

Getting cross about a chat

Emily Macmillan and Anne Watson

Emily Macmillan is a Department Lecturer at Oxford University Department of Education, Anne Watson is retired from the same place.

A colleague of ours suggested that this prompt would give us something to laugh at: 'please create some resources about functions for GCSE students. I would like something that uses the principles of variation similar to those used by Anne Watson'.

The response was, in some ways, familiar with ideas that could be used critically as a starting point for learning about functions: notations, inputs/outputs and mappings with a goal of grasping 'what a function is, how it behaves, and how different representations (algebraic, graphical, tabular) connect'. Our immediate critical reaction is that this ignores any functions that learners might already have experienced, perhaps without the word 'function' attached, and also implies simplicity about the behaviour of functions. It would be interesting for teachers to think about whether the starting point should be graphical or algebraic but ChatGPT does not offer a choice.

1. What Makes a Function?

Look at each mapping. Decide: Is it a function? Explain what stayed the same and what changed between examples.

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What must always be true for a mapping to be called a function? (Answer space below)

Figure 1: ChatGPT generated question in the style of Anne Watson with the prompt: 'Show these mappings, and ask students to decide which are functions and why.'

There were some input/output tables to identify as 'functions or not' according to whether they were one-to-one, many-to-one, one-to-many. However, while noticing the differences by comparison would be possible you would need to already know what made them functions or not; a better question would be 'in what ways are these input/output machines similar/different?' These were followed by four 'find the rule' tables, three linear and one for square numbers. Mathematically it isn't enough to deduce a rule that works for a few small positive integers in order to say 'that IS the rule'. We can draw wiggly lines through all of the given examples that are valid functions. Because the inputs in each case were 1,2,3,4 the claim was that these examples were varied 'systematically'. It was not clear to us what insights about functions these

variations were supposed to support – rather it was an exercise in spotting changes in output behaviour.

2. Function Machines: What's the Rule?

Each box shows a function machine. Describe the rule that produces the outputs.

Input	Output	Input	Output
1	3	1	2
2	5	2	4
3	7	3	6
4	9	4	8

Input	Output	Input	Output
1	0	1	1
2	2	2	4
3	4	3	9
4	6	4	16

How does changing the rule affect the outputs? What stays the same?

Figure 2: ChatGPT generated question in the style of Anne Watson with the explanation 'Each table keeps the same input pattern but changes the operation applied – helping students *notice change in output behaviour* as the rule changes.' (original emphasis)

Some of what ChatGPT offered were exercises to apply something learners had already been shown to do but there were some interesting suggestions for learners to vary elements of the examples, e.g. 'keep the rule but change the scale on the graph'; 'keep the table but change the algebraic expression'; 'keep the expression but show a graph with negative values'; 'design a family of functions'. These prompts set up an exploratory environment in which invariant aspects of relationships could be discerned across representations. This approach might be more effective if the idea of functions had been introduced by making connections between algebraic and graphical representations using graphing software in the first place. Anne remembers doing this with Omnigraph in the 80s!!!

Users would have to be fairly critical about using the ideas offered, and the suggestions did indeed use the idea of variation to (a) be exposed to a (limited) example space of functions and (b) encourage learners to vary elements in order to explore the behaviour of linear and quadratic functions.

But, and it is a big but, ChatGPT had been asked to use Anne Watson's principles of variation. Since one of us is Anne Watson we were quite surprised to learn very early on in the chat that her principles can be triaged as: conceptual variation, procedural variation and perceptual variation. This is a trio that Anne has never stated: the distinction between conceptual and procedural variation comes from Gu and his colleagues (e.g. Gu et al, 2017) slightly changed for use in NCETM and perceptual variation, while having an obvious meaning, was a phrase presented by Zoltan Dienes the inventor of number base materials (Dienes, 1971). Of course what matters is not who said what but whether it is helpful for learning, but we have always found that making distinctions can codify actions in ways that might over-simplify the connections between teaching and learning mathematics. This is particularly true if people depend on

reductions of ideas and assumptions about pedagogy rather than experiencing them directly for themselves. Related to this is the fact that ChatGPT was asked for Anne Watson's principles and blatantly used some ideas that Anne herself would avoid.

