Crossing Borders: Thinkers and Mathematical Ideas, 1100–1500 CE

Abstract:

The present paper investigates the transmission of Indo-Arabic numerals from India to the medieval West via Arabic sources, which provides an interesting intellectual and historical background to explore knowledge transfer across cultures. The goal of this paper is to shed light on Eastern and Western cultural exchanges and, in doing so, to add new interpretative layers to understand the Mediterranean world as the melting-pot of many histories. I provide a comparative perspective by juxtaposing Western and Eastern works belonging to different textual genres and which are commonly treated separately. From these works we can glimpse how individuals involved in cross-cultural interactions understood or chose to represent their own identity and that of ideas that originated beyond their cultural and linguistic milieu. In this regard, I emphasise the role played by translators, mediators of culture, and intellectuals in perceiving foreign knowledge and how this perception was communicated through language. The literature presented in the present study illustrates active links between cultural circuits across the Mediterranean world.

Keywords: Indo-Arabic numerals, al-Khwārizmī, algorismus, positional numeration, decimal arithmetic

1. Introduction

Otto Neugebauer once noted that mathematical-astronomical material often offers the clearest evidence of cultural transmission.[1] The present research explores the history of the transmission of Indo-Arabic numerals from India to the West during the medieval period—a phenomenon which, in fact, provides interesting insights to our understanding of knowledge transfer across the Mediterranean and beyond. The Indo-Arabic numeral system adopted universally across the globe is the most common system for the symbolic representation of numbers. This system of notation is based upon the Indian decimal numeration system.[2] Western numerals (1, 2, 3, etc.)—that is to say, the numerals most commonly used today worldwide— are ultimately of Indian origin; the Indian positional system was adopted and transmitted to the West by the Arabs sometime around the early medieval period. Western numerals are an adaptation of Arabic numerals based upon Indian digits. In scholarly and educational publications, Western numerals are also often referred to as 'Arabic numerals' or 'Indo-Arabic numerals'. In late

[1] Otto Neugebauer, *The Exact Sciences in Antiquity* (Princeton, N.J.: Princeton University Press, 1952), 1.

[2] All translations from primary sources are my own unless otherwise stated. The research for this paper has been funded by the Leverhulme Trust through the award of a Leverhulme Early Career Research Fellowship (Grant ECF 2018–338) at the Faculty of Linguistics, Philology and Phonetics, University of Oxford.

medieval and early Renaissance Europe, the transition process from the use of Roman (I, V, L, etc.) to Indo-Arabic figures stimulated the development of new textual traditions of mathematics. [3] The introduction of the new numeral system from the East was either initially looked at with some suspicion, as it threated a well-ordered world of things, or sparked curiosity and initiated scholarly enquiry. Nowhere did any such paradigm shift in pre-modern mathematical teaching and learning worthy of the name ever become more apparent. Judging from the substantial variety of sources, the scale of new knowledge production was extraordinary. It is often the case that when unfamiliar ways of thinking come to gradually percolate through cultural life, they awake a sense of self-awareness and open up an opportunity to address alterity. Languages categorise reality, and every culture acts as a frame within which external signs are interpreted. The texts presented in the following research articulate a complex and dynamic discourse in the receiving culture to construct images of 'the other'; unexpected meanings accompanied the spreading of new mathematical knowledge in other social spheres. In this corpus-based study, the story of positional numeration in the West serves as a pretext to explore the way different actors contributed to mathematical ideas crossing linguistic and religious boundaries, and taking new routes.

Here for the first time I provide a comparative perspective to identify knowledge transfer across the Mediterranean by juxtaposing textual traditions which are commonly treated separately. My aim is not to reconstruct each phase of the history of the transmission of numerals from India— it was clearly a multidimensional phenomenon; the emphasis is on processes of appropriation and adaptation of new scientific knowledge. In the next sections, I present passages from texts written in several languages (medieval Latin, Middle English, old Italian, and other Romance dialects of Italy) and which cover a 400 year period. From these works we can glimpse central developments in mathematical thought and society. For reasons of space, in such an overview of textual traditions, it has not been possible to treat the full range of details exhaustively. In writing this research, some of my leading questions have been: To what extent was the process of cultural exchange across boundaries conscious? How did individuals involved in these interactions understand or choose to represent their own identity and that of ideas that originated beyond their cultural and linguistic milieu?

In brief, my goals are fourfold: to bring together Eastern and Western textual traditions that are too often discussed in isolation from each other; to shed light on a significant but neglected phenomenon as a key to understanding the Middle Ages as a period of continual transformations;[4] to uncover the determining role played by individual actors in portraying a systemic change; and to reaffirm the

[3] As explained by Chrisomalis, in Europe for a long time Roman numerals were used side by side together with the newly introduced Indo-Arabic figures. The author provides a thorough comparative investigation of the history of numerations systems; see Stephen Chrisomalis, *Numerical notation: a comparative history*. (Cambridge: Cambridge University Press, 2010), 116–127.

[4] I use the term 'Middle Ages' to designate a time framework rather than a categorization of cultural formations.

centrality of a cross-cultural approach both to medieval and early modern studies.[5] As I cross the borders between cultural traditions, I focus on agents of exchange and literary representations of the 'other'. I emphasise the role played by translators, mediators of culture, and intellectuals in perceiving new knowledge and how this perception was communicated through language. In the first section, I present the main facts regarding the early transmission of numerals from India to Latin Europe via Arabic sources, and demonstrate the way some key thinkers contributed to accelerating the circulation of mathematical ideas. In the next section, I focus on texts that were the primary engines for spreading the knowledge of the decimal position number system. The third section examines the reception of the Indo-Arabic figures in the broader European learned circles. Lastly, I analyse a mathematical type of writing developed in Renaissance Italy to teach decimal and business arithmetic. The literature presented in the present study illustrates active links between cultural circuits across the Mediterranean world.

