Probing the Structure and Size of Dark Matter Couplings at the Large Hadron Collider

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Abstract

The mystery of dark matter (DM) is undeniably one of the greatest in modern physics. For decades, evidence has accumulated from astrophysical and cosmological experiments suggesting that the Universe contains a large amount of mass yet unaccounted for. In particular, present research indicates that approximately 84% of the total matter content in the Universe is non-baryonic DM. Together with the well-known limitations of the Standard Model of particle physics (SM), some of which could be remedied by theories which necessarily include additional particles, a particle physics solution to the DM problem is certainly well motivated. In this thesis the Large Hadron Collider (LHC) phenomenology of three models of DM is studied. In particular, the focus is on signals due to final states containing hadronic jets in association with a large amount of missing transverse energy, corresponding to DM which escapes detection. For the first model, it is found that studying events with one jet in the final state can allow constraints to be derived on the size of the couplings between DM and SM particles, but it is not possible to extract any information about the structure of the couplings. This motivates an analysis of events which feature two jets. For the second model, it is found that measurements of the azimuthal angular separation of these jets lead to contrasting distributions depending on the Lorentz structure of the interactions. These spectra are shown to be stable under various corrections and, more importantly, are clearly produced whether or not one performs the calculation using an effective field theory framework. For the third model, which differs from the first two in that DM now couples to the SM gauge bosons rather than the quarks, recent experimental searches are used to derive bounds on the fiducial cross sections for a variety of final states featuring missing transverse energy. This facilitates a comparison to be made between the various search strategies, identifying which of these most strongly constrain the operators under consideration. Considering the final state containing two jets, the azimuthal angular distributions are then plotted and are again found to be strongly dependent on the Lorentz structure of the underlying interactions. Prospects for the 14 TeV LHC run are then studied and it is found that a clear distinction between the spectra should be possible once 300 fb$^{-1}$ has been collected.
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Statement of Originality

This thesis is based on original research undertaken between October 2011 and September 2015, and contains no material that has already been accepted, or is concurrently being submitted, for any degree, diploma, certificate or any other qualification at the University of Oxford or elsewhere. To the best of my knowledge and belief this thesis contains no material previously published or written by another person, except where due reference is made in the text.

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indicates the present-day limit, while the dotted black line corresponds
to an assumed future bound on the electron EDM of $|d_e/e| \lesssim 3 \times 10^{-31}$ cm.
## Acronyms

<table>
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<th>Full Form</th>
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<tbody>
<tr>
<td>BBN</td>
<td>Big Bang Nucleosynthesis</td>
</tr>
<tr>
<td>CKM</td>
<td>Cabibbo-Kobayashi-Maskawa</td>
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<tr>
<td>CL</td>
<td>Confidence Level</td>
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<td>CMB</td>
<td>Cosmic Microwave Background</td>
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<td>DM</td>
<td>Dark Matter</td>
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<tr>
<td>EDM</td>
<td>Electric Dipole Moment</td>
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<tr>
<td>EFT</td>
<td>Effective Field Theory</td>
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<tr>
<td>EWSB</td>
<td>Electroweak Symmetry Breaking</td>
</tr>
<tr>
<td>FCNC</td>
<td>Flavour-Changing Neutral Current</td>
</tr>
<tr>
<td>KK</td>
<td>Kaluza-Klein</td>
</tr>
<tr>
<td>LHC</td>
<td>Large Hadron Collider</td>
</tr>
<tr>
<td>LKP</td>
<td>Lightest Kaluza-Klein Particle</td>
</tr>
<tr>
<td>LO</td>
<td>Leading Order</td>
</tr>
<tr>
<td>LSP</td>
<td>Lightest Supersymmetric Particle</td>
</tr>
<tr>
<td>MACHO</td>
<td>Massive Compact Halo Object</td>
</tr>
<tr>
<td>MC</td>
<td>Monte Carlo</td>
</tr>
<tr>
<td>MFV</td>
<td>Minimal Flavour Violation</td>
</tr>
<tr>
<td>MOND</td>
<td>Modified Newtonian Dynamics</td>
</tr>
<tr>
<td>MSSM</td>
<td>Minimally Supersymmetric Standard Model</td>
</tr>
<tr>
<td>NLO</td>
<td>Next-to-Leading Order</td>
</tr>
<tr>
<td>NWA</td>
<td>Narrow Width Approximation</td>
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<td>PDF</td>
<td>Parton Distribution Function</td>
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<td>PS</td>
<td>Parton Shower</td>
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<tr>
<td>QCD</td>
<td>Quantum Chromodynamics</td>
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<tr>
<td>QED</td>
<td>Quantum Electrodynamics</td>
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<tr>
<td>SD</td>
<td>Spin-Dependent</td>
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<tr>
<td>SHM</td>
<td>Standard Halo Model</td>
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<tr>
<td>SI</td>
<td>Spin-Independent</td>
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<tr>
<td>SM</td>
<td>Standard Model of Particle Physics</td>
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<tr>
<td>SUSY</td>
<td>Supersymmetry</td>
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<tr>
<td>UED</td>
<td>Universal Extra Dimensions</td>
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<tr>
<td>UV</td>
<td>Ultraviolet</td>
</tr>
<tr>
<td>VBF</td>
<td>Vector Boson Fusion</td>
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<tr>
<td>VEV</td>
<td>Vacuum Expectation Value</td>
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<tr>
<td>WIMP</td>
<td>Weakly Interacting Massive Particle</td>
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Chapter 1

Introduction and Motivation

The content and nature of the Universe has fascinated people for millennia. In many ways, the more it has been studied, the more mysterious it seems to be. Since roughly the start of the 20th century advances in science and technology have allowed huge progress to be made in the study of both the very large (astrophysics and cosmology) and the very small (particle physics). These fields have converged on one of the greatest mysteries of the Universe currently, dark matter (DM). A huge amount of evidence from many astrophysical and cosmological experiments indicates that there is a vast quantity of ‘missing mass’ in our Universe — that is, mass which must be present but whose source has not been identified. In fact, the majority of matter in the Universe is ‘dark’, in this sense, outweighing luminous matter by a factor of roughly 5. Much evidence also suggests that the dominant contribution to DM is not baryonic but is rather comprised of some unknown fundamental particle, or combination of particles. Simultaneously, while our understanding of physics at the most fundamental level has improved spectacularly in recent times, it is clear that the theories have considerable shortfalls and are far from complete. Many suggested extensions to the Standard Model of particle physics, which attempt to address (some of) these shortfalls, include new undiscovered particles, some of which meet the basic requirements for a DM particle candidate.

Thus a particle physics explanation for DM is extremely well motivated, not only
to explain the origin of this particular mystery, but also to provide us with some
direction of how to address the limitations of the Standard Model. Recent significant
advances in experimental particle physics, most notably the Large Hadron Collider
(LHC), greatly improve prospects for testing theories of physics beyond the Standard
Model. With all this in mind, there has never been a better time to conduct research
in particle physics, and those involved could be very close to making some monumental
findings including, perhaps, the discovery of a DM particle.

In this chapter evidence that points to the existence of DM is reviewed, start-
ing with the early astrophysical experiments and theory, in Section 1.1 and moving
on to more advanced research in astrophysics and cosmology, in Section 1.2. Section 1.3 reviews the Standard Model and its shortfalls, leading to a discussion about
the requirements of a DM particle, and potential candidates that are present in some
important theories of physics beyond the Standard Model. In Section 1.4, the current
major experimental searches for particle DM are introduced and discussed. The gen-
eral theoretical considerations for building models of DM are presented in Section 1.5.
The chapter concludes with Section 1.6, a summary of the structure of this thesis.

