID* estimator with Koenker and Bache (2014) in order to be able to apply the test introduced in Section 5.3. Table 6 reports the results of these tests. As can be seen, the null hypothesis of consistency of the *QID* estimator is rejected for all quantiles except $\tau = 0.25$, as one would expect given that the Anova-test shows that the coefficients differ across quantiles. If the individual-specific effects differ across quantiles, then the restrictions imposed by the quantile-independence hypothesis cause the coefficients of *QID* to be biased. It is interesting to note, however, that the two tests do not necessarily agree on their results, as can be seen from the test result for the $\tau = 0.25$ quantile. This is because the Anova-test may also be rejected due to differing values of the slope coefficients across quantiles. In this case the *QID* estimator, which does allow for the slope coefficients to vary across quantiles, may still be consistent, showing the importance of testing separately for the consistency of the *QID* estimator.

Table 6: Quantile-Independence Hypothesis Test

	S Statistic	p-value
$\tau = 0.25$	350.72	0.96
$\tau = 0.5$	1019.27	0.00
$\tau = 0.75$	1989.68	0.00
$\tau = 0.9$	2076.93	0.00
$\tau = 0.95$	2795.07	0.00
$\tau = 0.99$	1306.87	0.00

Source: OeNB. Own calculations.
Results for the Hausmann-type test in Eq. (5.12) for the models presented in Table 7 and Table 8.

Summarizing our findings so far, we can infer all coefficients including the individual-specific effects vary across quantiles. We do not report the values of the individual-specific effects for confidentiality reasons, but we can say that we observe a non-monotonic relation between the value of their coefficients and the quantile level for many banks.

In Table 7, the coefficient for Total assets is highly significant and positive for all quantiles. The coefficient increases for higher quantile, indicating that big banks simply cause more contagion losses and that this relationship is highly quantile dependent. However, this result is not only driven by a few (large) banks that cause high contagion losses but also holds true for banks that do not cause high contagion losses. As expected, private sector deposits reduce contagion losses, as these deposits are supplied outside the interbank market. Also private sector loans should not significantly influence contagion losses which holds true for most of the quantile estimation results. The coefficients of the face value of derivatives is not significant. In our data set only a few banks hold such derivatives and this balance sheet item does not contribute significantly to contagion losses. The interpretation of the coefficient of cross border deposits is very similar to that of private sector deposits, although it is economically less important but it still reduces contagion losses significantly. Cross-border loans tend to reduce contagion losses, especially for banks that cause large contagion losses. This is due to the fact that losses on such deposits are materialized outside the financial system under investigation. Regarding bank deposits, we find a positive coefficient that is however not significant. The coefficient for bank deposits decreases for higher quantiles, which makes sense when we consider that the highest contagion losses are rather driven by higher-order contagion rounds than first-round effects stemming directly from the deposits of the defaulted bank (see Section 5.5.1). For bank loans, we can find a similar explanation as for private sector loans. Finally, the coefficient of securitized debt is negative. Securitized debt is an outside refinancing source of banks that should reduce contagion losses in the financial system.

In Table 7 we also see that the fixed effects model appears to be largely driven by the higher quantiles of the loss distribution, unsurprisingly for highly skewed data. We also note that there are differences regarding the sign and significance for several variables, e.g. for bank deposits. Bank deposits are highly significant with a negative sign in the fixed effects model, while having a positive sign (albeit not being significant) for all quantiles. This could be explained by the decreasing contribution of bank deposits for higher quantiles discussed above. The fixed effects coefficient may in this case have been distorted by the extreme values of high losses, which in this case are however of particular importance and cannot be discarded as outliers. Quantile regression performs better at judging both the sign and the significance of these effects. Those variables that are significant with a negative under quantile regression are variables that indeed direct losses away from the financial system under consideration, such as cross border deposits. We conclude that quantile regression indeed is the better suited method to analyze financial contagion effects than least-squares regression.

Table 7: Dummy variable (unrestricted) model

	FE-Model	τ: 0.25	τ: 0.5	τ: 0.75	τ: 0.9	τ: 0.95	τ: 0.99
Total assets	0.9693^{***}	0.6223^{***}	0.7247^{***}	0.8136^{***}	0.8513^{***}	0.9641^{***}	1.1958^{***}
	(0.0078)	(0.1056)	(0.1264)	(0.1209)	(0.1433)	(0.1645)	(0.1998)
Private sector deposits	-1.0790^{***}	-0.7751^{***}	-0.7101^{***}	-0.8024^{***}	-0.7643^{***}	-0.8327^{***}	-1.1574^{***}
	(0.0198)	(0.1114)	(0.1417)	(0.1443)	(0.1638)	(0.1724)	(0.1991)
Private sector loans	0.4608^{***}	0.1620^{*}	0.0413	0.0291	0.0131	−0.0311	−0.0184
	(0.0192)	(0.0730)	(0.0613)	(0.0679)	(0.0822)	(0.0937)	(0.0974)
Face value of derivatives	-0.0084^{***}	−0.0045	−0.0036	−0.0042	−0.0019	−0.0059	−0.0077
	(0.0004)	(0.0044)	(0.0049)	(0.0042)	(0.0052)	(0.0052)	(0.0058)
Cross border deposits	-0.5432^{***}	-0.5765^{***}	-0.5427^{***}	-0.7153^{***}	-0.6569^{***}	-0.7017^{***}	-0.7011^{***}
	(0.0170)	(0.1384)	(0.1529)	(0.1561)	(0.1516)	(0.1524)	(0.1536)
Cross border loans	-0.3177^{***}	0.0116	−0.0634	−0.1003	−0.2323	−0.2416	-0.4899^{**}
	(0.0114)	(0.1154)	(0.1236)	(0.0924)	(0.1197)	(0.1494)	(0.1847)
Bank deposits	-0.2951^{***}	0.0219	0.1406	0.1431	0.1410	0.1083	0.0575
	(0.0119)	(0.0754)	(0.0910)	(0.0887)	(0.1027)	(0.0900)	(0.1101)
Bank loans	0.4170^{***}	0.0837	−0.0136	0.0502	−0.0257	−0.0773	−0.0465
	(0.0137)	(0.0725)	(0.0693)	(0.0634)	(0.0674)	(0.0777)	(0.0988)
Securitized debt	-0.4693^{***}	-0.2256^{*}	-0.2253^{*}	-0.3270^{***}	-0.3407^{**}	-0.3027^{*}	-0.3833^{*}
	(0.0132)	(0.0897)	(0.0972)	(0.0915)	(0.1268)	(0.1352)	(0.1650)
Goodness of fit	0.70	0.26	0.27	0.33	0.37	0.37	0.40

