

Traction Control for Electric Vehicles with Independently Driven Wheels



Nathan J. Ewin

Balliol College

University of Oxford

supervised by

Prof. Malcolm McCulloch and Prof. David Howey

Trinity Term, 2016

Acknowledgements

I would like to acknowledge my supervisors Prof. Malcolm McCulloch and Prof. David Howey for their guidance throughout my research, and their critical feedback which has challenged me to be a better researcher. The support of Delta Motorsport and its Directors' Nick Carpenter and Simon Dowson has been invaluable in enabling me to carry out the experiment parts of this research. Most importantly my parents and my girlfriend Jodie have provided continual support and encouragement, and without whom I would not have been able to make it to this stage.

Abstract

The necessity to reduce climate related emissions is driving the electrification of transportation. As well as reducing emissions Electric Vehicles (EV) have the capability of improving traction and vehicle stability. Unlike a conventional vehicle that uses a single Internal Combustion Engine (ICE) to drive one or both axles, an EV can have an electric machine driving each of the wheels independently. This opens up the possibility of using the electric machines as an actuator for traction control. In conventional vehicles the hydraulic brakes together with the ICE are used to actuate traction control. The advantages of electric machines over hydraulic brakes are precise measurable torque, higher bandwidth, bidirectional torque and kinetic energy recovery.

A review of the literature shows that a wide range of control methods is used for traction control of EVs. These are mainly focused on control of an individual wheel, with only a minority being advanced to the experimental stage of verification. Integrated approaches to the control of multiple wheels are generally lacking, as well as verification that tests the vehicle's directional stability. A large body of the literature uses the slip ratio of the wheel as the key control variable. A significant challenge for slip-based traction control is the detection of vehicle velocity together with the calculation of slip around zero vehicle velocity. A traction control method that does not depend upon vehicle velocity detection or slip ratio is Maximum Transmissible Torque Estimation (MTTE), after Yin et al. (2009).

In this thesis an MTTE based method is developed for a full size electric vehicle with independently driven rear wheels. The original MTTE method for a single wheel is

analysed using a simple quarter vehicle model. The simulation results of Yin et al. (2009) are in general reproducible although a lack of data in the original research prevents a quantitative comparison. A modification is proposed to the rate compensation term. Simulation results show that the proposed modification ensures that the torque demand is delivered to the wheel under normal driving conditions, this includes negative torque demand which is not possible for MTTE, Yin et al. (2009). Enabling negative torque demands means that the proposed traction control is compatible with higher level stability control such as torque vectoring.

The performance of the controller is verified through a combination of simulation and vehicle based experiments. Compared with experiments, simulations are fast and inexpensive and can provide greater insight as all of the variables are observable. To simulate the controller a high fidelity vehicle model is required. To achieve this it is necessary to initially validate the model against experimental data. Simulation verification using a validated vehicle model is lacking in the literature. A full vehicle model is developed for this thesis using Dymola, a multi-body system software tool. The model includes the full suspension geometry of the vehicle. Pacejka's "Magic Formula" is used for the tyre model. The model is validated using Delta Motorsport's E4 coupe.

The two Wheel Independent Drive (2WID) MTTE-based traction controller is derived from the equations of motion for the vehicle. This shows that the maximum transmissible torque for one driven wheel is dependent on the friction force of both driven wheels, which has not been shown before. An equal torque strategy is proposed to maintain vehicle directional stability on mixed- μ roads. For verification the 2WID-MTTE controller is simulated on the validated vehicle model described above. The proposed 2WID-MTTE controller is benchmarked against a similar method without the equal torque strategy, termed Independent MTTE, as well as a method combining Direct Yaw Control (DYC) and Independent MTTE. The three controllers are simulated for a vehicle accelerating onto a split- μ road. The results show that the proposed 2WID-MTTE controller prevents the vehicle spinning off the road when compared to Independent MTTE. 2WID-MTTE is found to be as effective as DYC+Independent MTTE but is simpler in design and requires

fewer sensors.

The proposed 2WID-MTTE controller is also simulated for a vehicle accelerating from a low- to high- μ road. This is done to assess the controller's ability to return to normal operation after a traction event, and because there are no simulations of this type for MTTE control on a high fidelity vehicle model in the literature. The results show that oscillations in the tyre-road friction force as the wheel transitions across the change in μ somewhat impede the return of the controller's output torque to the torque demand.

The 2WID-MTTE controller is implemented on Delta Motorsport's E4 coupe by integrating it into the vehicle's Powertrain Control Module (PCM). This is experimentally tested for the vehicle accelerating across a range of surfaces at the MIRA proving ground. The experimental tests include high- to low- μ , low- to high- μ and split- μ roads. The results for the high- to low- μ road tests show that 2WID-MTTE control prevents the vehicle spinning when compared to no control.

Similar to the simulation, the results of the low- to high- μ road experiment show that the controller output torque is also impeded from returning to the demand torque. Observation of the estimated friction force together with the on-board accelerometers confirm that this is due to tyre friction oscillating after the transition. This justifies the use of a tyre model with transient dynamics.

The proposed 2WID-MTTE controller uses wheel velocity and torque feedback to estimate friction torque. These signals are obtained from the vehicle's motor controllers via a Controlled Area Network (CAN) bus. The 2WID-MTTE controller is benchmarked against Independent MTTE that uses wheel velocity measured directly from the wheel hub sensors and the torque demand to estimate friction torque. The results show that the delays introduced by the CAN bus increase wheel slip for the 2WID-MTTE controller. However, the equal torque strategy means that 2WID-MTTE controller maintains greater vehicle directional stability, which is more important than the pursuit of greater acceleration.

Contents

1	Introduction	13
1.1	Tyre/road friction	14
1.1.1	Friction-slip relationship	16
1.2	Conventional traction control	19
1.3	Electric machines: an actuator for traction control	20
1.3.1	Electric vehicle powertrains	21
1.4	Research aims	23
1.5	Thesis outline	23
2	Literature Review	25
2.1	Methodology	27
2.2	Slip-based traction control	29
2.2.1	Vehicle velocity estimation	30
2.2.2	Slip control with vehicle velocity	31
2.2.3	Slip control without vehicle velocity	36
2.3	Torque-based traction control	39
2.3.1	Model following control	39
2.3.2	Maximum Transmissible Torque Estimation	41
2.4	Summary	47
2.5	Gap analysis	49
2.6	Research objectives	50

3	Single Wheel Traction Controller Analysis	52
3.1	Maximum Transmissible Torque Estimation	53
3.1.1	Simulation verification	57
3.2	Rate compensation	62
3.2.1	Control modification	63
3.2.2	Simulation results	64
3.2.3	Discussion	67
3.3	Friction force observer	68
3.3.1	Simulation results	70
3.4	Sensitivity analysis	75
3.4.1	Variability of parameters	75
3.4.2	Wheel inertia sweep simulations	77
3.4.3	Rolling resistance simulations	81
3.5	Summary	83
4	Vehicle Modelling	85
4.1	Model requirements	87
4.2	Model description	90
4.2.1	Test vehicle: Delta E4 coupe	90
4.2.2	Dymola vehicle model	92
4.2.3	Pacejka tyre model	94
4.3	Model calibration	95
4.3.1	Chassis mass, inertia and centre of gravity	96
4.3.2	Driveline inertia	98
4.3.3	Resistive forces	100
4.4	Model validation	104
4.4.1	Experimental data measurement	105
4.4.2	Post-processing tasks	106
4.4.3	Coast down test	108
4.4.4	Constant radius test	109

4.4.5	Straight-line acceleration test	116
4.4.6	Double lane change test	119
4.5	Summary	121
5	2WD Traction Controller Design and Verification	123
5.1	MTTE for two independently driven wheels	124
5.2	Split- μ road simulation	128
5.2.1	Simulation results	130
5.2.2	Discussion	135
5.3	Direction yaw moment control comparison	136
5.3.1	PI gain tuning	137
5.3.2	Split- μ simulation results	138
5.3.3	Discussion	141
5.4	Low- to high- μ road simulation	142
5.4.1	Simulation results	143
5.4.2	Discussion	147
5.5	Summary	148
6	Experimental Verification	150
6.1	Test vehicle	151
6.1.1	Controller implementation	154
6.1.2	Low pass filtering of differentiated wheel velocity	156
6.2	Test plan	157
6.2.1	Test track	159
6.2.2	Test schedule	160
6.2.3	Experimental setup	161
6.3	Experimental results	162
6.3.1	High- to mid- μ road test	162
6.3.2	High- to low- μ road test	165
6.3.3	Low- to high- μ road test	168

6.3.4	Comparison to Independent MTTE	172
6.4	Summary	176
7	Conclusion	178
7.1	Contributions to knowledge	184
7.2	Further work	185
	Bibliography	187
	Appendices	194
A	Vehicle Model parameters	195
B	Dymola Model Statistics	201
C	Tyre Model parameters	203
D	Split- μ road test	209

Nomenclature

α	Lateral/side slip angle
α	Relaxation factor
ω	Angular velocity vector
$\mathbf{a}$	Acceleration vector
$\mathbf{D}$	Gyroscope transformation matrix
$\mathbf{R}$	Accelerometer transformation matrix
δ	Steer angle
$\hat{\lambda}$	Slip ratio estimate
$\hat{F}_x$	Estimated longitudinal friction force
$\hat{T}_x$	Estimated friction torque
λ	Longitudinal slip ratio
λ^*	Slip ratio reference
μ	Tyre/road coefficient of friction (CoF)
ω	Wheel angular velocity
ω_0	Wheel angular velocity, free rolling
ω_x	Roll velocity
ω_y	Pitch velocity

ω_z	Yaw velocity
ϕ	Roll angle
ρ	Air density
τ_1	Wheel velocity low pass filter time constant
τ_2	Torque low pass filter time constant
τ_c	Rate compensation time constant
τ_m	Electric machine time constant
θ	Pitch angle
Θ_R	rotor temperature ($^{\circ}\text{C}$)
A	Frontal area
A_x	Longitudinal acceleration, average
a_x	Longitudinal acceleration
A_y	Lateral acceleration, average
a_y	Lateral acceleration
A_z	Vertical acceleration, average
a_z	Vertical acceleration
C_D	Drag coefficient
C_{Fx}	Longitudinal tyre force coefficient
F	Friction force
F_x	Longitudinal friction force
F_y	Lateral friction force
F_z	Vertical tyre force
F_{dr}	Aerodynamic drag force

F_{rr}	Rolling resistance force
G	Rate compensation gain
$i = l, r$	subscripts: l for left, r for right
I_q	q-axis current (A)
I_{xx}	Chassis inertia about x-axis
I_{yy}	Chassis inertia about y-axis
I_{zz}	Chassis inertia about z-axis
J	Effective inertia
J_c	Wheel inertia used by controller
J_d	Driveline inertia about axis of rotation
J_w	Wheel inertia about axis of rotation
K	Understeer gradient
k	Resistive forces coefficients
K_i	Integral gain
K_p	Proportional gain
k_{rr}	Rolling resistance coefficient
L	Wheel base
M	Vehicle mass
M_z	Yaw moment demand
R	Turn radius
r	Tyre rolling radius
r_e	Effective rolling radius
T'_{max}	Maximum transmissible torque estimate with rate compensation

T_m	Electric machine torque
T_{lim+}	Positive torque demand limit
T_{lim-}	Negative torque demand limit
T_{lim}	Torque demand limit
T_{max}	Maximum transmissible torque estimate
V, V_x	Vehicle longitudinal velocity
V_s	Slip velocity
V_w	Wheel longitudinal velocity
T^*, T_d	Torque demand
T_{comp}	Modified torque rate compensation

Chapter 1

Introduction

The need to reduce climate related emissions is leading to the electrification of road vehicles. Vehicles driven with electric machines not only offer improvements in energy efficiency but also open up greater opportunities in the field of vehicle dynamics control.

Automotive systems that control vehicle dynamics for the purposes of enhancing safety and improving performance are classed as Electronic Stability Control (ESC). A review by Ferguson (2007) on range of international studies on the benefits of ESC finds that for single-vehicle crashes, ESC reduces fatal crashes by 30-50 % for cars and 50-70 % for SUVs. Ferguson (2007) also highlights that a number of these studies find that ESC is effective at reducing crashes when road conditions are slippery. An important part of ESC is Traction Control (TC), as this aims to maintain grip between the tyres and the road during acceleration. This is fundamental to vehicle stability, as the tangential forces between the tyre and the road are the primary forces acting on the vehicle. The challenges in designing any traction control system are due to these tangential friction forces being non-linear, highly variable with road conditions, and very difficult to measure.

Conventional traction control is actuated using a combination of the engine and the hydraulic brakes. This relies on a single Internal Combustion Engine (ICE) to provide positive torque to the wheels via a differential and gearbox, while the individually actuated

hydraulic brakes are used to provide negative torque. Replacing these actuators with electric machines can simplify the controller design as the electric machine can provide both positive and negative torque therefore only a single actuator per driven wheel is required. Furthermore, the precise and fast torque response of electric machines has the potential to significantly improve the performance of traction control on electric vehicles.

1.1 Tyre/road friction

It is not possible to study traction control without also considering the nature of adhesion (friction) between the tyre and the road. The complexity in developing traction control is directly related to the complexity of the tangential friction forces generated between the tyre and the road.

When a vertical load is applied to a pneumatic tyre, a contact patch is formed between the tyre and road, as shown in Figure 1.1. The tangential friction forces are generated across this patch. Gillespie (1992) gives two primary mechanisms for this friction: surface adhesion due to intermolecular bonding between the rubber and the road aggregate, and hysteresis in the rubber as it slides over the aggregate. Both of these mechanisms depend upon a small amount of slip between the tyre and the road, as the rubber needs to be stretched or compressed to generate a force.

The longitudinal slip, or slip ratio, is the ratio between the slip velocity and the vehicle velocity, where the slip velocity is the velocity difference between the forward wheel velocity and the vehicle velocity. This is the definition given by Pacejka (2006)

$$\lambda = \frac{r_e \omega - V_x}{V_x}, \quad (1.1)$$

where

- V_x longitudinal/forward velocity of wheel centre
- ω angular velocity of wheel
- r_e effective rolling radius

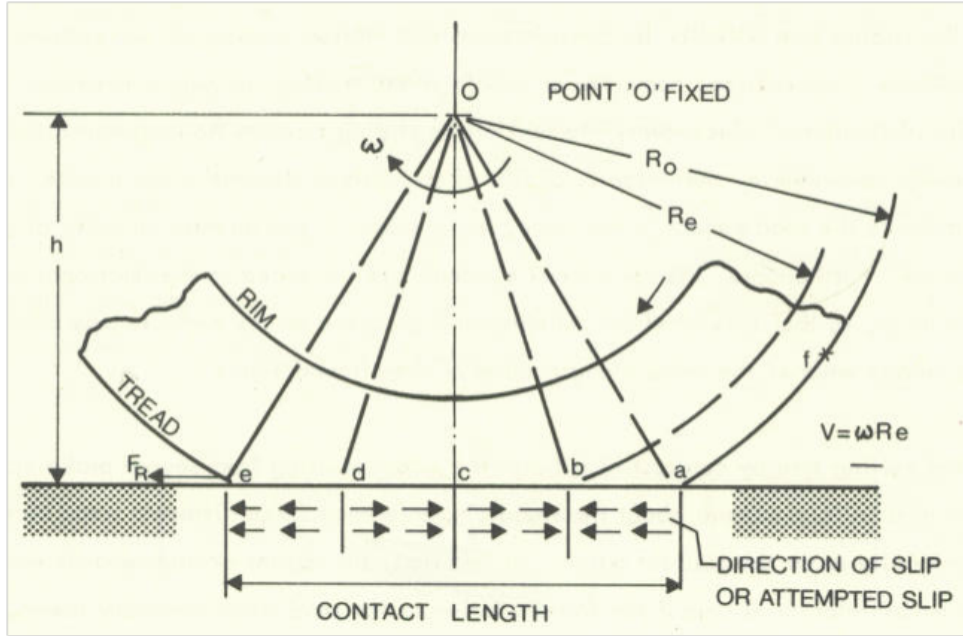


Figure 1.1: Longitudinal deformation along the contact length of free rolling tyre, copied from Moore (1975).

The effective rolling radius needs to be defined as the rolling tyre radius continually varies across the contact patch, due to the deformation of the tyre, see Figure 1.1. The effective rolling radius is defined by Pacejka (2006) as

$$r_e = \frac{V_x}{\omega_0}, \quad (1.2)$$

where ω_0 is the angular velocity of the wheel during free rolling. In subsequent chapters the symbol for the effective rolling radius is abbreviated to r .

By definition the longitudinal slip or slip ratio is zero at free rolling. When a positive torque is applied to the wheel a positive slip occurs and a positive force is generated at the contact patch. Similarly, when a negative (brake) torque is applied a negative slip occurs generating a negative force. When $\lambda = -1$ the wheel angular velocity is zero and the wheel is said to be locked. For $\lambda = 1$ the wheel is said to be fully sliding (wheel spin) although much higher values are possible.

1.1.1 Friction-slip relationship

“The complexity of the structure and behaviour of the tyre are such that no complete and satisfactory theory has yet been propounded”, after Temple (1956). Over the last 60 years there have been many advances in this field although no single theory that encompasses the entire behaviour of the tyre. One of the most notable contributions is Pacejka (2006) for the widely used semi-empirical tyre model known as the ‘Magic Formula’. This consists of fitting a curve to steady-state tyre data measured under special conditions where the the wheel velocity and vehicle velocity are controlled independently. There are physical elements to the model to account for transient effects. A more detailed definition of the Magic Formula tyre model can be found in Section 4.2.3. The Magic Formula is used here for illustrative purposes to describe some of the key characteristics of tyre-road friction that affect traction control.

There is a non-linear relationship between longitudinal tyre friction force and slip ratio, as shown in Figure 1.2. The friction force exhibits a distinct peak, and this peak separates the curve into stable and unstable regions of operation. In the stable region the relationship between friction force and slip ratio is approximately linear, which means that the more the wheel slips the greater the friction force that resists its motion. By contrast, in the unstable region, as the slip ratio increases (wheel velocity increases) the friction force reduces which leads to further increase in slip ratio and further reduction in friction force.

The non-linear relationship between friction force and slip ratio described above presents the first challenge to traction control design. When the drive torque of the actuator exceeds the tyre-road friction limit, the traction control must constrain the torque to prevent wheel spin from occurring, and keep the longitudinal force close to the peak.

The second challenge to designing traction control is that the friction-slip relationship can greatly vary depending upon the operating conditions. In Figure 1.2 three curves are shown for three different nominal coefficients of friction (μ) between the tyre and the road. These three curves can be considered representative of dry asphalt ($\mu = 1.0$), wet

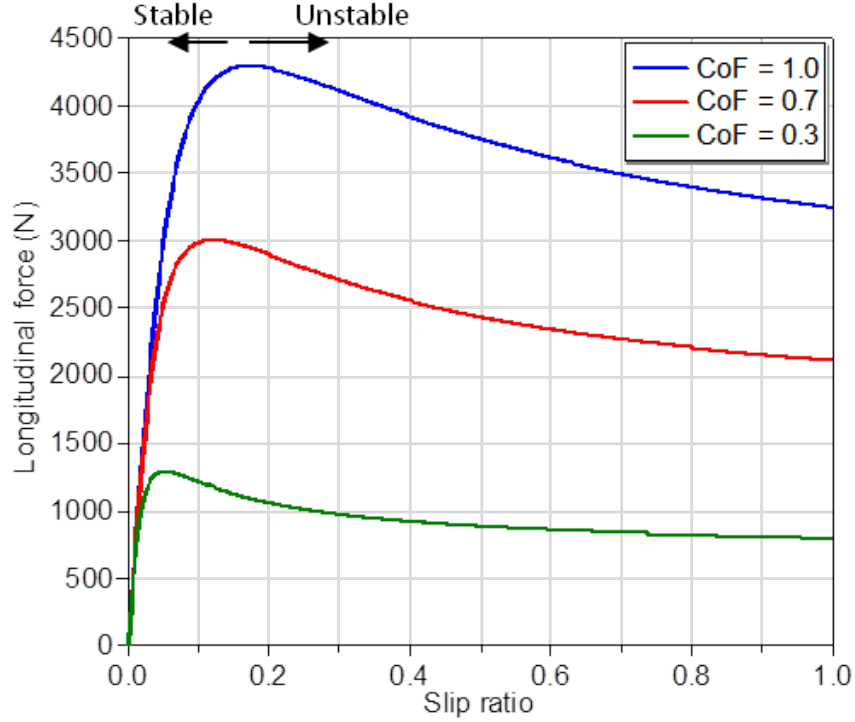


Figure 1.2: Longitudinal tyre friction force versus slip ratio for different nominal coefficients of friction (CoF), μ .

asphalt ($\mu = 0.7$), and snow ($\mu = 0.3$). Where the coefficient of friction is defined as the ratio of friction force (F) to vertical tyre force (F_z)

$$\mu = \frac{F}{F_z}. \quad (1.3)$$

Subsequently the coefficient of friction is referred to as μ . The shape of the friction-slip curve may vary, as will the slip ratio corresponding to peak friction, see Harned and Johnston (1969). Furthermore, the curves in Figure 1.2 are only representative of steady-state conditions; transient conditions will add further variation.

The vector sum of the longitudinal and lateral forces is limited to μF_z . This characteristic is referred to as the friction circle (or friction ellipse), after Radt and Milliken (1960). The coupled nature of longitudinal and lateral forces is shown in Figure 1.3. Figure 1.3a shows that the lateral force decreases with slip ratio, while an increase in slip angle will increase the lateral force but decrease the longitudinal force. Figure 1.3b shows the lateral force against the longitudinal force for different slip angles. The extremities of all of the curves

give the friction circle.

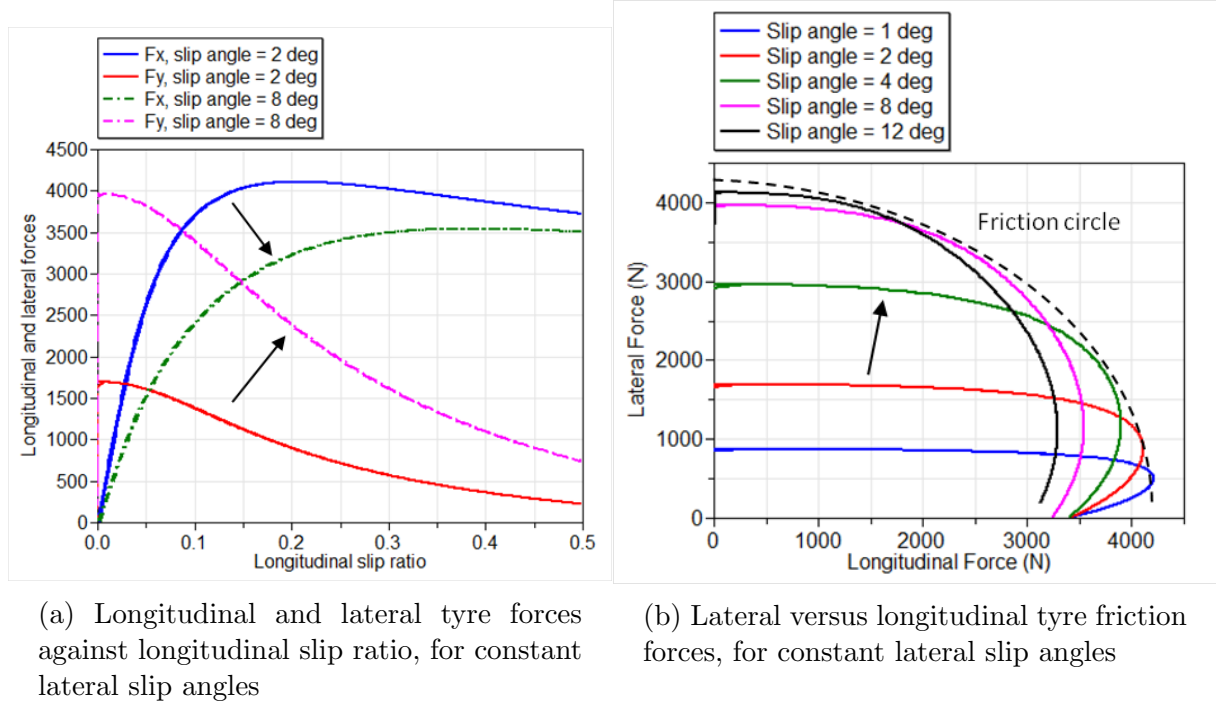


Figure 1.3: Combined lateral and longitudinal tyre force characteristics. Arrows show increase in slip angle.

While a vehicle is in motion the lateral tyre forces enable it to turn as well as determine the handling characteristics of the vehicle (understeer, neutral steer or oversteer¹). As the lateral and longitudinal forces are coupled, to maintain vehicle stability the traction control design needs to ensure the lateral tyre forces are not degraded in pursuit of maximising the longitudinal acceleration (longitudinal friction forces). This defines the third challenge to traction control.

The fourth challenge for traction control design is that the friction forces are very difficult to measure, as noted by De Castro et al. (2013), “Among the model uncertainties, the tyre-road friction (μ) is one of the most difficult variables to predict or measure. For this reason, ensuring robustness against this variation is an important requirement in the controller design”. Methods that can more accurately estimate the tyre friction forces are likely to lead to improvements in traction control performance.

¹See Section 4.4.4 for definitions

1.2 Conventional traction control

A brief summary of conventional traction control for ICE vehicles is given to aid understanding the benefits of using electric machines for traction control, as discussed in the next section. A diagram of the hardware components of a conventional traction control system is shown in Figure 1.4. Wheel speed sensors mounted in the wheel hubs feed into a control unit, to provide it with information on the state of the wheels. The traction control is actuated by modulating either the throttle or ignition of the engine, as well as using the individual brake caliper and disk at each wheel.

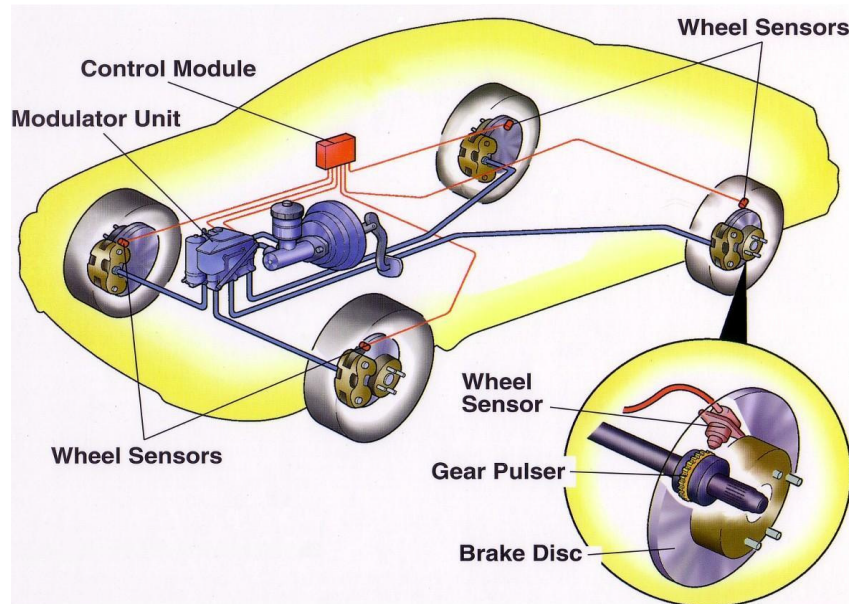


Figure 1.4: Hardware components of a conventional traction control system, copied from Yin (2009).

An early example of traction control in production vehicles is the MaxTrac system introduced by Buick (1971). The MaxTrac system senses wheel slip by comparing the front non-driven wheel speed to the transmission speed. The ignition to the engine is modulated when the difference between the two speeds exceeds a threshold.

Traction control is similar to Anti-lock Braking Systems (ABS) in that they are both designed to maintain the grip between the tyre and the road. Whereas traction control is used when the vehicle is accelerating, ABS is used when the vehicle is braking. Bosch introduced its Anti-Blockieren System in 1978, Johnson (2001), which became the market

leader. The Bosch system was the first to use a micro-processor and high speed valves with 60 Hz cycle time, to brake the wheels independently. Although the valves have a high bandwidth the overall response of the braking system is constrained by the hydraulic circuitry, Sakai and Hori (2001) give the time constant of a hydraulic brake system between 50-100 ms. The introduction of ABS allowed traction control systems to make use of the brake actuators in combination with the modulation of the engine torque.

1.3 Electric machines: an actuator for traction control

The advantages of using electric machines for traction control when compared to an ICE with hydraulic brakes have been propounded by Sakai and Hori (2001), Hori (2004), Crolla and Cao (2012) and Ivanov et al. (2015b). These advantages can be summarised as:

1. The electric machine has a wide and flat torque-speed curve that can be well matched to the operating range of a road vehicle.
2. Four quadrant operation of the electric machine means that it is bi-directional and can be used to accelerate as well as brake in either direction.
3. The recent advances in the power density of electric machines, YASA Motors (2016), enable multiple electric machines to be packaged in the same volume as a single ICE.
4. The electric machine torque response is of the order of 1-10 ms according to Sakai and Hori (2001) and Ivanov et al. (2015b), which makes it 5-50 times faster than hydraulic brakes.
5. Electric machines can be controlled using a continuous control variable such as a torque or speed demand, instead of the on-off control of a solenoid valve for hydraulic brakes.
6. The torque delivered by the electric machine can be measured more accurately and precisely than the torque of a hydraulic brake. This is because there is a

well understood relationship between the electric machine phase current and torque output.

The first three advantages allow the powertrain of electric vehicles to be designed in radically different ways to conventional vehicles. The torque-speed curve and bi-directional operation mean that the multi-speed transmission that is essential to an ICE can be eliminated. The use of multiple electric machines means that each wheel can be driven independently, which eliminates the need for mechanical differential. These advantages simplify the design of the powertrain by eliminating components and allow for greater packaging flexibility. From the perspective of traction control, and indeed vehicle dynamics control in general, having multiple wheels driven by electric machines independently increases actuator redundancy and allows vehicle stability control without the use of hydraulic brakes.

The last three advantages mean that traction control actuated by electric machines, instead of an internal combustion engine and hydraulic brakes, provides more accurate information on the state of the wheel and allows faster and smoother control of each wheel.

1.3.1 Electric vehicle powertrains

There are many electric powertrain configurations possible within the class of independently driven wheels. Possibilities include: the number of driven wheels, whether the electric machine is mounted within the chassis or the wheel, and the coupling between the electric machine and the wheel. The classes are summarised here to highlight the range of possible configurations that traction control could be applied to.

The primary distinction in a powertrain configuration is the number of driven wheels. For four wheeled road vehicles these can be Rear Wheel Drive (RWD), Front Wheel Drive (FWD) and four Wheel Drive (4WD). This can have significant effect on the handling of the vehicle. As noted above, the friction circle concept means that the greater the drive force the less lateral friction capacity there is in the tyres. This means the handling of a

RWD vehicle during acceleration will be more oversteer, whereas the handling of a FWD during acceleration will be more understeer. The handling of a 4WD vehicle will depend upon how the drive force is distributed front to rear.

The next configuration option is whether the electric machine is mounted in the chassis or the wheel. Electric machines mounted in the chassis are termed On-Board Motors (OBMs), while electric machines mounted in the wheels are termed In-Wheel Motors (IWM). Much attention has been given to IWMs in academia, Hori (2004) Crolla and Cao (2012) and Murata (2012). IWMs have the advantage of packaging efficiency; integrating the electric machine within the wheel frees up space within the chassis allowing greater design freedom. However, IWMs will increase the unsprung mass which is generally considered to have a negative impact on ride. There are challenges in connecting the cooling and high voltage cables through to the wheel, which will reduce serviceability: changing a wheel becomes much more technical if cooling loops and high voltage circuits need to be disconnected. Lastly, the size and therefore power of an IWM will be constrained by the size of the wheel hub. OBMs do not have the challenges related to IWMs, but do take up space in the chassis.

The last configuration option from above is the coupling used to connect the electric machine to the wheel. For OBMs there needs to be a half shaft between the electric machine and the wheel. For both IWMs and OBMs, the wheel can be direct drive or driven via a gearbox. The choice between the two is affected by the motor design and packaging constraints, because smaller higher speed machines often have greater power density but require gears to match the speed range of the wheel. According to Bottiglione et al. (2012), OBMs with high gear ratios may have unfavourable shaft dynamics; this is due to the shaft stiffness and the effective inertia of the electric machine, with respect to the wheel, which increases with the square of the gear ratio.

As no single configuration has a clear advantage it is likely that there will remain a wide range of electric powertrain configurations within electric vehicles for years to come. The focus of this research is on electric vehicles with individually driven wheels as they offer

the greatest benefit for traction and vehicle stability. The target vehicle for this research is Delta Motorsport's E4 coupe. Therefore this research focuses specifically on the E4 coupe powertrain configuration which has two electric machines that directly drive the rear wheels independently.

1.4 Research aims

The aims of this thesis are:

1. To investigate traction control applied to electric vehicles with independently driven wheels.
2. Address the challenges of controlling tyre forces which are non-linear, highly variable with road conditions, longitudinally and laterally coupled, and very difficult to measure.
3. Utilise the precise measurable torque and fast response of the electric machine for determining the level of traction of each driven wheel.
4. Develop a traction controller that improves vehicle's directional stability while accelerating on asymmetric road conditions.

1.5 Thesis outline

This thesis is structured as follows. In Chapter 2 the literature is reviewed. The review considers the control methods used, the electric vehicle it is applied to and the process by which the control is verified. The methods in the literature are categorised by whether they use slip ratio or torque for the control variable. From the review a gap analysis is carried out to help define the research objectives.

Chapter 3 analyses the traction control method of Yin and Hori (2008) termed Maximum Transmissible Torque Estimation (MTTE). In Section 3.1 simulation tests are carried out

by the Author to check the repeatability of results of Yin and Hori (2008). In Section 3.2 a modification to the rate compensation term is proposed to make MTTE compatible with higher level stability controllers. In Section 3.3 the friction force observer is analysed for robustness to changes in operating conditions. Finally in Section 3.4 a sensitivity analysis of the control parameters is carried out.

In Chapter 4 a full vehicle model of Delta Motorsport's E4 coupe is developed. The model requirements are discussed in Section 4.1. Section 4.2 gives a description of the vehicle model in the Dymola software environment. The model is calibrated using manufacturer and experimental data in Section 4.3. Finally, the vehicle model is validated in Section 4.4 using a range of experimental tests.

In Chapter 5 a MTTE-based traction control method is proposed for an electric vehicle with two independently driven wheels. In Section 5.2 the proposed traction controller is verified through simulation for the vehicle model from Chapter 4, on a split- μ road. In Section 5.3 the proposed control is benchmarked against a vehicle stability control method from the literature, using the same split- μ road simulation. Lastly, the proposed traction control is simulation tested on a high- to low- μ road.

Chapter 6 describes the experimental verification of the proposed traction control from Chapter 5. In Section 6.1 the implementation of the controller within the Powertrain Control Module (PCM) of Delta Motorsport's E4 coupe, is described. The details of the type of tests and the specific conditions for each test are given in Section 6.2. The results from the experimental tests are presented and discussed in Section 6.3.

In Chapter 7 conclusion are drawn from the preceding chapters. The contributions to research of this thesis are identified. Lastly, suggestions for further work are put forward.

Chapter 2

Literature Review

In this chapter a review of the available research literature on Traction Control (TC) for Electric Vehicles (EVs) is carried out. This is done to determine the state-of-the-art in terms of controller design, implementation, and testing. The aim is to identify gaps in knowledge that would benefit from further research.

The fundamental characteristics of TC, much like any control system, are: the target control variable; the input(s); the control method (control law); and control output (control action). Traction control is specifically concerned with maintaining good adhesion between the tyre and the road. As slip ratio is a good proxy for the level of adhesion it is often used as the target control variable. Torque is another often used target control variable as it is a common throughout the wider topic of powertrain control. Therefore TC methods can be primarily classified as being torque-based or slip-based. As vehicle velocity is difficult to measure, the control methods can be sub-categorised into those that require vehicle velocity detection and those that do not. These categories broadly agree with a survey by Ivanov et al. (2015b) on Traction Control (TC) and ABS for electric vehicles. In this chapter greater attention is given to model-based methods and sliding mode control as these are most prevalent in the literature.

The control output of a traction controller is sent to the motor controller(s) that drive the electric machines. Motor controllers can either be operated in torque control mode

or speed control mode. In general on-road vehicles are operated in torque control mode whereas speed control mode is more common for off-road vehicles. Because of this the control output is predominantly torque and only the exceptions to this are highlighted in the literature review.

The scope of this review will be confined to TC methods applicable to on-road EVs with independently driven wheels. This is because the regulation of individual wheel torques allows these EV topologies to realise advances in vehicle dynamics control not available to more conventional powertrains that use an Internal Combustion Engine (ICE) or single electric machines to drive the wheels through a differential. TC methods that rely on the electric machines' key features of fast response, precise torque feedback, multi-quadrant operation and distributed actuation are likely to yield substantial advantages.

Electric machines can deliver both positive and negative torque which means there is cross over between TC and Anti-lock Braking Systems (ABS) for EVs. The primary focus of this review is on TC systems but ABS is considered where appropriate. It is also beyond the scope of this research to review yaw moment control and control allocation/torque distribution. However, these topics are touched upon when considering vehicle stability and controller integration.

This chapter is structured as follows. The methodology for reviewing the literature is laid out in Section 2.1. The traction control literature is reviewed by category, as discussed above. Section 2.2 covers slip-based methods. Section 2.3 covers torque-based methods. Section 2.2 starts with a summary of the challenges of vehicle velocity estimation in Section 2.2.1. The remainder of Section 2.2 is divided between Section 2.2.2 for methods that require vehicle velocity detection and Section 2.2.3 for methods that do not. Torque-based methods are dominated by Model Following Control (MFC), which is covered in Section 2.3.1. One of the more advanced methods of MFC is Maximum Transmissible Torque (MTTE) which is given special attention in Section 2.3.2. The state-of-the-art in traction control is summarised in Section 2.4. This is followed by a gap analysis of the literature in Section 2.5. Finally the research objectives for this thesis are set out in

Section 2.6.

2.1 Methodology

In this section a methodology is set out for how the research literature is reviewed. The review methodology consists of three elements. These are the traction controller itself, the powertrain architecture of the EV it is applied to, and how the performance of the controller is verified.

The methodology of reviewing the traction controller itself follows the generic control structure as laid out at the start of this chapter. A particular approach uses a control method (control law) or number of methods which will define how it operates. The inputs the controller requires and how these are measured or estimated are useful in determining the practicality of the approach, as is whether the control method depends upon any *a priori* knowledge. For a traction controller to operate it needs to take over authority from the driver for the longitudinal acceleration of the vehicle. Therefore consideration should be given to how and when a traction controller takes authority, and how and when it returns authority to the driver.

To understand the limitations of a traction control method and its wider applicability the powertrain architecture is considered. The topology of EVs with independently driven wheels can either be Rear Wheel Drive (RWD), Front Wheel Drive (FWD) or 4-Wheel Drive (4WD). The actuator location can either be On-Board Motor(s) (OBM) or an In-Wheel Motor(s) (IWM). For OBMs this will include a driveline which could be direct (to the wheel) or via a gearbox. Aside from the physical aspects of the powertrain architecture the refresh time of individual control units and any signals between them affects the overall delay of the system.

How the controller is verified indicates its technology readiness. This ranges from simulation verification (low readiness) to vehicle based experimental verification (high readiness). This can be further broken down for simulation verification depending on how complex

the vehicle model is and whether it is validated. Midway between simulation and experimentation is hardware-in-the-loop (HIL) testing where only part of the vehicle is physical and the rest is simulated.

The wider the range of tests used to verify a traction controller the more confidence there can be in its design. At the wheel level this can encompass: different tyre-road friction coefficients (μ), representing different road conditions e.g. dry, wet, or ice; transitions in μ both high- to low- μ and low- to high- μ , representing sudden changes in road condition; different longitudinal velocities; and different torque demands. At a vehicle level additional tests can include split- μ roads where the road conditions are asymmetric between left and right wheels, and cornering as well as straight line manoeuvres. It is generally the case that testing the most extreme conditions is of most importance.

How well the traction controller performs is very much dependent on the test conditions and vehicle it is implemented on. This makes a quantitative comparison between methods in the literature difficult. For this reason Ivanov et al. (2015b) suggest more effort needs to be put into benchmarking different TC methods against each other. In the absence of benchmarking, a particular TC method can be reviewed against how well the controller compares to the same test without control.

To summarise the literature review methodology is:

- For the traction controller:
 - Target control variable: wheel torque or wheel slip (*covered by categorisation*)
 - Control method
 - Required inputs and their source
 - Control output (if it is not torque)
 - Means of transferring authority between the controller and the driver
- For powertrain architecture of the target EV:
 - Number of driven wheels

- Actuator location: in-wheel or on-board
- Driveline: direct drive or geared
- Maximum refresh time of powertrain controller(s)
- For the verification testing of the controller:
 - Technology readiness: simulation tests, hardware-in-the-loop (HIL) tests, or vehicle based experiments
 - Range of tests: tyre/road friction, manoeuvre, velocity range, road gradient
 - Benchmarking against other controllers
- Overall strengths and weaknesses

2.2 Slip-based traction control

To repeat Equation 1.1 for clarity, the slip ratio is typically defined as

$$\lambda = \frac{r\omega - V_x}{V_x}. \quad (2.1)$$

The challenges of using slip ratio as the control parameter are that a suitable slip ratio reference needs to be determined, the slip ratio becomes infinite at zero vehicle velocity, and the vehicle velocity needs to be estimated.

Many slip control methods in the literature use a fixed value for slip ratio reference. Notable exceptions are Drakunov et al. (1995) and Ivanov et al. (2014). As a first approximation for a proof of concept this maybe satisfactory as for many road conditions the friction-slip relationship exhibits a distinct peak typically between 0.1-0.2. However this is a non-optimal solution as the peak varies with road conditions and vehicle velocity, see experiments of Harned and Johnston (1969). Underestimating the slip ratio can reduce performance significantly due to the generally steep gradient to the left of the friction-slip

peak, see Figure 1.2. Overestimating the slip ratio means the tyre will operate in its non-linear region which may lead to instability in the controller.

By definition the slip ratio becomes infinite at zero vehicle velocity. This can be partially mitigated by including a small constant in the denominator of Equation 2.1 as in Fujii and Fujimoto (2007). This does not resolve what slip ratio reference to set at zero vehicle velocity, and further to this noise in the wheel velocity or vehicle velocity measurement at low speeds can render the slip ratio measurement unusable. This has lead Ivanov et al. (2014) to switch to a velocity reference instead of a slip reference below 30 km/h.

2.2.1 Vehicle velocity estimation

Estimation of vehicle velocity is a challenge as at present no single cost effective sensor provides a good estimate under all conditions. Wheel velocity provides a good approximation to longitudinal vehicle velocity but only while the wheels are not driven or braked.

Sensors that measure the vehicle velocity directly include GPS, optical and radar. These sensors measure the velocity over ground therefore the longitudinal component will need to be extracted. The accuracy of GPS is dependent on satellite visibility therefore the signal can be lost in built up areas and tunnels. In addition low cost GPS can introduce a lag in the velocity measurement which will result in an error in the slip ratio calculation that increases with acceleration.

Both optical and radar work by reflecting a signal (either light or sound) off the ground. Because of this they can become inaccurate when the surface is wet due to the change in reflectivity. Optical sensors are also sensitive to dirt ingress. Magnetic markers have also been proposed by Lee et al. (1995) but require large infrastructure change therefore are not a practical solution.

Accelerometer(s) mounted on the vehicle provide an indirect method of measuring vehicle velocity by integration of acceleration. However, misalignment and noise introduce errors which are also integrated meaning that the error of the velocity estimate rapidly increases

with time. This means that the integral of acceleration is only useable for a few seconds. In addition when an accelerometer is not level it will measure a component of gravitational acceleration. Methods using three orthogonal accelerometers are proposed to compensate for this by Gustafsson et al. (2001).

As explained above no individual sensor provides a good estimate of vehicle velocity under all conditions. This leads many to explore sensor fusion approaches where signals from multiple sensors are combined to give an estimate that is better than each individual sensor measurement. The most common sensor fusion approach is to use a Kalman filter such as Gustafsson et al. (2001) and Antonov et al. (2011) although other approaches are proposed such as fuzzy logic by Daiss and Kiencke (1995) and recursive least squares by Zhang et al. (2007).

Sensor fusion approaches often rely on knowing the validity of each signal at any instance to determine the weighting for each signal in the overall estimate. In the case of GPS this can be obtained from the additional information transmitted in the NEMA messages such as the number of satellites and the quality of the fix. The validity of the other sensors is less straightforward to infer. Vehicle velocity estimation through sensor fusion may also introduce lags due to the filtering process. Whereas this may be acceptable for other vehicle stability functions, any additional lag of the vehicle velocity compared to the wheel velocity will increase the error of the slip ratio during transient manoeuvres.

All of the above highlight that vehicle velocity estimation is non-trivial, and requires serious investigation for a traction controller to be reliant upon it. Therefore a traction control approach that requires vehicle velocity without specifying how it estimates vehicle velocity has a low readiness level.

2.2.2 Slip control with vehicle velocity

Within the context of slip control the majority of the approaches in the literature require vehicle velocity to calculate slip. A number of slip control methods have been proposed including different variants of Proportional-Integration (PI) control Hori et al. (1998),

Akiba et al. (2007), Ivanov et al. (2014), Linear Quadratic Regulator (LQR) Petersen et al. (2001), and Sliding Mode Control (SMC). Of these SMC is the most prevalent in the literature, and therefore is reviewed in detail below.

Early work by Hori et al. (1998) proposes slip ratio traction control based on linearisation of longitudinal vehicle dynamics using a PI controller. An off-line method is given for determining the optimal slip ratio target based on constructing $\mu-\lambda$ curve. This estimates the friction coefficient (μ) either with or without vehicle velocity, although the vehicle velocity is still required to calculate slip. Hori et al. (1998) experimentally benchmark their optimal slip ratio control against Model Following Control. Akiba et al. (2007) also benchmark slip ratio method against MFC torque-based method. Both claim slip ratio control performs better than MFC, because it achieves its reference slip. Akiba et al. (2007) is an early example of a control method that considers the dynamics of left and right wheels to be interrelated.

Petersen et al. (2001) propose slip control using a Linear Quadratic Regulator for ABS with hydraulic brakes. Implementation is relatively advanced as experimental tests are conducted on a Mercedes E220, for straight-line braking on dry asphalt, snow, and wet asphalt partially covered by plastic sheeting. The vehicle speed is estimated using a Kalman filter with acceleration and wheel speed inputs. The weakness of this approach is that it uses a fixed slip ratio reference that is adjusted by the researchers depending upon the test surface.

Ivanov et al. (2014) present a slip based approach to electric and hydraulic ABS. This uses a PI controller with feed-forward and feedback terms. Ivanov et al. (2014) take a novel approach to slip ratio reference. A slip ratio reference is calculated that is proportional to friction coefficient and normal wheel load. An online adaptation is also proposed to account for errors in the proportional method. The online method uses the longitudinal vehicle acceleration and measured slip ratio to estimate the sign of the gradient of the friction-slip curve. However, the researchers note that it can only be activated when the slip ratio measurement is considered to be accurate. Estimation of the friction coefficient,

normal wheel load, and vehicle velocity are not presented. Experimental tests are carried out on a Range Rover Evoque, see Ivanov et al. (2015a). It is interesting to note that in such a production prototype EV, the wheel speed sensors are connected to the brake boost unit and then transmitted to the powertrain controller over a Controlled Area Network (CAN) bus.

Sliding mode control

The goal of Sliding Mode Control (SMC) is to reach a sliding surface. For slip control this can be defined as

$$s = \lambda - \lambda^* \quad (2.2)$$

such that s remains zero, where λ is the measured slip and λ^* is the slip reference. Sliding mode control typically uses a model based observer together with a switching term to account for modelling errors. This discontinuous term has the drawback of introducing high frequency chattering which can limit its practical application.

Sliding mode control has been considered by Schinkel and Hunt (2002), Colli et al. (2006), Harifi et al. (2008), Amodeo et al. (2010), Xu et al. (2011), De Castro et al. (2013), and Kuntanapreeda (2015) for ABS and TC. Of these De Castro et al. (2013) is the most advanced.

De Castro et al. (2013) applies Seshagiri and Khalil (2005) method of a Continuous Sliding Mode Control (CSMC) with a Conditional Integrator (CI) to slip control. To reduce chattering of conventional SMC the discontinuous function is replaced with a saturation function. The limits of saturation function are termed the “boundary layer”. The steady-state tracking error that this introduces by the saturation function is overcome by using an integrator within the boundary layer. As the integrator is only active within the boundary layer, it is said to be conditional. Seshagiri and Khalil (2005) show that this type of control reduces to a PI controller with anti-windup followed by saturation.

The control is applied to both ABS and TC by summing the torque demand from the

brake and throttle pedals. The driver's total torque demand is equally allocated between left and right wheels. This torque demand is multiplied by the slip control output (u). The slip control for each wheel acts independently. A diagram of the slip controller for one wheel is shown in Figure 2.1.

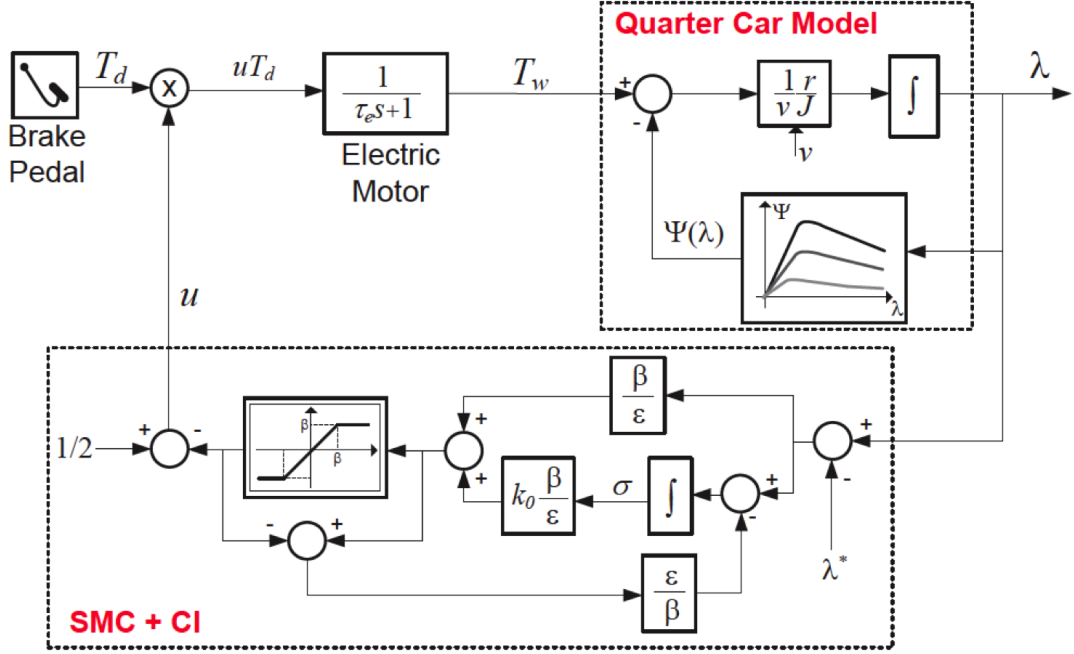


Figure 2.1: Continuous Sliding Mode Control (CSMC) with Conditional Integrator (CI) including anti-windup, after De Castro et al. (2013)

The slip controller requires vehicle velocity to calculate wheel slip. In their experiments De Castro et al. (2013) use the non-driven wheels to measure vehicle velocity, although alternatives are suggested. The controller requires a reference slip ratio, De Castro et al. (2013) use a fixed reference of $\lambda^* = 0.2$, although an optimal slip reference method is proposed in a separate paper by the same authors, De Castro et al. (2011).

The slip controller is enabled when the driver's torque demand exceeds the maximum load torque. The load torque is calculated as

$$\Psi(\lambda) = \left(\frac{J_w}{Mr^2}(1 - \lambda) + 1 \right) r F_z \mu(\lambda), \quad (2.3)$$

where

M	Vehicle mass
J_w	Wheel inertia
r	Tyre rolling radius
λ	Slip ratio
F_z	Vertical tyre force
μ	Tyre friction coefficient

Equation (2.3) is similar to the Maximum Transmissible Torque Equation 2.6 proposed by Yin and Hori (2008). However it is formulated for wheel slip instead of acceleration ratio.

The controller is tested on a 600kg experimental vehicle with an induction motor driving each of the front wheels through a 7:1 gearbox and driveshaft. The powertrain employs a single FPGA to implement the traction control and motor control functionality for both wheels. The outputs of the FPGA are the PWM signals to the two inverters. The inputs to the FPGA are the motor velocities and currents, the rear wheel velocity, and the throttle and brake pedal signals. The FPGA has a refresh time of 2 ms. The driver's torque demand is limited to ± 80 Nm.

The controller is simulated for step changes in μ , λ , and torque demand while the vehicle is braking. The controller is tested experimentally for a vehicle braking and accelerating on a homogeneous surface of $\mu = 0.15$. This surface is obtained using wet plates of an unspecified material.

De Castro et al. (2013) compare CSMC to CSMC+CI. The experimental results show that CSMC constrains slip but has a steady state error of 10 %, whereas CSMC+CI ensures the slip converges to the reference slip within 0.5 s. The steady state error for CSMC+CI is quoted as ± 4 %. The control parameters are tuned experimentally to eliminate chattering and achieve acceptable transient response. This is achieved through increasing the boundary layer, however simulations show that this increases slip overshoot.

