

Communication Through a Large Reflecting Surface With Phase Errors

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Abstract—Assume the communication between a source and a destination is supported by a large reflecting surface (LRS), which consists of an array of reflector elements with adjustable reflection phases. By knowing the phase shifts induced by the composite propagation channels through the LRS, the phases of the reflectors can be configured such that the signals combine coherently at the destination, which improves the communication performance. However, perfect phase estimation or high-precision configuration of the reflection phases is unfeasible. In this paper, we study the transmission through an LRS with phase errors that have a generic distribution. We show that the LRS-based composite channel is equivalent to a point-to-point Nakagami fading channel. This equivalent representation allows for theoretical analysis of the performance and can help the system designer study the interplay between performance, the distribution of phase errors, and the number of reflectors. Numerical evaluation of the error probability for a limited number of reflectors confirms the theoretical prediction and shows that the performance is remarkably robust against the phase errors.

Index Terms—Reflect-array, intelligent reflecting surface, meta-surface, phase errors, large system analysis, Nakagami fading, error probability.

I. INTRODUCTION

The wireless propagation environment is traditionally viewed as a detrimental medium of communication that distorts the transmitted signals in an uncontrollable manner. Wireless systems have improved consistently over the years through the development of increasingly sophisticated transmission and reception techniques that counteract the undesirable effects of propagation. An alternative to this conventional reactive approach is currently gaining interest, as several works propose to achieve a level of control over the propagation of electromagnetic waves by placing in the environment ‘smart’ passive devices that manipulate the impinging waves according to a desired functionality, such as reflection in an intended direction, full absorption, polarization, or filtering [1], [2], [3]. Furthermore, by endowing such devices with interconnection capability, they can be introduced in control loops that enable adaptation to the changing conditions of the wireless network, thus creating a smart radio environment. Examples of reconfigurable devices that can manipulate EM waves include tunable reflect-arrays [4], [1], frequency-selective surfaces [5], or meta-surfaces consisting of a thin layer of metamaterials [6].

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The latter have particularly attractive properties and are expected to be a key enabler of smart wireless environments [2], [3].

The focus of this paper is on the concept of a large reflective surface (LRS) which, irrespective of the building technology, can be abstracted into an array of passive reflector elements that induce adjustable phase shifts on the reflected signals. An LRS placed somewhere between a source and a destination can greatly enhance the communication between the two when the phase shifts are appropriately configured such that the reflected waves combine coherently at the destination [1], [7], [8], [9]. Most of the studies assume the phase shifts induced by the composite propagation channel through the LRS are perfectly known and the reflector phases are set to the ideal values, with no errors. However, perfect phase estimation or high-precision configuration of the reflection phases is unfeasible. In [10], [11], the LRS-based system is optimized over low-resolution phase shifts.

In this paper, we study the communication through an LRS whose induced phase shifts deviate from the ideal values according to a generic probability distribution. This set-up could, for example, represent imperfect phase estimation, quantized reflection phases, or both. We show that the transmission through a large number n of imperfect reflectors is equivalent to a point-to-point communication over a Nakagami fading channel with the following characteristics: the diversity order grows with n , the average SNR scales with n^2 relative to the average single-reflector SNR, and both parameters are attenuated by the phase uncertainty. Numerical results for a limited number of reflectors validate the theoretical prediction and show that accurate knowledge or representation of the ideal phase shifts is not necessary to closely approach the ideal performance.