1.1 Early Evidence for Dark Matter

The very first evidence for a considerable amount of non-luminous matter came in
the early 1930s, when J. H. Oort conducted a study of the velocities of stars in the
galactic plane of the Milky Way [1]. By measuring the Doppler shifts of these stars he
found that many were moving so fast that they should not be confined to the galaxy
by the gravitational potential due to visible matter. That these stars were, in fact,
orbiting around the centre of the galaxy led Oort to suggest that there must be a
great deal of invisible mass in the Milky Way to provide the required gravitational
field. Oort noted himself that another possible explanation for his findings was that
much of the light from the galactic centre was obscured by interstellar dust. This was
a considerable uncertainty, rendering his findings inconclusive, but his work can be
considered at least in part responsible for prompting others to investigate the idea
further.

In the same decade, F. Zwicky was studying the redshifts of galaxy clusters and
their constituents [2,3]. Focusing on the Coma cluster, he observed that there was a
huge variation in the velocities of nebulae in the cluster. Assuming the system was
in a mechanically stationary state, Zwicky applied the virial theorem, which provides
the following relation between the average kinetic and potential energies, \( \langle T \rangle \) and
\( \langle U \rangle \), of a system,

\[
\langle T \rangle = -\frac{1}{2} \langle U \rangle.
\]  

Using the measured velocities of nebulae, one can use (1.1) to obtain an estimate of
the mass of the cluster. Zwicky compared these estimates to those obtained using
standard mass to light ratio (\( M/L \)) methods. His results were consistent with the
vast majority of the mass in the Coma cluster being non-luminous.

In the 1970s and ’80s, extensive experimental and theoretical work was done study-
ing the rotation curves of a variety of different nearby galaxies [4–10]. A rotation
curve is a plot of the orbital speeds of stars and gas clouds in a galaxy as a function
of the distance from the centre. The resulting profile reflects the mass distribution
in the galaxy. It was originally expected that galaxies would demonstrate Keplerian
behaviour, similarly to the Solar System. From Newtonian gravity, the centripetal
acceleration of an object at radius \( R \) is provided by the gravitational acceleration
due to the mass \( M(R) \) enclosed by the orbit,

\[
\frac{v^2(R)}{R} = \frac{GM(R)}{R^2} \Rightarrow v(R) = \sqrt{\frac{GM(R)}{R}},
\]  

where \( G \) is the gravitational constant. The vast majority of luminous matter in galax-
ies is concentrated towards the centre. Thus, in the absence of any other contributions
to the mass, \( M(R) \) away from the centre is roughly constant and equal to the mass of
the central bulge. Consequently, if it were only luminous matter providing the mass in a galaxy, one would expect the orbital velocities in the disk to drop off as $\sqrt{1/R}$. The observed profile is quite different from this, however. Figure 1.1 shows the rotation curves for seven spiral galaxies, none of which demonstrate the expected Keplerian behaviour. In fact, there is a rapid approximately linear increase in orbital velocities at small $R$, while they are roughly constant at large $R$. The former observation is due to the extremely high density of matter towards the centre of the galaxy, which rotates in a similar way to a solid body. From (1.2) we can see that the latter observation is consistent with the enclosed mass $M(R)$ increasingly approximately linearly with $R$, such that $v(R)$ is almost constant. Such ‘flat’ rotation curves are obtained for the vast majority of galaxies that have been surveyed, with very few demonstrating significant deviations from this shape. Since the increasing mass away from the central bulge cannot be accounted for by visible matter, the inescapable conclusion is that there is some non-luminous matter present in abundance throughout the entire galaxy.

More than just implying that DM exists, these results show that it is not generally
distributed in the same way as luminous matter. The shape of the rotation curves is such that DM must be present throughout the galaxy, forming a ‘halo’ or ‘corona’ [8]. This is the first hint that DM is particularly mysterious; if it were mostly baryonic, one would naturally expect it to be distributed in a similar way to the visible matter.

In an attempt to explain these observations, one proposal was that gravitational interactions are not well understood over cosmological distances. Consequently, theories of modified gravity were developed, most notably MOND (modified Newtonian dynamics), to describe the motion of stars in galaxies. While these theories enjoyed some success in solving this particular problem, properties of galaxy clusters and evidence of DM from cosmological observations have not yet been fully addressed. A detailed discussion of modified gravity is beyond the scope of this work, but a good review can be found in [11].

1.2 Recent Evidence for Dark Matter

The convincing evidence for the presence of a large amount of missing mass in galaxies and galaxy clusters prompted physicists from the late 20th century up to the present day to conduct more extensive research in this direction. Experimental and theoretical advances allowed more targeted work to be undertaken, yielding not only further substantial evidence for the existence of DM, but also valuable insights regarding its nature.

1.2.1 Gravitational Lensing

One of the most notable predictions of Einstein’s theory of general relativity is that the path of light rays is bent around massive objects. Consequently, light from distant sources is distorted around massive foreground objects, an effect known as ‘gravitational lensing’. The size and shape of the gravitational potential due to foreground objects can be determined by analysing lensing events. The usefulness of this ex-
Figure 1.2: The distributions of mass and baryonic matter in the Bullet cluster. The green contours illustrate the weak lensing reconstruction of the gravitational potential. The coloured regions indicate the areas of X-ray emission, corresponding to the distribution of intergalactic gas—the dominant component of baryonic matter in the cluster [14].

Experimental technique to DM research is essentially twofold: firstly, it can be used to survey the non-luminous macroscopic objects in the Milky Way and, secondly, the gravitational potential — and therefore the mass distribution — in galaxy cluster collisions can be mapped and compared to the baryonic matter distribution.

It was suggested that the missing mass in galaxies could be accounted for by simply summing the contributions from ‘dark’ astrophysical objects such as black holes, neutron stars, brown dwarfs and planets, collectively termed MACHOs (massive compact halo objects). Extensive surveying of such objects in the Milky Way was carried out using gravitational microlensing events [12][13]. The results indicate that the density of MACHOs is far too low for these to form a significant component of the missing mass.

Particularly impressive evidence for DM comes from examining galaxy cluster collisions. In this context, the most famous of these is the Bullet cluster, the product of a collision of two smaller galaxy clusters [14]. The distribution of mass can be
mapped using weak gravitational lensing of distant sources, while the distribution of baryonic matter can be inferred from the X-ray emissions produced by the hot colliding intergalactic gas, which forms the majority of the baryonic matter in a cluster. The results, displayed in Figure 1.2, show that these two distributions are not the same; the majority of the mass has passed through unaffected by the collision, while the baryonic matter has been halted towards the centre. This is just one example of several which demonstrate a significant discrepancy between the locations of the gravitational potential and the luminous matter [15–17]. The conclusion is that the majority of the mass in galaxy clusters is very weakly interacting and non-baryonic. Furthermore, this evidence clearly compromises the plausibility of modified gravity theories as a solution to the DM problem.

1.2.2 The Cosmic Microwave Background

Famously discovered by Penzias and Wilson in 1965 the Cosmic Microwave Background (CMB) is radiation originating from the early Universe [18]. Following the Big Bang, the Universe contained a dense plasma of photons and charged particles. As the Universe expanded, the plasma cooled, leading to the formation of neutral atoms, a process known as ‘recombination’. Consequently the Universe became transparent to radiation and photons could travel freely with an effectively infinite mean free path. These photons form what is measured today as the CMB. It has a temperature of about 2.72 K and is highly uniform in all directions [19]. The fact that the fluctuations are very small implies that the density perturbations in the Universe at the time of photon decoupling were very small — too small, in fact, to account for the observed large scale structure to have formed in the time that has passed. In order to reproduce observations, charge-neutral matter is therefore required to have been present since before recombination to initiate structure formation [20].

The small-scale fluctuations in the CMB temperature are attributed to oscillations
of the photon-baryon plasma of the Universe in the period before recombination. To analyse these oscillations one typically studies the power spectrum which, in essence, indicates the size of temperature fluctuations as a function of the angular scale. Technically, the power spectrum characterises the multipole moments, labelled by $l$, of the temperature field. $l$ is then inversely proportional to the angular scale on the sky. Figure 1.3 depicts an example of the CMB power spectrum from WMAP data [22]. Three major peaks, corresponding to large temperature fluctuations, are observed. The positions and relative amplitudes of these peaks, in particular, yield a great deal of information about the cosmological densities of baryons and DM [23].