Amodeo et al. (2010) suggest that robustness is lost through using a boundary layer, therefore propose a second order SMC that generates a continuous control action to alleviate the chattering problem. This control method assumes that the vehicle velocity is known. A first order sliding mode observer is used to estimate the friction coefficient

online and seek the optimum slip, however this relies on *a priori* knowledge of the $\mu - \lambda$ characteristic of the tyre/road. The control method is only advanced to the simulation stage for a half vehicle model with an unspecified tyre model.

Kuntanapreeda (2015) proposes using a second order SMC known as Super-Twisting Algorithm (STA), for slip control. A nonlinear observer is used to estimate the tyre friction forces. A fixed slip ratio reference of $\lambda^* = 0.2$ is used. The STA control is compared to conventional SMC through simulation on half vehicle model with pitch dynamics and Burckhardt tyre model. The simulation results for a step change in friction coefficient show that STA converges to the reference slip in 1 s whereas conventional SMC takes 5-10 s which results in a larger overshoot. The controller is also tested on a wheel and drum HIL rig. Changes to the drum inertia are used to simulate changes in road condition. For the tests the refresh time of the controller is 20 ms. The experimental results show that the slip converges to the reference in 10 s for STA whereas it takes 30 s for SMC. The increase in convergence time from simulation to experiment suggest that for a practical application of the controller the refresh time should be reduced.

2.2.3 Slip control without vehicle velocity

Due to the difficulties in estimating vehicle velocity, highlighted in Section 2.2.1, some researchers investigate slip control without vehicle velocity detection. These include Deur and Pavković (2011), Liang and Lin (2012), and Fujii and Fujimoto (2007). Deur and Pavković (2011) propose a bi-directional saw-tooth excitation slip control method. This uses a quarter vehicle model to estimate the vehicle velocity, which means it is also classified as model based control. Unconventionally, the estimated vehicle velocity is scaled by the reference slip ratio to obtain the wheel velocity reference. This reference feeds into a conventional PI speed controller. Deur and Pavković (2011) verify their controller on a scale model vehicle. Liang and Lin (2012) propose a fuzzy sliding mode slip control. This uses a Kalman-like slip ratio observer based on wheel velocity and integration of vehicle acceleration. The errors due to the integration of acceleration over

time are corrected by using the wheel velocity when the vehicle coasts. Of these methods, Fujii and Fujimoto (2007) is the most advanced and is reviewed in greater detail here.

Fujii and Fujimoto (2007) propose a PI slip controller that does not require vehicle velocity detection. Instead slip is estimated using a Slip Ratio Observer (SRO). This is derived from the longitudinal equations of motion and the definition of slip ratio as

$$\dot{\hat{\lambda}} = -\frac{\dot{\omega}}{\omega}\hat{\lambda} + \left(1 + \frac{J_w}{Mr^2}\right)\frac{\dot{\omega}}{\omega} - \frac{T}{Mr^2\omega} + k\left(a_x - \frac{F_z}{M}\mu(\hat{\lambda})\right), \quad (2.4)$$

where

$\hat{\lambda}$	Slip ratio estimate
ω	Angular wheel velocity
M	Vehicle mass
J_w	Wheel inertia
r	Tyre rolling radius
T^*	Motor torque demand
a_x	Vehicle acceleration
k	Observer gain
F_z	Vertical tyre force
μ	Tyre/road friction coefficient

The last term in brackets on the right hand side of the equation is included to ensure the error converges to zero. This uses an accelerometer to measure a_x and estimates the friction coefficient by assuming a fixed linear relationship between μ and λ . The observer also requires wheel angular velocity (ω) and motor torque demand (T^*) as inputs. The observer is integrated into a controller with non-linear compensation as shown in Figure 2.2. A fixed slip ratio reference of $\lambda^* = 0.2$ is used.

The controller and observer are verified through simulations and experiments for a vehicle accelerating in a straight line on a road with a friction coefficient $\mu = 0.2$. The simulations use a quarter vehicle model. The experiments are carried out on a 420 kg EV with two independently driven wheels. The wheels are driven by IWMs through a single gear (6.267:1). The slip control for each wheel is implemented in the motor controllers.

The slip ratio observer is benchmarked against two simplified versions of the observer.

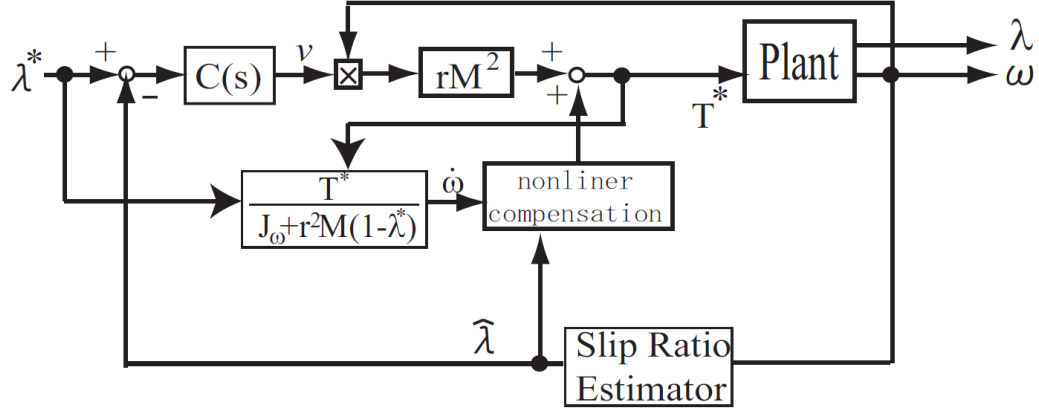


Figure 2.2: PI slip controller with Slip Ratio Estimator (SRE) and nonlinear compensation, copied from Fujii and Fujimoto (2007)

The simulation and experimental results show that the proposed observer converges to the measure slip ratio, whereas the simplified observers either do not converge or have a steady state error. The experimental tests of the controller and observer show that the slip ratio estimate converges to the measured slip ratio in 0.8 s. The results show that the slip ratio is controlled to its reference value, although it oscillates with a high frequency ($> 20 \text{ Hz}$) of amplitude 0.1. There are no experimental results in the literature showing the response of the controller to a change in road- μ . The common slip control problems of adapting the slip ratio reference to the road conditions, transitioning from open-loop to close-loop control and operating around zero vehicle velocity are not addressed.

Maeda et al. (2012) make use of the above SRO for four-wheel drive force distribution control. This investigates redistributing the torque to each wheel depending on the road conditions, to maintain longitudinal acceleration while suppressing yaw moment. Experiments are carried out on split- μ roads but the length of the low- μ section is limited to less than wheel base of vehicle, therefore one wheel per axle is always on the high- μ road.

2.3 Torque-based traction control

2.3.1 Model following control

Modelling Following Control (MFC) is investigated by a number of authors: Hori et al. (1998), Sakai and Hori (2001), Hori (2004), Akiba et al. (2007), Kawabe (2012), and Yin et al. (2009). It is often used as a benchmark for other controllers. Hori et al. (1998) propose a simple Model Following Control (MFC) based on the principle that the effective inertia of the vehicle, from the perspective of the wheel, reduces as slip increases. Hori et al. (1998) define the effective inertia as

$$J = J_w + Mr^2(1 - \lambda), \quad (2.5)$$

where

M	Vehicle mass
J_w	Wheel inertia
r	Tyre rolling radius
λ	Slip ratio

A model of the vehicle inertia is defined from Equation (2.5) for $\lambda = 0$ which is used to formulate the controller. A generic form of the controller diagram is shown in Figure 2.3. When the wheel slips the difference between the actual wheel speed and the wheel speed of the model is used for proportional feedback to reduce the torque demand to the electric machine. The controller receives an acceleration demand from the throttle pedal, and the wheel velocity feedback from the electric machine. The controller output is a torque demand to the electric machine.

Hori et al. (1998) use a converted Nissan micra (UOT Electric March I) to experimentally test MFC. The front two wheels are driven by an electric machine through a 5-speed manual transmission and a differential. The traction control is tested experimentally by accelerating the vehicle in a straight line onto a low- μ surface constructed from an iron

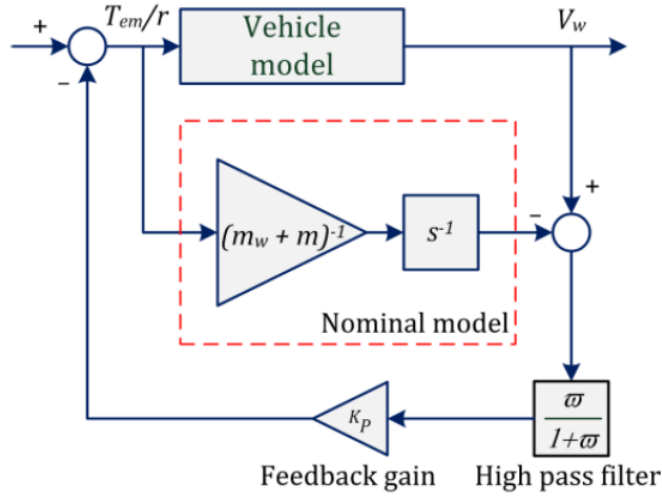


Figure 2.3: Model Follow Control (MFC) diagram, copied from Ivanov et al. (2015b).

plate of length 0.8 m covered in water. The results show that the controller can reduce the wheel slip from 0.4 to 0.1. The control method will not maximise tractive forces as it will attempt to reduce torque whenever slip is not zero. Without independently driven wheels it would be difficult to control vehicle stability.

Sakai and Hori (2001) extend the testing of MFC using the same vehicle as Hori et al. (1998). The experimental tests use a 14 m aluminium plate covered in water for the low- μ surface. This has a peak friction coefficient of 0.5. The results show that the wheel slip is reduced compared to the no control case. However, the slip ratio tends to increase across the low- μ surface. The best performance ($\lambda < 0.35$) is achieved for a gain of $K_p = 20$, although this is above the indicated optimum range of $0.1 < \lambda < 0.2$.

Hori (2004) applies Model Following Control (MFC) to a second converted Nissan micra (UOT Electric March II). This EV has four independently driven wheels, with IWMs at the rear and OBMs at the front. The MFC is applied independently to each wheel. A straight line deceleration on a test track ($\mu = 0.5$) shows that MFC can be used for ABS and prevents wheel lock when compared to no control. A second experiment is used to test the vehicle's lateral stability, in this case only the rear wheels are driven. The experimental test consists of applying a step torque demand while the vehicle is in a constant radius turn at 40km/h on a skid pad. The results show for the no control case the inner driven wheel spins, yaw rate increases and the vehicle oversteers. On the

other hand for the test with MFC the wheel slip is controlled, the average yaw rate stays constant, and the vehicle maintains its trajectory. A down side to the control is that there are very fast oscillations (>20 Hz) in the driven wheel velocity and torque, with magnitudes of 5 m/s and 1500 N¹ respectively.

2.3.2 Maximum Transmissible Torque Estimation

Maximum Transmissible Torque Estimation (MTTE) is researched in Yin and Hori (2008), Yin et al. (2009), Yin (2009), and Yin and Hori (2010). The friction force observer is adapted in Hu et al. (2009) and Hu et al. (2011). MTTE is integrated with Direct Yaw Control (DYC) in Hu and Yin (2011), Yin and Hu (2014), and Hu et al. (2015).

MTTE is a type of model-following control that does not not require vehicle velocity detection. MTTE control aims to maintain traction through managing the ratio of wheel acceleration to vehicle acceleration. This ratio is defined as a target control parameter termed the relaxation factor, α . From longitudinal equations of motion of a vehicle the maximum transmissible torque is derived as

$$T_{max} = \left(\frac{J_w}{\alpha M r^2} + 1 \right) r \hat{F}_x, \quad (2.6)$$

the tractive force is obtained from a kinematic friction force observer

$$\hat{F}_x = \frac{T_m - J_w \dot{\omega}}{r}, \quad (2.7)$$

where

M	Vehicle mass
J_w	Wheel inertia
ω	Wheel angular velocity
T_m	Motor torque demand
α	Relaxation factor
r	Tyre rolling radius

¹These are the units given by Hori (2004).

A diagram of the controller is shown in Figure 2.4a. The inputs to the controller are the driver's torque demand and the wheel velocity from the motor controller. The wheel velocity is low pass filtered to reduce noise. The motor torque demand, used for the friction observer, is low pass filtered to remain in phase with the angular velocity. The maximum transmissible torque T_{max} is used to saturate the driver's torque demand.

The controller is tested on a 360 kg single seater experimental EV. Each of the rear wheels are actuated independently by a permanent magnet IWM, although only one is driven during tests. The refresh time of the traction controller is 10 ms and the refresh time of the motor controller is 2 ms.

The controller is verified through simulations by accelerating a quarter vehicle model on high- to low- μ road. The controller is verified through experiments by accelerating the EV on high- to low- to high- μ surface. The low- μ is constructed from an acrylic sheet of length 1.2 m covered in water. The tests are conducted for vehicle velocities between 2-4 m/s.

The simulation results show that the acceleration ratio converges to its designed value of 0.9. The experimental results show the wheel slip is significantly reduced by MTTE when compared to the no control case. Yin et al. (2009) experimentally compare MTTE to MFC from Fujimoto et al. (2004a). The results show MTTE reduces slip when compared to MFC ($\lambda < 0.5$ compared to $\lambda < 2$). Alternatively, MFC with a higher gain has comparable slip but has reduced stability (motor torque oscillations by 90 % of demand torque). Yin (2009) and Yin and Hori (2010) investigate the sensitivity of MTTE to changes in α , filter time constants, controller mass estimate, and road μ .

Research by Hu et al. (2009) and Hu et al. (2011) replace the open loop friction force observer with a closed loop observer to account for model uncertainties such as drag force or a change in wheel inertia. The closed loop observer is realised with a PI compensator. The modified MTTE controller diagram is shown in Figure 2.4b. An advantage of this method is that the wheel velocity does not need to be differentiated, which can increase noise. MTTE with closed-loop and open-loop observers are compared using the same EV

Research by Hu and Yin (2011), Yin and Hu (2014) and Hu et al. (2015) combines MTTE with DYC for front wheel independent drive. The controller diagram is shown in Figure 2.5. Steering angle and throttle pedal demand are used by the electronic differential to proportion the torque demand. The Traction Distribution uses a PID control to adjust the left-right torque demand based on the yaw rate error. MTTE is applied independently for each driven wheel. The yaw rate reference is determined by a single track “bicycle” model. The yaw rate reference requires vehicle velocity and friction coefficient estimation but no solutions are specified for either.

The controller is simulated on a CarSim 8.03 vehicle model. The vehicle velocity and friction coefficient are obtained directly from the model. Constant steer, step steer, and sine wave steer manoeuvres on a homogeneous surface ($\mu = 0.4$) are used to verify the controller.

The DYC+MTTE control is benchmarked against DYC on its own and no control. The results show that the DYC+MTTE control has greater yaw rate stability than the DYC on its own for all three manoeuvres. However, for the step steer and sine steer simulations the DYC on its own performs worse than no control, which suggests there is little benefit of using DYC in these situations. No simulation results are presented for MTTE control on its own. Neither are there any simulations with asymmetric road friction conditions.

Experimental tests are carried out on a Opel Corsa converted to front wheel independent drive, using two permanent magnet synchronous motors. The refresh time of the controller is not given. The vehicle velocity is obtained from the rear right (non-driven) wheel. The value and source of the friction coefficient are not specified. A constant radius manoeuvre including a low- μ section is used to test the controller. A plastic plate lubricated with engine oil is used for the low- μ section. Insufficient data is presented to evaluate the results.

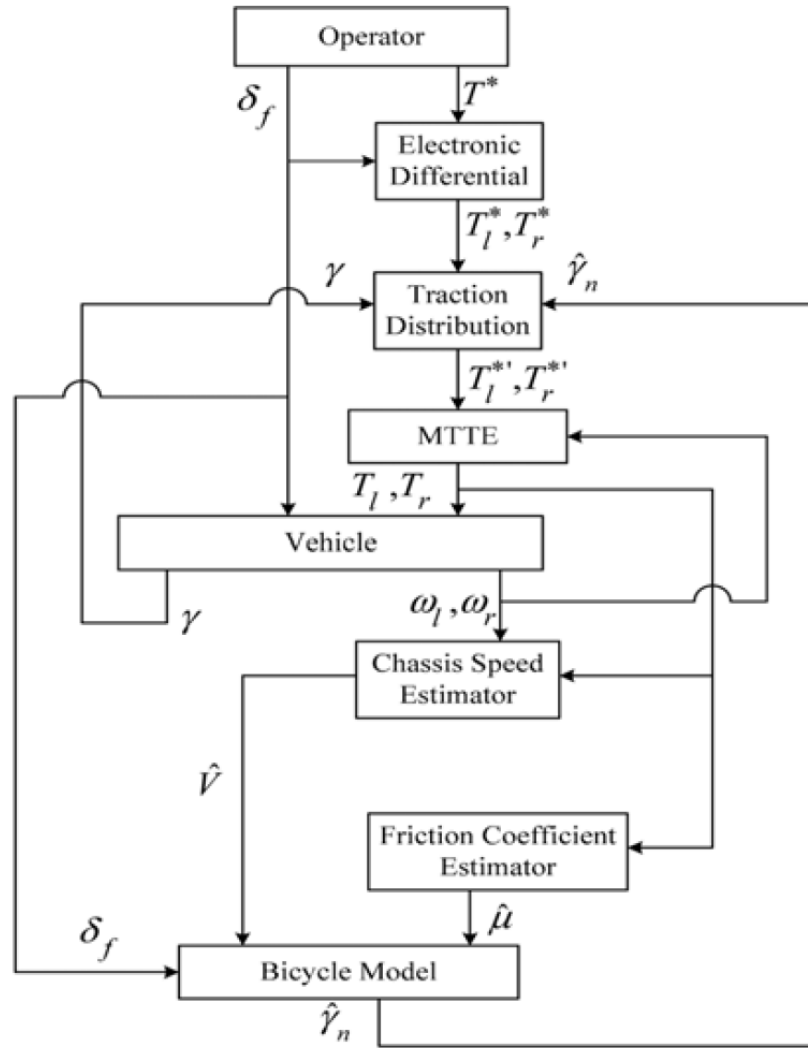


Figure 2.5: Combined DYC and MTTE control copied from Yin and Hu (2014)

Alternative MTTE with sliding mode observer

An alternative approach to maximum transmissible torque using a sliding mode observer with a LuGre tyre model is proposed by Magallan et al. (2011). Their controller is similar to MTTE after Yin and Hori (2008) in that it saturates the torque demand to the electric machines. The controller is derived from a full vehicle model as opposed to a quarter vehicle model. A diagram of the controller is shown in Figure 2.6. A sliding mode observer uses all four wheels speeds and the torque demands to each electric machine to estimate the state of the LuGre tyre model. This is then used to calculate the maximum torque for each wheel. The parameters of LuGre model require calibration to empirical data. This represents *a priori* knowledge of the tyre-road interaction which limits the practical implementation of such a controller.

The controller is applied to a 600 kg EV. Each rear wheel is driven independently by an on-board induction motor through an unspecified reduction gear. The controller is verified through simulation on a Dymola vehicle model using a LuGre tyre model. The controller is tested for a vehicle accelerating in a straight line on a high- to low- μ road and on a split- μ road. The results show that the controller constrains the wheel slip in 0.1 s; however, sensor noise is not considered in the simulation. The split- μ test did not have a larger enough difference in μ to produce significant yaw motion.

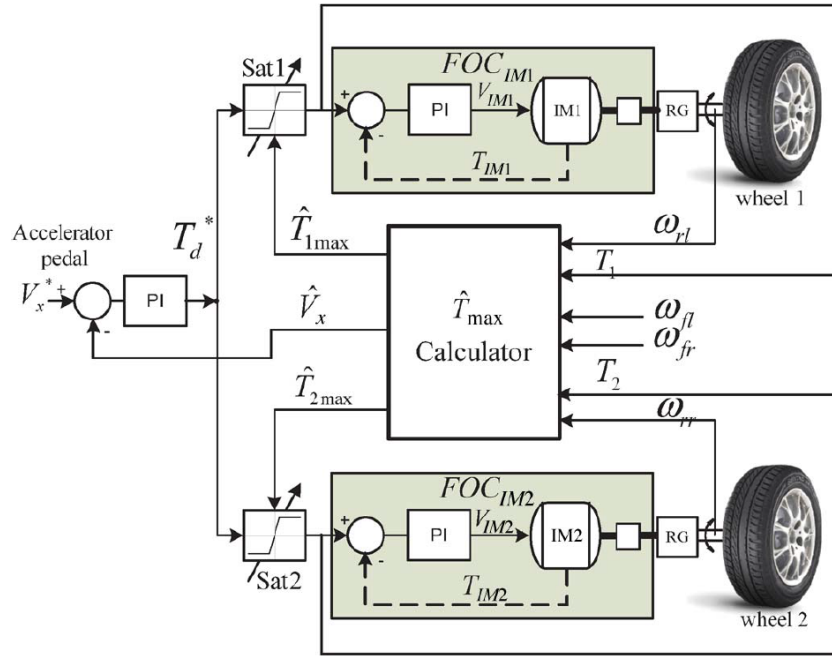


Figure 2.6: 2WD traction control using SMC and LuGre friction observer, copied from Magallan et al. (2011)

2.4 Summary

In the literature a wide range of control methods are applied to traction control of electric vehicles with independently driven wheels. The technology readiness of these controllers varies with respect to how it is verified (simulation or experiment) and under what conditions (range of tests). The research generally focuses on control of a single wheel.

Both Sliding Mode Control (SMC) and Model Following Control (MFC) methods appear to be more advanced, most notably De Castro et al. (2013) for SMC and Yin et al. (2009) for MTTE, a form of MFC. SMC is shown to be robust to model uncertainties and is shown to converge to a reference slip reasonably fast (0.5 s, De Castro et al. (2013)). These properties are likely to be due to the use of integral control action, which is not present in MTTE. In addition a number of solutions are presented to the chattering problem inherent with first order SMC. However, it is often assumed that vehicle velocity is known even though its estimation is non-trivial, and the slip reference tends to be a fixed value whereas the peak slip can vary with road condition. Both of which mean that many approaches are incomplete.

MTTE after Yin et al. (2009) is a form of model following or load following control. MTTE aims to manage the ratio of acceleration between the vehicle and the wheel, as opposed to optimising the slip ratio. It also maybe less robust to model uncertainties than SMC. On the other hand as MTTE does not use slip ratio it avoids the challenges of: an undefined slip ratio at zero vehicle velocity; determining a slip ratio reference; and estimating the vehicle velocity which is found to be non-trivial. MTTE makes use of wheel torque and wheel velocity measurement to estimate friction force. Importantly, this is a kinematic friction observer that does not depend upon *a priori* knowledge of the road condition, which is a common weakness of many control methods. The saturation block enables smooth transition between driver control and closed loop control.

The majority of traction control research focusing on the control of a single wheel as this is fundamental starting point for any research. However, the wider perspective is that traction control should improve safety and performance, therefore vehicle directional stability should be paramount. Consequently there is an overlap with other areas of Electronic Stability Control (ESC) such as Direct Yaw Control (DYC). DYC has been widely investigated in the literature, Sakai et al. (1999), Fujimoto et al. (2004b), Hu and Yin (2011) and De Novellis et al. (2015). A comprehensive review of such methods is outside the scope of this research. However many of the more advance designs depend on low-level traction control to prevent excessive wheel slip. The benefits to vehicle stability of traction control on it own are highlighted by Hori (2004).

A common trait of the most advanced traction control research is the use of customised powertrain controllers that have fast refresh times (2 ms) and use directly fed signals between the traction controller, motor controllers and sensors. The aim of this is reduce the system time constant and therefore improve the response of the controller. While this is desirable, it may not be possible in a production vehicle. An example of this is the use of a CAN bus for sensor feedback as in Ivanov et al. (2014) where traction control is applied to an Range Rover Evoque. The use of a CAN bus can introduce additional delays.

Existing TC methods have generally only been experimentally tested on low- μ surfaces and high- to low- μ transitions where the low- μ surface is constructed from a section of metal or acrylic sheet. The size of the sheet has often prevented quasi-steady state conditions from being observed, which would provide a clearer indication of controller stability. This necessitates the need for specialist test facilities with large homogeneous low- μ surfaces. There is insufficient research that also includes low- to high- μ tests, split- μ tests and as noted by Ivanov et al. (2015b) take-off tests.

2.5 Gap analysis

Traction control of a single wheel has been considered in detail. This makes up a fundamental part of higher level stability control. There is a gap in the research literature in developing a traction control method that takes an integrated approach to the control of multiple driven wheels, to improve vehicle directional stability. A notable exception is Magallan et al. (2011) although this research has similar drawbacks to SMC mentioned above. Furthermore, detailed investigation is needed into how such low level traction control interacts with higher level stability control.

Ivanov et al. (2015b) identify that the majority of the research is applied to small EVs that have In-Wheel Motor (IWM) and permanent magnet electric machines, and there is a lack of experimental testing on full size EVs with powertrain topologies such as On-board Motors (OBMs). To which it can be added that OBMs are the preferred option in production vehicles due to size and packaging constraints. This therefore warrants further research.

Several researchers note the use of fast refresh times (2 ms) and directly fed signals in traction controller implementation. However, automotive vehicles often use CAN buses to communication between control units and sensors. There does not appear to be any research to date specifically looking at the trade offs between feedback from direct-fed sensors and feedback via CAN.

Experimental verification has focused on high- to low- μ road tests, as this tests the primary function of single wheel traction control. The gaps in experimental tests are as follows. Tests on low- to high- μ roads are needed to show how authority is returned to the driver. Tests on split- μ roads are needed to investigate vehicle directional stability. Take-off tests are particularly important for slip-based traction control as slip becomes undefined at zero vehicle velocity. Beyond changes in road condition, there is limited testing of traction control for steering manoeuvres and road gradients.

2.6 Research objectives

Based on the review of the literature the objectives of this research are:

1. Fully utilise capabilities of electric machine actuator: fast response, precise torque feedback, positive/negative torque, distributed actuation.
 - (a) Make use of both torque and speed feedback, as opposed to just speed.
 - (b) Ensure approach is compatible with negative torque demands, for integration with higher level stability control (torque vectoring).
 - (c) Make use of distributed actuation to enhance vehicle stability.
 - (d) Investigate the effects of a CAN bus within the control feedback loop.
2. Develop single wheel MTTE control for a two wheel independent drive EV.
 - (a) Determine improvements for single wheel MTTE control.
 - (b) Combine information from both electric machines.
 - (c) Design a vehicle level control strategy.
 - (d) Verify controller under harsh conditions.
 - (e) Benchmark the controller against other vehicle stability control methods.
 - (f) Implement on a full size production prototype EV.

These objectives are explored as follows:

- A detailed analysis of single wheel MTTE is carried out in Chapter 3 to determine improvements, this includes a sensitivity analysis and adaptation for negative torque demands.
- In Chapter 5 MTTE control is extended to the control of multiple wheels to improve vehicle stability.
- In Chapter 3 and Chapter 5 the controller is verified through simulations on high- to low- μ road, low- to high- μ road and split- μ road for the largest anticipated change in μ .
- In Chapter 5 the proposed control is compared through simulation to Direct Yaw Control (DYC).
- The above necessitates the need for a high fidelity vehicle model. This is described and validated in Chapter 4.
- Finally in Chapter 6 the controller is implemented on Delta's E4 coupe. This enables comparison between direct-fed wheel velocity feedback, and wheel velocity feedback over CAN. Experimental tests are carried out at a specialist test facility, which enables testing of controller stability and vehicle directional stability.

Chapter 3

Single Wheel Traction Controller Analysis

From the literature review in Chapter 2 Maximum Transmissible Torque Estimation (MTTE) was identified as an attractive method of traction control for electric vehicles with independently driven wheels. In particular it makes use of the relatively accurate torque measurement and fast response of an electric machine, traits that stand out when compared to an Internal Combustion Engine (ICE). This chapter consists of a comprehensive assessment of MTTE control for a single driven wheel. It therefore only employs a simple quarter vehicle model. MTTE control of two independently driven wheels is developed in Chapter 5, using a high fidelity vehicle model validated in Chapter 4.

For clarity this chapter starts by repeating work by Yin et al. (2009) on the controller derivation and stability results. This is followed by a simulation comparison to the results of Yin et al. (2009) to assess their repeatability. Next, two parts of the controller are analysed, namely the rate compensation and the friction force observer to assess their robustness and to propose improvements. Lastly a sensitivity analysis is carried out of the parameters used in the controller. This draws on similar work in the literature and the author's own simulation work. Other than the first section where a direct comparison is made to work by Yin et al. (2009), the vehicle parameters used are representative of

Delta's E4 coupe which is the target vehicle for this research.

3.1 Maximum Transmissible Torque Estimation

In this section the work of Yin et al. (2009) is reproduced for the purpose of clarity and to investigate its repeatability. This starts with the derivation of the MTTE controller equations for single wheel control. Next, the outcome of the stability analysis by Yin et al. (2009) is presented as this determines the parameter constraints. Lastly, the results from two simulations carried out by Yin et al. (2009) are repeated.

Considering longitudinal motion only, the equations of motion for a quarter vehicle model are

$$M\dot{V}_x = F_x - F_{dr} - F_{rr}, \quad (3.1)$$

$$J_w\dot{\omega} = T_m - F_x r, \quad (3.2)$$

where

M	Vehicle mass
V_x	Longitudinal vehicle velocity
F_x	Tractive force generated at contact patch
F_{dr}	Aerodynamic drag force
F_{rr}	Rolling resistance force
J_w	Wheel inertia
ω	Wheel angular velocity
T_m	Electric machine torque
r	Tyre rolling radius

The work by Yin et al. (2009) does not include rolling resistance force. This force is significant as Gillespie (1992) notes that it is the dominant road load for on-road vehicles up to approximately 50 mph.

From Equation (3.2) Yin et al. (2009) derives a tractive force observer

$$\hat{F}_x = \frac{T_m - J_w\dot{\omega}}{r}. \quad (3.3)$$

This is a kinematic inversion of Equation (3.2) where all of the variables and parameters on the right of the equation are known. This is valid for an electric vehicle with independently driven wheels. This allows the friction force between the tyre and the road to be estimated without depending upon knowledge of the complex relationship between friction (μ) and slip (λ). The robustness of this observer against uncertainty is determined through simulation in Section 3.3.

A small difference between the wheel and vehicle velocity is required to allow a friction force to develop. However a large difference will result in a loss of traction. According to Yin et al. (2009) traction of a wheel can be maintained through controlling the ratio of vehicle to wheel accelerations. They define a relaxation factor α as

$$\alpha = \frac{\dot{V}_x}{\dot{V}_w}, \quad (3.4)$$

where V_x is the longitudinal chassis velocity, and V_w is the longitudinal wheel velocity. The relaxation factor is similar to slip ratio, as in Equation 2.1, as they both define a ratio between wheel velocity and vehicle velocity. The relaxation factor α will need to be close to one to prevent excessive slip but be less than one to allow some slip to occur.

Similar to Yin et al. (2009), the equation for α can be derived from Equations (3.3), (3.4) and (3.1)

$$\alpha = \frac{(F_x - F_{dr} - F_{rr})/M}{(T_{max} - rF_x)r/J_w}. \quad (3.5)$$

Rearranging Equation (3.5) gives the maximum transmissible torque

$$T_{max} = \left(\frac{J_w}{\alpha M r^2} + 1\right)r\hat{F}_x - \frac{J_w}{\alpha M r}(F_{dr} + F_{rr}), \quad (3.6)$$

where the friction force estimate $\hat{F}_x$ is obtained from Equation (3.3).

Yin et al. (2009) assume the aerodynamic force to be zero, leading to an over-evaluation of

T_{max} . This is acceptable at low speeds where the force is small as it is proportional to the square of the aero velocity of the vehicle. The aero velocity of the vehicle can be measured using a pitot tube or otherwise approximated by the vehicle velocity. This research focuses on traction controller performance at low speed, therefore the aerodynamic force is ignored.

The rolling resistance force acts on the wheel as soon as it is in motion. The rolling resistance force is the product of the vertical tyre force (F_z) and the rolling resistance coefficient (k_{rr})

$$F_{rr} = F_z k_{rr} . \quad (3.7)$$

The rolling resistance coefficient k_{rr} can be influenced by many factors; Gillespie (1992) gives examples of tyre temperature, pressure, velocity, surface type and slip. These factors also affect the tyre-road friction coefficient, however an empirical relationship between friction coefficient and rolling resistance coefficient is lacking, therefore they are assumed to be independent. As a first approximation the rolling resistance coefficient is assumed to be a constant. In Chapter 4 the rolling resistance coefficient is experimentally determined to be 0.021. As this is a quarter vehicle analysis load transfer is not considered and the vertical tyre force F_z can be approximated by the static load.

The maximum transmissible torque T_{max} is used by the controller to constrain the driver's torque demand (T^*) as follows

$$T = \begin{cases} T^*, & \text{for } -|T_{max}| < T^* < |T_{max}| \\ |T_{max}|, & \text{for } T^* \geq |T_{max}| \\ -|T_{max}|, & \text{for } T^* \leq -|T_{max}|. \end{cases} \quad (3.8)$$

A diagram of the controller is shown in Figure 3.1. The wheel velocity is passed through a linear first order filter with time constant τ_1 , to reduce noise. The wheel torque is filtered in the same way with $\tau_2 = \tau_1$ so that the signals remain in phase, after Yin et al. (2009).

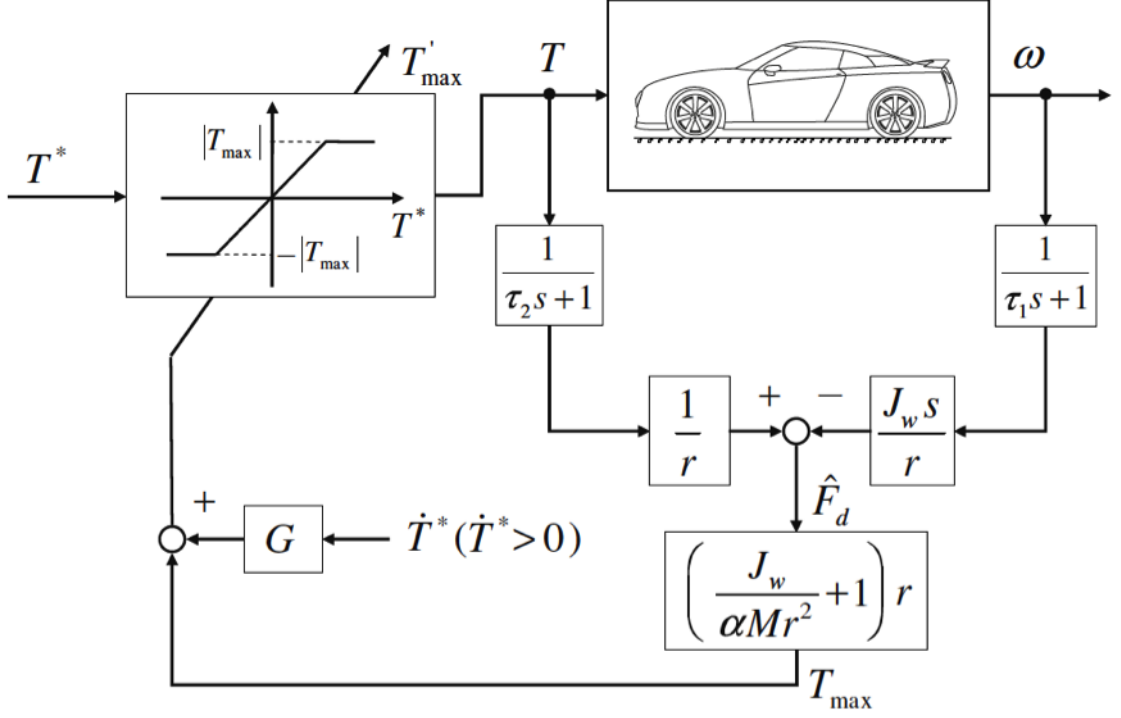


Figure 3.1: MTTE Controller diagram, copied from Yin and Hori (2010)

When the wheel begins to slip the maximum transmissible torque T_{max} effectively becomes the input, forming a closed-loop system. Yin et al. (2009) analyse the closed-loop stability based on the concept of effective inertia after Hori et al. (1998), whereby the effective inertia of the system from the perspective of the wheel varies depending upon the amount of slip. For no slip the effective inertia is equal to the wheel inertia plus vehicle inertia. At a theoretical maximum slip the effective inertia equals that of the wheel. This effective inertia is defined by Yin et al. (2009) as

$$J = \frac{J_w + Mr^2}{1 + \Delta}, \quad (3.9)$$

where the variation in inertia is

$$\Delta \in [0, Mr^2/J_w]. \quad (3.10)$$

The results of the analysis by Yin et al. (2009) give the stability criteria as

$$\alpha > \frac{1 - J_w \Delta / Mr^2}{1 + \Delta}, \quad (3.11)$$

$$\tau_1 > \frac{J_w \tau}{Mr^2}, \quad (3.12)$$

where τ is the system time constant. From the above equations stability is achieved for all values of Δ with $\alpha > 1$, although as previously stated α needs to be less than one to allow the vehicle to accelerate. Yin et al. (2009) set $\alpha = 0.9$ based on there being three possible operating states: no-slip, slight slip, and excessive slip. In the no-slip condition T_{max} is greater than T^* so the controller output follows the driver's demand. In the slight slip case the controller is theoretically unstable but allows T_{max} to increase which allows the friction force to increase while within the linear region of the tyre friction-slip curve. When there is excessive slip Δ tends to Mr^2/J_w therefore Equations (3.11) and (3.12) are satisfied, the controller is stable, and T_{max} decreases so that the vehicle to wheel acceleration ratio converges to the designed α .

Yin et al. (2009) find it necessary to add a compensation gain (G) to the limiter, as it is found that T_{max} would prevent the vehicle accelerating under normal conditions due to the delays in the system. They define the modified maximum torque

$$T'_{max} = T_{max} + G\dot{T}^* \quad \text{for} \quad T^* > 0. \quad (3.13)$$

3.1.1 Simulation verification

In this section simulations carried out by Yin et al. (2009) are redone to review their repeatability. These consist of a vehicle accelerating in a straight line from a high- μ to a low- μ road. The simulation is used to compare MTTE control to no control, and to assess the effect of changing different control parameters.

The simulations is carried out in Simulink using the controller presented in Section 3.1

together with a quarter vehicle model, a diagram of which is shown in Figure 3.2. The electro-mechanical response of the electric machine is modelled by a first order transfer function with time constant τ_m . The tyre is modelled using the Magic Formula, Pacejka (2006) along with a first order transfer function to represent transient dynamics. This has a time constant of 1 ms, after Yin (2009). The friction-slip relationships for the two road surfaces are shown in Figure 3.3. The curves are derived from tyre model coefficients given in Appendix C, with nominal road friction coefficient of 1.0 for the high- μ road and 0.3 for the low- μ road. The shape of the friction-slip curve has a significant effect on the simulation. As Yin et al. (2009) only state the peak coefficient (0.3) of the low- μ road used in their simulation, it is only possible to make a qualitative comparison with the original simulations.

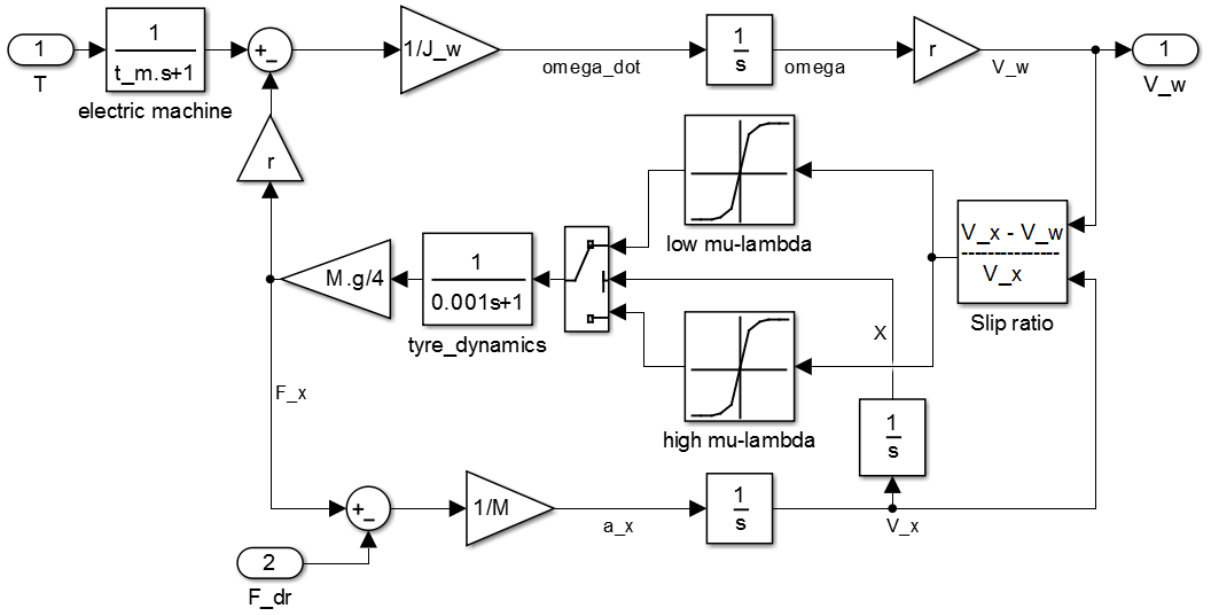


Figure 3.2: Simulink quarter vehicle model

The parameters for the quarter vehicle model, chosen to match the original simulations are shown in Table 3.1. For all the simulations the time constants of the low pass filters of the torque and wheel speeds signals are the same, $\tau_2 = \tau_1$.

In the first simulation the time constant of the filters is 20 ms and the electric machine response is instantaneous to show primary behaviour of the controller. Figure 3.4 shows the results from Yin et al. (2009) alongside the author's own results. From Figure 3.4b

Parameter	Symbol	Value	Unit
Vehicle mass	M	360	kg
Wheel radius	r	0.22	m
Wheel inertia	J	0.5	kg.m ²
Relaxation factor	α	0.9	

Table 3.1: Quarter vehicle model parameters, after Yin et al. (2009)

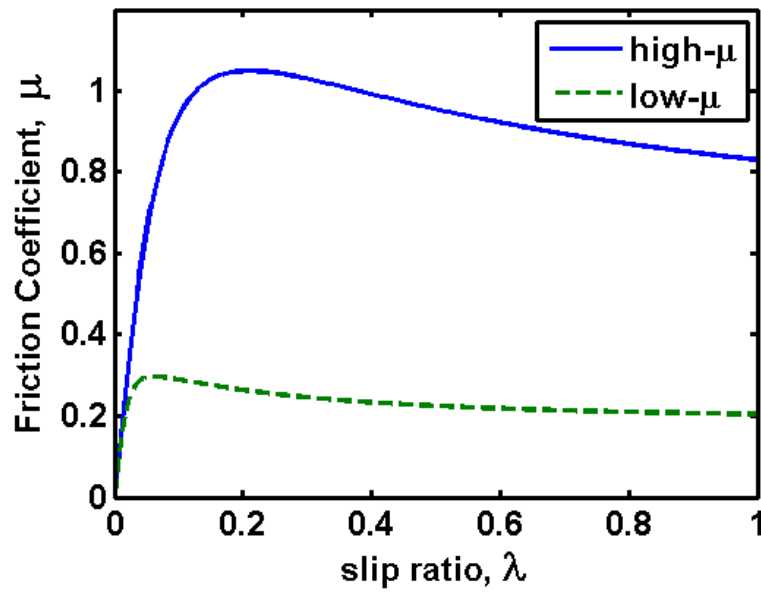
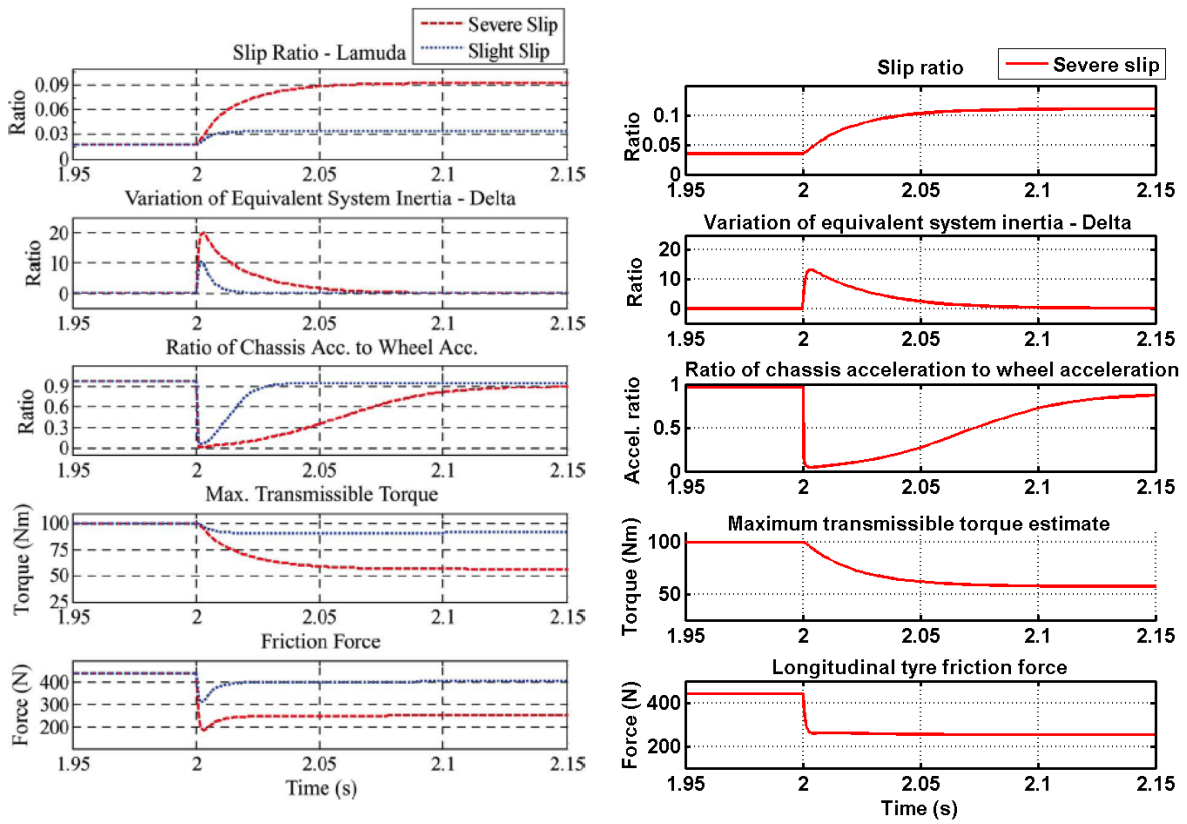


Figure 3.3: friction-slip relationship for the two surfaces used in the simulation.

it can be observed that the acceleration ratio between vehicle and wheel converges to the designed value of $\alpha = 0.9$. The final friction force and slip ratio are similar to that obtained by Yin et al. (2009) and do not appear to be very sensitive to the time constants of the filters. The shape of the friction force is somewhat different to the original research as it did not dip to 200 N on entry to the low- μ road. This is to be expected as the tyre model parameters are not identical. The response of both the acceleration ratio and slip ratio are similar to the results of Yin et al. (2009) and these are found to be sensitive to the filter time constants.



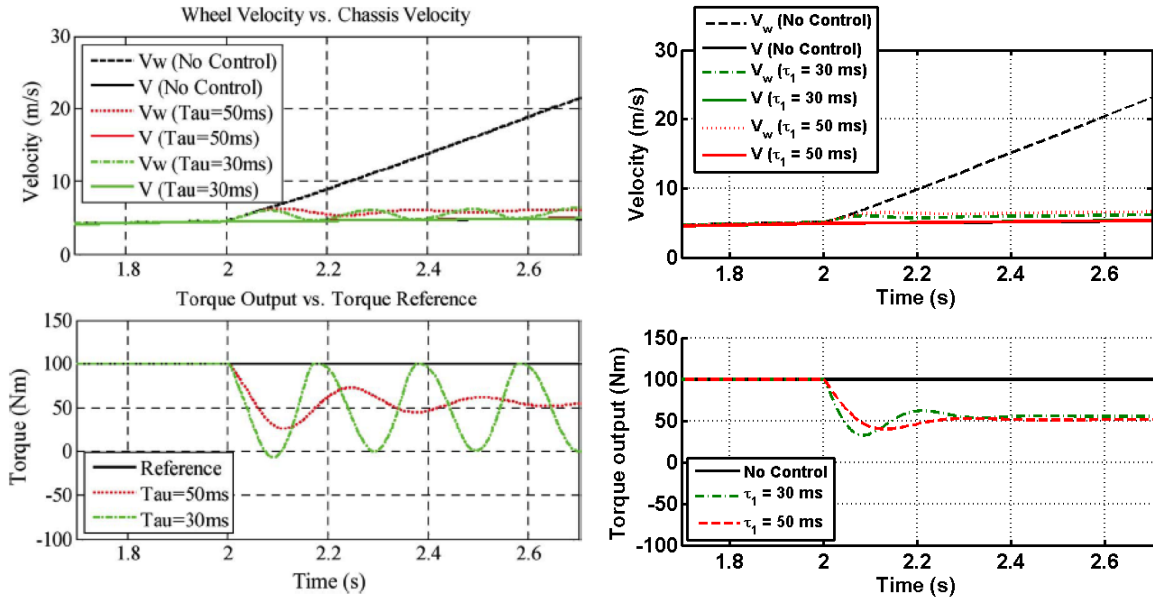
(a) Simulation results copied from Yin et al. (2009)

(b) Author's simulation results for comparison to severe slip

Figure 3.4: high- to low- μ simulation results with LPF time constant $\tau_1 = 20ms$ and no system delay.

A second set of simulations are carried for the MTTE controller with a filter time constant of $\tau_1 = 30$ ms, the MTTE controller with a filter time constant of $\tau_1 = 50$ ms, and a simulation without any traction control. The electro-machine time constant is $\tau_m = 40$ ms. Figure 3.5 shows the author's results compared with those of Yin et al. (2009). Figure 3.6a

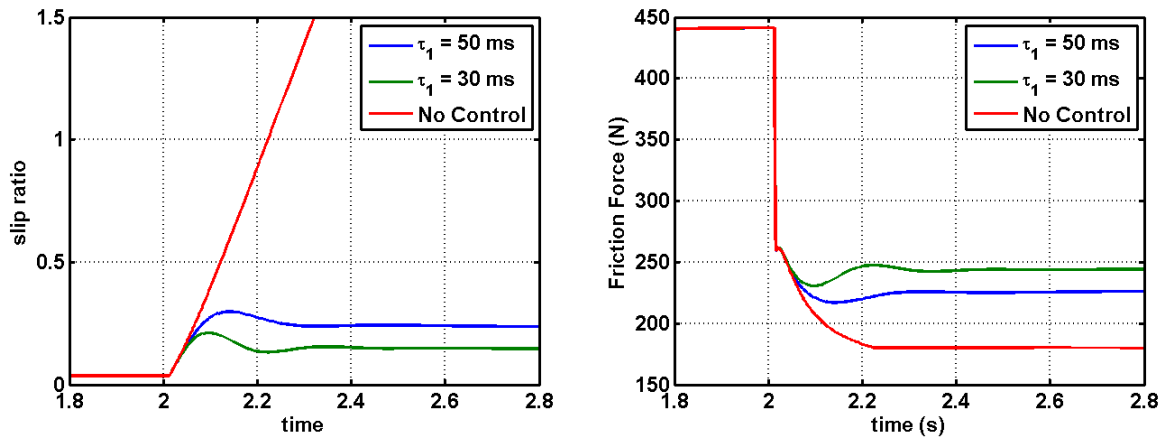
shows that the MTTE controller limits the wheel slip when compared to the no control case and therefore delivers a higher tyre friction force, Figure 3.6b.



(a) Simulation results copied from Yin et al. (2009)

(b) Author's simulation results

Figure 3.5: High- to low- μ simulations with controller filter time constant $\tau_1 = 50\text{ms}$, $\tau_1 = 30\text{ms}$, and no control. Where V_w is wheel speed and V is vehicle speed.



(a) Slip ratios from author's simulations

(b) Friction forces from author's simulations

Figure 3.6: High- to low- μ simulations with controller filter time constant $\tau_1 = 50\text{ms}$, $\tau_1 = 30\text{ms}$, and no control

The simulation with filter time constants of 30 ms has a damped oscillatory torque response with a peak to peak amplitude of 30 Nm, see Figure 3.5b. This differs from the results of Yin et al. (2009) which has undamped torque oscillations of 100 Nm peak-to-peak. Yin et al. (2009) attribute this to τ_1 being less than τ_m . However the stability

condition of Equation (3.12) evaluated with the vehicle parameter values for this simulation is

$$\tau_1 > 0.029\tau_m, \quad (3.14)$$

which is satisfied by the filter time constants values used in the simulation; therefore this alone does not cause the instability. The difference in responses can be explained by the difference in the part of the friction curve the controller operates in. It can be seen from Figure 3.6a that on the low- μ surface ($t > 2s$) the slip ratio is higher than the slip ratio corresponding to the peak friction coefficient from Figure 3.3. This means the friction force will change less rapidly and therefore produce a less oscillatory response. It can be concluded from this that it is the combination of having filter time constants less than the time constant of the system delay, along with operating at the peak of the friction-slip curve, that leads to instability in the controller. It is desirable to minimise the filter time constant as this results in a high friction force, see Figure 3.6b.

3.2 Rate compensation

In this section the rate compensation proposed by Yin et al. (2009) is developed to handle all conditions of changing torque. To allow a vehicle to accelerate under normal conditions the controller must allow the torque demand to the motors to increase. As mentioned in Section 3.1 MTTE control will overly restrict an increasing torque demand as T_{max} is under-evaluated due to system delays. To prevent this Yin et al. (2009) propose the addition of a rate compensation term to the maximum transmissible torque T_{max}

$$|T'_{max}| = |T_{max} + G\dot{T}^*| \quad \text{for} \quad \dot{T}^* > 0, \quad (3.15)$$

where T_{max} is the maximum transmissible torque estimated from Equation (3.6), T^* is the torque demand and G is the rate compensation gain. This is used to limit the torque demand.

