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Monopolistic Competition and Exclusive Quality

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Abstract:
In this paper I adapt a classic model of monopolistic competition where products are differentiated by quality, in order to study a market in which high-quality products can only be enjoyed by users with sufficient ability. Casting the model in the context of higher education – where selective colleges pledge quality by excluding low-ability students –, I show that there are two equilibrium market segmentations: one in which highly selective colleges serve high-income high-ability students, and another in which highly selective colleges are *cheaper* than the less selective competitors that cater to low-ability high-income students. I provide an example to illustrate the welfare implications of these two market configurations.

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1 Introduction

In many markets firms rely on the characteristics of their customers to provide their services. Higher education is a notable case in point: colleges pledge quality by excluding potential buyers with insufficient academic ability. Rothschild and White (1995)provide many other examples of commercially sold services where customer characteristics determine the quality of the provided output. In this paper I adapt a classic model of monopolistic competition where products are differentiated by quality (Gabszewicz and Thisse 1979; Shaked and Sutton 1982) to analyze the possible market structures that may arise if high-quality products can only be enjoyed by users with sufficient ability. Casting the model in the context of higher education, I show that there are two equilibrium market segmentations: one in which highly selective colleges serve high-income high-ability students, and another in which highly selective colleges are *cheaper* than the less selective competitors that cater to low-ability high-income students.

I consider here a market for higher education where two colleges compete for students by choosing prices and quality, the latter in the form of academic standards implemented through cut-off academic admissions. Since the admission cut-off is by assumption the college quality, this model allows an interpretation in the context of peer effects: the students' utility from attending a particular college depends on the lowest ability of their peers in that college. A natural interpretation of this assumption is that colleges set cut-off admission policies in order to provide tuition of a certain degree of academic complexity, because the teaching can only be as advanced as the least able student that is admitted in the class. This can be contrasted with the approach developed in Epple and Romano (1998)and other papers in the peer-effects literature, where the interaction between admission policies and tuition fees is modeled by assuming that the students' utility from attending a particular college depends on the *average* ability of entrants in that college.

A general human-capital model of education would allow for the fact that students benefit from learning outcomes, and those outcomes are affected by the quality of peers in the classroom, both directly through student-to-student spill-overs, as well as indirectly, by affecting the overall level of teachers' effort and teachers' choice of the level at which to target instruction.[1] The traditional approach in the peer effects literature (Epple and Romano 1998; Epple, Romano, and Sieg 2003; 2006 ; etc.) has been to shut off the indirect mechanism and assume that the students utility from attending a particular college depends on the average ability of entrants in that college. In the model I propose here, I do the opposite: I shut off the direct mechanism, and assume instead that the students utility from attending a particular college depends on the level at which teachers target instruction, which is in turn determined by the ability of students at the bottom of the class.

As pointed out by Duflo, Dupas, and Kremer (2011), the peer effects literature has focused on the direct effect of peer quality operating through student-to-student spillovers, mainly because of data reasons. One exception in the literature can be found in Lavy, Paserman, and Schlosser (2012), who show that the proportion of low-ability students in a high-school class lowers the scholastic achievements of the other students, in part

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due to deterioration of the teacher's pedagogical practices. The approach I propose in this paper takes such mechanism of peer effects seriously, and applies it to the market for higher education.

Focusing on the indirect effect of peer quality, rather than on the direct effect, has important consequences for the predictions of the theory. On the one hand, if utility depends on the marginal ability of other consumers (the indirect effect), then there are no incentives for firms to engage in price discrimination by ability; and as I show in this paper, this implies that it is possible to sustain in equilibrium, not only a hierarchical market segmentation where high-quality goods are purchased by the high-ability high-income segment of the population, but also a non-hierarchical segmentation where low-quality goods are relatively expensive, as the firms producing them cater to low-ability high-income customers. On the other hand, if utility is affected by the average ability of other individuals enjoying the same good (the direct effect), then firms do have an incentive to engage in price discrimination, and as discussed in Epple and Romano (1998), the only market structure that is sustainable in equilibrium is hierarchical and features "pricing by ability".[2]In Epple, Romano, and Sieg (2006)and other traditional models of market competition focusing on direct peer-effects, pricing by ability takes the form of tuition discounts: colleges charge a "sticker price" from which they offer reductions to talented students who improve the learning experience of all students in the classroom. Such model of pricing captures well what we observe in the market for higher education in the U.S., where merit scholarships are very common. Indeed, Epple, Romano, and Sieg (2006)and others have found econometric evidence supporting pricing by ability in the U.S. Yet pricing by ability is not observed in all countries with similarly deregulated markets for higher education. In Colombia, for instance, the private sector is similarly large (about 40% of students enrolled in undergraduate degrees study in private universities in Colombia compared to 60% in the U.S.), and while universities are also free to set tuition fees and admission standards, merit scholarships are rare.[3] The model I propose in this paper points out an alternative formulation of peer-effects that is consistent with lack of "pricing by ability" in the competitive equilibrium.

The structure of this paper is as follows. The notation and set-up are introduced in Section 2; a benchmark case with a single firm and a fixed outside option is explored in Section 3; the equilibrium market segmentation under monopolistic competition is explored in Section 4; and Section 5 concludes. Although the main contribution of the paper is developed in Section 4, the benchmark case in Section 3 provides a clean discussion of the key tension driving the main results: high academic standards reduce the size of the market that can be potentially served, while increasing the willingness to pay of potential students with sufficient ability.

2 Set-up and Definitions

Consider a market consisting of two ex-ante identical profit-maximizing firms (colleges), indexed $i \in \{1, 2\}$. The two colleges choose quality standards a_i and prices p_i. By assumption, all customers (students) buying services from a given college pay the same fees and perceive the same quality. The colleges' technologies are such that the cost of serving an extra student is c, regardless of the choice of quality or the total number of students.[4] On the other side of the market, consider a continuum of students making indivisible and mutually exclusive purchases from the colleges. A student either makes no purchase and takes a fixed outside option, or else buys exactly one unit from the most preferred college. Students are characterized by academic ability $\theta \in [0, 1]$ and income $\omega \in [0, 1]$. The student ability-income type (θ, ω) is publicly observable and drawn with equal probability from the unit square.

Students have preferences over disposable income and tuition quality, but as mentioned before, quality here excludes potential buyers with insufficient academic ability. More specifically, the utility achieved by a student of type $t = (\theta, \omega)$ attending college $i \in \{1, 2\}$ is given by

$$U^t(p_i, a_i) = \begin{cases} (\omega - p_i)a_i & \text{if} \quad \theta \geq a_i \\ 0 & \textit{otherwise} \end{cases}$$

Quality is implemented as a minimum academic requirement, so these preferences assume that conditional on being admitted, wealthier students are willing to pay more for higher academic standards than poorer students of comparable ability (i.e. preferences feature a positive income elasticity of demand conditional on admission).

