Fragmented Truth

A thesis submitted for the degree of Doctor of Philosophy

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Abstract

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This thesis comprises three main chapters—each comprising one relatively standalone paper. The unifying theme is *fragmentalism* about truth, which is the view that the predicate "true" either expresses distinct concepts or expresses distinct properties.

In Chapter 1, I provide a formal development of alethic pluralism. Pluralism is the view that there are distinct truth properties associated with distinct domains of subject matter, where a truth property satisfies certain truth-characterizing principles. On behalf of pluralists, I propose an account of logic and semantics that shows how they can answer central conceptual and logical challenges for their view.

In Chapter 2, I motivate and develop a modal account of propositions on the basis of an iterative conception of propositions, where the modality is logico-mathematical. The modal account of propositions takes the conception to motivate an inherently potential hierarchy of propositions. I show that the account helps provide satisfying solutions to the intensional paradoxes of Russell-Myhill, Kaplan, and Prior.

In Chapter 3, I propose that "true" is polysemous. I suggest that "true" is initially polysemous between *correspondence truth* and *disquotational truth*, and further polysemous between the meanings corresponding to the subconcepts of the concept *truth* generated by the indefinite extensibility of that concept. I show that the proposal provides satisfying solutions to the semantic paradoxes.

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Contents

Chapter 0

Unifying the three fragments

> The word “true,” like other words from our everyday language, is certainly not unambiguous. ...[W]e are confronted, not with one concept, but with several different concepts which are denoted by one word[.]
>
> — Tarski (1944, 342)

> [T]he question of which...*properties* of propositions, or sentences, realize the concept [of truth] can still sensibly be raised for every discourse in which truth has application. [emphasis in the original]
>
> — Wright (2001, 752)

In this integrative chapter, I explain how the following three chapters—each comprising one relatively standalone paper—relate to each other and lie at the intersection of philosophical logic and the philosophy of language.

* * *

Language, thought, and the world are systematically linked: linguistic entities such as predicates are linked to both mental entities—concepts (or meanings)—and worldly entities—properties. Strawson (1987, 404) puts it aptly: “to talk of the concept is to talk of the *idea* or *thought* of the thing, the property, whereas to talk of the property is to talk of the thing itself” [emphasis in the original]. On this standard characterization, concepts might still be in something like Frege’s mind-independent “third realm” of causally inert abstract objects.[1] But, as I understand them, concepts are more mind-dependent than properties are, in that what concepts there are and what they are like depend somewhat more on our thoughts than what properties there are and what they are like. Nonetheless, I take it, both concepts and properties are objectively real.

At least relative to context, meaningful predicates typically express both concepts and—in another sense of "express"—properties. For simplicity, let us abstract away from context for now, and let us focus on meaningful predicates. The predicate "electromagnetic" expresses both the concept *electromagnetism* and the property of being electromagnetic. Plausibly, the concept mediates the property: "electromagnetic" expresses the property of being electromagnetic because the concept *electromagnetic* picks out the property of being electromagnetic. But concepts are a fallible guide to properties. What we know about concepts can fall short of what we know about corresponding properties. To use a standard example, "water" expresses the concept *water*, which picks out the property of being water, and this property is in turn identical to the property of being H_2O. We might know much about what the concept *water* is like prior to empirical investigation—such as that it is associated with colorless and odorless liquid. But what the property of being H_2O is like and related facts—such as that "water" expresses the property of being H_2O, and that the property of being water is identical to the property of being H_2O—is only knowable after empirical investigation.

Generalizing from "electromagnetic," for a wide range of predicates *P*, it is natural to assume, perhaps by default, the following principle:[2]

(CP) The predicate *P* expresses the concept *P-ness* and the property of being *P*.[3]

Despite its benign appearance, however, (CP) is not trivially true. Crucial to the principle is the legitimacy of the existence and uniqueness claims—due to the definite ar-

[1]See Putnam (1969); Armstrong (1978); Peacocke (1992); Zalta (2001); Künne (2003, 2006) for standard characterizations of concepts and properties. And see Frege (1956) on his "third realm."

[2]Throughout this thesis, where there is little risk of confusion, I am loose on the distinction between use and mention.

[3]I use "expresses" zeugmatically in order to emphasize that, although the senses of "expresses" are different, they are related: a predicate expresses (in one sense) a property *because* the predicate expresses (in another sense) a concept that picks out that property.

ticle "the"—to the effect that there is exactly one concept and exactly one property expressed by a given predicate. But then not all predicates obviously conform to (CP). Consider that the predicate "light" is ambiguous between the meanings *pale in color* and *low in weight*.[4] Given this, a natural correspondence between meanings and concepts—perhaps meanings are even identical to concepts—suggests that if a predicate is ambiguous between various meanings, then the predicate expresses concepts corresponding to those meanings. So the predicate "light" expresses both the concept *pale in color* and the concept *low in weight*, where these are distinct concepts.[5] Thanks to the mediation of properties by concepts, "light" also expresses the property of being pale in color and the property of being low in weight, where these are distinct properties.

Let us now focus on the predicate "true." Does it express a unique concept? And does it express a unique property? The default answer to both questions would seem to be in the affirmative. Perhaps the principle of charity suggests that a given predicate expresses at least one concept and at least one property, while the principle of parsimony suggests that a given predicate expresses at most one concept and at most one property. In the absence of reason to maintain otherwise, we should presumably assume that a given predicate expresses a unique concept and a unique property. Thus, we have the following instance of (CP):

(CPT) The predicate "true" expresses the concept *truth* ("the concept true-ness") and the property of truth ("the property of being true").

However, in this thesis, I explore the possibility of rejecting the default answer. Specifically, I see how a view that rejects the default answer might be motivated and developed.

One way of rejecting the default assumption is to be a *nihilist*, in maintaining that

[4]Throughout this thesis, I assume that expressions—including predicates—are strings of symbols (of a given natural language). Given that strings of symbols can have distinct meanings, expressions can be ambiguous. I focus on expressions rather than "words" in order to avoid confusion. Confusion might arise if we just talk about words, because on a view according to which words are lexical items, and represent a single meaning, the expression "light" corresponds to the distinct words *light*$_1$ and *light*$_2$, which are spelled and pronounced the same way but differ in meaning. On such a view, words are not ambiguous.

[5]See Sennet (2016). A more controversial example maintains that "mass" expresses both the concepts *rest mass* and *relativistic mass*, where these are distinct concepts; see Field (1973, 1986, 1994a, 2001).

either "true" does not express any concept at all or it does not express any property at all. The former claim—that "true" does not express a concept—is perhaps too implausible, in manifestly conflicting with the fact that it does express a concept, since "true" is undoubtedly meaningful.[6] The latter claim—that "true" does not express a property—is more plausible, depending on how properties are construed. A slogan popular in early characterizations of deflationism was that truth is not a property, which is plausibly equivalent to the slogan that "true" does not express a property.[7] However, in this thesis, I am not explicitly concerned with deflationism, or with nihilism.

In this thesis, I focus on the other way of rejecting the default assumption, namely, by being a *fragmentalist*.[8] A fragmentalist denies the instance (CPT) of (CP), and instead maintains the following:

(FT) Either "true" expresses distinct concepts or it expresses distinct properties.

This disjunctive formulation of fragmentalism naturally suggests two kinds of fragmentalism, where each kind corresponds to accepting one of the two disjuncts. In the following three chapters, I investigate each of the two kinds of fragmentalism.

In Chapter 1, I focus on *property fragmentalism*, which accepts the first disjunct of (FT), in maintaining the following:

(FTP) "True" expresses distinct properties.

Property fragmentalism is manifested in the literature by what is known as "alethic pluralism" (or "pluralism about truth"). The idea is that, although there might be just one concept of truth, this concept picks out distinct properties.[9] According to pluralists, there is a "horizontal" fragmentation of truth across domains of subject matter: on a standard motivating picture, the truth property associated with the scientific domain is distinct from the truth properties associated with the mathematical and ethical

[6] But see Badici and Ludwig (2007) for a dissenting view, according to which "truth predicates in natural languages fail to express the concept of truth, or any other concept" (635).

[7] See Wright (1992); Alston (1996) for such characterizations of deflationism.

[8] What I am calling "fragmentalism" should be clearly distinguished from what Fine (2005) calls "fragmentalism." The views are not (obviously) related.

[9] See Alston (2002) on the gap between concepts and properties in the case of truth.

domains, where the truth properties associated with the mathematical and ethical domains are also distinct. Property fragmentalism in the form of pluralism has its historical origins in Wright (1992), and is motivated by the traditional debate between realism and antirealism with respect to domains of scientific, mathematical, and ethical subject matters. Although property fragmentalism remains far from mainstream, it has gained traction in recent years, and provides an alternative to deflationism.

In Chapters 2 and 3, I focus on *concept fragmentalism*, which accepts the second disjunct of (FT), in maintaining the following:

(FTC) "True" expresses distinct concepts.

One might be a concept fragmentalist in maintaining that there are two main concepts of truth—a more inflationary and a more deflationary concept of truth.[10] Such a concept fragmentalist takes the concept of truth to be fragmented because it is *bifurcated*. While I think such bifurcation is worth exploring, I focus more on concept fragmentalists who maintain that there is a hierarchy of concepts of truth—the concept of truth is fragmented because it is *hierarchical*. Since the concepts are not coextensive, assuming that each concept picks out at most one property, properties the concepts pick out are also not coextensive, and so distinct. Accordingly, there is a "vertical" fragmentation of truth across levels of hierarchies. Concept fragmentalism has its historical origins in Tarski (1944)'s account of truth and is more generally motivated by considerations having to do with the paradoxes. Although concept fragmentalism also remains far from mainstream, it is recommended by the phenomenon of indefinite extensibility—whereby certain concepts can somehow become more and more inclusive.

Specifically, in Chapter 2, I focus on concept fragmentalism involving the truth of propositions. A hierarchy of concepts of propositional truth is motivated by the intensional paradoxes of Russell-Myhill, Kaplan, and Prior. And in Chapter 3, I focus on concept fragmentalism involving the truth of sentences. A hierarchy of concepts of sen-

[10]Naess (1938, 2014) discusses a form of concept fragmentalism according to which, among nonphilosophers, "true" does not express a unique concept. However, I set aside such a view in this thesis.

tential truth is motivated by the semantic paradoxes, especially the contextual liar and associated revenge paradoxes.

In characterizing pluralism as a form of property fragmentalism, and hierarchical accounts as forms of concept fragmentalism, I follow the standard characterizations in the literature in terms of properties and concepts, respectively. These characterizations are not hard and fast. What motivates pluralism might also motivate concept fragmentalism.[11] For if pluralism is correct, the recognition that there are distinct truth properties suggests the formation of correspondingly distinct concepts. Similarly, what motivates a hierarchical account might also motivate property fragmentalism.[12] For if there is a hierarchy of concepts of truth, then since concepts mediate properties and the concepts are not coextensive, the concepts pick out distinct truth properties. Nonetheless, my characterizations place the focus on what is presumably explanatorily prior—properties in the case of pluralism and concepts in the case of hierarchical accounts.

*

At this point, let me address two initial challenges to fragmentalism. While I cannot reasonably claim to settle them once and for all here, I offer possible answers to them. At the least, I hope, the possible answers suggest that the initial challenges are inconclusive, and so do not undermine the legitimacy of investigating fragmentalism.

The first initial challenge is that a simple argument purports to show that "true" expresses at most one concept and at most one property, so that fragmentalism does not even get off the ground. Let $\langle \cdot \rangle$ be a name-forming device (e.g., quotation marks or "the proposition that…"), so that $\langle p \rangle$ is a truth bearer, such as the proposition that p or the sentence "p". A key premise is that any predicate Tr worth the honorific "truth predicate" is simply any predicate that conforms to the following equivalence principle:

(E) $\langle p \rangle$ is Tr iff p.

[11] See Lynch (2009, 54–59) and Kölbel (2013, 293) for characterizations of pluralism in terms of concepts.
[12] See Horsten (2011, 55) for a characterization of a hierarchical account of truth in terms of properties.

By the symmetry and transitivity of "iff," a straightforward consequence of (E) is that if Tr_i and Tr_j are truth predicates, then we have the following:

(E=) $\langle p \rangle$ is Tr_i iff $\langle p \rangle$ is Tr_j.

There are two versions of the simple argument, depending on whether "iff" expresses the biconditional or logical equivalence. In the case where "iff" expresses logical equivalence, (E=) maintains that "$\langle p \rangle$ is Tr_i" is logically equivalent to "$\langle p \rangle$ is Tr_j"—or equivalently that one can infer each from the other. According to one version of the simple argument, the *One Concept Argument*, this shows not only that concepts of truth are coextensive, but also that there is at most one concept of truth. But then concept fragmentalism is false. In the case where "iff" expresses the biconditional, (E=) maintains that Tr_i and Tr_j are coextensive. According to the other version of the simple argument, the *One Property Argument*, this shows that there is at most one property of truth. But then property fragmentalism is false.

Although the simple argument presents an apparent hurdle to the legitimacy of investigating fragmentalism, I think both versions of the argument are inconclusive. The One Concept Argument seems to infer from the claim that given concepts are unique up to logical equivalence that the concepts are identical.[13] But this inference is controversial. For consider that, where the truth bearers are sentences, the sentences "Socrates is mortal" and "Socrates is mortal and self-identical" are logically equivalent, but perhaps the concepts *mortal* and *mortal and self-identical* are distinct.

Similarly, the One Property Argument seems to infer from the coextensiveness of given properties—even if the coextensiveness is necessary (because it is secured by purely logical considerations)—to the identity of the properties. But the inference from the coextensiveness of properties to the identity of properties is invalid, since the property of having a heart and the property of having a kidney are coextensive but distinct. Granted, perhaps necessitated versions of (E) and (E=) are as plausible as (E) and (E=)

[13] See Belnap (1962); Harris (1982); Williamson (1988); McGee (2000, 2006); Williamson (2006); Dorr (2014) for related arguments involving "collapse" theorems.

themselves, and yield the result that truth predicates are necessarily coextensive. But the inference from the necessary coextensiveness of properties to the identity of properties is arguably still invalid, since arguably the property of having three sides and the property of having three angles are necessarily coextensive but distinct.

For the purposes of this thesis, however, the above answers are of limited effectiveness. The problem is that the above answers only seem helpful for fragmentalist views according to which "true" expresses concepts or properties that are coextensive. The above answers do not contest the key premise (E), and so concede the possibility that distinct concepts or properties of truth are coextensive. But some fragmentalist views—among them ones that interest me in this thesis—maintain that "true" does not even express concepts or properties that are coextensive. For such fragmentalists, pointing out that coextensive concepts or properties can be distinct seems beside the point. Can fragmentalists who maintain noncoextensiveness answer the challenge as well?

I suggest that they can. More importantly than the above, for the purposes of this thesis, the key premise (E) in the simple argument is in question. Despite its initial plausibility, it remains far from unassailable, with its plausibility depending partly on what the truth bearers are. A broadly inferentialist approach to truth might suggest that (E) is nonnegotiable, in that it is somehow analytic.[14] However, such an approach to truth in particular and to meaning in general is controversial.[15] As I note in Chapter 1, perhaps (E) needs to be restricted to truth bearers associated with certain domains. Similarly, as I explain in Chapter 3, classical solutions to the semantic paradoxes universally surrender the version of (E) where truth bearers are sentences and "iff" expresses either the biconditional or logical equivalence. Instead of (E), proponents of such solutions instead maintain, or at least should maintain, a restriction to "healthy" truth bearers. And even where truth bearers are propositions, (E) is compatible with readings where "true" is ambiguous or schematic, and so expresses distinct concepts.

[14] See Tarski (1944, 1983); Horwich (1990); Gupta and Belnap (1993); Field (1994a, 2001, 2008); Eklund (2002, 2007); Priest (2002, 2006); Beall (2009) on the significance of such a principle.

[15] See Williamson (2007) for an extended critique of inferentialism.

The second initial challenge is that, if fragmentalism is correct, then—given the intertwining of truth with other central notions in philosophy—unacceptably widespread revision of much of philosophy is required.

Granted, if fragmentalism is correct, then some revision is likely. However, I suspect that the challenge exaggerates the degree of revision required. The challenger might cite the link between truth and knowledge due to the principle of factivity, a version of which is the following:

(K) If one knows $\langle p \rangle$, then $\langle p \rangle$ is true.

For presumably, if "true" either expresses distinct concepts or expresses distinct properties, then (K) needs to be revised to reflect this. But, against the challenger, it is unclear that the presence of "true"—or a predicate resembling it—is essential here. Notwithstanding the extent to which (E) holds in general, and as deflationists may be wont to stress, much of what is said with "true" could be said without it. Another version of the principle of factivity that seems to serve just as well is the following:

(K′) If one knows $\langle p \rangle$, then p.

To the extent that the strategy to dispense with "true" succeeds in general, the correctness of fragmentalism does not require widespread revision.[16]

Further, while revision could be a cost, it does not follow that the cost is unacceptable. A cost provides at best a pragmatic reason, rather than an epistemic one, to avoid embracing fragmentalism. For consider, by analogy, that the discovery of Russell's paradox required revision of naive set theory, but the revision was justified. If fragmentalism is correct, then any required revision is presumably no less justified.

In light of the above, I conclude that the initial challenges are inconclusive. At the least, they do not undermine the legitimacy of investigating fragmentalism.

[16]Perhaps (K′) needs to be revised due to Montague's paradox: Montague (1963) shows that any system closed under $M\ulcorner\varphi\urcorner \to \varphi$ and necessitation—the principle that if φ is a theorem, then so is $M\ulcorner\varphi\urcorner$—is inconsistent. If so, then perhaps (K′) should be restricted to "healthy" truth bearers. Alternatively, perhaps one can treat "knows" and other intensional verbs—and even "true"—as *operators* rather than predicates. Operator approaches seem less vulnerable to paradox; see Künne (2003); Mulligan (2010).

*

Having introduced fragmentalism as the unifying theme and discussed some initial challenges to fragmentalism, let me now set out the scope of this thesis. This thesis as a whole, which seeks to contribute to the philosophy of truth, lies at the intersection of philosophical logic and the philosophy of language.

Chapters 1 and 2 are more on the side of philosophical logic, in employing formal methods to sharpen the discussion.

In the literature, pluralism has often been developed only informally. Although there is a place for such informal development, a more formal development is also worth pursuing. Besides more clearly articulating what pluralism is and is not committed to, it seems that only a more formal development is able to show that pluralists can answer central logical challenges for their view. Some of the existing accounts in the literature that attempt to answer these challenges are developed in too informal a way, so as to be hard to evaluate. Following the example set by Cotnoir (2013b)'s most recent algebraic account on behalf of pluralists, in Chapter 1, I provide a formal development of pluralism, and specifically, a unified account of logic and semantics that is pluralist-friendly. This account, I suggest, answers at least the central logical challenges for pluralism.

Similarly, at least in the earlier literature on indefinite extensibility, it has been unclear exactly how we should state the claim that a given concept is indefinitely extensible. Noting that some philosophers take the indefinite extensibility of certain concepts to motivate the denial of the availability of absolutely unrestricted quantification, unclarity about how we should state the claim that a given concept is indefinitely extensible manifests itself as unclarity about how we should state the claim that absolutely unrestricted quantification is unavailable.[17] Such unclarity has put such philosophers in an uncomfortable position, with Lewis (1991); Williamson (2003) criticizing their view for being self-refuting or somehow incoherent. A clear articulation of the claim that a given concept is indefinitely extensible is accordingly worthwhile. Following recent

[17] See Rayo and Uzquiano (2006) on the availability of absolutely unrestricted quantification.

examples set by Fine (2006); Linnebo (2013); Studd (2013); Uzquiano (2015d), in Chapter 2, I provide a clear and precise statement of the claim that the concepts *proposition* and *propositional truth* are indefinitely extensible. I exploit the resources of plural and modal languages in the development of a modal account of propositions.

Chapter 3, meanwhile, straddles the divide between philosophical logic and the philosophy of language, in developing the proposal that "true" is polysemous, and thus ambiguous, while being also concerned with more formal treatments of the semantic paradoxes. Some of the literature on the semantic paradoxes prioritizes the technical over the philosophical. Given the high level of clarity, precision, and rigor that an adequate treatment of these paradoxes demands, such work is welcome. But in Chapter 3, I emphasize the philosophical. Instead of attempting to develop a technically sophisticated theory, I provide more philosophical motivation for a technically simple theory. Specifically, I suggest that the underdeveloped proposal that "true" is ambiguous is worth taking seriously. While many philosophers are reluctant to embrace such ambiguity, I think such ambiguity is much less objectionable when it is polysemy in particular. Crucially, such polysemy provides a natural view that welcomes the ascent from an object language to a metalanguage that pervades theorizing about the semantic paradoxes.

This thesis also touches on related issues in metaphysics. Pluralism in Chapter 1 raises issues about the "substantiality" of properties, while the modal account of propositions in Chapter 2 suggests a distinctive view of the metaphysics of propositions and properties that is couched in terms of a logico-mathematical modality. This thesis also touches on related issues in the philosophy of mind. The modal account of propositions in Chapter 2 is motivated by an account of propositions that is fine-grained enough to accommodate the role of propositions as the objects of mental attitudes. And the proposal that "true" is polysemous in Chapter 3 assumes a natural correspondence between meanings and concepts, where concepts apparently change over time.[18]

[18] One might suspect that a view according to which concepts change undermines concept fragmentalism, since there might only be just one concept. However, as I explain in Chapter 3, on my understanding of such a view, concepts that change leave behind stages of their development, which I identify as subconcepts—and thus concepts. These subconcepts form the basis of concept fragmentalism.

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I close this integrative chapter by outlining the two aims of this thesis. The thesis has a more modest and a more ambitious aim.

The more modest aim of this thesis is to show that some kind of fragmentalism is worth considering, whether or not one ultimately endorses some kind of fragmentalism.

As Chapter 1 explains, property fragmentalism, in the form of pluralism, provides a potentially fruitful way of formulating the traditional debate between realism and antirealism, and deflationism about truth. But there are instructive parallels between, e.g., the pluralist and the expressivist formulations of antirealism about ethical sentences.

Consider the pluralist who maintains that, while the truth of true scientific sentences—such as "Water is H_2O"—is, as Wright would put it, "metaphysically heavyweight," the truth of true ethical sentences—such as "Murder is wrong"—is, as Wright would put it, "metaphysically lightweight." And consider the expressivist who maintains that, while scientific sentences are truth-apt, ethical sentences are not truth-apt; roughly, sentences are truth-apt just if they are either true or false. In each case, it is unclear how one should treat the mixing of scientific sentences with ethical sentences. For the pluralist: if "Water is H_2O" is true in one way and "Murder is wrong" is true in another way, in what way or ways is the conjunction "Water is H_2O and murder is wrong" true? For the expressivist: if "Water is H_2O" is true and "Murder is wrong" is untrue (because it is not truth-apt), what is the semantic status of "Water is H_2O and murder is wrong"? And for both the pluralist and the expressivist: assuming a standard account of logical consequence, according to which logical consequence necessarily preserves truth, what is preserved in the valid inference from the two conjuncts to the conjunction?

The overall challenge for the expressivist—which has received much attention—is known as the "Frege-Geach challenge," and there appears to be no consensus answer to it.[19] But pluralists clearly face a parallel challenge, so an answer on behalf of pluralists promises to transpose to an answer on behalf of expressivists. Chapter 1, which I

[19] See Stevenson (1937); Hare (1952); Blackburn (1984, 1993); Gibbard (1990, 2003); Schroeder (2008a,b) on the Frege-Geach challenge.

suggest answers the overall challenge on behalf of pluralists, might thus provide a general strategy available to any other view—expressivist or otherwise—that maintains that sentences about different subject matters have different kinds of semantic status.

Chapters 2 and 3 motivate concept fragmentalism by focusing on providing satisfying solutions to various paradoxes of a broadly logical nature. Whether or not one endorses the proposed solutions, I hope they add to the range of solutions currently on offer. Since the range of solutions to the semantic paradoxes is already very extensive, and there still appears to be no consensus solution, it is unlikely that the proposed solutions to them will be the last word on the matter. But the range of solutions to the intensional paradoxes is much less extensive. In any case, with both kinds of paradox, I am optimistic that the proposed solutions at least enrich our understanding of the relevant phenomena—involving not only truth, but also concepts and meaning.

The more ambitious aim of this thesis is to show that some kind of fragmentalism can be *both* well-motivated *and* technically feasible. In the literature, property fragmentalism, in the form of pluralism, may seem well-motivated but leaves open questions about its technical feasibility. Chapter 1 aims to fill this lacuna by providing a formal development of pluralism. By contrast, concept fragmentalism, in the form of hierarchical accounts of the truth of propositions or sentences, may seem technically feasible but leaves open questions about its motivations. Chapters 2 and 3 aim to fill this lacuna by supplying complementary motivations: Chapter 2 motivates a potential hierarchy of propositions, and thus of propositional truth; and Chapter 3 motivates a potential hierarchy of concepts and corresponding meanings, where semantic ascent is understood in light of conceptual change, semantic change, and polysemy in general.

Chapter 1

Logic and semantics for alethic pluralists

> The proposal is...that any predicate that exhibits certain very general features qualifies...as a truth predicate. That is quite consistent with acknowledging that there may, perhaps *must* be more to say about the content of any predicate that does have these features. But it is also consistent with acknowledging that there is a prospect of *pluralism*—that the more that there is to say may well vary from discourse to discourse[.] [emphasis in the original]
>
> — Wright (1992, 37–38)

Alethic pluralism, as I understand it, is the view that there are distinct truth properties associated with distinct domains of subject matter, where a truth property satisfies certain truth-characterizing principles. On a standard motivating picture, the truth property associated with the scientific domain is distinct from the truth properties associated with the mathematical and ethical domains, where the truth properties associated with the mathematical and ethical domains are also distinct.

In this chapter, I provide a formal development of pluralism, by developing a unified account of logic and semantics that is pluralist-friendly. Although pluralism remains far from mainstream, it has gained traction in recent years, beginning with Wright (1992)'s seminal contribution. In light of questions about the details of the view, the work of

Wright and other pluralists has developed the view to some extent. But these developments seem limited insofar as pluralists remain vulnerable to two kinds of central challenges. There are conceptual challenges to clarify the conceptions of truth properties, domains, and the association between them. There are also logical challenges to provide a unified logic and semantics for claims, whether or not they are "mixed," in that they involve distinct subject matters. I aim to show how pluralists can answer the central logical challenges, and explain how I think they should accordingly answer the central conceptual challenges.

This chapter is structured as follows. In Section 1, I present pluralism. In Section 2, I present central conceptual and logical challenges for the view, and propose constraints and desiderata to guide answers to the challenges. In Section 3, I argue that representative examples of existing accounts in the literature provide inadequate answers to the challenges. In Section 4, on behalf of pluralists, I propose a novel account, which I submit adequately answers the central logical challenges. I close the section by explaining how I think pluralists should accordingly answer the central conceptual challenges.

1 Pluralism

In this section, I present pluralism. In the first subsection, I present and clarify pluralism through Wright's work. In the second subsection, I provide a canonical formulation of pluralism and distinguish it from superficially similar but distinct views.

1.1 Pluralism in Wright's work

In his seminal book, *Truth and Objectivity*, Wright (1992) reacts to the traditional debate between realism and antirealism, as well as deflationism.[1] Why, he begins by asking, has there been limited progress on whether realism is true about domains of subject matter—"regions of discourse" (2)—such as mathematics and ethics? He diagnoses the

[1] See Jackson (1994); Williamson (1994a); Pettit (1996); Sainsbury (1996); Blackburn (1998); Horton and Poston (2012) for responses to Wright (1992).

problem as due to unclarity in the formulation of debate, and proposes his minimalism in general and pluralism in particular to help clarify what is at stake in the debate.[2]

Given that the debate between realism and antirealism sometimes—e.g., when the kind of antirealism about a given domain is expressivism about that domain—seems to involve whether "statements" (12) associated with the relevant domain are truth-apt, roughly in that they are either true or false, Wright first considers deflationism. He agrees with the deflationist view that truth-aptitude is a metaphysically "lightweight" (13) matter, but disagrees with the deflationist view that the predicate "true" does not express a "substantial" (*ibid*) property.[3]

Wright then proposes minimalism, and in particular pluralism, as a successor to deflationism. His minimalism maintains that any truth predicate—which for him is just any predicate that satisfies certain "platitudes" about truth—expresses a truth property, but that the truth property expressed *may* vary according to the domain. He suggests that a debate between realism and antirealism about a given domain should be a debate about the nature of the truth property expressed by the truth predicate associated with the domain. His pluralism maintains that the truth property expressed by the truth predicate not only may, but *does*, vary according to the domain.[4]

Pluralism in Wright (1992)

Let me present pluralism, as it is introduced in Wright (1992), in more detail. Since it is hard to adequately characterize realism and antirealism, I abstract away from Wright's characterizations of them. Roughly, we can characterize realism and antirealism as follows: realism maintains mind-independence, while antirealism maintains mind-dependence, about the claims, objects, and properties associated with a given domain. Wright suggests that the attractiveness of realism varies according to the domain, with realism being attractive in the case of scientific subject matter, but less attractive in the case of

[2]What Wright calls "minimalism" should be clearly distinguished from another view often called "minimalism," namely, Horwich (1990)'s version of deflationism.

[3]In Section 2.1, I address how pluralists should understand the "substantiality" of a property.

[4]In Section 2.1, I address how pluralists should understand truth *simpliciter*, or generic truth.

ethical, aesthetic, and comical subject matters (1–9). Accordingly, one might maintain realism about some domains while maintaining antirealism about other domains.

How should the debate between realism and antirealism be formulated? After rejecting three previous formulations of the debate as inadequate, Wright proposes his minimalism. A key thought is that claims associated with a given domain—whether one maintains realism or antirealism about that domain—are truth-apt, but that truth-aptitude is compatible with either realism or antirealism about the domain, so that the debate between realism and antirealism about that domain should be formulated as a debate about the nature of the truth property expressed by the truth predicate associated with the domain.

Wright initially considers deflationism, which is to serve as a foil to minimalism and pluralism. Wright takes deflationism to consist in a negative claim and a positive claim. The negative claim is that "true" does not express a "substantial" property. The positive claim is that the concept *truth* is completely fixed, and "true" is completely explained, by the "Disquotational Schema" (14). The Disquotational Schema maintains that, where "*p*" is a declarative sentence,

(DS) "*p*" is true iff *p*.

As Wright characterizes deflationism, claims about a given domain are truth-apt, but such truth-aptitude is compatible with antirealism about the domain. Wright agrees with deflationists on the importance ascribed to the Disquotational Schema, but he finds deflationism to be "inherently unstable" (21). He summarizes his argument—the details of which do not concern us—thus:

> [D]eflationism shows a tendency to inflate under pressure, to the point where its principal negative contention, that truth is not a substantial property of sentences or thoughts at all, comes to seem to run contrary to its principal explanatory contention, that the concept of truth may be regarded as fixed by the famous Disquotational Schema. (13)

He then proposes his "minimalism," which drops the deflationist commitment that truth is not a "substantial" property and supplements the deflationist commitment to the Disquotational Schema with other principles:

> The proposal is simply that any predicate that exhibits certain very general features qualifies, just on that account, as a truth predicate. That is quite consistent with acknowledging that there may, perhaps *must* be more to say about the content of any predicate that does have these features. But it is also consistent with acknowledging that there is a prospect of *pluralism*—that the more that there is to say may well vary from discourse to discourse—and that whatever may remain to be said, it will not concern any *essential* features of truth. The essential features are exhausted, rather, by the platitudes. [emphasis in the original] (37–38)

On Wright's minimalism, it is necessary and sufficient for a predicate to be a truth predicate that it has certain features, and in particular, satisfies certain truth-characterizing "platitudes"—"very general, very intuitive principles" (34). Claims associated with a given domain are then truth-apt just if there is a truth predicate associated with the domain. But, Wright maintains, there is such a truth predicate just if the domain exhibits basic standards of syntax and discipline (29), in that the sentences associated with the domain embed under the logical operators and "are subjected to communally acknowledged standards of proper use" (35–36).

Wright suggests the following truth-characterizing principles, although he does not claim that they are exhaustive and is open to the possibility that they are not:

> [T]hat to assert is to present as true; that any truth-apt content has a significant negation which is likewise truth-apt; that to be true is to correspond to the facts; that a statement may be justified [or warrantedly assertible (35)] without being true, and vice versa[.] (34)
>
> Any predicate is a truth predicate which satisfies certain basic principles. One such is the Disquotational Schema.... But lurking behind [this] is the more fundamental thesis that to assert is to present as true. Other relevant principles include: that to every assertible content corresponds an assertoric negation; that a content is true just in case it corresponds to the facts, depicts things as they are, and so on; that truth and warrant are distinct, and...that truth is absolute (there is, strictly, no being more or less true) and, more contentiously, that it is stable (if a content is ever true, it always is). (72)

One might suspect that the correspondence principle imposes metaphysical weight. But Wright clarifies that, for him, it does not: expressions such as "corresponds" and "facts" are to be understood as metaphysically lightweight, in a deflationary spirit (83–85). More pressingly, one might suspect that the inclusion of the Disquotational Schema among the principles makes no room for pluralism.[5] For the Schema entails that all truth properties are extensionally equivalent, and the necessitation of it—which pluralists should plausibly accept—entails that all truth properties are intensionally equivalent. Wright (1992) does not seem to explicitly address this suspicion, but perhaps he maintains that there remains room for pluralism, because even truth properties that are intensionally equivalent can be hyperintensionally inequivalent, and that this suffices for there to be distinct truth properties. An alternative is to follow Edwards (2011, 2013) in restricting the truth-characterizing principles, including the Disquotational Schema, to domains. Wright (1992, 63) seems to suggest such restrictions.[6]

On Wright's minimalism, the truth-characterizing principles are silent on the number of truth predicates there are. So, provided that the above suspicion involving the Schema can be addressed, Wright suggests that there is room for pluralism:

> [T]here is no reason to expect that the minimal platitudes will constrain their interpretation to within uniqueness. A variety of predicates may qualify as truth predicates, and we have to be receptive to the possibility that the truth predicates across different regions of discourse may differ in important respects. (75)

Wright offers as an example of a truth predicate "superassertible," which he defines as "warranted and some warrant for it would survive arbitrarily close scrutiny of its pedigree and arbitrarily extensive increments to or other forms of improvement of our information" (48).[7] This is an example of a truth predicate associated with antirealism about a given domain. While he does not seem to have a name for it, he also writes of a truth predicate associated with realism about some domain, which gives a more "robust" (84) interpretation of the correspondence principle. For Wright, such a truth predicate is associated with domains that satisfy two constraints. The first constraint, "Cognitive

[5] Although the Schema is not included among the principles on page 72, Wright (1992, 34) maintains that the other principles, together with plausible assumptions, give rise to the Schema.

[6] Wright (2013) explicitly endorses such restrictions.

[7] Wright offers an alternative definition of superassertibility as "assertibility which would be durable under any possible improvement to one's state of information" (75).

Command" (93), maintains that it is *a priori* that disagreement over claims—that are not due to vagueness—associated with the domain involves cognitive shortcoming. The second constraint, "Width of Cosmological Role" (196), maintains that facts associated with the domain are somehow independent and "substantial."

With his minimalism and pluralism in mind, Wright suggests that what realists and antirealists about a given domain should debate is the extent to which the truth property expressed by the truth predicate associated with the domain satisfies principles beyond the platitudes:

> A basic anti-realism...about a discourse contends that nothing further is true of the local truth predicate which can serve somehow to fill out and substantiate an intuitively realist view of its subject matter. ...
>
> [R]ealism...must try to make good its case, by...showing that the minimal platitudes leave out features of the local truth predicate which...justify the rhetoric of independence, autonomy and full-fledged cognitive interaction by which realism pretheoretically defines itself. (174)

Clarifications of pluralism

Having presented pluralism, as it is introduced in Wright (1992), I now clarify pluralism, drawing on subsequent clarifications from Wright (1994a,b, 1996, 1998a,b, 2001, 2013).[8]

First, what is the difference between minimalism and pluralism? Minimalism creates room for pluralism, in that minimalism's characterization of truth in terms of whatever satisfies the truth-characterizing principles leaves open the possibility that there are distinct satisfiers of those principles. Pluralism is minimalism supplemented with the view that there are distinct satisfiers of the principles according to the domain.

Second, focusing on pluralism from now on, what are truth predicates predicates of—sentences or propositions? Wright (1992) is not explicit about this. He writes of "statements," and does not seem to clearly identify them as either sentences or propositions. Still, it is implicit in his discussion that the relevant claims ("statements") are either sentences or propositions. For the principles he offers as those that characterize

[8] Wright's 1990s work on pluralism is collected in Wright (2003a) for easy reference.

truth on his minimalism employ both truth predicates of sentences and truth predicates of propositions. The Disquotational Schema employs a truth predicate of sentences. But other principles, which mention "assertible contents" and so presumably propositions, employ a truth predicate of propositions. Wright (2001) clarifies that he is concerned with the truth of "propositions, or sentences" (752).

Throughout this chapter, I assume for simplicity that truth bearers are sentences. This assumption is compatible with propositions and other entities also being truth bearers. There is also no loss of generality, since claims about sentences transpose to claims about propositions, especially if propositions have a sentence-like structure.

Third, what is it that Wright's pluralism is pluralist about—truth predicates or truth properties? We have seen that Wright writes of a "variety of truth predicates," which suggests that his pluralism maintains that there are distinct truth predicates. But since predicates are linguistic expressions, it is unlikely that this is all he maintains. For there is a simple and straightforward sense in which there are distinct truth predicates, independently of concerns having to do with realism and antirealism, minimalism, or pluralism: there are at least the truth predicates "true," "correct," and "right." But Wright's view is not the uncontroversial one that there are distinct truth predicates in this sense.

Wright's view is more contentious: there are distinct *properties* of truth. For as we have seen, he writes of the "content" of truth predicates varying as well, and suggests more generally that truth predicates inherit features of the properties the predicates express; Wright (1992, 74) writes of truth predicates being "metaphysically lightweight," which presumably reflects the claim that the properties expressed are "metaphysically lightweight." Wright (2001, 752) clarifies that "what property serves as truth may vary from discourse to discourse," so that "some regions of discourse may be subject to a truth property congenial to broadly realist thinking about them, while in other regions the character of the truth property may be more congenial to antirealism."

The view that there are distinct truth properties may, but need not, be supplemented by either the view that the single predicate "true" expresses distinct truth properties or the view that there are distinct truth predicates that express correspondingly distinct truth properties. Wright seems to formulate pluralism in both of these ways. For he not only writes of a "variety of truth predicates," but also later clarifies that "the question of which *property* or *properties* of propositions, or sentences, realize the concept [what is expressed by "true"] can still sensibly be raised for every discourse in which truth has application" [emphasis in the original] (752).

Fourth, are there also distinct concepts of truth? If so, then on the plausible assumptions that "true" expresses them and that meanings correspond to concepts, "true" is ambiguous. Sainsbury (1996) reads Wright (1992) as suggesting that "true" is ambiguous. However, as the passage from Wright just quoted suggests, Wright does not maintain that there are also distinct concepts of truth. Rather, he maintains, in line with his minimalism, that there is a single concept of truth that is characterized by certain principles. It is just that this concept may pick out distinct properties, and according to pluralism, the concept does pick out distinct properties. Wright (2001, 752) clarifies the relation between truth properties and the concept of truth as follows:

> [M]inimalism rejects the idea that the analysis of the concept of truth exhausts the philosophy of truth: rather, even if the *concept* may be fully characterized by reference to certain basic a priori principles concerning it, the question of which *property* or *properties* of propositions, or sentences, realize the concept can still sensibly be raised for every discourse in which truth has application. [emphasis in the original]

Wright (1994a,b, 1998a), explicitly denies that "true" is ambiguous.

Finally, what exactly are the principles that characterize truth? We have seen that even Wright (1992) presents two somewhat different lists of principles, on pages 34 and 72. But the lists there seem to only partially reflect Wright's considered list of principles. The most comprehensive list Wright offers is in Wright (1998b, 60).[9] I reproduce this list in full for completeness:

- the transparency of truth—that to assert is to present as true and, more generally, that any attitude to a proposition is an attitude to its truth—that to

[9] This is essentially the same list offered in Wright (2001, 760).

believe, doubt, or fear, for example, that *P* is to believe, doubt, or fear that *P* is true. (*Transparency*)

- the epistemic opacity of truth—incorporating a variety of weaker and stronger principles: that a thinker may be so situated that a particular truth is beyond her ken, that some truths may never be known, that some truths may be unknowable in principle, etc. (*Opacity*)
- the conservation of truth-aptitude under embedding: aptitude for truth is preserved under a variety of operations—in particular, truth-apt propositions have negations, conjunctions, disjunctions, etc., which are likewise truth-apt. (*Embedding*)
- the Correspondence Platitude—for a proposition to be true is for it to correspond to reality, accurately reflect how matters stand, "tell it like it is," etc. (*Correspondence*)
- the contrast of truth with justification—a proposition may be true without being justified, and vice versa. (*Contrast*)
- the timelessness of truth—if a proposition is ever true, then it always is, so that whatever may, at any particular time, be truly asserted may—perhaps by appropriate transformations of mood, or tense—be truly asserted at any time. (*Stability*)
- that truth is absolute—there is, strictly, no such thing as a proposition's being more or less true; propositions are completely true if true at all. (*Absoluteness*) [emphasis in the original]

Again, the correspondence platitude imposes no metaphysical weight:

> As a platitude it thus carries no commitment to a real ontology of facts...nor to any seriously representational construal of "correspondence," but merely claims that talk of truth may be paraphrased by any of a variety of kinds of correspondence idiom. (67)

Wright (1998b) does not maintain that the principles provide a "reductive" (59) account of truth. But he does maintain that they are "*a priori*" and "collectively constrain and locate the target concept and sufficiently characterize some of its relations with other concepts and its role and purposes" (59). Accordingly, for Wright, a property is a truth property iff it satisfies all the truth-characterizing principles. But what satisfies all these principles may and does, for him, vary according to the domain.

1.2 Canonical formulation of pluralism

To sharpen our focus, I now provide a canonical formulation of pluralism and distinguish it from superficially similar but distinct views.

Formulation and illustration of pluralism

My formulation of pluralism, as inspired by the work of Wright, is bare-bones. The minimalist component of pluralism takes a property to be a truth property iff it satisfies all the truth-characterizing principles. The distinctively pluralist component takes there to be distinct truth properties associated with distinct domains, provided that the domains meet basic standards of syntactic discipline. As nothing I discuss hinges on whether the domains meet these standards, from now on I omit the qualification, and simply assume that the domains meet the standards. Thus, I focus on the following formulation of pluralism:

> *Pluralism.* There are distinct truth properties associated with distinct domains of subject matter, where a property is a truth property iff it satisfies all the truth-characterizing principles.

The following examples, which are based on Wright's own, illustrate pluralism.[10] In the scientific domain, what satisfies the principles is the property of correspondence truth, where correspondence truth is understood to be metaphysically weighty, so that this property is associated with the scientific domain. But in the mathematical domain, what satisfies the principles is the property of coherence truth, so that this property is associated with the mathematical domain. And in the ethical domain, what satisfies the principles is the property of superassertible truth, so that this property is associated with the ethical domain.

Now consider the following three sentences:

(1) Water is H_2O.

[10]Wright (1998b, 57) suggests that "truth is only *sometimes* to be conceived as...a property bestowed upon it by its relations of coherence with certain other propositions, while in other cases the structure of truth is best conceived as by correspondence" [emphasis in the original]. The correspondence truth property is associated with the scientific domain on p. 40, the coherence truth property (specifically, cohering with the axioms of Peano arithmetic) is associated with the (first-order arithmetical) mathematical domain on pp. 68–71, and the superassertible truth property is associated with the ethical domain on p. 72.

(2) $1 + 1 = 2$.

(3) Murder is wrong.

Let us take (1) to be (just) a scientific sentence—i.e., a sentence that is associated with scientific subject matter (and not associated with any other subject matter). Similarly, let us take (2) to be a mathematical sentence, and (3) to be an ethical sentence. Moreover, let us take all these sentences to be true. Accordingly, (1) is correspondence true, in that it has the property of correspondence truth, (2) is coherence true, in that it has the property of coherence truth, and (3) is superassertible true, in that it has the property of superassertible truth.

As I understand pluralism, it is essential that there are distinct ways of being true, in that there are distinct *truth properties*, where these distinct ways of being true are associated with distinct domains of subject matter. Accordingly, a view that does not maintain that there are distinct truth properties, or does but does not associate them with distinct domains of subject matter, is not pluralism on my understanding.

But other claims are not essential to pluralism. Notably, the specific truth-characterizing principles Wright offers—which I have noted differs in different places—are not essential to pluralism. Not least, it might be hard to determine exactly the right principles.[11] Similarly, it is not essential exactly which domains there are, which truth properties there are, and which domains are associated with which truth properties. Little is lost if mathematical truth is taken to be superassertible truth rather than coherence truth, or if ethical truth is taken to be something other than superassertible truth. What matters more is that truth varies according to the domain, where there are distinct domains.

[11]Perhaps it is not even essential to pluralism that principles characterize truth. Depending on how exactly the truth-characterizing principles are understood, the insistence that truth properties satisfy them are more or less controversial. Notably, the insistence is controversial if the principles are understood as analytic of the concept of truth, as Wright (2001, 759) suggests; see Williamson (2007) for general arguments against the identification of certain principles as analytic of some concept. A key idea is that conceptual competence is a holistic matter irreducible to the acceptance of certain principles; see Nolan (2009) for related criticisms regarding the appeal to platitude-style principles in metaphysics. Presumably, little is lost if truth properties are primitive and unanalyzable; see Moore (1899); Russell (1904); Frege (1956); Davidson (1996) on primitivism about truth.

So much for the bare-bones formulation. A less bare-bones formulation might mention truth predicates as well. Pluralists might maintain that the single truth predicate "true" expresses the distinct domain-specific truth properties. Indeed, I think this is a natural claim for pluralists to make. Or, pluralists might maintain that distinct truth predicates express correspondingly distinct domain-specific truth properties. Nonetheless, on my understanding, pluralism is not committed to any particular view of truth predicates, and so is not committed to any particular view about the relation between truth predicates and truth properties.

Distinguishing pluralism from other views

As I understand pluralism, it is distinguished from relativism, which is the view that the truth value of sentences is determined only relative to an individual or group of individuals.[12] Indeed, pluralism is compatible with truth's being a monadic property, and "true"'s being a monadic predicate. Pluralism is also distinguished from the view that "true" is context-sensitive, as well as the view that "true" is ambiguous.[13] Pluralism may or may not be combined with relativism, or the view that "true" is context-sensitive or ambiguous. As I understand it, pluralism is primarily a view about the number of truth properties there are, and only secondarily if at all a view about the nature of truth properties, or the number or nature of truth predicates.

More importantly, pluralism—as I understand it—is distinguished from other views in the literature that are often described as pluralist. These other views, like pluralism, maintain that the way in which a sentence is true varies according to the domain under consideration. Crucially, however, unlike pluralism, they do not maintain that there are distinct truth properties. Rather, they maintain that there are distinct properties that stand in some relation—such as "realization"—to exactly one truth property, or that there are distinct kinds of correspondence—but again exactly one truth property, which is uniformly identified with the correspondence truth property.

[12] See MacFarlane (2014) for a recent development of relativism.
[13] I discuss these views in Chapter 3.

Pluralism is distinguished from *functionalism* about truth, which is the view that truth is a functional property.[14] What is distinctive to functionalism is the claim that the truth property is multiply realizable: it is *the* property that satisfies certain truth-characterizing principles, but distinct properties can realize this property across distinct domains. A property R_i realizes the truth property T just if (φ has T iff φ has R_i and is associated with the domain with which R_i is associated). Functionalism is sometimes described as a version of pluralism, since distinct properties can realize the truth role, even though truth is identified with the playing of the role itself. Granted, functionalism resembles Wright's pluralism, in that truth is characterized by the satisfaction of certain truth-characterizing principles, and properties that ground the truth of sentences are domain-specific. However, as Pedersen and Wright (2010, 2013b) observe, functionalism is not a version of pluralism, on the present understanding, in that according to functionalism there are no distinct *truth* properties but only distinct properties that realize the truth role. Indeed, Lynch (2009), the main proponent of functionalism, explicitly maintains that "there is only one property of truth" (85). And Wright (2013, 138–145) denies the characterization of pluralism in terms of functionalism.

Similarly, pluralism is distinguished from *correspondence pluralism* about truth, which is the view that, although the truth property just is the property of correspondence truth, there are distinct kinds of correspondence.[15] What is distinctive to correspondence pluralism is the claim that there are distinct kinds of correspondence—either one relation of correspondence but distinct kinds of correspondence or distinct relations of correspondence—associated with distinct domains. Granted, correspondence pluralism also resembles Wright's pluralism, in that truth varies according to the domain. However, correspondence pluralism is not a version of pluralism, on the present understanding, in that according to correspondence pluralism there are no distinct truth properties but only distinct kinds of correspondence.

[14]The most prominent proponent of functionalism is Lynch (2000, 2001a, 2004a,b, 2005a,b, 2006, 2009, 2013). Other proponents include Pettit (1996); Devlin (2003).

[15]The most prominent proponent of correspondence pluralism is Sher (1998, 2004, 2005, 2013). Other proponents include Horgan (2001); Barnard and Horgan (2013).

2 Central challenges

In this section, I discuss central challenges for pluralism. In the first subsection, I present conceptual challenges to clarify the conceptions of truth properties, domains, and the association between them. In the second subsection, I present logical challenges to provide a unified account of the logic and semantics of sentences, whether or not they are "mixed," in that they involve distinct subject matters. The logical challenges set the stage for the rest of this chapter. In the third subsection, I propose constraints and desiderata to guide answers to the challenges.

Although there are also challenges about the motivations for pluralism in the first place, I set them aside in this chapter.[16] There is not enough space to discuss these challenges, and discussing them would detract from my interest in developing pluralism as I have presented it, through Wright. Whether or not one is motivated to maintain pluralism in the end, I take it to be an interesting view worth seriously investigating.

2.1 Conceptual challenges

I present three conceptual challenges. They are to clarify the conceptions of truth properties, domains, and the association between them.

Domains

The first conceptual challenge is to clarify the conception of the domains, and especially the relation between sentences and domains.

Is every sentence associated with some domain, or are some sentences not associated with any domain? Wright seems to assume that every sentence is associated with some domain, because every sentence seems to be about some subject matter.[17] But log-

[16]The most serious motivational challenge doubts the appropriateness of attributing apparent differences across distinct domains to distinct truth properties, rather than to something else. See Sainsbury (1996, 900) and Quine (2013, 131).

[17]Wright does not say much about what it is for a sentence to be about some subject matter. But see Yablo (2014) for a recent development of a comprehensive theory of aboutness.

ical truths challenge this assumption. Consider a logical truth such as "Water is H_2O or water is not H_2O." On the one hand, there is inclination to take this sentence to be about whatever its subsentences are about—scientific subject matter. If so, then perhaps it is associated with the scientific domain. On the other hand, there is also inclination to take this sentence to be about nothing in particular, perhaps because logical truths are general and topic-neutral. If so, then perhaps it is associated with a domain of general subject matter, which may or may not subsume all other domains, or alternatively, a domain of logical subject matter. Then again, perhaps it is not associated with any domain. Similarly, consider a quantified logical truth such as "Everything is self-identical." On the one hand, there is inclination to take this sentence to be about everything, or alternatively, nothing in particular. If so, then perhaps it is associated with a domain of general subject matter, or alternatively, a domain of logical or metaphysical subject matter. Then again, perhaps it is not associated with any domain.

Relatedly, are some sentences associated with more than one domain? Wright seems to assume that every sentence is associated with at most one subject matter. But sentences that are about distinct subject matters challenge this assumption. Consider sentences such as the following:

(4) The number π is beautiful.

Presumably, (4) is about two subject matters, namely, mathematical and aesthetic subject matters. Similarly, consider the following sentence from Williamson (1994a, 141):

(5) Yesterday only five of the twelve torturers inflicted pain, but the other seven who amused them with funny stories were equally bad.

Apparently, (5) is about mathematical subject matter ("five" and "seven"), ethical subject matter ("bad"), temporal subject matter ("yesterday"), comical subject matter ("funny"), and physiological subject matter ("pain"). But if some sentences are about distinct subject matters, then perhaps such sentences are associated with distinct domains as well.[18]

[18]Even sentences that are apparently straightforward, in that they are apparently about exactly one

Truth properties and domains

The second conceptual challenge is to clarify the association between truth properties and domains. Wright seems to assume that there is a one-to-one correspondence between them. But his view also seems compatible with the failure of this correspondence.

Perhaps even in a given domain, there are distinct satisfiers of the truth-characterizing principles, so that the domain is associated with distinct truth properties. For example, perhaps in the scientific domain, both the correspondence truth property and the coherence truth property satisfy the principles, so that the domain is associated with both the correspondence truth property and the coherence truth property. Indeed, perhaps being correspondence true entails being coherence true. Conversely, perhaps a given truth property is associated with distinct domains. For example, perhaps the coherence truth property satisfies the principles in both the mathematical domain and the ethical domain, so that the coherence truth property is associated with both the mathematical domain and the ethical domain.

Truth properties

The third conceptual challenge is to clarify the conception of truth properties. Wright's truth-characterizing principles might be informative with respect to which properties are truth properties, but they are less informative with respect to what it takes for something to be a property in the first place. Wright is not explicit about what he takes a property to be.

