Suppressing ripple artifact while preserving accuracy in the computation of acoustic fields with the angular spectrum method using spatial filters

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Abstract: The angular spectrum method (ASM) is an effective tool for propagating wavefields between parallel planes through decomposition of the field into a series of independent plane waves. One source of error is interference from mirror sources introduced through the inherent periodicity of the fast Fourier Transform (FFT) used to implement this method numerically. Here, spatial filters were applied to attenuate waves propagating at large angles which are sensitive to mirror sources. Simulations show that this suppresses the ripple artifact while preserving the accuracy of the ASM-computed fields. To achieve comparable performance without filtering requires a 13.5-fold increase in computation time.

Keywords: Angular spectrum; spatial filtering; field propagation

1. Introduction

Angular spectrum methods (ASM) are widely used in acoustics and other areas to propagate wave fields. Based on Fourier acoustics, the technique can be viewed as the decomposition of the wave field into a series of plane waves that are propagated independently and then superposed to obtain the field at the required axial depth. The approach can be utilised to characterise the pressure fields from a range of different source configurations including planar[1] and spherical[2] single-element transducers as well as multi-element arrays[3,4]. It has also been extended to model wave propagation in layered homogeneous[1,5] and inhomogeneous[6] media. Propagation can be carried out in both the forward and backward directions where the former has applications to transcranial ultrasound focusing[7,8] and thermal ablation treatment planning[3,9] and the latter, so-called back-propagation, can be used to predict surface vibrations of acoustic sources often referred to as acoustic holography[10].

The ASM approach is computationally advantageous over Rayleigh-Sommerfeld integral methods computed in the spatial domain as the convolution becomes a multiplication in the frequency domain which is computationally efficient through the use of the FFT[11]. However, the FFT results in mirror sources caused by the its inherent periodicity which can distort the calculated field and result in ripple artifacts appearing in the resulting field. These errors can be minimised by the careful selection of the spatial discretisation and domain size which in turn restricts the angular range. This has been previously discussed extensively[12–15] where too large an angular range, resulting from increasing the number of grid points, or decreasing the domain size, results in a Fourier transformed Green's function that is undersampled and hence introduces ripples into the solution. Conversely, limiting the angular range and thus truncating the homogeneous wave components to prevent undersampling will increase the accuracy in the far field at the expense of the near field by removing the evanescent components[14–16]. Moreover, rather than controlling the angular range through the choice of sampling parameters, the removal of higher frequency plane wave components, termed angular restriction, can also increase accuracy[17].

Here, we explore the use of different spatial filters to attenuate plane wave components propagating above a particular depth dependent critical angle. Two different filters: a top-hat filter and a Hamming window (HW), are compared and the mean squared errors relative to analytical or numerical solutions between filtering methods are compared.

2. Methods

2.1 Angular spectrum implementation

We assume a time-harmonic velocity potential of the form $\tilde{\phi} = \phi(x, y, z)e^{-i2\pi ft}$, which results in the Helmholtz equation in terms of the spatial velocity potential, ϕ:

$$\nabla^2\phi + k^2\phi = 0, \tag{1}$$

where $k = 2\pi f/c_0$. Taking 2D spatial Fourier Transforms (FTs) and assuming propagation in the $+z$ direction, the solution for the field at any other parallel plane is given by:

$$\Phi(u, v, z) = G(u, v)e^{ik_z z}, \tag{2}$$

where u, v are the spatial frequencies in the x, y directions, $G(u, v)$ is the FT of the xy source plane (i.e. the Angular Spectrum of the source) and $e^{ik_z z}$ is the propagation factor which independently propagates each plane wave through a distance, z, based on the projection of the wavenumber in the z-axis, $k_z = \sqrt{k^2 - (2\pi)^2(u^2 + v^2)}$. Source conditions can either be entered as pressure or normal velocity distributions, which are then converted into the velocity potential, ϕ through $p = -i2\pi f\phi$ and $u = ik_z\phi$.

2.2 Filtering

The geometry of the domain is shown Fig. 1. The angle of each plane wave, θ, with respect to the axial coordinate, z, following plane wave decomposition can be calculated as follows,

$$\theta = \arccos\left(\frac{k_z}{k}\right). \tag{3}$$

A critical angle, θ_c, can then be defined as the cutoff frequency for the filters based on the geometry of the domain, as shown in Fig. 1. Here, we have chosen the cutoff angle to correspond to a quarter of the domain width, w, from the centreline in each direction. As such, the critical angle decreases with increasing propagation depth and more components of the angular spectrum are filtered out at large propagation distances. For all simulations, no filtering was employed for $z < 2\lambda$ so as not to remove the exponentially-decaying evanescent components in the near field.

