

# Information design through scarcity and social learning<sup>☆</sup>

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## Abstract

We show that a firm may benefit from strategically creating scarcity for its product, in order to trigger herding behaviour from consumers in situations where such behaviour is otherwise unlikely. We consider a setting with social learning, where consumers observe sales from previous cohorts and update beliefs about product quality before making their purchase. Imposing a capacity constraint directly limits sales but also makes information coarser for consumers, who react favourably to a sell-out because they infer only that demand must exceed capacity. Consumer learning is then limited even with large cohorts and unbounded private signals, because the firm acts strategically to influence the consumers' learning environment. Our results suggest that in suitable environments capacity constraints can serve as a useful tool to implement optimal information design in practice: if private signals are not too precise and capacity can be changed

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over time, then in large markets the firm's optimal choice of capacity delivers the same expected sales as the Bayesian persuasion solution.

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## 1. Introduction

This paper shows that firms may want to strategically create product scarcity, to influence consumer learning in a way that can effectively persuade consumers to buy. As is standard in the literature on social learning (see foundational papers by Banerjee (1992) and Bikhchandani et al. (1992)), each consumer receives a noisy private signal about product quality, but also infers some information from observing earlier sales. To illustrate the mechanism at work, consider a cohort of ten consumers who visit a firm, where each buys if and only if her signal was good. A consumer who then arrives and observes initial sales of three may refuse to buy, even if her own signal was good, because she infers that only three out of the ten others had a good signal. But if the firm, without knowing product quality, had initially limited capacity to three sales per period, then the consumer would observe a sell-out, and only infer that there were *at least* three good signals. That may well convince the consumer to buy even if her own signal was bad, and trigger a positive purchase cascade.

Now suppose that in our example, one out of ten consumers per period is perfectly informed about quality, and that the firm has unlimited capacity. If quality is low, but say seven consumers in period 1 receive good signals, then everyone will buy in period 2 except for the informed consumer. Provided that the number of potential buyers is perfectly understood, the informed consumer's choice not to buy will perfectly reveal low quality and lead to zero sales in later periods. However, this same choice would not reveal any information if the firm had restricted capacity, because the informed consumer would be effectively pooled with those who were unable to buy due to rationing.<sup>2</sup>

Thus, consumer learning is limited despite large cohort size (as in the 'guinea pigs' considered by Sgroi (2002)) and unbounded private signals (as in Smith and Sørensen (2000)). The reason is that here, the consumers' learning environment is endogenous, and the firm can manipulate this environment by restricting capacity. The result is that consumers may fail to learn, in particular in situations where learning would hurt the firm.

The attractiveness of restricting capacity as a tool to manipulate learning can be understood more broadly in terms of information design. A firm's choice of capacity affects the structure of consumers' public information, analogous to the way a sender determines the structure of a receiver's private signal in models of Bayesian persuasion. Bayesian persuasion mechanisms generally raise two practical concerns: how the signal space is determined and how a sender can commit to a particular signal structure. Neither concern is an issue in our setting, as both the

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<sup>2</sup> In the spirit of Smith and Sørensen (2000), we use the word 'cascade' to refer to a situation where all consumers with boundedly informative signals take the same action regardless of their private information.

signal space and commitment power follow naturally from the firm's choice of capacity. In this sense, our paper brings out a close connection between Bayesian persuasion and social learning despite apparent differences between the two approaches.

Our focus on sell-outs fits in with evidence that demand for some products seems to persistently outstrip supply and that suggests a plausible cause is seller strategic behaviour. Examples include restaurants (e.g. 'Noma', Damon Baehrel' or 'Club 33'), music festivals (tickets for Glastonbury 2016 selling out in just 30 minutes.<sup>3</sup>), professional sports (e.g. Real Madrid vs. Juventus match tickets were sold out in 8 minutes<sup>4</sup>; the Boston Red Socks experienced sell-outs from 2003-2013 in Fenway Park), concert halls (Courty and Pagliero (2012) argue that concert promoters believe that empty seats would reveal negative information to consumers, and therefore select venues and prices to make sellouts more likely), and the infamous 'Beanie Babies'. Relatedly, the effect of boosting demand via social learning has been well documented in environments such as movies (Moretti (2011), Cabral and Natividad (2016)) and restaurants (Cai et al. (2009)). In all these examples, when demand is high, firms may lose out on potential sales as their product is rationed, but nonetheless frequent price adjustments tend to be the exception rather than the rule.

We start in Section 2 by presenting a simple two-period model of social learning, in which all consumers have boundedly informative signals. Consumers arrive in cohorts of size  $2n$  in each period and period-2 consumers observe sales from period 1. The first period is interpreted as trial sales, and the second period as the continuation sales in the main market. All social learning takes place in between these two periods, and the seller looks to maximize period-2 sales.

Our analysis in Section 3 shows that, for any  $K \leq n$ , parameters exist for which the seller would rather restrict capacity to  $K$  in both periods rather than remain unconstrained. The capacity constraint directly limits period-2 sales and also reduces willingness to pay via a 'winner's curse' effect, since each consumer realises she is more likely to be served in the bad state. However, restricting capacity also censors the true level of demand following a sellout, which affects consumer learning. Sellouts effectively pool high demand events with events that would otherwise have triggered a negative purchase cascade, and now instead drive up willingness to pay. We show that in a large market, holding the value of consumers' outside option fixed, the seller prefers to restrict capacity when the prior that the state is good is not too high, but still makes buying a priori more attractive than the outside option, while the private signal precision takes intermediate values.

We then proceed in Section 4 to explore how optimal capacity constraints perform relative to the benchmark of optimal Bayesian persuasion (Kamenica and Gentzkow (2011)), i.e. an ex ante commitment to a rule that maps product quality to a binary purchase recommendation, where each consumer decides whether to buy based on the recommendation and on her private signal. If private signals are relatively imprecise, then the benchmark is familiar: always recommend that the customer buy when the state is good, and sometimes when it is bad. We show that in large markets, where the seller can adjust capacity over time, the optimal capacity constraint can attain these benchmark payoffs by effectively implementing the Bayesian persuasion outcome. Restricting capacity then results in seller-optimal information provision, where the probability of

<sup>3</sup> See <http://www.glastonburyfestivals.co.uk/glastonbury-2016-tickets-sell-out-in-30-minutes/>, accessed on August 18, 2022.

<sup>4</sup> See <https://sport.news.am/eng/news/87567/real-madrid-vs-juventus-tickets-sold-out-in-8-minutes.html> accessed on August 18, 2022.

a period-1 sellout in each state is equal to the corresponding probability of a buy recommendation under the benchmark.

Finally, Section 5 considers a fully dynamic model with an infinite time horizon and informed consumers with unbounded private signals, where two new phenomena arise. First, an infinite time horizon opens a possibility for gradual learning, where multiple sell-outs are required to trigger a cascade. Second, the presence of informed consumers can further increase the attractiveness of restricting capacity due to the possibility of cascade reversal. With unrestricted capacity, an informed consumer's decision not to buy will immediately reverse an incorrect positive cascade, where all boundedly informed consumers buy despite the state being bad. However, when the seller restricts capacity, an incorrect positive cascade can be maintained, since consumers cannot distinguish between those in earlier cohorts who chose not to buy and those who were not served due to rationing.

Our paper contributes to the literature on social learning with imperfect observability of past actions. Different work has assumed that agents can observe a random sample of actions that is anonymous (Banerjee and Fudenberg (2004), Smith and Sorensen (2013), Monzón and Rapp (2014), Monzón (2017)) or non-anonymous (Acemoglu et al. (2011), Lobel and Sadler (2015)), the aggregate total of all past actions (Callander and Hörner, 2009), the aggregate total of one particular action (Guarino et al. (2011), Herrera and Hörner (2013)), or only the choice of an agent's immediate predecessor (Çelen and Kariv, 2004). Unlike these papers, the information structure in our setting is endogenous, so consumers may fail to learn about low quality despite two features that the literature suggests should promote learning: multiple consumers who do not have access to social information (see Banerjee (1992), Sgroi (2002), Acemoglu et al. (2011), Smith and Sorensen (2013), Golub and Sadler (2017)); and unbounded private signals (see, e.g., Smith and Sørensen (2000), Banerjee and Fudenberg (2004)).

Our paper also relates to the recent literature on Bayesian persuasion pioneered by Kamenica and Gentzkow (2011). As our seller can only influence consumers' information through its choice of capacity, the paper complements work looking at a sender's choice from a restricted set of signal structures: Tsakas and Tsakas (2021) consider noise that distorts signal realizations, Perez-Richet and Skreta (2022) study sender manipulation of test results, and Ichihashi (2019) explores which signal-structure restrictions are optimal for the receiver. In terms of costly persuasion, Gentzkow and Kamenica (2016, 2017) and Mensch (2021) assume a direct cost associated with each experiment, whereas our cost of restricting capacity (i.e. foregone sales) is implicit and depends on consumer behaviour. Other papers share our focus on dynamics (see Au (2015), Ely (2017), Renault et al. (2017), Best and Quigley (2017), Bizzotto et al. (2021), Orlov et al. (2020)), but none consider scarcity or social learning.<sup>5</sup>

Our results also present a novel rationale for firms to strategically restrict capacity. Work on scarcity strategies has mainly focused on discouraging consumer strategic delay (DeGraba (1995), Nocke and Peitz (2007), Möller and Watanabe (2010)). Creating scarcity for low-valuation consumers can also help a monopolist to price discriminate, and may be optimal if revenues are non-concave (Wilson (1988), Bulow and Roberts (1989), Ferguson (1994), Lortscher and Muir (2022)). Both Debo et al. (2012) and Stock and Balachander (2005) consider

<sup>5</sup> The signal structure associated with a capacity constraint in our setting involves upper-tail censoring: revealing precise information about the state if news is sufficiently bad (i.e. demand below a threshold value), and coarse information otherwise. Kolotilin et al. (2017), Kolotilin and Zapechelnnyuk (2018), and Dworczak and Martini (2019) all show that upper-tail censoring can at times be optimal in persuasion problems where the state is drawn from a continuous distribution.

scarcity and social learning but in a setting with a privately informed seller, where scarcity does not help hide information from consumers, but instead may help reveal it.<sup>6</sup>

The broader literature on influencing consumer learning has mainly focused on pricing (Welch (1992), Bose et al. (2006), Bose et al. (2008), Sayedi (2018)), which affects learning differently. The main issue in Bose et al. (2006) and Bose et al. (2008) is under what conditions the firm will set a low pooling price to stop all social learning, whereas our seller decides how much information to reveal via its optimal capacity choice. In terms of results, Bose et al. (2006) and Bose et al. (2008) show that the best way to maximize expected future profits is to reveal as much information as possible, which is not the case in our setting. Indeed, the possibility of increasing future profits is precisely what motivates the firm to hide information by restricting capacity. We show in Online Appendix D that this motive still prevails when the firm can choose both capacity and price, provided that neither can be changed between periods.<sup>7</sup>

## 2. Model

Suppose there is a product or service of unknown quality and two possible states of the world,  $\Omega = \{G, B\}$ . In state  $G$ , quality is good and each consumer who buys obtains  $u_G = 1$ . In state  $B$ , quality is bad and each consumer who buys obtains  $u_B = 0$ . A consumer who does not buy gets reservation utility  $r \in (0, 1)$ .

The actual state is not initially known, neither to the seller nor to consumers. Prior beliefs of all players are that  $P(G) \equiv \beta$  and  $P(B) = 1 - \beta$ . There are two periods in the game and in each period there are  $2n$  potential buyers. Before making her purchase decision, each consumer receives a noisy private signal,  $s \in \{g, b\}$ , where  $P(g|G) = P(b|B) \equiv \alpha \in (1/2, 1)$ . By  $\alpha < 1$ , signals are *boundedly informative*. We focus on situations where consumers without further information would follow their signals, i.e.  $P(G|s = g) > r > P(G|s = b)$ .

At the start of the game,  $t = -1$ , the seller can set a *capacity constraint*  $1 \leq K \leq 2n$ . This capacity choice is irreversible and limits potential sales in each period (i.e. how many consumers can buy), which cannot exceed capacity.<sup>8</sup> The state is realized at  $t = 0$ , so the constraint itself does not reveal any information.<sup>9</sup> We will refer to  $K = 2n$  as *unrestricted or full capacity*, and  $K < 2n$  as *restricted or limited capacity*.

In each period  $t \in \{1, 2\}$ ,  $2n$  consumers arrive. We interpret consumers in period 1 as those in a trial sales round and consumers in period 2 as those from the main market. The seller receives a fixed profit per consumer who buys in the **second period**, normalized to 1. This profit specification is an approximation of the discounted stream of future profits in a fully dynamic model with an infinite time horizon, which we discuss in Section 5.

Consumers arriving in period 2 observe both capacity and total sales from the consumers in period 1. That is, consumers do not directly observe quantity demanded in the first period, but

<sup>6</sup> Vikander (2019) considers a privately informed firm that may limit capacity to influence consumer beliefs, but assumes bounded rationality and social image concerns.

<sup>7</sup> Other differences in our analysis include the link with optimal Bayesian persuasion, the winner's curse effect, and the fact that a firm's strategic choice of capacity can help an incorrect positive cascade to be maintained.

<sup>8</sup> We use irreversible capacity in our main analysis, and later explicitly relax this assumption by allowing the seller to adjust capacity over time.

<sup>9</sup> Parsa et al. (2005) document that about 60% of new restaurants fail within three years, which suggests that their owners had imprecise information about quality when opening and setting capacity. We require that the seller and consumers hold the same prior, as is common in the literature on social learning, see, e.g., Bose et al. (2006), Bose et al. (2008) and Bhalla (2013).

only quantity sold. Notice that a sell-out in period 1, where sales equal capacity, need not imply that demand precisely equalled capacity.

### 3. Analysis

We start by describing how consumers' behaviour will depend on their private signals, previously observed sales, and the seller's capacity choice. Given consumer behaviour, we then examine under what conditions the seller may profit from restricting capacity.

Period-1 consumers facing a seller with full capacity follow their private signals. The behaviour of period-2 consumers will depend on period-1 sales. Sales of at least  $n + 1$  out of  $2n$  will trigger a positive purchase cascade where all period-2 consumers buy, since these sales are sufficiently informative to outweigh a bad private signal.<sup>10</sup> Similarly, sales of at most  $n - 1$  will trigger a negative cascade where no period-2 consumer buys. Sales of exactly  $n$  are uninformative, so period-2 consumers then follow their private signals.<sup>11</sup> This result is formally stated in Lemma B.2 in the Appendix.

