

# TECHNICAL NOTE

## Undrained bearing capacity factors for conical footings on clay

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**KEYWORDS:** bearing capacity; clays; plasticity; theoretical analysis

### INTRODUCTION

The bearing capacity of circular foundations on undrained clay is of fundamental importance in many geotechnical problems. In particular there are a number of designs of offshore foundations where the foundation can be treated approximately as a large circular footing, for instance some gravity bases, the spudcan foundations of jack-up units, and the more recently developed caisson foundations. In most cases the footing is not placed at the ground surface, and it is important to take into account the depth of embedment. Furthermore, the base of a spudcan is generally not flat, but approximates a shallow cone. For foundations on soft clays, the effect of the increase of strength of the soil with depth needs to be taken into account, and this is particularly important for large foundations.

The purpose of this note is to present calculations of bearing capacity factors for shallow circular foundations, accounting for embedment, cone angle, rate of increase of strength with depth, and surface roughness of the foundation. The results have widespread application, particularly in the offshore industry.

The soil is assumed to be rigid-plastic, with yield determined by the Tresca condition with an undrained strength  $s_u$ . The method of characteristics is used for the bearing capacity calculation, as described by Shield (1955), Eason & Shield (1960), Houlsby (1982) and Houlsby & Wroth (1982a) for application to undrained axisymmetric problems. Some previous results have been published for this problem using similar numerical techniques (e.g. Houlsby & Wroth, 1982b; Salençon & Matar, 1982; Houlsby & Wroth, 1983; Tani & Craig, 1995; Martin, 2001), but the study presented here involves a much more comprehensive coverage of the parameters. Where comparisons can be made with the previous solutions, the factors differ by up to about 0.5%, which gives some indication of the level of accuracy attainable with this numerical technique. Exceptionally, the rough footing results given by Tani & Craig (1995) are higher by up to about 5%, but this may be due to a problem with their numerical integration procedures (see Martin & Randolph, 2001).

### CALCULATIONS

The soil is taken to be isotropic but non-homogeneous, with the undrained strength defined as varying linearly with depth:

$$s_u = s_{u0} + \rho z \quad (1)$$

where  $s_{u0}$  is the undrained strength at the ground surface,  $z$  is the depth below the surface, and  $\rho$  is the rate of increase of strength with depth. It is convenient to define the strength at the level of the footing as  $s_{u0} = s_{um} + \rho h$ , as shown in Fig. 1(b). The average bearing pressure  $q$  (for weightless soil) is then expressed in terms of this strength:

$$q = N_{c0} s_{u0} \quad (2)$$

The remainder of this note is concerned with the value of the dimensionless factor  $N_{c0}$ , which is a function of the cone angle, cone roughness, depth of embedment and the rate of increase of strength with depth of the clay. Each of these variables is expressed through a dimensionless parameter, and a parametric study has been made of the problem in which the following cases were examined (see Fig. 1):

- six values of the cone angle ( $\beta = 30^\circ, 60^\circ, 90^\circ, 120^\circ, 150^\circ$  and  $180^\circ$ )
- six values of the cone roughness factor  $\alpha = a_u/s_u$ , where  $a_u$  is the maximum shear stress that can be mobilised at the cone surface and  $s_u$  is the local value of the undrained shear strength ( $\alpha = 0.0, 0.2, 0.4, 0.6, 0.8$  and  $1.0$ )
- six values of the dimensionless depth of embedment ( $h/2R = 0.0, 0.1, 0.25, 0.5, 1.0$  and  $2.5$ )
- six values of the dimensionless rate of increase of strength with depth ( $2R\rho/s_{um} = 0.0, 1.0, 2.0, 3.0, 4.0$  and  $5.0$ ).

The bearing capacity factor is expressed as  $N_{c0} = N_{c0}(\beta, \alpha, h/2R, 2R\rho/s_{um})$ . In order to explore all the above cases 1296 analyses were required in total. The values calculated are given in Tables 1–6.

In all analyses the soil was assumed to be weightless, as it can be shown that the values of  $N_{c0}$  are independent of the soil unit weight,  $\gamma$ . Note, however, that when using these bearing capacity factors in practice, the ‘cohesive’ bearing capacity,  $q$ , given by equation (2) should be augmented by a surcharge term  $\gamma h$  (or  $\gamma' h$  for a submerged footing on the seabed). If there is complete backfilling of the hole above the foundation, as is often the case with a deeply penetrated spudcan, then equation (2) can be used to give the net available bearing capacity directly.