Students and colleges interact in three successive stages. First colleges simultaneously choose $a_i \in [0, 1]$; then, after quality standards are announced, colleges simultaneously choose $p_i \in [0, 1]$; and finally, after observing the available price-quality menus, students decide which (if any) of the colleges to attend. If students take the outside option, their utility is $U^t(0, q_0) = \omega q_0$ (i.e. the outside option is by assumption equivalent to a non-selective college offering quality q_0 to any student at no cost).

3 Single-College Benchmark

Before tackling the strategic competition problem in stages, consider the static two-dimensional monopoly problem that arises if there is only one college choosing p and a. In this "single-college benchmark" students with ability below the required standard take the outside option, while students with ability $\theta \geq a$ decide between attending college or taking the outside option. Moreover, for any given menu (p, a) there is an income level y for which students of sufficient ability are indifferent between attending college and taking the outside option. Specifically, $(y-p)a = yq_0$, so

$$y = \frac{pa}{a - q_0} \tag{1}$$

Now it follows on inspection of $U^t(p, a)$ that students endowed with ability $\theta \geq a$ and income $\omega \geq y$ will strictly prefer the college over the outside option; so demand is given by

$$x = \begin{cases} (1-y)(1-a) & \text{if} \quad 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

which in turn means that the problem faced by the monopolist consists in choosing $p \in [c, 1]$ and $a \in [q_0, 1]$ to maximize profits $\pi(p, a; q_0, c) = (p-c)(1-a)\left(1 - \frac{pa}{a-q_0}\right)$ subject to $0 \leq \frac{pa}{a-q_0} \leq 1$. The necessary first-order conditions for this problem give

$$p = \frac{(a-q_0)^2}{a^2 - 2aq_0 + q_0} \tag{3}$$

$$a = \frac{q_0}{c - 2p + 1} \tag{4}$$

which yield an interior solution if q_0 and c are sufficiently small. Specifically, notice that substituting eq. (4) into eq. (1) gives $y = \frac{a(1+c)-q_0}{2(a-q_0)}$; and from this, since positive demand requires $y \in [0, 1]$, it must then be the case that $\frac{q_0}{1-c} < a$. An interior solution thus requires $0 < \frac{q_0}{1-c} < a < 1$, so "sufficiently small" formally means $q_0 + c < 1$. In other words, the benchmark problem has an interior solution only if $0 < q_0 + c < 1$.

The first-order conditions above show that profits are non-monotonic in a, so there is a tension between raising quality and excluding potential customers – and this tension feeds back into the pricing rule. Indeed, it can be checked that these simultaneous equations have at most one solution satisfying $a \in (q_0, 1)$ and $p \in (c, 1)$; so the menu that optimally balances this tension in profits is unique whenever it exists. The following proposition formalizes this result, and Figure 1presents a graphical numerical example for $c = q_0 = 0.1$.

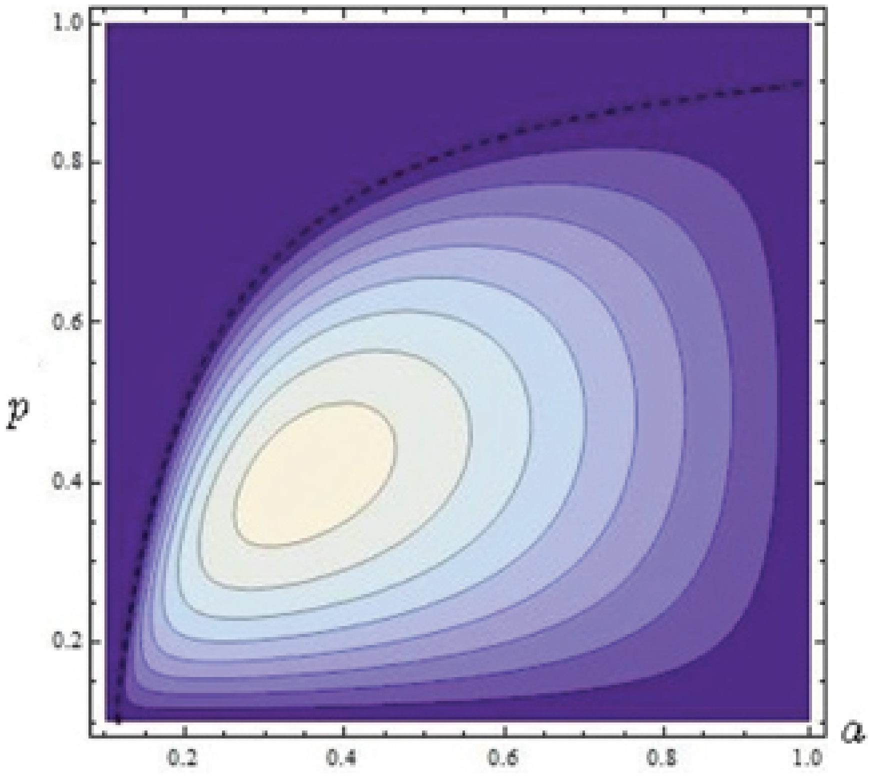


Figure 1 Iso-profit contours as a function of p and a, for $c = q_0 = 0.1$. Higher profit levels are depicted in lighter shades.

Proposition 1. *Suppose* $0 < q_0 + c < 1$. *Then the necessary first order conditions* (3) *and* (4) *have at most one solution satisfying* $a \in (q_0, 1)$ *and* $p \in (c, 1)$. *Hence, the interior solution* (p^*, a^*) *that maximizes* $\pi(p, a; q_0, c)$ *in the benchmark model is unique, whenever it exists.*

Proof See Appendix A.

The comparative statics of the benchmark problem are intuitive given that the monopolist relies on a combination of prices and ability requirements to maximize profits. Higher marginal costs imply that the college must operate at a smaller scale with higher marginal benefits – so price, quality and threshold income are all raised when c goes up. A more valuable outside option, on the other hand, reduces the monopolist's power and implies lower prices and higher quality – yet because of complementarities between income and quality, a higher outside option actually translates into a *lower* threshold income. The following proposition provides a characterization of the comparative statics in the benchmark model.

