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Benchmark of three-dimensional numerical models of subduction against a laboratory experiment[☆]

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Abstract

In this paper, we validate three-dimensional numerical models of subduction against a laboratory experiment. The design of our numerical models follows that of the laboratory, and employs a single subducting plate fixed at its trailing edge without an overriding plate. Rheology and buoyancy are not thermally dependent, but importantly the model has a free surface boundary. All the models include a sticky air layer, which is an approximation to a free surface boundary condition, as opposed to a free-slip condition that unrealistically binds the plate to the upper surface. We use a parallel, Python, particle-in-cell, finite-element scheme, and present 6 different models that differ in their viscosity at the interfaces of the subducting plate. We consider harmonic and arithmetic averaging of the viscosity together with a plate composite rheology associated with arithmetic averaging, which similarly to the harmonic averaging results in two lubricated sub-layers at the plate lower and upper interfaces. Despite showing a similar subduction-induced poloidal and toroidal flow in the mantle, the agreement of the models with the experiment is overall ordinary. The rates at which the slab tip sinks and the plate hinge retreats are approaching the laboratory counterparts to within 7 to 18%, at best, when using the composite plate models. Errors in the characterization of the laboratory parameters can account for 5% of such differences. As the composite plate sub-layers are made stiffer, the plate hinge also begins to depart from the steady retreat observed in

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the laboratory experiment. All numerical models exhibit a secondary Rayleigh-Taylor instability along the plate edge that is not observed in the laboratory experiment. The development of this instability is further delayed when the numerical spatial resolution is increased, or when the composite plate sub-layers are made stiffer. This benchmark exercise shows that 1) the glucose surface tension in the laboratory, not taken into account in the numerical modelling, is sufficient to prevent the flank instability during the retreat, and 2) numerical models are usually free from it due to their use of stiffer plate sub-layers and/or their display of a limited amount of retreat compared to this study.

10 *Keywords:* Computational modelling, Subduction, Analogue modelling,

11 **1. Introduction**

12 The validation of numerical models is an essential part of their development.
 13 It can be accomplished through either comparison with field measurements or
 14 laboratory experiments. In geodynamic modelling, the use of field measurements
 15 is often limited, if not impossible, whereas specifically designed laboratory ana-
 16 logues are scarce. Numerical models of subduction are part of those models
 17 difficult to benchmark, and most benchmarks have thus occurred between nu-
 18 merical models against themselves (e.g., van Keken et al., 2008; OzBench et al.,
 19 2008). Benchmarking against a three-dimensional laboratory experiment is a
 20 useful alternative because it allows to test one of the key ingredients of the
 21 subduction: the free-surface and development of topography. This can be in-
 22 vestigating with a minimal numerical model design removing complexities such
 23 as multiple subductions, overriding plates, and any thermal dependency on the
 24 rheology and buoyancy. Such a benchmark exercise is thus an important as-
 25 set in the development of realistic regional scale three-dimensional subduction
 26 models. This motivated the present study.

27 Numerical modelling of subduction faces a few challenges to attain a sub-
 28 stantial degree of realism. Among others, these challenges include the inherent
 29 three-dimensionality of the problem and the range of length scales, the presence

30 of a free surface, sharp viscosity changes with contrasts up to 10^5 , non-linear
 31 and temperature-dependent rheologies, the coupling between adjacent plates,
 32 and last but not least the initiation of the subduction. As a result, different
 33 simplifications in the models and various numerical approaches have emerged,
 34 each having their own strengths and shortfalls. Compared to the finite element
 35 method (FEM) Kaus et al. (2010), the boundary element method (BEM) Ribe
 36 (2010) or hybrid FEM-BEM method Morra and Regenauer-Lieb (2006), the
 37 particle-in-cell (PIC) method, a combined Eulerian - Lagrangian scheme, has
 38 been demonstrated to be a robust method as it permits: the representation of
 39 strong material contrasts; large deformation over the time-scale of simulations
 40 without the need for remeshing; the full time- and space-dependence; execution
 41 of distributed memory computers, thereby delivering scalable and computation-
 42 ally efficient results Gerya and Yuen (2003); Moresi et al. (2007); Schmeling et al.
 43 (2008). We thus chose this method as our method of choice. However, the sharp
 44 viscosity changes due to composition result in approximations as a PIC-based
 45 code needs to project the viscosity carried by markers to the finite element grid.
 46 To do so, models rely on averaging methods that include arithmetic, geometric,
 47 and harmonic averaging. Alternatively, they associate arithmetic averaging to
 48 a composite plate in which an isoviscous core is embedded between two visco-
 49 plastic upper and lower layers. The visco-plastic layers use a von Mises yield
 50 criterion such that the pre-yielded viscosity is replaced by an effective viscosity
 51 and stresses do not exceed a maximum allowable stress. In principle, such “rhe-
 52 ologically stratified” plates differ from the plates of Newtonian viscosity used
 53 in laboratory experiments, but it can simply be viewed as another viscosity
 54 averaging method.