3.2.1 Control modification

A traction controller may receive negative torque demands from a higher level controller, for example for torque vectoring. The four possible cases are: torque demand is positive and increasing; torque demand is positive and decreasing; torque demand is negative and decreasing; and torque demand is negative and increasing. These cases are illustrated in Figure 3.7. Only the top two cases in the graph require rate compensation. For the bottom two cases an under-evaluated T_{max} will not prevent the torque output of the controller decreasing.

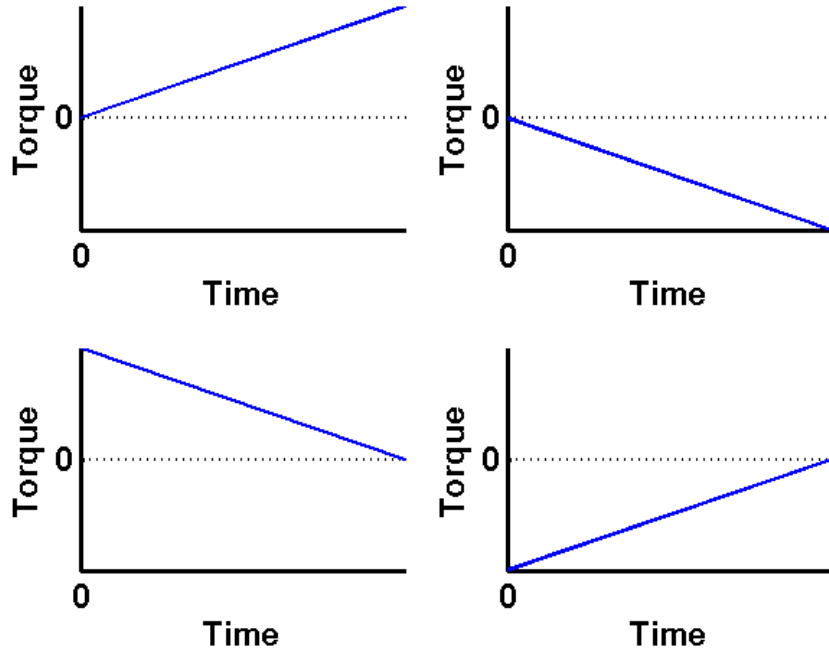


Figure 3.7: Torque demand into the traction controller can change by being a) positive and increasing; b) positive and decreasing; c) negative and decreasing; or d) negative and increasing.

A comparison of the generalised cases in Figure 3.7 shows that for both of the top two cases the sign of the torque and the sign of the rate of torque are the same. This is used to adapt the rate compensation Equation (3.15) to

$$|T'_{max}| = |T_{max}| + \left| \frac{G}{\tau_c s + 1} T_{comp} \right|, \quad (3.16)$$

with

$$T_{comp} = \begin{cases} \dot{T}^*, & \text{for } \text{sign}(\dot{T}^*)\text{sign}(T^*) > 0 \\ 0, & \text{otherwise,} \end{cases} \quad (3.17)$$

where τ_c is the linear first order filter time constant. This equation has three important new properties. Firstly the rate compensation is applied to the correct cases mentioned above which allows for negative torque demands. Secondly a low pass filter is added which smooths the transition between periods with and without rate compensation. Thirdly the magnitude of the two terms is taken separately to achieve the correct behaviour at zero crossing.

3.2.2 Simulation results

The performance of the proposed rate compensation from Equation 3.16 and 3.17 is compared to Equation 3.15 through simulation of MTTE control on a quarter vehicle model in Simulink. The parameters used in the simulation are shown in Table 3.2. These are representative of Delta's E4 coupe. The rate compensation gain is set to 0.1, after Yin et al. (2009). The vehicle model is simulated on a high friction surface with an initial velocity of 5 m/s. At $t = 1$ s a trapezoid torque demand is input to the controller. The torque demand curve includes both positive and negative values and crossings of the x-axis to verify the new capabilities of MTTE control with modified rate compensation.

Parameter	Value	Unit
Total vehicle mass	1005	kg
Wheel radius	0.3	m
Wheel inertia	1.0	kg.m ²
Relaxation factor	0.9	
Time constant (speed and torque)	30	ms
Time constant (torque demand)	30	ms
Rate compensation gain	0.1	

Table 3.2: Quarter vehicle model parameters for rate compensation simulation.

The results of the simulation with the original rate compensation of Equation (3.15), after Yin et al. (2009) are presented in Figure (3.8). The results of the simulation with

the modified rate compensation of Equation 3.15 are presented in Figure 3.9. For each simulation the torque demand along with the controller's upper torque limit, T_{lim+} lower torque limit, T_{lim-} and the friction limits calculated from the peak friction of the tyre/road are plotted. The torque output from the controller is not shown for clarity, although it is clear that the torque demand will be limited when it crosses either the upper or lower limits. The vehicle velocity for both simulations is shown in Figure 3.10.

It is found through simulation that the time constant of the rate compensation should be equal or greater than the time constants of the filters, to ensure the torque output is not unnecessarily limited. The rate compensation gain of 0.1 gives satisfactory results for this simulation, tuning of this gain to other driving conditions is left to further work.

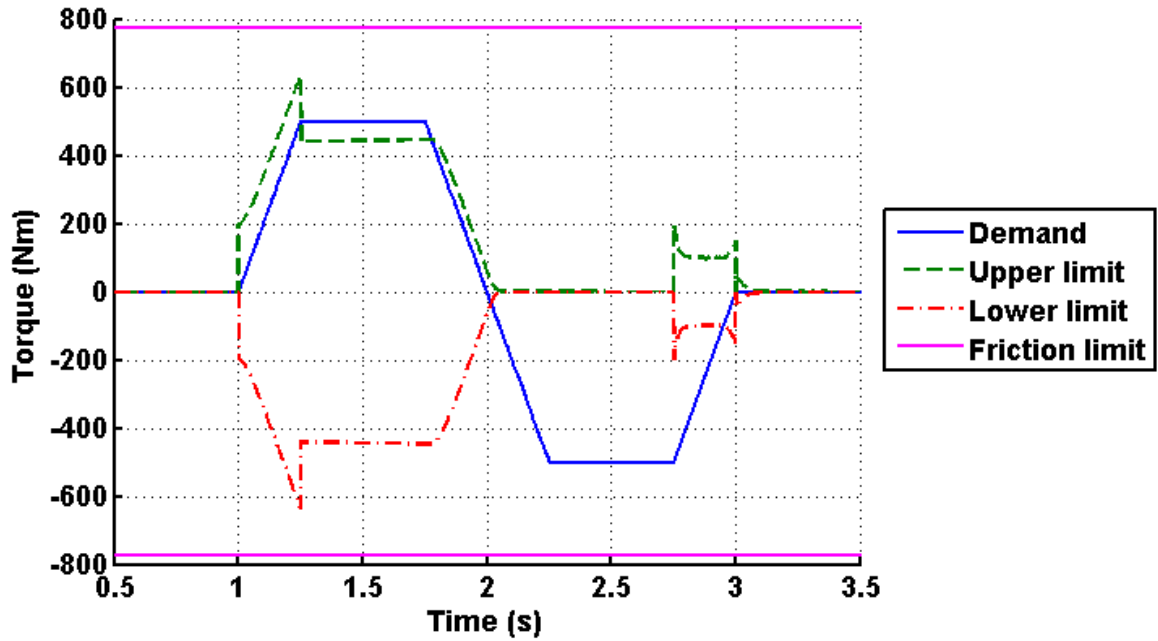


Figure 3.8: Results of MTTE with original rate compensation simulated with a trapezoid torque demand. The graph shows the torque demand input to the controller, the upper and lower limits of the saturation block that constrains the torque demand, and the friction limit calculated from the peak friction force of the tyre/road.

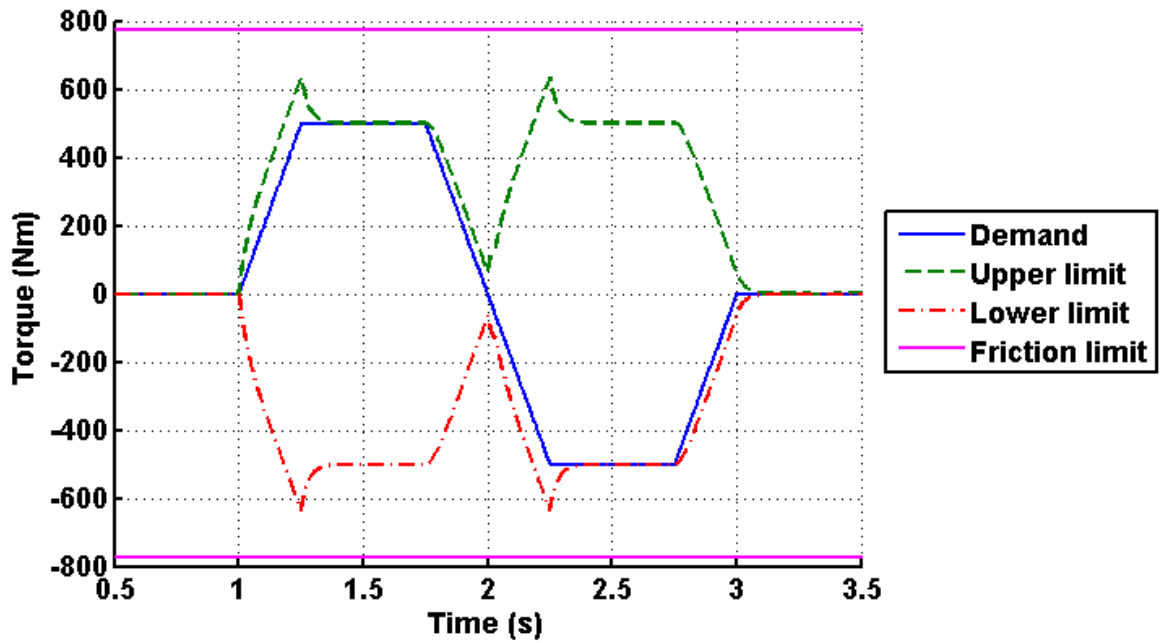


Figure 3.9: Results of MTTE with modified rate compensation simulated with a trapezoid torque demand. The graph shows the torque demand input to the controller, the upper and lower limits of the saturation block that constrains the torque demand, and the friction limit calculated from the peak friction force of the tyre/road.

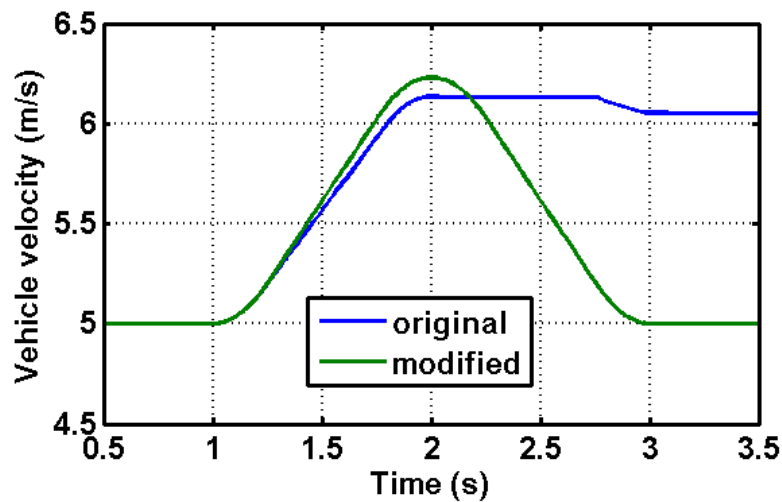


Figure 3.10: Graph shows vehicle velocity for two simulations with a trapezoid torque demand: MTTE with original rate compensation; and MTTE with modified rate compensation.

3.2.3 Discussion

Figure 3.8 shows that MTTE control with rate compensation after Yin et al. (2009) will prevent almost any negative torque demand. This is because rate compensation is only non zero when the rate of torque demand is positive. In contrast the results of the modified rate compensation simulation show that a negative torque demand is allowed to develop in the same way as a positive torque demand, thus allowing the vehicle to decelerate as well as accelerate, Figure 3.10.

In the original rate compensation simulation when the torque demand stops increasing at $t = 1.25$ s the rate compensation is instantaneously removed as there is no low pass filter. This results in the torque demand between $1.25 \text{ s} < t < 1.78 \text{ s}$ being overly restricted. This is not the case for the modified rate compensation simulation, where the upper limit is always greater than the torque demand. This can be explained more easily by plotting the initial part of both simulations side by side, with separate curves for T_{max} and the rate compensation term that sum together to give the upper limit, see Figure 3.11. The maximum transmissible torque, T_{max} is calculated from the filtered torque demand out of the controller; therefore in both simulations it lags the torque demand into the controller. For the modified rate compensation simulation, shown in the right hand graph of Figure 3.11, the rate compensation is filtered with the same time constant thus allowing the T_{max} to be greater than the torque demand in steady state.

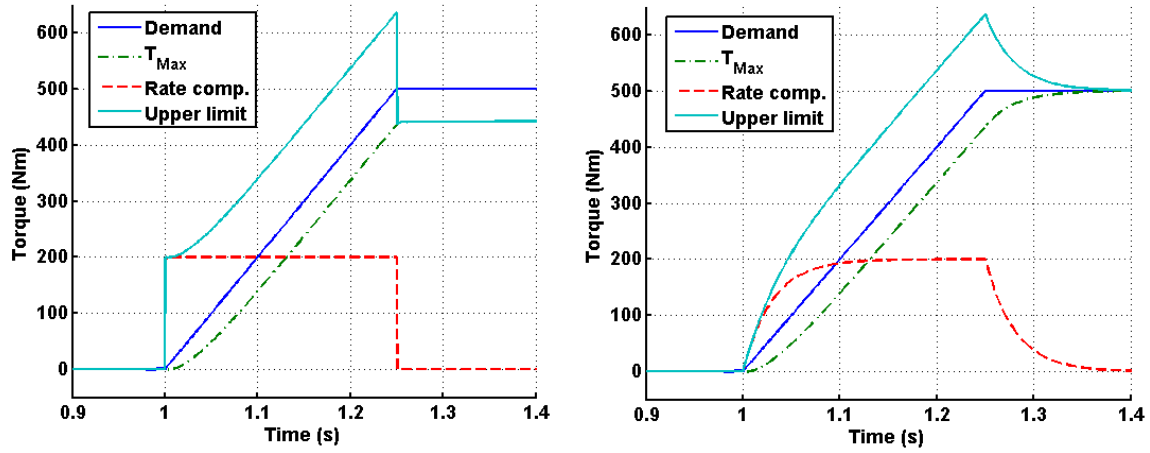


Figure 3.11: Comparison of initial part of simulations: MTTE with original rate compensation (Left); MTTE with modified rate compensation (Right). Curves shown for T_{max} and the rate compensation which sum together to give the upper limit curve.

3.3 Friction force observer

MTTE after Yin et al. (2009) uses a friction force observer, see Equation (3.3), to estimate the instantaneous longitudinal friction force of the tyre. The friction force observer should be robust to changes in longitudinal and lateral slip, vertical load, vehicle speed, and road condition. In this section the robustness of this observer is analysed through simulation comparison with the Magic Formula tyre model, Pacejka (2006).

A virtual tyre test rig is used to simulate the tyre through its full range of operation (acceleration and braking), a visualisation is shown in Figure 3.12. The lateral slip angle, vertical load, vehicle (rig) speed, and nominal road friction coefficient are set at the start of the test. During the test the rig moves along the road while sweeping the longitudinal slip.

The simulation parameters are shown in Table 3.3. Where applicable, these parameters are representative of Delta's E4 coupe. Unless otherwise stated these are the parameters used in each of the simulations. The tyre model used in the simulation is the Pacejka tyre model described in Chapter 4, with parameters given in Appendix C.

The friction observer requires the torque and angular velocity of the wheel to be measured. In this section sensors for these measurements are considered to be ideal. Real sensors

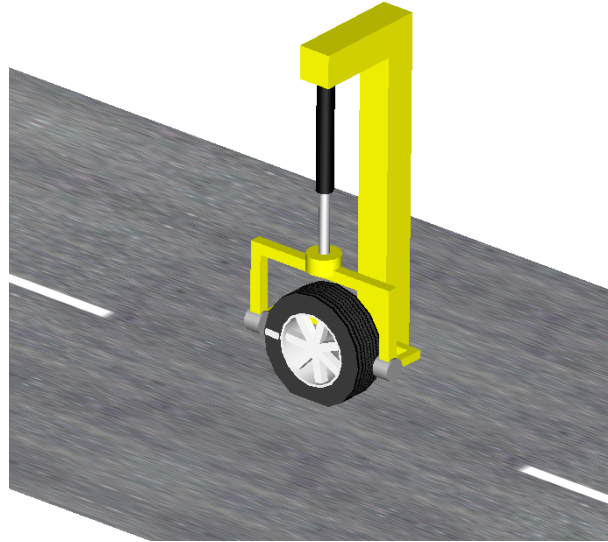


Figure 3.12: Visualisation of the virtual tyre test rig used to simulate a single wheel. The velocity of the rig, the vertical tyre force and the side slip are held constant while the slip ratio is swept.

Parameter	Value	Unit
Vertical tyre force	2500	N
Lateral slip angle	0	degrees
Vehicle (rig) speed	10	m/s
Friction coefficient	1.0	
Loaded tyre radius	0.296	m

Table 3.3: Virtual tyre test rig parameters

are considered in Chapter 6. The virtual tyre test rig is set up to provide the vertical load as an input to the tyre model. This means that the vertical dynamics are not simulated. Although less realistic this ensures both the tyre model and the estimator use the same wheel radius. Variation of the wheel radius is considered later in Section 3.4 for the MTTE controller as a whole.

3.3.1 Simulation results

The tyre test rig and friction observer are simulated with the parameters from Table 3.3. A time series plot of the torque, angular velocity, friction force, friction estimate, and difference between model and estimate are shown in Figure 3.13. There is 68 N difference between longitudinal friction force of the model and the estimate due to the rolling resistance force in the tyre model. This is not included in the observer as the rolling resistance is included in Equation (3.6) of the MTTE controller.

In Figure 3.14 the longitudinal friction force and friction estimate are plotted against slip ratio. This shows that the tyre model contains hysteresis for the slip ratio region of ± 0.15 . As the difference between the model and estimated friction force in Figure 3.13 varies by less than 1 N across the simulation the observer can be said to capture the hysteresis effect well.

The simulation results for a $+/- 40\%$ change in vertical force are shown in Figure 3.15. For a vertical force of 1000 N the difference between the estimate and tyre model is 27 N, and for a 4000 N vertical force the difference is 108 N. The difference within each simulation varies by less than 1 N. In each case the difference is due to the rolling resistance force, which from Equation (3.7) is proportional to the vertical tyre force. This shows the observer is robust to changes in vertical load.

Figure 3.16 shows the results for a $+/- 10$ degrees variation in lateral slip angle. The difference between the friction force and estimate is 68 N with less than 1 N variation. Again as expected there is a difference between the friction force of the model and the observer as the model includes the rolling resistance force which is accounted for elsewhere

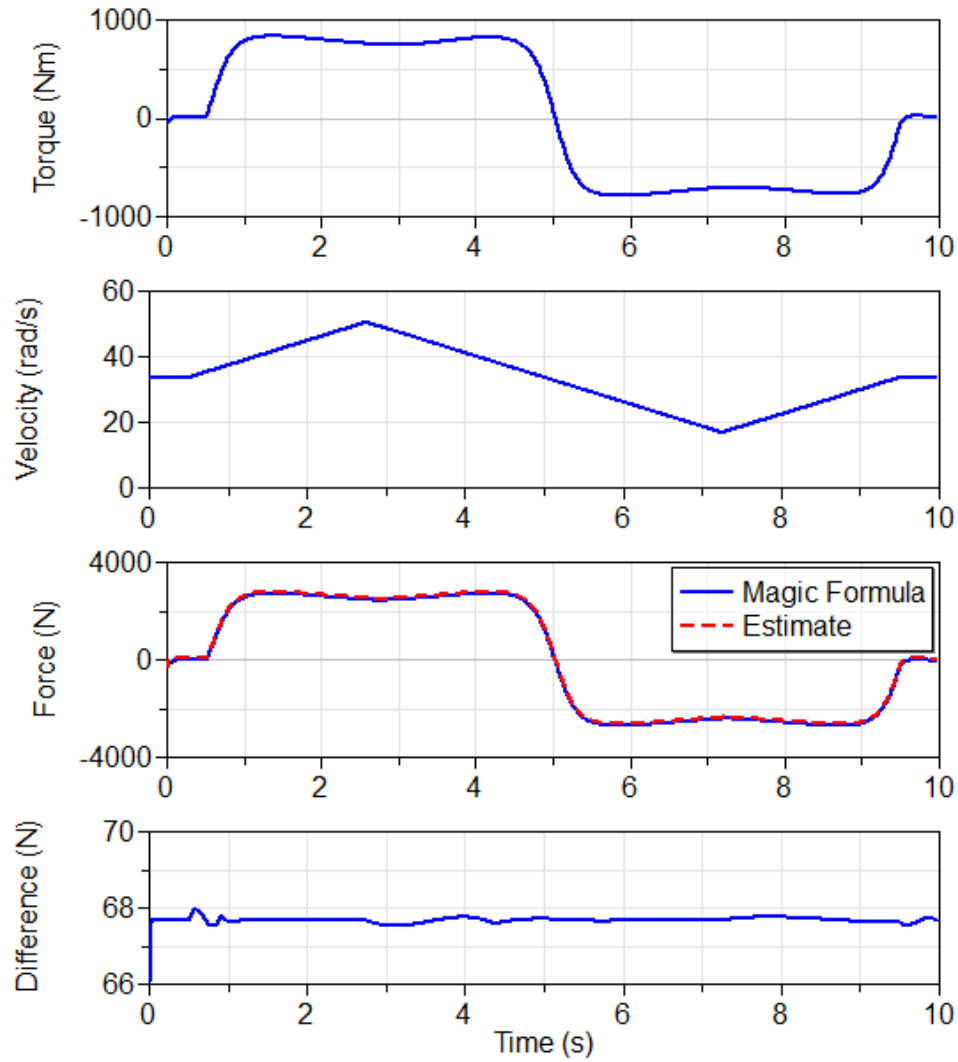


Figure 3.13: Results of longitudinal slip sweep simulation on the virtual tyre test rig showing: torque about the rotational axis of the wheel; angular velocity of the wheel; tyre model friction force and friction force estimate; and the difference between model and the estimate.

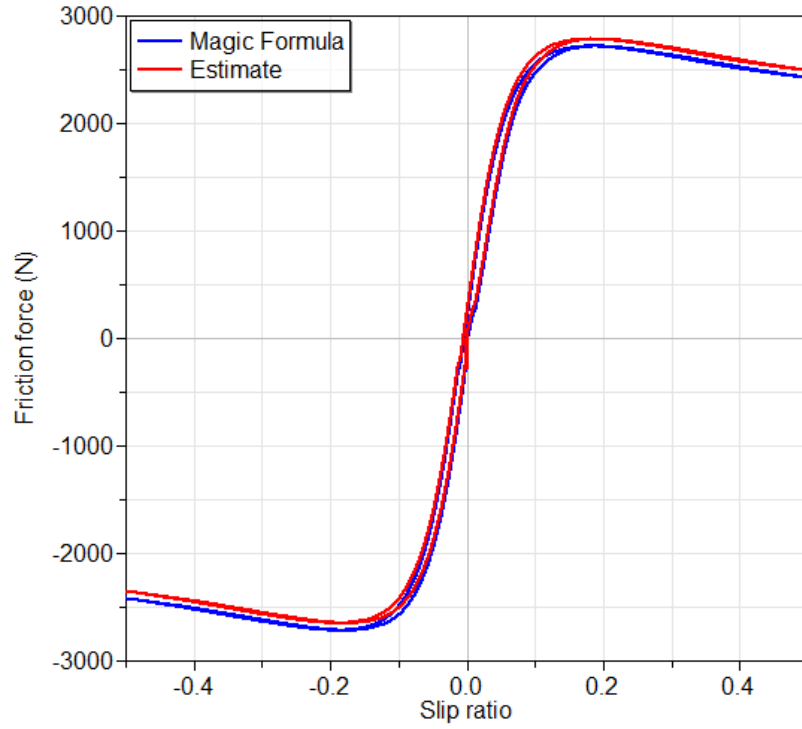


Figure 3.14: Tyre longitudinal friction force and friction force estimate against slip ratio using parameters from Table 3.3.

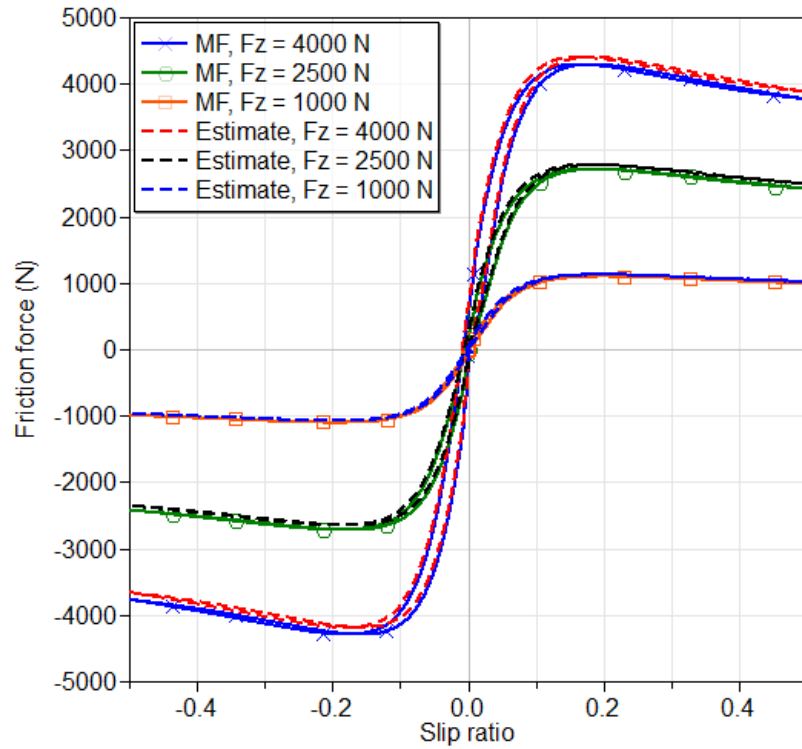


Figure 3.15: Simulation comparison of friction force observer and magic formula tyre model for a vertical force $F_z = 1000, 2500, 4000$ N.

in the MTTE controller.

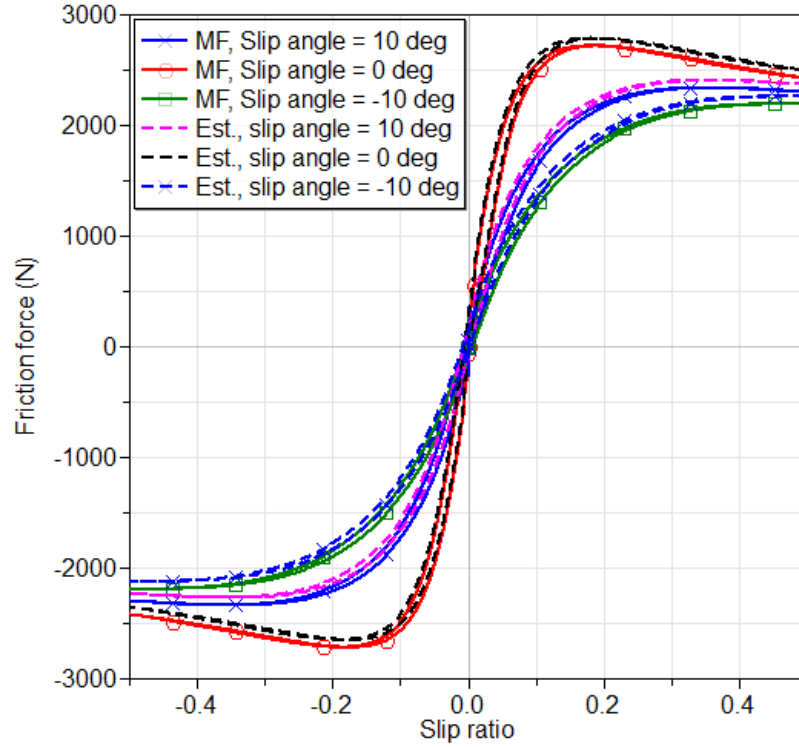


Figure 3.16: Simulation comparison of friction force observer and magic formula tyre model for a side slip angle $\alpha = -10, 0, 10$ degrees.

Figure 3.17 shows the friction force and estimate against slip ratio for a vehicle (rig) velocity of 30 m/s. At this velocity the hysteresis becomes negligible. The difference between the friction force and the estimate is still 68 N with less than 1 N variation, therefore the observer captures the velocity dependence of the hysteresis. The spike observed at a slip ratio of 0.0 is due to the simulation initialisation.

Figure 3.18 shows the friction force and estimate against slip ratio for a nominal tyre-road friction coefficient of $\mu = 0.1$. The difference remains 68 N with less than 1 N variation.

The results show that the friction force observer closely matches the Pacejka tyre model. Other than the rolling resistance force which is accounted for elsewhere in the MTTE controller, the difference between the two is less than 1 N for all simulation tests. The observer is robust to changes in vertical load, lateral slip and friction coefficient. The observer also captures the velocity dependent hysteresis exhibited by the model.

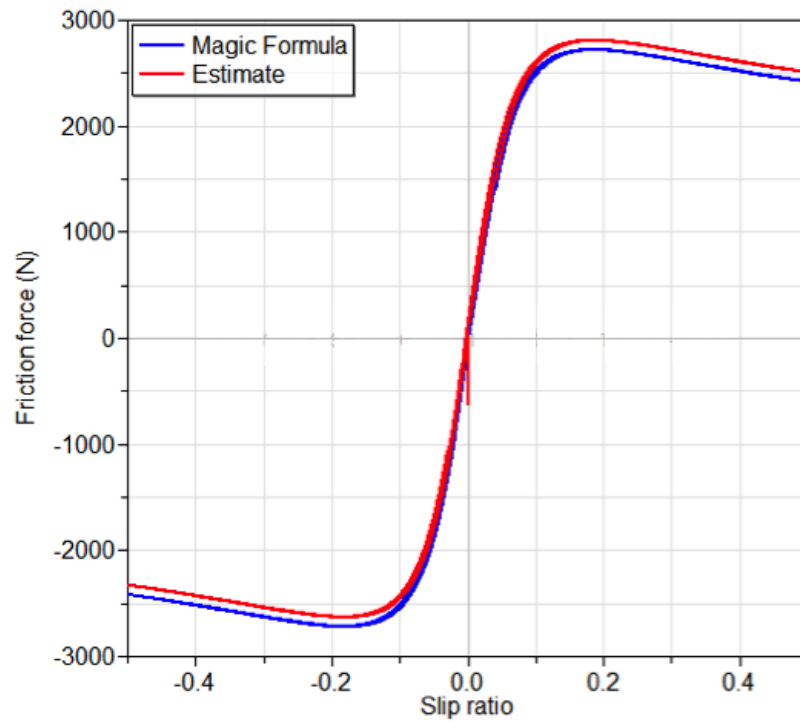


Figure 3.17: Simulation comparison of friction force observer and magic formula tyre model for a longitudinal velocity of 30 m/s.

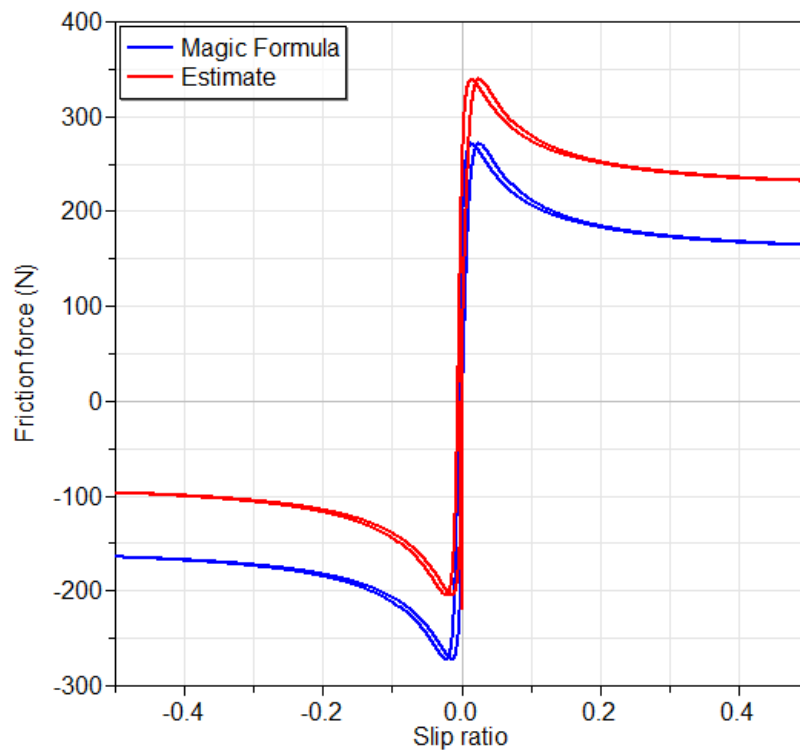


Figure 3.18: Simulation comparison of friction force observer and magic formula tyre model with a $\mu = 0.1$.

3.4 Sensitivity analysis

In this section the sensitivity of MTTE control to parameter variation is investigated. The parameters used by Yin et al. (2009) in their MTTE controller are vehicle mass, wheel radius and wheel inertia. The likely variation of each of these parameters is estimated. Where there is prior research for a similar variation the literature is drawn upon to review the controller's robustness. Where there are gaps in the literature simulation tests are carried out. As in previous sections the vehicle under consideration is the Delta E4 coupe.

3.4.1 Variability of parameters

The vehicle parameters can be classed by how accurately they are known, what their expected variation is and over what time period this variation occurs.

In the context of the MTTE controller the vehicle mass, M , refers to the total mass of the vehicle including its occupants. The kerb mass of the Delta E4 is 1005.5 kg. The additional weight due to the driver, 1-3 passenger(s) and luggage could change with every journey. This is estimated to be between 50 kg and 400 kg. For the E4 this represents a possible increase of 5 - 40 % of the nominal mass.

Yin and Hori (2010) experimentally test MTTE control where the value of vehicle mass used in the controller is as much as 33 % less than the real vehicle mass. Their results show that the peak wheel slip is unchanged by vehicle mass variation. From the single set of results that are presented MTTE appears to be robust to vehicle mass variation. Without multiple sets of results it is not easy to distinguish between a change in the parameter value and the experimental variability.

The nominal radius of the E4 coupe tyres is 308 mm. For a static corner load of 251 kg and assuming a vertical tyre stiffness of 200 N/mm the loaded radius will be 296 mm. The loaded tyre radius will depend on the tread depth, vertical load and pressure. A new

tyre has typically 8 mm of tread and the minimum legal tread depth is 1.6 mm according to UK law. A reduction of 6.4 mm in tread depth gives a reduction of 2.2 % from the loaded wheel radius. The vertical load on the tyre will change dynamically depending on the acceleration of the vehicle. For the worst case a dynamic manoeuvre could cause the vertical load to be completely removed i.e. wheel lift off. In that case the wheel radius would be equal to its nominal radius. This gives an increase of 4.2 % from the loaded wheel radius.

Wheel radius variability does not appear to have been investigated in the literature. From inspection of the MTTE controller diagram in Figure 3.1 it can be seen that a change in wheel radius will have the same effect as a change in vehicle mass, albeit with a squared dependency. The percentage variation in tyre radius is an order of magnitude smaller than the variation in vehicle mass; therefore if MTTE is robust to changes in vehicle mass it will also be robust to changes in wheel radius.

Wheel inertia is used to estimate the friction force, Equation (3.3) and to estimate the maximum transmissible torque, Equation (3.6). Yin et al. (2009) consider a vehicle driven by small in-wheel electric machines; therefore it is valid to approximate driveline inertia to wheel inertia. For a more conventional vehicle the other driveline components make up a significant proportion of the driveline inertia. The Delta E4 has two in-board electric machines that directly drive the wheels via a driveshaft. The E4's driveline consists of the electric machine rotor, the drive shaft and the wheel.

The driveline inertia is measured indirectly to be 1.04 kg.m^2 , see Chapter 4. The inertia of the tyre will vary with vertical load due to the variation in wheel radius. For an unloaded and unworn tyre of mass 9 kg and radius 308 mm, assuming its mass is concentrated at its outer radius (thin walled cylinder geometry) the inertia will be 0.85 kg.m^2 . With the tyre loaded and worn (radius 290 mm) the inertia could be as low as 0.75 kg.m^2 . This is 9.6 % less than the total driveline inertia.

Hu et al. (2011) investigate changes in wheel inertia estimate of -40 % through experimental tests. This wheel inertia variation is far greater than the estimate above either

because Hu et al. (2011) do not consider the inertia of the other driveline components or because they consider the extreme case of a flat tyre.

MTTE control uses the wheel inertia, or more correctly the driveline inertia, to estimate maximum transmissible torque and to estimate the friction force. The conclusion of Hu et al. (2011) that MTTE is sensitive to wheel inertia variation does not appear to be justified by their results. A reduction of -20 % has no noticeable effect and a reduction of -40 % gives an increase in wheel speed of ≤ 1.5 m/s.

3.4.2 Wheel inertia sweep simulations

As the experiments conducted by Yin and Hori (2010) use a low- μ surface that is not long enough for the controller to reach steady-state, simulations are carried out to determine the effect on MTTE control to a variation in wheel inertia and a variation in the inertia estimate used by the controller.

The simulation test used is similar to that used in Section 3.1.1: the MTTE controller is simulated accelerating on a high- to low- μ road using a quarter vehicle model. The simulation conditions are given in Table 3.4. The model and controller parameters are given in Table 3.5.

Condition	Value
Initial vehicle speed	5 m/s
Torque demand	300 Nm @ t=1s
Coordinates of high- μ	$0 < x < 10$ m
Coordinates of low- μ	$10 \text{ m} < x$

Table 3.4: High- to low- μ road simulation parameters

A ± 10 % error in the wheel inertia estimate used by the controller is simulated while the actual wheel inertia is held constant. A ± 10 % error is chosen as it is similar to the expected variation of -9.6 %. The results are presented in Figure 3.19. It is observed that an increase in the estimated inertia over the actual inertia reduces the quasi-steady state wheel speed and increases the quasi-steady state tyre friction force. The transient overshoot in the friction force estimate also increases. The increase in the

Parameter	Symbol	Value	Unit
Vehicle mass	M	1005	kg
Wheel radius	r	0.3	m
Actual wheel inertia	J_w	1.0	kg.m ²
Estimated wheel inertia	J_c	1.0	kg.m ²
Relaxation factor	α	0.9	
Time constant (speed and torque filters)	τ_1, τ_2	30	ms
Time constant (system delay)	τ	20	ms

Table 3.5: Vehicle and controller parameters for wheel inertia variation simulation

inertia estimate means that the wheel acceleration has a greater influence on the friction estimate which improves the steady-state performance of the controller at the cost of reducing stability.

A $\pm 10\%$ variation in the actual wheel inertia used in the vehicle model is simulated while the inertia estimate is held constant. An increase in wheel inertia could be caused by mud, whereas a decrease could be due to tyre wear or a decrease in pressure. The results are presented in Figure 3.20. The results show that variation in wheel inertia only has a transient effect, primarily on the friction estimate.

Based on these simulations the expected variation in wheel inertia will not have a significant influence on an MTTE controller. It is been found that to improve performance it is better to over estimate the wheel inertia value used by the controller although there is a trade off with stability.

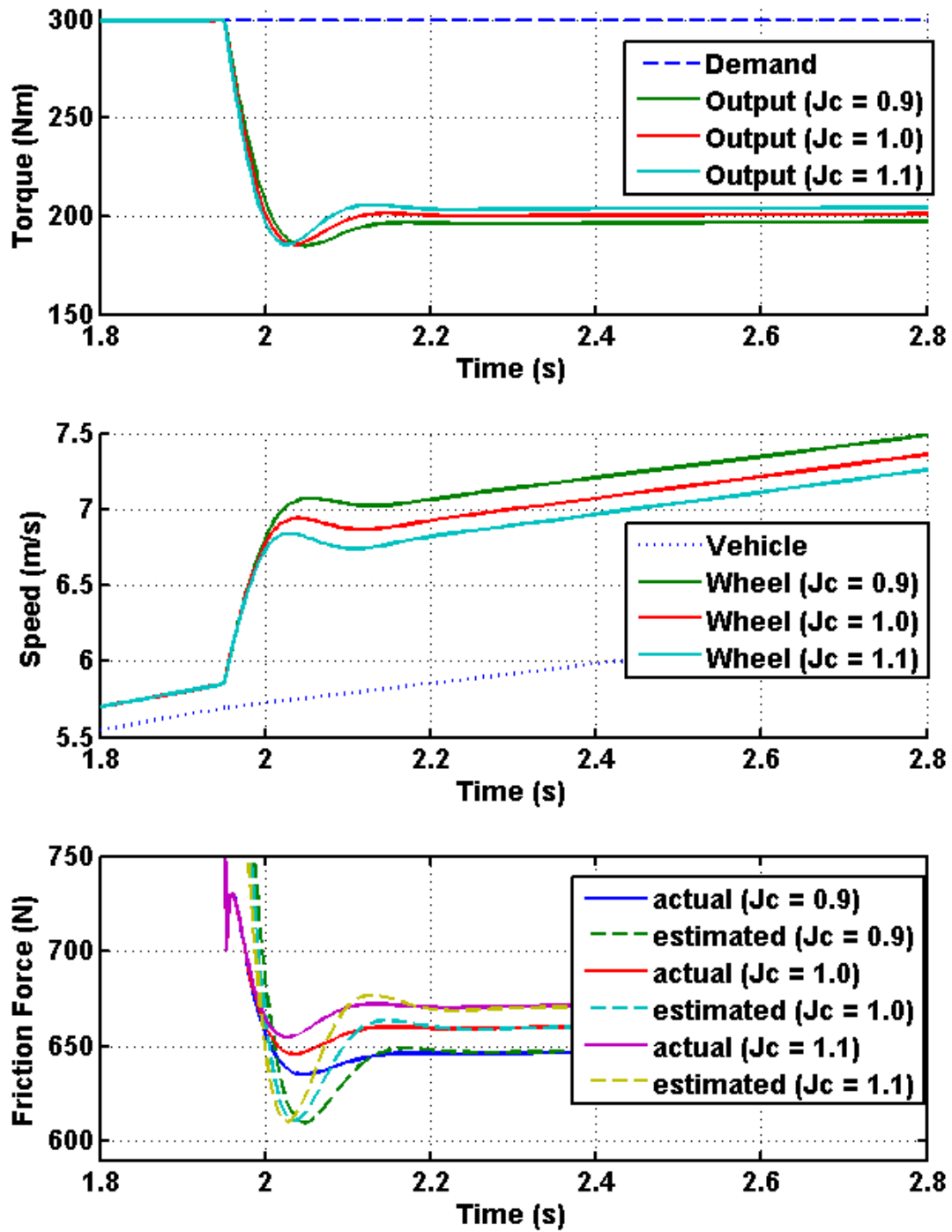


Figure 3.19: High- to low- μ simulation results for different values of controller inertia estimate, with a fixed wheel inertia of $J_w = 1.0 \text{ kg.m}^2$.

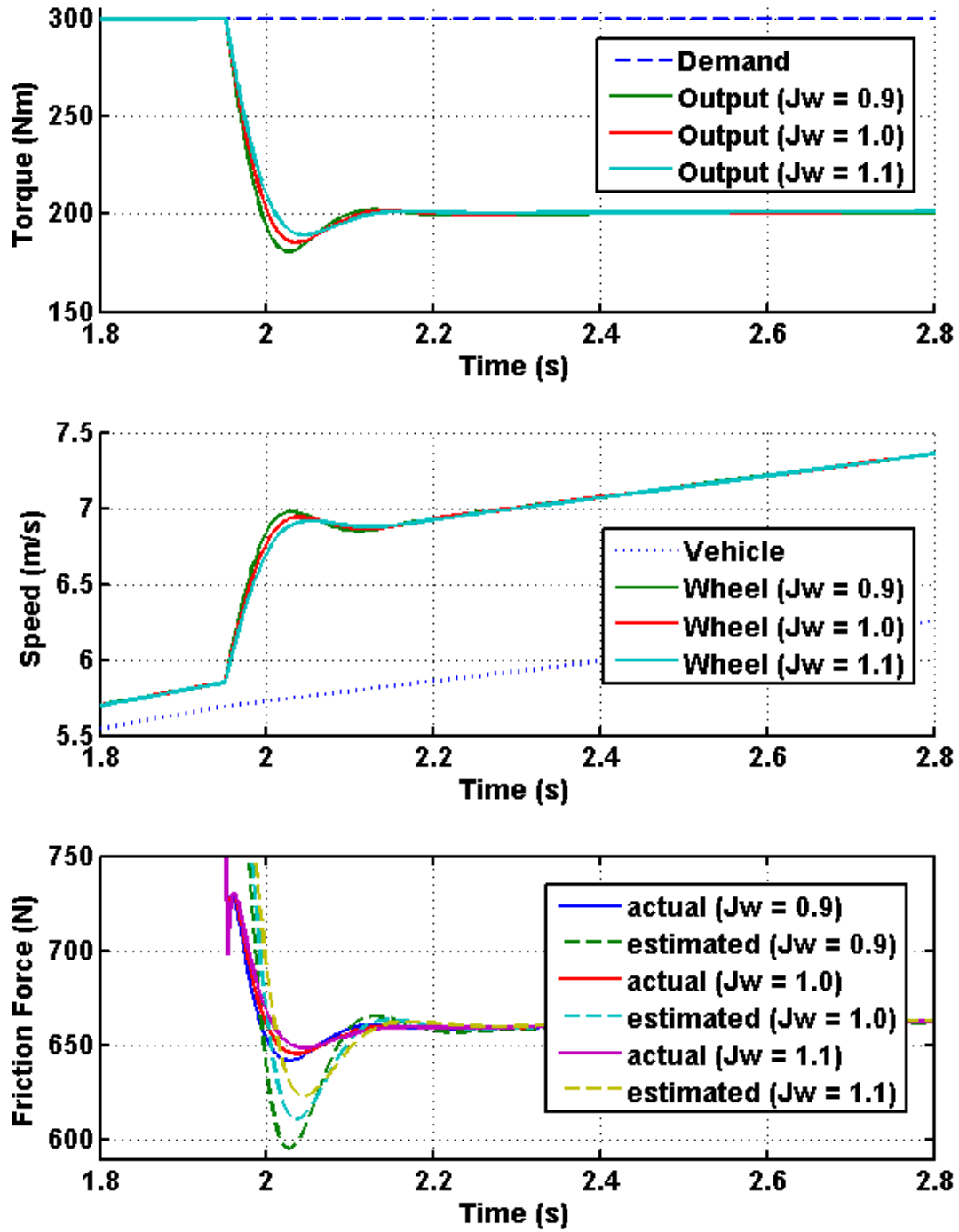


Figure 3.20: High- to low- μ simulation results for different values of wheel inertia, with a fixed controller inertia estimate of $J_c = 1.0 \text{ kg.m}^2$.

3.4.3 Rolling resistance simulations

According to Equation (3.6), the maximum transmissible torque T_{max} will be reduced by the rolling resistance force. If the MTTE controller does not measure this force then it will overestimate T_{max} . The aerodynamic drag force will affect the controller in the same way but it is likely to be less significant at low speed (< 50 mph), Gillespie (1992).

Previous work has only considered aerodynamic drag force acting on the vehicle without being accounted for in the controller. An experiment is conducted by Yin and Hori (2010) for MTTE control of a vehicle with an additional aerodynamic drag of 230 N, which they note as equal to a BMW 8-series at 86 km/h. Simulations are carried out by Yin (2009) for an aerodynamic drag force of 150 N and 300 N in the vehicle model. It is difficult to draw conclusions from their results, as varying the road loads on the vehicle (or vehicle model) produces different operating conditions. A better approach would be to vary the road loads estimated by the controller while keeping the actual road load constant between tests.

To analyse the benefit of including the rolling resistance in the controller equations simulations are carried out for a vehicle accelerating from a high- to low- μ road. The vehicle model has a fixed rolling resistance of 100 N. The rolling resistance estimated by the controller varies between tests to be 0, 100 and 200 N. The results from these three simulations are shown in Figure 3.21.

The results show that not estimating the rolling resistance ($F_{rr} = 0$ N) means that T_{max} is overestimated and the wheel accelerates more after entering the low- μ road at 2.8 s, Figure 3.21c. This is a relatively minor effect as after 1 s on the low- μ road the wheel velocity is only 0.2 m/s higher than the simulation where the controller estimates the rolling resistance ($F_{rr} = 100$ N). What is more significant, is the simulation that overestimates rolling resistance ($F_{rr} = 200$ N) as the torque output in Figure 3.21b can be seen to decrease before the vehicle reaches the low- μ road. T_{max} is under-evaluated to the extent that it is continually decreasing the torque output under normal operating conditions which is undesirable.

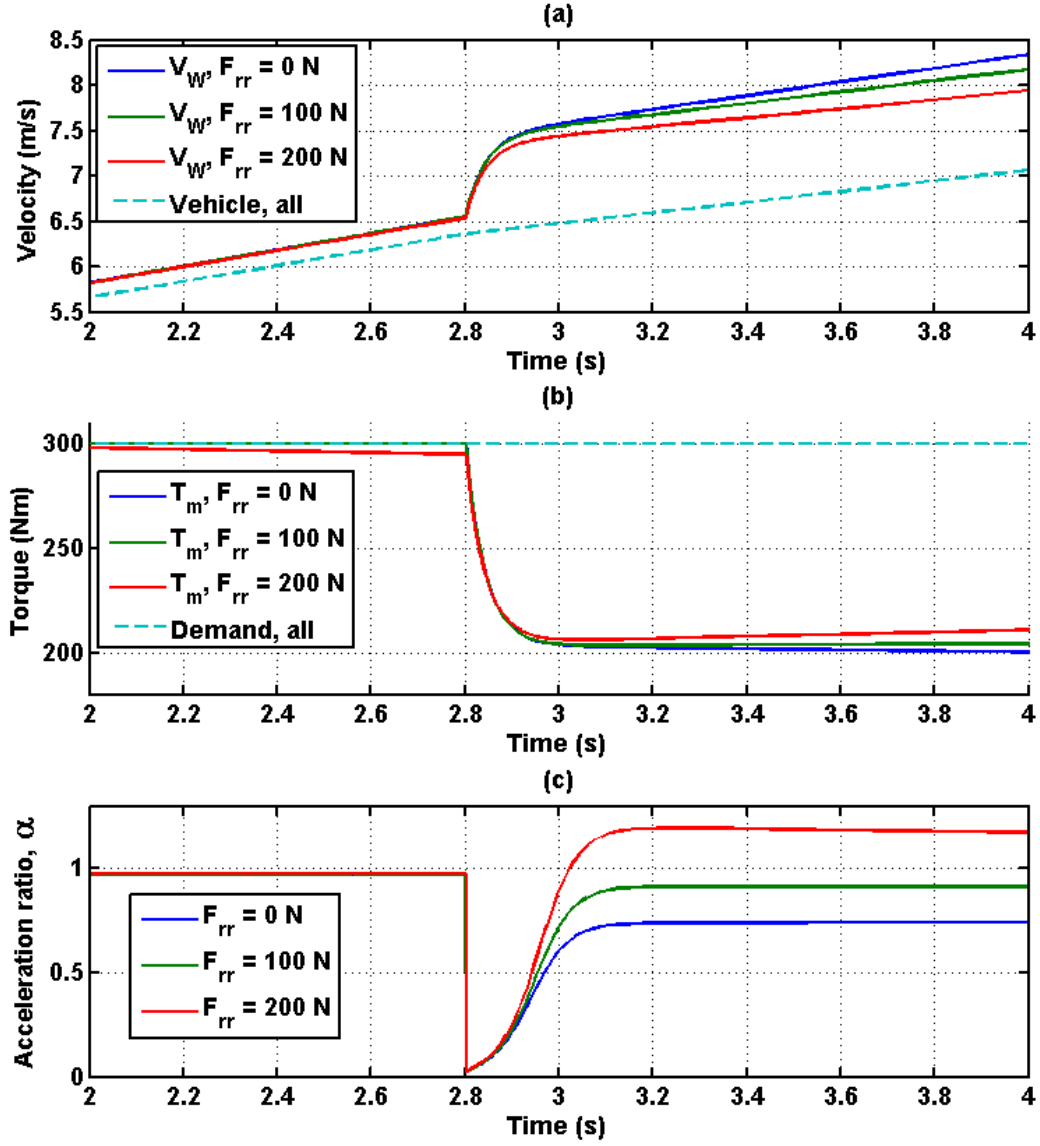


Figure 3.21: high- low- μ simulation results for a vehicle model rolling resistance $F_{rr}=100$ N, and controller rolling resistance estimate as indicated. Shows (a) vehicle and wheel (V_w) velocities, (b) torque demand to and torque output from the controller, and (c) acceleration ratio between vehicle and wheel.

3.5 Summary

A derivation of the MTTE control equations is presented that follows the work of Yin et al. (2009) but also includes the rolling resistance force which is omitted. This force is higher than aerodynamic drag at low speeds and so warrants inclusion first.