One issue is whether the conception of properties is *sparse* or *abundant*. The distinction between the two conceptions of properties comes from Lewis (1986), who takes the distinction to be perhaps primitive (63):

subject matter, may in fact be about distinct subject matters. For example, (2) may be about not just mathematical subject matter, but arithmetical subject matter as well. Similarly, (3) may be about not just ethical subject matter, but normative subject matter as well. More generally, it is unclear what pluralists should say about sentences that are apparently about subject matters that are properly contained in other subject matters. Still, in order to focus on the issues that concern me most, let us grant for the sake of argument that (2) is just about mathematical subject matter, and (3) is just about ethical subject matter.

> Sometimes we conceive of properties as *abundant*, sometimes as *sparse*. The abundant properties may be as extrinsic, as gruesomely gerrymandered, as miscellaneously disjunctive, as you please. They pay no heed to the qualitative joints, but carve things up every which way. Sharing of them has nothing to do with similarity. ...The abundant properties far outrun the predicates of any language we could possibly possess. There is one of them for any condition we could write down.... In fact, the properties are as abundant as the sets themselves, because for any set whatever, there is the property of belonging to that set. ...
>
> The sparse properties are another story. Sharing of them makes for qualitative similarity, they carve at the joints, they are intrinsic, they are highly specific, the sets of their instances are *ipso facto* not entirely miscellaneous, there are only just enough of them to characterise things completely and without redundancy. [emphasis in the original] (60–61)

On an abundant conception of properties, properties come for cheap, while on a sparse conception of properties, properties are rarer and more valuable commodities—in uniting their instances by resemblance, qualitative similarity, or the like. Lewis' presentation of the distinction between the two conceptions, however, suggests several versions of the distinction, depending on the formulation of the abundant conception. On the version I adopt for my purposes in this chapter, the abundant conception accepts but the sparse conception rejects the following principle, which maintains that properties are closed under the Boolean operations:[19]

> *Abundance.* If P and Q are properties, then so are $\neg P$ (whose extension contains all and only the things that do not instantiate P), $P \wedge Q$ (whose extension contains all and only the things that instantiate P and instantiate Q), and $P \vee Q$ (whose extension contains all and only the things that instantiate P or instantiate Q).

Whether the conception of properties is sparse or abundant may matter for pluralism. For on an abundant conception of properties, pluralism risks being trivially true. Perhaps there are distinct truth properties simply because there are distinct properties such as the property of being true and about trees, and the property of being true and about trees or true and about cars. Granted, perhaps the relevant properties are not

[19]On another version of the distinction, the abundant conception maintains that every predicate expresses a property. On still another version of the distinction, the abundant conception maintains that every set is the extension of a property.

truth properties, because they do not satisfy the truth-characterizing principles. Still, perhaps some of the relevant properties satisfy the truth-characterizing principles, and so are truth properties. To avoid the charge of triviality, pluralists might consider maintaining a sparse conception of properties instead, since doing so would help rule out what in their view might be spurious properties.[20]

A related issue is whether there is, in addition to domain-specific truth properties, also a generic truth property. Plausibly, a property whose extension contains all and only the true sentences, regardless of their associations with domains, is a generic truth property.[21] By contrast, a domain-specific truth property—such as the correspondence truth property—is a truth property that is associated with a specific domain.

There is a compelling argument for the conclusion that, whether or not pluralists explicitly posit it, they are committed to the existence of a generic truth property.[22] The *Generalization Argument* is the following. Let $T_{D_1}, T_{D_2}, \ldots$ be distinct domain-specific truth properties associated with, respectively, domains $D_1, D_2, \ldots$. Similarly, for each D_i, let A_{D_i}, be the property of being associated with domain D_i. By *Abundance*, we have that:

(GA1) There are distinct properties $T_{D_i} \wedge A_{D_i}$, for each D_i.

To illustrate, by (GA1), there is the property of being correspondence true and associated with the scientific domain. By *Abundance* again, we have that:

(GA2) There is a property $T_D = (T_{D_1} \wedge A_{D_1}) \vee (T_{D_2} \wedge A_{D_2}) \vee \ldots$.

That is, there is a property whose extension contains all and only the sentences that have a domain-specific truth property and are associated with the relevant domain. But pre-

[20]However, Wright (2001, 753) suggests that he maintains an abundant conception of properties: "For once the currency of a concept of truth is granted, it ought to be allowed that all truths have at least the following property in common: the property of falling under this concept."

[21]If there is a domain that all sentences are associated with—an intuitively generic domain—then a truth property associated with such a domain is a generic truth property. However, since the existence of a generic truth property is compatible with the nonexistence of a generic domain, I assume for simplicity that there are no generic domains, so that a domain-specific truth property is not a generic truth property. Also, if distinct properties can be coextensive, and even cointensive, there might be distinct generic truth properties. However, I assume for simplicity that there is at most one generic truth property. What is in contention with respect to pluralism is whether there is even one generic truth property.

[22]See Pedersen (2006, 2010); Edwards (2012a); Wright (2012); Pedersen and Wright (2013a,b); Cotnoir (2013a) for versions of this argument.

sumably, a sentence that is domain-specifically true in all the ways associated with all the domains it is associated with is generically true, so that T_D's extension contains all and only the true sentences. Assuming that T_D satisfies the truth-characterizing principles, we have that:

(GA3) T_D is a generic truth property.

So we conclude that:

(GAC) There is a generic truth property.

Does the existence of a generic truth property threaten pluralism? Strictly speaking, it does not. For pluralism is just the view that there are distinct truth properties, where a truth property is a property that satisfies the truth-characterizing principles. Accordingly, pluralism is at least *consistent* with the existence of a generic truth property.

Still, perhaps the existence of a generic truth property is undesirable for pluralism. Even if it does not threaten pluralism itself, it might threaten the spirit of pluralism. For the motivation for pluralism is that there are (explanatorily or metaphysically) significant distinctions between truth properties, in that there are distinct domain-specific truth properties. A generic truth property competes with such properties for significance: a generic truth property threatens to make domain-specific truth properties redundant, so that it is unclear why one should maintain pluralism in the first place. Indeed, because of this worry, pluralists might take the Generalization Argument to threaten the stability of pluralism.[23] The worry might motivate what is known as "strong" pluralism, which denies that there is also a generic truth property; by contrast, "weak" or "moderate" pluralism accepts that there is also such a property.[24]

To neutralize the threat from the existence of a generic truth property, pluralists might reject either (GA1) or (GA2). For they might have independent reason to maintain a sparse rather than an abundant conception of properties. Or, they might reject

[23] See Tappolet (1997, 2000); Lynch (2005a); Haack (2005, 2008); Pedersen (2006, 2010); Wright (2012) on versions of what is known as the "instability challenge."

[24] See Pedersen (2012); Pedersen and Wright (2010, 2013b,c); Cotnoir (2013b) on the distinction between strong pluralism and weak or moderate pluralism.

(GA3). For they might contend that the resulting property does not satisfy the truth-characterizing principles, and so is not a truth property at all.

Alternatively, pluralists might accept the Generalization Argument but neutralize the threat from the argument by downplaying the significance of a generic truth property. The idea is that, even though there is a generic truth property in addition to there being domain-specific truth properties, the domain-specific truth properties are properties that are more "substantial" than any generic truth property.[25]

One way to do this is to maintain a hybrid conception of properties according to which there are both abundant and sparse properties, but that sparseness comes in degrees.[26] Lewis (1986) suggests just such a conception:

> If we have the abundant properties then we have one of them for each of the sparse properties. So we may as well say that the sparse properties are just some — a very small minority — of the abundant properties. We need no other entities, just an inegalitarian distinction among the ones we've already got. When a property belongs to the small minority, I call it a *natural property*. Probably it would be best to say that the distinction between natural properties and others admits of degree. Some few properties are *perfectly* natural. Others, even though they may be somewhat disjunctive or extrinsic, are at least somewhat natural in a derivative way[.] [emphasis in the original] (60–61)

On a hybrid conception, pluralists can accept the existence of a generic truth property, but downplay the significance of such a property by maintaining that it is less natural than the distinct domain-specific truth properties.

Another way to downplay the significance of a generic truth property is to maintain a conception of properties according to which the instantiation of some properties ground the instantiation of other properties.[27] A conception of properties couched in terms of grounding, fundamentality, and related ideology has become increasingly widely accepted in recent years.[28] The specific conception need not concern us here. What matters is that there is some relation or other of priority that can be invoked.

[25]See Edwards (2012b) on the "substantiality" of properties.

[26]See Cotnoir (2009); Pedersen (2006, 2010); Edwards (2012a,b); Pedersen and Wright (2013a).

[27]See Pedersen (2010, 2012); Pedersen and Wright (2013a).

[28]Prominent proponents of such a conception include Fine (2001); Schaffer (2009); Rosen (2010); Sider (2012).

Given that there is such a conception, pluralists can accept the existence of a generic truth property but downplay the significance of such a property by maintaining that its instantiation is grounded in the instantiation of domain-specific truth properties. Indeed, Wright (1992, 143) considers a suggestion along these lines.

I defer answering the other two central conceptual challenges for pluralism until later. But let me now suggest that—notwithstanding a more developed answer on behalf of pluralists to the present challenge to clarify the conception of truth properties—the abundant conception of properties is independently attractive, and even pluralists should maintain it. For the desire to maintain sufficiently rich theories of semantic and mental content motivates maintaining an abundant conception of properties. But then, given that pluralists should accept that the property T_D resulting from the Generalization Argument satisfies the truth-characterizing principles and so is a truth property, the Generalization Argument shows that there is a generic truth property.[29] So even pluralists are committed to the existence of a generic truth property.

In my view, there is no good reason—even for pluralists—to deny that a generic truth property at least exists. Probably the best reason that pluralists could offer is that doing so helps mark out pluralism as a clear and distinct view. At least this is the kind of reason often mentioned, with some pluralists worrying that the acceptance of the existence of a generic truth property is too big a concession. But even this reason is weak, and more importantly outweighed by good reasons to accept the existence of such a property.

First, pretheoretic intuition strongly suggests that truth is unified in a significant way. Even pluralists such as Wright (1992) endorse this point, in taking the concept of truth to be uniquely characterized by certain principles. Second, generic truth seems theoretically important—even indispensable—in standard theories of assertion, mental attitudes, and semantic and mental content. Notably, it is posited that one should assert only what is true, and that to believe something is to believe that it is true. Such theories might be in jeopardy without generic truth. While pluralists might develop al-

[29] Pluralists such as Pedersen (2010); Edwards (2012a); Pedersen and Wright (2013a) suggest that a property like T_D satisfies the truth-characterizing principles and so is a truth property.

ternatives to such theories that do not invoke generic truth, it is unclear to what extent pluralists should embrace such revisionism. Third, standard conceptions of predicates and properties all countenance the definability and existence of a generic truth predicate or property. Again, on the abundant conception of properties, the Generalization Argument helps show that there is a generic truth property.

To be sure, perhaps considerations arising from the semantic and intensional paradoxes suggest that there is no such thing as generic truth, as either a predicate or property, after all.[30] Notably, Tarski suggests that there is no generic truth predicate, but rather only a hierarchy of typed truth predicates. However, both the motivation and the resulting kind of pluralism are radically different from the motivation and associated standard kind of pluralism that I am focusing on in this chapter. If one maintains some kind of pluralism because of considerations arising from the paradoxes, then the resulting kind of pluralism is radically different from pluralism as I am understanding it, and so is beyond the scope of this chapter.[31]

In sum, the three central conceptual challenges for pluralism are to clarify the conceptions of domains, truth properties, and the associations between them.

2.2 Logical challenges

I now turn to the logical challenges for pluralism, which I focus on in this chapter. They are linked to the conceptual challenges. For the logical challenges arise from considering mixtures of sentences about distinct subject matters. Considering such sentences brings questions about the associations between sentences, domains, and subject matters, as well as the significance of generic truth, to the forefront.

Throughout this chapter, I assume that pluralists reject truth as it is represented in the familiar account of logic and semantics in favor of an alternative representation that captures its distinctness and domain specificity. I set aside the suggestion that plu-

[30]See Tarski (1944); Parsons (1974); Burge (1979); Glanzberg (2001, 2004a,c); Cook (2008, 2009); Beall (2013); Cotnoir (2013a). Also see Chapters 2 and 3 in this thesis.

[31]See Beall and Glanzberg (2008); Cook (2011); Beall (2013); Cotnoir (2013a); Wyatt and Lynch (2016) on possible links between alethic pluralism and the semantic paradoxes.

ralists can unqualifiedly grant truth as it is represented in the familiar account, as just T ("truth"). Not least, pluralists themselves concede that they should say something more. Crucially, the suggestion does not do justice to pluralists' commitment that there are distinct domain-specific truth properties. For the commitment suggests that something is missing from the standard representation of truth, so that the familiar account stands in need of revision. Given the intertwining of truth with logic and semantics, alethic commitments motivate corresponding logical and semantic commitments.

I present four related logical challenges. The first challenge, concerning atomic sentences, has received modest attention in the literature. The second challenge, concerning quantified sentences, has received little attention in the literature. The third and fourth challenges, concerning complex sentences and logical consequence, have received most of the attention in the literature. I view the third and fourth challenges as twin challenges, because the standard accounts of complex sentences and of logical consequence are complementary. Any answer to one of the twin challenges should complement an answer to the other, rather than leave one clueless as to how the answer to one challenge helps answering the other. But the point about complementarity generalizes. I view all four challenges as components of a broader challenge to maintain a unified account of the semantics of atomic sentences, complex sentences, and quantified sentences, as well as the logic.

Atomic sentences

The challenge to maintain a unified account of atomic sentences arises from considering atomic sentences, which can be about distinct subject matters.[32]

Pluralism is often illustrated using straightforward sentences. For example, recall that the scientific sentence (1) is correspondence true. Such examples are straightforward in that the sentences are about exactly one subject matter, and so—given that they are true—are true in exactly one way. But what about when matters are less straight-

[32] This challenge is discussed in Lynch (2005b); Sher (2005, 2013); Wyatt (2013); Pedersen and Wright (2013b).

forward? Not all sentences are about exactly one subject matter. Setting aside complex sentences for now, there are atomic sentences that are mixed, in that the sentences are about distinct subject matters; these contrast with sentences that are pure, which are not about distinct subject matters. But if mixed sentences are true, it is unclear in what way or ways they are true.

Consider the following mixed atomic sentences:

(4) The number π is beautiful.

(5) Yesterday only five of the twelve torturers inflicted pain, but the other seven who amused them with funny stories were equally bad.

(6) Causing pain is bad.

Recall (4) and (5) from before. They as well (6) are about distinct subject matters. For example, (6) seems to be about scientific subject matter ("Causing pain") and ethical subject matter ("bad").[33] Before, I used such sentences to present the challenge of whether some sentences are associated with distinct domains. Now, my focus is on truth properties rather than domains. It is unclear in what way or ways true mixed sentences, such as (4)–(6), are true. For example, given that (6) is true, in what way or ways is it true? Perhaps it is both correspondence true and superassertible true. Alternatively, perhaps it is just superassertible true. Finally, perhaps it is true in neither of these ways, but if so, then in what way or ways is it true?

The challenge is for pluralists to maintain an account of mixed atomic sentences, and more generally, a unified account of atomic sentences, whether pure or mixed. We can put the challenge thus:

First logical challenge. Maintain a unified account of atomic sentences.

[33] One might, following Sher (2005, 328), break down scientific subject matter into physical ("Causing") and mental ("pain") subject matters. However, doing so introduces unnecessary complication, so for simplicity, I assume that (6) is about scientific subject matter more generally (and ethical subject matter).

This challenge is linked to the conceptual challenges. If every sentence is associated with exactly one domain, and every domain is associated with exactly one subject matter, then presumably, true mixed sentences are true—domain-specifically true—in exactly one way. But if some sentences are associated with distinct domains, then presumably, true mixed sentences are domain-specifically true in distinct ways.

Quantified sentences

The challenge to maintain a unified account of quantified sentences arises from considering quantified sentences, which can be about distinct subject matters.[34]

Consider the following mixed quantified sentences:[35]

(7) Everything is self-identical.

(8) Everything the Pope utters is true.

As with (4)–(6), (7) also seems to be about distinct subject matters. Straightforwardly, (7) seems to be about everything. Crucially, (7)–(8) also seem to call for a generic truth property. For if (7) is about everything, and so is associated with a generic domain, then the domain-specific property associated with the generic domain seems to be a generic truth property. While I have suggested that pluralists should grant the *existence* of a generic truth property, it remains unclear to what extent they should grant such a property theoretical *significance*. What about (8)?[36] Since the Pope could utter sentences about scientific, mathematical, and other subject matters, the truth property ascribed to the sentences must be sufficiently general. But a sufficiently general truth property seems to be no less than a generic truth property. Indeed, Wright (2013, 133) explicitly

[34]This challenge is discussed in Lynch (2009); Shapiro (2009, 2011); Cotnoir (2013b); Pedersen and Wright (2013b). Cotnoir (2013b) describes the challenge as "a particularly difficult variant" (563) of the challenge to maintain a standard account of complex sentences.

[35]Pure quantified sentences, such as "Every number is either even or odd," do not seem to pose a similar challenge. For they can be treated similarly to other pure sentences, such as (1), (2), and (3).

[36]One might worry that the presence of a truth predicate in (8) raises concerns about the semantic paradoxes. But nothing crucial hinges on the predicate's being a truth predicate. Other predicates, such as "fun to entertain," are substitutable for "true."

maintains that true quantified sentences such as (8), at least, are generically true. Again, it is unclear to what extent pluralists should grant a generic truth property significance.

The challenge is for pluralists to maintain an account of mixed quantified sentences, and more generally, a unified account of quantified sentences, whether pure or mixed. We can put the challenge thus:

> *Second logical challenge.* Maintain a unified account of quantified sentences.

Since this challenge is bound up with the issue of the significance of a generic truth property, it is also linked to the conceptual challenges.

Complex sentences

The challenge to maintain a standard account of complex sentences arises from considering logical combinations of sentences, which can be about distinct subject matters.[37]

Recall the following sentences, which are pure:

(1) Water is H_2O.

(2) $1 + 1 = 2$.

Now consider the following mixed complex sentences:

(9) Water is H_2O and $1 + 1 = 2$.

(10) Water is H_2O or $1 + 1 = 2$.

Given that the scientific sentence (1) is correspondence true and the mathematical sentence (2) is coherence true, their conjunction (9) and their disjunction (10) is also true. But in what way or ways are (9) and (10) true? The options available here are similar to the options available when considering mixed atomic sentences. Perhaps (9) is both

[37] This challenge is discussed in Williamson (1994a); Tappolet (2000); Lynch (2001a, 2004b, 2005b, 2008, 2009, 2013); Edwards (2008, 2009); Cotnoir (2009); Pedersen and Wright (2013b); Wright (2013). These philosophers often raise the challenge only for conjunction or disjunction, typically focusing only on conjunction.

correspondence true and coherence true. Alternatively, perhaps it is just correspondence true. Finally, perhaps it is true in neither of these ways, but if so, then in what way or ways is it true? Here, one version of this last option is salient. Given that a conjunction is true iff both conjuncts are true and a disjunction is true iff either disjunct is true, perhaps there is a truth property common to (1), (2), (9), and (10). But such a property seems to be a generic truth property. Again, it is unclear to what extent pluralists should grant such a property significance.

The challenge is for pluralists to maintain an account of mixed complex sentences, and more generally, a standard account of complex sentences, whether pure or mixed. We can put the challenge thus:

> *Third logical challenge.* Maintain a standard account of complex sentences, according to which the following obtains:
>
> (i) a negation is true iff the negand is false;
>
> (ii) a conjunction is true iff each conjunct is true;
>
> (iii) a disjunction is true iff either disjunct is true.

Plausibly, what counts as a standard account should be understood in a looser rather than a stricter sense. To permit a reasonable amount of flexibility, the standard account of complex sentences sought need not be identical to the account from the familiar account of logic and semantics.

Let me elaborate. Throughout this chapter, let us assume a sentential language $\mathscr{L}$, with atomic sentences $Atomic(\mathscr{L})$ and sentences $Sent(\mathscr{L})$. Also, let us adopt the standard convention of using the metavariables $\alpha, \beta, \gamma, \ldots$ (with or without subscripts) to range over atomic sentences, $\varphi, \psi, \chi, \ldots$ (with or without subscripts) to range over all sentences, and Γ to range over sets of sentences. For simplicity, we focus on the logical operators that are negation, conjunction, and disjunction. The material conditional and the material biconditional are defined in terms of these operators in the usual way, as follows: $\varphi \rightarrow \psi := \neg\varphi \vee \psi$ and $\varphi \leftrightarrow \psi := (\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$.

On the familiar account of complex sentences, a model $M : Atomic(\mathscr{L}) \to \{T, F\}$ is a function from atomic sentences to truth values, where T represents the generic truth property and F represents the generic falsity property.[38] Given a model M, we get the semantic value function $|\cdot|_M : Sent(\mathscr{L}) \to \{T, F\}$ from sentences to truth values based on the model M. We define $|\cdot|_M$ by $|\alpha|_M = M(\alpha)$ (where α is atomic) and the following:

(LO) (i) $|\neg\varphi|_M = T$ iff $|\varphi|_M = F$;

(ii) $|\varphi \wedge \psi|_M = T$ iff $|\varphi|_M = T$ and $|\psi|_M = T$;

(iii) $|\varphi \vee \psi|_M = T$ iff $|\varphi|_M = T$ or $|\psi|_M = T$.

Recalling the point about permitting a reasonable amount of flexibility, it is enough that the account of complex sentences sought is similar enough to the familiar account to count as standard. Granted, whether an account is similar enough to the familiar one is vague. Still, examples show that we have enough of a grip of what counts as standard for the notion to be fruitfully employed. The strong Kleene account of complex sentences is a standard account, but a noncompositional account probably is not.

Like the first logical challenge, this challenge is linked to the conceptual challenge to clarify the conception of domains. And like the second logical challenge, this challenge is also bound up with the conceptual challenge to clarify the conception of truth properties.

Logical consequence

The challenge to maintain a standard account of logical consequence arises from considering mixed arguments, where the premises and conclusion can be about distinct subject matters.[39]

[38] Throughout this chapter, where there is little risk of confusion, I am somewhat loose with the distinction between truth properties themselves and truth values that formally represent them. For example, I may write of T as either being a truth property or representing a truth property. When discussing pluralism informally, it is more natural to talk about truth properties, but when discussing the provision of a formal semantics, it is more natural to talk about truth values.

[39] This challenge is discussed in Sainsbury (1996); Tappolet (1997); Beall (2000); Lynch (2000, 2004b, 2005a, 2008, 2009); Pedersen (2006); Cotnoir (2013b); Wright (2013).

Consider the following argument, which has premises (1) and (2) and conclusion (9):

(1) Water is H_2O.

(2) $1 + 1 = 2$.

(9) Water is H_2O and $1 + 1 = 2$.

Clearly, the argument from (1) and (2) to (9) is valid: (9) is a logical consequence of (1) and (2). On the standard account of logical consequence, logical consequence necessarily preserves truth. So perhaps some single truth property is preserved from (1) and (2) to (9). But what property is that? Perhaps the truth property is common to (1), (2), and (9). If so, then the property seems to be a generic truth property. Indeed, Wright (1992, 74–75) seems to accept the suggestion that logical consequence necessarily preserves a generic truth property. And Wright (2013, 133) explicitly maintains that logical consequence necessarily preserves an abundant generic truth property. Once again, it is unclear to what extent pluralists should grant a generic truth property significance.

The challenge is for pluralists to maintain an account of mixed arguments, and more generally, a standard account of logical consequence. We can put the challenge thus:

> *Fourth challenge.* Maintain a standard account of logical consequence, according to which logical consequence necessarily preserves truth.

To permit a reasonable amount of flexibility, the standard account of logical consequence sought need not be identical to the account from the familiar account of logic and semantics. On the familiar account of logical consequence, we have the following:

(LC) $\Gamma \vDash \varphi$ iff for every model M, if $|\gamma_i|_M = T$ for every γ_i in Γ, then $|\varphi|_M = T$.

Logical consequence preserves truth across all models, and is classical. It is enough that the account sought is similar enough to the familiar account to count as standard. Examples show that we have enough of a grip of what counts as standard for the notion to be fruitfully employed. A many-valued account of logical consequence in terms of designation is standard, but not an account that validates the inference from φ to $\neg\varphi$.

In sum, the four central logical challenges for pluralism are to maintain unified accounts of atomic sentences and quantified sentences, and standard accounts of complex sentences and of logical consequence.

Before continuing, let me address the suggestion that the logical challenges—or at least the twin challenges—are merely apparent. Perhaps pluralists need not answer them, because the standard accounts of complex sentences and of logical consequence in particular are too wedded to a view of truth that pluralists maintain is mistaken to warrant taking those accounts to impose genuine constraints on pluralism. However, I think this suggestion is too pessimistic. The standard accounts are compelling enough that pluralists should want to maintain them. Moreover, it is independently interesting to investigate the extent to which pluralists can answer the challenges. If pluralists cannot answer them at all, then that is a significant result that clarifies what is at stake—namely, compelling accounts of complex sentences and of logical consequence. But if they can answer all the central logical challenges, then perhaps any case against pluralism should be made on broadly linguistic or metaphysical, rather than logical, grounds.

2.3 Constraints and desiderata

I now pave the way for evaluating answers to the central challenges, and especially the logical ones—which I focus on, because I think they should inform the answering of the conceptual challenges. Specifically, I propose two constraints and three desiderata to guide answers to the central logical challenges.

I propose two related constraints on a standard account of complex sentences, which accordingly constrain answers to the third challenge. To motivate the first and second constraints, consider that it is natural for pluralists to maintain two further commitments. First, just as truth properties are theoretically significant (for logico-semantic theorizing), so domains are theoretically significant. Second, just as there are distinct domain-specific truth properties, there are corresponding distinct domain-specific falsity properties. For convenience, let us say that a domain-specific truth property and a domain-specific falsity property are *duals* iff both are associated with the same domain. To illustrate, given that the correspondence truth property and the correspondence falsity property are both associated with the scientific domain, they are duals.

The first and second constraints are principles of conservativity:

> *Constraint 1.* All logical combinations—i.e., negations, conjunctions, and disjunctions—of sentences associated with the same domain are also associated with the same domain.
>
> *Constraint 2.* The negation of a domain-specifically true sentence is domain-specifically false, the negation of a domain-specifically false sentence is domain-specifically true, and conjunctions and disjunctions of domain-specifically true sentences are domain-specifically true—where the relevant domain-specific truth and falsity properties are duals.

These constraints are very plausible. To illustrate, Constraint 1 holds that if "$1+1=2$" and "$2+2=4$" are sentences associated with just the mathematical domain, then so are "$1+1\neq 2$," "$1+1=2$ and $2+2=4$," and "$1+1=2$ or $2+2=4$." Similarly, Constraint 2 holds that if "$1+1=2$" and "$2+2=4$" are just mathematically true, then "$1+1\neq 2$" is mathematically false, and "$1+1=2$ and $2+2=4$" and "$1+1=2$ or $2+2=4$" are mathematically true. The constraint rules out the possibility that "$1+1\neq 2$" is scientifically false, or that "$1+1=2$ and $2+2=4$" is ethically true.

Next, I propose two related desiderata for a standard account of complex sentences. To motivate the first and second desiderata, consider that it is natural for pluralists to have the notion of a pure domain—which parallels the notion of an atomic sentence—and the corresponding notion of an impure domain. Just as atomic sentences are the fundamental constituents of all sentences, so pure domains are associated with the most basic kinds of subject matter. A pure domain is associated with one subject matter that is irreducible to other subject matters, while an impure domain is associated with several subject matters. To illustrate, if the mathematical domain is associated with mathematical subject matter, which is somehow basic, then that domain is pure. A domain associated with both scientific and mathematical subject matters is, by contrast, impure.

The first and second desiderata are principles of novelty:

> *Desideratum A.* All logical combinations of sentences associated with distinct pure domains are associated with domains distinct from those associated with any of the operands.

> *Desideratum B.* Conjunctions and disjunctions of sentences that are domain-specifically true in distinct ways, where the associated domains are pure, are domain-specifically true in ways distinct from any of the ways in which the operands are domain-specifically true.

These desiderata are plausible: mixed sentences, at least when the subsentences are associated with pure domains, arguably induce new domains. To illustrate, Desideratum A holds that if "$1 + 1 = 2$" is as before and "Louis CK is funny" is associated with the comical domain, where the mathematical and comical domains are pure, then "$1+1 = 2$ and Louis CK is funny" is associated with a domain distinct from either domain; the conjunction is associated with neither the mathematical domain nor the comical domain. Similarly, Desideratum B holds that if "$1 + 1 = 2$" is mathematically true and "Louis CK is funny" is comically true, then "$1+1 = 2$ and Louis CK is funny" is domain-specifically true, but neither mathematically true nor comically true.

I also propose a third, more general, desideratum. To motivate this desideratum, consider that pluralists maintain that domain-specific truth properties are very different. As Cotnoir (2009, 478) puts it, "Pluralists often cite the varied and radical differences between propositions as their main motivation for positing multiple truth properties." This suggests that, for pluralists, there should be no strong "intuitive" ordering, such as one by metaphysical weight, on the set of domain-specific truth properties. The third desideratum is a principle of diversity:

> *Desideratum C.* There is no intuitive ordering, such as one by metaphysical weight,

on the set $\mathscr{T}$ of domain-specific truth properties that is total, where a total order $\leq$ on $\mathscr{T}$ is such that, for any T_1, T_2 in $\mathscr{T}$, either $T_1 \leq T_2$ or $T_2 \leq T_1$.

This desideratum is plausible. Given how diverse pluralists maintain that domain-specific truth properties are, pluralists should reject any total ordering on them—that has an intuitive basis, such as one based on metaphysical weight—as unacceptably strong. While pluralists can still maintain a *partial* ordering, even by metaphysical weight, they should not allow too strong an ordering—such as a *total* one—by metaphysical weight.[40]

3 Existing accounts

In this section, I argue that existing accounts provide inadequate answers to the central challenges, and especially the logical ones. In the first subsection, I examine representative examples of all but one of the existing accounts in the literature, which I argue are insufficiently systematic, precise, and unified. In the second subsection, I examine Cotnoir (2013b)'s algebraic account, which I grant is sufficiently systematic, precise, and unified, but I argue is still inadequate.

3.1 Initial accounts

I now consider representative examples of all but one of the existing accounts in the literature that attempt to answer the central logical challenges. I argue that they are inadequate, with my general criticism being that they are insufficiently systematic, precise, and unified. All the accounts are insufficiently unified, in failing to indicate that an integrated account of atomic sentences, complex sentences, quantified sentences, and logical consequence is available. Some accounts are also inadequate for other reasons—notably, because they violate one or more of the proposed constraints or desiderata.

In what follows, I do my best to extract the most systematic and precise account possible from the suggestions in the literature. I formalize the accounts where doing

[40]Granted, the axiom of choice guarantees the existence of some well-ordering or other. But it does not guarantee that the well-ordering is one that has any intuitive basis.

so is possible, noting that the formalizations are mine, and so might deviate—hopefully innocuously—from the original suggestions.

I also note that, although Lynch's functionalism and Sher's correspondence pluralism are not pluralism, the logical challenges I have posed for pluralism pose parallel challenges for these views. The parallel challenges concern, e.g., the truth-realizing properties or kinds of correspondence rather than domain-specific truth properties. Lynch and Sher recognize these parallel challenges, and so provide accounts that attempt to answer them. These accounts transpose to accounts that attempt to answer the present challenges for pluralism, and are therefore relevant in what follows.

Atomic sentences

The main accounts that attempt to answer the first challenge, to maintain a unified account of atomic sentences, are due to Lynch (2005b); Sher (2005); Pedersen and Wright (2013b); Wyatt (2013).

Consider, as a representative example, the following mixed atomic sentence:

(6) Causing pain is bad.

Given that (6) is true, in what way or ways is it domain-specifically true?

A first account, due to Lynch (2005b, 340–341), maintains that mixed sentences are paraphrasable as pure sentences. Given a mixed atomic sentence φ, consider a paraphrase φ' of φ, where φ' is a pure sentence. Assuming that φ' is associated with a pure domain, φ is then associated with whatever domain φ' is associated with. To illustrate, consider the following paraphrase of (6):

(6′) We ought not cause pain.

Assuming that (6′) is associated with the ethical domain, which is pure, (6) is then associated with the ethical domain as well. Despite being a mixed sentence, (6) is associated with a pure domain. So given that (6) is true, it is superassertible true. However, this account is inadequate, because it is insufficiently systematic. It is unclear whether an adequate paraphrase is always available. Indeed, it is unclear that even (6′) is an adequate paraphrase of (6), since the two sentences appear to differ in meaning.

A second account, due to Pedersen and Wright (2013b), maintains that a certain subject matter dominates and so determines the domain a given mixed atomic sentence is associated with.[41] Given a mixed atomic sentence $\Phi^n\tau_1 \ldots \tau_n$, consider the predicate Φ^n, which is assumed to be associated with a pure domain. The sentence φ is then associated with whatever domain Φ^n is associated with. To illustrate, assuming that the predicate "bad" is associated with the ethical domain, which is pure, (6) is then associated with the ethical domain as well. Given that (6) is true, it is superassertible true.

However, this account is inadequate. First, it inappropriately assumes that the domain associated with the predicate always dominates. Assuming that the predicate "beautiful" is associated with the (pure) aesthetic domain, the account predicts that (4)—"The number π is beautiful"—is also associated with the same domain. But plausibly, since (4) is also about numbers, it is also associated with mathematical subject matter, in which case perhaps it should instead, or also, be associated with the mathematical domain. The relevant objects can help determine the domain a sentence is associated with.[42] Second, the account inappropriately assumes that the predicate is always associated with exactly one pure domain. But some predicates, such as "pleasing equation," are about distinct subject matters, and so presumably either associated with impure domains or associated with distinct pure domains; "pleasing equation" is associated with mathematical and aesthetic subject matters.

A third account, due to Wyatt (2013), also maintains that a certain subject matter dominates and so determines the domain a given mixed sentence is associated with, but in a different way. Let a sentence's *candidate truth properties* be the truth properties associated with the pure domains where the subject matters are the ones the sentence is about. Given a mixed atomic sentence φ, consider how the candidate domain-specific

[41] Lynch (2009, 78–82) suggests a similar account.

[42] See Cotnoir and Edwards (2016, 131–133) for such a view. On their account, equivalence classes of objects are associated with domains, so that the relevant objects completely determine the domain a sentence is associated with.

truth properties compare to a default pure domain-specific truth property $T_\natural$, which is dominated by specified domain-specific truth properties.[43] If true, φ then has $T_\natural$ if it is not dominated, and has a dominating domain-specific truth property otherwise.

To use an example based on Wyatt's own (S233–S235), suppose that $T_\natural$ is the correspondence truth property. Given that (6) is about scientific and ethical subject matters, its candidate truth properties are the correspondence truth property and the superassertible truth property. Then, supposing that superassertible truth dominates correspondence truth, given that (6) is true, it is superassertible true.[44]

However, this account is inadequate, because it is insufficiently precise. It is unclear what dominating domain-specific truth properties are, intuitively, and risks violating Desideratum C. For the account maintains that, since mixed atomic sentences are about distinct subject matters and so can be associated with distinct (pure) domains, for any set of domain-specific truth properties, there is a property in the set that dominates the others. But the most natural interpretation of such a dominating property is that it is the most metaphysically weighty; it is unclear what other interpretations are available. So the account risks assuming that there is a total order on the set of domain-specific truth properties by metaphysical weight, in violation of Desideratum C.

A fourth and quite different account, due to Sher (2005), maintains that mixed atomic sentences are associated with impure domains and associated domain-specific truth properties. To illustrate, (6) is associated with an impure domain of scientific and ethical subject matters. Given that (6) is true, it has the domain-specific truth property associated with this impure domain. I think this account is on the right track. However, this account—as it stands—is still inadequate, because it is insufficiently unified, in failing to indicate integration with complementary accounts of quantified sentences, complex

[43]For Wyatt, the default pure domain-specific truth property is the correspondence truth property, which is dominated by the coherence truth property and certain other domain-specific truth properties.

[44]This example differs from Wyatt's in a minor way. Wyatt assumes that although every domain is pure and mixed sentences are associated with distinct pure domains, true mixed sentences have exactly one domain-specific truth property. In my example based on Wyatt's, I claim that mixed sentences are associated with exactly one pure domain, which is strictly speaking not what he maintains. But the difference is immaterial, as what matters for my purposes is that there is exactly one pure domain relevant for the purpose of determining which domain-specific property truth is had, which is what Wyatt maintains.

sentences, and logical consequence.

Quantified sentences

To my knowledge, there have only been two—relatively undeveloped—accounts that attempt to answer the second challenge, to maintain a unified account of quantified sentences. Consider, as representative examples, the following mixed quantified sentences:

(7) Everything is self-identical.

(8) Everything the Pope utters is true.

Given that (7) is true, in what way or ways is it domain-specifically true? Similarly, assuming—for the sake of illustration—that (8) is true, in what way or ways is it domain-specifically true?

One account, due to Pedersen and Wright (2013b), maintains a disjunctive generic truth property along the lines of T_D. The idea is that (8) has this truth property to accommodate the Pope's uttering things on disparate subject matters. A related account, due to Wright (2013), similarly maintains a generic truth property, but does not insist that it be one along the lines of T_D. The idea is that (7) and (8), and true quantified sentences more generally, have this generic truth property.

However, the first account is inadequate, because it is insufficiently systematic. It is silent on the treatment of quantified sentences more generally, such as (7), which do not ascribe truth to anything and quantify over entities other than sentences. Also, both accounts are insufficiently unified, in failing to indicate integration with complementary accounts of atomic sentences, complex sentences, and logical consequence.

Complex sentences

The main accounts that attempt to answer the third challenge, to maintain a standard account of complex sentences, are due to Edwards (2008, 2009); Cotnoir (2009); Pedersen and Wright (2013b).

Recall (1) and (2):

(1) Water is H_2O.

(2) $1 + 1 = 2$.

Now consider, as a representative example, the following mixed complex sentence:

(9) Water is H_2O and $1 + 1 = 2$.

Given that (9) is true, in what way or ways is it domain-specifically true?

One account, due to Edwards (2008), maintains that atomic sentences and complex sentences, whether pure or mixed, have distinct truth properties. It maintains that we are to read the truth-functional principle for conjunction—a conjunction is true iff both conjuncts are true—with an order of determination, from right to left, so that a conjunction is true *because* both conjuncts are true. (Edwards only considers conjunction.) Supposing that a model M assigns each atomic sentence to a domain-specific truth value (which represents a domain-specific truth property or a domain-specific falsity property), the semantic value function is defined as follows (147–148):[45]

(EC) $|\varphi \wedge \psi|_M = T_k$ iff $|\varphi|_M = T_i$ and $|\psi|_M = T_j$.

At least when φ are ψ are atomic sentences, Edwards emphasizes that the truth of the conjunction is distinct from the truth of either conjunct (148), so that k is distinct from each of i and j: "The further way that the conjunction is true is such that the truth of the conjunction is *dependent* on the truth of the individual conjuncts" [emphasis in the original] (148). Given that (1) is correspondence true and (2) is coherence true, (9) is then still domain-specifically true but neither correspondence true nor coherence true.

However, the account is inadequate, because it is insufficiently precise. Edwards (2008)'s account as stated omits crucial details. Cotnoir (2009) charges Edwards (2008)'s

[45]My formulation is more general, in not assuming that there are only finitely many domain-specific truth values, or truth properties.

account with an objectionable proliferation of truth properties that arises from considering iterated conjunctions. Cotnoir (2009) understands the claim that $|\varphi|_M = T_i$ in (EC) to be equivalent to the claim that $|\varphi|_M = T_1$ or $|\varphi|_M = T_2$ or ..., and similarly for $|\psi|_M = T_j$, so that (EC) as a whole is equivalent to the following:

(EC′) $|\varphi \wedge \psi|_M = T_k$ iff $|\varphi|_M = T_1$ or $|\varphi|_M = T_2$ or ...and $|\psi|_M = T_1$ or $|\psi|_M = T_2$ or ....

But then, Cotnoir asks, what if either φ or ψ is a complex sentence? Edwards (2008, 147) admits in a footnote the following:

> The analysis will work just as well for 'level 2' conjunctions where one or more of the conjuncts is *itself* a conjunction. In this case, we still hold that the conjunction is true just in case each of its conjuncts are true, but, when one of the conjuncts is a conjunction, what makes *that* conjunct true (having each of *its* conjuncts being true) will be different from what makes a conjunct consisting of a singular proposition true (having the relevant truth property for its domain of discourse). [emphasis in the original]

Cotnoir suggests that when either φ or ψ is a complex sentence, T_k is distinct from $T_1, T_2, \ldots$. So, he concludes, by considering iterated conjunctions, "[t]he result is a proliferation of truth properties" (476). Such a proliferation might be objectionable, since it departs significantly from pluralism's standard motivating picture.[46]

In reply to Cotnoir's charge, Edwards (2009, 686) suggests that his account does not result in a proliferation of truth properties:

> When we talk about the relationship between a compound and its components, it is plausible to think that we are operating within the domain of logical discourse: it is the appropriate logical rules which dictate the correct relationships. Thinking along alethic pluralist lines, the truth *property* here will be whatever property it is that is identified with truth in the logical domain. It is this property that a conjunction has when both of its conjuncts are true, and that a disjunction has when one or more of its disjuncts are true.
>
> There is thus not a proliferation of truth *properties* in the case of compounds. All true compounds are in possession of the *same* truth property: whatever property is identified with truth in the logical domain. [emphasis in the original]

[46] Interestingly, Cook (2011) endorses an account of complex sentences that resembles the one Cotnoir (2009) attributes to Edwards (2009). On Cook's account, there are infinitely many domains and associated domain-specific truth properties.

Let T_L represent the "logical" truth property—i.e., the domain-specific truth property associated with the logical domain (which is associated with logical subject matter). Where a model M is as before, the semantic value function is then defined as follows:[47]

(EC′) $|\varphi \wedge \psi|_M = T_L$ iff $|\varphi|_M = T_i$ and $|\psi|_M = T_j$.

However, this account is inadequate, because it remains insufficiently precise. Notably, it is unclear how the account extends to disjunction. Are disjunctions of atomic sentences that are domain-specifically true in exactly the same way also "logically" true in the *same* way in which conjunctions are true, or are such disjunctions "logically" true in *another* way? Further, the account violates Constraint 2. For it predicts that the conjunction of atomic sentences that are domain-specifically true in exactly the same way is "logically" true. But given that "Murder is wrong" and "Charity is good" are ethically true, the account predicts that the conjunction "Murder is wrong and charity is good" is not ethically true but rather "logically" true—just like "There are no true contradictions." This violates Constraint 2.

Another account, considered but not endorsed by Cotnoir (2009), also uniformly treats complex sentences, whether pure or mixed. It differs from Edwards' account(s), in that it avoids a proliferation of truth properties. The account assumes that there are domain-specific falsity properties associated with domain-specific truth properties, and allows for sentences to have multiple such properties. Where a model M assigns each atomic sentence to a set of one or more domain-specific truth values, the semantic value function is defined as follows (477–478):[48]

(NLO) (i) $|\neg\varphi|_M \ni T_i$ iff $|\varphi|_M \ni F_i$;

(ii) $|\varphi \wedge \psi|_M \ni T_i$ iff $|\varphi|_M \ni T_i$ or $|\psi|_M \ni T_i$, provided that $|\varphi|_M, |\psi|_M \neq \{\}$;

(iii) $|\varphi \vee \psi|_M \ni T_i$ iff $|\varphi|_M \ni T_i$ or $|\psi|_M \ni T_i$.

[47] This account has notable affinities with those of Lynch (2005b, 396–397), Lynch (2008, 124–132), Lynch (2009, 86–91), and Lynch (2013, 34–40).

[48] The symbol $\ni$ indicates set containment: $x \ni y$ means that x contains y, or equivalently that y is in x.

Thus, a conjunction inherits the domain-specific truth of its conjuncts provided that both conjuncts are domain-specifically true in some way, and a disjunction inherits the domain-specific truth of its disjuncts. Given that (1) is correspondence true and (2) is coherence true, (9) is then both correspondence true and coherence true.

However, this account is inadequate, because it is insufficiently unified, in failing to indicate integration with complementary accounts of atomic sentences, quantified sentences, and logical consequence. Also, the account violates Desideratum B. For it predicts that the conjunction of sentences that are domain-specifically true in distinct ways, where the domains are pure, is domain-specifically true in all of those ways. Given that "$1+1=2$" is mathematically true and "Louis CK is funny" is comically true, the account predicts that the conjunction "$1+1=2$ and Louis CK is funny" is both mathematically true and comically true, even though it is arguably *neither* (purely) mathematically true *nor* (purely) comically true. This violates Desideratum B.

Yet another account, due to Pedersen and Wright (2013b), parallels a standard many-valued account of complex sentences. This account also uniformly treats complex sentences, whether pure or mixed. Conjunction and disjunction are minimizing and maximizing operations, respectively:

(MLO) (i) $|\varphi \wedge \psi|_M = \min\{|\varphi|_M, |\psi|_M\}$;

(ii) $|\varphi \vee \psi|_M = \max\{|\varphi|_M, |\psi|_M\}$.

Suppose that, in the set containing the coherence truth property and the correspondence truth property, the former is the minimum and the latter is the maximum. Given that (1) is correspondence true and (2) is coherence true, their conjunction (9) is then coherence true, and their disjunction is then correspondence true.

However, this account is inadequate, because it is insufficiently precise. It is unclear what minimum and maximum domain-specific truth properties are, intuitively, and risks violating Desideratum C. For the account maintains that for every set of pairs of domain-specific truth properties, there are properties in the set that are the minimum and the maximum. But the most natural interpretation of such a minimum is that it is the least metaphysically weighty, and similarly the most natural interpretation of such a maximum is that it is the most metaphysically weighty; it is unclear what other interpretations are available. So the account risks assuming that there is a total order on the set of domain-specific truth properties, in violation of Desideratum C.

Logical consequence

The main accounts that attempt to answer the fourth challenge, to maintain a standard account of logical consequence, are due to Beall (2000); Pedersen (2006); Cotnoir (2013b).

Consider, as a representative example, the argument from (1) and (2) to (9), which is valid, in that (9) is a logical consequence of (1) and (2):

(1) Water is H_2O.

(2) $1 + 1 = 2$.

(9) Water is H_2O and $1 + 1 = 2$.

Given that the argument from (1) and (2) to (9) is valid, and logical consequence necessarily preserves truth, what truth properties are preserved from the premises (1) and (2) to the conclusion (9)?

An account, due to Beall (2000) and endorsed by Wright (2013), parallels a standard many-valued account of logical consequence. This account maintains a set of designated properties, where designated properties are just domain-specific truth properties. So the set of designated values is just the set $\mathscr{T}$ of domain-specific truth values that represent domain-specific truth properties. The account takes logical consequence to necessarily preserve designation (381–382):

(BLC) $\Gamma \vDash \varphi$ iff for every model M, if $|\gamma_i|_M = T_i$ for some T_i in $\mathscr{T}$ for every γ_i in Γ, then $|\varphi|_M = T_i$ for some T_i in $\mathscr{T}$.

Given that (1) is designated, in being correspondence true, and (2) is also designated, in being coherence true, (9) is also designated—or so we may assume. However, this account is inadequate, because it is insufficiently unified, in failing to indicate integration with complementary accounts of atomic sentences, quantified sentences, and especially complex sentences. As stated, it is mysterious why the designation of (1) and (2) guarantees the designation of (9). We want at least a complementary account of the semantics of sentences such as (1), (2), and (9), but Beall does not provide one.

Potentially, Beall's account is also inadequate because it seems to grant a generic truth property significance. As Tappolet (2000); Pedersen (2006) suggest, designation resembles truth, since it plays one of the key roles truth is supposed to be play—in being what is necessarily preserved by logical consequence. It is natural to maintain that the property of being designated is a generic truth property. But, again, it is unclear to what extent pluralists should grant a generic truth property significance. With the potential inadequacy in mind, Pedersen (2006) amends Beall's account to avoid the apparent employment of a generic truth property. Pedersen's suggestion is to take logical consequence to necessarily preserve domain-specific truth, where quantification over domain-specific truth properties in (BLC) does not involve commitment to there being a set of domain-specific truth properties. The idea is to employ plural quantification, as defended by Boolos (1984), to quantify over just the domain-specific truth properties themselves, without commitment to there being a corresponding set or property.

I think Beall's account is on the right track. But even if Pedersen's amendment helps avoid granting a generic truth property significance, the account—as it stands—is still inadequate, because it is insufficiently unified.

The most recent, and most well developed, account that attempts to answer this challenge is due to Cotnoir (2013b). This account is sufficiently systematic, precise, and unified. However, it too is inadequate—or so I argue. I now turn to this account.

3.2 Cotnoir's algebraic account

Cotnoir (2013b)'s most recent account—which should be clearly distinguished from his older 2009 account—is a notable exception to the criticism of insufficient systematicity, precision, and unity. However, the account focuses only on answering the fourth challenge, and founders as an attempt to answer the third challenge. The account's effective answer to the third challenge makes implausible predictions about negations and conjunctions, where these predictions can only be modestly improved at the cost of a very nonclassical logic. I conclude that the account is on the whole inadequate.

On the account, semantic values are n-tuples, where a 1 in the ith place represents a sentence's being domain-specifically true in the way associated with the ith domain and being in the ith domain, and a 0 in the ith place represents a sentence's being domain-specifically false in the way associated with the ith domain and being in the ith domain.[49] The account of complex sentences is componentwise classical, in that negation inverts values in each place, and conjunction and disjunction are minimizing and maximizing operations, respectively, on pairs of places. More formally, where a model M assigns each atomic sentence to an n-tuple with a 1 in at most one place, the semantic value function $|\cdot|_M$ based on M satisfies $|\alpha|_M = M(\alpha)$.[50] And where $|\varphi|_M = \langle a_1, \ldots, a_n\rangle$ and $|\psi|_M = \langle b_1, \ldots, b_n\rangle$, we have the following:

(CLO) (i) $|\neg\varphi|_M = \langle 1 - a_1, \ldots, 1 - a_n\rangle$;

(ii) $|\varphi \wedge \psi|_M = \langle \min(a_1, b_1), \ldots, \min(a_n, b_n)\rangle$;

(iii) $|\varphi \vee \psi|_M = \langle \max(a_1, b_1), \ldots, \max(a_n, b_n)\rangle$.

A partial order on semantic values is defined by the following:

(COR) $|\varphi|_M \leq |\psi|_M$ iff $|\varphi \vee \psi|_M = |\psi|_M$.

Given this, the account of logical consequence is then the following:

(CLO) $\Gamma \vDash \varphi$ iff $\bigwedge_{\gamma_i \in \Gamma} |\gamma_i|_M \leq |\varphi|_M$ for all models M.[51]

[49] My presentation of the account differs from Cotnoir's in two minor ways. First, Cotnoir formulates his account in terms of propositions rather than sentences; but since he assumes that propositions have sentential structure, this difference is immaterial. Second, I use different notation, so as to aid comparison between his account and others.

[50] As Cotnoir notes, the simplifying assumption that atomic sentences are assigned n-tuples with a 1 in at most one place can be dropped (569).

[51] To accommodate the case where Γ is infinite, it is implicitly assumed that $\bigwedge$ is a conjunction-like minimizing operation on semantic values defined for finite and infinite sequences of semantic values. Specifically, the operation $\bigwedge$ on semantic values is constrained to meet the following identity: $\bigwedge_{\gamma_i \in \Gamma} |\gamma_i|_M = |\bigwedge_{\gamma_i \in \Gamma} \gamma_i|_M$. Note the abuse of notation: the first occurrence of $\bigwedge$ expresses an operation on semantic values, while the second occurrence of $\bigwedge$ expresses an operation on sentences.

Cotnoir argues that this account answers the fourth challenge, to maintain a standard account of logical consequence. To do this, he shows that the resulting account of logical consequence is classical; the algebra comprising semantic values, conjunction, disjunction, negation, top value $\top = \langle 1, \ldots, 1 \rangle$, and bottom value $\bot = \langle 0, \ldots, 0 \rangle$ forms a complemented distributive lattice, and thus a Boolean algebra. Cotnoir suggests that the account is accordingly "*standard*, *non-revisionary*, [and] *semantic*" (571) and yet "does not appeal to any *single* truth property being preserved" [emphasis in the original] (572). Let us grant, for the sake of argument, that the account successfully answers the fourth challenge, to maintain a standard account of logical consequence. For I want to focus on other aspects of the account, which I argue are problematic.

Although Cotnoir does not argue that the account answers the third challenge, to maintain a standard account of complex sentences, the account does offer an account of complex sentences. So we can consider how it fares as an attempt to answer the third challenge. Unfortunately, I submit, the account makes implausible predictions. I suggest that the following three implausible predictions, taken together, render the account of complex sentences highly nonstandard, and so render Cotnoir's account as a whole inadequate.

First, the account implausibly predicts that every sentence is associated with every domain. This prediction follows immediately from the fact that every sentence is assigned either a 1 or a 0 in the ith place of the n-tuple that is its semantic value, where a 1 entails being associated with the ith domain and a 0 also entails being associated with the ith domain. Here is how Cotnoir puts it: "If 1 appears in the ith place, this represents that the proposition is in domain$_i$ and is true$_i$; 0 in the ith place represents that the proposition is in domain$_i$ and is false$_i$" (568). The prediction is implausible, since having every sentence be associated with every domain vitiates the supposed theoretical significance of positing the ideology of domains in the first place.

Second, the account incorrectly predicts that the negation of a sentence—such as "$1+1 = 2$"—that is domain-specifically true in exactly one way—say, it is just mathematically true—is domain-specifically true in every other way—so that the negation is also scientifically true and ethically true. To see this, note that where $|\varphi|_M = \langle 0, \ldots, 1, \ldots, 0\rangle$, we have $|\neg\varphi|_M = \langle 1, \ldots, 0, \ldots, 1\rangle$. The prediction is implausible, since if pluralists are persuaded by the thought that even *true* mathematical sentences do not correspond (to the facts, or reality, or...), then they surely should not accept that *false* mathematical sentences can correspond.