55 Many previous subduction models using PIC methods do not consider the
 56 air layer, but instead rely on a free-slip boundary condition at the top surface
 57 associated with yielding of the plate for it to subduct (e.g., Capitanio et al., 2010;
 58 Stegman et al., 2006; Király et al., 2017). While the use of such a boundary
 59 condition combined with the non-linear rheology of the plate has allowed the
 60 study of numerous aspects of the subduction dynamics, it implicitly ignores

the development of any topographic feature during subduction. It also biases the behaviour of subduction as a free-slip top boundary condition is bound to facilitate the plate hinge horizontal motion. Two alternatives to the free-slip top boundary condition are being increasingly used to more realistically represent the upper boundary condition: 1) a direct treatment of the free surface with a zero normal stress condition applied on this surface, and 2) a free-surface proxy referred to as the sticky-air layer approach. Compared to the free surface condition, the sticky-air method is easily implemented as it only requires the addition of a fluid “air” layer of low density and a sufficiently small viscosity at the top of the model domain. Small normal stresses at the surface ensured by the physical properties of the layer (density, viscosity) then allow for the development of a realistic interface. Such an approach has been shown to behave similarly to a free surface Quinquis et al. (2011); Schmeling et al. (2008).

Benchmarking of various numerical models of subduction against a laboratory experiment was first conducted by Schmeling et al. Schmeling et al. (2008) for two-dimensional models, in which the subducting plate (slab) was isoviscous and fixed to the box or tank at one edge, and included either a sticky-air layer or a free surface. The study showed that the Eulerian-based codes with a sticky-air layer surface layer reproduced to some degree the laboratory plate sinking and retreat of its hinge, when harmonic or geometric but not arithmetic averaging of the viscosity was used. Now, being two-dimensional, this study could not investigate how the models would reproduce the three-dimensional slab-induced toroidal flow (which increases as the slab lengthens), and its impact on a subduction dynamics controlled by slab rollback and hinge retreat due to the plate fixed trailing edge.

In this paper, we benchmark three-dimensional numerical models of subduction against a laboratory experiment using the particle-in-cell, finite-element framework *Underworld2*, version 2.3.0 (<http://www.underworldcode.org/>). Apart from the physical dimensions, our study differs from that of Schmeling et al. Schmeling et al. (2008) as our models do not consider geometric averaging. Besides, we go beyond the limits of a *stricto-sensu* benchmarking

exercise in considering a composite and not entirely iso-viscous plate, despite the fact that the plate in the laboratory experiment is a single and iso-viscous layer. The rationale for doing so is that the two methods in practice serve the same purpose. In the composite approach, commonly used in numerical models of subduction, the upper and lower visco-plastic sublayers sandwiching an iso-viscous core layer lubricate the interfaces of the plate with its surrounding, in the same way as harmonic averaging.

We present a set of 8 numerical models that are discussed and compared against the laboratory experiment. Our study provides a two-way conclusion in highlighting the influence on the dynamics of surface tension effects in the laboratory, and the lubrication of the interfaces of the subducting plate with its surroundings in the numerical models.

2. Model definitions

2.1. The laboratory experiment

The laboratory experiment that was performed for the benchmark was conducted at a temperature of $(20 \pm 0.5)^\circ\text{C}$ (Figure 1). We used an Acrylic tank 100 cm long, 62 cm wide, and 60 cm deep filled with glucose syrup (analogue mantle) up to 7.5 cm, in which we inserted a 61.5 cm wide wall of height 10 cm at 70 cm to reduce the tank length. The plate was made of high-viscosity linear-viscous silicone (Dow Corning Silastic) mixed with fine iron powder. The plate was fixed at one of its lateral edges using a rigid Acrylic corner of the same width as the plate against which the plate was adhering over an inward length of 6 cm from the tank wall. Subduction of the plate was initiated by downward bending 3 cm length of its free edge at an angle of ~ 30 degrees. Black neutrally buoyant spheres were spread at the top surface to serve as slab-induced flow tracers. The density and viscosity of the glucose were similar 1417 kg m^{-3} and 110 Pa s to within errors of 0.5 kg/m^3 and 6 Pa s , respectively. The density and viscosity of the plate were 1617 kg m^{-3} and $57,0000 \text{ Pa s}$ within errors of 4 kg/m^3 and 400 Pa s , respectively. This model setup with its fixed plate edge,

121 shallow depth and absence of an overriding plate is especially suited to look at
 122 a long-term slab rollback motion and hinge retreat at the difference of free plate
 123 subduction models for which motion is directed towards the hinge.

Figure 1: Two juxtaposed views of the laboratory experiment 40 seconds after initiation. From top to bottom are the side and top views, respectively. Note the Acrylic corner against the tank wall on top of which is an orange level.

