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Vulnerability to Poverty

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Abstract

Standard poverty analysis makes statements about deprivation after the veil of uncertainty has been lifted. Nonetheless, the term 'vulnerability' has been used as a tool to remark that uncertainty and risk do matter. In this paper, we define 'vulnerability to poverty' as the magnitude of the threat of poverty, measured ex-ante, before uncertainty is resolved. We describe the desirable properties of a vulnerability measure as a set of axioms, and present a family of measures satisfying our desiderata at the individual level. We also propose a family of measures of aggregate vulnerability, a concept which has remained largely unexplored thus far.

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1. Motivation

It may come as a surprise that the vast, long-lived literature on poverty measurement has until recently unfolded under the assumption of full certainty. Little regard has been given to the implications of exposure to risks. For instance, work on 'multi-dimensional' poverty has strived to develop poverty concepts and measures exhibiting sensitivity not only to consumption deficits, but also to meagre education levels and poor health conditions. However, the fact that wellbeing is also impoverished by a feeling of insecurity, uncertainty and defencelessness remains largely unexplored. In this paper, we explore the notion of vulnerability to poverty, defined as the magnitude of the threat of poverty, measured *ex-ante*, before uncertainty has been resolved. We identify desirable properties of a vulnerability measure as a set of axioms. We also present a family of measures satisfying these properties at the individual level. Finally, we address aggregation, by proposing a family of measures of aggregate vulnerability.

It is surprising that the calculus of risk has not systematically entered normative economic analysis of poverty until fairly recently. Even Sen's (1981) seminal contribution on famines is in its welfare analysis concerned with the ex-post consequences of the crisis in terms of poverty and destitution. Policy analysis is done with the benefit of hindsight, even though the sequence of events unfolding during the Bangladesh famine in 1974 and the realised outcomes were just one set among a number of possible scenarios *ex-ante*.

Still, the fact that poverty analysis has been typically backward-looking and oblivious of risks regarding the future does not have to be a serious problem. For example, when assessing the impact of a new transfer scheme after it has been introduced, data on its actual impact and the resulting poverty outcomes are obviously relevant. However, when deciding to commit resources to competing schemes *ex-ante*, evaluating which one will be more effective to reduce poverty will have to take into account potential outcomes in different states of the world. Furthermore, the possibility of serious hardship contains information relevant for assessing low wellbeing. For example, consider two families, both with the same expected consumption, above some accepted norm, but one with a positive probability of hardship, and the other one facing no such risk. Neither is expected to be poor, and *ex-post* we may observe them to have the same consumption, but surely the threat of future hardship for the former has some bearing on the forward-looking, *ex-ante* analysis of welfare.

The term 'vulnerability' has been invoked frequently, not least by practitioners.[1]. Yet its meaning seems to vary across the literature, and also across the few measures which have been advanced to date. This paper proposes a new class of measures of individual vulnerability. To the best of our knowledge, it constitutes the first one giving a formal axiomatic treatment to this concept, which allows us to be more accurate as to what we mean by 'vulnerability'. The analytical work in Ravallion (1988) provides one of the closest references to our paper, in spite of his alternative, but still related interest in the effect of risk on expected aggregate poverty. At a second stage, as we turn to aggregate vulnerability, we insist both on an axiomatic approach and on the need to allow uncertainty to impinge on wellbeing in a direct manner, and not only through the churning of individuals in and out of poverty and the resulting changes in aggregate poverty measures.

Poverty is a crucial element of the notion of vulnerability.[2] In the literature, vulnerability is always more than mere exposure to risk – it is also about deprivation and shortfalls. Furthermore, it is always more than mere expected poverty. The role of risk must not be confined to an 'informational' dimension, that is to the difficulties it entails for the prediction of future living conditions. In our view, uncertainty has a direct bearing on wellbeing. People suffer and feel wary of the future if they have no certain knowledge of what it will bring.

Thus we differ both from measures where vulnerability is some form of low expected utility (as in Ligon and Schechter 2003) and from those where it is *a priori*

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[1] Prominent among its earlier advocates is Chambers (1989), where vulnerability "refers to *exposure to contingencies and stress,* (…) which is *defencelessness*, meaning a lack of means to cope without damaging *loss*" (p. 1. Italics are ours). The World Bank's World Development Report 2000/01 emphasizes addressing vulnerability as one of the three pillars to attack world poverty (World Bank 2001).

[2] Throughout this paper, we understand 'vulnerability' as vulnerability to poverty. For instance, we construe 'vulnerability to an epidemic' as a shortcut to 'vulnerability to poverty due to an epidemic'.

equated to expected poverty (as frequently assumed in empirical work, e.g. Chaudhuri et al. 2002).[3] We hope to strike a proper balance by envisaging vulnerability as *the burden of the threat of future poverty*. As such, it relates both a) to the likelihood of future poverty episodes, and b) to the severity of poverty in such cases. Individuals dread the possibility of suffering poverty in the future, and they are said to be vulnerable to the extent that varying degrees of poverty cannot be ruled out as possible scenarios.

Section 2 proposes our preferred axioms and the resulting class of individual measures. Section 3 discusses issues arising as we shift to the aggregate level. In particular, drawing on literature on multidimensional poverty, we propose a set of axioms leading to an aggregate measure sensitive to the threat of widespread, simultaneous hardship. Lastly, Section 4 provides an empirical illustration based on data from Ethiopia.

2. A family of individual vulnerability measures

Let individual vulnerability (V) be measured by $V=v(z,\mathbf{p},\mathbf{y})$, where z is the poverty line, and $\mathbf{p}$ and $\mathbf{y}$ are k-dimensional vectors, containing state-of-the-world probabilities and outcomes, respectively – i.e., p_i is the probability of the i-th state occurring, with outcome y_i. We impose $y_i \geq 0$. It may be easiest to think of these outcomes as consumption levels in each possible state of the world, especially if poverty is defined as usual as a shortfall in consumption. We remark that we mean outcomes *after all consumption-smoothing efforts have been deployed*. In other words, their variability across states is taken as a final word, with no scope for reducing it further, e.g. by formal insurance, risk-sharing, or precautionary savings.