As shown in Fig. 1(a), it has been assumed for the purposes of analysis that the space above the footing is occupied by a rigid, smooth-sided shaft. The results are thus not strictly applicable to cases where there is an unsupported sidewall above footing level. For cases where backfill soil or a caisson-type structure is present, however, the stress fields obtained from the method of characteristics (Fig. 2) are statically admissible. Calculations to demonstrate the extensibility of these ‘partial’ stress fields (and thus confirm their status as strict lower bound solutions) were not undertaken as part of this exercise, but Martin & Randolph (2001) have shown that acceptable extension fields can be constructed for many combinations of the parameters examined here. It therefore seems reasonable to adopt the  $N_{c0}$  factors in Tables 1–6 as lower bound collapse loads.

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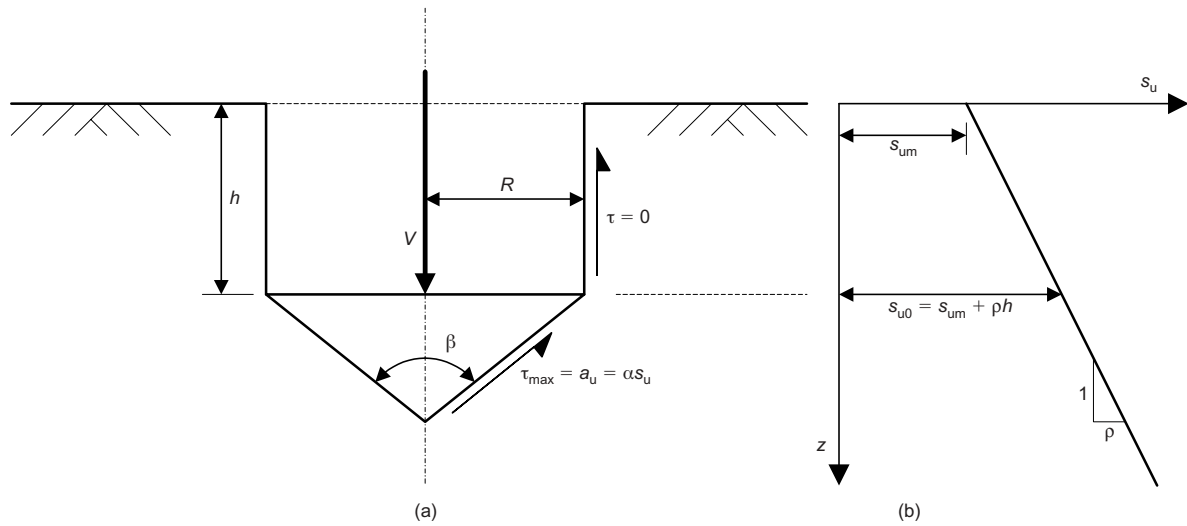