Period-1 consumers facing a seller with limited capacity should take into account an additional effect: how the probability of being served may depend on the state. Suppose a consumer who chooses to buy believes she is  $\lambda$  times more likely to be served in the bad state than in the good one. Let  $P(G|s, \lambda)$  denote the belief of such a buyer of the state being good, conditional on receiving private signal  $s$  and being served. Then for consumers to follow their private signals, the relevant condition is  $P(G|b, \lambda) < r < P(G|g, \lambda)$ , where  $P(G|b, \lambda) < P(G|g, \lambda)$  since signals are informative. The exact value of  $\lambda$  will depend on capacity  $K$  but always satisfies  $\lambda > 1$ , as long as each consumer believes that others will follow their private signals. Each consumer understands that others are more likely to receive good signals in the good state, which then results in higher demand and a lower probability of being served.<sup>12</sup> As such, whenever  $\lambda > 1$ , we have  $P(G|b, \lambda) < P(G|b)$  and  $P(G|g, \lambda) < P(G|g)$ , where this *winner's curse effect* reduces willingness to pay.

While restricting capacity can reduce willingness to pay via the winner's curse effect, and can directly limit sales, it can also lead to a sellout in period 1, which will increase willingness to pay in period 2. Let  $Q_\omega(j)$  denote the probability of exactly  $j$  period-1 consumers receiving good signals,

$$Q_G(j) = \binom{2n}{j} \alpha^j (1 - \alpha)^{2n-j}, \quad Q_B(j) = \binom{2n}{j} \alpha^{2n-j} (1 - \alpha)^j. \quad (1)$$

Now consider a period-2 consumer who receives a bad signal, observes a sellout in period 1, believes that period-1 consumers followed their private signals, and thinks that she is  $\lambda$  times more likely to be served in the bad state than in the good one. This consumer's willingness to pay is equal to her belief that the state is good, conditional on being served, which is equal to

<sup>10</sup> Since all signals are equally precise, evidence of  $n + 1$  good signals and  $n - 1$  bad signals from period 1-consumers, combined with one bad private signal in period 2, is informationally equivalent to having one single good signal, as  $(n + 1) - (n - 1) - 1 = 1$ .

<sup>11</sup> In the fully dynamic models discussed in Section 5, consumers might follow their own signals for a number of periods, but a cascade will eventually occur.

<sup>12</sup> If all period-1 consumers ignore their private signals, then the probability of being served will be independent of the state, resulting in  $\lambda = 1$ .

$$\gamma(K, \lambda) = \frac{P(G \cap b, \text{sell-out})}{P(G \cap b, \text{sell-out}) + P(B \cap b, \text{sell-out})} = \frac{1}{1 + \frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha} \left[ \frac{\sum_{j=K}^{2n} Q_B(j)}{\sum_{j=K}^{2n} Q_G(j)} \right] \lambda}. \quad (2)$$

Clearly a sellout is more likely in the good state than in the bad state if consumers follow their private signals:  $\sum_{j=K}^{2n} Q_G(j) > \sum_{j=K}^{2n} Q_B(j)$ . Thus, for any given  $\lambda$ , we have  $\gamma(K, \lambda) > P(G|b, \lambda)$ . The winner's curse effect of restricting capacity can also lead to the possibility of multiple equilibria, since the relevant values of  $\lambda$  in  $P(G|g, \lambda)$  and  $P(G|b, \lambda)$  are not always uniquely defined. In particular, multiple equilibria can exist in period 2 when  $\gamma(K, \lambda > 1) < r < \gamma(K, \lambda = 1)$ , where consumers will then only ignore their private signals following a sellout if they expect others to do the same. The following Lemma shows that there is a value of the outside option  $r$  such that there is an equilibrium where (i) all period-1 consumers follow their private signals and (ii) a sell-out in period-1 triggers a purchase cascade among period-2 consumers. It also shows that this equilibrium is unique when signals are precise.

**Lemma 1.** *For any  $(\alpha, \beta) \in (1/2, 1) \times (0, 1)$ , there exists  $r$  such that, for any capacity constraint  $K$  there is an equilibrium in which period-1 consumers follow their private signals and period-2 consumers buy after observing a sell-out. Moreover, there exists  $\hat{\alpha} < 1$  such that, for all  $\alpha > \hat{\alpha}$ , this equilibrium is unique.*

Given a value of  $r$  for which Lemma 1 applies, we consider how restricting capacity affects seller profits. Let

$$Q(j) = \beta Q_G(j) + (1 - \beta) Q_B(j),$$

denote the probability that  $j$  consumers receive good signals in period 1. For a seller with unrestricted capacity, profits are

$$\pi_u = 2n[\beta\alpha Q_G(n) + (1 - \beta)(1 - \alpha) Q_B(n)] + 2n \sum_{j=n+1}^{2n} Q(j). \quad (3)$$

That is, initial sales of exactly  $n$  lead consumers to follow their private signals, giving period-2 expected sales of  $2n\alpha$  in the good state and  $2n(1 - \alpha)$  in the bad state. Initial sales exceeding  $n$  trigger a positive cascade with period-2 sales of  $2n$ , whereas initial sales less than  $n$  trigger a negative cascade with period-2 sales of zero. For a seller with capacity  $K \leq n$ , profits are

$$\pi_c(K) = K \sum_{j=K}^{2n} Q(j), \quad (4)$$

where an initial sellout triggers a positive cascade with period-2 sales of  $K$ , and any failure to sell out triggers a negative cascade with period-2 sales of zero. Note that it is never optimal to restrict capacity to  $K > n$ , as sales higher than  $n$  would trigger a positive cascade even if the seller had full capacity (see, e.g., footnote 10). Thus, the only impact of restricting capacity to  $K > n$  would be to limit sales in period 2. Comparing (3) and (4) gives us the following result.

**Theorem 1.** *For any  $n > 1$ ,  $K \leq n$  there are  $(\alpha, \beta) \in (1/2, 1) \times (0, 1/2)$  and  $r \in (0, 1/2)$  for which the seller can increase its profits above the full-capacity level by restricting capacity to  $K$ , and where a sellout triggers a positive cascade.*

However, if  $\beta \geq 1/2$ , then the seller prefers not to restrict capacity:  $\pi_u > \pi_c(K)$  for all  $K \leq 2n$ .



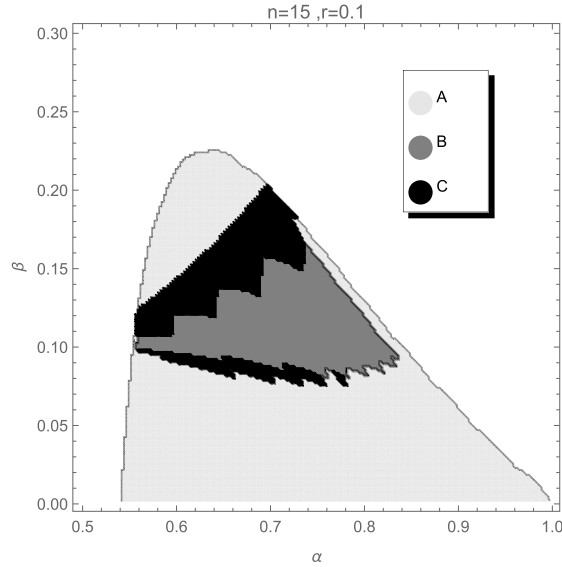


Fig. 1. When it is optimal to restrict capacity, for different values of the prior  $\beta$  and signal precision  $\alpha$ , given outside option  $r = 0.1$ .

The seller faces a tradeoff, as restricting capacity to  $K \leq n$  directly limits sales but can also increase the probability of a positive cascade. A key difference between (3) and (4) is that below-average period-1 demand, between  $K$  and  $n - 1$ , will trigger a negative cascade if the seller has unrestricted capacity, but will trigger a positive cascade if the seller restricted capacity to  $K$ . Intuitively, a capacity constraint will tend to help the seller when the state turns out to be bad by pooling intermediate outcomes with moderate demand with more favourable outcomes with high demand. This obfuscation decreases the probability of an incorrect negative cascade (where no consumer buys despite the state being good), and can increase the probability of an incorrect positive cascade (where all consumers buy despite the state being bad).<sup>13</sup>

The condition  $\beta \geq 1/2$ , which is sufficient for restricting capacity to be suboptimal, is simple and intuitive. When the state is good, initial sales are likely high enough to trigger a positive cascade regardless of the seller's choice of capacity, and a seller with unrestricted capacity then enjoys higher sales. The proof shows that for  $\beta \geq \frac{1}{2}$ , the good state is sufficiently likely that expected period-2 sales for such a seller exceed  $n$ . These sales are more than the seller could possibly enjoy by restricting capacity to  $K \leq n$ , which are the only capacity levels that can make a positive (negative) cascade more (less) likely.

Theorem 1 shows that setting capacity constraint  $K \leq n$  is better than having full capacity for *some* parameter values  $(\alpha, \beta, r)$ . We now investigate for *which* parameter values it is profitable to restrict capacity, first numerically in Fig. 1 and then analytically in large markets, i.e. when  $n \rightarrow \infty$ .

When describing Fig. 1, we will first focus on the bell-shape (combined areas A, B and C) which corresponds to the parameter region described in Theorem 1: the values of  $(\alpha, \beta)$  for which restricting capacity increases profits, for at least one value of  $r$ . We then explain why restricting

<sup>13</sup> The proof of Theorem 1 establishes existence for sufficiently high  $\alpha$ , while Lemma 1 guarantees that for high enough  $\alpha$ , the unique equilibrium of the consumer game involves a purchase cascade after a single sellout.



capacity is not optimal in area A for the specific value of  $r$  considered in the figure. Finally, we turn to areas B and C, where restricting capacity is optimal for that specific value of  $r$ , and describes when a sellout will trigger a purchase cascade.

The bell-shaped region shows, consistent with Theorem 1, that restricting capacity cannot be optimal if the prior is too high. It also suggests that for given  $\beta$ , restricting capacity can only be optimal when signal precision  $\alpha$  is moderate. Intuitively, with very imprecise signals, restricting capacity is unattractive because a positive cascade is already quite likely in the bad state, even with full capacity. The same conclusion applies with very precise signals because the very low capacity required to sell out in the bad state dramatically limits subsequent sales.

Area A in the bell-shaped region shows where restricting capacity is optimal for some values of the outside option, but not for the specific value  $r = 0.1$  considered in the figure. Unsurprisingly, restricting capacity cannot be profitable if the prior is high enough for all consumers to buy regardless of their private signals, or so low that nobody buys. More interesting is that for  $\beta$  moderately low, the very fact that the seller restricts capacity can result in zero sales, as the winner's curse effect makes consumers with good signals refuse to buy.<sup>14</sup>

Restricting capacity is optimal when  $r = 0.1$  in both Area B and Area C, but for different reasons. In area C, the seller sets capacity sufficiently high for a sellout to trigger a cascade, and profits are given by (4). In area B, it turns out that the seller prefers to restrict capacity more aggressively, which makes a sellout more likely but also less convincing. Period-2 consumers then follow their private signals following a sellout, giving profits

$$\begin{aligned} \tilde{\pi}_c(K) = & \beta \left( \sum_{j=K}^{2n} Q_G(j) \right) \times \left( \sum_{j=0}^{2n} \min\{j, K\} Q_G(j) \right) + \\ & (1 - \beta) \left( \sum_{j=K}^{2n} Q_B(j) \right) \times \left( \sum_{j=0}^{2n} \min\{j, K\} Q_B(j) \right). \quad (5) \end{aligned}$$

Thus, in area B, the seller's purpose in restricting capacity is not to start a positive cascade but only to prevent a negative one. We will return to this issue in our fully dynamic setting in Section 5, where the seller may sometimes set a capacity so low that a long sequence of sellouts is required to trigger a cascade.

Our numerical results give a sense of the parameter region for which the seller prefers to restrict capacity. When the market is large, we can describe this region precisely.

**Proposition 1.** *Consider a tuple of parameters  $(\alpha, \beta, r)$ . Then there exists a threshold  $n(\alpha, \beta, r)$ , such that for all  $n > n(\alpha, \beta, r)$*

1. *for  $K \leq n$ , the maximal profit is achieved by setting a capacity constraint for which a sell-out does not trigger a cascade, i.e.  $\max_{K \leq n} \tilde{\pi}_c(K) > \max_{K \leq n} \pi_c(K)$*
2. *restricting capacity yields higher profit than being unconstrained, i.e.  $\max_{K \leq n} \tilde{\pi}_c(K) > \pi_u$ , if*

$$\frac{\beta(1 - \alpha)}{\beta(1 - \alpha) + (1 - \beta)\alpha} < r < \beta < 1 - \alpha \quad (6)$$

*and  $\max_{K \leq n} \tilde{\pi}_c(K) \leq \pi_u$  if any inequality in (6) is reversed.*

<sup>14</sup> For more on the winner's curse effect, please see our discussion of large markets and Fig. 2.

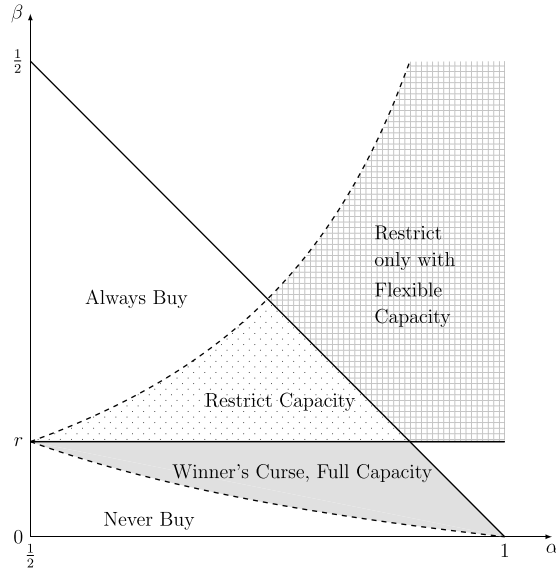


Fig. 2. When it is optimal to restrict capacity in large markets, for different values of the prior  $\beta$  and signal precision  $\alpha$ , given outside option  $r = 0.1$ .