1. East and West Intellectual Exchanges

The ten Indian figures, which in *devanāgarī* are १·२·३·४·५·६·७·८·९·०, are the base of the decimal place value system.[6] In positional numeration, a number is determined not just by its constituent signs but by the place of each sign. Before the transmission of decimal arithmetic from India, in scientific Arabic the standard system for representing numerals was a non-place-value alphanumeric notation. In the early medieval period, the symbolic notation invented by the Indians was taken over by Syrian and Arabic writers, and eventually passed to Western Europeans. Most of the Indian mathematics that came to early medieval Western Asia and Europe was transmitted by Muslim intermediaries. Arabs established realms in northern and western India during the Islamic expansion of the early 8th century; the Indian base-ten place value system reached Baghdad by the 9th century if not early. After the rise of Islam, Muslim scholars may have received the transmission of decimal mathematics in Iran or elsewhere from other West Asians, or perhaps directly through the translation of Indian astronomical texts.[7] The complexity of sources, the tangled networks of different and sometimes parallel literary traditions, and contaminations in manuscript transmission make it impossible, however, to fully understand the Indian influence on Arabic mathematics. In the 11th century CE, several Arabic texts expounding decimal arithmetic started to appear containing the expression *ḥisāb al-hind* or 'Indian

[5] For other examples of 'systemic change' and an overview of the shifts in thinking which characterized the period, see Harald Kleinschmidt, *Understanding the Middle Ages: The Transformation of Ideas and Attitudes in the Medieval World* (Rochester: New York: Boydell Press, 2000).

[6] Sanskrit is an old Indo-Aryan language that has been widely used over two thousand years; for a long time it was the main lingua franca in use across South Asia. The *devanāgarī* script is the script in which Sanskrit has been set down. A classic on the history of Indo-Arabic numerals is by David. E. Smith and Louis. C. Karpinski, *The Hindu-Arabic Numerals* (Boston; London: Ginn, 1911).

[7] See Kim Plofker, *Mathematics in India* (Princeton, N.J.; Oxford: Princeton University Press, 2009), 255.

calculation'. Among early medieval Arabic works elucidating Indian arithmetic is that of al-Khwārizmī (9th century CE). Muḥammad ibn Mūsā al-Khwārizmī was a renown Persian polymath who worked at the House of Wisdom (Dār al-Ḥikma) in Baghdad;[8] scholars at the House of Wisdom were dedicated to translating scientific and philosophic treatises, mainly from Greek, as well as producing original research. Al-Khwārizmī wrote numerous scientific treatises on mathematics, astronomy, and geography; his work on elementary algebra, *Al-Kitāb al-mukhtaṣar fī ḥisāb al-jabr waʾl-muqābala*, was translated into Latin in the 12th century CE.

In the West, the standard numeration system was based on Roman numerals before the introduction of Indo-Arabic figures via Arabic sources. Roman numerals did not represent a positional system and had no symbol of zero. By the end of the 10th century CE, Indo-Arabic numerals had made their appearance in Western Europe but had no representation of decimal values. Numerical symbols inevitably assumed varying forms as they travelled from place to place, from language to language, and from scribe to scribe before being standardised through printing.[9] A quick inspection of Western manuscripts demonstrates that there were various transitional and blended versions of Indo-Arabic numerals used from the 12th through 17th centuries. Scholars argue that two conventions were developed in early times: West-Arabic numerals and East-Arabic numerals. Western Arabic numerals, also known as Maghribi numerals or *ghubar* ('dust-reckoning) numerals, are a set of ciphered-positional numerals, quite distinct from the regular Arabic system, and were commonly used in North Africa and southern Spain.[10] A large part of Greek and Italian mathematical manuscripts contain Arabic Eastern positional numerals at least up to the 12th century.[11] Only in the 13th century, when many of the important Toledan scholars moved to Italy, were these numeral-signs fully abandoned.

The oldest known examples of the nine symbols are in two Latin manuscripts written in monasteries in Northern Spain in the 10th century: the *Codex Vigilanus* (976 CE) and the *Codex Emilianus* (992 CE). In these two manuscripts, numerals from 1 to 9 appear in the western Arabic form and are written from right to left in accordance with the Arabic way of writing. In the *Codex Vigilanus*, written in the monastery of Albelda near the town of Logroño in northern Spain, the numerals are described as 'Indian figures'. The text attributes the numerals to the Indians and praises them for their 'most subtle talent',

[8] The 'House of Wisdom' was an important center of learning in medieval Iraq.
[9] On numerals and arithmetic in the Middle Ages, see Charles Burnett, *Numerals and arithmetic in the Middle Ages* (Farnham: Ashgate, 2010).
[10] A study on the role played by al-Andalus in the adoption and adaptation of numerals is Richard Lemay, "The Hispanic origin of our present numeral forms," *Viator* 8(1977): 435–477 and, by the same author, "Arabic numerals," in *Dictionary of the Middle Ages*, vol. 1 Aachen–Augustinism, ed. Joseph R. Strayer (New York: Charles Scribner's Sons 1982), 382–398.
[11] See Charles Burnett, "Indian numerals in the Mediterranean Basin in the twelfth century, with special reference to the 'eastern forms' ", in *From China to Paris: 2000 years transmission of mathematical ideas*, eds. Yvonne Dold-Samplonius, Joseph W. Dauben, Menso Folkerts, and Benno van Dalen (Stuttgart: Steiner, 2002), 237–288. Reprinted as Chapter 5 in Burnett 2010.

stating that 'all other nations yield to them in arithmetic and geometry and the other liberal arts'.[12] In Latin Europe, from the end of the 10th to the 12th centuries several treatises were dedicated to teach how to use the calculating board (Latin *abacus*), the oldest being by the Benedectine Gerbert of Aurillac (ca. 940–1003). Early forms of Indo-Arabic numerals found their way into slightly more widespread usage through the writings of Gerbert, who was to become Pope Sylvester II in 1000 CE. Gerbert traveled extensively and studied arithmetic in Islamic Spain, at which time he almost certainly learned positional numeration and the Indo-Arabic numerals.[13] Gerbert's exposure to Arabic mathematics and astronomy stimulated his pedagogical innovations as master of the Rheims cathedral school in France; imported from Muslim Spain, devices like the astrolabe and the abacus complemented the study of the texts of the four classical liberal arts constituting the *quadrivium*. Gerbert embodied the type of master who stretches the cathedral-school curriculum beyond its original purpose. Later authors, such as his biographer Richer, credited Gerbert with having introduced the use of the *formae* or *notae*, which indicate the nine numeral signs (excluding zero), on the counting board, thus replacing a large number of tokens placed in any column with a single token bearing one of these signs. No zero-sign was needed because counting boards are positional by their nature, without the need for a placeholder.[14]