Fig. 1. (a) Outline of footing; (b) variation of undrained strength with depth

Table 1. Values of  $N_{c0} = V/\pi R^2 s_{u0}$  for  $\beta = 30^\circ$

$2R\rho/s_{um}$	$h/2R$	Roughness factor, $\alpha$					
		0.0	0.2	0.4	0.6	0.8	1.0
0.0	0.0	4.608	5.513	6.383	7.224	8.027	8.787
	0.1	4.795	5.697	6.561	7.398	8.199	8.954
	0.25	5.051	5.941	6.803	7.631	8.427	9.178
	0.5	5.405	6.290	7.144	7.967	8.757	9.501
	1.0	5.982	6.853	7.695	8.508	9.290	10.027
	2.5	7.124	7.981	8.810	9.612	10.376	11.102
1.0	0.0	7.531	9.020	10.456	11.843	13.190	14.467
	0.1	7.451	8.891	10.270	11.606	12.897	14.127
	0.25	7.376	8.731	10.046	11.318	12.550	13.722
	0.5	7.276	8.550	9.784	10.981	12.137	13.239
	1.0	7.205	8.378	9.514	10.614	11.676	12.682
	2.5	7.344	8.386	9.394	10.369	11.300	12.192
2.0	0.0	10.448	12.510	14.511	16.442	18.308	20.097
	0.1	9.653	11.532	13.334	15.083	16.785	18.399
	0.25	8.891	10.577	12.191	13.757	15.271	16.716
	0.5	8.197	9.665	11.089	12.469	13.805	15.079
	1.0	7.598	8.871	10.104	11.298	12.451	13.542
	2.5	7.365	8.438	9.476	10.479	11.437	12.354
3.0	0.0	13.360	15.980	18.561	21.032	23.419	25.706
	0.1	11.512	13.765	15.925	18.022	20.052	21.998
	0.25	9.979	11.891	13.718	15.492	17.206	18.849
	0.5	8.745	10.329	11.866	13.357	14.799	16.178
	1.0	7.792	9.114	10.395	11.636	12.834	13.977
	2.5	7.396	8.457	9.507	10.522	11.491	12.423
4.0	0.0	16.269	19.460	22.574	25.617	28.524	31.318
	0.1	13.102	15.676	18.142	20.537	22.860	25.079
	0.25	10.827	12.874	14.862	16.791	18.657	20.441
	0.5	9.109	10.770	12.383	13.961	15.470	16.906
	1.0	7.906	9.258	10.568	11.838	13.063	14.231
	2.5	7.399	8.467	9.523	10.545	11.523	12.457
5.0	0.0	19.177	22.938	26.610	30.200	33.632	36.921
	0.1	14.480	17.331	20.063	22.715	25.289	27.748
	0.25	11.461	13.637	15.749	17.800	19.783	21.680
	0.5	9.368	11.085	12.768	14.382	15.942	17.434
	1.0	7.982	9.354	10.683	11.972	13.214	14.402
	2.5	7.400	8.473	9.533	10.559	11.548	12.481

**Table 2.** Values of  $N_{c0} = V/\pi R^2 s_{u0}$  for  $\beta = 60^\circ$ 

$2R\rho/s_{um}$	$h/2R$	Roughness factor, $\alpha$					
		0.0	0.2	0.4	0.6	0.8	1.0
0.0	0.0	4.446	4.964	5.450	5.897	6.315	6.687
	0.1	4.677	5.193	5.672	6.119	6.531	6.898
	0.25	4.981	5.495	5.960	6.404	6.813	7.177
	0.5	5.414	5.900	6.370	6.809	7.208	7.566
	1.0	6.066	6.545	7.010	7.434	7.838	8.181
	2.5	7.327	7.813	8.245	8.659	9.049	9.391
1.0	0.0	5.808	6.510	7.150	7.768	8.343	8.870
	0.1	5.916	6.593	7.228	7.827	8.378	8.885
	0.25	6.040	6.698	7.299	7.880	8.420	8.910
	0.5	6.199	6.835	7.406	7.964	8.475	8.943
	1.0	6.431	7.046	7.582	8.122	8.590	9.033
	2.5	6.974	7.546	8.079	8.539	8.979	9.387
2.0	0.0	7.139	8.017	8.840	9.600	10.324	10.988
	0.1	6.916	7.729	8.493	9.212	9.884	10.504
	0.25	6.741	7.496	8.177	8.845	9.461	10.028
	0.5	6.591	7.290	7.912	8.526	9.086	9.608
	1.0	6.548	7.198	7.763	8.334	8.827	9.299
	2.5	6.986	7.492	8.033	8.504	8.955	9.370
3.0	0.0	8.486	9.537	10.495	11.423	12.292	13.099
	0.1	7.774	8.697	9.565	10.383	11.145	11.847
	0.25	7.239	8.025	8.799	9.528	10.198	10.819
	0.5	6.822	7.559	8.211	8.860	9.448	10.002
	1.0	6.605	7.271	7.850	8.436	8.943	9.429
	2.5	6.989	7.467	8.011	8.488	8.943	9.360
4.0	0.0	9.830	11.018	12.161	13.242	14.259	15.181
	0.1	8.507	9.524	10.480	11.382	12.219	12.996
	0.25	7.611	8.442	9.264	10.038	10.748	11.410
	0.5	6.974	7.736	8.410	9.080	9.688	10.262
	1.0	6.637	7.312	7.902	8.495	9.011	9.506
	2.5	6.864	7.453	8.000	8.478	8.936	9.354
5.0	0.0	11.174	12.522	13.827	15.062	16.197	17.264
	0.1	9.142	10.232	11.263	12.249	13.149	13.991
	0.25	7.899	8.784	9.631	10.432	11.174	11.868
	0.5	7.083	7.845	8.550	9.237	9.859	10.446
	1.0	6.658	7.323	7.936	8.534	9.056	9.557
	2.5	6.854	7.444	7.992	8.472	8.931	9.350