Third, the account incorrectly predicts that the conjunction of two sentences each domain-specifically true in exactly one but distinct ways is not domain-specifically true in any way, and so is not true in any way. To see this, suppose that $|\varphi|_M = \langle 0, \ldots, 1, \ldots, 0\rangle$, where the 1 is in the *i*th place, and $|\psi|_M = \langle 0, \ldots, 1, \ldots, 0\rangle$, where the 1 is in the *j*th place. Then, where $i \neq j$, we have that $|\varphi \wedge \psi|_M = \langle 0, \ldots, 0, \ldots, 0\rangle$. The prediction is implausible, since a conjunction of true sentences is surely itself true. Indeed, it is a requirement of the standard account of complex sentences.

As Cotnoir notes, the incorrect prediction regarding negations can be corrected (574–575). The correction involves allowing a $\frac{1}{2}$ in the *i*th place of an *n*-tuple that is a sentence's semantic value, where a $\frac{1}{2}$ in the *i*th place represents a sentence's being undefined in the *i*th domain. The negation of a sentence φ that is domain-specifically true in exactly one way can now be undefined in every other domain: where $|\varphi|_M = \langle \frac{1}{2}, \ldots, 1, \ldots, \frac{1}{2}\rangle$, we have that $|\neg\varphi|_M = \langle \frac{1}{2}, \ldots, 0, \ldots, \frac{1}{2}\rangle$. However, this correction comes at the high cost of a logic that is both paracomplete and paraconsistent. As Cotnoir points out, the logic is paracomplete, in that $\not\models \varphi \vee \neg\varphi$, and paraconsistent, in that $\not\models \neg(\varphi \wedge \neg\varphi)$. To see this, consider the case where $|\varphi|_M = \langle \frac{1}{2}, \ldots, \frac{1}{2}\rangle$. In this case, it follows that $|\varphi \vee \neg\varphi|_M = |\varphi \wedge \neg\varphi|_M = |\neg(\varphi \wedge \neg\varphi)|_M = \langle \frac{1}{2}, \ldots, \frac{1}{2}\rangle$. But, I suggest, such a nonclassical logic is surely too high a price to pay for the correction. Besides, no correction seems available for the incorrect prediction regarding conjunctions.

I conclude that all the existing accounts that attempt to answer the central logical challenges, including Cotnoir (2013b)'s, are inadequate.

4 A novel account

In this section, on behalf of pluralists, I propose a novel account of logic and semantics. In the first subsection, I propose an account for a sentential language, which I show answers three of the four central logical challenges—all except the challenge to maintain a unified account of quantified sentences. In the second subsection, I extend the account to a first-order language, and show that it answers all four of the central logical challenges. The resulting account, I submit, meets all the proposed constraints and desiderata on adequate answers to the challenges. In the third subsection, I explain how I think pluralists should accordingly answer the central conceptual challenges.

4.1 Logic and semantics for a sentential language

I now propose an account of logic and semantics for a sentential language.

The proposed account, informally

On the account I propose on behalf of pluralists, there is a one-to-one correspondence between domains, domain-specific truth properties, and domain-specific falsity properties. Pure domains are associated with exactly one subject matter, while impure domains are associated with two or more subject matters. Where domains are either pure or impure, pure domains generate all domains.

The basic picture is the following. Each atomic sentence is assigned to exactly one domain. Negations are always assigned to the same domain as the negand, while conjunctions and disjunctions may or may not be assigned to the same domain as each operand, depending on whether or not the operands are assigned to the same domain. Each atomic sentence is then assigned a domain-specific truth value, where the relevant domain is the one it is assigned. The domain-specific truth values of negations, conjunctions, and disjunctions are determined by the domain-specific truth values of the operands. Logical consequence necessarily preserves domain-specific truth.

The proposed account, more formally

To incorporate talk of domains, we assume a structure $\mathfrak{D} = \langle \mathscr{D}, \odot \rangle$, where $\mathscr{D}$ is a nonempty set and $\odot$ is a binary operation on $\mathscr{D}$. Intuitively, $\mathscr{D}$ is the set of domains, and $\odot$ is a mixing operation on domains. To capture the idea that there are distinct domains (and associated domain-specific truth properties, as well as domain-specific falsity properties), we require that there are at least two members in $\mathscr{D}$: $|\mathscr{D}| \geq 2$.[52] Let D_i denote a member of $\mathscr{D}$. Intuitively, $D_i \odot D_j$ is the mixed domain for D_i and D_j. Since we want the mixed domain for identical domains to be the same domain and do not want the order in which we mix domains to matter, we require that $\mathfrak{D}$ is a semilattice:

(DS) (i) $D_i \odot D_i = D_i$ (idempotency);

(ii) $D_i \odot D_j = D_j \odot D_i$ (commutativity);

(iii) $(D_i \odot D_j) \odot D_k = D_i \odot (D_j \odot D_k)$ (associativity).

As usual, a partial order $\leq$ on $\mathscr{D}$ is induced by setting the following:

(PO) $D_i \leq D_j$ iff $D_i \odot D_j = D_j$.

Note that $\leq$ in turn induces a strict partial order $<$, where $D_i < D_j$ iff $D_i \leq D_j$ and $D_i \neq D_j$. The following desirable feature follows from the fact that $\mathfrak{D}$ is a semilattice:

(SG) If $D_i \not\leq D_j$ and $D_j \not\leq D_i$, then $D_i < D_i \odot D_j$ and $D_j < D_i \odot D_j$.

For suppose that $D_i \not\leq D_j$ and $D_j \not\leq D_i$. Then $D_i \neq D_i \odot D_j$ and $D_j \neq D_i \odot D_j$. But since we always have that $D_i \odot (D_i \odot D_j) = (D_i \odot D_i) \odot D_j = D_i \odot D_j$ (by associativity and idempotency), we always have that $D_i \leq D_i \odot D_j$; similarly, we always have that $D_j \leq D_i \odot D_j$. So $D_i < D_i \odot D_j$ and $D_j < D_i \odot D_j$. This feature captures the idea that distinct domains where neither domain is induced by the other induce new domains.

[52] If there is exactly one domain, there is exactly one "domain-specific" truth property, where the domain is a generic one that all sentences are associated with. In this case, the account is effectively monist. This suggests that one way of understanding the dispute between monists and pluralists concerns the number of domains there are—for monists, exactly one, but for pluralists, at least two.

To incorporate talk of subject matter, we assume that there is a distinguished subset $Pure(\mathscr{D})$ of pure domains from which all domains are generated. Accordingly, $\mathscr{D}$ is the smallest superset of $Pure(\mathscr{D})$ closed under the mixing operation $\odot$. Intuitively, pure domains are associated with exactly one subject matter, while impure domains are associated with two or more subject matters. Pure domains are incomparable, in that for distinct pure domains D_i and D_j, we have both $D_i \nless D_j$ and $D_j \nless D_i$. To determine which pure domains, or subject matters, a given domain is associated with, we define the subject matter extraction function $m : \mathscr{D} \rightarrow \wp(Pure(\mathscr{D}))$ from domains to sets of pure domains by $m(D_i) = \{D_j \in Pure(\mathscr{D}) \mid D_j \leq D_i\}$.

To incorporate talk of domain-specific truth and domain-specific falsity, we associate with $\mathfrak{D}$ two isomorphic structures, $\mathfrak{T} = \langle \mathscr{T}, \oplus \rangle$ and $\mathfrak{F} = \langle \mathscr{F}, \ominus \rangle$. The semantic values, and more specifically domain-specific truth values, are drawn from $\mathscr{T} \cup \mathscr{F}$. Intuitively, $\mathscr{T}$ is the set of domain-specific truth properties, and $\mathscr{F}$ is the set of domain-specific falsity properties. Let T_{D_i} as well as T_i—both notations are useful—denote the domain-specific truth value that represents the truth property associated with domain D_i. Similarly, let F_{D_i} as well as F_i denote the domain-specific truth value that represents the falsity property associated with domain D_i. Since $\mathfrak{T}$ and $\mathfrak{F}$ are isomorphic to $\mathfrak{D}$, the partial orders on domain-specific truth values mirror the partial order on domains:

(OR) $D_i < D_j$ iff $T_i < T_j$ iff $F_i < F_j$.

A model M is a pair $M = \langle d, v \rangle$, where $d : Sent(\mathscr{L}) \rightarrow \mathscr{D}$ is a domain assignment function from sentences to domains and $v : Atomic(\mathscr{L}) \rightarrow \mathscr{T} \cup \mathscr{F}$ is a valuation function from atomic sentences to domain-specific truth values.[53] Intuitively, d assigns a sentence to the domain it is associated with. To ensure that the account meets Constraint 1 and Desideratum A, we define d by the following:

(DO) (i) $d(\alpha) \in \mathscr{D}$;

[53] Following Field (2008, Chap. 2), one might worry about the apparent divergence between (real) truth and model-theoretic truth. If so, then we can at least understand model-theoretic truth instrumentally. The model-theoretic formulation might not be essential, but has the advantage of showing that the account can be made sufficiently systematic and precise.

(ii) $d(\neg\varphi) = d(\varphi)$;

(iii) $d(\varphi \wedge \psi) = d(\varphi) \odot d(\psi)$;

(iv) $d(\varphi \vee \psi) = d(\varphi) \odot d(\psi)$.

This definition permits even atomic sentences to be assigned to any domain, whether pure or impure. Intuitively, v assigns an atomic sentence to a domain-specific truth value. Accordingly, we require that $v(\alpha) \in \{T_{d(\alpha)}, F_{d(\alpha)}\}$.

Given a model M, we get the semantic value function $|\cdot|_M : Sent(\mathscr{L}) \to \mathscr{T} \cup \mathscr{F}$ from sentences to domain-specific truth values based on M. To capture the idea that a sentence is either domain-specifically true or domain-specifically false, we first require that $|\varphi|_M \in \{T_{d(\varphi)}, F_{d(\varphi)}\}$. Then, to ensure that the account meets Constraint 2 and Desideratum B, we define $|\cdot|_M$ by $|\alpha|_M = v(\alpha)$ and the following:

(LO*) (i) $|\neg\varphi|_M = T_{d(\neg\varphi)}$ iff $|\varphi|_M = F_{d(\varphi)}$;

(ii) $|\varphi \wedge \psi|_M = T_{d(\varphi\wedge\psi)}$ iff $|\varphi|_M = T_{d(\varphi)}$ and $|\psi|_M = T_{d(\psi)}$;

(iii) $|\varphi \vee \psi|_M = T_{d(\varphi\vee\psi)}$ iff $|\varphi|_M = T_{d(\varphi)}$ or $|\psi|_M = T_{d(\psi)}$.

Intuitively, a negation is domain-specifically true iff the negand is domain-specifically false, a conjunction is domain-specifically true iff each conjunct is domain-specifically true, and a disjunction is domain-specifically true iff either disjunct is domain-specifically true.

Finally, we define logical consequence as follows:

(LC*) $\Gamma \vDash \varphi$ iff for every model M, if $|\gamma_i|_M = T_{d(\gamma_i)}$ for every γ_i in Γ, then $|\varphi|_M = T_{d(\varphi)}$.

Intuitively, an argument is valid iff in every model where each premise is domain-specifically true, the conclusion is also domain-specifically true. Logical consequence preserves domain-specific truth across all models.[54]

[54] A nonstandard feature is that, on the accounts of complex sentences and of logical consequence here, identity of semantic value may diverge from logical equivalence. Notably, φ and $\varphi \wedge (\psi \vee \neg\psi)$ may have distinct domain-specific truth values, and so distinct semantic values, even though $\varphi \dashv\vDash \varphi \wedge (\psi \vee \neg\psi)$. But pluralists can embrace this feature, by noting that semantic values may incorporate more information than that needed for logical consequence. While semantic values incorporate extralogical information about domains, logical consequence abstracts away from such information and looks only at formal or structural properties of sentences in relation to one another.

Answering the central logical challenges

Working through some simple examples shows that the account makes intuitively plausible predictions and should be taken seriously by pluralists. Let us assume for the sake of illustration that there are exactly four pure domains: D_1 is the scientific domain, D_2 is the mathematical domain, D_3 is the ethical domain, and D_4 is the aesthetic domain.

Take the account of atomic sentences. To illustrate with a pure atomic sentence and a mixed atomic sentence, consider the following sentences:

(3) Murder is wrong.

(4) The number π is beautiful.

Suppose that (3) is an atomic sentence associated with D_3 and is domain-specifically true in the associated way T_3, and that (4) is an atomic sentence associated with the impure domain $D_2 \odot D_4$—because the number π is associated with the mathematical domain, while "beautiful" is associated with the aesthetic domain—and is domain-specifically true in the associated way $T_2 \odot T_4$. On the intended model, $M_@ = \langle d_@, v_@ \rangle$, the domain assignment function $d_@$ assigns (3) to D_3, and (4) to $D_2 \odot D_4$, while the valuation function $v_@$ assigns (3) to T_3, and (4) to $T_2 \odot T_4$. Intuitively, (3) and (4) are true, and the account vindicates this intuition: $| \cdot |_{M_@}$ assigns (3) to T_3 and (4) to $T_2 \odot T_4$.

Now take the account of complex sentences. To illustrate with just pure sentences, consider the following sentences:

(1) Water is H_2O.

(11) Salt is NaCl.

(2) $1 + 1 = 2$.

(12) $2 + 2 = 4$.

Suppose that (1) and (11) are atomic sentences associated with domain D_1 and are domain-specifically true in the associated way T_1, and that (2) and (12) are atomic sentences associated with domain D_2 and are domain-specifically true in the associated way T_2. On the intended model, $d_@$ assigns each of (1) and (11) to D_1, and each of (2) and (12) to D_2, while $v_@$ assigns each of (1) and (11) to T_1, and each of (2) and (12) to T_2. Thus, $|\cdot|_{M_@}$ assigns each of (1) and (11) to T_1, and each of (2) and (12) to T_2. Now consider the conjunction (13) of (1) and (11), and the conjunction (14) of (2) and (12):

(13) Water is H_2O and salt is NaCl.

(14) $1+1=2$ and $2+2=4$.

Intuitively, (13) and (14) are true, and the account vindicates this intuition. Thanks to the way they are defined to treat complex sentences, $d_@$ assigns (13) to D_1 and (14) to D_2, and $|\cdot|_{M_@}$ assigns (13) to T_1 and (14) to T_2.

To illustrate with a mixed sentence, consider the conjunction (9) of (1) and (2):

(9) Water is H_2O and $1+1=2$.

Intuitively, (9) is true, and the account vindicates this intuition. On the intended model, $d_@$ assigns (9) to $D_1 \odot D_2$, which is distinct from each of D_1 and D_2. Nonetheless, (9) is true in that it is domain-specifically true in the associated way: $|\cdot|_{M_@}$ assigns (9) to $T_1 \odot T_2$.

Next, take the account of logical consequence. Intuitively, (1) and (11) mutually entail (13). (Analogously, (2) and (12) mutually entail (14).) Again, the account vindicates this intuition. Consider the argument from (1) and (11) to (13) in schematic form:

$$\varphi, \psi \therefore \varphi \wedge \psi.$$

Fixing a model M, assume that $|\varphi|_M = T_{d(\varphi)}$ and $|\psi|_M = T_{d(\psi)}$. By the clause for conjunction, we have that $|\varphi \wedge \psi|_M = T_{d(\varphi\wedge\psi)}$. So (1) and (11) entail (13). Conversely, consider the argument from (13) to, say, (1):

$$\varphi \wedge \psi \therefore \varphi.$$

Fixing a model M, assume that $|\varphi \wedge \psi|_M = T_{d(\varphi\wedge\psi)}$. Again by the clause for conjunction, we have that $|\varphi|_M = T_{d(\varphi)}$. So (13) entails (1). By analogous reasoning, (13) entails (11).

Just as intuitively, (1) and (2) mutually entail (9), even though (1) and (2) on the one hand and (9) on the other hand are not associated with the same domain on the intended model. But since the argument in the previous paragraph does not make any assumptions about what domains the relevant sentences are associated with, it is perfectly general, and shows that (1) and (2) also mutually entail (9). Thus, $\varphi, \psi \vDash \varphi \wedge \psi$ and $\varphi \wedge \psi \vDash \chi$ for $\chi = \varphi, \psi$.

More generally, by construction, the account of complex sentences meets all of Constraints 1–2 and Desiderata A–B. Since there need not be an intuitive ordering, such as one by metaphysical weight, on $\mathscr{D}$—or $\mathscr{T}$—that is total, the overall account also meets Desideratum C. Further, the account of logical consequence is classical:

Theorem. *The logical consequence relation defined by (LC*) is classical.*

Proof. We first observe that every plural model $M = \langle d, v \rangle$ corresponds to a classical model M' by setting $M'(\alpha) = T$ iff $v(\alpha) = T_{d(\alpha)}$. It follows (by induction) that $|\varphi|_{M'} = T$ iff $|\varphi|_M = T_{d(\varphi)}$. Similarly, every classical model N corresponds to a plural model $N^* = \langle d^*, v^* \rangle$ by setting $v^*(\alpha) = T_{d^*(\alpha)}$ iff $N(\alpha) = T$. It follows that $|\varphi|_{N^*} = T_{d^*(\varphi)}$ iff $|\varphi|_N = T$.

We then show that since the logical consequence relation defined by (LC) is classical, the logical consequence relation defined by (LC*) is also classical: Γ classically entails φ iff Γ plurally entails φ. We consider the contrapositive associated with each direction. For the left-to-right direction, suppose that Γ does not plurally entail φ. Then there is a plural model M such that $|\gamma_i|_M = T_{d(\gamma_i)}$ for every γ_i in Γ, but $|\varphi|_M = F_{d(\varphi)}$. By the above correspondence, there is a classical model M' such that $|\gamma_i|_{M'} = T$ for every γ_i in Γ, but $|\varphi|_{M'} = F$. So Γ does not classically entail φ. For the right-to-left direction, suppose that Γ does not classically entail φ. Then there is a classical model N such that $|\gamma_i|_N = T$ for every γ_i in Γ, but $|\varphi|_N = F$. By the above correspondence, there is a plural model N^* such that $|\gamma_i|_{N^*} = T_{d(\gamma_i)}$ for every γ_i in Γ, but $|\varphi|_{N^*} = F_{d(\varphi)}$. So Γ does not plurally entail φ. $\square$

I offer the account as a proof of concept, and for this purpose, it is highly desirable to show that it can vindicate classical logic, due to its familiarity. Also, it is independently interesting that the account can vindicate classical logic, because it shows that, while pluralists might have independent motivation for maintaining a nonclassical logic, they are not forced to do so. For pluralists who want to maintain a nonclassical logic, I show in Appendix A.1 that the account can be extended to accommodate a nonclassical logic.

In light of the above, I submit that the account proposed on behalf of pluralists answers three of the four central logical challenges—to maintain a unified account of atomic sentences and standard accounts of complex sentences and of logical consequence—in a sufficiently systematic, precise, and unified way that meets all the proposed constraints and desiderata. Clearly, the account of atomic sentences, whether pure or mixed, is unified. Atomic sentences, like complex sentences, can be associated with either pure or impure domains. Further, inspection of the clauses for complex sentences shows that the account of them is indeed standard. The account is consonant with the accounts reviewed in Section 3.1. Notably, it vindicates Wright (2013, 135)'s suggestion that "the right thing to say is that what makes the conjunction true is that the first conjunct is true AND the second conjunct is true." Similarly, inspection of the definition of logical consequence shows that the account of logical consequence is also standard. The account at least resembles a many-valued account of logical consequence, according to which logical consequence necessarily preserves one or more truth properties.

4.2 Logic and semantics for a first-order language

The provision of an account of logic and semantics for a sentential language on behalf of pluralists is significant progress. But such progress is limited insofar as pluralists lack an account of quantified sentences. Can the account of logic and semantics be extended to a first-order language? I now show that the answer is in the affirmative. Specifically, I extend the account of logic and semantics for a sentential language to a first-order setting. I show that the resulting account provides a unified account of quantified sentences as well, and conclude that it answers all the central logical challenges for pluralism.

The proposed first-order extension, informally

The proposed first-order extension is conservative with respect to the accounts of complex sentences and of logical consequence, but introduces richer models, and variable assignment functions. Following Sher (2005); Shapiro (2009, 2011), I suggest that pluralists about truth should also be pluralists about satisfaction. Here is Sher (2005, 322):

> [T]he plurality of truth is rooted not in differences between complete propositions but in differences between sub-propositional units: names, predicates, and functions. As a result, the plurality of truth is reduced to a plurality of reference, satisfaction, and fulfilment.

Similarly, here is Shapiro (2011, 41):

> [T]here are issues concerning quantified propositions, as the atomic components of quantified sentences are not themselves sentences, and thus do not express propositions. Those may be handled by invoking satisfaction instead of truth.

Taking the suggestion on board on behalf of pluralists, there are distinct domain-specific satisfaction relations. The main task is to formally develop this suggestion.

The proposed first-order extension, more formally

Let us move to a countable first-order language $\mathscr{L}+$, which includes quantified formulas. In this setting, the metavariables $\varphi, \psi, \chi, \ldots$ range over all formulas; $\tau_1, \tau_2, \ldots$ range over all terms (i.e., constants and variables); υ ranges over variables; and Φ^n ranges over n-ary predicates. The structures $\mathfrak{D}$, $\mathfrak{T}$, and $\mathfrak{F}$ are much as before.

A model is now a triple $M = \langle d, U, I\rangle$, where d is a domain assignment function, U is a nonempty set of objects, and I is an interpretation function. I discuss domain assignment functions and interpretation functions in more detail in what follows.

An interpretation function maps constants to members of U and n-ary predicates to functions from sets of n-tuples of objects to domain-specific truth values:

(I′) (i) $I(\varepsilon) \in U$ if ε is a constant;

(ii) $I(\Phi^n) \in \{f \mid f : U^n \to \mathscr{T} \cup \mathscr{F}\}$.

To motivate the definition of domain assignment functions, consider that the domain that a formula is assigned is determined by both the predicates and the relevant objects. The predicates are not enough to determine the domain. While "Water is H_2O" and "$0 = \{\}$" contain exactly the same predicates (i.e., the identity predicate), they are assigned distinct domains. That the former sentence is about scientific subject matter suggests that it is associated with the scientific domain (but it is not associated with the mathematical domain). And that the latter sentence is about mathematical subject matter suggests that it is associated with the mathematical domain (but it is not associated with the scientific domain). Similarly, the relevant objects are not enough to determine the domain. While "2 is a prime number" and "The number 2 is beautiful" are about exactly the same objects (i.e., the number 2), they are associated with distinct domains. That the former sentence involves a mathematical predicate suggests that it is associated with the mathematical domain (but it is not associated with the aesthetic domain). And that the latter sentence involves an aesthetic predicate suggests that it is associated with the aesthetic domain.

To capture the above, a domain assignment function now needs to assign formulas together with variable assignment functions to domains, where a variable assignment function a based on a model M assigns variables to members of U. Since only the terms' denotations (rather than the terms themselves) matter for determining the domain, we go through denotation functions and auxiliary domain assignment functions. A denotation function $[\cdot]_a$ based on variable assignment function a is defined by the following:

(DN) (i) $[\varepsilon]_a = I(\varepsilon)$ for all constants ε;

(ii) $[v]_a = a(v)$ for all variables v.

An auxiliary domain assignment function x from n-ary predicates and n-tuples of objects to domains, meanwhile, is required to satisfy the following, where $\langle u_1, \ldots, u_n \rangle \in U^n$:

(XD) $x(\Phi^n, \langle u_1, \ldots, u_n \rangle) \in \mathscr{D}$.

So, setting aside quantified formulas for now, we define d by the following:

(DO′) (i) $d(\Phi^n\tau_1 \ldots \tau_n, a) = x(\Phi^n, \langle[\tau_1]_a, \ldots, [\tau_n]_a\rangle)$;

(ii) $d(\neg\varphi, a) = d(\varphi, a)$;

(iii) $d(\varphi \wedge \psi, a) = d(\varphi, a) \odot d(\psi, a)$;

(iv) $d(\varphi \vee \psi, a) = d(\varphi, a) \odot d(\psi, a)$.

In the special case where $n = 0$, so that an atomic formula Φ^0 is just a sentence letter, d assigns Φ^0 to a domain irrespective of the variable assignment function.

The way d works with quantified formulas needs more explanation. The idea is that the domain assigned to a given quantified formula is determined by the domains assigned to the embedded formula. For example, the domain assigned to "Every x is such that x is identical to x" is determined by the domains assigned to the formula "x is self-identical" relative to all variable assignment functions that differ at most in their assignment to the variable x. Since there might be infinitely many such variable assignment functions, we assume that the mixing operation $\odot$ on domains is defined for infinite sets of domains. Let $a \sim_v b$ mean that variable assignment function a differs from variable assignment function b at most in the assignment to the variable v. Then, now focusing on quantified formulas, given that the universal quantifier and the existential quantifier are duals, and negation preserves domain membership, we define d by the following:

(DO′) (v) $d(\forall v\varphi, a) = \bigodot\{d(\varphi, a_i) \mid a_i \sim_v a\}$;

(vi) $d(\exists v\varphi, a) = \bigodot\{d(\varphi, a_i) \mid a_i \sim_v a\}$.

Given a model M, the semantic value function $|\cdot|_{M,a}$ based on M and variable assignment function a treats terms and predicates as usual:

(SV) (i) $|\tau|_{M,a} = [\tau]_a$ for all terms τ;

(ii) $|\Phi^n|_{M,a} = I(\Phi^n)$ for all predicates Φ^n.

To treat atomic formulas, we require that $|\cdot|_{M,a}$ satisfies the following:

(ATM) $|\Phi^n\tau_1 \ldots \tau_n|_{M,a} = T_{d(\Phi^n\tau_1\ldots\tau_n,a)}$ iff $|\Phi^n|_{M,a}(|\tau_1|_{M,a}, \ldots, |\tau_n|_{M,a}) \in \mathscr{T}$.

The account of negation, conjunction, and disjunction is much as before:

(LO*′) (i) $|\neg\varphi|_{M,a} = T_{d(\neg\varphi,a)}$ iff $|\varphi|_{M,a} = F_{d(\varphi,a)}$;

(ii) $|\varphi \wedge \psi|_{M,a} = T_{d(\varphi\wedge\psi,a)}$ iff $|\varphi|_{M,a} = T_{d(\varphi,a)}$ and $|\psi|_{M,a} = T_{d(\psi,a)}$;

(iii) $|\varphi \vee \psi|_{M,a} = T_{d(\varphi\vee\psi,a)}$ iff $|\varphi|_{M,a} = T_{d(\varphi,a)}$ or $|\psi|_{M,a} = T_{d(\psi,a)}$.

The account of the universal and existential quantifiers is as follows:

(LO*′) (iv) $|\forall\upsilon\varphi|_{M,a} = T_{d(\forall\upsilon\varphi,a)}$ iff $|\varphi|_{M,b} = T_{d(\varphi,b)}$ for all b where $b \sim_\upsilon a$;

(v) $|\exists\upsilon\varphi|_{M,a} = T_{d(\exists\upsilon\varphi,a)}$ iff $|\varphi|_{M,b} = T_{d(\varphi,b)}$ for some b where $b \sim_\upsilon a$.

Finally, the account of logical consequence is much as before:

(LC*′) $\Gamma \vDash \varphi$ iff for every model M, if $|\gamma_i|_{M,a} = T_{d(\gamma_i,a)}$ for every γ_i in Γ, then $|\varphi|_{M,a} = T_{d(\varphi,a)}$.

Answering the central logical challenges

Working through some simple examples shows that the resulting account makes intuitively plausible predictions and should be taken seriously by pluralists. Again, let us assume for the sake of illustration that there are exactly four pure domains: D_1 is the scientific domain, D_2 is the mathematical domain, D_3 is the ethical domain, and D_4 is the aesthetic domain.

To illustrate with a pure atomic sentence and a mixed atomic sentence, consider the following sentences:

(3) Murder is wrong.

(4) The number π is beautiful.

Suppose that (3) is an atomic sentence associated with D_3, and that (4) is an atomic sentence associated with the impure domain $D_2 \odot D_4$. On the intended model, $M_@ = \langle d_@, U_@, I_@\rangle$, the domain assignment function $d_@$ assigns (3) to D_3 and (4) to $D_2 \odot D_4$.

Meanwhile, the interpretation function $I_@$ assigns "Murder" to murder, "wrong" to a function f_1 from objects to domain-specific truth values, "π" to π, and "beautiful" to a function f_2 from objects to domain-specific truth values. Intuitively, (3) is true, and assuming—for the sake of illustration—that (4) is also intuitively true, the account vindicates these intuitions. Supposing that f_1 maps murder to a domain-specific truth value in $\mathscr{T}$, $|\cdot|_{M_@,a_@}$ assigns (3) to T_3. And supposing that f_2 maps π to a domain-specific truth value in $\mathscr{T}$, $|\cdot|_{M_@,a_@}$ assigns (4) to $T_2 \odot T_4$.

To illustrate with a quantified sentence, consider the following sentence:

(7) Everything is self-identical.

Suppose that (7) is a sentence associated with the impure domain $D_\top = D_1 \odot D_2 \odot D_3 \odot D_4$. On the intended model, $d_@$ assigns (8) to $D_\top$. Meanwhile, $I_@$ assigns "self-identical" to a function f_3 from pairs of objects to domain-specific truth values. Intuitively, (7) is true, and the account vindicates this intuition: $|\cdot|_{M_@,a_@}$ assigns (7) to $T_\top$ iff $|\cdot|_{M_@,a_i}$ assigns "x is self-identical" to the domain-specific truth value associated with truth in the relevant domain for all variable assignments a_i such that $a_i \sim_x a_@$. Supposing we have this—i.e., for every assignment of x to something o, the function f_3 maps the pair $\langle o, o \rangle$ to a truth value in $\mathscr{T}$—$|\cdot|_{M_@,a_@}$ assigns (7) to $T_\top$.

I submit that the account of logic and semantics—for a first-order language—proposed on behalf of pluralists answers all four of the central logical challenges for pluralism in a sufficiently systematic, precise, and unified way that meets all the proposed constraints and desiderata. Accordingly, pluralists can answer all the central logical challenges for their view. It is straightforward to show that predictions analogous to those involving (1)–(6) and (9)–(14) in the original setting carry over to this first-order setting, so the overall account answers three of the four central logical challenges. Clearly, since the account of quantified sentences is unified, the overall account also answers the remaining challenge, to maintain a unified account of quantified sentences.

4.3 Domains and truth properties

In closing, let me address a salient issue concerning the number and nature of domains and domain-specific truth properties in the account, and then explain how I think pluralists should answer the central conceptual challenges.

Domains and domain-specific truth properties

A salient issue concerns the number and nature of domains and domain-specific truth properties. A worry is that there is an objectionable proliferation of domains and domain-specific truth properties, by reasoning as follows. Given just two pure domains, D_1 and D_2, there is an impure domain $D_3 = D_1 \odot D_2$, another impure domain $D_4 = D_1 \odot D_3$, still another impure domain $D_5 = D_2 \odot D_3$, and so on. More generally, for distinct domains D_i and D_j, $D_i \odot D_j$ is distinct from each of D_i and D_j, so that two pure domains suffice to generate infinitely many domains. Since each domain is associated with a domain-specific truth property, there are also infinitely many domain-specific truth properties. This proliferation of domains and domain-specific truth properties is objectionable, because it requires a significant departure from the pure domains and associated domain-specific truth properties on the motivating picture for pluralism. Also, the nature of impure domains and associated domain-specific truth properties is unclear.

However, I think this worry is not serious. The reasoning for the conclusion that just two pure domains generate infinitely many domains is flawed. For there is no presumption that D_3 is distinct from either D_4 or D_5. All that is required is that, e.g., if $D_1 \not\leq D_3$ and $D_3 \not\leq D_1$, then $D_4 = D_1 \odot D_3$ is distinct from each of D_1 and D_3. But since $D_1 < D_3$, this is trivially satisfied even when $D_4 = D_3$. To control the number of domains and domain-specific truth properties, the following desirable requirement follows from (PO):

> *Requirement.* If $D_i \leq D_j$ or $D_j \leq D_i$, then $D_i \odot D_j = \max\{D_i, D_j\}$.

For suppose that $D_i \leq D_j$ or $D_j \leq D_i$. In the former case, by (PO), $D_i \odot D_j = D_j$; and in the latter case, by (PO), $D_j \odot D_i = D_i$. In either case, it follows that $D_i \odot D_j = D_j \odot D_i =$

$\max\{D_i, D_j\}$. This requirement holds, e.g., that the mixed domain induced by the pure domain associated with scientific subject matter and the mixed domain associated with scientific and mathematical subject matters is just the latter domain.

In principle, two pure domains only call for one impure domain, for a total of three domains. What if there are three or more pure domains? In general, the account is neutral on exactly how many impure domains there are, and permits coarser-grained as well as finer-grained accounts of impure domains. In the case where there are three pure domains D_1, D_2, and D_3, the account is neutral on whether the impure domains $D_1 \odot D_2$, $D_2 \odot D_3$, $D_1 \odot D_3$, and $D_1 \odot D_2 \odot D_3$ are identical. On a maximally coarse-grained account, they are all identical, while on a maximally fine-grained account, they are all distinct. There are also intermediate accounts in between these two extremes. Pluralists may endorse each of these accounts, and it is a virtue that the proposed account of logic and semantics can accommodate each. On a maximally coarse-grained account of impure domains, there is exactly one impure domain, while on a maximally-fine grained account, there are four impure domains. More generally, where the cardinality of the set of pure domains is κ, the cardinality of the set of impure domains ranges from 1 to $2^\kappa - 1$, so that the cardinality of the set of all domains ranges from κ to $2^\kappa - 1$.

Granted, perhaps the commitment to impure domains and associated domain-specific truth properties alone is objectionable, since there is still a departure from pure domains. Yet there might be no real alternatives. Desiderata A and B simply seem to call for the commitment. There are apparently insurmountable challenges to maintaining a standard account of even just complex sentences if only pure domains are admitted. With only pure domains, on one natural account of complex sentences, conjunction and disjunction are minimizing and maximizing operations on domain-specific truth values, respectively. But as I argued in Section 3.1, this account risks violating Desideratum C. On another natural account, conjunctions and disjunctions inherit one or more of the domain-specific truth values of their operands. But as I argue in Appendix A.2, on plausible assumptions, this account fails to be compositional.

To avoid the charge of obscurity, we can maintain the following about the metaphysics of impure domains and domain-specific truth properties. The existence of impure domains is grounded in—or at least supervenes on—the existence of pure domains. Similarly, the existence of impure domain-specific truth properties is grounded in—or supervenes on—the existence of pure domain-specific truth properties.

Answering the central conceptual challenges

I close by explaining how I think pluralists should, given the proposed account, accordingly answer the central conceptual challenges.

Answering the first two conceptual challenges, to clarify the conceptions of domains and the association between truth properties and domains, is straightforward. The answer to the first conceptual challenge, to clarify the conception of domains, is that every sentence is associated with exactly one domain. Given that domains can be impure, even mixed atomic sentences and quantified sentences are easily accommodated. Mixed atomic sentences are associated with impure domains. Quantified sentences can also be associated with impure domains—perhaps the psuedo-generic domain, $D_\top$, which is associated with all subject matters. Meanwhile, the answer to the second conceptual challenge, to clarify the conception of the association between truth properties and domains, is that there is a one-to-one correspondence between them: every domain is associated with exactly one domain-specific truth property, and every domain-specific truth property is associated with exactly one domain.

Answering the third conceptual challenge, to clarify the conception of truth properties, is less straightforward but still feasible. In Section 2.1, I suggested that even pluralists should grant that a generic truth property exists. But I kept it open to what extent pluralists should grant such a property significance. Does the proposed account grant a generic truth property too much significance?

Before I answer this question more fully, let me first set aside a mistaken worry. The worrier reasons as follows. Given pure domains $D_1, D_2, \ldots$, the account guarantees that there is an impure domain $D_\top = D_1 \odot D_2 \odot \ldots$ associated with all subject matters, which is the top element in the partial order on domains. Since there is a one-to-one correspondence between domains and domain-specific truth properties, it follows that there is a domain-specific truth property $T_\top$ that is associated with associated with the domain $D_\top$, so that $T_\top$ is the top element in the partial order on domain-specific truth properties. But this special truth property, $T_\top$, seems to be a generic truth property. Crucially, $T_\top$ seems significant, since it is the relevant truth property when determining the truth or falsity of certain quantified sentences, such as (8).

This worrier is mistaken: $T_\top$ is not a generic truth property. For if $T_\top$ were a generic truth property, then domain-specifically true sentences would all be assigned $T_\top$. But this need not be—domain-specifically true sentences can be assigned any domain-specific truth property, not only $T_\top$. At most, $T_\top$ is only a *pseudo-generic* truth property, in that it is associated with a domain that is in turn associated with all subject matters.

Still, the above does not fully answer questions about the significance of a generic truth property. A more serious worry is that the account permits the recoverability of generic truth from domain-specific truth, and might even ascribe significance to it. Given a model M and associated semantic value function $|\cdot|_M$, we can define the alternative semantic value function $||\cdot||_M$ from sentences to $\{T, F\}$ by the following:

(TG) $||\varphi||_M = T$ iff $|\varphi|_M = T_{d(\varphi)}$.

Perhaps—the worry goes—generic truth is even "doing all the work," and so is ascribed too much significance.

However, I think this worry does not pose a serious threat to the proposed account. The recoverability of generic truth from domain-specific truth is not a special feature of the account. It only relies on the core pluralist commitment that there are distinct domain-specific truth properties, and holds whether or not one is considering the account. The Generalization Argument already shows that even pluralists are committed to the existence of a generic truth property. While the account vindicates the recoverability of generic truth, it does so not because it introduces further commitments not compulsory to pluralism, but rather because it makes explicit what pluralists are already implicitly committed to. Insofar as the recoverability of generic truth is a problem at all, it is a problem for pluralists, rather than for the account itself.

More importantly, as I have suggested, pluralists can downplay the significance of a generic truth property. They can maintain that a generic truth property is not a natural property. Even if it is a natural property, perhaps because it satisfies the truth-characterizing principles, pluralists can still maintain that it is less natural than any domain-specific truth property, or that it is explanatorily or metaphysically posterior to any domain-specific truth property.

Further, pluralists need not be embarrassed by the recoverability of generic truth from domain-specific truth. If anything, they can view the recoverability of generic truth from domain-specific truth as a feature rather than a bug. For even if pluralism is true, pluralists arguably owe an explanation of why the familiar accounts of complex sentences and of logical consequence are so compelling. Pluralists can provide such an explanation by exploiting the proposed account and (TG). For together, they show how a pluralist-friendly account vindicates the attractions of the familiar accounts: (LO) is generated by (LO*), and (LC) is generated by (LC*). In each case, the former is less natural than, or somehow posterior to, the latter.

For those suspicious of the appeal to explanatory or metaphysical priority, the account at least clarifies exactly what pluralists' commitments are (or might be), and provides a systematic framework for evaluating pluralism—especially with respect to its take on the standard way of doing logic and semantics.[55] The development of the view in a transparent way alone warrants taking the account seriously, I suggest, if only as a tool to see how pluralism might be further developed.

[55]See Dorr and Hawthorne (2014a, 72–73) on skepticism about taking notions of priority too seriously.

Chapter 2

A modal account of propositions

> The totality of all logical objects, or of all propositions, involves, it would seem, a fundamental logical difficulty.
>
> — Russell (2010, 540)

An intuitively appealing conception of propositions has it that at least some propositions are determined by objects and properties ascribed to objects. But propositions are presumably themselves objects of a certain kind, so we can iterate the determination of propositions indefinitely.

In this chapter, I motivate and develop a modal account of propositions on the basis of an iterative conception of propositions. On the iterative conception of propositions, propositions are generated from more basic entities such as objects and properties—or more generally, sequences of objects and relations. The conception suggests that the existence of a given proposition is posterior to the prior existence of the objects the proposition is about. The modal account of propositions takes the conception to motivate an inherently potential hierarchy of propositions, so that every object and property could but need not determine a proposition, where the modality is irreducibly logico-mathematical. Accordingly, the concept *proposition* is indefinitely extensible, so that the concept *propositional truth* is also indefinitely extensible. As applications, I show that the proposed account helps provide satisfying solutions to the intensional paradoxes of Russell-Myhill, Kaplan, and Prior. These paradoxes threaten theories of intensionality as much as the semantic paradoxes threaten theories of meaning, but have received relatively little attention in the literature.

This chapter is structured as follows. In Section 1, I articulate the background conception of propositions I assume. In Section 2, I present the intensional paradox of Russell-Myhill, as well as Russell's paradox and how it motivates a modal account of sets. In Section 3, I present a modal account of propositions, and show that it provides a satisfying solution to the Russell-Myhill paradox. In Section 4, I develop the account in more detail. In Section 5, I motivate and develop a combined modal account of propositions and possible worlds, and show that it provides satisfying solutions to two further intensional paradoxes—Kaplan's and Prior's paradoxes.

1 Propositions

In this section, I articulate the background conception of propositions I assume. This conception is to guide the development of an account of propositions. In the first subsection, I highlight the roles of propositions I take to be nonnegotiable. In the second subsection, I endorse a neo-Russellian account of propositional identity.

1.1 Roles

Standardly, propositions are supposed to play several roles: they are supposed to be the fundamental bearers of truth and falsity, the objects of mental attitudes, the contents of assertion, the meanings of sentences, the referents of "that"-clauses, and more.[1] I maintain that the first two roles are nonnegotiable, and that the third role is nonnegotiable at least for my purposes in this chapter. Let me elaborate.

Uncontroversially, a first role of propositions is to be the fundamental bearers of truth and falsity. Accordingly, the truth or falsity of other kinds of truth bearers—

[1]See Richard (1990, 2013, 2015); McGrath (2012); King et al. (2014).

sentences in context, utterances, and so on—are true or false in a way derivative of the truth or falsity of associated propositions. On one natural view, a sentence is true in a given context iff the sentence expresses a true proposition in that context.

Granted, on some conceptions, propositions can be neither true nor false, or have a truth value other than just truth or falsity. Such conceptions are often motivated by the role of propositions (which I mention next) as the objects of mental attitudes. For perhaps one believes a vague proposition or some other "nonclassical" proposition that is neither true nor false. However, in this chapter, I assume for simplicity that propositions are just either true or false.

Relatively uncontroversially, a second role of propositions is to be at least the possible objects of mental attitudes such as belief, hope, and desire. Accordingly, if I believe that grass is green, then I believe a proposition, and in particular, the proposition that grass is green. Similarly with attitudes other than belief.

Granted, some philosophers deny that the having of mental attitudes entails the existence of even possible objects of such attitudes.[2] However, such philosophers probably prefer to theorize without propositions at all. In this chapter, I assume that propositions have a place in our theorizing, and so are worth theorizing about. But given this assumption, it is almost universally agreed that propositions are at least the possible objects of mental attitudes.

Somewhat more controversially, a third role of propositions is to be the contents of assertion.[3] On a simplified version of a standard account of assertion, a speaker's assertive utterance expresses a unique proposition, and if all goes well a hearer comes to believe that proposition. Accordingly, my assertive utterance of "Whales are mammals" expresses the proposition that whales are mammals, and if all goes well you come to believe this proposition. However, this role is somewhat controversial, for at least two reasons. First, it is unclear whether the contents of assertion are true or false only relative to a time (as temporalists maintain), and whether they are true or false only relative

[2]See Quine (1956, 1961, 1986, 2013); Field (1978).

[3]See Stalnaker (1978).

to some more general parameter that includes a person as well as a time (as relativists maintain). Second, according to some theories, the contents of assertion can be either multiple propositions rather than individual propositions, or "incomplete" propositions that must be somehow enriched to become complete propositions.[4]

Other than the three roles just mentioned, propositions are also supposed to play at least two other roles. A fourth role of propositions is to be the meanings—specifically, "semantic contents," or "semantic values"—of declarative sentences, at least relative to context. Accordingly, relative to context, the meaning of the sentence "Grass is green" is a proposition, namely, the proposition that grass is green. However, this other role is more controversial than the three roles just mentioned. First, some theorists maintain that the meaningfulness of expressions does not entail the existence of meanings.[5] Second, some natural language semantic theories suggest that, on the basis of considerations involving compositionality and modal operators, the meanings of declarative sentences need not be propositions. All that matters, according to such theories, is that propositions are recoverable from the meanings of declarative sentences, but this does not entail that propositions are themselves the meanings.[6]

A fifth role of propositions is to be the referents of "that"-clauses. Accordingly, the referent of "that grass is green" in the belief ascription "John believes that grass is green" is a proposition, namely, the proposition that grass is green. However, this other role is also more controversial. For consider what Moltmann (2003) calls the "substitution problem": the role suggests that expressions of the form "that p" and expressions of the form "the proposition that p" are coreferential and so presumably intersubstitutable *salva veritate*, but the two forms of expressions are not intersubstitutable *salva veritate*. For example, in "John fears that the bear will attack him," the expression "that the bear will attack him" does not seem intersubstitutable *salva veritate* with the expres-

[4]See Lormand (1996); Soames (2002, 2009); Cappelen and Lepore (2005); Braun and Sider (2007); Cappelen (2008); Dorr and Hawthorne (2014b) for views according to which the contents of assertion can be multiple propositions. And see Bach (1994); Soames (2005) for views according to which the contents of assertion can be "incomplete" propositions.

[5]See Quine (1956, 1961, 1986, 2013); Schiffer (1987).

[6]See Lewis (1980); Dummett (1991b); Ninan (2010).

sion "the proposition that the bear will attack him." Presumably, "John fears that the bear will attack him" may be true in some context, but "John fears the proposition that the bear will attack him" may not be true in that same context. After all, John's fearing the bear is one thing, and John's fearing a proposition is another thing.

In addition to the five roles just mentioned, propositions are supposed to play some further roles, in being the fundamental bearers of alethic modal properties (such as necessary truth and possible truth), the relata of the entailment relation, and the constituents of possible worlds.

We have seen that propositions are supposed to play several roles. However, I take it to be an open question whether a single kind of entity can play all these roles. As Lewis (1986, 54) suggests, "the conception we associate with the word 'proposition' may be something of a jumble of conflicting *desiderata*." Accordingly, I follow McGrath (2012) in maintaining a somewhat minimalist conception of propositions:

> The best way to proceed, when dealing with quasi-technical words like 'proposition', may be to stipulate a definition and proceed with caution, making sure not to close off any substantive issues by definitional fiat.
>
> Propositions, we shall say, are the sharable objects of the attitudes and the primary bearers of truth and falsity.

As I understand them, propositions are abstract objects that are either true or false and that we can bear attitudes to. I take the roles of propositions as the fundamental bearers of truth and falsity and as the objects of attitudes to be nonnegotiable. For my purposes in this chapter, I also take the role of propositions as the contents of assertion to be nonnegotiable. I am neutral on whether propositions must also play the other roles they are often supposed to play, as these other roles do not concern me in what follows.

1.2 Identity conditions

Notoriously, Quine (1956, 180) repudiated propositions as "creatures of darkness," in part because he found it unclear what the identity conditions for propositions are. The contrast is with sets, which have clear identity conditions: sets are identical iff they have the same members. Now contemporary philosophers are much less reluctant than Quine to help themselves to talk of propositions, as I am doing here. Still, I take it that it remains controversial what the identity conditions for propositions are.

In what follows, I discuss various accounts of propositional identity, arguing that a finer-grained account best serves my purpose of providing an account of propositions. In doing so, I reject accounts according to which propositions are too coarse-grained, such as the material equivalence and necessary equivalence accounts. I endorse a neo-Russellian account of propositional identity, according to which at least some propositions are determined by objects and properties ascribed to objects. Such an account leaves it open exactly how fine-grained an account of propositional identity should be, but provides at least a necessary condition for propositional identity, or equivalently a sufficient condition for propositional distinctness.

Propositional identity as material equivalence

On a maximally coarse-grained account of propositional identity, materially equivalent propositions are identical:

> *Propositional identity as material equivalence.* Proposition p is identical to proposition q iff (p is true iff q is true).

This account is maximally coarse-grained in that it is the minimum needed for propositions to be bearers of truth and falsity. There is a sense in which Frege maintained this account of propositional identity.[7] For he maintained that sentences refer to truth values, namely, either truth or falsity. Insofar as sentences refer to propositions, propositions are truth values. But to be fair, the sense in which Frege maintained this account is misleading. Frege probably would not have put things this way, since for him propositions are not truth values, but rather structured meanings; he would deny that sentences refer to propositions. In any case, according to the account, given that the proposition

[7] See Frege (1948).

that grass is green is true but the proposition that snow is red is false, the proposition that grass is green is distinct from the proposition that snow is red.

However, the material equivalence account of propositional identity is, I take it, clearly too coarse-grained. On this account, there are only two propositions: a single true proposition and a single false proposition. Accordingly, given that the proposition that grass is green and the proposition that $1+1=2$ are both true, the proposition that grass is green is identical to the proposition that $1+1=2$. On virtually any plausible account of propositional identity, this prediction is unacceptable.

This is not, of course, to deny that the material equivalence account might be acceptable for some theoretical purposes. Notably, the account, which is implicit in the standard provision of a classical semantics for a sentential language, might be acceptable for the purpose of theorizing about the logical consequence relation when we restrict our attention to a simple fragment of natural language. But the account is not acceptable for the purpose of theorizing about intensional rather than extensional matters, which calls for a finer-grained account of propositional identity.

Thus, I reject the material equivalence account of propositional identity for the present purposes.

Propositional identity as necessary equivalence

On a standard account of propositional identity, which is less coarse-grained than the material equivalence account but still coarse-grained, necessarily equivalent propositions are identical:

> *Propositional identity as necessary equivalence.* Proposition p is identical to proposition q iff (necessarily, p is true iff q is true).

Stalnaker (1976) maintains such an account of propositional identity. Here, necessity is understood as metaphysical necessity. On a standard account of metaphysical necessity in terms of truth in all (metaphysically) possible worlds, the necessary equivalence account is equivalent to the following:

> Proposition p is identical to proposition q iff (in every possible world, p is true iff q is true).[8]

Accordingly, given that there is a possible world at which the proposition that grass is green is true but the proposition that snow is white is false, the proposition that grass is green is distinct from the proposition that snow is white. Similarly, given that there is a possible world at which the proposition that grass is green is false but the proposition that $1 + 1 = 2$ is true, the proposition that grass is green is distinct from the proposition that $1 + 1 = 2$.

However, the necessary equivalence account of propositional identity is, I take it, still too coarse-grained.[9] Although it correctly distinguishes between the proposition that grass is green and the proposition that $1 + 1 = 2$, it founders when it comes to necessarily equivalent but intuitively distinct propositions. The account predicts that all necessary propositions, and so all mathematical truths, are identical. But this, I take it, is unacceptable. For one can believe some but not all mathematical propositions: one can believe that $1 + 1 = 2$ but not believe that not all continuous functions are differentiable. Even necessarily equivalent propositions can be distinct. Granted, proponents of the necessary equivalence account—including Stalnaker (1976)—are aware of the apparent problem and have attempted to show that it is not a genuine problem. However, it is controversial whether such defenses of the account succeed.

Again, this is not, of course, to deny that the necessary equivalence account might be acceptable for some theoretical purposes. Notably, the account might be acceptable for the purpose of theorizing about assertion and communication when we restrict our attention to a simple fragment of natural language. But the account seems to fall short in general: it seems to fare poorly when we move beyond contingently true propositions, such as the proposition that grass is green, to necessarily true propositions, such as the proposition that not all continuous functions are differentiable.

[8] Quantification over possible worlds might be either in the object language or the metalanguage. Also, propositions might be identified with sets of possible worlds, so that a proposition p is true at a possible world w iff w is a member of p.

[9] See Soames (1987) for this and related criticisms.

Thus, I also reject the necessary equivalence account of propositional identity for the present purposes.

Propositional identity as finer-grained than necessary equivalence

I have suggested that both the material equivalence and necessary equivalence accounts are too coarse-grained. If the only role propositions are supposed to play is to be the fundamental bearers of truth and falsity, then admittedly perhaps either the material equivalence account or the necessary equivalence account suffices. Accordingly, perhaps for some distinct sets A and B, the proposition that set A is consistent is identical to the proposition that set B is consistent, as claimed by both accounts, because the two propositions are necessarily equivalent and so materially equivalent as well. But given that another role propositions are supposed to play is to be the objects of attitudes such as belief, I take it that propositions must be finer-grained even than on the necessary equivalence account. On reflection, the identification of the proposition that A is consistent with the proposition that B is consistent seems much less plausible once we consider that one might believe the proposition that set A is consistent without believing the proposition that set B is consistent.

We want an account that is finer-grained than the necessary equivalence account.[10] But how fine-grained an account do we want?

It is worth proceeding with caution. We should not overemphasize the role of propositions as the objects of attitudes. Appealing to the role to motivate a fine-grained account of propositional identity is tricky business. For a risk is that the role motivates an extremely—and perhaps, excessively—fine-grained account, such as the following:[11]

Propositional identity as belief equivalence. Proposition p is identical to proposition q

[10] One way of putting it is that, in popular terminology, we want an account according to which propositions are structured, in that they have constituents. But, given that I do not think propositions must be structured—at least according to the standard conception of structured propositions, where propositions are ordered tuples with abstract or concrete constituents—in order to be fine-grained, I avoid speaking of propositions as structured and prefer to just speak of them as being more or less fine-grained.

[11] See Peacocke (1992, 102) for such an account.

iff (necessarily, everyone is such that they believe that *p* iff they believe that *q*).