During the 12th century CE, Spain and Italy witnessed a multifaceted process of knowledge transmission through the translation of Arabic mathematical texts into Latin.[15] Toledo was one of the centers from which Arabic and Greek works were translated into Latin and disseminated throughout Western Europe.[16] The Toledo School of Translators denotes a group of scholars who worked in Toledo during the 12th and 13th centuries; Gerard of Cremona, Adelard of Bath, Robert of Chester, Plato of Tivoli, Gugliemo de Lunis, John of Seville, and Hermann of Carinthia were among the most prolific translators of scientific works from Classical Arabic into Latin.[17] Most of the medieval translations of Aristotle and Avicenna were made in Toledo, many probably under the patronage of Archbishop

[12] *Scire debemus in Indos subtilissimum ingenium habere et ceteras gentes eis in arithmetica et geometrica et ceteris liberalibus disciplinis concedere*. Translation in Charles Burnett, "The semantics of Indian numerals in Arabic, Greek and Latin," *Journal of Indian Philosophy*, 34 (2006): 17.

[13] For a study on the abacus tradition developed by Gerbert, see Menso Folkerts, "The names and forms of the numerals on the abacus in the Gerbert tradition", in *Gerberto d'Aurillac, da Abate di Bobbio a Papa dell'Anno 1000*, ed. Fabio G. Nuvolone (Bobbio: Associazione culturale Amici di Archivum Bobiense, 2001b), 245–265. Modified reprint as Chapter 6 of Folkerts, *Essays on early medieval mathematics: the Latin tradition* (Aldershot: Ashgate, 2003).

[14] See Gillian R. Evans, "From abacus to algorism: theory and practice in medieval arithmetic," *British Journal for the History of Science* 10 (1977): 114–131, and Craig A. Gibson and Francis Newton, "Pandulf of Capua's De calculatione: An illustrated abacus treatise and some evidence for the Hindu-Arabic numerals in eleventh-century South Italy", *Mediaeval Studies* 57 (1995): 293–335.

[15] On translations of Arabic mathematical texts into Latin in Latin Europe, see Charles Burnett, "The coherence of the Arabic-Latin translation programme in Toledo in the twelfth century," *Science in Context* 14, 1–2 (2001): 249–288.

[16] An examination of medieval Latin mathematical texts is found in Edward Grant, ed., *A Source Book in Medieval Scienc*e (Cambridge, Mass.: Harvard University Press, 1974) and Menso Folkerts and Barnabas Hughes, "The Latin mathematics of medieval Europe," in *Sourcebook in the mathematics of medieval Europe and North Africa*, ed. Victor J. Katz (NJ: Princeton University Press, 2016), 4–223.

[17] See Charles Burnett, "A group of Arabic-Latin translators working in Northers Spain in the midtwelfth century," *Journal of the Royal Asiatic Society* 109 (1977): 62–108.

Raimundo (1124–1252). The main sciences cultivated in al-Andalus, which indicates the areas of medieval Spain that were under Islamic rule, were mathematics, astronomy, and medicine.[18] Al-Andalus was a major exporter of knowledge and active participant in the making of the Mediterranean intellectual tradition; important players included Italian scholars and translators, cultural intermediaries, go-between, Andalusi-Jewish intellectuals, Arabized Jews, and converted Jews. Authors had to adapt their language to meet the challenges posed by changes in knowledge and practice, such as the introduction of a symbol for zero and new procedures of calculations based on it. [19]

The slow introduction of Indo-Arabic numerals, and the major breakthrough in scientific and cultural practices they caused, coincided with treatises employing the noun *algorismus*, a neologism which is the Latin adaptation of the personal name al-Khwārizmī.[20] *Algorismus* came to denote the art of computing by means of the decimal number system and Indo-Arabic figures. Written arithmetic based on Indo-Arabic numerals entered Latin Europe in the first half of the 12th century as part of a wave of translations of Arabic scientific works. A seminal work in this regard is al-Khwārizmī's 9th century text *Book of Indian Reckoning*, which became available in a number of Latin adaptations. Al-Khwārizmī's work is preserved only in four partial Latin translations dating from the 12th to the 13th century CE. The *Dixit Algorismi*,[21] the *Liber Ysagogarum Alchorismi*,[22] the *Liber Alchorismi*,[23] and the *Liber Pulueris* [24] are precious pieces of evidence of the birth of positional numeration in the West.[25] The authorship of these works remain uncertain. Notably, the latter attributes the origin of the new arithmetic—meaning

[18] Al-andalus was the Arab-controlled part of the Iberian Peninsula between 711 CE and 1492 CE.

[19] On Latin medieval mathematical terminology is André Allard, "La formation du vocabulaire latin de l'arithmétique médiévale," in *Méthodes et instruments du travail intellectuel au moyenâge: études sur levocabulaire* , ed. Olga Weijers, (Turnhout: Brepols, 1990), 137–181.

[20] On the influence of Arabic mathematics in medieval Europe, see André Allard, "The Arabic origins and development of Latin algorisms in the twelfth century," *Arabic Sciences and Philosophy* 1(1991): 233–283; Nadia Ambrosetti, *L'eredità arabo-islamica nelle scienze e nelle arti del calcolo dell'Europa medievale* (Milano: Edizioni Universitarie di Lettere Economia Diritto, 2008). The mathematics of medieval Islam is investigated in two volumes by Len Berggren, *Episodes in the Mathematics of Medieval Islam* (New York: Springer, 1986), and *Episodes in the Mathematics of Medieval Islam*, 2nd ed., (New York: Springer, 2016).

[21] Transmitted in two manuscripts. Also known as *Algoritmi de numero Indorum*. The Latin text based on MS Ii.VI.5 (Cambridge University Library) is available in Baldassarre Boncompagni, *Trattati d'aritmetica. I. Algoritmi de numero Indorum* (Roma:Tipografia delle scienze fisiche e matematiche, 1857). English translation and analysis by John N. Crossley and Alan S. Henry, "Thus Spoke al-Khwarizmi: a Translation of the Text of Cambridge University Library Ms. Li. Vi. 5," *Historia Mathematica* 17 (1990):130–131 and by Menso Folkerts, *Die älteste lateinische Schriftüber das indische Rechnen nach al-Ḫwārizmī. Edition Übersetzung und Kommentar von M. Folkerts unter Mitarbeit von Paul Kunittzsch* (München: Verlag der Bayerischen Akademie der Wissenschaften, 1997). See also the analysis in Menso Folkerts, "Early Texts on Hindu-Arabic Calculation," *Science in Context* 14, 1–2 (2001): 13–38.