### CURVE FITTING

It is convenient to develop an algebraic expression that fits the calculated bearing capacity factors, and this is done in the following way. First it can be noted that a substantial part of the effect of cone roughness is accounted for by the vertical component of the shear force developed on the inclined surface of the cone, so that we can write:

$$N_{c0} = N_{c0\alpha} + \frac{\alpha}{\tan(\beta/2)} \left[ 1 + \frac{1}{6 \tan(\beta/2)} \frac{2R\rho}{s_{u0}} \right] \quad (3)$$

where  $N_{c0\alpha} = N_{c0\alpha}(\beta, \alpha, h/2R, 2R\rho/s_{u0})$  is the contribution of the normal stresses on the cone face only, and the second term is the contribution of the shear stresses, which for fully

developed roughness can be expressed analytically as above. It is then found empirically that  $N_{c0\alpha}$  can be expressed in the following form, where  $N_{c00} = N_{c00}(\beta, h/2R, 2R\rho/s_{u0})$  is the value of  $N_{c0}$  for a smooth footing:

$$N_{c0\alpha} = N_{c00} \left[ 1 + (f_1\alpha + f_2\alpha^2) \left( 1 - f_3 \frac{h}{2R + h} \right) \right] \quad (4)$$

Suitable values of the empirical constants are  $f_1 = 0.212$ ,  $f_2 = -0.097$  and  $f_3 = 0.53$ .

Furthermore,  $N_{c00}$  can be expressed approximately as a linear expression in  $2R\rho/s_{u0}$ :

**Table 3.** Values of  $N_{c0} = V/\pi R^2 s_{u0}$  for  $\beta = 90^\circ$ 

$2R\rho/s_{um}$	$h/2R$	Roughness factor, $\alpha$					
		0.0	0.2	0.4	0.6	0.8	1.0
0.0	0.0	4.643	5.022	5.364	5.672	5.946	6.172
	0.1	4.904	5.277	5.609	5.913	6.182	6.405
	0.25	5.223	5.594	5.927	6.226	6.490	6.710
	0.5	5.680	6.033	6.363	6.657	6.915	7.138
	1.0	6.372	6.714	7.047	7.324	7.581	7.787
	2.5	7.649	8.028	8.320	8.604	8.859	9.049
1.0	0.0	5.568	6.046	6.470	6.867	7.222	7.535
	0.1	5.741	6.206	6.619	7.005	7.356	7.653
	0.25	5.938	6.377	6.788	7.162	7.496	7.794
	0.5	6.164	6.605	6.993	7.355	7.679	7.972
	1.0	6.499	6.931	7.298	7.644	7.948	8.208
	2.5	7.246	7.575	7.941	8.248	8.535	8.776
2.0	0.0	6.463	7.028	7.539	8.013	8.445	8.824
	0.1	6.410	6.944	7.434	7.878	8.281	8.648
	0.25	6.409	6.883	7.345	7.756	8.144	8.645
	0.5	6.401	6.879	7.293	7.692	8.031	8.347
	1.0	6.539	6.991	7.372	7.735	8.057	8.328
	2.5	7.157	7.494	7.863	8.176	8.469	8.715
3.0	0.0	7.359	7.995	8.592	9.142	9.648	10.083
	0.1	6.993	7.573	8.104	8.599	9.055	9.447
	0.25	6.699	7.239	7.726	8.174	8.587	8.939
	0.5	6.540	7.040	7.469	7.880	8.240	8.568
	1.0	6.557	7.018	7.407	7.778	8.108	8.386
	2.5	7.118	7.458	7.828	8.145	8.441	8.686
4.0	0.0	8.223	8.964	9.644	10.254	10.820	11.334
	0.1	7.489	8.111	8.678	9.222	9.704	10.139
	0.25	6.940	7.505	8.013	8.485	8.917	9.293
	0.5	6.632	7.145	7.584	8.006	8.378	8.715
	1.0	6.567	7.032	7.427	7.803	8.137	8.420
	2.5	7.048	7.437	7.809	8.127	8.425	8.670
5.0	0.0	9.110	9.934	10.664	11.354	11.998	12.564
	0.1	7.874	8.550	9.174	9.744	10.261	10.746
	0.25	7.124	7.710	8.236	8.722	9.171	9.567
	0.5	6.696	7.219	7.666	8.088	8.475	8.819
	1.0	6.573	7.042	7.440	7.820	8.156	8.442
	2.5	7.034	7.425	7.797	8.116	8.415	8.660