Two filtering methods were employed in order to suppress the interference from image sources: a top-hat function and a HW of order 192 as shown in Fig. 1B. The top-hat function (T) was implemented as shown in Eq. (4)

$$T(\theta) = \begin{cases} 1, & \text{if } \theta < \theta_c, \\ 0, & \text{if } \theta \geq \theta_c. \end{cases} \tag{4}$$

$$\theta_c = \arctan\left(\frac{0.25 * w}{z}\right), \tag{5}$$

and the HW was generated using the MATLAB function `fir1`.

2.3 Error estimation

The accuracy of the ASM simulations was compared to expressions from the Rayleigh integral assuming a time-harmonic source of the form $u = u_0 e^{j\omega t}$. The on-axis solution is given by[18]

$$p_r = \rho_0 c_0 u_0 [e^{-ikz} - e^{-ik\sqrt{z^2+a^2}}]. \tag{6}$$

For radial distributions, the Hutchins solution was used which requires numerical evaluation of a line integral[19].

The mean squared error, MSE, was calculated between the simulated ASM pressure ($\bar{p}$) and the expected pressure value (p), summed and then averaged over the total number of grid points, N, in either the axial direction (up to the Rayleigh distance) or in the radial direction (out to the second null in the field in the transverse plane at the Rayleigh distance):

$$MSE = \frac{1}{N} \sum_{N} \left(\bar{p} - p\right)^2. \tag{7}$$

The error function was not normalised.

2.4 Simulations

ASM simulations were computed for two different piston source conditions with uniform normal velocities:

1. 1 MHz, $a = 10\lambda$.
2. 1 MHz, $a = 2.5\lambda$.

For each source condition, simulations were conducted with a fixed grid size of 1024^2 and domain width (w) of either $4a$, $10a$ or $20a$. Wavefronts were propagated in steps of $z_{rayl}/1000$ up to z_{rayl} (the Rayleigh distance) with either no filtering, a top-hat filter or a HW filter as defined in Section 2.2. Further simulations without filtering were also carried out with the same grid spacing but with increased zero-padding corresponding to domain widths of $40a$, $80a$ and $160a$.

3. Results

Fig. 2A–C displays typical pressure distributions for an unfocused, circular piston source vibrating with a uniform normal velocity. Diffraction from the source is evident in the xz-plane (Fig. 2A) and manifests as a series of rings in the two radial planes (Fig. 2B–C).

Consequently, for planes further into the farfield it is apparent that more energy will leak into the solution from mirror sources. Below each of the planar images the corresponding line scans are depicted comparing ASM with and without filtering to the analytical expression. It can be seen that for the unfiltered ASM simulation, ripples creep into the on-axis field at approximately a third of the Rayleigh distance and into the radial field at approximately one source radius out from the centreline, whereas the filtered method agrees well with the analytical expression throughout.

Fig. 3A–C show the MSE for the $a = 10\lambda$ source for a fixed number of grid points whilst varying the domain width for the unfiltered, top-hat filtered and HW filtered ASM simulations. For domain widths of $4a$, the spacing between mirror sources is insufficient to prevent large errors being introduced into the field even with filtering. However, at larger domain sizes, filtering proves an effective tool at reducing the errors with the HW filter performing better than the top-hat filter reducing the mean squared error by 4-fold for the axial, 2.2-fold for the radial slice at z_1 and 1.02-fold at the second radial slice z_2 for the $20a$ domain width. The addition of the filters does introduce an additional computational cost of 1.7 times and 2.5 times for the top-hat and HW filters, respectively, as shown in Fig. 3D.

In general, it is possible to improve the accuracy of the unfiltered ASM by increasing the domain size and hence the spacing of the mirror sources. The question arises then, of how large a domain size is necessary to match the performance of the filtered ASM results? To that end, unfiltered ASM simulations with constant grid spacing but increasing domain width were computed, until the errors were comparable to the HW filtered results for a $20a$

width. It was determined that a width of $80a$ was required to achieve a similar level of accuracy for the on-axis distributions (Fig. 3E). However, in the two radial planes the errors in the unfiltered results were still 2.6-fold and 1.6-fold greater, respectively, (Fig. 3F–G) in spite of the 4-fold increase in mirror source spacing. Also, the unfiltered $80a$ case took 2.3 times longer to calculate than the HW filtered $20a$ computation (Fig. 3H).

