Aspects of wave dynamics and statistics on the open ocean

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A thesis submitted for the degree of

Doctor of Philosophy

Trinity Term, 2009

Abstract

Water waves are an important design consideration for engineers wishing to design structures in the offshore environment. Designers need to know the size and shape of the waves which any structure is likely to encounter. Engineers have developed approaches to predict these, based on a combination of field and laboratory measurements, as well theoretical analysis. However some aspects of this are still poorly understood; in particular there is growing evidence that there are rare 'freak' waves which do not fit with our current understanding of wave physics or statistics.

In the first part of this thesis a new approach is developed for measuring the directional spreading of a sea-state, when the free surface time-history at a single point is the only available information. We use the magnitude of the second order 'bound' waves to infer this information. This is validated using fully non-linear simulations, for random waves in a wave-basin, and for field data recorded in the North Sea. We also apply this to the famous Draupner wave, which our analysis suggests was caused by two wave systems, propagating at approximate $120°$ to each other.

The second part of the thesis looks at the non-linear evolution of Gaussian wave-groups. Whilst much work has previously been done to investigate these numerically, we instead derive an approximate analytical model for describing the non-linear changes to the group, based on the conserved quantities of the non-linear Schrödinger equation. These are validated using a numerical model. There is excellent agreement for uni-directional waves. The analytical model is generally good for predicting change in shape of directionally spread groups, but less good for predicting peak elevation. Nevertheless, it is still useful for typical sea-state parameters.

Finally we consider the effect of wind on the local modeling of extreme waves. We insert a negative damping term into the non-linear Schrödinger equation, and consider the evolution of 'NewWave' type wave-groups. We find that energy input accentuates the non-linear dynamics of wave-group evolution which suggests it may be important in the formation of 'freak' waves.

Acknowledgements

Firstly, I would like to express my gratitude to my supervisor, Prof. Paul Taylor, for his support and encouragement throughout this work. His knowledge and enthusiasm for his subject, as well as his dedication to his students' learning, is what made this work possible. Further, I must thank him for his patience when I have made mistakes (particularly with my famously atrocious spelling) and for his tolerance of my not infrequent absences from the lab when I have been off running New College MCR or playing sport.

I also thank Prof. Rodney Eatock Taylor who read and gave valuable advice on the papers which form chapters 3 and 4. Prof. Alistair Borthwick co-supervised me for the first term and has been supportive throughout this work.

I would like to thank those who have provided data which is analysed in this work. Dr. Richard Gibbs gave me access to the results of the fully non-linear simulations he carried out whilst at Oxford. Dr. Kevin Ewans (Shell) provided the data from the Canadian Hydraulics Centre. Dr. Sverre Haver (Statoil) provided the data recorded at the Draupner platform.

This work was funded by an EPSRC departmental studentship and, in its final stages, by New College.

Various friends have helped keep my sanity in tact over the past few years – principally by partaking of various caffeinated and alcoholic beverages with me. In particular I would like to thank: Chris 'Gonzo' Smith, Andrew McLennan, Bex Inglis, Dan Thomas, 'Oz', Natalie Lundsteen, Jana Orszaghova, Chris 'Wolfie' Smith, Dominique van de Sompel, Lisa Kwan and Kai-Ting Chuang.

Finally, and most importantly, I thank my family – my Mother and David Wood – for their invaluable support throughout this undertaking.

Contents

Nomenclature

Symbol		Units
α	A damping parameter	s^{-1}
β	The amount of energy input per period	
γ	JONSWAP peak parameter	
ϵ	Steepness of the sea-state	
$\epsilon_{1,2}$	Perturbation parameters	
η	Free surface	m
$\bar{\eta}$	Mean shape of largest wave	m
$\hat{\eta}$	Hilbert transform of free surface	m
η_2^+	Free surface of second order sum components	m
η_2^-	Free surface of second order difference components	m
η_{crest}	Free surface recorded for crest focused run	m
η_{free}	Free surface of the freely propagating (linear) components	m
η_{trough}	Free surface recorded for crest focused run	m
θ	Angle relative to mean wave direction	degrees
Θ	Angle between interacting wave components	degrees
κ^+	Interaction kernel for second order sum terms	
κ^-	Interaction kernel for second order difference terms	
Λ	Scaling parameter	
ν	A discretisation parameter	

Symbol		Units
ξ	Relative phase of components	
σ_η	Standard deviation of the free surface	m
σ_θ	RMS spreading parameter	degrees
ς_x	The spatial bandwidth in the mean wave direction	m^{-1}
ς_x	The maximum spatial bandwidth in the mean	
	wave direction were the evolution linear	m^{-1}
τ	Non-linear timescale	
ϕ	Phase – given by $\omega t + \xi$	
Φ	Velocity potential	$m^2 s$
χ	A small error in the amplitude of a component	m
ω	Natural frequency	rad s^{-1}
ω_0	Natural frequency of carrier wave	rad s^{-1}
a	Wave amplitude, or amplitude of wave component	m
a_{lin}	The maximum amplitude of a group under linear evolution	m
a_{lin+}	The maximum amplitude of a group under linear	
	evolution but with energy input	m
a_{max}	The maximum amplitude during a run	m
A	Non-dimensional amplitude	
A_f	Non-dimensional amplitude at focus	
A_{s1}	Amplitude of the first solition produced	
A_{s2}	Amplitude of the second solition produced	
A_∞	Sum of non-dimensional amplitude components	
	of a fully dispersed group	
b_x	Constant related to the linear dispersion of a	
	Gaussian in x direction	s^{-1}
b_y	Constant related to the linear dispersion of a	

Symbol		Units
	Gaussian in y direction	s^{-1}
BF-index	Benjamin Feir index	
d	Water depth	m
D	Directional distribution of energy	
f	Frequency	s
Δf	Frequency bin	s
g	Acceleration due to gravity	$m\,s^{-1}$
H_s	Significant wave-height	m
i	$\sqrt{-1}$	
$I2$	A conserved quantity of the NLSE, equivalent to energy	
$I4$	A conserved quantity of the NLSE,	
	from conservation of the Hamiltonian	
$I6$	A conserved quantity of the NLSE	
$I6mod$	A modification of $I6$	
k	Wavenumber	m^{-1}
k_0	Wavenumber of carrier wave	m^{-1}
k_p	Peak wavenumber	m^{-1}
K	A constant	
m_n	The n_{th} spectral moment	m^2s^{-n}
q	Complex wave envelope for form of NLSE	
R	Ratio of bandwidths in X and Y given by $S_{X\infty}/S_{Y\infty}$	
s_i	Spreading index	
s_x	Bandwidth of wavenumber spectrum	
	in mean wave direction	m^{-1}
s'_ω	Normalised bandwidth of frequency spectrum	
$s_{x,lin}$	Bandwidth of wavenumber spectrum	

Symbol		Units
	in x direction for linear evolution	m^{-1}
s_y	Bandwidth of wavenumber spectrum in y direction	m^{-1}
$s_{y,lin}$	Maximum bandwidth of wavenumber spectrum	
	in y direction were evolution linear	m^{-1}
S_η	Spectral density	m^2s
S	Non-dimensional bandwidth of wavenumber spectrum	
S_f	Non-dimensional bandwidth at focus of wavenumber spectrum	
S_∞	Non-dimensional bandwidth of wavenumber	
	spectrum for completely dispersed group	
S_X	Non-dimensional bandwidth of wavenumber	
	spectrum in X direction	
S_{Xf}	Non-dimensional bandwidth at focus in X	
	direction of wavenumber spectrum	
$S_{X\infty}$	Non-dimensional bandwidth of wavenumber spectrum	
	in X direction for completely dispersed group	
S_Y	Non-dimensional bandwidth of wavenumber	
	spectrum in Y direction	
S_{Yf}	Non-dimensional bandwidth at focus in Y direction of	
	wavenumber spectrum	
$S_{Y\infty}$	Non-dimensional bandwidth of wavenumber spectrum	
	in Y direction for completely dispersed group	
SNR	Signal to noise ratio	
t	Time	s
T	Non-dimensional time	
T_p	Peak period	s
T_z	Period of zero up-cross	s

Symbol		Units
u	Complex wave envelope	m
U	Non-dimensional complex wave envelope	
U_{max}	The absolute maximum of the complex wave envelope over a run	
x	Horizontal co-ordinate in mean wave direction	m
X	Non-dimensional horizontal co-ordinate in mean wave direction	
y	Horizontal co-ordinate perpendicular to x	m
Y	Non-dimensional horizontal co-ordinate perpendicular to X	
z	Vertical co-ordinate relative to undisturbed free surface	m

Chapter 1

Introduction

1.1 The ocean environment

1.1.1 Surface gravity waves

In the ocean, waves are generated by a number of physical phenomena. Tides are caused by the gravitational pull of the moon and the sun; tsunamis are caused by the rapid displacement of water, often caused by tectonic activity or landslides; and gravity waves which are generated by wind, and which are the subject of this thesis. For all these waves, the restoring force is the earth's gravity, except for the very shortest waves where surface tension becomes significant. These different types of waves have different characteristic periods from each other (see table in Mei (1989), page 2) and can often be considered separately from each other.

Wind generated waves are non-deterministic. Wind generated waves may be combination of locally generated waves, and 'swell' waves, generated some time earlier, possibly thousands of miles away which have propagated across the ocean to the present location. As waves propagate into shallower water and into the coastal regime, their properties change, but this aspect of ocean wave science is not the subject of this thesis.

1.1.2 Offshore engineering

The ocean covers roughly 70% of the earth's surface and, as engineers, we need to devise ways to travel across it and exploit its resources. For millenia ships have been designed to venture into the most hostile marine enviroments. More recently, we have wished to place more permanent structures in deep water, primarily for the extraction of hydrocarbons. There is also growing interest in offshore windfarms, and marine renewables, which may be located in intermediate water depths. More exotic ideas for development in the offshore enviroment are proposals like offshore military bases.

Wind generated waves are an important design factor for any engineering project in the offshore environment. Waves are typically the dominant environmental load for both ships and offshore structures (Tucker & Pitt (2001)). Engineers need to know size and shape of waves their structures are likely to encounter at a particular location, and how the structure will respond. This thesis looks at aspects connected with the first of these.

1.1.3 Brief history of ocean wave science

Ever since man ventured out of sheltered coastal waters, and out into the open sea, which they must have done $40,000$ to $60,000$ years ago when Australia was first inhabited, there must have been interest in waves and designing for these. As mankind developed, so did man's ambition for exploration and trade, which meant designing larger and more advanced ships which could cope with more onerous marine conditions.

The scientific study of water waves commenced once the mathematical tools were in place to derive and analyse the governing equations. Important work by Poisson (1816) and Cauchy (1827) developed the understanding of waves in infinitely deep water. Airy (1845) and Stokes (1847) carried out the classic analysis describing the properties of regular waves on finite depth, and knowledge and understanding of wave motions grew. See Craik (2004) for a fuller account of the development of water wave theory.

Gradually, this scientific understanding started to be applied to engineering problems. Sainflou (1928) used second order theory to calculate the forces on a vertical breakwater. The need for amphibious landings during the Second World War was one of the major driving forces which led to increased investigation of ocean waves. Penny & Price (1952) did early work on diffraction, out of analysis of the Mulberry Harbours in the Second World War. The early work of Munk (Sverdrup & Munk (1946)) arose from work on wave prediction in the Pacific theatre of war.

Work continued after the War, and progress started to be made at taking into account the non-deterministic nature of ocean waves. Data of sufficient quality for scientific study started to be recorded (Sverdrup & Munk (1946) and Barber & Ursell (1948)), who identified where waves reaching a Cornish beach originated – in one example they identified these as having been generated by a storm in the Southern Ocean. Statistical models were then developed to explain the distribution of wave-heights in these data, such as by Longuet-Higgins (1952).

