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BIDDING FOR NETWORK SIZE: PLATFORM COMPETITION WHEN QUALITY AND NETWORK SIZE ARE COMPLEMENTS

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Bidding for Network Size: Platform Competition when Quality and Network Size are Complements*

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Abstract

We study two platforms competing for members by investing in network quality. Quality is complementary to the network size: the marginal utility generated by an additional member increases with the network's quality. Platforms are imperfect substitutes: a share of the potential members are biased toward each of the platforms and some are indifferent ex ante. We assume that, in case of multiple equilibria, consumers use the investment in quality as a coordination device. We find that, in equilibrium, platforms randomize over two disconnected intervals of investment levels, corresponding to competing for either the entire population or the mass of ex-ante indifferent members. While the "prize" of winning the competition for members is identical for both platforms, the value of the outside option "not investing" depends on a platform's share of ex-ante biased members. The platform with the smallest share of ex-ante biased members bids more aggressively to compensate for its lower outside option and achieves a monopoly network with higher probability than its competitor.

Keywords: Internet, Platforms, Investment, Network Effects
JEL-Code: D43, D44, M13

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1 Introduction

In 2012, the average Internet user devoted 27% of her time to social media sites. The two most popular components of social media are social networks and photo/video sharing sites.[1] When choosing to visit such an Internet platform, a consumer is concerned with a combination of three elements: (i) her intrinsic preference for each platform (that we treat as an ex-ante bias) (ii) the number of other users, and (iii) the quality of the platform. Platforms compete to attract members because their revenues are an increasing function of their audience.[2] The particularity of such Internet platforms is that two dimensions are complementary: the marginal impact of a platform's investment in quality is increasing in the number of members of the platform.

For instance, a platform like YouTube could be just a large server hosting videos. But it is not: in 2010, 60% of all video clicks from the home page resulted from user-based recommendations (Davidson *et al.*, 2010). Recommendations are based on complex and costly algorithms that extract information from the consumption behavior of all users. What users value is not the investment in developing and implementing an algorithm directly, though. Instead, users value the information generated, which is a function of the quality of the algorithm and of the number of other users providing content and information on viewing habits. The idea of firms' investment directly entering users' utility function is not new (this is, for instance, the case for complementary advertising as in Becker and Murphy, 1993 or Bagwell, 2007). Neither is the idea new that investment can be used to coordinate consumers in the presence of network externalities (Pastine and Pastine, 2002). But the conjunction of both leads to the following result: the platform with the smallest share of ex-ante biased members wins the highest share of members with a higher probability.

We analyze how two platforms compete for members by simultaneously choosing investments that increase positive network effects. Platforms are assumed to be imperfect substitutes for parts of the population whereas others are ex-ante indifferent. We also assume that when there are multiple equilibria, members coordinate toward the platform that invests the most. There is a unique equilibrium in mixed strategies. This is because the coordination assumption makes our game similar to an all-pay auction (Baye *et al.*, 1996): for every given level of investment of the winner, the competitor could win instead

[1]Experian, 2013 marketer survey.

[2]The intuition is that of a two-sided market (e.g., Rochet and Tirole, 2006, and Rysman, 2009), where advertisers are willing to pay more to reach more consumers. We rule out the possibility for the platform to generate revenue by charging users.

by marginally overbidding. In equilibrium, two platforms coexist (attract a strictly positive share of the members) with positive probability. This probability is decreasing in the substitutability between platforms because lower substitutability increases the platforms' market power. If the platforms are sufficiently different, the probability that two platforms coexist is one. If substitutability is high, however, it pays to invest more to win the other platforms' biased members with positive probability. However, the support of the equilibrium mixed strategy exhibits a gap just below the minimum level necessary to attract these biased members. This is because investing above this threshold does not only increase the probability of winning, but also the prize of winning, which is then the whole population instead of only the indifferent mass.

Cost effectiveness and popularity affect the equilibrium in different ways. Having lower costs makes a platform more competitive and weakly increases its profits and chances of winning. Popularity, on the other hand, makes the outside option of not investing at all more valuable to a platform. Indeed, for a given investment of the other platform, a platform's payoff of not investing is proportional to its share of biased members. Hence, the "prize" when winning the whole population is identical for both platforms, while the "prize" when keeping their ex-ante biased members is higher for the more popular platform. Therefore, a more popular platform decides to invest less on average, and wins less often in expectation.[3] The complementarity between quality and the network externality in members' utility is the key for this result. With only network externalities, the "prize" to be won would be the same regardless of the level of investment: either the whole population or the ex-ante indifferent members, depending on the substitutability. With investment in quality only, the winner would not be guaranteed to win the entire population even for high levels of investment: the competitor could keep its ex-ante biased members by underbidding by a sufficiently small amount.

When considering network externalities, the economic literature has studied various roles from producers (Economides, 1996; Shy, 2011) but not the possibility to endogenize the intensity of network effects. In our paper, this possibility comes from the complementarity between quality and the network externality in members' utility, and it is the key for our results. With exogenous network externalities, the "prize" to be won would be the same regardless of the level of investment: either the whole population or the ex-ante indifferent members, depending on the substitutability. With investment in quality only, the winner would not be guaranteed to win the entire population even for high levels of

[3]This does not mean that the more popular platform makes a lower profit than the less popular one.

investment: the competitor could keep its ex-ante biased members by underbidding by a sufficiently small amount.

We proceed as follows: we introduce the model in Section 2 and solve for the equilibrium of the simultaneous investment game in Section 3. We conclude in Section 4. For those results that do not follow directly from the text, formal proofs are collected in the appendix.

2 Model setup and preliminaries

There are two platforms, A and B, which compete for members from a population of mass one. This population consists of three types of individuals, a, b, and m. Types a and b occur with frequency α and β, respectively, in the population and the remaining part are of type m, $\mu = 1 - \alpha - \beta$. The structure of the game and frequencies of types are common knowledge.

All three types of individuals derive utility from the share of individuals choosing the same platform (positive network effect). The utility from the network effect does not depend on the identity or type of individuals joining the platform. The strength of this network effect depends on an investment K_i made by the platform. Types a and b favor platform A or B, respectively, in the sense that they receive extra utility when joining the platform. We call them 'ex-ante fans' of A and B or 'biased members'. Type m is ex ante indifferent between the two platforms and is called 'mass member'. For simplicity, we assume that the reservation utility of individuals is equal to 0, so that everyone joins a platform in equilibrium. We assume throughout that no platform has a majority to start with, $\alpha, \beta \leq \frac{1}{2}$, and that mass members exist, $\mu > 0$.