The source of the temperature fluctuations is photons that were released from compressed (hotter) or rarefied (cooler) regions of the plasma at the time of recombination. Oscillations occurred because density perturbations, amplified during the inflationary period, cause the plasma fall into gravitational wells. The compression is resisted by radiation pressure and oscillations are generated, in a similar way to a mass on a spring under gravity. The peaks observed in the power spectrum correspond to modes of the oscillations: the first peak is generated by the mode for which
the plasma had time to compress once inside the gravitational wells before recombi-
nation, while the second peak is generated by the mode for which the fluid had time
to compress and rarefy, and so on.

The position of the first peak, in particular, is indicative of the curvature of
the Universe. This is because the angular scale of fluctuations in the CMB that
we observe is measures the exact path that light travelled from the surface of last
scattering towards Earth. The exact position of the first peak indicates that the
Universe is very close to being spatially flat \cite{24}. Negative (positive) curvature would
cause the peaks to be shifted to the right (left).

The relative amplitudes of the peaks is sensitive to the baryon density due to
a mechanism called ‘baryon loading’. The greater the baryon density, the deeper
the plasma falls into the gravitational potential wells due to the additional mass.
This, in turn, enhances the maximal compression under gravity and therefore the
heights of the odd-numbered peaks, relative to the even-numbered ones. The high-$l$
behaviour of the distribution is also sensitive to the baryon density. At very large
$l$, the physical scale of fluctuations is comparable to the distance of the random
walk that is undertaken by photons during recombination, causing a damping of the
oscillations. The distance of this random walk is dependent on the baryon content of
the plasma; the higher the baryon density, the shorter this path will be, shifting the
damping tail to higher multipoles.

Earlier in the history of the Universe, when radiation dominated, perturbations
of the gravitational potential were in fact generated by the radiation itself. As the
Universe expanded these potentials decayed during the phase when the plasma was
maximally compressed. As the oscillations turned around, the radiation pressure no
longer had to fight those gravitational potentials, leading to an enhancement of the
amplitude. This process did not occur after matter became dominant, since the poten-
tial wells were then primarily due to perturbations in the DM density. We therefore
expect to only see this effect in modes which started oscillating before radiation-matter equality. The peaks at higher $l$ are consequently sensitive to the ratio of radiation to matter such that their heights can be used to infer the total matter density. Combining this with the baryon density measurement, as outline in the previous paragraph, allows a determination of the DM density.

Recent analysis of the CMB power spectrum has obtained \cite{24},

$$\Omega_b h^2 \approx 0.022, \quad \Omega_{\text{dm}} h^2 \approx 0.119,$$

(1.3)

where the reduced Hubble constant, $h$, is defined by $H_0 = 100h$ km sec$^{-1}$ Mpc$^{-1}$, and the exact values depend on the analysis. These imply that about 84% of the total matter content of the Universe is non-baryonic.

### 1.2.3 Big Bang Nucleosynthesis

In the first few minutes after the Big Bang, the Universe cooled to a temperature at which protons and neutrons could undergo nuclear fusion reactions to produce deuterium and helium nuclei, as well as small amounts of other light nuclei. This process is called ‘Big Bang nucleosynthesis’ (BBN). The instability of deuterium means that its abundance after BBN is very sensitive to the initial conditions, including the baryon density. Additionally, any deuterium produced in stars is rapidly fused to helium, meaning the amount present in the Universe today can be essentially traced back to BBN. The abundances of light elements, particularly in regions of the Universe with small amounts of elements heavier than lithium (since these areas have changed least since BBN), agree very precisely with theoretical predictions. Consequently the baryon density can be measured \cite{25},

$$0.021 \leq \Omega_b h^2 \leq 0.025.$$  

(1.4)

This is in excellent agreement with CMB results \cite{1.3}. Combined, these cosmological measurements provide the leading constraints on the density of baryonic matter in the Universe, and show that it forms a small component of the total matter density.
1.3 The Standard Model and Particle Candidates for Dark Matter

As we have seen, looking to particle physics for a solution to the DM problem is very well motivated. The Standard Model (SM) has enjoyed great success as a theory of physics at a fundamental level, but it is not complete. Most importantly for this work, none of the particles of the SM are suitable DM candidates. One must therefore look beyond the SM for a DM particle. Conveniently, some of the proposed extensions to the SM, which attempt to address other shortfalls, necessarily include hypothetical particles, some of which satisfy the basic requirements of a DM candidate.

This section begins with a review of the SM and a discussion of its main limitations. The requisite properties of a DM particle are outlined, before an exploration of potential candidates provided by some notable theories of physics beyond the SM.

1.3.1 The Standard Model\footnote{This subsection contains a brief summary of the Standard Model — for a full review see e.g.\[26\]28.}

The SM classifies the known fundamental particles and predicts how they interact via three of the fundamental forces: electromagnetic, strong and weak. The fourth fundamental force, gravity, does not feature due to the challenges of reconciling general relativity with quantum field theory, the mathematical framework of the SM. Quantum electrodynamics (QED) describes interactions of charged elementary particles, mediated by the photon. All SM fermions participate in weak interactions, which occurs via exchange of the $Z$ and $W^\pm$ gauge bosons. The electroweak theory unifies QED and the weak interaction at high energies (around 246 GeV, the vacuum expectation value (VEV) of the Higgs field). The $Z$ and $W^\pm$ bosons acquire their masses through electroweak symmetry breaking (EWSB). This is induced by mixing of the gauge bosons with three of the four components of the Higgs field, with the remaining component becoming the scalar Higgs boson. The non-zero masses of the
SM fermions are due to their interactions with the Higgs field. Quantum chromodynamics (QCD) describes how colour charged particles (quarks and gluons) interact via the strong force. Much of the theoretical formulation of the SM was done in the mid to late 20th century and extensive experimental testing has shown it to be extremely successful.

The SM, however, has some major shortfalls as a complete theory of physics at the fundamental level:

- **Gravity:** As briefly mentioned, the SM does not include gravitation since there is no known way to describe general relativity as a quantum field theory. Attempts to do so result in a theory which is not renormalisable, and often cannot make useful predictions. Furthermore, the weakness of gravitational interactions poses extreme difficulties for experimentally testing theories of gravity at the quantum level. The most notable attempts to describe quantum gravity come from string theory and its derivatives, which include other SM extensions such as supersymmetry (SUSY) and extra dimensions. A detailed discussion is beyond the scope of this work, but there is extensive literature available on the subject, e.g. [29][30]

- **Hierarchy Problem:** It is not understood why there is such a huge difference of $\mathcal{O}(10^{32})$ between the strength of gravity and the weak interaction. Specifically, the Higgs boson is expected to have a mass of order of the Planck scale ($\mathcal{O}(10^{19})$ GeV) due to large radiative corrections, in the (desirable) absence of a large amount of fine-tuning between the bare mass and these corrections. SUSY attempts to solve this problem by introducing bosonic superpartners to the SM fermions. The radiative corrections due to the new ‘sfermions’ cancel with those from the SM fermions, protecting the Higgs boson mass [31]. SUSY will be discussed in more detail in the following subsection.
• **Matter-Antimatter Asymmetry:** CP is the combined operation of charge conjugation (which reverses the charge and all internal quantum numbers of a particle) and parity inversion (which inverts the spatial coordinates). The Universe is observed to contain significantly more matter than antimatter, which implies that CP symmetry must have been violated in times shortly after the Big Bang, a process known as ‘baryogenesis’, which occurred before Big Bang nucleosynthesis [32]. There are known sources of CP violation in the SM: the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which describes the mixing of quark flavours, is generated by introducing Yukawa couplings between the Higgs boson and the quarks. The complex nature of the CKM matrix means that there is a CP violating phase for three generations of quarks. This type of CP violation, however, can only account for a small proportion of the amount required to produce the observed asymmetry. Furthermore, simply a large amount of CP-violation is not sufficient to explain baryogenesis; the set of ‘Sakharov conditions’ must be met [33]:

1. Baryon number violation.
2. C-violation and CP-violation.
3. Interactions out of thermal equilibrium.

In order to fully meet these conditions, new physics beyond the SM is required.