An important development was the appreciation of a wave spectrum and its use to characterise a sea-state. The theory for the non-linear evolution of the spectrum was developed by Hasselmann (1962) based on the work of Phillips (1960). Advances on this theory, coupled with a largely empirical understanding of how wind and waves interact, is now used to predict the offshore wave-climate.

The requirement in the 1960s and 70s to develop offshore oil and gas reserves lead to increased interest in ocean waves, and a significant increase in the quantity and quality of field measurements. Important developments included the incorporation of 'second order' non-linearity into the statistics describing wave amplitude (Tayfun (1980)). This reconciled the vast majority of field observations with theory. However, there is some evidence pointing to the existence of waves which do not fit with such models, so called 'freak' or 'rogue' waves (see 2.5). Recently, much work has gone into investigation of these (see for instance Olagnon & Athanassoulis. (2000), Olagnon & Prevosto (2004)).

The localised modeling of the evolution of water waves has advanced largely at the same rate as the available computing power, since the non-linear evolution equations (see section 2.4) generally need to be solved numerically. Only very recently has it become possible to model random sea-states using fully non-linear schemes, but issues such as the local effect of wind and wave-breaking are still unresolved.

1.2 Thesis overview

This thesis has two main parts. In the first part, a new methodology is developed for analysing field data. The second part consists of analytical and numerical modeling of the non-linear evolution of wave-groups. The unifying theme throughout this thesis is the effect of non-linearity on the local properties of waves.

In chapter 2 the standard theory used to model water waves, and used throughout this thesis is reviewed. The parameters used to describe a sea-state are presented and we consider different evolution equations for water waves. A comparison is made between the non-linear Schrödinger equation (NLSE) and the full water wave equations. Finally, we review the topic of 'freak' waves.

Chapter 3 develops a method for estimating the directional spread of waves when the only available information is a single point surface elevation time-history. This uses the low frequency 'second-order difference' waves. We apply this to fully non-linear simulations of wave-groups, to random waves in a wave-basin, and to field data. In all cases our results are in agreement with other measurements.

In the next chapter we extend the approach in chapter 3 to look at the Draupner wave. One of the unusual aspects of this wave is the low-frequency set-up beneath the giant crest. We show that this is consistent with the wave being formed by two wave-groups whose mean directions are $\sim 120^\circ$ apart. Other evidence is consistent with this conclusion.

In chapter 5 we consider the non-linear focusing uni-directional NewWave type wave-groups. We use the conserved quantities of the NLSE to derive an approximate analytical model for the evolution of the wave-groups, by making the assumption that the group remains close to Gaussian in form. We find that evolution is dependent on the amplitude to bandwidth ratio of the group, and that groups over a limiting non-linearity cannot be formed from an initially dispersed group. We validate the analytics using a numerical scheme and find good agreement in nearly all cases.

This is then extended to directionally spread waves in chapter 6. We take the same approach for deriving the analytical approximation, but have to use a new, pseudo-conserved quantity which is validated numerically. Again we find a limiting non-linearity for a group which is accessible from an initially dispersed group. We compare the analytics to numerical simulations: we observe reasonable agreement for fairly symmetric groups which are mildly non-linear, but the approach is not applicable to groups with high levels of asymmetry.

Finally, in chapter 7, we consider the effect of wind on the evolution of wave-groups. Rather than try and model the complexities of wind/wave interaction we introduce an excitation term into the NLSE which puts energy into the system. We show that this accentuates the non-linear dynamics of wave-group evolution.

Chapter 2

Theory and literature review

2.1 Introduction

In this chapter, some of the basic ideas and equations used in this study of ocean waves are set out and discussed. Parts of this are considered in further detail in subsequent chapters.

2.2 Water wave theory

2.2.1 Water wave equations

The standard approach to modeling water waves makes the following assumptions:

1. The flow is irrotational.

2. Water is incompressible, inviscid and of constant density.

3. Surface tension is negligible.

4. There is no current other than flows caused by the waves.

5. There is no interaction between waves and the wind.

The first of these allows us to use a potential flow solution, where the velocity is given by gradient of a velocity potential function $\Phi(x, y, z, t)$. The second assumption means that to satisfy continuity the velocity potential must satisfy Laplace's equation

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0. \tag{2.1}$$

If the z axis is vertical we can define the undisturbed free surface elevation as $z = 0$, and the disturbed free surface as $\eta = (x, y, t)$. We now apply the boundary conditions. The condition that there must be no flow across the free surface gives

$$\frac{\partial \Phi}{\partial z} = \frac{\partial \eta}{\partial t} + \frac{\partial \Phi}{\partial x}\frac{\partial \eta}{\partial x} + \frac{\partial \Phi}{\partial y}\frac{\partial \eta}{\partial y} \qquad \text{on } z = \eta. \tag{2.2}$$

The pressure on the free surface is assumed to be a constant. Therefore, we can apply Bernoulli's equation to the free surface

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}\left|\nabla \Phi\right|^2 + g\eta = 0 \qquad \text{on } z = \eta. \tag{2.3}$$

Finally, there must be no flow through the sea-bed, which is at $z = -d$, a constant distance below the undisturbed free surface

$$\left.\frac{\partial \Phi}{\partial z}\right|_{z=-d} = 0 \tag{2.4}$$

The difficulty in solving these equations arises from the free surface boundary conditions (equations 2.2 and 2.3) which must be evaluated at the free surface, which is not known *a priori*.

2.2.2 Linear waves

To solve the water wave equations, we may expand the free surface boundary conditions using a Taylor series around $z = 0$. If we retain only the first term in the Taylor series, the problem becomes linear, and has a solution

$$\eta = a \cos\left(\mathbf{k}.\mathbf{x} + \mathbf{k}.\mathbf{y} - \omega t + \xi\right), \tag{2.5}$$

where ξ gives the phase and the wavenumber, $|\mathbf{k}|$ and frequency, ω are related by the linear dispersion relation

$$\omega^2 = |\mathbf{k}|\, g \tanh\left(|\mathbf{k}|\, d\right). \tag{2.6}$$

2.2.3 Stokes waves

Simple first order theory was extended by Stokes (1847), in the format of a power series solution in wave steepness ($a\,|\mathbf{k}|$), to the third order for arbitrary depth and to fifth order for deep-water (where $d \to \infty$, which can be applied in practice when $\tanh\left(|\mathbf{k}|\, d\right) \to 1$). For the regular progressive wave in deep water, propagating in the $\mathbf{x}$ direction, a solution is

$$\begin{aligned} \eta \quad = \quad & \left(a - \frac{1}{8} a^3 \left|\mathbf{k}\right|^2\right) \cos\left(\mathbf{k}.\mathbf{x} - \omega t + \xi\right) + \\ & \frac{a^2 \left|\mathbf{k}\right|}{2} \cos\left(2\left(\mathbf{k}.\mathbf{x} - \omega t + \xi\right)\right) + \\ & \frac{3}{8} a^3 \left|\mathbf{k}\right|^2 \cos\left(3\left(\mathbf{k}.\mathbf{x} - \omega t + \xi\right)\right). \end{aligned} \tag{2.7}$$

This has the effect of making crests taller and troughs flatter. This may be extended to higher orders, see for instance Fenton (1985).

The dispersion relation for a regular wave is modified at higher orders. To the third order, for deep water, it is

$$\omega^2 = |\mathbf{k}|\, g \left(1 + a^2 |\mathbf{k}|^2 + O\left(a^4 |\mathbf{k}|^4\right)\right). \tag{2.8}$$

This means that tall waves will travel faster than small waves. Thus the evolution of water waves is no longer linear.

If multiple 'linear' frequencies are present, then they will interact to give 'bound' waves, which occur, to the second order, at the sum and difference of the frequency and wavenumber of the interacting waves. These interactions were calculated for deep water by Longuet-Higgins (1963) and for finite depth by Dean & Sharma (1981), although this contains some errors which were corrected in Dalzell (1999) and Forristall (2000). Further corrections for third order effects are given in Janssen (2008). The kernels are discussed later and are given in 3.3.1. The effect of these is interactions is again to make crests steeper and troughs flatter. These interactions form the basis for the work in Chapters 3 and 4.

2.3 Sea-states

2.3.1 Omnidirectional wave spectra

Engineers generally assume that over a short period of time, of the order of hours, that the statistical parameters describing waves remain stationary in time. The distribution of wave-height, and other quantities of interest, may then be estimated. Of particular use in describing a sea-state is the wave-spectrum, from which other parameters may then be derived.

If the free-surface time-history at a point is recorded, then it can be written as the sum of sinusoids

$$\eta(t) = \sum_n a_n \cos(\phi_n), \tag{2.9}$$

where

$$\phi_n = \omega_n t + \xi_n, \tag{2.10}$$

where ξ_n gives the relative phase of the component. The omnidirectional spectral density function is then given by

$$S_\eta(f_n) = \frac{1}{2} a_n^2 \Delta f, \tag{2.11}$$

where Δf is the size of the frequency bin used. $S_\eta(f)$ remains finite as $\Delta f \to 0$. A typical spectrum recorded for a 20 minute storm is shown in figure 3.1. From the spectral density function a number of parameters may be found by taking moments (see Tucker & Pitt (2001)), where the n^{th} moment is defined by

$$m_n = \int_0^\infty f^n \, S_\eta(f) \mathrm{d}f. \tag{2.12}$$

Of particular use are the significant wave-height,

$$H_s = 4\sqrt{m_0}, \tag{2.13}$$

and, for a narrow-banded random process, 'zero-crossing period', the average period of the zero-upcross waves, see Rice (1944)

$$T_z = \sqrt{\frac{m_0}{m_2}}. \tag{2.14}$$

The variance of the sea-surface displacement with time is

$$\sigma_\eta^2 = m_0. \tag{2.15}$$

A number of standard spectra may be fitted to measured data, or used to model a sea-state. In this work a JONSWAP spectrum is used (see Hasselmann et al.

(1976)). For some of the numerical simulations we approximate this using a Gaussian wavenumber spectrum as done in Gibbs & Taylor (2005).

$$S_\eta(k) = \exp\left(\frac{-(k-k_p)^2}{2s_x^2}\right) \tag{2.16}$$

where k_p is the peak wavenumber and s_x is the bandwidth parameter. This can be converted into a frequency spectrum (using the dispersion relation for deep water) to give

$$S_\eta(f) = \frac{8\pi^2 f}{g}\exp\left(\frac{-8\pi^4\left(f^2-f_p^2\right)^2}{s_x^2 g^2}\right) \tag{2.17}$$

Gibbs (2004) fits this to a JONSWAP spectrum and finds $s_x = 0.0046\text{m}^{-1}$ for $t_p = 12\text{s}$ and $\gamma = 3.3$ as shown in figure 2.1.

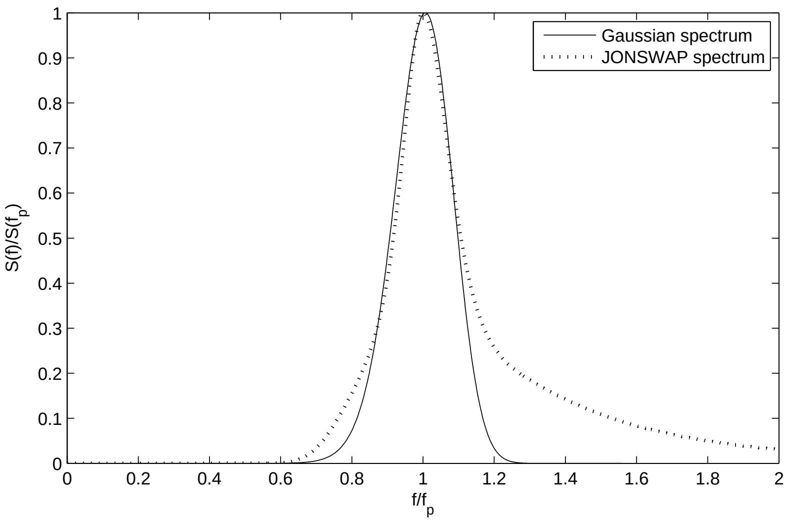

Figure 2.1: The Gaussian spectrum fitted to a JONSWAP spectrum with $\gamma =$ 3.3.