An interesting anomaly was encountered when increasing the domain size to $160a$ for the unfiltered simulation. In this case, the errors increased substantially relative to the $80a$ case particularly for the on-axis distribution.The cause of this anomaly was found to rest with the conversion of the velocity source condition into the velocity potential through spatial differentiation, or division by ik_z, in the frequency domain. Consequently, at the evanescent wave boundary the magnitude of the transverse wavenumber equals the wavenumber itself, and thus $k_z = 0$ and a singularity occurs. In most cases, the sampled wavenumber space does not directly coincide with the evanescent boundary and so a singularity is avoided but there is still amplification of nearby wavenumber components which introduces the observed errors. This problem can be averted by implementing an additional band-reject filter or mask to remove components close to the evanescent wave boundary. It is also noted that these problems do not occur for a pressure-source where division by k_z is not required.

Similar trends were observed for a smaller source radius, $a = 2.5\lambda$ as shown in Fig. 4 which shows the data at z_1 in which the error was 1.7 times lower for the HW filter than the top-hat filter. When increasing the domain size for the unfiltered simulations, the anamolous error rise for $160a$ seen in Fig. 3E–G was not observed and this domain width offered comparable radial errors to the $10a$ HW filter but at an increased computational expense of 13.5-fold (Fig. 4D).

4. Conclusion

This paper has described the use of spatial filters with the ASM to supress the ripple artifact from mirror sources by restricting the angular range of the ASM in a depth dependent manner, to remove components that propagate at large angles. Two filters were considered, a top-hat filter, and a Hamming window filter, with the latter providing improved performance. To match the performance achieved using a Hamming window filter with an unfiltered ASM approach, domain widths of up to 8 times larger are required resulting in an increased computational cost of between 2.3 and 13.5-fold. Therefore, this motivates the use of spatial filters as an effective method to improve the accuracy of ASM simulations whilst reducing computational times.

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Figure Captions

Figure 1: Schematic of ultrasound propagation from circular piston source (shown in grey). (A) Transverse (xy) plane of width, w. (B) Axial (yz) plane from source to propagation planes, $z_1 = 0.334 z_{rayl}$ and $z_2 = z_{rayl}$. Filter cutout frequency is defined based on the angle of the wavevector, θ, with two different low-pass filters: a top-hat function (black) and a Hamming window (grey).

Figure 2: Typical pressure distributions relative to the source pressure, $p_0 = \rho_0 c_0 u_0$, for an unfocused piston source with and without Hamming window (HW) filtering. Filtered fields: (A) xz-plane, (B) xy-plane at $z_1 = 0.334 z_{rayl}$, (C) xy-plane at $z_2 = z_{rayl}$. (D) Axial field comparison between on-axis Rayleigh solution, Eq. (6), and ASM method with and without HW filtering. (E,F) Radial field comparison between Hutchins solution and ASM method with and without HW filtering at z_1, z_2 (log-scale).

Figure 3: Mean squared errors for a circular piston source of radius, $a = 10\lambda$ for unfiltered (N), top-hat (T) filtered and Hamming window (HW) filtered ASM simulations. Top row: fixed number in grid points for each plane, 1024^2, for three domain widths, $4a$, $10a$ and $20a$. Bottom row: comparison of $20a$ domain width simulations with larger unfiltered simulations of $40a$, $80a$ and $160a$. (A, E) On-axis field errors. (B, F) Radial field errors at $z_1 = 0.334 z_{rayl}$. (C, G) Radial field errors at $z_2 = z_{rayl}$. (D, H) Computational times normalised to HW $20a$ case. The reason for the anamolous result for $160a$ in E–G is discussed in the text.

Figure 4: Mean squared errors for a circular piston source of radius, $a = 2.5\lambda$ for unfiltered (N), top-hat (T) filtered and Hamming window (HW) filtered ASM simulations. Top row: fixed number in grid points for each plane, 1024^2, for three domain widths, $4a$, $10a$ and $20a$. Bottom row: comparison of $20a$ domain width simulations with larger unfiltered simulations of $40a$, $80a$ and $160a$. (A, C) Radial field errors at $z_1 = 0.334 z_{rayl}$. (B, D) Computational times normalised to HW $20a$ case.