Each platform $i \in \{A, B\}$ has the goal to maximize its network size n_i, corresponding to the share of its members. To attract members, each platform chooses an investment $K_i \geq 0$ which influences how much utility its members derive from its network size. The unit cost of investing is c for both platforms.[4] Denote by γ the strength of ex-ante bias or ex-ante attachment.[5] The payoff of platform i is:

$$U_i(n_i, K_i) = n_i - c \cdot K_i, \quad \text{for } i \in \{A, B\} \tag{1}$$

[4] Allowing for asymmetric costs complicates the analysis but leaves our main finding intact. Having a cost advantage makes a platform more likely to monopolize the market but unless the cost difference is very large, both platforms invest positive amounts in equilibrium and there are still cases where the less favored platform wins more often.

[5] The inverse of the bias can be understood as the substitutability between the two alternatives for biased members.

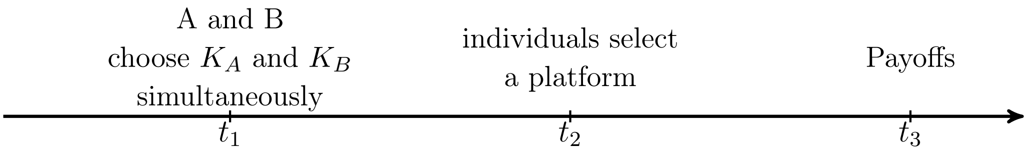

Figure 1: Timing of the game.

Utilities of the different types in the population are:

	join A	join B	abstain
type a	$U_a(A) = \gamma + K_A n_A$	$U_a(B) = K_B n_B$	0
type b	$U_b(A) = K_A n_A$	$U_b(B) = \gamma + K_B n_B$	0
type m	$U_m(A) = K_A n_A$	$U_m(B) = K_B n_B$	0

Figure 1 illustrates the timing of the game. Platforms A and B choose their investments simultaneously. After both platforms have chosen their investments, individuals decide which platform to join ('member subgame').

Each member type joins the platform which maximizes her utility. We assume that if indifferent between abstaining or joining a platform, members join. Since utility depends on the network size of the platform, i.e., depends on which platform is joined by the other members, members face a coordination game. Therefore, we cannot expect to find a unique equilibrium without an assumption of coordination. Following the literature on advertising as a coordination device (Pastine and Pastine, 2002), we make the following assumption:

Assumption 1. ***(Equilibrium Selection)*** *If for given investments there exist multiple equilibria in the member subgame, we select the equilibrium in which each type of member joins platform i if platform i's investment exceeds that of platform j, i.e., $K_i > K_j$. If both platforms choose the same investment, $K_i = K_j$, coordination occurs on either platform with equal probability.*[6]

For ease of the following exposition, we name the network outcomes associated with pure-strategy equilibria in the member subgame as follows:

Definition 1. *If the entire population joins the same platform, a* ***single network*** *obtains with corresponding network sizes $n_A = 1$, $n_B = 0$ or $n_A = 0$, $n_B = 1$. If instead biased members join their ex-ante preferred platform, we obtain* ***competing networks*** *where network sizes are $n_A = \alpha$, $n_B = \beta + \mu$ or $n_A = \alpha + \mu$, $n_B = \beta$.*

[6] This mirrors the commonly used assumption that ties at the same bid in the all-pay auction are broken randomly.

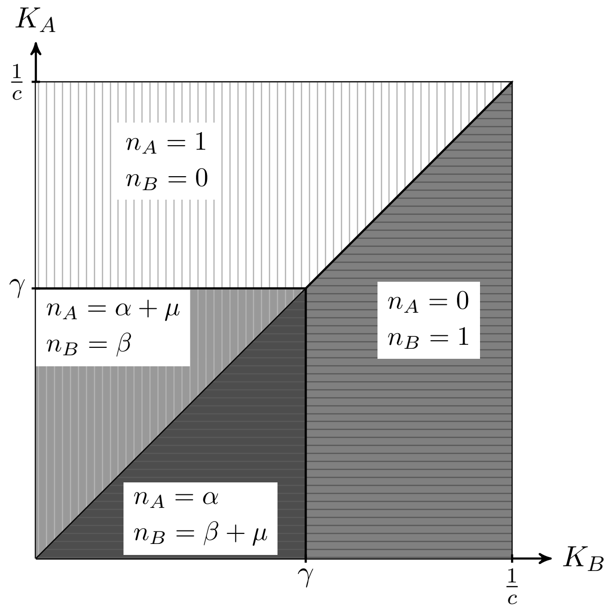

Figure 2: Network sizes for given investments under Assumption 1

3 Equilibrium analysis

We start by solving the last stage of the game, the **member subgame**. Lemma 1 proves that Assumption 1 also yields a generically unique equilibrium prediction. The equilibrium outcomes are depicted in figure 2.

Lemma 1. *Under Assumption 1, the member subgame has a unique equilibrium except for cases where platform investments tie. The corresponding network sizes are:*

(i) $n_i = 1$ *and* $n_j = 0$ *if* $K_i > K_j$ *and* $K_i \geq \gamma$.

(ii) $n_A = \alpha + \mu$ *and* $n_B = \beta$ *if* $K_B < K_A < \gamma$.

(iii) $n_A = \alpha$ *and* $n_B = \beta + \mu$ *if* $K_A < K_B < \gamma$.

If $K_i = K_j$ *two equilibria exist in which members coordinate and join one or the other platform. Under Assumption 1, both are played with equal probability.*

For each combination of investments by platforms A and B, Figure 2 states the network sizes in the equilibrium resulting in the member subgame under Assumption 1. If both platforms choose investments within the square in the lower left of the figure, $K_A < \gamma$, $K_B < \gamma$, the investments are not high enough for biased members to join the network against which they are biased even under perfect coordination. Thus, the resulting equilibrium features competing networks. The platform which invests more obtains the larger network independent of the relative shares of biased members. If at least one of the two platforms invests γ or more, the equilibrium partition is a single network. An investment at γ or above by platform B is sufficient to compensate an individual biased toward platform A for joining platform B if all others join platform B too (and vice versa). By Assumption 1, members coordinate on the platform with the higher investment so that even individuals biased toward the other platform do not have an interest in deviating unilaterally. The competitor with the lower investment does not attract any member in this case.

We now derive the **equilibrium investments** for platforms A and B. The game faced by the two platforms resembles an all-pay auction where the bids are the investment levels and the prize of winning is the share of members joining the platform. If the investment (i.e., the bid) exceeds the threshold γ, the network size of the winning platform and thereby the valuation of winning increases discontinuously because at this point the investment level is just high enough to attract members biased toward the competitor in addition to mass members.

