Matching Markets with Contracts for Electric Vehicle Smart Charging

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***Abstract*—This paper proposes the novel application of matching markets with contracts for electric vehicle smart charging. The flexibility inherent in electric vehicle charging presents the new opportunity for automation and coordination to shape overall demand. Mechanisms that incentivise coordination are a promising approach for integrating the individual preferences and energy requirements of electric vehicle owners. However, these mechanisms require careful design, since they directly influence power system operation. This paper proposes a new market design, allowing owners of electric vehicles and aggregators to competitively negotiate contracts specifying the time and price of charging at discrete power levels. It is shown that the agents' preferences over contracts satisfy the condition of substitutability, establishing the existence of a stable outcome – an agreed set of contracts no group of agents wish to mutually deviate from. A distributed price-adjustment process is then presented for finding stable outcomes, which only requires local agent decisions and agent-to-agent negotiation. An advantage of the proposed market design is that contracts specify discrete charging power levels, rather than requiring continuously controllable charging powers. This makes it suitable for standard charging infrastructure, and could potentially increase efficiency compared with strategies that operate chargers at low power levels.**

***Index Terms*—Bilateral contracts, discrete optimization, electric vehicle, game theory, market design, matching market, peer-to-peer trading, smart charging, transactive energy.**

I. Introduction

Transport electrification is seen as a key part of energy system decarbonisation [1]. The cost of lithium-ion battery packs has fallen by approximately 77% over the last six years, making electric vehicles increasingly cost-competitive [2].

Uncoordinated electric vehicle charging could cause significant power system challenges. In particular, a significant increase in evening peak demand is expected when drivers arrive home from work and plug in their cars [3]. However, smart charging – coordinated scheduling of the charging time and/or power of electric vehicles – could alleviate the need for generation and transmission infrastructure investments, increase network efficiency and increase energy security. Optimisation strategies for electric vehicle smart charging are presented in [4]–[6]. The aim of these strategies is to shift charging to reduce maximum demand by filling a nighttime demand valley. However, these strategies are not directly applicable when electric vehicles have different owners, since the owners need to be incentivised to charge in a coordinated manner.

Time-of-use energy prices have been proposed to incentivise charging during periods with low expected demand [7]. A limitation is that new worse demand peaks can be created if all electric vehicles take advantage of the same low price periods. In [8], an iterative price-based strategy is proposed, with a central authority that sends price signals based on aggregate demand to electric vehicles. This is extended in [9] to consider vehicles with different constraints, assuming the constraints are drawn randomly from a probability distribution. With a suitable update algorithm, the electric vehicles converge to a Nash Equilibrium – a set of charging schedules no individual can benefit by unilaterally deviating from. With a large number of electric vehicles (each with negligible individual impact) the Nash Equilibrium will minimise a quadratic system cost function, achieving the goal of valley filling.

These strategies have several limitations. First, valley filling is only achieved when a large number of electric vehicles are coordinated together. Second, it is assumed there is a central authority with the aim of minimising system cost. Finally, the strategies are only valid for electric vehicle chargers capable of operating over a continuously controllable range of charging powers, and the electric vehicles tend to converge towards a common normalised profile, with significant time spent at low charging power levels.

In [10], it is noted existing electric vehicle battery charging standards do not provide for continuously controllable charging power levels, and instead are only able to vary between a discrete set of values. In addition, a coordination strategy that primarily adjusts vehicle charging powers, rather than charging times, tends to result in vehicles operating at low power levels for significant periods. However, this can increase losses, due to power converters having low efficiency when operated at low power levels [11]. Discrete optimisation strategies have been proposed to address this issue for electric vehicle fleets with a single owner [12], [13].

Recently, there have been significant developments in the theory of matching markets with contracts, which analyses agent-to-agent negotiation mechanisms, where agents have competing interests and trade non-homogeneous goods [14]–[16]. Key considerations are the existence of stable outcomes – sets of contracts which agents do not wish to mutually deviate from – and mechanisms for finding them. This presents a new mathematical framework for designing a scalable market for electric vehicle smart charging, which needs to account for time-dependent system impacts.