Another quantity of relevance to the work in this thesis is the Benjamin-Feir index (see Janssen (2003)). This relates the steepness of the waves to the bandwidth of the spectrum. The BF-index (for a sea-state) is given by

$$\text{BF-index} = \frac{\sqrt{2}\epsilon}{s'_\omega} \tag{2.18}$$

where ϵ is the steepness parameter given by $\sqrt{k_0^2 m_0}$ and s'_ω is the relative bandwidth. This parameter has a large influence on the stability of the spectrum and on the probability of tall waves (Onorato, Osborne, Serio, Cavaleri, Brandini & Stansberg (2006)).

2.3.2 Directional spectra

Another important parameter in describing a sea-state is its directionality as not all components will move in the same direction. The directional spreading of a sea-state may be described by a frequency-directional spectrum $S(f, \mu)$. We assume this is the product of a power spectrum and a directional distribution

$$S_\eta(f, \theta) = S_\eta(f) D(f, \theta), \tag{2.19}$$

this may be simplified by assuming that the directional distribution is independent of frequency. Various forms may be used, most of which are very similar in form. Here we model the directional distribution using a wrapped normal spreading function

$$D(\mu) = \frac{1}{\sigma_\theta \sqrt{2\pi}} \exp\left(-\left(\frac{\theta^2}{2\sigma_\theta^2}\right)\right), \tag{2.20}$$

where θ is the angle relative to the mean wave direction and σ_θ the standard deviation of spreading around the mean wave direction.

In reality, spreading is not found to be independent of frequency. Studies of field data such as Mitsuyasu et al. (1975) and Ewans (1998) find a considerable variation with frequency, and that the distribution is typically bi-modal at wavenumbers above the peak. Even in a physical wave-tank where waves were generated with frequency independent spreading, Cornett et al. (2002) found the measured spreading was found to have this property. It was also observed in the numerical results of Banner & Young (1994), and for the more non-linear cases in Gibbs & Taylor (2005).

2.3.3 Statistics of large waves

Given the sea-state parameters described above, engineers need to be able to predict the likelihood of a particular size of wave being exceeded at a particular location. For a linear, narrow-banded sea the wave-crest distribution was described by Longuet-Higgins (1952). This was extended to allow for second-order effects by Tayfun (1980). This distribution is generally considered adequate when compared to measured data (Forristall (2000)). Socquet-Juglard et al. (2005) found the Tayfun distribution fitted numerical data for directionally spread seas generated with a higher order Dysthe equation (see section 2.4), although it had to be extended using extreme value theory for the most extreme cases.

2.3.4 The shape of extreme waves

It is useful for those designing against extreme waves to know the shape of the largest waves in a sea-state. Lindgren (1970) showed that for a linear random Gaussian signal the average shape of a large peak or trough tends to the auto-correlation function. This was applied to water waves by Boccotti (1983) and introduced to offshore engineering practice by Tromans et al. (1991) where it became known as NewWave. Thus the shape around an extreme wave crest event is given by

$$\bar{\eta} = a \frac{\int S(k) \cos(kx)\, \mathrm{d}k}{\int S(k) \mathrm{d}k} \tag{2.21}$$

where a is the amplitude of the wave. Thus, this is a localised wave-group with all the spectral components in phase under the crest. This approach was validated by comparison against field data by Phillips et al. (1993) and Jonathan & Taylor (1997). For a sea-state with a Gaussian spectrum the NewWave will also be approximately Gaussian in form (Gibbs (2004)), and given by

$$\eta(x) = a.\exp\left(-\frac{1}{2}s_x^2x^2\right)\exp\left(-\frac{1}{2}s_y^2y^2\right)\cos\left(k_p\,x\right) \tag{2.22}$$

In this work, the same parameters are used as in Gibbs & Taylor (2005). These are representative for a storm in the North Sea.

$$s_x = 0.0046\,\mathrm{m}^{-1} \tag{2.23}$$

$$\sigma = 15^\circ \tag{2.24}$$

$$t_p = 12\,\mathrm{s} \tag{2.25}$$

unless otherwise stated, these are used throughout this work.

The effect of second order bound waves, which modify the free surface but not the underlying dynamics of the waves was included in this model by Jonathan & Taylor (1997) and Taylor & Williams (2004).

However, ocean waves are non-linear beyond the second-order and these interactions can modify their dynamics. One approach to modeling this is to start with a focused NewWave group, which has been established to be the most likely shape for an extreme wave-group on a linear basis. We then run this backward in time linearly (equation 2.6) until it is sufficiently dispersed that the waves are small enough that the evolution is essentially linear. It is then run forward again, using a non-linear evolution equation (see section 2.4) to find the shape and elevation of non-linear waves. This approach has been used by various authors, using different numerical and physical wavetanks including

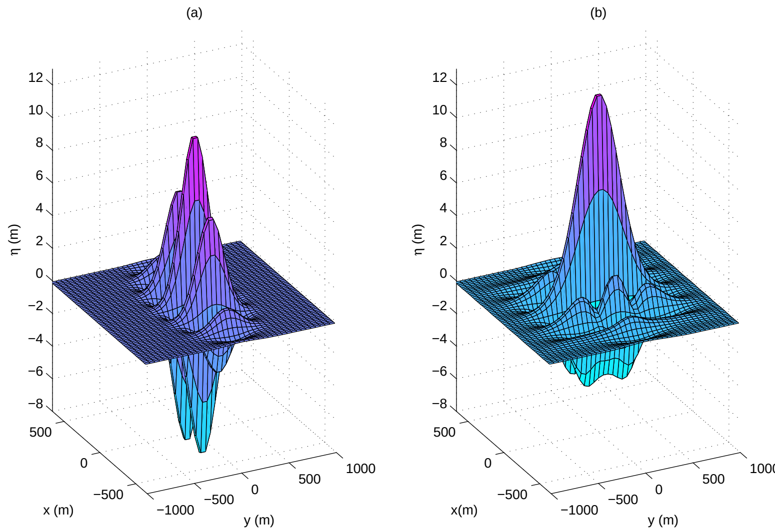

Figure 2.2: A NewWave group with $ak = 0.3$. (a) Linear focus. (b) Non-linear.

Taylor & Haagsma (1994), Baldock et al. (1996), Taylor & Vijfvinkel (1998), for uni-directional waves and Johannessen & Swan (2001), Gibbs & Taylor (2005) and Gibson & Swan (2007) for directionally spread groups. These latter find that the wave-envelope contracts in the mean wave direction whilst expanding in lateral direction. This is shown in figure 2.2, where the evolution equation used for the non-linear case is the non-linear Schrödinger equation (see section 2.4).

2.4 Evolution equations

If waves evolve on a linear basis (equation 2.6) then the magnitude of the linear wavenumber spectrum will remain fixed with only the phase changing. If we account for the bound waves (subsection 2.2.3) then we still do not see any permanent change in the distribution of the energy in the wavenumber domain over time. The frequency and wavenumber of the bound waves do not satisfy the linear dispersion equation and therefore remain bounded in time. However, when three, or more, waves interact, the resulting wave component may satisfy the linear dispersion relation and therefore will result in a permanent change to the spectrum. The theory for this was developed by Phillips (1960) and Hasselmann (1962); four-wave resonance, which may include one of the components interacting with itself, is of particular importance to the evolution of the spectrum in deep water. It is this which gives rise to the bi-modal spreading at high wave numbers discussed in section 2.3.2.

Of particular importance to the evolution of wave-groups over timescales $\sim 10 - 100$ periods are the results of Lighthill (1967) and the generalisation of these by Benjamin & Feir (1967). If one considers the evolution of a wavegroup with shorter waves in front of a central peak, and longer waves at the back, then the linear dispersion relation means that the longer waves travel faster than the short waves and so tend to catch them up. However, equation 2.8 implies that taller waves will travel faster than smaller ones, thus increasing the speed of the waves in the middle of the group relative to the ones at the front meaning they catch up quicker. Thus a positive feedback situation is created leading to a local increase in peak amplitude.

2.4.1 Approximate evolution equations

The simplest non-linear evolution equation for deep water is the Non-linear Schrödinger equation (NLSE), first derived by Zakharov (1968)

$$i\left(\frac{\partial u}{\partial t}\right) = \left(\frac{\omega_0}{8k_0^2}\right)\frac{\partial^2 u}{\partial x^2} - \left(\frac{\omega_0}{4k_0^2}\right)\frac{\partial^2 u}{\partial y^2} + \frac{1}{2}\omega_0 k_0^2 \left|u\right|^2 u. \tag{2.26}$$

This models the evolution of the complex wave envelope u, where the frame of reference moves at the group velocity. The free surface may be recovered by multiplying by the carrier wave

$$\eta\left(x,y,t\right)=\mathrm{Re}\left(u\,\mathrm{e}^{i(\mathbf{k}_0\mathbf{x}-\omega_0 t)}\right), \tag{2.27}$$

where $|\mathbf{k}_0|$ and ω_0 are related by the linear dispersion relation. As it is relatively easy to manipulate, the NLSE is used extensively in this thesis, and its properties are discussed in more detail in subsequent chapters. The NLSE is a good approximation for the basic physics of real water waves, including the non-linear dispersion given in equation 2.8 and some aspects, though not all, of Phillips 4-wave resonance (Yuen & Lake (1982)). However it does not capture the full non-linear dynamics and is limited to waves of small steepness and narrow bandwidth, where waves are propagating in one dominant direction, although it has been extended to wave-trains crossing by Onorato, Osborne & Serio (2006). The NLSE may also produce an unphysical transfer of energy to wave-numbers which are a multiple of the carrier wave (Martin & Yuen (1980)). A general comparison of the NLSE with the full water wave equations by Henderson et al. (1999) and a comparison with experimental results is given in Yuen & Lake (1975). Much of this thesis examines the evolution of focused wave-groups (section 2.3.4) and a short comparison with the fully non-linear results for this particular problem is given in section 2.4.3. A thorough review of the NLSE is given in Yuen & Lake (1980).

The NLSE is limited to waves of limited steepness and narrow bandwidths. More complicated evolution equations have been derived which give a more accurate approximation to the evolution of real water waves. The Davey-Stewartson equation (Davey & Stewartson (1974)) is similar to equation 2.26 but includes an additional term, coupling a second differential equation, to account for the return flow under the wave-group. Dysthe (1979) extended the NLSE to higher order, and there have been various subsequent extensions to this, relaxing the constraints on steepness and bandwidth (for a review see Dysthe & Trulsen (2001)).

The Zakharov equation (Zakharov (1968)) gives complete representation to Phillips resonant interactions, without the assumption of narrow bandwidth.

2.4.2 Fully non-linear models

The full water wave equations (2.1 to 2.4) may be solved numerically to give the exact evolution of water waves given the assumptions under which the equations are derived. A review of different numerical schemes for solving this problem is given in Bateman (2000). Fully non-linear models are generally very computationally demanding but are needed if one wishes to explore the full complexities real ocean waves.