But data concerning attitude ascriptions are delicate. As Mates (1952) points out, such data suggest that one cannot even freely substitute synonyms for synonyms in belief contexts. Since presumably one can believe that Sam chews without believing that Sam masticates, if one does not know that "chews" is synonymous with "masticates," it is not the case that, necessarily, everyone is such that they believe the former proposition iff they believe the latter proposition. So, in light of the fact that it is possible for one to believe one proposition without believing the other proposition, the belief equivalence account predicts that the proposition that Sam chews is distinct from the proposition that Sam masticates. But perhaps these propositions are not only necessarily equivalent, but also identical. If the propositions are indeed identical, then the account is probably too fine-grained.

Still, I think the appeal to the role of propositions as the objects of attitudes has some force. For what it is worth, the appeal has standardly been one of the main motivations for maintaining an account of propositional identity that is at least finer-grained than the necessary equivalence account. While I suppose that it is conceivable that the many accounts that have been motivated on this basis are ill-conceived, I am more inclined to think that such accounts are worth pursuing, perhaps with the proviso that they are not excessively fine-grained.

More importantly, however, one need not slip on the slope Mates suggests. Not every deviation from the necessary equivalence account is committed to taking the role of propositions as the objects of attitudes to require propositions so fine-grained that the proposition that Sam chews is distinct from the proposition that Sam masticates. On the contrary, I take it, there are principled reasons for maintaining an account of propositional identity that is finer-grained than the necessary equivalence account but coarser-grained than the belief equivalence account.

Plausibly, the proposition that $1 + 1 = 2$ is distinct from the proposition that not all continuous functions are differentiable, even though the two propositions are necessarily equivalent. We can grant this even in the absence of an account of propositional identity, just as we can grant that Obama is distinct from Xi even in the absence of an account of personal identity. Indeed, I take the distinctness of these two propositions to be a basic datum that any plausible account of propositional identity must account for.

The basic datum that the proposition that $1 + 1 = 2$ is distinct from the proposition that not all continuous functions are differentiable is neutral with respect to neo-Fregean and neo-Russellian accounts of propositions. Both accounts take propositions to be structured complexes of some kind. The difference lies in what kinds of entities the accounts take the complexes to involve. The neo-Russellian account takes the kinds of entities involved to be representation-independent entities—objects, properties, and relations. The neo-Fregean account, meanwhile, takes the kinds of entities involved to be representation-dependent entities: senses—or modes of presentation—of objects, properties, and relations. The senses might be objective, as Frege thought of them, but the neo-Fregean account still provides more fine-grained an account of propositional identity than that provided by the neo-Russellian account, since many senses can correspond to the same referent.

Clearly, the neo-Fregean account can account for the basic datum. For the senses involved in the proposition that $1 + 1 = 2$ are distinct from the senses involved in the proposition that not all continuous functions are differentiable. But the neo-Russellian account can account for the basic datum equally well. For the objects, properties, and relations involved in each proposition are distinct. The ascription of identity to the number obtained by adding one to one and the number two is distinct from the ascription of nonemptiness to the extension of the property of being a continuous but nondifferentiable function.

A neo-Russellian account

I endorse a neo-Russellian account that takes propositions to ascribe properties to objects—or more generally, relations to sequences of objects. Accordingly, the proposition that grass is green ascribes the property of being green to grass. Similarly, the proposition that Jamie likes Liann ascribes the relation of liking to Jamie and Liann, in that order. For simplicity, I restrict my attention to propositions that predicate monadic properties.

Crucially, the account is not excessively fine-grained. Given that ascriptions of properties are relatively representation-independent, and that the property of chewing is identical to the property of masticating, the ascription of the property of chewing to Sam can still be identical to the ascription of the property of masticating to Sam, so that the proposition that Sam chews is identical to the proposition that Sam masticates.

The account provides necessary conditions for propositional identity, or equivalently, sufficient conditions for propositional distinctness. These conditions account for the basic datum that the proposition that $1 + 1 = 2$ is distinct from the proposition that not all continuous functions are differentiable. There are two more specific conditions, depending on the relevant kind of proposition. On the present, neo-Russellian account of propositions, a proposition ascribes a property to an individual object. But this account of propositions is a simplification. One consideration is that propositions can be complex. For consider the proposition that grass is green and snow is white. The account, as I have presented it so far, does not straightforwardly account for complex propositions. But let us set aside this complication for now, since it is not important for my purposes in this chapter. Another consideration is that some propositions ascribe a property not to an individual object but rather to a plurality of objects.[12] For consider the proposition that all and only the trees are green, or the proposition that all and only the British monarchs are revered. It is important for my purposes that we extend the account to accommodate such propositions as well.

Following standard usage, let us say that a *singular* proposition is a proposition that ascribes exactly one property P to an (individual) object o, and let us say that a *plural* proposition is a proposition that ascribes exactly one property P to a plurality oo of ob-

[12] See McKay (2006) on plural predication. Following the standard understanding, the locution "the plurality of objects" is meant to pick out just the objects themselves, and is not meant to entail the existence of something over and above the objects themselves.

jects. Further, let us say that a singular proposition that ascribes a property P to an object o is *about* o, and similarly let us say that a plural proposition that ascribes a property P to a plurality oo of objects is *about* oo. Accordingly, the proposition that Fido is a dog is a singular proposition that ascribes the property of being a dog to Fido, and is about Fido. Similarly, the proposition that all and only the the British monarchs are revered ascribes the property of being revered to all and only the British monarchs, and is about all and only the British monarchs. The present notion of aboutness is straightforward: propositions are just about the objects they ascribe properties to.

Keeping in mind the distinction between singular propositions and plural propositions, and the notion of aboutness, I can now state the two sufficient conditions for propositional distinctness—or equivalently, the two necessary conditions for propositional identity. The relevant condition depends on whether we are dealing with singular propositions or plural propositions. First, we have that if the singular proposition p is about the object a and the singular proposition q is about the object b, where a is distinct from b, then p is distinct from q; equivalently, where p and q are singular propositions, if p is identical to q, then if p is about a and q is about b, then a is identical to b. Second, we have that if the plural proposition p is about the plurality aa of objects and the plural proposition q is about the plurality bb of objects, where aa is distinct from bb, then p is distinct from q; equivalently, where p and q are plural propositions, if p is identical to q, then if p is about aa and q is about bb, then aa is identical to bb.

These conditions are intuitively compelling. Given that the proposition that Fido is a dog is about Fido and the proposition that Lassie is a dog is about Lassie, where Fido is distinct from Lassie, the former proposition is distinct from the latter proposition. Similarly, given that the proposition that all and only the British monarchs are revered is about all and only the British monarchs and the proposition that all and only the Thai monarchs are revered is about all and only the Thai monarchs, where the plurality comprising all and only the British monarchs is distinct from the plurality comprising all and only the Thai monarchs, the former proposition is distinct from the latter proposition.

2 Propositions, sets, and paradox

In this section, I show how the background conception of propositions just articulated risks leading to paradox. In the first subsection, I present what I call the "naive conception of propositions," and show that it leads to (a version of) the Russell-Myhill paradox of propositions. In the second subsection, I draw an analogy between this paradox of propositions and Russell's paradox of sets. I mention what I think is a satisfying solution to Russell's paradox due to existing modal accounts of sets that I find attractive.

2.1 The Russell-Myhill paradox

Propositions seem easy to generate. With at least some propositions, the propositions' identities are determined by some objects, or pluralities of objects, and properties ascribed to those objects, or pluralities of objects.[13] Focusing on such propositions, propositions that ascribe the same properties to the same objects, or pluralities of objects, are identical, and posterior to those objects, or pluralities of objects. Accordingly, Fido and the property of being self-identical determine the proposition that Fido is self-identical. Similarly, a plurality of propositions and the property of being true determine the proposition that all and only the propositions among that plurality are true. In what follows, however, I show that a naive way of capturing these intuitions about propositions leads to the Russell-Myhill paradox.

[13] The qualification that this is with respect to "some propositions" is in place, because I am focusing on singular and plural propositions, as explained in the previous section. So far, I have said nothing about quantified propositions. But considering such propositions might complicate the picture. For example, consider the quantified proposition that ascribes the property P to every individual object x. And consider the plural proposition that ascribes the property P to the plurality xx comprising all and only the objects. These might be distinct propositions, even though there seems to be a sense in which they involve the ascription of the same property and ascribe that property to all and only the same objects, namely, everything. I set quantified propositions aside for now, but I briefly discuss them in Appendix B.1.

Notation

Given an (individual) object o and a property P, o and P generate a proposition, namely, the proposition that o is P, where this proposition is about o. Let $y \triangleright x$ mean that y is a proposition about x, and $\pi(y)$ mean that y is a proposition.[14] For simplicity, let us assume that only propositions are about objects:

(AP) $y \triangleright x \rightarrow \pi(y)$.

Since propositions can also be about pluralities of objects, let us extend the notion of propositional aboutness to pluralities of objects as well. Given a plurality oo of objects and a property P, oo and P generate a proposition, namely, the proposition that all and only the objects among oo are P, where this proposition is about all and only the objects among oo. By abuse of notation, let $y \triangleright xx$ mean that y is a proposition about all and only the objects among xx:

(APL) $y \triangleright xx := \forall x(x \prec xx \leftrightarrow y \triangleright x)$.

The predicate taking a plural term is understood distributively, in that it holds of an object y and a plurality of objects xx iff the corresponding predicate taking a singular term holds of y and all and only the objects among xx. To illustrate, $y \triangleright xx$ when y is the proposition that all and only the objects among xx are believed by me, y is the proposition that all and only the objects among xx are self-identical, or y is the proposition that all and only the objects among xx are true.

Let me clarify that $y \triangleright xx$ means that y is about all and *only* the objects among xx. While y's being about all and only the objects among xx entails that y is about every x among xx, the converse entailment does not hold: that y is about every x among xx does not entail that y is about all and only the objects among xx, since y might also be about some x' not among xx. A related point is that its being the case that $y \triangleright x$ does not entail

[14] It is natural to assume that propositions are not about themselves in any way: $\forall x \neg x \triangleright x$. Accordingly, the proposition that Yuvraj is happy is not about itself. Similarly, the proposition that snow is white and grass is green is not about any of its conjuncts.

its being the case that $y \rhd xx$, where xx is the plurality comprising just x: that y is about x does not entail that y is about all and only the objects among xx, where xx is the plurality comprising just x, since y might also be about some x' not among xx.

The naive conception of propositions

How should we capture our intuitions about propositions? Let me articulate what I call the "naive conception of propositions"; the reason for the qualification "naive" will become clear in what follows.

First, propositions not only seem, but *are*, easy to generate. Specifically, every plurality of objects determines a proposition that is about all and only the objects among that plurality:

(PD-) $\forall xx \exists y(y \rhd xx)$.

The legitimacy of plural quantification, employed in (PD-) and elsewhere in this chapter, is defended in Boolos (1984, 1985); Cartwright (1994); Rayo and Yablo (2001); Rayo (2002); Oliver and Smiley (2013). To illustrate, every plurality of objects determines the proposition that all and only the objects among that plurality are self-identical, or determines the proposition that all and only the objects among that plurality are true.

Second, propositions are fairly fine-grained. Specifically, if the pluralities of objects are distinct, then any propositions about them are also distinct:

(FG) $\forall xx \forall yy \forall x \forall y(xx \neq yy \wedge x \rhd xx \wedge y \rhd yy \rightarrow x \neq y)$.

To illustrate, the proposition that these players are good is distinct from the proposition that those players are good. Of course, the converse principle—according to which if the propositions are distinct, then the pluralities of objects they are about are also distinct—is not at all plausible. On the present understanding of aboutness, the proposition that these players are good is distinct from the proposition that these same players are bad, but the two propositions are nonetheless about the same plurality of objects, namely, the plurality comprising the relevant players. But the principle that is (FG) is plausible.

We can define the identity predicate that takes plural rather than singular terms as arguments by the following:

(IPL) $xx = yy := \forall z(z \prec xx \leftrightarrow z \prec yy)$.

This definition is natural, especially given the standard assumption that—like sets—pluralities' identities are completely determined by the objects among the pluralities. Crucially, given this definition, (FG) falls out as a theorem. For suppose that *aa* and *bb* are distinct pluralities, where *p* is about all and only the objects among *aa* and *q* is about all and only the objects among *bb*. Since *aa* and *bb* are distinct, there is some object *o* that is among *aa* but not among *bb* or among *bb* but not among *aa*. So either *p* but not *q* is about *o*, or *q* but not *p* is about *o*. Either way, *p* is distinct from *q*; for if they were identical, the substitution of one for the other would yield a contradiction of the form "*z* is and is not about about *o*."

Third, propositions and other objects can be picked out for consideration by means of a condition. Specifically, given any condition, there is a plurality comprising all and only the objects that satisfy the condition:

(PC) $\exists xx \forall x(x \prec xx \leftrightarrow \varphi(x))$.

To illustrate, we can consider the instance of (PC) where $\varphi(x)$ is $x = x$, which guarantees that there is a plurality comprising all and only the self-identical objects. To accommodate the case where nothing satisfies a given condition, let us assume that there is an empty plurality, so that in such a case the plurality guaranteed by the instance of (PC) for the relevant condition is empty.

The Russell-Myhill paradox

Unfortunately, the naive conception of propositions is inconsistent due to the Russell-Myhill paradox; see Russell (2010, Appendix B) for the original formulation of the paradox.[15] By the instance of (PC) where $\varphi(x)$ is $\exists yy(x \rhd yy \wedge x \not\prec yy)$, there is the Russell-

[15]See Myhill (1958)'s reformulation in Church (1951)'s intensional logic. See Frege (1980b); Klement (2001, 2010a,b); Tucker (2010, 2013); Tucker and Thomason (2011); Uzquiano (2015a,b,c); Goodman (2016)

Myhill plurality *mm* of all and only the propositions that are about some plurality of propositions and not among that plurality of propositions:

(RM1) $\forall x(x \prec mm \leftrightarrow \exists yy(x \rhd yy \wedge x \not\prec yy))$.

By (PD-), there is a proposition about *mm*:

(RM2) $\exists y(y \rhd mm)$.

By existential elimination on (RM2), we have that:

(RM3) $p \rhd mm$.

Further, by universal elimination on (RM1) [on *p*], we have that:

(RM4) $p \prec mm \leftrightarrow \exists yy(p \rhd yy \wedge p \not\prec yy)$.

Now, is *p* among *mm*? If *p* is among *mm*, then it is about distinct pluralities of propositions, in violation of (FG). To see this, assume for *reductio* that $p \prec mm$. By conditional elimination on this and the left-to-right direction of (RM4), we have that $\exists yy(p \rhd yy \wedge p \not\prec yy)$. By existential elimination on this, we have that $p \rhd nn \wedge p \not\prec nn$. Since $p \prec mm$ but $p \not\prec nn$, we have that $mm \neq nn$. But since $p = p$, $p \rhd mm$, and $p \rhd nn$, by (FG), $mm = nn$. Contradiction. But if *p* is not among *mm*, then it satisfies *mm*'s defining condition, and so it is among *mm*. To see this, assume for *reductio* that $p \not\prec mm$. By conjunction introduction on this and (RM3), we have that $p \rhd mm \wedge p \not\prec mm$. By existential introduction on this, we have that $\exists yy(p \rhd yy \wedge p \not\prec yy)$. By conditional elimination on this and (RM4), we have that $p \prec mm$. Contradiction. In short, we have that *p* is among *mm* iff *p* is not among *mm*:

(RMI) $p \prec mm \leftrightarrow p \not\prec mm$.

for discussion. The paradox can be formulated in terms of sets, classes, or pluralities; I formulate the paradox in terms of pluralities here. Also, the paradox can be formulated as a cardinality argument—by appealing to Cantor's theorem, in the case of sets, or Bernays' theorem (which generalizes Cantor's theorem), in the case of classes or pluralities. However, I avoid appealing to these theorems here, since such appeals are unnecessary to generate the paradox.

Standard principles of reasoning show that this yields a contradiction.

More informally, we can state the Russell-Myhill paradox as involving the inconsistency of the following three principles:

> *Association.* For every plurality of objects, there is a proposition that is associated with that plurality.
>
> *Grain.* If two pluralities of objects are distinct, then any proposition that is associated with one plurality is distinct from any proposition that is associated with the other plurality.
>
> *Comprehension.* Given any condition, there is a plurality comprising all and only the objects that satisfy the condition.

Note that I am taking a proposition that is associated with a given plurality of objects to be a proposition that is *about* that plurality. On my formalization, *Association* is formally captured by (PD-), *Grain* is formally captured by (FG), and *Comprehension* is formally captured by (PC).

For comparison with my formulation, here is Russell (2010, 539)'s original formulation of the paradox:

> If *m* be a class of propositions, the proposition "every *m* is true" may or may not be itself an *m*. But there is a one-one relation of this proposition to *m*: if *n* be different from *m*, "every *n* is true" is not the same proposition as "every *m* is true". Consider now the whole class of propositions of the form "every *m* is true", and having the property of not being members of their respective *m*'s. Let this class be *w*, and let *p* be the proposition "every *w* is true". If *p* is a *w*, it must possess the defining property of *w*; but this property demands that *p* should not be a *w*. On the other hand, if *p* be not a *w*, then *p* does possess the defining property of *w*, and therefore is a *w*. Thus the contradiction appears unavoidable.

Whereas Russell formulates the paradox in terms of classes, I formulate it in terms of pluralities. Russell's assumption that, where *m* is a class of propositions, there is a proposition that every proposition in *m* is true corresponds to *Association*.[16] His assumption

[16]However, *Association* is more general, in that it does not assume that the relevant proposition is one that ascribes the property of truth. Clearly, this assumption is unnecessary to generate the paradox.

that, where *m* and *n* are distinct classes, the proposition that every proposition in *m* is true is distinct from the proposition that every proposition in *n* is true corresponds to *Grain*. And his assumption that there is a class of propositions where each member of the class is a proposition that says that every proposition in some class *m* is true and is not in *m* is an instance of what corresponds to *Comprehension*. Russell (2010, 540) concluded the following:

> The totality of all logical objects, or of all propositions, involves, it would seem, a fundamental logical difficulty. What the complete solution of the difficulty may be, I have not succeeded in discovering; but as it affects the very foundations of reasoning, I earnestly commend the study of it to the attention of all students of logic.

The Russell-Myhill paradox shows that the naive conception of propositions must be abandoned. What should we replace it with? In what follows, I sketch an iterative conception of propositions, which motivates a modal account of propositions. Crucially, I show that this account provides a satisfying solution to the Russell-Myhill paradox. Since the account is in the spirit of recently-developed modal accounts of sets, however, let me first review the case with sets, which provides a helpful analogy. I focus on the iterative conception of sets, which motivates a modal account of sets, and such a modal account of sets in turn provides a satisfying solution to Russell's paradox.

2.2 Russell's paradox

Like propositions, sets seem easy to generate. In fact, sets seem even easier to generate than propositions, given their clear and uncontroversial identity conditions. Sets' identities are completely determined by pluralities of objects. Sets that are formed by the same pluralities of objects are identical, and posterior to those objects. Accordingly, the plurality comprising all and only the chairs forms the set containing all and only the chairs. In what follows, however, I show that a naive way of capturing these intuitions about sets leads to Russell's paradox.

Notation

Let $\sigma(y)$ mean that y is a set. For simplicity, let us assume that only sets have members:

(MS) $x \in y \to \sigma(y)$.

For convenience, let us extend the notion of set membership from individual objects to pluralities of objects. Given a plurality *oo* of objects, *oo* generates a set, namely, the set containing all and only the objects among *oo*. By abuse of notation, let $xx \in y$ mean that y is a set containing all and only the objects among xx:

(MPL) $xx \in y := \forall x(x \prec xx \leftrightarrow x \in y)$.

Much as before, let me clarify that $xx \in y$ means that y contains all and *only* the objects among xx. Also, its being the case that $x \in y$ does not entail its being the case that $xx \in y$, where xx is the plurality comprising just x.

The naive conception of sets

How should we capture our intuitions about sets? The naive conception of sets consists of two principles.

First, sets not only seem, but *are*, easy to generate. Specifically, every plurality of objects forms a set that contains all and only the objects among that plurality:

(SF-) $\forall xx \exists y(xx \in y)$.

To illustrate, the plurality comprising all and only the natural numbers forms the set containing all and only the natural numbers.

Second, sets and other objects can be picked out for consideration by means of a condition. Specifically, given any condition, there is a plurality comprising all and only the objects that satisfy the condition:

(PC) $\exists xx \forall x(x \prec xx \leftrightarrow \varphi(x))$.

This is the same principle from the naive conception of propositions.

Russell's paradox

However, the naive conception of sets is inconsistent due to Russell's paradox.[17] By the instance of (PC) where $\varphi(x)$ is $x \notin x$, there is the Russell plurality *rr* of all and only sets that are not members of themselves:

(R1) $\forall x(x \prec rr \leftrightarrow x \notin x)$.

By (SF-), *rr* form a set:

(R2) $\exists y(rr \in y)$.

By existential elimination, we have that:

(R3) $rr \in r$.

Now *r* is a set, so the question arises whether *r* is a member of itself. If *r* is a member of *r*, then it does not satisfy its defining condition, and so is not a member of *r*. To see this, assume for *reductio* that $r \in r$. By this and the left-to-right direction of the conditional resulting from universal elimination on (R1) [on *r*], we have that $r \not\prec rr$. By the definition (MPL), we have that $r \notin r$. Contradiction. But if *r* is not a member of *r*, then it satisfies its defining condition, and so it is a member of *r*. To see this, assume for *reductio* that $r \notin r$. By conditional elimination on this and the right-to-left direction of the conditional resulting from universal elimination on (R1) [on *r*], we have that $r \prec rr$. By the definition (MPL), we have that $r \in r$. Contradiction. In short, we have that *r* is a member of *r* iff *r* is not a member of *r*:

(RI) $r \in r \leftrightarrow r \notin r$.

Standard principles of reasoning show that this yields a contradiction.

More informally, we can state Russell's paradox as involving the inconsistency of the following two principles:

[17] I follow the formulation of Uzquiano (2003, 2009); Linnebo (2010) in terms of pluralities.

> *Containment.* For every plurality of objects, there is a set containing all and only the objects among that plurality.
>
> *Comprehension.* Given any condition, there is a plurality comprising all and only the objects that satisfy the condition.

On my formalization, *Containment* is formally captured by (SF-), and *Comprehension* is formally captured by (PC).

The iterative conception of sets

Russell's paradox shows that the naive conception of sets must be abandoned. What should we replace it with? Standardly, the naive conception of sets is replaced with the iterative conception of sets, which is equally natural and, for all we know, consistent.[18] On the iterative conception of sets, sets are formed at different levels of a hierarchy, where sets at lower levels are prior to sets at higher levels, which contain sets at lower levels.[19] Specifically, at level 0, all the nonsets, or "urelements," are formed; at level 1, all sets that can be formed from pluralities of objects at level 0 are formed; at level 2, all sets that can be formed from pluralities of objects at level 0 or 1 are formed; and in general, at level n, all sets that can be formed from pluralities of objects at levels $0, 1, \ldots, n-1$ are formed.

A potential hierarchy of sets

Following Parsons (1983); Hellman (1993); Fine (2006); Linnebo (2010, 2013); Studd (2013), the iterative conception of sets motivates a potential hierarchy of sets. Crucially, every plurality of objects *potentially* forms a set.

Accordingly, the concept *set* is indefinitely extensible, in that from purportedly all and only the sets we can generate a set that is not among them. The indefinite extensi-

[18] See Boolos (1971).

[19] For simplicity, I omit discussion of transfinite levels.

bility of *set* captures the intuition that every plurality of objects could but need not form a set—and in particular the set containing all and only the objects among that plurality.

The question is how we should capture the relevant potentiality. As Parsons (1983); Hellman (1993); Fine (2006); Linnebo (2010, 2013); Studd (2013); Uzquiano (2015d) suggest, we can do so by appealing to modal vocabulary, and thus formulate a modal account of sets.

The modality as logico-mathematical

The relevant modality is distinctively logico-mathematical. Here is Linnebo (2010, 155):

> The existence of a set is potential relative to its elements: if the elements exist, their set may be formed. To make explicit this potential character of the hierarchy of sets, we introduce two modal operators, $\Diamond$ and $\Box$. Officially, these operators are new primitives governed only by a modal logic. For heuristic purposes, however, it will be useful to interpret $\Diamond\varphi$ as "it is possible to go on to form sets so as to make it the case that φ," and $\Box\varphi$ as "no matter what sets we go on to form it will remain the case that φ."

Clearly, this modality is not ordinary metaphysical modality, not least because the existence of pure sets is metaphysically necessary, but not logico-mathematically necessary.[20] What matters is that this modality supports the following principle: the existence of a set is potential relative to the plurality of objects from which the set is formed, in that if a plurality of objects exists (i.e., the objects themselves exist), it is possible for there to exist a set containing all and only the objects among that plurality.

While this modality might be less familiar, I think we should not be too skeptical of its intelligibility. One might develop suggestions in the literature on how this modality

[20]Let me clarify. On the present understanding of logico-mathematical modality, it is forward-looking, in the expansive direction: at a given logico-mathematically possible level, there are certain sets the existence of which is merely logico-mathematically possible. The modifier "logico-mathematically" in "logico-mathematically possible" expands the range of possibility (relative to metaphysical possibility); by contrast, the modifier "physically" in "physically possible" contracts the range of possibility. If a pure set exists at a given logico-mathematically possible level, then its existence is not only metaphysically necessary but also logico-mathematically necessary. But given a logico-mathematically possible level at which a pure set does not exist but potentially exists, its existence remains metaphysically necessary but is not logico-mathematically necessary.

is to be understood. Notably, Fine (2006)'s suggestion is that the relevant modality involves the postulation of objects into existence. This suggestion is developed in some detail, and is distinctive in introducing a third parameter—relevant for the determination of truth value—corresponding to an ontology, where this new parameter is distinct from more familiar parameters corresponding to content or circumstance. Meanwhile, Linnebo (2010, 158)'s suggestion is that the modality involves the individuation of mathematical objects by providing clear and determinate identity conditions:[21]

> I understand the above modalities in terms of a process of individuating mathematical objects. To individuate a mathematical object is to provide it with clear and determinate identity conditions. This is done in a stepwise manner, where at any stage we can make use of any objects already individuated in our attempts to individuate further objects. In particular, at any stage we can consider a plurality of objects already individuated and use this to individuate the set with precisely these objects as elements. A situation is deemed to be possible relative to one of these stages just in case the situation can be obtained by some legitimate continuation of the process of individuation.

Alternatively, one might simply take the logico-mathematical modality as primitive, and jettison the purported need to reduce it to other kinds of modality. Indeed, I am myself inclined to take the modality as primitive. We have enough of a grip on the modality, thanks to modalized versions of certain principles, where the modalized principles supplement nonmodalized principles—such as (SF-)—with judicious insertions of modal expressions. (I elaborate on modalized versions of certain principles below.) The situation parallels the situation with metaphysical modality: we have enough of a grip on metaphysical modality, thanks to principles such as that whatever is metaphysically necessary is metaphysically actual and so metaphysically possible.

The indefinite extensibility of set

This logico-mathematical modality lets us formulate a modal account sets. We want to formulate the indefinite extensibility of a concept C as the following claim: no matter what objects fall under C, there *could* be some object that falls under C that is not among

[21] Also see Linnebo (2012).

them. But a nonmodal account is expressively inadequate, since the best we can do without modal vocabulary is to formulate the claim as the following:

(IES-) $\forall xx(\forall x(x \prec xx \rightarrow \sigma(x)) \rightarrow \exists x(\sigma(x) \wedge x \not\prec xx))$.

However, (IES-) is false. To see this, consider that, by (PC), there is a plurality *ss* comprising all and only the sets, so that $\forall x(x \prec ss \rightarrow \sigma(x))$. By conditional elimination on this and the result of universal elimination on (IES-) [on *ss*], there is a set that is not among *ss*. But then *ss* is not a plurality comprising *all* and only the sets. Contradiction. By contrast, a modal account lets us say that the concept *set* is indefinitely extensible, as follows:

(IES) $\Box\forall xx(\forall x(x \prec xx \rightarrow \sigma(x)) \rightarrow \Diamond\exists x(\sigma(x) \wedge x \not\prec xx))$.

A satisfying solution to Russell's paradox

An application of a modal account of sets is that it provides a satisfying solution to Russell's paradox. The account diagnoses the problem with Russell's paradox as owing to the falsity of the principle (SF-) that every plurality of objects forms a set. Although (SF-) is *prima facie* attractive, it is false. Russell's paradox is solved by noting that (SF-) is false. Nonetheless, the intuitive motivation for (SF-) is preserved by a weaker principle. This weaker principle arises because quantifiers in set-theoretic claims are often implicitly modalized. Here is Linnebo (2010, 155):

> [T]he quantifiers used in set theory should be seen as having an implicit modal character. When a set theorist writes $\forall$ and $\exists$, she typically should be analyzed as meaning $\Box\forall$ and $\Diamond\exists$, respectively. Thus, when a set theorist says that a formula holds for "all sets," she typically should be understood as claiming that no matter how far the hierarchy of sets is continued, the formula will hold of all the sets formed by then. Likewise, when a set theorist says that a formula holds for "some set," she typically should be understood as claiming that it is possible to continue the hierarchy of sets such that there is some set for which the formula holds.

Crucially, $\Box\forall$ and $\Diamond\exists$ behave logically like quantifiers. The weaker, modalized version of (SF-) says that (at all levels of the hierarchy), for every plurality of objects, it is possible that there is a set containing all and only the objects among that plurality:

(SF) $\Box\forall xx\Diamond\exists y(xx \in y)$.

3 A modal account of propositions

In this section, I propose a modal account of propositions that parallels a modal account of sets in both motivation and development. In the first subsection, I present an iterative conception of propositions. In the second subsection, I suggest that the conception motivates a potential hierarchy of propositions, and so a modal account of propositions. In the third subsection, I show that the account provides a satisfying solution to the Russell-Myhill paradox. In the fourth subsection, I discuss features of the solution.

3.1 The iterative conception of propositions

I propose that, just as the naive conception of sets is replaced with the iterative conception of sets, so the naive conception of propositions should be replaced with what I call the "iterative conception of propositions," which is equally natural and, for all we know, consistent.

A key motivation for this conception is the compelling principle that the existence of a proposition about some objects somehow depends on the prior existence of those objects.[22] This principle is a version of what Plantinga (1983) calls "existentialism," according to which "a singular proposition is ontologically dependent on the individuals it is directly about" (160). Versions of this principle are endorsed by others. Notably, Williamson (2002) maintains the principle that, "[n]ecessarily, if the proposition that $P(o)$ exists then o exists" (240), which he describes as expressing a "kind of object-dependence" (242). Similarly, Stalnaker (2012), who explicitly endorses Plantinga's existentialism (42), maintains the principle that "there are some propositions—singular propositions—that are object-dependent in the sense that the proposition would not exist if the individual did not" (22).

[22] See Fine (1995); Lowe (2001) on ontological dependence.

On the iterative conception of propositions, propositions are determined at different levels of a hierarchy, where objects, propositions, and pluralities of them at lower levels are prior to propositions at higher levels, which are about objects, propositions, and pluralities of them at lower levels. Specifically, at level 0, there are all the urelements that are not propositions, perhaps plus some (e.g., qualitative) propositions; at level 1, all propositions that can be determined from objects and pluralities of objects at level 0 are determined; at level 2, all propositions that can be determined from objects and pluralities of objects at level 0 or 1 are determined; and in general, at level n, all propositions that can be determined from objects and pluralities of objects at levels $0, 1, \ldots, n-1$ are determined.

To illustrate, at level 0, we have the Taj Mahal, Socrates, Plato, Fido, and perhaps qualitative propositions such as the proposition that every puppy is cute, the proposition that some plants have fruits, and proposition that the president is powerful; at level 1, we have the proposition that all and only Socrates and Plato have a beard, as well as the proposition that the proposition that the president is powerful is a proposition; and at level 2, we have the proposition that the proposition that all and only Socrates and Plato have a beard is true.

3.2 The indefinite extensibility of *proposition*

I also suggest that, just as the iterative conception of sets motivates a potential hierarchy of sets, so the iterative conception of propositions motivates a potential hierarchy of propositions. Crucially, every plurality of objects *potentially* determines a proposition.

Accordingly, the concept *proposition* is indefinitely extensible, in that from what are purportedly all and only the propositions we can generate a proposition that is not among them.[23] The indefinite extensibility of *proposition* captures the intuition that every plurality of objects could but need not determine a proposition—and in particular a

[23]Notably, Glanzberg (2001, 2004a,c); Lindström (2003, 2009) also develop accounts of the indefinite extensibility of *proposition*. However, neither of these accounts is a modal account. Also, Glanzberg's interest is in solving the liar paradox rather than the Russell-Myhill paradox.

proposition about all and only the objects among that plurality.

What matters is that the relevant modality supports the following principle: the existence of a (plural) proposition is potential relative to the plurality of objects that the proposition is about, in that if a plurality of objects exists (i.e., the objects themselves exist), it is possible for there to exist a proposition about all and only the objects among that plurality. As in the set-theoretic case, we capture the relevant potentiality by appealing to modal vocabulary, where the relevant modality is distinctively logico-mathematical.

Clearly, it is an open question whether the logico-mathematical modality in the modal account of sets is the same as that in the modal account of propositions. I do not mean to foreclose the possibility that they are not the same. But for simplicity, at least in this chapter, I assume that they are the same: the way in which the formation of sets is potential is the same as the way in which the determination of propositions is potential.[24]

In any case, the logico-mathematical modality lets us formulate a modal account of propositions, according to which the concept *proposition* is indefinitely extensible, as follows:

(IEP) $\Box\forall xx(\forall x(x \prec xx \rightarrow \pi(x)) \rightarrow \Diamond\exists x(\pi(x) \wedge x \not\prec xx))$.

3.3 A satisfying solution to the Russell-Myhill paradox

An application of the modal account of propositions is that it provides a satisfying solution to the Russell-Myhill paradox. The account diagnoses the problem with the Russell-Myhill paradox as owing to the falsity of principle (PD-) that every plurality of objects determines a proposition. Although (PD-) is *prima facie* attractive, it is false. The Russell-Myhill paradox is solved by noting that (PD-) is false. Nonetheless, the intuitive motivation for (PD-) is preserved by a weaker, modalized version of principle. This weaker principle arises because proposition-theoretic claims are often implicitly modalized, as

[24]I return to discuss the logico-mathematical modalities in the two accounts in Section 4.1.

we might say in the following speech emulating Linnebo:

> When a proposition theorist writes $\forall$ and $\exists$, she typically should be analyzed as meaning $\Box\forall$ and $\Diamond\exists$, respectively. Thus, when a proposition theorist says that a formula holds for "all propositions," he typically should be understood as claiming that no matter how far the hierarchy of propositions is continued, the formula will hold of all the propositions formed by then. Likewise, when a proposition theorist says that a formula holds for "some proposition," he typically should be understood as claiming that it is possible to continue the hierarchy of propositions such that there is some proposition for which the formula holds.

The weaker, modalized version of (PD-) says that (at all levels of the hierarchy), for every plurality of objects, it is possible that there is a proposition about all and only the objects among that plurality:

(PD) $\Box\forall xx\Diamond\exists y(y \rhd xx)$.

3.4 Features of the solution to the Russell-Myhill paradox

I now discuss key features of the proposed solution to the Russell-Myhill paradox in comparison to alternative solutions.

Fine-grained propositions

A feature of the proposed solution is that it maintains *Grain*, and so the following consequence of it, which concerns pluralities of propositions in particular:

> *Grain*.* If two pluralities of propositions are distinct, then any proposition that is associated with one plurality is distinct from any proposition that is associated with the other plurality.

Since *Grain* is formally captured by (FG), *Grain** is formally captured by the following:

(FG′) $\forall xx\forall yy\forall x\forall y(xx \neq yy \wedge \pi(xx) \wedge \pi(yy) \wedge x \rhd xx \wedge y \rhd yy \rightarrow x \neq y)$.

The proposed solution maintains (FG) and its consequence (FG′).

What matters for my purposes is that the informal principle *Grain* captured by (FG)—or at least the informal principle *Grain** captured by (FG′)—is attractive enough that we should seriously consider maintaining it. The proposed solution in this chapter, based on the modal account of propositions, does just this. By contrast, however, three alternative solutions do not.

A first alternative solution, which Russell considers but dismisses, maintains that materially equivalent propositions are identical. Thus, this solution rejects *Grain**—since *Grain** requires some materially equivalent propositions to be distinct—and thus *Grain*. To see that *Grain** requires some materially equivalent propositions to be distinct, consider the plurality cc comprising just the proposition p, and the plurality dd comprising p together with the proposition $p(cc)$ that all and only the propositions among cc are true. Now consider the proposition $p(dd)$ that all and only the propositions among dd are true. According to *Grain**, since the propositions $p(cc)$ and $p(dd)$ are about distinct pluralities, the propositions are distinct. But the propositions are materially equivalent: $p(cc)$ is true iff $p(dd)$ is true. Russell (2010, 539) puts the solution thus:

> In order to deal with this contradiction, it is desirable to reopen the question of the identity of equivalent propositional functions and of the nature of the logical product of two propositions. These questions arise as follows. If m be a class of propositions, their logical product is the proposition "every m is true", which I shall denote by $\wedge' m$. If we now consider the logical product of the class of propositions composed of m together with $\wedge' m$, this is equivalent to "Every m is true and every m is true," i.e. to "every m is true" i.e. to $\wedge' m$. Thus the logical product of the new class of propositions is equivalent to a member of the new class, which is the same as the logical product of m. Thus if we identify equivalent propositional functions ($\wedge' m$ being a propositional function of m), the proof of the above contradiction fails, since every proposition of the form $\wedge' m$ is the logical product both of a class of which it is a member and of a class of which it is not a member.

However, Russell (2010, 539) dismisses this solution—correctly, in my view:

> But such an escape is, in reality, impracticable, for it is quite self-evident that equivalent propositional functions are often not identical. Who will maintain, for example, that "x is an even prime other than 2" is identical with "x is one of Charles II"s wise deeds or foolish sayings"? Yet these are equivalent, if a well-known epitaph is to be credited. The logical product of all the propositions of the class composed of m and $\wedge' m$ is "Every proposition which either is an m or asserts that every m is

> true, is true"; and this is not identical with "every *m* is true", although the two are equivalent. Thus there seems no simple method of avoiding the contradiction in question.

The problem with this first alternative solution is that identifying materially equivalent propositions conflates intuitively distinct propositions.

A second alternative solution, which is natural to consider in light of the first alternative solution, maintains that necessarily equivalent propositions are identical. Thus, this solution also rejects *Grain**—since *Grain** requires some necessarily equivalent propositions to be distinct—and thus *Grain*. To see that *Grain** requires some necessarily equivalent propositions to be distinct, consider the plurality *ee* comprising the proposition p together with the negation p' of p, and the plurality *ff* comprising q together with the negation q' of q, where p is distinct from q. Now consider the proposition $p(ee)$ that all and only the propositions among *ee* are true, and the proposition $p(ff)$ that all and only the propositions among *ff* are true. According to *Grain**, since the propositions $p(ee)$ and $p(ff)$ are about distinct pluralities, the propositions are distinct. But the propositions are necessarily equivalent: since each plurality comprises a pair of contradictory propositions, it is impossible for all—let alone all and only—the propositions in each plurality to be true, so, necessarily, $p(ee)$ is true iff $p(ff)$ is true. The problem with this second alternative solution, however, is similar to the problem with the first alternative solution. This time, the problem is that identifying necessarily equivalent propositions conflates intuitively distinct propositions.

A third alternative solution allows that necessarily equivalent propositions may be distinct, but still rejects *Grain**, and thus *Grain*. Recall the informal statement of the Russell-Myhill paradox, in terms of the mutual inconsistency of *Association*, *Grain*, and *Comprehension*. The proposed solution, based on the modal account of propositions, maintains *Grain*—and its consequence *Grain**—and *Comprehension* but not *Association*. On my formalization, this amounts to maintaining (FG) and (PC) but not (PD-).

However, Uzquiano (2015b) points out that on an alternative formalization of the

Russell-Myhill paradox, the rejection of *Grain** is the only solution available.[25] This formalization provides the basis for the third alternative solution. Using the following principles, Uzquiano formalizes the paradox in Morse-Kelley class theory with urelements (MKU), where classes are interpreted as pluralities, and (U1)–(U3) are mutually inconsistent:

(U1) $\forall xx(\forall x(x \prec xx \rightarrow \pi(x)) \rightarrow \exists x(x = \alpha(xx)))$.

(U2) $\forall xx \forall yy(\forall x(x \prec xx \rightarrow \pi(x)) \wedge \forall x(x \prec yy \rightarrow \pi(x)) \rightarrow (xx \neq yy \rightarrow \alpha(xx) \neq \alpha(yy)))$.

(U3) $\exists xx \forall x(x \prec xx \leftrightarrow \exists yy(\forall y(y \prec yy \rightarrow \pi(y)) \wedge x = \alpha(yy) \wedge x \not\prec yy))$.

Here, plural variables such as xx, yy, zz, ...range over pluralities, objectual variables such as x, y, z, ...range over individuals and sets, $\pi(x)$ means that x is a proposition, and the functional symbol α combines with a plural variable xx to yield a singular term, $\alpha(xx)$, which refers to the proposition that every proposition among xx is true. On Uzquiano's formalization, *Association* is formally captured by (U1), *Grain** is formally captured by (U2), and the relevant instance of *Comprehension* is formally captured by (U3).

On Uzquiano's formalization, (U1) is nonnegotiable. Since α is a functional symbol, $\exists x(x = \alpha(xx))$ is a theorem.[26] Similarly, (U3) is nonnegotiable, since it is just an instance of impredicative class comprehension of MKU; given the interpretation of classes as pluralities, this comprehension principle is just (PC). Only (U2) is negotiable. Unsurprisingly, Uzquiano proposes rejecting (U2)—which he attempts to justify on the basis of certain logical results:

> We have suggested that Russell's paradox of propositions is best viewed as a constraint on propositional granularity: no matter how fine-grained one may like propositions to be, they cannot all be as fine-grained as to be able to discriminate between any two classes [pluralities]. (343)

Thus, Uzquiano's solution maintains *Association* and *Comprehension* but not *Grain**. However, as he concedes, although logical results speak against (U2), on this formalization,

[25] To maintain consistency throughout, I alter Uzquiano's notation.

[26] As Uzquiano (2015b, 333) admits, with respect to this formalization, "the choice of regimentation is not completely innocent."

there is little to no intuitive motivation for rejecting the informal claim captured by (U2), that is *Grain**, or explanation of why that claim might be false:

> The suggestion that, on pain of contradiction, *some* propositions conflate classes [pluralities] with different members difficult to fathom. How could a proposition, in addition, conflate such a vast range of classes of propositions...? It may be fine to adopt some coarse-grained account of propositions or another, but it seems difficult to accept that the choice is handed down to us by logic alone. Desperate times call for desperate measures. [emphasis in the original] (341)

Yet, I suggest, the solution being considered is wanting insofar as the denial of *Grain** seems insufficiently unmotivated.

Unrestricted comprehension

A second feature of the proposed solution is that it maintains *Comprehension*. A comparison with other solutions is not straightforward, because whereas the proposed solution is formulated in terms of pluralities, most other solutions are not so formulated, and are rather formulated in terms of sets or classes. Nonetheless, we can consider what other solutions amount to when formulated in terms of pluralities, by reinterpreting talk of sets or classes as talk of pluralities, and I do just that.

Unlike Walsh (2016), we do not impose any predicative restrictions on the condition that we use to pick out some objects for consideration. Unlike Menzel (2012), we do not impose the restriction that the objects we pick out for consideration are members of sets; we can pick out for consideration all and only the sets, even though they are not (but could be) members of a set. And unlike Deutsch (2014), we do not impose the restriction that objects are members of classes or among pluralities.

Unrestricted quantification

A third feature of the proposed solution is that—unlike a closely-related solution based on a ramified account of propositions—it maintains some kind of unrestricted quantification over all propositions.

A modal account of propositions is notably similar to a ramified account of propositions, as in Russell (1906, 1908); Russell and Whitehead (1910), in that both take seriously the idea that there is some kind of hierarchy of propositions.[27] However, I suggest, the former is considerably more natural, whereas the latter seems too artificial and *ad hoc*. Moreover, an apparently severe cost of a ramified account of propositions is that it makes unrestricted quantification over all propositions unavailable. Specifically, on a ramified account, quantified propositions only quantify over propositions of certain orders, and in particular orders lower than the order of the quantified proposition itself. Accordingly, talk of "all propositions" is rendered as talk of "all propositions of order n" for some level n of the hierarchy of propositions. Does a modal account of propositions have a similar cost?

A worry is as follows. Given the similarities with a ramified account of propositions, a modal account of propositions also makes unrestricted quantification over all propositions unavailable. But such quantification seems to be both available and useful. Presumably, there are propositions such as those expressed by the following sentences:

(1) All propositions are self-identical.

(2) Every proposition is either true or false.

(3) No proposition is both true and false.

Such propositions allow us to make useful generalizations about propositions. To bolster the worry, one might point out that the following modalized version of (PC) is false:

(PC′) $\Diamond\exists xx\Box\forall x(x \prec xx \leftrightarrow \varphi(x))$.

For consider the following false instance:

($\text{PC}'_{\pi(x)}$) $\Diamond\exists xx\Box\forall x(x \prec xx \leftrightarrow \pi(x))$.

[27] See Church (1974, 1976, 2017); Anderson (1980); Tucker (2013); Whittle (2016) for other ramified accounts of propositions. See Hodes (2013, 2015) for recent discussions of such accounts.

This instance of (PC′) is false, because as we go on to form more propositions, there will be more and more propositions—so that it cannot be that there is a plurality of objects such that, no matter the level of the hierarchy, it comprises all and only the propositions.

In response, I suggest that the worry might not be as damning as it is sometimes taken to be. Granted, unrestricted quantification over all propositions seems available. But it is a fact of life that not everything that seems available and useful is in fact available. Various paradoxes, of a logical or mathematical nature, demonstrate this point forcefully. Paradoxes are just that because they compel us to reconsider one or more intuitively compelling principles.

Further, on the modal account of propositions, at least, it is far from clear that the availability of unrestricted quantification is really in doubt. For consider that ordinary modal claims, such as the claim that there could be talking donkeys, where the relevant modality is metaphysical modality, do not seem to call unrestricted quantification into doubt. While there indeed *could* be talking donkeys, unrestricted quantification is only in doubt if there really *are* talking donkeys. But whether or not there really are talking donkeys is a highly contentious claim, one that is true on certain accounts of metaphysical modality but certainly not all or even just standard accounts. Similarly, the proposition that there could be such-and-such a proposition does not seem to call unrestricted quantification into doubt. While there indeed *could* be such-and-such a proposition, unrestricted quantification is only in doubt if there really *is* such-and-such a proposition. But whether or not there really is such-and-such a proposition is a contentious claim, depending on what the relevant proposition is supposed to be. The claim is one that the proposed account rejects with respect to certain purported propositions.

More importantly, we maintain some kind of unrestricted quantification over all propositions after all. Specifically, on the account, unrestricted quantifiers are emulated by means of modalized quantifiers, such as $\Box\forall$ and $\Diamond\exists$, where the former is the modalization of the universal quantifier and the latter is the modalization of the existential quantifier. Accordingly, we can express (1), (2), and (3) via, respectively:

(1′) $\Box\forall x(\pi(x) \rightarrow x = x)$.

(2′) $\Box\forall x(\pi(x) \rightarrow (T(x) \vee F(x)))$.

(3′) $\Box\forall x\neg(T(x) \wedge F(x))$.

Here, $T(x)$ means that x is true, and similarly $F(x)$ means that x is false. According to (1′), no matter the level of the hierarchy, every proposition is self-identical. According to (2′), no matter the level of the hierarchy, every proposition is either true or false. And according to (3′), no matter the level of the hierarchy, nothing is both true and false. Thanks to the availability of modalized quantifiers, we can make meaningful, true, and general claims about all propositions.

Having introduced talk of the truth of propositions on the account, let me now also mention an important consequence of the indefinite extensibility of *proposition*. A minimal assumption is that among merely possible propositions are ones that could be true—perhaps because a (merely possible) proposition that ascribes the property of being self-identical to one or more of those propositions would be true. Given this, the concept *propositional truth* is also indefinitely extensible:

(IET) $\Box\forall xx(\forall x(x \prec xx \rightarrow T(x)) \rightarrow \Diamond\exists x(T(x) \wedge x \not\prec xx))$.

4 Developing the account

In this section, I develop the account in more detail. In the first subsection, I discuss the logico-mathematical modality in the account. In the second subsection, I discuss the prospects of adding further principles to the account. In the third subsection, I discuss alternative formulations.

4.1 Logico-mathematical modality

Let me explain why I take the logico-mathematical modality in the modal account of propositions to be the same as that in a modal account of sets, address the issue of where we are in the hierarchy of propositions, and suggest that context helps determine when quantifiers are and are not modalized.

Number of logico-mathematical modalities

I have suggested that the logico-mathematical modality in the modal account of propositions is the same as that in a modal account of sets: the way in which the existence of sets is potential is the same as the way in which the existence of propositions is potential, so that we have the same logico-mathematical modality in both cases. For simplicity, I have just assumed this, while not wanting to foreclose the possibility that the logico-mathematical modalities in the accounts are distinct. Let me now provide two reasons to maintain that the assumption not only simplifies the discussion but is also independently desirable.

First, the assumption is more ideologically parsimonious. In principle, the proposed modal account of propositions can stand on its own, independently from modal accounts of sets. In practice, however, because the motivations and ideological commitments are so similar, one who embraces a modal account in the case of sets is likely to do so in the case of propositions as well, and conversely. Given that one embraces a modal account in one case, and so is already committed to the legitimacy and fruitfulness of a logico-mathematical modality, one is likely to embrace a modal account in the other case too; not least, given that not all sets exist at level 0 of some hierarchy, the priority of objects—sets and other propositions among them—to the propositions about them suggests that not all propositions are at level 0 of the hierarchy either. But the same modality seems to be involved in both cases. In the absence of compelling reasons to maintain that the relevant logico-mathematical modalities are different in each case, it is reasonable to assume that they are the same. Put another way, if one is unsure whether it is worth accepting the legitimacy and fruitfulness of a logico-mathematical modality in order to adopt a modal account in just the case of sets, or propositions, then one can be more sure that it is worth doing so in order to adopt a modal account in both cases. Taking the cost of the modality to be fixed, the relative benefit of the modality, so to speak, is doubled. Where the logico-mathematical modality is the same, a combined modal account of sets and propositions is attractive.

Second, and relatedly, given a modal account of sets, the assumption is encouraged by the neo-Russellian account of propositional identity I am assuming in this chapter. On a certain version of the neo-Russellian account, which takes propositions to be identical to certain sets, and so structured accordingly, the proposition that grass is green is the ordered pair consisting of the property of being green and grass:[28]

$$\text{the proposition that grass is green} = \langle \text{property of being green}, \text{grass} \rangle.$$

More generally, atomic propositions are identical to ordered pairs consisting of a property (or relation) and an object (or sequence of objects), negative propositions are identical to ordered pairs consisting of the truth function for negation and the negated proposition, conjunctive propositions are identical to ordered pairs consisting of the truth function for conjunction and an ordered pair consisting of the two propositions that are the conjuncts; and quantified propositions are also identical to certain ordered pairs. Since ordered pairs are—or at least are definable in terms of—sets, on this account, propositions are also sets.[29]

Crucially, given a modal account of sets, a neo-Russellian account of propositional identity suggests a modal account of propositions where the logico-mathematical modalities are the same. For if, in general, sets—not only pure ones but impure ones too—exist at different levels of the hierarchy of sets, and in particular the sets that are identical to propositions also exist at different levels of the hierarchy, then propositions exist at different levels of the same hierarchy (of sets) as well.

[28]See Soames (1985, 1987, 1989); Salmon (1986) for such a conception.

[29]On the standard definition due to Kuratowski, an ordered pair is defined as follows: $\langle x, y \rangle := \{\{x\}, \{x, y\}\}$. Analogous definitions hold for sequences of objects more generally.

Location in the hierarchy

The logico-mathematical modality in the modal account of propositions might be the same as that in a modal account of sets, but consideration of the modality still raises a pressing question. Consider that just as there is a distinguished metaphysically actual world in the space of possible worlds, it is natural to suppose that there is a distinguished logico-mathematically actual level of the potential hierarchy of propositions. But then are we somewhere in the hierarchy, and if so, where?

In considering this question for his modal account of sets, Linnebo (2010, 158–159) downplays its importance:

> A related question is which sets are actual with respect to the modality invoked above. One answer is that precisely those sets are actual that have been formed at our current stage of the process of forming sets. But this is uninformative. For what *is* our current stage of this process? The most plausible response to this follow-up question is, I think, that set theorists generally do not regard themselves as located at some particular stage of the process of forming sets but rather take an external view on the entire process. It therefore would be wrong to assign to ourselves any particular stage of the process. [emphasis in the original]

One might maintain a similar stance in considering the question for the proposed modal account of propositions. Such a stance maintains that proposition theorists generally do not regard themselves as at some level of the hierarchy of propositions but rather take an external view of the entire hierarchy, so that it would be at least unimportant to determine which level of the hierarchy we are at.

However, I suspect that it is unsatisfactory to leave matters here. For consider that even if metaphysicians tend to take an external view of the space of possible worlds, the question of where we are in the space of possible worlds is legitimate. Similarly, even if proposition theorists tend to take an external view of the hierarchy of propositions, the question of where we are in the hierarchy is legitimate.