This paper proposes the novel application of matching markets with contracts for electric vehicle smart charging. A new market design is presented, allowing owners of electric vehicles and aggregators to competitively negotiate charging contracts specifying the time and price of charging at discrete power levels. It is shown that the agents' preferences over contracts satisfy the condition of substitutability, establishing the existence of a stable outcome. A distributed price-adjustment process is then presented for finding stable outcomes, which only requires local agent decisions and agent-to-agent negotiation. An advantage of the proposed market design is that contracts specify discrete charging power levels, rather than requiring continuously controllable charging powers.

The rest of this paper is organised as follows. Section II establishes the proposed matching market design. Section III presents the agent preferences. The agent-to-agent price-adjustment process is presented in Section IV. Section V presents a case study verifying the operation of the proposed market design. Section VI concludes the paper. In the Appendix, it is shown that the agent preferences satisfy substitutability, establishing the existence of a stable outcome.

II. Matching Market Design for Smart Charging

The market consists of two types of agents: 1) electric vehicle owners, which need to charge their vehicles by buying energy between their arrival and departure times from home, and 2) aggregators which sell energy to the electric vehicle owners, considering increasing marginal costs of supplying energy (e.g. due to losses or fuel-based generation).

The potential trading relationships are described by a set of contracts between aggregators and electric vehicle owners. Each contract specifies a price for charging an electric vehicle at a discrete power level during a particular interval of time. Note that all contracts have the same power level and interval duration, and therefore are the same size in terms of energy. The contracts are negotiated ahead of time over a particular time period (e.g. one day). In this paper, only forward trading is considered, but in practice an additional real-time reconciliation procedure would be required to account for inaccurate load and electric vehicle usage predictions.

Let $\mathcal{A}$ be a set of agents, let X be a set of contracts between them and let $\mathcal{T} = \{1, \ldots, T\}$ be the set of time intervals during which charging can occur. A contract $x \in X$ is a tuple $(\omega, t_\omega, p_\omega)$, where $\omega \in \Omega$ is the underlying trade, $t_\omega \in \mathcal{T}$ is the time interval of the trade and p_ω is the contract price. For a set of contracts X, let the underlying set of trades be given by $\tau(X) := \{\omega \in \Omega | (\omega, t_\omega, p_\omega) \in X\}$.

Each contract (and trade) has a buyer and a seller. For $x = (\omega, t_\omega, p_\omega)$, the buyer is denoted $b(x) = b(\omega) \in \mathcal{A}$ and the seller is denoted $s(x) = s(\omega) \in \mathcal{A}$. The agents involved with a set of contracts X are given by $a(X) := \{\cup_{x \in X} b(x), s(x)\}$. The contracts and trades agent i is associated with are given by $X_i := \{x \in X | i \in b(x) \cup s(x)\}$ and $\Omega_i := \{\omega \in \Omega | i \in b(\omega) \cup s(\omega)\}$ respectively.

Each agent i has a utility function U_i. The agent's utility function gives rise to a choice correspondence, which specifies the agent's preferences over sets of contracts,

$$C_i(X) = \underset{X_i' \subseteq X_i}{\operatorname{argmax}} \; U_i(X_i'). \tag{1}$$

The complementary rejected set of contracts is given by,

$$R_i(X) = X_i \setminus C_i(X_i). \tag{2}$$

Let X be the full set of contracts in the market. A set of contracts X' is feasible if each trade $\omega \in \tau(X')$ is associated with at most one contract, and therefore has a unique price. A market outcome is a feasible set of contracts, $X' \subseteq X$. An arrangement $[\Psi|p]$ specifies a set of trades $\Psi \subseteq \Omega$ and a vector of prices p for all trades in the market $\omega \in \Omega$. The contracts associated with arrangement $[\Psi|p]$ are given by $\kappa([\Psi|p])$.