Proposition 2. *Provided that an interior solution exists to the benchmark model, the optimal price-quality menu is such that*:

$$(i) \quad \frac{dp^*(q_0, c)}{dq_0} < 0 \qquad (ii) \quad \frac{da^*(q_0, c)}{dq_0} > 0 \qquad (iii) \quad \frac{dy(p^*, a^*; q_0)}{dq_0} > 0$$

Proof *See Appendix B.*

Regarding student welfare, the monopolist's segmentation of the market leads to surplus contributions across three groups: (i) the high-ability students who have high incomes and attend college; (ii) the high-ability students who take the outside option because of low income; and (iii) the students who, independently of income, take the outside option because of low ability. Adding up the expected utility of students across these groups (weighing by the corresponding mass of students in each group) gives

$$SW(p, a; q_0) = \left(\tfrac{y+1}{2} - p\right) a(1-a)(1-y) + \tfrac{y^2}{2} q_0 (1-a) + \tfrac{q_0}{2} a$$

Replacing eq. (1) above, it can be checked that student welfare is non-increasing in prices – so it is maximized for $p = c$. Tuition quality, in contrast, implies a trade-off because an increase in a is associated with higher welfare via higher utility of college entrants, but at the same time lower welfare via a reduction in the number of such entrants. This means that a Utilitarian social planner would also face a tension between raising quality standards and making college more accessible. As an example, consider the case where $c = q_0 = 0$, so the monopolist has complete power over the market (assume that students always prefer to attend the college if they can afford it). In this example $y = p$, and a Utilitarian planner would choose

$$\begin{aligned} p^{sw} &= c = 0 \\ a^{sw} &= \underset{a}{\text{argmax}} \left[SW(0,a)\right] = \underset{a}{\text{argmax}} \left[\tfrac{a(1-a)}{2}\right] = \tfrac{1}{2} \end{aligned}$$

which means that only the most able half of the students would go to college (irrespective of income). Yet in this same example profits are $\pi(p,a;0,0) = p(1-a)(1-p)$, so a monopolist would choose $a^* = 0$ and $p^* = \frac{1}{2}$. Thus, at the menu offered by the monopolist only the richest half of the student population would attend college – so the student welfare would be entirely lost, yet the size of served demand would be at the welfare-maximizing level.

4 Monopolistic Competition

4.1 Demand

Following a backward-induction approach, the first step to solve the model is to derive demand functions taking the menus (p_1, a_1) and (p_2, a_2) as given. Clearly, since prices are a best response to quality choices, any menu entailing $a_1 = a_2$ leads to zero profits due to price competition (suppose that students randomize if colleges offer the same menus). I hence concentrate here on the more interesting asymmetric case, and label colleges such that $a_1 < a_2$.[5] The fact that there is asymmetry in qualities implies that there are three distinct groups of students: those with ability $\theta < a_1$ (who attend the outside option); those with ability $a_1 \leq \theta < a_2$ (who choose between the low-quality college and the outside option); and those with ability $\theta \geq a_2$ (who contemplate all alternatives). Analogously to the analysis of the benchmark problem, choices can be now broken down in terms of income thresholds for pairwise comparisons between alternatives.

- Students prefer the menu (p_1, a_1) over the outside option if they have ability $\theta \geq a_1$ and income $\omega \geq y_{1,0}$, where $(y_{1,0} - p_1)(a_1) = y_{1,0}q_0$ and hence

$$y_{1,0} = \frac{p_1 a_1}{a_1 - q_0}$$

- Students prefer the menu (p_2, a_2) over the outside option if they have ability $\theta \geq a_2$ and income $\omega \geq y_{2,0}$, where $(y_{2,0} - p_2)(a_2) = y_{2,0}q_0$ and hence

$$y_{2,0} = \frac{p_2 a_2}{a_2 - q_0}$$

- Students prefer the menu (p_2, a_2) over (p_1, a_1) if they have ability $\theta \geq a_2$ and income $\omega \geq y_{2,1}$, where $(y_{2,1} - p_2)(a_2) = (y_{2,1} - p_1)(a_1)$ and hence

$$y_{2,1} = \frac{p_2 a_2 - p_1 a_1}{a_2 - a_1}$$

In this setting, any student with ability below a_1 will choose the outside option regardless of income, while students with ability between a_1 and a_2 will prefer the low-quality college if and only if $\omega \geq y_{1,0}$. For students with ability above a_2, demand will depend on whether the threshold $y_{2,0}$ is higher or lower than the threshold $y_{2,1}$, which in turn is determined by the relative prices of the two colleges. If it is the case that the low-quality college is relatively expensive, then $y_{2,0} > y_{2,1}$ and the low-quality college fails to attract any students with ability $\theta \geq a_2$, irrespective of income. This is intuitive, since all high-ability students would either attend the high-quality college, or else prefer the outside option at no cost. However, if it is the case that the low-quality college is relatively cheap, then $y_{2,0} < y_{2,1}$, and the high-ability students are sorted by income across the three possible alternatives. This leads to two possible demand structures illustrated diagrammatically in Figure 2. As it can be appreciated, deriving demand functions for each of these two segmentations is a simple geometric exercise.

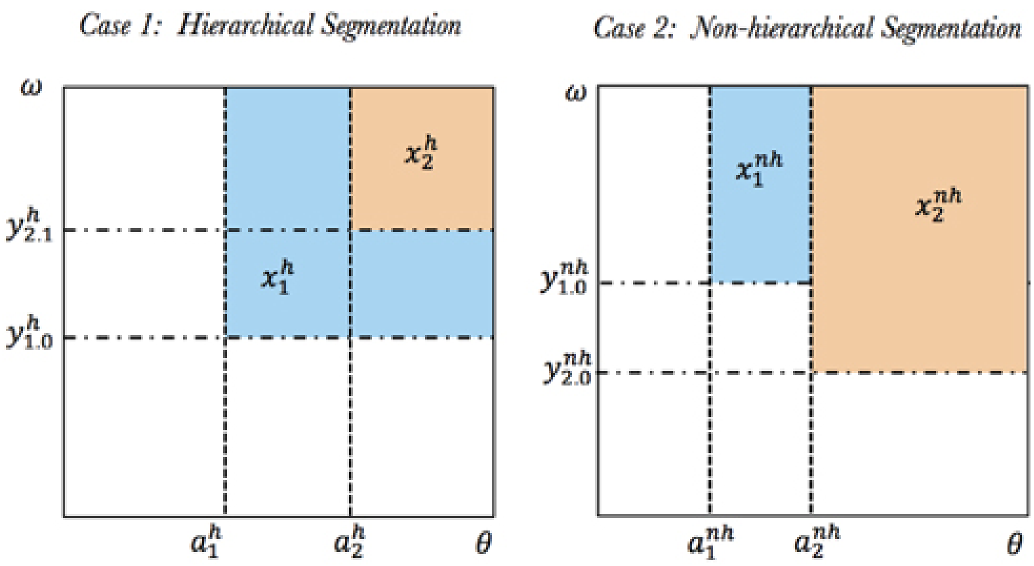


Figure 2 Characterization of market segmentation by cases.

Definition *A hierarchical segmentation is a market structure featuring student stratification by ability and income.*

Proposition 3. *Suppose* $0 < q_0 < a_1 < a_2 < 1$. *Then a hierarchical segmentation obtains in equilibrium if and only if*

$$\frac{p_1}{p_2} < \frac{a_2(a_1 - q_0)}{a_1(a_2 - q_0)}$$

Proof There are six different ways in which the thresholds $y_{2,1}$, $y_{2,0}$, and $y_{1,0}$ can be ordered along the unit line. Any ordering in which $y_{2,0}$ is the highest or the lowest is not feasible under the assumption that $a_1 < a_2$. This leaves only two cases, namely $y_{2,1} > y_{2,0} > y_{1,0}$ and $y_{1,0} > y_{2,0} > y_{2,1}$. Stratification by ability and income requires $y_{2,1} > y_{2,0}$. It can be checked that $y_{2,1} > y_{2,0}$ only holds whenever $\frac{p_1}{p_2} < \frac{a_2(a_1-q_0)}{a_1(a_2-q_0)}$.□

4.2 Equilibrium Menus

Using Proposition 3, the model can be solved by assuming that $\frac{p_1}{p_2}$ is either larger or smaller than $\frac{a_2(a_1-q_0)}{a_1(a_2-q_0)}$, and then checking whether the equilibrium menus that result are indeed consistent with the assumption. Making use of numerical simulations, it is thus possible to show that both hierarchical and non-hierarchical segmentations can arise consistently in equilibrium.