$$N_{c00} = N_1 + N_2 \frac{2R\rho}{s_{u0}} \quad (5)$$

where the coefficients are expressed in the form  $N_1 = N_1(\beta, h/2R)$ ,  $N_2 = N_2(\beta, h/2R)$ . The values of  $N_1$  and  $N_2$  are given in Tables 7 and 8. If curve-fitted expressions are required for these values then the following procedure can be used, with only slight loss of accuracy. The value of  $N_2$  is well approximated by

$$N_2 = f_4 + f_5 \left[ \frac{1}{\tan(\beta/2)} \right]^{f_6} + f_7 \left( \frac{h}{2R} \right)^2 \quad (6)$$

with the empirical constants  $f_4 = 0.5$ ,  $f_5 = 0.36$ ,  $f_6 = 1.5$  and  $f_7 = -0.4$ .

The factor  $N_1$  gives the bearing capacity of smooth cones in homogeneous soil. It is less easy to fit with a simple expression than the other variables described above, but it can be reasonably approximated by

$$N_1 = N_0 [1 - f_8 \cos(\beta/2)] \left( 1 + \frac{h}{2R} \right)^{f_9} \quad (7)$$

where  $N_0 = 5.69$  is the bearing capacity factor for a smooth flat footing at the surface of a homogeneous soil, and the remaining empirical factors are  $f_8 = 0.21$  and  $f_9 = 0.34$ . Using the values for  $N_0$  and  $f_1$  to  $f_9$  given above, all 1296

**Table 4.** Values of  $N_{c0} = V/\pi R^2 s_{u0}$  for  $\beta = 120^\circ$ 

$2Rp/s_{um}$	$h/2R$	Roughness factor, $\alpha$					
		0.0	0.2	0.4	0.6	0.8	1.0
0.0	0.0	4.959	5.253	5.509	5.732	5.918	6.053
	0.1	5.228	5.516	5.769	5.987	6.170	6.298
	0.25	5.570	5.852	6.100	6.312	6.489	6.617
	0.5	6.037	6.310	6.550	6.756	6.934	7.047
	1.0	6.737	7.006	7.243	7.441	7.614	7.718
	2.5	8.068	8.322	8.551	8.746	8.899	8.988
1.0	0.0	5.687	6.043	6.362	6.646	6.893	7.092
	0.1	5.887	6.237	6.547	6.824	7.065	7.259
	0.25	6.117	6.454	6.756	7.022	7.258	7.451
	0.5	6.393	6.719	7.010	7.266	7.485	7.659
	1.0	6.797	7.097	7.367	7.615	7.816	7.970
	2.5	7.521	7.817	8.080	8.294	8.493	8.615
2.0	0.0	6.375	6.787	7.161	7.495	7.795	8.038
	0.1	6.413	6.803	7.155	7.473	7.751	7.973
	0.25	6.465	6.833	7.167	7.461	7.719	7.935
	0.5	6.561	6.909	7.220	7.493	7.741	7.918
	1.0	6.805	7.119	7.396	7.654	7.867	8.034
	2.5	7.427	7.721	7.989	8.207	8.411	8.534
3.0	0.0	7.043	7.509	7.932	8.312	8.658	8.930
	0.1	6.838	7.267	7.653	7.998	8.307	8.570
	0.25	6.710	7.094	7.447	7.761	8.046	8.271
	0.5	6.658	7.018	7.340	7.625	7.882	8.077
	1.0	6.805	7.115	7.407	7.671	7.890	8.062
	2.5	7.385	7.680	7.948	8.173	8.376	8.507
4.0	0.0	7.696	8.217	8.685	9.108	9.488	9.814
	0.1	7.201	7.657	8.071	8.441	8.771	9.031
	0.25	6.876	7.285	7.654	7.985	8.274	8.525
	0.5	6.721	7.085	7.416	7.715	7.971	8.178
	1.0	6.805	7.117	7.413	7.681	7.902	8.078
	2.5	7.385	7.658	7.925	8.152	8.356	8.487
5.0	0.0	8.349	8.911	9.429	9.894	10.310	10.668
	0.1	7.521	7.991	8.427	8.819	9.176	9.450
	0.25	7.012	7.433	7.814	8.153	8.453	8.718
	0.5	6.765	7.134	7.471	7.775	8.034	8.250
	1.0	6.804	7.119	7.417	7.687	7.910	8.088
	2.5	7.341	7.643	7.911	8.139	8.343	8.475