A primary market design objective is to create a mechanism that can find a *competitive equilibrium* – a set of prices specified for all potential trades, such that at these prices there is a balance between supply and demand, based on the agents' preferences. A second objective is to ensure the agents agree to a *stable outcome* – a set of contracts no group of agents is incentivised to mutually deviate from and renegotiate. In [14], competitive equilibria and stable outcomes for matching markets with contracts are defined as follows:

Definition 1. A *competitive equilibrium* is an arrangement $[\Psi^*|p^*], \Psi^* \subseteq \Omega$, such that $\kappa([\Psi_i^*|p^*]) = C_i(\kappa([\Omega|p^*]))$ for all $i \in \mathcal{A}$.

Definition 2. An outcome X is *stable* if it is:

1) *Individually Rational*: $X_i \in C_i(X)$ for all $i \in \mathcal{A}$;
2) *Unblocked*: There is no feasible non-empty blocking set $B \subseteq X$. B is a blocking set if: a) $B \cap X = \varnothing$, and b) for all $i \in a(X)$, $B_i \subseteq C_i(B \cup X)$.

A key result from [14] is that when the agents have *substitutable* preferences over contracts, competitive equilibria are guaranteed to exist, and will coincide with stable outcomes. Also, when the agent preferences are not substitutable, stable outcomes and competitive equilibria may not exist.

Definition 3. The preferences of agent i are *substitutable* if, for all contracts $X' \subseteq X$, the agent's choice correspondence satisfies $R_i(X') \subseteq R_i(X)$.

In other words, an agent's preferences are substitutable if, when offered additional contracts, the agent continues to reject all previously rejected contracts.

III. Agent Preferences

This section presents the electric vehicle owner and aggregator preferences. In the Appendix, it is shown that the preferences satisfy substitutability, establishing the existence of a stable outcome.

1) Electric Vehicle Owner Preferences: Let $\mathcal{V} \subseteq \mathcal{A}$ be the set of electric vehicle owners. It is assumed that the electric vehicle owners' preferences are to buy energy to recharge their vehicle at the lowest possible cost, considering their arrival time, departure time and the power limit of their charging infrastructure. Electric vehicle owner i's preferences are described by the following utility function,

$$U_i(X_i) = \begin{cases} -\sum_{(\omega,t_\omega,p_\omega)\in X_i} p_\omega, & \text{if } |X_{ti}| \le e_{ti}^{max} \; \forall\, t \in \mathcal{T} \\ & \text{and } \sum_{t=t_{si}}^{t_{fi}} |X_{ti}| \ge e_i^{req} \\ -\infty, & \text{otherwise,} \end{cases}$$

$$X_{ti} = \{(\omega, t_\omega, p_\omega) \in X_i | t_\omega = t\}, \quad (3)$$

where $t_{si}, t_{fi} \in \mathcal{T}$ are the arrival and departure times ($T \ge t_{fi} \ge t_{si} \ge 1$), $e_i^{req} \ge 0$ is the required number of contracts to recharge the vehicle and $e_{ti}^{max} \ge 0$ is the maximum number of contracts that can be bought in a time interval due to the maximum charging power.

Theorem 1. The electric vehicle owner preferences, described by $U_i, i \in \mathcal{V}$, are substitutable.

The electric vehicle owner preferences can be extended in several ways, for example with a maximum price beyond which contracts will be refused, or with multiple charging periods.

2) Aggregator Preferences: Let $\mathcal{S} \subseteq \mathcal{A}$ be the set of aggregators. It is assumed that the aggregators' preferences are to maximise profits, considering quadratically increasing marginal costs of supplying energy at each time period, which depend on an underlying base load. If the aggregator obtains energy in an upstream market, increasing marginal costs are a feature of the additional power that must be obtained to account for distribution losses [17]. Also, fuel-based generation sources are often modelled with quadratically increasing marginal costs [18].

Aggregator i's preferences are described by the following utility function,

$$U_i(X_i) = \sum_{(\omega,t_\omega,p_\omega)\in X_i} p_\omega - \sum_{t\in\mathcal{T}} (c_{1i}|X_{ti}| + c_{2i}(|X_{ti}| + L_t)^2) \quad (4)$$

where $c_{1i}, c_{2i} \ge 0$ are linear and quadratic cost coefficients and $L_t \in \mathbb{Z}$ is the base load (normalised by the contract energy) for interval $t \in \mathcal{T}$.