Proposition 4. *There is a non-empty set of parameters* q_0 *and* c *for which hierarchical and non-hierarchical segmentations are sustainable in equilibrium.*

Proof The statement in this proposition can be proved by example. I thus proceed by solving the strategic-interaction problem for each of the two market configurations, assuming $q_0 = c = 0.1$. Other parametric examples are provided in Appendix D for comparison.

Hierarchical Segmentation

The problems for the low- and high-quality colleges at the pricing stage under the hierarchical segmentation are:

$$\underset{p_1 \in [c,1]}{\text{MAX}} \; (p_1 - c)((a_2 - a_1)(1 - y_{1,0}^h) + (1 - a_2)(y_{2,1}^h - y_{1,0}^h))$$
$$\text{s.t.} \quad 0 \le y_{1,0}^h \le y_{2,1}^h \le 1$$
$$\underset{p_2 \in [c,1]}{\text{MAX}} \; (p_2 - c)(1 - a_2)(1 - y_{2,1}^h)$$
$$\text{s.t.} \quad 0 \le y_{1,0}^h \le y_{2,1}^h \le 1$$

The necessary first-order conditions for these problems yield:

$$\begin{aligned} p_2(p_1, a_1, a_2; q_0, c) &= A_0 + A_1 p_1 \\ p_1(p_2, a_1, a_2; q_0, c) &= B_0 + B_1 p_2 \end{aligned}$$

where the slope and intercept of the reaction functions are

$$B_0 = \frac{a_1^3(1+c) - a_2^2 q_0 - a_1^2(2a_2(1+c) + q_0) + a_1\left(a_2^2 - cq_0 + a_2(c + 2q_0 + cq_0)\right)}{2a_1\left(a_1^2 + a_2 - 2a_1 a_2 - q_0 + a_2 q_0\right)}$$

$$B_1 = \frac{a_2(1-a_2)(a_1 - q_0)}{2a_1\left(a_1^2 + a_2 - 2a_1 a_2 - q_0 + a_2 q_0\right)}$$

$$A_0 = \frac{a_2(1+c) - a_1}{2a_2}$$

$$A_1 = \frac{a_1}{2a_2}$$

Through algebraic manipulation it can be shown that A_0, A_1, B_0, B_1 are strictly positive and smaller than 1 whenever $q_0 < a_1 < a_2 < 1$; so the reaction functions must intersect exactly once in the unit square. The resulting unique intersection of these pricing reaction functions can be then substituted back into the profits function to derive new reaction functions for the quality choices. Figure 3presents the iso-profit contours for both colleges as a function of quality standards for $c = q_0 = 0.1$. To check that the solution is indeed interior, the second panel in Figure 3presents again the same plot, but cropping the region to include only combinations of quality where the threshold incomes are consistent with $\frac{p_1}{p_2} < \frac{a_2(a_1 - q_0)}{a_1(a_2 - q_0)}$, as required for the hierarchical segmentation.

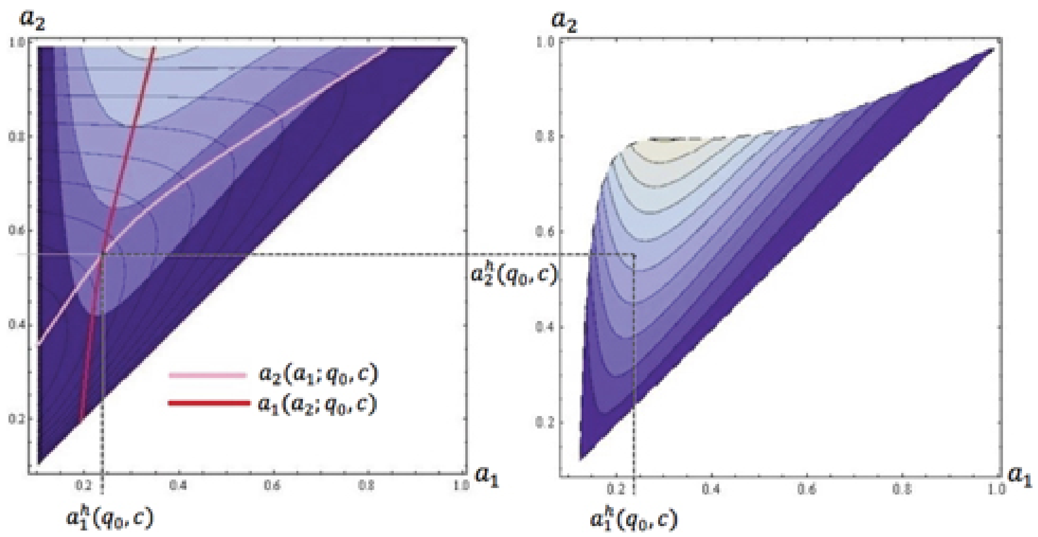


Figure 3 Superimposed iso-profit contours and optimal quality choices for $c = q_0 = 0.1$. Right panel restricted to case-consistent combinations of (a_1, a_2).

Non-hierarchical Segmentation

The problems for the low and high quality colleges at the pricing stage under the non-hierarchical segmentation are:

$$\begin{aligned} \underset{p_1 \in [c,1]}{\text{MAX}} \quad & (p_1 - c)(a_2 - a_1)(1 - y_{1,0}^{nh}) \\ \text{s.t.} \quad & 0 \le y_{1,0}^{nh} \le y_{2,1}^{nh} \le 1 \end{aligned}$$

$$\begin{aligned} \underset{p_2 \in [c,1]}{\text{MAX}} \quad & (p_2 - c)(1 - a_2)(1 - y_{2,0}^{nh}) \\ \text{s.t.} \quad & 0 \le y_{1,0}^{nh} \le y_{2,1}^{nh} \le 1 \end{aligned}$$

Clearly, since demand only depends on own prices, the necessary first-order conditions yield two independent equations; so there is no strategic interaction in prices and reaction functions are flat. More specifically, $p_i^{nh} = \frac{a_i(1+c)-q_0}{2a_i}$ for $i \in \{1,2\}$

Again, these equilibrium prices can be then substituted back into the profits function to derive reaction functions for the quality choices. Figure 4presents the iso-profit contours for both colleges as a function of quality standards for $c = q_0 = 0.1$ in the non-hierarchical case. It can be checked that now all values of a_1 and a_2 for which $q_0 < a_1 < a_2 < 1$ are consistent with $\frac{p_1}{p_2} > \frac{a_2(a_1-q_0)}{a_1(a_2-q_0)}$.