124 *2.2. The generic numerical model*

125 The subduction of the buoyancy-driven plate fixed at its trailing edge in the
 126 shallow-mantle was considered with an overlying sticky air layer (Figure 2). The
 127 subducting plate consisted of either an entirely iso-viscous layer of Newtonian
 128 viscosity μ_s , or a composite plate made of three sublayers: a core of thickness
 129 d_c and Newtonian viscosity μ_s embedded in an upper and a lower visco-plastic
 130 layers of thicknesses d_{lp} and d_{lp} , respectively, following a von Mises criterion.
 131 In practice, the von Mises criterion is enforced by assigning to the viscosity of
 132 the upper and lower layers the minimum between the Newtonian viscosity μ_s
 133 and an effective von Mises viscosity μ_{vM} defined by

$$\mu_{vM} = \frac{C}{2\dot{\varepsilon}}, \quad (1)$$

134 where C is the cohesion and $\dot{\varepsilon}$ is the second invariant of the strain-rate
 135 tensor given by $\dot{\varepsilon} = \left(1/2 \sum_{ij} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij}\right)^{1/2}$, in which the components of the
 136 strain-rate tensor ε_{ij} are derived from the velocity field $\mathbf{u}$ according to
 137 $\varepsilon_{ij} = 1/2 (\partial u_i / \partial x_j + \partial u_j / \partial x_i)$.

138 Subduction is initiated by assuming a pre-existing instability of the plate,
 139 namely a slab tip. The subsequent evolution is self-sustained by the negative
 140 buoyancy of the slab. Table 1 details all the numerical model parameters aligned
 141 with those of the laboratory experiment. We note that the width of the numer-
 142 ical box was taken 2 cm shorter than the width of the tank in the laboratory
 143 after tracers within 1 cm from the tank walls had shown no more than 4 cm of
 144 advection compared to ~ 30 cm for tracers in the mantle wedge region during
 145 the laboratory experiment. This choice was made to reduce the computational
 146 cost.

Figure 2: Generic numerical model. The two quantities $y_t(t)$ and $x_h(t)$ are the two metrics that will be used for the quantitative comparison with the laboratory experiment for the sinking of the slab tip to the bottom boundary, and the plate hinge retreat.

	Variable	Values	Units
<u>Box dimensions</u>			
Box height	L_y	7.88	cm
Box length	L_x	70	cm
Box width	L_z	60	cm
<u>Mantle fluid</u>			
Gravitational acceleration	g	9.8	m s^{-2}
Mantle fluid density	ρ_m	1417	kg m^{-3}
Mantle fluid viscosity	μ_m	110	Pa s
Mantle fluid depth	H	7.5	cm
<u>Sticky layer</u>			
Sticky Layer density	ρ_a	0.0	kg m^{-3}
Sticky Layer viscosity	μ_a	1.0	Pa s
Sticky Layer thickness	h	0.38	cm
<u>Plate</u>			
Plate density	ρ_s	1617	kg m^{-3}
Plate density contrast	$\Delta\rho_s = \rho_s - \rho_a$	200	kg m^{-3}
Plate Newtonian viscosity	μ_s	57,000	Pa s
Core plate cohesion	C	1-5	Pa
Plate length (prior to imposing initial slab tip)	L	37.6	cm
Plate width	W	20	cm
Isoviscous plate thickness	d	0.76	cm
Core plate thickness	d_c	0.38	cm
Upper plate thickness	d_{up}	0.19	cm
Lower plate thickness	d_{lp}	0.19	cm
<u>Initial tip</u>			
Slab initial tip length	$l(0)$	3	cm
Initial dip angle of the slab	θ_0	30	degrees

Table 1: Numerical model parameters. Lengths of the model were scaled so that 1 cm represents 100 km in nature. The timescale was scaled as $t^{\text{nature}}/t^{\text{model}} = (H^{\text{nature}}/H^{\text{model}}) ((\Delta\rho_s g d^2 / \mu_m)^{\text{model}} / (\Delta\rho_s g d^2 / \mu_m)^{\text{nature}})$, where g is the gravitational acceleration. Hence 1 s represents 3 kyr when the following nature parameters are used: plate thickness $d = 100$ km, plate density contrast $\Delta\rho_s = 100 \text{ kg m}^{-3}$, and upper mantle depth H km and viscosity μ_m of 750 km and 10^{20} Pa s , respectively.

3. Numerical method

To solve the incompressible Stokes flow problem presented above, we used the *Underworld2* code (<https://github.com/underworldcode/underworld2>). *Underworld2* is a Lagrangian integration point finite element code that includes a parallel 3D finite element discretization. It is the newest Python-friendly version of *Underworld* Moresi et al. (2007), which utilises the LavaVu library for inline visualisation (<https://github.com/OKaluza/LavaVu>). The discretized equations were solved using the multigrid approach with the PETSc package (Balay et al. (2017a,b, 1997)).

We took advantage of the system symmetry about the hinge-perpendicular z axis (see Figure 1) to halve the computational domain in this dimension. As a result, the physical dimensions of the model were $0.7 \times 0.0788 \times 0.3 \text{ m}^3$ in the x , y and z directions, respectively, including the top thin lubrication layer employed to simulate a free surface Crameri et al. (2012); Schmeling et al. (2008). The results were obtained using two primary grids of dimensions $256 \times 128 \times 128$ and $128 \times 64 \times 64$ (Models 1-6), with mesh refinement applied to these grids via mapping functions in the x and z directions, resulting in a $\sim 20\%$ reduction of the grid size in the plate region along x and z . Two intermediary grids of dimensions $160 \times 112 \times 112$ and $128 \times 96 \times 96$ (Models 7 & 8), with more aggressive mesh refinement in the plate region, were also employed for the only purpose of a convergence test of the harmonic averaging. In the plate region, the grid resolutions Δx , Δy and Δz in the respective x , y and z directions were thus $\Delta x/d = 0.288, 0.575$, $\Delta y/d = 0.081, 0.162$ and $\Delta z/d = 0.246, 0.493$ for the primary grids, and $\Delta x/d = 0, 403, 0.504$, $\Delta y/d = 0.092, 0.108$ and $\Delta z/d = 0.246, 0.288$ for the intermediary grids.