To answer the question more satisfactorily, I suggest that where we are in the hierarchy is a somewhat empirical matter.[30] A linguistic community whose language has the

[30] See Linnebo (2012) for related discussion.

resources to talk about only objects that are not propositions or sets—roughly, urelements—is at level 1 of the hierarchy. But a more sophisticated linguistic community whose language has the resources to talk about propositions about level 0 objects is at level 2 of the hierarchy. So where we are in the hierarchy depends on the linguistic (and perhaps conceptual) resources available to us. What about once we have the resources to talk about arbitrary finite levels of the hierarchy? The answer might be that, in such a case, we are at a transfinite level of the hierarchy. And what about once we have the resources to talk about arbitrary transfinite levels of the hierarchy? The answer might be that, in such a case, we are at still higher levels of the hierarchy.

Modalization of quantifiers

Considering that quantifiers are sometimes modalized, as in (PD), and sometimes unmodalized, as in (PC), another question arises. When are quantifiers modalized, and when are they not modalized? A principled answer is crucial to providing a principled solution to the Russell-Myhill paradox. For on the proposed solution to the paradox, it is crucial to explain why *Association* is best understood as the modalized claim (PD), while *Comprehension* is best understood as the unmodalized claim (PC). I suggest that, perhaps together with the principle of charity, context helps determine when quantifiers are and are not modalized. The key is to understand the motivations for various principles in light of the potential hierarchy of propositions.

Association is best understood as the modalized claim (PD), because a context in which this principle is true is likely to be one in which we are interested in the potential existence of certain propositions based on the pluralities of objects the propositions are about. But *Comprehension* is best understood as the unmodalized claim (PC), because a context in which this principle is true is likely to be one in which we are interested in picking out whatever is actually available for consideration; we cannot pick out what is not yet available for consideration. There is a plurality of objects comprising all and only the objects that satisfy some condition, but there need not be some level of the hierarchy where there is a plurality of objects comprising all and only the objects that at any level satisfy some condition; in particular, there is never a plurality of objects comprising all and only the propositions at any level. For pluralities of objects are modally rigid, in that their identities are completely determined by what they comprise at a given level, but as we go up the hierarchy there are more and more propositions.[31]

4.2 Prospects of adding further principles

Having said more about the logico-mathematical modality in the account, let me also discuss the prospects of adding further principles to the account.

The formulation of the account in this chapter has been bare-bones, in that it exploits only minimal assumptions to generate paradox. These assumptions are minimal, in requiring little beyond the assumption that propositions are fine-grained enough to support a principle like *Grain*, or at least *Grain**. Such assumptions are plausible on many accounts of propositional identity, and virtually all accounts of structured propositions. An advantage of this bare-bones formulation of the account in this chapter is that it gets right to the heart of the issue, in remaining neutral on a number of controversial issues often central to accounts of propositional identity.

But the focus on the simplest of propositions—singular propositions, which ascribe properties to individual objects, and plural propositions, which ascribe properties to pluralities of objects—might be thought to come at the apparent disadvantage of neglecting to treat more complex propositions—such as negative propositions, conjunctive propositions, and quantified propositions. So perhaps the bare-bones formulation of the account has the corresponding disadvantage of falling short of a more full-fledged account of propositions.

One might think that, while the proposed account of propositions is promising, it is too limited and otherwise hard to evaluate insofar as it falls short of yielding a complete

[31]See Bricker (1989); Forbes (1989); Williamson (2003, 2010, 2013); Rumfitt (2005); Uzquiano (2011); Linnebo (2013) on the modal rigidity of pluralities of objects. Modal rigidity can be characterized by the following two principles: (i) $x \prec xx \rightarrow \Box(Exx \rightarrow x \prec xx)$ and (ii) $x \not\prec xx \rightarrow \Box((Ex \wedge Exx) \rightarrow x \not\prec xx)$; here, Exx means that the plurality xx exists, and Ex means that x exists.

account of propositions. As it stands, the proposed account is rather weak, in not even assuring us of the existence of any propositions; it only assures us of the potential existence of certain propositions. Relatedly, one might think that, just as a modal accounts of sets may motivate a complete axiomatic account of sets, so perhaps a modal account of propositions should motivate a complete axiomatic account of propositions.[32] This might be so, notwithstanding it being a considerable challenge that, while we can help ourselves to a standard theory of sets, we cannot help ourselves to a standard theory of propositions, because it remains controversial exactly what an axiomatic theory of propositions should look like.

Still, I think we can be more optimistic, in focusing on what the proposed account does achieve. Even if the proposed account is not particularly strong, in that it does not guarantee the existence of any propositions, it would be unreasonable to conclude that the account says too little of significance. Despite the bare-bones formulation, there is enough to show that minimal assumptions generate paradox. That such minimal assumptions generate paradox is significant. Just as significant is that an account of propositions in terms of a logico-mathematical modality can solve paradox.

This is of course not to deny that it is worth seeking a stronger, more complete axiomatic account of propositions. A natural first step towards developing the proposed account, I suggest, is to provide a deeper analysis of propositional aboutness. To do so, instead of taking propositional aboutness to be primitive, we can define it in terms of the ascription of properties to objects. Let us take as primitive the ternary predicate $\circledcirc$, which intuitively relates propositions, properties, and (individual) objects. Specifically, $\circledcirc(y, z, x)$ means that the proposition y ascribes property z to object x. For simplicity, let us assume that only propositions ascribe properties to objects, and that propositions only ascribe properties to objects:

(CPR) $\circledcirc(y, z, x) \to (\pi(y) \land \rho(z))$.

[32]See Linnebo (2010, 2013); Studd (2013) on the suggestion that a modal account of sets motivates a complete axiomatic account of sets.

Here, $\rho(z)$ means that z is a property. The predicate $\circledcirc$, which takes singular terms as arguments, can be extended to take plural terms as arguments as well. We define the following:

(CPL) $\circledast(y, z, xx) := \forall x(\circledcirc(y, z, x) \leftrightarrow x \prec xx)$.

This is in terms of our original $\circledcirc$ predicate. With the ascription predicate in place, we can define propositional aboutness in terms of it as follows:

(ABT) $y \rhd x := \exists z \circledcirc (y, z, x)$.

I have suggested nothing more than a first step here, and undoubtedly much more work remains to be done. Such work, I think, is best left for the future. But see Appendix B.1 for some further principles I tentatively suggest adding to the account in this chapter.

4.3 Alternative formulations

The proposed account is formulated in terms of a plural language, but what about alternative formulations? A salient alternative is to formulate it in a higher-order language, as in Goodman (2016).[33] Not least, as Boolos (1984) shows, there is a natural correspondence between plural languages and second-order languages—at least in the special case where the concern is with monadic predicates.

Of course, for the higher-order formulation to be a serious alternative, it cannot merely be a notational variant of the plural formulation. A notational variant is exactly what we would have if we were to just substitute higher-order expressions for plural expressions, so that, e.g., Xx substitutes $x \prec xx$, and $\forall X$ substitutes $\forall xx$. For the higher-order formulation to be a serious alternative, the higher-order framework must be embraced more fully, so that propositions and properties are treated as higher-order entities. On the envisaged higher-order formulation, propositions and properties are

[33] Another alternative formulation is one in terms of a first-order language with two modal operators rather than a single one, as in Studd (2013). Unfortunately, I cannot discuss this alternative formulation in any detail here.

not objects, in that they are not in the range of first-order quantifiers. Rather, propositions are the semantic values of sentences, and in the range of *sui generis* propositional quantifiers—higher-order quantifiers. Similarly, properties are the semantic values of predicates, and in the range of *sui generis* higher-order quantifiers. Higher-order quantification, on this formulation, is irreducible to first-order quantification.

There are two reasons one might offer for adopting the higher-order formulation, which taken together make a case worth taking seriously. First, a higher-order formulation apparently avoids commitment to the existence of propositions and properties, where such commitment amounts to admitting propositions and properties into the domains of first-order quantifiers, and accordingly seems to permit a cleaner ontology. If so, then the formulation makes clear that the motivations put forth for a modal account of propositions do not depend on Platonism about propositions. Second, the formulation more naturally treats higher-order analogues of the Russell-Myhill paradox that concern properties of propositions rather than pluralities of them. A crucial difference is that whereas the higher-order formulation treats properties intensionally, the plural formulation treats pluralities extensionally; the identities of pluralities, but not properties, are completely determined by the objects among the pluralities at a given level.

However, I do not think these reasons are decisive. First, consider Linnebo and Rayo (2012)'s suggestion that the ontological commitments of set theory and those of type theory—which generalizes higher-order logic—are not as different as they might initially appear. Perhaps, I suggest, the ontological commitments of an account of propositions that treats propositions and properties as objects (such as that sketched above) and those of type theory are similarly not as different as they might initially appear. Besides, even if the higher-order formulation avoids ontological commitment involving propositions and properties in letter, it might not do so in spirit. For perhaps it maintains metaphysical commitment more generally that is no more and no less contentious than ontological commitment.[34] Accordingly, the higher-order formulation

[34] Williamson (2013, Chap. 5.9) suggests that higher-order logic comes with nonontological but still metaphysical commitments, which are as substantive as ontological commitments.

does not clearly avoid commitment to the existence of propositions and properties in a way that the plural formulation does not. The two formulations might be quite similar with respect to their ontological—or at least metaphysical—commitments.

Second, while the higher-order formulation might more naturally treat higher-order analogues of the Russell-Myhill paradox, such analogues—despite structural similarities—are arguably crucially different and so do not demand the same treatment. For consider that Russell's paradox of sets is structurally similar to Russell's barber paradox—which involves considering a barber who shaves all and only those people who do not shave themselves, and asking whether this barber shaves himself; the barber shaves himself iff he does not shave himself. Despite the structural similarity, the paradox of sets poses a genuine puzzle, whereas the barber paradox does not seem to do so, since we can simply conclude that there is no such barber. More generally, Priest (1991, 1994, 2006) argues that a range of set-theoretic, intensional, and semantic paradoxes are structurally similar, and so require the same treatment. But, *pace* Priest, despite the structural similarities, it still seems appropriate to treat the various paradoxes differently, as the literature usually does. The property-theoretic version of the Russell-Myhill paradox presents a challenge that deserves its own treatment. While a higher-order formulation of the proposed account could be appropriate for treating it, it does not follow that such a formulation is more appropriate than the plural formulation for treating the version of the paradox that this chapter focuses on, which is formulated in terms of pluralities.

To some extent, the adoption of a plural formulation in this chapter is more a matter of preference than one of principle. Still, consider that, in an account that focuses on sets, it is natural to adopt an ontology of sets. Notably, Zermelo-Fraenkel set theory is at least *prima facie* committed to an ontology of sets. Given the proposed account's focus on propositions, I think it is similarly natural to adopt an ontology of propositions and properties, which is what we have on the plural formulation.

5 Propositions, possible worlds, and paradox

In the previous sections, I have motivated a modal account of propositions and shown that it has at least one application: the account provides a satisfying solution to the Russell-Myhill paradox of propositions. In this final section, I show that the account has further applications, by showing that it helps provide satisfying solutions to two more intensional paradoxes, which concern both propositions and possible worlds. In the first subsection, I articulate the background conception of propositions and possible worlds I assume. In the second subsection, I show how this conception risks leading to Kaplan's and Prior's paradoxes. In the third subsection, I show that a modal account of propositions and possible worlds provides satisfying solutions to these paradoxes.

5.1 Propositions and possible worlds

I begin by articulating the background conception of propositions and possible worlds I assume.

Quantification over possible worlds

I remain as neutral as possible on the metaphysics of possible worlds. But I make a nontrivial semantic assumption, which might or might not bear on the metaphysics of possible worlds, depending on the extent to which semantic commitments involve metaphysical commitments. The assumption is that it is appropriate to quantify over possible worlds in the object language. In making this assumption, I follow the provision of a semantics for metaphysical modal sentences in Lewis (1968). The treatment of possible worlds as objects we can quantify over parallels the standard treatment of numbers, sets, and other entities we can apparently quantify over—as well as the treatment of propositions on the modal account of propositions in this chapter. However, it is not the prevailing treatment in the standard provision of a semantics for modal sentences:

> We can conduct formalized discourse about most topics perfectly well by means of our all-purpose extensional logic, provided with predicates and a domain of quantification suited to the subject matter at hand. That is what we do when our topic is numbers, or sets, or wholes and parts, or strings of symbols. That is not what we do when our topic is modality: what might be and what must be, essence and accident. Then we introduce modal operators to create a special-purpose, nonextensional logic. Why this departure from our custom? Is it a historical accident, or was it forced on us somehow by the very nature of the topic of modality?
>
> It was not forced on us. We have an alternative. Instead of formalizing our modal discourse by means of modal operators, we could follow our usual practice. We could stick to our standard logic (quantification theory with identity and without ineliminable singular terms) and provide it with predicates and a domain of quantification suited to the topic of modality. That done, certain expressions are available which take the place of modal operators. (Lewis, 1968, 113)

On the prevailing treatment, a metaphysical necessity sentence "Necessarily, φ" is formalized as a sentence where a box symbol precedes φ, while a metaphysical possibility sentence "Possibly, φ" is formalized as a sentence where a diamond symbol precedes φ. These boxes and diamonds may or may not be abbreviations for object language quantifiers over possible worlds, on the prevailing treatment. Modalists prefer to take the boxes and diamonds at face value, while antimodalists—often, modal realists—are happy to take them to be abbreviations.[35] On the treatment I adopt, following Lewis and the latter approach, however, we can dispense with boxes and diamonds altogether in favor of explicit quantifiers over possible worlds.

A treatment where we quantify over possible worlds in the object language seems fine to me. Not least, if we are already happy to provide a standard possible worlds semantics for modal sentences, then we are already granting that it is appropriate to quantify over possible worlds in the metalanguage. But insofar as this amounts to implicit acceptance of the appropriateness of quantification over possible worlds, I see no principled reason against making such acceptance explicit in the object language.

More importantly, for my purposes, I opt for such a treatment because I think it is conducive to the presentation of the two further intensional paradoxes that I discuss, namely, Kaplan's paradox and Prior's paradox. Kaplan's paradox in particular depends

[35] See Peacocke (1980); Forbes (1989); Fine (2005) on modalism, and Lewis (1968); Melia (1992) on antimodalism.

on assumptions about possible worlds themselves, so treating possibility sentences as overtly quantifying over possible worlds makes the significance of possible worlds conspicuous, and avoids having to bridge the gap between possibility claims and claims about the existence of certain possible worlds. Relatedly, I think there is insight to be gained by formulating the relevant paradoxes in terms of object language quantification over possible worlds. It helps us see that the paradoxes have bite in a variety of settings.

Possible worlds and propositions

I assume two key theoretical links between possible worlds and propositions. My two assumptions are more modest versions of standard assumptions. Instead of formulating the assumptions in terms of sets of propositions, I formulate them in terms of pluralities of propositions. I do this not only to be consistent with my formulation of the modal account of propositions from earlier, but also emphasize that the paradoxes I discuss do not depend on the assumption that we are dealing with sets—rather than pluralities or classes—of propositions. Notably, Kaplan's paradox, as we will see, does not assume that propositions are determined by—let alone identical to—sets of possible worlds.[36]

My assumptions are the following. First, every plurality of possible worlds determines a proposition that is true at all and only the possible worlds among that plurality.[37] Accordingly, the plurality comprising all and only the possible worlds in which grass is green determines a proposition—say, the proposition that grass is green—that is true at all and only the possible worlds in which grass is green. Second, every maximally consistent plurality of propositions determines a possible world at which all and only the propositions among that plurality are true.[38] A maximally consistent plurality of propositions is one where two conditions are fulfilled: first, for every proposition, either it or its negation is among the plurality; and second, all the propositions among

[36]In this respect, I sympathize with Bueno et al. (2014, 816), who maintain that "overreliance upon...the assumption that there is a set of all worlds and the identification of propositions with sets of worlds...[leads] to a misdiagnosis regarding the true source of the problem [Kaplan's paradox]."

[37]See Stalnaker (1976).

[38]See Adams (1974).

the plurality could be true, in some perhaps primitive sense of "could." Accordingly, the maximally consistent plurality of propositions comprising the proposition that grass is green, the proposition that snow is white, and so on, determines a possible world at which the proposition that grass is green is true, the proposition that snow is white is true, and so on.

5.2 Kaplan's and Prior's paradoxes

Not only do propositions seem easy to generate, but possible worlds also seem easy to generate. Possible worlds' identities are determined by maximally consistent pluralities of propositions. Further, possible worlds at which the same propositions are true are identical, and are posterior to those propositions. In what follows, however, I show that a naive way of capturing these intuitions about propositions and possible worlds leads to Kaplan's and Prior's paradoxes.

Notation

Let $\theta(y)$ mean that y is a possible world. Since propositions can be either true or false at possible worlds, let $T(x,y)$ mean that the proposition x is true at possible world y, and $F(x,y)$ mean that the proposition x is false at possible world y. For simplicity, let us assume that only propositions are true or false, and that they are only true or false at possible worlds:

(BPW) $(T(x,y) \vee F(x,y)) \to (\pi(x) \wedge \theta(y))$.

For convenience, let us extend the notion of truth at a possible world from individual possible worlds to pluralities of possible worlds. By abuse of notation, let $T(x,yy)$ mean that the proposition x is true at all and only the possible worlds among the plurality yy:

(TPL) $T(x,yy) := \pi(x) \wedge \forall y(y \prec yy \leftrightarrow (\theta(y) \wedge T(x,y)))$.

Much as before, let me clarify that $True(x,yy)$ means that x is true at all and *only* the possible worlds among yy. Also, its being the case that $True(x, y)$ does not entail its being the case that $True(x, yy)$, where yy is the plurality comprising just y.

Since propositions are either true or false, and not both, at possible worlds, we maintain world-relative versions of the principles of bivalence and noncontradiction:

(WBIV) $\forall x \forall y((\pi(x) \wedge \theta(y)) \rightarrow (T(x, y) \vee F(x, y))$.

(WNC) $\forall x \forall y \neg(T(x, y) \wedge F(x, y))$.

We could even maintain modalized versions of these two principles, but these unmodalized versions suffice for the present purposes.

Next, let $Bel(x)$ mean that the proposition x is believed by a particular person at a particular time. To avoid clutter, we leave the person and time parameters implicit. For simplicity, let us assume that only propositions are believed:

(BP) $Bel(x) \rightarrow \pi(x)$.

By abuse of notation, let $Bel(x, y)$ mean that the proposition x is believed at possible world y by a particular person at a particular time. Again, to avoid clutter, we leave the person and time parameters implicit. For simplicity, let us assume that only propositions are believed, and that they are only believed at possible worlds:

(BPW) $Bel(x, y) \rightarrow (\pi(x) \wedge \theta(y))$.

The naive conception of propositions and possible worlds

How should we capture our intuitions about propositions and possible worlds? Let me articulate what I call the "naive conception of propositions and possible worlds"; the reason for the qualification "naive" will become clear in what follows.

First, propositions are easy to generate. Specifically, every plurality of possible worlds determines a proposition that is true at all and only the possible worlds among that plurality:

(WD) $\forall xx(\forall x(x \prec xx \rightarrow \theta(x)) \rightarrow \exists y T(y, xx))$.

To illustrate, the plurality comprising all and only the possible worlds in which grass is green determines a proposition that is true at all and only the possible worlds in which grass is green.

Second, possible worlds are also easy to generate. Specifically, every proposition determines a possible world at which that proposition is uniquely believed:[39]

(PW-) $\forall x(\pi(x) \rightarrow \exists y(Bel(x,y) \wedge \forall z(Bel(z,y) \rightarrow z = x)))$.[40]

To illustrate, the proposition that there are infinitely many twin prime numbers determines a possible world at which that proposition is uniquely believed.

Third, possible worlds and other objects can be picked out for consideration by means of a condition. Specifically, given any condition, there is a plurality comprising all and only the objects that satisfy the condition:

(PC) $\exists xx \forall x(x \prec xx \leftrightarrow \varphi(x))$.

This is the same principle from the naive conception of propositions and the naive conception of sets.

Fourth, propositions can be picked out for consideration by means of a condition. Specifically, given a sentence, for every possible world, there is a proposition that is true at that possible world iff that sentence obtains at that possible world, and that is false at that possible world iff that sentence does not obtain at that possible world:

(RC) $\forall y \exists x(\theta(y) \rightarrow (\pi(x) \wedge (T(x,y) \leftrightarrow \varphi(y)) \wedge (F(x,y) \leftrightarrow \neg\varphi(y))))$.

To illustrate, consider the sentence "Grass is green." By (RC), at every possible world, there is a proposition that is true at that possible world iff grass is green at that possible world, and that is false at that possible world iff grass is not green at that possible world.

[39]By "that proposition is uniquely believed," I mean "that proposition, and no other, is believed."

[40]Compare this to a more standard formalization in terms of propositional quantifiers and modal operators: $\forall p \Diamond(Bp \wedge \forall q(Bq \rightarrow q = p))$, where $\Diamond$ indicates metaphysical possibility, p and q are propositional variables, and B is a sentential operator interpreted as "it is believed that ...." Formalized in this way, this principle is inconsistent in modal propositional logic supplemented with propositional quantifiers, where propositional quantification is objectual quantification over sets of possible worlds.

Kaplan's paradox

Unfortunately, the naive conception of propositions and possible worlds is inconsistent due to Kaplan's paradox; see Kaplan (1995) for the original formulation of the paradox.[41] By the instance of (PC) where $\varphi(x)$ is $\exists y(Bel(y,x) \wedge \forall z(Bel(z,x) \rightarrow z = y) \wedge F(y,x))$, there is the Kaplan plurality *kk* comprising all and only the possible worlds at which a false proposition is uniquely believed.

(K1) $\forall x(x \prec kk \leftrightarrow \exists y(Bel(y,x) \wedge \forall z(Bel(z,x) \rightarrow z = y) \wedge F(y,x)))$.

Clearly, *kk* is a plurality of possible worlds. So by (WD), there is a proposition that is true at all and only the possible worlds among *kk*:

(K2) $\exists y T(y, kk)$.

By existential elimination, we have that:

(K3) $T(n, kk)$.

Clearly, *n* is a proposition. So by (PW), there is a possible world at which it is uniquely believed:

(K4) $\exists y(Bel(n,y) \wedge \forall z(Bel(z,y) \rightarrow z = n))$.

By existential elimination, we have that:

(K5) $Bel(n,d) \wedge \forall z(Bel(z,d) \rightarrow z = n)$.

Further, by universal elimination on (K1) [on *d*], we have that:

(K6) $d \prec kk \leftrightarrow \exists y(Bel(y,d) \wedge \forall z(Bel(z,d) \rightarrow z = y) \wedge F(y,d))$.

[41] See Anderson (2009); Lindström (2009); Whittle (2009); Chalmers (2011); Kripke (2011) for discussion. The paradox can be formulated in terms of irreducibly propositional quantification. However, I formulate it in terms of objectual quantification over propositions here. Also, the paradox can be formulated as a cardinality argument—by appealing to Cantor's theorem, in the case of sets, or Bernays' theorem, in the case of classes or pluralities. However, I avoid appealing to these theorems here, since such appeals are unnecessary to generate paradox. In this respect, my formulation follows Whittle (2009).

Now, is d among kk? If d is among kk, then it is a possible world at which a proposition that is uniquely believed is both true and false, which is impossible due to (WNC). To see this, assume for *reductio* that $d \prec kk$. By conditional elimination on this and the left-to-right direction of (K6), we have that $\exists y(Bel(y,d) \wedge \forall z(Bel(z,d) \rightarrow z = y) \wedge F(y,d))$. By existential elimination on this, we have that $Bel(l,d) \wedge \forall z(Bel(z,d) \rightarrow z = l) \wedge F(l,d)$—i.e., l is uniquely believed and false at d. But by conjunction elimination on (K5), we have that $Bel(n,d)$—i.e., n is believed at d. Since l is uniquely believed at d, we have that $n = l$. By conjunction elimination on the previous conjunction, we have that $F(l,d)$. By identity elimination on this and the identity $n = l$, we have that $F(n,d)$. But since we have that $d \prec kk$, by (K3) and the definition (TPL), we have that $T(n,d)$. So n is both true and false at d, in violation of (WNC). But if d is not among kk, then it satisfies kk's defining condition, and so it is among kk. To see this, assume for *reductio* that $d \not\prec kk$. By this and (K3), n is not true at d, so by (WBIV), n is false at d: $F(n,d)$. By conjunction introduction on this and (K5), we have that $Bel(n,d) \wedge \forall z(Bel(z,d) \rightarrow z = n) \wedge F(n,d)$. By existential introduction on this, we have that $\exists y(Bel(y,d) \wedge \forall z(Bel(z,d) \rightarrow z = y) \wedge F(y,d))$. By conditional elimination on this and the right-to-left direction of (K6), we have that $d \prec kk$. Contradiction. In short, we have that n is among kk iff n is not among kk:

(KI) $n \prec kk \leftrightarrow n \not\prec kk$.

Standard principles of reasoning show that this yields a contradiction.

More informally, we can state Kaplan's paradox as involving the inconsistency of the following three principles:

> *Truth.* For every plurality of possible worlds, there is a proposition that is true at all and only the possible worlds among that plurality.

> *Belief.* For every proposition, there is a possible world at which that proposition is uniquely believed.

> *Comprehension.* Given any condition, there is a plurality comprising all and only the objects that satisfy the condition.

On my formalization, *Truth* is formally captured by (WD), *Belief* is formally captured by (PW-), and *Comprehension* is formally captured by (PC).

Prior's paradox

The naive conception of propositions and possible worlds is also inconsistent due to Prior's paradox; see Prior (1961) for the original formulation of the paradox.[42] Consider an arbitrary possible world w: $\theta(w)$. By conditional elimination on this and the result of universal elimination on the instance of (RC) where $\varphi(y)$ is $\forall x(Bel(x, y) \rightarrow F(x, y))$ [on w], there is a proposition q that is true at w iff everything believed at w is false, and that is false at w iff not everything believed at w is false:

(PR0) $\pi(q) \wedge (T(q, w) \leftrightarrow \forall x(Bel(x, w) \rightarrow F(x, w))) \wedge (F(q, w) \leftrightarrow \neg\forall x(Bel(x, w) \rightarrow F(x, w)))$.

By conjunction elimination on (PR0), we have that:

(Pq) $\pi(q)$.

(Tq) $T(q, w) \leftrightarrow \forall x(Bel(x, w) \rightarrow F(x, w))$.

(Fq) $F(q, w) \leftrightarrow \neg\forall x(Bel(x, w) \rightarrow F(x, w))$.

Now assume (for conditional introduction later) that q is believed at the given possible world:

(PR1) $Bel(q, w)$.

By (WBIV), either $T(q, w)$ or $F(q, w)$. Suppose for *reductio* that:

(PR2) $T(q, w)$.

By conditional elimination on the left-to-right direction of (Tq) and (PR2), we have that:

(PR3) $\forall x(Bel(x, w) \rightarrow F(x, w))$.

[42] See Tucker and Thomason (2011); Tucker (2013); Uzquiano (2015a,c); Bacon et al. (2016) for discussion. The paradox can be formulated, as Prior formulates it, in terms of irreducibly propositional quantification. However, I formulate it in terms of objectual quantification over propositions here. Also, unlike Prior, but like Uzquiano (2015a,c), I formulate the paradox in terms of possible worlds.

By universal elimination on (PR3) [on q], we have that:

(PR4) $Bel(q, w) \rightarrow F(q, w)$.

By conditional elimination on (PR1) and (PR4), we have that:

(PR5) $F(q, w)$.

But then by conjunction introduction on (PR2) and (PR5), q is both true and false at w, in violation of (WNC). So by *reductio* on (PR2), q is not true at w, in which case by (WBIV), q is false at w:

(PR6) $F(q, w)$.

By conditional elimination on the left-to-right direction of (Fq) and (PR6), we have that:

(PR7) $\neg\forall x(Bel(x, w) \rightarrow F(x, w))$.

Now (PR7) classically entails that there is something believed at w that is not false at w. Since, by (BPW), only propositions are believed at w, there is a proposition believed at w that is not false at w. So by (WBIV), there is a proposition that is true and believed at w:

(PR8) $\exists x(Bel(x, w) \wedge T(x, w))$.

But by conjunction introduction on (PR1) and (PR6), we have that:

(PR9) $Bel(q, w) \wedge F(q, w)$.

By existential introduction on (PR9), there is a proposition that is false and believed at w:

(PR10) $\exists x(Bel(x, w) \wedge F(x, w))$.

By conjunction introduction (PR8) and (PR10), there is a proposition that is true and believed at w as well as a proposition that is false and believed at w:

(PR11) $\exists x(Bel(x, w) \wedge T(x, w)) \wedge \exists x(Bel(x, w) \wedge F(x, w))$.

By conditional introduction on (PR1) and (PR11), if q is believed at w, then there is a proposition that is true and believed at w as well as a proposition that is false and believed at w:

(PR12) $Bel(q, w) \rightarrow \exists x(Bel(x, w) \wedge T(x)) \wedge \exists x(Bel(x, w) \wedge F(x, w))$.

Since by (WNC), no proposition is both true and false at a possible world, it follows from (PR12) that if q is believed at w, then there are at least two distinct propositions that are believed at w:

(PR13) $Bel(q, w) \rightarrow \exists x \exists y(Bel(x, w) \wedge Bel(y, w) \wedge x \neq y)$.

But (PR13) classically entails that q is not uniquely believed at w:

(PR14) $\neg(Bel(q, w) \wedge \forall z(Bel(z, w) \rightarrow z = q)$.

Since w is arbitrary, by universal introduction on (PR14), we have that:

(PR15) $\forall y \neg(Bel(q, y) \wedge \forall z(Bel(z, y) \rightarrow z = q)$.

But (PR15) classically entails that there is no possible world at which q is uniquely believed there:

(PR16) $\neg \exists y(Bel(q, y) \wedge \forall z(Bel(z, y) \rightarrow z = q)$.

By conjunction introduction on (Pq) and (PR16), we have that:

(PR17) $\pi(q) \wedge \neg \exists y(Bel(q, y) \wedge \forall z(Bel(z, y) \rightarrow z = q))$.

By existential introduction on (PR17), we have that:

(PR18) $\exists x(\pi(x) \wedge \neg \exists y(Bel(x, y) \wedge \forall z(Bel(z, y) \rightarrow z = x)))$.

But (PR18) classically entails that not every proposition is such that there is a possible world at which it is uniquely believed:

(PR19) $\neg \forall x(\pi(x) \rightarrow \exists y(Bel(x, y) \wedge \forall z(Bel(z, y) \rightarrow z = y)))$.

This contradicts (PW-).

More informally, we can state Prior's paradox as involving the inconsistency of the following two principles:

> *Belief.* For every proposition, there is a possible world at which that proposition is uniquely believed.
>
> *Propositions.* For every possible world, there is a proposition that is true at that possible world iff a given sentence is true at that possible world, and that is false at that possible world iff the given sentence is false at that possible world.

On my formalization, *Belief* is formally captured by (PW), and *Propositions* is formally captured by (RC).

5.3 Satisfying solutions to Kaplan's and Prior's paradoxes

I have shown that the naive conception of propositions and possible worlds leads to Kaplan's and Prior's paradoxes. I now suggest that a modal account of propositions and possible worlds provides satisfying solutions to these two further intensional paradoxes.

A modal account of propositions and possible worlds

I propose that, just as the naive conception of propositions is replaced with the iterative conception of propositions, so the naive conception of propositions and possible worlds should be replaced with what I call the "iterative conception of propositions and possible worlds," which is equally natural and, for all we know, consistent.

A key motivation for this conception is the principle that the existence of possible worlds at which certain propositions are true somehow depends on the existence of those propositions. This principle has precedent in Adams (1974); Kaplan (1995); Chalmers (2011)'s conceptions of possible worlds. The idea is that possible worlds depend on maximally consistent pluralities of propositions, and such propositions in turn depend on objects they are about, so that possible worlds also depend on those objects.

On the iterative conception of propositions and possible worlds, propositions and possible worlds are determined at different levels of a hierarchy, where objects, propositions, and possible worlds at lower levels are prior to propositions and possible worlds at higher levels, which are about propositions, possible worlds, and other objects at lower levels. Specifically, at level 0, there are all the urelements that are not propositions or possible worlds; at level 1, all propositions that can be determined from objects at level 0 are determined, and all possible worlds that can be determined from propositions at level 0 are determined; at level 2, all propositions that can be determined from objects at levels 0 or 1 are determined, and all possible worlds that can be determined from propositions at level 0 or 1 are determined; and in general, at level n, all propositions that can be determined from objects at levels $0, 1, \ldots, n-1$ are determined, and all possible worlds that can be determined from propositions at levels $0, 1, \ldots, n-1$ are determined.

To illustrate, at level 0, we have the Taj Mahal, Socrates, and Fido; at level 1, we have the proposition that Socrates has a beard, and possible worlds at which maximally consistent pluralities of level 1 propositions are true; at level 2, we have the proposition that the proposition that Socrates has a beard is true, and possible worlds at which maximally consistent pluralities of level 2 propositions are true.

A satisfying solution to Kaplan's paradox

An application of the modal account of propositions and possible worlds is that it provides a satisfying solution to Kaplan's paradox. The account diagnoses the problem with Kaplan's paradox as owing to the falsity of the principle (PW-) that every proposition determines a possible world at which it is uniquely believed. Although (PW-) is *prima facie* attractive, it is false. Kaplan's paradox is solved by noting that (PW-) is false. Nonetheless, the intuitive motivation for (PW-) is preserved by a weaker, modalized version of the principle. This weaker principle arises because claims about propositions and possible worlds are often implicitly modalized. The weaker, modalized version of (PW-) says that (at all levels of the hierarchy) every proposition could determine a possible world at which it is uniquely believed:

(PW) $\Box\forall x(\pi(x) \rightarrow \Diamond\exists y(Bel(x, y) \wedge \forall z(Bel(z, y) \rightarrow z = y)))$.

A feature of the proposed solution is that it maintains *Truth*, which is formalized by (WD). This principle is worth preserving, since—in line with the iterative conception of propositions and possible worlds—given that possible worlds are objects, every plurality of possible worlds and the property of truth could determine a proposition that is true at all and only the possible worlds among that plurality.

By contrast with the proposed solution, an alternative solution, due to Deutsch (2014), does not maintain *Truth*. This solution takes propositions to be functions from possible worlds to truth values (which are equivalent to sets of possible worlds), and maintains that there are some propositions and possible worlds for which it does not make sense to ask whether the proposition is true or false at the possible world: there is a possible world—such as *d* in my presentation of Kaplan's paradox—that "is not an argument of any function and hence not an argument of any function from worlds to truth values" (7). Thus, against *Truth*, there is at least one plurality of possible worlds—any plurality among which a possible world such as *d* is among—such that there is no proposition that is true at all (let alone all and only) the possible worlds among that plurality. But Deutsch offers little to no independent motivation for rejecting *Truth*. He seems to just take the fact that his formalization of the principle—I formalize it as (WD)—is inconsistent with his rather peculiar background assumptions to be a *reductio* of the principle. The solution's claim that some possible worlds are ones at which it does not make sense to ask whether certain propositions are true or false seems odd and insufficiently motivated.

A second feature of the proposed solution is that it provides independent motivation for rejecting *Belief*, which is formalized by (PW-). Instead, it maintains the modalization (PW), which preserves the intuitive motivation for (PW-). This preserves the spirit if not the letter of *Belief*. But the spirit is worth preserving, since *prima facie* there is no conceptual—rather than metaphysical—obstacle to each proposition's being uniquely believed at any possible world. As Kaplan (1995) suggests, one might maintain that even if we should reject *Belief* due to metaphysical considerations, it should not be because logic alone requires us reject it. Even if some propositions are not believed at any possible world, it does not seem appropriate for logic to tell us that. Of course, Kaplan's suggestion assumes a conception of logic according to which logic remains relatively neutral on metaphysical issues—logic is said to be formal, general, and topic-neutral. But while such a conception is not indisputable, many philosophers find it attractive.[43] Crucially, on such a conception, logical truths should be clearly distinguished from metaphysical truths and contingent truths. For while it is a contingent truth that grass is green, it is not a logical truth. Similarly, while it may be a metaphysically necessary truth that water is H_2O and also a metaphysically necessary truth that some propositions are not uniquely believed at any possible world, these truths are not logical truths. The point for us here is that, while perhaps it is a truth that some propositions are not uniquely believed at any possible world, it does not seem to be a *logical* truth.

By contrast with the proposed solution, an alternative solution, due to Lewis (1986, 104–108), does not maintain *Belief*, but seems to inadequately motivate doing so:

> Most sets of worlds [propositions], in fact all but an infinitesimal minority of them, are not eligible contents of thought. It is absolutely impossible that anybody should think a thought with a content given by one of these ineligible sets of worlds. (105)

According to Lewis, at least when propositions are identical to sets of possible worlds, it is not the case that for every proposition there is a possible world at which it is *uniquely* believed, because it is not the case that for every proposition there is even one possible world at which it is believed.[44] The idea is that some propositions cannot be believed, because there are more propositions than there are objects of belief—or more generally,

[43] See Ryle (1954); Haack (1978); Wright (1981); Etchemendy (1990); Sainsbury (2001) on the conception of logic as topic-neutral. But see Williamson (2013) for a dissenting view.

[44] Actually, Lewis has two solutions, depending on what propositions are stipulated to be. He maintains that when propositions are stipulated to be identical with sets of possible worlds, not all propositions are uniquely believed at a possible world. But he maintains that when propositions are stipulated to be the possible objects of belief, not all sets of possible worlds are propositions. I am presently considering the former solution, which is his own focus. The latter solution is more like the one that I am proposing.

attitudes. And there are more propositions than there are objects of belief, not because of human limitations, but due to Lewis' "analytic functionalism":

> A man or a beast or a god, or anything that is a thinker at all, has a thought with a certain content in virtue of being in a state which occupies a certain functional role. (106)

The functionalism is analytic in that the relation between objects of belief and functional roles is necessary rather than contingent. What is crucial for Lewis is that, according to his functionalism, there are only as many objects of belief as there are functional roles, but there is a relatively low upper bound on the number of functional roles despite a much larger number of propositions:

> I cannot see the slightest *prima facie* reason to think that there are even uncountably many of the definitive roles, let alone that there are as many as there are propositions (beth-three, on the lowest reasonable estimate). (107)

However, against Lewis, Chalmers (2011, 91–92) suggests that Lewis neglects to consider arbitrarily complex beings, the consideration of which suggests that there are arbitrarily many functional roles:

> Given κ states (each corresponding to a thought, for example), one can straightforwardly define 2^κ functional roles in terms of those states (each corresponding to a conjunction of some of the original thoughts, for example).

Besides, Lewis' functionalism—and functionalism more generally—is controversial.[45] So resting the case against *Belief* on the basis of functionalism alone seems inadequate.

Similarly, Bueno et al. (2014); Uzquiano (2015a,c)'s solutions do not maintain *Belief*. But they offer little to no independent motivation for simply rejecting the principle. For example, Uzquiano (2015c, 12) suggests that "[t]he real problem with (A) [Uzquiano's formalization of *Belief*]...is that it is plainly inconsistent." While it might be fine to reject a principle because it is inconsistent, these solution are wanting insofar as they leave matters at that, and so do not explain why *Belief* is at least initially plausible. By

[45]See Schwarz (2015) for criticism of Lewis' functionalism.

contrast, I contend, the proposed solution, based on the modal account of propositions and possible worlds, provides independent motivation for rejecting the principle while preserving the intuitive motivation for the principle.

A third feature of the proposed solution is that it maintains *Comprehension*. As with the Russell-Myhill paradox, a comparison with other solutions is not straightforward, because whereas the proposed solution is formulated in terms of pluralities, most other solutions are not so formulated, and are rather formulated in terms of sets or classes. Nonetheless, as before, we can consider what other solutions amount to when formulated in terms of pluralities, by reinterpreting talk of sets or classes as talk of pluralities, and I do just that. Unlike Oksanen (1999); Chalmers (2011), we do not impose the restriction that the objects we pick out for consideration are members of sets or even classes.[46]

A fourth feature of the proposed solution is that it maintains unrestricted quantification over all propositions and possible worlds. A combined modal account of propositions and possible worlds is notably similar to a ramified account of propositions and possible worlds, as in Kaplan (1995), in that both take seriously the idea that there is some kind of hierarchy of propositions.[47] However, I suggest, the former is considerably more natural, whereas the latter seems too artificial and *ad hoc*. Moreover, an apparently severe cost of a ramified account is that it makes unrestricted quantification over propositions and possible worlds unavailable. Specifically, on a ramified account, quantified propositions only quantify over propositions and possible worlds of certain orders, and in particular orders lower than the order of the quantified proposition itself. Accordingly, talk of "all propositions" is rendered as talk of "all propositions of order n for some level n of the hierarchy of propositions, and talk of "all possible worlds" is rendered as talk of "all possible worlds of order n for some level n of the hierarchy of pos-

[46]By contrast, Oksanen (1999) claims that propositions form only a "non-Cantorian" set, where a non-Cantorian set is characterized by being bigger than or equal to its powerset and so is not one for which Cantor's theorem holds, and unrestricted comprehension does not hold for non-Cantorian sets. Meanwhile, Chalmers (2011) claims that propositions do not form a set, which seems to entail restricting *Comprehension* to objects that are members of sets.

[47]Chalmers (2011) maintains a stratified account of scenarios that resembles Kaplan's ramified account. For Chalmers, scenarios are essentially epistemically possible worlds, and he is concerned to avoid problematic features of ramified accounts.

sible worlds. As on the modal account of propositions on its own, the combined account maintains some kind of unrestricted quantification over all propositions and possible worlds. Unrestricted quantifiers are emulated by means of modalized quantifiers, such as $\Box\forall$ and $\Diamond\exists$. With the help of modalized quantifiers, we can make meaningful, true, and general claims about all propositions and possible worlds.

A fifth feature of the proposed solution is that it is nonrevisionary with respect to the standard conception of metaphysical modality and possible worlds. By contrast, Fritz (2016); Roberts (ms) offer more revisionary proposals. Fritz, who seems to tie metaphysical modality to possible worlds, suggests that Kaplan's paradox, together with other paradoxes involving modality, casts doubt on the coherence of the concept of metaphysical modality. He calls the supposition that there is a single concept of metaphysical modality a "dogma of current metaphysics" and suggests that "the puzzle [generated by the paradoxes] at least chips away at the support for the assumption that the notion of metaphysical necessity is in good standing" (10). Meanwhile, Roberts suggests that metaphysical modality is indefinitely extensible, and proposes an account of metaphysical modality that resembles the account I am proposing—except that my account maintains that only logico-mathematical modality is indefinitely extensible and so does not also maintain that metaphysical modality is such.

A satisfying solution to Prior's paradox

Another application of the modal account of propositions and possible worlds is that it provides a satisfying solution to Prior's paradox. As with Kaplan's paradox, the account diagnoses the problem with Prior's paradox as owing to the falsity of (PW-). Although (PW-) is *prima facie* attractive, it is false. Prior's paradox is solved by noting that (PW-) is false. Nonetheless, the intuitive motivation for (PW-) is preserved by a weaker, modalized version of the principle, namely, (PW).

A feature of the proposed solution is that it maintains *Propositions*, which is formalized by (RC). This principle is hard to deny, since on minimal assumptions it is true: the existence of a true proposition as well as a false proposition ensures the truth of (RC).

A second feature of the proposed solution is that it provides independent motivation for rejecting *Belief*, which is formalized by (PW-). Instead, it maintains the modalization (PW), which preserves the intuitive motivation for (PW-). As before, this preserves the spirit if not the letter of *Belief*. As some observe, a principle such as *Belief* is plausible, and so hard to deny without explaining why it fails. The significance of *Belief* is general, since versions of the principle plausibly hold for attitudes—and other relations towards propositions—other than belief. Even if we accept that some propositions cannot be uniquely believed, it might be harder to accept that some propositions cannot be uniquely entertained—hoped, wondered, and so on—or asserted. And even if we accept that some propositions cannot be uniquely entertained, or asserted, it might be harder to accept that some propositions cannot be uniquely *attempted* to be entertained, or asserted. Prior (1961, 29) observes that we can also consider *uttered with words that conventionally signify that*.

By contrast, alternative solutions, due to Prior (1961); Uzquiano (2015a,c), do not maintain *Belief*. But they offer little to no intuitive motivation for simply rejecting the principle. Even Prior (1961, 32) is reluctant to embrace his own solution:

> I have been driven to my conclusion very unwillingly.... So far as I can see, we must just accept the fact that thinking, fearing, etc. ...must from time to time be oddly blocked..., and we must just let Logic teach us where these blockages will be encountered.

Other features of the proposed solution to Prior's paradox resemble the features of the proposed solution to Kaplan's paradox. This is unsurprising, given the similarities between the two paradoxes. On the proposed solution to Prior's paradox, we again maintain unrestricted quantification over all propositions and possible worlds.[48] Further, we are again nonrevisionary with respect to the standard conception of metaphysical modality and possible worlds.

[48]Prior (1961, 23, 32) considers but rejects the solution offered by a ramified account of solutions due to the expressive limitations. As I have suggested, the modal account of propositions and possible worlds resembles but avoids the expressive limitations of a ramified account of propositions and possible worlds.

Chapter 3

'True' as polysemous

> [T]he concept of truth (as well as other semantical concepts) when applied to colloquial language in conjunction with the normal laws of logic leads inevitably to confusions and contradictions. Whoever wishes, in spite of all difficulties, to pursue the semantics of colloquial language with the help of exact methods...will find it necessary to define its structure, to overcome the ambiguity of the terms which occur in it, and finally to split the language into a series of languages of greater and greater extent, each of which stands in the same relation to the next in which a formalized language stands to its metalanguage.
>
> — Tarski (1983, 267)

Plausibly, the predicate "true" is ambiguous. For we speak of true sentences, true propositions, true friends, and true works of art. In each case, "true" seems to mean something different. Of course, it is not just that "true" seems to apply to different kinds of entities. Since there are different kinds of objects, "object" applies to different kinds of objects, but "object" is not ambiguous. For a predicate to be ambiguous, it should be able to both apply and not apply to something. For example, "light" is ambiguous: "light" may both apply to dark-colored feather, since "light" may mean (roughly) *low in weight*, and may not apply to it, since "light" may also mean *pale in color*.[1] Similarly with "true": "true" in "true works of art" means *genuine*, and may apply to the proposition that grass is blue (but not entities that are not propositions), while "true" in "true

[1]Throughout, the locutions "*P* means *m*" and "*P* has the meaning *m*" should be construed as providing a rough analysis of the meaning of *P* that suffices for the purposes at hand.

propositions" may not apply to the proposition that grass is blue.

But let us set aside the question of whether "true" is ambiguous in being a predicate of different kinds of entities. Let us focus on "true" as a predicate of sentences—of a single natural language, namely, English—relative to context, with the caveat that claims about "true" are also claims about cognate expressions, such as "truth," as well.

In increasing order of controversiality, it has been suggested that the predicate "true" of sentences is vague, context-sensitive, or ambiguous. In this chapter, I propose that "true" is polysemous, and thus ambiguous. I provide independent motivation for the proposal on the basis of an existing distinction between the concepts *correspondence truth* and *disquotational truth*, as well as the indefinite extensibility of the concept *truth*. I suggest that "true" is initially polysemous between *correspondence truth* and *disquotational truth*, and further polysemous between the meanings corresponding to the subconcepts of the concept *truth* generated by the indefinite extensibility of that concept. I show that the proposal provides satisfying solutions to the semantic paradoxes.

As a general methodological principle, the default assumption is that "true" is unambiguous. As Kripke (1977, 268) warned, "[d]o not posit an ambiguity unless you are really forced to, unless there are really compelling theoretical or intuitive grounds to suppose that an ambiguity really is present." Similarly, Grice (1978)'s "modified Occam's razor" maintains that "*[s]enses are not to be multiplied beyond necessity*" [emphasis in the original].[2] What about considerations specific to "true"? In addressing apparent differences in kind between the truth of logical claims and the truth of mathematical claims, Quine (2013, 118) maintains that "true" is "unambiguous but very general."[3]

Nonetheless, these methodological considerations do not rule out any kind of ambiguity proposal from the outset. Even if we should avoid ambiguity in systematic theorizing, we should recognize ambiguity to the extent that it is present in what we are

[2]In the linguistics literature, "senses" are reserved for the closely related meanings of polysemous expressions, while "meanings" is reserved for the meanings of homonymous expressions. However, I do not conform to this usage here. Throughout, to emphasize that polysemy is a kind of ambiguity, I take the relevant (singular or plural) forms of these expressions to be interchangeable.

[3]Quine's concern here is not with those who take "true" to be ambiguous due to the paradoxes.

theorizing about—in the present case, natural language. Such ambiguity is to be explained, not explained away. Grice's modified Occam's razor is accordingly of limited significance. The razor suggests that, all else equal, we should not posit ambiguity. But, I contend, not all else is equal.

This chapter is structured as follows. In Section 1, I present Bacon (2015)'s general challenge to classical solutions to the liar paradox, and outline an ambiguity solution that I suggest is not vulnerable to the challenge. In Section 2, I present the polysemy proposal, and develop the first part of the proposal, according to which "true" is initially polysemous between *correspondence truth* and *disquotational truth*. In Section 3, I develop the second part of the proposal, according to which "true" is further polysemous between the meanings corresponding to the subconcepts of the concept *truth* generated by the indefinite extensibility of that concept. In Section 4, I suggest that the proposal provides satisfying solutions to the semantic paradoxes. (In Appendix C.1, I suggest that various tests for polysemy corroborate the proposal.)

1 Motivation from the semantic paradoxes

In this section, I argue that the semantic paradoxes motivate taking "true" to be ambiguous. In the first subsection, I rehearse the liar paradox and outline classical solutions to the paradox. In the second subsection, I discuss Bacon (2015)'s general challenge to classical solutions. In the third subsection, I suggest that a classical solution that posits ambiguity, which Bacon does not consider, is not vulnerable to Bacon's challenge.

1.1 Classical solutions to the liar paradox

Straightforwardly, it seems that if "$1+1=2$" is true, then $1+1=2$, and conversely, if $1+1=2$, then "$1+1=2$" is true:

(*) $\mathit{True}(\ulcorner 1+1=2 \urcorner) \leftrightarrow 1+1=2$.

More generally, Tarski's disquotational principle of truth is plausible:

(T) $True(\ulcorner\varphi\urcorner) \leftrightarrow \varphi$.

After all, (T) vindicates (*), and other facts like it. However, it also seems that (T) does not hold in general. For consider λ, which names the liar sentence "$\neg True(\lambda)$," and intuitively reads "λ is not true":

(λ) $\lambda = \ulcorner\neg True(\lambda)\urcorner$.

The instance of (T) where φ is $\neg True(\lambda)$ is the following:

(T_λ) $True(\ulcorner\neg True(\lambda)\urcorner) \leftrightarrow \neg True(\lambda)$.

Given (λ), by an application of identity elimination on (T_λ), we have the following:

(X) $True(\lambda) \leftrightarrow \neg True(\lambda)$.

Standard principles of reasoning show that this yields a contradiction.

The liar paradox is a paradox in that intuitively compelling principles—(T) and the classical logical principles of reasoning employed in the derivation of (X)—result in a contradiction. Any theory of truth that maintains these principles is therefore inconsistent, and thus—in most logics—trivial, in that every sentence is true. But, I take it, the arrival at a contradiction by itself is already absurd.

To solve the paradox, either (T) or classical logic must be surrendered.[4] Nonclassical solutions surrender classical logic. Notably, paracomplete solutions surrender the principle of the excluded middle, according to which $\varphi \vee \neg\varphi$ is valid, while paraconsistent solutions surrender the principle of explosion, according to which the inference from φ and $\neg\varphi$ to ψ is valid.[5] They may but need not also deny the classical semantic principle of bivalence, according to which every sentence is either true or false but not both. Notably, gappy solutions maintain that some sentences are neither true nor false, while glutty solutions maintain that some sentences are both true and false.

[4]In the coming three paragraphs, I loosely follow the discussion in Bacon (2015, 209–305).

[5]See Kripke (1975); Field (2008) for paracomplete solutions, and see Priest (2002, 2006); Beall (2009) for paraconsistent solutions.

However, I mention nonclassical solutions only to set them aside. This is not the place to adjudicate between classical and nonclassical solutions. But, suffice to say, it is controversial to what extent solutions to the semantic paradoxes, such as the liar paradox, are compatible with maintaining classical logic. In this chapter, I simply adopt as a working hypothesis that a classical solution is tenable. Thus, I restrict my attention to classical solutions. As Williamson (2016) suggests, in his provision of an abductive justification of classical logic in light of the semantic paradoxes, the simplicity, elegance, and power of classical logic suggest that investigating classical solutions is worthwhile.

Classical solutions surrender (T). It seems incumbent upon proponents of such solutions, however, to explain why the instance (T_λ) in particular and perhaps other instances do not hold. Arguably, it is not unreasonable to ask and expect them to provide at least sufficient conditions for when (T) holds. As Tarski (1966, 66) puts it, "the appearance of an antimony is...a symptom of disease." A typical strategy is to specify a property of sentences the having of which exemplifies disease.[6] The specified property is such that, if a given sentence does not have it, and so is not diseased, then the relevant instance of (T) holds. Examples of such properties include the property of containing a predicate that is introduced by an impredicative definition (Russell, 2010), the property of containing a predicate that is context-sensitive in a special way (Parsons, 1974), the property of being ungrounded (Kripke, 1975), the property of containing a predicate that is introduced by an inconsistent definition (Chihara, 1979), the property of not following analytically from nonlinguistic facts (McGee, 1990), and the property of being semantically unstable (Gupta and Belnap, 1993).