II. SYSTEM MODEL

The considered system comprises a single-antenna source S , a single-antenna destination D and an LRS with n reflector elements R_1, \dots, R_n , which assist the communication between S and D . Since we want to study the effect of the phase errors at the LRS, we restrict our focus on the transmission through the LRS only and assume there is no direct link between S and D . Assuming slow and flat fading, let H_{i1} and H_{i2} be the complex fading coefficients of the S -to- R_i and, respectively, R_i -to- D channels. The reflectors are sufficiently spaced apart such that all the $2n$ coefficients are mutually independent. Furthermore, they have unit power and average magnitudes

$$a_1 = E[|H_{i1}|] < 1 \quad \text{and} \quad a_2 = E[|H_{i2}|] < 1, \quad (1)$$

for all $i = 1, \dots, n$. In the following, we use

$$a = \sqrt{a_1 a_2}. \quad (2)$$

Denote by ϕ_i the phase shift induced by R_i . The signal received by the destination over the equivalent baseband channel is¹

$$Y = \sqrt{\gamma_0} \sum_{i=1}^n H_{i2} e^{j\phi_i} H_{i1} X + W, \quad (3)$$

where X is the transmitted symbol with mean zero and $E[X^2] = 1$, $W \sim \mathcal{CN}(0, 1)$ models the normalized receiver noise and γ_0 is the average SNR when only one reflector is present and accounts for path loss, shadowing and reflection loss.

Ideally, given the phases $\arg(H_{i1})$ and $\arg(H_{i2})$, the phase ϕ_i is set so as to cancel the overall phase shift $\arg(H_{i1}) + \arg(H_{i2})$, which maximizes the SNR at the receiver [9]. Assuming the phase shifts induced by the channels are not estimated perfectly and/or the desired phases cannot be set precisely (e.g., when only a discrete set of phases is technologically possible), we model the deviation of ϕ_i from the ideal setting by the phase noise Θ_i , which is randomly distributed on $[-\pi, \pi)$ according to a certain circular distribution [12].

Now, we express the received signal as

$$Y = n\sqrt{\gamma_0} H X + W, \quad (4)$$

where

$$H = \frac{1}{n} \sum_{i=1}^n |H_{i1}| |H_{i2}| e^{j\Theta_i} \in \mathbb{C}. \quad (5)$$

All the variables in the r.h.s. of (4) and (5) are mutually independent. We assume Θ_i , $i = 1, \dots, n$, are i.i.d. with common characteristic function expressed as a sequence of complex numbers $\{\varphi_p\}_{p \in \mathbb{Z}}$,

$$\varphi_p = E[e^{jp\Theta}], \quad (6)$$

which are also called trigonometric (or circular) moments [12]. Note that $|\varphi_p| \leq 1$, for all $p \in \mathbb{Z}$; for $|p| \geq 1$ equality holds if, and only if, the distribution is degenerate (Dirac delta). A distribution that is broad on the circle (signifying high uncertainty) gives small magnitudes of the trigonometric moments (for the uniform distribution, $\varphi_p = 0$ for $p \geq 1$). It is reasonable to assume that the distribution of the error Θ_i has mean direction zero ($\arg E[e^{j\Theta_i}] = 0$) and its pdf is symmetric around zero. Due to the symmetry, the characteristic function takes real values, i.e., $\varphi_p \in \mathbb{R}$, for all $p \in \mathbb{Z}$.

The receiver performs ideal coherent detection based on the sufficient statistic

$$e^{-j \arg(H)} Y = n\sqrt{\gamma_0} |H| X + W', \quad (7)$$

where $W' = e^{-j \arg(H)} W \sim \mathcal{CN}(0, 1)$.

¹The signal is normalized by the standard deviation of the receiver noise.

III. EQUIVALENT SCALAR FADING CHANNEL

In this section, we find an equivalent representation of the transmission through an LRS as a point-to-point fading channel. First, we provide the following result.

Lemma 1. *For large n , the coefficient H given by (5) has a (non circularly symmetric) complex normal distribution. Its real and imaginary parts, $U = \Re(H)$ and $V = \Im(H)$, are independent, and $U \sim \mathcal{N}(\mu, \sigma_U^2)$ and $V \sim \mathcal{N}(0, \sigma_V^2)$ with*

$$\mu = \varphi_1 a^2 \quad (8)$$

$$\sigma_U^2 = \frac{1}{2n} (1 + \varphi_2 - 2\varphi_1^2 a^4) \quad (9)$$

$$\sigma_V^2 = \frac{1}{2n} (1 - \varphi_2). \quad (10)$$

Proof: See Appendix A. ■

The ideal case where the phase shifts induced by the LRS-augmented channel are perfectly estimated and the reflector phases can be set precisely is instantiated as follows.