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$
Source: OeNB. Own Calculations.
The dependent variable is the systemic contagion loss (see Section 5.5.1). The independent variables are described in Table 5.
The FE-Model is estimated with the least squares dummy variable estimator. All other models are estimated with the DV quantile estimator given in Eq. (5.4) for the 0.25, 0.5, 0.75, 0.9, 0.95 and the 0.99 quantile.
For the FE Model, we use the within R^2. For the quantile regressions, we calculate the goodness of fit measure suggest by Koenker and Machado (1999) which is defined in Eq. (5.14).

Table 8: Quantile-independent dummies (restricted) model

	τ: 0.25	τ: 0.5	τ: 0.75	τ: 0.9	τ: 0.95	τ: 0.99
Total assets	0.7261^{***}	0.7547^{***}	0.8069^{***}	0.8444^{***}	0.8794^{***}	1.0743^{***}
	(0.1155)	(0.1249)	(0.1338)	(0.1544)	(0.1664)	(0.1589)
Private sector deposits	-0.7704^{***}	-0.7404^{***}	-0.8285^{***}	-0.8729^{***}	-0.8926^{***}	-0.9873^{***}
	(0.1622)	(0.1399)	(0.1495)	(0.1708)	(0.1842)	(0.2099)
Private sector loans	0.0780	0.0163	0.0540	0.0673	0.0501	0.0225
	(0.0904)	(0.0838)	(0.0848)	(0.0856)	(0.0941)	(0.1843)
Face value of derivatives	−0.0051	−0.0054	−0.0063	−0.0065	−0.0069	-0.0075^{*}
	(0.0040)	(0.0037)	(0.0038)	(0.0039)	(0.0036)	(0.0034)
Cross border deposits	-0.5144^{*}	-0.4882^{*}	-0.4904^{*}	−0.4338	−0.3869	−0.0895
	(0.2186)	(0.2156)	(0.2254)	(0.2226)	(0.2116)	(0.2117)
Cross border loans	−0.1216	−0.1321	−0.1097	−0.1202	−0.1311	-0.3391^{*}
	(0.0932)	(0.1135)	(0.1091)	(0.1126)	(0.1182)	(0.1400)
Bank deposits	0.0115	0.1113	0.0895	0.0665	0.0736	0.0105
	(0.1009)	(0.1006)	(0.1048)	(0.1272)	(0.1445)	(0.1795)
Bank loans	0.0720	0.0152	0.0584	0.0702	0.0637	−0.0127
	(0.0987)	(0.0824)	(0.0917)	(0.0987)	(0.1086)	(0.1931)
Securitized debt	-0.3066^{*}	-0.2444^{*}	-0.3031^{**}	-0.3187^{*}	-0.3276^{*}	−0.2756
	(0.1437)	(0.1173)	(0.1137)	(0.1279)	(0.1369)	(0.1647)
Goodness of fit	0.22	0.15	0.32	0.61	0.76	0.92

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$
The dependent variable is the systemic contagion loss (see Section 5.5.1). The independent variables are described in Table 5. All models are estimated with the QID quantile estimator given in Eq. (5.7) for the 0.25, 0.5, 0.75, 0.9, 0.95 and the 0.99 quantile. We calculate the goodness of fit measure as defined in Eq. (5.17).

5.5.3. *Policy Implications*

One of the main benefits of the analysis conducted herein is that it allows a quantitative assessment of the current regulatory practice for assigning SII risk scores. This process is defined in Article 131(3) of Directive 2013/36/EU (CRD) for domestic systemically important institutions (OSII). It works by computing a weighted sum of scores of individual indicators, which are averaged over the sum of the values of that indicator in the same country:

$$\text{OSII-Score}_i = 10,000 * \sum_{Ind. \in \text{OSII-Indicators}} w^{Ind.} \frac{Ind._i}{\sum_{j=1}^{n} Ind._j} \tag{5.19}$$

This normalization is meant to allow for comparing OSII-Scores across countries. The indicators are grouped into four categories and can be seen together with the respective weights in Table 9.

Table 9: Scoring Process

Criterion	Indicators	Weight
Size	Total assets	25%
Importance	Value of domestic payment transactions	8.33%
	Private sector deposits from depositors in the EU	8.33%
	Private sector loans to recipients in the EU	8.33%
Complexity/Cross-border activity	Value of OTC derivatives (notional)	8.33%
	Cross-jurisdictional liabilities	8.33%
	Cross-juridictional claims	8.33%
Interconnectedness	Intra financial system liabilities	8.33%
	Intra financial system assets	8.33%
	Debt securities outstanding	8.33%

Source: EBA (2014).