[22] Transmitted in several versions and eight manuscripts. It has been hesitantly attributed to Adelard of Bath (a 12th century English translator and philosopher) and to Petrus Alfonsus (a 12th century Jewish Spanish scholar).

[23] Transmitted in ten manuscripts.

[24] Transmitted in three manuscripts. See Kurt Vogel, *Mohammed ibn Musa Alchwarizmi's Algorismus, das früheste Lehrbuch zum Rechnen mit indischen Ziffern. Nach der einzigen (lateinischen) Handschrift (Cambridge Un. Lib. Ms. Ii. 6.5) in Faksimile* (Aalen : O. Zeller, 1963) and André Allard, *Muhammad Ibn Mūsā al-Khwārizmī: Le calcul indien (Algorismus)* (Paris: Blanchard, 1992). It seems that John of Seville (early 12th century) also wrote an *algorismus*.

[25] See André Allard, "The influence of Arabic mathematics n the medieval West," In *Encyclopedia of the History of Arabic Science*, ed. Roshdi Rashed (London, New York: Routledge, 1996) vol. 2, 539–580.

the new calculation methods based on Indo-Arabic figures— to the Egyptians, while the *Dixit Algorismi* and the *Liber Alchorismi* mention the 'Indian figures', and the *Liber Ysagogarum Alchorismi* avoids the topic altogether. These *algorismi* are considered the first surviving Latin versions of Arabic works expounding place value notation based on Indo-Arabic figures.[26] They elucidate new procedures of calculations and, in doing so, mark a significant departure from the traditional methods of the abacus. The 12th and 13th centuries represent a momentous change, a turning point for medieval mathematics in the West; new knowledge from Arabic sources became available through the works of translators. The impact of the translation movement was enormous; the vast corpus of Arabic mathematical works which was made available to Latin readers revolutionized western thought and culture.

2. Mediators of Culture

Over the course of the 12th and 13th centuries, European society confronted rapid monetization. In Italy the transition from the individual merchant-adventurer to the sedentary enterprise, the partnership form of business organization, the development of an international network of trade and banking, and the rise of merchant-bankers revolutionized the organization of commerce.[27] The success of the sedentary merchant depended upon the development of certain new sophisticated techniques in business procedures, such as double-entry bookkeeping. Sedentary merchants specialised in the financing and organisation of trade. It is in light of the early stages of these new economic forces and intellectual climate that the deep influence of the *Liber abaci* of Leonardo Pisano (also known as Fibonacci) should be understood.[28] Written in medieval Latin, this work teaches methods of computation that make use of Indian positional numeration. The *Liber abaci* (first redaction 1202, second redaction 1228) is an encyclopaedic work comprising 15 chapters that treats much of the known mathematics of the time on arithmetic, algebra, and problem solving. Also, it provides a large section on business mathematics involving financial transactions, money exchange, barter, and other trade-related practices. The author mentions Mediterranean cities, units of money, weight, and products which depict typical situations occurring in the marketplace.[29] In the *Liber abaci* for the first time, the term *abacus* refers to the art of performing calculations using the Indo-Arabic numerals. In the dedicatory letter, Leonardo explains that he became acquainted with the new science of numbers during his training and travels in the

[26] Two early *algorismi* have been published and studied by Louis Charles Karpinski, "Two Twelfth Century Algorisms," *Isis* 3, (1921): 396–413.

[27] Richard A. Goldthwaite, *The Economy of Renaissance Florence* (Baltimore, Md: Johns Hopkins University Press, 2009).

[28] Leonardo Pisano is the author of other mathematical works, such as the *Pratica geometriae* and the *Liber quadratorum.* See Laurence Sigler, *The Book of Squares of Leonardo Pisano Fibonacci: An Annotated Translation into Modern English* (Orlando; Boston: Academic Press, 1987) and Barnabas B. Hughes, *Finobacci's De practica geometrie* (New York: Springer, 2008).

[29] See Eva Caianiello, "Leonardo of Pisa and the Liber Abaci: biographical elements and the project of the work", in *Scientific sources and teaching contexts throughout history: problems and perspectives,* ed. Alain Bernard and Christine Proust (Dordrecht: Springer, 2014), 217–246.

Mediterranean basin, as his father was the scribe responsible for the customs in Béjaïa (in modern-day Algeria). In the preface, Leonardo states that he considers the Indian number system and its methods of calculation to be superior to all other methods, and that he wishes to bring these to the Italian people.[30] In the first chapter, the nine Indian figures and zero are presented in descending order following the Arabic convention. He explicitly mentions the 'Indian figures' (see *novem figure indorum*) and that the sign 0 is called *zephirum*. Notably, Leonardo employs a rich vocabulary constituted of both old Latin terms and neologisms to explain new computation techniques, including words resulting from processes of lexical borrowing, transliteration, and translation from Arabic. For example, the terms *algebra* and *zephirum* ('zero') are instances of transliteration from Arabic *al-jabr* and *ṣifr*.[31] As sources have survived, Fibonacci's treatise represents a type of writing in which commercial arithmetic is cultivated to such a degree for the first time in a Latin mathematical work. It would be hard to underestimate the impact that Leonardo's work had in popularising business arithmetic and Indo-Arabic figures. Leonardo's own name and passages from his work are commonly cited in the vernacular mathematical tradition known as *abbaco* or 'the art of computation' developed in Italy in the 14th century.[32]

The mathematics of Latin Europe was taught within the framework of the seven liberal arts. When university teaching emerged in the 13th century, the curricula were rather innovative as most of early lectures and texts included recent imports into Latin culture from the Arabic.[33] Universities stimulated the writing of new works for teaching purposes, which is the case of Sacrobosco's *De sphaera mundi* (circa 1230 CE) for elementary astronomy and *Algorismums vulgaris* for basic arithmetic. [34] Both works were part of the 14th century university teaching curriculum at Oxford and Paris. *De sphaera mundi* is based heavily on Ptolemy's Almagest and draws additional ideas from Islamic astronomy; it was one of the most influential works of pre-Copernican astronomy in Europe. Sacrobosco's *Algorismus vulgaris* and the *Carmen de Algorismo* (circa 1220 CE) by the French scholar Alexander de Villedieu[35] were extremely influential *algorismi.* The significant number of extant manuscripts of both works attest to their popularity and wide circulation.[36] The *Algorismus vulgaris* by Sacrobosco was the first Latin work

[30] The dedicatory letter is addressed to Michael Scot (circa 1175–1232), who was considered the leading intellectual of his time and became the dominate figure among the scholars supported by the Emperor Frederick II at his court in Sicily.