Corollary 1. *When there are no phase shift errors, i.e., $\Theta_i \equiv 0$, for all $i = 1, \dots, n$, the coefficient H in (5) is real and distributed as $\mathcal{N}(a^2, (1 - a^4)/n)$.*

Proof: Since the phase errors are zero, we have $\varphi_1 = \varphi_2 = 1$. This implies that $\Im(H) \equiv 0$ in Lemma 1 and the result follows. ■

The ideal case of no phase errors and Rayleigh fading (i.e., $a = \sqrt{\pi}/2$) is studied in [9], which Corollary 1 recovers.

Corollary 2. *When the phase shift errors are uniformly distributed on $[-\pi, \pi)$, which corresponds to a complete lack of knowledge about the phases of H_{i1} and H_{i2} , $i = 1, \dots, n$, the coefficient H in (5) has a circularly-symmetric complex normal distribution with mean zero and variance $1/n$, which means the equivalent channel resembles Rayleigh fading.*

Proof: For the uniform circular distribution, $\varphi_1 = \varphi_2 = 0$ and the result follows from Lemma 1. ■

When $\varphi_1 > 0$, even though U and V are Gaussians with non-zero and, respectively, zero means, the magnitude $|H| = \sqrt{U^2 + V^2}$ does not have a Rice distribution because $\sigma_U^2 \neq \sigma_V^2$ in general. We obtain the following result instead.

Theorem 1. *For large n and $\varphi_1 > 0$, the magnitude of H in (5) has a Nakagami distribution with pdf*

$$f_{|H|}(x) = \frac{2m^m}{\Gamma(m)\mu^{2m}} x^{2m-1} \exp\left(-\frac{m}{\mu^2} x^2\right), \quad (11)$$

where the fading parameter m is given by

$$m = \frac{n}{2} \frac{\varphi_1^2 a^4}{1 + \varphi_2 - 2\varphi_1^2 a^4}. \quad (12)$$

Proof: See Appendix B. ■

Thus, the transmission through the LRS is statistically equivalent to a direct transmission over a Nakagami fading channel.² The equivalent channel has n^2 larger receive power

²The magnitude of the fading coefficient has a Nakagami distribution, although its phase is not uniformly distributed as is sometimes assumed for Nakagami fading. However, this is irrelevant, since we consider phase coherent detection.

relative to the transmission through one reflector. The average single-reflector SNR γ_0 can be very low because of the attenuation of the constituent channels and reflection. Next, we make a few remarks.

Remark 1. The full distributions of the $2n$ coefficients H_{i1} , H_{i2} , $i = 1, \dots, n$, are irrelevant to the distribution of $|H|$, as this depends only on the average magnitudes (1). The individual coefficients may even have different distributions, as long as they are mutually independent and have the respective means.

Furthermore, the phase errors manifest in (11) through the first two trigonometric moments (6) of their common distribution, which are subunitary reals for the considered zero-mean symmetric distributions.

Remark 2. The average SNR, $\bar{\gamma} = n^2\gamma_0 \mathbb{E}[|H|^2] = n^2\gamma_0\varphi_1^2 a^4 < n^2\gamma_0$, is attenuated by φ_1^2 , such that the broader the phase distribution (larger errors), the lower $\bar{\gamma}$.

The average SNR does not grow indefinitely with n , because it cannot be larger than the transmit signal power divided by the receiver noise.³

Remark 3. The quantity $n^2\gamma_0$ is $O(1)$ w.r.t. n .