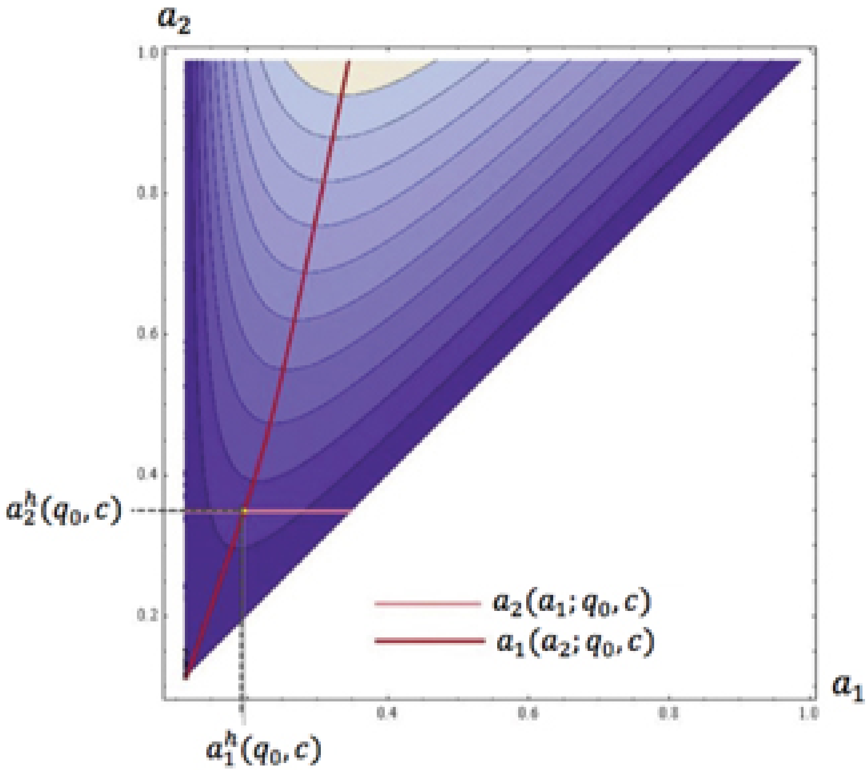


Figure 4 Equilibrium qualities derived graphically from superimposing iso-profit contours of the high-quality college (for $c = q_0 = 0.1$).

To illustrate equilibrium outcomes, I present here the complete solution for $q_0 = c = 0.1$. The proof to Proposition 4 above shows how this solution is obtained. Table 1presents a summary of the key market outcomes, while Figure 5shows the equilibrium segmentations graphically.

Table 1 Market equilibrium outcome for $q_0 = c = 0.1$.

Case 1: Hierarchical equilibrium							
College	p_i^h	a_i^h	$y_i(p^h,a^h)$	$x_i(p^h,a^h)$	$\pi_i(p^h,a^h)$	$SW_i(p^h,a^h)$	$SW(p^h,a^h)$
1	0.21	0.23	0.37	0.21	0.02	0.019	0.086
2	0.35	0.46	0.49	0.27	0.07	0.050	
Outside	–	–	–	0.52	–	0.017	
Case 2: Non-hierarchical equilibrium							
College	p^{nh}	a^{nh}	$y_i(p^{nh},a^{nh})$	$x_i(p^{nh},a^{nh})$	$\pi_i(p^{nh},a^{nh})$	$SW_i(p^{nh},a^{nh})$	$SW(p^{nh},a^{nh})$
1	0.31	0.20	0.60	0.06	0.01	0.006	0.066
2	0.41	0.35	0.57	0.28	0.09	0.037	
Outside	–	–	–	0.56	–	0.023	

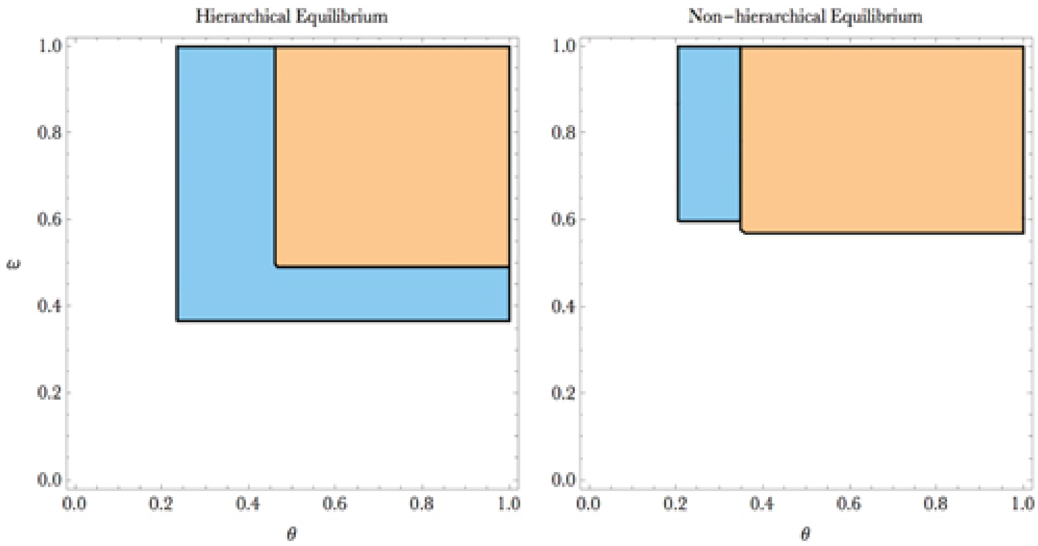


Figure 5 Hierarchical and non-hierarchical equilibrium segmentations for $q_0 = c = 0.1$.

The following points from this numerical exercise are worth emphasizing.

-- The hierarchical segmentation leads to lower prices, higher quality, and lower threshold incomes.

-- The total number of students attending college is higher under the hierarchical segmentation, and this is mostly due to the demand served by the low-quality college.

-- In the non-hierarchical case, the low-quality college (which is by definition expensive for the relative quality it offers) is still cheaper than the high-quality alternative; this makes the average income of students attending college very similar across colleges in the non-hierarchical case.

-- Students attending the high-quality college under the non-hierarchical case are, on average, richer than students attending the high-quality college under the hierarchy.