Theorem 2. The aggregator preferences, described by $U_i, i \in \mathcal{S}$, are substitutable.

Increasing quadratic marginal supply costs is a common assumption for electric vehicle smart charging strategies [4], [5], [8], [9], [12], [13]. Substitutability will additionally be maintained for any aggregator utility function with increasing marginal costs, and can accommodate maximum power limits, allowing for a wider range of possibilities than valley filling. However, substitutability will not be satisfied if the aggregator has diminishing marginal costs.

IV. Price-Adjustment Process

Since the agents have substitutable preferences, the distributed price-adjustment process from [14] can be used to find a stable outcome. The price-adjustment process is constructed with the electric vehicle owners offering progressively higher priced contracts to aggregators, and aggregators choosing to accept or reject the offers they receive.

Let Δp be the increment of the price-adjustment process, which is the minimum difference in price that can be specified between contracts.

1) Each electric vehicle owner $i \in \mathcal{V}$ starts by specifying the trades it may be willing to buy. Let the full set of trades in the market be given by Ω.
2) Each trade $\omega \in \Omega$ is assigned a buyer price p_ω^b and a seller price p_ω^s. Initially, $p_\omega^b = p_\omega^s = 0$.
3) Iteratively:
   a) Each electric vehicle owner $i \in \mathcal{V}$ constructs a set of contracts to choose from: $X_i = \{\cup_{\omega\in\Omega_i}(\omega, p_\omega^b)\}$. Each aggregator $i \in \mathcal{S}$ constructs a set of contracts to choose from: $X_i = \{\cup_{\omega\in\Omega_i}(\omega, p_\omega^s)\}$.
   b) Each agent $i \in \mathcal{A}$ selects its favourite set of contracts $X_i^* = C_i(X_i)$.
   c) For each trade $\omega \in \Omega_i$, the buyer and seller prices are adjusted as follows:
      if $\omega \in X_{b(\omega)}^*$ and $\omega \in X_{s(\omega)} \setminus X_{s(\omega)}^*$ **then**
        if $p_\omega^b > p_\omega^s$ **then**
          $p_\omega^s \leftarrow p_\omega^s + \Delta p$
        else
          $p_\omega^b \leftarrow p_\omega^b + \Delta p$
        end if
      end if
   d) The price-adjustment process is complete when no price changes occur during an iteration.
4) $[\tau(\cup_{i\in\mathcal{A}} X_i^*)|p^b]$ is a competitive equilibrium, where p^b is the vector of buyer prices for the trades in Ω. This coincides with a stable outcome, $\cup_{i\in\mathcal{A}} X_i^*$.

Note that the price-adjustment process can start at $p_\omega^b = p_\omega^s = \min\{c_{1i}|i \in \mathcal{S}\}$, the lowest price any aggregator would sell a contract for.

V. Results

To verify the operation of the proposed market design, a simulation case study was completed with a single electric vehicle aggregator and 200 electric vehicles. This was completed using a data set made available by the NREL for studying the impact of residential electric vehicle charging [19]. The data set includes synthetic 10 minute resolution electricity demand profiles for 200 households validated using meter data from the Midwest region of the United States, and associated electric vehicle usage data generated using the modelling approach presented in [20]. Fig. 1 shows the charging energy required during a 24 hour period for 200 electric vehicles. It has been assumed that the electric vehicles only charge at home, and have access to 3kW-rated charging infrastructure.

Fig. 2 shows the number of electric vehicles arriving and departing from home during each 10 minute interval over a 24 hour period (12pm to 12pm). These were generated randomly, assuming normal distributions with mean arrival and departure times of 6pm and 8am respectively, and standard deviations of 1.5 hours.