The key point for the seller's capacity choice in large markets is that the ratio of period-1 demand to market size is concentrated around its conditional expected values,  $\alpha$  in the good state and  $1 - \alpha$  in the bad state. As the market size increases, the seller can let capacity approach a fraction  $1 - \alpha$  of the market size from below, in a way that almost always yields a period-1 sellout. Period-2 consumers then follow their own signals, giving profits per consumer close to  $1 - \alpha$ .<sup>15</sup> By comparison, for a seller who does not restrict capacity, increased market size makes period-1 sales increasingly informative about the state, and profits per consumer become close to  $\beta$ . Restricting capacity can therefore pay off when  $\beta < 1 - \alpha$ . The constraint  $\beta > r$  arises from the winner's curse effect: for the optimal capacity constraint we have that  $\lambda \rightarrow \frac{\alpha}{1-\alpha}$  as  $n \rightarrow \infty$  and, therefore,  $P(G|g, \lambda) \rightarrow \beta$ . Thus, when the market becomes large, period-1 consumers with good signals facing a capacity-constrained seller will only buy if  $\beta > r$ . Condition  $\frac{\beta(1-\alpha)}{\beta(1-\alpha) + (1-\beta)\alpha} < r$  simply ensures that consumers all follow their signal given full capacity.<sup>16</sup>

Fig. 2 shows for what points  $(\alpha, \beta)$  the seller will restrict capacity in a large market, given  $r = 0.1$ . The area between the dashed curves is where period-1 consumers facing a seller with unrestricted capacity would follow their own signals. The seller will restrict capacity in the dotted part of this area, which lies both above the horizontal line  $\beta = r$  and below the downward-sloping line  $\beta = 1 - \alpha$ . Restricting capacity would also be optimal in the grey part of this area if period-1 consumers followed their private signals, but they refuse to buy due to the winner's curse effect. Clearly, allowing  $r$  to vary in the spirit of Theorem 1 implies that restricting capacity will be optimal whenever  $\beta < 1 - \alpha$  for at least one value of  $r$ . That is, the large triangular region in Fig. 2 is the limiting case of the bell-shaped region from Fig. 1 when markets are large.

<sup>15</sup> These are the same profits the seller would earn if the sellout had triggered a positive cascade.

<sup>16</sup> The winner's curse is severe under the optimal capacity in large markets, since the probability of being served in the bad state is close to one.

Fig. 2 also highlights that for given outside option  $r$ , restricting capacity will be optimal in large markets when both signal precision  $\alpha$  and the prior  $\beta$  take on intermediate values.<sup>17</sup> Signal precision should not be so high that a very low capacity is required to generate sellouts in the bad state, which would strongly limit period-2 sales. Signal precision should also not be so low that consumers ignore their private signals, which would leave no scope for capacity constraints to influence social learning. Relatedly, the prior should not be so high that a positive cascade is likely even in the absence of capacity constraint, or so low that the winner's curse effect prevents consumers from buying. The fact that the ratio of capacity to market size equals  $1 - \alpha$  in the limit  $n \rightarrow \infty$  also implies that the size of the optimal capacity constraint, when the seller chooses to restrict capacity in large markets, will be decreasing in signal precision and insensitive to changes in the prior.

We now continue to look at large markets but assume flexible capacity, where the seller can freely adjust its capacity constraint in period 2 after observing sales in period 1. The seller's optimal strategy given flexible capacity will serve as a useful benchmark for the subsequent section on information design.

**Proposition 2.** Consider a tuple of parameters  $(\alpha, \beta, r)$ . Then there exists a threshold  $n(\alpha, \beta, r)$ , such that for all  $n > n(\alpha, \beta, r)$

1. for  $K \leq n$ , the maximal profit is achieved by setting a capacity constraint for which a sell-out triggers a cascade, i.e.  $\max_{K \leq n} \pi_c(K) > \max_{K \leq n} \tilde{\pi}_c(K)$
2. restricting capacity yields higher profit than being unconstrained, i.e.  $\max_{K \leq n} \pi_c(K) > \pi_u$  if

$$\frac{\beta(1 - \alpha)}{\beta(1 - \alpha) + (1 - \beta)\alpha} < r < \beta \quad (7)$$

and  $\max_{K \leq n} \pi_c(K) \leq \pi_u$  if either inequality in (7) is reversed.

The seller now prefers to restrict capacity for a wider range of parameter values than in Proposition 1, corresponding to both the dotted and the grid regions in Fig. 2, because the proportion of period-2 consumers who buy following a sellout is no longer bounded by  $(1 - \alpha)$ . Moreover, when restricting capacity, the seller always sets  $K$  high enough for a sellout to trigger a positive cascade,  $\gamma(K, 1) > r$ . Thus, in the dotted region of Fig. 2, the seller now restricts capacity less aggressively than under Proposition 1, even though restricting capacity no longer directly limits period-2 sales. Triggering a cascade yields a larger reward when capacity is flexible,  $2n$  instead of  $K$ , so the seller prefers to set capacity large enough to convince consumers to ignore their private signals upon a sell-out, even though doing so makes a sellout less likely.

As  $\lim_{n \rightarrow \infty} \sum_{j=K}^{2n} Q_G(j) = 1$ , the seller's per-consumer profit at the optimal capacity in a large market approaches

$$\pi = \beta + (1 - \beta) \left[ \frac{\beta}{1 - \beta} \frac{1 - r}{r} \frac{1 - \alpha}{\alpha} \right], \quad (8)$$

provided that period-1 consumers follow their private signals. These limiting profits exceed both those under full capacity,  $\beta$ , and those under a capacity that would not trigger a positive cascade,  $\beta\alpha + (1 - \beta)(1 - \alpha)$ .

<sup>17</sup> This conclusion is also consistent with Fig. 1, where we assumed market size  $2n = 30$ .

#### 4. Optimal information design

Having shown that the seller may want to restrict capacity, we now compare the resulting profits to those from the benchmark of optimal information design. We consider the limiting case of large markets,  $n \rightarrow \infty$ , to facilitate comparison with the profits that follow from Proposition 1 and Proposition 2.

There is a close connection between the seller's choice of capacity in our setting and Bayesian Persuasion. Our seller (sender) chooses capacity without knowing the state, and each consumer (receiver) then receives information that depends on the realized state and on capacity. In this sense, capacity constraints serve as a natural example of a Bayesian persuasion mechanism that can be easily implemented in practice. As capacity is chosen ex ante, and market participants do not observe excess demand, any commitment problem in implementing the desired information structure is avoided. The seller simply cannot serve demand that exceeds capacity. Consumers then directly observe the resulting sell-out; if they did not, the seller could easily disclose that a sell-out occurred.

To derive the 'persuasion mechanism' in our setting, and find the associated profits, we assume the seller commits to a rule that maps the binary state into a purchase recommendation. The state is realized, and each consumer receives a recommendation according to the chosen rule. Each consumer then makes a purchase decision based on the recommendation and her own private signal, and the seller serves all consumers who want to buy. Thus, consumers do not learn from one another but rather from the seller's recommendation. The seller's persuasion mechanism therefore substitutes for the social learning process studied in the previous section.

When the receiver is privately informed, the optimal persuasion mechanism can take one of two forms: either consumers all follow the seller's recommendation, or consumers with bad private signals follow the recommendation but those with good signals always buy. Following Bergemann and Morris (2016, 2019), we refer to the former as an obedient mechanism and the latter as being non-obedient. Let  $p_\omega$  be the probability of a buy-recommendation in state  $\omega$ .

**Proposition 3.** *For any  $r \in [0, 1]$ , there exist values  $\underline{\beta}(r)$ ,  $\bar{\beta}(r)$ ,  $\bar{\alpha}(r)$  and a function  $\underline{\alpha}(r, \beta)$  such that if  $\beta \in [\underline{\beta}(r), \bar{\beta}(r)]$  and  $\alpha \in [\underline{\alpha}(r, \beta), \bar{\alpha}(r)]$  then the optimal persuasion mechanism is obedient, with*

$$p_G = 1, \quad p_B = \frac{\beta(1 - \alpha)(1 - r)}{(1 - \beta)\alpha r}. \quad (9)$$

*Otherwise, the optimal persuasion mechanism is non-obedient, with both  $p_G, p_B < 1$ , as long as  $\alpha \geq \underline{\alpha}(r, \beta)$ .<sup>18</sup>*

Proposition 3 says that as long as signal precision is not too high, i.e.  $\alpha \leq \bar{\alpha}(r)$ , then a no-buy recommendation under the optimal mechanism fully reveals the state, and all consumers follow the seller's recommendation. The restriction  $\alpha \geq \underline{\alpha}(r, \beta)$  corresponds to our initial condition that consumers follow their own signals in the absence of any other information. Finally, the restriction  $\beta \in [\underline{\beta}(r), \bar{\beta}(r)]$  guarantees that the interval  $[\underline{\alpha}(r, \beta), \bar{\alpha}(r)]$  is non-empty.<sup>19</sup>

<sup>18</sup> Expressions for these probabilities can be found in the proof of the Proposition in the Appendix.

<sup>19</sup> Proposition 3 is similar to the result obtained in Kolotilin (2018), the only difference being that he also allows for the case  $\bar{\alpha}(r) < \alpha < \underline{\alpha}(r, \beta)$ . This case is ruled out in our setting by the assumption that consumers follow their private signals in the absence of any other information.

In contrast, if signal precision is high,  $\alpha > \bar{\alpha}(r)$ , then a no-buy recommendation under the optimal mechanism is sufficiently noisy for consumers with good signals to ignore it, whereas consumers with bad signals always follow the seller's recommendations. Compared to the obedient mechanism, the seller experiences higher sales from consumers with good signals, but lower sales from consumers with bad signals, who may now receive (and follow) a no-buy recommendation in the good state. The higher the signal precision, the less likely a no-buy recommendation must be sent in the good state for consumers with good signals to ignore it, which makes the non-obedient mechanism more attractive.

The seller's expected profits per consumer under the obedient mechanism are

$$\pi = \beta + (1 - \beta) \left[ \frac{\beta}{1 - \beta} \frac{1 - r}{r} \frac{1 - \alpha}{\alpha} \right]. \quad (10)$$

These profits coincide with (8), which are the profits the seller may achieve by restricting capacity in a large market, if capacity can be adjusted over time.

In general, restricting capacity can underperform relative to the Bayesian Persuasion benchmark for two reasons: the direct effect from limiting period-2 sales, and the indirect effect of using potentially suboptimal information design. The direct effect is enough to push expected profits below those from the benchmark if capacity, once set, is fixed. That is, the limiting profits from restricting capacity according to Proposition 1, namely  $1 - \alpha$ , are strictly lower than (10).

The direct effect from limiting period-2 sales vanishes if capacity is flexible, in which case restricting capacity in large markets performs equally well to mechanism (9) under the conditions described in Proposition 2. The seller sets capacity so that in the good state a sell-out almost surely occurs, and in the bad state it occurs with a probability equal to  $p_B$  from (9), i.e. the probability of a no-buy recommendation under the obedient mechanism.

We conclude that restricting capacity can result in optimal information design in large markets if the optimal mechanism is obedient. If the optimal mechanism is non-obedient, where consumers with good private signals always buy, then restricting capacity will underperform relative to the benchmark. The reason is that capacity then gives the seller too few degrees of freedom.<sup>20</sup> For the obedient mechanism, the seller need only approximate the buy-recommendation probability in the bad state, by setting capacity close to the conditional expected level of demand. The probability of a sellout in the good state then approaches one due to the Law of Large Numbers.

We formally state our result in the following proposition.

**Proposition 4.** *If market size is large and the seller can adjust capacity in period 2 after observing sales in period 1, then for any  $r \in [0, 1]$ , if  $\beta \in [r, \bar{\beta}(r)]$  and  $\alpha \in [\underline{\alpha}(r, \beta), \bar{\alpha}(r)]$ , the seller can implement optimal persuasion through its choice of capacity constraint.*

Fig. 3 illustrates the scope of Proposition 4, for when restricting capacity can result in optimal information provision. The obedient mechanism is optimal in the region where  $\alpha < \bar{\alpha}(r)$  (note that  $\alpha > \underline{\alpha}(r, \beta)$  holds in the region to the right of either dashed line). However, this mechanism can only be implemented by restricting capacity if  $\beta > r$  (dotted area), as for  $\beta < r$  (grey area) the winner's curse results in zero sales. In the gridded area where  $\alpha > \bar{\alpha}(r)$  and  $\beta > r$ , the

<sup>20</sup> Both recommendations under the non-obedient mechanism should be noisy, so that consumers with good signals will always buy ( $p_B < 1$ ,  $p_G < 1$ ). The seller's choice of capacity  $k$  effectively determines the probability of buy-recommendations in both the good and bad state simultaneously. As such, the seller cannot approximate both the probability  $p_B < 1$  and the probability  $p_G < 1$  under the optimal non-obedient mechanism via this single instrument.

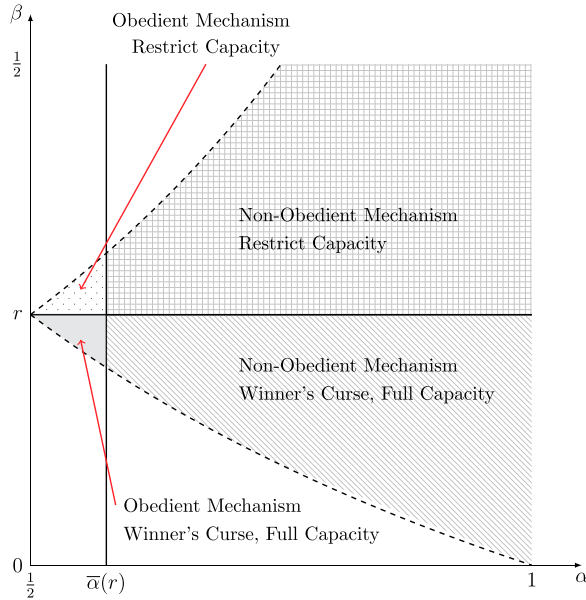


Fig. 3. Optimal persuasion and restricting capacity in large markets, for different values of the prior  $\beta$  and signal precision  $\alpha$ , given outside option  $r = 0.25$ .

seller chooses to restrict capacity but earns lower profits than under the Bayesian persuasion benchmark, where the mechanism is not obedient.