2.4.3 Comparison between fully non-linear and non-linear Schrödinger equation

This thesis makes extensive use of the NLSE for modeling the evolution of compact NewWave type wave-groups. Before commencing this work, we examine the similarity and differences between the NLSE and the full-water wave equations for this problem. This subsection makes use of the numerical results of Gibbs & Taylor (2005). They used the fully non-linear code of Bateman et al. (2001), which extended to 2-D the work of Vijfvinkel (1996) in improving the numerical efficiency of the Craig & Sulem (1993) non-linear scheme. Gibbs & Taylor (2005) used the parameters in equations 2.23 to 2.25 and used a Gaussian group based on the linear evolution of a Gaussian given in Kinsman (1965), where the frame of reference is moving at the group velocity.

$$\begin{aligned} u(x,y,t) &= \frac{a}{\sqrt{1+is_x^2b_xt}\sqrt{1+is_y^2b_yt}} \times \\ & \exp\left(-\frac{1}{2}\left(\frac{s_x^2x^2}{1+s_x^4b_x^2t^2}+\frac{s_y^2y^2}{1+s_y^4b_y^2t^2}\right)\right) \times \\ & \exp\left(\frac{i}{2}\left(\frac{s_y^2y^2}{1+s_y^4b_y^2t^2}-\frac{s_x^2x^2}{1+s_x^4b_x^2t^2}\right)\right), \end{aligned} \tag{2.28}$$

where a is the amplitude at focus, u the complex wave envelope, and

$$s_y = k_p\sigma \tag{2.29}$$

$$b_x = \frac{\omega_p}{4k_p^2} \tag{2.30}$$

$$b_y = \frac{\omega_p}{2k_p^2}, \tag{2.31}$$

where σ, the spreading angle of the group at focus is given in radians. The simulations were started at -20 periods. Simulations were classified by the steepness, ak_p, of the focused linear group.

In modeling extreme wave-groups we are interested in the change in the group shape as it evolves, as well as in the magnitude and lifetime of the tallest wave or waves. Gibbs & Taylor (2005) found, in agreement with experimental results of Johannessen & Swan (2001), that as waves focused there was significant contraction in the mean wave direction and lateral expansion along the wave crests, reducing the local directional spreading. At least qualitatively, the NLSE gives the same effects. Figures 2.3, 2.4 and 2.5 compare the wave envelope for linear, fully non-linear and the equivalent evolution in the NLSE. The fully non-linear results have been 'linearised' i.e. the 'bound waves', which will be present in any fully non-linear simulation, have been removed (at least approximately) to leave only the 'free' wave components. All the fully non-linear results presented in this section are linearised as set out below.

The odd and even harmonics can be separated by running simulations which are π out of phase (equivalent to replacing a_n by $-a_n$ in 2.9) but have the same linear envelope, producing 'crest' and 'trough' focused events. The odd and even harmonics are then given by

$$\eta_{odd} = \frac{\eta_{crest} - \eta_{trough}}{2} \quad , \quad \eta_{even} = \frac{\eta_{crest} + \eta_{trough}}{2}. \tag{2.32}$$

The inversion of the wave group permits the straightforward extraction of the spectral contributions from different orders, because the non-linear contributions to the surface profile scale as powers of the linear components a_n. As an example, the third-order (cubic) terms are inverted by inverting the linear term $((-a_n)^3 = -a_n^3)$, whereas second order (quadratic) terms remain the same $((-a)_n^2 = a_n^2)$. The free waves can then be extracted by filtering in the wavenumber domain, although for the most non-linear cases ($ak_p >\sim 0.27$) the movement of energy to high wave numbers means that the free waves and the bound third harmonic overlap in wavenumber and cannot be straightforwardly separated. There will also be third order difference bound waves with wavenumber comparable to the peak of the spectrum which cannot be removed – for the most non-linear case considered ($ak_p = 0.33$) these will be $\sim 1\%$ the magnitude of free waves.

It can be seen in figures 2.3, 2.4 and 2.5 that the NLSE does capture the most important non-linear changes in the group shape. Compared to linear evolution, the NLSE group is far more compact in the x direction, and much broader in the y direction. However, the NLSE has not modeled the shift in the position of the peak towards the front of the wave which is seen in the fully non-linear case. This is discussed in Lo & Mei (1987) where they find the Dysthe equation does, at least qualitatively, display this behaviour. However, for much of this thesis we are trying to identify the gross global behaviour of a group, rather than the exact evolution.

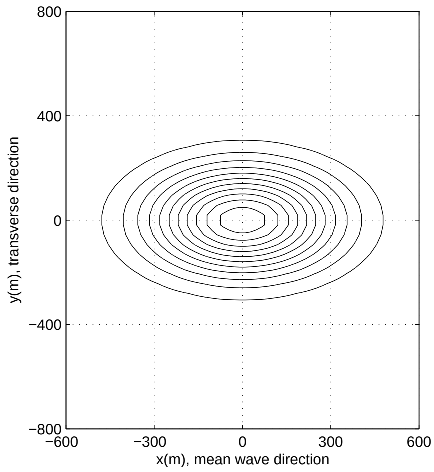

Figure 2.3: The wave-group envelope at peak amplitude for linear evolution. $ak_p = 0.3$

We can quantify the contraction of the group by fitting a Gaussian to the wave-envelope, as done by Gibbs & Taylor (2005). For the fitting we minimise the square of the difference between the wave-group and the fitted Gaussian, centered on the peak of wave-group. The values presented here are different from those given in Gibbs & Taylor (2005) as a different distance from the peak is considered. The results of this analysis, for the point at which the size of the wave is a maximum, are shown in table 2.1. We use the parameters: a_{max} - the maximum amplitude in the run, a_{lin} - the maximum amplitude if evolution were linear, s_x - here the bandwidth parameter in the mean wave direction at the time of a_{max} and s_{lin} - the bandwidth parameter on for linear evolution.

We can see that the NLSE over-estimates the non-linear dynamics over the evolution. For amplitude, the fully non-linear results give very little increase in crest height, whereas the NLSE predicts significant extra elevation. The NLSE does rather better at estimating the change in the bandwidth of the group. The contraction of the group is considerable, and even for the highly non-linear and very steep group the NLSE only over-estimates this by 20%.

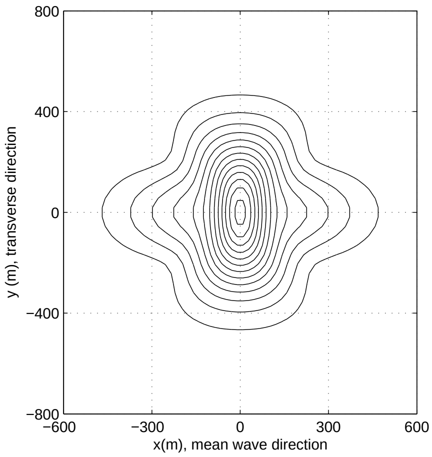

Figure 2.4: The wave-group envelope at peak amplitude for NLSE evolution. $ak_p = 0.3$

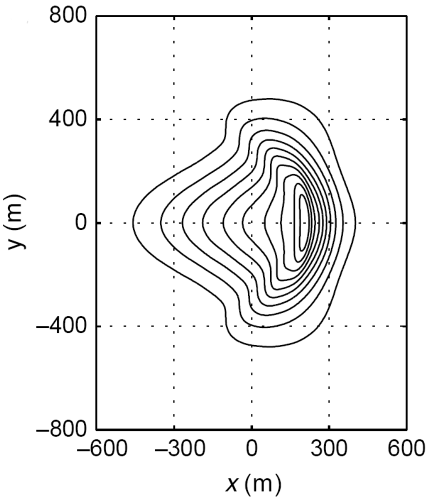

Figure 2.5: The wave-group envelope at peak amplitude for fully non-linear equations (from Gibbs). $ak_p = 0.3$

Linear steepness	Non-linear		NLSE	
	a_{max}/a_{lin}	s_x/s_{lin}	a_{max}/a_{lin}	s_x/s_{lin}
0.1	0.99	1.06	0.99	1.06
0.21	0.97	1.34	1.02	1.40
0.24	0.98	1.50	1.06	1.63
0.27	0.99	1.75	1.12	1.90
0.3	1.02	2.04	1.21	2.37
0.33	1.06	2.42	1.30	2.90

Table 2.1: Comparison of maximum surface elevation and group-shape in fully non-linear evolution, compared with NLSE evolution, for 2-D cases.

We can also consider the shape of the evolution in time. For the moment, we consider the shape of the wave envelope, rather than the individual waves. Figure 2.6 shows the time-history of a run with linear steepness $ak_p = 0.27$. The surface elevation is recorded at the point where a linear group would focus. Again we see that the NLSE does not capture the full dynamics of the non-linear run, but has approximated the changes to this time-history around the peak.

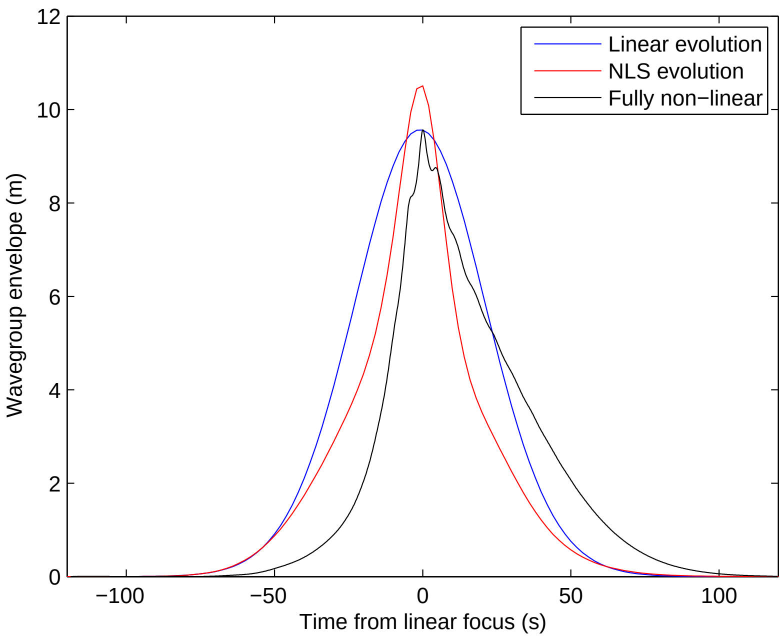

Figure 2.6: The surface elevation recorded at the point of linear focus for NLSE and fully non-linear evolution. The runs started at −240s.

The NLSE has some well known scaling properties. This allows us to scale our wave-group thus:

$$u = \Lambda u \tag{2.33}$$

$$s_x = \Lambda s_x \tag{2.34}$$

$$\sigma = \Lambda \sigma \tag{2.35}$$

$$t = \Lambda^{-2} t \tag{2.36}$$

where Λ is the scaling parameter. Gibbs (2004) used this scaling and applied it to fully non-linear evolution. He found that the major changes to the group shape scaled well using this approach, although some aspects, such as the amount of wave energy behind the peak, do not scale. However, that this scaling works approximately is in itself evidence that the NLSE can capture the major group changes in non-linear evolution.

Thus the NLSE captures the dominant dynamics of wave-group evolution without representing all details of the non-linear changes. In much of this work, we set out to examine and approximate these dominant changes, rather than model the detailed evolution. For this, the NLSE is adequate, and its simplicity means that it is easier to manipulate than higher order equations.

2.5 Freak waves

Over the last sixty years, oceanographers and engineers have built up physical and statistical models of ocean waves, some of which have been described in the first sections of this chapter. With these we can predict the size and characteristics of extreme wave events. However, there is evidence from a variety of sources that there may be waves which do not fit with our current understanding – either large waves occur more often than they should or they have characteristics which do not fit with our current modeling of wave physics. Such waves we describe as 'freak' or 'rogue' waves. Thus freak waves are not necessarily tall waves but are waves which differ from the waves surrounding them.

2.5.1 Evidence of freak waves

The evidence for or against their existence is unsatisfactory. For many accounts of freak waves we rely on accounts of mariners, and whilst many mariners are extremely skilled in judging sea-states and waves, their observations, particularly under the stress of an encounter with a giant wave, must be treated with caution. Various reports of freak waves are given in Lawton (2001), Kharif & Pelinovsky (2003), Didenkulova et al. (2006), Liu (2007) and Dysthe et al. (2008). Several characteristics are often reported:

- The giant wave is very long-crested, forming a 'wall of water' and it persists over many wave periods.
- Three large waves may arrive in succession, the 'three sisters'.
- Individual waves may be propagating in a direction completely different to that of the rest of the waves.