For each state, define ‘censored outcome’ $\tilde{y}_i$ by $\tilde{y}_i \equiv \mathrm{Min}(y_i,z)$, and the ‘rate of coverage of basic needs’ x_i by $x_i \equiv \tilde{y}_i/z$, so that $0 \leq x_i \leq 1$. Vectors $\tilde{\mathbf{y}}$ and $\mathbf{x}$ are defined correspondingly. $\mathbf{e}_i$ stands for a k-dimensional vector whose elements are 0, except

[3] We ignore here a third, smaller stream in the literature, which understands vulnerability as inability to isolate wellbeing from income shocks, e.g. as in Amin, Rai and Topa (2003). For instance, in a regression of consumption on income and other variables, the income coefficient would be construed as a vulnerability measure.

for the i-th one, which equals 1. We close our notation with vectors $\hat{\mathbf{y}}$ and $\tilde{\mathbf{y}}^c$. Their elements are all equal to $\hat{y}$ and $\tilde{y}^c$, respectively, which in turn are defined by $\hat{y}=\sum_{i=1}^{k} p_i \tilde{y}_i$ and $v(z,\mathbf{p},\tilde{\mathbf{y}})=v(z,\mathbf{p},\tilde{\mathbf{y}}^c)$. Note that $\tilde{y}^c$ can be written as a function $\tilde{y}^c(z,\mathbf{p},\tilde{\mathbf{y}})$ and will shortly be called the risk-free equivalent to the set of prospects described by $(z,\mathbf{p},\mathbf{y})$, in the sense that it yields the same degree of vulnerability. $\hat{y}$ is the expected value of $\tilde{y}_i$.

We propose eight desiderata. The first is the FOCUS AXIOM, which imposes $v(z,\mathbf{p},\mathbf{y})=v(z,\mathbf{p},\tilde{\mathbf{y}})$. Our measure will thus disregard outcome changes above the poverty line. If vulnerability is understood as a burden caused by the threat of future poverty, it should not be compensated by simultaneous (ex-ante) possibilities of being well-off. In consequence, high vulnerability is not necessarily tantamount for grim overall expected wellbeing (as arguably in Ligon and Schechter), since the 'promise' of richness in some states can raise welfare expectations, with no bearing on vulnerability.

Imagine that a farmer faces two scenarios: rain (no poverty) or drought (poverty). Does she become less vulnerable if the harvest in the rainy scenario improves? Our answer is 'no'. *Poverty is as bad a threat as before.* It is as likely as before, and it is potentially as severe as before.

According to this axiom, 'excess' outcomes $y_i-z>0$ are 'wasteful' and can be ignored, as far as vulnerability is concerned. Taking this for granted, the remaining axioms can be presented as follows:

SYMMETRY OVER STATES: $v(z,\mathbf{p},\tilde{\mathbf{y}})=v(z,\mathbf{Bp},\mathbf{B}\tilde{\mathbf{y}})$, where $\mathbf{B}$ is any $k\times k$ permutation matrix. All states receive the same treatment, and the only relevant difference between two states of the world i and j is the difference in their outcomes (y_i, y_j) and probabilities (p_i, p_j).

CONTINUITY AND DIFFERENTIABILITY. Function $v(z,\mathbf{p},\tilde{\mathbf{y}})$ is continuous and twice-differentiable in $\mathbf{y}$, for tractability and to preclude abrupt reactions to small changes in outcomes.

SCALE INVARIANCE. $v(z,\mathbf{p},\tilde{\mathbf{y}})=v(\lambda z,\mathbf{p},\lambda\tilde{\mathbf{y}})$ for any $\lambda>0$. Our measure will not depend on the unit of measure of outcomes.

NORMALISATION. $\text{Min}_{\tilde{\mathbf{y}}}[v(z,\mathbf{p},\tilde{\mathbf{y}})]=0$ and $\text{Max}_{\tilde{\mathbf{y}}}[v(z,\mathbf{p},\tilde{\mathbf{y}})]=1$. We impose closed boundaries to facilitate interpretation and comparability.

PROBABILITY-DEPENDENT EFFECT OF OUTCOMES. For $-c<\tilde{y}_i<z$ and $p_i p'_i\neq 0$, $v(z,\mathbf{p},\tilde{\mathbf{y}})-v(z,\mathbf{p},\tilde{\mathbf{y}}+c\mathbf{e}_i)=v(z,\mathbf{p}',\tilde{\mathbf{y}}')-v(z,\mathbf{p}',\tilde{\mathbf{y}}'+c\mathbf{e}_i)$ if and only if $p_i=p'_i$ and $\tilde{y}_i=\tilde{y}'_i$. Should $\tilde{y}_i$ change, the consequent effect on vulnerability is not allowed to depend on the outcomes or probabilities of other states of the world – for a given p_i, the change in vulnerability depends only on $\tilde{y}_i$.[4] In the opposite direction, the effect must be sensitive to the likelihood of that particular state of the world. Note that $p_i p'_i\neq 0$ discards 'impossible' states ($p_i=p'_i=0$).

PROBABILITY TRANSFER. For every $p_j\geq d>0$, $v(z,\mathbf{p}+d(\mathbf{e}_i-\mathbf{e}_j),\tilde{\mathbf{y}})\begin{Bmatrix}\leq\\ \geq\end{Bmatrix}v(z,\mathbf{p},\tilde{\mathbf{y}})$ if $\tilde{y}_i\begin{Bmatrix}\geq\\ \leq\end{Bmatrix}\tilde{y}_j$.

If $\tilde{y}_i$ is greater than or at least equal to $\tilde{y}_j$, then vulnerability cannot increase as a result of a probability transfer from state j to state i. Likewise, if $\tilde{y}_i$ is lower than or at most equal to $\tilde{y}_j$, then vulnerability cannot decrease. Going back to the example of the farmer facing rain and drought, we say that she becomes more vulnerable if a drought becomes more likely, at the expense of the rainy scenario (or at least, her vulnerability does not lessen as a result).

RISK SENSITIVITY. $v(z,\mathbf{p},\tilde{\mathbf{y}})>v(z,\mathbf{p},\hat{\mathbf{y}})$. Vulnerability would be lower if the expected (censored) outcome $\hat{y}$ were attained in all states of the world and uncertainty were thus removed. In other words, greater risk raises vulnerability.[5] Thus we link up with our first intuition about vulnerability, as a concept aiming to capture the burden of insecurity, the fact that hardship is also related to fear of future threats.