The Author's simulation results of MTTE control on a quarter vehicle model accelerating on high- to low- μ road shows the simulations by Yin et al. (2009) are in general repeatable. However the response varies somewhat due to the choice of tyre model. The Author's simulation results do not find system stability is compromised by the filter time constant of 30 ms, as Yin et al. (2009) do. This is backed up by analysis of the stability criterion. The effectiveness of control is dependent on filter time constants which in turn are dependent on system delay, so it is important to minimise this where possible.

The torque rate compensation proposed by Yin et al. (2009) is found to under-evaluate the torque as soon as the rate of change of torque demand returns to zero, and is found not to handle negative torque demands. Both of these are required for effective traction control and integration with higher level stability control. A modified rate compensation is proposed that can handle positive and negative torque demands, together with positive and negative rate of change of torque demands.

The friction force observer is found to be robust to changes in lateral and longitudinal slip, vertical load, vehicle speed and tyre/road friction coefficient, when compared with the Pacejka tyre model. It is also shown to capture the velocity dependent hysteresis exhibited by the model. The only difference between the observer and the tyre model is that the tyre model includes the rolling resistance force. This is to be expected as the rolling resistance is accounted for elsewhere in the MTTE equations. The results prove the worth of a kinematic friction observer which is not dependent on *a priori* knowledge of the friction-slip characteristics of the tyre.

From the literature it is found that MTTE control is robust to variation in the vehicle mass estimate up to 40 %. It is reported in the literature that MTTE is sensitive to wheel

inertia variation, although the experimental results in the literature do not clearly show this. This is in part due to the short duration that the vehicle spends on the low- μ surface, which is insufficient for the controller to reach steady state. Therefore this is identified as a key criterion for future experimental tests and justification to carry out simulation tests. The simulation results show that quasi-steady state response to be slightly sensitive to an error in wheel inertia estimate, but not to a variation in the actual wheel inertia. The transient response and controller stability are affected by both. Simulations that vary the rolling resistance force estimated in the controller show that MTTE controller is robust to underestimation of the rolling resistance. However, over-estimation of the rolling resistance can lead to under-evaluation of the torque which will degrade performance over time.

In this chapter single wheel MTTE control is simulated on a full size vehicle (2x2 seater, >1000 kg). The controller is adapted to handle varying positive and negative torque demands, essential for integrating with other stability controllers. From here the controller can be developed for a two wheel independent drive EV, which is discussed in Chapter 5. To verify a full vehicle controller requires a high fidelity full vehicle model, validation of which is discussed in Chapter 4.

Chapter 4

Vehicle Modelling

“Remember that all models are wrong, the practical question is how wrong do they have to be to not be useful.”

— Box, G. E. P., and Draper, N. R.,

Empirical Model Building and Response Surfaces

Verifying a traction controller through real world testing alone presents the following challenges:

- It is expensive and time consuming.
- There are certain road conditions such as very low road friction that are difficult to find under controlled conditions.
- Experimental test data is often noisy and there is a limit to what variables can be measured directly, both of which can obscure the findings.

A solution is to first verify the controller through simulation. This allows many tests to be carried out quickly, at lower cost, and with little restriction on the variables that can be measured. This enables the designer to prove a concept, giving confidence that it will work in reality, before proceeding to the test track to carry out refinement. The rise in computer power along with the developments in multi-body systems (MBS) software has made this approach more attractive, Kortüm (1993).

To verify a controller through simulation necessitates the need for a vehicle model of suitable fidelity. The model needs to represent the tyres as traction control is primarily concerned with the grip between the tyres and the road and the electric machines as these actuate the control. The parts of the vehicle that have a significant effect on the tyre forces also need to be modelled such as the vehicle chassis and suspension.

To have confidence in the simulation results the vehicle model should be calibrated and validated. Calibration involves tuning the physical parameters of the model to known values. This increases the model's plausibility and makes it easier to validate. There are many ways to validate a model, including validating against experimental data from the literature or validating against vehicle models from the literature. However, first hand experimental data from the vehicle of interest is possibly the best approach.

This research uses Delta Motorsport's E4 coupe to obtain the experimental validation data. The data is recorded from a number of dynamic vehicle manoeuvres including coast down, straight line acceleration, constant radius and double lane change tests. The vehicle manoeuvres are selected to excite the longitudinal, lateral and yaw dynamics of the vehicle. The vehicle model is simulated completing the same manoeuvres. The model is validated by comparing the longitudinal, lateral and yaw time responses from the simulation with that of the experiment. It should be noted that the validation process is often iterative, in that a particular test will highlight the need for tuning a parameter, after which the simulation needs to be repeated. The aim of this chapter is to produce a sufficiently valid vehicle model to be useful for verifying a traction controller.

The chapter's structure is now be described. In Section 4.1 the requirements of the vehicle model are laid out. These define what parts of the vehicle need to be modelled and to what accuracy.

In Section 4.2 a description of the E4 coupe is given and the available manufacturer data is listed. The vehicle model and its main components are summarised. The full list of vehicle parameters are contained within the appendices.

In Section 4.3 experimental data combined with vehicle manufacturer's data is used to

calibrate some of the parameters of the vehicle model. This differs from the use of experimental data for validation as the data is used to derive physical parameters as opposed to comparing the vehicle and model responses. The chassis component masses and locations are used to estimate the chassis inertia. The driveline inertia is estimated from a wheel spin test. Finally, vehicle coast down tests are used to calibrate the rolling resistance coefficients.

In Section 4.4 the simulated vehicle model is compared to experimentation data from a variety of vehicle manoeuvres to validate the model as a whole. A combination of steady-state and transient manoeuvres are used. A constant radius test is used to determine the vehicle's understeer gradient. A coast down test and a straight line acceleration test are used to validate the longitudinal response of the model. A double lane change test is used to validate lateral and yaw rate response of the model.

4.1 Model requirements

Brooks and Tobias (1996) gives the following criteria for evaluating the suitability of a model

- Extent to which the model output describes the behaviour of interest.
- The strength of the theoretical basis of the model including the quality of input data (credibility).
- The accuracy with which the model fits the known historical data (validity).
- Time and cost to build, run and analyse the model.
- The ease with which the model and its result can be understood.
- The probability of the model containing errors.

Zendri et al. (2010) notes there is a trade off between the representativeness of a model and its complexity. Complexity can affect the last three bullet points above by increasing time to build the model, making the model less easy to understand and increasing the

probability of errors in building the model. From this it can be summarised that the general model requirements are that a model should be: representative, credible, valid, and not overly complex.

To develop specific model requirements the problem of interest must be analysed. The purpose of the vehicle model is to verify the traction controller for an on-road vehicle. The traction controller's aim is to maintain grip between the tyre and road for any road condition while the vehicle is accelerating. Selecting the harshest road conditions to simulate the controller will assure its performance. These road conditions will include: very low friction e.g. ice; sudden changes in condition; asymmetric conditions between left and right tyres; and combinations of the above. This model is restricted to flat non-deformable road surfaces as the research is focused on on-road vehicles.

As the primary forces acting on the vehicle, other than aerodynamic forces, are those generated between the tyre and the road, Gillespie (1992), it is critical to model the tyre accurately. Simulation of (relatively) high acceleration on wet or icy roads will mean the tractive forces will exceed the tyre-road friction limits, therefore the tyre model needs to be non-linear. Asymmetric conditions can result in lateral and rotational motion (vehicle spinning) which means both longitudinal and lateral tyre forces need to be modelled. Sudden changes in road condition will mean the tyre model must account for transient effects.

The deformation of the pneumatic tyres gives rise to a rolling resistance force that opposes the motion of the vehicle. Although this force is generally small compared to the tractive forces, it may be significant when simulating icy roads, so it also needs to be accounted for in the model.

The tangential tyre forces are influenced by the vertical force on the tyre. The static vertical tyre forces are determined by the vehicle mass. Given that the simulations are restricted to flat roads and the focus of the research is on traction control rather than ride or comfort, the vehicle chassis can be modelled as a lumped mass. The acceleration of the vehicle gives rise to the so called D'Alembert's force that acts from the centre of

gravity of the chassis to produce a dynamic vertical force on the tyres. The suspension influences these forces so it needs to be included in the model. Shim and Ghike (2007) note that wheel travel can change camber angle and spring orientation which affects tyre force generation and equivalent suspension stiffness respectively. This means the suspension geometry needs to be included in the model. To model rotational motion the inertia of the chassis needs to be accounted for.

For the wheels of the vehicle model to be driven, the driveline of the vehicle needs to be modelled. This will include the electric machines and the coupling between the electric machines and the wheels. The energy usage is not of interest and so energy storage and conversion do not need to be modelled.

The communication of the torque demands and feedback signals between the traction controller and the electric machine controller will introduce some delays. Preliminary research suggests that this does not have a significant influence on the controller if refresh times are kept to 10 ms, Ewin et al. (2013), while it increases the simulation run time. Therefore it is not included in the model.

In a traction event the controller will need to respond much faster than a typical driver can, and in ways that are not normally accessible to a driver i.e. control of individually driven wheels. For this reason the driver reactions do not need to be modelled, so the driver model can be a simple open loop of the steering and acceleration inputs.

From the above assessment the vehicle model will need to include a chassis with lumped properties and 6 degrees of freedom (DOF). The Delta E4 coupe is front wheel steer so the front wheels will need 4-DOF to account for steering, spin rotation, camber rotation and vertical displacement. The rear wheels will need 3-DOF. A non-linear tyre model is required that can generate tangential forces, moments in the plane of the contact patch, and transient effects. The suspension geometry and any springs, dampers and roll bars need to be modelled, as will the driveline from the wheel to the electric machine.

4.2 Model description

This section starts with a description of Delta’s E4 coupe that is used as the experimental test vehicle. The data provided by the manufacturer is listed. Next the Dymola vehicle model is summarised along with the Pacejka Tyre model.

4.2.1 Test vehicle: Delta E4 coupe

The Delta E4 coupe is an electric vehicle with two independently driven rear wheels. Each rear wheel is driven by an in-board mounted YASA750 electric machine via Constant Velocity (CV) joints and a half shaft. The chassis is a light-weight carbon composite. The E4 is powered by a 31 kWh LiFePO₄ battery, mounted in the floor of the vehicle. The E4 has double wishbone suspension at the front and rear, with a spring-damper at each corner and an anti-roll bar at the front. A Controlled Area Network (CAN) bus is used to communicate between the Powertrain Control Module (PCM) and the Sevcon Gen4 Size 8 inverters. CAN messages are transmitted and received at every 10 ms, with a baud rate of 500 kbit/s. An image of the E4 is shown in Figure 4.1. The vehicle parameters are summarised in Table 4.1. In Appendix A the vehicle component masses and locations are listed in Table A.1 and the suspension geometry coordinates are given in Table A.2.



Figure 4.1: Photo of Delta Motorsport’s E4 coupe.

Parameter	Value	Units
Dimensions		
Wheel base	2500	mm
Track width	1374	mm
Frontal area	1.68	m ²
Aerodynamic drag coefficient	0.3	m ²
Suspension		
Front springs rate	39.5	N/mm
Rear springs rate	48.3	N/mm
Front dampers coefficient	3000	N.s/m
Rear dampers coefficient	3000	N.s/m
Front anti roll bar stiffness	777	N.m/rad
Steering		
Rack to pinion travel	8.0	mm/rad
Overall steering ratio	0.062	
Powertrain		
Drive	2x YASA750	
Peak torque (per machine)	685	Nm
Peak power (per machine)	82	kW
Max speed	2200	rpm
Battery		
Chemistry	LiFePO ₄	
Capacity	31	kWh
Voltage (nominal)	317	V

Table 4.1: Delta E4 coupe vehicle parameters.

4.2.2 Dymola vehicle model

The vehicle model is constructed in Dymola using Standard Modelica and Vehicle Dynamics libraries. Dymola is based on Modelica, which is a physical modelling language. Dymola models are constructed from blocks and subsystems much like Simulink although connections represent physical links (mechanical, electrical, etc) rather than simple input/output signals. The use of Dymola's library blocks and templates reduces the time to build a model and its simulation interface makes it straightforward to analyse results, meaning that the model requirement to reduce complexity is satisfied.

The top level of the model defines the vehicle, driver and road and is extended from the DriverVehicle template in the Vehicle Dynamics library. The sign convention used in the model is positive forward/left/upwards, which follows ISO 8855 standard. The origin of the vehicle model is the point equidistant between the front wheel centres. An open loop model is used for the driver which simply passes the acceleration and steering inputs to the vehicle subsystem. In the road subsystem the road geometry and nominal road friction are defined.

The vehicle model is extended from the Car template in the Vehicle Dynamics Library. The two main subsystems of the vehicle model are the chassis which includes the suspension and tyres and the powertrain which includes the electric machines. A diagram of the model hierarchy is shown in Figure 4.2.

In the chassis model the sprung mass of the vehicle is modelled as a lumped mass with inertia properties. This also includes separate lumped masses for the driver and passenger.

For the front and rear suspension the steerable independent suspension (IndependentSTT) and independent suspension (IndependentTT) templates are used respectively. The A-frames, king pin and wheel hub at each corner are modelled with DoubleWishboneTTE2 block. This uses the E4 suspension geometry data from Table A.2, and models the members rigidly with spherical joints and mass. The springs and dampers are modelled

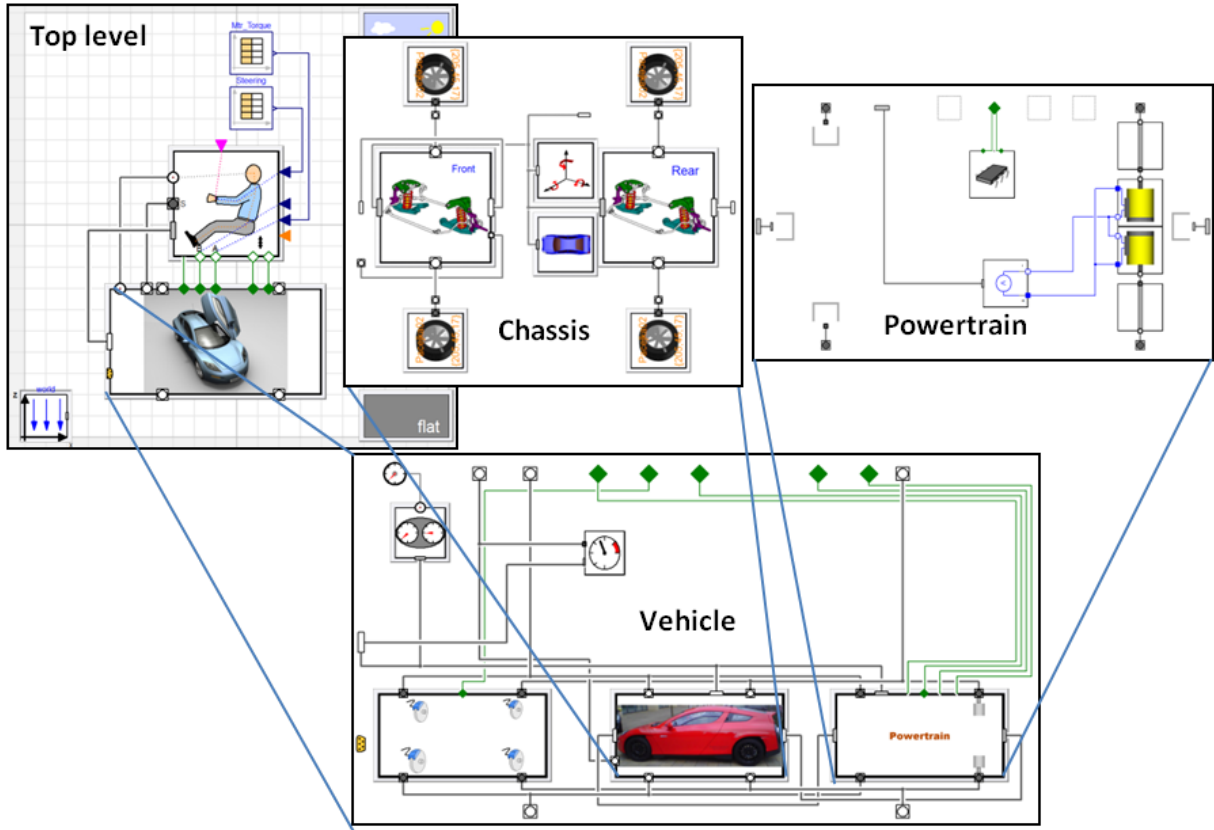


Figure 4.2: Dymola vehicle model diagram

with linear properties and one degree of freedom. Likewise the front anti-roll bar geometry and mass are modelled and the stiffness is represented using a linear rotational spring. The stiffness and damping coefficients are taken from Table 4.1. The tyre model is described later in Subsection 4.2.3.

The electric machines in the powertrain are modelled using the GenericDC model. The electric machines are considered ideal in terms of efficiency as the electrical losses are not of interest and the mechanical losses are lumped with the rolling resistance losses measured in Subsection 4.3.3. The electric machine models use the power and torque limits from Table 4.1. The battery is modelled as a constant voltage source as the electrical properties of the vehicle are not of interest and this is the simplest model that allows the vehicle model to be simulated.

The vehicle model has 135 continuous time states. For more detailed statistics on the composition of model see Appendix B. The main non-linearities in the model are the tyre-road forces which are described in the next section.

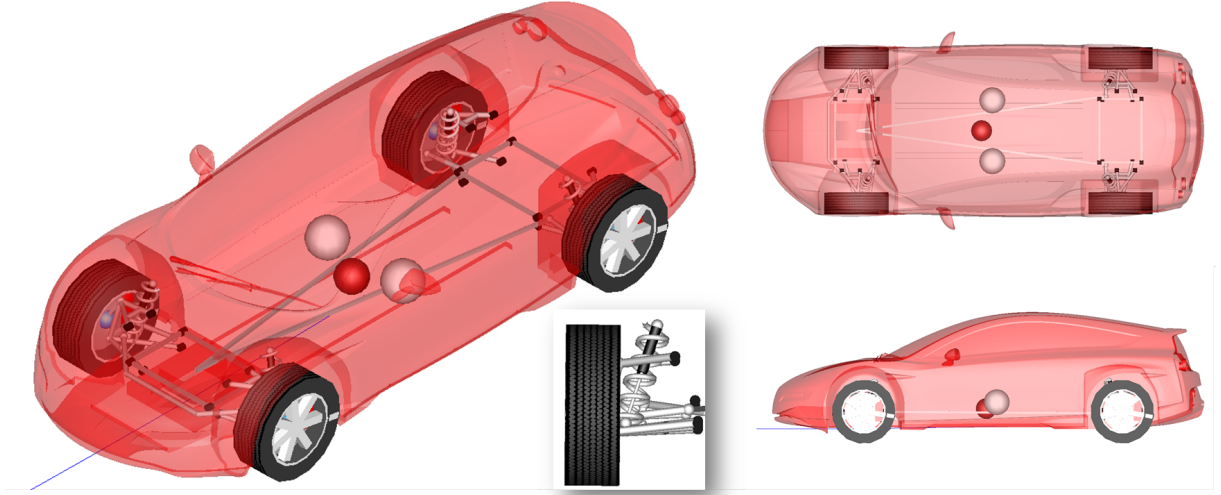


Figure 4.3: Dymola vehicle model visualisation.

4.2.3 Pacejka tyre model

The tyre model is based on the Magic Formula from chapter 4 of Pacejka (2006). This model is widely used in academia and industry for vehicle simulations, Lutz et al. (2007). The general form of the equation relating force to slip is

$$y = D \sin[C \arctan(Bx - E[Bx - \arctan(Bx)])], \quad (4.1)$$

with

$$Y(X) = y(x) + S_V, \quad (4.2)$$

$$x = X + S_H, \quad (4.3)$$

where

Y	longitudinal or lateral force
X	slip (ratio) or slip angle
B	stiffness factor
C	shape factor
D	peak value
E	curvature factor
S_H	horizontal shift
S_V	vertical shift

The model also includes aligning torque, combined lateral and longitudinal slip, and transient characteristics. Graphs of the lateral and longitudinal tyre forces for varying slip ratios and slip angles are shown in Figure 4.4. The complete list of tyre parameters is given in Appendix C. The tyre model is adapted from the Dymola Pacejka02 model using the mass and geometry of the E4 tyre (205/55 R17 Bridgestone Potenza RE50).

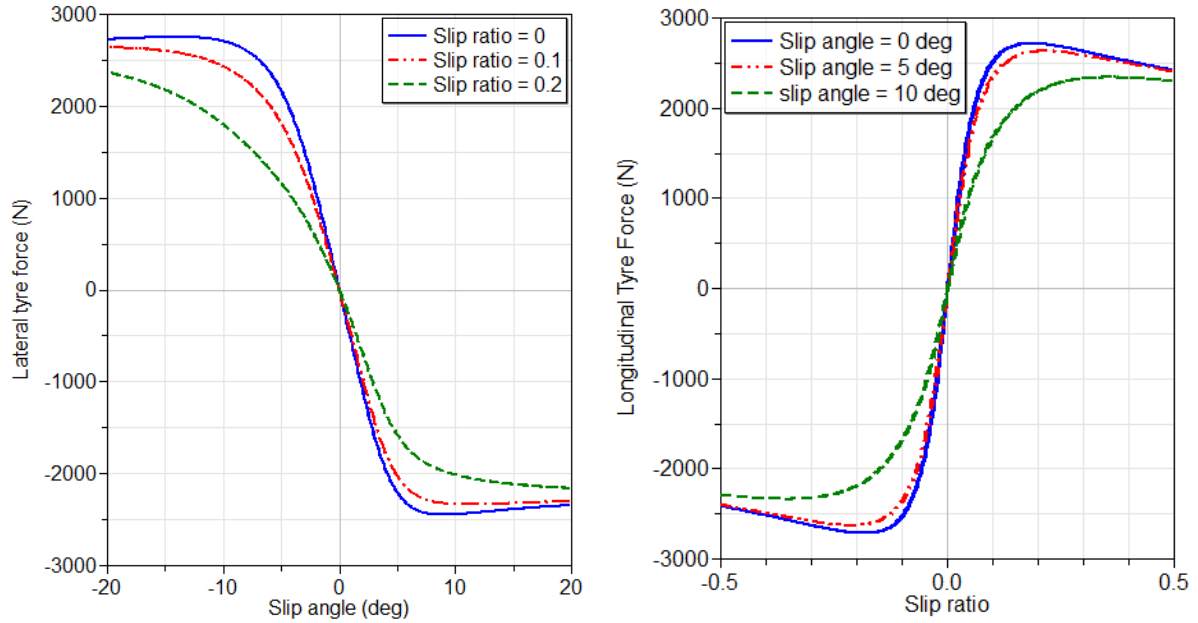


Figure 4.4: Lateral and longitudinal tyre friction forces for a vertical load of 2500 N.

4.3 Model calibration

In the previous section a multi-body vehicle dynamics model of the E4 coupe is described. This fulfils the first modelling requirement of representativeness. In this section calibration of key parameters of the model is described. This uses a combination of manufacturer data together with experimental test data. The data is used to calculate physical values within the model, to satisfy the second modelling requirement of credibility.

The calibration of the following model parameters is described in this section: chassis mass, chassis inertia, chassis centre of gravity (CoG), driveline inertia, rolling resistance coefficients, and drag coefficient. These parameters are calibrated as they directly affect the forces that act on the vehicle and tyres. This is somewhat influenced by the availability

of the data. Ideally the Pacejka tyre model parameters would also be calibrated to the specific tyre through independent testing. However, the cost is not justifiable for the purposes of this research. It is shown in Section 4.4 that the vehicle model as a whole is validated, therefore tyre testing is not necessary.

In the first subsection the process of calibrating the chassis model's mass, inertia, and centre of gravity are described. This uses measurements from corner scales and manufacturer data on vehicle part masses and locations to estimate these values. In the second subsection results from a wheel spin test are presented which are used to estimate the driveline inertia. The last subsection explains how the rolling resistance coefficients are derived from experimental vehicle coast down tests.

4.3.1 Chassis mass, inertia and centre of gravity

The chassis body is defined in the vehicle model as a lumped mass with inertia about its centre of gravity. As it is not possible to measure these parameters separately without disassembling the car, they are inferred from vehicle mass measurements and manufacturer data on vehicle components.

The vehicle's mass is measured using digital corner scales. This gives corner measurements of: 253.5 kg front left; 261 kg front right; 244 kg rear left; and 247 kg rear right. The total vehicle mass without the driver or passenger (kerb mass) is 1005.5 kg. The total vehicle mass with driver and passenger mass is 1163.5 kg. The corner scales measurements give a static load distribution of 51.2 % to the front axle and 49.5 % to the left side.

Data from Delta on vehicle component masses and locations is given in Appendix A, Table A.1. The sum of all of the vehicle component masses is 973.8 kg. This is 31.7 kg less than the measured vehicle mass. It is assumed that this difference is due to parts on the chassis such as additional wiring harnesses that are not accounted for in the manufacturer's data. The chassis mass is calculated from Table A.1 by subtracting the masses for the unsprung components that are included elsewhere in the model and adding the offset from above, and is found to be 878.5 kg.

The centre of gravity (CoG) of the vehicle determines its static and dynamic load distribution. As the tyre's tangential friction forces are proportional to its vertical load, the CoG will have a significant effect on the primary forces acting on the vehicle. The E4's CoG position in the x- and y-axes is calculated from the corner masses and vehicle dimensions and is given in Table 4.2. The CoG in the z-axis is estimated from the chassis component masses and locations data of Table A.1, together with the vehicle dimensions. The values from Table 4.2 are used for the chassis model centre of gravity. This can be justified as the static load distribution of the vehicle model when simulated at rest is within 0.1 % of the measured value.

Parameter	Value	Units
x-axis	-1.221	m
y-axis	-0.007	m
z-axis	0.031	m

Table 4.2: Vehicle centre of gravity with respect to the origin equidistant between the front wheel centres

The vehicle's inertia about its principal axes are important as they are used in the rotational equations of motion. Using data from Delta on chassis component masses and locations in Table A.1 along with the centre of gravity from Table 4.2, the inertia of the chassis about the vehicle's principal axes is estimated¹. This uses the parallel-axis theorem and assumes point masses for each of the components. The estimated x,y,z chassis inertia is listed in Table 4.3.

Parameter	Value	Units
I_{xx}	131	kg.m ²
I_{yy}	688	kg.m ²
I_{zz}	756	kg.m ²

Table 4.3: Chassis inertia about centre of gravity.

¹An alternative is to measure the vehicle inertia, Watson (2014), however this requires specialist test equipment

4.3.2 Driveline inertia

The driveline consists of the rotor of the electric machine, the drive shaft, the brake disk, the wheel hub and the tyre. It is important to know the driveline inertia as it determines the response of the wheel to a torque input from the electric machine. It is justifiable to lump these parameters together as it is a direct drive without any gears.

The dimensions and masses of the driveline components are provided by the manufacturer, see Table 4.4. From this data an inertia range for the driveline is estimated assuming the parts are homogenous discs in the case of the rotor, drive shaft and brake disc, or somewhere between homogenous discs and thin wall cylinders for the wheel hub and tyre. This helps assess the validity of the indirect measurement of the driveline inertia described below.

Component	Radius (m)	Mass (kg)	Inertia Est. (kg.m ²)
Rotor	0.17	10	0.145
Drive shaft	0.03	3	0.001
Disc brake	0.10	5	0.025
Wheel hub	0.22	5	0.121-0.242
Tyre	0.308	9	0.427-0.854
Total Inertia			0.72-1.27

Table 4.4: Expected range of driverline inertia.

The total inertia of the driveline (J_d) can be measured by rotating it at a constant speed and known torque (T_m), then removing the torque and measuring the deceleration ($\dot{\omega}$). At constant speed the applied torque will exactly match the resistive torques. At the instance the applied torque is removed ($t = t_0$), only the resistive torques will act on the driveline therefore the inertia can be determined from

$$J_d \dot{\omega} = T_m|_{t=t_0}. \quad (4.4)$$

The driveline inertia of the E4 coupe is measured by jacking the rear driven wheels off the ground. A torque is applied to the the driveline using the electric machine. This

is used to accelerate the driveline and then hold it at a constant angular speed. The applied torque is then removed. The torque and speed are measured from the electric machine at 100 Hz using a Vector VN8900 data logger. See below for details of the torque measurement.

The results of a single test are shown in Figure 4.5. A 2nd order polynomial is fitted to the coast down section of the rotor speed. The differential of this curve gives the driveline acceleration at the instance the torque is removed. From three such tests the inertia is measured as 1.03, 1.03, and 1.07 kg.m². This gives an average of 1.04 kg.m².

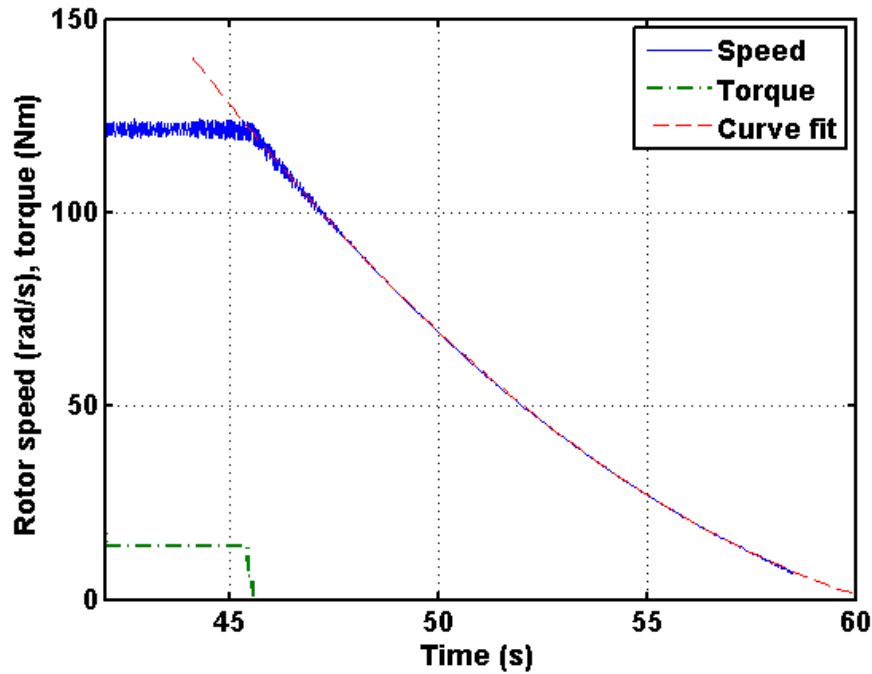


Figure 4.5: Torque and speed from driveline inertia measurement test along with the polynomial curve fit to the coast down section.

The measured value can be trusted as it is approximately in the middle of the range estimated in Table 4.4. A key advantage of this method is that the complete driveline inertia is measured in situ on the vehicle without removing any parts.

Torque measurement

The torque output of each electric machine is calculated from the following equation provided by the electric machine manufacturer Yasa Motors

$$T_m = (-0.00086685 \operatorname{sign}(I_q) I_q^2 + 2.1446 I_q) \left(1 - \frac{\Theta_R - 65}{1000} \right), \quad (4.5)$$

where

I_q q-axis current (A)
 Θ_R rotor temperature ($^{\circ}\text{C}$)

Figure 4.6 shows the relationship between torque and current for a range of rotor temperatures.

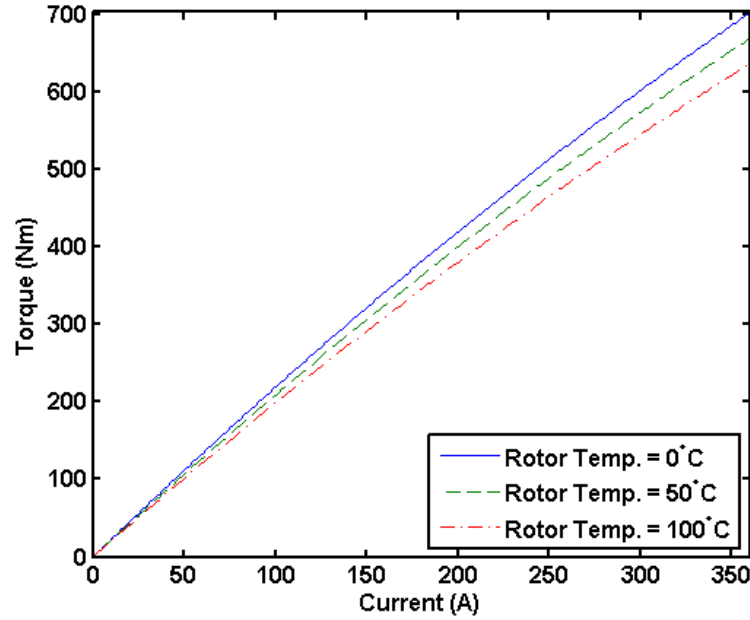


Figure 4.6: Torque against current and rotor temperature for YASA750 electric machine.

4.3.3 Resistive forces

The resistive forces that act to decelerate a car are primarily the rolling resistance of the tyres and the aerodynamic drag acting on the body. In as much as vehicle dynamics is concerned with the forces that act on the car and the motion they produce, it is important to quantify these resistive forces.

Rolling resistance force is primarily due to deformation of the tyre, and is known to be proportional to load:

$$F_{rr} = k_{rr}F_z, \quad (4.6)$$

where k_{rr} is the rolling resistance coefficient and F_z is the vertical force on the tyre. There are many factors affecting the rolling resistance coefficient including inflation pressure, temperature, material, design, velocity and tyre slip. An extensive evaluation of factors affecting rolling resistance can be found in Gillespie (1992). This work only investigates how resistive forces vary with velocity, as the other variables are assumed to remain constant for the calibration test.

The speed dependent rolling resistance coefficient is given by Pacejka (2006) as

$$k_{rr} = k_0 + k_1V + k_4V^4, \quad (4.7)$$

where V is vehicle velocity and $k_{0,1,4}$ are coefficients. Pacejka (2006) notes that the fourth order term is only significant at very high speeds close to the rated speed of the tyre, and so is ignored.

As a car moves the air in front of it will be displaced, this gives rise to pressure drag and viscous friction which result in an aerodynamic drag force acting to resist the motion of the car. The general equation for drag force on a car is

$$F_{dr} = \frac{1}{2}\rho C_D A V^2, \quad (4.8)$$

where

ρ Air density
 C_D Drag coefficient
 A Frontal area

Equations 4.6, 4.7 and 4.8 are combined to give an equation for total resistive forces

$$F_{res} = k_0F_z + k_1F_zV + k_2V^2, \quad (4.9)$$

where $k_2 = \frac{1}{2}\rho C_D A$. When a vehicle is coasting on a flat road these are the only forces acting on the vehicle. From Newton's second law

$$(M + \frac{J}{r^2})a_x = -F_{res}, \quad (4.10)$$

where M is the mass of the vehicle, J is the total inertia of all of the wheels, r is the rolling radius of the wheels and a_x is the longitudinal acceleration of the vehicle. By measuring a_x from a coast down test all of the left hand side of Equation 4.10 is known so the resistive forces can be determined.

The calibration coast down tests are carried out at the same time as the validation tests as described in Section 4.4, where the data measurement is described fully. The coast down test consists of driving the vehicle in a straight line up to an initial velocity, then placing it in neutral (zero torque) and recording the rear wheels' velocity as the vehicle decelerated. The test area size restricted the speed and duration of the tests which mean that multiple tests are carried out to capture a larger speed range.

The vehicle velocity time traces from three coast down tests used for calibration are shown in Figure 4.7a. The vehicle velocity is calculated from the average of the rear wheel velocities. As zero torque is applied this gives a good approximation of vehicle velocity. The three test results are each low pass filtered with a time constant of 0.1 s to remove noise. Through visual inspection the filtered test results are combined into one curve by shifting them along the time axis, shown in Figure 4.7b.

The acceleration is obtained analytically by fitting a polynomial curve to the velocity in Figure 4.7b and differentiating its polynomial equation. To not over fit the data it is only realistic to fit a second order polynomial. This is due to the low velocity range of the tests. The acceleration against speed for each of the tests is plotted in Figure 4.8. A first order polynomial is fitted to this curve. This gives the rolling resistance coefficients from Equation (4.9) as

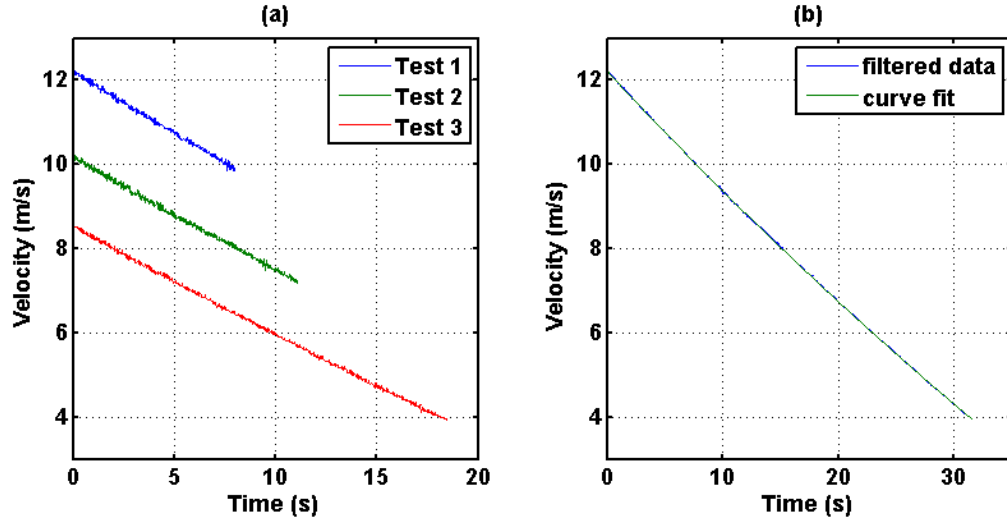


Figure 4.7: (a) Vehicle velocity for each coast down test. (b) Filtered vehicle velocity for all tests plus second order polynomial curve fit.

$$k_0 = 0.021, \quad k_1 = 0.0008, \quad (4.11)$$

which are used to calibrate the tyre model. It is not possible to distinguish between velocity dependent rolling resistance and other forces such as viscous friction in the driveline. However, what is important is that the total resistive forces of the model match the E4 coupe.

Where possible these values are compared against those found in the literature. For speed independent rolling resistance Gillespie (1992) gives a typical value for a passenger car on concrete as $k_0 = 0.015$. The measured k_0 is somewhat higher than this although given that the concrete test area is noticeably worn this is not an unreasonable value. A value for k_1 is not available in the literature.

Due to the low velocity of the test the vehicle model's drag coefficient is not calibrated experimentally. Instead data provided by Delta in Table 4.1 is used, which gives a drag coefficient of $C_D = 0.3$ and $A = 1.68 \text{ m}^2$.

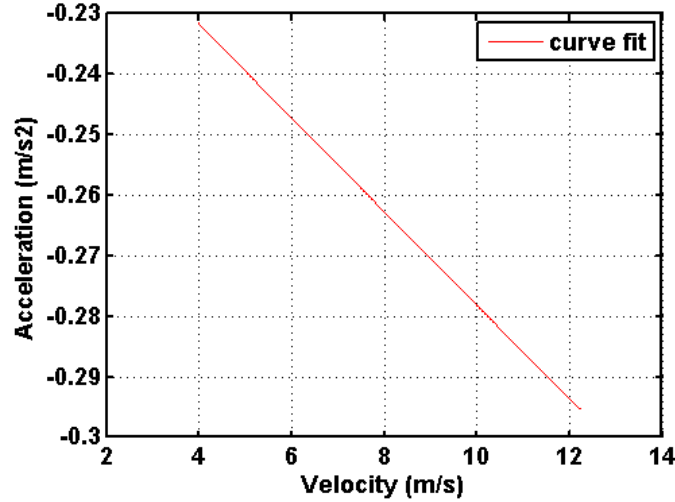


Figure 4.8: first order polynomial curve fit to acceleration against velocity data.

4.4 Model validation

In the last section some key parameters of the vehicle model are calibrated. In this section the complete vehicle model is validated against the author's own experimental data for the E4 coupe. This is the best type of data to validate a model against. This is preferable to validating against experimental data from the literature, as, without first hand experience of the experiment, interpreting the signal from the noise can be more difficult. It is also preferable to validating against another vehicle model from the literature, as this presents a layer of abstraction between the model to be validated and the experimental data used by the literature model.

The purpose of validation is to ensure that the vehicle model is an accurate representation of reality, namely that its response closely approximates to that of the E4 coupe. This allows the new control method to be verified through simulation with a higher degree of confidence that the results are reproduced on the test track.

In this section four experimental tests are used for validation. These are coast down, constant radius, straight-line acceleration, and double lane change. These tests cover longitudinal and lateral motion, together with steady-state and transient manoeuvres. The constant radius test also enables the vehicle model's understeer gradient to be tuned,

which is a key measure of the vehicle's handling. The tests are carried out at the north end of Bruntingthorpe proving ground.

For each test the experimental data is recorded and processed as described in Subsections 4.4.1 and 4.4.2 respectively. The electric machine torque and the steering wheel angle are used as inputs to the vehicle model simulation. The outputs of the simulation are then compared to the data recorded from the experiments. In the case of the constant radius test it requires an iterative process to tune the understeer gradient.

4.4.1 Experimental data measurement

This section gives a description of how data is measured for the experimental tests with the Delta E4 coupe.

The data is recorded using two devices, a Vector VN8900 and a mbed NXP LPC1768, because of the incompatible data sources. The VN8900 is used to record the steering wheel angle, break light signal, and each electric machine's current and angular velocity, via the vehicle's CAN bus at 1 kHz. An additional CH Robotics UM6 inertia measurement unit (IMU) with 3-axis gyroscope and 3-axis accelerometer is mounted on the vehicle dash. The IMU data is recorded via a serial connection to the LPC1768 with SD card breakout board at 100 Hz. The signals that are recorded are listed in Table 4.5, along with how often their values are updated. A 10 Hz GPS is also integrated with the LPC1768, but not used due to poor satellite signal.

The two data loggers are synchronised by transmitting a digital pulse from the LPC1768 to the VN8900 at the start of each recording. The 1 kHz sample rate of the VN8900 ensures the maximum offset between the two data sets is 1 ms. During post-processing the time stamps are homogenised and the data re-sampled at 100 Hz.

Signal	Source	Rate (Hz)
Wheel torque	Estimated from motor current	100
Wheel angular velocity	Rotor position sensor	100
Steering wheel angle	Steering column angle sensor	100
Brake signal	Brake lights	1000
Chassis acceleration (3-axis)	UM6 accelerometers	20
Chassis angular velocity (3-axis)	UM6 gyroscopes	20

Table 4.5: Measured signals for experimental tests along with their source and the rate at which they are updated.

4.4.2 Post-processing tasks

Due to the position and orientation of the IMU it is necessary to transform and translate the sensor's data so that the accelerations and angular velocities are expressed in the the vehicle's reference frame. The transformation and translation processes are described below.

IMU orientation compensation

The IMU is orientated so that its x axis is in the vehicle's x-z plane. Due to the angle of the dash it is not possible to physically align the sensor any more than this. To account for the offset in orientation of the gyroscopes and accelerometer sensors, the vectors of their signals are transformed to align with the vehicle axis. The vector transformation uses accelerometer data from a 5 s period at the start of each experimental test when the vehicle is stationary. The mean is calculated for each axis (A_x, A_y, A_z). From these values the roll angle offset (ϕ) is determined as

$$\tan(-\phi) = \frac{-A_y}{A_z}, \quad (4.12)$$

and the pitch angle offset (θ) is determined as

$$\tan(-\theta) = \frac{-A_x}{A_z}. \quad (4.13)$$

These offset angles are used to calculate the transformation matrix ($\mathbf{R}_2(-\phi)$) from the sensor reference frame to an intermediate reference frame

$$\mathbf{R}_2(-\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-\phi) & \sin(-\phi) \\ 0 & -\sin(-\phi) & \cos(-\phi) \end{bmatrix}, \quad (4.14)$$

the transformation matrix ($\mathbf{R}_1(-\theta)$) from the intermediate reference frame to the vehicle reference frame

$$\mathbf{R}_1(-\theta) = \begin{bmatrix} \cos(-\theta) & 0 & -\sin(-\theta) \\ 0 & 1 & 0 \\ \sin(-\theta) & 0 & \cos(-\theta) \end{bmatrix}, \quad (4.15)$$

and the gyroscope transformation matrix ($\mathbf{D}$)

$$\mathbf{D}(\phi, \theta) = \begin{bmatrix} 1 & \cos(\phi)\tan(\theta) & \sin(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{bmatrix}. \quad (4.16)$$

The accelerometer output ($\mathbf{a}_s$) for the whole experimental test is transformed to give the acceleration in the vehicle frame of reference

$$\mathbf{a}_{vf} = \mathbf{R}_1(-\theta)\mathbf{R}_2(-\phi)\mathbf{a}_s, \quad (4.17)$$

where

$$\mathbf{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}. \quad (4.18)$$

Similarly the gyroscope output ($\boldsymbol{\omega}_s$) is transformed to give vehicle angular velocity

$$\boldsymbol{\omega}_v = \mathbf{D}(\phi, \theta) * (\boldsymbol{\omega}_s - \boldsymbol{\omega}_o), \quad (4.19)$$

where

$$\boldsymbol{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \quad (4.20)$$

and $(\boldsymbol{\omega}_o)$ is the offset at the start of the experiment due to gyroscope drift.

IMU position compensation

The position of the IMU is $r_x = 0.81 \text{ m}$, $r_y = 0.00 \text{ m}$, $r_z = 0.48 \text{ m}$ from the vehicle's centre of gravity (CoG). This means that the accelerometers output will be non-zero for a pure angular velocity about the vehicle's CoG. The angular velocity measured by the gyroscope is used to compensate this by

$$\mathbf{a}_v = \mathbf{a}_{vf} + \mathbf{R}(x, y, z) * \boldsymbol{\omega}_v^2, \quad (4.21)$$

where

$$\mathbf{R}(x, y, z) = \begin{bmatrix} 0 & r_x & r_x \\ r_y & 0 & r_y \\ r_z & r_z & 0 \end{bmatrix}. \quad (4.22)$$

Using equations 4.17, 4.19, and 4.21 the accelerations and angular velocities in the vehicle reference frame are calculated for each test.

4.4.3 Coast down test

The vehicle model's rolling resistance and aerodynamic drag coefficients are calibrated earlier in the chapter. To validate that these coefficients produce the expected deceleration the model is simulated coasting (zero torque) in a straight line from an initial velocity of 12.8 m/s. Figure 4.9a shows the simulation result together with the wheel velocity from the experiment, and Figure 4.9b shows the difference between the two. The experimental data is from a separate test to those used for calibration. The simulation is found to be within the noise tolerance of the experiment. There is also no discernible trend in Figure 4.9b which suggests the difference can be ascribed to noise. Therefore the resistive forces

in the model are valid.

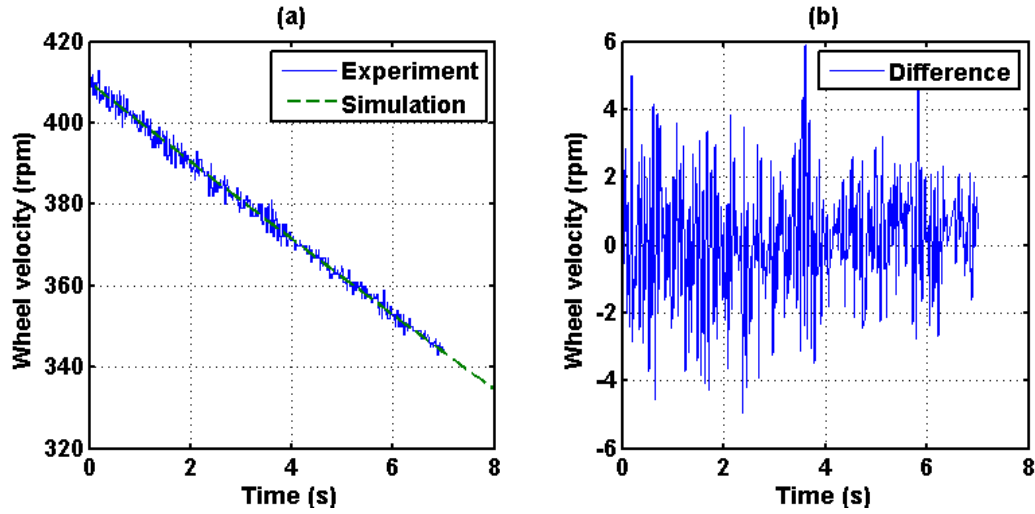


Figure 4.9: Comparison of coast down simulation and experiment showing angular velocity for the rear left wheel.

4.4.4 Constant radius test

In this section the constant radius test is used to measure the understeer gradient of the Delta E4 coupe. This allows the vehicle model’s understeer gradient to be tuned which is a key measure of a vehicle’s handling. This section starts with an explanation of understeer gradient based on Gillespie (1992). Next the constant radius experimental test is described and the results are presented. After this the raw data is processed and a curve is fit to determine the understeer gradient. Finally two tyre/road friction parameters in the vehicle model are tuned through iterative simulation steps to obtain the desired understeer gradient.

The SAE (1976) define the understeer gradient as “The quantity obtained by subtracting the Ackerman steer angle gradient from the ratio of the steering wheel angle gradient to the overall steering ratio”. More simply put, it defines the relationship between steering wheel angle and lateral acceleration. The steady state understeer gradient equation, after Gillespie (1992), is

$$\delta = (180/\pi)L/R + Ka_y, \quad (4.23)$$

where,

δ	Front wheel steer angle (deg)
L	Wheel base (m)
R	Turn radius (m)
K	Understeer gradient (deg/g)
a_y	Lateral acceleration (g)

The first term on the right hand side of the equation is the Ackermann steering angle, this is determined solely by the geometry of the turn and is independent of acceleration. To summarise the meaning of the understeer gradient from Gillespie (1992)

1. For $K = 0$ a change in lateral acceleration does not require a change in steer angle of the front wheels. This is because there is an equal amount of side slip at the front and rear tyres. The vehicle is said to be neutral steer (NS).
2. For $K > 0$ an increase in lateral acceleration will require a corresponding increase in the steer angle to maintain the radius of turn. In this case the front tyres have greater side slip than the rear tyres. The vehicle is said to be understeer (US).
3. For $K < 0$ an increase in lateral acceleration will require a reduction in the steer angle to keep the vehicle on its current radius. For this the tyres at the rear have greater side slip than at the front. The vehicle is said to be oversteer (OS).

A vehicle can vary between NS, US and OS over the full range of lateral acceleration. A neutral steer vehicle is desirable as the attitude of a vehicle in a turn will not vary with the turn radius or speed, making it easier to control. In general, passenger vehicles tend to be designed to be slightly understeer as oversteer can lead to instability at the tyre friction limit.

Experimental test

One of the methods of measuring understeer gradient experimentally is by a constant radius test. For this the vehicle is driven around a fixed radius circle at a constant speed. The steady state steering wheel angle and lateral acceleration are measured. The speed

is increased incrementally and the process is repeated. The understeer gradient can be determined from the gradient of the curve of steering wheel angle to steering ratio against lateral acceleration. The steering ratio is the gain between the angle at the steering wheel and the angle of the front tyres.

In the experiment a radius of 21 m is used due to the available test area. Given that Gillespie (1992) notes that for two-axle vehicles the understeer gradient is not affected by the radius of the turn, this radius is acceptable.

The vehicle forward velocity is calculated from the average of the left and right driven wheels. Given that this is a steady state manoeuvre and the torque applied to the driven wheels is small, the longitudinal slip is small which means the average wheel velocity will closely approximate the longitudinal vehicle velocity. Due to the noise in the accelerometer measurements the lateral acceleration is calculated from the product of the yaw velocity and vehicle velocity.

Results

A set of measurements from a single test run is shown in Figures 4.10. Observation of the raw data shows that the test driver needs to apply small steering corrections to follow the fixed radius path. As each test lasts for at least one revolution of the circular path, the average steer angle gives a good approximation to the steady state steer angle.

For each test the average steer angle (steering wheel angle/steering ratio) and lateral acceleration are calculated for the steady-state section of the test. This data is shown in Figure 4.11 with one standard deviation error bars. This does not show any discernible trend. One would expect the data to follow a linear trend at low to moderate acceleration while the tyres are in their linear region. If each test is not carried out at exactly the same path radius, which is likely to be the case, then the Ackermann steer angle (L/R) varies between each test and skews the data.