1.2 Bacon's challenge to classical solutions

Unfortunately, classical solutions face a general challenge recently posed by Bacon (2015). The challenge takes the form of a revenge paradox, in exploiting vocabulary a given

[6]Bacon maintains that these classical solutions are "linguistic" (300), in that they specify a property of sentences rather than the propositions they express. As a classical theorist himself, he prefers a "non-linguistic" (301) solution that specifies a property of propositions.

solution employs to solve the liar paradox in order to construct a structurally similar paradox. The revenge paradox might result in outright contradiction, as with the liar paradox, but even if it just leads to incoherence, that is arguably already absurd.

Recall that proponents of classical solutions who adopt the typical strategy above specify a property the having of which exemplifies disease, so that if a given sentence does not have it, then the relevant instance of (T) holds. Given that sentences are either healthy or diseased, what is equivalently specified is a property the having of which exemplifies healthiness, so that if a given sentence has this property, then the relevant instance of (T) holds. Suppose that the predicate *Healthy* expresses the property of being healthy, and that the predicate *Diseased* expresses the property of being diseased, where $\mathit{Diseased}(x) := \neg \mathit{Healthy}(x)$. Sentences that satisfy *Healthy* include "$1 + 1 = 2$," while sentences that do not satisfy *Healthy* and so satisfy *Diseased* include the liar sentence λ. Bacon (2015, 306) suggests the following interpretations of *Healthy* according to various classical solutions (the citations of examples of such solutions are his):

- *Healthy is "grounded".* This is the interpretation for solutions based on grounding, such as those due to Kripke (1975); Yablo (1982); Leitgeb (2005); Maudlin (2004).

- *Healthy is "semantically stable".* This is the interpretation for solutions based on the revision theory or theory of circular definitions, such as those due to Herzberger (1982a); Gupta and Belnap (1993); Yaqüb (1993).

- *Healthy is "definitely true or definitely not true".* This is the interpretation for solutions based on definite and indefinite truth, such as that due to McGee (1990).

- *Healthy is "every utterance expresses a proposition" or "every utterance expresses a proposition with a definite truth value".* This is the interpretation for contextualist solutions, such as those due to Parsons (1974); Burge (1979); Gaifman (1992); Simmons (1993); Williamson (1998).[7]

[7] There are typos in the presentation of contextualist solutions on pp. 306 and 312 with respect to the contextualist construal of *Healthy* and *True* in Bacon (2015). I have confirmed with Bacon in personal correspondence that my presentation here is correct.

As Bacon suggests, the proponents in question should maintain the following restriction of (T) to what according to their solution are healthy sentences:

(HT) $Healthy(\ulcorner\varphi\urcorner) \to (True(\ulcorner\varphi\urcorner) \leftrightarrow \varphi)$.

However, Bacon proves that if a given theory satisfies two minimal constraints—it is classical and contains (HT)—then it proves that it has theorems that it deems diseased. Specifically, he proves the following theorem, which presents a trilemma (308–309):

Theorem (Bacon). *Let T be a set of sentences of an arithmetical language $\mathscr{L}$ supplemented with the primitive predicates Healthy and True. Then one of the following is true:*

(i) T does not contain some axiom of classical logic, does not contain some axiom of Peano arithmetic, or is not closed under the classical rules of inference.

(ii) T does not contain every instance of (HT).

(iii) There is a sentence γ of T such that $\neg Healthy(\ulcorner\gamma\urcorner)$ is also a sentence of T.

Proof sketch. Assume that neither (i) nor (ii) hold. Then consider the sentence γ which, by the diagonal lemma, is such that $\gamma \leftrightarrow (Healthy(\ulcorner\gamma\urcorner) \to \neg True(\ulcorner\gamma\urcorner))$.[8] We show using classical logic that $\neg Healthy(\ulcorner\gamma\urcorner)$ is a theorem of *T*. By the above equivalence, we conclude that γ is also a theorem of *T*. □

Intuitively, γ in the theorem is a revenge sentence that says "γ is either not healthy or not true." It is worth explaining what is supposed to be bad about each horn of the trilemma in the theorem. Horns (i) and (ii), recall, are minimal constraints: horn (i) is supposed to be bad, because we are confining our attention to classical solutions, while horn (ii) is supposed to be bad, because failing to contain every instance of (HT) amounts to maintaining an inadequate diagnosis of the disease, since perhaps there are true instances of (HT) that the theory does not recognize are true. Horn (iii) is supposed to be bad, meanwhile, because it seems to commit the theorist to a certain kind of incoherence: the theorist asserts a sentence—namely, γ—that the theory deems diseased and

[8]The diagonal lemma guarantees that, given $\varphi(x)$, there is a sentence ψ such that $\psi \leftrightarrow \varphi(\ulcorner\psi\urcorner)$.

so presumably unassertible. (The minimal assumption here is that one who accepts a given theory asserts or should assert the theorems of that theory.)

Bacon invokes his theorem to argue against solutions based on grounding, the revision theory, or definite and indefinite truth (311–315). However, I am most interested in his invocation of the theorem in arguing against contextualist solutions, because the kind of solution I wish to consider resembles such solutions in key respects. Bacon suggests that contextualists are committed to the following principle:

(BC) If some utterance of $\ulcorner \varphi \urcorner$ is true, then φ.

He then argues that contextualist solutions "are susceptible to a particularly simple revenge paradox, an instance of our general theorem" (313). His argument is the following, although the numbering of the claims is mine:

(B1) If an utterance is true, then it says something.

(B2) If an utterance is true and says that φ, then φ.

(B3) If an utterance of "no utterance of L is true" says anything at all, it says that no utterance of L is true.

(B4) If an utterance of "no utterance of L is true" is true, then no utterance of L is true.

(B5) If some utterance of L is true, then no utterance of L is true.

(B6) No utterance of L is true.

Above, (B1)–(B3) are premises, (B4) is an intermediate conclusion, (B5) is an intermediate conclusion obtained from (B4) by the identity that $L =$ "no utterance of L is true," and (B6) is the final conclusion obtained from (B1)–(B5) by *reductio* on the assumption that some utterance of L is true. Here, (B4) is just an instance of (BC). In a footnote, Bacon remarks that instances of (BC) are false when $\ulcorner \varphi \urcorner$ is context-sensitive: "Of course, not every instance of this schema is true. Ordinary context-sensitive sentences that express different propositions in different contexts do not conform to this schema" (313).

However, it is reasonable to suppose that L is not context-sensitive, so that (B4) is a legitimate instance of (BC). Given the reasoning (B1)–(B6), Bacon (2015, 313) suggests that contextualist solutions face an expressibility problem:

> Anyone who endorses these premises is committed to the claim that no utterance of L is true, and therefore should surely be able to assert the fact that no utterance of L is true. Yet when the theorist tries to express this commitment, by uttering the sentence 'no utterance of L is true' he fails. Since according to his own view there are no true utterances of the sentence 'no utterance of L is true' — even his very own utterances of this sentence fail to be true...by failing to express a proposition.

As mentioned, Bacon takes the foregoing argument—let us call it his "anticontextualist argument"—to be "an instance" of his theorem. Admittedly, I am unsure exactly what he means by this. After all, his theorem (and his proof of it) does not appeal to the notion of an utterance saying something, a notion that seems crucial to his anticontextualist argument. What I think he means, however, is that his argument provides intuitive support for why contextualist solutions are supposed to be threatened by his theorem. Contextualist solutions satisfy the two minimal constraints of being classical and containing all instances of (HT), but assert a sentence γ that they deem diseased and so presumably unassertible. His anticontextualist argument provides intuitive support by providing a specific example of γ. The sentence L, which is just the sentence "no utterance of L is true," is presumably asserted in expressing the conclusion (B6) that no utterance of L is true. But L is also deemed diseased, according to contextualist solutions, because no utterance of L says anything (expresses a proposition).

Before continuing, let me briefly mention contextualist responses to anticontextualist arguments like Bacon's. Assume, as is plausible, that utterances correspond to sentences in context. Contextualists are likely to object to the legitimacy of revenge sentences such as "no utterance of L is true," which we are assuming is equivalent to "L is not true in any context," on the basis that they illegitimately quantify over all contexts. Notably, Burge (1979); Glanzberg (2004a,b, 2006) maintain that quantification over all contexts is illegitimate. However, against such contextualist responses, it is unclear whether there is independent motivation for denying the legitimacy of such quantification. *Prima facie*, such quantification is perfectly legitimate. Thus, it is at least desirable to search for alternative responses to anticontextualist arguments like Bacon's.

1.3 Outline of an ambiguity solution

Bacon's general challenge to classical solutions targets what he calls "linguistic" solutions, which specify a property of sentences the having of which exemplifies disease, in contrast to what he calls "nonlinguistic" solutions (which he prefers), which specify a property of propositions the having of which exemplifies disease. I will not discuss his own preferred nonlinguistic solution here, however. Instead, in this chapter, I will explore an intriguing possibility that is left undiscussed and—I suggest—untouched by his challenge. The solution I explore is a classical solution that is linguistic in Bacon's sense, and resembles the contextualist solutions he considers, but differs in positing ambiguity, perhaps in addition to context sensitivity.

What does an ambiguity solution look like? Like contextualist solutions, ambiguity solutions specify a property of sentences the having of which exemplifies disease, where the having of this property depends on whether the sentence says something (expresses a proposition). Similarly, ambiguity solutions satisfy the two minimal constraints of being classical and containing (HT), where *Healthy* expresses the property of not being diseased. This suggests that ambiguity solutions are also threatened by Bacon's theorem. In a sense, this is correct—his theorem applies to ambiguity solutions as well. However, in my view, it is less clear that ambiguity solutions face the general challenge posed, in the sense of being threatened with incoherence to the effect that a sentence γ deemed diseased and so presumably unassertible is asserted.

On an ambiguity solution, "true" and related expressions, such as "says" and "false," are ambiguous—they have distinct meanings. But then a sentence that does not say something, and so is not true, in one sense and in one context can still say something, and be true, in another sense and in perhaps another context. An ambiguity solution thus has the resources to convincingly answer Bacon's general challenge to classical solutions. The idea is that although γ is deemed diseased in one sense, it is nonetheless healthy in another sense, and so the assertion of it does not—or at least need not—result in incoherence. In what follows, I outline an ambiguity solution in more detail. I provide some principles for the theory, formalize Bacon's anticontextualist argument, and show how the ambiguity theory neutralizes the threat from his argument.

Principles

As is standard, let us take expressions—including sentences and subsentential expressions such as predicates—in a given formal language to be strings of symbols. What principles govern the truth of sentences in context?[9]

Recall the initial plausibility of (T). The semantic paradoxes motivate restricting (T) in some way. But there is motivation independent of considerations having to do with the semantic paradoxes to maintain that (T) does not hold in general. The plausibility of (T) seems to depend on the plausibility of the following two principles, which jointly entail (T):

(S-) $\mathit{Says}(\ulcorner\varphi\urcorner, \varphi)$.

(ST-) $\mathit{Says}(\sigma, \varphi) \to (\mathit{True}(\sigma) \leftrightarrow \varphi)$.

Straightforwardly, the following instance (S-*) of (S-) and the following instance (ST-*) of (ST-) hold:

(S-*) $\mathit{Says}(\ulcorner 1+1=2 \urcorner, 1+1=2)$.

(ST-*) $\mathit{Says}(\ulcorner 1+1=2 \urcorner, 1+1=2) \to (\mathit{True}(\ulcorner 1+1=2 \urcorner) \leftrightarrow 1+1=2)$.

But plausibly, (S-) does not hold in general. For perhaps $\ulcorner\varphi\urcorner$ is context-sensitive, as when it is "I am happy." But then the following instance of (T) does not hold:

(T**) "I am happy" is true iff I am happy.

[9] See Ramsey (1927); Prior (1971); Williamson (1994b, 1998); Andjelković and Williamson (2000); Bacon (2015); Bacon and Goodman (2015) for further discussion of many of the principles I discuss here.

And that is presumably because the following instance of (S-) does not hold:

(S-**) "I am happy" says that I am happy.

The problem is that "I am happy" is context-sensitive: what it says, and so whether it is true, depends on the context—who "I" refers to, or who the speaker is. In a context where Otavio is the speaker, "I am happy" does not say that I am happy. In general, $\ulcorner\varphi\urcorner$ need not say that φ, and so $\ulcorner\varphi\urcorner$ need not be true iff φ. Sentences say different things, and so are true or false, depending on the context.

Despite the initial plausibility of (S-) and so (T), I suggest that an adequate semantic theory need not maintain them, since it is an empirical matter what expressions say. Here is Bacon (2015, 317), who I think is correct on this point:

> [W]e shouldn't require an adequate theory to tell us what every sentence means. Even the cases in which sentences say what they seem to say record empirical facts. For example, while the sentence 'snow is white' says that snow is white, this fact depends on how we use the words 'snow' and 'white'.

We should reject (S-) and (T) as they stand.

A response one might pursue in light of the above is to restrict (S-) and (T) to cases where $\ulcorner\varphi\urcorner$ is not context-sensitive. However, a more general response is available. Let us consider the truth of sentences in context, so as not to prejudge the significance of context for truth, where sentences in context correspond to utterances, or sentence tokens.[10] Formally, let us represent the truth of sentences in context by means of a binary predicate *True* of pairs of sentences σ and contexts c. We can, as I will, maintain the following plausible pair of principles, which associate the truth or falsity of a given sentence in a given context with what the sentence says in that context:

(ST) $\mathit{Says}(\sigma, \varphi, c) \rightarrow (\mathit{True}(\sigma, c) \leftrightarrow \varphi)$.

(SF) $\mathit{Says}(\sigma, \varphi, c) \rightarrow (\mathit{False}(\sigma, c) \leftrightarrow \neg\varphi)$.

[10]This follows the treatment of truth bearers in Williamson (1998). Even if sentences in context are not identical to utterances, or sentence tokens, there is arguably a one-to-one correspondence between them, which suffices to render my treatment here appropriate.

According to (ST), a sentence that says something in a given context is true in that context iff what the sentence says in that context obtains. Similarly, according to (SF), a sentence that says something in a given context is false in that context iff what the sentence says in that context does not obtain. Accordingly, given that "$1+1=2$" in a given context says that $1+1=2$, "$1+1=2$" is true in that context iff $1+1=2$. Equivalently, where what is said is identified with a proposition expressed, according to (ST), a sentence that expresses a proposition in a given context is true iff that proposition obtains. Similarly, according to (SF), a sentence that expresses a proposition in a given context is false iff that proposition does not obtain. Given classical logic, (ST) and (SF) entail an appropriate principle of bivalence:

(BIV) $\mathit{Says}(\sigma, \varphi, c) \rightarrow (\mathit{True}(\sigma, c) \vee \mathit{False}(\sigma, c))$.

According to (BIV), a sentence that says something in a given context is either true or false in that context.

Next, we assume an appropriate converse of (BIV):

(CBIV) $\neg\exists\varphi \mathit{Says}(\sigma, \varphi, c) \rightarrow \neg(\mathit{True}(\sigma, c) \vee \mathit{False}(\sigma, c))$.

According to (CBIV), if a sentence does not say anything at all in a given context, then it is neither true nor false in that context. Note that—unlike (ST), (SF), and (BIV)—(CBIV) quantifies over what is said: quantification is into sentence position, or over propositions.[11] The universal generalizations of (ST) and (SF) are, respectively, the following:

(ST-A) $\forall\sigma\forall\varphi\forall c(\mathit{Says}(\sigma, \varphi, c) \rightarrow (\mathit{True}(\sigma, c) \leftrightarrow \varphi))$.

(SF-A) $\forall\sigma\forall\varphi\forall c(\mathit{Says}(\sigma, \varphi, c) \rightarrow (\mathit{False}(\sigma, c) \leftrightarrow \neg\varphi))$.

Taken together, (ST-A), (SF-A), and (CBIV) entail the following pair of principles, which provide explicit definitions of truth and falsity in context:[12]

[11]The legitimacy of such quantification is defended in Prior (1971).

[12]For the left-to-right direction of (SPT), suppose that $\mathit{True}(\sigma, c)$. By (CBIV), we have that $\exists\psi \mathit{Says}(\sigma, \psi, c)$. By (ST), we also have that ψ. By existential introduction, we have that $\exists\varphi(\mathit{Says}(\sigma, \varphi, c) \wedge \varphi)$. For the right-to-left direction of (SPT), suppose that $\exists\varphi(\mathit{Says}(\sigma, \varphi, c) \wedge \varphi)$. Then by (ST-A), we have that $\mathit{True}(\sigma, c)$. A similar argument establishes (SPF).

(SPT) $True(\sigma, c) \leftrightarrow \exists\varphi(Says(\sigma, \varphi, c) \wedge \varphi)$.

(SPF) $False(\sigma, c) \leftrightarrow \exists\varphi(Says(\sigma, \varphi, c) \wedge \neg\varphi)$.

The left-to-right directions of (SPT) and (SPF) help capture the intuition that the truth and falsity of sentences in context presuppose the saying of something in context.

Finally, consider the following principle:[13]

(UNQ) $Says(\sigma, \varphi, c) \wedge Says(\sigma, \psi, c) \rightarrow (\varphi \leftrightarrow \psi)$.

According to (UNQ), anything said by a sentence in a given context is materially equivalent to any other thing said by the sentence in that context. (The idea is that the notion of saying is the notion of *unique* saying.) This rules out the possibility that a given sentence says more than one thing in a given context, and in particular, something that obtains as well as something that does not obtain.[14] *A fortiori*, this rules out the possibility that a sentence is both true and false in a given context; for if a sentence is both true and false in a given context, then by (SPT) and (SPF), it both says something that obtains and says something that does not obtain in that context, so that it is not the case that anything said by the sentence in that context is materially equivalent to any other thing said by that sentence in that context.

The contextual liar and revenge paradoxes

I now formalize Bacon's anticontextualist argument, and show how the ambiguity theorist neutralizes the force of the argument.

I begin by reconstructing contextualist reasoning for the conclusion that a certain sentence is deemed diseased by the theory, in that it does not say something in any context, even though it is also asserted as a theorem. Given that a sentence in context corresponds to an utterance of that sentence (where utterances take place in context), we read "σ is true in every context c" as "every utterance of σ is true."

[13]If we assume (UNQ), then together with (SPT) and (SPF), we can derive (ST) and (SF).

[14]See Lormand (1996); Soames (2002, 2009); Cappelen and Lepore (2005); Braun and Sider (2007); Cappelen (2008); Travis (2008); Read (2009); Dorr and Hawthorne (2014b) on this possibility.

Consider the contextual liar sentence λ_γ, which names the contextual liar sentence "$\forall c \neg True(\lambda_\gamma, c)$," and intuitively reads "λ_γ is not true in any context":

(λ_γ) $\lambda_\gamma = \ulcorner \forall c \neg True(\lambda_\gamma, c) \urcorner$.

Presumably, $\ulcorner \forall c \neg True(\lambda_\gamma, c) \urcorner$ is not context-sensitive. So if $\ulcorner \forall c \neg True(\lambda_\gamma, c) \urcorner$ says anything at all in a given context, $\ulcorner \forall c \neg True(\lambda_\gamma, c) \urcorner$ says that λ_γ is not true in any context. For after all, what else could $\ulcorner \forall c \neg True(\lambda_\gamma, c) \urcorner$ say? We have that:

(L1) $\exists \varphi Says(\ulcorner \forall c \neg True(\lambda_\gamma, c) \urcorner, \varphi, c) \rightarrow Says(\ulcorner \forall c \neg True(\lambda_\gamma, c) \urcorner, \forall c \neg True(\lambda_\gamma, c), c)$.

Consider an arbitrary context c_0. Assume for *reductio* that:

(L2) $\exists \varphi Says(\ulcorner \forall c \neg True(\lambda_\gamma, c) \urcorner, \varphi, c_0)$.

By conditional elimination on (L1) and (L2), we have that:

(L3) $Says(\ulcorner \forall c \neg True(\lambda_\gamma, c) \urcorner, \forall c \neg True(\lambda_\gamma, c), c_0)$.

By conditional elimination on (ST) and (L3), we have that:

(L4) $True(\ulcorner \forall c \neg True(\lambda_\gamma, c) \urcorner, c_0) \leftrightarrow \forall c \neg True(\lambda_\gamma, c)$.

By identity elimination on (λ_γ) and (L4), we have that:

(L5) $True(\lambda_\gamma, c_0) \leftrightarrow \forall c \neg True(\lambda_\gamma, c)$.

By the left-to-right direction of (L5) and universal elimination, we have that:

(L6) $True(\lambda_\gamma, c_0) \rightarrow \neg True(\lambda_\gamma, c_0)$.

By propositional logic, (L6) entails that:

(L7) $\neg True(\lambda_\gamma, c_0)$.

Since c_0 is arbitrary, by universal introduction on (L7), we have that:

(L8) $\forall c \neg True(\lambda_\gamma, c)$.

By biconditional elimination on (L5) and (L8), we have that:

(L9) $True(\lambda_\gamma, c_0)$.

But (L7) and (L9) contradict each other. So we conclude, by *reductio*, that:

(L10) $\neg\exists\varphi Says(\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner, \varphi, c_0)$.

Since c_0 is arbitrary, by universal introduction on (L10), we have that:

(L11) $\forall c\neg\exists\varphi Says(\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner, \varphi, c)$.

Recall Bacon's suggestion that contextualist theories should take *Healthy* to mean "every utterance expresses a proposition" (or "says something in every context") so that *Diseased* means "not every utterance expresses a proposition" (or "does not say something in every context"). I think this suggestion is odd, because it seems to render healthiness too demanding a notion, and correlatively, disease too undemanding a notion. Plausibly, some healthy sentences, such as "She is in the room," do not say something in *every* context, simply because not every context supplies the required referents of context-sensitive expressions. It seems that healthy sentences only need to say something in *some* context, and correlatively, that diseased sentences need to not say something in any context. Accordingly, I suggest modifying Bacon's suggestion for my purposes, so that *Healthy* means "says something in some context," and that *Diseased* means "does not say something in any context":

(H) $Healthy(\sigma) := \exists c\exists\psi Says(\sigma, \psi, c)$.

(D) $Diseased(\sigma) := \neg\exists c\exists\psi Says(\sigma, \psi, c)$.

From (L11), we conclude that λ_γ is not healthy, that is, it is diseased:

(BR) $Diseased(\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner)$.

Now, we have the revenge paradox. Below, c_0 is an arbitrary context and each (Bi') formalizes each (Bi), respectively, in Bacon's argument:

(B1′) $True(\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner, c_0) \to \exists\varphi Says(\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner, \varphi, c_0)$.

(B2′) $True(\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner, c_0) \wedge Says(\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner, \varphi, c_0) \to \varphi$.

(B3′) $\exists\varphi Says(\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner, \varphi, c_0) \to Says(\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner, \forall c\neg True(\lambda_\gamma, c), c_0)$.

(B4′) $True(\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner, c_0) \to \forall c\neg True(\lambda_\gamma, c)$.

(B5′) $True(\lambda_\gamma, c_0) \to \forall c\neg True(\lambda_\gamma, c)$.

(B6′) $\forall c\neg True(\lambda_\gamma, c)$.

The ambiguity theorist, like the contextualist, should grant the premises: (B1′) is entailed by the relevant instance of (CBIV), (B2′) is entailed by the relevant instance of (ST), and (B3′) is entailed by the relevant instance of (L1). The conclusions then follow: (B4′) is propositionally entailed by (B1′), (B2′), and (B3′); (B5′) follows by identity elimination on (λ_γ) and (B4′); and (B6′) is classically entailed by (B5′).

In sum, we have as a theorem that λ_γ is not true in any context (equivalently, no utterance of λ_γ is true). Presumably, we want to express this result by asserting the sentence "λ_γ is not true in any context." However, we also have from (BR) that "λ_γ is not true in any context" is diseased, and so presumably unassertible.

So far, we have just rehearsed Bacon's anticontextualist argument in a more formal setting. The problem is that we have arrived at a conclusion that we presumably want to express, but we have also shown that the prime candidate we would like to assert in order to express that conclusion is diseased, and so presumably unassertible. Now, I want to show how the ambiguity theorist neutralizes the threat from his argument.

Although we have shown that there is *a* sense in which $\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner$ does not say anything in any context, it is also hard to deny that there is still *some* sense in which $\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner$ says something, and in particular, that λ_γ is not true in any context. For the ambiguity theorist, the response to this apparent quandary is straightforward. There *is* a sense in which $\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner$ says that λ_γ is not true in any context, but it is a distinct sense of "says"—one that extends the previous sense of "says."

Let *Says′* express this extended sense of saying, and consider some context c_1:

(A1) $Says'(\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner, \forall c\neg True(\lambda_\gamma, c), c_1)$.

By (B6′), what λ_γ says′ is true. So by an appropriate version of (SPT) involving *Says′* and *True′*, $\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner$ is true in c_1, but in a distinct sense *True′*:

(A2) $True'(\ulcorner\forall c\neg True(\lambda_\gamma, c)\urcorner, c_1)$.

I leave it open whether c_0 is the same context as c_1. What matters now is that we can express the truth of the claim that λ_γ is not true in any context, by asserting the sentence that is the prime candidate for expressing that very claim, provided that we grant that expressions such as "says" and "true" can have distinct senses.

On my understanding, an ambiguity theory is distinguished from contextualist theories in maintaining that the liar and associated revenge paradoxes involve shifts in *meaning*—not just shifts in context. Of course, on a sufficiently liberal conception of contexts, shifts in meaning are just shifts in context of a special kind. But, I take it, such a conception is at odds with a more standard conception, according to which shifts in meaning are not mere shifts in context. Throughout, I assume that shifts in meaning are not mere shifts in context. But in order to avoid a terminological dispute with one who insists otherwise, an ambiguity theory can if necessary be taken to be a theory that takes the paradoxes to involve shifts in context of a special kind.

2 Initial polysemy

In this section, I present and develop the polysemy proposal. The proposal is that "true" and related expressions, such as "says" and "false," are polysemous, where polysemy is ambiguity when the meanings are related. On the proposal, the polysemy of "says" is linked to the polysemy of "true" and "false" via versions of principles such as (ST) and (SF). The proposal comes in two parts. The first part of the proposal maintains that "true" is initially polysemous between *correspondence truth* and *disquotational truth*, and that this polysemy accounts for the shifts in the meanings of "says" and "true" in the contextual liar and associated revenge paradoxes we saw in the previous section. The second part of the proposal maintains that "true" is further polysemous between the meanings corresponding to the subconcepts of the concept *truth* generated by the indefinite extensibility of that concept. Crucially, such polysemy accounts for further shifts in the meanings of "says" and "true" in further revenge paradoxes.

My focus in this section is on the first part of the proposal. In the first subsection, I clarify my conceptions of vagueness, context sensitivity, and ambiguity. In the second subsection, I endorse suggestions that "true" is vague and context-sensitive. In the third subsection, I suggest that "true" is also ambiguous, and in particular, polysemous.

2.1 Vagueness, context sensitivity, and ambiguity

Throughout, let us identify "meaning"—when unqualified—with what is known as "conventional linguistic meaning," or "Kaplanian character." With this in mind, I adopt standard conceptions of vagueness, context sensitivity, and ambiguity.

Vagueness, context sensitivity, and ambiguity in general

My conception of ambiguity is a standard conception in philosophy. An ambiguous expression is characterized by the having of two or more meanings.[15] To take a paradigmatic example, the expression "bank" is ambiguous: "bank" has the meanings *financial institution* and *riverside*.

Ambiguity can be present at the level of presupposition. On standard understanding, a sentence of the form "The F is G" presupposes the existence of a unique F. Accordingly, in "The nearest bank is 15 minutes away," the ambiguous expression "bank" comes with the presupposition "there is a unique bank 15 minutes away." But since this presupposition is ambiguous between *there is a unique financial institution 15 minutes away*

[15] I focus on lexical ambiguity, which contrasts with syntactic ambiguity. A syntactically ambiguous expression is characterized by the having of distinct logical forms, due to, e.g., ambiguities in the scope of logical operators and quantifiers.

and *there is a unique riverside 15 minutes away*, "The nearest bank is 15 minutes away" is ambiguous between the meanings *the nearest financial institution is 15 minutes away* and *the nearest riverside is 15 minutes away*. On a natural understanding of the polysemy proposal, the polysemy of "true," which presupposes the saying of something, inherits the polysemy—which I take to be a kind of ambiguity—of "says."

Next, my conception of context sensitivity is the standard conception in the philosophical literature.[16] A context-sensitive expression is characterized by the having of at least one meaning that varies in content across contexts. In general, a meaning determines a content relative to context. A context-insensitive expression only has a constant meaning, which determines the same content relative to every context, while a context-sensitive expression has at least one variable meaning, which does not determine the same content relative to every context. To take a paradigmatic example, the expression "I" is context-sensitive: "I" has a single meaning that varies in the person it refers to across contexts, depending on the speaker. In this case, the content of the context-sensitive expression is just the extension, and specifically the referent.

Context sensitivity, on this conception, is distinct from a related phenomenon that involves something *like* context sensitivity, but is not, on the present conception, context sensitivity. The related phenomenon arises when an ambiguous expression's meanings are determined relative to context. For example, "Sierra went to the bank" exhibits this phenomenon: in a context where it is salient that Sierra withdrew cash, its meaning is just *Sierra went to the financial institution*; in a context where it is a salient that Sierra collected some samples of sand, its meaning is just *Sierra went to the riverside*; and in contexts that cannot resolve the ambiguity of "bank," its meanings are both.

Finally, my conception of vagueness is the standard conception in the philosophical literature.[17] On this conception, a vague expression is characterized by the having of borderline cases where it is indeterminate whether the cases fall into the extension

[16] See Kaplan (1989) for a standard conception of context sensitivity.

[17] See Williamson (1994b); Keefe and Smith (1996); Keefe (2000); Graff and Williamson (2002); Dietz and Moruzzi (2010) for standard conceptions of vagueness.

of the expression. To take a paradigmatic example, the expression "bald" is vague: it is indeterminate whether "bald" applies to Kim, who has 400 hairs.[18] To take another paradigmatic example, the expression "tall" is vague: it is indeterminate whether "tall" applies to someone who is 5' 8".

Various accounts of vagueness differ on the kind of indeterminacy involved, in taking the indeterminacy to be either semantic, epistemic, or metaphysical. I only mention the two accounts that concern me most, because they somehow maintain classical logic. These two accounts take the indeterminacy to be epistemic and semantic, respectively.

The epistemicist account maintains that the indeterminacy is merely epistemic.[19] Accordingly, it is indeterminate whether "bald" applies to Kim, who has 400 hairs, merely because we do not—and perhaps cannot—know whether it applies or not. There is nonetheless a fact of the matter as to whether it applies or not. A distinctive feature of the epistemicist account is that it maintains both classical logic and bivalence.

The supervaluational account, by contrast, maintains that the indeterminacy is semantic.[20] Accordingly, it is indeterminate whether "bald" applies to Kim because the linguistic usage of "bald" does not settle whether or not it applies. There is no fact of the matter as to whether Kim is bald. A distinctive feature of the supervaluational account is that it maintains classical logic, in that its theorems are all and only the theorems of classical logic. On the account, supertruth is truth on all precisifications, where a precisification assigns every sentence to either truth or falsity, and *mutatis mutandis* for superfalsity; intuitively, a precisification is conservative, in the sense of agreeing in truth value, exactly with respect to clear cases. Truth is identified with supertruth, and falsity is identified with superfalsity.[21] Given this, and the claim that vague sentences are neither supertrue nor superfalse, the account takes sentences to be either true, false, or indeterminate. Accordingly, the account denies bivalence.

[18] Here and elsewhere, I abstract away from context sensitivity when it does not affect the point.
[19] See Williamson (1994b), whose epistemicist account I focus on. Also see Sorensen (2001).
[20] See Fine (1975); Lewis (1982); McGee and McLaughlin (1994); Keefe (2000, 2008).
[21] At least for the most part. McGee and McLaughlin (1994) do not do this, and instead identify "determinate" truth with supertruth, and "determinate" falsity with superfalsity.

Given my focus on classical solutions, I focus on the accounts of vagueness that maintain classical logic, namely, the epistemicist and supervaluational accounts. I assume for the purposes of this chapter the epistemicist account, for two reasons. First, while both accounts somehow maintain classical logic, only the epistemicist account maintains classical logic in a robust manner. The supervaluational account maintains classical logic in that its theorems are all and only the theorems of classical logic. But, as it has been pointed out, the account does not maintain all the rules of inference of classical logic: given the addition of a determinacy operator, which supervaluationists should grant, the classically valid rules that are negation introduction, conditional introduction, and contraposition are invalid.[22] Second, the principles I introduced in the previous section include bivalence, but the supervaluational account rejects bivalence.

Vagueness, context sensitivity, and ambiguity are distinct phenomena. But they are not mutually exclusive. Plausibly, many ordinary expressions exhibit two or more of these phenomena. The expression "big" is ambiguous between the meanings *big in size* and *large in influence*, vague (in admitting of borderline cases relative to each meaning), and context-sensitive (since each meaning is variable).

Homonymy and polysemy as kinds of ambiguity

Following Aristotle, and a standard conception, I take homonymy and polysemy to be kinds of ambiguity.[23] The difference between them depends on the relatedness of the multiple meanings: with homonymy, the multiple meanings are unrelated, but with polysemy, the meanings are related.[24] Thus, the proposal that "true" and related expressions are polysemous refines the proposal that they are ambiguous.

Granted, whether or not meanings are related can be a vague matter. But just as the presence of borderline cases of tallness should not yield skepticism about there being

[22] See McGee (1990); Williamson (1994b).

[23] See Aristotle (1984), especially the *Categories*, as well as the *Metaphysics*.

[24] In principle, on this characterization, expressions may be both homonymous and polysemous, since they may have some unrelated meanings and some related meanings. This is, as Vetter and Viebahn (2016) suggest, unproblematic. But in practice, I focus on only certain meanings of interest, so that it is appropriate to ask whether an expression is *either* homonymous or polysemous.

a genuine distinction between tallness and non-tallness, so the presence of borderline cases of polysemy should not yield skepticism about there being a genuine distinction between homonymy and polysemy. The expression "bank" is homonymous between the meanings *financial institution* and *riverside*, while the expression "book" is polysemous between the meanings *concrete book made of paper* and *abstract work of literature*.

Further, certain relations that are characteristic of the relations between the meanings of polysemous expressions suggest that relations between meanings are discernible. Vetter and Viebahn (2016) suggest that the meanings of polysemous rather than homonymous expressions exhibit one or more of the following relations: constitutive relations between what constitutes something and the thing itself (e.g., with "paper," between *newspaper* and *paper*); causal relations between cause and effect (e.g., with "cut," between *the activity of cutting* and *the result of cutting*); instantiating relations between concrete tokens and abstract types (e.g., with "letter," between *the letter inscribed on a page* and *the letter type*); metaphorical extension relations between original literal meanings and extended meanings (e.g., with "see," between *visual perception* and *intellectual understanding*); and pragmatic strengthening relations between original implicatures and new literal meanings (e.g., with "since," between *temporally prior to* and *because*).

Representing vagueness, context sensitivity, and ambiguity

I focus on representations of vagueness, context sensitivity, and ambiguity in first-order languages, where expressions in a formal language are stipulated to correspond to exactly one meaning, one concept, and one property. Restricting attention to predicates, since they are what concern me, the idea is to distinguish between the predicate in natural language, the meanings the predicate in natural language corresponds to, and the predicates in a formal language that formally represent these meanings.

Take ambiguity first. Following a standard representation, I represent the ambiguity of a predicate P in natural language by taking it to correspond to multiple meanings m_i for each i in an index set M, where each m_i in turn corresponds to a predicate Φ_i in a formal language, and each Φ_i formally represents one of the multiple meanings of P. Accordingly, "bank" corresponds to the meanings $bank_1$ (= *financial institution*) as well as $bank_2$ (= *riverside*), where $bank_1$ corresponds to $Bank_1$ in a formal language and $bank_2$ corresponds to $Bank_2$ in a formal language.

Next, take context sensitivity. Following a standard representation, I represent the context sensitivity of an n-ary predicate P in natural language by taking it to correspond to a variable meaning m, where m in turn corresponds to an $n+1$-ary predicate Φ in a formal language, and Φ formally represents m and has an extra argument place for a contextual parameter. This is what we have with "true": the context-sensitive unary predicate "true" of sentences of English corresponds to, say, a meaning *true(c)*, which in turn corresponds to the binary predicate *True* of pairs of sentences and contexts.

Finally, take the epistemicist account of vagueness. Following a standard representation of vagueness on the account, I represent the vagueness of a predicate P in natural language by taking it to correspond to a meaning m, where m in turn corresponds to a predicate Φ in a formal language. The indeterminacy due to vagueness is then the indeterminacy in whether, for any meaning m, P corresponds to m. For the epistemicist, the indeterminacy is ignorance. Accordingly, "bald" may in fact correspond to $bald_{439}$, where the subscript indicates the cutoff point for its jointly exhaustive extension and antiextension, but it is indeterminate whether "bald" corresponds to $bald_{439}$ or whether it corresponds to $bald_{438}$, or some other meaning.

2.2 'True' as vague and context-sensitive

I endorse the suggestion in the literature that "true" is vague.[25]

The suggestion is *prima facie* plausible, since "true" seems to have borderline cases where it is indeterminate whether it applies. Liar-like sentences, such as λ and λ_γ, are such borderline cases. The indeterminacy exhibits the kind of indeterminacy we find in paradigmatic examples of vagueness. For consider that, where Kim is a borderline case

[25] See McGee (1989, 1990); Tappenden (1993); Field (2003, 2008). But see Maudlin (2004, 194) for a dissenting view.

of "bald," we have some inclination to judge that "Kim is bald" is true and some inclination to judge that it is false, but we are reluctant to outright judge that it is either true or false. At least some suggest that vagueness results in such inclinations.[26] And it is uncontroversial that vagueness results in such reluctance. Similarly, it seems to me, even relative to context, we have some inclination to judge that liar-like sentences are true and some inclination to judge that they are false, but we are reluctant to outright judge that they are either true or false. Indeed, these inclinations and judgments are taken to be data that gappy and glutty theories of the semantic paradoxes seek to account for.

A related argument provides independent motivation for the suggestion that "true" is vague. The idea is that "true" inherits the vagueness of vague predicates occurring in sentences.[27] In general, given that P is a vague predicate with the borderline case o, so that "o is P" is vague, "'o is P' is true" is also vague. To illustrate, suppose Kim is a borderline case of "bald." Plausibly, "true" inherits the vagueness of "bald": since "Kim is bald" is vague, "'Kim is bald' is true" is also vague. Vague predicates other than "bald" illustrate the same point.

In addition to endorsing the suggestion in the literature that "true" is vague, I also endorse the more controversial suggestion in the literature that "true" is context-sensitive.[28] Notably, Burge (1979) maintains that "true" is an indexical, much like "I." Meanwhile, Glanzberg (2001) maintains that the predicate "true" of sentences is context-sensitive in that the truth of a given sentence depends on there being a true proposition that the sentence expresses, but it is a context-sensitive matter what propositions are available to be expressed and what proposition is expressed.[29]

There is independent motivation for maintaining the suggestion. Recall from Section 1.3 that whether or not a sentence is true can depend on the context, since whether or not a given sentence says anything in the first place depends on the context. For ex-

[26] See Wright (1987); Sainsbury (1990); Shapiro (2003); Wright (2003b); Shapiro (2006).

[27] Versions of this argument are in McGee (1990, 216–217) and Williamson (1997, 947–948).

[28] See Parsons (1974); Burge (1979); Barwise and Etchemendy (1987); Gaifman (1992); Koons (1992); Simmons (1993); Glanzberg (2001, 2004a,c). But see Gauker (2006); Zardini (2008) for dissenting views.

[29] Glanzberg (2001, 239) maintains that "for any context, there [is] another context relative to which...there are...more propositions available."

ample, "He is happy" does not say anything in a context where there is no one available to be the referent of the pronoun "he," but says something when there is such a referent available. And even granting that a given sentence says something, what it says in particular can also depend on the context. For example, granting that "I am happy" says something, what it says in a context where I am the speaker is that Andy is happy, but what it says in a context where Shivani is the speaker is that Shivani is happy.

Moreover, that "true" is vague suggests that "true" is also context-sensitive. On several prominent accounts of vagueness, vagueness is often accompanied by context sensitivity.[30] The suggestion is at least initially plausible. Given that Adam is ordinarily a borderline case of "tall," the sentence "Adam is tall" can be true in a context where young children are salient but false in a context where basketball players are salient. Similarly, then, it is plausible that, given that "true" is vague, it is also context-sensitive.

2.3 'True' as ambiguous and in particular polysemous

I additionally suggest that the semantic paradoxes show that "true" is also ambiguous, and in particular, polysemous. This controversial suggestion has precedent in the literature, but is rarely enthusiastically endorsed. Tarski (1944) was among the first to make the suggestion, on the basis of the semantic paradoxes:[31]

> The word "*true*," like other words from our everyday language, is certainly not unambiguous. ...In works and discussions of philosophers we meet many different conceptions of truth and falsity, and we must indicate which conception will be the basis of our discussion. [emphasis in the original] (342)

Granted, many view the ambiguity Tarski posits as a cost to be avoided. Still, similar suggestions to the effect that "true" is ambiguous have also been made on the basis of the soritical paradoxes, as well as considerations unrelated to the paradoxes.

Notably, Field, McGee, and McLaughlin (henceforth, "FMM") suggest that the soritical paradoxes show that there are two ordinary concepts of truth—namely, a disquo-

[30]See Lewis (1979); Raffman (1994); Soames (1999); Graff (2000); Shapiro (2006).
[31]Also see Tarski (1983).

tational concept ("truth") and a correspondence concept ("definite truth" or "determinate truth").[32] However, McGee and McLaughlin (1994) stop short of characterizing "true" as ambiguous, and even distance themselves from the characterization:

> We see no reason to believe [that ordinary usage is ambiguous between two different uses of 'true.'] Instead, there is a single notion of truth whose use is governed by conflicting principles. The correspondence principle and the disquotation principle are both integral components of our ordinary usage of 'true'. (216).

In my view, their characterization as quoted risks collapsing their view into an inconsistency theory of truth, according to which truth is governed by inconsistent principles. But, if radicalness is the worry, an inconsistency theory is probably more radical than an ambiguity theory. So I prefer to be more open about characterizing the existence of two ordinary concepts of truth in terms of ambiguity.[33] Once we grant that "true" expresses distinct concepts, we should also grant that it is ambiguous. After all, the result follows from the minimal—and apparently innocuous—assumption that an expression that expresses a concept accordingly means something corresponding to that concept.

Unconcerned with the paradoxes, Lynch (2006); Kölbel (2008, 2013) suggest that "true" is ambiguous, also in that it expresses both a disquotational concept and a correspondence concept, with the idea being that concepts of truth can be more or less deflationary or inflationary. Meanwhile, Williamson (1998) does not explicitly suggest that "true" is ambiguous, but as I explain shortly, I think he comes close to making the suggestion.

Why maintain that "true" is ambiguous? A key motivation arises from the consideration that just as the vagueness of "true" suggests the context sensitivity of "true," so the vagueness of "true" also suggests the ambiguity of "true." But then, given that "true" is vague, it may also be ambiguous.

Recall that on the epistemicist account, it is indeterminate whether "bald" applies to Kim because we do not—and perhaps cannot—know whether or not it applies. Vagueness is due to ignorance. But on the face of it, the claim of ignorance demands more

[32]See Field (1986, 1994a, 2001); McGee and McLaughlin (1994, 2000); McGee (1990, 1993, 2005). As FMM have roughly the same view on the matter, I treat their views as one.

[33]McGee (2006, 190) is more receptive to the characterization in terms of ambiguity.

explanation. There are many cases where we do not know whether a given predicate applies to some object, and so we are ignorant, but the predicate is not vague. For consider that I do not know whether the predicate "divisible by three without remainder" applies to the number of people on earth right now, because I am ignorant of the number of people there are on earth right now, but the predicate is not vague.

To distinguish between indeterminacy due to ordinary ignorance and indeterminacy due to the kind of ignorance that is distinctive of vagueness, epistemicists such as Williamson maintain that vague expressions are *semantically plastic*, in that slight and unnoticed—and perhaps unnoticeable—changes in facts about use induce slight and unnoticed changes in meaning.[34] Accordingly, what we are ignorant of with vague predicates is what specifically they mean. We do not know whether the predicate "bald" means $bald_{439}$, $bald_{438}$, or something else, even though there is a fact of the matter as to whether or not "bald" means $bald_i$ for any natural number i.

The epistemicist account almost maintains that "true" is ambiguous, and—given the presumed relatedness of the various meanings one cannot distinguish between—in particular polysemous. A gap is that, according to the account, "true" *could* easily mean something different from what it actually means, but it does not follow from this that "true" *actually* means more than one thing. Still, I think this gap can be closed.

Recall the contextual liar and associated revenge paradoxes as I presented them in Section 1.3. In an initial sense of "says" and associated sense of "true," λ_γ does not say in a given context that it is not true in any context; it does not say in that context anything at all. In these initial senses of "says" and "true," λ_γ is not true in any context. But in an extended sense of "says," λ_γ says that it is not true in any context, and in the sense of "true" associated with the extended sense of "says," λ_γ is true in some context. While there is a sense of "true" on which λ_γ is true in a context, and there is a sense of "true" on which it is not true in a context, the senses are distinct.

Now, for there to be ambiguity, it must be that "true" and related expressions al-

[34] See Williamson (1997); Hawthorne (2006); Dorr and Hawthorne (2014b) on semantic plasticity. Also see Larsson (2006) for discussion in the linguistics literature.

ready have multiple meanings that they can shift between. Is there independent motivation for maintaining this? I submit that there is. Plausibly, "true" has had slightly different meanings over time, and now retains two or more of these meanings, and similarly for related expressions. Which meanings, specifically?

I suggest exploiting FMM's suggestion, made independently on the basis of the soritical paradoxes, that "true" is ambiguous in that it expresses both a disquotational concept and a correspondence concept.[35] As they formulate the distinction, *disquotational truth* characterizes the truth of sentences by some disquotational principle, such as (T) or (ST). By contrast, *correspondence truth* characterizes the truth of sentences in terms of the having of truth conditions that are established by the activities—thoughts, actions, and speech acts—of the speakers of the language; in particular, a true sentence is true in virtue of the activities of the speakers together with nonlinguistic reality.

The concepts *correspondence truth* and *disquotational truth* are equally legitimate: McGee and McLaughlin (1994, 217) write that "[b]oth the disquotation principle and the correspondence principle are integral to our ordinary usage of the word 'true.'" The concepts serve different purposes. The correspondence concept captures the intuition that the truth of sentences is grounded in nonlinguistic reality. Meanwhile, the disquotational concept is an expressively useful logico-linguistic device for expressing infinite conjunctions and a device for expressing the acceptance of one or more of what Field (1994b) calls "factually defective" utterances—involving vague expressions, nonreferring terms, conditionals, or ethical and aesthetic judgments—when the correspondence concept might be inappropriate. McGee (2005, 77–78) suggests that the starkest difference between the two concepts arises when we consider a pair of contradictory vague sentences, such as "Kim is bald" and "Kim is not bald," where Kim is a borderline case of "bald." According to McGee, one of the two sentences is disquotationally true, but neither is correspondence true.[36]

[35] Williamson (2004), who defends epistemicism, maintains that the *soritical* paradoxes do not suffice to motivate following FMM in distinguishing between the disquotational and correspondence concepts. But perhaps the *semantic* paradoxes suffice to do so.

[36] The claim that every sentence (that expresses a proposition) is either true or has a true negation en-

Notably, McGee (1990) suggests the distinction on the basis of the semantic paradoxes as well. The idea is that correspondence truth is what is sometimes also called "determinate truth," where determinate truth is truth in the minimal fixed point of Kripke (1975)'s now standard method for constructing a self-applicable truth predicate—i.e., a truth predicate that can apply to sentences with occurrences of that same predicate. Correspondence truth also resembles Herzberger (1970)'s conception of grounded truth, where groundedness consists in being "nonsemantic" (149).[37]

Crucially, I suggest exploiting the suggestion that "true" is ambiguous, and in particular polysemous, between *correspondence truth* and *disquotational truth* in order to motivate the ambiguity required to solve the contextual liar and associated revenge paradox. The ambiguity is polysemy, because despite being distinct concepts, *correspondence truth* and *disquotational truth* are closely related: as McGee (2005, 73, 78) suggests, they largely coincide, and the distinction between them is hard to discern. The polysemy helps explain why the shifts in "true," as one goes through the contextual liar and associated revenge paradoxes, are slight and unnoticed.

Specifically, "true" goes from meaning *correspondence truth* to meaning *disquotational truth*. Perhaps "true" starts off meaning *correspondence truth*, because this meaning is more natural—in the Lewisian sense—and so is the default concept.[38] But then "true" goes on to mean *disquotational truth*, because the arrival at contradiction when considering the correspondence concept leads us, by charity considerations, to move to the disquotational concept; charity inclines us to interpret expressions as meaningful and

tails bivalence on the minimal assumption that the truth of the negation entails the falsity of the negand. McGee's idea that *disquotational truth* but not *correspondence truth* is bivalent is one that I return to shortly.

[37]Indeed, the proposal that distinct senses of "true" are associated with distinct senses of "says" maps onto something like Herzberger's distinction between the determination of a statement and the expression of a proposition (152). For him, the former is more stringent than the latter, in that a statement is both meaningful and truth-evaluable, but a proposition can be meaningful but non-truth-evaluable. On the polysemy proposal, the initial sense of "says" according to which λ_γ does not express a proposition captures Herzberger's characterization of liar-like sentences as not determining a statement, while the extended sense of "says" according to which λ_γ expresses a proposition captures his characterization of liar-like sentences as expressing a proposition.

[38]On the Lewisian conception of naturalness, certain properties and relations are more objectively natural, and so more eligible to be the meanings of expressions; see Lewis (1986). See Dorr and Hawthorne (2014a) for discussion.

in particular true.[39] In the correspondence sense of "true," the contextual liar sentence does not say anything true. But in the disquotational sense of "true," the contextual liar sentence does say something true. The suggestion here is in line with the thought that *correspondence truth* is more stringent, and more tightly bound up with the activities of speakers and nonlinguistic reality, than *disquotational truth*.

How does my conception of the distinction between *correspondence truth* and *disquotational truth* compare to FMM's? Ultimately, I do not think it is that important to what extent the two conceptions are the same. What matters for my purposes is that the distinction between these two concepts of truth is independently motivated in the literature, where *correspondence truth* is more stringent a concept than *disquotational truth*. On my conception at least, the former is extended by the latter.

Still, let me note two ways in which my conception of the distinction differs from FMM's. First, I maintain that *correspondence truth* is more basic than *disquotational truth*, in that the former concept is properly extended by the latter concept, which is derivative.[40] I take the hierarchy of concepts of truth to be a more Tarskian one, in that the concepts of truth become weaker and weaker—more and more sentences count as being true. Thanks to classical construals of Kripke's fixed point construction, this is compatible with a concept of truth with a high degree of self applicability.[41] By contrast, FMM seem to maintain that *disquotational truth* is more basic than *correspondence truth*. Notably, Field (2008, 222, 242) suggests a hierarchy of concepts of truth where the concepts of truth become stronger and stronger—going from *disquotational truth* to *determinate truth*—to form a "reverse" Tarski hierarchy.[42]

Second, I maintain that both *correspondence truth* and *disquotational truth* are bivalent

[39]See Lewis (1975) for related discussion.

[40]For what it is worth, the etymology of "truth" indicates that the expression originally—from its Old English origins—suggested faithfulness and loyalty, in line with the correspondence concept. Only from the 1560s onwards did the expression come to suggest accuracy and correctness, in line with the disquotational concept. See Harper (2016).

[41]Classical construals consider the "closed off" Kripke construction, where all the sentences not in the extension of the truth predicate are thrown into the antiextension of the truth predicate. See Kripke (1975); McGee (1990); Halbach (1997); Glanzberg (2001, 2004a,c) for discussion.

[42]Field (2008), like many others, writes of truth "predicates," but I prefer to formulate the discussion in terms of concepts of truth, which is what truth predicates are meant to correspond to.

concepts, in that they either apply or fail to apply to a given object. Given epistemicism, the activities of speakers and nonlinguistic reality can still determine a sharp extension for "true."[43] By contrast, FMM maintain that *correspondence truth* is a nonbivalent concept, purportedly because the activities of speakers and nonlinguistic reality can fail to make even a sentence that says something either true or false. Still, there is a natural way for me to translate what FMM might maintain about the truth or falsity of liar-like sentences in the correspondence sense. While FMM might maintain that liar-like sentences are neither true nor false in the correspondence sense, even though such sentences say something, I maintain that liar-like sentences are neither true nor false in the correspondence sense because such sentences do not say anything—compatibly with the bivalence of *correspondence truth*.

3 Further polysemy

In this section, I focus on the second part of the polysemy proposal. In the first subsection, I present further revenge paradoxes that motivate positing further shifts in meaning and so further polysemy. In the second subsection, I suggest that expressions can change in meaning over time. In the third, fourth, and fifth subsections, I explain how the indefinite extensibility of the concept *truth* generates further subconcepts of the concept *truth*, and so further meanings corresponding to such subconcepts that "true" is polysemous between.

3.1 Handling further revenge paradoxes

I have suggested that the polysemy of "true" between the meanings *correspondence truth* and *disquotational truth* provides motivation for an ambiguity solution to the contextual

[43]At least the *vagueness* of "true" does not threaten bivalence, even if the *ambiguity* of "true" does. But the ambiguity of "true," does not threaten bivalence in a robust manner, since presumably even the epistemicist maintains that the ambiguous sentence "Sierra went to the bank" is only bivalent in that its disambiguations are bivalent: "Sierra went to the bank" is not the proper subject of semantic theorizing; only its disambiguations "Sierra went to the bank$_1$" and "Sierra went to the bank$_2$" are.

liar and associated revenge paradoxes. But a recurring lesson of attempts to provide satisfying solutions to these paradoxes is that we should be on the lookout for further revenge paradoxes. Are there further revenge paradoxes? If so, how does the polysemy proposal handle them? To answer these questions, we should distinguish between two kinds of attempts to construct further revenge paradoxes. One kind of attempt involves a sentence that quantifies over all interpretations, or meanings. Another kind of attempt involves a sentence that involves a given interpretation, or meaning. I suggest, however, that the polysemy proposal can handle such further revenge paradoxes.