Obviously, it is never a best response for either platform to invest more than $\frac{1}{c}$, the utility from attracting all members normalized by the cost. If one platform invested above $\frac{1}{c}$, the other platform would best respond by investing zero. For investments up to $\frac{1}{c}$, overbidding is in general profitable. Thus, if $\gamma \geq \frac{1}{c}$, the best response functions for both platforms are equal to the identity for investments up to $\frac{1}{c}$ and zero thereafter. Suppose instead $\gamma < \frac{1}{c}$. If one platform invests at γ or above, the other platform best responds by slightly overbidding so as to attract the entire population. If one platform invests below γ, the other platform again prefers to slightly overbid the given investment to any investment below or equal to it. In this case, both platforms attract members biased toward them, respectively, and mass members join the network offering the larger investment. However, for investments closely below the threshold γ, a platform might do even better by investing a discretely higher amount and capturing the entire population. Specifically, platform A is better off attracting everyone than slightly overbidding platform B's investment if $K_B < \gamma$ and

$$1 - c\gamma > \alpha + \mu - cK_B \Leftrightarrow K_B > \gamma - \frac{\beta}{c}. \tag{2}$$

An analogous inequality holds for platform B. It implies that the best responses are flat at γ for investments within a certain interval just below γ. As illustrated in Figure 3,

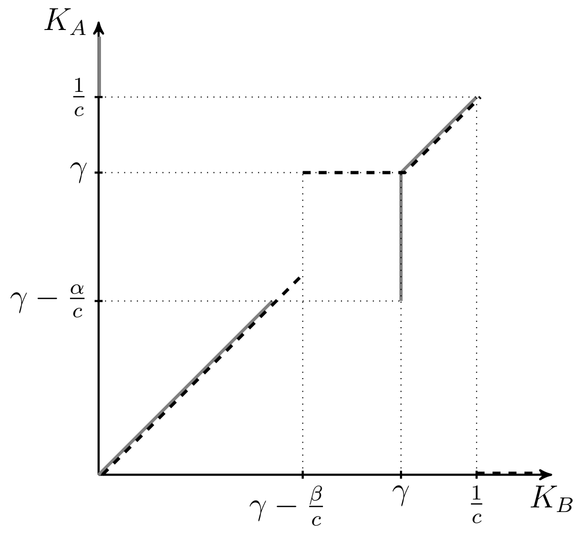

Figure 3: Best responses under Assumption 1 with $\alpha > \beta$ and $\gamma < \frac{1}{c}$. Black dashed: platform A, gray solid: platform B.

the best responses of platforms A and B do not intersect and the game does not have a pure-strategy equilibrium.

Lemma 2. *There is no equilibrium in pure strategies.*

Before we continue, note that a platform has a positive reservation utility, i.e., utility from not investing at all, if its competitor invests below γ with a positive probability. The reservation utility is equal to utility from the share of biased members multiplied by the probability that the competitor invests below γ, i.e., it is $\text{Prob}(K_B < \gamma)\alpha$ for platform A and $\text{Prob}(K_A < \gamma)\beta$ for platform B. This implies that platforms do not invest up to the level at which they just break even. Instead they invest up to the point where the competition gives them in expectation the same utility as they derive from their share of biased members when taking into account the probability of keeping those without investing anything.

The mechanics of the game imply that the outside option is higher for the platform that has more ex-ante biased members for any given investment distribution of the opponent. For the following we assume without loss of generality that platform A has a higher share of biased members than platform B, $\alpha > \beta$, and find that this makes platform B bid more aggressively under certain circumstances.

Proposition 1. *Suppose $\alpha > \beta$.*

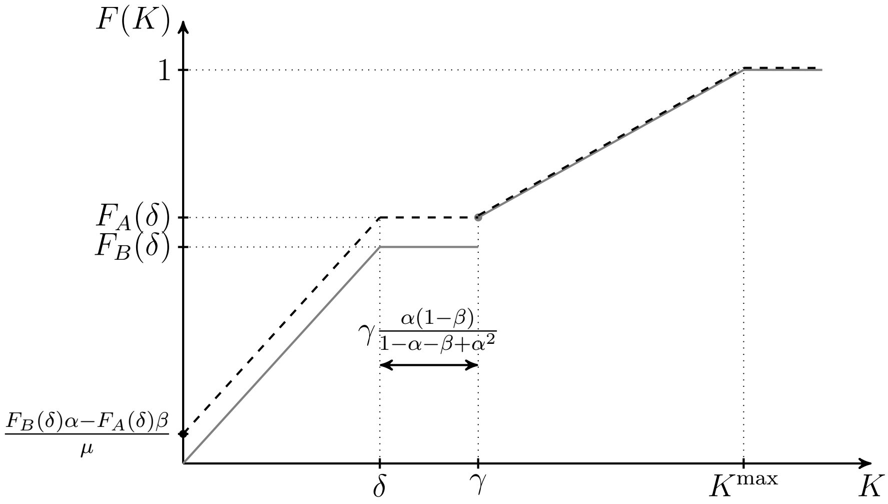

Figure 4: Cumulative distribution functions if $\gamma < \frac{1}{c}\frac{1-\alpha-\beta+\alpha^2}{1-\beta}$. $K^{\max} = \frac{1}{c} - \gamma\frac{1-\alpha}{1-\alpha-\beta+\alpha^2}$. Dashed: platform A, Gray solid: platform B.

(i) *If $\gamma < \frac{1}{c}\frac{1-\alpha-\beta+\alpha^2}{1-\beta}$ there exists $\delta \in (0, \gamma)$ such that in equilibrium, both platforms randomize uniformly over $(0, \delta]$ and $(\gamma, \frac{1}{c} - \gamma\frac{\alpha}{1-\beta})$ where $\delta = \frac{(1-\alpha)(1-\alpha-\beta)}{1-\alpha-\beta+\alpha^2}$. The density is $f(K) = \frac{c}{\mu}$ for $0 \leq K \leq \delta$ and $f(K) = c$ for $\gamma \leq K \leq \frac{1}{c} - \gamma\frac{\alpha}{1-\beta}$. Platform B invests γ with probability $\frac{c(\alpha-\beta)\gamma}{1-\alpha-\beta+\alpha^2}$, platform A invests 0 with probability $\frac{c(\alpha-\beta)\gamma}{1-\alpha-\beta+\alpha^2}$. Both platforms make an expected profit of $F_B(\delta)\alpha$. Platform B invests more in expectation and wins with higher probability.*