While we show that restricting capacity may implement the optimal mechanism whenever this mechanism is obedient, any other mechanism consisting of two recommendations, where one recommendation perfectly reveals the state, is also implementable.<sup>21</sup> A non-obedient mechanism of this form may be optimal in a setting with a binary state but a richer signal structure than considered in our paper. For example, in such a setting, a buy recommendation under the optimal mechanism should convince some consumers to buy, but plausibly not those with very accurate and negative private signals, just as the latter group might refuse to buy after observing a sellout.

## 5. General case

In this section we consider whether restricting capacity can also be optimal in a fully dynamic setting with potentially unbounded signals. This setting allows us to examine how consumer beliefs evolve over time, including after multiple sellouts, and explore whether sellouts now always trigger purchase cascades. It also allows us to address how the presence of ‘informed’ consumers affects the seller’s incentives to restrict capacity, including via their ability to reverse incorrect cascades.

The results show that the main insights from our two-period model carry over to this new setting: if consumers arrive in cohorts of equal size in each period and observe previous sales,

<sup>21</sup> In sufficiently large markets, demand will be concentrated in the neighbourhoods of two values. The seller can therefore set capacity such that a sell-out occurs with probability one in the good state and with optimal intermediate probability in the bad state, and implement the recommendation probabilities under such a mechanism, provided consumers are willing to buy despite the winner’s curse.

while the seller cares about the discounted value of future sales, it can still be optimal for some parameter values to restrict capacity. Moreover, although the seller may set capacity low enough so that consumers follow their private signals after an initial sellout, a sufficiently long sequence of sellouts will always trigger a cascade. Informed consumers' choices always eventually reveal the state, but their presence generates another advantage to restricting capacity, namely to extend the length of an incorrect purchase cascade, with repeated sellouts in the bad state.

The first change we make in order to proceed with a fully-fledged dynamic analysis is that we consider an infinite time horizon, where  $2n$  consumers arrive in each period. Consumers observe sales from previous cohorts, make their purchase decisions, and then leave the market.<sup>22</sup> The seller is interested in the net present value of future sales and discounts them with discount factor  $\delta$ .

The second change is that we allow for unbounded private signals. There are two types of consumers: 'uninformed' and 'informed'. Uninformed (or boundedly informed) consumers are identical to those in the two-period model from Section 2, whereas informed consumers receive unbounded signals that effectively reveal the state. We assume that each of the  $2n$  consumers is informed with probability  $\varepsilon > 0$ .<sup>23</sup> We now use the term cascade to refer to a situation where all uninformed consumers either buy or do not buy, regardless of their private signals.

Cascades are triggered in a similar way as in Section 3. If the seller does not restrict capacity, then sales of over (under)  $n$  will immediately trigger a positive (negative) cascade. If the seller restricts capacity, then a failure to sell out once at capacity  $K \leq n$  will trigger a negative cascade, whereas multiple consecutive sellouts may now be required to trigger a positive cascade.

Specifically, uninformed consumers' beliefs following sellouts evolve according to a Bayesian updating process. Let  $Q_\omega(j)$  again denote the probability of having  $j$  out of  $2n$  good signals in state  $\omega$ . Then, in a similar way to Section 3, we can define the belief of uninformed consumers with bad private signals who observe a sequence of  $l$  consecutive sell-outs at capacity  $K$ , and conjecture all others have followed their signals, as

$$\gamma(l, K, \lambda) = \frac{P(G \cap b, l \text{ sell-outs, served})}{P(G \cap b, l \text{ sell-outs, served}) + P(B \cap b, l \text{ sell-outs, served})} = \frac{1}{1 + \frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha} \left[ \frac{\sum_{j=K}^{2n} Q_B(j)}{\sum_{j=K}^{2n} Q_G(j)} \right]^l} \lambda, \quad (11)$$

where  $\lambda$  is again the relative probability of being served in the bad state compared to the good one. A sufficiently long sequence of sell-outs will eventually trigger a positive cascade:  $\gamma(l, K, \lambda)$  is increasing in  $l$  and approaches 1 as  $l \rightarrow \infty$ , so it eventually exceeds  $r$ .<sup>24</sup>

An additional issue in our fully dynamic setting, that could not be addressed in a two-period model, is that not all cascades once started will be maintained. An incorrect negative cascade, where no uninformed consumer buys despite the state being good, will be reversed by any subsequent period with positive sales, since these sales reveal that informed consumers chose to buy.

<sup>22</sup> It does not matter how long a history is observed, provided that consumers observe sales from at least two previous cohorts.

<sup>23</sup> Our approach of modelling unboundedly informative signals, through the presence of fully informed consumers, differs from the more common approach of assuming continuous signals, and dramatically helps with tractability. The analysis of a setting when  $0 \leq m \leq n$  consumers are informed about the state leads to qualitatively similar results, as demonstrated in Online Appendix C.

<sup>24</sup> See Lemma B.3 in the Appendix.



However, an incorrect positive cascade, where all uninformed consumers buy despite the state being bad, will only be reversed when sales drop below capacity  $K$ , i.e. if at least  $2n - K + 1$  informed consumers arrive in the same period. The implication is that incorrect negative cascades are quickly reversed, but incorrect positive cascades, though eventually reversed, are long-lived if the seller restricts capacity.

In order to derive the profit functions for when the seller restricts capacity, suppose that period-1 consumers follow their signals given capacity  $K$ . Let  $L$  denote the smallest value of  $l$  such that  $\gamma(l, k, 1) > r$ , where  $\gamma(l, k, 1)$  is given by (11) for  $\lambda = 1$ . That is,  $L$  consecutive sellouts at capacity  $K$  will trigger a purchase cascade. Moreover, let  $\eta_\omega$  denote the probability of a sell-out, and  $S_\omega$  denote expected sales as of a period where a sellout does not occur, given state  $\omega \in \{G, B\}$ . Then expected profits given capacity  $K$  and  $L \geq 1$  are

$$\pi_c(K) = \beta \left[ \frac{1 - (\delta\eta_G)^L}{1 - \delta\eta_G} (S_G + \eta_G K) + (\delta\eta_G)^L \frac{K}{1 - \delta} \right] + (1 - \beta) \left[ \frac{1 - (\delta\eta_B)^L}{1 - \delta\eta_B} (S_B + \eta_B K) + (\delta\eta_B)^L R \right], \quad (12)$$

where  $R$  are expected sales in the bad state in the event of a purchase cascade.<sup>25</sup> The first term in each of the square brackets, which correspond to the good and the bad state, represents the expected sales if the sellers fails to sell-out for  $L$  consecutive periods. The second term represents expected sales when the sellers sells out for  $L$  periods, where it keeps receiving  $K$  per period in the good state, and receives expected discounted sales of  $R < K/(1 - \delta)$  in the bad state (where the arrival of  $2n - K + 1$  informed consumers will eventually reverse the positive cascade).

Although it might be optimal to set a capacity such that multiple sell-outs are required to trigger a cascade, our existence result focuses on an equilibrium where period-1 consumers follow their private signals and a cascade occurs after a single sellout. Similar to Section 3, we show that this equilibrium exists for sufficiently high signal precision  $\alpha$ , and is unique when  $\alpha$  is even higher.

**Lemma 2.** *There are thresholds  $\hat{\alpha}_0 \leq \hat{\alpha}_1 < 1$  (possibly functions of parameters) and a value of the outside option  $r \in (0, 1)$  such that for any capacity constraint  $K \leq n$ :*

1. *for  $\alpha > \hat{\alpha}_0$ , consumers follow their private signals in the first period;*
2. *for  $\alpha > \hat{\alpha}_0$ , there is an equilibrium in which one sell-out triggers a cascade;*
3. *for  $\alpha > \hat{\alpha}_1$ , there is a unique equilibrium in which one sell-out triggers a cascade.*

Lemma 2 allows us to set  $L = 1$  in (12) and show that the result of Theorem 1 can be extended to a fully dynamic setting with informed consumers.

**Theorem 2.** *For any  $n > 1$ ,  $K \leq n$  and  $\delta > \sqrt{\frac{2n-K}{2n}}$ , there are  $(\alpha, \beta, \varepsilon) \in (1/2, 1) \times (0, 1/2) \times (0, 1)$  and  $r > 0$  for which the seller can increase its profits above the full-capacity level by restricting capacity to  $K$ .*

<sup>25</sup> Explicit expressions for  $R$ ,  $\eta_\omega$ , and  $S_\omega$ , for  $\omega \in \{G, B\}$ , are presented in the proof of Theorem 2 in Appendix B.

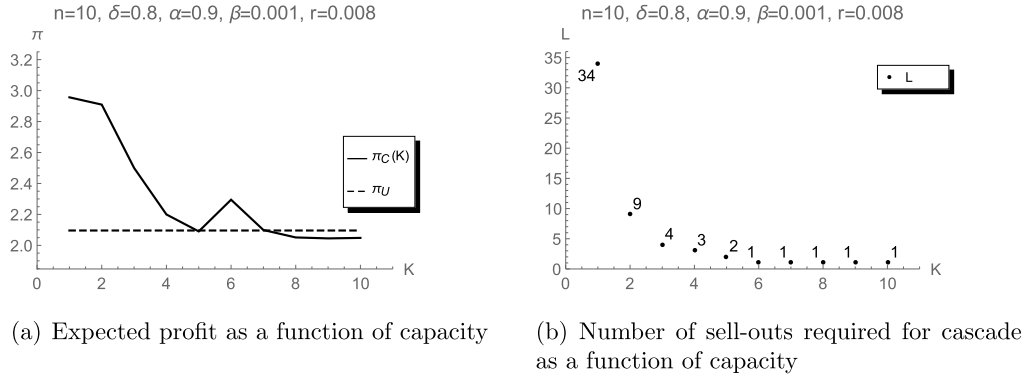


Fig. 4. Seller profits and optimality of delaying consumer learning to postpone a cascade,  $\varepsilon = 0$ .

Intuitively, restricting capacity can help the seller by increasing the probability of a positive cascade and by helping incorrect positive cascades (once triggered) to be maintained. The former channel works in a similar way to our two-period model, where a period-1 sellout can induce subsequent cohorts to buy regardless of their private signals. The latter channel follows from the presence of informed consumers, whose actions will quickly reverse an incorrect positive cascade if the seller operates at full capacity.

Incorrect positive cascades are also eventually reversed if the seller restricts capacity, but typically only after a substantially longer time. Formally, given an incorrect positive cascade, the expected number of periods until the bad state is revealed is  $1/P$ , where  $P = 1 - \sum_{i=0}^{2n-K} \binom{2n}{i} \varepsilon^i (1-\varepsilon)^{2n-i}$  if the seller sets capacity constraint  $K$ , and  $P = 1 - (1-\varepsilon)^{2n}$  if the seller has unrestricted capacity. For example, if  $2n = 6$  and  $\varepsilon = 0.1$ , and if initial sales trigger an incorrect positive cascade, then it requires on average of two periods to reveal the bad state under full capacity, but 787 periods if  $K = 3$  and about one million periods if  $K = 1$ .

The result from our two-period model that the seller sometimes set capacity in such a way that period-2 consumers followed their private signals following a period-1 sellout, has a natural counterpart in our fully dynamic setting. The purpose of restricting capacity to such a low level in Section 3 was to reduce the probability of revealing bad news, i.e. to prevent a negative cascade rather than triggering a positive one. Here, selling out at a particularly low capacity may not trigger a positive cascade immediately, but it can still serve to hide potentially low demand. In contrast to Section 3, however, a cascade will eventually occur, either a positive cascade after a sufficiently long sequence of sellouts or a negative cascade in the first period without a sellout. Fig. 4 shows that such a strategy of setting low capacity, in order to delay consumer learning and postpone the start of a cascade, is indeed optimal for certain parameters values. For the parameter values assumed in the Figure, the seller finds it optimal to set capacity  $K = 1$ , so that  $L = 34$  sellouts are required to trigger a cascade.

Finally, we numerically investigate how the presence of informed consumers affects the seller's incentive to restrict capacity. Fig. 5 compares two parameter regions: the set of points  $(\alpha, \beta)$  for which there exists some outside option  $r$  and capacity  $K$  such that it is profitable to restrict capacity when there are no informed consumers,  $\varepsilon = 0$ ; and the corresponding set of points  $(\alpha, \beta)$  when each consumer is informed with probability  $\varepsilon = 0.1$ . In the former case restricting capacity is optimal in the bell-shaped region comprised of areas A and C. In the latter case restricting capacity is optimal in areas B and C, but is no longer optimal in area A.

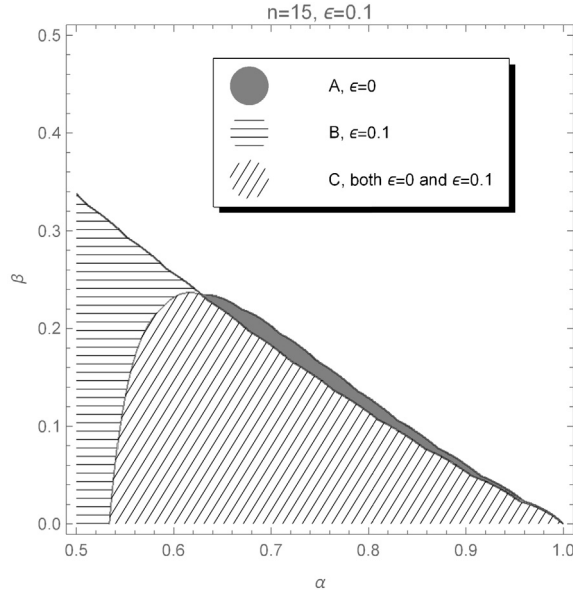


Fig. 5. When it is optimal to restrict capacity, for different values of the prior  $\beta$  and signal precision  $\alpha$ , given  $r = 0.1$ , with informed consumers (regions B, C) and without informed consumers (regions A, C).

The presence of informed consumers means that demand is now more likely to reflect the true state, which has an ambiguous effect on the seller's incentive to restrict capacity. The bell-shape in Fig. 5 shifts to the left, in a way that is broadly similar to a change in signal precision, so it is optimal to remain unconstrained in area A. However, the presence of informed consumers also means that incorrect positive cascades are more quickly reversed under full capacity than under capacity  $K \leq n$ , which unambiguously makes restricting capacity more attractive. This latter effect is particularly important if signals are imprecise, because incorrect cascades are then more likely, leading the seller to restrict capacity in area B.