IV. PERFORMANCE ANALYSIS

We investigate how the performance of the system is impacted by the system size and phase errors. We consider the following types of errors:

- *phase estimation errors*: Θ_i is modelled as a zero-mean von Mises variable whose concentration parameter κ captures the accuracy of the estimation. In this case, the characteristic function is $\varphi_p = \frac{I_p(\kappa)}{I_0(\kappa)}$, where I_p is the modified Bessel function of the first kind and order p [12].
- *quantization errors*: when only a discrete set of 2^q phases can be configured, $q \geq 1$, the error Θ_i is assumed to be uniformly distributed over $[-2^{-q}\pi, 2^{-q}\pi]$. For this distribution, we obtain $\varphi_1 = \frac{\sin(2^{-q}\pi)}{2^{-q}\pi}$ and $\varphi_2 = \frac{\sin(2^{-q+1}\pi)}{2^{-q+1}\pi}$.

We study these types of errors separately, although the case with both estimation and quantization errors can be easily treated because, assuming they are independent, the resulting φ_1 and φ_2 are given by the product of the respective trigonometric moments.

A. Distribution of the SNR

The performance of the system is fundamentally determined by the distribution of the instantaneous SNR, which equals $n^2\gamma_0|H|^2$.

Corollary 3. For large n , the instantaneous SNR, $n^2\gamma_0|H|^2$, is gamma distributed. Its pdf can be expressed as

$$f_{\text{snr}}(\gamma) = \frac{m^m}{\Gamma(m)\bar{\gamma}^m} \gamma^{m-1} \exp\left(-\frac{m}{\bar{\gamma}}\gamma\right), \quad (13)$$

where m is given by (12) and $\bar{\gamma} = n^2\varphi_1^2 a^4 \gamma_0$ is the average SNR.

³The upper bound would correspond to the contrived case when the emitted waves all arrive at the receiver without any absorption.

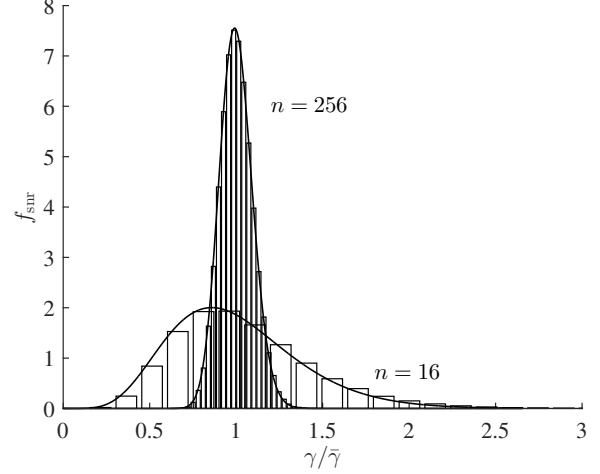


Fig. 1. The pdf of the instantaneous SNR for $n = 16$ and $n = 256$, Rician fading on the source-LRS channels, Rayleigh fading on the LRS-destination channels and von Mises distributed phase errors. The normalized histograms are obtained through Monte Carlo simulation with 10^5 trials, whereas the solid curves represent the gamma pdf (13).

Proof: The results follows from the fact that $|H|^2$ has a gamma distribution for n large, see (20), and the scaling factor $n^2\gamma_0$ is $O(1)$, as pointed out in Remark 3. ■

As an example, the pdf (13) is displayed in Fig. 1 for the following scenario: the phase errors have a zero-mean von Mises distribution with concentration $\kappa = 8$; the S-to- R_i channels exhibit Rician fading with unit power and a Rice factor of $K = 1$, which gives $a_1 = \sqrt{\frac{\pi}{4(K+1)}} {}_1F_1(-1/2; 1; K)$, while the fading over the R_i -to-D channels is Rayleigh with unit power, for which $a_2 = \sqrt{\pi}/2$. We observe a very good agreement between the theoretical approximation and the Monte Carlo estimate even for a moderate value of n . Also, when n becomes large, the pdf concentrates around the mean $\bar{\gamma}$ and approaches a Gaussian pdf.