(ii) *If $\frac{1}{c}\frac{1-\alpha-\beta+\alpha^2}{1-\beta} < \gamma < \frac{1-\beta}{c}$, both platforms randomize uniformly over $(0, \delta)$, where $\delta = \frac{\mu}{c} - \frac{1-\beta}{c} + \gamma$. The density is $f(K) = \frac{c}{\mu}$. Platform B invests γ with probability $\frac{1-\beta}{\mu} - \frac{c}{\mu}\gamma$ and platform A invests zero with probability $\frac{1-\beta-c\gamma}{\mu}$. The expected profit of platform B is $1 - c\gamma$ and the expected profit of platform A is $\alpha > 1 - c\gamma$. Platform B invests more in expectation and wins with a higher probability.*

(iii) *If $\gamma > \frac{1-\beta}{c}$ both platforms randomize continuously over the interval $[0, \frac{\mu}{c}]$. The density is $f(K) = \frac{c}{\mu}$. Platform A (B) makes an expected profit of $\frac{\alpha}{c}$ ($\frac{\beta}{c}$). Expected investments are $\frac{1}{2}\frac{\mu}{c}$ for each platform, i.e., in total $\frac{\mu}{c}$. Each platform wins with probability $\frac{1}{2}$.*

The range of investments for which the density is zero is $[\delta, \gamma]$. Substitutability increases competition: the lower substitutability is as implied by a greater bias γ, the less probability mass both platforms assign to investments above γ. As long as $\gamma < \frac{1-\beta}{c}$, the highest investment below γ which is still included in the mixed strategy, δ, is lower than the maximum investment which would be chosen if the two platforms agreed to compete only for mass members. Hence, there is always a gap in the support. The probability mass on higher investment levels overcompensates this so that in total investments are higher when platforms compete for the entire population. As γ approaches the threshold at which platforms decide to only compete for mass members, the expected investments in both types of equilibrium are the same and the expected investment is continuous in γ.

For the highest levels of substitutability, when the bias γ is small and both platforms compete for members biased toward their competitors, a new twist comes in. If platforms were to compete for members biased toward their competitors with certainty, both would be willing to invest up to $K = \frac{1}{c}$ to win the competition. However, a platform might choose an investment which is not high enough to divert biased members from the competitor who would therefore attain a network with a positive size even for an investment of zero. Thus, both platforms have to take into account the probability that the competitor chooses investments from this low range when deciding on their own investment strategies. The expected utility of a platform therefore depends on its share of biased members and on the probability that the competitor invests below γ. It turns out that expected utilities are the same for both platforms because they behave identically for very high investment levels. At low investment levels, platform B compensates for its lower share of biased members by investing more aggressively to attract members biased toward its competitor. Consequently, platform B has in expectation higher investments than platform A. Therefore, platform B attracts more members than platform A in expectation even though A's share of biased members is greater.

Corollary 1. *In expectation, the platform with the smallest share of ex-ante biased members B establishes a single network more often than platform A.*

The higher winning probability of platform B comes about by B choosing on average higher investments than platform A. The weakest platform B knows that platform A has fewer incentives to bid aggressively, and uses this information.[7] In the last two cases (ii) and (iii) identified in proposition 1, platform A still benefits from its advantage in biased members and makes a higher expected profit, even though it does not win more often. Mechanically, as $\alpha > \beta$, the aggregate surplus of members would be higher if platform A were to use the bidding strategy of platform B and vice versa, at the same aggregate expected cost of investment for the platforms.

[7] We should point out that this result depends on the assumption that the parameters of the game are common knowledge. If we included imperfect information on member types the game would be different.

4 Conclusion

It is well known that in the presence of network externalities the "best" platform is not necessarily the one that becomes the equilibrium standard. Instead coordination failures occur. In this paper, we show that even when we allow consumers to use firms' investments as a coordination device, which platform dominates in equilibrium remains to a certain extent random. Moreover, we find that if there are differences in the probabilities of one or the other platform dominating, the one that has the lowest support ex-ante has the higher chance of being successful. This platform invests more aggressively because its outside option of remaining a niche for biased members only is less attractive than it is for the competitor.

In our model, we assume that platform investment and network externalities enter members' utility complementary and this is crucial for our results. We believe this assumption (and therefore the results) to be particularly relevant in the case of Internet platforms offering "social" content. A necessary condition for consumers to enjoy network externalities is the existence of a good quality platform. And the only way to enjoy the quality of a platform is that it is widely used by other members. Further applications could also extend to other types of industries with a similar link between platform quality and network externalities, such as complementary advertising for products for which network externalities matter. This is, for instance, the case of game consoles or technological accessories that *stage* the release of new products, so that the information released in the media is the number of consumers willing to buy the product on the first day. Such advertising is costly and only works if consumers are actually queuing to buy the new product. Further work on this issue could address alternative coordination assumptions for members, an explicitly dynamic environment or a micro-foundation of the reduced-form objective function of platforms.

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Appendix

A Proofs

A.1 Proof of Lemma 1

Definition 2. *Define the level of investment which must be exceeded to attract members which are favorably biased ex ante given the other platform's investment level as* $\underline{K}_A(K_B)$ *and* $\underline{K}_B(K_A)$. *Define further the level of investment which must be exceeded to attract members biased toward the competitor given the competitor's investment as* $\underline{\underline{K}}_A(K_B)$ *and* $\underline{\underline{K}}_B(K_A)$. *Finally, define the level of investment which must be exceeded to attract mass members under the assumption that biased members do not move as* $\underline{\mu}_A(K_B)$ *and* $\underline{\mu}_B(K_A)$.