The obvious point should be made, that ships may not survive the most extreme encounters, and so these may go unreported.

In offshore areas where oil has been extracted, particularly the North Sea, wave data have been accurately recorded for the past $\sim$ 30 years. It should however be pointed out the sensors are designed to record the sort of waves one expects to see and may not be accurate in recording unusual events, which thus exacerbates the errors and noise which are present in all data. Forristall et al. (2004) gives a review of the performance of different sensors.

Studies of the statistical distribution of wave crest heights are inconsistent in reaching conclusions on freak waves. For instance, Olagnon (2008) and Stansell

(2004) both analyse data sets measured at the Alwyn platform: the former found second-order statistics satisfactory for describing extreme event whereas the latter found a number of waves which did not fit theory. These are by definition very rare events and it is difficult to satisfactorily sample sufficient data, as discussed in Haver & Andersen (2000).

A few individual waves have been recorded which appear highly unusual. The best known of these is the Draupner wave (see Haver (2004)). This wave is considered in chapter 4 where its properties are considered in more detail.

2.5.2 Physical mechanisms for unusual waves

A number of mechanisms are known to produce large waves which do not fit with a Tayfun type distribution (see section 2.3.3).

If a current opposes the direction of wave propagation, waves may exhibit unusual characteristics (Smith (1976), and Lavrenov (1998)). The numerical simulations of the latter suggest H_s may be increased by up to 100%. This effect is well documented for the Agulhas Current of South Africa, see for instance Liu & MacHutchon (2008) who recordered some very unusual waves in this area. The Author has observed (and kayaked through) a similar phenomenon on the Thames just below King's Lock, where waves of around 15cm in height met a strong current coming around a bend just below a weir. The speed of the current reduced rapidly as it moved downstream, but is estimated to have been about 1ms^{-1} as it rounded the bend in the river. Furthest downstream the waves appeared to be unaffected by the current. As they propagated upstream they noticably steepened over about a 3 m stretch, before, over about 1 m, the waves became visibly unstable. Further upstream than this the water surface was almost completely flat.

Another cause of unusually large waves is wave refraction due to sea-bed topography. As waves enter shallower water, they may be steered by refraction.

Where waves are steered towards each other, a concentration of energy takes place leading to an increase in wave amplitude. Adcock & Taylor (2009) suggest this effect may have led to the otherwise unexplained failure of one of the components of the Mulberry Harbour during the Second World War.

In this work, we only look at waves in the 'open ocean', i.e. where interaction with the shore, with currents and with changes in the water depth are insignificant. For waves in this category, two main physical mechanisms have been suggested which might lead to unusually large waves occurring. One is the non-linear interactions briefly discussed in section 2.4 and considered further in chapters 5 and 6; the other is local interaction with wind which is discussed in further detail in chapter 7.

2.6 Aims and objectives

This thesis sets out to examine various non-linear problems associated with ocean waves.

- To establish a robust method of determining the directional spreading of a sea-state from a point surface elevation time-history.
- To find a simple analytical expression to describe the non-linear changes in the shape of a wave-group as it focuses.
- To examine the changes in wave-group evolution due to energy input.

Chapter 7

Evolution of wave-groups with energy input

7.1 Introduction

The modelling of waves in previous chapters, and most localised modelling of waves referenced up to here in this thesis, assumes that the energy of the waves is conserved. These models aim to solve the hydrodynamic problem set out in section 2.2.1. However, a system of waves on the open ocean is not conservative. Waves are formed by wind, and if there is a wind blowing there will be an energy transfer between the wind and the waves. Energy will also be dissipated in waves due to viscous effects and wave breaking. All of these processes are likely to be more significant in extreme waves. Tall waves will protrude further into the wind-field, disturbing the wind field more and exposing the waves to faster winds. This chapter focusses on the case where energy input from wind exceeds the energy dissipation.

Unfortunately, the mechanisms of energy transfer between wind and waves are poorly understood at a wave by wave scale. To paraphrase Tucker & Pitt

(2001), whilst we know qualitatively the physical processes involved we cannot, as yet, quantify these accurately. There are two physical processes thought to be important in the energy exchange: pressure fluctuations on the free surface (Phillips (1957)) and the effect of drag between the wind and the waves (Miles (1957)). Kinsman (1965) argues that these are "complementary rather than exclusive" and thus can be considered separately.

For steep waves, Jeffreys (1925) proposed a sheltering mechanism, with separation of the air flow over the waves. This separation was demonstrated for waves on the point of breaking by Banner & Melville (1976) although they did not find separated air-flow for less steep cases. The Jeffreys' sheltering mechanism has been applied to the numerical modeling of waves by Kharif et al. (2008), although they admit that the results are sensitive to some of the experimentally derived parameters which may not be accurate. However, in some cases, they find that the presence of wind accentuates the non-linear dynamics of waves.

Quantifying the local energy transfer between wind and waves is, at present, impractical for anything but idealised cases. At large scale (i.e. in estimating sea-state parameters, see for instance Hasselman et al. (1973), Janssen (1991)) empirical relationships have been established, but at the small scale we are not yet able to satisfactorily model the wind/wave interaction. In this chapter we will therefore simplify the problem by introducing a negative damping term into the evolution equations. This approach has also been applied by both Wong (2006) in an undergraduate project at Oxford, and by Leblanc et al. (2008), and is similar to that of Miles (1984). A damping term is also used by Segur et al. (2005) to model dissipation, where they show that this can stabilise the Benjamin-Feir instability. To generate unusually tall waves we wish to accentuate the instability, so this is potentially a good model for doing this. However, no claims are made as to the accuracy of this model in representing the actual physics of wind/wave interaction.

7.2 The excited non-linear Schrödinger equation

Taking the non-linear Schrödinger equation (equations 2.26) as a simple model for non-linear wave evolution. Into this an extra term is added representing an energy input which is proportional to local wave-height

$$i\left(\frac{\partial u}{\partial t} - \alpha u\right) = \left(\frac{\omega_0}{8k_0^2}\right)\frac{\partial^2 u}{\partial x^2} - \left(\frac{\omega_0}{4k_0^2}\right)\frac{\partial^2 u}{\partial y^2} + \frac{1}{2}\omega_0 k_0^2 \left|u\right|^2 u, \tag{7.1}$$

where α may be varied to give the amount of damping required. For this work, the energy input per period, β under linear evolution will be used as the parameter as this has more obvious physical significance than α. These two parameters are related by

$$\beta = e^{2\alpha t_p} - 1, \tag{7.2}$$

where t_p is the period of the carrier wave. Thus there is energy input if α is positive. The total energy of of the system will also vary as $e^{2\alpha t}$, see Segur et al. (2005).

Introducing a new variable q, so that

$$u = qe^{\alpha t}, \tag{7.3}$$

the excited NLS then becomes

$$i\left(\frac{\partial q}{\partial t}\right) = \left(\frac{\omega_0}{8k_0 2}\right)\frac{\partial^2 q}{\partial x^2} - \left(\frac{\omega_0}{4k_0 2}\right)\frac{\partial^2 q}{\partial y^2} + e^{2\alpha t}\frac{1}{2}\omega_0 k_0^2 \left|q\right|^2 q, \tag{7.4}$$

which is refered to as the "q" form of the equation, as opposed to the "u" form of equation 7.1. We can observe from this equation that the energy input modifies the non-linear term in the NLSE. Without the non-linear term, the spectral shape will not change as the group evolves – so the group would just grow. However, when coupled with the non-linear term, the energy input will accentuate this term, in turn accentuating the non-linear changes to the group.

It is useful that the first conserved quantity is also conserved in this form of the equation. Thus

$$0 = \frac{d}{dt} \int_{-\infty}^{\infty} |q|^2 \, dx. \tag{7.5}$$

The "q" form is also useful for numerical modelling as it is found to be more stable than the "u" form for large excitation and it produces smaller inaccuracies when the model is run forwards and backwards in time, for nearly identical computational effort.

7.3 1-D modelling

Although in the open ocean the free surface is 2 dimensional, let us start by examining the properties to the damped NLSE in 1 dimension. As discussed in chapter 5, the soliton is a solution to the undamped 1-D NLSE. The soliton is given by

$$u = a \,\mathrm{sech}\left(\sqrt{2} a k_0^2 x\right) e^{-\frac{1}{2} i \omega_0 k_0^2 a^2 t}, \tag{7.6}$$

where a is the amplitude of the soliton. Using the soliton as the initial condition in a numerical wavetank we can observe how this evolves in the positively damped case. A 4th order Runge-Kutta scheme is used, as in chapter 5. Typical evolution is shown in figure 7.1.

It can be seen that the wave envelope remains soliton like in form. This is confirmed if we plot a soliton of the same total energy against the numerical

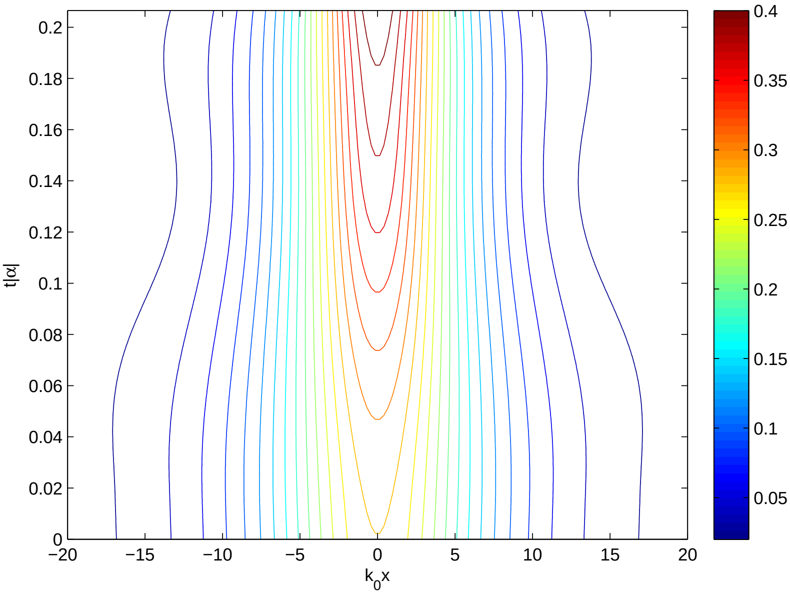

Figure 7.1: The evolution of a soliton with energy input of 1% per cycle. The contours are in $|u|k_0$. The computational domain is much larger than that shown here.

solution (figure 7.2). The energy of the soliton here is given here by

$$\text{Energy} \sim \int_{-\infty}^{\infty} u^2 dx = \frac{a\sqrt{2}}{k_0^2} \tag{7.7}$$

Thus the approximate evolution is given by

$$u \sim a_0 e^{2\alpha t} \text{sech}\left(\sqrt{2} a_0 k_0^2 e^{-2\alpha t}\right) \tag{7.8}$$

implying that the maximum amplitude of a soliton will grow faster than the amplitude of a regular wave.

$$Umax_{soliton} = a_0 e^{2\alpha t} \tag{7.9}$$

$$Umax_{regular} = a_0 e^{\alpha t} \tag{7.10}$$

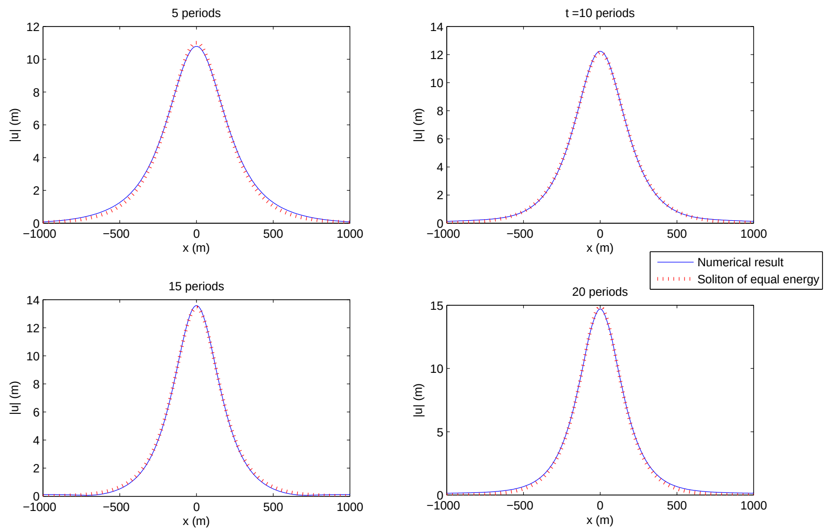

Figure 7.2: The envelope of solition evolution with energy input of 1% per period at various times. Also plotted is the solition of equal energy.