[31] I have explained this term in Alessandra Petrocchi, "Medieval Literature in Comparative Perspective: Language and Number in Sanskrit and Latin," *Journal of Medieval Worlds* 1–2 (2019): 57–76.

[32] On the *abbaco* tradition, see more in the next sections.

[33] See Michael H. Shank, "Schools and Universities in Medieval Latin Science," and "Islamic mathematics," in *The Cambridge History of Science*, vol. 2, ed. David C. Lindberg and Michael H. Shank (Cambridge: Cambridge University Press, 2013), 207–239 and 62–83.

[34] On Sacrobosco, see Olaf Pedersen, "In quest of Sacrobosco," *Journal for the History of Astronomy* 16 (1985): 175– 221.

[35] On *the Carmen de algorismo*, see the study by Nadia Ambrosetti, "Algorithmic in the 12th century: the Carmen de Algorismo by Alexander de Villa Dei," in *History and philosophy of computing*, eds. Fabio Gadducci and Mirko Tavosanis (Cham: Springer, 2016) 71–86.

[36] *Algorismus vulgaris* was among the first printed books on mathematics. Early English translations of Alexander de Villedieu and John of Sacrobosco's texts were published by Robert Steele, *The Earliest Arithmetic in English* (London: Milford, 1922); Latin texts by James O. Halliwell-Phillipps, *Rara-mathematica: Or, A collection of treatises on the mathematics and subjects connected with them, from ancient inedited manuscripts* (London:

to be widely adopted in universities to teach decimal arithmetic by means of Indo-Arabic; it was one of the most popular Latin textbook on arithmetic in the Middle Ages and the standard university text for several centuries.[37] In the incipit the author introduces the subject matter of his work and provides an explanation of the neologism *algorismus*, where the reference to the philosopher Algus alludes (directly or indirectly) to the Persian scholar al-Khwārizmī:[38]

> 'A philosopher called Algus declared this concise science of numbering that is therefore named algorismo (*algorismus*), which translates as the "art of numbering" or the "introductory art into number" (*ars introductoria in numerum*).'

The *Liber abaci,* which was almost certainly written before Sacrobosco and de Villedieu's works differs from the *algorismus* tradition in several aspects. The two *algorismi* were university textbooks written in a concise and formal style to facilitate teaching and learning; their emphasis is on theoretical explanations and they do not provide practical examples. Leonardo's book addresses a diverse readership, which includes intellectuals (such as members of Frederick II's court), municipal officials, merchants, architects, and everyone who needed mathematics in their work or for education. The *Liber abaci* is not, however, a practical manual for merchants, as one there finds highly developed theoretical aspects which are missing in later mercantile works, such as in Italian *abbaci*.[39] Also, whereas the two *algorismi* follow a terse style of presentation which clearly favours mnemonic learning (de Villedieu's text is a versified work), the *Liber abaci* is a voluminous and detailed work which provides hundreds of sample problems on business arithmetic.

3. The Story of a Word

Some mon-mathematical text offers evidence of reception mechanics in the slow process of the spread of Indo-Arabic figures. This next section focuses on the interesting story of the etymology constructed around the word *algorismus* to map the ideological representations and social conceptions attached to it. It seems that two major fanciful etymologies of *algorismus* were circulating: i) one interpreted *algo* as 'art, introduction', and *rismus* as 'number', perhaps deriving the latter term from Greek *rhythmos*, and

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Samuel Maynard, 1841), 2nd ed. Another popular *algorismus* was the *Algorismus de minuciis* (1320 CE) by Johannes de Lineriis, which focuses on fractions. Johannes de Lineriis studied and taught at the University of Paris and also wrote several astronomical works.

[37] Peter of Dacia wrote a comprehensive Latin commentary on it in 1291 CE. He lectured at the University of Bologna and Paris.

[38] While the *Carmen de algorismo* does mention that 'this art' is called *algorismus* and that it involves the 'ten Indian figures' (including zero), it does not provide the explanation of the term *algorismus*.

[39] Chapters 14th and 15th have a highly speculative content, with the first explaining quadratic and cubic radicals. With respect to *abbaco* treatises, see more in the next sections. On Leonardo Pisano and merchantile mathematics, see the essays collected in Enrico Giusti and Raffaella Petti, eds. *Un ponte sul Mediterraneo. Leonardo Pisano, la scienza araba e la rinascita della matematica in Occidente* (Firenze: Edizioni Polistampa, 2002).

ii) the other reads *algo* as the name of the person who invented the new art of computing, most likely denoting al-Khwārizmī. Curious anecdotes associate the proper name and it various spellings, such as Algus/Algo/Algor, to the identity of this mysterious figure. He is often described as a 'philosopher', for instance in Sacrobosco's *algorismus*; other fascinating accounts include that by John of Norfolk who describes Algus as a *rex quondam castelliae* or a 'Castillian king'. [40] In a 14th century work from England, Algor is a king and inventor of the new art of computing invented by the Indians: *haec praesens ars dicitur algorismus ab Algore rege ejus inventore, vel dicitur ab algos quod est ars, et rodos quod est numerus; quae est ars numerorum vel numerandi, ad quam artem bene sciendum inveniebantur apud Indos bis quinque (id est decem) figurae.*[41] In his commentary on Sacrobosco's *Algorismus vulgaris*, Peter of Dacia describes Algus as a philosopher with an Arabic name (*quidam philosophus Algus nomine Arabicus*).[42] Another manuscript states that 'the work of the philosophus Algus was translated by Sacrobosco '*de graeco in latinum'* ('from Greek into Latin'). [43] A 13th century *algorismus* written in Old French states that *algorismus* was made in Greece, and that it is so called after the philosopher Algus who invented it. [44] In *Il Fiore*, an Italian vernacular poem most likely written between 1282 and 1297 by a certain Durante who some scholars believed to be the renowned author Dante Alighieri, the confusion between the spelling *argus* and *algus* may be responsible for the association between the 'tens figures' (*le diece figure*) and the mythical Greek Argus who built the ship Argo.[45] The French author Jean de Meun (circa 1240–1305 CE), best known for his continuation of the *Le Roman de la Rose*– an extremely influential 13th century medieval French poem belonging to the genre of courtly literature– describes Algus as a Greek and mentions his name next together with Plato, Aristotle, Euclid and Ptolemy.[46] In the same work, one of the characters (la Vieille) associates Algus with arithmetic and states: [...] *se mestre Algus, li bien contanz, i voussist bien metre ses cures wished et venist o ses .X. figures par quoi tout certefie et nombre* [...] or '[...] If Master Algus, the expert counter, wished to put his mind to it and came with his .x. figures with which he certifies and numbers