Proof. Assuming coordination on the higher investment on part of the members, we specify $\underline{K}_i(K_j) = \max\{0, K_j - \gamma\}$ and $\underline{\underline{K}}_i(K_j) = \max\{K_j, \gamma\}$ and $\underline{\mu}_i(K_j) = K_j$. Suppose first $K_i > \max\{K_j, \gamma\}$. Since $K_i > K_j$, mass members do not want to deviate even if all members were collectively deviating to joining platform j. Since $K_i > \gamma$, j-fans are better off joining platform i if all others do so. Trivially, i-fans are better off staying with platform i too. Thus, this is an equilibrium. Under Assumption 1, members expect that others join the platform that chooses a higher investment. Thus, members cannot split differently than what was proposed. Note that j-fans might be better off joining platform j if they did so collectively but such an equilibrium would be inconsistent with Assumption 1. Suppose now that $K_j < K_i < \gamma$. Since $K_i < \gamma$, j-fans are better off joining platform j even if all others join platform i. However, for $K_j < K_i$ and using Assumption 1, mass members expect that all others join platform i and therefore compare $K_A(\alpha + \mu)$ with $K_B\beta$ if $i = A, j = B$ or $K_B(\beta + \mu)$ with $K_A\alpha$ if $i = B, j = A$. Joining platform A (or B) is consistent with equilibrium behavior as long as $K_A > K_B\frac{\beta}{\alpha+\mu}$ ($K_B > K_A\frac{\alpha}{\beta+\mu}$, respectively), which always holds because we have assumed $\alpha, \beta \leq \frac{1}{2}$. Note again, that there are other equilibrium candidates where groups of members might be better off but none of those is consistent with Assumption 1. $\square$

A.2 Proof of Lemma 2

Proof. For $\mu = 1 - \alpha - \beta > 0$, both platforms profit from entering the competition for mass members. Suppose first $\gamma > \max\{\frac{1}{c_A}, \frac{1}{c_B}\}$. Suppose platform A chooses any positive investment $K < \frac{\mu+\beta}{c_B}$ with certainty $P(K_A = K) = 1$. Then, platform B can invest at any $K_B = K + \varepsilon$ and win with certainty. The payoff from doing so is strictly positive for ε small enough. But in this case, platform A would be better off by investing zero. Obviously choosing any investment above $\frac{\mu+\beta}{c_B}$ does not pay off for platform A either since it could marginally reduce its investment without reducing the probability of winning. Suppose now that A invests $\frac{\mu+\beta}{c_B}$ with certainty. Then, a best response for platform B would be to invest zero with certainty so that this is not an equilibrium either. Finally, also investing zero with certainty cannot be part of the equilibrium. Suppose it were and platform A played zero with probability 1. This would give her zero profit. Then, platform B would profit from investing $\varepsilon > 0$: It could make a profit arbitrarily close to $\frac{\mu+\beta}{c_B} = \frac{1-\alpha}{c_B}$. However, if platform B invests $\varepsilon < \frac{\mu+\beta}{c_B}$, platform A would be better off investing $\delta > \underline{K}_A(\varepsilon)$ since for $\underline{K}_A(\frac{\mu+\beta}{c_B}) \leq \frac{\mu+\alpha}{c_A}$ it would thereby make a positive profit. Suppose now that $\gamma \leq \min\{\frac{1}{c_A}, \frac{1}{c_B}\}$, so that both platforms would compete for members biased toward their competitor. Without loss of generality let A be at least as strong as platform B, $\underline{\underline{K}}_A(\frac{1}{c_B}) < \frac{1}{c_A}$. Now, we can repeat the same arguments as in the preceding paragraph. □

A.3 Proof of Proposition 1

Proof. Suppose that $\alpha > \beta$. The resulting equilibrium is asymmetric due to the asymmetry in shares of biased members. We distinguish three cases according to the three parts of the proposition. For expositional reason we consider first part (i), then part (iii), and then finally part (ii). Note that:

$$\frac{1-\beta}{c} > \frac{1}{c}\frac{1-\alpha-\beta+\alpha^2}{1-\beta} \Leftrightarrow \alpha - \beta > \alpha^2 - \beta^2. \tag{3}$$

The latter expression is true as can be seen by rewriting the right-hand side as the third binomial formula and noting that $\alpha + \beta < 1$.

Part (i) Consider first the case $\gamma < \frac{1}{c}\frac{1-\alpha-\beta+\alpha^2}{1-\beta} < \frac{1}{c}$. Both platforms are in principle willing to choose investments high enough to attract members biased toward the opposing platform. Both platforms must be indifferent between all investments which are contained in the support of their equilibrium mixed strategy. We posit and verify an equilibrium in which both platforms randomize over investments in a range not high enough to attract members biased toward the opposing platform and a range where all biased members join the platform with higher investment. Moreover, in this equilibrium, platform A invests zero with positive probability.

For every investment of platform B below γ which is contained in the support of the equilibrium strategy, the following condition has to hold:

$$F_B(K)\mu + \lim_{\varepsilon\to 0} F_B(\gamma-\varepsilon)\alpha - cK = \lim_{\varepsilon\to 0} F_B(\gamma-\varepsilon)\alpha \quad \Rightarrow \quad F_B(K) = \frac{c}{\mu}K \tag{4}$$

and for every investment equal to or above γ

$$F_B(K) - cK = \lim_{\varepsilon\to 0} F_B(\gamma-\varepsilon)\alpha \quad \Rightarrow \quad F_B(K) = cK + \lim_{\varepsilon\to 0} F_B(\gamma-\varepsilon)\alpha \tag{5}$$

If platform A chooses zero with positive probability, platform B's mixed strategy must not contain zero. However, platform B must also be indifferent between all investment levels in the support of its equilibrium mixed strategy. Denote B's expected profit by $E[\Pi_B]$. Then, for all $K < \gamma$

$$\begin{aligned} & F_A(K)\mu + \lim_{\varepsilon\to 0} F_A(\gamma-\varepsilon)\beta - cK = E[\Pi_B] \\ \Rightarrow \quad & F_A(K) = \frac{c}{\mu}K + \frac{E[\Pi_B] - \lim_{\varepsilon\to 0} F_A(\gamma-\varepsilon)\beta}{\mu} \end{aligned} \tag{6}$$

For every investment at γ or above having a lower investment than the competitor implies also losing their share of favorably biased members.

$$F_A(K) - cK = E[\Pi_B] \quad \Rightarrow \quad F_A(K) = cK + E[\Pi_B] \tag{7}$$

From Lines (4) to (7) follows that the slope of both distribution functions is identical for low and high investment ranges. Since the slope is higher for investments below γ than for investments above γ, there exists $\delta \in (0, \gamma)$ such that for both platforms

$$F_A(K) = F_A(\delta) \text{ and } F_B(K) = F_B(\delta) \text{ for all } K \in [\delta, \gamma) \tag{8}$$

and therefore $\lim_{\varepsilon\to 0} F_A(\gamma-\varepsilon) = F_A(\delta)$ and $\lim_{\varepsilon\to 0} F_B(\gamma-\varepsilon) = F_B(\delta)$.