• **Number of Generations and Parameters:** It is not explained why there are three generations of quarks and leptons in the SM, nor is it clear how the masses of particles relate to their counterparts in other generations. More generally, there are a large number of parameters in the SM which need to be inserted by hand and cannot currently be predicted. These include the masses of all twelve fermions, the coupling constants of the three forces, the Higgs boson mass and vacuum expectation value, the three flavour mixing angles and CP-violating
phases of both the quark and neutrino mixing matrices and a CP-violating parameter of the strong interaction. This is an undesirable feature of a theory attempting to describe fundamental physics completely, and in order to explain why these parameters take their observed values, we must again look for physics beyond the SM.

- **Strong CP Problem:** The QCD Lagrangian contains a term which allows for a large amount of CP violation. Measurements of the neutron electric dipole moment, however, constrain this term to be extremely small, implying that the CP symmetry is extremely well respected in the strong sector. This hints at fine tuning, a good indicator that the physics in question is not well understood, since the SM has no explanation for why the offending term should be so small (or perhaps zero). One proposed solution to this problem introduces an additional symmetry which, when broken, forces the term to be zero. The excitations of the field are called ‘axions’, which will be discussed further in the next section.

- **Dark Energy:** The accelerating expansion of the Universe is typically attributed to the existence of so-called ‘dark energy’, which permeates all of space and is found to comprise 69.0\% of the mass-energy content of the observable Universe. The SM contains no explanation for this observation, and there is currently no generally accepted way of incorporating theories of dark energy.

- **Dark Matter:** The mass-energy density of DM constitutes 26.1\% of the observable Universe. Combined with the equivalent measurement for dark energy above, only 4.9\% of the mass-energy content of the Universe is baryonic matter. The SM falls short of accounting for DM since, as will be discussed further below, it contains no suitable DM particle candidate.

\[2\]While CP violation has not yet been observed in the neutrino sector, the corresponding phase can in general be present in the mixing matrix.
This list is not complete, but clearly highlights that the SM is far from a final theory of fundamental physics. The remainder of this work will be focused on the last item.

### 1.3.2 Dark Matter Particle Candidates

If the solution to the DM problem is some fundamental particle or particles, it is crucial to first understand the properties that they must have. We have seen that there are strong constraints from astrophysics and cosmology on the amount of missing mass that can be baryonic, but there is very little information about the (dominant) non-baryonic component of DM. It is useful, therefore, to consider exactly which particles must be excluded. The basic requirements are immediately obvious: a viable candidate must possess mass, and must not interact very strongly with SM particles. The latter tells us that DM particles must be electrically neutral and cannot possess colour charge since, if this was not the case, they would be very easy to detect via their electromagnetic and strong interactions. These basic requirements therefore exclude all SM fermions except the three neutrinos. DM particles must also be stable on cosmological timescales, otherwise they would have decayed by now. This constrains candidates provided by extensions to the SM, such as supersymmetry and extra dimensional theories.

Studies of the large-scale structure of the Universe also rule out neutrinos. Specifically, in a universe dominated by relativistic (‘hot’) DM, large structures form first before fragmenting into smaller objects. This process of structure formation is inconsistent with cosmological observations, which suggest a ‘bottom-up’ process, whereby stars were formed, then galaxies, then galaxy clusters. Since neutrinos are relativistic, they therefore cannot form a significant contribution to DM \[36\]. Additionally, neutrinos are very light: the sum of their masses is constrained to be \( \mathcal{O}(1 \text{ eV}) \) or less \[37\]. This in turn constrains the cosmological neutrino density to be very small, and so neutrinos can only make up a tiny fraction of the total missing mass in the
Since the SM does not contain a possible DM particle, new suitable candidates must be hypothesised if the particle physics solution is to be pursued. The following list summarises the basic properties, motivation and current status of notable DM particle candidates.

- **Weakly Interacting Massive Particles (WIMPs):** These are fundamental particles characterised by interactions of comparable strength to the weak force and masses around the weak scale. WIMPs have the major advantage of being able to fully account for the observed amount of non-baryonic DM. Under the assumption that they were in equilibrium with SM particles after cosmological inflation, the WIMP relic density can be calculated. Once the temperature of the Universe drops below approximately $m_\chi/20$, where $m_\chi$ is the WIMP mass, the WIMPs ‘freeze out’ of equilibrium and their comoving density becomes constant, assuming subsequent adiabatic evolution of the Universe. The relic density is given by

$$\Omega_\chi h^2 \approx 3 \times 10^{-27} \text{ cm}^3 \text{ s}^{-1} \langle \sigma_{\text{ann}} v \rangle.$$  \hspace{1cm} (1.5)

In order to generate the value for $\Omega_{\text{dm}} h^2$ in equation (1.3), the thermally averaged total annihilation cross section, $\langle \sigma_{\text{ann}} v \rangle$, must be approximately equal to $3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$. For weak interactions, $\langle \sigma_{\text{ann}} v \rangle \sim G_F^2 m_\chi^2 / 2\pi$ [39], which reproduces the required value with $m_\chi \sim 10 \text{ GeV}$. More generally, particles with masses $10 \text{ GeV} \lesssim m_\chi \lesssim 1 \text{ TeV}$ which undergo interactions comparable in strength to the weak force are able to satisfy the required relic density of non-baryonic DM.

Consequently, WIMPs are the most extensively studied DM candidate, and are the focus of this work. Good WIMP candidates are also included in various popular extensions to the SM which attempt to address some of the issues
listed in Subsection 1.3.1. SUSY, the most notable of these, introduces a new symmetry relating fermions and bosons [31]. As a result, the addition of SUSY to the SM doubles the particle content, providing each SM particle with a new superpartner. SUSY is a broken symmetry, resulting in the superparticles having much greater masses than their SM counterparts. In versions of SUSY which feature ‘R-parity’, the lightest supersymmetric particle (LSP) is stable, as it is not permitted to decay to SM particles. Section 1.5 discusses the link between SUSY and DM in more detail.

Theories of extra spatial dimensions, first postulated by Kaluza and Klein in the early 20th century as a move towards a grand unified theory [40,41], re-emerged recently as a mechanism to solve the hierarchy problem. In the framework of universal extra dimensions (UED) all SM particles are free to propagate into compactified extra dimensions [42]. These particles then have an associated tower of Kaluza-Klein (KK) states, the lightest of which could be stable due to KK parity conservation. Furthermore, it has been shown that, under certain assumptions, the DM relic density can be accounted for by the first excited state of the photon or the neutrino, depending on which is the lightest KK particle (LKP) [43]. Thus, the neutral, stable and weakly interacting LKP is a good WIMP candidate.

- **Axions:** A proposed solution to the strong CP problem involves introducing an additional global $U(1)$ symmetry to the SM, while the CP violating term, $\bar{\theta}$, in the Lagrangian is replaced with a dynamic field. Spontaneous breaking of this symmetry forces $\bar{\theta}$ to be zero, and axions are the pseudo-Nambu-Goldstone bosons of the broken symmetry [35,44,45].

Axions are extremely light but, since their interactions are so weak, they are produced non-thermally. They would therefore be classified as cold DM, avoid-
ing the structure formation problems associated with relativistic DM particles \[46,47\]. Considering this, with a certain choice of parameters, axions could potentially be the dominant contribution to \(\Omega_{\text{dm}}\).