Comparing these scores to the results in Table 7, the first thing that we notice is that while the regulatory score only has positive weights, the coefficients in all the models for different quantiles and fixed effects have negative weights. The negative weights for cross-border deposits and loans are likely due to the fact that these are not part of contagion simulation for the given (national) banking system. We conjecture that the negative signs would become positive if one had access to the data on the international banking network and could include those foreign nodes in the simulations as well. We thus focus on the coefficient for total assets, which receive a far higher coefficient than its weight in the OSII score, and this effect is exacerbated when one moves to higher higher quantiles of the loss distribution. This hints at the fact that interconnectedness and size are more strongly interrelated than the debate on whether banks are "too big" or "too interconnected" to fail (see chapter 1) suggests: total assets are even more strongly correlated with systemic risk impact for highly systemic institutions than for less important ones. Seen from a different angle, this real-world phenomenon of very high heterogeneity in terms of the network position of large vs. small banks induces strong constraints on the structure of the network, if one were to try to estimate the full network from marginal sums. This might explain why methods that aim to reconstruct the network from such partial data, which might seem like a highly challenging task at first sight, tend to perform relatively well in practice (see e.g. Anand et al. (2018)).

One interesting question is to what extent the over-emphasis on other components than size distorts the explanatory power of the OSII indicator for systemic contagion effects. For this purpose we reconstructed the OSII-score in Eq. (5.19) for the available data and performed the same regressions using this score. Table 10 reports the results of these regressions. As can be seen from Table 11 the quantile independence hypothesis is rejected for all quantiles, so we focus our analysis on the unrestricted model. We observe that the value of the OSII coefficient is barely significant at all, and mostly for the highest quantiles. We further observe a far lower explanatory power from the model induced by the OSII weights as compared to the models in Table 7. Both findings suggest that the weighting of the OSII scores could be improved by moving closer to the coefficients in Table 7. In particular, our analysis suggests that size, as measured by Total Assets, should be given a higher weight. We also note the negative coefficients of some of the variables, which raises the question whether these coefficients should be kept in the indicator set at all.

Looking further at the results of the tests for quantile independence, we see that the assumption of constant intercepts across quantiles is generally rejected, and the consistency of the restricted estimator is rejected for all quantiles except the lowest one $\tau = 0.25$. The individual-specific intercepts describe i.a. the influence of unobserved time-constant effects. In the context of our analysis, we argue that these effects capture at least some of the contribution of the network effects that cannot be explained by the OSII indicators. These indicators only look at node-specific characteristics, without accounting for the structure of the interbank network. It would in fact be surprising if this contribution was constant across quantiles. The importance of network effects would be expected to increase with higher losses, because as the size of the cascade grows more second-round and higher-order contagion effects are observed. We cannot show the values of the individual-specific effects for confidentiality reasons, but we can report that their contribution increases as one moves to higher quantiles, confirming the above reasoning. Hence the rejection of the quantile-independence hypothesis highlights the particular importance of network effects.

Table 10: Contagion losses vs. OSII Score: Unrestricted vs. Restricted Models

Unrestricted Models	FE Model	tau: 0.25	tau: 0.5	tau: 0.75	tau: 0.9	tau: 0.95	tau: 0.99
OSII Score	3169.93*** (297.38)	1081.34 (1070.35)	2656.78 (2870.32)	4596.66 (4580.90)	9913.70* (4159.86)	9913.70* (4010.04)	9063.73* (4437.10)
Goodness of fit	0.01	0.01	0.02	0.02	0.04	0.05	0.06
Restricted Models							
OSII Score		1367.11 (3901.28)	4859.60 (4936.98)	8670.85 (5127.06)	13303.28** (4397.19)	17852.00*** (4068.11)	28303.68*** (4923.81)
Goodness of fit		0.12	0.03	0.21	0.50	0.67	0.89

*** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$.
Source: OeNB. Own calculations.
All unrestricted models are estimated with the DV quantile estimator given in Eq. (5.4) for the 0.25, 0.5, 0.75, 0.9, 0.95 and the 0.99 quantile.
All restricted models are estimated with the QID quantile estimator given in Eq. (5.7) for the 0.25, 0.5, 0.75, 0.9, 0.95 and the 0.99 quantile.

Table 11: Quantile-Independence Hypothesis Test results for OSII score

	S Statistic	p-Value
$\tau = 0.25$	491.08	0.00
$\tau = 0.5$	1087.36	0.00
$\tau = 0.75$	932.87	0.00
$\tau = 0.9$	1240.10	0.00
$\tau = 0.95$	980.57	0.00
$\tau = 0.99$	922.15	0.00

Source: OeNB. Own calculations.
Results for the Hausmann-type test in Eq. (5.12) for the models presented in Table 10.

In this chapter, we present a new framework for systemic risk analysis combining the strengths of two different approaches to systemic risk analyses. The popularity of the CoVaR method based on quantile regression shows that the focus on the tails of the loss distribution helps to identify the most systemically important banks. However, CoVaR as well as similar methods relying on market-based information, are not applicable for the vast majority of banks, who are generally not listed. We thus combine the idea of focussing on the tails of the loss distribution with a model based on network theory that allows computing losses for all banks in a given system. The resulting systemic risk measure describes the risk of large crises in the financial system being triggered by initially idiosyncratic shocks. While we are not able to elaborate on individual results for confidentiality reasons, we observe that non-listed institutions can cause and transmit high contagion losses.