Neither platform has an incentive to strictly exceed the maximum investment of the other platform. This would increase cost but not increase the probability of winning. Thus, the maximum investment chosen by each platform must be identical in equilibrium, i.e., there exists a unique $\overline{K}$ such that $F_A(\overline{K}) = F_B(\overline{K}) = 1$ and for all $\varepsilon > 0$, $F_A(\overline{K}-\varepsilon) < 1$

and $F_B(\overline{K} - \varepsilon) < 1$. Since the distribution functions of platforms A and B also have identical slopes for $K \geq \gamma$, the distribution functions of both platforms are identical for $K \geq \gamma$:

$$F_A(K) = F_B(K) \text{ for all } K \geq \gamma \tag{9}$$

Combining Equations (5), (7), and (9) yields $E[\Pi_B] = F_B(\delta)\alpha$. Starting with Line (6) and plugging in yields for $K < \gamma$

$$F_A(K) = \frac{c}{\mu}K + \frac{F_B(\delta)\alpha}{\mu} - \frac{F_A(\delta)\beta}{\mu} \tag{10}$$

We solve (10) for $F_B(\delta)$ and obtain

$$F_B(\delta) = F_A(\delta)\frac{\mu + \beta}{\alpha} - \frac{c}{\alpha}\delta$$

We plug in from Line (4) and solve for $F_A(\delta)$ to obtain

$$F_A(\delta) = c\delta \left(\frac{\alpha}{\mu(\mu + \beta)} + \frac{1}{\mu + \beta} \right) \tag{11}$$

The flat part in the distribution functions (Equation (8)) implies together with the different shares of biased members that platform B chooses an investment equal to γ with a positive probability while platform A's strategy has an atom at zero. Since the two platforms cannot have an atom at the same investment level, and since neither platform has an incentive to choose δ with positive probability, the distribution function of platform A must be continuous in δ and γ. In addition, at γ the distribution functions of both platforms take identical values. Thus, the following holds

$$F_A(\delta) = F_A(\gamma) = F_B(\gamma) \tag{12}$$

Since $F_B(K)$ is linear for $K \leq \delta$, we can rewrite (5) as

$$F_B(\gamma) = c\gamma + \frac{c}{\mu}\delta\alpha \tag{13}$$

Taking Line (12) and plugging in from Line (11) on the left-hand side and from Line (13) on the right-hand side, we arrive at

$$
\begin{aligned}
& c\delta\left(\frac{\alpha}{\mu(\mu+\beta)}+\frac{1}{\mu+\beta}\right)=c\gamma+\frac{c}{\mu}\delta\alpha \\
\text{(14)} \qquad \Leftrightarrow \qquad & \delta=\gamma\frac{\mu(\mu+\beta)}{\mu+\alpha-\alpha(\mu+\beta)}=\gamma\frac{(1-\alpha)(1-\alpha-\beta)}{1-\alpha-\beta+\alpha^2}
\end{aligned}
$$

It is easily verified that

$$(\mu+\alpha)(\mu+\beta)<\mu+\alpha \Rightarrow \mu(\mu+\beta)<\mu+\alpha-\alpha\mu-\alpha\beta \Rightarrow \delta<\gamma$$

Finally, we have to derive the maximum investment levels. Suppose $K>\gamma$. Since the distribution functions stay constant at one for all investment levels above the maximum level chosen, we obtain the following condition

$$\text{(15)} \qquad c\overline{K}+F_B(\delta)\alpha=1 \Leftrightarrow c\overline{K}=1-\frac{c}{\mu}\delta=1-c\gamma\frac{(1-\alpha)(1-\alpha-\beta)}{(1-\alpha-\beta)(1-\alpha-\beta+\alpha^2)}$$

where δ has been derived in Equation (14). Rewriting (15) yields the maximum investment level

$$\overline{K}=\frac{1}{c}-\gamma\frac{1-\alpha}{1-\alpha-\beta+\alpha^2}$$

For the derivation of the maximum investment, we have assumed $\overline{K}>\gamma$. This is indeed verified if

$$\text{(16)} \qquad \frac{1}{c}-\gamma\frac{1-\alpha}{1-\alpha-\beta+\alpha^2}>\gamma \Leftrightarrow \gamma<\frac{1}{c}\frac{1-\alpha-\beta+\alpha^2}{1-\beta}$$

Combining the above, we derive the following equilibrium distribution functions:

$$F_A(K) = \begin{cases} \frac{c}{\mu}K + \frac{c(\alpha-\beta)(1-\alpha-\beta)\gamma}{(1-\alpha-\beta+\alpha^2)\mu} & \text{if } K \in [0,\delta] \\ \frac{c(1-\beta)\gamma}{1-\alpha-\beta+\alpha^2} & \text{if } \delta < K < \gamma \\ cK + \frac{c(1-\alpha)\alpha\gamma}{1-\alpha-\beta+\alpha^2} & \text{if } \gamma \leq K \leq \overline{K} \\ 1 & \text{if } K \geq \overline{K} \end{cases}$$

$$F_B(K) = \begin{cases} K\frac{c}{\mu} & \text{if } K \in (0,\delta] \\ \frac{c(1-\alpha)\gamma}{1-\alpha-\beta+\alpha^2} & \text{if } \delta \leq K \leq \gamma \\ cK + \frac{c(1-\alpha)\alpha\gamma}{1-\alpha-\beta+\alpha^2} & \text{if } \gamma < K \leq \overline{K} \\ 1 & \text{if } K \geq \overline{K} \end{cases}$$

The probability that platform B invests γ is $\text{Prob}(K_B = \gamma) = F_B(\gamma) - F_B(\delta) = \frac{c(\alpha-\beta)\gamma}{1-\alpha-\beta+\alpha^2}$ and the probability that platform A invests zero is identical, i.e. $\text{Prob}(K_A = 0) = F_A(0) = \frac{c(\alpha-\beta)\gamma}{1-\alpha-\beta+\alpha^2}$.

Expected investments in equilibrium are computed from these distributions as

$$\begin{aligned} E[K_A] &= \int_0^\delta \frac{c}{(1-\alpha-\beta)} x dx + \int_\gamma^{\overline{K}} cx dx \\ &= \tfrac{c(1-\alpha)^2(1-\alpha-\beta)\gamma^2}{2(1-\alpha-\beta+\alpha^2)^2} - \tfrac{1}{2}c\left(\gamma^2 - \tfrac{(1-\beta-(1-\alpha)\alpha(1+c\gamma))^2}{c^2(1-\alpha-\beta+\alpha^2)^2}\right) \\ E[K_B] &= \int_0^\delta x\frac{c}{(1-\alpha-\beta)} dx + \int_\gamma^{\overline{K}} cx dx + \text{Prob}(K_B = \gamma)\gamma \\ &= \tfrac{c(\alpha-\beta)\gamma^2}{1-\alpha-\beta+\alpha^2} + \tfrac{c(1-\alpha)^2(1-\alpha-\beta)\gamma^2}{2(1-\alpha-\beta+\alpha^2)^2} - \tfrac{1}{2}c\left(\gamma^2 - \tfrac{(1-\beta-(1-\alpha)\alpha(1+c\gamma))^2}{c^2(1-\alpha-\beta+\alpha^2)^2}\right) \end{aligned}$$

It is easily verified that $E[K_A] < E[K_B]$.