- **Sterile Neutrinos:** The see-saw mechanism is the leading explanation for the non-zero masses for the SM neutrinos, and for why these masses are so small compared to the other SM fermions \[48,49\]. The theory necessarily includes right-handed Majorana neutrinos, dubbed ‘sterile’ because they do not interact via any of the SM forces\(^3\). The small masses of the ‘active’ SM neutrinos can be generated for a large range of sterile neutrino masses. Of particular interest is the setup where one sterile neutrino has a mass of a few keV. In this case it can function as ‘warm’ DM \[50\], and potentially alleviate some of the discrepancies between predictions of cold DM models and observations of small-scale structure \[51\].

- **Exotic Candidates:** Various more exotic DM candidates have been hypothesised. SuperWIMPs, which interact considerably more weakly than conventional WIMPs, include particles such as the graviton and gravitino\(^4\) in models with SUSY and UED \[52\]. They may be able to recover the DM relic density but are essentially impossible to detect experimentally. Asymmetric DM was proposed to explain the similarity between the relic densities of baryons and DM. The theory posits that the observed baryon asymmetry in the Universe is reflected in the dark sector, providing an explanation for the similarity of \(\Omega_b\) and \(\Omega_{\text{dm}}\) for a DM mass between around 5 and 15 GeV \[53\]. Little Higgs models, through conservation of T-parity, can provide a lightest stable state which could be a WIMP \[54\]. Other candidates include ‘techni-baryons’

\(^3\)Since the weak interaction only affects left-handed particles, and neutrinos only interact via the weak force, the SM neutrinos must be left-handed.

\(^4\)The graviton is the hypothetical force carrier of gravity, while the gravitino is its superpartner.
from theories of new strong dynamics \[55\], non-topological solitons, ‘Q-balls’, contained in SUSY models \[56\], non-thermally produced heavy WIMPs, ‘WIMPZILLAs’ \[57\], particles from extra dimensional theories which only interact gravitationally, ‘GIMPs’ \[58\], and new fields in the brane-world scenario, ‘branons’ \[59\].

This work is focused on properties of interactions between a generic WIMP candidate and SM particles. As such, little emphasis is placed on the origin of this new particle in order to keep the findings as model-independent as possible. Nevertheless, it is useful to see that DM candidates can emerge out of many theories of physics beyond the SM. In fact, this is often a side-effect of these theories, originally proposed to solve other problems in particle physics. It is, of course, possible that several of the above candidates contribute to DM, or even that there is a ‘dark sector’ of particles and forces, much like the SM, and rich in new physics. Recalling that there is roughly five times as much DM as baryonic matter in the Universe, this idea is rather compelling. It could also serve to remedy some of the other major issues in physics, such as dark energy. It makes sense, however, to assume a degree of simplicity in the quest to find DM particles. More specifically, it is assumed here that there is at least one new DM particle and one new force connecting it to SM particles. Whether or not this is the extent of the dark sector, and irrespective of the model from which the DM particle arises, this seems like a good place to begin.

1.4 Experimental Searches for Dark Matter

There are currently three major classes of experiments searching for interactions of DM with the SM: collider searches, direct detection and indirect detection. The processes that are probed by these experiments illustrated schematically by the diagram in Figure 1.4. Assuming that DM interacts with SM particles, production should be possible at particle colliders. A typical signal of this would be SM particles in
Figure 1.4: A pictorial representation of the processes studied by the three primary DM particle searches. The shaded circle corresponds to some unknown interaction(s) that take place between SM and DM particles.

association with a large amount of missing transverse energy corresponding to the DM that escapes the detector. Since collider searches are the focus of this thesis, a more detailed discussion is postponed until later chapters. In this section, a summary is provided of the mechanisms and current status of direct and indirect detection experiments.

1.4.1 Direct Detection

Presented here is a brief overview of direct detection, following [60]. For a more detailed discussion the reader is encouraged to consult that reference, or other dedicated works, e.g. [61]–[64]. These experiments seek to identify the recoil of target nuclei from elastic collisions with WIMPs that pass through the detector, utilising the fact that the Earth is moving through the Milky Way DM halo. For a recoil energy $E$, the differential counting rate for events, measured in counts per kg per keV per day, can be expressed as

$$\frac{dR}{dE}(E,t) = \rho_0 \frac{m_\chi m_A}{m_\chi m_A} \int_{v_{\text{min}}}^{v_{\text{esc}}} d^3v \ |v| \ f(v,t) \ \frac{d\sigma}{dE}(E,v),$$

(1.6)
where $\rho_0$ is the local DM density, $m_\chi$ is the WIMP mass, $m_N$ is the mass of the target nucleus, $f(v, t)$ is the WIMP velocity distribution as measured in the rest frame of the detector, and $\frac{d\sigma}{dE}(E, v)$ is the differential cross section (discussed below). The dependence of (1.6) on local characteristics of the DM halo clearly enters through $\rho_0$ and $f(v, t)$, and these are therefore sensitive to the halo model used. The time dependence of the velocity distribution (and therefore the event rate) is due to the motion of the Earth around the Sun.

For elastic collisions, the minimum velocity required for a DM particle to deposit energy $E$ in the detector is

$$v_{\text{min}}(E) = \sqrt{\frac{m_NE}{2\mu_{\chi N}}}$$

(1.7)

where $\mu_{\chi N}$ is the reduced mass of the DM-nucleus system, $\mu_{\chi N} = m_\chi m_N/(m_\chi + m_N)$. The velocity integral in (1.6) also has an upper limit, stemming from a truncation of the velocity distribution above the DM escape velocity. The value used for $v_{\text{esc}}$ is dependent again on the details of the DM halo model. The Standard Halo Model (SHM) assumes an isotropic Maxwell-Boltzmann velocity distribution, as well as $\rho_0 = 0.3 \text{ Gev cm}^{-3}$ and $v_{\text{esc}} = 544 \text{ km s}^{-1}$ [65, 66].

The form of the differential cross-section for scattering of DM particles off atomic nuclei is dependent on whether the interaction is spin-independent (SI) or spin-dependent (SD). For an energy $E$ deposited in the detector, the SI contribution can be written as

$$\frac{d\sigma_{\text{SI}}}{dE} = C_T^2(A, Z)F^2(E)\frac{m_N}{2\mu_{\chi p}v^2}\sigma_{p,\text{SI}},$$

(1.8)

where $C_T(A, Z) \equiv (f_p/f_n)Z + (A - Z)$ with $Z$ and $A$ the atomic number and mass number of the nuclei, respectively, and $f_{n,p}$ the effective coupling of DM to neutrons and protons. $F(E)$ is the nuclear form factor, $\mu_{\chi p}$ is the DM-proton reduced mass, and $\sigma_{p,\text{SI}}$ is the SI cross-section for scattering of a DM particle off a proton, at zero
Figure 1.5: The LUX 90% confidence limit on the WIMP-nucleon cross section (blue line), compared to results from other direct detection experiments. The inset shows the bounds at low WIMP masses. See [71] for further details.

momentum transfer. For the SD case,

$$\frac{d\sigma_{\text{SD}}}{dE} = \frac{4\pi}{3(2J+1)} \left[ a_0^2 S_{00}(E) + a_0 a_1 S_{01}(E) + a_1^2 S_{11}(E) \right] \frac{m_N}{2\mu_p^2 v^2} \sigma_{p,\text{SD}},$$

(1.9)

where $J$ is the spin of the nucleus, $a_0 = a_p + a_n$ and $a_1 = a_p - a_n$ for DM couplings to protons ($a_p$) and neutrons ($a_n$), and the $S_{ij}$ are spin-dependent nuclear structure form factors. The Helm form factor is typically used in (1.8), while for (1.9) the form factors depend on the target material [67–70].

