BOOK REVIEW

A review of *Theory of Simple Glasses: Exact Solutions in Infinite Dimensions*
By Giorgio Parisi, Pierfrancesco Urbani and Francesco Zamponi
Cambridge University Press, 2020, 349 pages, (£62.99 hardcover)
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Scope: Textbook
Level: Researcher

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While I was preparing this review, Giorgio Parisi, the lead author of this book was awarded the Nobel Prize for Physics "for groundbreaking contributions to our understanding of complex physical systems". Parisi shared his 2021 prize with two scientists who had, via modelling the Earth's climate, predicted global warming. Certainly, the Earth's atmosphere and oceans comprise a complex physical system, and a critically important one at that, but Parisi's work has focussed on much more general aspects of complexity, rather than applying the ideas to just one single physical system.

Parisi, together with Urbani and Zamponi, has written a book which discusses these more general aspects of complexity for which Parisi received his prize. Condensed matter physics has been developed to study periodic systems, such as crystals, so the theory works quite naturally in reciprocal space. However, lots of condensed phases in nature are not crystalline but are, instead, *amorphous* (such systems include "glasses, foams, pastes, granulars and plastics"). How do you study those when you can't use all the familiar tricks that have been developed for periodic crystals? A clear message of the book is that it helps to go to an infinite number of dimensions. The authors show how to treat phase transitions as the number of dimensions $d\to\infty$, in which case the mean field model applies; but then, you can step back from infinity and see how the corrections work for finite dimension.

They show that a particularly interesting case occurs for the liquid phase as $d\to\infty$, because then all the dynamics seize up and the system becomes trapped in a restricted portion of phase space, unable to escape by diffusion. In short, we have the formation of a metastable state, and this captures many of the properties of a glass. These metastable states are particularly difficult to treat theoretically, but one method developed by Parisi and his colleagues is called "state following". You add a new term to the Hamiltonian to select a particular metastable state, and then gradually remove it.

Disorder breaks translational invariance and hence stymies most attempts to perform calculations that would be straightforward in periodic systems (that have translational invariance automatically). The trick is to work with a system containing an integer number of replicas of the original disordered system, restoring some translational and rotational invariances, and allowing one to compute the partition function and hence extract many of the usual statistical mechanical observables. Then, the number of replicas can be reduced to zero at the very end of the calculation. This can work well, but deep in the glass phase, replica symmetry breaking can sometimes occur where the replica generation process becomes unstable and this leads to what is known as a Gardner transition (although not mentioned in the book, this phenomenon is named in honour of Elizabeth Gardner, a brilliant condensed matter theorist who died tragically young). The book provides a detailed treatment of the Gardner transition in glasses. The ideas developed in the book are then applied to problems such as the packing of spheres, jamming (where the crystallisation time diverges as $d \to \infty$) and the effect of strain on amorphous systems (rheology). Each chapter in this detailed treatise is finished off with a helpful summary of the key ideas and ample suggestions for further reading.