It is assumed the aggregator obtains energy upstream with linear costs of c_1 = \$0.10/kWh and quadratic costs of

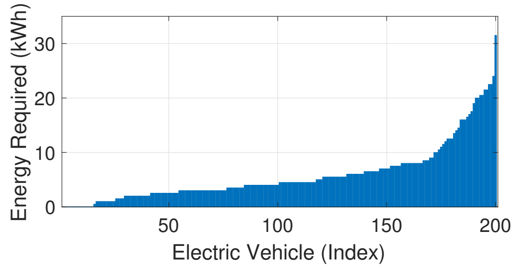

Fig. 1. The charging energy required by each of the 200 electric vehicles over a 24 hour period, ordered from lowest to highest.

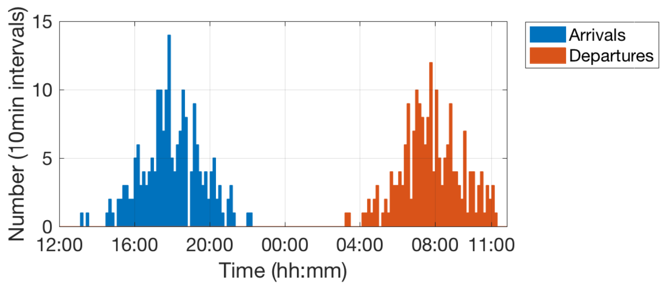

Fig. 2. The number of electric vehicles arriving and departing from home during each 10 minute interval of a 24 hour period (12pm to 12pm).

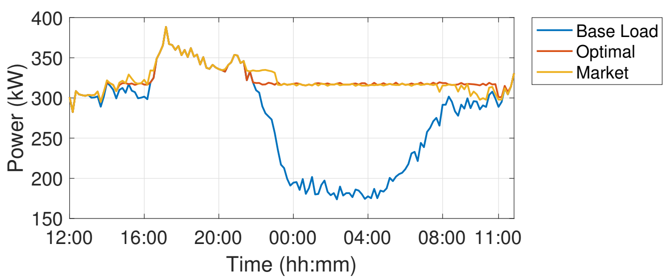

Fig. 3. The base load (the load for the 200 homes without electric vehicle charging), the total load with optimally coordinated charging (using mixed integer quadratic programming) and the total load with charging coordinated by the proposed market design.

$c_2 = \$3.4{\times}10^{-5}/(\text{kWh})^2$ (equivalent to 10% losses at 300kW). The proposed market design is designed with contracts specifying 3kW charging during 10 minute intervals (0.5kWh). The price increment is selected as $\Delta p = \$2{\times}10^{-4}$/kWh.

With a single aggregator, centralised optimisation can provide a baseline to check the performance of the proposed market design. For the optimisation problem, it is assumed the aggregator can freely schedule the electric vehicles (without payments), to minimise its cost function while meeting their charging requirements. This can be formulated as a mixed-integer quadratic program (similar to [13]),

$$\begin{aligned} \underset{e_{ti}, t\in\mathcal{T}, i\in\mathcal{V}}{\text{minimize}} \quad & \sum_{t\in\mathcal{T}} \Big(c_1 \sum_{i\in\mathcal{V}} e_{ti} + c_2 \big(\sum_{i\in\mathcal{V}} e_{ti} + L_t \big)^2 \Big) \qquad (5) \\ \text{subject to} \quad & e_{ti} \le e_{ti}^{max}, e_{ti} \in \mathbb{Z}_{\ge 0}, \ \forall i \in \mathcal{V}, t \in \mathcal{T} \\ & \sum_{t=t_{si}}^{t_{fi}} e_{ti} \ge e_i^{req}, \ \forall i \in \mathcal{V}, \end{aligned}$$

where e_{ti} is the number of contract-sized energy units assigned to electric vehicle owner i for time interval t.

Fig. 3 shows the base load (the load for the 200 homes without electric vehicle charging), the total load when charging is optimised by the mixed-integer quadratic program and the total load under the proposed market design. The proposed market design gives a load profile that is very close to the one obtained through centralised optimisation, and in both cases there is no increase in the maximum demand.

The size of the price increment introduces a trade-off between the required number of iterations and the optimality of the market outcome. With the selected price increment of $\$2{\times}10^{-4}$/kWh, the price-adjustment process requires 3200 iterations, and the market outcome gives 0.0368% higher losses than the optimal solution. During each iteration, the agents require at most 0.002s to select their preferred contracts, and these decisions can be made in parallel. This opens the possibility for a high speed implementation using parallel processing.