The left, right, bottom and front walls of the domain were prescribed a no-slip boundary condition, while the back wall was applied a symmetry condition and the top was open (i.e. prescribed a zero normal stress). Furthermore, to mimic the effect of the rigid Acrylic corner fixing the plate in the laboratory, a no-slip velocity condition was applied to the top of the plate over an inward

length of 6 cm from its fixed edge, and over its entire width. Thirty particles carrying the composition (density and viscosity) of the materials were initially distributed in each grid cell, which resulted in a swarm of up to 1.26×10^8 Lagrangian particles. A second order Runge-Kutta scheme was used for particle advection with particles able to escape through the top boundary. Population control mechanisms ensured sufficient particle counts per element. In addition, passive tracers were initially distributed randomly throughout the domain at the height of the plate base in all the domain, and also within the plate interior in a series of 10 laterally spaced lines across the plate half-width. The tracers were used to image the slab-induced flow and extract $y_t(t)$ and $x_h(t)$ shown in Figure 1 for the analysis of section 4.2.

At material interfaces, either harmonic or arithmetic averaging of the viscosities was employed, while only arithmetic averaging was used for the composite plate models. The harmonic mean has the form $1/\langle \mu \rangle = 1/\mu_i + 1/\mu_j$, where i and j refer to neighbouring particles. At the high viscosity contrast interfaces of the plate-air ($\mu_s/\mu_a = 57,000$) and plate-mantle ($\mu_s/\mu_m = 518$), harmonic averaging has the effect of damping the viscosity contrasts ($\langle \mu \rangle/\mu_a = (1 + \mu_a/\mu_s)^{-1} \equiv 1$ and $\langle \mu \rangle/\mu_m = (1 + \mu_m/\mu_s)^{-1} \equiv 1$). This results in a plate with a core of high viscosity μ_s and two thin layers of viscosity $\equiv \mu_m$ and $\equiv \mu_a$. As a result, harmonic averaging bears similarities with the composite plate as shown in Figure 3, especially at the interface plate/mantle. The noticeable deviation is the difference in the viscosity reduction at the interface air/plate, that is more drastic following harmonic averaging. Full details about the algorithm used for casting the harmonic mean function to the nodes can be found at http://underworld2.readthedocs.io/en/latest/underworld.utils.html#underworld.utils.MeshVariable_Projection.

At the working resolution $256 \times 128 \times 128$, and when using the harmonic averaging, the calculation is expected to be closed to convergence as the ratio of the grid size to the characteristics length d is less than 0.35 (Figure 21 of Schmeling et al. (2008)). Table 2 lists the numerical runs that have been performed.

Model 2

Model 4

Figure 3: Initial map of the slab viscosity for model 2 and model 4. Note the similarities of the viscosity layering at the slab/air and mantle interfaces.

Model Id.	Grid size	Plate layers	Plate rheology	Viscosity averaging	$ u_s $ mm/min	$ u_r $ mm/min	$\Delta u_s/u_s$ %	$\Delta u_r/u_r$ %	R cm
1	$256 \times 128 \times 128$	1	isoviscous	harmonic	10.06	9.31	36	19	16
2	$128 \times 64 \times 64$	1	isoviscous	harmonic	9.07	9.64	23	23	11
3	$128 \times 64 \times 64$	1	isoviscous	arithmetic	10.06	1.06	36	86	5.5
4	$128 \times 64 \times 64$	3	isoviscous & von Mises ($C = 1$ Pas)	arithmetic	8.62	7.26	17	7	10
5	$128 \times 64 \times 64$	3	isoviscous & von Mises ($C = 2.5$ Pas)	arithmetic	8.67	7.09	18	9	11
6	$128 \times 64 \times 64$	3	isoviscous & von Mises ($C = 5$ Pas)	arithmetic	8.21	6.4	11	18	14
7	$160 \times 112 \times 112$	1	isoviscous	harmonic	9.08	8.83	23	20	13
8	$128 \times 96 \times 96$	1	isoviscous	harmonic	9.16	8.57	24	16	12

Table 2: Numerical experiments described in this paper. The table includes the grid size, the plate structure and rheology, the applied viscosity averaging, and the measured slab sinking and retreat absolute rates, $|u_s|$ and $|u_r|$, respectively. The value of the cohesion C used in each run is indicated. $\Delta u_s/u_s$ and $\Delta u_r/u_r$ are the relative errors to the experimental velocities that are defined by $\Delta u_s/u_s = (|u_s| - |u_s^l|)/|u_s^l|$ and $\Delta u_r/u_r = (|u_r| - |u_r^l|)/|u_r^l|$. The absolute values of the slab sinking and retreat rates of the laboratory experiment were $|u_s^l| = 7.37$ mm/min and $|u_r^l| = 7.80$ mm/min, respectively. R is the amount of slab retreat in cm that was achieved before the secondary instability developed.