An important observation that is closely related to the remarks above (and which can be verified directly from the last column in Table 1), is that in this example student welfare is higher under the hierarchical segmentation. Intuitively, however, since the utility of individuals attending college is always decreasing in prices, a Utilitarian planner would choose $p_1 = p_2 = c$, which in turn implies a first-best non-hierarchical allocation of students to colleges. It can be checked that in the numerical example above, student welfare is maximized where $p_1^{SW} = p_2^{SW} = 0.1$ and $\left(a_1^{SW}, a_2^{SW}\right) = (0.41, 0.7)$ (see Appendix C for details); so a Utilitarian planner would choose a segmentation where students are divided into segments of similar size along the ability dimension, in order to implement the first-best allocation of students to colleges. In fact, if there was no outside option and education had no cost, such that $q_0 = c = 0$, then a planner would implement $\left(p_1^{SW}, p_2^{SW}\right) = (0, 0)$ and $\left(a_1^{SW}, a_2^{SW}\right) = \underset{(a_1, a_2)}{\text{argmax}} \left[\frac{1}{2}(1 - a_2)a_2 + \frac{1}{2}a_1(a_2 - a_1)\right] = \left(\frac{1}{3}, \frac{2}{3}\right)$; so quality would split the student space in equidistant segments along the ability dimension.

4.3 Policy Implications

The model developed here assumes that colleges share identical fundamentals. This implies that asymmetric equilibrium outcomes featuring a non-hierarchical market structure can be thought of as the result of coordination. Consider the equilibrium outcomes from Table 2.

Table 2 Market equilibrium outcomes for $q_0 = c = 0.25$.

	Hierarchy	Non-hierarchy
College 1 Profits	0.003	0.004
College 2 Profits	0.014	0.019
Student welfare	0.143	0.132

As it can be seen from Table 2, for this parametric specification ($q_0 = c = 0.25$), equilibrium outcomes are such that both colleges would benefit from coordinating in the asymmetric non-hierarchical segmentation – this would yield higher profits to both colleges. The implication, therefore, is that coordination can plausibly explain why different equilibrium segmentation structures may arise in otherwise identical markets.[6] Another interesting point that follows from the example above is that coordination may be at the detriment of student welfare: in this example students would be better off if the prevailing market structure was hierarchical; yet, as discussed, colleges would have incentives to coordinate in the non-hierarchical equilibrium. Evidently, this implies that policy interventions aimed at reducing the potential for coordination in the non-hierarchical structure may have scope for improving student welfare. This is particularly relevant to the extent that implementing the first-best allocation of students to colleges (i.e. the combination of prices and admission requirements that maximizes student welfare) is not trivial, since marginal-cost pricing is difficult to enforce by governments with restricted information about particular production technologies in private institutions.

5 Conclusions

I proposed a model of monopolistic competition with product differentiation, where firms choose quality-price menus but only customers with sufficient ability enjoy high quality. I argued that this is relevant in the context of higher education, where quality is implemented through an academic admission cut-off. The model was first explored in a benchmark case with only one college and a fixed outside option, in order to show that the tension between making standards high and excluding potential customers, makes it optimal for a monopolist to use a combination of prices and admission requirements to maximise profits.

Strategic interaction between ex ante identical colleges was then introduced, and it was shown that there are two market segmentations that may be sustainable in equilibrium, and in only one of them there is a hierarchical allocation of students into colleges; so it is not necessarily the case that more academically selective colleges are more expensive. I provided a numerical example to illustrate the welfare implications of these two market configurations.

Appendix

A Benchmark: Uniqueness

Note that eqs (3) and (4) can be solved for p to obtain

$$p - \frac{a(1+c)-q_0}{2a} = 0$$
$$p - \frac{(a-q_0)^2}{a^2+q_0-2aq_0} = 0$$

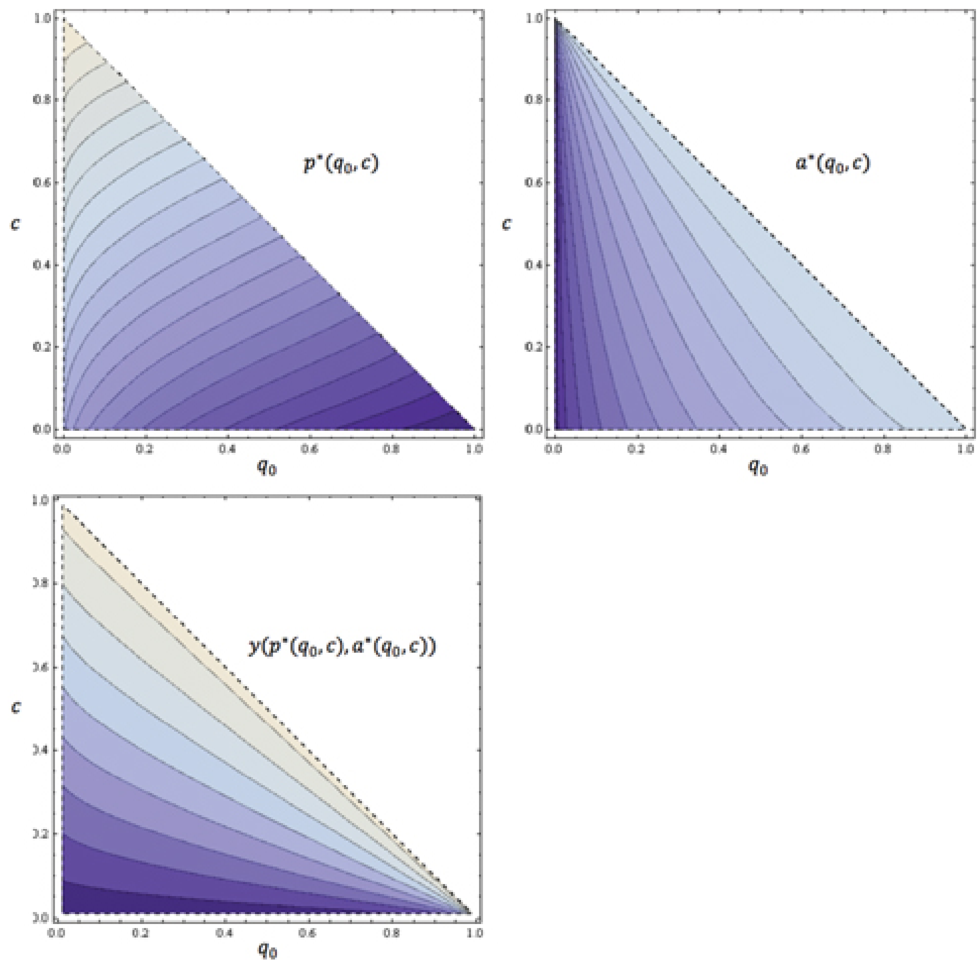
So the optimal value of quality corresponds to a solving

$$\frac{a^3(-1+c) + a(1+c)q_0 - q_0^2 + a^2(q_0 - 2cq_0)}{2a\left(a^2 + q_0 - 2aq_0\right)} = 0 \tag{5}$$

The denominator in eq. (5) is positive for $0 < q_0 < a < 1$. So the solution must correspond to the roots of the cubic polynomial in the numerator. It can be checked that, from the three possible roots of this polynomial, whenever $0 < q_0 + c < 1$ there is only one real root $a^* \in (q_0, 1)$. So the interior solution is unique.

