

# Outage Probability Calculation for Two-Ray Ground Reflection Scenarios with Frequency Diversity

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**Abstract**—We consider a two-ray ground reflection scenario with only statistical knowledge about the distance between transmitter and receiver. Additionally, two frequencies are used in parallel to mitigate possible destructive interference. In the context of ultra-reliability, it is important to be able to quantify the reliability of such communication systems. In this work, we analyze the outage probability of the described system and present a semi-closed form expression to calculate it. Based on this, results on optimizing the spacing between the two frequencies will be derived.

## I. INTRODUCTION

Many applications of modern wireless communication systems have strict reliability requirements [1]. It is therefore important to develop techniques to both improve the reliability as well as analyze and calculate it for a given scenario. In this work, we investigate the outage probability and its worst-case bound for wireless communication systems that can be described by the two-ray ground reflection model. Additionally, we assume a frequency diversity where two frequencies are used in parallel. This model is relevant for scenarios where an unmanned aerial vehicle (UAV) is flying above large flat terrain, e.g., large concrete areas at airports [2] or water [3]. In particular, this includes high frequency bands like millimeter wave (mmWave) [4].

In [5], the authors consider a similar scenario, where they focus on the optimization of the frequency spacing  $\Delta\omega$  for maximizing the minimum receive power without knowledge of the distribution of the distance  $d$ . In contrast, we assume knowledge of the distribution  $F_d$  in this work, which allows us to solve the problem of calculating the outage probability for a fixed frequency spacing  $\Delta\omega$ .

## II. SYSTEM MODEL AND PROBLEM FORMULATION

Throughout this work, we consider the classical two-ray ground reflection model [6, Chap. 4.6]. In this scenario, the propagation environment is approximated as a plane reflecting ground surface. A transmitter is located at height  $h_{\text{Tx}}$  above the ground. At distance  $d$ , the receiver is placed at height  $h_{\text{Rx}}$ . Based on the setup, it can be seen that the transmitted signal is propagated via two separate paths to the receiver. First, there is a line-of-sight (LoS) propagation with path length  $\ell$ . Additionally, the signal is reflected by the ground, which leads to a second component which superimposes with the LoS component at the receiver. The total length of the second ray

is  $\tilde{\ell} > \ell$ . From basic trigonometric considerations, the two path lengths can be calculated as

$$\ell^2 = (h_{\text{Tx}} - h_{\text{Rx}})^2 + d^2 \quad (1)$$

$$\tilde{\ell}^2 = (h_{\text{Tx}} + h_{\text{Rx}})^2 + d^2. \quad (2)$$

Depending on the distance  $d$  between the transmitter and the receiver, the two received signal components can interfere constructively or destructively. Destructive interference leads to drops in the receive power at certain distances, which in turn can cause outages in the data transmission. It is therefore of great interest to mitigate such local minima of the receive power, especially in the context of ultra-reliable communications.

One way to achieve this is the use of a second frequency in parallel [5]. The resulting total sum receive power  $P_s$  is shown in (3) at the top of the next page. The total transmit power  $P_t$  is equally distributed over the two used frequencies  $\omega_1 = 2\pi f_1$  and  $\omega_2 = 2\pi f_2 = \omega_1 + \Delta\omega$ .

For applications that require an ultra-high reliability, it is important that a certain performance can always be guaranteed, even in the worst-case scenario. Therefore, it is a valid approach to work with a worst-case bound on the performance metric instead of the actual quantity. In the following, we will consider the lower bound  $\underline{P}_s$  of the actual sum receive power which is shown in (4) at the top of the next page, cf. [5, Lem. 1]. As the performance metric to quantify the reliability of the described system, we use the *outage probability*  $\varepsilon$ , which we define as the probability that the received power is less than the receiver sensitivity  $s$ , i.e.,  $\varepsilon = \Pr(P_s < s)$ .

With the above preliminaries, we are now able to formulate the problem that will be addressed in this work.

**Problem Statement** (Outage Probability  $\varepsilon$ ). For the described communication system with a fixed frequency spacing  $\Delta\omega$  and a known distribution of the distance  $d$ , what is the outage probability  $\varepsilon$ ?

