

### Supplementary Fig. S1. Benchmarking nonlinear learners on corticospinal features

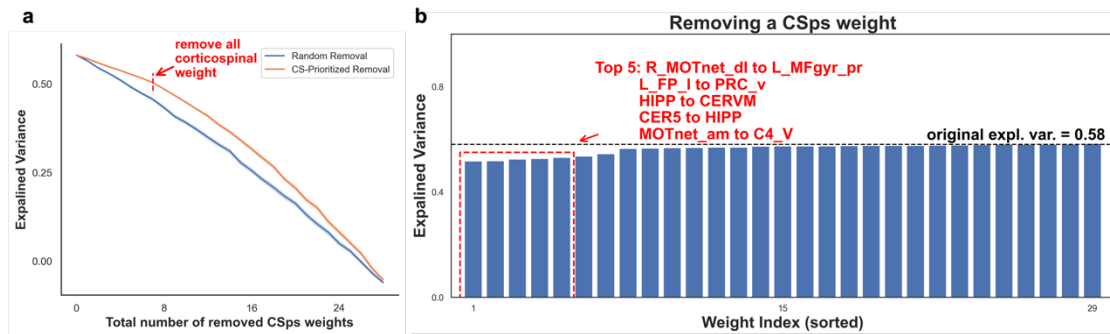
Panels show out-of-sample predictions of thermal pain sensitivity for two alternative learners trained on Dataset 1 and evaluated without refitting on Dataset 2 ( $n=35$ ) and Dataset 3 ( $n=36$ ). All models used the same feature space as in the main analysis: vectorized partial-correlation edges from the 139-ROI corticospinal atlas. Hyperparameters were tuned by nested grid search within the training pipeline; performance was quantified on the held-out datasets. The solid line is the least-squares fit; the shaded band shows the 95% CI around the fit; the measure of center is the fitted mean; dashed lines indicate the MAE.

**a, rbf-kernel support vector regression (SVR).** Scatterplots show predicted versus observed scores separately for Dataset 2 (red) and Dataset 3 (blue). Solid lines denote least-squares fits with 95% CIs. Reported  $r$  and  $p$  values are permutation-derived (10,000 iterations).

**b, Random Forest regressor.** Same display and evaluation as in a.

**Abbreviations.** SVR, support vector regression; RF, random forest; ROI, region of interest.

Source data are provided as a Source Data file.



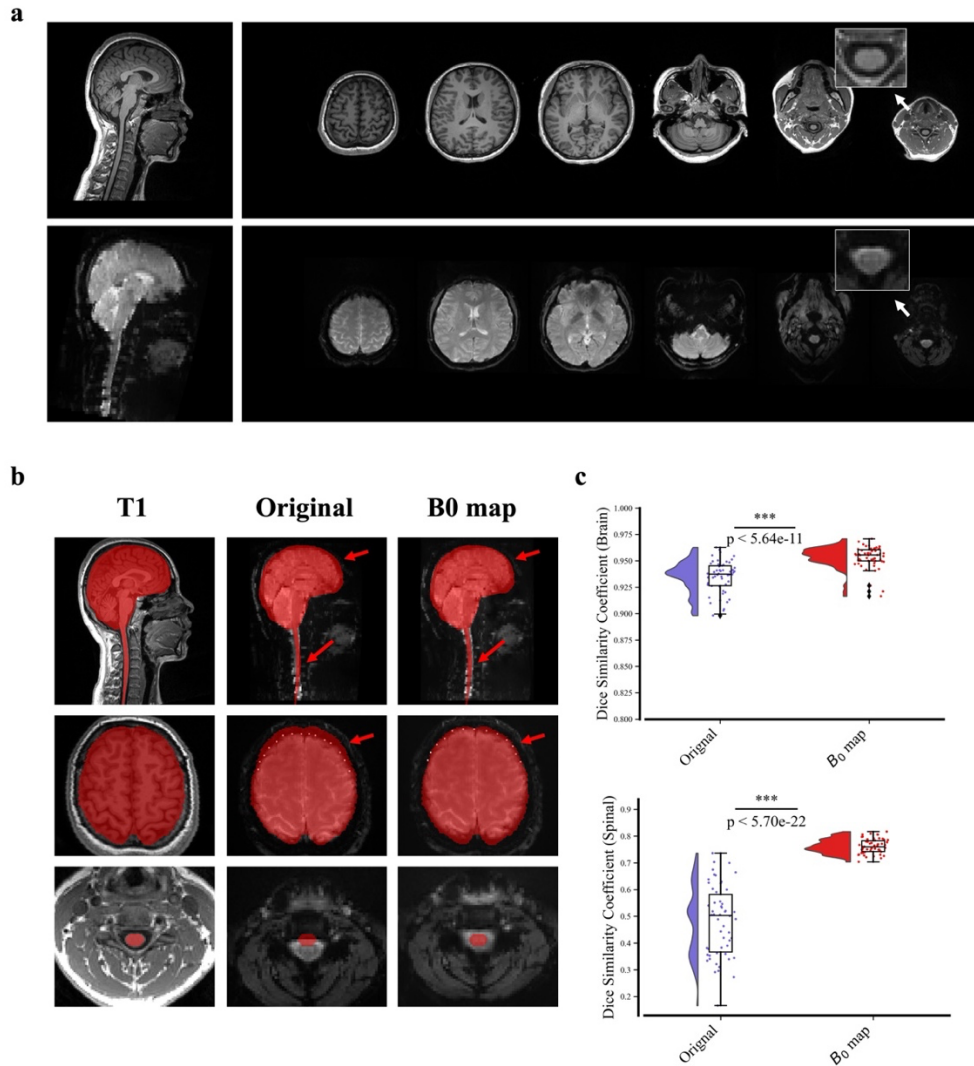
**Supplementary Fig. S2. CSps' Stability to region drop-out.**