Our analysis in this section has considered an infinite-time horizon and unbounded private signals but continues to not explicitly consider pricing. Pricing is markedly different than restricting capacity in that it simultaneously serves two functions, namely influencing consumer learning and extracting surplus. In terms of influencing learning, pricing is a very coarse tool for a seller with full capacity in a binary-signal setting, as sales will then either reveal all information (separating price) or reveal nothing (pooling price). A seller who could freely adjust its price over time, as in Bose et al. (2006) and Bose et al. (2008), would have a strong temptation to first reveal information and then extract surplus. In particular, fully revealing the state would eliminate any difference in willingness to pay based on consumer private information, and fully flexible pricing would then allow the seller to capture all surplus. Naturally, hiding information via capacity constraints, or any other form of information design, would then be suboptimal. That being said, our analysis in Online Appendix D shows that restricting capacity can still increase profits when the seller has discretion over the initial price but cannot adjust it over time, i.e. when the surplus extraction function of the price is limited. In this sense, restricting capacity can be seen as a tool for information management in situations like those described in the Introduction, where sellers cannot capture all surplus via dynamic pricing.

## 6. Conclusion

In this paper, we show that a seller may benefit from restricting capacity, so as to create scarcity for its product and increase future sales. Limiting capacity results in coarser information, as consumers who observe a sell-out attach positive probability to all levels of demand that exceed capacity. The results show that two main mechanisms the literature suggests may help avoid pathological social learning outcomes, ‘guinea pigs’ and unbounded private signals, can fail to do so, if the seller is able to manipulate the learning environment by a simple instrument such as limiting capacity. We also show that this simple instrument can serve a practical tool for persuading consumers, in the sense of implementing optimal information design in large markets.

Although we assume throughout our analysis that the seller can strategically set capacity, our mechanism can also shed light on situations where capacity is exogenous. Our main results will then have a slightly different interpretation; namely, that a seller that must limit production, or use a small venue, may do just as well (or better) than a seller that is not similarly constrained. In particular, this outcome will tend to occur in situations where the exogenous capacity happens to be close to the optimal level.

Our results rely on the idea that consumers can observe sales and capacity, which is reasonable in many markets, e.g. restaurants, sports and concert tickets, and limited edition products. In these markets, sales and capacity are often widely known, but the extent of any excess demand is not. Product scarcity should also affect learning in other settings, but in a way that depends precisely on what consumers can observe. For example, it will matter if precise sales figures for certain products are only observed by consumers if a sellout occurs, say due to the sellout being widely reported in the press. A common point is that seller may still have an incentive to act strategically to influence the social learning process.

Relatedly, the key point for our mechanism is that firms cannot costlessly claim to sell out regardless of the true level of sales. If sell-out claims were simply cheap talk, then consumers would fully discount them, and sellouts would not affect willingness to pay. These type of false claims are generally not feasible in relation to restaurants, sports events, or limited edition products, where consumers can directly observe very low sales figures. For performances, there are longstanding reports of ‘papering the house’, where promoters quietly give away a limited number of tickets to inflate attendance figures, but there are limits to the effectiveness of this strategy. That is, it is not feasible to fill up a large venue for a concert that nobody wants to attend. Moreover, even attempting to do so would be costly, both directly and indirectly, due to the danger that consumers may learn what the seller is up to.

Our mechanism can also apply more broadly to non-market settings, where one party wants others to take an action with positive externalities, but where there is uncertainty as to whether this action is privately optimal. For example, a government may want to promote vaccine uptake amongst its citizens, who have some sense of how their private benefits from vaccination compare to the costs associated with side effects. A small-scale vaccine roll-out, with high-take up rates to limited groups, may then convince others to take the vaccine when it is rolled out more broadly.<sup>26</sup>

<sup>26</sup> A small-scale roll-out may be largely due to production and distribution issues, rather than strategic behaviour. Nonetheless, the link between early take-up and vaccine confidence is a matter of interest for both commentators and policy makers. For example, for the case of France, see <https://www.euronews.com/2021/01/18/why-do-so-few-people-in-france-want-to-take-the-covid-19-vaccine>, accessed on August 18, 2022.

## Data availability

No data was used for the research described in the article.

## Appendix A. Proofs for two-period setting

**Proof of Lemma 1.** We first show that period-1 consumers follow their private signals for some  $r$ . We start with demonstrating that

$$P(G|b) = \frac{(1-\alpha)\beta}{(1-\alpha)\beta + \alpha(1-\beta)} < \frac{1}{1 + \frac{1-\beta}{\beta} \frac{1-\alpha}{\alpha} \lambda} = P(G|g, \lambda),$$

or equivalently

$$\lambda < \left( \frac{\alpha}{1-\alpha} \right)^2, \quad (13)$$

given

$$\lambda = \frac{P(\text{served}|B)}{P(\text{served}|G)} = \frac{\sum_{i=0}^{2n-1} \binom{2n-1}{i} (1-\alpha)^i \alpha^{2n-1-i} \min \left\{ 1, \frac{K}{i+1} \right\}}{\sum_{i=0}^{2n-1} \binom{2n-1}{i} \alpha^i (1-\alpha)^{2n-1-i} \min \left\{ 1, \frac{K}{i+1} \right\}}, \quad (14)$$

which is the relevant value of  $\lambda$  if period-1 consumers follow their private signal. To establish (13), it is sufficient to show

$$\lambda < \frac{\alpha}{1-\alpha},$$

since  $\alpha > 1/2$ . Moreover, as the distribution of sales in the good state first order stochastically dominates the distribution of sales in the bad state,  $\lambda < \alpha/(1-\alpha)$  must hold for all  $K$  if it holds for  $K = 1$ . For  $K = 1$  we get

$$\begin{aligned} \alpha P(\text{served}|G) &= \sum_{i=0}^{2n-1} \frac{(2n-1)!}{i!(2n-i-1)!} \frac{1}{i+1} \alpha^{i+1} (1-\alpha)^{2n-i-1} \\ &= \frac{1}{2n} \sum_{i=0}^{2n-1} \frac{2n!}{(i+1)!(2n-i-1)!} \alpha^{i+1} (1-\alpha)^{2n-i-1} = \frac{1}{2n} [1 - (1-\alpha)^{2n}]. \end{aligned}$$

Similarly  $(1-\alpha)P(\text{served}|B) = \frac{1}{2n}[1 - \alpha^{2n}]$ , so since  $\alpha > 1/2$  we get  $\alpha P(\text{served}|G) > (1-\alpha)P(\text{served}|B)$ , or equivalently  $\lambda < \alpha/(1-\alpha)$ . Thus, we obtain  $P(G|b) < P(G|g, \lambda)$ . This means we can find  $r$  for which  $P(G|b, \lambda) \leq P(G|b) < r < P(G|g, \lambda)$ , and period-1 consumers follow their private signal for any  $\lambda \geq 1$ , i.e. for any capacity  $1 \leq K \leq 2n$ .

Now we look at period-2 consumers. We show that there are values of  $\alpha$  such that, for some  $r$ , consumers follow their private signals in the first period regardless of capacity, and one sell-out at capacity  $K$  triggers a cascade in any equilibrium. To show this we establish that  $\gamma(K, \lambda) > P(G|b)$ , as then choosing  $r \in (P(G|b), \min\{P(G|g, \lambda), \gamma(K, \lambda)\})$  would deliver the result. Note that  $\gamma(K, \lambda) > P(G|b)$  is equivalent to

$$\frac{\sum_{j=K}^{2n} Q_B(j)}{\sum_{j=K}^{2n} Q_G(j)} \lambda < 1, \quad (15)$$

which holds for any  $\alpha > 1/2$  when  $\lambda = 1$ , i.e. consumers expect everyone else to herd after a sellout in the first period. Thus, we get  $\gamma(K, 1) > P(G|b)$ , so the required value of  $r$  exists.

For uniqueness, notice that  $\lambda \leq 2n$  must hold regardless of capacity  $K \geq 1$  or signal precision  $\alpha$ , since  $P(\text{served}|B) \leq 1$  and  $P(\text{served}|G) \geq K/2n$ . Moreover, since

$$\lim_{\alpha \rightarrow 1} \sum_{j=K}^{2n} Q_B^i(j) = 0,$$

there is  $\alpha$  high enough such that (15) holds for any  $\lambda \leq 2n$ , so all period-2 consumers buy regardless of their beliefs about others' behaviour.  $\square$

**Proof of Theorem 1.** From (1), we have  $Q_G(n) = Q_B(n)$ , so that (3) gives

$$\pi_u = 2n[\beta\alpha + (1 - \beta)(1 - \alpha)]Q_n + 2n \sum_{j=n+1}^{2n} Q(j).$$

Moreover, for  $K \leq n$  we have  $\pi_c(K) = K \sum_{j=K}^{2n} Q(j) \geq K \sum_{j=n}^{2n} Q(j)$ . Thus, a sufficient condition for  $\pi_c(K) > \pi_u$  is

$$Q(n)[K - 2n(\beta\alpha + (1 - \beta)(1 - \alpha))] > (2n - K) \sum_{j=n+1}^{2n} Q(j). \quad (16)$$

The left-hand side of (16) is positive if  $\beta < [K/2n - (1 - \alpha)]/(2\alpha - 1)$ . Define

$$\hat{\beta}(\alpha) \equiv \frac{\frac{K}{2n} - (1 - \alpha)}{2\alpha - 1}.$$

Now we show that there are parameters  $(\alpha, \beta)$  such that (16) holds. In order to do that we show that for any number  $M$  there are parameter values  $(\alpha, \beta)$  such that  $\frac{Q(n)}{\sum_{j=n+1}^{2n} Q(j)} > M$ . This can be rewritten as

$$\frac{A_\alpha - C_\alpha M}{M(B_\alpha - C_\alpha)} > \beta,$$

where

$$A_\alpha = \binom{2n}{n} \alpha^n (1 - \alpha)^n, \quad B_\alpha = \sum_{i=n+1}^{2n} \binom{2n}{i} \alpha^i (1 - \alpha)^{2n-i}, \quad C_\alpha = \sum_{i=n+1}^{2n} \binom{2n}{i} \alpha^{2n-i} (1 - \alpha)^i.$$

Now take some sequence  $\alpha_s \rightarrow 1$ . Then,  $\lim_{s \rightarrow \infty} \frac{A_{\alpha_s}}{C_{\alpha_s}} = \infty$  and therefore there exists  $T_0$  such that for all  $s > T_0$  we get  $A_{\alpha_s} > M C_{\alpha_s}$ . Moreover, there exists  $T_1$  such that  $\hat{\beta}(\alpha_s) > 0$  for all  $s > T_1$ . Choose  $\beta_s = \min \left\{ \hat{\beta}(\alpha_s), \frac{1}{2} \frac{A_{\alpha_s} - C_{\alpha_s} M}{M(B_{\alpha_s} - C_{\alpha_s})} \right\}$  (note that  $B_{\alpha_s} > C_{\alpha_s}$  as  $\alpha_s > 1/2$ ). Then the sequence  $\{\alpha_s, \beta_s\}_{s=0}^\infty$  yields  $\lim_{s \rightarrow \infty} \frac{Q^s(n)}{\sum_{j=n+1}^{2n} Q^s(j)} = \infty$  and therefore for  $s > \max\{T_0, T_1\}$  inequality (16) holds. To complete the proof note that due to Lemma 1 there is  $r$  such that consumers follow their private signals and for  $\alpha_s$  large enough there is a unique equilibrium in which one sell-out triggers a positive cascade.

Now consider the second-period profits of a seller who does not restrict capacity. First, notice that (1) directly implies  $Q_B(j) = Q_G(2n - j)$ . Combined with  $Q(j) = \beta Q_G(j) + (1 - \beta) Q_B(j)$ , we can write

$$\begin{aligned}
Q(j) - Q(2n - j) &= \\
&\beta Q_G(j) + (1 - \beta)Q_B(j) - \beta Q_G(2n - j) - (1 - \beta)Q_B(2n - j) \\
&= \beta Q_G(j) + (1 - \beta)Q_G(2n - j) - \beta Q_G(2n - j) - (1 - \beta)Q_G(j) \\
&= (2\beta - 1)[Q_G(j) - Q_G(2n - j)]. \quad (17)
\end{aligned}$$

Moreover, (1) implies

$$\frac{Q_G(j)}{Q_G(2n - j)} = \frac{Q_B(j)}{Q_B(2n - j)} = \alpha^{2(j-n)}(1 - \alpha)^{2(n-j)},$$

which equals 1 when  $j = n$  and which is increasing in  $j$  by  $\alpha > 1/2$ . Thus,  $Q(j) - Q(2n - j) > 0$  for all  $j \geq n + 1$ , by  $\beta \geq 1/2$ , yielding

$$\sum_{j=n+1}^{2n} 2n Q(j) \geq n \left[ \sum_{j=0}^{n-1} Q(j) + \sum_{j=n+1}^{2n} Q(j) \right] = n[1 - Q(n)].$$

We now combine with (3) to obtain

$$\begin{aligned}
\pi_u &= 2n[\beta\alpha + (1 - \beta)(1 - \alpha)]Q(n) + 2n \sum_{j=n+1}^{2n} Q(j) \geq \\
&2n[\beta\alpha + (1 - \beta)(1 - \alpha)]Q(n) + n[1 - Q(n)],
\end{aligned}$$

which implies  $\pi_u \geq n$  by  $\beta\alpha + (1 - \beta)(1 - \alpha) > 1/2$ .

Turning to profits when the seller restricts capacity, note that setting capacity  $K \geq n$  can never be optimal, since it will neither increase the probability of a positive cascade nor decrease the probability of a negative cascade. For any  $K \leq n$ , we have  $\pi_c(K) \leq K \leq n < \pi_u$ , which completes the proof.  $\square$

**Proof of Proposition 1.** First, note that the condition  $\frac{\beta(1-\alpha)}{\beta(1-\alpha)+(1-\beta)\alpha} < r$  simply implies that consumers with bad private signals do not buy in period 1. If this condition is violated, then the seller prefers not restricting capacity, as consumers always buy.