Quantificational interpretational revenge paradox

Suppose we can quantify over interpretations—of "says," "true," and so on—where we formally represent an interpretation of "says" as $Says_i$, "true" as $True_i$, and so on. And suppose we revise principles such as (ST) and (SF) to take this into account. One kind of attempt to construct a further revenge paradox considers all interpretations of "true." Consider the quantificational interpretational liar sentence λ_ι, which names the quantificational interpretational liar sentence "$\forall i \forall c \neg True_i(\lambda_\iota, c)$," and intuitively reads "λ_ι is not true in any context under any interpretation":

(λ_ι) $\lambda_\iota = \ulcorner \forall i \forall c \neg True_i(\lambda_\iota, c) \urcorner$.

The kind of attempt to construct a further revenge paradox being considered attempts to run an argument parallel to the one in the contextual revenge paradox. However, a crucial assumption fails. The problem is that $\ulcorner \forall i \forall c \neg True_i(\lambda_\gamma, c) \urcorner$ is not interpretation-insensitive. Perhaps $\ulcorner \forall c \neg True(\lambda_\gamma, c) \urcorner$ says something in some context under some interpretation, but $\ulcorner \forall i \forall c \neg True_i(\lambda_\gamma, c) \urcorner$ does not say that λ_ι is not true in any context under any interpretation. After all, $\ulcorner \forall i \forall c \neg True_i(\lambda_\gamma, c) \urcorner$ could say something quite different when we consider varying interpretations, or meanings. We have that the following assumption analogous to (L1) is false:

(I1) $\exists \varphi Says_i(\ulcorner \forall i \forall c \neg True_i(\lambda_\iota, c) \urcorner, \varphi, c) \to Says_i(\ulcorner \forall i \forall c \neg True_i(\lambda_\iota, c) \urcorner, \forall i \forall c \neg True_i(\lambda_\iota, c), c)$.

So the attempt to construct a further revenge paradox by considering the quantificational interpretational liar sentence fails.

Interpretational revenge paradox

Another, more potent, kind of attempt to construct a revenge paradox considers a given interpretation of “true.” It resembles the original contextual revenge paradox but for an interpretation of “says” and “true” that is not the initial one we considered. I suggested that the contextual liar sentence is not true, but is true′. But then we can consider a contextual liar sentence involving “true′” rather than “true,” and we can consider more liar-like sentences, but involving “true″,” “true‴,” and so on. If “true” is “correspondence true” and “true′” is “disquotationally true,” as I have implicitly been suggesting, then for the ambiguity solution to be sufficiently general, we need to consider further meanings, *truth″*, *truth‴*, *truth⁗*, and so on. Such meanings are *further* meanings, because they are more inclusive. It cannot be that, e.g., *truth″* is identical to *truth′′′′′*.

Is there any basis for considering such further meanings, or concepts? I submit that the answer is in the affirmative. Meanings of “true,” or the concepts expressed by “true,” can extend to become more inclusive. Thus, the multiple meanings of “true” that arise as we consider further iterations of the revenge paradox on new senses of “says,” “true,” and so on are related in ways that are characteristic of the meanings of polysemous expressions more generally—the multiple meanings are related by relations of metaphorical extension. The inspiration here is from Williamson (1998):

> Encountering a semantic paradox might prompt us to enlarge what we mean by ‘say’. We start with one set of correlative meanings for ‘say’, ‘true’ and ‘false’; we use them to construct a sentence that says nothing in that sense of ‘say’; but reflection on that sentence causes normal speakers to give ‘say’, ‘true’ and ‘false’ a new set of correlative meanings, much like the previous ones except that the sentence in question says something in the new sense of ‘say’; the process can be repeated indefinitely. Normal speakers are not aware of the change, just as they are not aware of many ordinary processes of gradual semantic change. They feel themselves to be going on in the same way, but they are not. They are not applying the old rule to a new case; they are applying a very slightly different rule. (15)

> [A] word such as 'true' stands for slightly different concepts (has slightly different meanings) in successive languages. We no more grasp a single super-concept that encapsulates all the concepts for which 'true' might come to stand than we can give a word a single super-meaning that encapsulates all its possible metaphorical extensions. (20)

I suggest that the multiple meanings engender polysemy, and thus ambiguity.[44] As I explain in the following subsections, general considerations involving semantic change suggest that expressions in general can acquire extended meanings over time.

3.2 Further polysemy from semantic change

On the polysemy proposal, further polysemy of "true" and related expressions arises due to widening and a process akin to metaphorical extension.[45] To support the proposal, I now motivate two claims. First, the shifts in meaning posited fit general patterns of semantic change—the gaining, losing, or other changing of meanings over time—and are otherwise unexceptional.[46] Second, speakers' apparent ignorance of the polysemy and shifts in meaning posited has much in common with speakers' apparent ignorance of polysemy and shifts in meaning more generally, and is similarly unexceptional.

The shifts in meaning fit general patterns of semantic change and are otherwise unexceptional. That expressions shift in meaning across communities and times is uncontroversial. "Awful" used to have the meaning *inspiring awe* but now has the meaning *very bad*, and "chip" has the meaning *fries* in British English but has the meaning *crisps* in American English. Bloomfield (1933, 426–427) provides a standard classification of the main kinds of semantic change—among them widening and metaphor. Straightforwardly, the shifts in meaning that take place in further revenge paradoxes involve some combination of widening and metaphor. The meanings of "say" and "true" become

[44]The positing of such ambiguity has earlier precedent in Russell and Whitehead (1910)'s "typical ambiguity" and Parsons (1974)'s "systematic ambiguity"; see Russell (1908); Quine (1937, 1938); Feferman (2004); Glanzberg (2004b, 2006); Hellman (2006); Lavine (2006). It echoes Herzberger (1970)'s claim that "[t]here is something schematic in the concept of truth, which requires filling in" (150).

[45]See Rakova (2003) for a view according to which an expression's many apparently metaphorical meanings are really literal meanings of an ambiguous expression.

[46]See Bloomfield (1933, 1987); Blank (1999); Traugott and Dasher (2002) for the linguistics literature on semantic change.

wider. The shifts also remind one of metaphorical extension: while liar-like sentences do not say anything in one sense of "say," and so are not true in the associated sense, in an extended sense of "say," they say something, and are true in the associated sense.[47]

Perhaps one grants that semantic change takes place, but suspects that it can only happen gradually over extended periods of time. But semantic change can happen over relatively short periods of time, almost instantaneously within a conversation, as well. That speakers ordinarily use expressions in a novel way is well-attested.[48] Clark and Clark (1979), who offer a host of examples where expressions that usually function as nouns are used innovatively to function as verbs, provide as two examples the novel meaning of "porch" in "to porch a newspaper" and "Houdini" in "to Houdini one's way out of a closet." Nunberg (1995) discusses examples of predicate transfer, where a given expression is used to refer to different kinds of things that are related by metaphorical extension. Examples include "I" in "I am parked out back," where "I" refers to the speaker's car rather than the speaker themselves, and "the ham sandwich" in "The ham sandwich is at table 7," where "the ham sandwich" refers to the customer who ordered the ham sandwich rather than the ham sandwich itself. Recanati (2004) also discusses the widening of a predicate's extension, offering the example of "swallow" in "The ATM swallowed my credit card." Similarly, Camp (2006) offers the examples of "bulldozer" in "Bill is a bulldozer" and "jail" in "My job is a jail."

Further, speakers' apparent ignorance of the polysemy and shifts in meaning posited is also unexceptional. Bloomfield (1987, 238–239) observes that shifts in meaning are often unnoticed and hard to notice:

> By far the most semantic changes...are unconscious shiftings of meaning...and are made independently and simultaneously by numerous speakers. It is only the observer afterwards looking back over history who sees that a change has taken place.

[47] See Lyons (1977); Cooper (1986); Sweetser (1990); Saka (2007) for the linguistics literature on semantic change arising from metaphorical extension.

[48] Notably, Barsalou (1983, 1993); Barsalou et al. (1993) discuss *ad hoc categorization*, Nerlich and Clarke (2001) discuss "'live polysemy' as it is created and exploited in linguistic interaction" (3), and Carston (2002) discusses "*ad hoc* concepts," which "refer to concepts that are constructed pragmatically by a hearer in the process of utterance comprehension" (322). Also see Pustejovsky (1995); Traugott and Dasher (2002); Falkum (2015).

And in discussing polysemy (“name transfer”), Blank (1999, 77) notes that:

> [S]peakers make transfers without being aware of it, because their knowledge about the limits of these concepts and the respective categories is momentarily or permanently blurred. Confusions of this kind happen every day to almost everybody.

The vagueness and thus semantic plasticity of “true” help explain why it is that speakers might be ignorant in the way posited.

3.3 Concepts and conceptual change

For the polysemy proposal to be sufficiently general, it must be that “true” can mean, in addition to *correspondence truth* and *disquotational truth*, further things. Granted, this is compatible with a view according to which meanings, or concepts, do not change. For those who are sympathetic to the thought that meanings and concepts are mind-independent abstract objects that do not change, “true” just means something slightly different, or expresses a slightly different concept, so that all that changes is the relation between the expression “true” and meanings, or concepts. I have no objection to the thought that “true” can come to mean something slightly different, or express a slightly different concept, as suggested. Indeed, I think this thought is correct. Still, the reader might share my suspicion that there is more to the story than this: the meanings or concepts themselves seem to change as well. For those who are sympathetic to this thought, let me explain how the relevant meanings or concepts might have changed and continue to change.

I suggest that the concept *truth* can extend further, so that “true” can acquire still wider extensions, or more inclusive meanings. I motivate this suggestion by exploiting suggestions in the literature to the effect that the concept *truth* is indefinitely extensible, in that no matter what its extension may be, it could have a wider extension. I suggest that the indefinite extensibility of *truth* generates further subconcepts of the concept *truth*, and so further meanings—corresponding to such subconcepts—“true” is polysemous between. To develop this suggestion, let me now say more about concepts and conceptual change. What I say about concepts suffice for my purposes, but does not purport to be anything like a complete account of concepts. For example, I say little about the individuation of concepts, and more generally hope that—despite not being uncontroversial—what I say is compatible with a range of views about concepts.[49]

Consider that we create, change, or destroy concrete objects, as well as social objects, such as countries and corporations (and perhaps fictional objects as well), without compromising their objective reality. On the "intersubjective" conception of the objectivity of concepts I assume, concepts are likewise objectively real, although mind-dependent in such a way that we can create, change, or destroy them.[50] This conception contrasts with the Platonic conception, which maintains that concepts are mind-independent abstract objects.[51] On the Platonic conception, we may acquire concepts but we do not create, change, or destroy concepts.[52]

Crucially, as I understand concepts, they can change. Concepts are flexible, in admitting of extension via a combination of widening and metaphorical extension. There are presumably other kinds of conceptual change as well, of course, since changes in concepts need not involve widening or metaphorical extension. But for reasons that should be clear, I focus on conceptual change due to extension.

For simplicity, I focus on individual episodes of conceptual change, where a concept undergoes a single change. I conceive of a concept C as a temporally extended continuant with extensions relative to past, present, or future times, where stages of C at a time t are *subconcepts* of C, and subconcepts are concepts.[53] Let us say that a concept C *changes*

[49]For some of the main accounts of concepts: see Fodor (1987) for the mental representations conception; see Dummett (1993) for the abilities conception; and see Peacocke (1992); Zalta (2001) for the abstract objects conception.

[50]See Perlman (2000, 405): "the concepts do undergo change: We build and rebuild them, and change them, both consciously and unconsciously." Also see Wilson (1982); Sainsbury and Tye (2001, 2012); Cocchiarella (2007); Carey (2009).

[51]See Moore (1899); Frege (1980a,c) on the Platonic conception.

[52]I add that the intersubjective conception is compatible with some kind of idiolectical conception of concepts as well, in that perhaps each individual who we might say has a given concept has their own copy of a shared concept, just as each individual who we might say has the same book has their own copy of the book. Perhaps each individual has a token of the concept type.

[53]See Sainsbury and Tye (2012, Chap. 3) for a conception of concepts as temporally extended continuants. More realistically, we should consider stages of C at *intervals* of time rather than instants of time. However, for simplicity, we can consider representative instants of time over intervals of time where

from time t to time t' iff its extension at t is distinct from its extension at t'.

I talk about concepts as though they are in time, but how seriously should we understand the temporal characterization of concepts? Skepticism about conceptual change (which I discuss in the next subsection) might suggest that the temporal characterization—if taken too literally—is objectionable. Still, I find it natural to take at least some concepts to change, or have changed, in time. Again, conceptual change is plausible on the intersubjective conception of the objectivity of concepts I am assuming.

Given the intricacy of the issues, I leave it open how seriously we should understand the temporal characterization of concepts. Perhaps the temporal characterization is dispensable in favor of an atemporal, quasi-logical characterization—much as the temporal characterization of the iterative conception of *set* is often taken to be dispensable in favor of an atemporal, quasi-logical characterization.[54] To render my discussion compatible with an atemporal characterization, I assume that the development of concepts in time mirrors the "logical" development of concepts, so that the extension (in the sense of expansion) of a concept over time mirrors the logical extension of that concept. This rules out the possibility of there being a mismatch so that a concept becomes more restricted over time even though the concept has logically extended, or *vice versa*.

I make three minimal assumptions about concepts and extensions. First, as is standard, I assume that extensions are sets. Perhaps extensions are instead classes or pluralities, but it is convenient to make claims about sets. Second, I assume that all concepts have extensions, and in particular, unique extensions at a given time. This assumption is more substantive, since one might maintain that certain—perhaps defective or vague—concepts have either no extension at all or multiple extensions, even relative to a time. Third, and relatedly, I assume that extensions are determinate, in that a given object either is in or is not in a given concept's extension at a time. This assumption is similarly more substantive, since one might maintain that certain—perhaps defective

extensions do not change.

[54] See Boolos (1971); Studd (2013).

or vague—concepts have indeterminate extensions.[55] Still, these last two assumptions are encouraged by my simplifying assumptions of classical logic and bivalence.

With this in mind, I suggest in what follows that the concept *truth* is both extended and extensible, and that "true," which expresses the concept, is polysemous between the meanings corresponding to the subconcepts of *truth*. The indefinite extensibility of *truth* helps ensure that there are some subconcepts—and thus concepts—of *truth* that go beyond just *correspondence truth* and *disquotational truth*, and that there are some merely potential subconcepts that await actualization by running through arguments for the indefinite extensibility of *truth*. Thus, "true," which expresses the concept *truth*, is polysemous between the meanings corresponding to the various subconcepts.

I develop this suggestion in the following subsections. In the next subsection, I consider extended concepts. Certain concepts are extended, where a concept is extended in that a past or present extension is more inclusive than a previous extension. Given an extended concept, the less inclusive, previous extension can be identified as the extension of a subconcept, while the more inclusive, subsequent extension can be identified as the extension of another subconcept. A key claim is that an expression that expresses an extended concept is polysemous between the meanings corresponding to the subconcepts. In the subsection after the next one, I consider extensible concepts in general and indefinitely extensible concepts in particular. Certain concepts are extensible, in that they can become extended. And certain concepts are indefinitely extensible, in that they can always become extended. Crucially, the indefinite extensibility of a given concept generates further subconcepts, and so further meanings corresponding to these subconcepts for an expression that expresses the concept to be polysemous between.

3.4 Extendedness and polysemy

I now consider the extendedness of certain concepts. I suggest that expressions that express extended concepts are polysemous between the meanings corresponding to the

[55] See Waismann (1968)'s "open texture" conception of concepts, according to which certain concepts do not have determinate extensions.

subconcepts of the extended concepts.

Extendedness

Throughout this chapter, let $t_@$ denote the present time. Let us say that a concept C is (properly) *extended* iff there is a time t earlier than a past or present time t' ($t' \leq t_@$) such that C's extension $X(C, t')$ at t' is (strictly) more inclusive than its extension $X(C, t)$ at t: $X(C, t) \subsetneq X(C, t')$.

I take the mathematical concept *number* to be a paradigmatic example of an extended concept. Granted, it is hard to know the full historical development of this concept. The etymology of the expression "number" provides at best an imperfect guide to the historical development of the concept, since the concept is presumably language-independent, whereas the etymology of the English expression "number" is tied to the historical development of the English language. The mathematical concept *number* might also deviate from the ordinary concept *number*. Nonetheless, the literature on mathematical concepts and the history of mathematics suggests that the mathematical concept *number* is an example of an extended concept.[56]

For simplicity, let me focus on a single episode of extension of the mathematical concept *number*, that from a previous concept that applied only to the real numbers to a subsequent concept that applied to the complex numbers more generally. At least as a historical matter, mathematicians initially only applied *number* to the real numbers, and only later applied it to the complex numbers. But how mathematicians applied, or intended to apply, the concept is one thing, and what the concept actually applied to is another thing. Here, I suggest that the matter had to do with the concept itself, not just mathematicians' applications of the concept: the concept initially only applied to the real numbers, so that it only later applied to the complex numbers. *Number* is extended, in that it went from having an extension comprising all and only the real numbers to having a more inclusive extension comprising all and only the complex numbers.

[56]See Hilbert (1983); Gödel (1990); Tait (2000); Buzaglo (2001); Mancosu (2009).

But in addition to *number*, there are at least two other concepts in play, which are more or less inclusive subconcepts of *number*. The first is the concept *real number*, and the second is the concept *complex number*.[57] The less inclusive extension of *number*, which comprises all and only the real numbers, can be identified with the extension of *real number*, which we can call the "pre-extension concept" *number*. Similarly, the more inclusive extension of *number*, which comprises all and only the complex numbers, can be identified with the extension of *complex number*, which we can call the "post-extension concept" *number*. The concepts *real number* and *complex number* are subconcepts of the concept *number*, in that they are stages of *number*.[58]

More generally, consider an extended concept C, which has had an extension more inclusive than a previous extension. The less inclusive, previous extension of C can be identified with the extension of a subconcept C_1 ("the pre-extension concept") of C, where C_1 is identical to an earlier stage of C. Similarly, the more inclusive, subsequent extension of C can be identified with the extension of a subconcept C_2 ("the post-extension concept") of C, where C_2 is identical to a later stage of C.

Skepticism about conceptual change and thus extendedness

In taking the concept *number* to be extended, and so have changed, I am taking conceptual change to be a genuine phenomenon. But doing so is not uncontroversial. Buzaglo (2001, 2) suggests that skepticism about conceptual change has been the dominant view about conceptual change in logic since Frege. Skepticism is found in Frege, as follows:

> [C]oncept-like constructions that are still in flux and have not yet received final and sharp boundaries, logic cannot recognise as concepts[.] (Frege, 2013, 71)
>
> A logical concept does not develop and it does not have a history[.] ...If we said instead 'history of attempts to grasp a concept' or 'history of the grasp of the concept', this would seem to me much more to the point; for a concept is something objective: we do not form it, nor does it form itself in us, but we seek to grasp it[.] (Frege, 1984, 133)

[57] A third concept in play is the concept *imaginary number*, but this does not concern us here.

[58] Buzaglo (2001, 5) suggests that "the range of a concept is not given all at once but is composed of stages that are connected in a treelike structure."

As Brown (1986, 302) puts it, "[m]any philosophers find the suggestion that concepts undergo change to be virtually unintelligible."

For skeptics about conceptual change, two alternative characterizations of apparent conceptual change are available, both inspired by Frege (1979, 1980a,c, 1984): "What is known as the history of concepts is really a history either of our knowledge of concepts or the meanings of words" (Frege, 1980a, VII). On the first alternative characterization, apparent conceptual change is best understood as mere change in belief about the relevant concept. According to this characterization, while the concept *number* has never changed, we went from believing (falsely) that the extension excluded the imaginary numbers to believing (truly) that the extension includes them. In discussing the apparent change of the concept *cardinal number* from having an extension comprising all and only the finite cardinal numbers to having a more inclusive extension comprising all and only the finite and transfinite cardinal numbers, Frege (1980a, 98) writes that "our concept of [cardinal] Number has from the outset covered infinite numbers as well." This suggests that, while the concept *cardinal number* has never changed, we went from believing (falsely) that the extension excluded the transfinite cardinal numbers to believing (truly) that the extension includes them.

On the second alternative characterization, apparent conceptual change is merely change in the concept expressed by an expression. As Frege (1979, 242) puts it:

> What suggests itself is that we should, as people say, extend our concepts. Of course this is an inexact way of speaking, for when you come down to it, we do not alter a concept; what we do rather is to associate a different concept with a concept-word or concept-sign—a concept to which the original concept is subordinate. The sense does not alter, nor does the sign, but the correlation between sign and sense is different. ...[I]nstead of providing old expressions or signs with new meanings, we [should] introduce wholly new signs for the new concepts we have introduced. But this is not usually what happens; we continue instead to use the same signs.

According to this characterization, the expression "number" merely went from expressing the concept *real number*, whose extension excluded the imaginary numbers, to expressing the concept *complex number*, whose extension includes the complex numbers.

Why be skeptical in the first place? Conceptual change certainly seems intelligible.

Of course, even if conceptual change is intelligible, perhaps it is still not a genuine phenomenon, because concepts are abstract objects, and abstract objects cannot change. Relatedly, perhaps concepts are objectively real, and this entails the unchangeability of concepts. For consider the concept *apple*. Surely, when children acquire this concept, they do not change the concept but only discover it. But, one might maintain, what holds of the concept *apple* holds for all concepts.

However, I do not find such skepticism compelling. First, it is unclear that abstract objects cannot change. If fictional objects, or social objects, are abstract objects, then insofar as they are changeable, abstract objects can change. Second, the objective reality of concepts does not entail the unchangeability of concepts. Social objects such as countries and corporations are presumably objectively real, but can change. Even the qualification that concepts are mind-independent does not help skepticism: the sun is mind-independent and as objectively real as anything can be, but it can and does change.

Further, I think the alternative characterizations of apparent conceptual change provide incomplete characterizations of conceptual change.

Consider the first alternative characterization, according to which apparent conceptual change is best understood as mere change in belief about the relevant concept. Granted, some apparent conceptual changes are merely changes in belief. Plausibly, the apparent change of the (legal) concept *person*, according to which its extension went from excluding women to including women, is merely change from the (false) belief that the extension excluded women to the belief that it included women.[59] But other apparent conceptual changes do not seem to be mere changes in belief. Consider the apparent conceptual narrowing of *meat*. Taking its etymology as a guide to the concept, the concept's extension went from including food that was not edible flesh to excluding such food.[60] It is implausible to characterize this apparent conceptual change as mere change from the belief that the extension included food that was not edible flesh to the

[59]In 1971, the U.S. Supreme Court ruled in *Reed v. Reed* that the U.S. Constitution's 14th Amendment's protection of persons' rights applies to women as well, because "persons" applies to women.

[60]See Harper (2016).

belief that the extension excluded such food. For doing so suggests that older applications of the concept to food that was not edible flesh were systematically mistaken.[61]

Now consider the second alternative characterization, according to which apparent conceptual change is best understood as mere change in the concept expressed by an expression. This characterization accommodates apparent conceptual changes that are not conceptual extensions, including the one with *meat*. But while I do not think that the characterization is mistaken in maintaining that conceptual change involves change in the concept expressed by an expression, I think that it is mistaken in maintaining that nothing changes in addition to the concept expressed by an expression.

Take an analogy involving the name "Alex" of the person Alex. Although "Alex" always picks out Alex, the person that is Alex can change. The view analogous to the alternative characterization of conceptual change being considered suggests, implausibly, that instead of Alex changing as well, all we have is that "Alex" goes from picking out an earlier timeslice of Alex to a later timeslice of his. Similarly, it seems, the problem with maintaining that all we have is that "number" goes from expressing *real number* to expressing *complex number* is that it fails to capture the straightforward sense in which there is continuity and unity in the concepts expressed. The change in what is expressed is not as wild as the change from having an expression stipulated to express *cup* to its expressing, again by stipulation, *rock*.[62]

Despite skepticism about conceptual change, I maintain that the phenomenon is genuine. The view that certain concepts change is not only intelligible, but natural and plausible. The concept *number* is extended, and so has changed.

[61] Sellars (1973, 89–92) suggests that *triangle* changed from from *Euclidean triangle* to *Riemannian triangle*, while *length* changed from *classical length* to *relativistic length*. Meanwhile, Brigandt (2004, 2010), suggests that *gene* changed from *classical gene* to *molecular gene*. His proposal resembles the one I am about to make: "the term 'gene'...can be viewed as corresponding to two different concepts at the same time" (Brigandt, 2004, 9); see also Field (1973, 476–477). Finally, focusing on *acid*, Thagard (1990) argues against the alternative characterization in terms of mere change in belief.

[62] Sellars (1973, 91–92) makes a similar point in arguing against skepticism about conceptual change.

Polysemy

Recall from earlier the suggestion that an extended concept C is associated with two subconcepts (or stages), C_1 and C_2, which are respectively the pre- and post-extension concepts. Crucially, a natural correspondence between concepts and meanings motivates taking expressions that express extended concepts to be polysemous between the meanings corresponding to the subconcepts of the extended concepts.[63] Given that "number" expresses the extended concept *number*, which has the subconcepts *real number* and *complex number*, "number" is polysemous between the meanings corresponding to the subconcepts *real number* and *complex number* of *number*. More generally, consider an extended concept C whose pre-extension concept is C_1 and post-extension concept is C_2, where C_1 and C_2 are subconcepts, or stages, of C. An expression e that expresses C is polysemous between the meanings corresponding to the subconcepts C_1 and C_2 of C.[64]

The present suggestion has precedent in the literature on conceptual change. Citing Feyerabend (1987, 269)'s claim that we can "change *concepts* while retaining the associated *words*" [emphasis in the original], Perlman (2000, 204) writes the following:

> If we understand both the old and new concepts, part of the way we explain their interrelation is by using the same words to describe them. ...[W]hen we merely change our use of a word...we signify the similarity by using the same word for both old and new, as in 'temperature', 'mass', etc.

Indeed, this quotation explains why "number" can continue to mean *real number*; *real number* is sufficiently similar to *complex number*.

As with ambiguous expressions more generally, polysemous expressions can be disambiguated. Context, theoretical interests, and practical interests help determine which meaning is relevant. Depending on the context, the sentence "There is a number x such that $x^2 = -1$" can be either true or false. In a context where "number" expresses the concept *real number*, the sentence is false; consider a context prior to the year 200 B.C.E.

[63] See Elbourne (2011); Glanzberg (2011) on the correspondence between concepts and meanings.

[64] Perhaps there are cases where, even given the conditions, e fails to mean something corresponding to C_1, because some subconcepts—stages—of C are unavailable as meanings. However, for simplicity, I set such cases aside in this chapter.

(before the first known broaching of the concept *imaginary number* by Heron of Alexandria sometime between 100 B.C.E. and 100 C.E.; see Roy 2007). But in a context where "number" expresses the concept *complex number*, the sentence is true; consider a context in a university-level course on complex analysis.

More examples

Besides the concept *number*, I consider two more examples from mathematics. Among others, Buzaglo (2001, 2) suggests that the contemporary concept *cardinal number*, which applies to the infinite cardinal numbers such as $\aleph_3$ and $\beth_5$ as well, extended a previous concept that only applied to the finite cardinal numbers—i.e., the natural numbers. For what it is worth, Gödel (1990, 254) writes that "the concept of '[cardinal] number' had been *extended* to infinite sets" [emphasis added].[65] Accordingly, "cardinal number" can express either of these concepts, and is polysemous between the corresponding meanings. Similarly, the contemporary concept *ordinal number*, which applies to the transfinite ordinals as well, extended a previous concept that only applied to the natural numbers. Accordingly, "ordinal number" can express either of these concepts, and is polysemous between the corresponding meanings.

Now, for some examples from philosophy. Citing cross-linguistic surveys, Vetter and Viebahn (2016) suggest that modals, such as "might" and "must," are polysemous between nonepistemic modalities and epistemic modalities, where the corresponding concepts expressed originated with the former and developed into the latter:

> [M]odals have (or used to have) a 'core' meaning, which is in each case a particular root meaning: ability for 'may' and 'can', obligation for 'must'. These core meanings provide the starting point for the historical development. This historical priority strongly suggests that there is explanatory priority as well: modals have certain epistemic meanings *because* they have certain root meanings in the first place. And indeed, the relation between these 'core' meanings and the derived (in particular, the epistemic) meanings can be classified in one (or both) of (at least) two ways that are typical of polysemy: as *metaphorical extension*, or as *pragmatic strenghtening [sic]*. [emphasis in the original] (16)

[65] See Mancosu (2009) for discussion of Cantor's extension of the concept *cardinal number* and alternative ways of extending the concept.

I close with three examples from ordinary life. I appeal to the etymologies of expressions as guides to the concepts the expressions express.[66] With each concept, an expression that expresses the concept is polysemous between the meanings corresponding to the subconcepts that are stages of the concept, where subconcepts are themselves concepts. First, the contemporary concept *crazy*, whose extension includes cool and exciting things as well, extended a previous concept that only applied to diseased, sickly people; the meaning *cool and exciting* of "crazy" was first recorded in 1927. Second, the contemporary concept *hierarchy*, whose extension includes rankings of governmental and military officials as well, extended a previous concept that only applied to the ranking of angels; the meaning *ranked organization of persons or things* of "hierarchy" was first recorded in the 1610s. Third, the contemporary concept *surf*, whose extension includes the moving from one internet site to another as well, extended a previous concept that only applied to the riding of the crest of a wave; the meaning *moving from one internet site to another* of "surf" was first recorded in 1993.

3.5 Extensibility, indefinite extensibility, and polysemy

Since actuality implies possibility, extended concepts were extensible. But what is it for a concept to be extensible? I now consider the extensibility of certain concepts in general, the indefinite extensibility of certain concepts in particular, and how extensibility generates new subconcepts and corresponding meanings, so that expressions that express the concepts become polysemous between these meanings.

Extensibility

Let us say that a concept C is (properly) *extensible* iff there is a future time t' ($t_@ < t'$) such that its extension $X(C, t')$ at that time is (strictly) more inclusive than its present extension $X(C, t_@)$: $X(C, t_@) \subsetneq X(C, t')$.

Since the concept *number* became extended, the concept was extensible just before

[66] See Harper (2016).

the episode of extension. What about after the episode of extension that brought the concept's extension to comprise the complex numbers? It turns out that even when *number* was extended to coincide with *complex number*, it could be extended further, to coincide with the subconcept *quaternion*. And even when *number* was extended to coincide with the subconcept *quarternion*, it could be extended further, to coincide with the subconcept *octonion*, and further still, to coincide with the subconcept *sedenion*. More generally, the Cayley-Dickinson construction shows that each subconcept can be extended further: where the complex numbers form a two-dimensional algebra over the field of real numbers, the quaternions form a two-dimensional algebra over the field of complex numbers, the octonions form a two-dimensional algebra over the field of quaternions, and so on. This suggests that the concept *number*—whose subconcepts, or stages, include *real number*, *complex number*, *quaternion*, and so on—remains extensible.

Crucially, I take extensibility to be genuinely expansive. The pre-extension concept is not a mere restriction of an antecedently given concept that is only purportedly the post-extension concept. Instead, the pre-extension concept can become a concept that genuinely extends the post-extension concept. In Aristotelian terminology, the extensibility is potential rather than actual. The distinction I have in mind between expansion and mere restriction resembles Fine (2006)'s distinction between "expansionist" and "restrictionist" conceptions of indefinite extensibility:[67]

> Under the restrictionist account, the old and new domains are also to be understood as restrictions; but these restrictions are not themselves to be understood as restrictions of some broader domain. Under the expansionist account, by contrast, the new domain is not to be understood as a restriction at all but as an expansion. What we are provided with is not a new way of seeing how the given domain might have been restricted but with a way of seeing how it might be expanded. We might say that the new domain is understood from 'above' under the ...restrictionist [account], in so far as it is understood as the restriction of a possibly broader domain, but that it is understood from 'below' under the expansionist account, in that it is understood as the expansion of a possibly narrower domain. (38)

Fine is concerned with how the indefinite extensibility of concepts—an apparent phe-

[67]Fine (2006)'s distinction has earlier precedent in Fine (2002)'s distinction between "creative" and "standard" definitions.

nomenon that I am about to take up—motivates the denial of the availability of unrestricted quantification, but in my view his distinction bears on the extensibility of concepts more generally. Just substitute "concept extension" for "domain extension," and "extension" for "domain." I generalize Fine's distinction to conceptions of extensibility.

What is crucial to the expansionist conception of extensibility is that prior to extension, a given concept does not apply to some object that, after the extension, the concept applies to, so that the post-extension subconcept has a more inclusive extension than the pre-extension subconcept.[68]

Indefinite extensibility

Let us say that a concept C is (properly) *indefinitely extensible* iff at all present or future times t' ($t_@ \geq t'$), there is a subsequent time t'' such that C's extension $X(C, t'')$ at t'' is (strictly) more inclusive than its extension $X(C, t')$ at t': $X(C, t') \subsetneq X(C, t'')$. Accordingly, an indefinitely extensible concept is a kind of necessarily extensible concept: any indefinitely extensible concept is also an extensible concept.[69]

Three remarks about this characterization of indefinite extensibility are in order. First, the temporal characterization is nonstandard, but again, might be dispensable in favor of a more standard, quasi-logical characterization.[70] I maintain the temporal characterization, because that is the more natural one when it comes to extensible concepts in general, and I want to emphasize the commonalities between extensible concepts in general and indefinitely extensible concepts in particular. Second, the temporal characterization might misleadingly suggest an upper bound on the extent of the indefinite extensibility of concepts, where the number of possible extensions is the number of possible times, i.e., $2^{\aleph}_0$—the cardinality of the continuum. But we should idealize and

[68] Although Fine (2006, 37–39) adds that expanded domains can include new objects, for my purposes, it suffices that the extensions of extended concepts include objects that the extensions, pre-extension, do not apply to; the objects need not be new.

[69] The characterization of an indefinitely extensible concept as a kind of necessarily extensible concept follows Fine (2006, 30)'s characterization.

[70] Given the quasi-temporal characterization of indefinite extensibility in Studd (2013), perhaps talk of extensions at future times can be understood in terms of talk of possible extensions where the modality is primitive or logico-mathematical, as in Fine (2006); Linnebo (2010); Studd (2013).

allow that there are many more possible times. Since the times are just indexing stages in the development of concepts, we need not take them—as indices—to be constrained by the metaphysics of the distribution of times, taken literally. Third, an indefinitely extensible concept is in practice often an extensible concept that can be systematically extended by means of a rule or argument. The provision of a general rule or argument to the effect that a concept is indefinitely extensible is probably the only way to demonstrate indefinite extensibility.

The concept most widely suggested to be an example of an indefinitely extensible concept has been the concept *set*. Other concepts suggested to be indefinitely extensible include the concepts *cardinal number*, *ordinal number*, *truth*, *language*, *interpretation*, *proposition*, *property*, and *object*. In what follows, I focus on the concepts *set* and *truth*.

Sets and indefinite extensibility

The suggestion that the concept *set* is indefinitely extensible is in Dummett (1993, 441) (who offers a slightly different formulation on p. 454):[71]

> An indefinitely extensible concept is one such that, if we can form a definite conception of a totality all of whose members fall under the concept, we can, by reference to that totality, characterize a larger totality all of whose members fall under it.

See Rayo and Uzquiano (2006); Studd (2013); Uzquiano (2015d) for recent discussions of the indefinite extensibility of *set*. To some extent, it remains somewhat controversial whether there are indefinitely extensible concepts, and so whether in particular the concept *set* is indefinitely extensible. I discuss skepticism about indefinite extensibility below, but I think it is plausible that there are indefinitely extensible concepts.

I focus on two arguments for the indefinite extensibility of the concept *set*. The first is a Cantorian argument, which appeals to Cantor's theorem:

Theorem (Cantor). *For any set A, the powerset $\wp(A)$—the set of all subsets of A—is strictly larger than A: $|A| < |\wp(A)|$.*

[71] Earlier suggestions are in Dummett (1981, 1991a), as well as Russell (1906).

Consider the extension $X_0 = X(S, t_0)$ of *set* ("S") at an arbitrary present or future time t_0. By Cantor's theorem, $\wp(X_0)$ is strictly larger than X_0. But every member of $\wp(X_0)$ is also a set. Letting X_1 be the set obtained by taking the union of X_0 with $\wp(X_0)$, we have that $X_1 = X(S, t_1)$ is the extension of *set* at a subsequent time t_1. Clearly, every set in X_0 is also a set in X_1, and there is at least one set in $\wp(X_0)$ that is not a set in X_0. So X_0 is a proper subset of X_1. Since t_0 is arbitrary, we have that at all present or future times t', there is a subsequent time t'' such that the extension of *set* at t'' is strictly more inclusive than the extension of *set* at t'.

The second argument for the indefinite extensibility of the concept *set* is a Russellian argument, which appeals to the set of all sets that satisfy a certain condition.[72] Consider the extension $X_0 = X(S, t_0)$ of *set* at an arbitrary present or future time t_0. Then consider the set $R = \{x \in X_0 \mid x \notin x\}$. Letting X_1 be the set obtained by taking the union of X_0 with $\{R\}$, we have that $X_1 = X(S, t_1)$ is the extension of *set* at a subsequent time t_1. Clearly, every set in X_0 is also a set in X_1, and R is not in X_0; for otherwise, $R \in R$ iff $R \notin R$, which is inconsistent.[73] So X_0 is a proper subset of X_1. Since t_0 is arbitrary, we have that at all present or future times t', there is a subsequent time t'' such that the extension of *set* at t'' is strictly more inclusive than the extension of *set* at t'.

Truth and indefinite extensibility

The concept *truth* is also indefinitely extensible. See Grim (1984, 1991); Williamson (1998); Cook (2008, 2009) for recent discussions of the indefinite extensibility of *truth*. Cook in particular suggests that the related concepts *truth value* and *language* are indefinitely extensible. I myself have argued, in Chapter 2, that the Russell-Myhill paradox of propositions and an iterative conception of propositions motivate taking the concept *propositional truth* to be indefinitely extensible.

Again, I focus on two arguments for the indefinite extensibility of the concept *truth*.

[72] See Dummett (1993, 441–443) for discussion of a version of this argument.

[73] Assume for *reductio* that $R \in X_0$. Now suppose that $R \in R$. Since $R \in X_0$ and $R \in R$, R does not satisfy the defining condition of R, so that $R \notin R$. But if $R \notin R$, then $R \in X_0$ and $R \notin R$. So R satisfies the defining condition of R, in which case $R \in R$.

These two arguments require two assumptions about the existence and fine-grainedness of truths, which are worth making explicit. Truths here may be propositions. In what follows, we assume the following two principles:[74]

(T1) For every set X, there is a truth $t = t(X)$ about X.

(T2) For distinct sets X_1 and X_2, the truth $t_1(X_1)$ about X_1 is distinct from the truth $t_2(X_2)$ about X_2.

According to (T1), there is a truth associated with every set. To motivate (T1), consider that we can take the truth $t = t(X)$ about X to be the truth that 0 is in X if 0 is in X and the truth that 0 is not in X if 0 is not in X. According to (T2), truths associated with distinct sets are themselves distinct. To motivate (T2), consider that truths are plausibly fine-grained enough so as to discriminate on the basis of sets that they are about. In my view, (T1) is very plausible, and (T2) is at least plausible enough for us to want to maintain it, all things equal. However, (T2) is more controversial; Uzquiano (2015b) concedes its plausibility but suggests surrendering it on the basis of the intensional paradoxes.

The first argument for the indefinite extensibility of *truth* is a Cantorian argument.[75] Consider the extension $X_0 = X(T, t_0)$ of *truth* ("T") at an arbitrary present or future time t_0. By Cantor's theorem, $\wp(X_0)$ is strictly larger than X_0. But by (T1) and (T2), every set in $\wp(X_0)$ is associated with a distinct truth. Let T be the set whose members are all and only the truths associated with a set in $\wp(X_0)$. Letting X_1 be the set obtained by taking the union of X_0 with T, we have $X_1 = X(T, t_1)$ is the extension of *truth* at a subsequent time t_1. Clearly, every truth in X_0 is also a truth in X_1, and there is at least one truth in T that is not a truth in X_0. So X_0 is a proper subset of X_1. Since t_0 is arbitrary, we have that at all present or future times t', there is a subsequent time t'' such that the extension of *truth* at t'' is strictly more inclusive than the extension of *truth* at t'.

The second argument is a Russellian argument.[76] Consider the extension $X_0 = X(T, t_1)$

[74] As Menzel (2012); Uzquiano (2015b) observe, the arguments can be formulated in terms of classes or pluralities rather than sets.

[75] See Grim (1991, 91–124) for discussion of a version of this argument.

[76] See Uzquiano (2015b, 339–340) for discussion of a version of this argument.

of *truth* at an arbitrary present or future time t'. Then consider the set $T^* = \{t \in X_0 \mid \text{for some } A \subseteq X_0, t = t(A) \text{ and } t \notin A\}$. Further, consider the truth $m = m(T^*)$ about T^*, whose existence (T1) guarantees. Letting X_1 be the set obtained by taking the union of X_0 with $\{m\}$, we have that $X_1 = X(T, t_1)$ is the extension of *truth* at a subsequent time t_1. Clearly, every truth in X_0 is also a truth in X_1, and m is not in X_0; for otherwise, $m \in T^*$ iff $m \notin T^*$, which is inconsistent.[77] So X_0 is a proper subset of X_1. Since t_0 is arbitrary, we have that at all present or future times t', there is a subsequent time t'' such that the extension of *truth* at t'' is strictly more inclusive than the extension of *truth* at t'.

One unified concept and many subconcepts

At this point, let me clarify my conception of indefinite extensibility. There are two apparently competing conceptions of indefinite extensibility. On what I call the "Dummettian conception," there is just one concept whose extension expands from pre-extension to post-extension. This conception is that in Dummett (1993), and is close to being the standard one. By contrast, on what I call the "Williamsonian conception," there are at least two concepts—the pre-extension concept and the post-extension concept. This conception is that in Williamson (1998).[78]

A reason to adopt the Williamsonian conception is that it readily accommodates classical logic and bivalence, whereas the Dummettian conception, which suggests that the relevant concept has an indeterminate extension, less readily accommodates classical logic and bivalence. Dummett (1993, 441) writes that sentences that quantify over objects falling under an indefinitely extensible concept lack "determinate truth-conditions," and adds that "[t]hey will not then satisfy the laws of classical logic." Similarly, Rayo and Uzquiano (2006) introduce indefinitely extensible concepts as ones that "are usually taken to be ones lacking definite extensions" (4), and later reiterate that "[m]ost pro-

[77] Assume for *reductio* that m is a member of X_0. Now suppose that $m \in T^*$. Then there is some subset $B \subseteq X_0$ such that $m = m(B)$ is about B and $m \notin B$. But since $m \in T^*$, we have that $B \neq T^*$. But then m is about distinct sets T^* and B, in violation of (T2). So $m \notin T^*$. But then $m = m(T^*)$, where $T^* \subseteq X_0$, and $m \notin T^*$. So m satisfies the defining condition of T^*, in which case $m \in T^*$.

[78] Also see Frege (1980c, 145, 148).

ponents of indefinite extensibility take…supplying a definite extension for the concept *set* [to be] impossible" (5). Meanwhile, a reason to adopt the Dummettian conception is that it more straightforwardly maintains that there is a unified concept, whereas the Williamsonian conception suggests that there are two or more concepts.

This suggests a dilemma whose horns are the Dummettian and Williamsonian conceptions. However, I suggest, the dilemma is a false one. The conception I adopt is a hybrid between these two: there is a unified concept as well as subconcepts that fall under that unified concept. There is a unified concept of truth, but there are also recognizable subconcepts—the concepts *correspondence truth* and *disquotational truth*, as well as the subconcepts generated by the arguments for the indefinite extensibility of *truth*. The unified concept of truth is coarser-grained, while the various subconcepts are finer-grained. Still, each of these concepts deserves the honorific "concept of truth." And given that the extensions of all these concepts are determinate, this hybrid conception is compatible with classical logic and bivalence.

On the hybrid conception I adopt, the unified concept of truth resembles what Glanzberg (2015) calls the "stratified concept of truth," where a stratified concept—what he elsewhere seems to identify with a "Kreiselian concept"—for him is a unified concept that resists a unified definition or theory (31); also see Glanzberg (2004a,c). For Glanzberg (2015, 34), stratifications correspond to "sub-concepts." In my view, we can grant that there are different subconcepts as well as a unified concept, where subconcepts are concepts, provided that we recognize such concepts as forming a unified cluster.

Skepticism about indefinite extensibility

Granted, one might be skeptical of the foregoing arguments for the indefinite extensibility of the concepts *set* and *truth*. Let me address three reasons for skepticism.

First, the arguments for the indefinite extensibility of *truth* assume the principles (T1) and (T2), but one might reject either assumption.

Second, the arguments assume that the concepts *set* and *truth* have an extension, but one might maintain that they have no extensions at all, because the assumption that they do yields contradiction. Where extensions are sets, the arguments for the indefinite extensibility of *set* assume that there is a set containing all and only the sets, but standard set theory proves that there is no such set. Similarly, the arguments for the indefinite extensibility of *truth* assume that there is a set containing all and only the truths, but perhaps a version of Tarski's theorem shows that there is no such set.[79]

Third, there is an expressibility worry to the effect that indefinite extensibility threatens absolute generality—the availability of unrestricted quantification and so the expressibility of general claims. In the case of sets, the indefinite extensibility of the concept *set* might suggest the unavailability of unrestricted quantification over all sets, and so the inexpressibility of general claims about all sets.[80] If, as suggested, we cannot unrestrictedly quantify over all sets, then it seems that we cannot express the intended generality of claims such as "Every set is self-identical." Similarly, in the case of truth, the indefinite extensibility of the concept *truth* might be taken to suggest the unavailability of unrestricted quantification, and so the inexpressibility of general claims. If we cannot unrestrictedly quantify over all truths, then it seems that we cannot express the intended generality of claims such as "Every truth is not false."

However, I suggest that we should not be skeptical. First, although one might reject (T1) or (T2), I think the onus is on one who does so to justify the rejection. They strike me as about as pretheoretically plausible as the naive comprehension principle for sets. Thus, just as one should justify the rejection of the naive comprehension principle for sets, one should justify the rejection of (T1) or (T2). Without such justification, one is entitled to assume them. Granted, just as one might take the need to solve the set-theoretic paradoxes to justify the rejection of the naive comprehension principle for sets, so one might take the need to solve the intensional paradoxes to justify the rejection of (T1) or (T2). However, such a justification is a theoretical one, and so should be compared to the rival suggestion that the need to solve the intensional paradoxes is

[79]See Grim (1991); Priest (1994) for set-theoretic versions of the liar paradox.
[80]See Rayo and Uzquiano (2006).

rather a justification for taking the concept *truth* to be indefinitely extensible. Indeed, I argued for this rival suggestion in Chapter 2.

Second, perhaps extensions need not be sets. If extensions are instead classes or pluralities, we might still be able to employ appropriate versions of the indefinite extensibility arguments, where these versions appeal to Bernays' theorem—which applies to classes and pluralities, and so generalizes Cantor's theorem—rather than Cantor's theorem. More importantly, even if there *is* no set of all and only the sets, there remains a strong intuition to the effect that there *could* be a set of all and only the sets. On the temporal characterization, even if at a given time t_0 there is no set of all and only the sets at t_0, there is a subsequent time t_1 where there is a set of all and only the sets at t_0. Following Dummett (1993, 440), Uzquiano (2015b, 1) writes the following:

> Absent some independent reason to doubt that we can collect the non-self-membered sets in the totality into a set, to claim that there is no such set is merely "to wield the big stick," not to provide an explanation.

It is also hard to maintain that, while there are sets of many things, there is no sense in which there is a set of all and only the sets. But this already motivates taking the concept *set* to be indefinitely extensible. Similarly, even if there is no set of all and only the truths, there is still an intuition to the effect that there could be such a set. This helps motivate taking the concept *truth* to be indefinitely extensible.

Third, indefinite extensibility does not clearly threaten absolute generality. Granted, earlier developments of indefinite extensibility tended to suggest that the threat is genuine, and accordingly faced embarrassing charges to the effect that the suggestion is self-defeating or otherwise incoherent.[81] The naive formulation "We cannot unrestrictedly quantify over everything" seems to do exactly what it claims we cannot do, namely, unrestrictedly quantify over everything. But more recent developments suggest that the threat is not genuine. Recalling the distinction between genuine expansion and mere restriction, and following Fine (2006), we should understand indefinite extensibility in terms of the expansion rather than the restriction of concepts. Given a suitable

[81] See Lewis (1991); Williamson (2003) for such charges.

modality, we can express the claim "Every truth is not false" via the claim "Necessarily, every truth is not false." Where the modality is interpretational, the claim becomes "In every interpretation, every truth is not false." The interpretations might be subject to certain constraints. Consider the following suggestion from Parsons (2006, 217):

> [N]o matter how our conception of 'object' or 'entity' might be expanded, the truth of '$\forall x(x = x)$' will be preserved. It is...a constraint on possible expansions of these conceptions. In this particular case, it also cuts across the different ways in which even the objects of the familiar world are conceived in science and metaphysics; ...whatever 'criteria of identity' we deploy, the law of self-identity will hold.

Parsons' conception of indefinite extensibility might not be the present one, but we can still adopt his suggestion that expansions are constrained. Transposing his suggestion to the present conception of indefinite extensibility and the case of *truth*, interpretations might be subject to the constraint that they render "Every truth is not false" true. Other constraints include principles such as (ST), (SF), and (BIV). The details are best left for another occasion. What matters hers is that we can be optimistic that indefinite extensibility does not threaten absolute generality.

I conclude that the above arguments succeed in establishing that the concepts *set* and *truth* are indefinitely extensible.

The preceding discussions of the extendedness, extensibility, and indefinite extensibility of concepts clarify my suggestion that the concept *truth* is both extended and extensible, and "true" is polysemous between the meanings corresponding to the subconcepts of *truth* generated due to indefinite extensibility. The concept *truth* is indefinitely extensible, and thus extensible. It is also extended, thanks to its indefinite extensibility; theorists of truth have rehearsed the argument for the indefinite extensibility of *truth* before, and we can do so ourselves. On the present conception, concepts are intersubjective, and are collectively shaped by us—the many possessors and users of concepts.

4 Solving the semantic paradoxes

In this section, I show that the polysemy proposal provides satisfying solutions to the semantic paradoxes. In the first subsection, I summarize the proposed solutions. In the second subsection, I explain why the proposed solutions are satisfying.

4.1 The proposed solutions

To recap, the polysemy proposal comes in two parts. The first part of the proposal maintains that "true" is initially polysemous between *correspondence truth* and *disquotational truth*, where the latter concept extends the former concept, and that this polysemy accounts for the shifts in the meanings of "says" and "true" in the original contextual liar and associated revenge paradoxes. The second part of the proposal maintains that "true" is further polysemous between the meanings corresponding to the subconcepts of the concept *truth* generated by the indefinite extensibility of that concept. The indefinite extensibility of the concept suggests that the concept is both extended and extensible. Crucially, further polysemy accounts for further shifts in the meanings of "says" and "true" in further revenge paradoxes.

On the proposal, there is a hierarchy of actual and merely potential subconcepts—and thus concepts—of truth, where each concept in the hierarchy is extended or could be extended by a concept higher up in the hierarchy. The sequence of actual and merely potential concepts of truth is, say, the following: *truth*, *truth′*, *truth″*, and so on. We can, I suggest, identify *truth* with *correspondence truth*, and *truth′* with *disquotational truth*. At least *truth* and *truth′* are actual concepts of truth, but there are likely other actual concepts of truth higher up in the hierarchy. The other concepts are merely potential.

The polysemy proposal justifies positing polysemy to solve the semantic paradoxes. The indefinite extensibility argument ensures that there are as many concepts, and so meanings, as needed to provide a general solution to the semantic paradoxes.

4.2 Satisfying solutions to the semantic paradoxes

The polysemy proposal provides satisfying solutions for three reasons.

First, the proposed solutions are independently motivated on the basis of the existing distinction in the literature between the concepts *correspondence truth* and *disquotational truth*—which are both subconcepts of *truth*—as well as the indefinite extensibility of the concept *truth*. Taken together, and noting that concepts correspond to meanings, the distinction in the literature between the two concepts and indefinite extensibility supply as many meanings as needed to motivate the shifts in meaning that take place when going through further revenge paradoxes. "True" and related expressions are polysemous between the various meanings.

Second, the proposed solutions vindicates intuitions that liar-like sentences are somehow defective.[82] A widespread intuition, which gappy theories seek to capture, is that liar-like sentences are somehow neither true nor not true.[83] A less widespread intuition, which glutty theories seek to capture, is that liar-like sentences are somehow both true and not true.[84] A related intuition is that liar-like sentences are somehow unstable in truth value. This intuition, which contextualist and revision theories seek to capture, maintains that liar-like sentences either change in truth value across contexts or receive nonstandard truth values that represent their unstable semantic status.[85] We vindicate these intuitions by maintaining that while liar-like sentences may not be true in an initial sense of "true," they may be true in an extended sense of "true." However, unlike gappy, glutty, and revision theories, we maintain both classical logic and bivalence.