B Benchmark: Comparative Statics

The following are contour plots corresponding to $p^*(q_0, c)$, $a^*(q_0, c)$ and $y^*(p^*, a^*)$ for $q_0 \in [0, 1]$ and $c \in [0, 1]$, such that $c + q_0 < 1$. Lighter shades represent higher levels.



In these figures it can be checked that: (i) $\frac{dp^*}{dc} > 0$, (ii) $\frac{da^*}{dc} > 0$, (iii) $\frac{dy^*}{dc} > 0$, (iv) $\frac{dp^*}{dq_0} < 0$, (v) $\frac{da^*}{dq_0} > 0$, (vi) $\frac{dy*}{dq_0} > 0$

C Monopolistic Competition: Student Welfare

Analogously to the welfare analysis from the benchmark case, aggregate student welfare under monopolistic competition can be calculated as the expected utility associated with a given pair of menus. For the hierarchical and non-hierarchical equilibria, student welfare is hence given respectively by

$$\begin{aligned} SW^{nh}(a_1, a_2, p_1, p_2) = \quad & a_1\left(a_2 - a_1\right)\left(1 - y_{1,0}\right)\left(\frac{1+y_{1,0}}{2} - p_1\right) \\ & + a_2\left(1 - a_2\right)\left(1 - y_{2,0}\right)\left(\frac{1+y_{2,0}}{2} - p_2\right) \\ & + q_0 \frac{y_{1,0}}{2}\left(a_2 - a_1\right) y_{1,0} + q_0 \frac{y_{2,0}}{2}\left(1 - a_2\right) y_{2,0} + \frac{q_0}{2} a_1 \end{aligned}$$

and

$$\begin{aligned} SW^{h}(a_1, a_2, p_1, p_2) = \quad & a_2(1 - a_2)(1 - y_{2,1})\left(\frac{1+y_{2,1}}{2} - p_2\right) \\ & + a_1(1 - a_2)(y_{2,1} - y_{1,0})\left(\frac{y_{2,1}+y_{1,0}}{2} - p_1\right) \\ & + a_1(a_2 - a_1)(1 - y_{1,0})\left(\frac{1+y_{1,0}}{2} - p_1\right) \\ & + q_0 \frac{y_{1,0}}{2}(1 - a_1) y_{1,0} + q_0 \frac{a_1}{2} \end{aligned}$$

Both of these expressions are decreasing in p_1 and p_2. So a utilitarian planer would choose $\left(p_1^{SW}, p_2^{SW}\right) = (c, c)$, which would in turn mean that the resulting allocation would necessarily be non-hierarchical (any student who is sufficiently skilled would take the high-quality college instead of low-quality alternative for the same price). This also means that $y_{1,0} = c\frac{a_1}{a_1 - q_0}$ and $y_{2,0} = c\frac{a_2}{a_2 - q_0}$, so the planner would choose (a_1, a_2) to maximise

$$\begin{aligned} SW(a_1, a_2) = \quad & a_1 (a_2 - a_1) \left(1 - \frac{a_1 c}{a_1 - q_0}\right)\left(\frac{1}{2}\left(1 + \frac{a_1 c}{a_1 - q_0}\right) - c\right) \\ & + a_2 (1 - a_2) \left(1 - \frac{a_2 c}{a_2 - q_0}\right)\left(\frac{1}{2}\left(1 + \frac{a_2 c}{a_2 - q_0}\right) - c\right) \\ & + \frac{q_0 \frac{a_1 c}{a_1 - q_0}}{2} (a_2 - a_1) \frac{a_1 c}{a_1 - q_0} + \frac{q_0 \frac{a_2 c}{a_2 - q_0}}{2} (1 - a_2) \frac{a_2 c}{a_2 - q_0} + \frac{q_0}{2} a_1 \end{aligned}$$

Figure 6presents a plot of iso-welfare levels for $q_0 = c = 0.1$ (higher contours depicted in lighter shades). As it can be seen, aggregate student welfare is maximized for $\left(a_1^{SW}, a_2^{SW}\right) = (0.41, 0.71)$.

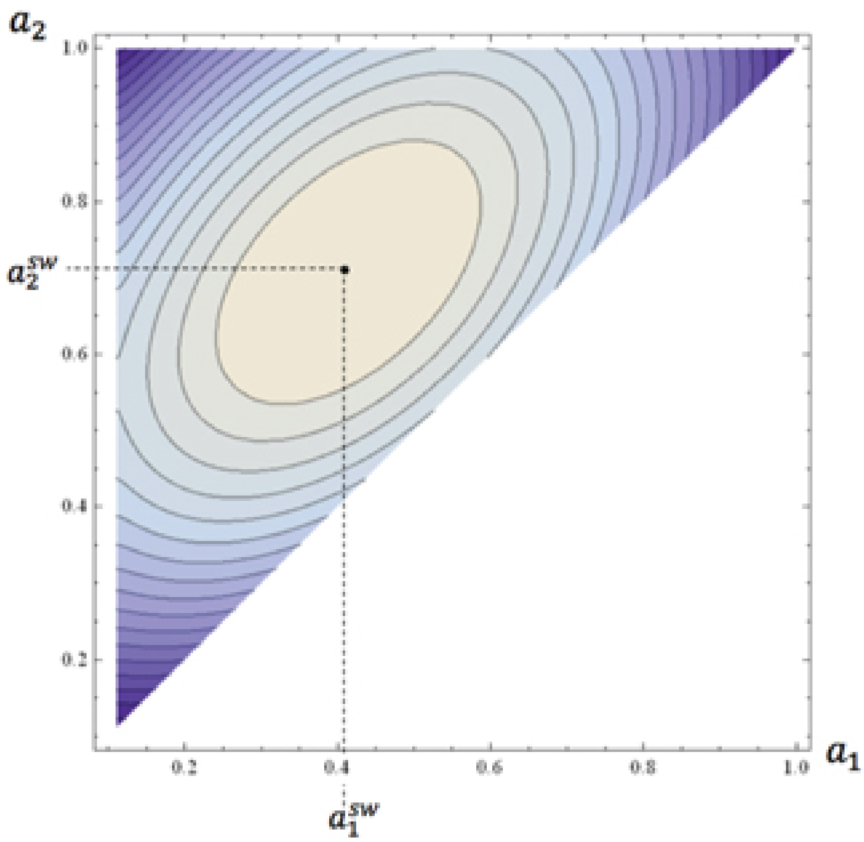


Figure 6 Contour plot of student welfare as a function of quality standards, for $p_1 = p_2 = c$ and $q_0 = c = 0.1$.

This exercise shows that a social planner would choose a segmentation where students are divided into segments of similar size along the ability dimension. In fact, if there was no outside option and providing education had no cost, such that $q_0 = c = 0$, then a planner would choose $\left(p_1^{SW}, p_2^{SW}\right) = (0, 0)$ and

$$\left(a_1^{SW}, a_2^{SW}\right) = \underset{(a_1, a_2)}{\text{argmax}} \left[\frac{1}{2}(1 - a_2)a_2 + \frac{1}{2}a_1(a_2 - a_1)\right] = \left(\frac{1}{3}, \frac{2}{3}\right)$$

so quality would split the student space in equidistant segments along the ability dimension.