[40] See MS. Harl. Mus. Brit. 3742 in Halliwell-Phillipps, *Rara-mathematica*, 1.
[41] Quoted in Halliwell-Phillipps, *Rara-mathematica*, 73.
[42] Edited by Maximilian Curtze, *Petri Philomeni de Dacia in Algorismum vulgarem Johannis de Sacrobosco Commentarius, una cum Algorismo ipso* (Hauniae, 1897).
[43] Quoted in Curtze , *Petri Philomeni*, IX: *explicit algorismus, quem quidam philosophus nomine Algus primitus invenit, sed Iohannes de Sacro Busco, considerans arismetricam a Boetio et ab aliis arismetricis nimis late et diffuse seu confuse traditam, hunc tractatum de Graeco in Latinum transtulit et sub compendio collegit.*
[44] Edited by Louis C. Karpinski and E. G. R. Waters, "A Thirteenth Century Algorism in French Verse," *Isis* 11, 1 (1928): 45-84. See *argorisme fu fait [en] Grece* [...] *et li mestre argorisme* (p. 56 and 58). See *argorisme fu fait [en] Grece* [...] *et li mestre argorisme* in pp. 56 and 58.
[45] *Se mastro Argus[so] che fece la nave in che Giason ando' per lo tosone, e fece a conto regole e ragione e le diece figure, com'on save [...].* See *Il Fiore e il Detto d'Amore attribuibili a Dante Alighieri*, a cura di G. Contini (Milano: Mondadori, 1984), LIV.
[46] See Félix Lecoy, ed. *Le Roman de la rose by Guillaume de Lorris and Jean de Meun*, 3 vols (Classiques françaises du moyen âge. Paris: Champion, 1965–75). English translation by Frances Horgan, trans. and annot. *The Romance of the Rose*. Oxford World's Classics (Oxford: Oxford University Press, 1999).
See lines 16138–56. The first section was written by Guillame de Lorris (circa 1200 – 1240 CE), a French scholar and poet. Lorris' poem was translated into Middle English by Chaucer.

everything [...]'.[47] The same passage was somewhat reworked by Chaucer in the *Book of the Duchess* (written between 1369 and 1372), which is the first of Chaucer's major poems. In verse 435–442, Chaucer also associates Argus with the arts of computing and mentions 'Argus, the noble contour' and the 'ten figures':[48]

> *Shortly, hit was so ful of bestes,*
> *That thogh Argus, the noble countour,*
> *Sete to rekene in his countour,*
> *And rekened with his figures ten*
> *For by tho figures mowe al ken,*
> *If they be crafty, rekene and noumbre,*
> *And telle of every thing the noumbre*
> *Yet shulde he fayle to rekene even*
> *The wondres, me mette in my sweven.*

In several Middle English manuscripts, *algorym* and *augrym* denote Indo-Arabic numerals. In this regard, Acker (1994: 296) emphasises that the new arithmetic involving Indo-Arabic numerals was not encouraged by the clergy; the 13th century *Ancrene Wisse* is an anonymous monastic manual for female anchoresses written sometime roughly between 1225 and 1240 in Middle English. In this text, one finds the *figures of augrim* to be involved in 'the calculations of the devil's accountant'. [49] In *A treatise on the Astrolabe* (1391 CE), Chaucer's uses the expressions *noumbres of augrym* (Part 1, 7), *signes be writen in augrym* (Part 1, 8), and *nombre in augrym* (Part 1, 9) to indicate Indo-Arabic figures. The John Rylands University Library MS English 80 is a 15th century English New Testament in the so-called later version of the Wycliffite translation of the Bible. [50] The prologue contains an explanation of the place-value system (ff. 22v–23r) and refers to the '*augrym* numbers'. The explanation occurs because the Bible contained in the manuscript is numerated in both Roman and Indo-Arabic numerals, which prompted the author to explain place value notation to readers. The term *augrym* is also found at the beginning of the *Kalendarium* in MS Ashmole 39, II, circa 1440 CE, housed in the Bodleian Library (Oxford), which is a work on calendar written by John Somer (circa 1340–1409 CE), a Franciscan friar and one of the leading English astronomers of the late 14th century. The author also uses the expression *augrym nombre* to indicate to Indo-Arabic figures. In the medieval English work *The Court of Sapience*,

[47] See lines 12760–12766.
[48] On the use of the expression *augrim-stones* in Chaucer, see Louis C. Karpinski, "Augrim-Stones", *Modern Language Notes*_7 (1912): 206–209.
[49] See in Eric J. Dobson and Richard Dance, eds. *Ancrene Wisse: A Corrected Edition of the Text in Cambridge*, vol. 1 (Oxford: Oxford University Press 2005), 82.
[50] See Matti Peikola "Instructional aspects of the calendar in later medieval England, with special reference to The John Rylands University Library MS English 80", in *Instructional Writing in English Studies in honour of Risto Hiltunen*, eds. Matti Peikola, Janne Skaffari and Sanna-Kaisa Tanskanen (Amsterdam: John Benjamins Publishing, 2009), 83–104.