In the case of SI scattering, the best constraints for $m_\chi \gtrsim 6$ GeV are provided by the Large Underground Xenon (LUX) experiment [71–73]. The results are derived from an analysis of 85.3 live days of data, with a fiducial target mass of 118 kg of liquid xenon. These are illustrated on Figure 1.5. The experiment reports an upper limit at 90% CL of $7.6 \times 10^{-46}$ cm$^2$ on the WIMP-nucleon cross section, for a WIMP mass of 33 GeV. The SI cross section scales with the size of the target nuclei, while this is not the case for SD scattering. Consequently the bounds on the SD WIMP-nucleon cross section are considerably weaker. Results are typically presented separately for the two cases of couplings to protons or neutrons. Over the majority of the DM mass range, for the proton coupling the current best limits have been obtained by...
the PICO experiment \cite{74}, while for the neutron coupling XENON100 provides the strongest constraints \cite{75}. These experiments place limits of $O(10^{-39}) \text{ cm}^2$ on the SD cross section. In later chapters, the direct detection bounds on the specific simplified DM models under consideration will be discussed.

1.4.2 Indirect Detection

Indirect detection experiments search for the products of DM particle annihilations. In principle, SM particles should be produced in these annihilations, and the important signals will come from searches for gamma rays and antimatter. Experiments typically target regions of space of high DM density — and therefore more annihilation events — such as the Sun, the galactic centre, dwarf spheroidal galaxies and galaxy clusters.

Gamma ray signals can arise either due to annihilations to quarks and gauge bosons, leading to a particle jet which radiates high energy photons, or due to direct annihilation to photons. The latter case would lead to gamma rays of specific energy (a ‘spectral line’) dependent on the DM mass. For typical WIMP masses, such a signal would be a strong indicator of DM annihilation, however the process cannot occur at tree level (since DM has no electric charge) and thus the cross section will be suppressed by a loop factor. Gamma rays have the advantage that they do not scatter significantly on their journey to Earth, meaning that signals are expected to be focused in the direction of the source. The galactic centre is expected to provide the largest flux of gamma rays from DM annihilation, however making definitive statements about a signal relies upon a good understanding of both astrophysical sources and the DM distribution in the central region of the Milky Way. Reducing the associated uncertainties is one of the major challenges facing gamma ray searches over the coming years. Neighbouring dwarf elliptical galaxies contain high proportions of DM and the astrophysical gamma ray background is typically negligible. For
these reason, studies of dwarf ellipticals provide robust constraints on DM from indirect detection. The satellite-based Fermi LAT and Earth-based HESS experiments currently place the best bounds on DM annihilations to gamma rays.

The presence of antimatter in cosmic rays is another important indirect detection signal. Positrons and antiprotons ought to be SM products of DM annihilations, and excesses of these would therefore be indicative of such processes. AMS-02 is a detector mounted on the International Space Station designed to study the constituents of cosmic rays, focusing in particular on antimatter. The collaboration report an excess of positrons, supporting similar earlier findings by the PAMELA experiment. The primary difficulty involved with interpreting these results is identifying and modelling possible astrophysical sources, such as pulsars and supernova remnants. A promising avenue for future antimatter searches is the antideuteron channel, which benefits from considerably smaller predicted backgrounds. The GAPS experiment is designed specifically to search for antideuteron signals in cosmic rays, while AMS-02 also has antideuteron detecting capabilities. The prospects for searches in this channel are studied in.  

1.5 Theoretical Considerations

Broadly speaking, there are three main approaches to the theoretical study of DM:

- **Effective Field Theory (EFT) Approach:** This can be considered as the simplest case; mediating particles are assumed to be heavy enough, compared to the momentum transfer, that they can be safely integrated out and the SM-DM interactions are considered as point-like. In such cases the most important variable is the suppression scale which contains all information regarding the strength of the interaction. The EFT approach has been useful in analysing data from direct detection and collider searches, but its limitations are clear. With large energy transfers, the approach fails to provide an accurate description
of the underlying physical process. Furthermore, if one wishes to investigate particular physical parameters more deeply (the DM mass, for example) it is necessary to go beyond the EFT.

- **Simplified Models:** Motivated by the drawbacks to the EFT approach, simplified models typically assume that there is an additional accessible state which mediates interactions between the SM and DM. These have the advantage over an EFT description of being able to describe the kinematics of collider production of DM. The cost, naturally, is that there are additional parameters involved such as the mediator mass and couplings strengths for DM and SM particles.

- **Complete Theories:** At this end of the spectrum lie the theoretical extensions to the SM which contain DM candidates. Examples, which have already been mentioned, include SUSY, UED and Little Higgs models. These are phenomenologically rich and often contain many other additional states and associated parameters. One advantage of studying such models is that they are often able to explain associations between observables that may otherwise appear accidental. On the other hand, the larger number of degrees of freedom with such models means that it becomes more difficult to make useful and unambiguous predictions.

For illustrative purposes, we consider DM in the Minimally Supersymmetric Standard Model (MSSM). As mentioned previously, the LSP in SUSY theories is naturally a viable DM particle. The neutralino (a mixture of bino, wino and Higgsino) is the most viable and extensively studied candidate, being thermally produced in the early Universe and able to account for the observed relic density for a WIMP mass in the appropriate range. Other possibilities that have been considered include, in particular, the gravitino and the sneutrino. The former requires additional physics beyond the MSSM and its interactions are too small to detect, while the latter has
been ruled out by direct detection searches. A more detailed discussion of SUSY candidates for DM can be found in e.g. [83,84].

The complete theory, in this case, is the MSSM with its full particle content and all of the additional parameters. Alternatively, one could consider a simplified model, in which the neutralino is taken to be the DM candidate and interactions with the SM proceed via the $t$-channel exchange of squarks or sleptons. In fact, this DM model is very typical of the MSSM. Simplifying further, one could assume that the mediating sfermion could be integrated out to obtain a point-like interaction. This would be the EFT description.

This thesis focuses largely on simplified models, although the EFT approach is employed where it is useful. Simplified models permit an accurate study of collider phenomenology without committing to any single complete theory; it is therefore a more agnostic approach to the study of DM.

1.6 Summary and Thesis Structure

In this chapter, the wealth of astrophysical and cosmological evidence for a large amount of missing non-baryonic mass in the Universe has been reviewed. We have seen that a particle physics explanation is well motivated, firstly since searches indicate a very small contribution from macroscopic objects, secondly because theories of modified gravity have struggled to explain the behaviour of galaxy clusters as well as some cosmological observations, and thirdly because of the well known incompleteness of the SM and the need for new physics to resolve this. The SM and its shortfalls were reviewed, leading to a discussion of the notable DM particle candidates and the theories from which they originate. A brief overview of direct and indirect searches for DM was presented before the chapter concluded with an overview of the ‘theory space’ of DM models.
In Chapter 2 a simplified model is considered which introduces a new Dirac or Majorana fermion, $\chi$, as the the DM particle, in addition to a new spin-1 $s$-channel particle, $Z'$, which mediates interactions between SM and DM particles. The details of this model are presented, and constraints on the parameter space are analysed by considering LHC searches for events which have either two charged leptons or two hadronic jets in the final state. For a then well-motivated choice of parameters, it is found that the dominant decay mode of the $Z'$ is to DM pairs. Consequently, the leading collider signature for DM production in the model will be missing transverse energy, corresponding to the DM pairs which escape detection, in association with hadronic jets. Simulations are then performed of LHC searches for $E_T + j$ or ‘monojet’ events. The results of these simulations are studied, firstly to derive further constraints on the parameter space for the model, and secondly to make a comparison between the cases in which the mediator is a pure vector or a pure axial vector. The latter observation motivates the work in the following chapter.

In Chapter 3 a model with a spin-0 $s$-channel mediator is considered. Again, the model details are presented and it is verified that missing transverse energy searches are an appropriate method for studying this model. Here, final states which feature two final jets are considered, with calculations being performed in the EFT framework. The kinematics of these processes are examined, supporting the study of events containing two jets in the final state as a means to determine the CP nature of the mediating particles, and also guiding the choice of event selection criteria applied in the analysis. The differential cross section as a function of the azimuthal angular separation of the two jets is considered as the relevant observable, and the corresponding results for scalar and pseudoscalar mediators are compared to each other, as well as to the dominant SM background. The impact of taking a heavy top quark approximation is considered, and the stability of the results is then discussed when including next-to-leading order (NLO) QCD corrections, hadronisation effects and
parton showering (PS). To conclude this chapter, the validity of the EFT approach is studied.