calculated factors are fitted to within better than 5% by the use of just 10 empirical constants.

## DISCUSSION

Apart from the comparisons with other solutions using the method of characteristics, there are few comparisons that can be made with other published work. The calculations presented here do, however, invite comparison with the well-known curve presented by Skempton (1951) for the effect of depth on the bearing capacity factor,  $N_c$ , for homogeneous clays. In Fig. 3 we compare Skempton's curve with our

results for a rough circular flat footing. These are the figures in the top right hand corner of Table 6. As Fig. 3 shows, the results are indistinguishable from Skempton's curve up to  $h/2R$  of about 2. At greater depths our figures are slightly higher. It is worth noting that, although over the years Skempton's curve has been very widely used and is known to be of practical use, in his original paper he presented only five case records that were relevant to the curve for circular footings. These records are shown on the figure, and in fact those at high  $h/2R$  values fall below his curve. He did, however, present other data supporting an asymptotic value of the bearing capacity factor of about 9 at great depth.

**Table 5.** Values of  $N_{c0} = V/\pi R^2 s_{u0}$  for  $\beta = 150^\circ$ 

$2R\rho/s_{um}$	$h/2R$	Roughness factor, $\alpha$					
		0.0	0.2	0.4	0.6	0.8	1.0
0.0	0.0	5.320	5.548	5.738	5.894	6.007	6.064
	0.1	5.599	5.818	6.004	6.157	6.262	6.319
	0.25	5.942	6.158	6.339	6.486	6.588	6.613
	0.5	6.412	6.621	6.797	6.936	7.030	7.054
	1.0	7.125	7.323	7.494	7.624	7.708	7.727
	2.5	8.463	8.653	8.807	8.925	8.991	9.003
1.0	0.0	5.937	6.220	6.464	6.672	6.844	6.965
	0.1	6.161	6.434	6.671	6.872	7.037	7.152
	0.25	6.407	6.672	6.899	7.091	7.246	7.357
	0.5	6.705	6.959	7.177	7.359	7.507	7.599
	1.0	7.126	7.357	7.568	7.728	7.863	7.946
	2.5	7.909	8.116	8.309	8.444	8.561	8.609
2.0	0.0	6.500	6.824	7.111	7.353	7.569	7.733
	0.1	6.593	6.895	7.163	7.396	7.594	7.743
	0.25	6.689	6.977	7.230	7.446	7.628	7.761
	0.5	6.837	7.102	7.336	7.537	7.703	7.817
	1.0	7.107	7.351	7.567	7.744	7.890	7.982
	2.5	7.806	8.010	8.206	8.350	8.470	8.528
3.0	0.0	7.033	7.396	7.715	7.980	8.240	8.430
	0.1	6.938	7.266	7.560	7.811	8.032	8.207
	0.25	6.879	7.183	7.453	7.684	7.881	8.030
	0.5	6.908	7.183	7.428	7.635	7.811	7.936
	1.0	7.233	7.347	7.567	7.749	7.900	7.997
	2.5	7.761	7.966	8.164	8.309	8.431	8.493
4.0	0.0	7.548	7.943	8.295	8.579	8.876	9.097
	0.1	7.225	7.575	7.889	8.155	8.395	8.585
	0.25	7.019	7.336	7.617	7.857	8.066	8.226
	0.5	6.954	7.234	7.485	7.701	7.881	8.014
	1.0	7.091	7.339	7.565	7.749	7.904	8.005
	2.5	7.719	7.942	8.133	8.287	8.409	8.472
5.0	0.0	8.046	8.476	8.858	9.193	9.483	9.736
	0.1	7.463	7.833	8.163	8.437	8.693	8.903
	0.25	7.125	7.450	7.739	7.987	8.196	8.373
	0.5	6.986	7.271	7.526	7.742	7.929	8.067
	1.0	7.086	7.336	7.563	7.751	7.906	8.009
	2.5	7.703	7.927	8.118	8.273	8.396	8.459

Even so, Skempton's original data would provide only weak support for preferring his empirical curve to our theoretically derived factors.