We estimate these contagion losses using panel quantile estimation. We discuss two types of panel quantile estimators, an unrestricted dummy-variable estimator, as well as a restricted estimator, that forces the individual-specific intercepts to be constant across quantiles. In our analysis, we interpret the intercept as part of the network effects that cannot be explained by the bank-specific regulatory variables alone. Hence, the rejection of the quantile independence hypothesis can be seen as a confirmation of the importance of network effects. We further compare the results of the quantile regression with a standard fixed-effects regression and find that the latter appears to be distorted by extreme losses, which in our context cannot be discarded as outliers. These distortions lead to (i) highly significant coefficients and to (ii) economically less meaningful signs compared to the quantile estimation results. We therefore prefer the results of the quantile estimation for describing systemic risk effects, in line with the approaches in the literature mentioned above.

We use this framework to perform an assessment of the current approach to regulating systemic risk regulations. In particular, we look at the OSII regulations by EBA (2014) for capital buffers for systemically important institutions, based on the global recommendations by BIS (2012, 2013). We find that the OSII score alone has very weak explanatory power for the contagion losses in the contagion model. This can be explained by comparing the weightings used in the OSII scoring system with the coefficients in the resulting models. We find in particular that the size of the bank, as measured by Total Assets, is given a too low weight in the regulatory score. Giving a lower score to size may be motivated by the desire to focus on too highly connected banks rather than just too large ones. However, as our results from a network-based contagion model show, the two concepts may be more strongly related than the regulators thought.

Putting the contribution in this chapter in a broader context, the metamodeling approach presented herein is one approach to dealing with partial observability problems for financial networks: clearing algorithms require knowledge of the full network in order to be used, however, these data are often not available in practice. One way of dealing with this problem that has been pursued in the literature is to resort to *network reconstruction* algorithms. Such algorithms use partial data, such as the marginal sums of total interbank assets and liabilities, which are usually broadly available. They then aim to reconstruct the full network from these partial data, using further assumptions. Given the fact that the largest institutions in terms of total assets also tend to the those with the most connections, as documented in this chapter, algorithms that impose sparsity constraints on the network often perform reasonably well at this challenging task. Anand et al. (2018) provide a broad overview and direct comparison of many network reconstruction methods. Given the fact that clearing algorithms are highly sensitive to the presence or absence of even a single link, relying on such methods still poses risks of mis-estimating systemic risk. Metamodeling, as presented in this chapter, presents an alternative approach to network reconstruction for dealing with missing data. The idea here is to compute contagion results for a system where the network is known, learn a metamodel using data that is available for a target system where the network is not known, and then use the predictions from the metamodel as systemic risk estimates for the target system. This approach has been pioneered by Greenwood et al. (2015) and Siebenbrunner et al. (2017), the contribution in this chapter is to extend this approach by providing a new statistical methodology that is better able to provide capture the highly skewed distribution of contagion losses.

The model presented herein can be used in an application in two main ways: (i) directly use the learned weights from Table 7 with the OSII indicators for the target system. Or (ii) use another system with known data that would be expected to generalize better to the desired target system, and repeat the estimation using the methodolgies laid out in both chapters. In both cases one would be making two main assumptions: that the learned weights will generalize to the target system, and that the clearing model used does provide a good characterization of systemic risk. The latter approach offers more flexibility, both in terms of choosing a model system that is as close as possible to the desired target system, as well as replacing the clearing algorithm with a different one such as Cont et al. (2019), Bardoscia et al. (2019), Barucca et al. (2020) or many others. In both cases, the main contribution of the study in this chapter consists in providing a statistical methodology for learning a metamodel that allows measuring tail risk. An interesting avenue to explore would be to try to validate the above mentioned clearing algorithms with real-world data on financial crises, which would require much larger data sets and to the best of our knowledge is still has not been addressed in the literature so far.

6. Conclusion & Outlook

This thesis presents multiple contributions relating to the use of clearing algorithms and statistical methods for systemic risk assessment. Chapter 2 presents the formal foundations of clearing algorithms and shows that the solution of a standard clearing model commonly used in contagion analyses for financial systems can be expressed as a specific form of a generalized Katz centrality measure under conditions that correspond to a system-wide shock. This result provides a formal explanation for earlier empirical results which showed that Katz-type centrality measures are closely related to contagiousness (Puhr et al., 2014). It also allows assessing the assumptions that one is making when using such centrality measures as systemic risk indicators. The analysis shows that the assumptions that one is making when using even a highly specific, specially derived form of a Katz centrality are strong and unrealistic for typical applications (most notably that the entire banking system is in fundamental default). Based on these results we conclude that clearing algorithms should be given preference over centrality measures in systemic risk analyses.

Having established why we prefer clearing algorithms for systemic risk assessment, chapter 3 then presents an original extension of such clearing models. We build on the model of Elsinger (2009), and extend it to present algorithms that allow computing clearing in the presence of contingent convertible and bail-in-able debt, two instruments of recapitalizing banks through debt-to-equity conversion. We combine these with recently proposed methods of network valuation under stochastic external assets (Fischer, 2014; Barucca et al., 2020), allowing for the pricing of debt instruments in each seniority layer, the calculation of default probabilities, and the consideration of 'pre-default contagion' in systemic risk analyses. We show that there exist well-defined valuations for all financial assets cross-held within the system. The full model constitutes an extension of classic asset pricing models that accounts for cross-holdings of debt securities. Our contribution is to add convertible debt to this framework. Potential outputs of the model in the context of systemic stress tests include: (i) loss 'add-ons' to microprudential stress tests to capture the effects of contagion and make them more macroprudential. (ii) Rankings of financial institutions by systemic contagiousness and vulnerability by considering idiosyncratic defaults of individual institutions or subgroups. (iii) Merton (1974)-type default probabilities and loss rates for banks. Furthermore, (iv) the model allows assessing the sensitivity of all of the aforementioned outputs to bail-in decisions and CoCo parameters.