The turn radius is estimated from the forward vehicle velocity (V_x) over the yaw velocity

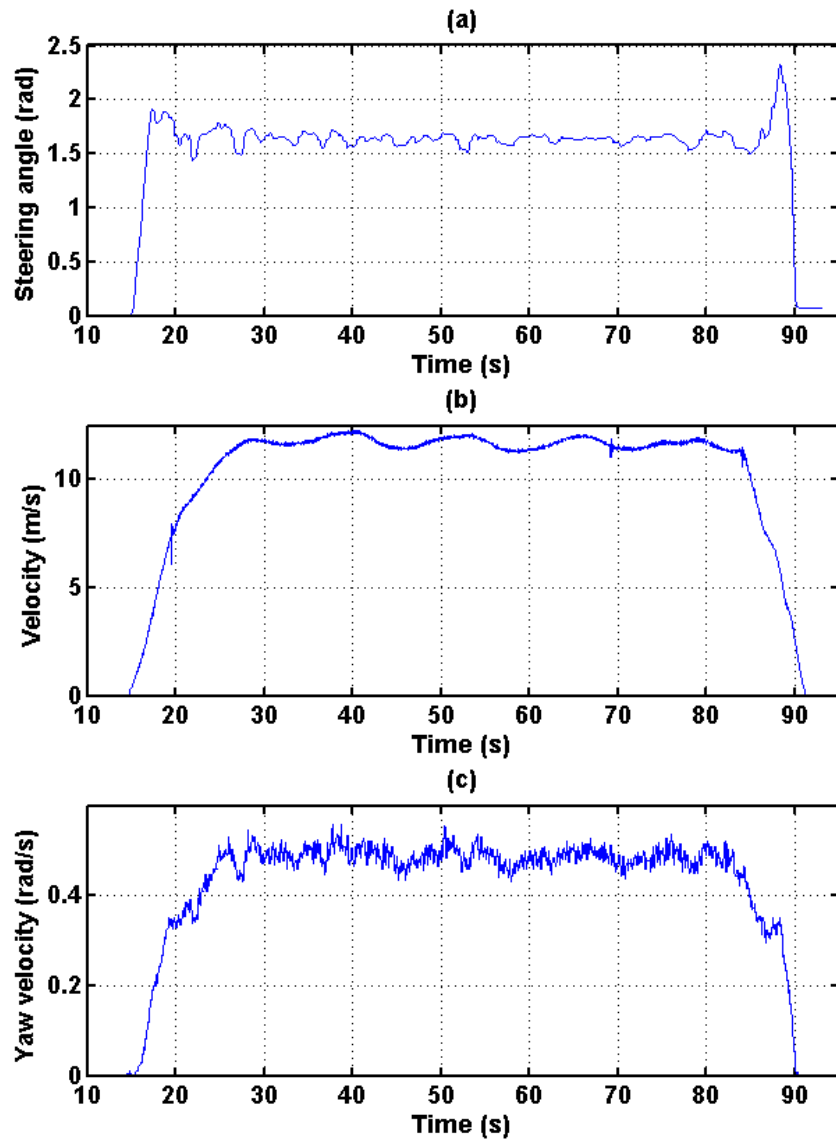


Figure 4.10: Results from a single constant radius test showing: (a) steering wheel angle (b) average rear wheel velocity which approximates longitudinal vehicle velocity (c) vehicle yaw velocity.

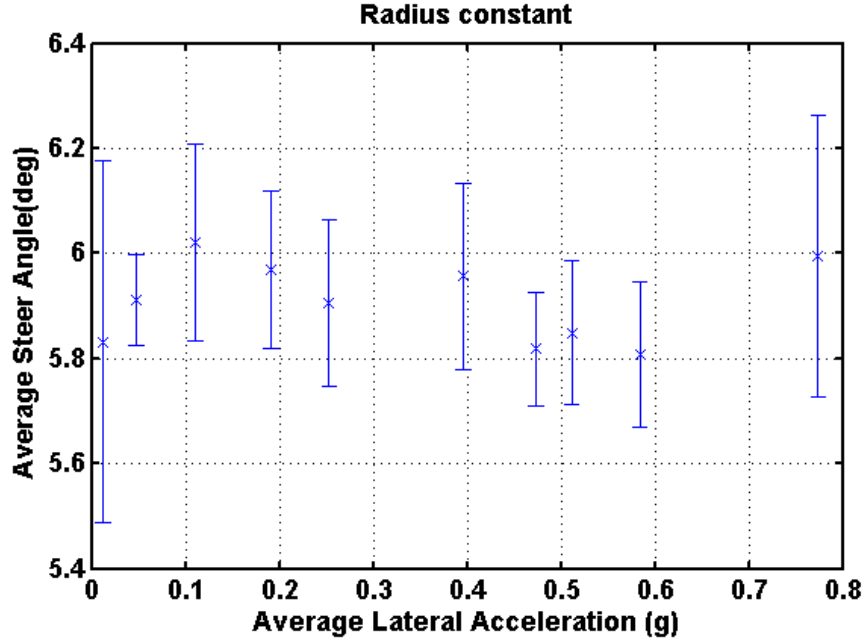


Figure 4.11: Average steer angle at the front wheels against average lateral acceleration for each constant radius test run. Error bar shows one standard deviation.

(ω_z) ,

$$R = \frac{V_x}{\omega_z}. \quad (4.24)$$

The estimated turn radii for all of the tests are shown in Figure 4.12. The turn radius increases with lateral acceleration. This is possibly because the test driver found it difficult to hold the original path at higher lateral accelerations. As the radius varied between tests so will the Ackermann angle, to account for this Equation 4.23 is rearranged to

$$(\delta - L/R) = K a_y. \quad (4.25)$$

A graph of the steer angle less the Ackermann angle against lateral acceleration is given in Figure 4.13. A cubic spline is fit to all of the data points. The type of fit is chosen as it provides a smooth function that closely fits the data. The gradient of the curve gives the understeer gradient (K) which is shown in Figure 4.14. This shows that the vehicle is close to neutral steer for low to medium lateral acceleration, but increasingly understeer as it approaches the friction limit, as is expected for an on-road passenger vehicle. The bump in the curve at 0.5 g is due to noise combined with the proximity of the measurement

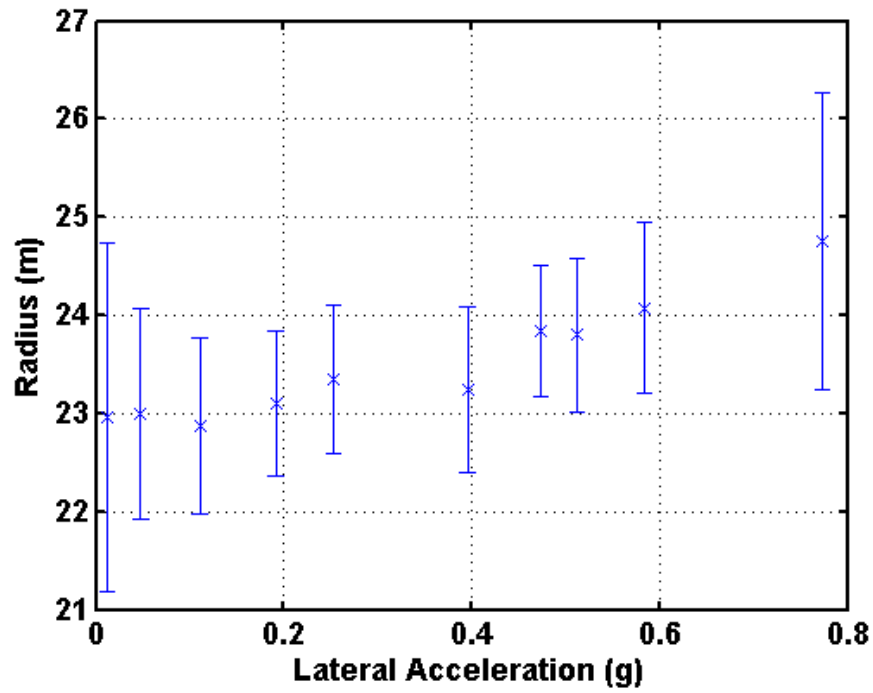


Figure 4.12: Estimation of turn radius against lateral acceleration for each constant radius test. Error bar shows one standard deviation.

points, see Figure 4.13. The increase in understeer gradient at an acceleration less than 0.2 g is a real effect.

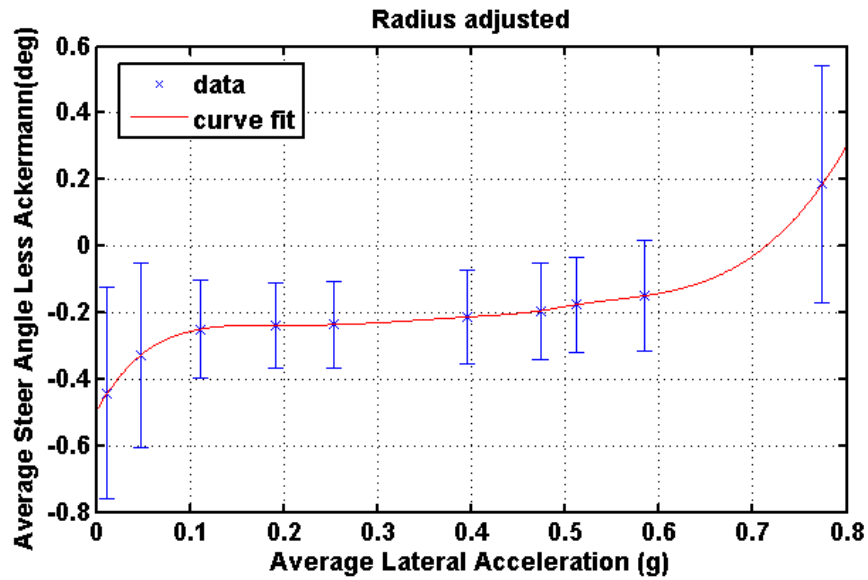


Figure 4.13: Average steer angle less the Ackermann angle against lateral acceleration with cubic spline curve fit. The error bars show one standard deviation.

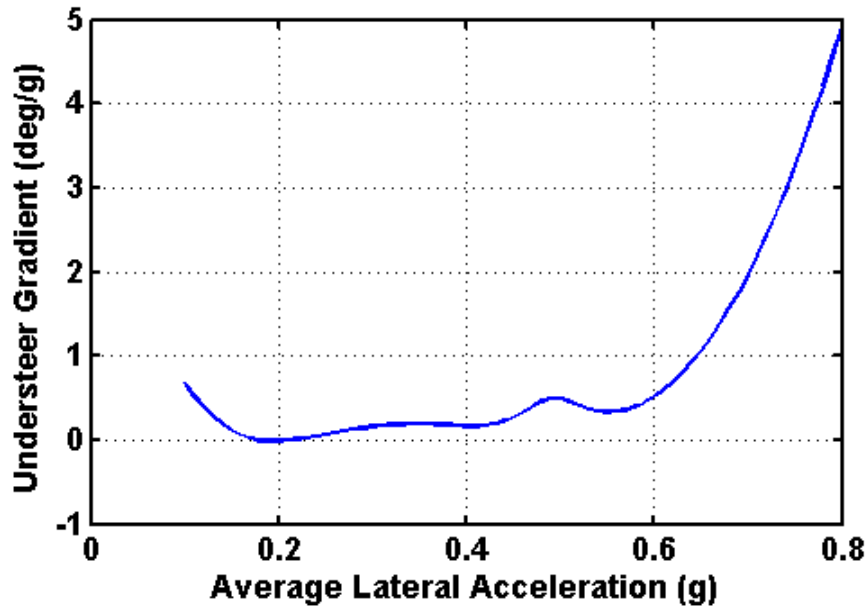


Figure 4.14: Understeer gradient (K) calculated from the curve fit of steer angle against lateral acceleration.

Simulation test

The Pacejka tyre model used for simulation matches the geometry of the actual tyres but is otherwise generic. The tyre model includes a number of user scaling parameters for the purpose of tuning the model to a specific tyre. These parameters along with the nominal tyre-road friction coefficient are used to tune the model to fit the experimental data. The results from three simulation tests are shown in Figure 4.15. A rear tyre cornering stiffness scaler of $L_{Ky} = 0.88$ and a tyre-road friction coefficient between 1.0 and 1.1 give the best fit to the data.

The understeer gradient is found to be sensitive to the tyre cornering stiffness in the linear region of tyre (low acceleration), and the tyre-road friction coefficient near the tyre saturation limit (high acceleration). The value found for the tyre-road friction coefficient is in line with the value for dry concrete from Harned and Johnston (1969). The tyre-road friction coefficient varies with road condition; therefore there is little benefit in tuning further. To calibrate the model's understeer gradient the rear tyres cornering stiffness is reduced even though the front and rear tyres are the same. This is justifiable as the rear wheels are driven and therefore have experienced greater wear.

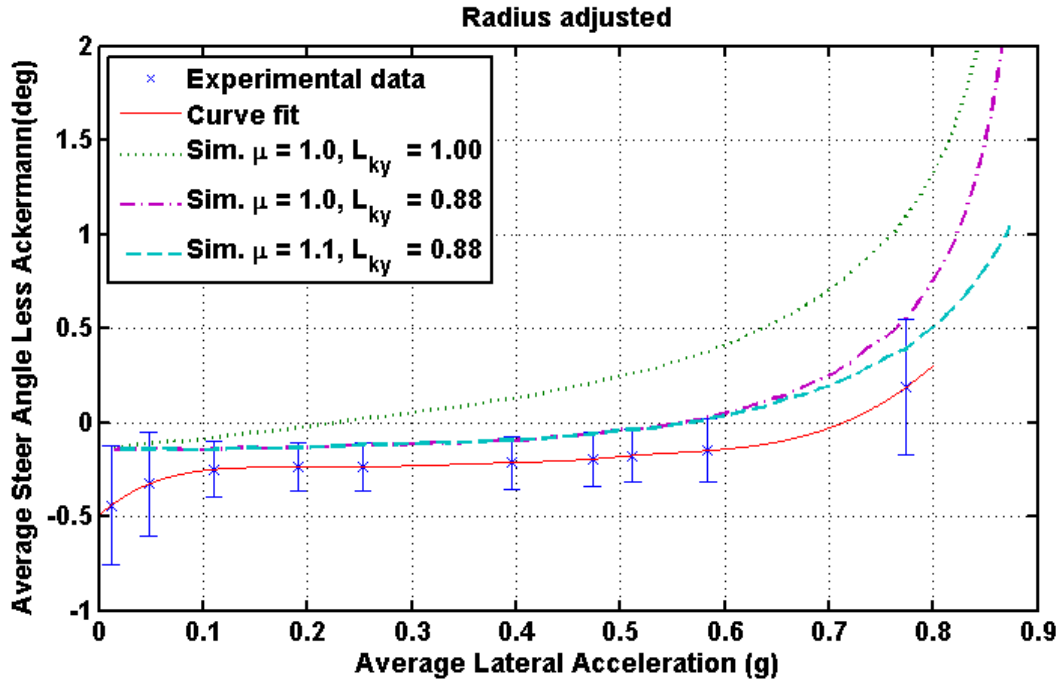


Figure 4.15: Understeer gradient comparison of vehicle model to experimental data. The vehicle model is simulated for a variation in the coefficient of tyre-road friction and for a variation in the rear wheel cornering stiffness scaling.

In Figure 4.15 there is a constant offset between the simulation and experimental results; however, it is the gradient that is important. The vehicle model captures the main vehicle characteristics of being close to neutral steer for low to mid acceleration, and increasingly understeer at high acceleration as it approaches the limit. The vehicle model does not capture increased understeer gradient below 0.1 g, although the very low speed nature of this reduces its significance.

4.4.5 Straight-line acceleration test

A traction controller's primary aim is to prevent excessive wheel slip during longitudinal acceleration. A vehicle model used to test traction control should therefore closely match the longitudinal acceleration and velocity of the real vehicle for a given torque delivered by the motors. The longitudinal acceleration response is determined experimentally by accelerating the E4 in a straight line from stationary on a flat road. The driver starts with his foot on the brake to prevent the creep torque moving the vehicle forward. At 1.0 s he releases the brake pedal and fully presses the accelerator pedal. After several

seconds the driver releases the accelerator pedal. The measured motor torque from the experiment is used as the input for the simulation of the vehicle model.

The results for both the experiment and simulation are shown in Figure 4.16. The top graph shows the measured torque from the left motor, the right motor torque is nearly identical. In the experiment the initial torque of 68 Nm correspondes to the creep torque. As the brakes are not included in the vehicle model the torque input to the simulation is zeroed while the brakes are applied. The brakes are released at 1.0 s and the throttle is applied at 1.3 s.

The middle graph of Figure 4.16 shows the longitudinal vehicle acceleration. In the period between 2.2 s and 7.3 s when the torque is constant the acceleration is expected to be flat. Instead there is significant noise of up to $\pm 1.5 \text{ m/s}^2$ about the mean value. This can be attributed to the combination of a rough concrete road surface with the stiff suspension of the vehicle. The acceleration from the simulation appears to approximate the moving average of the acceleration from the experiment.

The bottom graph of Figure 4.16 shows the angular velocity of the rear left driven wheel. In the experiment this is measured from the position sensor in the motor. The simulation shows good agreement with the experiment. Up to 7.8 s the difference is $< 1.5 \text{ rad/s}$. At 7.8 s the torque decreases resulting in an oscillation in the wheel velocity that is not captured by the vehicle model. This is possibly due to shaft compliance not accounted for in the model. By the end of the test there is a 2.4 rad/s difference between the experiment and simulation which equates to 3.4 % error.

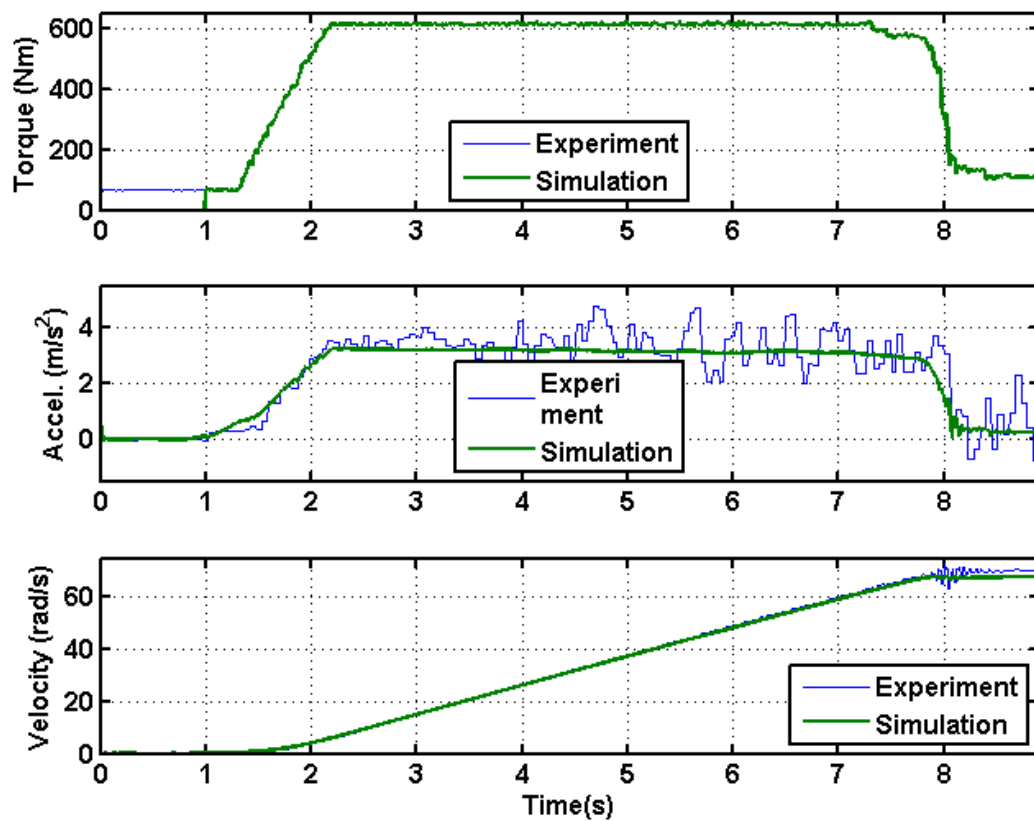


Figure 4.16: Comparison of simulation to experimental results for a longitudinal acceleration test. The graphs show the measured torque for the left motor (top), longitudinal vehicle acceleration (middle), and angular velocity of the left wheel (bottom).

4.4.6 Double lane change test

A loss of traction while a vehicle is accelerating on asymmetric road conditions can lead to the vehicle spinning. This will excite its lateral and yaw dynamics. For this reason it is necessary to validate the vehicle model's yaw velocity and lateral acceleration response against experimental data. A double lane change manoeuvre is selected for this purpose.

The test consists of the driver accelerating the vehicle in a straight line up to an initial velocity, after which the driver releases the throttle and then executes a double lane change steering manoeuvre. The measured steering wheel angle is used as an input in the simulation. The initial velocity, taken as the average wheel velocity after the throttle is released (zero torque), is used as the initial vehicle velocity in the simulation.

The results from the double lane change experiment along with the simulation are shown in Figure 4.17. Figure 4.17a shows the steering wheel angle. Figure 4.17b shows the rear wheel velocities. The wheel velocities in the simulation follow the shape and left-right separation of the velocities in the experiment, although the average deceleration is greater in the simulation. This results in a velocity error of 4.0 % by the end of the test. This is unexpected given the close agreement in wheel velocities between simulation and experiment for the coast down test in Figure 4.9.

Figure 4.17c shows the vehicle's yaw velocity. The results of the simulation closely approximate the experimental results for most of the test, although there is up to 10.0 % error at the peaks and troughs.

Figure 4.17d shows the vehicle's lateral acceleration. As in the longitudinal acceleration test the acceleration in the experiment is very noisy which can be attributed to the combination of a rough concrete road surface and the stiff suspension of the vehicle. The acceleration from the simulation appears to approximate the moving average of the acceleration from the experiment.

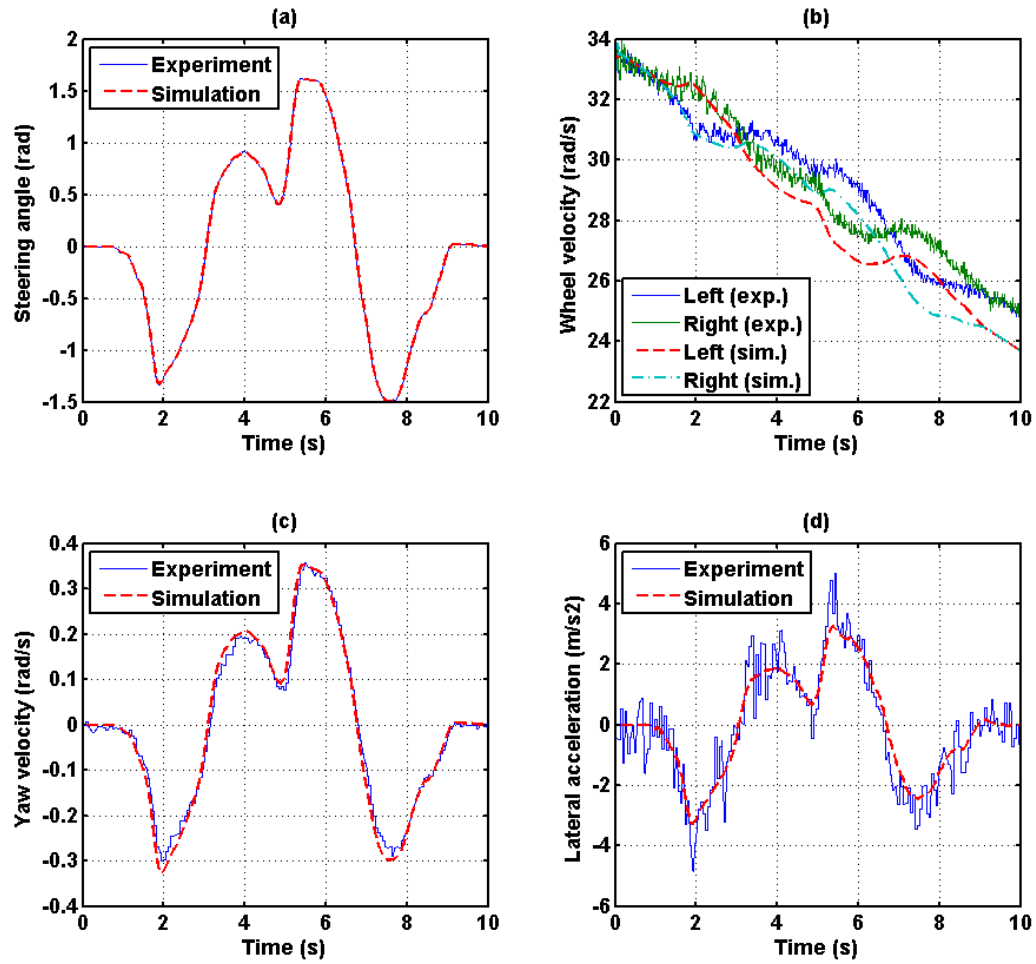


Figure 4.17: Comparison of simulation to experimental results for a double lane change test. The graphs show: (a) steering wheel angle (b) angular velocity for the rear left and rear right wheels (c) vehicle yaw velocity (d) vehicle lateral acceleration.

4.5 Summary

There are many examples in the literature of low order models, but few of high order models that are described in full and validated against experimental data. This chapter describes the construction, calibration and validation of a comprehensive vehicle dynamics model of Delta Motorsport's E4 coupe.

The approach has been to model the vehicle to a high level of detail based on the available data rather than to presuppose that only part of the dynamics are relevant. The methods used for calibration and validation do not require specialist test equipment or disassembly of the vehicle, therefore present an approach that can be widely adopted. The use of detailed vehicle manufacturer data together with the calibration of key parameters such as chassis CoG, chassis inertia and driverline inertia, have meant that few iterations of tuning model parameters are necessary during validation.

The experimental calibration of the vehicle model's resistive forces are slightly unconventional due to the shortness of the test track. This means several runs are used to cover a large enough speed range for the curve fit to be accurate, although this did have the advantage that the same section of track was used for all of the calibration and validation tests. The measured rolling resistance coefficients are similar to those found in the literature. It was not possible to measure the aerodynamic drag coefficient due to the low speed of the test, however this indicates it is not significant at these speeds. A drag coefficient provided by Delta Motorsport is used instead.

The coast down validation test shows that the deceleration of the vehicle model is within the error tolerance of the experimental data, meaning that the longitudinal forces on the model are accurate when the wheels are not driven and the vehicle is travelling in a straight line. Likewise, the straight-line acceleration test, shows that this is also the case when the wheels are driven. However a small oscillation in the wheel velocity when the torque is removed from the wheel is not captured.

The constant radius test shows that the E4 coupe is close to neutral steer for low to mid

lateral acceleration, while at high lateral acceleration becomes increasingly understeer. With limited tuning of the rear tyres cornering stiffness and the tyre/road friction coefficient the vehicle model is able to capture these handling characteristics. There is a slight increase in the measured understeer gradient at very low lateral acceleration, which the model does not capture, however this is less significant.

The two difficulties found in measuring the understeer gradient are the noise in the accelerometer measurement and the variation in turning radius between test runs. The first is overcome by calculating the lateral acceleration from the product of vehicle velocity and yaw velocity. This is particular to this experiment due to the combination of a rough road and stiff suspension, although it highlights the need for careful placement and mounting of accelerometers. The turning radius variation is eliminated by calculating the radius for each test and subtracting the Ackermann angle from the steer angle. This method does not appear in the literature although the problem it overcomes could be common to any constant radius test. A cruise controller capable of low speed operation is expected to improve this test by allowing the driver to focus solely on the steering.

The double lane change test shows that the yaw rate of the vehicle model closely matches (error $< 10\%$) the measured yaw rate. This is a good validation of the steering system, lateral tyre forces and chassis inertia, which all effect the yaw dynamics. There is small offset (0.03 m/s^2) between the average wheel deceleration of the model and the experiment. This suggests that the steering angle has a small effect on rolling resistance that is not captured by the model.

In this chapter a high fidelity vehicle model is developed. Some minor effects such as increased understeer at low lateral acceleration and oscillation in the wheel velocity after torque removal are not captured. However, across a range of validation tests the model shows good agreement with experimental data and proves it is a useful tool for controller verification.

Chapter 5

2WD Traction Controller Design and Verification

Based on the review of the literature, MTTE represents a simple yet effective approach to slip control of a single wheel. In Chapter 3 the MTTE method is analysed for its strengths and weaknesses and a number of enhancements for single wheel traction control are put forward. As four wheel road vehicles are not normally driven through one wheel only, in this chapter the MTTE method is extended to the control of a full vehicle with two wheel independent drive (2WID-MTTE). This is important as not only must the controller maintain traction at each driven wheel under unknown road conditions, but the controller also must maintain directional stability of the vehicle. For a 2WID vehicle accelerating on a straight flat road the change to the tyre-road friction can be categorised into high- to low- μ , low- to high- μ and split- μ . The high- to low- μ change is considered extensively in the previous chapter. Split- μ road simulations are used in this chapter to test the vehicle's directional stability as there is potential for a large torque imbalance between left and right driven wheels. Low- to high- μ road simulations are used to test the controller's ability to return to normal operation after a traction event. The last two tests are found not to be thoroughly investigated in the literature.

In the first section of this chapter the controller equations are derived including a novel

finding in the relationship between friction estimation and maximum transmissible torque. In addition a control strategy is proposed to balance the forces acting on the vehicle. In the second section the performance of the proposed 2WID-MTTE controller is verified through simulation of a vehicle accelerating onto a split- μ road. A comparison is made to 2WID-MTTE without a strategy for balancing forces, which is referred to as independent MTTE. This is done to highlight the significance of this control strategy. In the third section a comparison is made to a more conventional method for stability control called Direct Yaw Control (DYC). In the last section 2WID-MTTE is simulated for a vehicle accelerating on a low- to high- μ road.

In this chapter it is assumed that the mass of the vehicle is known. Vehicle mass perturbations are dealt with in Chapter 3. As the mass of the driver and passenger(s) are considered to be perturbations they are not included in the model for the simulations in this chapter. Otherwise the vehicle model is identical to the model described in Chapter 4.

5.1 MTTE for two independently driven wheels

In this section the controller equations are derived from the equations of motion of the vehicle and of the two driven wheels. The derivation below is proof that the so called maximum transmissible torque of each driven wheel is dependent on the estimation of friction torque of both of the driven wheels. This is a novel contribution to the development of MTTE control, Ewin et al. (2013).

The equation for the rotational motion of a driven wheel about its axle is

$$J_w \dot{\omega} = T_m - F_x r, \quad (5.1)$$

where F_x is the longitudinal friction force, r is the loaded tyre radius, T_m is the motor torque, J_w is the wheel inertia, ω is the wheel angular velocity. Figure 5.1 shows the free body diagram for a driven wheel with arrows to indicate the positive direction of the

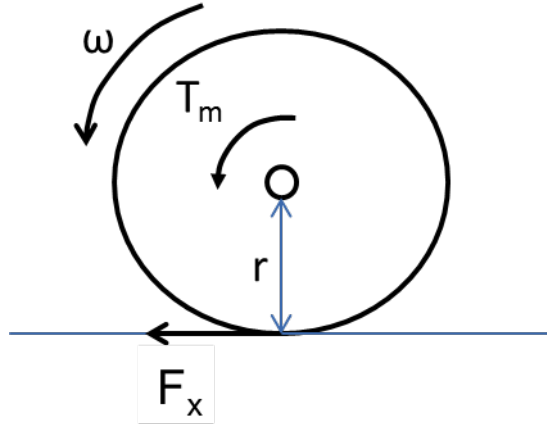


Figure 5.1: Free body diagram of a driven wheel

force, torque and velocity vectors.

The available friction torque between each wheel and the ground is estimated from

$$\hat{T}_{xi} = F_{xi}r = T_{mi} - J_w\dot{\omega}_i \quad \text{for } i=l,r, \quad (5.2)$$

where the subscript i denotes the left or right wheel. The inertia of each wheel is assumed to be equal.

The longitudinal equation of motion for a vehicle with two driven wheels is

$$M\dot{V}_x = F_{xl} + F_{xr} - F_{dr} - \sum_{i=1}^4 F_{rri}, \quad (5.3)$$

where M is the mass of the vehicle, V_x is the longitudinal vehicle velocity, F_{dr} is the aerodynamic drag force, and F_{rr} is the rolling resistance force at each wheel. Estimation of the rolling resistance and aerodynamic drag forces have been considered in the Chapter 3. These can be included for improved performance but for clarity are not considered further here.

A design parameter α is introduced by Yin et al. (2009), which can be defined for each wheel as

$$\alpha_i = \frac{\dot{V}_x}{\dot{V}_{wi}} \quad \text{for } i = l, r, \quad (5.4)$$

where V_w is the longitudinal velocity of the wheel. The design parameter α should be

set to prevent excessive slip between the tyre and the road, which reduces traction, so α should be ~ 1 . In addition to allow the vehicle to accelerate α should be < 1 as this enables the tyre to generate a friction force. Therefore a value of $\alpha = 0.9$ is used, as in Yin et al. (2009). For simplicity $\alpha_l = \alpha_r = \alpha$.

Substituting Equation (5.2) into (5.3) gives

$$\dot{V}_x = \frac{\hat{T}_{fl} + \hat{T}_{fr}}{Mr}. \quad (5.5)$$

The relationship between longitudinal velocity and angular velocity is

$$V_w = r\omega. \quad (5.6)$$

Differentiating (5.6) and substituting Equation (5.2) for the left wheel gives

$$\dot{V}_{wl} = \frac{r(T_{ml} - \hat{T}_{fl})}{J_w}. \quad (5.7)$$

Substituting (5.5) and (5.7) into Equation (5.4) for the left wheel gives

$$\alpha = \frac{(\hat{T}_{fl} + \hat{T}_{fr})J_w}{(T_{maxl} - \hat{T}_{fl})Mr^2}, \quad (5.8)$$

and rearranging gives the maximum transmissible torque for the left driven wheel of a 2WID vehicle

$$T_{maxl} = \left(\frac{J_w}{\alpha Mr^2} + 1 \right) \hat{T}_{fl} + \frac{J_w}{\alpha Mr^2} \hat{T}_{fr}. \quad (5.9)$$

A similar equation can be derived for the right wheel. The derivation of Equation (5.9) is important as it shows that T_{maxl} depends on the friction torque estimates from both wheels. This can be understood as allowing one wheel to accelerate with respect to the torque applied to the other. This means that the control of left and right wheels cannot be treated separately.

The equation for the maximum transmissible torque for the whole vehicle is given by

summing the equations for the left and right wheels

$$T_{max} = \left(\frac{J}{\alpha M r^2} + 1 \right) \hat{T}_f, \quad (5.10)$$

where

$$J = 2J_w, \quad (5.11)$$

$$\hat{T}_f = \hat{T}_{fl} + \hat{T}_{fr}, \quad (5.12)$$

which is equivalent to the MTTE equation from Yin et al. (2009) for a single wheel.

A diagram of the controller is shown in Figure 5.2. The controller uses Equation (5.2) to estimate the friction torque from the wheel speed and motor torque. These signals are low pass filtered to reduce noise while maintaining phase lag, after Yin et al. (2009). The controller uses Equation (5.9) to calculate the maximum transmissible torque from the friction torque estimates. The rate compensation term, Equation (3.16), developed in Chapter 3 is added to the maximum transmissible torque (not shown in Figure 5.2). This is used to limit the demand torque sent to each motor.

For straight line driving the following control strategy is adopted. The torque demand from the driver is split equally left and right. To ensure safe operation of the vehicle the saturation blocks are applied to the torque demand in series; that is, the output torques of the controller $T_{1,2}$ are limited by the lower of the two T_{max} values.

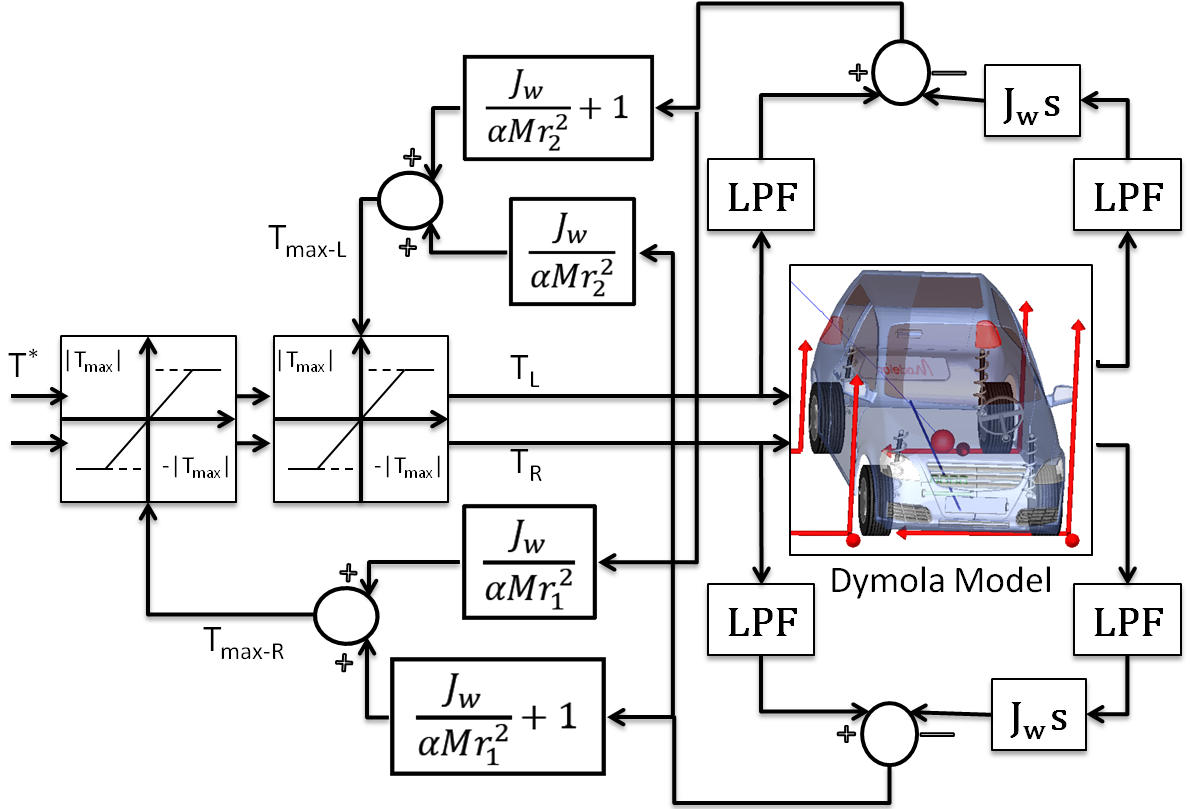


Figure 5.2: MTTE based traction controller diagram

5.2 Split- μ road simulation

In this section the performance of the proposed controller is verified through simulation with the vehicle model described in Chapter 4. The test case of the vehicle accelerating in a straight line onto a split- μ road is chosen as this is a good example of where controlling the traction of the left and right wheels together has the potential to improve vehicle directional stability. A comparison is made to a controller similar to 2WID-MTTE but without a strategy for balancing left and right motor torques, referred to as Independent MTTE. The purpose of the simulation comparison is to validate the strategy proposed in the last section to limit left and right motor torques to the value of the lower one. It is expected that the proposed controller will show improved vehicle directional stability when the road conditions for the left and right wheels vary. The controller parameters, which match the vehicle model parameters, are given in Table 5.1.

The split- μ road consists of a straight flat surface of width 10 m. The right hand side

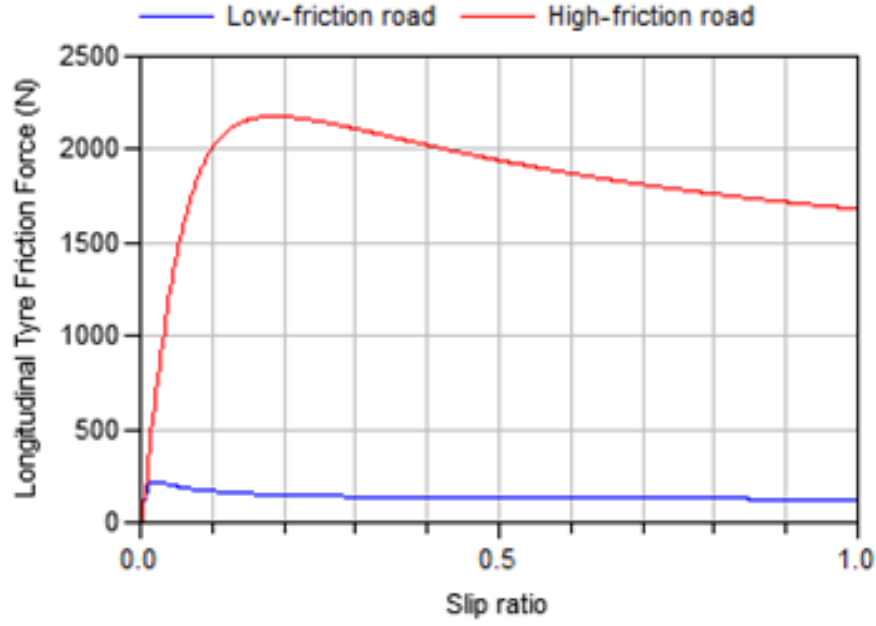


Figure 5.3: Longitudinal friction-slip relationship of tyre model for a vertical force of 2000 N

Parameter	Symbol	Value	Unit
Vehicle mass	M	1005.5	kg
Wheel static loaded radius	r	0.296	m
Driveline inertia	J_W	1.04	kg.m ²
Relaxation factor	α	0.9	
Speed and torque filters' time constants	τ_1, τ_2	30	ms
Rate compensation time constant	τ_c	30	ms

Table 5.1: Vehicle and controller parameters.

of the road had a nominal friction coefficient of $\mu = 1.0$ which is representative of dry asphalt, referred to as the high- μ surface. After 15 m the left hand side of the road changes from dry asphalt to a nominal friction coefficient of $\mu = 0.1$ which is representative of ice, referred to as the low- μ surface. The friction-slip relationship of the two surfaces is shown in Figure 5.3.

The simulation is started with the vehicle travelling at 5 m/s in the middle of the road with the steering wheel angle fixed at 0° . After one second a torque demand of 500 Nm is applied to each driven wheel. For the rest of the simulation no other driver action was taken. The simulation conditions are presented in Table 5.2.

Parameter	Value	Units
Initial vehicle speed	5	m/s
Torque demand (per wheel) @ $t = 1$ s	500	Nm
Low- μ	$x > 15$ and $y < 0$	m

Table 5.2: Split- μ simulation parameters.

5.2.1 Simulation results

A visualisation of the vehicle trajectory for each simulation is shown in Figure 5.4. The top trajectory shows the proposed 2WID-MTTE controller simulation with equal torque strategy. The bottom trajectory shows the Independent MTTE controller simulation. This shows that the response of the vehicle under Independent MTTE control was to deviate from its intended straight line path to the extent that it exited the road at 6.9 s.

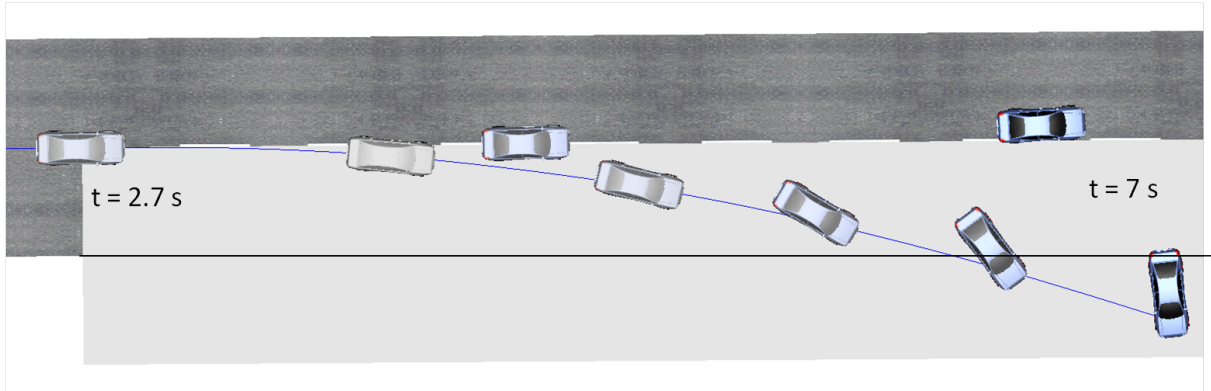


Figure 5.4: Vehicle trajectories for the proposed 2WID-MTTE controller simulation (top) and Independent MTTE controller simulation (bottom) on a split- μ road. The light grey area represents the low- μ part of the road.

The vehicle's yaw rate and body slip for both simulations are shown in Figure 5.5. For the proposed 2WID-MTTE controller simulation the yaw rate remains less than 0.1 rad/s in magnitude and the body slip remains less than 0.3 degrees for the whole time. There are small changes when the right wheel enters the low- μ surface at 2.8 s and again when it leaves the low- μ surface at 7.0 s. The yaw rate for Independent MTTE controller simulation remains less than 0.1 rad/s in magnitude up to 4.3 s. After this time the left wheel also enters the low- μ surface and the yaw rate steadily increases in magnitude. The yaw rate reaches -0.83 rad/s and the body slip angle reaches 62 degrees by the time all

four wheels exit the road.

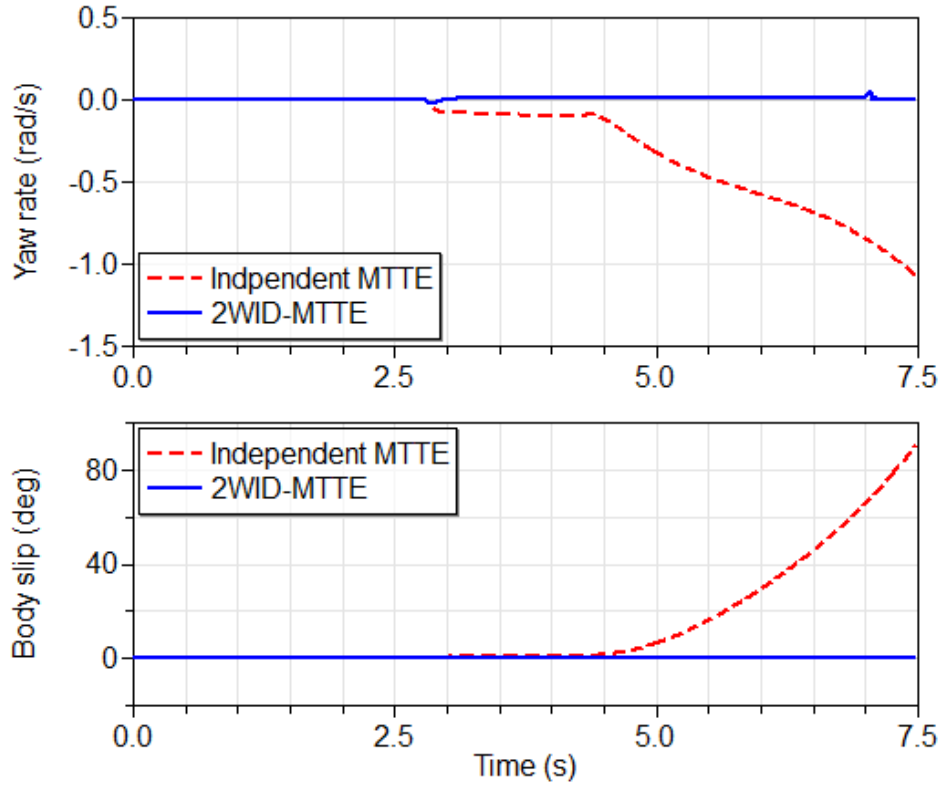


Figure 5.5: Vehicle yaw rate and body slip for the proposed 2WID-MTTE controller simulation and Independent MTTE controller simulation on a split- μ road.

Figure 5.6 shows the torques delivered by the left and right motors along with the torque demand input to the controller, for the Independent MTTE controller simulation. The rear right drive wheel enters the low- μ road at 2.8 s at which point the controller reduces the torque demand to the right motor. The rear left driven wheel enters the low- μ road at 4.3 s and the controller similarly reduces the torque demand to the right motor.

Figure 5.7 shows the same torque signals for the 2WID-MTTE controller simulation. When the rear right drive wheel enters the low- μ road at 2.8 s the controller reduces the torque demand to the right motor to maintain traction of the right wheel while the torque demand to left is reduced by the same amount. The rear right driven wheel returns to the high- μ road at 7.0 s.

Figure 5.8 shows the angular wheel velocities for the Independent MTTE controller and 2WID-MTTE controller simulations. As the front left wheel velocity is free rolling it provides a good approximation for the longitudinal vehicle velocity. For both simulations

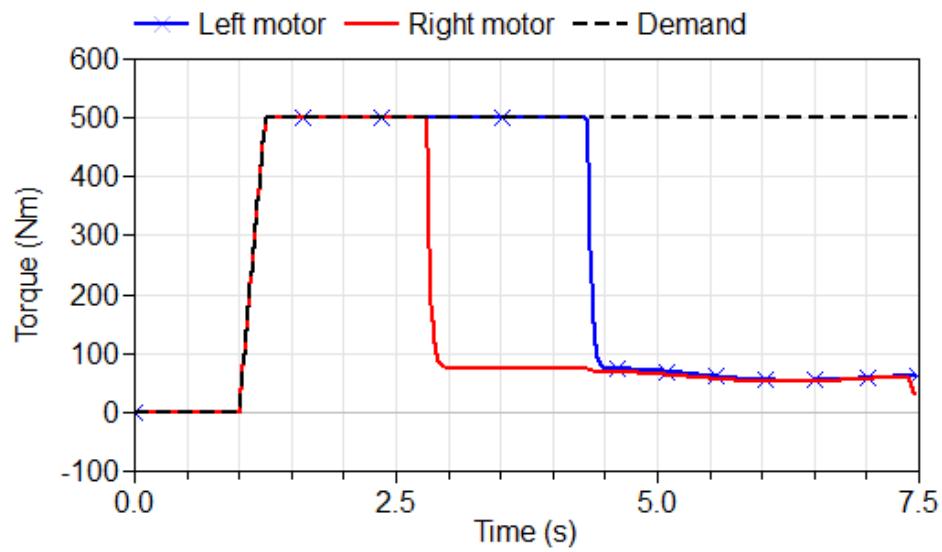


Figure 5.6: Torque demand input to the controller and torque delivered by the left and right motors for the Independent MTTE controller simulated on a split- μ road.

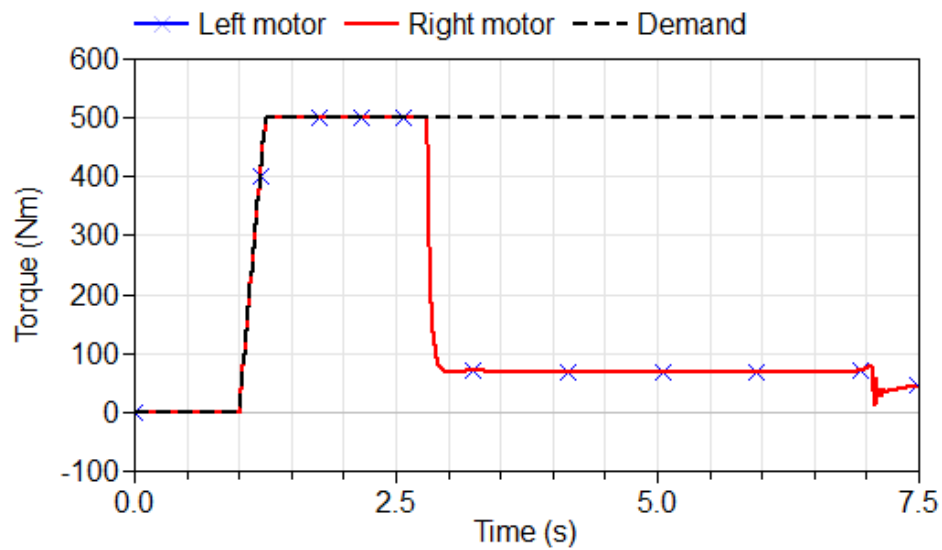


Figure 5.7: Torque demand input to the controller and torque delivered by the left and right motors for the 2WID-MTTE controller simulated on a split- μ road.

the rear right wheel velocity increases rapidly at 2.8 s due to the wheel slipping more as it enters the low- μ surface. For both simulations between 2.8 s and 4.3 s the right driven wheel velocity changes in proportion to the vehicle velocity as the traction controller aims to maintain a constant acceleration ratio. For the Independent MTTE simulation at 4.3 s the rear left wheel velocity rapidly increases as it also enters the low- μ surface. After this time the longitudinal vehicle velocity begins to decrease as the vehicle body slip increases. For the 2WID-MTTE simulation at 7.0 s the rear right wheel velocity returns approximately to the non-driven wheel velocity as the rear right wheel returns to the high- μ surface.

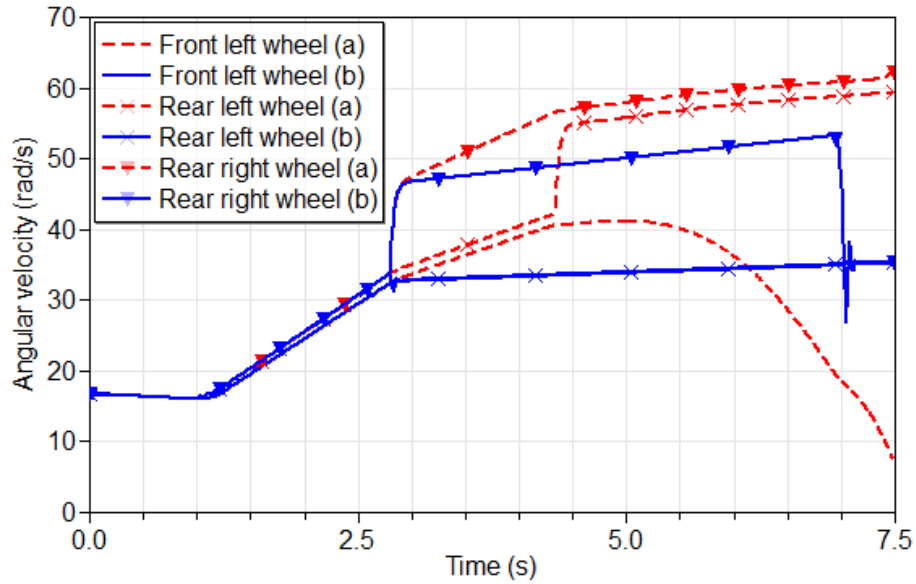


Figure 5.8: Wheel velocities of rear driven wheels and front left free rolling wheel (proxy for longitudinal vehicle velocity) for: a) Independent MTTE controller simulation; and b) proposed 2WID-MTTE controller simulation, on a split- μ road.

Figures 5.9 and 5.10 show the tangential tyre forces for Independent MTTE simulation and 2WID-MTTE simulation respectively. The top graph in each figure shows the longitudinal tyre forces, where the forces on the rear driven wheels for the most part are proportional to the torque delivered by the motors and the forces on the front non-driven wheels are near zero. The bottom graph in each figure shows the lateral tyre forces. For both simulations the left and right lateral forces are balanced up to 2.5 s while all the wheels are on the high- μ surface. For the Independent MTTE simulation after 2.7 s the rear left lateral force increases to counter the turning moment created by the longitudinal forces, but

results in a net negative lateral force which moves the vehicle to the right. Conversely, for the 2WID-MTTE simulation there is a net positive lateral force that acts to move the vehicle to the left.