B. Average error probability

We evaluate through simulations the average error probability, P_e , of the LRS system for BPSK transmission, $n = 32$ and the same fading models as in the previous example. In Fig. 2, the results for estimation and quantization errors are plotted, together with the theoretical performance [13] of the Nakagami fading channel given by Theorem 1. We observe a good agreement between the simulations and the theoretical prediction even for a limited n . Other results not included here show that the match is closer for $n = 64$, whereas for $n = 16$ the theoretical approximation is less accurate, although fairly good. As illustrated in Fig. 2a, even a vague knowledge of the ideal phase shifts can be beneficial: e.g., when $\kappa = 2$ (for which the distribution is broad) the gap to ideal performance is of about 5 dB, while for $\kappa = 8$ (for which the distribution is still not very concentrated) P_e is within 1 dB from the ideal case. It is also remarkable to observe in Fig. 2b that phase quantization of just one bit is already at about 5 dB from the ideal P_e , while with two bits the error probability becomes close to the ideal.

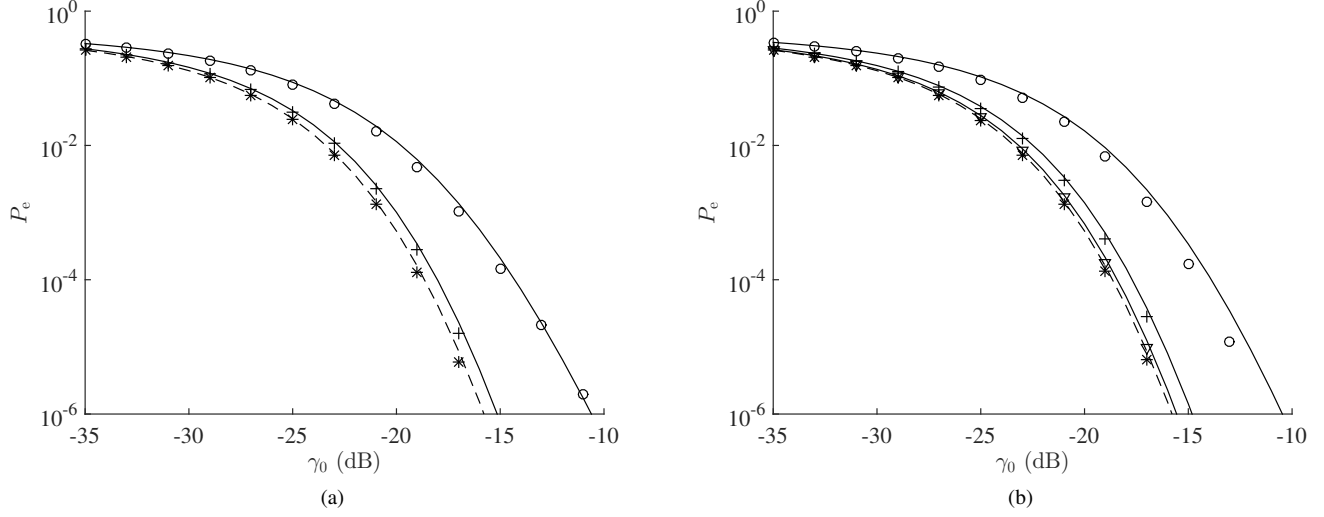


Fig. 2. Average probability of error for $n = 32$ under phase estimation errors (left) and quantization errors (right). The curves correspond to the theoretical performance for the Nakagami channel in Theorem 1, whereas the marked points are obtained via Monte Carlo simulations of the LRS system with 10^7 trials. The dashed curves represent the ideal performance with zero errors. Left: the concentration parameter of the von Mises distribution is $\kappa = 2$ and $\kappa = 8$, in increasing order of performance. Right: the number of quantization bits is $q = 1, 2$ and 3 bits, in increasing order of performance.