## III. PRELIMINARY RESULTS

As mentioned above, we will replace the actual outage probability  $\varepsilon$  by its worst-case bound  $\bar{\varepsilon}$ . For a given distribution of the distance  $d$  with probability density function (PDF)  $f_d$ , it can be calculated as  $\bar{\varepsilon} = \Pr_d(\underline{P}_s < s)$ . An illustration of this can be seen in Figure 1.

For a practical calculation, we need to determine the distance intervals in which  $\underline{P}_s < s$ . In order to do this, we consider the

$$P_s(d, \Delta\omega; \omega_1, h_{\text{Tx}}, h_{\text{Rx}}, P_t) = \frac{P_t}{2} \left(\frac{c}{2}\right)^2 \left[ \left(\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}\right) \left(\frac{1}{\ell^2} + \frac{1}{\tilde{\ell}^2}\right) - \frac{2}{\ell\tilde{\ell}} \left( \frac{\cos\left(\frac{\omega_1}{c}(\tilde{\ell} - \ell)\right)}{\omega_1^2} + \frac{\cos\left(\frac{\omega_2}{c}(\tilde{\ell} - \ell)\right)}{\omega_2^2} \right) \right] \quad (3)$$

$$\underline{P}_s(d, \Delta\omega; \omega_1, h_{\text{Tx}}, h_{\text{Rx}}, P_t) = \frac{P_t}{2} \left(\frac{c}{2}\right)^2 \left[ \left(\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}\right) \left(\frac{1}{\ell^2} + \frac{1}{\tilde{\ell}^2}\right) - \frac{2}{\ell\tilde{\ell}} \sqrt{\left(\frac{1}{\omega_1^2}\right)^2 + \left(\frac{1}{\omega_2^2}\right)^2 + \frac{2 \cos\left(\frac{\Delta\omega}{c}(\tilde{\ell} - \ell)\right)}{\omega_1^2 \omega_2^2}} \right] \quad (4)$$

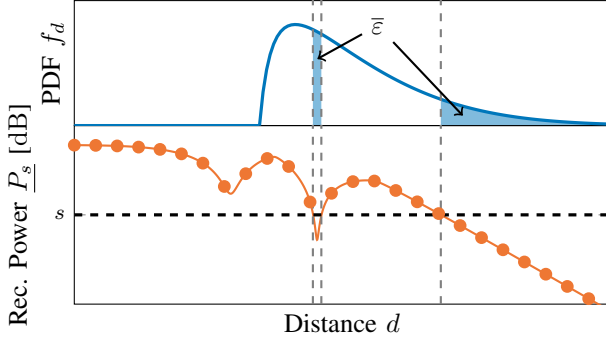


Figure 1. Illustration of the outage probability  $\bar{\epsilon}$  for a threshold  $s$  and a given PDF  $f_d$  of the distance  $d$ . The outage probability is determined by the probability mass of the distance intervals for which  $\underline{P}_s < s$  holds.

intervals of  $d$  in which  $\underline{P}_s$  is monotonic. Based on [5, Lem. 2], we know that the (local) minima of  $\underline{P}_s$  are approximately attained at distances  $\delta_k$  and the maxima at distances  $\delta_{\pi,k}$ ,  $k = 0, 1, \dots, k_{\max}$ , with

$$\delta_k^2 = \frac{(c^2 k^2 \pi^2 - \Delta\omega^2 h_{\text{Rx}}^2)(c^2 k^2 \pi^2 - \Delta\omega^2 h_{\text{Tx}}^2)}{c^2 \Delta\omega^2 k^2 \pi^2} \quad (5)$$

$$\delta_{\pi,k}^2 = \frac{(c^2 \pi^2 (2k+1)^2 - 4\Delta\omega^2 h_{\text{Rx}}^2)(c^2 \pi^2 (2k+1)^2 - 4\Delta\omega^2 h_{\text{Tx}}^2)}{4c^2 \Delta\omega^2 (2k+1)^2 \pi^2} \quad (6)$$

Based on this, the intervals in which  $\underline{P}_s$  is monotonically increasing are  $[\delta_{k+1}, \delta_{\pi,k}]$ . Similarly,  $\underline{P}_s$  is decreasing over the intervals  $[\delta_{\pi,k}, \delta_k]$ . It should be noted that we always have  $\delta_0 = \infty$ .