In Chapter 4, a more complex model is studied in which DM couples to the SM gauge bosons. Again, the EFT framework is used. First, experimental searches for a variety of final states containing missing transverse energy are used to derive constraints on the size of the interactions between the gauge bosons and DM. This motivates a discussion regarding the validity of the EFT approach, and possible ultraviolet (UV) completions are studied. Our attention then turns to the 14 TeV LHC run, considering how the previously obtained constraints might be improved, and studying simulated predictions of the azimuthal angular distributions of the two jets in $E_T + 2j$ events. The chapter concludes with a toy Monte Carlo (MC) study, analysing the results that might be obtained at the 14 TeV LHC once 300 fb$^{-1}$ and 3000 fb$^{-1}$ of data have been collected.

In Chapter 5, the key results presented in this thesis are summarised and discussed, with a particular emphasis on the implications for future collider searches for DM.
Chapter 2

Dark Matter Searches at the LHC

The previous chapter has demonstrated that the search for DM is very well motivated, and has provided ideas of how to proceed. Eventually, it would be desirable to have a theory (or set of theories) which predicts how DM interacts with the SM, and which is experimentally testable. As briefly touched upon, predictions would ideally be as model independent as possible — that is, they would not be dependent on the exact nature of the DM particle and mediator(s), nor constrained to a very specific region of parameter space, for example. The trade-off in building as general a model as possible is, of course, that it becomes increasingly difficult to extract useful predictions. Consequently, model building is typically a compromise between generality and predictability, and the challenge is in maximising both of these elements.

This chapter introduces a simplified model of DM and its interaction with the SM. In Section 2.1, the model is introduced. In Section 2.2, the flavour physics of the model is studied, and the motivation for imposing minimal flavour violation (MFV) is presented. In Section 2.3, predictions of contributions to dilepton and dijet production are compared to experimental data in order to constrain the model parameters. This also motivates searches for missing energy as a promising approach to studying the model in greater depth. In Section 2.4, production of DM at the LHC via the new interaction is simulated, and the signal corresponding to at least one hadronic jet in association with a large amount of missing transverse energy is analysed. The results
motivate more advanced work that is presented in the next chapter.

2.1 Model Description

The model under consideration introduces two new particles: a Dirac or Majorana fermion, $\chi$, corresponding to the DM particle, and a spin-1 $s$-channel mediator, $Z'$, which couples the DM particles to the SM. The Lagrangian for these interactions is

$$\mathcal{L}_{Z'} = (\bar{\chi} \gamma_\mu [g^V_\chi + g^A_\chi \gamma_5] \chi) \, Z'^\mu + \sum_{f=q, l} (\bar{f} \gamma_\mu [g^V_f + g^A_f \gamma_5] f) \, Z'^\mu,$$

(2.1)

where $g^V, A_\chi$ and $g^V, A_f$ are the vector/axial vector couplings of the DM particle $\chi$ and SM fermion $f$ to the mediator, and the sum runs over all of the SM fermions. In the Majorana case, $g^V_\chi = 0$ (see Appendix A). One method of introducing the mediator is by assuming the presence of a new $U(1)'$ gauge symmetry, which is spontaneously broken to generate the mass $M_{Z'}$ of the mediator. This could occur in a simple way due to the presence of a ‘dark Higgs’ with a non-zero VEV. Furthermore, in the case that $g^A_f \neq 0$, this mechanism would also generate the DM mass. The phenomenology and implications of extensions to the SM which incorporate a new $U(1)'$ symmetry have been extensively studied. See e.g. [85–90].

In order to study the collider phenomenology of the model given by (2.1), it is necessary to examine the possible decays of the $Z'$. From this Lagrangian one obtains for the total width

$$\Gamma_{Z'} = \frac{M_{Z'}}{12\pi} \sum_{f=q, l, \chi} N^f_C \left(1 - \frac{4m_f^2}{M_{Z'}^2}\right)^{1/2} \left[ (g^V_f)^2 + (g^A_f)^2 + \frac{2m_f^2}{M_{Z'}^2} ((g^V_f)^2 - 2(g^A_f)^2) \right],$$

(2.2)

where the sum runs over all final states that are kinematically accessible, and $N^f_C$ is the number of colours associated with the fermion $f$ (3 for SM quarks and 1 for SM leptons and DM). From here, we restrict ourselves to the scenario in which the interaction is purely vectorial ($g^A_f = 0$) or purely axial vectorial ($g^V_f = 0$), and the
couplings are universal in the quark and lepton sectors, such that the set is reduced to \( \{g_q, g_l, g_\chi\} \). In the case that that \( M_{Z'} \gg m_f \), the total width then simplifies to

\[
\Gamma_{Z'} = \frac{M_{Z'}}{12\pi}[18g_q^2 + 6g_l^2 + g_\chi^2].
\]

(2.3)

## 2.2 Minimal Flavour Violation

In order to prevent the appearance of excessive flavour- and CP-violating processes within this new model, the hypothesis of MFV is imposed, which states that the CKM matrix governs all CP-violating and flavour-changing processes in the quark sector. In particular, MFV requires that, similarly to the SM, only Yukawa interactions are permitted to break the global flavour symmetry of the SM quark sector, \( G_q = SU(3)_q \otimes SU(3)_u \otimes SU(3)_d \) \[91\]. Overall, enforcing MFV avoids the interactions between SM quarks and the new mediator from generating dangerous flavour-changing neutral currents (FCNCs), and also gives rise to a stable DM candidate \[92\].

In the model under consideration, the couplings of the \( Z' \) to DM are unrestricted, while those between the \( Z' \) and the SM fermions are forced to be flavour-independent (but can be dependent on chirality, such that \( g_f^A \neq 0 \) in general).

To illustrate the motivation for imposing MFV, we consider now a scenario in which it is not enforced. Here, the \( Z' \) couples to the quark gauge eigenstates vectorially (the axial vector current is taken to be zero) and the couplings to the SM leptons are ignored. For further simplicity, only couplings to down-type quarks are studied. It is assumed that the couplings to the second and third generation quarks are of the same size, and different to the size of the first generation coupling. In the gauge eigenstate basis, the Lagrangian then contains the following terms

\[
\mathcal{L} \supset \sum_i \left( \bar{d}_i \gamma_\mu \left[ g + \Delta \delta_{i1} \right] d_i \right) Z'\mu,
\]

(2.4)

\[\text{For the remainder of this chapter, the superscript } V \text{ or } A \text{ on the couplings is dropped as it will be clear from the context whether a vector or axial vector mediator is being considered.}\]
where $\Delta$ parametrises the difference in size of the couplings, and the sum runs over the three SM quark generations. In order to represent this in terms of the mass eigenstates of the quarks, the left- and right-handed fields must be rotated using the $3 \times 3$ unitary matrices $U_{L,R}^{u,d}$. The term proportional to $g$ will lead to flavour diagonal terms, which are not of interest here since we are concerned with the flavour violating effects of this model. The other term is first split into its left- and right-handed parts

$$
\Delta \delta_{11} (\bar{d}_i \gamma_{\mu} d_i) = \Delta \delta_{11} \left(\bar{d}_i^L \gamma_{\mu} d_i^L + \bar{d}_i^R \gamma_{\mu} d_i^R\right),
$$