D Monopolistic Competition Simulations

The following tables present a summary of market outcomes for different combinations of the underlying parameters. A selection of these outcomes is presented graphically in Figure 7to allow a visual comparison.

Table 3: Market outcomes for different values of q_0, holding $c = 0.1$.

Case 1. Hierarchical eq.							
q_0	a_1^h	a_2^h	p_1^h	p_2^h	$y_{1,0}(p^h, a^h)$	$y_{2,1}(p^h, a^h)$	$SW(p^h, a^h)$
0.01	0.05	0.20	0.27	0.45	0.33	0.52	0.028
0.1	0.23	0.46	0.21	0.35	0.37	0.49	0.086
0.3	0.49	0.69	0.16	0.26	0.43	0.47	0.174
Case 2. Non-hierarchical eq.							
q_0	a_1^{nh}	a_2^{nh}	p_1^{nh}	p_2^{nh}	$y_{1,0}(p^{nh}, a^{nh})$	$y_{2,0}(p^{nh}, a^{nh})$	$SW(p^{nh}, a^{nh})$
0.01	0.04	0.11	0.41	0.50	0.57	0.55	0.014
0.1	0.20	0.35	0.31	0.41	0.60	0.57	0.066
0.3	0.46	0.60	0.23	0.30	0.64	0.60	0.161

Table 4 Market outcomes for different values of c, holding $q_0 = 0.1$.

Case 1. Hierarchical eq.							
c	a_1^h	a_2^h	p_1^h	p_2^h	$y_{1,0}(p^h, a^h)$	$y_{2,1}(p^h, a^h)$	$SW(p^h, a^h)$
0.01	0.20	0.42	0.14	0.30	0.27	0.45	0.091
0.10	0.23	0.46	0.21	0.35	0.37	0.49	0.086
0.30	0.31	0.54	0.38	0.47	0.56	0.60	0.073
Case 2. Non-hierarchical eq.							
c	a_1^{nh}	a_2^{nh}	p_1^{nh}	p_2^{nh}	$y_{1,0}(p^{nh}, a^{nh})$	$y_{2,0}(p^{nh}, a^{nh})$	$SW(p^{nh}, a^{nh})$
0.01	0.18	0.32	0.23	0.35	0.51	0.51	0.069
0.10	0.20	0.35	0.31	0.41	0.60	0.57	0.066
0.30	0.27	0.42	0.46	0.53	0.74	0.70	0.059

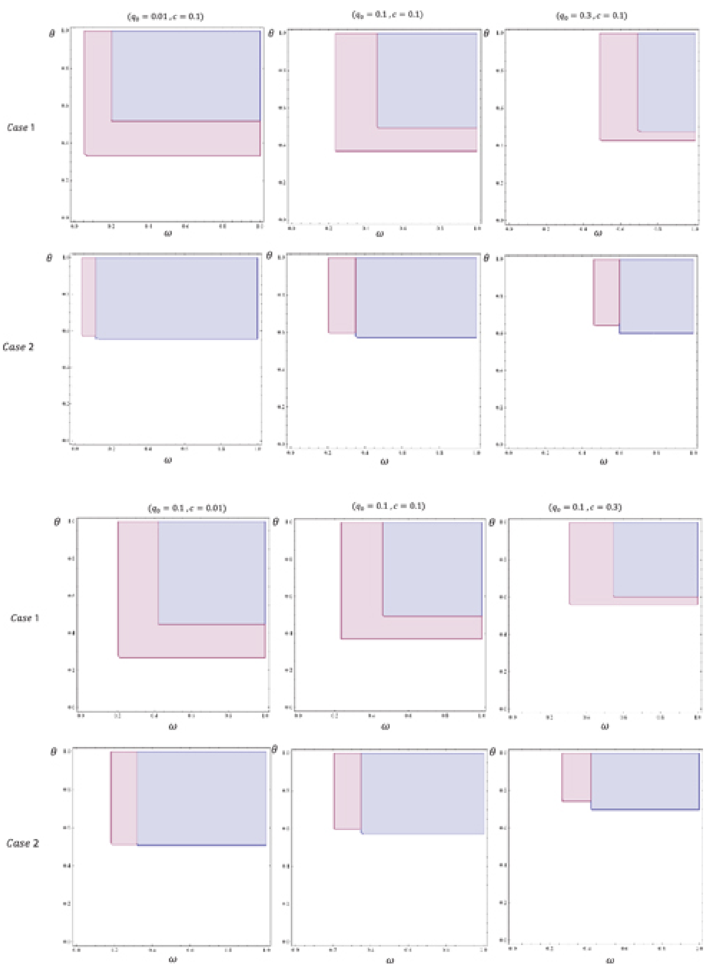


Figure 7 Market outcome by cases for different parameter specifications.

Notes

[1]A model along these lines is proposed and tested in Duflo, Dupas, and Kremer (2011).

[2]To be more specific, the unique equilibrium in Epple and Romano (1998)has a strict hierarchy of school qualities (where quality is defined by the average ability of students in each school) and two-dimensional student sorting (i.e. stratification by ability and income). They predict that such equilibrium is sustained by prices that depend both on ability and income within schools (high-ability, low-income students receive tuition "discounts", and low-ability, high-income students pay tuition "premia").

[3]It is important to note that Colombian public universities are on average more selective than private universities, in spite of being much cheaper. According to figures reported in OECD (2015), average fees in Colombian private universities in the academic year 2013–14 were 3,082 PPP USD, while in public universities they were 574 PPP USD. Public universities, however, score higher on the official university rankings (based on standardized tests administered to final-year undergraduate degree candidates across public and private institutions). Whether this is the result of a non-hierarchical equilibrium along the lines of my model is difficult to establish from the data, since it is of course likely that public universities are not attempting to maximize profits. Nevertheless, the fact remains that pricing by ability is not observed even within the large segment of competing private institutions that charge higher fees.

[4]One could alternatively assume that the marginal cost of tuition depends on quality only for high levels of selectivity – this would capture situations in which, for example, increasing the complexity and breadth of teaching after a certain point requires instructors that are more qualified. Under such alternative formulation, the analysis proposed here would still be relevant locally for those segments of the production function exhibiting constant marginal costs.

[5]Colleges are identical in terms of their fundamental parameters, so the model does not predict which college is more selective in equilibrium. In practice, however, asymmetry in the degree of selectivity could be explained by a number of factors. Heterogeneity in recruitment technologies, for instance, is likely to play a crucial role in determining academic selectivity in practice: observing student quality is typically costly at the point of admission, and some universities have endowments that enable them to rank candidates at the higher end of the spectrum more easily.

[6]The plausibility of this interpretation crucially depends on the underlying parameters since, as I show on Table 1, it is not generally the case that profits under a non-hierarchical equilibrium are larger for both colleges.

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