Part (iii) Suppose now that $\gamma > \frac{1-\beta}{c}$. The distribution functions derived above do not constitute an equilibrium anymore since by Lines (3) and (16), the maximum investment would lie below γ which is a contradiction. Instead, we proof in the following, that in equilibrium neither platform invests at γ or above so that $\lim_{\varepsilon\to 0} F(\gamma-\varepsilon) = 1$ and both platforms keep members biased toward them for sure.[8] Neither platform invests enough to attract members biased toward the opposing platform. The outside option for both

[8]For $\gamma > \frac{1}{c}$ competition for the entire population is not profitable even if the success probability was one. For $(1-I)\frac{1}{c} < \gamma < \frac{1}{c}$ competing for everyone is profitable if the success probability is high enough. However, in equilibrium, this is not the case. Thus, we analyze the two cases jointly.

platforms is to keep only their ideological members and get a payoff equal to its share of biased members α respectively β. The valuation of winning is then the value of getting the mass members in addition, i.e., μ, so that in equilibrium, both players randomize continuously on $[0, \frac{\mu}{c}]$ according to the following cumulative distribution function

$$F(K) = \begin{cases} \frac{c}{\mu}K & \text{for all } K \in [0, \frac{\mu}{c}] \\ 1 & \text{for } K \geq \frac{\mu}{c} \end{cases}$$

It is straightforward from part (i) that each platform is indifferent between all investments in $[0, \frac{\mu}{c}]$. None of the two platforms invests zero with positive probability by the same argument as in part (i). The expected payoff to platform A and B is equal to α and β, respectively. By deviating to an investment at γ, sufficient to capture the entire population, a platform would make an expected profit of $F(\frac{\mu}{c}) - c\gamma = 1 - c\gamma < 1 - (1 - \beta) = \beta < \alpha$ such that this deviation is not profitable.

The expected investment in equilibrium equals

$$E[K_i] = \int_0^{\frac{\mu}{c}} \frac{c}{\mu} x dx = \frac{1}{2}\frac{\mu}{c} = \frac{1}{2} 1 - 2Ic \text{ for } i = A, B$$

per platform. In total, the two platforms invest $\frac{\mu}{c}$.

Since equilibrium mixed strategies and investments are identical, both platforms have the same probability of winning which equals $\frac{1}{2}$.

Part (ii) Suppose finally that $\frac{1}{c}\frac{1-\beta}{1+\alpha-\beta} < \gamma < \frac{1-\beta}{c}$. In this case, the distribution functions from part (iii) do not constitute an equilibrium anymore since platform B would profit from deviating. As shown in part (i), the distributions where both randomize over two disconnected intervals are not an equilibrium either since the maximum investment would lie below γ.

In the following, we show that in this parameter range, we find $\delta \in (0, \gamma)$ such that there exists an equilibrium where both platforms randomize over $(0, \delta)$, platform A invests zero with positive probability and platform B chooses γ with positive probability. In this equilibrium platform A chooses investments below or equal to δ with certainty, i.e., $F_A(\delta)$ whereas platform B also invests at γ such that $F_B(\delta) < 1$.

Since platform B could ensure profit $1-c\gamma$ by deviating to investing γ, the distribution function of platform A must fulfill for all $K \leq \delta$

$$F_A(K)\mu + \beta - cK = 1 - c\gamma \Rightarrow F_A(K) = \frac{c}{\mu}K + \frac{1-\beta-c\gamma}{\mu} \tag{17}$$

By assumption $\gamma < \frac{1-\beta}{c}$ and thus $\frac{1-\beta-c\gamma}{\mu} > 0$. Note that investing γ also yields an expected profit equal to $1-c\gamma$ for platform B.

Platform A obtains an expected profit equal to its share of biased members multiplied by the probability that B invests less than γ, $F_B(\delta)\alpha$. For the distribution function of platform B and investments $K \leq \delta$ the following must hold:

$$F_B(K)\mu + \alpha - cK = \alpha \Leftrightarrow F_B(K) = \frac{c}{\mu}K$$

The investment level δ is such that the distribution function of platform A just reaches 1 at this level

$$\frac{c}{\mu}\delta + \frac{1-\beta-c\gamma}{\mu} = 1 \Leftrightarrow \delta = \gamma - \frac{\alpha}{c} \tag{18}$$

If $\gamma < \frac{1-\beta}{c}$, then $\delta < \frac{\mu}{c}$.

Finally, we derive the probability with which platform B invests γ.

$$\text{Prob}(K_B = \gamma) = 1 - \frac{c}{\mu}\delta = 1 - 1 + \frac{1-\beta}{\mu} - \frac{c}{\mu}\gamma = \frac{1-\beta}{\mu} - \frac{c}{\mu}\gamma$$

From Line (17) also

$$\text{Prob}(K_A = \gamma) = \frac{1-\beta}{\mu} - \frac{c}{\mu}\gamma = \text{Prob}(K_B = \gamma)$$

By $\gamma < \frac{1-\beta}{c}$, it holds that $\text{Prob}(K_B = \gamma) > 0$. Moreover,

$$\mu > 0 \Rightarrow \mu + \beta > \beta \Rightarrow (1-\alpha)^2 > \beta(1-\alpha) \Rightarrow 1-\alpha-\beta+\alpha^2 > \alpha - \alpha\beta \Rightarrow \frac{1-\alpha-\beta+\alpha^2}{1-\beta} > \alpha$$

and therefore

$$\gamma > \frac{1}{c}\frac{1-\alpha-\beta+\alpha^2}{1-\beta} \Rightarrow \gamma > \frac{\alpha}{c}$$

so that $\text{Prob}(K_B = \delta) < 1$.