Chapter 4 presents an application of the model developed in the previous chapter to addressing a real-world policy question. Financial regulation has introduced Bail-Ins as a tool for bank resolution. Regulators need to decide whether to perform a Bail-In (i.e. a forced debt-to-equity swap) for a failing bank or let the bank enter into insolvency. We present a framework for providing quantitative evidence to support this decision. We account for systemic feedback effects using state-of-the-art multilayer contagion models that we extend to account for liquidation losses during insolvencies. We study systemic risk impact both in the case of idiosyncratic and systemic shock events. We perform an empirical assessment using real-world data collected under the reporting requirements introduced by the Bank Recovery and Resolution Directive. Our results suggest that bail-ins generally reduce systemic risk in the case of idiosyncratic shocks, especially once liquidation losses are taken into account, but not necessarily in the case of systemic shocks. We find that the principle of checking whether any creditors are worse off under a bail-in than under an insolvency can provide useful guidance for the bail-in decision, if applied with caution.

Chapter 5 presents an alternative approach to measuring systemic risk, by learning a metamodel of the output of a contagion model. We use quantile regression as a tool here in order to account for the highly skewed nature of systemic contagion losses, where a small number of institutions can cause massive systemic crises, whereas a large number of institutions has very little importantance in the case of an idiosyncratic default. We use two types of quantile panel estimators to infer whether indicators used by regulatory authorities to identify systemically important banks are good predictors of the output of contagion models. We present a statistical test to decide between the two estimators as well as goodness-of-fit measures for these estimators. We find that regulators could substantially improve their identification of systemically important institutions by using different weights for their indicator sets. Our contribution can be used to estimate systemic risk for systems where the full network data is not available, either by applying our learned weights directly, or by replicating the empirical approach using the statistical methodology presented in the chapter.

The work leaves open several avenues for future research. The clearing algorithm presented in chapter 3 could be extended to account for the fact that bail-ins are not performed for every bank, but rather decided by the regulator on a case-by-case basis. The algorithm could further be extended to account for the presence of deposit guarantee schemes, institutional protection schemes and for single-point-of-entry resolution, where multiple entities are grouped together for the purpose of deciding on a resolution process. These extensions could then enhance the policy framework presented in chapter 4. Once sufficient historical data on bail-in-able debt networks is available, the analysis in chapter 5 could be extended to include these effects. More generally, if sufficient data on contractual linkages during financial crises becomes available, an empirical validation of the output of these contagion models could be performed.

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Appendix A. Measurability of the payoff function

Theorem 5. *[Measurability of the payoff function] The Elsinger-type payoff function Ψ is measurable. The extensions of the payoff function for CoCo's and bail-in are also measurable.*

Proof.

1. Firstly, consider the case of the clearing payment vector in a financial system with no seniority structure.

   Theorem 2 states that the sequence $p^{k+1} = (W^*(p^k, e) + \bar{p})^+ \wedge \bar{p}$, with $p^0 = \bar{p}$, converges to the largest clearing payment vector. Lemma 1 states that the sequence $W^k(p, e)$ defined by $W^0(p, e) = e + \Pi' p - \bar{p}$ and $W^{k+1}(p, e) = W^0(p, e) + \Theta\Lambda^k W^{k+1}$, where $\Lambda^k = diag(W^k > \mathbf{0})$, converges to $W^*(p, e)$. Hence, defining $\Phi_W(W)$ to be a solution of the linear equation $\Phi_W(W) = W^0(p, e) + \Theta * diag(W > \mathbf{0}) * \Phi_W(W)$, $W^*(p, e)$ can be expressed as

$$W^*(p, e) = \lim_{k \to \infty} \Phi_W^k(W^0(p, e)) \tag{A.1}$$

   The function Φ_W is a solution to a linear equation, hence it is measurable, and so is Φ_W^k by composing. Finally, $W^*(p, e)$ is measurable with respect to e almost surely, as it is a point-wise limit of measurable functions.

A similar argument can be made to show that the clearing payment vector $p^*(e)$ is a measurable function of e. Define $\Phi_p(p)$ by $\Phi_p(p) = (W^*(p, e) + \bar{p})^+ \wedge \bar{p}$. Because W^* is measurable with respect to e, so is Φ_p, and

$$p^*(e) = \lim_{k \to \infty} \Phi_p^k(\bar{p}) \tag{A.2}$$

is a measurable function of e.

2. Now, consider the case of seniority structure. The calculation of the clearing matrix with seniority structure was described through an iterative algorithm. Each step in the algorithm can be described as a function of the external assets e and the previous values of H. Moreover, in each iteration, all steps consist of transformations which are compositions of linear transformations, divisions and solutions to linear equations. All these are measurable. The algorithm has finite steps, so the resulting clearing matrix P^* is a measurable function of the input e.

3. The extension to CoCo's and bail-in is analogous. Each step of the algorithm for calculating the clearing matrix with CoCo's and bail-in consists of transformations which are compositions of linear transformations, divisions and solutions to linear equations, all of which are measurable with respect to the input e. A limiting argument analogous to (A.2) proves the measurability of the resulting clearing payment matrix with respect to e.

□

Appendix B. Conversion matrix properties

Lemma 4.

(i) *When* $\text{Equity}_j^n + \text{Bail-In}_j^n \leq 0$*, any bail-in is non-positive.*

(ii) *When* $\text{Equity}_j^n \leq 0, \text{Equity}_j^n + \text{Bail-In}_j^n \geq 0$ *and* $\varphi_i^p = \sum_{j,s}\left(C_{i,j,s}^{\text{fair, } y=1} - C_{i,j,s}\right), \forall\ i,$ *a fair conversion matrix implies a non-positive bail-in.*

(iii) *When* $\text{Equity}_j^n \geq 0$ *and* $\varphi_i^p = \sum_{j,s}\left(C_{i,j,s}^{\text{fair, } y=1} - C_{i,j,s}\right), \forall\ i$ *, a fair conversion matrix implies a neutral bail-in.*

Proof. (i) follows trivially from definition 9 by noting that $\text{Bail-In}_j^n \geq 0$ for all j, n.