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[4] A possible counterargument could run 'in fact, there could be some relief in considering that one could have done much better had the odds been more fortunate' (or to the contrary, 'she may rue having missed a better possible outcome, with no fault on her part, and thus her misery will be greater'). We ignore such counterarguments for the sake of tractability. In doing so, we simply adhere to the common concept of poverty as mere failure to reach a poverty line, with no regard for 'subjective' subtleties.

[5] We implicitly define the increase in risk as a probability transfer 'from the middle to the tails', in keeping with one of the Rothschild-Stiglitz senses of risk.

Alternatively, resorting to the risk-free equivalent $\tilde{y}^c$, the same axiom could be expressed as $\tilde{y}^c/\hat{y}<1$. Expected outcome is unevenly and 'inefficiently' spread across states of the world, in the sense that a similarly low degree of vulnerability would result from $\tilde{y}^c<\hat{y}$ being secured in every state. $\tilde{y}^c/\hat{y}$ reflects this 'efficiency loss'.

CONSTANT RELATIVE RISK SENSITIVITY. For $\kappa>0$, $\kappa\tilde{y}^c(z,\mathbf{p},\tilde{\mathbf{y}})=\tilde{y}^c(z,\mathbf{p},\kappa\tilde{\mathbf{y}})$. A proportional increase by κ in the outcomes of all possible states of the world leads to a similar proportional increase in the risk-free equivalent $\tilde{y}^c$. While risk sensitivity ensures $\tilde{y}^c/\hat{y}<1$, we now require this ratio (or 'efficiency loss') to remain constant if all state-specific outcomes increase proportionally.

As compared to the previous axioms, this final property seems less compelling. Still, we find it attractive for its contribution both to narrowing down the families of acceptable measures to only one, and to securing that risk sensitivities receive an appropriate treatment. As for this second point, Ligon and Schechter (2003) were the first to point out that some existing vulnerability measures hid some awkward assumptions, e.g. risk sensitivity increasing in initial income, at odds with most empirical findings on risk attitudes (e.g. Binswanger 1981).

Needless to say, we are avoiding here terms such as 'risk aversion' or 'utility'. We intend our choice of language to convey our view of vulnerability as distinct from expected utility, if only to stress our departure from proposals where vulnerability boils down to some form of bad 'overall' expectations (e.g. Ligon and Schechter). On the other hand, parallels should be obvious. In fact, the proof of the following theorem heavily draws on results from expected utility theory (mainly Pratt 1964), necessarily with some departures due to the specific traits of our vulnerability concept. For this reason and for brevity, it is not provided, but it is available on request.

THEOREM 1 – If all the axioms above are satisfied, then

$$V_{(\alpha)}=1-\mathrm{E}[x^{\alpha}], \text{ with } 0<\alpha<1. \qquad (1)$$

E is the expected value operator, and we recall $x_i \equiv \tilde{y}_i/z$ is the rate of coverage of basic needs, and $0 \leq x_i \leq 1$. We highlight the simplicity of this single-parameter family of measures $V_{(\alpha)}$.[6] Of course, α regulates the strength of risk sensitivity – as α rises to 1, we approach risk-neutrality.

A few remarks are in place. First, for those facing no uncertainty and with known $x_i = x^* < 1$ for all i, $V_{(\alpha)} > 0$. If vulnerability is about the threat of poverty, certainty of being poor is but a dominant, irresistible threat. The concept is not confined to those whom the winds might blow into poverty or out from it. Vulnerability is about risk, but not only about it.

Second, it is easy to prove that $V_{(\alpha)}$ is equal to the probability of being poor only if outcomes are expected to be zero in every state of the world where the individual is poor. If vulnerability were measured as expected FGT_0 (as in Chaudhuri et al. 2002), then vulnerability would be overestimated. Ligon and Schechter have pointed out the shortcomings of other FGT choices.[7]

Finally, $V_{(\alpha)}$ can still be assimilated into the expected-poverty approach to vulnerability, provided poverty is measured as in Chakravarty (1983). In some sense, one of the contributions of this paper is to identify the Chakravarty poverty index as the best choice if the poverty analysis moves from static poverty on to vulnerability.

3. Aggregation and the threat of widespread poverty

Given a measure of individual vulnerability, the level of aggregate vulnerability can be arguably measured as a convenient combination of individual levels. Simple averages is an option (as in Ligon and Schechter 2003, Suryahadi and Sumarto 2002), or less simplistically, some account for inequality can be brought on board, as first advocated by Basu and Nolen (2004). If a measure of individual

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[6] For instance, if our last axiom (constant relative risk sensitivity) were replaced by constant absolute risk sensitivity $[\kappa + \tilde{y}^c(z,\mathbf{p},\tilde{\mathbf{y}}) = \tilde{y}^c(z,\mathbf{p},\tilde{\mathbf{y}}+\boldsymbol{\kappa})$, for $\kappa > 0]$, the less attractive measure $V_{(\beta)} = 1 - \mathrm{E}[\{e^{\beta(1-x)} - 1\}/\{e^{\beta} - 1\}]$, with $\beta > 0$, would result.

[7] More precisely, we should speak about expected individual poverty, as measured by the function implicit in the corresponding aggregate FGT index, as in Foster, Greer and Thorbecke (1984).

vulnerability is available (which was not the case of Basu and Nolen), then it can be treated much in the same way as income is treated in distribution-sensitive measures of aggregate income.[8] This is a sensible path, yet we do not pursue it here.

Whatever the particular formula to combine individual vulnerability levels, the result is bound to imply that a society can only be made more (less) vulnerable if at least one of its members is made more (less) vulnerable in the first place. Much to the contrary, an alternative approach could allow society as a whole to be sensitive to issues which pass unnoticed at the individual level.

Take the threat of widespread poverty. According to $V_{(\alpha)}$, the individual is more or less vulnerable depending on her own outcomes in the possible states of the world – whether other individuals face poverty in the same states where she does is entirely irrelevant. Nonetheless, society could well be concerned with the possibility of a considerable section of the population suffering hardship *simultaneously*. It may help to imagine the position of a policymaker: it may not be enough to ensure that aggregate vulnerability will be low 'overall' as in 'on average', and the policymaker must pay attention to some particularly hard states of the world with a large part of the population suffering simultaneously, except perhaps for dictatorial governments with no need to respond to public needs. A relative bias towards averting famine, even at the cost of some persistent malnutrition may be a reflection of these principles.[9] In fact, a similar concern for the realisations of aggregate poverty underlies the analysis in Ravallion (1988), even though there, risk is deprived of any unmediated effect on wellbeing.