By $\gamma > \frac{1}{c}\frac{1-\beta}{1+\alpha-\beta}$ platform A does indeed not want to deviate to investing γ:

$\gamma > \frac{1}{c}\frac{1-\beta}{1+\alpha-\beta} \Rightarrow c\gamma(\mu+\alpha) > \mu+\alpha^2 \Leftrightarrow -\frac{\alpha^2}{\mu} + \frac{c}{\mu}\gamma\alpha > 1-c\gamma \Leftrightarrow F_B(\delta)\alpha > 1-c\gamma$

The distribution functions have been derived as

$$
\begin{aligned}
F_A(K) &= \begin{cases} \frac{c}{1-\alpha-\beta}K + \frac{1-\beta-c\gamma}{1-\alpha-\beta} & \text{if } K \in [0,\delta] \\ 1 & \text{if } K \geq \delta \end{cases} \\
F_B(K) &= \begin{cases} \frac{c}{1-\alpha-\beta}K & \text{if } K \in (0,\delta] \\ \frac{c}{1-\alpha-\beta}K & \text{if } \delta \leq K \leq \gamma \\ 1 & \text{if } K \geq \gamma \end{cases}
\end{aligned}
$$

where δ is given in Line (18) as $\delta = \gamma - \frac{\alpha}{c}$.

The resulting expected investments levels in part (ii) are

$$
\begin{aligned}
E[K_A] &= \int_0^\delta \frac{c}{(1-\alpha-\beta)}xdx = \frac{(c\gamma-\alpha)(2-2c\gamma-\alpha-2\beta+c\gamma)}{2c(1-\alpha-\beta)} \\
E[K_B] &= \int_0^\delta \frac{c}{(1-\alpha-\beta)}xdx + \text{Prob}(K_B=\gamma)\gamma = \gamma - \frac{c^2\gamma^2-\alpha^2}{2c(1-\alpha-\beta)}
\end{aligned}
$$

□

A.4 Proof of Corollary 1

Proof. The probabilities of winning the larger network are derived from the distribution functions as computed in the proof to Proposition 1

Part (i)

$$\text{Prob}(n_A = \alpha + \mu) = \int_0^{\delta} F_B(K)\frac{c}{1-\alpha-\beta}dx = \frac{1}{2}c^2\gamma^2\frac{(1-\alpha)^2}{(1-\alpha-\beta+\alpha^2)^2}$$

$$\text{Prob}(n_B = \beta + \mu) = \int_0^{\delta} F_A(K)\frac{c}{1-\alpha-\beta}dx = \frac{1}{2}c^2\gamma^2\frac{(1-\alpha)(1-\beta+(\alpha-\beta))}{(1-\alpha-\beta+\alpha^2)^2}$$

$$\text{Prob}(n_A = 1) = \int_{\gamma}^{\overline{K}} F_B(K)cdx = \frac{1}{2} - \frac{1}{2}c^2\gamma^2\frac{(1-\beta)^2}{(1-\alpha-\beta+\alpha^2)^2}$$

$$\text{Prob}(n_B = 1) = \int_{\gamma}^{\overline{K}} cF_A(K)dx + \text{Prob}(K_B = \gamma)F_A(\gamma)$$

$$= \frac{1}{2} - \frac{1}{2}c^2\gamma^2\frac{(1-\beta)(1-\alpha-(\alpha-\beta))}{(1-\alpha-\beta+\alpha^2)^2}$$

$$\text{Prob}(n_A > \alpha) = \int_0^{\delta} F_B(K)\frac{c}{(1-\alpha-\beta)}dx + \int_{\gamma}^{\overline{K}} F_B(K)cdx$$

$$= \frac{1}{2} - \frac{1}{2}c^2\gamma^2\frac{(\alpha-\beta)(2-\alpha-\beta)}{(1-\alpha-\beta+\alpha^2)^2}$$

$$\text{Prob}(n_B > \beta) = \int_0^{\delta} F_A(K)\frac{c}{(1-\alpha-\beta)}dx + \int_{\gamma}^{\overline{K}} cF_A(K)dx + \text{Prob}(K_B = \gamma)F_A(\gamma)$$

$$= \frac{1}{2} + \frac{1}{2}c^2\gamma^2\frac{(\alpha-\beta)(2-\alpha-\beta)}{(1-\alpha-\beta+\alpha^2)^2}$$

where δ is defined in Line (14) as $\delta = \gamma\frac{(1-\alpha)(1-\alpha-\beta)}{1-\alpha-\beta+\alpha^2}$.

Obviously, $\text{Prob}(n_B > \beta) > \text{Prob}(n_A > \alpha)$. Since $1-\alpha < 1-\alpha+(\alpha-\beta) < 1-\beta+(\alpha-\beta)$ it holds that $\text{Prob}(n_A = \alpha+\mu) < \text{Prob}(n_B = \beta+\mu)$. Moreover, since $1-\alpha-(\alpha-\beta) < 1-\beta-(\alpha-\beta) < 1-\beta$ it also holds that $\text{Prob}(n_A = 1) < \text{Prob}(n_B = 1)$. Platform B is more likely to establish a network larger than that of platform A when competing networks result as the equilibrium outcome, and platform B is more likely than platform A to obtain a single network. Overall, platform B establishes a larger network than platform A more often than A does a larger one than B.

Part (ii)

$$
\begin{aligned}
&\text{Prob}(n_A = \alpha + \mu) = \int_0^{\delta} F_B(K) \frac{c}{1-\alpha-\beta} dx = \frac{(c\gamma - \alpha)^2}{2(1-\alpha-\beta)^2} \\
&\text{Prob}(n_B = \beta + \mu) = \int_0^{\delta} F_A(K) \frac{c}{1-\alpha-\beta} dx = \frac{(c\gamma - \alpha)(2 - c\gamma - \alpha - 2\beta)}{2(1-\alpha-\beta)^2} \\
&\text{Prob}(n_A = 1) = 0 \\
&\text{Prob}(n_B = 1) = \text{Prob}(K_B = \gamma) = \frac{(c\gamma - \alpha)(2 - c\gamma - \alpha - 2\beta)}{2(1-\alpha-\beta)^2} \\
&\text{Prob}(n_A > \alpha) = \text{Prob}(n_A = \alpha + \mu) = \frac{(c\gamma - \alpha)^2}{2(1-\alpha-\beta)^2} \\
&\text{Prob}(n_B > \beta) = \text{Prob}(n_B = \beta + \mu) + \text{Prob}(n_B = 1) = 1 - \frac{(c\gamma - \alpha)^2}{2(1-\alpha-\beta)^2}
\end{aligned}
$$

where δ is defined in Line (18) as $\delta = \gamma - \frac{\alpha}{c}$.

Part (iii) Since both platforms invest only below γ, $\text{Prob}(n_A = 1) = \text{Prob}(n_B = 1) = 0$. Moreover, for $K < \gamma$ the distribution functions of platforms A and B are identical such that $\text{Prob}(n_A = \alpha + \mu) = \text{Prob}(n_B = \beta + \mu) = \frac{1}{2}$. $\square$