The average error probability for the Nakagami channel at high average SNR $\bar{\gamma}$ is [14]

$$P_e \approx \frac{m^{m-1} \Gamma(m + \frac{1}{2})}{2\sqrt{\pi} \Gamma(m)} \bar{\gamma}^{-m}. \quad (14)$$

Using $\bar{\gamma} = n^2 \varphi_1^2 a^4 \gamma_0$, we express $P_e(\gamma_0) = (G_c \gamma_0)^{-G_d}$, such that the diversity and coding gains w.r.t. the average single-reflector SNR γ_0 are $G_d = m$ and $G_c = n^2 \varphi_1^2 a^4 \left[\frac{m^{m-1} \Gamma(m + \frac{1}{2})}{2\sqrt{\pi} \Gamma(m)} \right]^{-1/m}$. These relationships may be useful to find the number of reflector elements required to achieve a certain G_d or G_c , given the distribution of the phase errors (e.g., κ for estimation errors). While n can be easily established in the case of the diversity gain based on (12), the relationship is more complicated in the case of the coding gain.

V. CONCLUSION

We showed that the composite channel through an LRS with phase errors is equivalent to a point-to-point channel with Nakagami fading whose parameters are affected by the phase uncertainty through the first two trigonometric moments of the phase error distribution. Despite the phase shift errors, the average SNR still grows with n^2 (w.r.t. the single-reflector SNR) and the diversity order grows linearly with n ; however, both are attenuated by the phase uncertainty. Also, the distributions of the fading coefficients of the two constituent channels manifest in the equivalent channel only through their average magnitudes. Numerical results for a limited number of reflectors showed good agreement with the theoretical prediction and that the performance is remarkably robust against the phase errors.

APPENDIX A PROOF OF LEMMA 1

The n complex variables $|H_{i1}| |H_{i2}| e^{j\Theta_i}$, $i = 1, \dots, n$, are i.i.d. with common mean $\mu = a^2 \varphi_1$, variance $\nu = 1 - a^4 |\varphi_1|^2$ and pseudo-variance $\rho = \varphi_2 - a^4 \varphi_1^2$ (see [15] for the second-order characterization of complex variables). According to the assumptions on the distribution of Θ_i , the trigonometric moments $\varphi_1, \varphi_2 \in \mathbb{R}$ and, consequently, $\mu, \rho \in \mathbb{R}$. By invoking the central limit theorem, we approximate the distribution of H in (5) for n large by a complex normal distribution, $\mathcal{CN}(\mu, \nu/n, \rho/n)$. Let $U = \Re(H)$ and $V = \Im(H)$. We have [15]

$$\text{Cov}[U, V] = \frac{1}{2} \Im \left(-\frac{\nu}{n} + \frac{\rho}{n} \right) = 0. \quad (15)$$

Being jointly Gaussian with zero covariance, it follows that U and V are independent. Furthermore, $U \sim \mathcal{N}(\mu, \sigma_U^2)$ and $V \sim \mathcal{N}(0, \sigma_V^2)$ with $\mu = a^2 \varphi_1$,

$$\sigma_U^2 = \frac{1}{2} \Re \left(\frac{\nu}{n} + \frac{\rho}{n} \right) = \frac{1}{2n} (1 + \varphi_2 - 2a^4 \varphi_1^2) \quad (16)$$

$$\sigma_V^2 = \frac{1}{2} \Re \left(\frac{\nu}{n} - \frac{\rho}{n} \right) = \frac{1}{2n} (1 - \varphi_2). \quad (17)$$

APPENDIX B PROOF OF THEOREM 1

We first consider the squared magnitude $|H|^2 = U^2 + V^2$. The normalized variable U^2/σ_U^2 has a non-central chi-squared distribution, while V^2 is gamma distributed with shape $1/2$ and scale $2\sigma_V^2$. Thus, $|H|^2$ is the sum of a *scaled* non-central chi-squared variable and a gamma variable. We characterize the distribution of $|H|^2$ through its cumulant generating func-

tion, $K_{|H|^2}(t) = \ln \mathbb{E}[e^{t|H|^2}]$. Since U and V are independent (Lemma 1),

$$K_{|H|^2}(t) = K_{U^2}(t) + K_{V^2}(t) = \frac{\mu^2 t}{1 - 2\sigma_U^2 t} - \frac{1}{2} \ln(1 - 2\sigma_U^2 t) - \frac{1}{2} \ln(1 - 2\sigma_V^2 t) \quad (18)$$