an incomplete and anonymous allegorical poem that enjoyed a certain popularity at the end of the 15th century, some stanzas refer to arithmetic. Verse 281, line 1963 states that Arithmetic' (see *Dame Arsmetryke*) 'taught the *ten fygures of augrym.*[51] The *Craft of nombrynge* ('Art of Numbering') is a 15th century CE Middle English anonymous commentary on the earlier mentioned popular *algorismus Carmen de algorismo* by de Villedieu.[52] The author of the *Crafte of Nombringe* provides two explanations of the term *algorismus*: in the first the author states that Algor was the name of an Indian king who invented the art of computing, thus he called it *Algorym* after his name. In the second account, the author explains that the Latin word is *algorismus*, and that *Algos* means *ars* or 'art' and *rides* is 'number', and in Latin he clarifies the morphology of the word: *inde dicitur Algorismus per addicionem huius sillabe mus and subtraccionem d and e*. Here the author elucidates that the syllable *mus* has been added and *d* and *e* have been subtracted to form *algorismus*. It is interesting to observe that the *Carmen de algorismo*, which is the text the *Crafte of Nombringe* comments upon, does not mention the semantics of the word *algorismus* but refers to the 'Indian figures'—a fact indicating that the author of the *Crafte of Nombringe* was borrowing from or relying on other sources. Notably, the expression *ars numerandi* used by the anonymous author of the *Crafte of Nombringe* is found in Sacrobosco's *algorismus.*

4. Vernacular Arithmetic in Renaissance Italy

The mathematical scenario in 14th century Italy is mainly dominated by treatises on *abbaco*.[53] So-called *abbaco* texts are training manuals produced in large numbers during the Italian Renaissance until the 16th century. *Abbaci* played a major part in teaching decimal numerical notation and training future merchants, bankers, and artisans; their purpose was to put mathematical knowledge into everyday use.

[51] Elizabeth Ruth Harvey, trans., *The Court of Sapience* (Toronto: Toronto University Press, 1984).
[52] MS Egerton 2622, ff. 136–165, British Museum. Another manuscript of this text is examined in Paul Acker. "The Crafte of Nombrynge in Columbia University Library, Plimpton MS 259," *Manuscripta* 37 (1993): 71–83.
[53] Van Egmond provides a ground-breaking investigation of the relationship between the commercial revolution of the Italian Renaissance and the *abbaco* textual tradition; see Warren Van Egmond, *The Commercial Revolution and the Beginnings of Western Mathematics in Renaissance Florence, 1300 –1500* (Ann Arbor, MI: London: University Microfilms International, 1976). Van Egmond also has published a catalogue of Italian *abbaco* manuscripts and printed books to 1600; see Warren Van Egmond, *Practical Mathematics in the Italian Renaissance: A Catalogue of Italian Abacus Manuscripts and Printed Books to 1600* (Florence: Istituto e museo di storia della scienza, 1980). On education and learning during the Italian Renaissance, see Paul F. Grendler, *Books and Schools in the Italian Renaissance* (Aldershot: Variorum, 1995). On abbaco schools see Paul F. Grendler, *Schooling in Renaissance Italy: Literacy and Learning, 1300–1600* (Baltimore: Johns Hopkins University Press, 1989). Vernacular and abbaco schools are investigated in Robert Black, *Education and society in Florentine Tuscany* (Leiden, Boston: Brill, 2007). Schools and masters of *abbaco* in Tuscany are investigated in Raffaela Franci, "Leonardo Pisano e la trattatistica dell'abaco in Italia nei secoli XIV e XV", *Bollettino di storia delle scienze matematiche* 23, 2 (2003): 33–54 and in Elisabetta Ulivi, "Maestri e scuole d'Abaco a Firenze: la Bottega di Santa Trinita", *Bollettino di storia delle scienze matematiche* 24, 1 (2004): 43–91 and Elisabetta Ulivi, *Gli abacisti fiorentini delle famiglie del maestro Luca, Calandri e Micceri e le loro scuole d'abaco* (secc. XIV–XVI) (Florence: Olschki, 2013).

The term *abbaco* denotes the art of computing and should not be confused with the medieval reckoning device. As mentioned earlier, *abbaco* as 'computation' by means of the Indo-Arabic numerals seems to appear for the first time in Leonardo Pisano's *Liber abaci*. *Abbaci* were written in Florentine and other Romance dialects of Italy— a fact which underlines the popular audience that *abbac*i addressed.[54] Interestingly, *abbaco* literature developed alongside the spread and popularity of abbaco schools and were, in fact, associated with the curriculum taught at the *botteghe d'abbaco*. Grendler (2006), who has studied Renaissance education in Italy extensively, demonstrates that *botteghe d'abbaco* were vernacular and commercially oriented schools attended by 10–12 years boys to learn commercial arithmetic. *Abbaco* schools were also attended by young men from wealthy families and by those who wished to continue their studies; moreover, they were the training camps for future *abbaco* masters. It has been well-documented that Tuscany, with Florence first, was home to some of the most important and prestigious *abbaco* schools and author-teachers.

An influential *abbaco*, which was written in 1307 in Montpellier by a certain Jacopo da Firenze, is the *Tractatus algorismi*.[55] This text contains one of the earliest extant algebra works in a vernacular. At the very beginning of the work, the author provides an explanation of the term *algorismus*:[56]

> *Et sapiate che si chiama algorismus perchè questa scienza fo principalmente facta in Arabia, et quelli che la troverono forono simigliantemente arabi. Et l'arte è dicta in lingua arabica algho, e'l numero è dicto rismus, et perciò è dicto alghorismus.*[57]
>
> 'You should know that it (i.e., this art) is called *algorismus* because this science was invented in the Arab world, and those who found it were similarly Arabs. And in Arabic "art" is called *algho*, and "number" is called *rismus*, and for this reason it is called *algorismus*.'