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[8] An obvious caveat will be the interpretation of the parameter of inequality aversion, since inequality raises aggregate vulnerability, whereas it lowers aggregate distribution-adjusted income. For instance, if we take $\frac{1}{n}\sum_{j=1}^{n}\frac{V_{(\alpha)}^{1-\gamma}}{1-\gamma}$, then $\gamma<0$ and its increase implies that *less* attention is paid to inequality. Also, we should note that under this approach, inequality aversion must refer to *vulnerability as such*, since the condition $0<\alpha<1$ already embeds some degree of aversion to inequality in *consumption* (below the poverty line) into the simple average of $V_{(\alpha)}$ (i.e. $\gamma=0$).

[9] An illustration can be taken from Sen (1999), who dwells on "the massive Chinese famines of 1958-1961. Even before the recent economic reforms, China had been much more successful than India in economic development in many significant respects. (…) Nevertheless, there was a major failure in China in its inability to prevent famines. The Chinese famines of 1958-61 killed, it is now estimated, close to thirty million people – ten times more than even the gigantic 1943 famine in British India" (p. 181). So to speak, a democratic government in China would have feared the prospect of a famine and put into practice effective preventive measures.

Let n be the number of individuals, and imagine n=3, k=3, z=10, and **p**=(1/3,1/3,1/3), i.e. all states of the world are equally likely. Two matrices **Y** and $\mathbf{Y}'$ (whose j-th columns are the outcome vectors $\mathbf{y}_j$ of the j-th individual) describe two alternative scenarios:

$$\mathbf{Y}=\begin{bmatrix}15 & 7 & 18\\ 25 & 12 & 8\\ 5 & 20 & 14\end{bmatrix}\begin{pmatrix}\text{drought}\\ \text{normal}\\ \text{floods}\end{pmatrix}, \text{ and } \mathbf{Y}'=\begin{bmatrix}5 & 7 & 8\\ 15 & 12 & 14\\ 25 & 20 & 18\end{bmatrix}\begin{pmatrix}\text{drought}\\ \text{normal}\\ \text{floods}\end{pmatrix}$$

For instance, **Y** implies that only the third state (floods) is a poverty threat for the first individual ($y_{31}<z$), whereas the second individual is highly dependent on generous rainfall ($y_{12}<z$) and the third one profits from others' distress ($y_{23}<z$), say because she is a speculative trader. Under $\mathbf{Y}'$, hardship hits them all together when a drought occurs, say because they are all farmers. Thus, even though the switch from **Y** to **Y**' does not make any individual more nor less vulnerable, society may well be said to be more vulnerable under **Y**'.

On a practical note, this concern for widespread poverty implies that *correlations of individual outcomes across states of the world* will matter. Hence, the formulation of a suitable aggregate measure can draw on the literature on multidimensional poverty (Atkinson and Bourguignon 1982, Bourguignon and Chakravarty 2003) and multivariate risk (Richard 1975, Epstein and Tanny 1980, Pollak 1967, 1979). The former is interested in correlations of wellbeing dimensions across individuals, while the latter deals with assessments of total risk when two or more factors covary.

Again, a set of desiderata can determine our measure of aggregate vulnerability $AV=\zeta(z,\mathbf{p},\mathbf{Y})$. The following axioms apply with only notational adjustments: FOCUS, SYMMETRY OVER STATES, CONTINUITY AND DIFFERENTIABILITY, SCALE INVARIANCE, and PROBABILITY-DEPENDENT EFFECT OF OUTCOMES. Two more can be added with no need of much explanation:

SYMMETRY OVER INDIVIDUALS. $\zeta(z,\mathbf{p},\mathbf{Y})=\zeta(z,\mathbf{p},\mathbf{Y}\mathbf{B}_n)$, where $\mathbf{B}_n$ is any $n\times n$ permutation matrix.

REPLICATION INVARIANCE. $\zeta(z,\mathbf{p},\mathbf{Y})=\zeta(z,\mathbf{p},\mathbf{Y}\otimes\mathbf{1}_r)$, where $\mathbf{1}_r$ is an r-dimensional row vector and $\otimes$ performs a Kronecker product.

The other axioms need more significant modifications. NORMALISATION must be weakened so that only the lower boundary $\mathrm{Min}_{\mathbf{Y}}\zeta(z,\mathbf{p},\mathbf{Y})=0$ applies. Since a set of n outcomes exist in each state of the world, the risk-related axioms have no direct analogues. As a remedy, we imagine a society where $\mathbf{Y}^u=\mathbf{y}^u\mathbf{1}_n$, i.e. any i-th state is characterised by the fact that all individuals reach the same outcome level y_i^u. We may think of this as an hypothetical instance of perfect risk-sharing. We can now require that *in all cases where* $\mathbf{Y}$ *can be written as* $\mathbf{Y}^u$, PROBABILITY TRANSFER, RISK SENSITIVITY, and CONSTANT RELATIVE RISK SENSITIVITY must apply.

Note that the condition $\mathbf{Y}^u=\mathbf{y}^u\mathbf{1}_n$ implies maximum correlation in individual outcomes. Thus, in the risk-related axioms above, differences in outcome correlations are inexistent, and hence unable to blur the effects of outcome changes, probability transfers, and increases in individual risks. To take account of the impact of correlations, we impose two additional axioms, which follow the multivariate risk literature and in particular, Richard (1975) and Epstein and Tanny (1980), respectively.

CONDITIONAL INDEPENDENCE. $\zeta(z,\mathbf{p},[\mathbf{Y}_1,\mathbf{1}_k^{\mathrm{T}}\overline{\mathbf{y}}_2])>\zeta(z,\mathbf{p},[\mathbf{Y}'_1,\mathbf{1}_k^{\mathrm{T}}\overline{\mathbf{y}}_2]) \Rightarrow \zeta(z,\mathbf{p},[\mathbf{Y}_1,\mathbf{1}_k^{\mathrm{T}}\overline{\mathbf{y}}_2])>\zeta(z,\mathbf{p},[\mathbf{Y}'_1,\mathbf{1}_k^{\mathrm{T}}\overline{\mathbf{y}}_2])$, where $\overline{\mathbf{y}}_2$ is an n_2-dimensional row vector and $\mathbf{1}_k^{\mathrm{T}}$ is a k-dimensional column vector. As far as aggregate vulnerability is concerned, the outcomes of (n_2) risk-free individuals do not intervene in the comparison of any two scenarios which only differ by the prospects faced by the $n_1=n-n_2$ individuals exposed to risk. The treatment of correlations remains tractable only by focussing on the outcomes actually covarying, and isolating other invariant factors (outcomes of risk-free individuals).[10]

[10] In fact, a similar condition typically exists in works on multivariate risk, e.g. Pollak (1967) and Keeney (1972).