Since the function  $\underline{P}_s$  is monotonic in each given interval, there is at most one distance in the interval such that  $\underline{P}_s = s$ . Throughout the following, we denote the distances at which  $\underline{P}_s = s$  as follows

$$\underline{P}_s(\alpha_k) = s \quad \text{with} \quad \alpha_k \in [\delta_{\pi,k}, \delta_k] \quad (7)$$

$$\underline{P}_s(\beta_k) = s \quad \text{with} \quad \beta_k \in [\delta_{k+1}, \delta_{\pi,k}] \quad (8)$$

Based on the distances  $\alpha_k$  and  $\beta_k$ , we can determine the intervals for which  $\underline{P}_s < s$  holds. This can in turn be used to calculate the outage probability as shown in the following.

**Theorem 1** (Calculation of the Outage Probability). *Consider the described communication system where two frequencies  $\omega_1$  and  $\omega_2 = \omega_1 + \Delta\omega$  are used in parallel. The worst-case outage probability  $\bar{\epsilon}$  can be calculated as*

$$\bar{\epsilon} = 1 - \sum_{\alpha_k} F_d(\alpha_k) + \sum_{\beta_k} F_d(\beta_k), \quad (9)$$

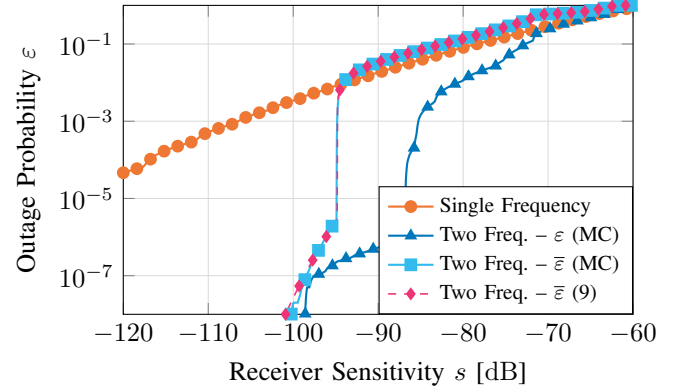


Figure 2. Outage probability of the receive power for a two-ray ground reflection scenario. (Example 1)

with  $F_d$  being the cumulative distribution function (CDF) of the distance  $d$  and the distances  $\alpha_k$  and  $\beta_k$  defined according to (7) and (8), respectively.

**Example 1.** In order to illustrate Theorem 1, we use the following numerical example. We set the system parameters to  $f_1 = 2.4$  GHz,  $\Delta f = 250$  MHz,  $h_{\text{Tx}} = 10$  m, and  $h_{\text{Rx}} = 1.5$  m. Additionally, we assume a shifted exponential distribution with  $d - 10 \text{ m} \sim \text{Exp}(15)$  for the distance  $d$ , i.e., the CDF is given as  $F_d(d) = 1 - \exp(-(d - 10)/15)$ . For this system, Figure 2 shows the outage probability over the power threshold  $s$ . First, we show both the outage probability  $\epsilon$  and its worst-case bound  $\bar{\epsilon}$  as evaluated by Monte Carlo (MC) simulations with  $10^8$  samples. Next, we indicate the calculated value of  $\bar{\epsilon}$  from (9). Furthermore, we show the outage probability for the single frequency case as comparison. First, it can be seen that the MC simulations confirm the analytically calculated values of  $\bar{\epsilon}$ . Next, there is a large increase in the outage probability for thresholds above a certain value. This is due to the additional outages at distances around the local minima in receive power at this value, cf. Figure 1. Overall, it should be emphasized that employing a second frequency can significantly improve the reliability over only using a single frequency, especially in the context of ultra-reliable communications.

#### IV. OUTLOOK

Since the calculation of (9) may require numerical inversion of a function, we will derive easy-to-calculate approximations in the final version of this paper. Additionally, we will take a closer look at the influence of the frequency spacing  $\Delta\omega$  on the outage probability and its optimization for a given tolerated outage probability.

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