Third, and most importantly, the proposed solutions provide a natural view that welcomes the seamless ascent from an object language to a metalanguage—and more generally from one language to another, that pervades theorizing about the semantic paradoxes. The ascent is that one engages in when reflecting on the semantic facts,

[82] See Tarski (1966); Chihara (1979); Sainsbury (2009); Badici and Ludwig (2007); Armour-Garb and Woodridge (2013) on the intuition that liar-like sentences are somehow defective.

[83] See Kripke (1975).

[84] See Priest (2002, 2006).

[85] See Belnap (1982); Herzberger (1982a,b); Gupta and Belnap (1993); Yaqüb (1993) for revision theories.

concerning—among other expressions—"true," about a given language. Given that each metalanguage is strictly more expressive, the ascent engenders a hierarchy of languages. As Glanzberg (2015, 21–22) puts it:

> [R]eflection [on the semantics of a language] ...must be incomplete, and so, hierarchies are not just unsurprising, they are inevitable. ...The main idea is that once we have our semantics in hand, it turns out we can imagine in further reflection on how it works.... Each such reflection introduces a new level of the hierarchy, and the process is open-ended and does not terminate.

On the polysemy proposal, we have something like Tarski's hierarchy of concepts of truth, where concepts of truth become weaker and weaker, in that they have an increasingly wide extension, the higher up in the hierarchy they are. I say "something like," because by exploiting classical construals of Kripke's fixed point construction, concepts of truth can be much more unrestricted in their application than in Tarski's hierarchy.[86]

A common worry about hierarchies of languages maintains that hierarchies prevent us from enjoying a universal language in Tarski's sense, even though natural language seems to be just such a language. A semantically closed language is a language that can express facts about that very language, in that it can refer to expressions of that very language and contains semantic expressions such as "true" that apply to expressions of that language (Tarski, 1944, 350). More generally, a universal language is a language that can express whatever is expressible in some language or other:

> A characteristic feature of colloquial language...is its universality. It would not be in harmony with the spirit of this language if in some language a word occurred which could not be translated into it[.] (Tarski, 1983, 164)

Assuming consistency, the existence of hierarchies of languages suggests that, despite appearances, a natural language such as English is not semantically closed, and thus not universal.

To be sure, some theorists conclude that English is inconsistent.[87] But even setting aside the possibility of inconsistency, in light of the polysemy proposal, it is not so clear

[86] See the references in Section 2.3.
[87] See Yablo (1993); Eklund (2002, 2007); Azzouni (2007); Patterson (2009); Scharp (2013a,b).

to me that the existence of hierarchies suggests this. Recall the distinction between expansionist and restrictionist conceptions of extensibility. The view that hierarchies show that English is not universal seems to assume the restrictionist conception. For it seems that only on this conception do we have that English cannot express everything that is in fact available for it to express; we cannot, e.g., use "λ_γ is not true in any context" to express the fact that λ_γ is not true in any context. By contrast, the view does not seem to be supported on the expansionist conception. For on this conception, perhaps it is initially merely a potential fact that λ_γ is not true in any context, and once it becomes a fact, we can express it using "λ_γ is not true in any context," where the sense of "true" extends the initial one. But the polysemy proposal endorses the expansionist rather than the restrictionist conception.

Tellingly, despite the attempt to eschew hierarchies in many contemporary theories of truth, including nonclassical ones, such theories often remain committed to a metalanguage that is strictly expressively richer.[88] This suggests that the existence of a hierarchy is a genuine phenomenon to be explained rather than explained away. As Williamson (1998, 18) puts it, "levels [hierarchies] have a habit of returning through the back door when they are kicked out of the front." To provide a representative example, in Kripke (1975)'s theory, there is a straightforward sense in which liar-like sentences are not true—they do not fall into the extension of "true." But, as Kripke remarks, the theory cannot express this fact about the theory in that very language:

> [T]here are assertions we can make about the object language which we cannot make in the object language. For example, Liar sentences are *not true* in the object language, ...but we are precluded from saying this in the object language by our interpretation of negation and the truth predicate. ...[T]he sense in which we can say, in natural language, that a Liar sentence is not true must be thought of as associated with some later stage in the development of natural language, one in which speakers reflect on the generation process.... It is not itself a part of that process. [emphasis in the original] (714)

[88] Gupta and Belnap (1993, 103–104, 256) concede the need for a hierarchy of concepts of truth. See Beall (2008) for arguments that Field (2008)'s paracomplete theory engenders a hierarchy of concepts of determinate truth to handle revenge paradoxes formulated in terms of determinate truth, and see Whittle (2004) for arguments that Priest (2002, 2006)'s paraconsistent theory engenders a hierarchy of concepts of logical consequence. Both these nonclassical theories are meant to avoid hierarchies of languages.

It is common now to maintain that what the theory cannot express is *determinate truth*: liar-like sentences are not determinately true, and the initial language cannot express this fact, but an extended language can.[89] What matters for my purposes is that while "true" in the object language can express an initial concept of truth but not an extended concept of truth, "true" in the metalanguage can express both. The ascent from the object language to the metalanguage is seamless. A good explanation of this, in my view, is that "true" is polysemous between the meanings corresponding to each concept. Further concepts, and so meanings, thanks to the indefinite extensibility of *truth*.

It is not unusual for theorists to be reluctant to embrace the envisaged ascent from one language to another and the resulting hierarchy of languages. Notably, Kripke (1975) laments, "The necessity to ascend to a metalanguage may be one of the weaknesses of the present theory. The ghost of the Tarski hierarchy is still with us" (714). Many contemporary theorists of truth share Kripke's frustration with the apparent unavoidability of a hierarchy. But, against such tendencies, I think we should reasonably expect rather than be frustrated by the apparent existence of a hierarchy. Not only are hierarchies a genuine phenomenon to be explained, as I have suggested, but reasons for denying the existence of a hierarchy might not be that compelling in the end.

One reason for eschewing hierarchies arises from the thought that taking "true" to be associated with a hierarchy of languages is unfaithful to "true" in natural language, which seems uniform.[90] However, this thought seems to me to assume an insufficiently general view of the relation between formal languages and natural languages according to which a single formal language represents a single natural language, so that a hierarchy of formal languages represents a hierarchy of natural languages. This view is insufficiently general, because it seems that to account for diverse semantic and pragmatic phenomena over time and across speakers, we must represent even one natural language by means of many formal languages. Indeed, on the polysemy proposal, the ambiguity and in particular polysemy of "true" in English is represented by means of

[89] See Burge (1979); McGee (1990); Maudlin (2004); Field (2008); Beall (2008).

[90] See Priest (2002, 19) and Priest (2006, 168–169).

distinct expressions corresponding to distinct (formal) languages.

Given that there is no straightforward relation between formal languages and natural languages, and that talk of object languages, metalanguages, and hierarchies of languages is almost entirely about formal languages, we cannot make straightforward inferences about what a given theory says about natural language from certain claims it makes about formal languages. In particular, it does not follow from the fact that the polysemy proposal appeals to a hierarchy of concepts of truth and a corresponding hierarchy of truth predicates in formal languages that, according to the proposal, English itself is a hierarchy of languages. Perhaps we should, as I suggest, accept a hierarchy in our formal theorizing, but such a hierarchy is only a rough representation of the flexible meanings across times and communities in natural language.

Another reason for eschewing hierarchies arises from the thought that extended truth-like concepts—sometimes, it is claimed, *determinate truth*, or *correspondence truth*—are not genuine concepts of truth, and so are not within the purview of theories of truth.[91] But this thought seems to me to simply defer judgment on the issue of whether hierarchies are legitimate. Also, it implausibly overlooks notable similarities and relations between, say, *correspondence truth* and *disquotational truth*. Notably, perhaps, correspondence truth entails disquotational truth. No matter what "true" is supposed to mean, it is hard to resist applying it to liar-like sentences in some sense or another. In my view, both *correspondence truth* and *disquotational truth* have enough of a claim to being concepts of truth to demand a unified treatment of both.

The case of truth parallels the case of sets. One might, following Halmos (1960), think that set-like concepts such as *class* are not genuine concepts of set, and so are not within the purview of theories of sets. But this thought seems to me to simply defer judgment on the issue of whether other set-like entities are legitimate. Also, it implausibly overlooks notable similarities and relations between *set* and *class*. Notably, sethood entails classhood. No matter what "set" is supposed to mean, it is hard to resist applying it to

[91] See Parsons (1984); McGee (1990); Soames (1999); Field (2008).

the von Neumann universe V of all sets in some sense or another.[92] *Class* has enough of a claim to being a concept of set to demand a unified treatment of *set* and *class*.

Nonetheless, I do not want to exaggerate the difficulty in resisting applying "true" and "set" to novel entities. Difficulty does not entail impossibility. While it might be natural to apply "true" to liar-like sentences and "set" to something whose members are all and only the non-self-membered sets, the applications are not forced. We can for certain purposes and in certain contexts restrict "true" to only grounded sentences, or "set" to only improper classes. Polysemy should not promote skepticism about the stability of meaning or otherwise threaten clear, precise, and rigorous theorizing.

[92] See Meadows (2016)'s "superset approach" for an endorsement of the idea. Meadows himself draws an analogy between the case of sets and the case of truth (1890–1894).

Appendix A

Supplement to Chapter 1

1 Further extensions of the proposed account

In this section, I outline logical pluralism and ontological pluralism, and then propose further extensions of the account that I proposed on behalf of pluralists in Section 4. Each extension accommodates one of logical pluralism and ontological pluralism, each of which one might maintain in addition to *alethic* pluralism. I focus on domain-specific versions of logical pluralism and ontological pluralism, which are motivated by considerations similar to the ones that motivate alethic pluralism. I emphasize, however, as those who suggest combining alethic pluralism with either logical pluralism or ontological pluralism emphasize, that alethic pluralism is independent of either.[1]

In the first subsection, I propose an extension that accommodates logical pluralism. In the second subsection, I propose an extension that accommodates ontological pluralism. While I do not pursue it, one might combine all three of alethic pluralism, logical pluralism, and ontological pluralism.[2] I hope the details I provide suffice to show how an extension that accommodates the combination of all three pluralisms might go.

Recall Wright (1992)'s motivation for alethic pluralism: one might maintain realism about some domains but antirealism about others. But perhaps domain-specific truth

[1]See Pedersen (2014); Cotnoir and Edwards (2016).

[2]See Pedersen (2014) for the suggested combination of all three pluralisms.

properties are linked to domain-specific logics, and domain-specific quantifiers.

Insofar as Wright's consideration regarding realism motivates alethic pluralism, it might also motivate *logical pluralism*, a salient version of which is the view that there are distinct logics associated with distinct domains.[3] To illustrate logical pluralism, classical logic is associated with the scientific domain, intuitionistic logic is associated with the mathematical domain, and some nonclassical logic is associated with the ethical domain. To illustrate the combination of alethic pluralism with logical pluralism, realist domains, which are associated with the correspondence truth property, are associated with classical logic, while antirealist domains, which are associated with the superassertible truth property, are associated with a weaker, nonclassical logic.[4]

Similarly, insofar as Wright's consideration motivates alethic pluralism, it might also motivate *ontological pluralism*, a salient version of which is the view that there are distinct logically or metaphysically "perspicuous" quantifiers associated with distinct domains.[5] To illustrate ontological pluralism, quantifiers that range only over concrete objects are associated with the scientific domain, quantifiers that range only over abstract objects are associated with the mathematical domain, and quantifiers that range only over mind-dependent entities of an appropriate kind are associated with the ethical domain.[6] To illustrate the combination of alethic pluralism with ontological pluralism, each domain comes with a distinguished ontology of its own, inducing distinctions in the associated domain-specific truth property.[7]

To clarify, I am only explaining why one who maintains alethic pluralism might also maintain logical pluralism or ontological pluralism. One might combine alethic plural-

[3]See Beall and Restall (2006); Shapiro (2014) on logical pluralism more generally. The versions they discuss are not all domain-specific versions of logical pluralism.

[4]Wright (1992, 44) seems to suggest this combination of alethic pluralism with logical pluralism. Lynch (2008, 2009); Pedersen (2014) both suggest the combination.

[5]See McDaniel (2009, 2010a,b, ms); Turner (2010, 2012) on ontological pluralism.

[6]In the scientific case, I assume for the sake of illustration that mathematics—or at least commitment to the existence of abstract mathematical objects—is dispensable; see Field (1980) for a defense of this view. In the ethical case, the relevant entities might be mind-dependent overall but have some basis in mind-independent entities as well. For example, perhaps the fact that an action constitutes murder is mind-dependent but has basis in the mind-independent fact that death is brought about.

[7]Pedersen (2014); Cotnoir and Edwards (2016) suggest this combination of alethic pluralism with ontological pluralism.

ism with logical pluralism or ontological pluralism. Then again, one might not. I do not think alethic pluralists are committed to any of the combinations here.

1.1 Accommodating logical pluralism

The proposed extension of the account for a sentential language accommodates logical pluralism, so that some domains but not others are associated with a nonclassical logic, by allowing certain domains to be associated with a third domain-specific property—an "indeterminacy" property that indicates indeterminacy—that is neither domain-specific truth nor domain-specific falsity, and employing (versions of) the strong Kleene clauses for complex sentences.

Recall that the pair of structures $\mathfrak{T}$ and $\mathfrak{F}$ are both isomorphic to $\mathscr{D}$ (and to each other), and provide domain-specific truth values that represent domain-specific truth properties and domain-specific falsity properties, respectively. We now associate with $\mathfrak{D}$ further isomorphic structures to provide further domain-specific truth values. In the simple case, we associate a single further isomorphic structure $\mathfrak{I}$, which provides domain-specific truth values that represent domain-specific indeterminacy properties. The semantic values, and more specifically domain-specific truth values, are then drawn from $\mathfrak{T} \cup \mathfrak{I} \cup \mathfrak{F}$. For convenience, let I_{D_i} as well as I_i denote the domain-specific truth value that represents the indeterminacy property associated with domain D_i. Since $\mathfrak{I}$ is isomorphic to $\mathfrak{D}$, the partial order on $\mathfrak{I}$ mirrors the partial order on $\mathfrak{D}$:

(OR*) $D_i < D_j$ iff $I_i < I_j$.

A model is a pair $M = \langle d, v \rangle$, much as before, except that now the valuation function $v : Atomic(\mathscr{L}) \to \mathfrak{T} \cup \mathfrak{I} \cup \mathfrak{F}$ is from atomic sentences to domain-specific truth values in $\mathfrak{T} \cup \mathfrak{I} \cup \mathfrak{F}$. We require that $v(\alpha) \in \{T_{d(\alpha)}, I_{d(\alpha)}, F_{d(\alpha)}\}$.

Given a model M, the semantic value function $|\cdot|_M : \mathscr{L} \to \mathfrak{T} \cup \mathfrak{I} \cup \mathfrak{F}$ is much as before, except that now it is a function from atomic sentences to domain-specific truth values in $\mathfrak{T} \cup \mathfrak{I} \cup \mathfrak{F}$. To capture the idea that a sentence is either domain-specifically true, domain-specifically indeterminate, or domain-specifically false, we first require that $|\varphi|_M \in \{T_{d(\alpha)}, I_{d(\alpha)}, F_{d(\alpha)}\}$. Then, to ensure that the account meets Constraint 2 and Desideratum B, we define $|\cdot|_M$ by $|\alpha|_M = v(\alpha)$ and the following:

(LO**) (i) $$|\neg\varphi|_M = \begin{cases} T_{d(\neg\varphi)} \text{ iff } |\varphi|_M = F_{d(\varphi)}; \\ F_{d(\neg\varphi)} \text{ iff } |\varphi|_M = T_{d(\varphi)}; \\ I_{d(\neg\varphi)} \text{ otherwise;} \end{cases}$$

(ii) $$|\varphi \wedge \psi|_M = \begin{cases} T_{d(\varphi\wedge\psi)} \text{ iff } |\varphi|_M = T_{d(\varphi)} \text{ and } |\psi|_M = T_{d(\psi)}; \\ F_{d(\varphi\wedge\psi)} \text{ iff } |\varphi|_M = F_{d(\varphi)} \text{ or } |\psi|_M = F_{d(\psi)}; \\ I_{d(\varphi\wedge\psi)} \text{ otherwise;} \end{cases}$$

(iii) $$|\varphi \vee \psi|_M = \begin{cases} T_{d(\varphi\vee\psi)} \text{ iff } |\varphi|_M = T_{d(\varphi)} \text{ or } |\psi|_M = T_{d(\psi)}; \\ F_{d(\varphi\vee\psi)} \text{ iff } |\varphi|_M = F_{d(\varphi)} \text{ and } |\psi|_M = F_{d(\psi)}; \\ I_{d(\varphi\vee\psi)} \text{ otherwise.} \end{cases}$$

Next, we define domain-specific logical consequence as follows:

(LC**) $\Gamma \vDash_{D_i} \varphi$ iff for all M such that $d(\gamma_j) = d(\varphi) = D_i$ for all γ_j in Γ, if $|\gamma_j|_M = T_{D_i}$ for all γ_j in Γ, then $|\varphi|_M = T_{D_i}$.

1.2 Accommodating ontological pluralism

The proposed extension of the account for a first-order language accommodates ontological pluralism, so that there are distinct quantifiers associated with distinct domains, by maintaining that quantifiers are domain-specific, in that they assign the relevant variables to objects in domain-specific partitions of a model's universe.

We assume that the language now contains domain-specific quantifiers $\forall_{D_i}$ and $\exists_{D_i}$, where D_i is a domain in $\mathscr{D}$. A preliminary revision to the account is to assume a partition of the universe U in each model on the basis of domains. Given a model $\langle d, U, I\rangle$, each cell U_{D_i} in the partition corresponds to a domain D_i in $\mathscr{D}$. Then we revise the clause for quantified formulas to the following:

(LO***) (i) $|\forall_{D_i} \upsilon\varphi|_{M,a} = T_{d(\forall\upsilon\varphi,a)}$ iff $|\varphi|_{M,b} = T_{d(\varphi),b}$ for all b where $b \sim_{\upsilon} a$ and $b(\upsilon) \in U_{D_i}$;

(ii) $|\exists_{D_i} \upsilon\varphi|_{M,a} = T_{d(\exists\upsilon\varphi,a)}$ iff $|\varphi|_{M,b} = T_{d(\varphi),b}$ for some b where $b \sim_{\upsilon} a$ and $b(\upsilon) \in U_{D_i}$.

2 An impossibility result

In Section 3.1, I argued that Cotnoir (2009)'s account is inadequate because it violates Desideratum B. But perhaps this violation is not fatal. Admittedly, even taking the violation into account, I think that an account in this spirit should be given due consideration. Such an account attempts to answer the third challenge, to maintain a standard account of complex sentences, by maintaining two motivating ideas. First, there are only domain-specific truth properties associated with pure domains. Second, complex sentences inherit one or more of the domain-specific truth values of their their subsentences. In this section, however, I argue that, on assumptions that should be attractive to pluralists, such an account fails to be compositional, let alone truth-functional.

Consider that on the familiar account of complex sentences, complex sentences are truth-functional, in that the truth values of complex sentences are completely determined by the truth values of the subsentences. As we have seen, attempts to answer the third challenge often focus on showing how pluralists can maintain a truth-functional account of complex sentences. But consider that the familiar account of complex sentences is also compositional, in that the semantic values of complex sentences are completely determined by the semantic values of the subsentences. Where semantic values are truth values, compositionality and truth functionality are equivalent. But in general, semantic values can incorporate more information in addition to truth values, so that compositionality is more general than truth functionality. The failure of truth functionality is compatible with compositionality. Such a situation might be tolerable insofar as compositionality is what matters. But the failure of compositionality is less tolerable.

The account that I argue fails to be compositional maintains three underlying assumptions that are initially attractive in light of pluralism's motivating picture.

The first assumption is the *subject matter assumption*: the pure domains include the scientific and mathematical domains, which are respectively associated with the pure domain-specific truth properties that are the correspondence truth property and the coherence truth property.[8] This assumption is natural, in that it helps pluralists preserve the core of their motivating picture.

The second assumption is the *purity assumption*: pure domains are all the domains there are, and pure domain-specific truth properties are all the truth properties there are. This assumption is natural, in that it helps pluralists preserve no more than the core of their motivating picture, and thereby avoid awkward questions about the number and nature of domains and domain-specific truth properties that are not pure. It rules out impure domains that are associated with more than one subject matter, such as a domain associated with both scientific and mathematical subject matters. It also rules out associated domain-specific truth properties, and truth properties that are not domain-specific.

The third assumption is the *semantic value assumption*: semantic values incorporate information about all and only truth values as well as domains, and specifically, information about all and only which truth values and domains sentences are assigned. This assumption is similarly natural, in that it helps pluralists maintain just enough and no more about semantic values than they would seem to need to maintain.

Given the purity assumption, there are two salient accounts available of the truth values of sentences. The first salient account maintains that all sentences are assigned exactly one truth value, as on the familiar account of semantics. But as I argued in my criticism of Pedersen and Wright (2013b)'s account (which parallels the many-valued account of complex sentences), this account is unpromising, because it risks violating Desideratum C. The second salient account, which I assume in what follows and is what we have on Cotnoir (2009)'s account, is more promising. On this account, sentences can

[8]Of course, pluralists can maintain that there are other pure domains—e.g., an ethical domain and an aesthetic domain—and associated domain-specific truth properties as well.

be assigned more than one domain-specific truth value. For example, "Water is H_2O," which is assigned the scientific domain, is assigned both correspondence truth and coherence truth, even though correspondence truth but not coherence truth is associated with the scientific domain. Perhaps the sentence corresponds as well as coheres, because corresponding entails cohering.

Given the above underlying assumptions, together with below further assumptions that should be natural for pluralists, the resulting account fails to be compositional: the semantic values of complex sentences are not completely determined by the semantic values of the subsentences.

Consider the following sentences:

(1) Water is H_2O.

(2) Salt is NaCl.

Suppose that (1) and (2) are both just scientific sentences—i.e., sentences associated with just the scientific domain—that are both just correspondence true and coherence true. Since (1) is a scientific sentence, by Constraint 1, $\neg(1)$ is also a scientific sentence (but is not associated with any other domain). By analogous reasoning, $\neg(2)$ is also a scientific sentence (but is not associated with any other domain). Further, since (1) is correspondence true, its negation $\neg(1)$ should not be correspondence true. But since (1) is coherence true and not a mathematical sentence, $\neg(1)$ should still be coherence true; failure of correspondence is compatible with coherence, and in any case $\neg(1)$ does seem at least coherent. So $\neg(1)$ should be coherence true but not correspondence true. By analogous reasoning, $\neg(2)$ should be coherence true but not correspondence true.

Now consider the conjunction $(1) \wedge \neg(1)$. Since the conjuncts are scientific sentences, by Constraint 1, this conjunction is also a scientific sentence. But since it is a logical falsehood, it should be neither correspondence true nor coherence true; presumably, a logical falsehood neither corresponds to anything nor coheres. By contrast, consider the conjunction $(1) \wedge \neg(2)$. Since the conjuncts are scientific sentences, by

Constraint 1, this conjunction is also a scientific sentence. But although it should not be correspondence true (since the conjunct $\neg(2)$ is not correspondence true), it should still be coherence true; again, failure of correspondence is compatible with coherence, and in any case $(1) \wedge \neg(2)$ does seem at least coherent. So the conjunction $(1) \wedge \neg(2)$ should be coherence true but not correspondence true.

The two conjunctions $(1)\wedge\neg(1)$ and $(1)\wedge\neg(2)$ have different semantic values, in that they are not assigned the same truth values and domains. However, the corresponding conjuncts have the same semantic values: the first conjunct is assigned both just correspondence truth and coherence truth, and just the scientific domain; while the second conjunct is assigned just coherence truth, and just the scientific domain.

Thus, pluralists should accept the *conjunction constraint*: there are corresponding conjuncts assigned the same truth values and domains, and so are assigned the same semantic values, but where the corresponding conjunctions are not assigned the same truth values, and so are not assigned the same semantic values. Given this constraint, the semantic values of conjunctions are not completely determined by the semantic values of the conjuncts. That is, conjunction is not compositional.

The foregoing argument can be formulated more formally. Let $\mathscr{L}$ be a sentential language with negation ($\neg$) and conjunction ($\wedge$). Let $\mathscr{D}$ be the set of all domains. Also, let $\mathscr{T}$ be the set of all truth values that represent domain-specific truth properties, and $\mathscr{F}$ be the set of all truth values that represent domain-specific falsity properties, where there is a one-to-one correspondence between $\mathscr{D}$, $\mathscr{T}$, and $\mathscr{F}$ in order to capture the associations between domains, domain-specific truth properties, and domain-specific falsity properties. The set $\mathscr{V} = \mathscr{T} \cup \mathscr{F}$ is the set of all truth values.

By the subject matter assumption, there are at least two pure domains in $\mathscr{D}$: D_S (the scientific domain) and D_M (the mathematical domain). Without loss of generality, let us assume that there are no other pure domains. By the purity assumption, all the domains are pure domains. So, given that there are no other pure domains, there are no other domains. We have that $\mathscr{T} = \{T_R, T_H\}$ and $\mathscr{F} = \{F_R, F_H\}$. Here, D_S (the scientific domain) is associated with T_R (the correspondence truth property) and F_R (the correspondence falsity property); similarly, D_M (the mathematical domain) is associated with T_H (the coherence truth property) and F_H (the coherence falsity property).

By the semantic value assumption, semantic values incorporate information about all and only truth values and domains. So, without loss of generality, let semantic values be pairs $v = \langle v_{\mathscr{V}}, v_{\mathscr{D}} \rangle$ of tuples. Specifically, $v_{\mathscr{V}}$ is a tuple with T_i in the ith place indicating the having of domain$_i$-specific truth and F_i in the ith place indicating the having of domain$_i$-specific falsity; $v_{\mathscr{D}}$ is a tuple with D_i in the ith place indicating association with domain i and $\overline{D_i}$ in the ith place indicating lack of association with domain i.

Given our simplifying assumption that there are exactly two domains, D_S and D_M, semantic values are pairs of pairs. Now let us fix an arbitrary order for the domain-specific truth values and domains; the specific order does not matter. In each pair within a pair that is a semantic value, let the first place be associated with the scientific domain, and let the second place be associated with the mathematical domain. To illustrate, a sentence with the semantic value $\langle [T_R, T_H], [D_S, \overline{D_M}] \rangle$ is assigned both correspondence truth and coherence truth, and just the scientific domain. A model M is a function from atomic sentences to semantic values, and a semantic value function $|\cdot|_M$ based on M is a function from sentences to semantic values.

Let us say that an n-place compositional function f is a function from ordered sequences of n semantic values to semantic values. Also, let us say that an n-place operator $*$ is compositional iff there is an n-place compositional function f such that, for all $\varphi_1, \ldots, \varphi_n$ the following holds: $|*(\varphi_1, \ldots, \varphi_n)|_M = f([|\varphi_1|_M, \ldots, |\varphi_n|_M])$. That is, the semantic value assigned to a sentence with $*$ occurring as the main connective is a function of the semantic values of the operands. In particular, the conjunction operator $\wedge$ is compositional iff there is a two-place compositional function $f_\wedge$ such that, for all φ, ψ the following holds: $|\varphi \wedge \psi|_M = f_\wedge([|\varphi|_M, |\psi|_M])$.

What we have shown in the argument is the following, where $M_@$ is the intended model. According to the conjunction constraint, for some sentences φ and ψ, such as

(1) and $\neg(1)$, respectively, we have that $|\varphi \wedge \psi|_{M_@} = \langle [F_R, F_H], [D_S, \overline{D_M}] \rangle$, but for other sentences χ and ρ, such as (1) and $\neg(2)$, respectively, we have that $|\chi \wedge \rho|_{M_@} = \langle [F_R, T_H], [D_S, \overline{D_M}] \rangle$, so that $|\varphi \wedge \psi|_{M_@} \neq |\chi \wedge \rho|_{M_@}$, even though we have that $|\varphi|_{M_@} = |\chi|_{M_@} = \langle [T_R, T_H], [D_S, \overline{D_M}] \rangle$ and $|\psi|_{M_@} = |\rho|_{M_@} = \langle [F_R, T_H], [D_S, \overline{D_M}] \rangle$. Clearly, there is no compositional function $f_\wedge$ that satisfies this constraint. Thus, $\wedge$ is not compositional.

Appendix B

Supplement to Chapter 2

1 Further principles

In this section, I suggest further principles for the proposed modal account of propositions. However, let me make two remarks about these principles. First, to err on the side of caution, I opt for fairly weak principles, although stronger versions of these principles—that do not generate inconsistency—might be tenable. The greater the number of principles, and the stronger the principles are, the more likely it is that revenge paradox will be encountered. Second, I do not claim that the principles constitute anything like a complete account of propositions. The considerably more ambitious project of developing such an account is an enormous undertaking that is notoriously fraught with difficulties. Needless to say, this project is beyond the scope of this thesis.

What propositions are there? I suggest taking the individuation of propositions to depend on the individuation of properties.[1] Plausibly, no matter the level of the hierarchy, whatever properties and objects there are, for every property P and object o, provided that o is not also a property, there could be a proposition that ascribes P to o. We make this plausible assumption explicit by means of the following principle:

(S) $\Box\forall z\forall x((\rho(z) \wedge \neg\rho(x)) \rightarrow \Diamond\exists y \circledcirc (y, z, x))$.

[1]See Zalta (1983, 1988) for a similar approach.

This principle has the proviso that the objects are not also properties, because one might worry about the tenability of the version of the principle without the proviso, which secures the potential existence of propositions that ascribe properties to properties—including propositions that ascribe a certain property to the property itself.

Assuming that there could be complex properties and that such properties are abundant, no matter the level of the hierarchy, for any properties P and Q and object o, provided that o is not also a property, there could be a proposition that ascribes the property of being $\neg P$ to o, a proposition that ascribes the property of being $P \wedge Q$ to o, and a proposition that ascribes the property of being $P \vee Q$ to o. But then, no matter the level of the hierarchy, provided that the objects are not also properties, there could be a property that is instantiated by all and only the objects that do not instantiate P, a property that is instantiated by all and only the objects that instantiate P and instantiate Q, and a property that is instantiated by all and only the objects that instantiate P or instantiate Q. The following principles secure the potential existence of such complex properties:

(NEG-PY) $\Box\forall x(\rho(x) \rightarrow \Diamond\exists y(\rho(y) \wedge \forall z(\neg\rho(z) \rightarrow (\iota(z,y) \leftrightarrow \neg\iota(z,x))))).$

(CON-PY) $\Box\forall x\forall y((\rho(x) \wedge \rho(y)) \rightarrow \Diamond\exists z(\rho(z) \wedge \forall z'(\neg\rho(z') \rightarrow (\iota(z',z) \leftrightarrow (\iota(z',x) \wedge \iota(z',y)))))).$

(DIS-PY) $\Box\forall x\forall y((\rho(x) \wedge \rho(y)) \rightarrow \Diamond\exists z(\rho(z) \wedge \forall z'(\neg\rho(z') \rightarrow (\iota(z',z) \leftrightarrow (\iota(z',x) \vee \iota(z',y)))))).$

Here, $\iota(x,z)$ means that x instantiates z. Stronger versions of these principles, where $\Box$ is inserted before the universal quantifier, might be tenable as well.

What we have from (S) are singular propositions, broadly construed. To secure the potential existence of quantified propositions as well, let us assume that, no matter the level of the hierarchy, for every property P, there could be a proposition that ascribes P to every object that is not a property:

(A) $\Box\forall z(\rho(z) \rightarrow \Diamond\exists y\forall x(\neg\rho(x) \rightarrow \circledcirc(y,z,x))).$

To extend (S) and (A) to handle pluralities as well, we also assume the following:

(SP) $\Box\forall z\forall xx((\rho(z) \wedge \forall x(x \prec xx \rightarrow \neg\rho(x))) \rightarrow \Diamond\exists y \circledast (y,z,xx)).$

(AP) $\Box\forall z(\rho(z) \to \Diamond\exists y \forall xx(\forall x(x \prec xx \to \neg\rho(x)) \to \circledast(y, z, xx)))$.

These principles generate complex and quantified propositions that ascribe properties to pluralities of objects rather than individual objects.

Given (NEG-PY), (CON-PY), and (DIS-PY), which secure the potential existence of complex properties, we assume parallel principles that secure the potential existence of complex propositions—namely, negative, conjunctive, and disjunctive propositions:

(NEG-PN) $\Box\forall x(\pi(x) \to \Diamond\exists y(\pi(y) \wedge (T(y) \leftrightarrow F(x))))$.

(CON-PN) $\Box\forall x \forall y((\pi(x) \wedge \pi(y)) \to \Diamond\exists z(\pi(z) \wedge (T(z) \leftrightarrow (T(x) \wedge T(y)))))$.

(DIS-PN) $\Box\forall x \forall y((\pi(x) \wedge \pi(y)) \to \Diamond\exists z(\pi(z) \wedge (T(z) \leftrightarrow (T(x) \vee T(y)))))$.

Stronger versions of these principles, where $\Box$ is inserted before the biconditional, might be tenable as well. As desired, these principles do not require the complex propositions to be unique. Take (CON-PN) as an example. Consider the proposition p that ascribes the property of being even to the number four, which is true iff four is even, and the proposition q that ascribes the property of being even to the number six, which is true iff six is even. The principle (CON-PN) ensures that there could be at least one proposition that is true iff p is true and q is true, but it does not require the proposition to be unique. On the neo-Russellian account I assume, the proposition that $1+1=2$ and the proposition that $2+2=4$ are distinct, but each is true iff four is even and six is even, and so iff p is true and q is true.

The above principles are conditional existence claims, in that they secure the potential existence of certain properties or propositions conditional on the existence of other properties or propositions. What about unconditional existence claims? It might be tempting to maintain the following comprehension principle for properties:

(YC-) $\Box\exists z(\rho(z) \wedge \forall x(\iota(x, z) \leftrightarrow \varphi(x)))$.

According to (YC-), given any condition, no matter the level of the hierarchy, there is a property that is instantiated by all and only the objects that satisfy the condition. However, we should not maintain this principle, because it yields a property-theoretic version of Russell's paradox. For consider the instance of (YC-) where the given condition $\varphi(x)$ is $\neg\iota(x,x)$. This instance guarantees the existence of a property that is instantiated by all and only the objects that do not instantiate themselves. This in turn entails that there is something that instantiates itself iff it does not instantiate itself. Standard principles of reasoning show that this yields a contradiction. Instead of (YC-), we should maintain a modalized version of (YC-), such as the following principle:

(YC) $\Box \uparrow \Diamond \exists z(\rho(z) \wedge \forall x(\iota(x,z) \leftrightarrow \downarrow \varphi(x)))$.

Here, $\uparrow$ and $\downarrow$ are the operators from Vlach (1973); $\uparrow$ is an operation that stores the current level of evaluation and $\downarrow$ is an operation that retrieves the currently stored level of evaluation.[2] According to (YC), given any condition, no matter the level of the hierarchy *l*, there could be a property that is instantiated by all and only the objects that satisfy the condition at *l*.

Further development of the proposed account might provide an underlying modal logic. A natural development follows Linnebo (2013) in maintaining system S4.2, which adds to system S4 a principle that ensures the directedness of the accessibility relation: for any two levels, there is a level that each of the two levels accesses.[3] The accessibility relation is then a partial order—i.e., reflexive, antisymmetric, and transitive—and directed. However, further development of the account is best left for another occasion.

[2]See Forbes (1989, 27–29) on these "Vlach" operators. Also see Williamson (2013, Chap. 7.9) for technical background.

[3]System S4 adds to a normal modal logic the following two axioms: (T) $\Box\varphi \rightarrow \varphi$; and (4) $\Box\varphi \rightarrow \Box\Box\varphi$. System S4.2 adds to S4 the following axiom: (G) $\Diamond\Box\varphi \rightarrow \Box\Diamond\varphi$.

Appendix C

Supplement to Chapter 3

1 Tests for polysemy

In this section, I suggest that, despite limitations, various tests for ambiguity in general and polysemy in particular corroborate the polysemy proposal. In the first subsection, I consider three standard tests for ambiguity in general. In the second subsection, I consider two tests for polysemy rather than homonymy. In the third subsection, I consider four tests for polysemy rather than mere context sensitivity.

I emphasize three limitations of these tests. First, the tests are meant to rely on pretheoretic judgments, whereas my applications of them often rely on theoretical judgments involving liar-like sentences. Second, the tests are heuristics at best. It can be tricky to apply them, and negative results do not provide conclusive evidence against polysemy.[1] Third, the tests for ambiguity in general are better understood as tests for homonymy rather than polysemy, and correlatively provide more reliable evidence about homonymy than about polysemy.[2] Consider that the expression "dog" is polysemous between the meanings *canine* and *male canine*, but since the distinct meanings are not easy to access, the tests might—depending on one's judgments—suggest that it is not

[1] See Geeraerts (2006, 2016), who maintains that ambiguity is hard to identify, and argues that the three standard tests for ambiguity make conflicting suggestions and are also subject to contextual variation.

[2] See Hawthorne and Lepore (2011, 471).

ambiguous, and so not polysemous. Given this limitation, for the tests for ambiguity, I take even weak positive results to provide evidence for polysemy, and negative results to provide evidence merely against homonymy—not polysemy.

1.1 Standard tests for ambiguity

There are three standard tests for ambiguity in general: the logical test, the linguistic test, and the definitional test.[3]

Logical test

The logical test for ambiguity considers whether apparent contradictions are "really" contradictory. Specifically, the test considers apparent contradictions of the apparent form "a is F and a is not F." (It is assumed that there are two occurrences each of a and F, so that we are dealing with two occurrences of the same expression.) Such apparent contradictions are uncontroversially *syntactically* contradictory, since they have the form of a contradiction. But what is still left open is whether they are *semantically* contradictory, in that there are no noncontradictory readings of them. If the sentence is indeed contradictory, in that on every reading of the sentence it is contradictory, this suggests that F is not ambiguous. But if there is at least one reading on which the sentence is not contradictory, this suggests that F is ambiguous. The idea is that, if an expression is ambiguous, then—perhaps thanks to charity considerations that lead us to gravitate towards true or at least noncontradictory readings—we should be able to access the multiple meanings in encountering an apparent contradiction containing the expression.

The contradiction test suggests that "international" is not ambiguous, as it indeed is not. For consider the following apparent contradiction:

(1) #The United Nations is international and it is not international.

[3] See Zwicky and Sadock (1975); Cruse (1986); Geeraerts (2006); Tuggy (2006); Geeraerts (2016).

Clearly, (1) is indeed contradictory, in that on every reading of the sentence it is contradictory. We cannot access multiple meanings of "international." Meanwhile, the contradiction test suggests that "light" is ambiguous, as it indeed is. For consider the following apparent contradiction:

(2) That block is light but it isn't light.

Clearly, there is at least one reading on which (2) is not contradictory—a reading that assigns the meaning *pale in color* to the first occurrence of "light" and the meaning *low in weight* to the second occurrence of "light." We can access multiple meanings of "light."

However, the logical test is limited, in that it might misleadingly suggest that vague but presumably unambiguous expressions, such as "tall," are ambiguous. For consider the following apparent contradiction, where 5' 5" Kyle is tall for an average 10-year-old but not for an adult basketball player:

(3) Kyle is tall but he is not tall.

Especially when focal stress is put on the second occurrence of "tall," (3) is arguably only apparently contradictory.[4] Standardly at least, "tall" is vague but unambiguous. If so, then the logical test misleadingly suggests that it is ambiguous. That said, if vagueness itself suggests ambiguity, then perhaps the suggestion that "tall" is ambiguous is not misleading after all.

What does the logical test suggest about "says" and "true"? For "says," consider the following apparent contradiction:

(4) "λ_γ is not true in any context" does not say anything and "λ_γ is not true in any context" says something.

Going through the contextual liar and associated revenge paradoxes, it seems that (4) is only apparently contradictory. We can access multiple meanings of "says": λ_γ does not say anything in one sense, since the liar paradox shows that the assumption that λ_γ says

[4]To be fair, putting focal stress on occurrences of some expressions often makes it easier to get non-contradictory readings. The context sensitivity of "tall" is likely playing a role here as well.

something leads to contradiction, but λ_γ also does say something in another sense, as the associated revenge paradox suggests. Similarly, for "true," consider the following apparent contradiction:

(5) "λ_γ is not true in any context" is not true and "λ_γ is not true in any context" is true.

Going through the liar and associated revenge paradoxes, it seems that (5) is only apparently contradictory. We can access multiple meanings of "true": λ_γ is not true in one sense, since there is a sense in which it does not say anything, but λ_γ is also true in another sense, since there is also a sense in which it says something true. Thus, the logical test suggests that "true" is ambiguous, which corroborates the polysemy proposal.

Linguistic test

The linguistic test, which comprises two tests, for ambiguity considers whether certain syntactic constructions are zeugmatic. The conjunction test considers constructions of the apparent form "*a* and *b* are *F*," while the ellipsis test considers constructions of the apparent form "*a* is *F* and *b* is too." With each test, if the construction is zeugmatic, this suggests that *F* is ambiguous. But if the sentence is not zeugmatic, this suggests that *F* is not ambiguous. The idea is that we should find the relevant constructions containing ambiguous expressions to be zeugmatic, because the constructions inappropriately assume that the expressions have just one meaning.

The conjunction and ellipsis tests suggest that "international" is not ambiguous, as it indeed is not. For consider the following constructions:

(6) The United Nations and the World Bank are international.

(7) The United Nations is international and the World Bank is too.

Clearly, (6) and (7) are not zeugmatic. Meanwhile, the conjunction and ellipsis tests suggest that "light" is ambiguous, as it indeed is. For, given the felicity of "The colors [of a certain heavy object] are light" and "The feathers [which are dark in color] are light," consider the following constructions:

(8) ?The colors and the feathers are light.

(9) ?The colors are light and the feathers are too.

Clearly, (8) and (9) are zeugmatic.

However, the linguistic test is limited. First, zeugma is hard to detect when the ambiguity involves closely related meanings, as in polysemy. Second, as Cruse (1986) points out, the presence or absence of zeugma is context-sensitive. For consider the application of the conjunction test to "dissertation," which is ambiguous and in particular polysemous, between the meanings *the content of the dissertation* and *the physical copy of the dissertation*:

(10) ?Judy's dissertation is thought provoking and yellowed with age.

(11) Judy's dissertation is still thought provoking although yellowed with age.

Since (10) is zeugmatic, this suggests that "dissertation" is ambiguous. But since (11) is not zeugmatic, this suggests that "dissertation" is not ambiguous. Third, like the logical test, the linguistic test might misleading suggest that vague but presumably unambiguous expressions, such as "tall," are ambiguous. For, where Yao Ming is 7' 6", consider the following constructions:

(12) ?Yao Ming and Kyle are tall.

(13) ?Yao Ming is tall and Kyle is too.

Both (12) and (13) are zeugmatic.

What does the linguistic test suggest about "says" and "true"? Consider the following constructions:

(14) ?"Grass is green" and "λ_γ is not true in any context" say something.

(15) ?“Grass is green” says something and “λ_γ is not true in any context” does too.

(16) ?“Grass is green” and “λ_γ is not true in any context” are true.

(17) ?“Grass is green” is true and “λ_γ is not true in any context” is too.

Clearly, all of (14)–(17) are zeugmatic. Thus, the linguistic test suggests that “says” and “true” are ambiguous, which corroborates the polysemy proposal.

Definitional test

The definitional test for ambiguity considers whether there is a unified definition of an expression *e* that encompasses multiple meanings. If there is such a definition, this suggests that *e* is not ambiguous. If there is not such a definition, this suggests that *e* is ambiguous. The idea is that an ambiguous expression does not admit of a unified definition, but an unambiguous one does. Of course, the definition should be natural rather than highly disjunctive.

The definitional test suggests that “international” is not ambiguous, as it indeed is not. For there is the following unified definition of “international”:

(DFI) *x* is international iff *x* involves two or more nations or their people.

Meanwhile, the definitional test suggests that “light” is ambiguous, as it indeed is. For there is no unified definition of “light.” Granted, one might consider the following attempt at a unified definition of “light”:

(DFL) *x* is light iff *x* is pale in color or *x* is low in weight.

But this definition is unnatural and too disjunctive to qualify as a satisfying definition.

However, the definitional test is limited. First, it is not always clear what a unified definition of the kind sought consists in. Second, as Fodor (1998) suggests, there are few satisfying definitions—that provide necessary and sufficient conditions for some expression—available. But then the definitional test implausibly suggests that many more expressions than we might initially suspect are ambiguous. Third, as Geeraerts (2006, 130) points out, an apparently unified definition of the form "x is F iff x ..." might in fact not be unified, if what occurs on the right-hand side is somehow ambiguous:

> If the definition of what is allegedly a single sense of a lexical item is itself polysemous, the definition may either leave in the dark which of the various polysemous alternatives actually applies to the definiendum, or it may cover up the actual polysemy of the latter.

What does the definitional test suggest about "says" and "true"? Since it seems hard to define "says"—perhaps it is a primitive expression—there seems to be no definition, let alone unified definition, of "says" that encompasses multiple meanings. But then the definitional test suggests that "says" is ambiguous, which corroborates the polysemy proposal. Similarly with "true." The most straightforward definition of "true" comes from Tarski's disquotational principle of truth, i.e., (T). But the liar paradox shows that this principle yields contradiction, whereas presumably a definition should not yield contradiction. What about other versions of (T)? A salient version maintains that σ is true iff, for some φ, σ says that φ and φ—this is essentially just (SPT), *modulo* context. This principle does not yield contradiction. Assuming it is indeed a unified definition of "true," the definitional test suggests that "true" is not ambiguous. However, matters are not straightforward. For if the polysemy proposal is correct, "says" on the right-hand side is ambiguous, so that "true" on the left-hand side is also ambiguous. It is unclear that we really have a unified definition of "true." Thus, if this is the best we can do, and it is not a unified definition of "true," then the definitional test suggests that "true" is ambiguous, which corroborates the polysemy proposal.

1.2 Two tests for polysemy rather than homonymy

Lyons (1977) suggests two tests for polysemy rather than homonymy: the reflection test and the etymology test.[5]

[5] See Falkum (2011, 16–19) for discussion.

Reflection test

The most straightforward test depends on simple linguistic reflection. According to this test, an ambiguous expression is homonymous if, on reflection, native speakers judge its meanings to be unrelated; and polysemous if, on reflection, native speakers judge its meanings to be related. This test suggests that "bank" is homonymous, as it indeed is. For presumably, on reflection, native speakers judge its meanings—among them, *financial institution* and *riverside*—to be unrelated. The test also suggests that "book" is polysemous, as it indeed is. For presumably, on reflection, native speakers judge its meanings *concrete book made of paper* and *abstract work of literature* to be related.

However, the reflection test is limited. As Falkum (2011, 18) points out, native speaker judgments might not be sufficiently robust. Consider the expression "ear," which has the meanings *organ of hearing* and *seed-bearing head or spike of a cereal plant*. Perhaps, even on reflection, some native speakers but not others judge the associated meanings to be distinct. Relatedly, the test is less reliable when the distinct meanings of polysemous expressions are only subtly distinct. Native speakers can miss subtly distinct meanings of certain expressions: they can be unaware that "dog" is polysemous between the meanings *canine* and *male canine*.

What does the reflection test suggest about "says" and "true"? After going through the contextual liar and associated revenge paradoxes, it is clear that, on reflection, we judge "says" and "true" to have related meanings. Thus, the reflection test suggests that "says" and "true" are polysemous.

Etymology test

Another test depends on expressions' etymologies. According to this test, an ambiguous expression is homonymous if its meanings are historically unrelated, and polysemous if its meanings are historically related. This test suggests that "file" is homonymous, as it indeed is.[6] For its meanings are historically unrelated: one meaning, *folder or box for holding loose papers*, comes from the French expression "fil," while the other meaning, *tool with roughened surface* comes from the Old English expression "féol." The test also suggests that "position" is polysemous, as it indeed is. For its meanings, *particular way in which someone or something is placed or arranged* and *person's particular view or attitude toward something*, are historically related via the Old French expression "ponere."

However, the etymology test is also limited. As Falkum (2011, 17) observes, etymology is a fallible guide to the relatedness of meanings—which is what matters for distinguishing between homonymy and polysemy. The test suggests that "cardinal" is polysemous, but it seems to be homonymous. For its meanings, *leader of the Roman Catholic church (who is not the Pope)* and *North American songbird of the bunting family*, are historically related; since male cardinals are mostly red, like the red cassocks worn by Roman Catholic leaders, the bird was named "cardinal" after the Roman Catholic leaders.

What does the etymology test suggest about "says" and "true"? It seems clear that multiple meanings of "says" and "true" are historically related. Given that, as the polysemy proposal maintains, new meanings of "says" and "true" arise due to the combination of widening and metaphor, the multiple meanings of "says" and "true" are historically related. Thus, the etymology test suggests that "says" and "true" are polysemous.

1.3 Four tests for polysemy rather than mere context sensitivity

Vetter and Viebahn (2016) suggest four tests for polysemy rather than mere context sensitivity.[7] Although each of polysemy and context sensitivity can be responsible for shifts in extension, only the former involves shifts in meaning.[8] A merely polysemous expression has distinct constant meanings (extensions), while a merely context-sensitive expression has a single variable meaning that determines distinct contents (extensions)

[6] See Falkum (2011, 16–17) on "file" and "position."

[7] I omit mention of the linguistic test Vetter and Viebahn discuss, since I have already discussed it above as one of the tests for polysemy rather than homonymy.

[8] To maintain consistency throughout, I use the term "extension" rather than Vetter and Viebahn's preferred term, namely, "semantic value." Nothing crucial hinges on this difference.

in distinct contexts. An expression that is both polysemous and context-sensitive has distinct variable meanings, each of which determines distinct contents in distinct contexts.

According to the first test, an expression is polysemous if it has a small number of candidate extensions, and merely context-sensitive if it has a large number of candidate extensions. This test suggests that "book" is polysemous, as it indeed is.[9] For it does not seem that "book" has many candidate extensions beyond the meanings *concrete book made of paper* and *abstract work of literature*. The test also suggests that "I" is merely context-sensitive, as it indeed is. For "I" has as many extensions as there are speakers of English. This test provides limited evidence for the polysemy proposal. For on the polysemy proposal, only some of the actual and potential concepts in the hierarchy are actual concepts of truth. Perhaps *truth*, *truth′*, *truth″*, and a few more are actual, but the remaining are merely potential. Accordingly, "true" may only have a small number of candidate meanings, and so a small number of candidate extensions, even though it may have a large number of *potential* meanings, and so a large number of *potential* extensions. Insofar as the potential meanings are not actual, they are not candidate meanings. But then the test suggests that "true" is polysemous.

According to the second test, an expression is polysemous if its candidate extensions are "clustered," and merely context-sensitive otherwise. Candidate extensions that are "clustered" form natural divisions that do not overlap with one another. This test suggests that "long" is polysemous, as it indeed is. For "long" has one cluster of candidate extensions, consisting of spatial distances, and another cluster of candidate extensions, consisting of temporal distances. The test also suggests that "I" is merely context-sensitive, as it indeed is. For its candidate extensions are not clustered—or, put another way, form only one cluster. This test also provides limited evidence for the polysemy proposal. For on the polysemy proposal, the candidate meanings and so the candidate extensions of "true" form at least two clusters. One cluster of candidate

[9] This example, and all the other examples that illustrate Vetter and Viebahn's tests, are theirs.

extensions naturally corresponds to what we might think of as correspondence truth, while another cluster naturally corresponds to what we might think of as disquotational truth. There seems to be a natural division between the initial concepts *correspondence truth* and *disquotational truth*. So the test suggests that "true" is polysemous.

According to the third test, an expression is polysemous if its candidate extensions are related by explanation or history, and merely context-sensitive otherwise; where the relation is historical, this test resembles the etymology test. This test suggests that "long" is polysemous, as it indeed is. For the meaning *spatially long* is explanatorily and historically prior to the meaning *temporally long*.[10] The test also suggests that "I" is merely context-sensitive, as it indeed is. For its candidate extensions seem historically and explanatorily on a par. This test provides evidence for the polysemy proposal. For on the polysemy proposal, the candidate extensions are related by explanation or history: an initial meaning corresponding to *correspondence truth* gives rise, by a combination of widening and metaphorical extension, to an extended meaning corresponding to *disquotational truth*. Meanings corresponding to concepts in the hierarchy give rise, by a similar process, to further extended meanings corresponding to further concepts. Thus, this test also suggests that "true" is polysemous.

According to the fourth test, an expression is polysemous if some of its candidate extensions occur only in one kind of construction at the level of logical form while others occur only in a different kind of construction at the level of logical form, and merely context-sensitive otherwise. The idea is that merely context-sensitive expressions have only one meaning, where this meaning must be appropriate to the logical type of the expression. This test suggests that "cut" and "run" are polysemous, as they indeed are. For each has distinct but related meanings appropriate to its occurrences as either nouns or verbs. The test also suggests that "I" is merely context-sensitive, as it indeed is. For it has uniform meanings—a function from contexts to speakers—that could not be assigned to occurrences as both, say, nouns and verbs.

[10] Or so Vetter and Viebahn (2016) maintain. The claim is not easy to verify on the basis of etymology alone. See Harper (2016).

Unlike the other three tests, this test does not provide evidence for the polysemy proposal. For on the polysemy proposal, there is no suggestion that any of the candidate meanings, or candidate extensions, can occur only in one kind of construction at the level of logical form. However, as Vetter and Viebahn concede, this test is very limited. Polysemous expressions need not occur as such different logical constructions as nouns and verbs. Perhaps they only occur as mildly different logical constructions as count nouns and mass nouns, as in the cases of "hair" and "cabbage." Indeed, polysemous expressions need not occur as different logical constructions at all. "Long" is polysemous, even though its meanings *spatially long* and *temporally long* are both appropriate to occurrences of "long" as just an adjective. So although this test does not suggest that "true" is polysemous, it does not show that "true" is not polysemous either.

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