Now assume that consumers follow their private signals in period 1 (we verify this later) and capacity is restricted to  $K$ . Let  $\eta_\omega$  be first period sales in state  $\omega$ . Fix some  $p \in (0, 1)$ . Let  $K_p$  be the largest  $K$  such that  $\eta_B(K) \equiv \sum_{j=K}^{2n} Q_B(j) > p$ . Let  $z_p^\omega = \sum_{j=0}^{2n} \min\{j, K_p(n)\} Q_\omega(j)$ . Applying the Law of Large Numbers we have that for any  $p$   $\lim_{n \rightarrow \infty} \frac{K_p}{2n} = 1 - \alpha$ . Therefore, we have  $\lim_{n \rightarrow \infty} \frac{z_p^\omega}{2n} = 1 - \alpha$  for both  $\omega \in \{G, B\}$ . Thus,

$$\lim_{n \rightarrow \infty} \frac{\tilde{\pi}_c(K_p)}{2n} = \lim_{n \rightarrow \infty} \left[ \beta \eta_G(K_p) z_p^G + (1 - \beta) \eta_B(K_p) z_p^B \right] = (1 - \alpha)[\beta + (1 - \beta)p].$$

Profits are therefore maximized when  $p \rightarrow 1$ , which implies that for sufficiently large  $n$ , a cascade is not triggered at the optimal capacity.<sup>27</sup> Thus, we get  $\lim_{n \rightarrow \infty} \frac{\tilde{\pi}_c(K^*(n))}{2n} = 1 - \alpha$ , where  $K^*(n)$  is the optimal capacity. Similarly, by applying the Law of Large Numbers we get  $\lim_{n \rightarrow \infty} \frac{\pi_u}{2n} = \beta$ . This gives

<sup>27</sup> This is due to  $\lim_{n \rightarrow \infty} \frac{\pi_c(K_p)}{2n} = (1 - \alpha)[\beta + (1 - \beta)p]$  and the fact that  $p$  must be less than one in order to trigger a cascade.



$$\lim_{n \rightarrow \infty} \frac{\tilde{\pi}_c(K^*(n)) - \pi_u}{2n} = 1 - \alpha - \beta,$$

which is positive if and only if  $1 - \alpha > \beta$ .

Now we check the fact that consumers follow their private signals given the optimal capacity constraint. We have  $\lim_{n \rightarrow \infty} \lambda(K^*(n)) = \frac{\alpha}{1-\alpha}$ . This gives the expected value conditional on being served

$$\lim_{n \rightarrow \infty} P(G|g, \lambda(K^*(n))) = \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1-\beta}{\beta} \frac{1-\alpha}{\alpha} \lambda(K^*(n))} = \beta.$$

It follows that consumers follow their good signals for sufficiently large  $n$  if  $\beta > r$ , and will not buy if  $\beta < r$  and  $n$  is sufficiently large, which completes the proof.  $\square$

**Proof of Proposition 2.** Similarly to Proposition 1, we have that  $\frac{\beta(1-\alpha)}{\beta(1-\alpha)+(1-\beta)\alpha} < r < \beta$  must hold for consumers to follow their private signals in period 1.

The Law of Large Numbers implies  $\lim_{n \rightarrow \infty} \frac{\pi_u}{2n} = \beta$ . Now, for any capacity constraint such that period-2 consumers follow their private signals after a sellout, we have

$$\lim_{n \rightarrow \infty} \frac{\tilde{\pi}_c}{2n} \leq \beta\alpha + (1-\beta)(1-\alpha).$$

Now consider the set of capacity constraints which trigger a cascade upon a sell-out. This requires that  $\eta_B(K) \equiv \sum_{j=K}^{2n} Q_B(j) \leq \frac{\beta(1-\alpha)(1-r)}{(1-\beta)\alpha r}$ . Let  $K^*$  be smallest  $K$  that this condition is satisfied. Then

$$\lim_{n \rightarrow \infty} \frac{\pi_c(K^*)}{2n} = \beta + \frac{\beta(1-\alpha)(1-r)}{\alpha r},$$

which is larger than  $\beta$  and  $\beta\alpha + (1-\beta)(1-\alpha)$ .  $\square$

**Proof of Proposition 3.** We first consider a mechanism where all consumers follow the seller's recommendation. Suppose the seller sends a *buy* recommendation with probability  $p_G$  in a good state and with probability  $p_B$  in bad state, and otherwise sends a *no-buy* recommendation. The belief of a consumer with a bad signal upon receiving a buy recommendation is

$$\gamma(s=b, \text{buy}) = \frac{\beta(1-\alpha)p_G}{\beta(1-\alpha)p_G + (1-\beta)\alpha p_B} \geq r,$$

which implies

$$\beta(1-\alpha)(1-r)p_G \geq (1-\beta)\alpha r p_B. \quad (18)$$

Clearly, if (18) is satisfied, then consumers with  $s=g$  prefer to buy. Both  $p_G$  and  $p_B$  enter opposite sides of (18) with positive signs, so sales are maximized by

$$p_G = 1, \quad p_B = \frac{\beta(1-\alpha)(1-r)}{(1-\beta)\alpha r}.$$

Our assumption that a consumer would not buy, given only a bad private signal,  $P(G|s=b) < r$ , implies  $p_B < 1$ , since

$$p_B < 1 \Leftrightarrow \beta(1-\alpha)(1-r) < (1-\beta)\alpha r \Leftrightarrow \frac{\beta(1-\alpha)}{\beta(1-\alpha) + (1-\beta)\alpha} < r.$$

The optimal profit in this case is

$$\pi^* = \beta + (1 - \beta)p_B = \beta + \frac{(1 - \alpha)(1 - r)\beta}{\alpha r}. \quad (19)$$

Now consider a mechanism where only consumers with bad signals follow the seller's recommendation, and again let  $(p_G, p_B)$  denote the probabilities of buy recommendations. The incentive compatibility constraint for a consumer with a good signal who receives a no-buy recommendation is

$$\gamma(s = g, \text{not buy}) = \frac{\beta\alpha(1 - p_G)}{\beta\alpha(1 - p_G) + (1 - \beta)(1 - \alpha)(1 - p_B)} \geq r,$$

or

$$\beta\alpha(1 - r)(1 - p_G) \geq (1 - \beta)(1 - \alpha)r(1 - p_B). \quad (20)$$

Both (20) and (18) must bind at the optimum, which implies values

$$p_G = \frac{\alpha(\alpha\beta - r(\alpha(2\beta - 1) - \beta + 1))}{(2\alpha - 1)\beta(1 - r)}, \quad p_B = \frac{(1 - \alpha)(\alpha\beta - r(\alpha(2\beta - 1) - \beta + 1))}{(2\alpha - 1)(1 - \beta)r} \quad (21)$$

under the optimal such mechanism. The probability  $p_B$  is well-defined as long as  $p_G \in [0, 1]$ . This is the case if  $\alpha \geq \underline{\alpha}(r, \beta) \equiv \frac{\max\{(1-r)\beta, r(1-\beta)\}}{r+\beta-2r\beta}$ , which is equivalent to  $P(G|b) < r < P(G|g)$ , that consumers follow their private signals in the absence of other information. The profits from this persuasion mechanism are

$$\tilde{\pi} = \beta[\alpha + (1 - \alpha)p_G] + (1 - \beta)[\alpha p_B + (1 - \alpha)] \quad (22)$$

with  $p_G, p_B$  given by (21). As long as  $p_G, p_B > 0$ , these profits exceed  $\beta\alpha + (1 - \beta)(1 - \alpha)$ , which is what the seller would earn if all consumers followed their private signals. Thus, the optimal mechanism either leads all consumers to follow the seller's recommendation, or only those with bad signals to do so.

To compare  $\tilde{\pi}$  and  $\pi^*$ , define

$$\Delta \equiv \pi^* - \tilde{\pi} = \frac{(1 - \alpha)(\alpha^2 + \alpha - 2\alpha r + r - 1)(\beta - \alpha\beta - r(\alpha(1 - 2\beta) + \beta))}{\alpha(2\alpha - 1)(1 - r)r}$$

The equation  $\Delta = 0$  has four roots:

$$\alpha_1 = \frac{(1 - r)\beta}{r + \beta - 2\beta r}, \quad \alpha_{2,3} = \frac{1}{2} \left( 2r - 1 \pm \sqrt{4r^2 - 8r + 5} \right), \quad \alpha_4 = 1.$$

Note that  $\alpha_2 = \frac{1}{2} \left( 2r - 1 - \sqrt{4r^2 - 8r + 5} \right) < 0$ ,  $\alpha_3 \equiv \bar{\alpha}(r) \leq 1 = \alpha_4$  and  $\Delta < 0$  in a left neighbourhood of  $\alpha = 1$ . Moreover,  $\alpha_3 \geq 1/2$  for all  $r$ .

Suppose that  $\beta \geq r$ . Then we have  $\underline{\alpha}(\beta, r) = \alpha_1 \geq 1/2$ . It is straightforward to verify that  $\alpha_1 > \bar{\alpha}(r)$  if and only if  $\beta > \frac{r}{1-r}[\bar{\alpha}(r) + 1 - 2r] \equiv \bar{\beta}(r)$ , where  $\bar{\beta}(r) > r$ . Since in the absence of any recommendation consumers follow their private signals, we require that  $\alpha \geq \alpha_1$ . Thus, we have  $\Delta > 0$  if  $r \leq \beta < \bar{\beta}(r)$  and  $\alpha_1 < \alpha < \bar{\alpha}(r)$ ; we have  $\Delta < 0$  if  $r \leq \beta < \bar{\beta}(r)$  and  $\alpha > \bar{\alpha}(r)$ , or if  $\beta > \bar{\beta}(r)$  and  $\alpha > \alpha_1$ .

Now suppose that  $\beta < r$ . Then we have  $\underline{\alpha}(\beta, r) = \frac{r(1-\beta)}{r+\beta-2r\beta} > \frac{1}{2} > \alpha_1$ . Moreover,  $\underline{\alpha}(\beta, r) < \bar{\alpha}(r)$  if and only if  $\beta > r \left[ \frac{2r-\bar{\alpha}(r)}{3r-1} \right] \equiv \underline{\beta}(r)$ , where  $\underline{\beta}(r) < r$ . Thus, we have  $\Delta > 0$  if  $\underline{\beta}(r) < \beta < r$  and  $\underline{\alpha}(\beta, r) \leq \alpha < \bar{\alpha}(r)$ ; we have  $\Delta < 0$  if  $\underline{\beta}(r) < \beta < r$  and  $\alpha > \bar{\alpha}(r)$ , or if  $\beta < \underline{\beta}(r)$  and  $\alpha \geq \underline{\alpha}(\beta, r)$ .

As for  $\beta \geq r$ , we have  $\alpha_1 = \underline{\alpha}(\beta, r)$ , so combining cases  $\beta \geq r$  and  $\beta < r$  gives the result in the proposition.  $\square$

**Proof of Proposition 4.** As we already established the equivalence of profits from the obedient mechanism and profits in large markets with flexible capacity, we now proceed by comparing the conditions in Propositions 2 and 3. First, note that  $\underline{\beta}(r) < r < \bar{\beta}(r)$  for all  $r \in (0, 1)$ . Moreover, if  $\beta > r$  we have

$$\underline{\alpha}(r, \beta) = \frac{(1-r)\beta}{r + \beta - 2\beta r}.$$

Thus, condition  $\alpha > \underline{\alpha}(r, \beta)$  is equivalent to condition  $\frac{\beta(1-\alpha)}{\beta(1-\alpha) + (1-\beta)\alpha} < r < \beta$  in Proposition 2, which completes the proof.  $\square$

## Appendix B. Proofs for general setting

This appendix contains formal results for the fully dynamic setting outlined in section 5. We start with a sequence of Lemmas describing sales probability distributions and the consumer learning process, see Lemmas B.1-B.3 and Lemma 2. We then turn to profit comparisons and the proof that restricting capacity can be beneficial, see Lemma B.4 and Theorem 2.

**Lemma B.1.** Let  $Q_\omega(j)$  be the probability of having  $j$  out of  $2n$  good signals in state  $\omega \in \{G, B\}$ . Then,

$$Q_G(j) = \binom{2n}{j} [1 - (\alpha + \varepsilon - \alpha\varepsilon)]^{2n-j} (\alpha + \varepsilon - \alpha\varepsilon)^j, \quad (23)$$

$$Q_B(j) = \binom{2n}{j} [1 - (\alpha + \varepsilon - \alpha\varepsilon)]^j (\alpha + \varepsilon - \alpha\varepsilon)^{2n-j}. \quad (24)$$

Moreover,

- (i)  $\frac{Q_B(j)}{Q_G(j)}$  is non-increasing in  $j$ .
- (ii)  $\frac{Q_B(n)}{Q_G(n)} = 1$ ,  $\frac{Q_B(n-1)}{Q_G(n-1)} \geq \left(\frac{\alpha}{1-\alpha}\right)^2$  and  $\frac{Q_B(n+1)}{Q_G(n+1)} \leq \left(\frac{1-\alpha}{\alpha}\right)^2$ .
- (iii)  $Q_B(j) = Q_G(2n-j)$  for all  $j \leq 2n$ .
- (iv)  $Q_G(j) > Q_G(2n-j)$  if  $j \geq n+1$ .

**Proof of Lemma B.1.** The expressions for probabilities are obtained directly by computing the number of good signals in each state of the world. Let  $\xi \equiv \alpha + \varepsilon - \alpha\varepsilon$ , and note that  $\xi \in (1/2, 1)$ . Then we have

$$\frac{Q_B(j)}{Q_G(j)} = \xi^{2(n-j)} (1-\xi)^{2(j-n)},$$

which is decreasing in  $j$  and equals to 1 when  $j = n$ . Moreover, for  $j = n-1$  we get

$$\frac{Q_B(n-1)}{Q_G(n-1)} = \frac{\xi^2}{(1-\xi)^2} > \frac{\alpha^2}{(1-\alpha)^2},$$

and for  $j = n+1$  we get

$$\frac{Q_B(n+1)}{Q_G(n+1)} = \frac{(1-\xi)^2}{\xi^2} < \frac{(1-\alpha)^2}{\alpha^2}.$$

Now,

$$Q_G(2n-j) = \binom{2n}{2n-j} \xi^{2n-j} (1-\xi)^j = \binom{2n}{j} \xi^{2n-j} (1-\xi)^j = Q_B(j).$$

Finally,

$$\frac{Q_G(j)}{Q_G(2n-j)} = \frac{Q_G(j)}{Q_B(j)} > 1,$$

for all  $j > n+1$  as the ratio  $Q_G(j)/Q_B(j)$  is increasing in  $j$  and  $\frac{Q_G(n)}{Q_B(n)} = 1$ .  $\square$

**Lemma B.2.** Suppose the seller does not restrict capacity and consider an uninformed consumer  $A$  acting in period  $t \geq 2$ , given sales  $(S_1, \dots, S_{t-1})$ .