MS Ash 1168, held by the Biblioteca Medicea in Florence, is an early 15th century manuscript presumably from Cremona (Lombardy); this work has neither been studied nor edited yet. The quality of workmanship and the use of the *littera textualis* instead of the *mercantesca* indicate that the treatise could have been written for practical learning for a wealthy family of merchants or commissioned by a wealthy reader for his private library.[58] The topics treated by the author are: dedication and stating the

[54] Arrighi, Franci, and Ulivi have published several important contributions on the development of the *abbaco* mathematical tradition (too many to be mentioned here).
[55] See an English translation and edition by Jens Høyrup, *Jacopo da Firenze's Tractatus Algorismi and early Italian abbacus culture* (Basel; Boston: Birkhäuser, 2007).
[56] Several other *abbaci* mention the term *algorismus* in various spellings.
[57] See Hoyrup, *Jacopo da Firenze,* 194; the English translation is my own.
[58] The *littera textualis* or gothic is a script that originated in University circles, and by the end of the 13th century had extended into all book genres. Until the 13th century, in Western Europe only one script was used in book production: the *littera textualis*. This script, which originated in north-eastern France between the end of the 11th and the beginning of the 12th century, soon spread southward. This new script typology developed from the caroline minuscule (the common West European handwriting from the 9th to the 11th centuries. By contrast, the *mercantesca* originated in a mercantile environment, mainly Tuscan. Until the 16th century, the *mercantesca* was the usual script of merchants, bankers, painters, sculptors, architects, artisans and working men who learned to read and write in vernacular only.

purpose of the work, numeration (*numeracio*), arithmetical operations with integers (*integri*), arithmetical operations with fractions (*rotti*), rule of three (*la regola del tre*), money exchange (lo *cambio de le monete*), barter (*baratti*), partnership (*compagnie*), false position (*posizione falsa*), algebra (*regole de lo alzibra*), and practical geometry. In the first folio (1r-v) the author states:

> [...] *si comenzaremo lo n(ost)ro tractado il quale fi ditto Algorismo. Et dueti sapere che noi lo chiamamo Algorismo però che questa scientia fo trovata principalmente i(n) Arabia, e quello che la atrovò fo similmente arabo, e sapiati che l'arte è detratta*[59] *in lingua arabica 'algo' et il nome*[60] *è ditto 'rismus', e perciò è ditto 'algorismus'*[...]
> '[...] and so we shall begin our treatise, which is called *Algorismus*. And you must know that we call it *Algorismus* because this science was discovered in the Arabic world, and the man who discovered it was, in fact, and Arab. And you must know that in Arabic the 'art' is called *algo* and "number" is called *rismus*, and for this reason [this art] is called *algorimus*.'

Below is a passage from *Tractato de l'algorisimo* by Giovanni de' Danti Aretino, MS Cod. Plut. 30.26, held by the Biblioteca Medicea di Firenze.[61] In this work, one finds the unusual spelling *algorisimus*. In 1v, the author of *Tractato de l'algorisimo* explains that one should write the 'following algorismus according to the Arabs' usage': writing backward but reading forward, that is to say, 'start writing from the smaller number and start reading from the larger [...]'

> [1v]: *Donqua dobiamo scrivare il decto algorisimus secondo il costume degli Arabici cioè a l'aretroso e legiere a ricto secono il costume loro cioè inchominciare a scrivere dal minore numero e incominciarle a leggiere dal magiore comme ordinatamente trovarete seguitata la materia secondo la verità. Il decto algorisimo descrive nove figure e la prima rilieva unità cioè tanti numero quanto dice la figura, la seconda rilieva dicine, la tertia centonaia [...]*
> *[...] Anco iscrivaremo qui da lato comme sono facte le figure del decto algorisimus, in prima porremo per ordine li fegure de l'arte vecchia e puoi quelle de la nova a lato esse [...]*

When putting side by side the incipit and the numeration section of several *abbaci*, it can be noticed that some *abbaci* describes the 'Arabs' as the inventors of the new science, others mention the 'Indian figures', and other skip the topic altogether to expound mathematical topics. The meanings attached to the work *algorismus,* the fact that the Arabs are mentioned as the inventors of the new computation techniques, and that the nine figures are clarified to be Indian in origin demonstrate that texts such as

[59] 1r 30: it should read *è ditta* instead of *è detratta.*
[60] 1v 1: it should read *numero* ('number') instead of *nome* ('name').
[61] See Gino Arrighi, ed., *Giovanni de' Danti Aretino, Tractato de l'algorisimo.* Dal Cod. Plut. 30. 26 (sec. XIV) della Biblioteca Medicea Laurenziana di Firenze. Atti e Memoria dell'Accademia Petrarca di Lettere, Arti e Scienze, Nuova Serie 47 (Arezzo: Accademia Petrarca1987).

Sacrobosco's *algorismus,* Leonardo Pisano's *Liber abaci*,[62] and Jacopo da Firenze's *abbaco* played an influential role in the process of knowledge exchange and production, as some ideas relating to Indo-Arabic figures were copied or reworked in several later *abbaci*.

Conclusion

The analysis developed in this paper invites conclusions to be drawn on several levels. The transmission of the decimal place value system represents one of the most fascinating examples of East/West cross-cultural contact. Although limited in scope, this paper has tried to bring to light some of the ways in which new knowledge from the East was received and adapted in medieval Europe. The comparative analysis of works written different languages has demonstrated the vibrant circulation of knowledge from India to Europe through the Mediterranean regions. These sources often feature cross-references and other data that can be used to shed light on knowledge transfer across cultures. They also testify to a Middle Ages full of people encountering the unfamiliar, grappling with changing of concepts, and redefining attitudes and learning. Many of those particular historical instances that I have chosen to light upon are prime examples of the complexity of events accompanying cultural exchange across the Mediterranean world. I have also tried to emphasise the role played by some cultural brokers who acted as mediators between different contexts. On the whole, the literature surveyed in this study shows that the transmission of Indo-Arabic numerals provides a means to highlight the entanglement of Asian, Mediterranean, and European histories and the various kinds of interactions which have shaped their shared past.

[62] Vernacularizations are listed in the catalogue by Van Egmond (1980). In Italy, several vernacularizations of chapters of the Liber Abaci were produced between the 14th and 15th centuries CE. Feola has provided a study of a work (first half of 13th century CE) written in the Romance dialect of Pisa which is a vernacular abridgment of the *Pratica Geometriae* of Leonardo Pisano; see Francesco Feola, *Gli esordi della geometria in volgare. Un volgarizzamento trecentesco della Practica Geometriae di Leonardo Pisano* (Firenze Accademia della Crusca, 2008). See also Enrico Giusti, *The Twelfth Chapter of Fibonacci's Liber Abaci in Its 1202 Version* (Pisa; Roma: Fabrizio Serra Editore, 2017).