4. Modelling results

4.1. Qualitative results

In the following, we only present qualitative results for models 1-6, as models 7 & 8 were essentially designed for examining the convergence of the harmonic models.

In all numerical models, the slab first steepened with time while sinking before rolling back. Except in model 3 using the arithmetic averaging, the slab tip reached the bottom boundary within about 600 s in good agreement with the laboratory experiment. However, in greater detail, models show differences in their evolution of the slab geometry, namely the slab dip angle, and deformation. Figure 4 shows the seven slabs 600 s after the initiation of subduction. All the models apart from model 3, whose slab tip has not touched the bottom, show a limited deformation of the free lateral plate edge, that is best seen from the top views thanks to the slab tracers. Such a deformation also developed in the laboratory experiment. Slabs of models 1 and 2 using the harmonic averaging hardly exhibit any concave curvature and have a lower dip angle than

224 their stratified counterparts (models 4-6), which compare more closely with the
225 laboratory experiment.

226 During the slab rollback and hinge retreat, the velocity as shown by the
227 four cross sections for model 1 in Figure 5, is consistent with that inferred in
228 this laboratory experiment and previous laboratory studies Strak and Schellart
229 (2014). In particular, the ambient shear profile and its change of direction below
230 the plate, as seen in the vertical cross section c, is similar to that inferred from
231 slab-plume experiment (see the supplementary information S2 of Mériaux et al.
232 (2015)). The ability of the mantle to flow over the plate edge and below the
233 sticky air layer is also noticeable in cross section d at $x = 16$ cm. Besides, the
234 slab-induced return flow combining a poloidal and toroidal component shows
235 a dependence on two length scales: the width of the plate (20 cm) in the z
236 direction, and the depth of the mantle (7.5 cm) in the x direction [see in Figure
237 5 the yellow velocity magnitude halo that laterally extends up almost as far as
238 the walls (z direction), but only half as much in the mantle wedge and sub-slab
239 corner (x direction)]. This is also consistent with the motion of the randomly
240 distributed tracers seen at the surface in both the laboratory experiment and
241 numerical models (see the Movies 0-6 provided in the Supplementary informa-
242 tion). The dependence of these two length scales on the scale of the return flow
243 appears to somewhat contradict the conclusions of Király et al Király et al.
244 (2017), for which the characteristic length, axis and the shape of the toroidal
245 cell mainly depend on the thickness of the mantle.

246 Now, at varying times but comparable amount of hinge retreat, the slab
247 lateral edge became unstable (see all numerical experiment panels in Figure 6),
248 before collapsing. The development of the instability appears to be sensitive to
249 the grid resolution as model 1 whose grid is twice as fine as model 2, everything
250 else being the same, has an instability that is delayed by 274 s. The instability
251 is also all the more delayed as the cohesion of the stratified models increases. It
252 thus takes almost two and a half times longer for the instability to develop in
253 model 4 compared to model 6 as the cohesion is increased by a factor 5. This
254 instability was not observed during the laboratory experiment.

Laboratory

Model 1

Model 2

Model 3

Model 4

Model 5

Model 6

Figure 4: Slab geometry at $t = 600$ s as the tip has reached the bottom. Side views are shown in the left hand side and top views are shown in the right hand side.

Figure 5: Cross-Sections in x , y and z of the velocity for Model 1 at time $t = 665$ s. The colour bars refers to the velocity magnitude. Velocity scales are in mm/min. The slab tracers (in red) are used to localize the slab.

Laboratory

$t=1320$ s

Model 1

$t=961$ s

Model 2

$t=687$ s

Model 4

$t=731$ s

Model 5

$t=875$ s

Model 6

$t=1778$ s

Figure 6: Early stage of the instability of the lateral edge for all numerical models except model 3, as the amplitude of the growth a is about twice the initial plate thickness, i.e. $a \sim 2d$. Side views are shown in the left hand side and top views are shown in the right hand side. Note also the decrease of the wavelength λ of the instability as the harmonic averaging is replaced by the composite plates and the cohesion is increased. For comparison, the slab from the laboratory experiment is shown at a comparable amount of retreat to model 1.

255 *4.2. Quantitative results*

256 To further investigate how close the numerical models agree with the lab-
 257 oratory experiment, we followed the slab tip sinking to the bottom boundary,
 258 and the plate hinge retreating over time using the Lagrangian swarm and the
 259 plate interior array of passive tracers, respectively. In practice, we recorded at
 260 each time step the vertical position y_t of the slab tip at the slab centreline, and
 261 the horizontal position x_h of the hinge of the plate relative to the plate fixing
 262 wall (see Figure 1). Figures 7 and 8 show the respective profiles as a function
 263 of time for the laboratory experiment and numerical models.