SENSITIVITY TO CORRELATIONS IN OUTCOMES. $\zeta(z,\mathbf{p},\mathbf{W})<\zeta(z,\mathbf{p}^*,\mathbf{W}^*)$ whenever a) $\mathbf{W}$ is a $k\times2$ outcomes matrix with $w_{s1}>w_{t1}$, $w_{s2}<w_{t2}$, and $w_{s2},w_{t1}<z$ for some pair of states s and t, b) $\mathbf{W}^*$ is $(k+2)\times2$, with the $(k+1)$-th and $(k+2)$-th states being described by (w_{s1},w_{t2}) and (w_{t1},w_{s2}) respectively, and c) $\mathbf{p}^*$ is $(k+2)$-dimensional probability vector with $p^*_{k+1}=p_s-p^*_s>0$ and $p^*_{k+2}=p_t-p^*_t>0$.

The switch from $(\mathbf{p},\mathbf{W})$ to $(\mathbf{p}^*,\mathbf{W}^*)$ must be understood as an increase in the likelihood of both individuals simultaneously facing low outcomes (respectively, w_{t1} and w_{s2}), at the expense of the probability of states where meagreness is confined to only one of them. Put it differently, correlation in individual outcomes is higher under $(\mathbf{p}^*,\mathbf{W}^*)$, and society is assumed to be in distress at the prospect of any two individuals falling into (deeper) poverty together.[11] In terms of the multidimensional poverty literature, we thus imagine society to see individuals as 'substitutes'.

We can now state our second result, whose proof is provided in the Appendix.

THEOREM 2 – If all the axioms above are satisfied, then

$$AV_{(\mu,\theta)}=\mu\mathrm{E}[(\textstyle\prod_{j=1,\ldots,n}x_j)^{\theta/n}-1],\text{ with }\mu>0\text{ and }\theta<0 \qquad (2)$$

4. An empirical illustration

To illustrate our approach, we use three rounds (1994, 1999 and 2004) of a rural household panel data survey from Ethiopia, on 15 villages and about 1400 households.[12] Possibly more than of any other country in the world, most people's perception of Ethiopia is shaped by risk, linked to drought and other climatic vagaries. The data requirements for a credible estimation of vulnerability measures are vast, since we need to construct predictions at t-1 for perceived welfare outcomes at t in different possible states of the world.

[11] The axiom of conditional independence allows us to focus solely on a *pair* of individuals. Richard (1975) very early clarified that "multivariate risk aversion is a pairwise property".

[12] Three other rounds were available for 1995, 1997 and 1998, but they were excluded to avoid seasonality issues (rounds were collected in different seasons) and to keep the gap between rounds constant.

A number of different approaches can be found in the literature. For example, Chaudhuri et al. (2002) use a cross-section prediction model of consumption, imposing multiplicative heteroscedasticity on the error term to 'estimate' the risk distribution based on the household level variance. A key assumption is that within the cross-section, information is contained on all possible states of the world, which is hard to accept for covariate risk. Ligon and Schechter (2003) use panel data to identify the sources of covariate and idiosyncratic variability in form directly useful for their utility-based measure of vulnerability.

Our approach is different in at least two respects. First, we will allow for an autoregressive structure in our prediction model for consumption at *t* based on information at *t*-1. Secondly, we identify shocks directly by using data on the historical rainfall distribution and reported shocks such as illness, price and market shocks, and asset losses. Details on specific features of the data (including on sampling, coverage and issues such as the low attrition in the data) can be found in Dercon et al. (2005).

The logarithm of consumption was regressed on lagged log consumption and a number of variables expressing shocks.[13] In particular, we first introduce rainfall data using a spline function, where a dummy exists for each decile of the locality-specific distribution of rainfall (usually a series of about 30 years), i.e. each dummy indicates whether local rainfall in the most recent agricultural year reached a particular decile. The questionnaire also contained other information on shocks, realized risk outcomes. Our regression includes dummy variables indicating market-related shocks (e.g. demand or price collapses), loss of assets (e.g. due to fire or theft), death of a household member and serious illness of the head or the spouse, or of other relatives. Further controls are introduced via village fixed effects (a set of dummies) and variables accounting for household composition changes over time.

[13] Consumption values were constructed using the total value of food and non-food consumption, based on purchased items, as well as from the own harvest and from gifts. They were deflated using a local food Laspeyres price deflator using 1994 as the base.

To account for the endogeneity of lagged consumption, we used lagged holdings of land and of livestock as identifying instruments.[14]

Appendix 2 shows the results, based on a random-effects estimation.[15] Beyond a strongly significant lagged dependent variable, rainfall shocks matter substantially, in fact with a non-linear impact. For instance, significant positive effects can be found for rainfall levels between 60 and 80 percent of the usual rainfall distribution, but too much rain is damaging as well. Other shocks do not appear to have a systematic negative effect (not jointly significant), but serious illness on its own does prove to be significant.[16] Unsurprisingly, risk is not insured but affects consumption considerably. The main source of risk identified in this model is rainfall risk, and to a lesser extent illness.

We can now use this model in each period *t*-1 to predict outcomes for possible states of the world. For rainfall, we will be able to use the village-specific distribution as implied by the rainfall patterns of the last 30 years. For other sources of risk, we assume for simplicity that these risks are idiosyncratic, and that for each year, the village-specific realizations in the data give the probability distribution of this risk. We assume that this village-level distribution is independent of rainfall risk. Alternative distributional assumptions were also explored, with only a limited impact on the findings.