We now approximate for large n the first term in the r.h.s. of (18). We develop its Maclaurin series expansion as

$$\begin{aligned} \frac{\mu^2 t}{1 - 2\sigma_U^2 t} &= \mu^2 t + \mu^2 (2\sigma_U^2) t^2 + \mu^2 (2\sigma_U^2)^2 t^3 + \dots \\ &= \frac{\mu^2}{4\sigma_U^2} \sum_{k=1}^{\infty} \frac{k}{2^{k-1}} \frac{(4\sigma_U^2 t)^k}{k} = \frac{\mu^2}{4\sigma_U^2} \underbrace{\sum_{k=1}^{\infty} \frac{(4\sigma_U^2 t)^k}{k}}_{=-\ln(1-4\sigma_U^2 t)} - g(t) \end{aligned}$$

where

$$\begin{aligned} g(t) &= \frac{\mu^2}{4\sigma_U^2} \sum_{k=3}^{\infty} \left(1 - \frac{k}{2^{k-1}}\right) \frac{(4\sigma_U^2 t)^k}{k} \\ &< \frac{\mu^2}{4\sigma_U^2} \sum_{k=3}^{\infty} \frac{(4\sigma_U^2 t)^k}{k} = 4\mu^2 \sigma_U^2 t^2 \sum_{k=1}^{\infty} \frac{(4\sigma_U^2 t)^k}{k+2} \\ &< 4\mu^2 \sigma_U^2 t^2 \sum_{k=1}^{\infty} \frac{(4\sigma_U^2 t)^k}{k} = -4\mu^2 \sigma_U^2 t^2 \ln(1 - 4\sigma_U^2 t) \end{aligned}$$

Given that $\sigma_U^2 = O(n^{-1})$, we have $g(t) = O(n^{-2})$, such that (18) is well approximated by⁴

$$K_{|H|^2}(t) = -\frac{\mu^2}{4\sigma_U^2} \ln(1 - 4\sigma_U^2 t) - \frac{1}{2} \ln(1 - 2\sigma_U^2 t) - \frac{1}{2} \ln(1 - 2\sigma_V^2 t) \quad (19)$$

Expression (19) corresponds to the cumulant generating function of the sum of three independent gamma variables with (shape, scale) parameters $(\frac{\mu^2}{4\sigma_U^2}, 4\sigma_U^2)$, $(1/2, 2\sigma_U^2)$ and respectively $(1/2, 2\sigma_V^2)$. While the resulting distribution can be characterized based on [16], we simplify by making a further approximation,

$$K_{|H|^2}(t) = -\frac{\mu^2}{4\sigma_U^2} \ln(1 - 4\sigma_U^2 t), \quad (20)$$

which has $O(n^{-1})$ error. Thus, for large n , $|H|^2$ has a gamma distribution with shape $\frac{\mu^2}{4\sigma_U^2}$ and scale $4\sigma_U^2$. It follows through variable transformation that $|H|$ has a Nakagami distribution with fading parameter (shape)

$$m = \frac{\mu^2}{4\sigma_U^2} = \frac{n}{2} \frac{\varphi_1^2 a^4}{1 + \varphi_2 - 2\varphi_1 a^4}$$

and spread μ^2 .

⁴Alternatively, the Maclaurin expansion of (18) can be truncated to second-order, which corresponds to a Gaussian distribution and incurs an error of $O(n^{-2})$ as well. However, for moderate values of n , the Gaussian approximation is less accurate than the one we developed.

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