We then note that when $\varphi_j^p = \sum_{i,s}\left(C_{j,i,s}^{\text{fair, } y=1} - C_{j,i,s}\right)$ we obtain:

$$\Theta_{j,i}^{n+1} = \left(1 - \sum_i \sum_s C_{j,i,s}^{\text{fair, } y=1}\right)\Theta_{j,i}^n + \sum_s C_{j,i,s} \tag{B.1}$$

We now consider the case $\text{Equity}_j^n \leq 0, \text{Equity}_j^n + \text{Bail-In}_j^n \geq 0$ (ii). We insert Eq. (B.1) into the definition of a non-positive bail-in and obtain:

$$\forall\, i,j:\ \left(\left(1-\sum_i\sum_s C^{\text{fair},\,y=1}_{j,i,s}\right)\Theta^n_{j,i}+\sum_s C^{\text{non-positive}}_{j,i,s}\right)\left(\text{Equity}^n_j+\text{Bail-In}^n_j\right) \tag{B.2}$$

$$\leq \sum_s \Pi_{j,i,s}\text{Bail-In}^n_{j,s}$$

$$\iff \forall\, i,j:\ \sum_s C^{\text{non-positive}}_{j,i,s}\left(\text{Equity}^n_j+\text{Bail-In}^n_j\right) \tag{B.3}$$

$$+\,\Theta^n_{j,i}\left(1-\sum_i\sum_s C^{\text{fair},\,y=1}_{j,i,s}\right)\left(\text{Equity}^n_j+\text{Bail-In}^n_j\right)\leq \sum_s \Pi_{j,i,s}\text{Bail-In}^n_{j,s} \tag{B.4}$$

If $\text{Bail-In}^n_j = 0$, the result clearly holds. Assume $\text{Bail-In}^n_j > 0$, and note that

$$\forall\, j:\ \left(1-\sum_i\sum_s C^{\text{fair},\,y=1}_{j,i,s}\right)\left(\text{Equity}^n_j+\text{Bail-In}^n_j\right)= \tag{B.5}$$

$$\left(1-\frac{\sum_i\sum_s \Pi_{j,i,s}\text{Bail-In}^n_{j,s}}{\left(\text{Equity}^n_j+\text{Bail-In}^n_j\right)}\right)\left(\text{Equity}^n_j+\text{Bail-In}^n_j\right)= \tag{B.6}$$

$$\text{Equity}^n_j+\text{Bail-In}^n_j-\sum_s\sum_i \Pi_{j,i,s}\text{Bail-In}^n_{j,s}=\text{Equity}^n_j\leq 0 \tag{B.7}$$

and obtain by inserting (B.5) into (B.2):

$$\forall\, i,j:\ \sum_s C^{\text{non-positive}}_{j,i,s}\left(\text{Equity}^n_j+\text{Bail-In}^n_j\right)+\Theta^n_{j,i}\left(\text{Equity}^n_j\right)\leq \sum_s \Pi_{j,i,s}\text{Bail-In}^n_{j,s} \tag{B.8}$$

Setting $C^{\text{non-positive}} = C^{\text{fair}}$, which in this case is given by y^{fair} (Eq. 3.9), yields:

$$\forall\, i, j\ :\ \gamma \frac{\sum_s \Pi_{j,i,s} \text{Bail-In}^n_{j,s}}{\text{Bail-In}^n_j} \left(\text{Equity}^n_j + \text{Bail-In}^n_j\right) + \Theta^n_{j,i} \left(\text{Equity}^n_j\right) \leq \sum_s \Pi_{j,i,s} \text{Bail-In}^n_{j,s} \tag{B.9}$$

$$\iff \forall\, i, j\ :\ \gamma \frac{\sum_s \Pi_{j,i,s} \text{Bail-In}^n_{j,s}}{\text{Bail-In}^n_j} \left(\text{Equity}^n_j\right) + \gamma \sum_s \Pi_{j,i,s} \text{Bail-In}^n_{j,s} + \Theta^n_{j,i} \left(\text{Equity}^n_j\right) \tag{B.10}$$

$$\leq \sum_s \Pi_{j,i,s} \text{Bail-In}^n_{j,s}$$

$$\iff \forall\, i, j\ :\ \gamma \frac{\sum_s \Pi_{j,i,s} \text{Bail-In}^n_{j,s}}{\text{Bail-In}^n_j} \left(\text{Equity}^n_j\right) + (\gamma - 1) \sum_s \Pi_{j,i,s} \text{Bail-In}^n_{j,s} + \Theta^n_{j,i} \left(\text{Equity}^n_j\right) \leq 0 \tag{B.11}$$

which is strictly less than zero in the case that $\text{Bail-In}^n_j > 0$ and $\text{Equity}^n_j \leq 0$.