Using these predictions, individual and aggregate vulnerability measures implied by (1) and (2) can be calculated. For comparison, table 1 also shows a number of other indicators. The first line shows the logarithm of consumption in each of the three periods considered. As can be seen, consumption increased considerably in these villages between 1994 and 1999, but remained constant on average between 1999 and 2004. The increases coincided with much improved weather and a continued

[14] Land is not privately owned, but user rights are allocated by local authorities, while livestock is both a factor of production for these mixed farmers, and the main liquid asset for accumulation and smoothing. Together they are by far the most important assets in this rural economy.
[15] The Hausman test provided no guidance in our case (as not infrequent in small samples), but the Breusch-Pagan test suggested the existence of random effects.
[16] Unsurprisingly, village fixed effects and household composition effects are also strongly jointly significant, but will be assumed to be perfectly predictable, so that uncertainty is confined to rainfall and idiosyncratic shocks. Detailed results can be obtained upon request.

recovery from very low levels in the late 1980s and early 1990s, a period coinciding with drought and the final stages of the civil war (which ended in 1991). The table also shows some suggestive differences between households with good (above median) versus bad road access and those owning livestock holdings above and below the village-level median. Road access and livestock are (significantly) correlated with substantially higher consumption levels. Calculating poverty levels (using FGT measures, with α=0,1,2 – i.e. the head count, the poverty gap and the squared poverty gap, Foster et al. 1984) shows similar patterns.

In contrast, we present two measures of aggregate vulnerability. First, we present V, the average value of the individual measure (1), with α=1/3. V does not follow the pattern above, since it focuses on expected outcomes. Note that the values reported are the values of V at t, forward looking to t+1. While poverty diminished between 1994 and 1999, we observe an increase in vulnerability. The bottom part of the table helps to understand this move: even though the average of expected consumption remained constant in this period, when moving to a narrower focus on the poor, the expected number of the poor rises in this period and their mean consumption declines.

According to this measure, vulnerability in these communities in the coming years, as seen from 2004, appears to have come down again to levels similar to 1994. However, V with no risk (i.e. valuing only expected shortfalls in resources) does remain above 1994 levels, suggesting that risk has become a less important component of the threat of future poverty. Moreover, this is not inconsistent with the fact that expected mean levels of consumption of the poor that are in 2004 lower than predicted ten years earlier (while V remains around 0.039), since this may point to lower inequality in the distribution of inequality (see footnote 8, *in fine*).

On the other hand, these results are mainly due to the use of a particular aggregation method (V averages). Using AV, calculating vulnerability based on (2), allowing society to fear wide-spread poverty, we do not observe a return of vulnerability to levels as in 1994.[17] Vulnerability also increased then considerably between 1994 and 1999, but after 1999, vulnerability is not coming down substantially. These data suggest that 2009 poses a greater threat of widespread poverty compared to 1999: in general, despite increased mean consumption, vulnerability in these communities across rural Ethiopia has increased over the last ten years.

[17] As for shock correlations across individuals, shocks were assumed to abide with any one of the observed set of shocks (i.e. possible states of the world preserve the way shocks are combined in the population).

Table 1. Poverty and Vulnerability in villages in rural Ethiopia

	Year			Road access (t=2004)		Livestock (t=2004)	
	1994	**1999**	**2004**	**Bad**	**Good**	**Poor**	**Rich**
y_t	0.219	0.500	0.501	0.229	0.586	0.229	0.709
$FGT_{(0)}$	0.393	0.260	0.253	0.349	0.222	0.390	0.148
$FGT_{(1)}$	0.153	0.085	0.086	0.118	0.076	0.140	0.046
$FGT_{(2)}$	0.082	0.039	0.042	0.055	0.038	0.072	0.019
$V_{(1/3)}$	0.039	0.049	0.039	0.043	0.038	0.059	0.025
$V_{(1/3)}$ [No risk]	0.027	0.039	0.036	0.040	0.035	0.055	0.021
$AV_{(1,-2)}$	0.125	0.217	0.182	0.268	0.157	0.322	0.086
$E[y_{t+1}]$	0.499	0.502	0.632	0.451	0.688	0.373	0.829
$E[FGT_{(0)}]$	0.220	0.266	0.223	0.281	0.206	0.337	0.137
$E[y_{t+1} \mid y_{t+1}<z]$	-0.350	-0.439	-0.499	-0.465	-0.514	-0.533	-0.436

Source: calculated using the Ethiopian Rural Household Survey and appendix 1.

Just as in the case of poverty analysis, aggregate vulnerability can only give headline figures, useful for broad comparison between groups and over time. Further insights can be gained from constructing a vulnerability profile, based on the (multivariate) correlations of household vulnerability with a set of basic characteristics, such as demographics, assets, and other general household- and village-level characteristics. As an illustration, table 2 presents an example of such a profile, as well as results for a squared poverty gap index, to show that not all characteristics are similarly reflected in poverty and vulnerability.

Many of the characteristics of vulnerability and poverty are shared, as table 2 shows for a profile in 2004, using a Tobit model. Adult labour, livestock and road access, for example, have similar (relative) effects. However, this does not hold for all variables. Distance to town is strongly significant as far as vulnerability is concerned. It has however no significant effect on poverty. This result is robust to using the head count or the poverty gap. We also find that permanent cropping areas are less vulnerable than the baseline (which is Kersa, a drought-prone area with chat), but not significantly less poor. Vulnerability is not just the same as poverty.

Table 2. Poverty and Vulnerability Profiles, rural Ethiopia 2004

	FGT$_{(2)}$	***V*$_{(1/3)}$**
Females [0-15]	8.709***	3.762**
	3.15	1.58
Females [16 and older]	11.602**	6.693***
	4.57	2.30
Males [0-15]	4.690	1.368
	2.99	1.52
Males [16 and older]	1.898	-1.060
	3.62	1.83
Males/Female ratio	2.396	1.509
	2.22	1.12
Livestock	-4.196***	-1.254***
	0.59	0.30
Road access	-1.444**	-0.649*
	0.70	0.35
Distance to town	0.481	2.434**
	1.90	1.00
Permanent cropping villages	-3.802	-10.311***
	4.35	2.25
Northern highland villages	-19.234***	-22.867***
	4.77	2.47
High-potential highland villages	-24.973***	-18.090***
	4.52	2.18
Resettlement villages	-8.131*	-17.511***
	4.92	2.60
Constant term	-5.814	2.420
	9.31	4.79
Observations	*1138*	*1138*
LR $\chi^2_{(12)}$	*[209.8]****	*[199.6]****

Standard deviations in smaller font. *, **, *** denote significance at 10%, 5%, and 1% significance levels.