Figure 7: Vertical position of the slab tip $y_t(t)$ measured at the slab centreline for the laboratory and numerical models.

Figure 8: Horizontal position of the hinge of the plate $x_h(t)$ to the plate fixing wall measured at the slab centreline for the laboratory and numerical models. The diamonds indicate the early stage of the lateral side instability as shown in Figure 6.

264 In all the models, between the heights y of 3.5 and 1.5 cm, the rates at which
265 the tip sink are reasonably steady, and are thus estimated by linear fits of the
266 tip vertical position y_{tip} as a function of time within this depth interval. Values
267 of the rates are given in Table 2. The tip sinking rates differ from the laboratory
268 rate by 11 to 36 %. Models 4-6 are in best agreement with the laboratory, as
269 opposed to models 1-3 and 7 & 8.

270 By contrast, while the retreat rate of the plate hinge in the laboratory ex-
271 periment is steady from a distance to the wall from 30 to 10 cm, the numerical
272 models exhibit more limited steady retreats. Models 1, 2, 7 and 8 based on
273 harmonic averaging show a steady retreat until they develop their instability,
274 but models 4-6 based on the stratified plate are quasi-steady before their insta-
275 bility. A slight non-linearity in the retreat actually emerges and increases as the
276 cohesion increases. Nevertheless, for comparison purposes, we could estimate
277 the retreat rates by linear fits of the hinge horizontal position x_h as a function
278 of time within the limited interval $x_h = [25, 29]$ cm, still representing 30-40% of
279 the total amount of retreat. Excluding model 3, the hinge retreat rates differ
280 from the laboratory rate by 7 to 23 %. Again, models 1-3 and 7 & 8 are the
281 models in worst agreement with the laboratory one as opposed to models 4-6,
282 aside from the non-linearity of the latter.

283 Models 1 and 2, which differ in their grid size by a factor two in the x , y and
284 z directions, show the importance of the resolution. Although the slab sinking
285 and retreat rates remain steady by only 10 and 3%, respectively, the amount
286 of retreat before the instability develops is reduced by a substantial 30% at
287 the lower resolution. Therefore, we anticipate that models 4-6, if simulated
288 again using $2\times$ higher the spatial resolution, would develop their instability
289 later, but it is not clear though if the higher spatial resolution would improve
290 the steadiness of the retreat. Testing this hypothesis was not computationally
291 feasible with our current resources, and thus was left for future investigation.
292 Besides, the change in cohesion by a factor 5 across models 4-6 has hardly any
293 impact on the sinking rate that varies by 4% only, but it affects the retreat rate
294 by 11%.

Models 1, 2, 7 and 8 give further insight into the dependence of the Rayleigh-Taylor instability on the resolution in the harmonic models. Figure 9 shows that, as the resolution is increased, the time of the lateral edge instability t_i is further delayed. A logarithmic fit suggests that t_i will not converge to a finite value as the resolution goes to infinity, but the rate of increase of t_i will go to zero. As shown in Appendix A, such a behaviour is expected when the Rayleigh-Taylor instability grows from numerical noise inherent to the model. When the Rayleigh-Taylor instability is seeded by a finite amplitude initial perturbation, the onset of the Rayleigh-Taylor instability does not depend on resolution.

Figure 9: Time of the lateral edge instability t_i in harmonic models 1,2, 7 & 8, defined as in Figure 4, as a function of resolution $d / \sqrt[3]{\Delta x \Delta y \Delta z}$ (blue diamonds). The black line is a logarithmic fit expected from the exponential growth rate of a Rayleigh-Taylor instability (see Appendix A).

5. Discussion

Our comparison of a series of numerical models of subduction with a laboratory experiment has shown various dynamics of the models. The numerical results obtained using the arithmetic averaging differ dramatically from the other models and the laboratory experiment. The lack of retreat is quite striking. This study thus agrees with the findings of Schmeling et al. (2008), and naturally extends their conclusions to the three-dimensional case.

Beyond this concordance, although all the models together with the laboratory experiment show a sinking rate that is steady over 27% of the total depth, the model rates do not match with the experimental rate, and appear to depend on the plate or plate sub-layer viscosity. This behaviour is actually not physically sound when compared to the analytic solution for the sinking of a sphere of diameter D , excess density $\Delta\rho$ and viscosity μ_s in a fluid of viscosity μ_m . As the viscosity contrast μ_s/μ_m increases and exceeds ~ 200 , the sinking rate U defined by $U = f(\mu_s/\mu_m)\Delta\rho g D^2/12\mu_m$ with $f(\mu_s/\mu_m) = (1 + \mu_s/\mu_m)/(1 + 1.5\mu_s/\mu_m)$, converges to that of a rigid sphere, and thus only depends on the ambient μ_m (Lamb (1932)). We should therefore expect that the sinking of the slab is independent of the plate viscosity, as if it was rigid, and only depends on the viscosity of the mantle. A similar and more specific conclusion was reached by Ribe (2010) and Li and Ribe (2012) using a dimensionless stiffness parameter S , measuring the ability of a viscous sheet to deform by bending in an ambient fluid of different viscosity. Defining the stiffness by $S = (\mu_s/\mu_m) \times (d/l_b)^3$, where l_b is the combined slab and flexural bulge lengths, these authors showed that, when $S \leq 1$, the sinking speed is controlled by the mantle viscosity, but, when $S \gg 1$, it is controlled by the viscosity of the sheet. As the bending length l_b is not constant but increases with time, S decreases with time and a transient may occur at the initiation of the subduction, when the sinking rate is controlled by the viscosity of the sheet. In our experiments and assuming that the initial bending length is the tip length $l = 3$ cm, S is