**a, Progressive removal of connection weights.** Explained variance (EV) on the pooled external datasets (Datasets 1–3; weighted by sample size;  $n=125$ ) as a function of the number of non-zero CSps connections removed (out of 29). Two removal policies are shown: a random order (median across many random sequences) and a CSps-informed order in which connections are removed according to a prespecified rule (e.g., by descending absolute weight). EV declines monotonically under both policies and approaches zero when most weights are removed, indicating that predictive information is distributed across connections rather than concentrated in a single edge.

**b, Single-edge ablations.** Bars show the EV obtained when one connection is removed at a time (weights sorted left→right by impact). The dashed line marks the original EV of the full CSps model. The five most impactful removals are listed above the panel (labels follow the ROI/network glossary in the Supplementary Table 6). Importantly, although some edges have larger marginal effects (e.g., the M1–C4 ventral horn link highlighted in the main text), no single connection is indispensable—removing any one edge produces only a modest EV reduction.

**Notes and abbreviations.** These edge-level analyses are descriptive and not used for formal inference (post-selection constraints). CSps, corticospinal pain signature; EV, explained variance; M1, primary motor cortex; C4 ventral horn, cervical spinal segment C4 (ventral).

Source data are provided as a Source Data file.



**Supplementary Fig. S3. Acquisition coverage and susceptibility-distortion correction for simultaneous brain–spinal imaging**

**a, Imaging coverage.** Representative structural T1-weighted images and echo-planar (EPI) functional images demonstrate whole-brain and cervical cord (C1–C7) coverage used for corticospinal analyses. Insets highlight regions prone to susceptibility-induced geometric distortion (e.g., spinal cord).

**b, Distortion correction improves anatomical agreement.** Example slices show the individual’s T1 reference, the original (uncorrected) EPI, and the B0-corrected EPI after susceptibility-distortion correction using reverse-phase-encoded field mapping (“B0 map”). Red overlays denote the T1-derived brain and cord masks. Arrows indicate areas where the B0 correction reduces bowing and misregistration, improving alignment to anatomy.

**c, Quantitative evaluation (Dice similarity coefficient, DSC).** Registration quality to the T1 reference increases markedly after correction for both brain (top; paired statistics as shown,  $p < 5.46 \times 10^{-11}$ ) and spinal cord (bottom;  $p < 5.70 \times 10^{-22}$ ) using Dataset 1 ( $n=54$ , after exclusion). Points represent participants; box-and-violin overlays summarize participant-level change scores (centre line = median; box = IQR; whiskers =  $1.5 \times \text{IQR}$ ; points are individuals).

**Abbreviations.** EPI, echo-planar imaging; B0 map, reverse-phase-encoded field map-based correction; DSC, Dice similarity coefficient. Source data are provided as a Source Data file.