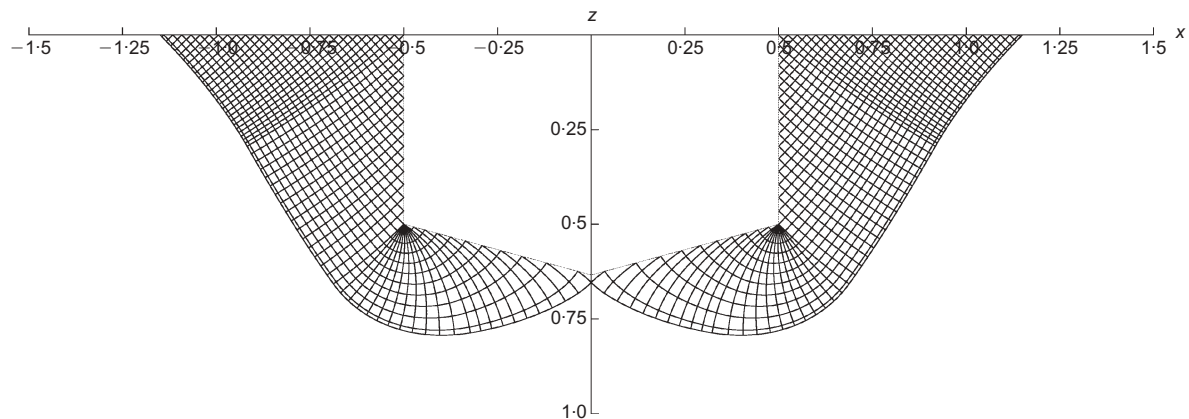
In Fig. 3 we also compare our calculations with the simplified formula suggested by Skempton,  $N_c = 6[1 + 0.2(h/2R)]$ , but subject to the cut-off value  $N_c = 9$ . We also show the result of applying the depth factors of Vesic (1975) (based on earlier work by Hansen) to the theoretical  $N_c$  value of 6.05 calculated at the surface. The depth factors are  $[1 + 0.4(h/2R)]$  for  $h/2R \leq 1$  and  $[1 + 0.4 \arctan(h/2R)]$  for  $h/2R \geq 1$ . It seems, however, more rational to apply this latter formula at smaller depths too. Our figures lie above or

close to Skempton's simplified formula, and below or close to Vesic's curve.

Finally we note that the effect of strength increasing with depth is often taken into account by applying bearing capacity factors calculated for constant strength, but using the strength  $s_{u1}$  at some appropriate depth below the base of the foundation. This implies, for a flat-based foundation, the use of the figures in the first six rows of Table 6, together with  $s_{u1} = s_{u0} + 2Rf\rho$ . With a value  $f \approx 0.09$  (that is, use of a strength 0.09 diameters below the base of the footing) the effect of strength increasing with depth is reproduced quite well, with the fit being within  $\pm 12\%$  of the correct values.

**Table 6.** Values of  $N_{c0} = V/\pi R^2 s_{u0}$  for  $\beta = 180^\circ$  (flat plate). (Final three rows from Hously & Wroth, 1983)