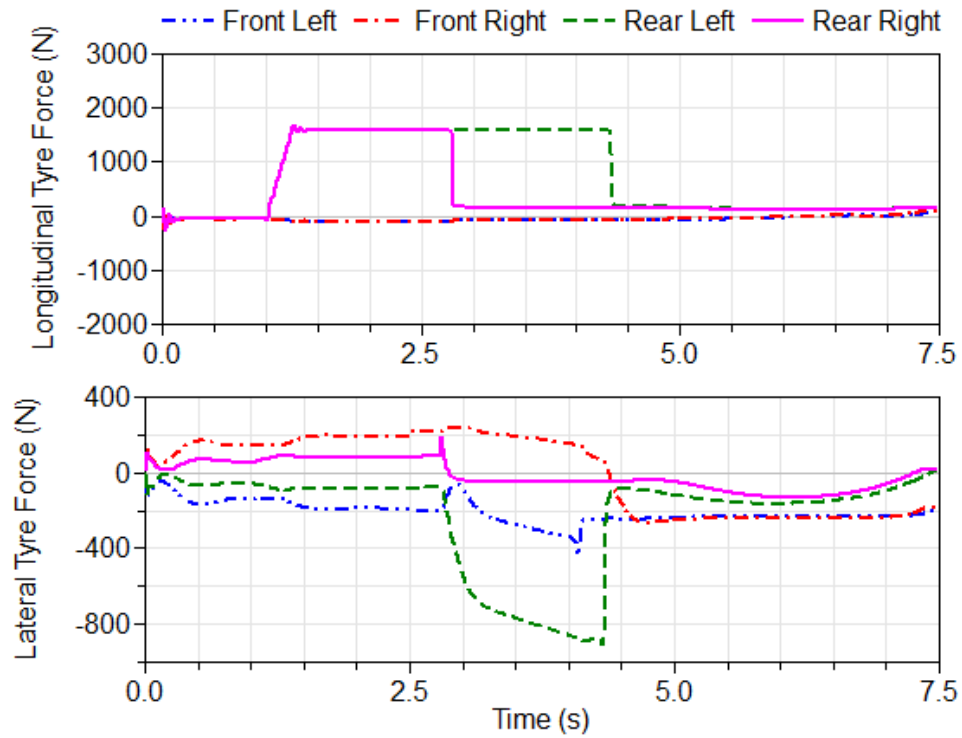


Figure 5.9: Longitudinal and lateral tyre forces for Independent MTTE controller simulation on a split- μ road.

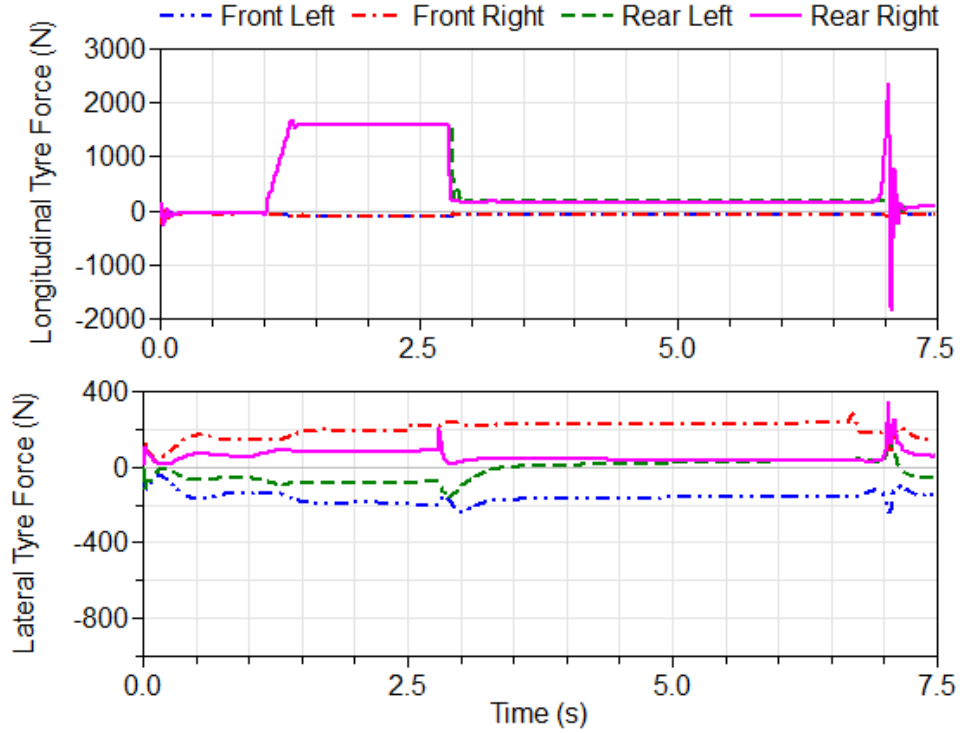


Figure 5.10: Longitudinal and lateral tyre forces for proposed 2WID-MTTE controller simulation on a split- μ road.

5.2.2 Discussion

The Independent MTTE control allows a torque difference ~ 440 Nm between left and right motors from 2.7 s to 4.3 s while they are on different surfaces. This generates a yaw moment that turns the vehicle towards the low- μ road. The rear left wheel enters the low- μ road at 4.3 s after which the controller reduces the torque demand to the left motor by a similar amount. This is insufficient to prevent the yaw rate of the vehicle from increasing which leads to the vehicle spinning off the road. The magnitude of the yaw rate and the body slip by the time the vehicle exits the road makes the vehicle very difficult for a driver to control.

This is in contrast to the proposed 2WID-MTTE controller that maintains vehicle directional stability, Figure 5.4: top trajectory. As the rear right wheel enters the low- μ road the controller limits the torque demand to both motors by the same amount, see Figure 5.7. This minimises yaw moment generation which can be seen from Figure 5.5 as the yaw rate remains < 0.1 rad/s for the whole simulation. The small deviation in trajectory

seen in Figure 5.4 is the result of lateral tyre forces, generated by the camber angle of the wheel, becoming imbalanced when the tyres enter the split- μ road, see Figure 5.10. It is plausible that a small amount of steering action from the driver could correct this.

For the 2WID-MTTE simulation at 5.2 s there is a large oscillation in the longitudinal tyre force, see Figure 5.10, and after this time the torque delivered by both motors only gradually increases, Figure 5.7. Both these results are due to the right driven wheel returning to the high- μ surface. Simulation of the vehicle accelerating from low- to high- μ road under 2WID-MTTE control is dealt with later on, see Section 5.4.

5.3 Direction yaw moment control comparison

In this section the 2WID-MTTE controller described above, which does not depend on yaw motion feedback, is compared to direct yaw moment control (DYC). DYC alters the yaw moment acting on a vehicle by applying different torques to the left and right wheels either through the brakes or through independent electric machines. Examples can be found in the literature by Shibahata et al. (1993), Hori (2004), and De Novellis et al. (2015). DYC is a high level stability control that generates a yaw moment demand. A torque distributor assigns torque demands to each of the wheels to satisfy the yaw moment demand and the driver's longitudinal acceleration demand. After which a traction controller determines whether these torque demands can be achieved. A diagram of this control architecture is shown in Figure 5.11.

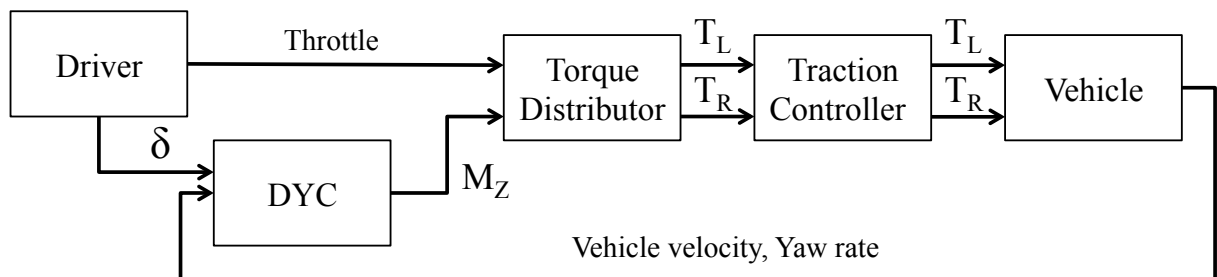


Figure 5.11: Diagram of a stability control system.

A yaw rate reference is required to drive the yaw controller. For this comparison a single

track vehicle model is used, this requires the steering wheel angle and the vehicle velocity as inputs. For the purposes of this simulation the vehicle velocity is assumed to be known. To actually implement this controller the velocity would need to be estimated which is non-trivial. A control error is calculated by subtracting the measured yaw rate from the yaw rate reference. The yaw controller calculates the yaw moment demand from this error. For this comparison a simple PI controller is used. A diagram of this is shown in Figure 5.12. The traction controller used is the Independent MTTE controller described earlier in this chapter.

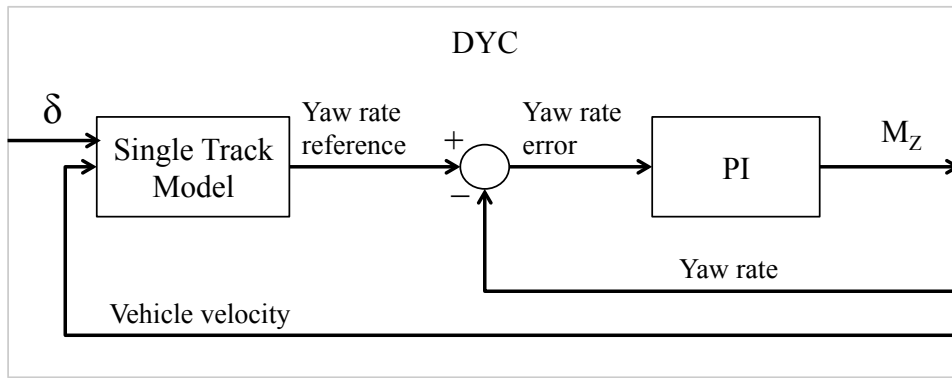


Figure 5.12: Diagram of a direct yaw moment controller.

5.3.1 PI gain tuning

The PI yaw controller gains are manually tuned using a step steer manoeuvre on a high- μ road. More systematic tuning methods exist such as using the Bode or Nyquist¹ plots, however these are beyond the scope of this thesis as the aim is to qualitatively compare the two control methodologies. The vehicle is simulated from an initial speed of 16 m/s. At $t = 0.5$ s a longitudinal acceleration demand of 450 Nm is applied. At $t = 1$ s a steering wheel angle of $\delta = \pi/2$ rad is applied. This simulation is repeated for a number of Proportional and Integral gain values. From Figure 5.13 it can be observed that gains of $K_p = 4000$ and $K_i = 20$ give a marked improvement over the non-control case with a satisfactory response time.

¹Nyquist plots also provide robustness margins.

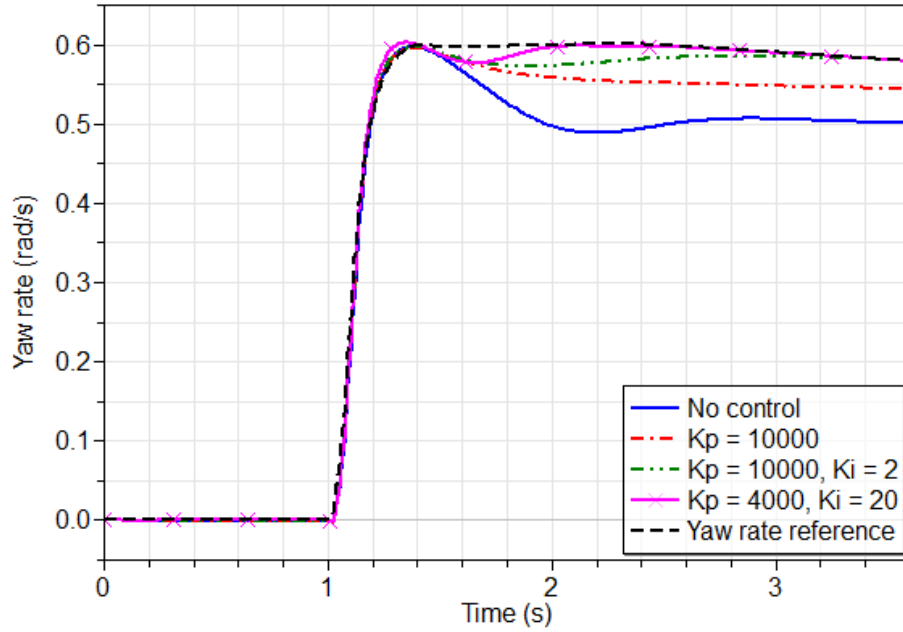


Figure 5.13: Tuning of PI controller gains through the yaw rate response of a step steer simulation.

5.3.2 Split- μ simulation results

The direct yaw moment controller described above is simulated on a split- μ road with test conditions described in Table 5.2.

Figure 5.14 shows the vehicle trajectories for the DYC + Independent MTTE simulation alongside the 2WID-MTTE simulation. Figure 5.15 shows the vehicle yaw rates and body slips for both simulations. Figure 5.16 shows the yaw moment demand out of the PI controller from the DYC + Independent MTTE simulation. From these graphs it appears that up to 7.0 s the DYC + Independent MTTE controller achieves a similar performance to the 2WID-MTTE controller, albeit that the trajectory goes slightly to the right rather than the left. The transient yaw rate error as the vehicle enters the split- μ road at 2.8 s is reduced to near zero in less than 1.0 s, apparently by a positive yaw moment demand with a steady-state value of 1620 Nm.

However, a comparison of the torque demands out of the Torque Distributor with the torque delivered by the motors, Figure 5.17, shows that although the yaw moment out of the PI controller is positive the traction controller reduces the torque to the right motor

to such an extent that the resulting yaw moment delivered by the motors is negative. The traction controller is effectively cancelling the action of the DYC. The fact that the vehicle maintains a stable trajectory, Figure 5.14, up to 7.0 s is due to the actual yaw moment produced by the longitudinal and lateral tyre forces being near zero. This is fortuitous rather than due to good controller design.

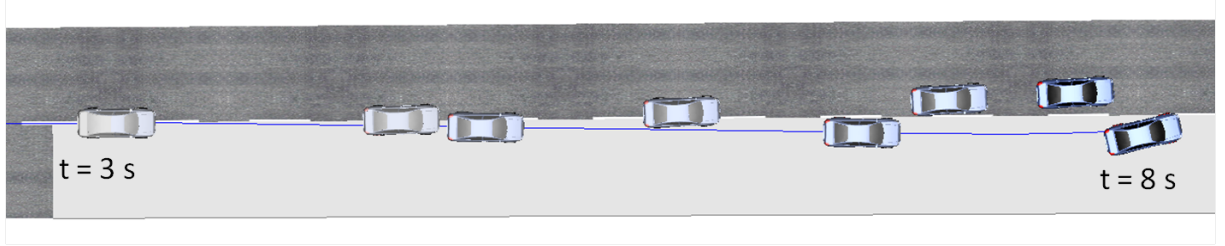


Figure 5.14: Vehicle trajectories for DYC + Independent MTTE controller simulation (bottom) and 2WID-MTTE controller simulation (top), on a split- μ road.

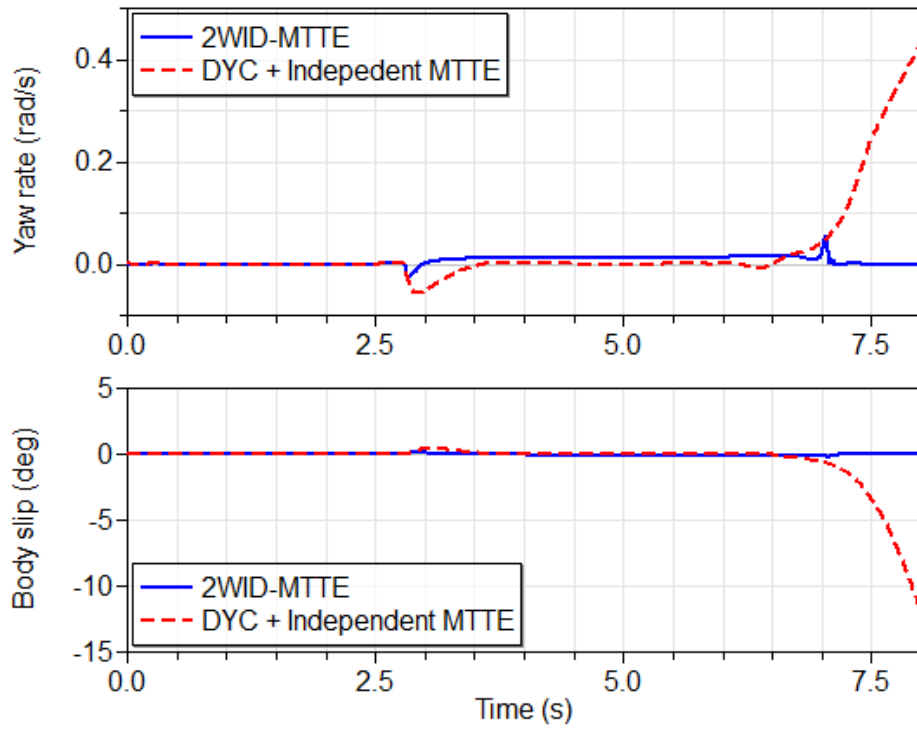


Figure 5.15: Vehicle yaw rate and body slip for 2WID-MTTE controller simulation and DYC + Independent MTTE controller simulation on a split- μ road.

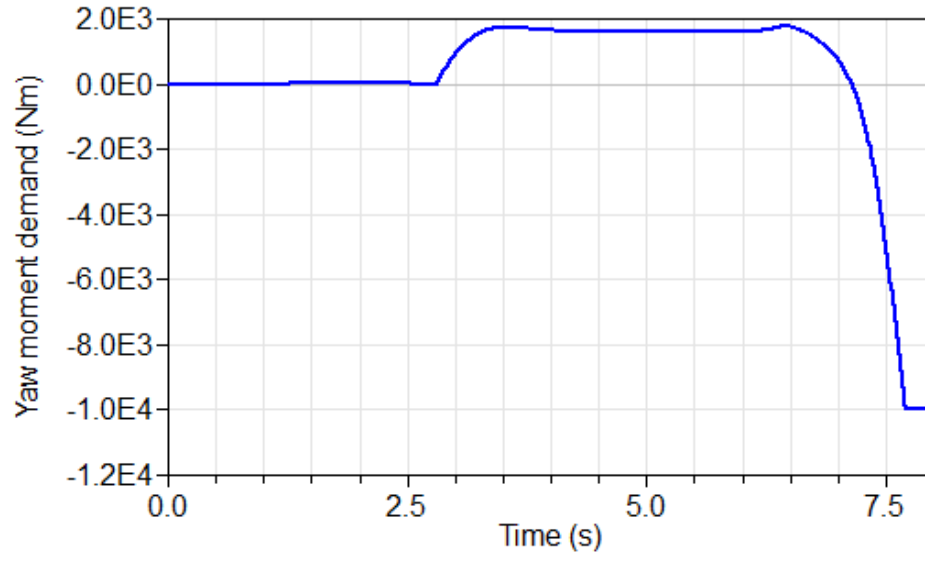


Figure 5.16: Yaw moment demand output from PI controller for DYC + Independent MTTE simulation on a split- μ road.

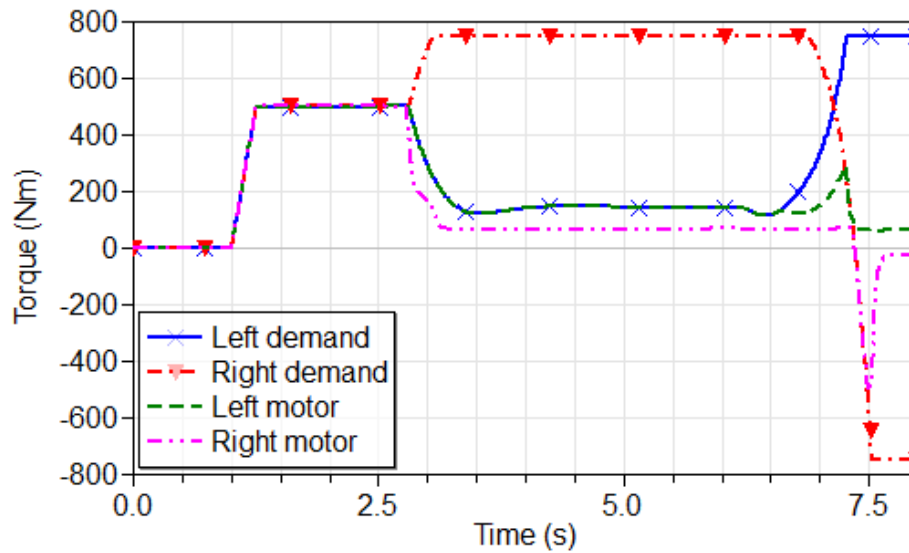


Figure 5.17: Torque demands from torque distributor and torque delivered by the motors for DYC + Independent MTTE split- μ road simulation.

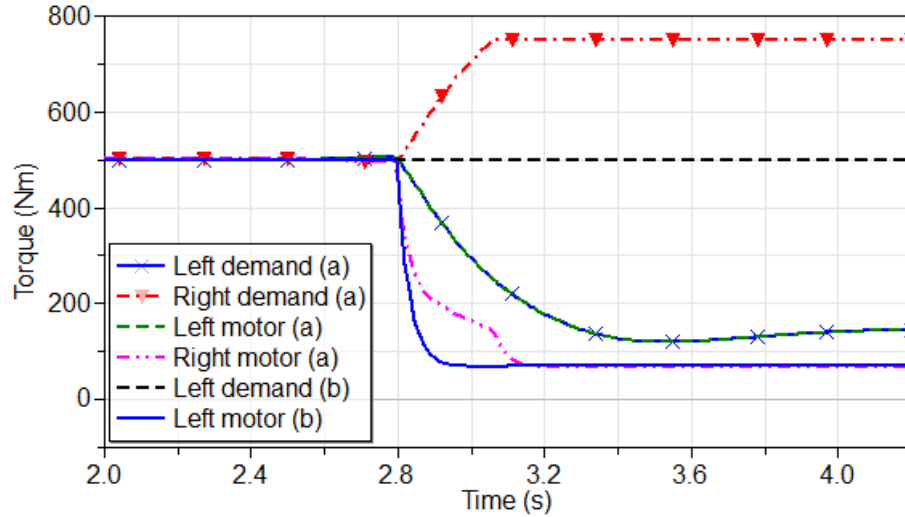


Figure 5.18: Torque demands and torque delivered by the motors at entry to split- μ road for: (a) DYC + Independent MTTE simulation (b) 2WID-MTTE simulation.

5.3.3 Discussion

It can be concluded from the simulation results that the proposed 2WID-MTTE controller is as effective as DYC combined with Independent MTTE at maintaining stability of a vehicle accelerating in a straight line on a split- μ road. The proposed controller is also simpler in design, needing fewer sensors and does not require vehicle velocity to be estimated. The DYC + Independent MTTE simulation highlights the need for the lower level traction controller to feed back when the torque demand is saturated to the high level controller and torque distributor, as in this instance the traction controller cancels out the action of the DYC. Sensing or estimating the actual yaw moment is also important as this can differ greatly from the yaw moment demand due to the lateral tyre forces.

It is not necessarily the case that the proposed control should replace DYC as the ability to “vector” torque from left to right can greatly aide a vehicle manoeuvre in many other situations. The proposed controller could be integrated with DYC by only activating the proposed controller in critical situations. Under normal conditions the DYC would provide torque demands to an independent MTTE traction controller as in the above simulation. Switching to 2WID-MTTE control would require detection of a critical situation such as the above split- μ road. This could be done by observing that one motor has gone into

closed loop control, while the yaw moment allowed by the traction control is opposite in sign to the yaw moment demand from the DYC.

5.4 Low- to high- μ road simulation

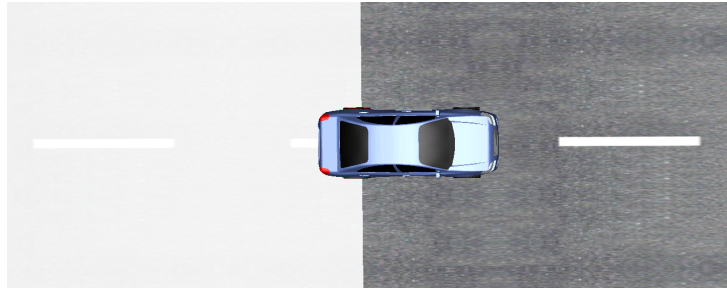


Figure 5.19: Low- μ to high- μ road simulation.

Traction control methods are often verified by straight line driving from a high- μ to a low- μ road, Yin et al. (2009), Magallan et al. (2011), De Castro et al. (2013) and Patil et al. (2003). This tests whether the torque demand to the wheel can be restricted sufficiently to maintain traction. It is also important to consider the opposite case of driving from a low- μ to high- μ road. When the tyre-road friction increases to the point where it is no longer the limiting factor on accelerating the vehicle, the controller should return torque to the demanded value within a reasonable time period.

Low- to high- μ road simulation and experiments are considered by Fujii and Fujimoto (2007) and Ivanov and Barber (2015) for a slip controller. The research by Fujii and Fujimoto (2007) shows considerable oscillation in the wheel slip at the point when the wheels return to the high- μ road during the experiment but not the simulation, although this is not discussed in the paper. The research by Ivanov and Barber (2015) presents simulation results showing wheel oscillations with a magnitude of 3-4 times the vehicle speed when a wheel transitions from a low- μ to high- μ surface. No mention is given of the fidelity of this effect other than that it is indicative of the change in road conditions.

The simulation conducted by Fujii and Fujimoto (2007) uses a rigid tyre model as did Yin et al. (2009) when simulating MTTE. Ivanov and Barber (2015) uses a IPG CarMaker

vehicle model but does not state the tyre model used. The vehicle model presented in chapter 4 uses a Magic Formula tyre model Pacejka (2006), that includes tangential transient tyre dynamics. This can be represented by a spring-damper as illustrated in Figure 5.20. The damper has a velocity dependent term and so at low speeds it will be stiff and the behaviour will be dominated by the spring.

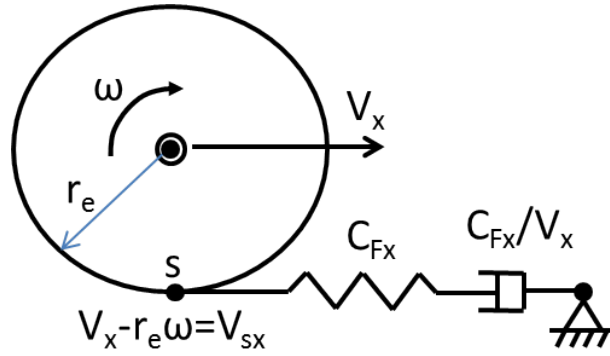


Figure 5.20: A mechanical representation of carcass compliance in the tyre model, after Pacejka (2006).

The vehicle is simulated accelerating from a low- μ to high- μ road to investigate the effect of carcass compliance on MTTE control. The simulation parameters are shown in Table 5.3. The tyre-road friction-slip relationship for the low- μ and the high- μ road surfaces are shown in Figure 5.3.

Parameter	Value
Initial vehicle speed	5 m/s
Torque demand (per wheel)	125 Nm @ t=1s
Coordinates of low- μ	$0 < x < 15$ m
Coordinates of high- μ	$15 \text{ m} < x$

Table 5.3: Low-high- μ simulation parameters

5.4.1 Simulation results

The results are presented in Figures 5.21, 5.22 and 5.23 for a simulation where the tangential tyre dynamics are modelled alongside a simulation where the tangential tyre dynamics are not modelled. The results show that as the wheel transitions from the low- μ to the high- μ surface at 3.48 s the actual torque applied to the wheel by the controller oscillates

with a frequency of 15-16 Hz. After the oscillation the torque ramps up slowly rather than returning quickly to its demand value as might be expected.

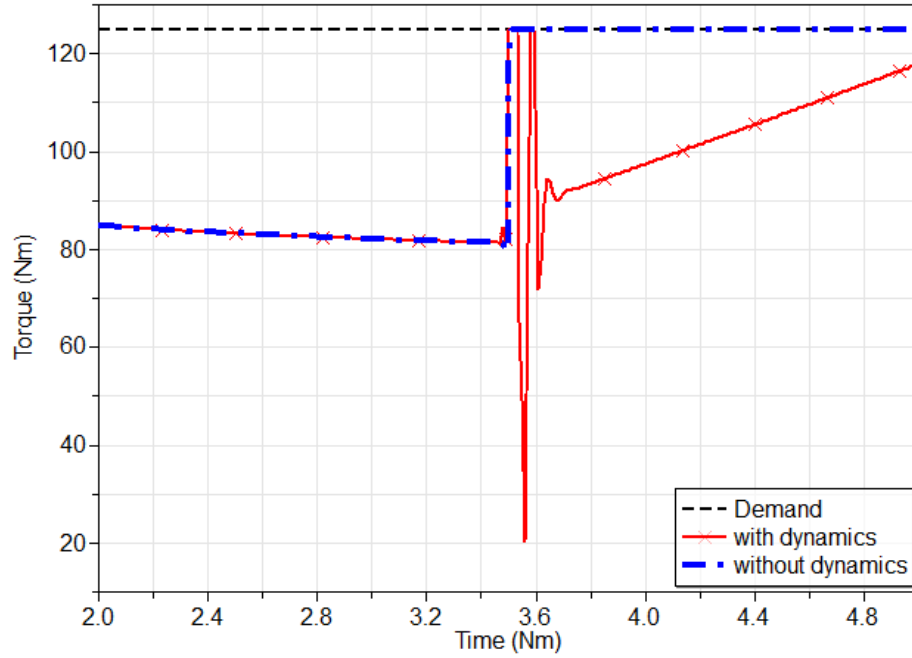


Figure 5.21: Torque demand and torque output of controller, for low-high- μ simulation with and without transient tangential tyre dynamics at 5 m/s.

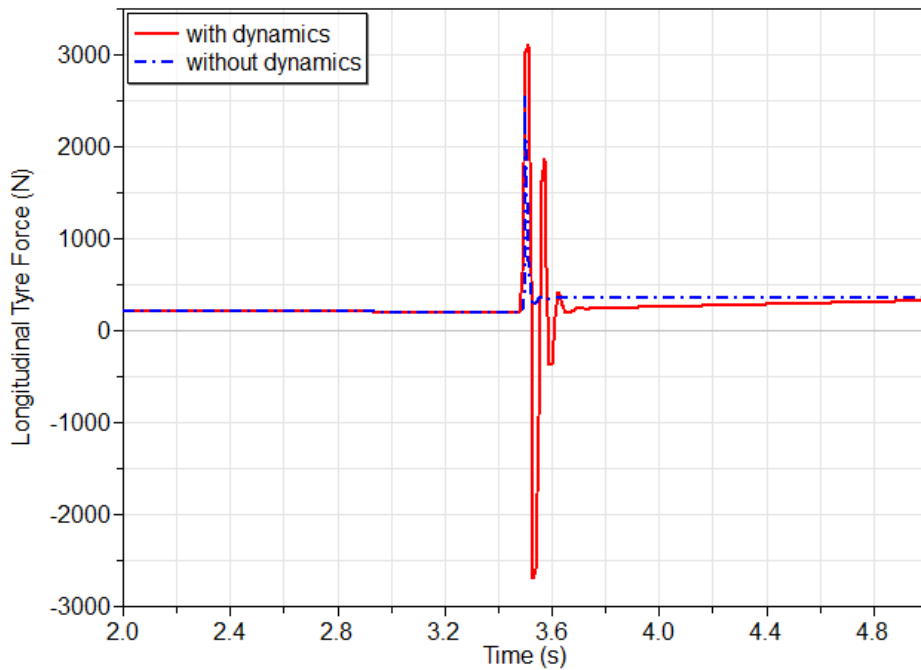


Figure 5.22: Wheel speeds for low-high- μ simulation with and without transient tangential tyre dynamics at 5 m/s. Key: RL, Rear Left; FL, Front Left.

To clarify that this oscillatory effect is due to the tyre model and not the MTTE controller,

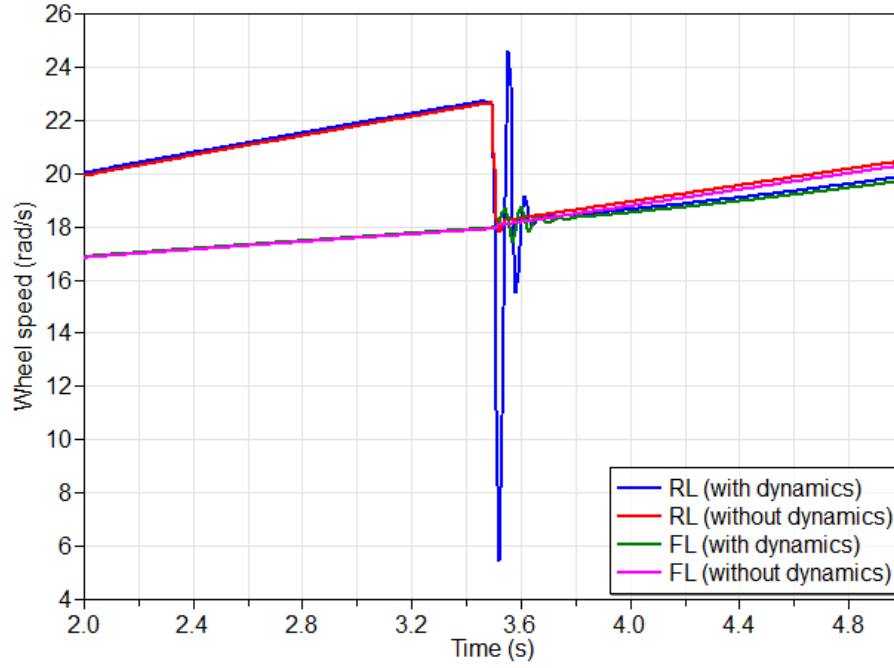


Figure 5.23: Low-high- μ simulation with and without transient tangential tyre dynamics at 5 m/s.

the simulation is repeated without any control, that is a constant torque demand is applied directly to the wheels. The longitudinal tyre friction force from this simulation is shown in Figure 5.24. The force oscillates after the vehicle returns to the high- μ surface at 3.48 s, therefore this is a tyre model not controller effect. By comparison to Figure 5.22 it can be seen that the controller is damping the oscillations in the friction force.

The simulation is repeated at a higher speed to demonstrate the speed dependency of the tyre dynamics. The simulation conditions are shown in Table 5.4. The road surfaces are longer than the low speed simulation to maintain consistency between the duration of time the vehicle spends on each surface. The friction-slip relationship of the low- μ and high- μ road surfaces are the same as the previous simulation.

Parameter	Value
Initial vehicle speed	15 m/s
Torque demand (per wheel)	125 Nm @ t=1s
Coordinates of low- μ	$0 < x < 45$ m
Coordinates of high- μ	$45 \text{ m} < x$

Table 5.4: Low-high- μ simulation parameters

The results presented in Figure 5.25 show that the number and magnitude of oscillations

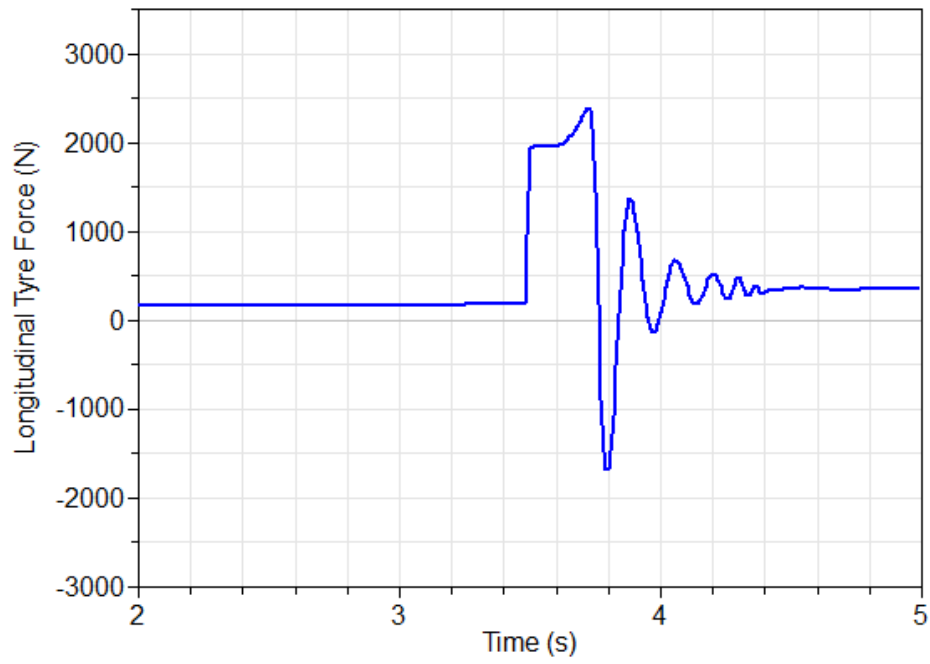


Figure 5.24: Low-high- μ simulation without controller but with transient tangential tyre dynamics at 5 m/s.

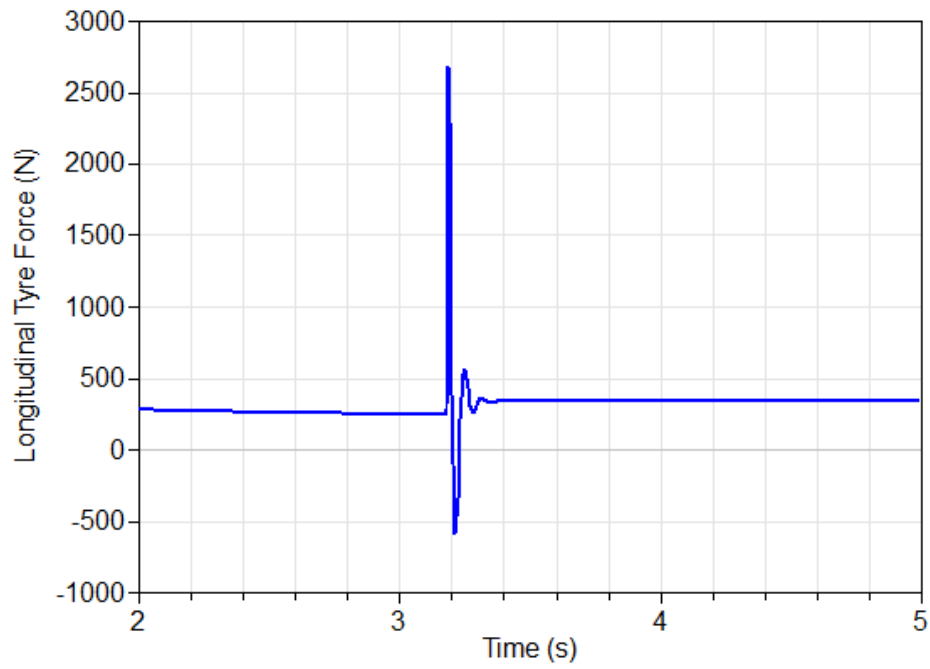


Figure 5.25: Low-high- μ simulation at 15 m/s with transient tangential tyre dynamics.

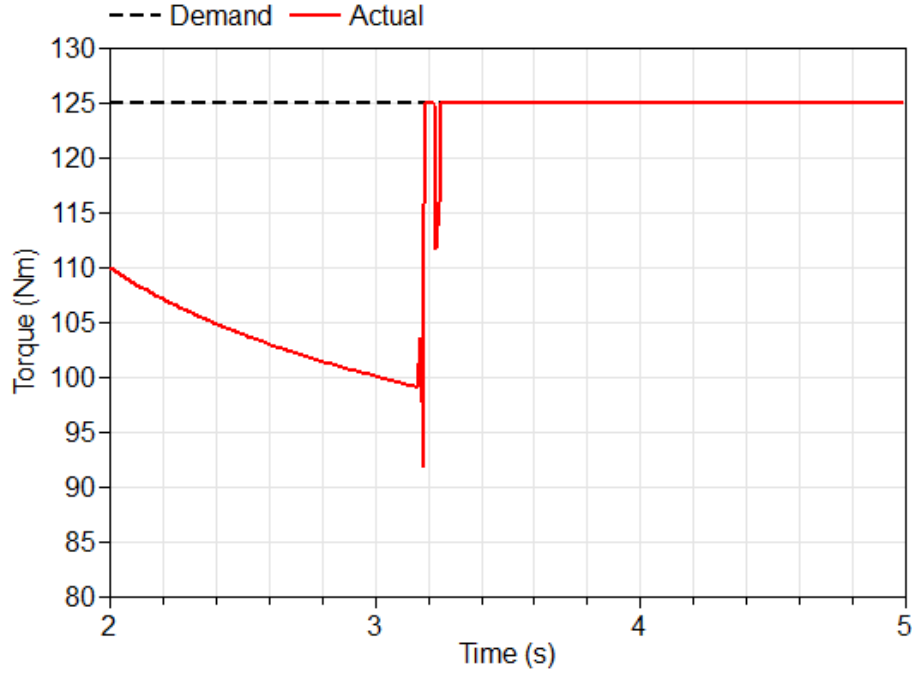


Figure 5.26: Torque demand and torque output (Actual) of controller, for low-high- μ simulation at 15 m/s with transient tangential tyre dynamics.

in the friction force decrease when compared to the low speed test. This is because the higher wheel speeds mean that the damper in the tyre model is less stiff and therefore has a greater effect on the dynamics. This means there is almost no oscillation in the torque from the control, see Figure 5.26.

5.4.2 Discussion

It is worth questioning whether the tyre model used in the above simulations is representative of a real tyre. Referring back to Figure 5.22 it can be seen that, even without modelling the transient tyre dynamics, there is a large spike in longitudinal friction force when the tyre transitions from a low- μ to a high- μ surface. By plotting this force against the relative slip velocity between the tyre and the road, Figure 5.27, it can be seen that the friction force follows the low- μ friction-slip curve of Figure 5.3; then at the transition from low- to high- μ it jumps to the high- μ curve. This behaviour is expected from the Pacejka tyre model as the change in slip velocity is limited by the wheel inertia while the change in friction force can be near instantaneous. Therefore a mechanism in the tyre

model exists to generate such a spike in friction force.

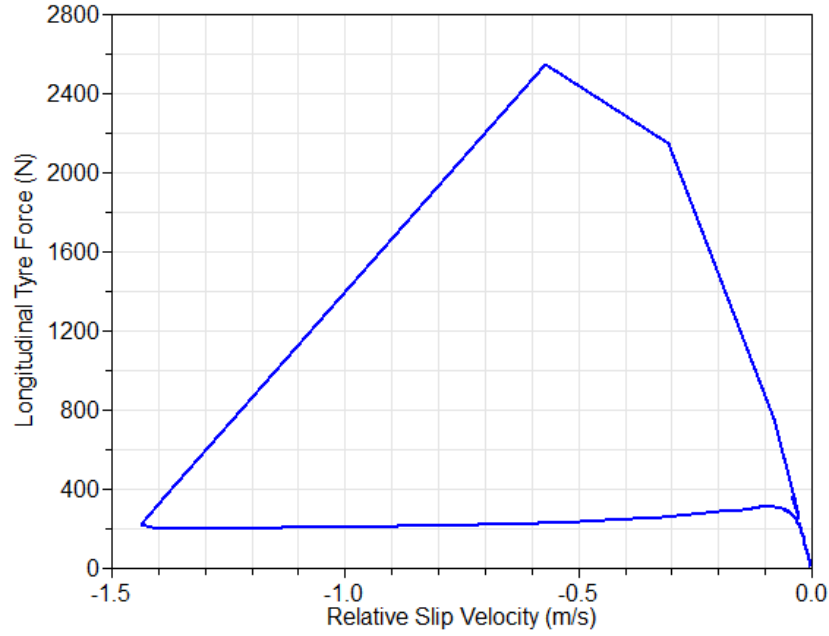


Figure 5.27: Friction against slip velocity for low- to high- μ road simulation at 5 m/s without tangential tyre dynamics

5.5 Summary

Through the derivation of the controller equations it is shown that the so called maximum transmissible torque for one driven wheel is dependent on the estimated friction torque of each of the driven wheels. This is a novel improvement to MTTE control.

The equal torque strategy for the 2WID-MTTE controller is verified in simulation as maintaining a vehicle's directional stability while driving on a severe split- μ road. This is significant given that under the same conditions an Independent MTTE controller results in the vehicle spinning off the road. The proposed 2WID-MTTE control is as effective as DYC in stabilising a vehicle on a split- μ road, but is simpler in design and requires fewer sensors. In other situations and manoeuvres it is beneficial for a stability controller such as DYC to apply asymmetric torques to the wheels, and a method for integrating these two controllers is discussed.

The low- to high- μ simulation demonstrates the need for a high fidelity tyre model when

verifying traction control through simulation. For a sudden increase in friction coefficient at low speed, carcass compliance (transient tyre dynamics) can cause oscillations in the longitudinal tyre friction force. This can excite torque oscillation in traction control that relies on friction estimation. MTTE control exhibits torque oscillations under these conditions although it is more damped than the no control case. The slow return to the demanded value after this transition represents a small reduction in performance, although it could also be considered advantageous if the coefficient of friction of the road reduces again.

Chapter 6

Experimental Verification



Figure 6.1: 4WD E4 coupe.

In the previous chapter the value of the 2WID-MTTE traction controller for a vehicle accelerating on mixed- μ roads is shown through simulation. In addition it is shown that the Pacejka tyre model with transient dynamics exhibits oscillatory behaviour while accelerating from a low- to high- μ road. In this chapter the proposed 2WID-MTTE traction controller is implemented and tested on Delta Motorsport's E4 coupe. This is to verify the performance of the traction controller and further validate the tyre model.

The traction controller is integrated into the Powertrain Control Module (PCM) of Delta

Motorsport's E4 coupe. Controlled Area Network (CAN) signals are used for inputs to the friction estimation as these are commonly available to a powertrain controller. Additional filtering in the friction estimation is required to accommodate these signals, which is specific to the motor controllers used in the vehicle.

Experimental tests are carried out to verify the performance of the traction controller across a range of different road conditions. The primary focus of these tests is to evaluate the traction controller's effect on the directional stability of the vehicle. This is assessed based on the vehicle's yaw dynamics and the level of the driver's steering intervention required during the test. Specific tests include straight line acceleration on high- to low- μ , low- to high- μ and split- μ roads. The 2WID-MTTE controller is benchmarked against no traction control for all tests, and against Independent MTTE control for tests where the vehicle's direction stability is affected. No benchmarking is done against DYC + Independent MTTE, as it is found through simulations in Chapter 5 that there is a conflict between the control action of the DYC and of the Independent MTTE. It is beyond the scope of this thesis to resolve this conflict.

This chapter is structured as follows. In Section 6.1 the E4 coupe used for experimental testing is described. This focuses on the powertrain architecture that the traction controller is integrated into. In addition modification to the friction estimation in the 2WID-MTTE controller is described. In Section 6.2 a test plan is defined based on the available surfaces at the MIRA proving ground. This section also includes a description of the experimental setup. In Section 6.3 the experimental results are presented and discussed, including benchmarking the proposed 2WID-MTTE controller against no traction control and Independent MTTE control.

6.1 Test vehicle

In this section the test vehicle used to verify the 2WID-MTTE traction controller is described. This includes a description of the powertrain used to actuate the control, as

well as how the traction controller is integrated into the powertrain controller.

The test vehicle used is Delta Motorsport's E4 coupe shown in Figure 6.1. This is a 4WD variant of the E4 coupe described in Chapter 4, which has two additional electric machines independently driving the front wheels. However only the rear wheels are driven during testing in order to replicate the 2WD E4 coupe. The main differences between the 2WD and 4WD E4s are that the 4WD E4 is slightly heavier, the kerb mass 1137 kg as opposed to 1005 kg (2WD), and its front weight distribution is further forward, 52.5 % instead of 51.2 % (2WD). This is due to the additional electric machines and inverters at the front axle. The other vehicle parameters of the 4WD E4 are largely the same as those defined in Chapter 4 for the 2WD E4.

The powertrain of the vehicle is structured as follows. Each of the rear wheels is driven by an electric machine (YASA750) and motor controller (Sevcon Gen4 Size8). The motor controllers receive torque demands from the Powertrain Control Module (PCM) over a Controlled Area Network (CAN) bus, and feed back the motor current and rotor velocity. The PCM receives data from across the vehicle including from the driver (throttle pedal position), from the battery (DC voltage and current), and from the Inertia Measurement Unit (IMU) (chassis acceleration and yaw rate). The PCM is implemented on a dSPACE MicroAutoBox II 1401/1511. A vehicle schematic of the E4 coupe is shown in Figure 6.2 which highlights the key powertrain components.

In the literature review, Chapter 2, the importance of the refresh times of the powertrain controllers and signals is highlighted. The refresh time of the PCM is 1 ms. The torque demands from the PCM are sent to the motor controllers every 10 ms over the CAN bus. The motor controllers transmit the rotor angular velocity and motor current every 10 ms over CAN.

The torque control of the electric machines is contained within the PCM software. A simplified diagram of the torque control structure is shown in Figure 6.3. A proprietary algorithm determines the Primary Torque Demand (T^*) from the driver's throttle pedal position among other signals. The Control Allocation determines the individual wheel

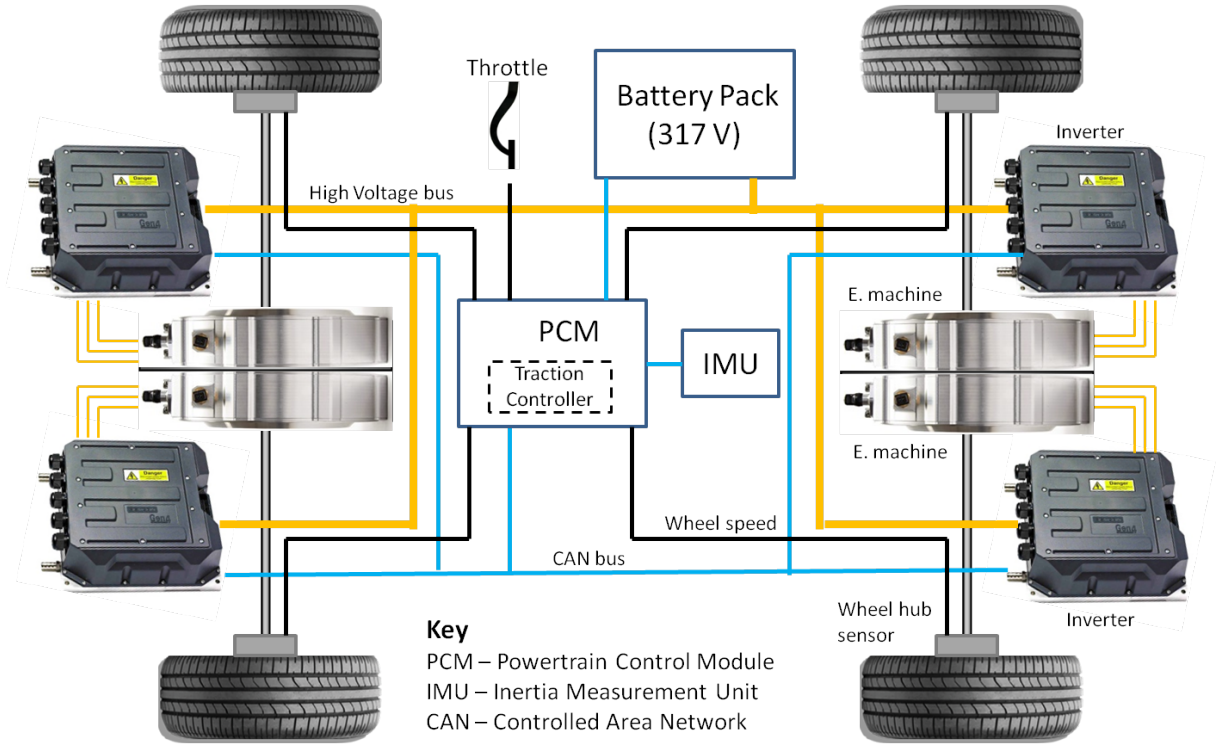


Figure 6.2: Vehicle schematic of 4WD E4 coupe featuring the key powertrain components.

torques. For the testing described in this chapter only the rear wheels are driven and the primary torque demand is allocated 50/50 left/right. The torque demands (T_{LX}, T_{RX}) from the Control Allocation feed into the Traction Controller which is the focus of this research. The output torques of the traction controller are T_{LX} , and T_{RX} . At the final stage of the torque structure there is the System Protection which limits the torque demands (T_{LB}, T_{RB}) sent to the motor controllers based on the power limits of the battery pack and electric machines.

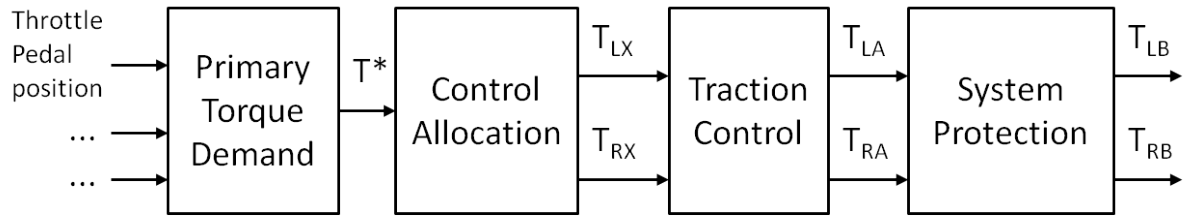


Figure 6.3: A simplified diagram of the torque control structure within the PCM.