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[Available upon request: Proof of Theorem 1]

FOCUS, SCALE INVARIANCE, PROBABILITY-DEPENDENT EFFECT OF OUTCOMES, CONTINUITY AND DIFFERENTIABILITY, and SYMMETRY OVER STATES lead directly to $v(z,\mathbf{p},\mathbf{y}) = \varphi(\mathbf{p}) + \sum_{i=1}^{k} w(p_i, x_i)$, where function w is continuous and differentiable. Next, PROBABILITY-TRANSFER implies $v(z,\mathbf{p},\mathbf{y}) = \sum_{i=1}^{k} p_i \breve{w}(x_i)$, with $\breve{w}' < 0$. RISK-SENSITIVITY further imposes $\breve{w}'' > 0$. Resorting to $x^c \equiv \frac{y^c}{z} = \breve{w}^{-1}\left(\sum_{i=1}^{k} p_i \breve{w}(x_i)\right)$, we can rewrite CONSTANT RELATIVE RISK SENSITIVITY as $\sum_{i=1}^{k} p_i \breve{w}(x_i) = \breve{w}\left(\frac{\breve{w}^{-1}\left(\sum_{i=1}^{k} p_i \breve{w}(\kappa x_i)\right)}{\kappa}\right)$. Let $h_i \equiv \breve{w}(\kappa x_i)$ and $g(h_i) \equiv \breve{w}\left(\frac{\breve{w}^{-1}(h_i)}{\kappa}\right)$, so that we can write $\sum_{i=1}^{k} p_i g(h_i) = g\left(\sum_{i=1}^{k} p_i h_i\right)$. From Jensen's inequalities, any concave (convex) segment in function g would turn this equality in to a lower-than (greater-than) inequality. Hence, since function $\breve{w}$ is continuous, it must be the case that $\frac{d^2 g}{dh_i^{\,2}} = \frac{\breve{w}'(x_i)}{[\kappa \breve{w}'(\kappa x_i)]^2 x_i}\left[\frac{\breve{w}''(x_i)}{\breve{w}'(x_i)} x_i - \frac{\breve{w}''(\kappa x_i)}{\breve{w}'(\kappa x_i)} \kappa x_i\right] = 0$. For the difference in brackets to be necessarily zero, $\frac{\breve{w}''(x_i)}{\breve{w}'(x_i)} x_i = \alpha - 1$, or equivalently, $\frac{d \ln \breve{w}'(x_i)}{dx_i} = \frac{\alpha - 1}{x_i}$. Integrating twice, and imposing NORMALISATION, we reach $v(z,\mathbf{p},\mathbf{y}) = 1 - \sum_{i=1}^{k} p_i x_i^{\alpha}$, with $0<\alpha<1$ to secure $\breve{w}' < 0$ and $\breve{w}'' > 0$.

Appendix 1. Proof of Theorem 2

Let $\bar{\mathbf{x}}_i$ stand for a row vector with n elements x_{ij}. FOCUS, SCALE INVARIANCE, SYMMETRY OVER STATES, and PROBABILITY-DEPENDENT EFFECT OF OUTCOMES lead directly to $\zeta(z,\mathbf{p},\mathbf{Y})=\varphi(\mathbf{p})+\sum_{i=1}^{k}\breve{\breve{\varsigma}}(p_i,\bar{\mathbf{x}}_i)$. Next, PROBABILITY-TRANSFER (for $\mathbf{Y}=\mathbf{Y}^u$) narrows function ζ down requires linearity in probabilities, so that

$$\zeta(z,\mathbf{p},\mathbf{Y})=\sum_{i=1}^{k}p_i\breve{\varsigma}(\bar{\mathbf{x}}_i) \qquad \text{(R1)}$$

and moreover, $\frac{\partial\breve{\varsigma}(\bar{\mathbf{x}}_i)}{\partial x_{ij}}<0$ for any j, provided CONTINUITY AND DIFFERENTIABILITY.

Given $\frac{\partial\breve{\varsigma}(\bar{\mathbf{x}}_i)}{\partial x_{ij}}<0$, RISK-SENSITIVITY (for $\mathbf{Y}=\mathbf{Y}^u$) requires convexity, i.e. $\frac{\partial^2\breve{\varsigma}(\bar{\mathbf{x}}_i)}{\partial {x_{ij}}^2}>0$ for any j. Next, let $\bar{\mathbf{x}}_{i(\sim g)}$ contain i-state outcomes for all individuals, except for g. CONDITIONAL INDEPENDENCE then requires that functions $\dot{w}(\bar{\mathbf{x}}_{i(\sim g)})$ and $\ddot{w}(\bar{\mathbf{x}}_{i(\sim g)})$ will exist such that for any individual g,

$$\breve{\varsigma}(x_{ig},\bar{\mathbf{x}}_{i(\sim g)})=\dot{w}(\bar{\mathbf{x}}_{i(\sim g)})+\ddot{w}(\bar{\mathbf{x}}_{i(\sim g)})\varsigma(x_{ig}) \qquad \text{(R2)}$$

To see why, imagine $\bar{\mathbf{x}}_{i(\sim g)}=\bar{\mathbf{x}}_{(\sim g)}$ for all i, such that individuals in this partition face no risk. Then, note that only affine transformations of $\varsigma(\bar{\mathbf{x}}_{ig})$ can be allowed if we intend to prevent changes in $\bar{\mathbf{x}}_{(\sim g)}$ from causing re-orderings of triplets (z,$\mathbf{p}$,$\mathbf{Y}$) in the vulnerability ranking, as determined by (R1). To exploit SYMMETRY OVER INDIVIDUALS, (R2) can be rewritten as in (R3) and (R4), letting either the g- or the j-individual act as reference point:

$$\breve{\varsigma}(x_{ig},x_{ih},\bar{\mathbf{x}}_{i(\sim g,h)})=\dot{w}(x_{ih},\bar{\mathbf{x}}_{i(\sim g,h)})+\ddot{w}(x_{ih},\bar{\mathbf{x}}_{i(\sim g,h)})\varsigma(x_{ig}) \qquad \text{(R3)}$$

$$\breve{\varsigma}(x_{ig},x_{ih},\bar{\mathbf{x}}_{i(\sim g,h)})=\dot{w}(x_{ig},\bar{\mathbf{x}}_{i(\sim g,h)})+\ddot{w}(x_{ig},\bar{\mathbf{x}}_{i(\sim g,h)})\varsigma(x_{ih}) \qquad \text{(R4)}$$