This is shown in figure 7.3. The soliton solution is not exact, and it can be seen that the numerical solution oscillates around the approximate solution (equation 7.9). This result is well known in nonlinear optics, where it is of considerable importance in that preferential energy input into the coherent solitons, rather than the optical noise, can help restore the structure of the pulse after many kilometers of propagation through an optical fiber (Mollenauer & Smith (1988)). Thus, amplification can help overcome the combined effects of dispersion and losses.

7.4 2-D modelling

7.4.1 General results

In this section we use the approach of Gibbs & Taylor (2005) to modelling extreme wave-groups, as described in section 2.3.4. Thus focused wave groups will be run backward in time on a linear basis, before running forward using non-

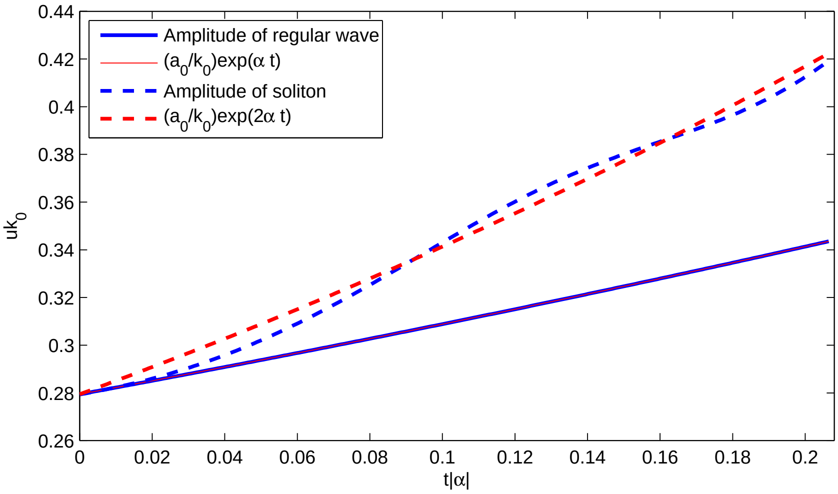

Figure 7.3: The maximum amplitude of the evolution of an initial solition with energy input of 1% per period, compared with an approximate solution. Also shown is the amplitude of a regular wave under energy input (both numerical and analytical), although these are very close in value and difficult to distinguish.

linear evolution. We choose to start the evolution of our groups at 20 periods before linear focus as was done in section 2.4.3.

Let us consider the evolution of wave-groups with energy input. Figures 7.4 and 7.5 show the evolution of wave-groups for various levels of energy input. Note that all values are normalised by the equivalent value at linear focus, denoted by a subscript lin. It is also instructive to show the same groups normalised against the maximum amplitude of a linear wavegroup evolving with the same energy input (equation 7.1 without the final term). In this case the waves evolve and grow with no change in the shape of the wavenumber spectrum. These are shown in figures 7.6 and 7.7.

There are a number of general observations which may be made from figures 7.4 through to 7.7. The energy input clearly accentuates the non-linear dynamics of the evolution, increasing the peak elevation, causing a contraction of the group in the x direction and an extension in the y. This behaviour was expected from the form of the equation. We also observe that some groups "split" – that is,

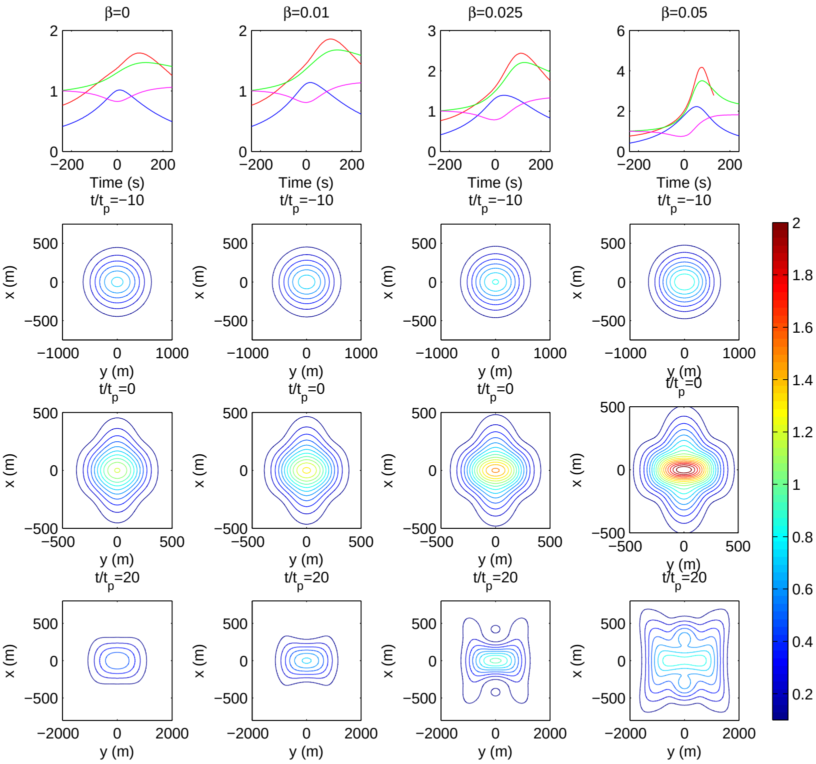

Figure 7.4: The evolution of wave-groups with linear steepness $ak_p = 0.2$. The evolution for different amounts of energy input (β) is in each column. The lines on the top graph are: blue - u_{max}/a_{lin}, red - $\varsigma_x/\varsigma_{x,lin}$, , green - $s_x/s_{x,lin}$, magenta - $s_y/s_{y,lin}$. In the contour plots, the envelope is divided by the amplitude at linear focus and the contours are at 0.1 intervals. Note the changes in the size of the observation window.

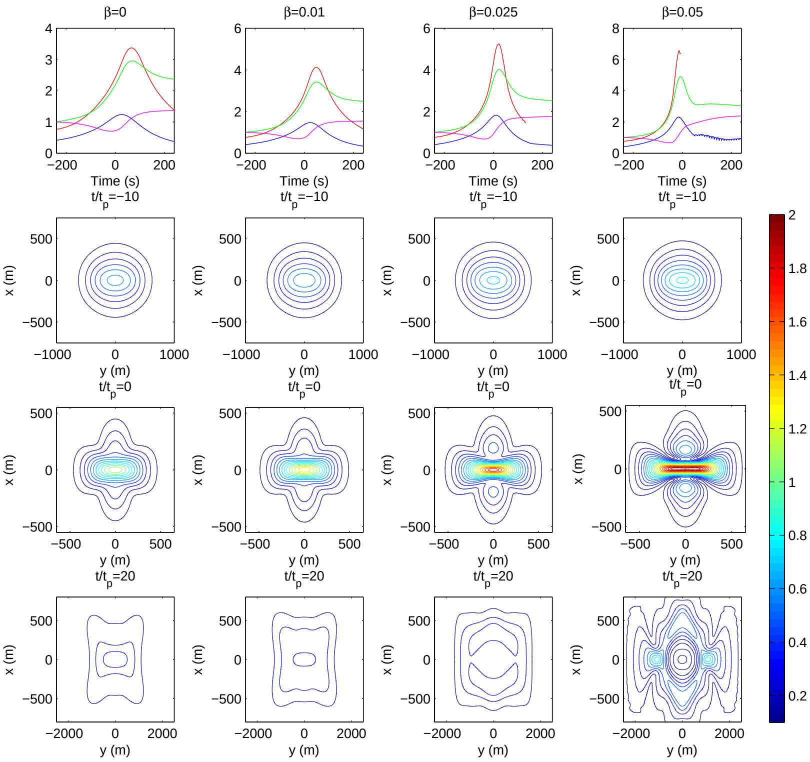

Figure 7.5: The evolution of wave-groups with linear steepness $ak_p = 0.3$. The evolution for different amounts of energy input (β) is in each column. The lines on the top graph are: blue - u_{max}/a_{lin}, red - $\varsigma_x/\varsigma_{x,lin}$, , green - $s_x/s_{x,lin}$, magenta - $s_y/s_{y,lin}$. In the contour plots, the envelope is divided by the amplitude at linear focus and the contours are at 0.1 intervals.

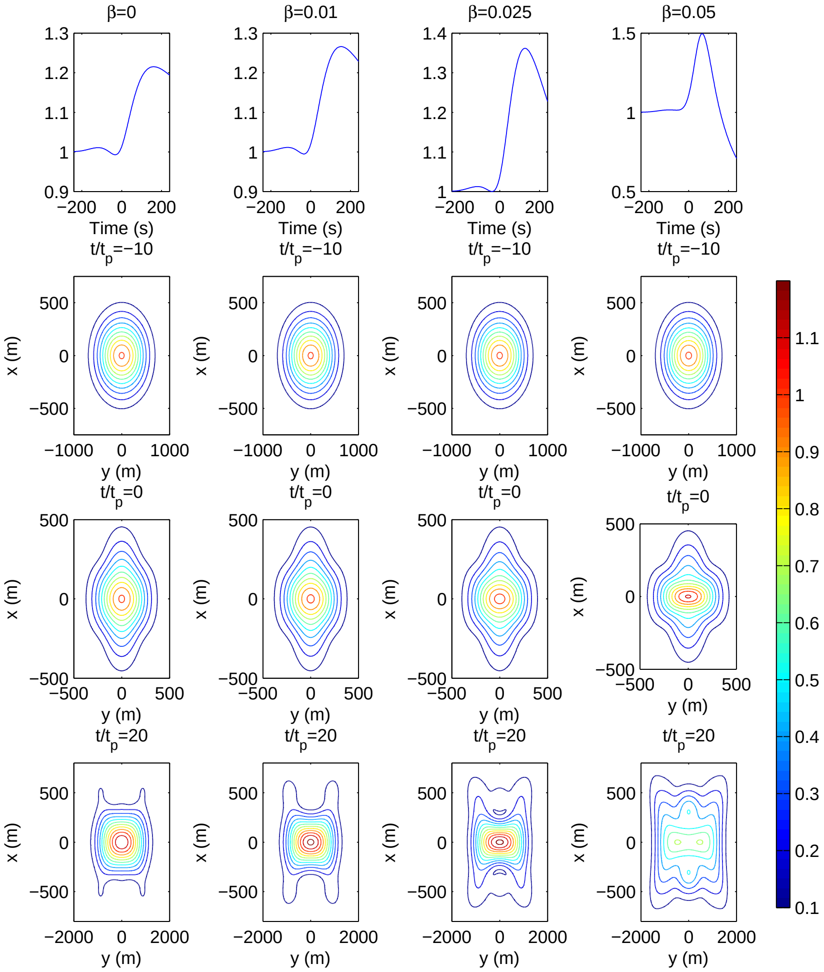

Figure 7.6: The evolution of wave-groups with linear steepness $ak_p = 0.2$. The evolution for different amounts of energy input (β) is in each column. The line on the top graph is the maximum amplitude of the non-linear evolution divided by the maximum amplitude in evolution without non-linearity but with energy input. In the contour plots, the envelope is divided by the maximum amplitude of the equivalent wave-group evolving with energy input but without non-linearity.