VI. Conclusion

A new market design has been presented for incentivising coordinated electric vehicle smart charging. Using agent-to-agent negotiation and local decision making, electric vehicle owners and aggregators are able to agree on a set of discrete charging contracts specifying charging times and prices, which no group of agents wish to mutually deviate from. A simulation case study has been presented, showing the market outcome closely approximates the charging profile resulting from a centralised optimisation strategy.

Future work will be needed to ensure the market design is suitable for practical implementation. In particular, a real-time reconciliation procedure will be required to deal with situations when contracts are not respected/honoured (e.g. if a vehicle departs earlier than expected). One promising approach could be to implement the market with a receding time horizon, so that contracts can be renegotiated if load or electric vehicle usage predictions are deviated from.

Appendix

In this appendix, it is shown that the agent preferences from Section III satisfy substitutability. The following definition for an agent's indirect utility function, from [16], is needed.

Definition 4. The indirect utility function of agent i is given by $V_i(p) := \max_{\Psi\subseteq\Omega_i}\{U_i(\kappa([\Psi|p]))\}$.

The preferences of an agent are substitutable, if and only if they induce a submodular indirect utility function V_i. To show V_i is submodular, it is enough to show that for any two trades $\phi, \psi \in \Omega_i$ and prices $p_{\Omega_i'} \in \mathbb{R}^{|\Omega_i \setminus \{\psi,\phi\}|}$, $p_\phi^h \ge p_\phi$, $p_\psi^h \ge p_\psi$ [16],

$$\begin{aligned} V_i(p_{\Omega_i'}, p_\phi, p_\psi^h) - V_i(p_{\Omega_i'}, p_\phi^h, p_\psi^h) \ge \\ V_i(p_{\Omega_i'}, p_\phi, p_\psi) - V_i(p_{\Omega_i'}, p_\phi^h, p_\psi). \qquad (6) \end{aligned}$$

The following contracts are introduced,
$x_\phi := (\phi, t_\phi, p_\phi), x_\psi := (\psi, t_\psi, p_\psi), x_\phi^h := (\phi, t_\phi, p_\phi^h), x_\psi^h := (\psi, t_\psi, p_\psi^h)$.

Proof of Theorem 1. Let $X''_i, X'_{\phi i}, X'_{\psi i}, X_i$ be sets of contracts, such that $x_\psi, x_\phi, x^h_\psi, x^h_\phi \notin X_i$, $X''_i \subseteq X'_{\psi i} \subseteq X_i$, $X''_i \subseteq X'_{\phi i} \subseteq X_i$ and $|X_i| = |X'_{\phi i}|+1 = |X'_{\psi i}|+1 = |X''_i|+2$. Since each contract has the same energy unit and a non-negative price, (3) implies that the electric vehicle owners always select e^{req}_i contracts, with time intervals between t_{si} and t_{fi}, such that the maximum number of contracts for each interval is less than or equal to e^{max}_{ti}. Therefore, to show (6) is satisfied, it is sufficient to show the following condition holds.

$$U_i(X'_{\phi i}, x_\phi) - U_i(X_i) \geq U_i(X''_i, x_\phi, x_\psi) - U_i(X'_{\psi i}, x_\psi), \tag{7}$$

Two cases need to be considered.

Case 1: $X'_{\phi i} = X'_{\psi i}$. Let $h_1 \in X'_i$ and $h_1 \notin X'_{\phi i}$, and let $h_2 \in X'_{\phi i}$ and $h_2 \notin X''_i$. Condition (7), results in the requirement that $p_{h_1} \geq p_{h_2}$, which is true, since h_2 is accepted when h_1 is rejected.