We now consider the case $\text{Equity}^n_j, \text{Bail-In}^n_j \geq 0$ (iii). We note that in this case $C^{\text{fair},\, y=1} = C^{\text{fair}}$ and insert Eq. (B.1) into the definition of a neutral bail-in to obtain:

$$\forall\, i,j:\ \left(\left(1-\sum_i\sum_s C^{\text{fair}}_{j,i,s}\right)\Theta^n_{j,i}+\sum_s C^{\text{neutral}}_{j,i,s}\right)\left(\text{Equity}^n_j+\text{Bail-In}^n_j\right)-\Theta^n_{j,i}\left(\text{Equity}^n_j\right) \tag{B.12}$$

$$=\sum_s \Pi_{j,i,s}\text{Bail-In}^n_{j,s} \tag{B.13}$$

$$\Longleftrightarrow \sum_s C^{\text{neutral}}_{j,i,s}\left(\text{Equity}^n_j+\text{Bail-In}^n_j\right)+\left(1-\sum_i\sum_s C^{\text{fair}}_{j,i,s}\right)\Theta^n_{j,i}\left(\text{Bail-In}^n_j\right)- \tag{B.14}$$

$$\left(\sum_i\sum_s C^{\text{fair}}_{j,i,s}\right)\Theta^n_{j,i}\left(\text{Equity}^n_j\right)=\sum_s \Pi_{j,i,s}\text{Bail-In}^n_{j,s} \tag{B.15}$$

$$\Longleftrightarrow \Theta^n_{j,i}\left(\left(1-\sum_i\sum_s C^{\text{fair}}_{j,i,s}\right)\left(\text{Bail-In}^n_j\right)-\left(\sum_i\sum_s C^{\text{fair}}_{j,i,s}\right)\left(\text{Equity}^n_j\right)\right)+ \tag{B.16}$$

$$\sum_s C^{\text{neutral}}_{j,i,s}\left(\text{Equity}^n_j+\text{Bail-In}^n_j\right)=\sum_s \Pi_{j,i,s}\text{Bail-In}^n_{j,s} \tag{B.17}$$

$$\Longleftrightarrow \sum_s C^{\text{neutral}}_{j,i,s}=\frac{\sum_s \Pi_{j,i,s}\text{Bail-In}^n_{j,s}}{\text{Equity}^n_j+\text{Bail-In}^n_j}=\sum_s C^{\text{fair}}_{j,i,s} \tag{B.18}$$

$$\Longleftarrow C^{\text{neutral}}_{j,i,s}=C^{\text{fair}}_{j,i,s} \tag{B.19}$$

where the last step follows from:

$$\forall\, j:\ \left(1-\sum_i\sum_s C^{\text{fair}}_{j,i,s}\right)\left(\text{Bail-In}^n_j\right)-\left(\sum_i\sum_s C^{\text{fair}}_{j,i,s}\right)\left(\text{Equity}^n_j\right)= \tag{B.20}$$

$$\text{Bail-In}^n_j-\left(\sum_i\sum_s C^{\text{fair}}_{j,i,s}\right)\left(\text{Equity}^n_j+\text{Bail-In}^n_j\right)= \tag{B.21}$$

$$\text{Bail-In}^n_j-\left(\sum_s\sum_i \frac{\Pi_{j,i,s}\text{Bail-In}^n_{j,s}}{\text{Equity}^n_j+\text{Bail-In}^n_j}\right)\left(\text{Equity}^n_j+\text{Bail-In}^n_j\right)= \tag{B.22}$$

$$\text{Bail-In}^n_j-\sum_s\sum_i \Pi_{j,i,s}\text{Bail-In}^n_{j,s}=0 \tag{B.23}$$

since $\sum_i \sum_s \Pi_{j,i,s} = 0 \Leftrightarrow \sum_s (\bar{P}^n_{\cdot,s})_j = 0 \Leftrightarrow \text{Bail-In}^n_j = 0$ for all j, n.

$\square$

Appendix C. Results for Bail-In analysis

Table C.12: Systemic losses in the bail-in regime for idiosyncratic shocks

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0	34.49	63.26	89.36	110.99	126.3	139.05	150.9	156.21	160.85
0.1	0	34.27	63.21	89.33	111.19	126.04	139.01	150.44	155.66	160.32
0.2	0	34.14	63.34	88.61	110.88	126.6	140.55	150.9	156.04	161.79
0.3	0	34.24	63.79	88.62	110.08	127.19	141.36	151.38	156.47	162.48
0.4	0	34.51	63.69	88.68	110.01	127.65	142.04	151.71	156.75	162.9
0.5	0	34.64	63.53	88.73	109.89	128.08	142.61	152.03	156.98	163.3
0.6	0	34.5	63.56	88.87	109.87	128.63	142.68	152.43	157.34	163.81
0.7	0	34.58	63.61	89.01	109.89	129.5	143.45	152.84	157.86	164.32
0.8	0	34.72	63.67	89.16	110.02	129.83	144.23	153.26	158.25	164.86
0.9	0	34.87	63.82	89.41	110.08	130.41	145	153.68	158.64	165.31

Columns: shock size. Rows: Haircuts. Numbers in EUR bn.

Table C.13: Systemic losses in the insolvency regime for idiosyncratic shocks

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0	43.83	73.15	96.93	115.92	132.54	142.54	149.71	152.59	154.68
0.1	0	55.79	81.4	104.95	122.47	137.94	147.24	153.64	156.25	158.25
0.2	0	67.82	91.19	113.2	128.7	143.54	152.17	157.81	160.17	162.08
0.3	0	79.84	100.93	121.4	135.6	149.04	157.01	161.91	164.03	165.83
0.4	0	91.71	110.54	129.54	142.38	154.45	161.79	165.95	167.84	169.56
0.5	0	103.45	119.95	137.5	148.97	159.64	166.36	169.81	171.48	173.03
0.6	0	115.4	129.46	145.67	155.73	165.03	171.14	173.89	175.33	176.81
0.7	0	127.27	138.86	153.83	162.47	170.37	175.9	177.96	179.19	180.54
0.8	0	139.19	148.3	161.95	169.16	175.66	180.61	181.99	183.02	184.22
0.9	0	151.06	157.67	170.03	175.78	180.89	185.27	186	186.82	187.87

Columns: shock size. Rows: Haircuts. Numbers in EUR bn.