initially ~ 8.4 and it is not until $l_b \sim 6$ cm (i.e. slab tip height of ~ 3.25 cm) that $S \leq 1$ and the sinking of the slab would be entirely controlled by the viscosity of the mantle. This is consistent with the results shown in Figure 5, where the linear steady sinking phase is found from a slab tip height of 3.5 cm.

In this regard, one would think that the harmonic averaging in reducing the plate viscosity at its interface with the mantle to the viscosity of the mantle would be a wise choice. However, the agreement between the measured sinking rate is, in general, poor. Eventually, models best matching the experimental results in terms of capturing the sinking rate, the dip angle, the concavity of the slab are those of the composite plates. The same trend applies to the retreat rate. While the overall mismatch is reduced compared to the sinking rate, models with the composite models have rates closest to the experimental rate. However, as the composite plate sub-layers are made stiffer, the retreat increasingly departs from a steady retreat as the one measured in the laboratory experiment.

The differences found in the sinking and retreat rates between the numerical and laboratory could result from the lack of accuracy in the determination of the experimental parameters. This would essentially arise from the uncertainty in the measurement of the viscosity of the glucose syrup as the sinking and retreat rates are independent of the plate or core plate viscosity given the high plate/mantle viscosity contrast, but inversely depend on the viscosity of the glucose. However, only 5% error in the rates could be explained in this way (see section 2.1), thus failing to explain the full extent of the differences.

All the models apart from the one relying on the arithmetic mean of the viscosity developed a secondary instability along the slab edge. Such instability began after 10 to 16 cm (i.e. 1000 to 1600 km) of retreat. To our knowledge, the secondary instability has not been reported in previous numerical studies of subduction, but there are a few possible explanations. Firstly, models using a free-slip upper boundary condition are expected to be *de-facto* devoid of this behaviour as the plate is then constrained to remain flat and attached to the upper surface of the model domain. Secondly, we have shown that the

instability development is substantially retarded as the cohesion is increased. Previous models typically use higher values of the cohesion than those of this study. Their dimensionless value of the cohesion or yield stress C^* is typically chosen in the range 0.04-0.06, whereas C^* is scaled using a reference stress σ given by $\sigma = (\rho_s - \rho_m)gH$. In our calculations, $\sigma = 147$ Pa. Hence, the equivalent dimensionless cohesions C^* are in a lower range [0.007-0.034]. Finally, studies usually simulate more limited retreat phase and less hinge retreat rate compared to this study due to the presence of an overriding plate suppressing hinge migration (see for instance Yamato et al. (2009)). In nature, the sinking of a plate is actually hampered by the presence of surrounding plates, and plate reconstructions more often identify plate hinge migration in the order of hundreds of kilometres rather than thousands. Only a few events major events in the Earth evolution have been marked by significant hinge migration of the order of 1000 km.

Conversely, the secondary instability was never seen in the laboratory experiment, which we explain by the surface tension of the glucose. While the inputs of the numerical and analogues experiments were closely matched within errors of at most 7%, the numerical models neglect a surface traction boundary condition intended to model the surface tension of the glucose, which is well known for suppressing or retarding the growth of instabilities Bellman and Pennington (1954). We also know that it is actually not negligible because of the initial floating of the plate above the glucose despite being 14% more dense. Evaluating the Bond number measuring the relative importance of the buoyancy forces to those of surface tension, which is defined by $Bo = \Delta\rho g d^2 / \Gamma$, where the surface tension Γ of glucose syrup is 0.08 N/m Borhan and Pallinti (1999); Llewellyn et al. (2002), gives $Bo = 1.4 \approx \mathcal{O}(1)$ for our experiment, which does not support the neglect of the surface tension.

The importance of the ratio of the typical buoyancy plate stress to the surface tension in the dynamics of subduction in the laboratory experiments was previously reported by Ribe in his two-dimensional numerical study of free subduction based on a boundary-integral representation of the subducting plate

associated with an upper thin lubrication layer, similar to the sticky air layer Ribe (2010), and more recently by Li and Ribe in the three-dimensional numerical equivalent Li and Ribe (2012). However, when their three-dimensional model produced a detachment of its plate edge (appendix B of Li and Ribe (2012)), they numerically suppressed it on the ground that the sinking of the plate edges had a negligible effect on the dynamics of the slab itself. In the first place, Li and Ribe's work shows that the edge instability described in this study is not inherent to the particle-in-cell, finite-element scheme that we used. Besides, their suppression of the flank instability was in effect equivalent to modelling subduction with surface tension, which could explain why these authors ultimately found a good agreement between their model and the laboratory experiments of Schellart (2008) while delimitating the modes or styles of subduction.