Case 2: $X'_{\phi i} \neq X'_{\psi i}$. In this case, (7) results in an equality. □

Proof of Theorem 2. Let $X''_i, X'_{\phi i}, X'_{\psi i}, X_i$ be sets of contracts, such that $x_\psi, x_\phi, x^h_\psi, x^h_\phi \notin X_i$, $X''_i \subseteq X'_{\psi i} \subseteq X_i$, $X''_i \subseteq X'_{\phi i} \subseteq X_i$, $|X_i| \leq |X'_{\psi i}| + 1 \leq |X''_i| + 2$ and $|X_i| \leq |X'_{\phi i}|+1 \leq |X''_i|+2$. From (4), the marginal utility for accepting a contract only depends on its price and the number of previously accepted trades with the same time interval. Therefore, to show (6) is satisfied, it is sufficient to show the following condition holds.

$$U_i(X'_{\psi i}, x^h_\psi) - U_i(X''_i, x^h_\phi, x^h_\psi) \geq U_i(X_i) - U_i(X'_{\phi i}, x^h_\phi), \tag{8}$$

Five cases need to be considered:

Case 1: $|X_i| = |X'_{\phi i}| = |X_{\psi i}| = |X''_i|$. If $t_\phi \neq t_\psi$, then (8) results in an equality. If $t_\phi = t_\psi$, (8) requires,

$$c_{2i}(|X_{t_\phi i}| + 2 + L_{t_\phi})^2 - c_{2i}(|X_{t_\phi i}| + 1 + L_{t_\phi})^2 \geq c_{2i}(|X_{t_\phi i}| + 1 + L_{t_\phi})^2 - c_{2i}(|X_{t_\phi i}| + L_{t_\phi})^2, \tag{9}$$

which is true for all $c_{2i} \geq 0$, $|X'_{t_\phi i}| \in \mathbb{Z}_{\geq 0}$ and $L_{t_\phi} \in \mathbb{Z}$.

Case 2: When $|X'_{\psi i}| = |X_i|$ and $|X'_{\phi i}| = |X''_i| = |X_i| - 1$. In this case, $t_\phi \neq t_\psi$ (since $|X_{\phi i}| \neq |X_{\psi i}|$), and condition (8) results in an equality.

Case 3: $|X'_{\phi i}| = |X_i|$ and $|X'_{\psi i}| = |X''_i| = |X_i| - 1$. In this case, $t_\phi \neq t_\psi$ (since $|X'_{\phi i}| \neq |X'_{\psi i}|$), and condition (8) results in an equality.

Case 4: $|X'_{\psi i}| = |X'_{\phi i}| = |X_i| - 1$ and $|X''_i| = |X_i| - 2$. For both $t_\phi \neq t_\psi$ and $t_\phi = t_\psi$, (8) results in an equality.

Case 5: $|X'_{\psi i}| = |X'_{\phi i}| = |X_i|$ and $|X''_i| = |X_i| - 1$. In this case, $t_\phi = t_\psi$ (since $|X''_i| = |X_i| - 1$). Condition (8) requires,

$$p_l - c_{1i} - c_{2i}(|X_{t_\phi i}| + 1 + L_{t_\phi})^2 + c_{2i}(|X_{t_\phi i}| + L_{t_\phi})^2 \geq 0, \tag{10}$$

where (l, t_ϕ, p_l) is the lowest price contract in $X_{t_\phi i}$. Since $(l, t_\phi, p_l) \in X_{t_\phi i}$ and $U_i(X'_{\phi i} \cup x_\phi) \geq U_i(X''_i \cup x_\phi)$, the contract $(l, t_\phi, p_l) \in X_{t_\phi i}$ must provide a net increase in utility when there are $|X_{t_\phi i}|$ other accepted contracts for time interval t_ϕ. Therefore, (10) is true. □

References

[1] M. Tran, D. Banister, J. D. K. Bishop, and M. D. McCulloch, "Realizing the electric-vehicle revolution," *Nature Climate Change*, vol. 2, no. 5, pp. 328–333, 2012.

[2] McKinsey, "Electrifying insights: How automakers can drive electrified vehicle sales and profitability," *Advanced Industries*, pp. 1–28, 2017.

[3] A. Lojowska, D. Kurowicka, G. Papaefthymiou, and L. Van Der Sluis, "From transportation patterns to power demand: Stochastic modeling of uncontrolled domestic charging of electric vehicles," *IEEE Power and Energy Society General Meeting*, pp. 1–7, 2011.