Table C.14: Systemic losses in the bail-in regime for systemic shocks

	0	1	2	3	4	5	6	7	8	9
0	0	0.12	0.31	0.96	2.21	2.99	5.3	7.93	10.54	14.35
0.1	0	0.12	0.31	0.96	2.08	3.13	5.51	8.14	10.75	12.8
0.2	0	0.12	0.31	0.96	1.97	3.3	5.71	8.26	10.98	12.91
0.3	0	0.12	0.31	0.95	1.86	3.52	5.87	8.35	11.67	13.14
0.4	0	0.12	0.31	0.99	1.75	4.14	6.11	8.59	11.84	13.47
0.5	0	0.12	0.31	0.97	1.63	4.37	6.36	8.85	12.09	13.65
0.6	0	0.12	0.31	0.96	1.54	4.62	6.59	9.07	12.17	13.82
0.7	0	0.12	0.31	0.93	1.39	4.86	6.86	9.38	12.31	13.96
0.8	0	0.12	0.31	0.91	1.42	5.1	7.05	9.48	12.43	14.09
0.9	0	0.12	0.31	0.9	1.41	5.3	7.22	9.61	14.65	16.77

Columns: shock size. Rows: Haircuts. Numbers in EUR bn.

Table C.15: Systemic losses in the insolvency regime for systemic shocks

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0.08	0.67	4.15	5.99	8.41	10.38	12.28
0.1	0	0.01	0.03	0.23	1.13	6.91	8.71	11.02	12.89	15.64
0.2	0	0.02	0.05	0.38	1.59	9.72	11.48	13.05	15.5	18.98
0.3	0	0.03	0.07	0.53	2.05	12.47	13.81	15.77	18.11	22.31
0.4	0	0.04	0.1	0.68	2.51	15.21	16.68	18.48	20.7	25.62
0.5	0	0.05	0.12	0.83	2.97	17.79	19.5	21.14	23.28	28.92
0.6	0	0.05	0.14	0.98	3.43	20.59	22.29	23.79	25.85	32.2
0.7	0	0.06	0.16	1.13	3.89	23.34	25.05	26.38	28.41	35.47
0.8	0	0.07	0.19	1.28	4.34	26.06	27.77	28.95	30.95	38.76
0.9	0	0.08	0.21	1.43	4.8	28.75	30.46	31.49	33.49	41.99

Columns: shock size. Rows: Haircuts. Numbers in EUR bn.

Appendix D. Anova Tables

In this section, we test if the coefficients for each contagion channel and tau-specific models are jointly different. The tests include all combinations of tau-specific models (also combinations of more than two.) These following Anova tables are estimated with Koenker (2018).

Table D.16: Anova test results

Models	ndf	ddf	Tn	pvalue
τ =0.25, τ =0.5	736	36,070	14,087.62	0.00
τ =0.25, τ =0.75	736	36,070	36,560.74	0.00
τ =0.25, τ =0.9	736	36,070	50,759.71	0.00
τ =0.25, τ =0.95	736	36,070	53,154.73	0.00
τ =0.25, τ =0.99	736	36,070	119,001.65	0.00
τ =0.5, τ =0.9	736	36070	17,898.26	0.00
τ =0.5, τ =0.95	736	36,070	16,900.33	0.00
τ =0.5, τ =0.99	736	36,070	53,313.77	0.00
τ =0.75, τ =0.9	736	36,070	12,579.44	0.00
τ =0.75, τ =0.95	736	36,070	14,958.96	0.00
τ =0.75, τ =0.99	736	36,070	30,577.63	0.00
τ =0.9, τ =0.95	736	36,070	1,258.42	0.00
τ =0.9, τ =0.99	736	36,070	3,258.30	0.00
τ =0.95, τ =0.99	736	36,070	1,555.21	0.00

Models refers to the set of models to be tested. ndf refers to the number of parameters. ddf refers to the degrees of freedom. Tn return an F-like statistic in the sense that the an asymptotically Chi-squared statistic is divided by its degrees of freedom and the reported p-value is computed for an F statistic based on the numerator degrees of freedom equal to the rank of the null hypothesis and the denominator degrees of freedom is taken to be the sample size minus the number of parameters of the maintained model.

Appendix E. Data Coverage and Summary Statistics

Table E.17: Summary statistics of included variables

Var.Name	Min.	1st Qu.	Median	Mean	3rd Qu.	Max	StD	Data C.
Contagion losses	0	1,699	8,896	370,402	33,863	79,732,168	2,619,878	90%
Total assets	301	67,580	147,461	1,271,509	343,526	157,220,135	7,277,256	90%
Private sector deposits	0	52,394	111,725	485,265	246,644	54,007,704	2,339,717	89%
Private sector loans	0	33,405	81,529	601,138	195,482	73,111,936	3,267,999	89%
Face value of deriv.	0	0	0	4,039,725	7,878	1,513,070,221	46,117,293	89%
Cross border deposits	0	901	2,825	198,405	11,051	43,498,252	1,515,197	89%
Cross border loans	0	844.50	4,065	378031	17,729	67,202,424	2,990,777	89%
Bank deposits	0	2,957	10,731	394,504	39,718	190,752,720	3,082,595	88%
Bank loans	0	16,291	29,847	371,660	64,939	162,070,592	2,857,894	89%
Securitized debt	0	0	0	258,292	0	37,764,072	1,875,005	89%

Source: OeNB.
Regulatory reporting data and credit registry data for the observation span 2008Q2 to 2016Q1. This table shows the summary statistics of the included variables. To improve readability, all variables are expressed in Tsd. EUR. Data C. refers to data coverage.