1. If  $t = 2$  or  $t > 2$  and  $S_\tau = n$  for all  $\tau \leq t-1$ :

- (a) if  $S_{t-1} > n$  then  $A$  buys regardless of her own signal;
- (b) if  $S_{t-1} = n$  then  $A$  follows her own signal;
- (c) if  $S_{t-1} < n$  then  $A$  does not buy regardless of her own signal.

2. If  $t > 2$  and  $\max_{\tau \leq t-2} S_\tau > n$ :

- (a) if  $S_{t-1} = 2n$ , then  $A$  buys regardless of her own signal;
- (b) if  $S_{t-1} < 2n$  then  $A$  does not buy regardless of her own signal.

3. If  $t > 2$  and  $\max_{\tau \leq t-2} S_\tau < n$ :

- (a) if  $S_{t-1} > 0$ , then  $A$  buys regardless of her own signal;
- (b) if  $S_{t-1} = 0$  then  $A$  does not buy regardless of her own signal.

**Proof of Lemma B.2.** We denote the belief that the state is  $\omega$ , conditional on past sales  $(S_1, \dots, S_t)$  and a private signal  $s$ , by  $P(\omega|S_1, \dots, S_t, s)$ . Consider  $t = 2$ . Note that

$$P(G|n+1, b) = \frac{1}{1 + \frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha} \frac{Q_B(n+1)}{Q_G(n+1)}} \geq \frac{1}{1 + \frac{1-\beta}{\beta} \frac{1-\alpha}{\alpha}} = P(G|g) > r,$$

where the first inequality follows from  $\frac{Q_B(n+1)}{Q_G(n+1)} \leq \left(\frac{1-\alpha}{\alpha}\right)^2$ . Thus, the belief of a consumer that quality is good after observing  $S_1 \geq n+1$  and  $s = b$  is better than  $P(G|g)$ , so the consumer should buy regardless of her private information. In a similar way we get

$$P(G|n-1, g) = \frac{1}{1 + \frac{1-\beta}{\beta} \frac{1-\alpha}{\alpha} \frac{Q_B(n-1)}{Q_G(n-1)}} \leq \frac{1}{1 + \frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha}} = P(G|b) < r,$$

due to  $\frac{Q_B(n-1)}{Q_G(n-1)} \geq \left(\frac{\alpha}{1-\alpha}\right)^2$ . Finally,  $P(G|n, b) = P(G|b)$  and  $P(G|n, g) = P(G|g)$  due to  $\frac{Q_B(n)}{Q_G(n)} = 1$ , so if  $S_2 = n$  then a consumer should follow her own signal. Now consider  $t > 2$ .

Suppose that for all  $t' < t - 1$ ,  $S_{t'} = n$  holds. Due to  $\frac{Q_B(n)}{Q_G(n)} = 1$ , this implies that in all cohorts consumers have followed their own signals. Thus, if  $S_{t-1} = n$ , then consumers in cohort  $t$  must also follow their signals. Suppose that  $S_{t-1} > n$ . In this case

$$P(G|S_1, \dots, S_{t-1}; b) = \frac{\beta(1-\alpha)Q_G(S_{t-1})[Q_G(n)]^{t-2}}{\beta(1-\alpha)Q_G(S_{t-1})[Q_G(n)]^{t-2} + (1-\beta)\alpha Q_B(S_{t-1})[Q_B(n)]^{t-2}} = \frac{1}{1 + \frac{1-\beta}{\beta} \frac{\alpha}{1-\alpha} \frac{Q_B(S_{t-1})}{Q_G(S_{t-1})}} > r,$$

so consumers should buy. Similarly, if  $S_{t-1} < n$  we get  $P(G|S_1, \dots, S_{t-1}; g) < r$  and consumers should not buy.

Now, suppose there exists a first  $t' < t - 1$  such that  $S_{t'} \neq n$ . If  $S_{t'} < n$  consumers in the next cohort should not buy and a negative cascade starts. If for all  $\tau \in [t' + 1, t - 1]$   $S_\tau = 0$ , then consumers in cohort  $t$  do not gain any additional information, and should also refuse to buy. If for some  $\tau \in [t' + 1, t - 1]$   $S_\tau > 0$  then the purchase must come from an informed consumer and consumers in later cohorts should buy. In a similar vein if  $S_{t'} > n$  then a positive cascade starts, it persists if for all  $\tau \in [t' + 1, t - 1]$   $S_\tau = 2n$ . It is otherwise reversed, as the decision not to buy comes from an informed consumer, so consumers in later cohorts should not buy.  $\square$

**Lemma B.3.** *If  $\lambda > 0$ , then for all  $1 \leq K \leq n$ , consumer beliefs  $\gamma(l, K, \lambda)$  given by (11) are increasing in  $l$ , with  $\lim_{l \rightarrow \infty} \gamma(l, K, \lambda) = 1$ .*

**Proof of Lemma B.3.** From (11) to show that  $\gamma(l, K, \lambda)$  is increasing it is sufficient to show that  $\sum_{j=K}^{2n} Q_B(j) < \sum_{j=K}^{2n} Q_G(j)$ . For all  $j \geq n + 1$ , we have  $Q_G(j) > Q_G(2n - j)$ , which implies  $\sum_{j=0}^{K-1} Q_G(j) < \sum_{j=2n-K+1}^{2n} Q_G(j)$ . By adding  $\sum_{j=K}^{2n-K} Q_G(j)$ , we obtain that  $\sum_{j=0}^{2n-K} Q_G(j) < \sum_{j=K}^{2n} Q_G(j)$ . Changing the summation order on the left-hand side gives  $\sum_{j=K}^{2n} Q_G(2n - j) < \sum_{j=K}^{2n} Q_G(j)$ . Finally, due to  $Q_B(j) = Q_G(2n - j)$ , we obtain  $\sum_{j=K}^{2n} Q_B(j) < \sum_{j=K}^{2n} Q_G(j)$ .  $\square$

**Proof of Lemma 2.** First we show that

$$P(G|b) = \frac{(1-\alpha)\beta}{(1-\alpha)\beta + \alpha(1-\beta)} < \frac{1}{1 + \frac{1-\beta}{\beta} \frac{1-\alpha}{\alpha} \lambda} = P(G|g, \lambda),$$

given

$$\lambda = \frac{\sum_{j=0}^{K-1} Q_B(j) + \sum_{j=K}^{2n-1} Q_B(j) \frac{K}{j+1}}{\sum_{j=0}^{K-1} Q_G(j) + \sum_{j=K}^{2n-1} Q_G(j) \frac{K}{j+1}}. \quad (25)$$

This is the relevant value of  $\lambda$  if period-1 consumers follow their private signal.

To establish  $P(G|b) < P(G|g, \lambda)$ , we require that the following condition holds<sup>28</sup>

$$\lambda < \left( \frac{\alpha}{1-\alpha} \right)^2. \quad (26)$$

<sup>28</sup> Note that it is exactly the same as condition (13) for period one in our two-period setting.

Recall that  $\lambda = P(\text{served}|B)/P(\text{served}|G)$ . Denote  $\xi = \alpha + \varepsilon - \alpha\varepsilon$  and write out

$$\alpha^2 P(\text{served}|G) = \frac{\alpha^2}{\xi} \sum_{i=0}^{2n-1} \frac{(2n-1)!}{i!(2n-i-1)!} \frac{1}{i+1} \xi^{i+1} (1-\xi)^{2n-i-1}$$

$$\frac{1}{2n} \frac{\alpha^2}{\xi} \sum_{i=0}^{2n-1} \frac{2n!}{(i+1)!(2n-i-1)!} \xi^{i+1} (1-\xi)^{2n-i-1} = \frac{1}{2n} \frac{\alpha^2}{\xi} [1 - (1-\xi)^{2n}],$$

and

$$(1-\alpha)^2 P(\text{served}|B) = \frac{1}{2n} \frac{(1-\alpha)^2}{1-\xi} [1 - \xi^{2n}].$$

Then, a sufficient condition for (26) is

$$\frac{\alpha^2}{\xi} > \frac{(1-\alpha)^2}{1-\xi},$$

which always holds provided that

$$\alpha > \hat{\alpha}_0(\varepsilon) \equiv \frac{1 - 2\varepsilon + \sqrt{1 + 4\varepsilon - 4\varepsilon^2}}{4(1-\varepsilon)}.$$

It is straightforward to check that  $\hat{\alpha}(\varepsilon)$  is an increasing function with  $\hat{\alpha}_0(0) = 1/2$  and  $\lim_{\varepsilon \rightarrow 1} \hat{\alpha}_0(\varepsilon) = 1$ , which completes this part of the proof. We have shown that  $P(G|b) < P(G|g, \lambda)$ . Thus, we can find  $r$  for which  $P(G|b) < r < P(G|g, \lambda)$ : period-1 consumers follow their private signals.

One sell-out is sufficient to trigger a cascade if  $\gamma(1, K, \lambda) > r$ . Thus, an appropriate choice of  $r$  is possible if

$$P(G|b) < \min \{P(G|g, \lambda), \gamma(1, K, \lambda)\}.$$

Note that if  $\lambda = 1$ , then  $\gamma(1, K, 1) > P(G|b)$  will hold for any  $\alpha > 1/2$ . This implies that if  $\alpha > \hat{\alpha}_0(\varepsilon)$ , there is an equilibrium where period-1 consumers follow their private signals and a single sell-out triggers a cascade.

For uniqueness, notice that  $\lambda \leq 2n$  must hold regardless of capacity  $K \geq 1$  or signal precision  $\alpha$ , since  $P(\text{served}|B) \leq 1$  and  $P(\text{served}|G) \geq K/2n$ . By  $\lim_{\alpha \rightarrow 1} \sum_{j=K}^{2n} Q_B^j(j) = 0$ , we have

$$\frac{\sum_{j=K}^{2n} Q_B(j)}{\sum_{j=K}^{2n} Q_G(j)} \lambda < 1,$$

and, as  $\lambda \leq 2n$ , there is  $\alpha_1(\varepsilon) < 1$  such that, if  $\alpha > \alpha_1(\varepsilon)$ , then  $P(G|b) < \gamma(1, K, \lambda)$ . It is then optimal for a consumer to ignore her private signal and always buy after one sell-out, even if she expects others to follow their own signals. To complete the proof, set  $\hat{\alpha}_1(\varepsilon) = \max\{\hat{\alpha}_0(\varepsilon), \alpha_1(\varepsilon)\}$ .  $\square$

**Lemma B.4.** *Let*

$$\pi_u^Q = \frac{1}{1 - \delta Q(n)} \left( \sum_{j=0}^{2n} j Q(j) + \frac{2n\delta}{1 - \delta} \sum_{j=n+1}^{2n} Q(j) \right), \quad (27)$$

and

$$\pi_c^Q(K) = \sum_{j=0}^{2n} \min\{j, K\} Q(j) + \frac{K\delta}{1-\delta} \sum_{j=K}^{2n} Q(j), \quad (28)$$

with  $\delta \in (0, 1)$ .

Consider a sequence of probability measures  $\{Q^s\}_{s=0}^\infty$  such that  $\lim_{s \rightarrow \infty} \frac{Q^s(n)}{\sum_{j=n+1}^{2n} Q^s(j)} = \infty$  and  $Q^s(n)$  is uniformly bounded below 1:  $Q^s(n) \leq \bar{Q} < 1$ . Then, there exist  $\delta$  and  $T_0$ , such that for all  $s > T_0$ ,  $n > 1$ ,  $K \leq n$

$$\pi_u^{Q^s} < \pi_c^{Q^s}(K)$$

Suppose furthermore that  $\lim_{s \rightarrow \infty} Q^s(n) = 0$ . Then, for any  $\delta \in \left(\sqrt{\frac{2n-K}{2n}}, 1\right)$ , there exists  $T_1$  such that for all  $s > T_1$  we have  $\pi_u^{Q^s} < \pi_c^{Q^s}(K)$ .

**Proof.** For simpler notation we omit the  $s$  superscript in our preliminary steps and work with generic  $Q$ . Rewrite (27) as

$$[1 - \delta Q(n)] \pi_u^Q = S_1 + \frac{\delta}{1-\delta} 2n S_2, \quad (29)$$

where  $S_1 = \sum_{j=1}^{2n} j Q(j)$  and  $S_2 = \sum_{j=n+1}^{2n} Q(j)$ . Note that as long as  $Q(n) < 1$ , the term  $[1 - \delta Q(n)]$  is bounded away from 0 for  $\delta < 1$ . In a similar way,

$$\begin{aligned} \pi_c^Q(K) &= S_1 - \sum_{j=K}^{2n} (j - K) Q(j) + \frac{\delta}{1-\delta} K \left[ \sum_{j=K}^{n-1} Q(j) + Q(n) + S_2 \right] > \\ &S_1 - (2n - K) \left[ \sum_{j=K}^{n-1} Q(j) + Q(n) + S_2 \right] + \frac{\delta}{1-\delta} K \left[ \sum_{j=K}^{n-1} Q(j) + Q(n) + S_2 \right], \end{aligned}$$

where the inequality follows from replacing all terms  $(j - K)$  with the larger term  $2n - K$ . Moreover, for  $\delta > \frac{2n-K}{2n}$  the penultimate term is smaller than the last one, and thus

$$\pi_c^Q(K) > S_1 + \frac{\delta K - (2n - K)(1 - \delta)}{1 - \delta} [Q(n) + S_2].$$

This implies that  $\pi_c^Q(K) > \pi_u^Q$  if

$$\left\{ S_1 + \frac{\delta K - (2n - K)(1 - \delta)}{1 - \delta} [Q(n) + S_2] \right\} [1 - \delta Q(n)] \geq S_1 + \frac{\delta}{1 - \delta} 2n S_2,$$

which can be rewritten as

$$\begin{aligned} -\delta Q(n) S_1 + \frac{\delta K - (2n - K)(1 - \delta)}{1 - \delta} Q(n) [1 - \delta Q(n)] \geq \\ \frac{S_2}{1 - \delta} \{2n\delta - [\delta K - (2n - K)(1 - \delta)][1 - \delta Q(n)]\}. \end{aligned}$$