Our experiments suggest that the presence of the surface tension in the laboratory influences the bending of the plate forcing the plate to sink more vertically than what it would do otherwise (see Figures 4 and 6), so that the force acting downwards due to the plate negative buoyancy is maximised and can overcome the additional resisting upward force. In doing so, the slab dip angle in the laboratory model is larger than that of the harmonic numerical model, lacking such resistance, but closer to the slab dip angle of the composite plate models 4–6, whose overall stiffness is lowered by reducing the thickness of the effective stiff layer (see, for instance, Capitanio et al. (2009)). This could explain the better fit between the dynamics of the composite plate models and the laboratory model.

To conclude, this benchmark study provides a two-way insight in the laboratory experiments and numerical models of subduction. On one hand, we show that a secondary instability of a single subducting plate, that is physically sensible, occurs along its edge in numerical models that include a free surface provided that the plate detachment from the surface is not suppressed by artificial means (e.g. via a flatness condition or plate rheology). On the other hand, we show that surface tension could play a more important role in the subduction

and slab deformation dynamics of the laboratory experiments than previously thought. When transposed to nature, this work provides a further incentive for building numerical and laboratory models in which the surrounding plates would be included, as those are realistically preventing plate edge instability.

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Appendix A. Implicit numerical noise and Rayleigh-Taylor instability

Implicit numerical noise can seed the development of a Rayleigh-Taylor instability. When this occurs, the onset of the instability depends the resolution of the model. By contrast, when an explicit perturbation is used to seed the instability development, the time to onset is independent of resolution (once the simulation is convergent).

Following Chandrasekhar (1961), the amplitude A of the growing mode k should have a solution whose dependence on time t is exponential and, as given by Mériaux et al. (2011) in appendix B, its form would

follow

$$A(t) = \dot{A}(0)\tau_G(k) \left(\exp \left(\frac{t}{\tau_G(k)} \right) - 1 \right), \quad (\text{A.1})$$

where $\tau_G(k)$ is the growth rate for mode k and $\dot{A}(0)$ is the initial vertical velocity of the disturbance, or equivalently

$$\ln \left(\frac{A(t)}{\dot{A}(0)\tau_G(k)} + 1 \right) = \frac{t}{\tau_G(k)}. \quad (\text{A.2})$$

By numerical noise, we postulate an amplitude related to resolution by $\dot{A}(0)\tau_G(k) \sim \Delta y$. This assumes that the initial particle distribution is the primary source of noise driving the simulation. The time t_i for the onset of the instability then depends on the resolution as

$$t_i = \tau_G(k) \ln \left(\frac{A(t_i)}{\Delta y} + 1 \right), \quad (\text{A.3})$$

giving

$$\frac{dt_i}{dx} = \frac{\tau_G(k)A(t_i)}{A(t_i)x + 1}, \quad (\text{A.4})$$

where $x = 1/\Delta y$.

In the case where the instability is seeded by an initial and finite disturbance $A_0 > \Delta y$, the time t_i for the onset of the instability becomes independent of the resolution and satisfies

$$t_i = \tau_G(k) \ln \left(\frac{A(t_i)}{A_0} + 1 \right). \quad (\text{A.5})$$

To demonstrate and characterize this behaviour, we used a two-dimensional unit box, in which a compositionally viscous buoyant layer lies underneath a denser layer of identical viscosity η_0 in an unstable configuration. The boundary conditions satisfied periodic conditions on the vertical boundaries, and a no-slip condition on the horizontal boundaries. The buoyant layer was half as thick as the box height. The densities of the light and dense layers were $\rho_l = 0$ and $\rho_d = 1$, respectively, so that the density contrast $\Delta\rho$ was 1. Time was made dimensionless with respect to $\eta_0/\Delta\rho g$. We employed the successive grid sizes $N \times N = 5 \times 5, 9 \times 9, 17 \times 17, 33 \times 33$, and 65×65 , so the resolutions were $\Delta x = \Delta y = 1/N$. We used odd numbers of N to ensure that the interface

463 was positioned in the middle of a grid cell. We ran two types of models at the
 464 successive resolutions until a given amplitude of a mode growth was reached. In
 465 the first case, no perturbation was prescribed. In the second case, we prescribed
 466 a cosine perturbation of finite amplitude (0.05) at the interface.

467 Figure A.10 shows the onset of the instability as a function of resolution
 468 in the two cases. In the unseeded perturbation case, the time for the onset of
 469 the instability increases with increasing resolution. The rate of increase of t_i
 470 eventually decreases as $1/\Delta y$ increases following equation A.4 but t_i will never
 471 reach a constant value as $1/\Delta y \rightarrow \infty$. In the seeded perturbation case, the
 472 mode growth does not depend on resolution and t_i reaches a constant value of
 473 ~ 20 .

Figure A.10: Dimensionless time of the Rayleigh-Taylor instability t_i as a function of resolution $1/\Delta y$. Unseeded perturbation case (i.e. implicit numerical noise case) is in blue. The black line is the logarithmic fit, whose form follows (equation A.3), for the latter case. Seeded perturbation case is in magenta.

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