[4] S. Sojoudi and S. H. Low, "Optimal charging of plug-in hybrid electric vehicles in smart grids," *2011 IEEE Power and Energy Society General Meeting*, pp. 1–6, 2011.

[5] Y. He, B. Venkatesh, and L. Guan, "Optimal scheduling for charging and discharging of electric vehicles," *IEEE Transactions on Smart Grid*, vol. 3, no. 3, pp. 1095–1105, 2012.

[6] E. Sortomme, M. M. Hindi, S. D. J. MacPherson, and S. S. Venkata, "Coordinated Charging of Plug-In Hybrid Electric Vehicles to Minimize Distribution System Losses," *IEEE Transactions on Smart Grid*, vol. 2, no. 1, pp. 198–205, mar 2011.

[7] H. Liu and S. Ge, "Optimization of TOU price of electricity based on Electric Vehicle orderly charge," *IEEE Power and Energy Society General Meeting*, 2013.

[8] Z. Ma, D. S. Callaway, and I. A. Hiskens, "Decentralized Charging Control of Large Populations of Plug-in Electric Vehicles," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 1, 2013.

[9] L. Deori, K. Margellos, and M. Prandini, "On the connection between Nash equilibria and social optima in electric vehicle charging control games," *20th World Congress of the International Federation of Automatic Control*, no. 2017, pp. 1–6, jul 2016.

[10] Q. Wang, X. Liu, J. Du, and F. Kong, "Smart Charging for Electric Vehicles: A Survey from the Algorithmic Perspective," *IEEE Communications Surveys and Tutorials*, vol. 18, no. 2, pp. 1500–1517, 2016.

[11] K. N. Kumar and K. J. Tseng, "Efficiency evaluation of coordinated charging methods used for charging electric vehicles," *IEEE PES Innovative Smart Grid Technologies Conference Europe*, pp. 270–275, 2016.

[12] L. Gan, U. Topcu, and S. H. Low, "Stochastic distributed protocol for electric vehicle charging with discrete charging rate," *IEEE Power and Energy Society General Meeting*, pp. 1–10, 2012.

[13] R. Wang, G. Xiao, and P. Wang, "Hybrid Centralized-Decentralized (HCD) Charging Control of Electric Vehicles," *IEEE Transactions on Vehicular Technology*, vol. 66, no. 8, pp. 6728–6741, 2017.

[14] J. W. Hatfield, S. D. Kominers, A. Nichifor, M. Ostrovsky, and A. Westkamp, "Stability and Competitive Equilibrium in Trading Networks," *Journal of Political Economy*, vol. 121, no. 5, pp. 966–1005, Oct. 2013.

[15] T. Fleiner, Z. Janko, A. Tamura, and A. Teytelboym, "Trading networks with bilateral contracts," in *Proceedings of the The Third Conference on Auctions, Market Mechanisms and Their Applications*. ACM, 2015, pp. 1–39.

[16] J. W. Hatfield, S. D. Kominers, A. Nichifor, M. Ostrovsky, and A. Westkamp, "Full Substitutability in Trading Networks," in *Proceedings of the Sixteenth ACM Conference on Economics and Computation - EC '15*. New York, New York, USA: ACM Press, 2015, pp. 39–40.

[17] H.-P. Chao and S. Peck, "A Market Mechanism For Electric Power Transmission," *Journal of Regulatory Economics*, vol. 10, pp. 25–59, 1996.

[18] D. Gregoratti and J. Matamoros, "Distributed Energy Trading: The Multiple-Microgrid Case," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 4, pp. 2551–2559, Apr. 2015.

[19] M. Muratori, "Impact of uncoordinated plug-in electric vehicle charging on residential power demand - supplementary data," *National Renewable Energy Laboratory*, 2017. [Online]. Available: https://data.nrel.gov/submissions/69

[20] M. Muratori, M. J. Moran, E. Serra, and G. Rizzoni, "Highly-resolved modeling of personal transportation energy consumption in the United States," *Energy*, vol. 58, pp. 168–177, 2013.