As  $S_1 \leq 2n$ , the above inequality holds as long as



$$\frac{Q(n)}{1-\delta} \{[\delta K - (2n - K)(1 - \delta)][1 - \delta Q(n)] - 2n\delta(1 - \delta)\} \geq \frac{S_2}{1-\delta} \{2n\delta - [\delta K - (2n - K)(1 - \delta)][1 - \delta Q(n)]\}.$$

Now note that for all  $\delta \in (0, 1)$ , the expression

$$f_R \equiv 2n\delta - [\delta K - (2n - K)(1 - \delta)][1 - \delta Q(n)] = \delta^2 K Q(n) + (2n - K)[1 - \delta Q(n)],$$

is strictly positive. Moreover, the expression

$$f_L \equiv [\delta K - (2n - K)(1 - \delta)][1 - \delta Q(n)] - 2n\delta(1 - \delta),$$

is also strictly positive if  $\delta > \frac{\sqrt{[(2n-K)Q(n)]^2 + 8n(2n-K)[1-Q(n)]} - (2n-K)Q(n)}{4n-4nQ(n)} \equiv \delta^*(Q(n); n, K)$ ,

where the critical value  $\delta^*(Q(n); n, K)$  is increasing in  $Q(n)$ , equals  $\sqrt{\frac{2n-K}{2n}}$  at  $Q(n) = 0$ , and approaches 1 as  $Q(n) \rightarrow 1$ . Moreover, as  $Q(n) \leq \bar{Q} < 1$ , we have that  $\delta^*(Q(n); n, K) < 1$ . Note that  $\sqrt{\frac{2n-K}{2n}} > \frac{2n-K}{2n}$ , and therefore the condition we used for the approximation of  $\pi_c^Q(K)$  is automatically satisfied as long as  $\delta > \sqrt{\frac{2n-K}{2n}}$ . Thus, for any  $Q(n) < 1$ , there exists  $\delta \in (\sqrt{(2n-K)/2n}, 1)$  such that the right-hand-side of

$$\frac{Q(n)}{S_2} = \frac{Q(n)}{\sum_{j=n+1}^{2n} Q(j)} \geq \frac{f_R}{f_L}, \quad (30)$$

is positive and finite. Thus, if  $\lim_{s \rightarrow \infty} \frac{Q^s(n)}{\sum_{j=n+1}^{2n} Q^s(j)} = \infty$ , there exists  $T_0$  such that for all  $s > T_0$  the left-hand-side of (30) is larger than the right-hand-side and therefore  $\pi_u^{Q^s} < \pi_c^{Q^s}(K)$ . Moreover,  $\delta$  can be chosen arbitrarily close to  $\sqrt{\frac{2n-K}{2n}}$ , as long as  $Q(n)$  is sufficiently close to zero, which proves the second statement of the lemma.  $\square$

**Proof of Theorem 2.** We start by providing explicit expressions for the different terms in the profit function (12). Let  $\eta_\omega = \sum_{j=K}^{2n} Q_\omega(j)$  denote the probability of a sell-out, given state  $\omega \in \{G, B\}$ . Let  $S_B = \sum_{j=0}^{K-1} j Q_B(j)$  denote expected sales as of a period where a sellout does not occur and the state is bad. Let  $S_G$  denote the corresponding expected sales when the state is good, where

$$S_G \equiv \sum_{j=0}^{K-1} Q_G(j) \left( j + \delta \frac{\sum_{i=1}^{2n} \binom{2n}{i} \varepsilon^i (1 - \varepsilon)^{2n-i} (\min\{i, K\} + \frac{K\delta}{1-\delta})}{1 - (1 - \varepsilon)^{2n}\delta} \right),$$

since a negative cascade in the good state is reversed in the first subsequent period that an informed consumer arrives. Let  $R$  denote expected discounted sales in the bad state in the event of a purchase cascade, where

$$R = \frac{K}{1-\delta} - \frac{\sum_{i=2n-K+1}^{2n} \binom{2n}{i} \varepsilon^i (1 - \varepsilon)^{2n-i} (i - 2n + K + \frac{K\delta}{1-\delta})}{1 - \sum_{i=0}^{2n-K} \binom{2n}{i} \varepsilon^i (1 - \varepsilon)^{2n-i}\delta},$$

where the cascade will be reversed upon the arrival of  $2n - K + 1$  informed consumers.

If the seller does not restrict capacity, then its profit can be rewritten as:

$$\pi_u = \sum_{j=0}^{2n} j Q(j) + \delta Q(n) \pi_u + \delta \sum_{j=n+1}^{2n} Q(j) \frac{2n}{1-\delta} + (2\beta - 1) \delta \sum_{j=0}^{n-1} Q_G(j) \left( \frac{\sum_{i=1}^{2n} \binom{2n}{i} \varepsilon^i (1-\varepsilon)^{2n-i} (i + \frac{2n\delta}{1-\delta})}{1 - (1-\varepsilon)^{2n}\delta} \right). \quad (31)$$

Thus, for  $\beta < 1/2$  we have  $\pi_u < \pi_u^Q$ .

From Lemma 2 it follows that for  $\alpha$  large enough there is a value of  $r$  such that a single sell-out triggers a cascade. Thus, by setting  $L = 1$  in (12) and using the expressions for  $R$ ,  $\eta_B$ ,  $\eta_G$ ,  $S_B$ , and  $S_G$ , we can rewrite the profit function (12) as

$$\begin{aligned} \pi_c(K) = & \sum_{j=0}^{2n} \min\{j, K\} Q(j) + \sum_{j=K}^{2n} Q(j) \frac{K}{1-\delta} \\ & + \beta \delta \sum_{j=0}^{K-1} Q_G(j) \left[ \frac{\sum_{i=1}^{2n} \binom{2n}{i} \varepsilon^i (1-\varepsilon)^{2n-i} (\min\{i, K\} + \frac{K\delta}{1-\delta})}{1 - (1-\varepsilon)^{2n}\delta} \right] \\ & - (1-\beta) \delta \sum_{j=K}^{2n} Q_B(j) \left[ \frac{\sum_{i=2n-K+1}^{2n} \binom{2n}{i} \varepsilon^i (1-\varepsilon)^{2n-i} (i - 2n + K + \frac{K\delta}{1-\delta})}{1 - \sum_{i=0}^{2n-K} \binom{2n}{i} \varepsilon^i (1-\varepsilon)^{2n-i}\delta} \right]. \end{aligned} \quad (32)$$

Thus,  $\pi_c(K) > \pi_c^Q(K)$  if and only if

$$\left( \frac{\beta}{1-\beta} \right) \left( \frac{\sum_{j=0}^{K-1} Q_G(j)}{\sum_{j=K}^{2n} Q_B(j)} \right) > \frac{\left[ \frac{\sum_{j=2n-K+1}^{2n} \binom{2n}{j} \varepsilon^j (1-\varepsilon)^{2n-j} (j - 2n + K + \frac{K\delta}{1-\delta})}{1 - \sum_{j=0}^{2n-K} \binom{2n}{j} \varepsilon^j (1-\varepsilon)^{2n-j}\delta} \right]}{\left[ \frac{\sum_{j=1}^{2n} \binom{2n}{j} \varepsilon^j (1-\varepsilon)^{2n-j} (\min\{j, K\} + \frac{K\delta}{1-\delta})}{1 - (1-\varepsilon)^{2n}\delta} \right]}. \quad (33)$$

Notice that  $\min\{j, K\} \geq 0$  and  $j - 2n + K \leq K$ . Thus, to establish (33), it will be enough to show that

$$\begin{aligned} \left( \frac{\delta\beta}{1-\beta} \right) \left( \frac{\sum_{j=0}^{K-1} Q_G(j)}{\sum_{j=K}^{2n} Q_B(j)} \right) > \\ \left[ \frac{\sum_{j=2n-K+1}^{2n} \binom{2n}{j} \varepsilon^j (1-\varepsilon)^{2n-j}}{1 - \sum_{j=0}^{2n-K} \binom{2n}{j} \varepsilon^j (1-\varepsilon)^{2n-j}\delta} \right] / \left[ \frac{\sum_{j=1}^{2n} \binom{2n}{j} \varepsilon^j (1-\varepsilon)^{2n-j}}{1 - (1-\varepsilon)^{2n}\delta} \right]. \end{aligned} \quad (34)$$

Note that  $\sum_{j=0}^{K-1} Q_G(j) > Q_G(0) = (1-\xi)^{2n} > (1-\alpha)^{2n}$  where  $\xi \equiv \alpha + \varepsilon - \alpha\varepsilon$ . Also,  $\sum_{j=K}^{2n} Q_B(j) < 1$ . This implies that

$$\left( \frac{\delta\beta}{1-\beta} \right) \left( \frac{\sum_{j=0}^{K-1} Q_G(j)}{\sum_{j=K}^{2n} Q_B(j)} \right) > \frac{\delta\beta}{1-\beta} (1-\alpha)^{2n}.$$

For all  $\alpha < 1$ ,  $\beta \in (0, 1)$ , the right-hand-side is strictly positive and independent from  $\varepsilon$ .

For  $\delta < 1$ , both  $1 - \sum_{j=0}^{2n-K} \binom{2n}{j} \varepsilon^j (1-\varepsilon)^{2n-j}\delta$  and  $1 - (1-\varepsilon)^{2n}\delta$  are strictly positive for all  $\varepsilon \in [0, 1]$ . Moreover,

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0} \frac{\sum_{j=2n-K+1}^{2n} \binom{2n}{j} \varepsilon^j (1-\varepsilon)^{2n-j}}{\sum_{j=1}^{2n} \binom{2n}{j} \varepsilon^j (1-\varepsilon)^{2n-j}} &= \\ \lim_{\varepsilon \rightarrow 0} \varepsilon^{2n-K} \frac{\sum_{j=2n-K+1}^{2n} \binom{2n}{j} \varepsilon^{j-2n+K} (1-\varepsilon)^{2n-j}}{\sum_{j=1}^{2n} \binom{2n}{j} \varepsilon^j (1-\varepsilon)^{2n-j}} &= \\ \lim_{\varepsilon \rightarrow 0} \varepsilon^{2n-K} \frac{\binom{2n}{2n-K+1}}{2n} &= 0. \end{aligned}$$

Thus, for all  $\alpha$  and  $\beta$  there exists  $\varepsilon_0(\alpha, \beta)$ <sup>29</sup> small enough, such that the inequality (34) is satisfied for all  $\varepsilon < \varepsilon_0(\alpha, \beta)$ .

Therefore,  $\pi_c(K) \geq \pi_u$  if for all  $\varepsilon$  there is a sequence  $\{\alpha_s, \beta_s\}$  such that  $\{Q_\omega^s(j)\}_{s=0}^\infty$  satisfies the requirements of Lemma B.4. That is, we have to show that there is a sequence such that, for any number  $M$ , there is  $T_0$  such that

$$Q^s(n) / \left( \sum_{j=n+1}^{2n} Q^s(j) \right) = Q^s(n) / \left( \beta_s \sum_{j=n+1}^{2n} Q_G^s(j) + (1-\beta_s) \sum_{j=n+1}^{2n} Q_B^s(j) \right) > M,$$

or

$$\beta_s < \frac{Q^s(n) - M \sum_{j=n+1}^{2n} Q_B^s(j)}{M \left( \sum_{j=n+1}^{2n} Q_G^s(j) - \sum_{j=n+1}^{2n} Q_B^s(j) \right)},$$

for all  $s > T_0$ .

Because of the properties of  $Q_\omega(j)$  established in Lemma B.1, we have that for all  $(\alpha_s, \beta_s)$ ,  $Q^s(n)$  is bounded away from 1 and  $\sum_{j=n+1}^{2n} Q_G^s(j) - \sum_{j=n+1}^{2n} Q_B^s(j) > 0$  holds. Now, take a sequence  $\alpha_s \rightarrow 1$ . Note, that for any bounded sequence  $\varepsilon_s$  we have that  $\lim_{\alpha_s \rightarrow 1} \xi_s = 1$ . Then,

$$\lim_{s \rightarrow \infty} \frac{Q^s(n)}{\sum_{j=n+1}^{2n} Q_B^s(j)} = \lim_{s \rightarrow \infty} \frac{\binom{2n}{n}}{\sum_{j=n+1}^{2n} \binom{2n}{j} \xi_s^{n-j} (1-\xi_s)^{j-n}} = \infty.$$

Then choose  $\beta_s = \frac{1}{2} \frac{Q^s(n) - M \sum_{j=n+1}^{2n} Q_B^s(j)}{M \left( \sum_{j=n+1}^{2n} Q_G^s(j) - \sum_{j=n+1}^{2n} Q_B^s(j) \right)}$ , which is positive for  $s$  large enough.

Therefore, for such a sequence  $(\alpha_s, \beta_s)$ , we have  $\lim_{s \rightarrow \infty} \frac{Q^s(n)}{\sum_{j=n+1}^{2n} Q^s(j)} = \infty$ . Moreover,  $\lim_{s \rightarrow \infty} Q^s(n) = 0$  (which also implies that for sufficiently large  $s$  we have  $\beta_s < 1/2$ ). Thus, all conditions of Lemma B.4 are satisfied, i.e. there is a sequence  $\alpha_s \rightarrow 1$  and corresponding sequence  $\beta_s$  such that  $\pi_u^{Q^s} < \pi_c^{Q^s}$  starting from some  $s$ . Choose  $\varepsilon_s = \frac{1}{2} \varepsilon_0(\alpha_s, \beta_s)$ . For such  $\varepsilon_s$ , starting from some  $s$ , we have  $\pi_c(K) > \pi_u$ .

Finally, we verify that for  $(\alpha_s, \beta_s, \varepsilon_s)$ , starting at some  $s$ , we can find  $r$  such that the following holds: period-1 consumers all follow their private signal regardless of  $K$ , and a single sell-out triggers a positive cascade. The relevant condition is  $P(G|b) < P(G|g, \lambda)$ . Lemma 2 shows that it holds for any value of  $\varepsilon$  if  $\alpha > \hat{\alpha}_1$ .

It is straightforward to verify that  $\lim_{\varepsilon \rightarrow 0} \hat{\alpha}_1 < 1$  and thus there is a sequence  $(\alpha_s, \beta_s, \varepsilon_s)$  and  $s_0$  such that period-1 consumers follow their private signals, one sell-out triggers a cascade and the firm prefers to be constrained with capacity constraint  $K$  if  $s > s_0$ .  $\square$

<sup>29</sup> We omit other arguments for simpler notation.

## Online appendices. Supplementary material

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.jet.2022.105586>.

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