For (R3) and (R4) to be equivalent, first we need $\dot{w}(x_{ig},\bar{\mathbf{x}}_{i(\sim g,h)})=\dot{w}(x_{ih},\bar{\mathbf{x}}_{i(\sim g,h)})$, which implies $\dot{w}(\bar{\mathbf{x}}_{i(\sim j)})=\mu_1$, where μ_1 stands for a constant. Second, we require $\ddot{w}(x_{ih},\bar{\mathbf{x}}_{i(\sim g,h)})\varsigma(x_{ig})=\ddot{w}(x_{ig},\bar{\mathbf{x}}_{i(\sim g,h)})\varsigma(x_{ih})$, which in turn imposes $\ddot{w}(\bar{\mathbf{x}}_{i(\sim j)})=\prod_{k\neq j}\varsigma(x_{ik})$. Thus we have

$$\breve{\varsigma}(\bar{\mathbf{x}}_i)=\mu_1+\prod_{j=1}^{n}\varsigma(x_{ij}) \tag{R5}$$

Next, we require $\zeta(z,\mathbf{p},\mathbf{Y})=\mu_1+\sum_{i=1}^{k}p_i\prod_{j=1}^{n}\varsigma(x_{ij})$ to satisfy CONSTANT RELATIVE RISK SENSITIVITY (for **Y=Y**^{u}) and REPLICATION INVARIANCE. They jointly impose

$$\varsigma(x_{ij})=\mu_2(x_{ij})^{\frac{\theta}{n}} \tag{R6}$$

To see why, note that in a uniform society, $\zeta(z,\mathbf{p},\mathbf{Y}^u)=\mu_1+\sum_{i=1}^{k}p_i[\varsigma(x_i)]^n$, and the certainty equivalent is $x^c=\varsigma^{-1}\left(\left[\sum_{i=1}^{k}p_i[\varsigma(x_i)]^n\right]^{\frac{1}{n}}\right)$, so that relative risk sensitivity $\left(\frac{x^c(\kappa\mathbf{x}_j)}{\kappa}=x^c(\mathbf{x}_j)\right)$ is constant if

$$\left[\varsigma\left(\frac{\varsigma^{-1}\left(\left[\sum_{i=1}^{k}p_i[\varsigma(\kappa x_i)]^n\right]^{\frac{1}{n}}\right)}{\kappa}\right)\right]^n=\sum_{i=1}^{k}p_i[\varsigma(x_i)]^n$$

which can be rewritten as

$$r\left(\sum_{i=1}^{k}p_iq_i\right)=\sum_{i=1}^{k}p_ir(q_i) \tag{R7}$$

if we let $q_i=[\varsigma(\kappa x_i)]^n$ and $r(q_i)=\left[\varsigma\left(\frac{\varsigma^{-1}\left([q_i]^{\frac{1}{n}}\right)}{\kappa}\right)\right]^n$.

From Jensen's inequalities, any concave (convex) segment in function *r* would turn the (R7) equality into a lower-than (greater-than) inequality. Hence, it must be the case that $\frac{d^2r}{dq_i^{\,2}}=0$, which after some algebra can be spelt out as

$$A_1\left[A_2\left\{\frac{\varsigma'\left(\frac{t_i}{k}\right)}{\varsigma\left(\frac{t_i}{k}\right)}\left[\frac{t_i}{k}\right]-\frac{\varsigma'(t_i)}{\varsigma(t_i)}[t_i]\right\}+\left\{\frac{\varsigma''\left(\frac{t_i}{k}\right)}{\varsigma'\left(\frac{t_i}{k}\right)}\left[\frac{t_i}{k}\right]-\frac{\varsigma''(t_i)}{\varsigma'(t_i)}[t_i]\right\}\right]=0 \tag{R8}$$

where $$A_1 = \left[\varsigma\left(\frac{t_i}{k}\right) \right]^{n-1} \frac{[q_i]^{\frac{1}{n}-2}}{n\kappa} \frac{\varsigma'\left(\frac{t_i}{k}\right)}{[\varsigma'(t_i)]^2}$$

$$A_2 = \frac{n-1}{t_i}[q_i]^{\frac{1}{n}}$$

$$t_i = \varsigma^{-1}\left([q_i]^{\frac{1}{n}}\right)$$

Since the two curly brackets in (R8) must be equal to zero, we have a pair of differential equations with common solution as in (R6). To see why, note e.g. that the first equation can be written as

$$\frac{\varsigma'(t)}{\varsigma(t)} = \frac{\theta^*}{t}$$

Integration (or double integration, for the second brackets) yields (R6), after defining the constant θ^*, such that REPLICATION INVARIANCE holds. NORMALISATION, together with PROBABILITY TRANSFER $\left(\frac{\partial \breve{\varsigma}(\bar{\mathbf{x}}_i)}{\partial x_{ij}} < 0\right)$, RISK SENSITIVITY $\left(\frac{\partial^2 \breve{\varsigma}(\bar{\mathbf{x}}_i)}{\partial {x_{ij}}^2} > 0\right)$ and SENSITIVITY TO CORRELATIONS IN OUTCOMES, sets the final restrictions on θ ($\theta<0$), μ_1 and μ_2 ($\mu_2=-\mu_1>0$), and completes the proof.

Appendix 2. Consumption model 1994-2004.

Dependent variable is the logarithm of consumption per capita, expressed in 1994 prices. (Random effects model)

	Coefficient	Standard error
Lagged consumption†	0.537***	(0.06)
Rainfall	***[83.2]****	
Decile 2	-0.175	(0.15)
Decile 3	1.075***	(0.16)
Decile 4	-0.797***	(0.12)
Decile 5	-0.312**	(0.14)
Decile 6	0.646***	(0.11)
Decile 7	0.304***	(0.08)
Decile 8	-0.600***	(0.12)
Idiosyncratic shocks	***[6.3]***	
Market-related	0.048	(0.04)
Asset losses	0.048	(0.04)
Death	0.031	(0.04)
Illness (head or spouse)	-0.034	(0.05)
Illness (others)	-0.099*	(0.06)
Village fixed-effects (not reported)	***[263.6]****	
Household composition (not reported)	***[137.0]****	
Observations	*2329*	
Breusch-Pagan [H_0: $\sigma_u^2=0$]	*10.6****	

Standard deviations in smaller font. *, **, *** denote significance at 10%, 5%, and 1% significance levels. Joint significance tests in square brackets. Deciles of rainfall ordered from low to high.

† instrumented, using lagged livestock values and land values as identifying instruments.