Automata, Logic and Games for the λ-Calculus*

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Automata, logic and games provide the mathematical theory that underpins the model checking of reactive systems:

- *automata* on infinite words and trees as models of computation for state-based systems,
- *logical systems* such as temporal and modal logics for specifying correctness properties, and
- *two-person games* as a mathematical model of the interactions between a system and its environment.

An elegant and fundamental result in the theory of automata, logic and games [3] is the correspondence between alternating parity tree automata (APT), the modal mu-calculus L_μ, and parity games, which are standard formalisms for algorithmic reasoning about trees. On the one hand, the mu-calculus model-checking problem and the PARITY decision problem (Does Verifier have a winning strategy in a given parity game?) are interreducible [17, 19]. On the other, modal mu-calculus and alternating parity tree automata are recursively equivalent for defining tree languages [2, 8, 19].

Research in model checking has traditionally concerned itself with the verification of properties of "ground type objects" such as words or trees. A recent trend in algorithmic verification is *higher-order model checking* [4, 9, 10], which is the model checking of infinite trees generated by the $\lambda\mathbf{Y}$-calculus (simply-typed lambda calculus extended with fixpoint operators) [13, 15] or, equivalently, recursion schemes. Higher-order model checking has been applied with some success to the verification of higher-type functional programs [5, 7, 14, 11]. In this model checking approach, the verification problem is reduced to the model checking of Böhm trees, which are the computation trees of functional programs.

Our work is motivated by the question: *what is the automata-logic-games correspondence for (higher-type) Böhm trees?* Simply-typed Böhm trees are ordered ranked trees extended with binders in the form of lambda-abstractions; which may be viewed as higher-order functions on trees. Indeed, one may well ask if such a correspondence is plausible, since Clairambault and Murawski [1] have considered a monadic second order logic over a class of binding structures and shown the model checking problem to be undecidable. However, we develop just such a correspondence for Böhm trees. In the following we discuss the main ideas.

The starting point is recent work by Tsukada and Ong [18] on the model checking problem for higher-type (possibly infinite) Böhm trees. They introduced

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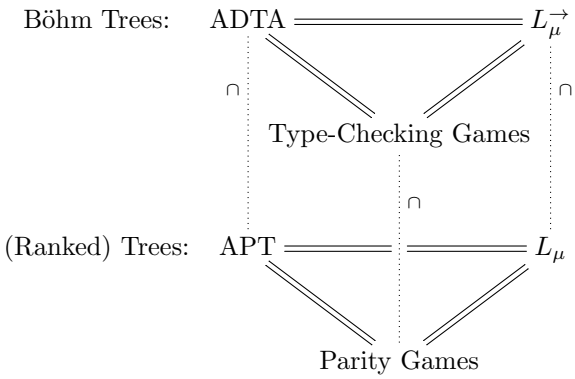
a notion of type [6] for Böhm trees: a Böhm tree u is said to have type σ if Verifier has a winning strategy in the corresponding *type-checking game*. Tsukada and Ong showed that the type checking of $\lambda\mathbf{Y}$-definable Böhm trees is decidable. This type-checking game is the games component of the new trio for higher types.

The automata component of the trio is *alternating dependency tree automata* (ADTA), which was first introduced by Stirling [16] for finite binding trees, in order to characterise solution sets of the Higher-Order Matching Problem. We extend ADTA to infinite binding trees with ω-regular winning conditions. ADTA are closed under union, intersection and complementation. The emptiness problem for nondeterministic dependency tree automata is decidable, but undecidable for alternating dependency tree automata [12]. A key result is that types and ADTA are effectively equivalent[1] for defining languages of Böhm trees. As a corollary, the ADTA acceptance problem for $\lambda\mathbf{Y}$-definable Böhm trees is decidable.

The logic for describing the corresponding correctness properties is *higher-type mu-calculus* $L_\mu^\rightarrow$, which extends ordinary modal mu-calculus with predicates for detecting variables, and corresponding constructs for detecting λ-abstractions. There is a characterisation of the set-theoretic semantics of $L_\mu^\rightarrow$ by a model checking game. Furthermore $L_\mu^\rightarrow$ and ADTA are recursively equivalent for defining languages of Böhm trees.

Thus there is an exact automata-logic-games correspondence for Böhm trees at higher types, which naturally extends the classical correspondence for ordinary trees, as illustrated by the following diagram.



[1] We suppress a delicate distinction between types and a subsystem of *parity permissive types*. There is a corresponding distinction between ADTA and a subclass of *parity permissive ADTA*. The expressive equivalence result holds both generally and when restricted to the parity permissive subsystems.

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