So instead of critically pursuing the tasks that ChatGPT suggested we pursued the source of the trio. We asked:

"where does Anne Watson distinguish between conceptual and procedural variation? I thought she eschewed these distinctions."

Reply: "Anne Watson does not formally distinguish conceptual, procedural and perceptual variation ... these categories have emerged as interpretations and simplifications used by teachers influenced by her work and others such as Gu, Marton and Runesson ... Let me unpack this carefully and trace the origins"

There follows a broadly correct suggestion of how these ideas have been turned into 'pragmatic labels' by various people so that they are usable for practical planning. But as we saw in the initial task suggestions by ChatGPT itself, merely applying variation to the construction of examples for teaching is not the same as using variation so that learners can discern similarities and differences between mathematical objects. These discernments shape the emergence of a new-to-them concept that they might gradually come to define for themselves, and hence understand the formal definitions, procedures and conventional representations that follow in the teaching. Instead, it offered a version of "I tell you - you do a few - what do you notice?" and then "make some of your own". This teaching heuristic is more interesting than "I do-we do-you do" where the end result is practice of procedures, but is not in itself the heart of what is possible when learners encounter varied examples of mathematical objects, which is curiosity about generalities, special cases, extensions of ideas and so on.

ChatGPT then presented a more authentic version of what Anne has written about variation:

"Dimensions of possible variation: what features can vary in a mathematical object

Range of permissible change: what features may vary without changing the essence

Learner discernment: the noticing of difference and sameness that supports concept formation"

When asked why it did not originally use Anne's work, ChatGPT replied that it chose to use 'widespread school-facing materials' familiar to teachers. It admitted that 'I had flattened and distorted the original theoretical depth of Watson's ideas'.

Emily, as a teacher educator who expects her students to base their thinking (and written assignments) on established literature and research, pushed further in a new chat and asked:

"where does Watson discuss her principles of variation saying they are conceptual, procedural and perceptual?"

Surprisingly, ChatGPT then replied that Anne Watson and John Mason have used that trio in more than one context. It referred to a few publications, mainly *Mathematics as a Constructive Activity* (Watson and Mason, 2006). It claimed that we use it on page 95 and makes up a completely spurious quotation that is nothing to do with what is actually on page 95 not even as a paraphrase (which is what it claims). It also refers to several earlier publications in which Anne is alleged to have laid foundations for the trio. Again, this is spurious. It offered to find explicit quotations, and Emily replied 'yes'. It failed to find any that explicitly mentioned the trio, but says that Watson and Mason discuss variation in multiple dimensions (which is true) 'notably conceptual, procedural and perceptual' (which is not true).

It seemed determined to say, again and again, that Watson is talking about this trio even though she doesn't mention it. It also suggests that Emily should do her own searching to find the exact quotation and gives several correct references, and one which is a complete invention. But in its fruitless search it does find all kinds of interesting things that Watson HAS said and even, towards the end of the chat, notes that there are places where she talks about multiple dimensions of variation and variation over an example space and does not use the trio. But, had the prompter not been an expert on Watson's work, Anne herself, how were we to discern the truth of these and the lies of the trio when they were each presented with quotes and references?

It was amusing at first to come face to face with the bare-faced lying about citations and quotations that we had heard about, but also worrying, highlighting that people entering the profession need to access core literature rather than depend on derivatives that might have been simplified. About the resources it provided: they are much the same as many others as a mechanistic and limited introduction to functions via mappings, and seem ignorant of research and digital possibilities that seek to bring functions education into the digital age.

References

Dienes, Z. (1971). *Building up mathematics*. London, Hutchinson Educational.

Gu, F., Huang, R. and Gu, L. (2017) "Theory and Development of Teaching Through Variation in Mathematics in China." in Li, Y. and Rongjin, L. (eds). *Teaching and Learning Mathematics through Variation*. Rotterdam, Sense Publishers, Pp 13-41.

Watson, A., & Mason, J. (2006). *Mathematics as a constructive activity: Learners generating examples*. Routledge.

(By the way, this book is dedicated "to the 1952 founders of the ATM, whose vision of mathematics as a creative and constructive endeavour, and of all learners as active constructors of meaning, inspired and informed the thinking and practice of generations of teachers, and whose influence is still felt by teachers, teacher educators, and curriculum designers throughout the world".)