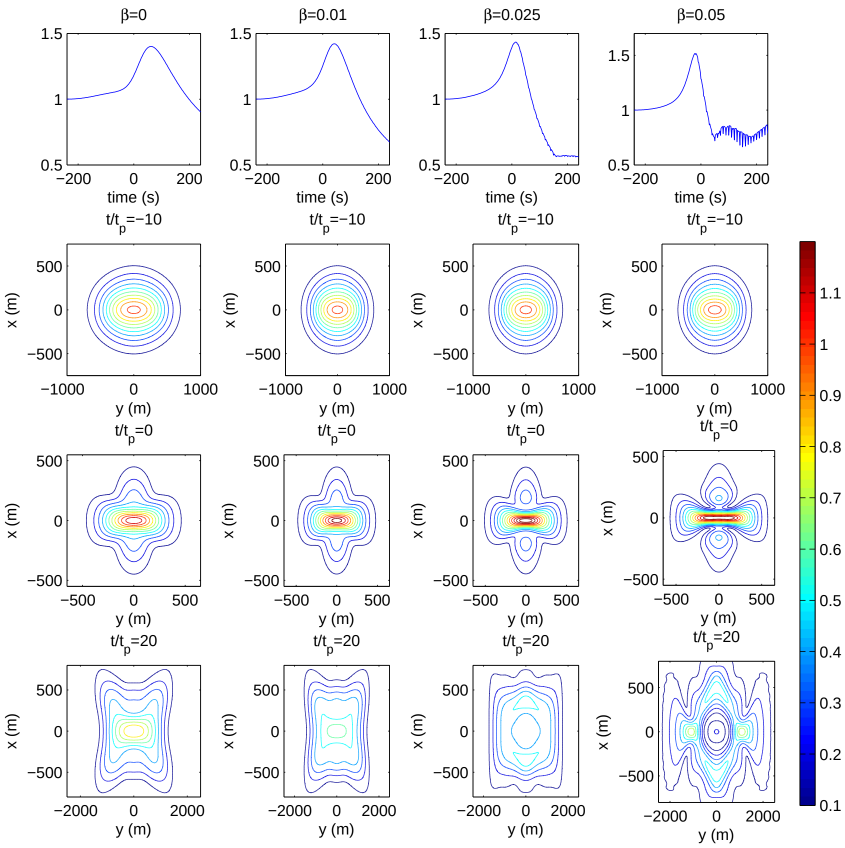

Figure 7.7: The evolution of wave-groups with linear steepness $ak_p = 0.3$. The line on the top graph is the maximum amplitude of the non-linear evolution divided by the maximum amplitude in evolution without non-linearity but with energy input. In the contour plots, the envelope is divided by the maximum amplitude of the equivalent wave-group evolving with energy input but without non-linearity.

rather than staying as a single group they form distinct packages and these persist as the group de-focuses.

7.4.2 The effect of energy input on group shape

The studies of wave-group evolution by Johannessen & Swan (2001) and Gibbs & Taylor (2005) showed that non-linear wave-groups contract in the mean wave direction compared to linearly evolving groups (see figures 2.3, 2.4 and 2.5). The same characteristic is observed for cases of energy input, where all these changes are exaggerated. We quantify these changes by examining changes in the maximum amplitude reached during a run, a_{max}, and also the spatial width of the group described by ς_x.

Firstly we consider the change in amplitude of the group. If energy is input into a group which is evolving linearly, the group will simply grow with no change to the spectral shape. The maximum amplitude of a group evolving thus is denoted a_{lin+}. In figure 7.8 we compare the maximum amplitude reached in non-linear evolution, to the value for linear evolution. This shows that the non-linear dynamics cause an increase in the amplitude over linear evolution. However, for very non-linear cases the group splitting will limit the maximum amplitude obtainable in a NLSE solution. For physical water waves breaking would probably occur before this value could be reached.

We now consider the changes to the width of the group. The group width is found by fitting a Gaussian (equation 7.11) to the peak of the group.

$$\text{envelope} = a \exp\left(-\frac{1}{2}\varsigma_x^2 x^2\right) \tag{7.11}$$

Rather than fit the whole group, we instead consider only three wavelengths centered at the envelope peak. The maximum value reached during various runs $\varsigma_{x,max}$ – when the group is most compact in the mean wave direction –

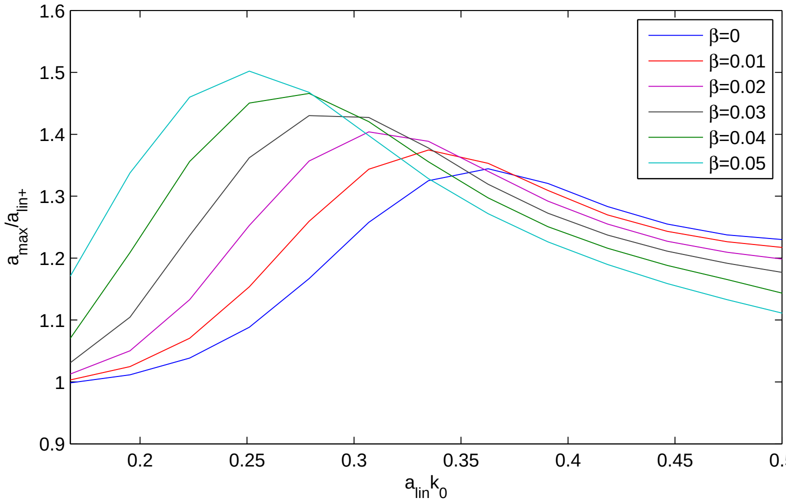

Figure 7.8: The maximum amplitude reached for various energy inputs. All runs have initial $s_x = 0.0046\text{m}^{-1}$ and $s_y = 0.0073\text{m}^{-1}$ and start 20 periods before linear focus.

is shown in figure 7.9. This again shows the energy input accentuating the non-linear dynamics and hence increasing the contraction of the group.

We can also examine the height to width ratio of the group, which is a measure of the maximum envelope steepness. This is given by $a.\varsigma_x$. We plot the maximum value of the envelope steepness recorded during a run in figure 7.10. Again, this ratio increases with initial amplitude and with energy input.

Some descriptions of freak wave events state the crest appears to persist for longer than normal (see for instance account in Lawton (2001)). It has already shown in chapter 5 that the non-linear dynamics can slow down the dispersion of a group relative to linear evolution. To investigate this further, we choose to take as our parameter the amount of time the group is within 90% of the maximum height it reaches during its evolution. For our input wave-group with linear evolution the envelope would be taller than this for 76 seconds, or around six periods. We plot the duration above our limit against energy input in figure 7.11.

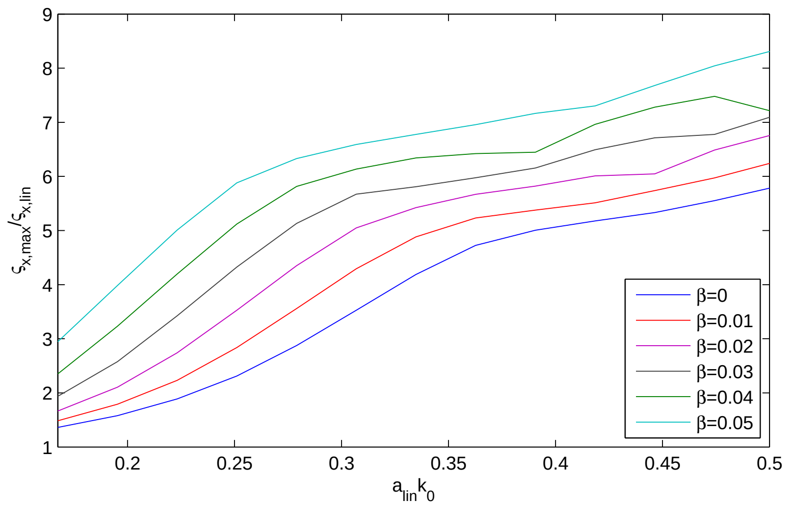

Figure 7.9: The maximum ‘compactness’ of the wave-group for various energy inputs. All runs have initial $s_x = 0.0046\text{m}^{-1}$ and $s_y = 0.0073\text{m}^{-1}$ and start 20 periods before linear focus.

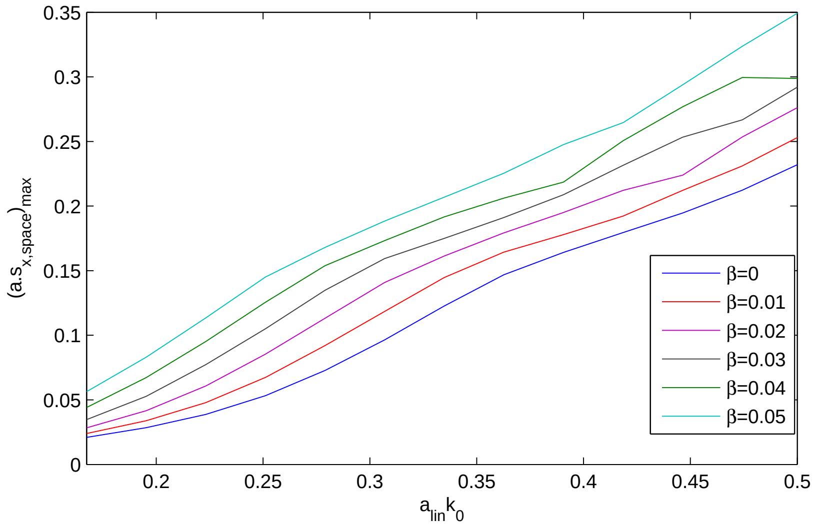

Figure 7.10: The maximum envelope steepness of the wave-group. All runs have initial $s_x = 0.0046\text{m}^{-1}$ and $s_y = 0.0073\text{m}^{-1}$ and start 20 periods before linear focus.

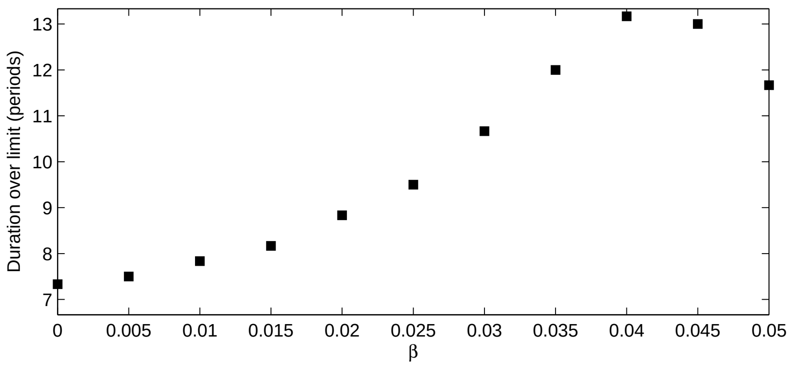

Figure 7.11: The amount of time for which a wave-group is greater than $0.9|u|_{max}$. Input conditions are $ak_p = 0.16$.

Thus we can see that energy input will lead to the group persisting longer, except for high values of energy input where the group splits. It should however be stressed that we are here considering the persistence of the envelope, while what is observed in the ocean is the persistence of the crest.

7.4.3 Splitting of groups

We investigate which groups split and which stay intact numerically. Again taking the start of the evolution at 20 periods before linear focus and run the group through focus. We record whether or not the group is observed to split. These results are presented in figure 7.12, where we classify each run by the maximum group amplitude to wavenumber bandwidth in the x direction ratio observed during the run.