6.1.1 Controller implementation

The proposed 2WID-MTTE traction controller from Chapter 5 is integrated into the torque control structure of the PCM described above. The Simulink model of the 2WID-MTTE traction controller is shown in Figure 6.4. The 2WID-MTTE controller receives RL_TqDmd_X and RR_TqDmd_X torque demand inputs from the Control Allocation. The RL_TqDmd_A and RR_TqDmd_A torque outputs are sent to the System Protection. The Friction Torque Estimator blocks are modified from Chapter 5, this is discussed below in Section 6.1.2. The model for the Rate Compensation block, defined by Equation (3.16), is shown in Figure 6.5. The Saturation L2 and Saturation R2 blocks in Figure 6.4 are used to implement the equal torque strategy previously defined in Chapter 5. The PCM software, including the 2WID-MTTE controller, is auto-coded in Simulink and then compiled onto the MicroAutoBox hardware. The parameters used in the 2WID-MTTE controller are given in Table 6.1, where the vehicle mass is the mass of the 4WD E4, driver and passenger.

Parameter	Symbol	Value	Units
Vehicle Mass (inc. driver and passenger)	M	1287	kg
Wheel radius	r	0.296	m
Driveline Inertia	J_w	1.04	kg.m ²
Relaxation factor	α	0.9	
Speed and torque filter time constant 1	τ_{lpf1}	10	ms
Speed and torque filter time constant 2	τ_{lpf2}	20	ms
Rate compensation gain	G	0.1	
Rate compensation time constant	τ_c	30	ms

Table 6.1: Parameters used in the 2WID MTTE controller

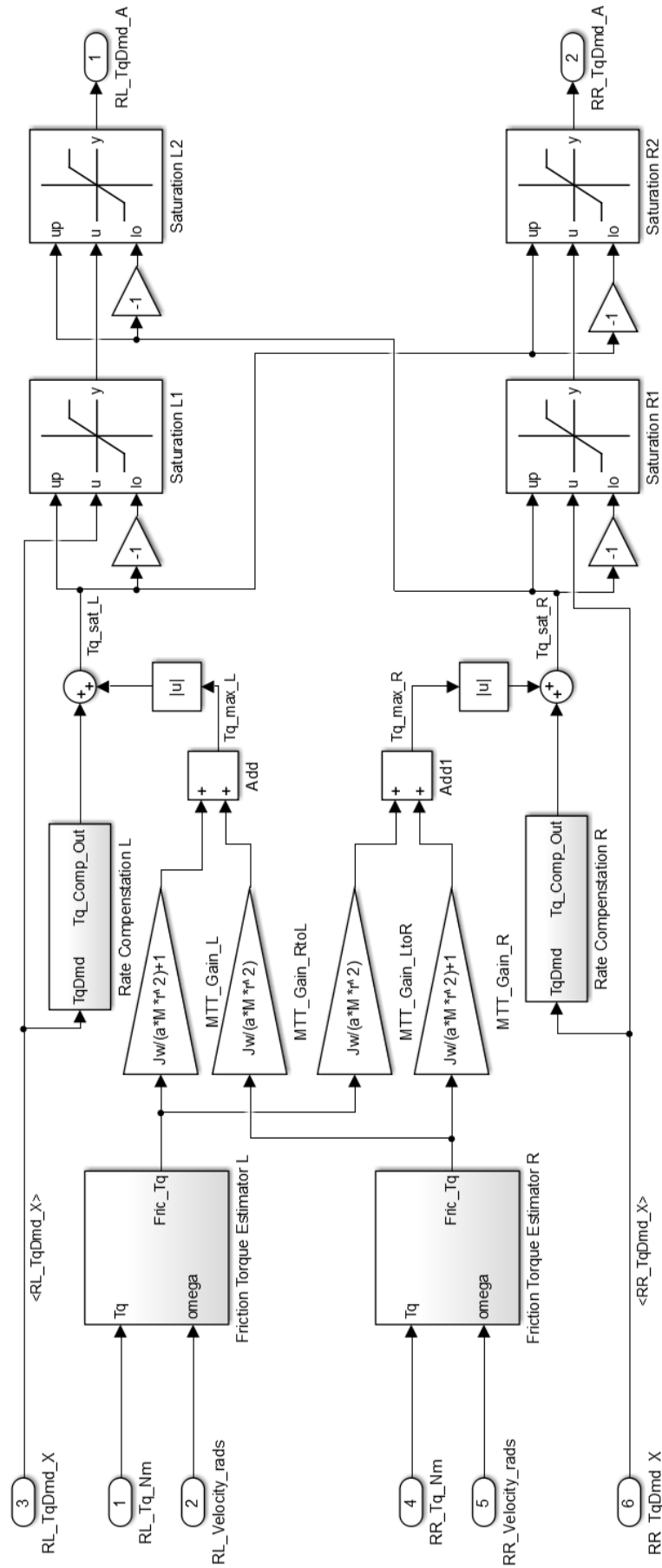


Figure 6.4: Simulink model of 2WID MTTE controller.

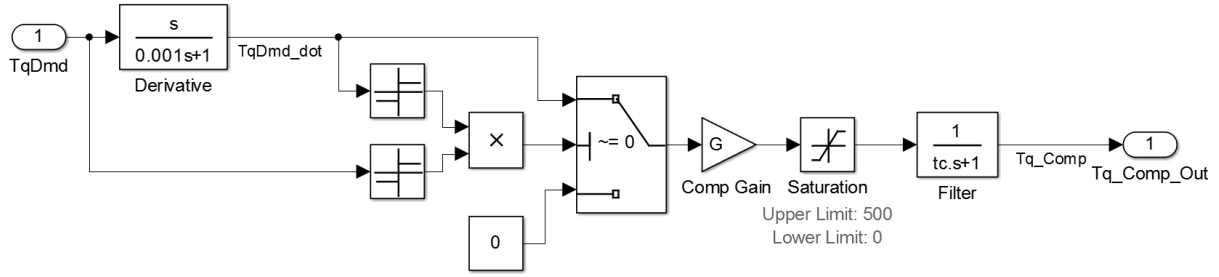


Figure 6.5: Simulink model of rate compensation from Equation (3.16).

6.1.2 Low pass filtering of differentiated wheel velocity

In the 2WID-MTTE controller the friction torque at each wheel is estimated from the wheel velocity and the wheel torque. The wheel velocity is obtained from the rotor velocity transmitted by the motor controller over CAN. This source of wheel velocity is chosen as it is generally available for any conventional powertrain controller, as opposed to using a direct-fed wheel hub sensor. The wheel torque is also obtained from the motor controller over CAN to ensure the two signals are in phase. The wheel torque is calculated from motor current and rotor temperatures as described in Chapter 4.

In the Friction Torque Estimator to calculate wheel acceleration the wheel velocity is differentiated. The rotor velocity is transmitted every 10 ms, however, it is found that the Sevcon motor controller only updates the actual value every 20 ms, as can be seen from Figure 6.6 (raw CAN data). If the raw signal in Figure 6.6 is differentiated it will show the wheel acceleration as being zero every 20 ms. To mitigate this an additional filter is added before the differentiator.

A linear first order filter¹ is used to filter the wheel velocity as it is straight forward to implement in the Simulink framework, and it smooths out the raw data albeit with an added delay. Figure 6.6 shows the rotor velocity low pass filtered with three different time constants. The 10 ms time constant is used as it removes the flat sections without losing too much of the transient information in signal. The wheel torque is filtered in the

¹Other techniques exists such as re-sampling the data every 20 ms. This has advantage of not adding a delay as long as the signal is sampled at the correct instance. Given more time this would be the preferable option.

same way to keep the two signals in phase. A diagram of the modified friction torque estimator is shown in Figure 6.7. The other low pass filter time constants are given in Table 6.1.

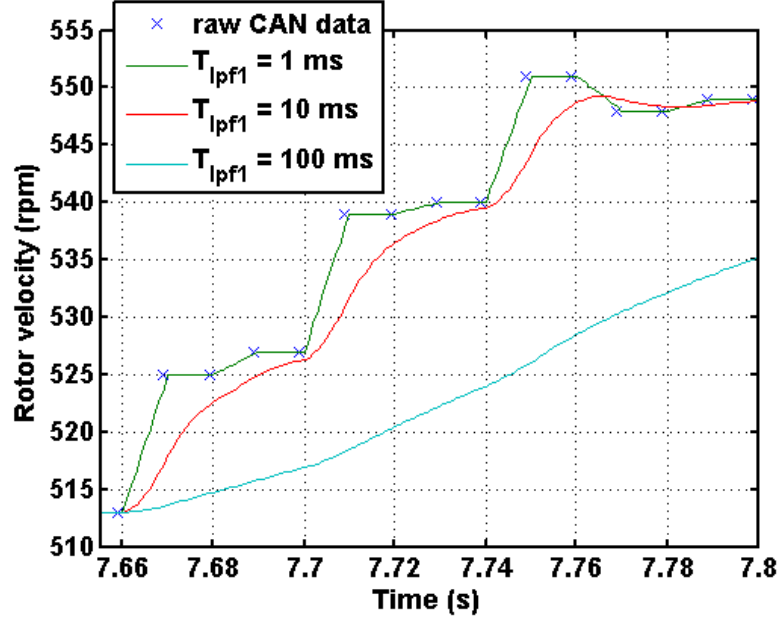


Figure 6.6: Rotor velocity transmitted by the motor controller over CAN.

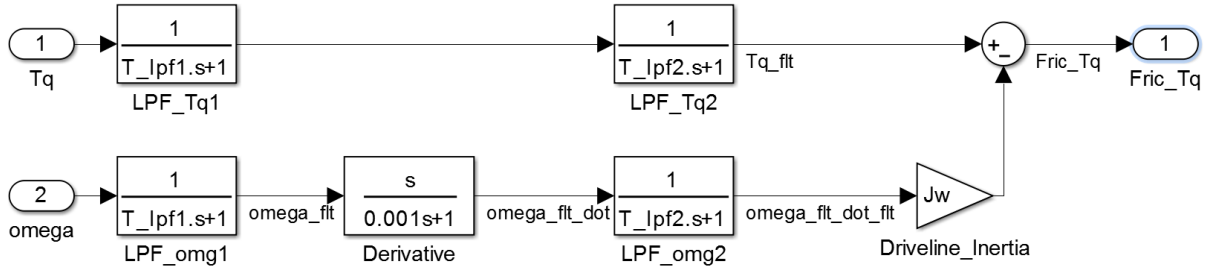


Figure 6.7: Modified friction torque estimator with pre- and post-differentiator filtering.

6.2 Test plan

In this section the experimental tests required to verify the traction controller are defined. The tests are carried out at MIRA proving ground, MIRA (2016). Based on the available surfaces at the test circuit and the required tests a test schedule is laid out. Finally details of the experimental setup are given, including the data that is recorded for each test.

The traction controller is experimentally verified by accelerating the vehicle in a straight line across the following changes in road condition:

- High-to low- μ road
- Low- to high- μ road
- Split- μ road

These tests are chosen because they cover the main changes in tyre/road friction a 2WD vehicle will experience when accelerating on a straight flat road. The aim of these experiments is to understand how real world road conditions affect the directional stability of the vehicle and how the implementation of the traction controller affects its performance.

A high-to low- μ road is used to test the situation where the driver encounters an unexpected change in road conditions (such as a patch of water or ice) after accelerating normally. This experimental test follows on from the simulation tests in Chapter 3 which investigate single wheel MTTE control on a quarter vehicle model. Even though the controller in the simulation differs from the experiment, they are both MTTE based methods. Therefore the comparison to the simulation helps assess the impact of real-world implementation on the 2WID-MTTE controller's performance.

The high-to low- μ road test is the main test case used for traction control in the literature. However the tests in the literature often use small sections of low- μ where the controller does not reach steady-state thus making the interpretation of the results more difficult. In addition the width of the low- μ surfaces used in the literature is often not much wider than the tyre, which prevents testing the directional stability of the vehicle. These two issues are addressed in this research by using a low- μ surface that is: wider than the track width of the vehicle so that vehicle stability can be properly tested; and long enough so that controller stability can be tested.

A split- μ road is used to test the vehicle's directional stability for asymmetric conditions between the left and right tyres. This follows on from the split- μ road simulations in

Chapter 5.

A low- to high- μ road is used to test two situations. Firstly, how the controller responds when the road surface changes so that the tyre/road friction is no longer the limiting factor. Secondly, whether at the transition from low- to high- μ oscillations occur in the tyre friction force that would validate the Pacejka tyre model with transient dynamics used in Chapter 5. As above, the low- μ surface used in the test is sufficiently large to investigate vehicle directional stability and controller stability.

For these tests the proposed 2WID-MTTE control is benchmarked against no traction control as well as Independent MTTE control (without equal torque strategy).

6.2.1 Test track

The experimental tests are carried out on the Straight Line Wet Grip (SLWG) circuit at MIRA proving ground. The SLWG circuit consists of a number of lanes with different surfaces. The layout of the circuit is shown in Figure 6.8. Each of the surfaces has a different friction coefficient when wet. The friction coefficients and dimensions of the surfaces provided by MIRA are given in Table 6.2. The test area is wetted with a built in sprinkler system.

It should be noted that although the ceramic surface has the lowest friction coefficient, at the time of testing its condition made it unusable. The aquaplaning section was also not available.

Surface	Dimensions	Nominal μ (wet)
Smooth concrete	100 m x 4 m	0.40
Bridport	100 m x 4 m	0.40
Asphalt - 2 lanes	100 m x 4 m	0.70
Delugrip	100 m x 4 m	0.75
Basalt tiles	200 m x 7 m	0.30
Ceramic tiles	100 m x 3 m	0.10
Aquaplaning	66 m x 4 m	/

Table 6.2: Road surfaces of SLWG circuit at MIRA proving ground, with dimensions and nominal road friction coefficient when wet.

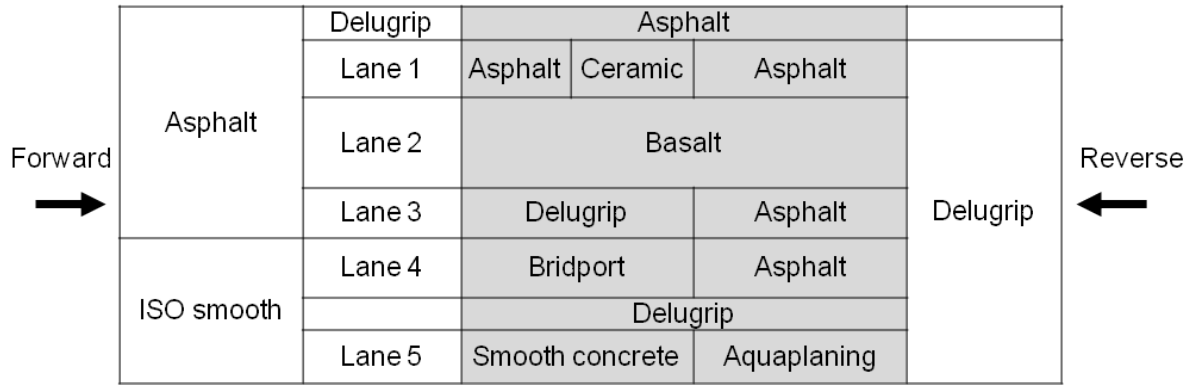


Figure 6.8: Lane layout for the Straight Line Wet Grip (SLWG) circuit at MIRA proving ground. Surfaces covered by sprinkler system shown in grey.

6.2.2 Test schedule

Based on the required tests and the available surfaces identified above a schedule of testing is described in Table 6.3. The high-to mid- μ road test is carried out as well as the high-to low- μ road test, to distinguish more precisely the road conditions which affect vehicle stability.

Split- μ road tests carried out on an asphalt/basalt tiles surface show that even without traction control or driver intervention the vehicle continues accelerating straight ahead. As the vehicle's directional stability without traction is unaffected the split- μ road test is not analysed further. For completeness the results are presented in Appendix D.

The high-to low- μ road from asphalt to basalt tiles is used to benchmark the the 2WID-MTTE controller against an Independent MTTE controller. This test is selected to compare the two controllers as it has a significant effect on the directional stability of the vehicle.

Test	Surfaces	Lane(s)	Direction	Benchmarked controllers
High-to mid- μ	Asphalt/Bridport	4	Forward	2WID-MTTE, no TC
High-to low- μ	Asphalt/Basalt tiles	2	Forward	2WID-MTTE, no TC
Low- to high- μ	Basalt tiles/Asphalt	2	Reverse	2WID-MTTE, no TC
High-to low- μ	Asphalt/Basalt tiles	2	Forward	2WID-MTTE, Independent MTTE

Table 6.3: Test schedule for experimental verification

6.2.3 Experimental setup

For each test the vehicle starts in the lane and faces the direction specified in Table 6.3. The driver accelerates the vehicle up to an initial velocity of 4 m/s. When the vehicle is 10 m in front of the change in surface, the driver applies full throttle. A torque limit of 500 Nm per wheel is set in the PCM so that a constant torque demand is achieved during the test. The driver continues to apply full throttle until the end of the test or until he losses control of the vehicle. The driver applies steering correction if he senses the vehicle yawing from its intended straight-line path.

For each test the variables are recorded from the PCM (MicroAutoBox) to laptop via Ethernet cable using Control Desk software at a rate of 5 ms. The following variables from within the PCM are recorded: torque demands into traction controller; torque outputs to the motor controllers; and friction torque estimates. The following variables are recorded from sensors on the vehicle for reference purposes only: wheel velocities from wheel hub sensors; steering wheel angle from a sensor mounted on the steering column, and acceleration and yaw rate from the Inertia Measurement Unit (IMU) mounted at the centre of the chassis. As the front wheels are not driven for these tests, an average of the front wheel velocities is used to approximate the vehicle velocity.

6.3 Experimental results

In this section the experimental results are presented for the tests described in Table 6.3. The proposed 2WID-MTTE controller is compared to no traction control for a high- to mid- μ road and high- to low- μ road. This helps understand the conditions that lead to the vehicle's directional stability. A low- to high- μ road test is carried out to help understand tyre friction oscillations observed in the low- to high- μ road simulations of the previous chapter. As the high- to low- μ road test is found to affect the vehicle's directional stability the most, this test is used to compare the proposed 2WID-MTTE controller against Independent MTTE control. This helps to assess the importance of the equal torque strategy for vehicle directional stability.

6.3.1 High- to mid- μ road test

The proposed 2WID-MTTE controller is tested on the E4 coupe accelerating from high- to mid- μ road. The high- μ road is a wet asphalt surface with a nominal road friction coefficient of $\mu = 0.7$. The mid- μ road is a wet Bridport surface with a nominal road friction coefficient of $\mu = 0.4$. This test aims to investigate the vehicle stability for a step down in μ of 0.3, where the road conditions are approximately equal for both sides of the vehicle. The same test is repeated for the vehicle with no traction control to benchmark the 2WID-MTTE controller against.

The results from both tests are shown in Figure 6.9. For each test the vehicle starts on the high- μ road at approximately 6 m/s, V_x Figure 6.9b. The initial torque demand is 500 Nm per driven wheel, T_X Figure 6.9a. For both tests the rear driven wheels of the vehicle enter the mid- μ road at $t = 2$ s, which can be observed from Figure 6.9b as the rear right wheel in each test begins to slip (V_{RR}).

In the 2WID-MTTE controller test the increase in acceleration of the rear right wheel after $t = 2$ s reduces the Maximum Transmissible Torque Estimate which leads to the controller limiting the torque output (T_B) for both driven wheels after $t = 2.03$ s, Figure

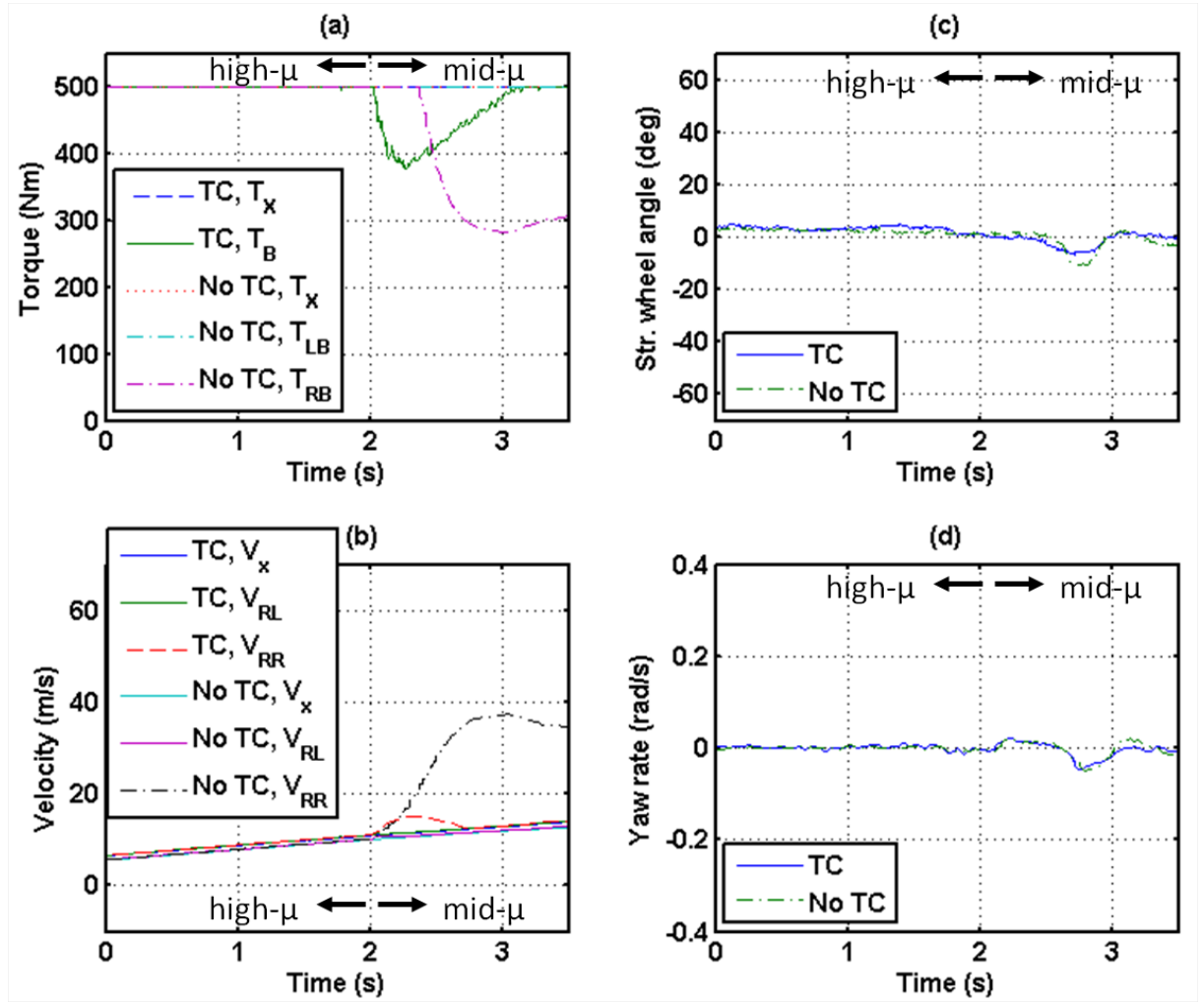


Figure 6.9: Experimental results for E4 coupe accelerating from high- to mid- μ road. Compares 2WID-MTTE control to no traction control. Key: TC, traction control; T_X , torque demand (per wheel); T_B , torque output (per wheel) V_x , vehicle longitudinal velocity; V_{RL} , rear left wheel velocity; V_{RR} , rear right wheel velocity.

6.9a. This prevents the velocity of the rear right wheel increasing further, and in this case it returns close to the vehicle velocity (V_x) from $t = 2.8$ s. The torque output (T_B) returns to the demand torque (T_X) at $t = 3.1$ s.

In 2WID-MTTE test the equal torque strategy reduces the torque to both driven wheels by the same amount. As the conditions for each tyre are not identical this means the left tyre does not slip, Figure 6.9b. The benefit of this is that the left tyre has greater capacity to generate a lateral force and stabilise the vehicle. This is evident from the yaw rate remaining small (± 0.05 rad/s), Figure 6.9d. In addition there is almost no driver steering intervention as there is little change in steering wheel angle (± 10 degrees), see Figure 6.9c. These results show that the vehicle continues to head in a straight line after entering the mid- μ road.

It is possible to make some comparison between this experiment and the simulation test results from Chapter 3 for the response of the controller. In Section 3.4.2 the simulation test uses similar vehicle parameters to the experimental test and also reduces torque by approximately the same amount (100 Nm) on transition to the lower μ road. Although it should be noted that in the simulation the MTTE control is of a single wheel only and the change in μ is 1.0 to 0.3. The simulation results in Figure 3.19 show the time between wheel slip starting to increase and reaching its peak is 100 ms. In the experiment this time is approximately 300 ms. This suggests there are larger delays in the experiment that lead to a slower controller response. In the simulation the wheel velocity and wheel torque are low pass filtered (30 ms time constants) and the system time constant is 20 ms. In the experiment the wheel velocity and wheel torque signal are each low pass filtered twice (10 ms, 20 ms time constants), see Section 6.1.2 above. As the lower pass filtering in the simulation and experiment are similar, this suggests the system time constant of the experiment is much greater than 20 ms.

For the test without traction control the rear right (V_{RR}) wheel spins after it enters the low- μ surface, see Figure 6.9b, which indicates a loss of traction, whereas the left wheel velocity remains close to the vehicle longitudinal velocity (maintains traction). This is

due to slightly different levels of grip as it is observed that the water covering the track is not completely even. The right wheel only stops accelerating after the torque output (T_{RB}) is limited, Figure 6.9a from $t = 2.4$ s. This is due to the power limit set by the System Protection in the PCM. This results in a torque difference of 200 Nm between the rear driven wheels, after $t = 2.7$ s. However, this has little effect on the vehicle's direction stability as there is little change in the steering wheel angle (± 10 degrees) or the yaw rate (± 0.05 rad/s) of the vehicle, Figure 6.9c and 6.9d respectively. This shows that under these test conditions a torque difference of 200 Nm, together with the reduced lateral force on the right rear wheel due to the loss of traction, is insufficient to destabilise the vehicle.

6.3.2 High- to low- μ road test

The 2WID-MTTE controller is tested on the E4 coupe accelerating from high- to low- μ road. The high- μ road is a wet asphalt surface with a nominal road friction coefficient of $\mu = 0.7$. The low- μ road is a wet basalt tile surface with a nominal road friction coefficient of $\mu = 0.3$. This test aims to investigate the vehicle stability for a step down in μ of 0.4, where the road conditions are approximately equal for both sides of the vehicle. The same test is repeated for the vehicle with no traction control.

The results from the two tests are shown in Figure 6.10. For each test the vehicle velocity starts at $V_x = 6$ m/s, Figure 6.10b. The initial torque demand is 500 Nm per driven wheel, T_X Figure 6.10a. For both tests the rear driven wheels enter the low- μ road at $t = 2$ s.

The results of the 2WID-MTTE controller test show the right wheel initially slips (V_{RR}) after entering the low- μ road, Figure 6.10b. The increase in right wheel acceleration means that the Maximum Transmissible Torque Estimate for the right wheel reduces. This causes the controller to reduce the output torque (T_B) to both driven wheels due to the equal torque strategy. In Figure 6.10a it can be seen that the output torque to each driven wheel (T_B) reduces to less than 200 Nm after entering the low- μ road. The steering

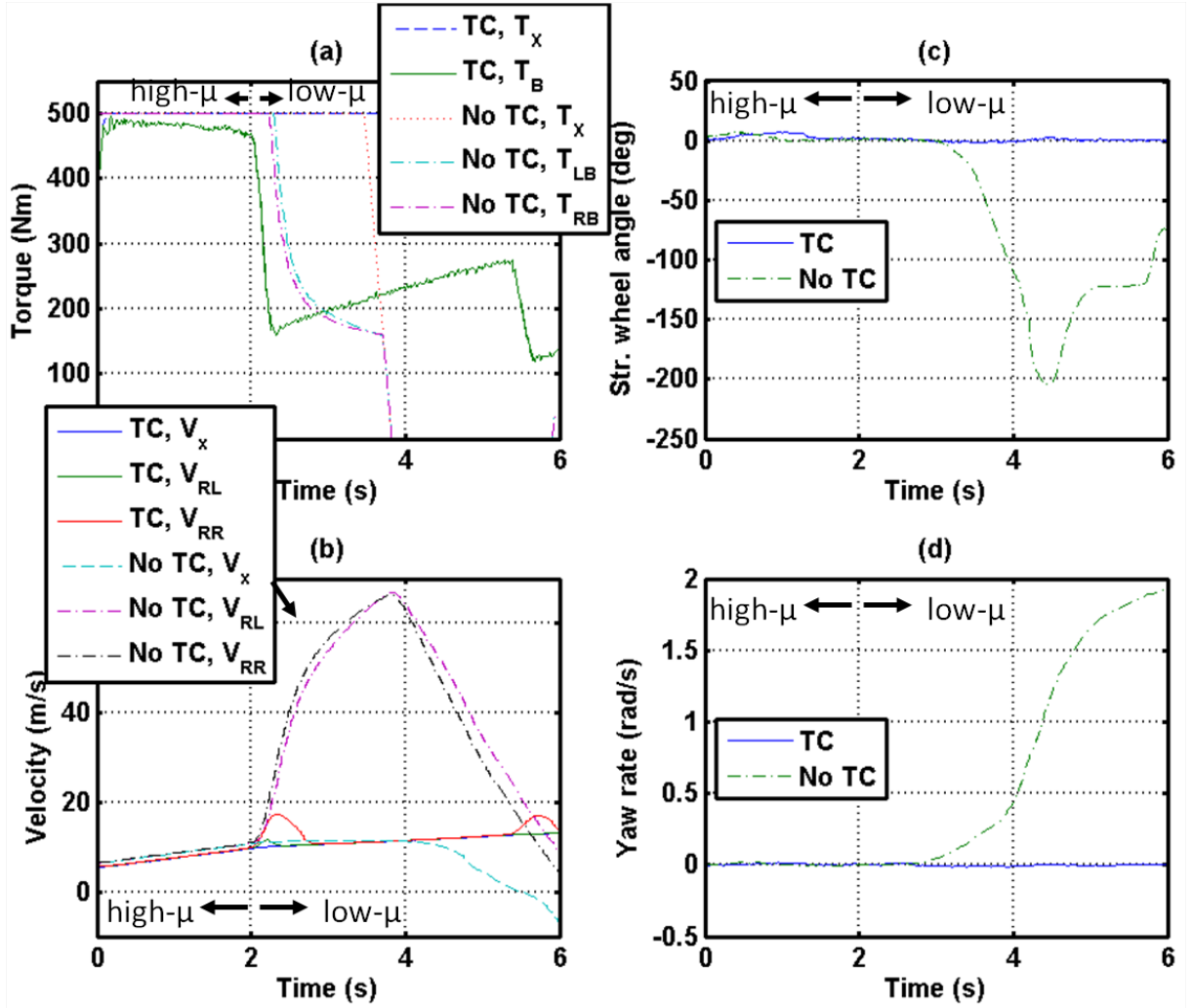


Figure 6.10: Experimental results for E4 coupe accelerating from high- to low- μ road. Compares 2WID-MTTE control to no traction control. Key: TC, traction control; T_X , torque demand (per wheel); T_B , torque output (per wheel) V_x , vehicle longitudinal velocity; V_{RL} , rear left wheel velocity; V_{RR} , rear right wheel velocity.

wheel angle in Figure 6.10b and yaw rate in Figure 6.10d are near zero, ± 7 degrees and ± 0.02 rad/s respectively. This shows that the vehicle continues to head in a straight line and therefore directional stability is maintained.

In Figure 6.10a between $t = 2.3$ s and $t = 5.4$ s the torque output (T_B) only increases by 115 N, even though after $t = 2.9$ s the right wheel velocity returns to within 0.1 m/s of the vehicle velocity. This is in contrast to the results of the previous section for the high- to mid- μ road test, Figure 6.9, where the torque output (T_B) returns to the torque demand (T_X) 0.3 s after the wheel velocity returns to close to the vehicle velocity. This suggests for the high- to low- μ road test the 2WID-MTTE controller may not be outputting the absolute maximum transmissible torque. However, this is beneficial if the level of grip reduces further as can be seen in Figure 6.10b at $t = 5.4$ s where the right rear wheel slips again and the controller further reduces its torque output (T_B). The benefit is that a smaller torque output results in less wheel slip.

For the test without traction control both driven wheels excessively slip (wheel spin) after entering the low- μ road, Figure 6.10b (No TC, V_{RL} and V_{RR}). The output torques (T_{LB} and T_{RB}) to the motor controllers are reduced from $t = 2.3$ s, Figure 6.10a, due to the System Protection power limits. At $t = 2.7$ s the yaw rate of the vehicle begins to increase, Figure 6.10d. From $t = 3.1$ s the driver attempts to counter steer, Figure 6.10c, and at $t = 3.5$ s the driver releases the throttle pedal which zeros torque demand, Figure 6.10a. Neither action by the driver is sufficient to prevent the vehicle spinning, which is evident from the reversal of the vehicle velocity after $t = 5.5$ s, Figure 6.10b, together with the very high yaw rate (> 1.5 rad/s), Figure 6.10d.

A comparison of the no control tests on high- to low- μ and high- to mid- μ roads indicates the most significant effect on vehicle directional stability is the loss of traction (wheel spin) on both rear wheels. In the high- to mid- μ road test of the previous section, only the rear right wheel spins and the power limit on the rear right wheel results in a torque difference of 200 Nm between the rear driven wheels. However, this does not result in a significant yaw moment acting on the vehicle, presumably because the lateral tyre forces

of the other three wheels are sufficient to balance the torque difference. On the other hand, in the high- to low- μ road test both rear wheels spin by a similar amount and the power limit results in both torque outputs (T_{LB} and T_{RB}) reducing by a similar amount. In this test the vehicle spins even though the torque difference is less than 80 Nm. An explanation for this is that as both rear wheels are spinning they will not be able to generate any significant lateral tyre forces to counter any torque difference. In effect this makes the vehicle behaviour extremely oversteer meaning it is very unstable, as can be seen from the results.

6.3.3 Low- to high- μ road test

The 2WID-MTTE controller is tested for the E4 coupe accelerating from a low- to high- μ road. The low- μ road is a wet basalt tile surface with a nominal road friction coefficient of $\mu = 0.3$. The high- μ road is a wet asphalt surface with a nominal road friction coefficient of $\mu = 0.7$. The same test is repeated for the vehicle with no traction control.

The results from the two tests are shown in Figure 6.11. For each test the vehicle starts on the low- μ road at a velocity of $V_x = 4$ m/s, Figure 6.11b. At $t = 1$ s the driver applies full throttle which gives a torque demand of $T_X = 500$ Nm per driven wheel, Figure 6.11a. The vehicle accelerates on the low- μ road for approximately 2.5 s before the rear wheels reach the high- μ road.

For the 2WID-MTTE test both driven wheels begin to slip after the torque demand (T_X) reaches 500 Nm at $t = 1.3$ s, Figure 6.11b. The increase in wheel acceleration reduces the Maximum Transmissible Torque Estimate which results in the controller reducing the output torque (T_B) to both wheels, Figure 6.11a. This prevents further wheel slip from occurring after $t = 1.7$ s, see Figure 6.11b. After this time it can be seen that the the gradient of the wheel velocities (wheel acceleration) are approximately equal to the gradient of the vehicle velocity (wheel acceleration). This is to be expected as the relaxation factor, in the controller, is set to $\alpha = 0.9$ to limit the ratio of acceleration between the vehicle and the driven wheels. The equal torque strategy, together with any

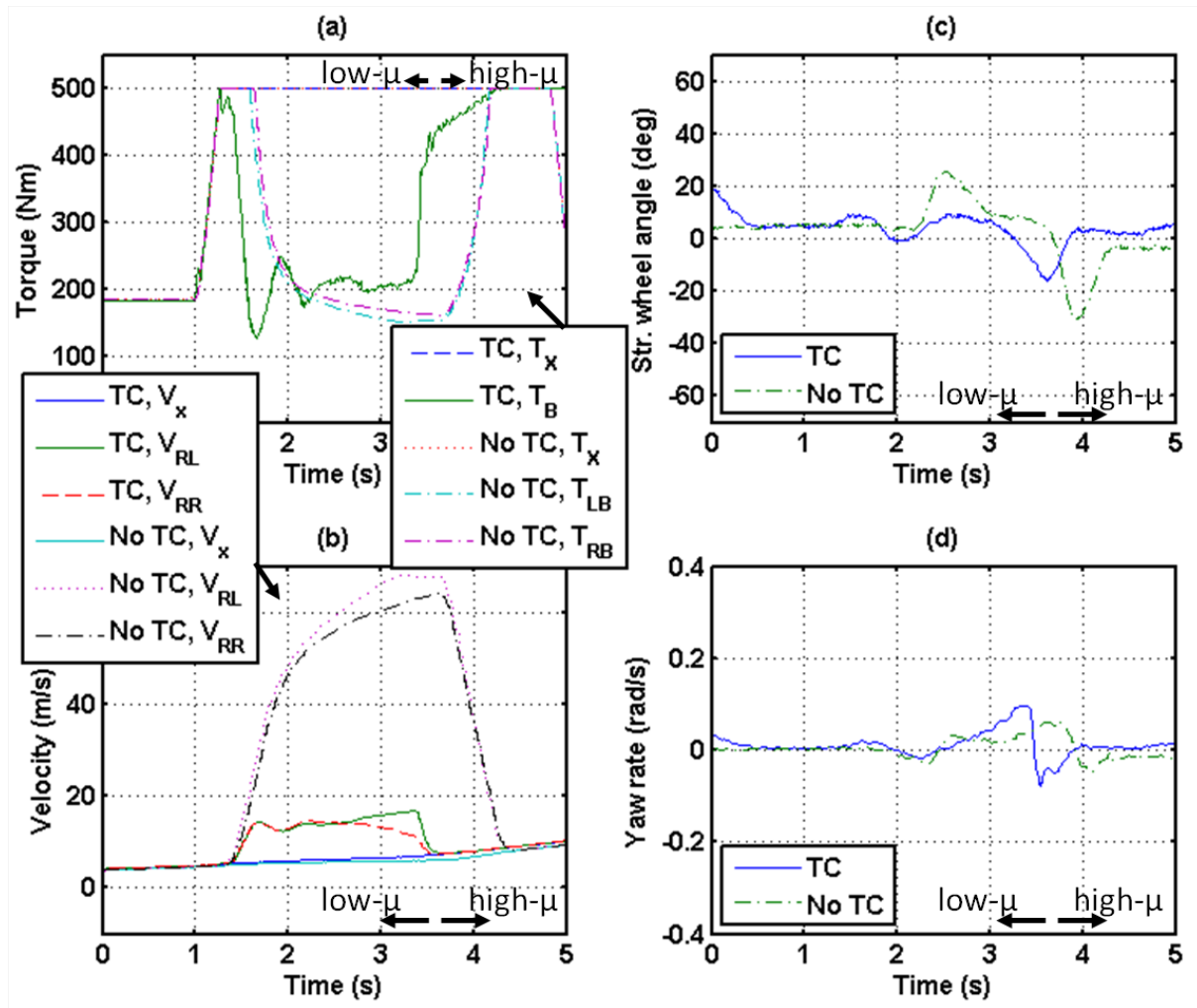


Figure 6.11: Experimental results for E4 coupe accelerating from low- to high- μ road. Compares 2WID-MTTE control to no traction control. Key: TC, traction control; T_X , torque demand (per wheel); T_B , torque output (per wheel) V_x , vehicle longitudinal velocity; V_{RL} , rear left wheel velocity; V_{RR} , rear right wheel velocity.

slight slight difference between road conditions for left and right driven wheels, means that the left wheel velocity (V_{RL}) reduces after $t = 2.3$ s, Figure 6.11b. At $t = 3.4$ s the rear wheel velocities return to the vehicle velocity, as the rear wheels enter the the high- μ road.

Throughout the test the vehicle continues to head in a straight line with little driver steering intervention. This can be seen from the small change in yaw rate ± 0.06 rad/s and the the small change in steering wheel angle ± 15 degrees, in Figure 6.11d and Figure 6.11c respectively.

For the no traction control test both wheels slip excessively (wheel spin) after the torque demand (T_X) reaches 500 Nm at $t = 1.3$ s, Figure 6.11a. At $t = 1.6$ s both torque outputs ((T_{LB}, T_{RB})) become power limited by the System Protection, Figure 6.11a. At $t = 3.6$ s both wheels decelerate as they enter the high- μ road, Figure 6.11b. For this test even though both wheels spin there appears to be no significant effect on the directional stability of the vehicle as the yaw rate remains small ± 0.1 rad/s see Figure 6.11d. The steering angle range of ± 31 degrees is nearly twice as great as the steering angle range of the test with traction control, see Figure 6.11c, although it is still relatively small.

Next the experimental results are compared to the low- to high- μ simulation of Chapter 5. This is to determine the evidence for transient tyre forces in such conditions, and therefore justify the use of a transient tyre model. For the TC test at $t = 3.4$ s the rear wheels decelerate as they transition to the high- μ road, see Figure 6.11b. At this point the output torque (T_B) jumps to 400 Nm before slowly returning to the demand torque (T_X). This slow ramp in output torque after the μ transition is similar to the low- to high- μ simulation result in Figure 5.21, although in the experiment there is no high frequency oscillation immediately before the torque ramp. However, observation of the chassis acceleration in Figure 6.12 does show an oscillation between $t = 3.4$ s and $t = 3.6$ s. The frequency of this oscillation is 17 Hz, which is very similar to the oscillation frequency in the simulation (15-16 Hz). The acceleration of the chassis is due to the forces acting on it which in this case will predominantly be the longitudinal friction forces. This

suggests the tyre model with transient dynamics used in Chapter 5 is valid.

In Figure 6.12 it is also significant that for both tests the acceleration at the μ transition peaks at a much higher value than is expected when comparing to the drive torques applied to the vehicle in Figure 6.11a. This gives an indication of the tyre/road friction behaviour, as it shows that when a tyre transitions from a low- to high- μ road the friction force ‘jumps’ between the friction-slip curves. Furthermore, the friction force then returns along the friction-slip curve through the peak even though this is greater than the force due to the electric machine torque.

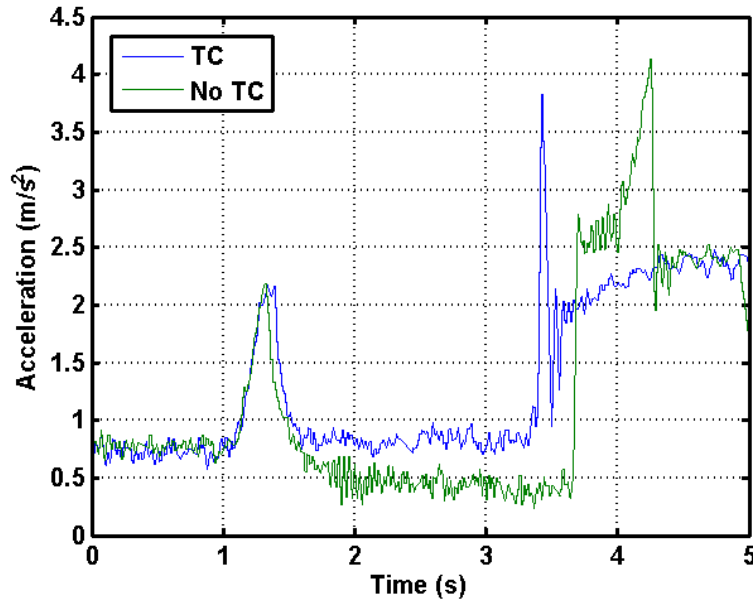


Figure 6.12: Chassis acceleration measured by accelerometer from low- to high- μ road tests. Key TC: Traction Control (2WID-MTTE)

Using the friction estimation for the left wheel from the 2WID-MTTE controller the friction against slip is plotted in Figure 6.13. The slip ratio requires vehicle velocity which is calculated using the average of the front wheel velocities, for analysis purposes only. As the friction estimation is recorded even when the controller is not active, the friction-slip is also shown for the no traction control test. In Figure 6.13 both tests start at 1), then move up the curve to 2) as torque is applied to the wheels. Both tests follow the same curve as the wheel begins to slip. The 2WID-MTTE reaches steady-state at 3) whereas the no traction control continues accelerating to 4). When the tyre enters the high- μ the friction jumps to the peak at 5) before returning to its steady-state value

around 2). It is of interest that the steady-state friction value for the TC curve at 3) is lower than the transient friction for the same slip ratio for no TC. This indicates the friction force differs between transient and steady state, further justifying the use of a transient tyre force model in the simulation.

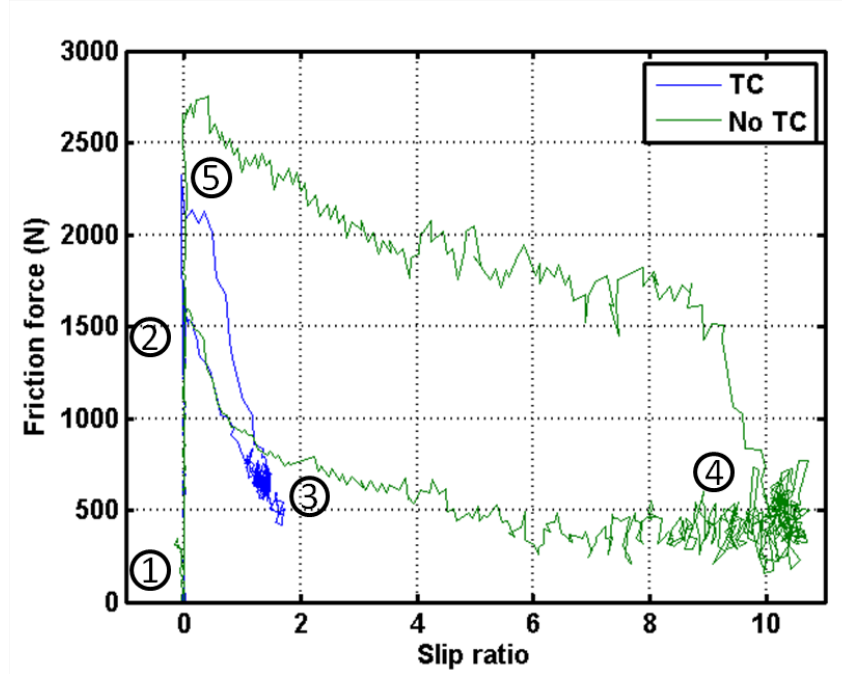


Figure 6.13: Estimation tyre friction force against slip ratio from low- to high- μ road tests. Key TC: Traction Control (2WID-MTTE)

6.3.4 Comparison to Independent MTTE

In this section the proposed 2WID-MTTE control is compared to Independent MTTE control. The Independent MTTE control differs from the 2WID-MTTE control in two respects. Firstly, the Independent MTTE control model does not use an equal torque strategy; Saturation L2 and Saturation R2 blocks, Figure 6.4, not included in controller. Secondly, the wheel velocities for the Friction Estimation are obtained from the direct fed wheel hub sensors. To ensure that the wheel torque and wheel velocity are in phase the torque for the Friction Estimation is obtained from the torque output sent to the motor controllers. High level diagrams of the proposed 2WID-MTTE control and Independent MTTE control are shown in Figure 6.14a and 6.14b respectively.

The use of the direct fed wheel velocity and torque output to estimate friction makes

the Independent MTTE control comparable to the implementation of other MTTE-based methods in the literature. The advantage of a direct-fed signal is that it does not have the additional processing delays from the motor controller or sampling delays from the CAN bus. The disadvantage of a direct-fed signal is that it is not always available to the vehicle's PCM, so the method is less widely applicable. By testing 2WID-MTTE control against Independent MTTE the significance of the equal torque strategy can be compared against the advantage of using direct-fed wheel velocity feedback.

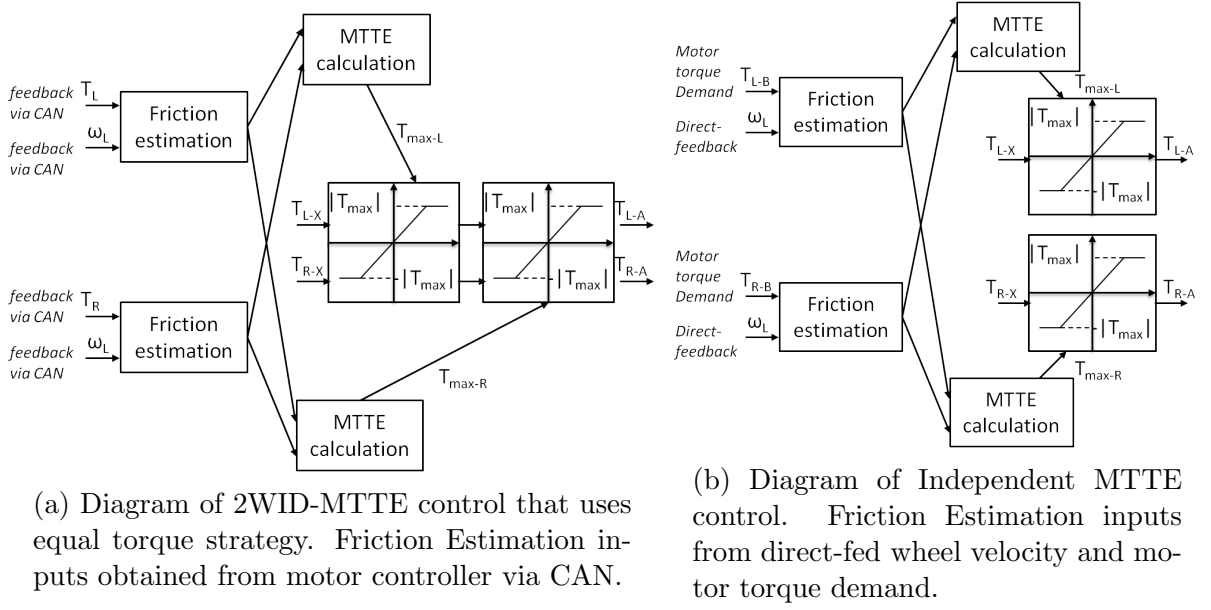


Figure 6.14: Comparison of 2WID-MTTE control to Independent MTTE.

The two controllers are compared experimentally using a high- to low- μ road test as described in Section 6.3.2. For both tests the initial torque demand is 500 Nm per driven wheel, and the rear wheels enter the low- μ road at $t = 1$ s. For the Independent MTTE test the driver releases the throttle, which zeros the torque demand after $t = 2.9$ s. For the 2WID-MTTE test the driver releases the throttle after $t = 4.2$ s. The results from both tests are shown in Figure 6.15. The results show that both controllers prevent excessive slip of the rear wheels from occurring.

From Figure 6.15a it can be seen that the torque response of Independent MTTE is faster than 2WID-MTTE. This results in the maximum wheel slip being less, see Figure 6.15b. For Independent MTTE the faster response comes at the price of increased oscillations. This reduction in controller stability is expected given that the delay in the wheel torque

and wheel velocity is reduced.

The equal torque strategy of the 2WID-MTTE control means that the lower torque limit is always applied to both wheels. As the road conditions are never exactly the same for each wheel this generally prevents both wheels slipping by the same amount, as seen in Figure 6.15b, where the rear left wheel velocity returns to within 0.1 m/s of the vehicle velocity after $t = 1.7$ s. This means that the rear left tyre can generate sufficient lateral forces to maintain directional stability of the vehicle without the driver's intervention. This is evident from the small change in steering angle (± 5 degrees) together with the small change in yaw rate (± 0.03 rad/s), see Figure 6.15c and d respectively.

For Independent MTTE the torque limits are applied separately, which results in both wheels slipping by a similar amount, Figure 6.15b. This has a significant effect on vehicle directional stability. The increase in slip of both rear wheels reduces the capacity of the rear tyres to generate lateral forces, which means the vehicle behaviour becomes more oversteer and less stable. This is evident from Figure 6.15d as the yaw rate increases after $t = 2.1$ s, and the driver counter-steers from $t = 2.5$ s, see Figure 6.15c.