$2Rp/s_{um}$	$h/2R$	Roughness factor, $\alpha$					
		0.0	0.2	0.4	0.6	0.8	1.0
0.0	0.0	5.690	5.855	5.974	6.034	6.052	6.052
	0.1	5.967	6.127	6.238	6.290	6.298	6.298
	0.25	6.314	6.467	6.570	6.611	6.613	6.611
	0.5	6.785	6.927	7.020	7.048	7.047	7.048
	1.0	7.492	7.627	7.703	7.709	7.714	7.714
	2.5	8.824	8.944	8.991	8.993	8.987	8.990
1.0	0.0	6.249	6.469	6.651	6.794	6.895	6.946
	0.1	6.482	6.692	6.867	7.003	7.095	7.138
	0.25	6.741	6.940	7.106	7.234	7.317	7.350
	0.5	7.048	7.237	7.393	7.509	7.577	7.599
	1.0	7.469	7.644	7.787	7.884	7.933	7.942
	2.5	8.264	8.319	8.525	8.595	8.608	8.615
2.0	0.0	6.725	6.983	7.203	7.385	7.529	7.632
	0.1	6.852	7.084	7.295	7.463	7.593	7.676
	0.25	6.979	7.203	7.394	7.547	7.660	7.725
	0.5	7.148	7.357	7.532	7.667	7.760	7.804
	1.0	7.447	7.628	7.782	7.897	7.963	7.984
	2.5	8.157	8.266	8.427	8.503	8.527	8.527
3.0	0.0	7.156	7.445	7.694	7.906	8.080	8.210
	0.1	7.132	7.395	7.622	7.813	7.965	8.072
	0.25	7.147	7.375	7.581	7.750	7.880	7.962
	0.5	7.211	7.422	7.605	7.751	7.856	7.912
	1.0	7.433	7.617	7.777	7.896	7.968	7.992
	2.5	8.134	8.227	8.385	8.462	8.491	8.493
4.0	0.0	7.560	7.872	8.145	8.382	8.583	8.734
	0.1	7.375	7.642	7.885	8.091	8.260	8.385
	0.25	7.258	7.497	7.714	7.892	8.033	8.127
	0.5	7.245	7.462	7.651	7.803	7.919	7.983
	1.0	7.442	7.609	7.772	7.894	7.971	7.995
	2.5	8.086	8.194	8.361	8.440	8.470	8.471
5.0	0.0	7.943	8.274	8.572	8.828	9.051	9.228
	0.1	7.555	7.847	8.103	8.321	8.504	8.641
	0.25	7.341	7.590	7.812	7.998	8.147	8.249
	0.5	7.269	7.490	7.683	7.839	7.956	8.025
	1.0	7.435	7.604	7.768	7.892	7.973	8.003
	2.5	8.069	8.180	8.346	8.428	8.456	8.461
6.0	0.0	8.33					9.67
8.0	0.0	9.03					10.54
10.0	0.0	9.67					11.33

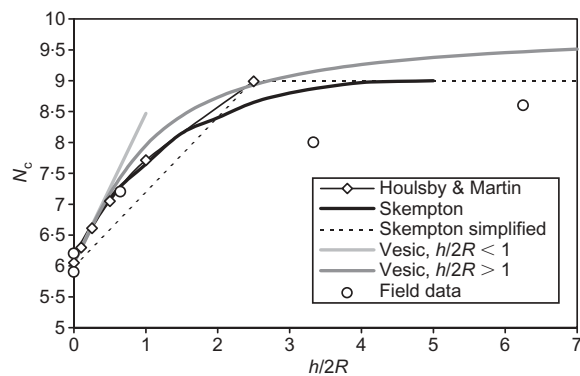
**Fig. 2.** Typical stress characteristic field ( $\beta = 150^\circ$ ,  $\alpha = 0.8$ ,  $h/2R = 0.5$ ,  $2Rp/s_{um} = 5$ )

**Table 7. Values of coefficient  $N_1$** 

$h/2R$	Cone angle, $\beta$					
	30°	60°	90°	120°	150°	180° (flat base)
0.0	4.608	4.446	4.643	4.959	5.320	5.690
0.1	4.795	4.677	4.904	5.228	5.599	5.967
0.25	5.051	4.981	5.223	5.570	5.942	6.314
0.5	5.405	5.414	5.680	6.037	6.412	6.785
1.0	5.982	6.066	6.372	6.737	7.125	7.492
2.5	7.124	7.327	7.649	8.068	8.463	8.824

**Table 8. Values of coefficient  $N_2$** 

$h/2R$	Cone angle, $\beta$					
	30°	60°	90°	120°	150°	180° (flat base)
0.0	2.918	1.349	0.906	0.699	0.576	0.497
0.1	2.912	1.346	0.905	0.703	0.585	0.514
0.25	2.887	1.318	0.872	0.664	0.552	0.490
0.5	2.787	1.173	0.718	0.520	0.417	0.359
1.0	2.417	0.719	0.247	0.095	0.006	-0.065
2.5	0.753	-1.153	-1.541	-1.923	-1.999	-1.996

**Fig. 3. Comparison with Skempton's formula and field data**

This procedure is, however, less satisfactory for conical footings.

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