We can see that there is a clear dividing line between the groups which split and those which do not. As would be expected, for increased energy input, lower values of amplitude to wavenumber bandwidth are needed to cause the group to split. It should be noted that the amplitude to wavenumber bandwidth is analogous to the Benjamin-Feir index used to describe groups in chapters 5 and 6. For most of the runs in figure 7.12 this was a maximum at the start of the

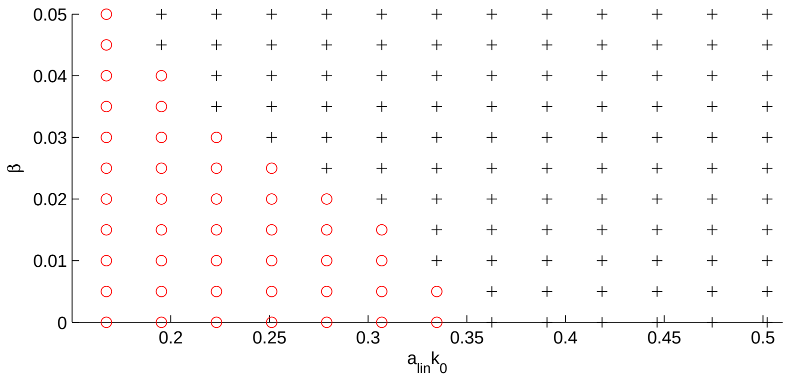

Figure 7.12: A comparison of whether a wave-group "splits" for various input parameters. Red circles indicate that for that run the group has not split, whereas the runs with black crosses have split. Initial $s_x = 0.0046$ and $s_y = 0.0073$.

run.

After a group splits, some unusual patterns form and energy appears to move in the y direction as well as the mean wave direction. We show an example in figure 7.13.

The amplitude spectrum of the actual free surface for this case is shown in figure 7.14. It can be seen that the spectrum has become quite broadbanded and has split in the mean wave direction, and has also expanded in the 'y' direction. The NLSE is a narrowbanded model and will not reproduce this accurately. Therefore we must treat with great caution the results of the evolution once the group has split.

7.5 Discussion

This chapter examines the results of numerical experiments looking at the effect of wind on the non-linear evolution of wave-groups. Rather than attempt to model the exceptionally complex details of this interaction, negative damping is introduced to the NLSE which has the effect of putting energy into the system.

In 1-D, the behaviour is fairly straightforward. An isolated soliton is ampli-

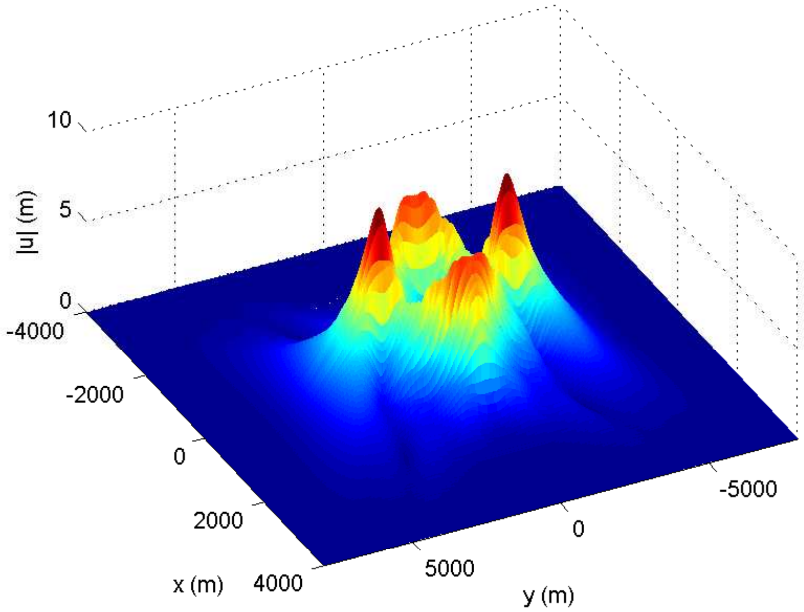

Figure 7.13: The spatial structure of the envelope at $t = 20$ periods for run $ak_p = 0.3$ and $\beta = 0.05$.

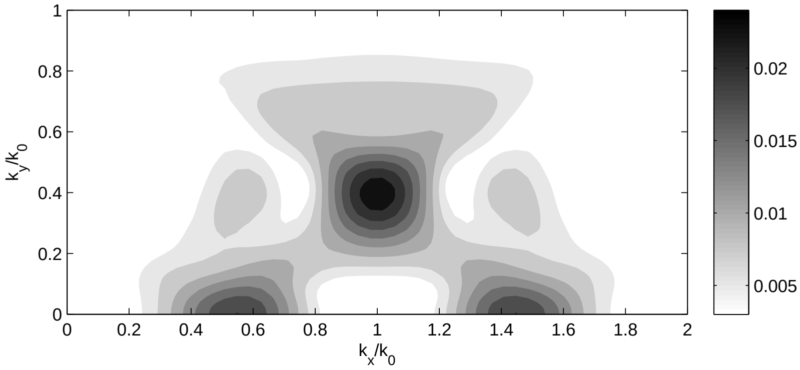

Figure 7.14: The amplitude-wavenumber spectrum of the free surface at $t = 20$ periods for run $ak_p = 0.3$ and $\beta = 0.05$. Note that the spectrum is symmetrical so negative k_y values are not plotted.

fied, with the soliton becoming taller and thinner but staying close in form to an undamped soliton of the same height. This amplification is greater for solitons than for regular waves. Thus, in a random uni-directional sea, any soliton-like structures are preferentially amplified.

In 2-D the behaviour is far more complex. Again the energy input has the effect of accentuating the non-linear dynamics, and thus enhancing the changes to the group discussed in 2.3.4. As the NLSE generally models these changes adequately, it is probable that similar results would be found if the full water wave equations were used.

However, any conclusion beyond this is more speculative. In these simulations there are some highly non-linear groups, which push the bandwidth and steepness limits of the NLSE. For cases where the group splits, there is significant energy travelling out laterally – the NLSE will not be able to model this accurately. For real water waves, breaking would probably occur before the splitting which we observe here could develop.

Some mention should be made of the numerics. These highly non-linear simulations are a very stern test of the numerical scheme. For the most non-linear case considered ($ak_0 = 0.3$, $\beta = 0.05$) there is a mis-match in energy of $\sim 1\%$ after forty periods. However, we believe the numerical scheme is still adequate for investigating the general behaviour of the NLSE. The constraints on the validity of the equation are a more severe limit as to which results can be accepted.

Chapter 8

Conclusions and recommendations

8.1 Main findings

In this thesis a number of topics related to the mechanics of offshore waves, in particular the effects of local non-linearity, are considered.

In chapter 3 a method is formulated and validated for determining the directional spreading of a sea-state, when the only available information is a single point free surface time-history. This uses the low frequency waves in the record, which are compared with the theoretical low frequency waves predicted by second order Stokes-type theory. We apply this to wave-groups in a wave-tank. A method is developed for non-deterministic data which is demonstrated to be robust in the presence of noise. This is then applied to measurements from a wave-basin and to field data recorded in the North Sea. In all cases the method is in good agreement with other measurements. This approach is thus a useful tool for analysis of rough sea-states in deep or intermediate water depths.

In chapter 4, the method of estimating directional spreading was applied to the famous Draupner wave, which exhibits an unusual low frequency set-up underneath it. We show that this can be explained if this was formed by two wave-groups travelling at 120° to each other. These results are consistent with other evidence.

Chapters 5 and 6 consider an analytical model for describing the non-linear evolution of Gaussian wave-groups. It is assumed that the group remains Gaussian in shape and use the conserved quantities of the non-linear Schrödinger equation to relate the parameters of a fully dispersed group to those of one at perfect focus. This gives a simple set of equations for describing the non-linear changes of a wave-group as it focuses. This works well for uni-directional waves, but has less general applicability for directionally spread waves, although it is still useful for many wave-groups which are of interest to ocean engineers, and predicts results which are consistent with known behaviour.

Chapter 7 considers energy transfer from wind to waves, using a relatively crude model of including an excitation term in the NLSE. It is found that this accentuates the non-linearity of the wave-group evolution – increasing the contraction in the mean wave direction and causing the group to expand in the lateral direction.

8.2 Further work arising from this thesis

The work on directional spreading should provide a useful tool for analysing field data. Further validation needs to be done, using field data where the directional spreading is measured independently at the same location. A straightforward extension is to include frequency dependent spreading in the model (some preliminary results are given in Taylor et al. (2006)). Attempts have also been made to improve the analysis of the discrepancy using more sophisticated statistical techniques (in collaboration with the machine learning group in Oxford) although we have no conclusive results as yet.

The analysis of the Draupner wave, though speculative, is potentially very important. In addition to the Draupner wave, there is other evidence that short wave-packets occasionally propagate in a direction different to the mean wave direction (see 4.7). However, the author knows of no study of the dynamics of waves in crossing seas other than Onorato, Osborne & Serio (2006). As waves propagate in different directions there will be resonant interactions between these wave systems and transfers of energy to new wave-numbers. Longuet-Higgins (1962) considered a simple case of two wave-trains traveling at 90°, but this was in the context of validating Phillip's interactions, rather than as a study of waves in the real ocean. Crossing seas certainly do exist, and work needs to be done to establish how these evolve, and further whether it is plausible for short wave-packets to propagate in one direction, with the rest of the waves traveling in a different direction.

To extend the analytical work on the evolution of wave groups is less straightforward. Further validation could be carried out using a fully non-linear scheme. For directionally spread waves, it might be possible to use the Davy-Stewartson equation, which is solvable by inverse scattering so has an infinite number of conserved quantities. Thus we would not have to rely on the pseudo-conserved quantity $I6$. This will complicate the analysis but would be expected to give a better representation of real water waves.

The work on energy input and the interaction of wind with waves should be regarded as a preliminary study. Our work predicts that energy input enhances the non-linear dynamics of wave-group evolution, which suggests that it is worth investigating further as a possible mechanism for 'freak' waves. The most obvious next step is to use a fully non-linear scheme to model the evolution. It should be relatively straightforward to modify the free-surface pressure in such a model. The difficulty is changing this pressure term so that it gives an adequate representation of the pressure fluctuations caused by wind. However, even a relatively crude model should contribute to our knowledge. Ultimately, a coupled air-flow/ water-wave model may be required, with all the difficulties of turbulence modeling, but this presents a formidable computational challenge.

Another area, indirectly arising from the work in this thesis which warrants further investigation, concerns the shape of extreme waves in real seas. The dominant behaviour of isolated wave-groups is well established (Johannessen & Swan (2001), Gibbs & Taylor (2005) and chapter 6) – that there is a contraction in the mean wave direction coupled with a lateral expansion. It is important to validate whether this happens in a realistic random sea. Whilst these studies find no significant increase in amplitude (over second order theory) there is a reduction in directional spread and hence increase in forces on structures for large waves. This is not currently taken into account in design. Thus, it would be interesting to observe if these changes are observed for random waves in a numerical wave-tank, and for field measurements in the open ocean.

A second issue is the effect of the random background within which the focused event is assumed to be occurring. Alber's NLSE predictions (Alber (1978)) suggest that a random background stabilises the non-linear dynamics, presumably by disrupting the phasing of the third order terms. However, it is possible that surrounding waves might act as an energy source for extreme wave events, with energy being sucked in, a phenomenon Peregrine (1983) observed for specific examples for uni-directional waves.

Another area which needs to be considered is the kinematics of large waves. Whilst the tools are in place to investigate this (for instance Bateman et al. (2001)) further study needs to be made for this to be of practical use to engineers. Studying the kinematics leads on to wave breaking, which is another area which merits further investigation.

Thus, there is much still to explore in developing a complete understanding of steep waves in the open ocean.

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