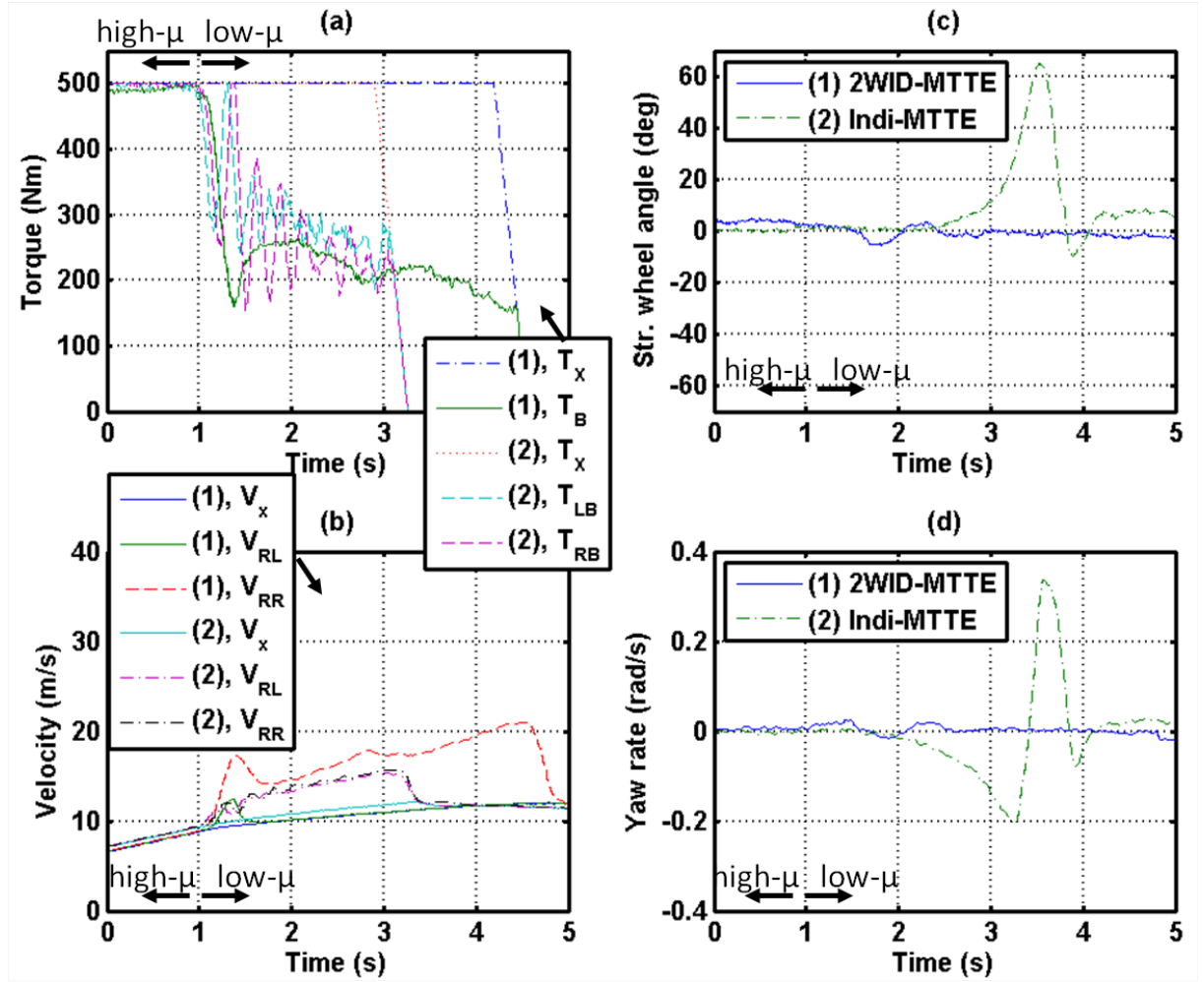


Figure 6.15: Experimental results for E4 coupe accelerating from high- to low- μ road. Compares proposed (1) 2WID-MTTE control to (2) Independent MTTE control. Key: TC, traction control; T_X , torque demand (per wheel); T_B , torque output (per wheel) V_x , vehicle longitudinal velocity; V_{RL} , rear left wheel velocity; V_{RR} , rear right wheel velocity.

6.4 Summary

For this chapter the proposed 2WID-MTTE traction controller is integrated into the Powertrain Control Module (PCM) of Delta Motorsport's E4 coupe. This is done by auto-coding the traction controller together with the PCM in Simulink, and downloading the compiled code onto a MicroAutoBox. The implementation of the 2WID-MTTE controller developed in Chapter 5 requires the wheel velocity to be low pass filtered before and after differentiation. This is due to how often the wheel velocity value is updated over CAN and is specific to the motor controller used in the E4 coupe.

The E4 coupe with the 2WID-MTTE traction controller is tested on the Straight Line Wet Grip circuit at MIRA proving ground. A range of harsh road conditions is used including high- to low- μ and low- to high- μ . The tests focus on evaluating the vehicle's directional stability as well as the transient nature of the tyre forces. The 2WID-MTTE controller is benchmarked against no traction control and Independent MTTE control with direct-fed wheel velocity sensors.

The results of the high- to mid- μ test show that with no traction control one driven wheel can have significant wheel slip while the vehicle still remains stable. On the other hand the high- to low- μ test with no traction control shows both driven wheels spinning which leads to loss of vehicle directional stability. This suggests that provided one rear wheel maintains traction it is still able to generate sufficient lateral friction force to stabilise the vehicle, whereas when both rear wheels spin there is little to no lateral friction force across the rear axle so the vehicle can be easily destabilised. In both these tests the 2WID-MTTE traction controller quickly reduces its output torque to both wheels which limits wheel slip and maintains the vehicle's directional stability. Comparison to similar simulations results from Chapter 3 reveal that the system delay is much larger than 20 ms, which is in part due to the communication of signals over CAN.

The low- to high- μ test results show that the 2WID-MTTE traction controller prevents excessive slip when starting to accelerate on a low- μ test. The test results also show

that the oscillations in the longitudinal tyre forces predicted by the tyre model do occur in reality, based on the chassis accelerometer measurements. Plotting the friction force estimate against slip ratio reveals that the tyre/road friction does indeed ‘jump’ between friction-slip curves, and explains how the transient friction force can be greater than the drive force. This justifies the use of a transient tyre force model when simulating step changes in μ .

The proposed 2WID-MTTE traction controller is compared to Independent MTTE control with direct-fed wheel velocity sensors. The results show that the faster response of the direct fed wheel velocities compared with the wheel velocities measured via CAN, do reduce the wheel slip. However, the equal torque strategy in 2WID-MTTE control proves more significant at maintaining vehicle directional stability as it generally ensures one of the driven wheels adheres very closely to the road. This allows sufficient lateral tyre force to be generated to stabilise the vehicle and is more important than reducing the slip of any individual wheel.

Chapter 7

Conclusion

As outlined in the introduction there are considerable challenges to traction control due to the non-linear and highly variable nature of the friction forces between the tyre and the road, together with the fact that these forces are very difficult to measure. The coupling between longitudinal and lateral friction forces (friction circle) means that when designing traction control there can be a trade-off between maximising longitudinal acceleration and maintaining vehicle directional stability.

Conventional traction control for ICE vehicles is actuated by the hydraulic brakes and the ignition or throttle of the engine. For traction control in EVs the electric machine can replace both of these actuators due to its four quadrant operation, flat torque-speed curve, and power density. It is widely anticipated that electric machines can improve traction control as their torque response is 5-50 times greater than hydraulic brakes, and they can provide precise and accurate torque measurement. The aim of this thesis is to exploit these attributes of electric machines for traction control and vehicle stability, for a vehicle with independently driven wheels.

The review of the literature on traction control for electric vehicles, in Chapter 2, shows there are a wide range of control methods applied to this topic. Due to the inherent difficulty of measuring the tyre friction many methods have used slip ratio¹ to estimate

¹This may also be due to conventional methods using only the wheel velocity as an input, as the wheel torque is difficult to measure on a ICE vehicle.

peak friction. It is highlighted that slip ratio control has a number of challenges, including detecting vehicle velocity, determining the optimum reference slip, and controlling slip at low speed. A method identified in the literature that does not require slip ratio or vehicle velocity is Maximum Transmissible Torque (MTTE). This is a form of Model Following Control (MFC) that uses the wheel torque and wheel velocity from the electric machine to estimate the longitudinal friction force kinematically.

The kinematic friction force observer used to estimate the longitudinal tyre force is fundamental to MTTE control. The observer's key strength is that it does not rely on *a priori* knowledge of tyre-road friction. However, there has been little investigation to date on how accurate this observer is. It is known that the longitudinal tyre force varies greatly with tyre/road friction coefficient, lateral and longitudinal slip, and vertical load. In Chapter 3 the robustness of the friction force observer to the above conditions is shown through simulation comparison with the Pacejka tyre model. The observer also captures the velocity dependent hysteresis of the tyre friction model. It is suggested in the literature that the friction force observer and MTTE controller are sensitive to driveline inertia variation. In this thesis simulation tests show that the sensitivity is small when considering the likely variation for a full size EV. Furthermore, the driveline inertia variation is much less significant than the friction force variation discussed above. Using this type of open-loop observer reduces complexity and requires less tuning compared with an equivalent closed-loop observer.

A significant gap in the literature is the lack of research in developing a control method from the traction control of a single wheel to the traction control of multiple wheels, in an integrated way, that considers the directional stability of the vehicle. This has also not been thoroughly considered for MTTE control. In Chapter 5 the MTTE equations are derived from first principles for a two wheel independent drive (2WID) vehicle. This shows, for the first time, that the maximum transmissible torque at each driven wheel is dependent on the longitudinal friction force at both driven wheels. An equal torque strategy is proposed for maintaining vehicle directional stability for asymmetric road conditions. These developments are combined with the modified rate compensation from

Chapter 3, to form the proposed 2WID-MTTE traction control.

Computer simulations play an important role in traction controller verification. Compared to experimental tests they are quick, inexpensive, exactly repeatable, and measured variables are not limited by sensor availability. Simulations require a vehicle model of suitable fidelity. Controllers verified through simulation in the literature often use low order vehicle models that do not capture all of the dynamics of interest, and generally these models are not validated; therefore there can be less confidence in their results. The vehicle model used in this thesis addresses both of the above concerns.

A credible high order model is developed using the latest multi-body systems software, and data provided by the target vehicle's manufacturer, see Chapter 4. The model includes detailed aspects of the vehicle such as the suspension geometry. The Pacejka "Magic Formula" is used to model the tyre, which includes transient tyre force properties. The fidelity of the model is ensured through calibration and validation of the model against a range of experimental tests.

The chassis Centre of Gravity (CoG), chassis inertia, driveline inertia, and resistive forces are calibrated with a combination of vehicle manufacturer data and experimental tests. The chassis CoG and inertia are estimated using a detailed list of chassis component masses and locations, together with the parallel axis theorem. This provides a cost effective alternative to measuring the chassis inertia, which requires specialist equipment. The total driveline inertia, which is a key parameter used by the MTTE controller, is calibrated using an in situ wheel spin test on the vehicle. This method does not require removal of any components and makes use of the torque measured by the electric machine.

The experimental validation of the vehicle model includes coast down, straight-line acceleration, constant radius and double lane change tests. The tests are designed to cover a range of transient and steady state manoeuvres. The constant radius test is used to calculate the understeer gradient of the vehicle. This is an important characteristic that defines the handling of the vehicle. The standard literature method requires adaptation to account for variations in path radius between test runs. The model's understeer gra-

dient is tuned using the road friction coefficient and rear tyre cornering stiffness. The model captures the close to neutral steer behaviour of the vehicle at low to mid lateral acceleration, and the exponentially increasing understeer behaviour at high lateral acceleration. The model does not capture the increase in understeer gradient at very low lateral acceleration; however, this is less significant.

The straight-line acceleration and coast down test the longitudinal response of the vehicle while the wheels are driven and while they are not, respectively. The vehicle model shows good agreement with both the measured wheel velocity and the measured longitudinal acceleration from these experiments. The dynamics of the driveline are not completely captured by the model, as there is a short oscillation in the experimental wheel velocity that does not appear in the simulation. The double lane change tests the lateral and yaw response of the vehicle. The vehicle model shows good agreement with the lateral acceleration and yaw rate of these experiments.

The proposed 2WID-MTTE traction controller is verified through simulation in Chapter 5 using the validated vehicle model from Chapter 4. A severe split- μ road is used to test the vehicle's directional stability while accelerating on asymmetric conditions. The 2WID-MTTE controller is benchmarked against a MTTE-based method without the equal torque strategy, termed Independent MTTE control. The simulation comparison shows that the 2WID-MTTE controller maintains directional stability, while the Independent MTTE controller cannot prevent the vehicle spinning off the road.

The 2WID-MTTE controller is also benchmarked against a higher level stability controller from the literature. This alternative method uses Direct Yaw Control together with Independent MTTE, this is termed MTTE+DYC. The simulation comparison on a severe split- μ road shows that 2WID-MTTE is as effective as MTTE+DYC but is simpler in design and requires fewer sensors.

The 2WID-MTTE controller is simulated on a low- to high- μ road. This is used to test the controller's ability to return to normal operation after a traction event. This type of test is not often used in the literature. The simulation results show that the μ transition excites

an oscillation in the tyre friction force that is due to the transient characteristics of the tyre model. Compared to no control the 2WID-MTTE is able to damp these oscillations. However, there is a slight reduction in performance as the torque output takes several seconds to return to the torque demand after the μ transition. This is advantageous if the road μ decreases rapidly again.

2WID-MTTE controller is tested experimentally on Delta Motorsport's E4 coupe. This is a prototype EV, although much more representative of production EVs than many of the test vehicles used in the literature. E4 coupe is a full size EV (>1000 kg, 2 by 2 seater), with two independent On-Board Motors (OBMs) driving the rear wheels. The 2WID-MTTE controller is implemented within the Powertrain Control Module (PCM) of the E4 coupe. The communication of torque demands and sensor feedback between the traction controller and motor controller is done via a Controlled Area Network (CAN) bus. CAN buses are widely used in the automotive industry as they reduce the size, cost and complexity of wiring harnesses, compared to direct-fed signals. An additional filter is used to compensate for the signals from motor controller only being updated every other message. Given more time, re-sampling the signals would be more appropriate.

In the literature low- μ surfaces for experiments are often constructed from sections of metal or acrylic sheet, covered in water, that are not much wider than a tyre's width. The experimental tests for this research are carried out at the MIRA proving ground. This specialist test facility provides a range of road surfaces with different levels of friction coefficient. The size of the road surfaces is sufficiently long to test controller stability and sufficiently wide to test vehicle directional stability.

The 2WID-MTTE traction controller is experimentally verified for the E4 coupe accelerating on the following road conditions: high- to mid- μ , high- to low- μ , and low- to high- μ . This is benchmarked against the same tests without Traction Control (TC). The high- to mid- μ test results show that 2WID-MTTE controller prevents excessive slip of both driven wheels, whereas in the no TC test the rear right wheel spins. However, in both the test with and the test without TC the vehicle continues in a straight line, meaning that

vehicle directional stability is unaffected.

The high- to low- μ tests results show that the 2WID-MTTE controller prevents excessive slip of both driven wheels. For the no TC test both rear wheels spin, and the vehicle spins. In comparison to this, the high- to mid- μ test shows the importance of maintaining good grip of at least one wheel per axle, for maintaining the directional stability of the vehicle.

The low- to high- μ test results show that the 2WID-MTTE traction controller prevents excessive slip when starting to accelerate on a low- μ road. Neither the test with or without TC has a significant effect on vehicle directional stability. At the transition to high- μ , oscillations in the longitudinal tyre forces occur. The oscillations in the experiment have a frequency within 1 Hz of the oscillations for the same simulation test. This suggests the transient tyre model used in the simulation is valid.

The proposed 2WID-MTTE traction controller is benchmarked against an Independent MTTE controller. The Independent MTTE controller uses direct-fed wheel velocity sensors and the torque output to estimate friction torque, instead of the wheel velocity and torque feedback from the motor controller via CAN. The use of these signals makes the Independent MTTE controller comparable to the implementation of other MTTE-based methods in the literature. The advantage of a direct fed signal is that it does not have the additional processing delays from the motor controller or sampling delays from the CAN bus. The experimental results show that the maximum wheel slip of Independent MTTE is less than that of 2WID-MTTE, as the delays in the system are less. However, the 2WID-MTTE maintains vehicle directional stability without driver intervention, which is ultimately more important. This is achieved as one wheel adheres very closely to the road, due to the equal torque strategy.

7.1 Contributions to knowledge

This thesis contributes to research by developing MTTE control for a two wheel independent drive electric vehicle. By deriving the controller equations from first principles it is shown for the first time that the estimate for maximum transmissible torque of a driven wheel is dependent upon the friction torque estimates of all of the driven wheels. This highlights the importance of taking an integrated approach to traction control, as opposed to considering each wheel in isolation.

An analysis of the rate compensation, first proposed by Yin et al. (2009), identifies that it is necessary to modify this term to allow correct handling of both positive and negative, increasing and decreasing torque demands. This is important if the traction controller is to be integrated with higher level stability functions such as torque vectoring. The effectiveness of the modified rate compensation is verified through simulation.

The simulation and experimental results show the importance of the equal torque strategy at maintaining vehicle directional stability. The experimental results show that equal torque strategy is more significant than faster feedback for improving vehicle directional stability.

To summarise, the contributions of this thesis are:

1. Extended MTTE to vehicle's with multiple driven wheels.
2. Proved that maximum transmissible torque of each wheel is dependent on the friction estimates of all the driven wheels.
3. Proposed a modified rate compensation for MTTE that allows both positive and negative torques to develop.
4. Proposed an equal torque strategy, and shown through simulation and experimentation how this can improve vehicle directional stability under harsh conditions.
5. Implemented 2WID-MTTE controller on full size production prototype EV.

6. Shown that delays due to CAN communication increase wheel slip.

7.2 Further work

There are several avenues that could be explored to extend the control method in this thesis. A straight forward development would be to derive a MTTE-based controller for four wheel independent drive (4WID), following the same principles used in Chapter 5. As noted by Ivanov et al. (2015b), there is very limited research into traction control for 4WD electric vehicles.

Parameter tuning is an area that could be explored more thoroughly. According to Yin et al. (2009) the time constant of the powertrain restricts the time constant of the filters, methods such as Ivanov et al. (2014) could be used to experimentally determine the time constant of a powertrain and therefore improve the tuning of the filter time constants. A new rate compensation is proposed in Section 3.2. Further simulation and experimentation is needed to determine the most suitable gain and time constant, in particular for the case when the torque demand is changing while the road μ is also changing.

The 2WID-MTTE with equal torque strategy has proven its worth for straight-line driving under asymmetric road conditions. However, to function with high level stability controllers that use torque vectoring, the 2WID-MTTE requires adaptation. The equal torque strategy together with the fact that the maximum transmissible torque follows the torque demand, mean that the torque difference that is allowed between left and right wheels is too small. To allow a greater torque difference to develop, which would allow an adequate yaw moment to be applied, the equal torque strategy needs to be conditional. This condition could be based on whether the vehicle is turning, and/or whether any of the wheels are traction limited. The 2WID-MTTE traction control could also feedback to the higher level controllers information on the longitudinal friction force estimate at each tyre, together with the allowable torque difference out of the traction controller.

In terms of verification testing this research has extensively explore straight-line acceler-

ation of flat roads. It would be interesting to test the controller for steering manoeuvres, and on road gradients. Steering manoeuvres could be combined with changes in road condition to further test the vehicle's directional stability. For steep road gradients the equal torque strategy may need to be adapted to give higher priority to longitudinal acceleration to allow the vehicle to climb the gradient.

From the experimental validation tests in Chapter 4, some practical recommendations can be made for sensing and test procedures. Considerable vibrations can be generated in the vehicle chassis during tests, particularly if the road surface is rough or of poor quality and the vehicle suspension is stiff. This necessitates careful placement of sensors such as accelerometers to reduce the road noise in their measurements. The constant radius test is a common test used to measure the understeer gradient of a vehicle. However, it is challenging for the test driver to produce consistent results as they are attempting to modulate the steering wheel angle and throttle pedal position simultaneously to follow a fixed path radius. A way to improve this test would be to use a cruise controller to allow the driver to focus on steering the vehicle. Alternative tests such as constant throttle and constant steer, Gillespie (1992), can also be used to determine understeer gradient.

Suggestions for further work are summarised as follows:

1. Apply MTTE to a four independent wheel drive vehicle.
2. Investigate tuning of rate compensation and filter parameters.
3. Adapt 2WID-MTTE so it can allow a difference in torque demands to left and right wheels, under suitable road conditions (torque vectoring compatibility).
4. Make use of longitudinal friction force estimate in a higher level stability controller.
5. Test 2WID-MTTE controller for steering manoeuvres and on road gradients.
6. For experimentation, where possible ensure accelerometers are aligned with the vehicle axes and adequate mechanical damping is present.
7. For validation testing, make use of automated control such as cruise control to improve the consistency of test results.

Bibliography

- Akiba, T., Shirato, R., Fujita, T., and Tamura, J. (2007). A Study of Novel Traction Control Method for Electric Motor Driven Vehicle. In *Power Conversion Conference*, number 6, pages 699–704, Nagoya, Japan. Ieee.
- Amodeo, M., Ferrara, A., Terzaghi, R., and Vecchio, C. (2010). Wheel slip control via second-order sliding-mode generation. *IEEE Transactions on Intelligent Transportation Systems*, 11(1):122–131.
- Antonov, S., Fehn, A., and Kugi, A. (2011). Unscented Kalman filter for vehicle state estimation. *Vehicle System Dynamics*, 49(9):1497–1520.
- Bottiglione, F., Sornioti, A., and Shead, L. (2012). The effect of half-shaft torsion dynamics on the performance of a traction control system for electric vehicles. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 226(9):1145–1159.
- Brooks, R. J. and Tobias, A. M. (1996). Choosing the best model: Level of detail, complexity, and model performance. *Mathematical and Computer Modelling*, 24(4):1–14.
- Buick (1971). Webpage. <http://www.buick-riviera.com/maxtrac.html>.
- Colli, V. D., Tomassi, G., and Scarano, M. (2006). “Single Wheel ” Longitudinal Traction Control for Electric Vehicles. *IEEE Transactions on Power Electronics*, 21(3):799–808.
- Crolla, D. and Cao, D. (2012). The impact of hybrid and electric powertrains on ve-

- hicle dynamics, control systems and energy regeneration. *Vehicle System Dynamics*, 50(Supplement):95–109.
- Daiss, A. and Kiencke, U. (1995). Estimation of vehicle speed: Fuzzy estimation in comparison with Kalman filtering. In *Control Applications, Proceedings of the 4th IEEE Conference on*, number 1, pages 281–284.
- De Castro, R., Araujo, R., and Freitas, D. (2011). Optimal linear parameterization for on-line estimation of tire-road friction. In *IFAC World Congress Proceedings*, volume 18, pages 8409–8414.
- De Castro, R., Araujo, R. E., and Freitas, D. (2013). Wheel slip control of EVs based on sliding mode technique with conditional integrators. *IEEE Transactions on Industrial Electronics*, 60(8):3256–3271.
- De Novellis, L., Sorniotti, A., Gruber, P., Orus, J., Rodriguez Fortun, J.-M., Theunissen, J., and De Smet, J. (2015). Direct yaw moment control actuated through electric drivetrains and friction brakes: Theoretical design and experimental assessment. *Mechatronics*, 26:1–15.
- Deur, J. and Pavković, D. (2011). A model-based traction control strategy non-reliant on wheel slip information. *Vehicle system dynamics*, 49(8):1245–1265.
- Drakunov, S., Ozguner, U., Dix, P., and Ashrafi, B. (1995). ABS control using optimum search via sliding modes. *IEEE Transactions on Control Systems Technology*, 3(1):79–85.
- Ewin, N., McCulloch, M. D., and Howey, D. (2013). MTTE-Based traction control for directional stability on mixed-mu roads. In *Hybrid and Electric Vehicles Conference 2013 (HEVC 2013)*, London, UK. IET.
- Ferguson, S. A. (2007). The effectiveness of electronic stability control reducing real-world crashes: a literature review. *Traffic Injury Prevention*, 8(4):329–338.
- Fujii, K. and Fujimoto, H. (2007). Traction control based on slip ratio estimation without

- detecting vehicle speed for electric vehicle. In *Power Conversion Conference*, pages 688–693, Nagoya, Japan. IEEE.
- Fujimoto, H., Saito, T., and Noguchi, T. (2004a). Motion stabilization control of electric vehicle under snowy conditions based on yaw-moment observer. In *The 8th IEEE International Workshop on Advanced Motion Control*, pages 35–40. Ieee.
- Fujimoto, H., Saito, T., Tsumasaka, A., and Noguchi, T. (2004b). Motion control and road condition estimation of electric vehicles with two in-wheel motors. In *Proceedings of the IEEE International Conference on Control Applications*, volume 2, pages 1266–1271. Ieee.
- Gillespie, T. (1992). *Fundamentals of vehicle dynamics*. Society of Automotive Engineers, Warrendale, PA.
- Gustafsson, F., Ahlqvist, S., Forssell, U., and Persson, N. (2001). Sensor fusion for accurate computation of yaw rate and absolute velocity. *SAE Technical Paper*, (2001-1-1064).
- Harifi, a., Aghagolzadeh, a., Alizadeh, G., and Sadeghi, M. (2008). Designing a sliding mode controller for slip control of antilock brake systems. *Transportation Research Part C: Emerging Technologies*, 16(6):731–741.
- Harned, J. L. and Johnston, L. E. (1969). Measurement of Tire Brake Force Characteristics as Related to Wheel Slip (Antilock) Control System Design. *SAE Technical Paper*, (690214).
- Hori, Y. (2004). Future vehicle driven by electricity and control-Research on four-wheel-motored UOT Electric March II. *IEEE Transactions on Industrial Electronics*, 51(5):954–962.
- Hori, Y., Toyoda, Y., and Tsuruoka, Y. (1998). Traction control of electric vehicle: basic experimental results using the test EV "UOT electric march". *IEEE Transactions on Industry Applications*, 34(5):1131–1138.

- Hu, J. and Yin, D. (2011). MTTE-based motion stabilization control for in-wheel motor electric vehicles. In *Proceedings of IEEE, SICE Annual Conference*, pages 312–317.
- Hu, J., Yin, D., and Hori, Y. (2011). Fault-tolerant traction control of electric vehicles. *Control Engineering Practice*, (19):204–213.
- Hu, J., Yin, D., Hori, Y., and Hu, F. (2009). A new MTTE methodology for electric vehicle traction control. In *IEEE International Conference on Electrical Machines and Systems*, pages 1–6. IEEE.
- Hu, J.-S., Lin, X.-C., Yin, D., and Hu, F.-R. (2015). Dynamic motion stabilization for front-wheel drive in-wheel motor electric vehicles. *Advances in Mechanical Engineering*, 7(12):1–11.
- Ivanov, V. and Barber, P. (2015). Wheel Slip Control for All-Wheel Drive Electric Vehicle. *Journal of Terramechanics*, 61:1–10.
- Ivanov, V., Savitski, D., Augsburg, K., and Barber, P. (2015a). Electric vehicles with individually controlled on-board motors: Revisiting the ABS design. In *IEEE International Conference on Mechatronics, ICM*, pages 323–328.
- Ivanov, V., Savitski, D., and Shyrokau, B. (2015b). A Survey of Traction Control and Antilock Braking Systems of Full Electric Vehicles with Individually Controlled Electric Motors. *IEEE Transactions on Vehicular Technology*, 64(9):3878–3896.
- Ivanov, V., Shyrokau, B., Savitski, D., Drive, F., Orus, J., Meneses, R., Theunissen, J., and Janssen, K. (2014). Design and Testing of ABS for Electric Vehicles with Individually Controlled On-Board Motor Drives. *SAE International Journal of Passenger Cars - Mechanical Systems*, 7(2):902–913.
- Johnson, A. (2001). Unpacking reliability: The success of Robert Bosch, GmbH in constructing antilock braking systems as reliable products. *History and Technology*, 17(3):249–270.
- Kalman, R. E. (1960). A New Approach to Linear Filtering and Prediction Problems. *ASME Transactions, Journal of Basic Engineering*, 82D:35–45.

- Kawabe, T. (2012). Slip Suppression of Electric Vehicles using Model Predictive PID Controller. *World Academy of Science, Engineering and Technology, International Journal of Electrical, Computer, Energetic, Electronic and Communication Engineering*, 6(7):649–654.
- Kortüm, W. (1993). Review of Multibody Computer, Codes for Vehicle System Dynamics. *Vehicle System Dynamics*, 22(S1):3–31.
- Kuntanapreeda, S. (2015). Super-twisting sliding-mode traction control of vehicles with tractive force observer. *Control Engineering Practice*, 38:26–36.
- Lee, H., Love, D. W., and Tomizuka, M. (1995). Longitudinal Maneuvering Control for Automated Highway Systems Based on a Magnetic Reference/Sensing System. In *IEE American Control Conference*, pages 150–154.
- Liang, B. and Lin, W. (2012). A new slip ratio observer and its application in electric vehicle wheel slip control. In *IEEE International Conference on Systems, Man, and Cybernetics*, pages 41–46.
- Lutz, A., Rauh, J., and Reinalter, W. (2007). Developments in vehicle dynamics and the tire model performance test. *Vehicle System Dynamics*, 45(S1):7–19.
- Maeda, K., Fujimoto, H., and Hori, Y. (2012). Four-wheel driving-force distribution method based on driving stiffness and slip ratio estimation for electric vehicle with in-wheel motors. In *IEEE Vehicle Power and Propulsion Conference, VPPC*, pages 1286–1291.
- Magallan, G., De Angelo, C., and Garcia, G. (2011). Maximization of the Traction Forces in a 2WD Electric Vehicle. *IEEE Transactions on Vehicular Technology*, 60(99):1–1.
- MIRA (2016). Website.
- Moore, D. (1975). *The Friction of Pneumatic Tyres*. Elsevier.
- Murata, S. (2012). Innovation by in-wheel-motor drive unit. *Vehicle System Dynamics*, 50(6):807–830.

- Pacejka, H. (2006). *Tyre and Vehicle Dynamics*. Butterworth-Heinemann.
- Patil, C. B., Longoria, R. G., and Limroth, J. (2003). Control prototyping for an anti-lock braking control system on a scaled vehicle. In *42nd IEEE Conference on Decision and Control*, number 5, pages 4962–4967, Maui, Hawaii, USA.
- Petersen, I., Johansen, T., Kalkkuhl, J., and Lüdemann, J. (2001). Wheel slip control in ABS brakes using gain scheduled constrained LQR. In *European Control Conference (ECC)*, pages 606–611.
- Radt, H. S. and Milliken, W. (1960). Motions of skidding automobiles. *SAE Technical Paper*, 600133:21.
- SAE (1976). Vehicle Dynamics Terminology. Technical report, Warrendale, PA.
- Sakai, S., Sado, H., and Hori, Y. (1999). Motion control in an electric vehicle with four independently driven in-wheel motors. *IEEE/ASME Transactions on Mechatronics*, 4(1):9–16.
- Sakai, S.-i. and Hori, Y. (2001). Advantage of electric motor for anti-skid control of electric vehicle. *European Power Electronics (EPE) Journal*, 11(4):26–32.
- Schinkel, M. and Hunt, K. (2002). Anti-lock braking control using a sliding mode like approach. In *Proceedings of the 2002 American Control Conference (IEEE Cat. No. CH37301)*, volume 3, pages 2386–2391. American Automatic Control Council.
- Seshagiri, S. and Khalil, H. K. (2005). Robust output feedback regulation of minimum-phase nonlinear systems using conditional integrators. *Automatica*, 41(1):43–54.
- Shibahata, Y., Shimada, K., and Tomari, T. (1993). Improvement of Vehicle Maneuverability by Direct Yaw Moment Control. *Vehicle System Dynamics*, 22(5-6):465–481.
- Shim, T. and Ghike, C. (2007). Understanding the limitations of different vehicle models for roll dynamics studies. *Vehicle System Dynamics*, 45(3):191–216.
- Temple, G. (1956). The dynamics of the pneumatic tyre. *Endeavour*, 15(60):200–205.
- Watson, J. (2014). A Unique Method of Measuring Road Vehicle Moments of Inertia and

- Centre of Gravity. In *73rd Annual Conference of Society of Allied Weight Engineers*, number 3624, Long Beach, CA , USA. Society of Allied Weight Engineers, Inc.
- Xu, K., Xu, G., Li, W., and Jian, L. (2011). Anti-skid for Electric Vehicles based on sliding mode control with novel structure. In *IEEE International Conference on Information and Automation*, number June, pages 650–655, Shenzhen, China.
- YASA Motors (2016). Webpage. <http://www.yasamotors.com/products/yasa-750/>.
- Yin, D. (2009). *General Approach to Traction Control for Electric Vehicles Based on Maximum Transmissible Torque Estimation*. PhD thesis, University of Tokyo.
- Yin, D. and Hori, Y. (2008). A new approach to traction control of EV based on maximum effective torque estimation. In *34th Annual Conference of IEEE Industrial Electronics (IECON)*, pages 2764–2769. Ieee.
- Yin, D. and Hori, Y. (2010). Traction Control for EV Based on Maximum Transmissible Torque Estimation. *International Journal of Intelligent Transportation Systems Research*, 8(1):1–9.
- Yin, D. and Hu, J.-s. (2014). Active approach to Electronic Stability Control for front-wheel drive in-wheel motor electric vehicles. *International Journal of Automotive Technology*, 15(6):979–987.
- Yin, D., Oh, S., and Hori, Y. (2009). A novel traction control for EV based on maximum transmissible torque estimation. *IEEE Transactions on Industrial Electronics*, 56(6):2086–2094.
- Zendri, F., Antonello, R., Biral, F., and Fujimoto, H. (2010). Modeling, identification and validation of an electric vehicle for model-based control design. In *11th IEEE International Workshop on Advanced Motion Control (AMC)*, pages 118–123. Ieee.
- Zhang, Q., Liu, G., Liu, B., and Xie, X. (2007). Sensor Fusion Based Estimation Technology of Vehicle Velocity in Anti-lock Braking System. In *IEEE International Conference on Information Acquisition*, pages 106–111.

Appendices

Appendix A

Vehicle Model parameters

The coordinates datum (x,y,z) is: 1475 mm in front of the front wheel centre; centre line of car; underside of chassis

Part	Mass (kg)	X (mm)	Y (mm)	Z (mm)
DRIVER	79	2762	288	251
PASSENGER	79	2762	-288	251
CHASSIS				
MONOCOQUE 1	15	3200	265	550
MONOCOQUE 2	15	3200	-265	550
MONOCOQUE 3	15	2150	265	500
MONOCOQUE 4	15	2150	-265	500
FT CRASH STRUCTURE	10	1160	0	355
BOOT ASSEMBLY	7	4120	0	370
SEAT LH	10	2600	-300	240
SEAT RH	10	2600	300	240
BODYWORK				
BUMPER FT	12	845	0	228
BONNET	3	1290	0	563
WHEEL ARCH FRT LH	3	1500	-675	500

Continued on next page

Table A.1 – *Continued from previous page*

Part	Mass (kg)	X (mm)	Y (mm)	Z (mm)
WHEEL ARCH FRT RH	3	1500	675	500
WHEEL ARCH LINER LH	2	1475	-675	180
WHEEL ARCH LINER RH	2	1475	675	180
DOOR LH	17.5	2670	-680	555
DOOR RH	17.5	2670	680	555
REAR QUARTER PANEL LH	3	3800	-745	400
REAR QUARTER PANEL RH	3	3800	745	400
RR WHEEL ARCH LINER LH	2	3975	660	400
RR WHEEL ARCH LINER RH	2	3975	-660	400
TAIL GATE	10	4500	0	726
BUMPER RR	12	4480	0	236
TRIM	5	2200	0	500
TRIM	5	4000	0	500
WINDSCREEN	12	2100	0	816
AXLE ASSY FT				
SUBFRAME	10	1475	0	55
DRIVESHAFT ASSY LH	3	1492	-300	173
DRIVESHAFT ASSY RH	3	1492	300	173
FLWB LH	1.5	1505	-435	46
FLWB RH	1.5	1505	435	46
FTWB LH	1	1450	-460	300
FTWB RH	1	1450	460	300
TRACK ROD LH	1	1321	-470	96
TRACK ROD RH	1	1321	470	96
STEERING RACK	4	1640	30	82
UPRIGHT FT LH	2	1475	-640	171

Continued on next page

Table A.1 – *Continued from previous page*

Part	Mass (kg)	X (mm)	Y (mm)	Z (mm)
UPRIGHT FT RH	2	1475	640	171
GEN 3 WHEEL BRG LH	1	1475	-710	171
GEN 3 WHEEL BRG RH	1	1475	710	171
COILOVER LH	3	1550	-480	230
COILOVER RH	3	1550	480	230
DISC LH	5	1475	-681	171
DISC RH	5	1475	681	171
CALLIPER LH	2	1595	-681	171
CALLIPER RH	2	1595	681	171
WHEEL LH	5	1475	-687	171
WHEEL RH	5	1475	687	171
TYRE LH	9	1475	-687	171
TYRE RH	9	1475	687	171
AXLE ASSY RR				
SUBFRAME	10	3975	0	45
MOTOR	60	4000	0	165
DRIVESHAFT ASSY LH	3	3972	-300	163
DRIVESHAFT ASSY RH	3	3972	300	163
FLWB LH	1.5	3965	-435	36
FLWB RH	1.5	3965	435	36
FTWB LH	1	3400	-460	290
FTWB RH	1	3400	460	290
TRACK ROD LH	1	3810	-470	86
TRACK ROD RH	1	3810	470	86
UPRIGHT FT LH	1.5	3975	-640	161
UPRIGHT FT RH	1.5	3975	640	161

Continued on next page

Table A.1 – *Continued from previous page*

Part	Mass (kg)	X (mm)	Y (mm)	Z (mm)
GEN 3 WHEEL BRG LH	1	3975	-710	161
GEN 3 WHEEL BRG RH	1	3975	710	161
COILOVER LH	3	3890	-480	220
COILOVER RH	3	3890	-480	220
DISC LH	5	3975	-681	161
DISC RH	5	3975	681	161
CALLIPER LH	2	4055	-681	161
CALLIPER RH	2	4055	681	161
WHEEL LH	5	3975	-687	161
WHEEL RH	5	3975	687	161
TYRE LH	9	3975	-687	161
TYRE RH	9	3975	687	161
POWER SYSTEMS				
LH BATTERY TRAY FT	49	2164	-467	65
LH BATTERY TRAY MID	49	2725	-467	65
LH BATTERY TRAY RR	49	3325	-467	65
RH BATTERY TRAY FT	49	2164	467	65
RH BATTERY TRAY MID	49	2725	467	65
RH BATTERY TRAY RR	49	3325	467	65
CTR BATTERY TRAY FT	49	2164	0	65
CTR BATTERY TRAY MID	49	2725	0	65
CTR BATTERY TRAY RR	49	3325	0	65
BATTERY CONTROL	3	3675	0	240
POWER CONTROLLER RR LH	8	3900	-270	280
POWER CONTROLLER RR RH	8	3900	270	280
WIRING HARNESS	10	2800	0	400

Continued on next page

Table A.1 – *Continued from previous page*

Part	Mass (kg)	X (mm)	Y (mm)	Z (mm)
COOLER	2	890	0	200
COOLANT PUMP	1	1050	234	340
DC/DC CONVERTER	8	3340	-640	210
CHARGERS	14.8	4400	0	270
CAR SYSTEMS				
BODY CONTROL UNIT	1	3360	-600	420
WIRING HARNESS	10			
HVAC UNIT	10	1792	0	420
HVAC CONDENSER	2	1150	194	200
HVAC POWER UNIT	5	1350	-60	427
HEAD LIGHTS	3	1110	0	500
TAIL LIGHTS	3	4440	0	500
PEDALS	5	1700	203	390
STEERING COLUMN	8	2230	284	544
AIRBAGS	3	2100	0	624

Table A.1: Vehicle parts' mass and location

Part	X (mm)	Y (mm)	Z (mm)
Front			
Top wishbone inner fwd	1337.0	376.4	330.5
Top wishbone inner rwd	1508.0	376.4	330.5
Top wishbone outer	1489.2	588.2	325.3
Lower wishbone inner rwd	1649.9	290.0	82.7
Lower wishbone inner fwd	1362.1	290.0	72.7
Lower wishbone outer	1465.6	648.9	62.2
Track rod inner	1640.0	335.0	130.0
Track rod outer	1602.5	607.5	125.0
Contact patch	1475.0	689.5	-120.0
Wheel centre	1475.0	687.5	168.0
Spring lwr wishbone	1517.4	561.8	93.5
Spring chassis	1593.0	-445.4	452.6
ARB lwr wishbone	1424.4	503.4	95.3
ARB lever pickup	1417.2	494.2	174.3
ARB ctr of chassis mtg	1212.0	271.0	138.2
Rear			
Top wishbone inner fwd	3942.0	376.1	324.7
Top wishbone inner rwd	4113.0	376.1	324.7
Top wishbone outer	3960.8	587.7	315.3
Lower wishbone inner rwd	4087.9	290.0	72.7
Lower wishbone inner fwd	3800.1	290.0	82.7
Lower wishbone outer	3984.4	648.3	52.6
Track rod inner	3838.0	325.0	126.0
Track rod outer	3838.5	607.5	110.0
Contact patch	3975.0	689.0	-130.0
Wheel centre	3975.0	687.0	158.0
Spring lwr wishbone	3932.3	562.0	86.2
Spring chassis	3854.9	-443.0	443.5

Table A.2: E4 coupe suspension geometry

Appendix B

Dymola Model Statistics

Below are the statistics for the Dymola Vehicle Model of the E4 coupe. The statistics are for before and after translation of the model, whereby symbolic manipulation is performed to produce more efficient code for simulation.

Original Model

Number of components: 4942

Variables: 37387

Constants: 62 (70 scalars)

Parameters: 15342 (30008 scalars)

Unknowns: 21983 (72089 scalars)

Differentiated variables: 1129 scalars

Equations: 15980

Nontrivial : 12100

Number of numerical Jacobians: 0

Appendix C

Tyre Model parameters

The tyre model was extend from the Conventional 205/55 R17 Pacejka-02 model in the Dymola Vehicle Dynamics Library for a Bridgestone Potenza RE50A 205/45 R17. The parameters i_{xx} , QSY1 and QSY3 were calibrated as described in Chapter 4, along with the tyre's mass, dimensions and nominal load.

Symbol	Value	Description
Mass and Inertia		
m	14	Mass of wheel [kg]
i_{xx}	1.04	Wheel inertia about the spin axis [$kg.m^2$]
i_{yy}	$i_{xx}/2$	Wheel inertia perpendicular to spin axis [kg.m2]
Out-of-plane		
Normal load		
CZ	0.2e6	Tyre vertical stiffness [N/mm]
KZ	0.5e3	Tyre vertical damping [N.s/mm]
Effective rolling radius		
FNOMIN	5500	Nominal wheel load [N]
BREFF	8.4	Low load stiffness effective rolling radius [3,12]

Continued on next page

Table C.1 – *Continued from previous page*

Symbol	Value	Description
DREFF	0.27	Peak value of effective rolling radius [0.2,0.4]
FREFF	0.07	High load stiffness effective rolling radius [0.03,0.25]
Overturning couple		
QX1	0.032	Lateral force induced overturning couple
QX2	0.49	Camber induced overturning couple
QX3	0.12	F_y induced overturning couple
Rolling resistance		
QY1	0.022	Rolling resistance torque coefficient
QY2	0	Rolling resistance torque depending on F_x
QY3	0.0004	Rolling resistance torque depending on speed
QY4	0	Rolling resistance torque depending on speed, V^4
LONGVL	1	Measurement speed [m/s]
Pure slip		
Pure slip longitudinal		
PCX1	1.63	Shape factor C_{fx} for longitudinal force
PDX1	1.06	Longitudinal friction μ_x at F_{znom}
PDX2	-0.0492	Variation of friction μ_x with load
PDX3	-2.29	Variation of friction μ_x with camber
PEX1	0.5	Longitudinal curvature E_{fx} at F_{znom}
PEX2	-0.11	Variation of curvature E_{fx} with load
PEX3	-0.06	Variation of curvature E_{fx} with load squared
PEX4	0	Factor in curvature E_{fx} while driving
PKX1	19.7	Longitudinal slip stiffness K_{fx}/F_z with load
PKX2	-0.15	Variation of slip stiffness K_{fx}/F_z with load

Continued on next page

Table C.1 – *Continued from previous page*

Symbol	Value	Description
PKX3	0.18	Exponent in slip stiffness K_{fx}/F_z with load
PHX1	-0.0005	Horizontal shift S_{hx} at F_{znom}
PHX2	8.50E-05	Variation of shift S_{hx} with load
PVX1	0	Vertical shift S_{vx}/F_z at F_{znom}
PVX2	0	Variation of shift S_{vx}/F_z with load
Pure slip lateral		
PCY1	1.28	Shape factor C_{fy} for lateral force
PDY1	-0.92	Lateral friction μ_y
PDY2	0.22	Variation of friction μ_y with load
PDY3	-4.55	Variation of friction μ_y with squared camber
PEY1	-1.1	Lateral curvature E_{fy} at F_{znom}
PEY2	0.65	Variation of curvature E_{fy} with load
PEY3	-0.65	Zero order camber dependency of curvature E_{fy}
PEY4	-12.41	Variation of curvature E_{fy} with camber
PKY1	-13.06	Maximum value of stiffness K_{fy}/F_{znom}
PKY2	1.77	Load at which K_{fy} reaches maximum value
PKY3	0.19	Variation of K_{fy}/F_{znom} with camber
PHY1	0.0034	Horizontal shift S_{hy} at F_{znom}
PHY2	-0.003	Variation of shift S_{hy} with load
PHY3	0.044	Variation of shift S_{hy} with camber
PVY1	0.044	Vertical shift in S_{vy}/F_z at F_{znom}
PVY2	-0.03	Variation of shift S_{vy}/F_z with load
PVY3	-0.176	Variation of shift S_{vy}/F_z with camber
PVY4	-0.44	Variation of shift S_{vy}/F_z with camber and load

Continued on next page

Table C.1 – *Continued from previous page*

Symbol	Value	Description
Pure slip aligning torque		
QBZ1	8.4	Trail slope factor for trail B_{pt} at F_{znom}
QBZ2	-2.96	Variation of slope B_{pt} with load
QBZ3	0.45	Variation of slope B_{pt} with load squared
QBZ4	-0.45	Variation of slope B_{pt} with camber
QBZ5	-0.45	Variation of slope B_{pt} with absolute camber
QBZ9	3.45	Slope factor B_r or residual torque M_{zr}
QBZ10	0	Slope factor B_r or residual torque M_{zr}
QCZ1	1.2	Shape factor C_{pt} for pneumatic trail
QDZ1	0.1	Peak trail $D''_{pt} = D_{pt}(F_z/F_{znom}R_0)$
QDZ2	-0.003	Variation of peak D''_{pt} with load
QDZ3	-0.55	Variation of peak D''_{pt} with camber
QDZ4	8.4	Variation of peak D''_{pt} with camber squared
QDZ6	-0.003	Peak residual torque $D''_{mr} = D_{mr}/(F_zR_0)$
QDZ7	0.005	Variation of peak factor D''_{mr} with load
QDZ8	-0.11	Variation of peak factor D''_{mr} with camber
QDZ9	0.11	Variation of peak factor D''_{mr} with camber and load
QEZ1	-2.9	Trail curvature E_{pt} at F_{znom}
QEZ2	-0.5	Variation of curvature E_{pt} with load
QEZ3	0	Variation of curvature E_{pt} with load squared
QEZ4	-0.12	Variation of curvature E_{pt} with sign of Alpha-t
QEZ5	-3.71	Variation of E_{pt} with camber and sign of Alpha-t
QHZ1	0.003	Trail horizontal shift S_{ht} at F_{znom}

Continued on next page

Table C.1 – *Continued from previous page*

Symbol	Value	Description
QHZ2	0.00084	Variation of shift S_{ht} with load
QHZ3	0.158	Variation of shift S_{ht} with camber
QHZ4	0.121	Variation of shift S_{ht} with camber and load
Combined slip		
Combined slip longitudinal		
RBX1	9	Slope factor for combined slip F_x reduction
RBX2	-8.6	Variation of slope F_x reduction with κ
RCX1	1.131	Shape factor for combined slip F_x reduction
REX1	0.081	Curvature factor of combined F_x
REX2	-0.15	Curvature factor of combined F_x with load
RHX1	-0.029	Shift factor for combined slip F_x reduction
Combined slip lateral		
RBX1	6.4	Slope factor for combined F_y reduction
RBX2	7.91	Variation of slope F_y reduction with κ
RBX3	-0.059	Shift term for alpha in slope F_y reduction
RCY1	1.16	Shape factor for combined F_y reduction
REY1	0.22	Curvature factor of combined F_y
REY2	0.43	Curvature factor of combined F_y with load
RHY1	0.0007	Shift factor for combined F_y reduction
RHY2	0.023	Shift factor for combined F_y reduction with load
RVY1	0	Kappa induce side force $S_{vyk}/M_{uy} * F_z$ at F_{znom}
RVY2	0	Variation of $S_{vyk}/M_{uy} * F_z$ with load
RVY3	0	Variation of $S_{vyk}/M_{uy} * F_z$ with camber
RVY4	10	Variation of $S_{vyk}/M_{uy} * F_z$ with alpha

Continued on next page

Table C.1 – *Continued from previous page*

Symbol	Value	Description
RVY5	1.94	Variation of $S_{vyk}/M_{uy} * F_z$ with kappa
RVY6	-50	Variation of $S_{vyk}/M_{uy} * F_z$ with atan(kapa)
Combined slip aligning torque		
SSZ1	0.023	Nominal value of s/R_0 effect of F_x on M_z
SSZ2	0.019	Variation of distance s/R_0 with F_y/F_{znom}
SSZ3	0.5	Variation of distance s/R_0 with camber
SSZ4	-0.28	Variation of distance s/R_0 with load and camber
Transient		
PTX1	1.98	Relaxation length σ_κ/Fz at F_{znom}
PTX2	0.0003	Variation of σ_κ/Fz with load
PTX3	-0.31	Variation of σ_κ/Fz with exponent of load
PTY1	1.8	Peak value of relaxation length σ_α
PTY2	1.8	Shape factor for σ_α
QTZ1	0.2	Gyroscopic torque constant
MBELT	4.1	Belt mass of the wheel [kg]

Table C.1: Magic Formula tyre model parameters

Appendix D

Split- μ road test

Figure D.1 shows the results from the split- μ road test without traction control. The split- μ road consists of high- μ road of wet asphalt (peak $\mu = 0.7$) on the right hand side and a low- μ road of wet basalt tiles (peak $\mu = 0.3$) on the left. In this test the vehicle starts on a wet asphalt surface with a torque of 500 Nm applied to each driven wheel, Figure D.1a. The rear wheel enters the split- μ road at $t = 2$ s and the rear left wheel begins to slip, see Figure D.1b. At $t = 2.4$ s the torque to the rear left wheel is reduced due to power limit from the System Protection. At $t = 4.0$ s the rear left wheel velocity reaches steady-state due to the speed limit of the motor controller. At $t = 8.3$ s the driver releases the throttle and the torque demand to both wheels is removed. Across the test the steering wheel angle varies by ± 10 degrees, see Figure D.1c, and the yaw rate varies by ± 0.04 rad/s, see Figure D.1d.

These results show that a split- μ of 0.7/0.3 was insufficient to destabilise the vehicle even without traction control.

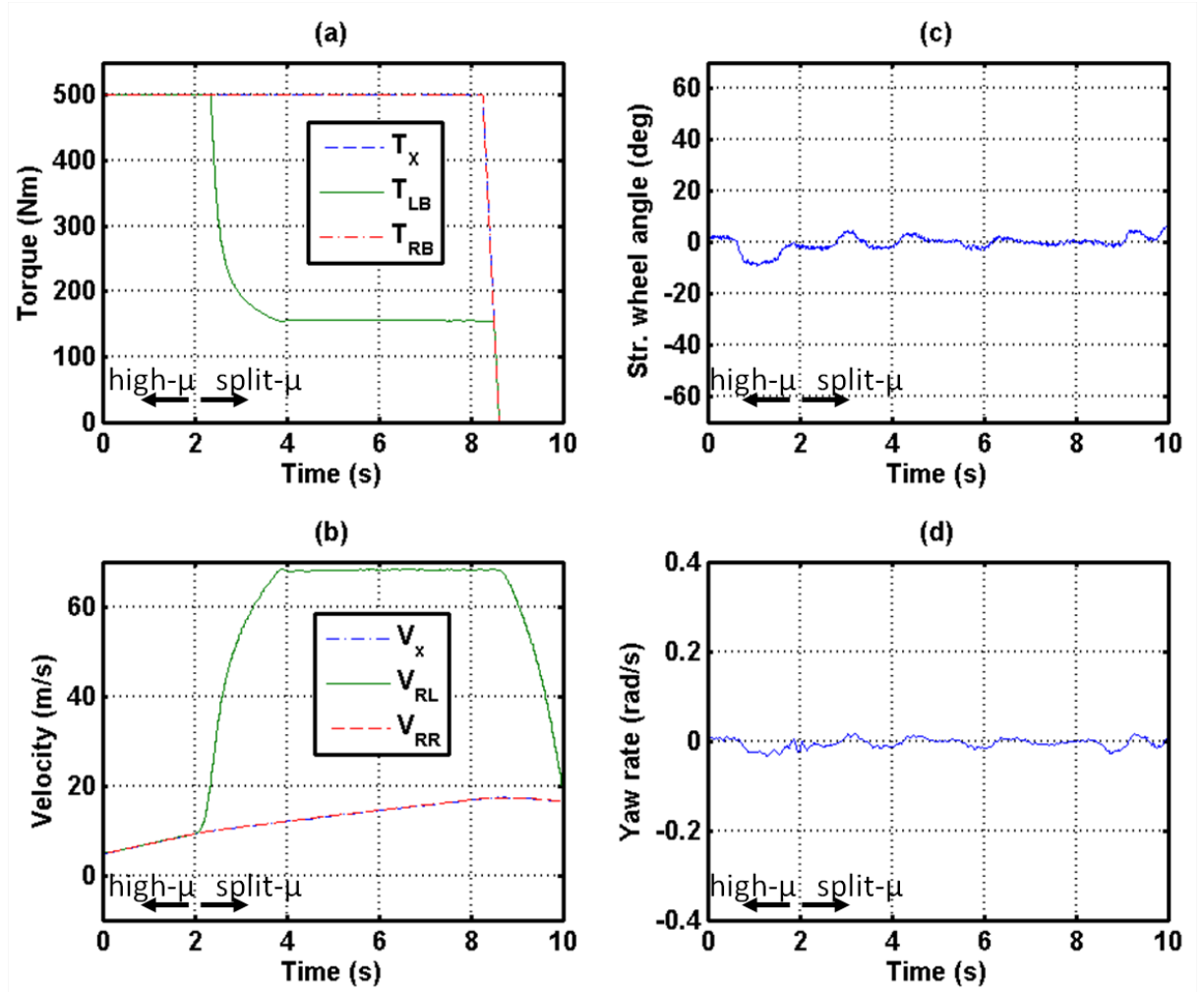


Figure D.1: Experimental results for E4 coupe accelerating onto a split- μ road without traction control. High- μ road on right side, low- μ road on left side.