(2.5)

where the left- and right-handed components of the fields are defined by $d_{L,R}^i = \mathcal{P}_{L,R} d_i^i$, $\bar{d}_{L,R} = \bar{d}_i \mathcal{P}_{R,L}$, using the projection operators $\mathcal{P}_{L,R} = (1 \mp \gamma_5)/2$. Under the rotations, $d_i^L \rightarrow (U_{d}^L)^{ij} d_j^L$ and $\bar{d}_i^L \rightarrow \bar{d}_i^L (U_{d}^L)^{ij}_{\bar{d}_j^L}$ which leads to

$$
\bar{d}_i^L \gamma_{\mu} d_i^L \rightarrow \bar{d}_i^L \left((U_{d}^L)^{ij}_{\bar{d}_j^L} \gamma_{\mu} (U_{d}^L)^{jk}_{\bar{d}_k^L} d_k^L\right)
= (U_{d}^L)_{ij} D_{\bar{d}_j^L} \bar{d}_i \gamma_{\mu} P_{L} d_k^L,
$$

(2.6)

and similarly for $L \rightarrow R$. Simplifying the notation, we arrive at the following flavour non-diagonal terms in the Lagrangian

$$
\mathcal{L} \supset \sum_{i,k} \Delta \left(L_{ik} \bar{d}_i \gamma_{\mu} P_{L} d_k + R_{ik} \bar{d}_i \gamma_{\mu} P_{R} d_k\right) Z^{\mu},
$$

(2.7)

with

$$
L = (U_{d}^L)^{\dagger} \text{diag}(1,0,0) U_{d}^L \quad \text{and} \quad R = (U_{d}^R)^{\dagger} \text{diag}(1,0,0) U_{d}^R.
$$

(2.8)

It is now necessary to specify the form of the matrices $U_{L,R}^{u,d}$ in order to obtain an estimate of the size of the FCNCs that can arise in this model. In the SM, mixing between right-handed quarks is not observable, which means that a suitable choice for $U_{R}^{u,d}$ is the $3 \times 3$ identity matrix, $I_3$. In contrast, the left-handed rotations are observable in the SM because they give rise to the CKM matrix, i.e. $V_{\text{CKM}} = (U_{d}^u)^{\dagger} U_{d}^d$. A simple choice therefore is either $U_{L}^{d} = V_{\text{CKM}}$ and $U_{L}^{u} = I_3$, or $U_{d}^{u} = I_3$ and $U_{L}^{u} = V_{\text{CKM}}^{\dagger}$. These will generate FCNCs in the left-handed down-type or up-type quark sectors,
Figure 2.1: Feynman diagrams that give rise to neutral kaon mixing. Left: the SM contribution, which proceeds at loop level via the exchange of $W$-bosons and up-type quarks. A second similar ‘box diagram’ contributes, in which the positions of the internal quarks and $W$-bosons are interchanged. Right: the new physics contribution, which proceeds at tree level via the exchange of the new $Z'$ mediator.

respectively. Since this discussion is restricted to the down-type quarks, we take the former option. The matrices defined in (2.8) are then given by

$$L = \begin{pmatrix} |V_{ud}|^2 & V^*_{us}V_{ub} & V^*_{ud}V_{ub} \\ V^*_{us}V_{ad} & |V_{us}|^2 & V^*_{us}V_{ub} \\ V^*_{ub}V_{ad} & V^*_{ub}V_{us} & |V_{ub}|^2 \end{pmatrix} \quad \text{and} \quad R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.9)$$

Thus, FCNCs are generated at tree level due to $Z'$-boson exchange. We consider the processes by which neutral kaons, $K^0$, oscillate into their antiparticles, $\bar{K}^0$, and vice versa. In the SM this proceeds via the ‘box diagram’ shown on the left in Figure 2.1. The amplitude for this process is given approximately by 

$$A(K_0 \leftrightarrow \bar{K}_0)_{\text{SM}} \sim \frac{g_w^4 (V^*_{ud}V_{us})^2 m_t^2}{2048 \pi^2 M_W^2 v^2} \sim 7 \times 10^{-17}, \quad (2.10)$$

where $g_w$ is the weak coupling constant, $m_t$ is the top quark mass, $M_W$ is the $W$-boson mass and $v$ is the Higgs boson VEV. Kaon oscillations proceed in our model via the tree-level diagram shown on the right in Figure 2.1. The amplitude for this process is approximately given by

$$A(K_0 \leftrightarrow \bar{K}_0)_{Z'} \sim \frac{(V^*_{ud}V_{us} \Delta)^2}{M_{Z'}^2} \sim 5 \times 10^{-2} \frac{\Delta^2}{M_{Z'}^2}. \quad (2.11)$$

By enforcing a simple requirement that the new-physics contribution (2.11) is smaller
than the SM contribution \((2.10)\), we obtain the bound
\[
\left| \frac{\Delta}{M_{Z'}} \right| \lesssim 4 \times 10^{-8}. \tag{2.12}
\]
If \(\Delta\) is \(\mathcal{O}(1)\), this translates to a bound on the mediator mass of
\(M_{Z'} \gtrsim 3 \times 10^4\) TeV, so that the \(Z'\) cannot be produced at the LHC. Consequently, it is clear that in order
to be able to actually study the properties of a simplified model of DM at the LHC,
we require \(\Delta\) to be very small. In the MFV framework this term is by definition set
to zero and it is for this reason that MFV is enforced.

### 2.3 Experimental Constraints on Simplified Model Parameters

Despite the simplifications that have been applied to this DM model, there are still a
number of free parameters. This section focuses on the constraints that can be applied
to the size of the SM lepton and quark couplings, \(g_l\) and \(g_q\), in \((2.2)\), as compared to
bounds on the mediator mass \(M_{Z'}\), by considering experimental searches for dilepton
and dijet production. The discussion in this section is restricted to the case in which
\(Z'\) is purely vectorial. The analogous calculations for an axial vector can also be
performed and lead to similar results.

#### 2.3.1 Dilepton Constraints

In this subsection, the cross section for dilepton production via a \(Z'\) mediator at the
LHC is calculated and compared to experimental data. This process is illustrated
at leading order (LO) by the Feynman diagram in Figure 2.2. The value of \(g_l\) is
varied, while the other couplings are set to \(g_\chi = 1\) and \(g_q = 0.25\). This choice for \(g_q\)
will be justified in the following subsection, in which dijet constraints are considered.
Inserting these values into \((2.3)\), one obtains \(\Gamma_{Z'}/M_{Z'} \lesssim 0.22\) for \(g_l \leq 1\), implying that
the narrow width approximation (NWA) can be employed in this analysis \([94]\). This
greatly simplifies the calculations since, in this case, the \(Z'\) production and decay
processes factorise and the total dilepton production cross section can be written as
\[ \sigma(pp \to Z' \to l^+l^-) = \sigma(pp \to Z') B(Z' \to l^+l^-). \]

The production cross section, which is hereafter denoted as \( \sigma \) for simplicity, is calculated for a range of values of \( M_{Z'} \) between 300 GeV and 5 TeV using \texttt{MadGraph 5} \cite{95,96} for proton collisions with a centre-of-mass energy \( \sqrt{s} \) of 8 TeV. Details of how this was done are presented in Appendix B.

The cross sections are then multiplied by the branching ratio for \( Z' \to l^+l^- \), denoted as \( B_{l^+l^-} \) for simplicity, which is given by

\[
B_{l^+l^-} = \frac{M_{Z'}}{6\pi\Gamma_{Z'}g_l^2}.
\]

(2.13)

Figure 2.3 shows the resulting \( \sigma B_{l^+l^-} \) curves, as a function of \( M_{Z'} \) for values of \( g_l \) in the range \([0.025, 1]\) compared to the 95% confidence level (CL) upper limit arising from the ATLAS search for dilepton resonances \cite{97}. Clearly, as \( g_l \) is increased, the lower limit on \( M_{Z'} \) also increases, as one would expect. It is found that, for \( g_l = 0.025 \),
which is equal to \( g_l/10 \) in this analysis, the bound is \( M_{Z'} \gtrsim 1.6 \text{ TeV} \). In the following subsection we will confirm that this is in line with dijet constraints and that, for the same choice of couplings, a slightly weaker bound is obtained. It is noted that for larger values of \( g_l \), the bound on \( M_{Z'} \) increases significantly. For example, with \( g_l = 0.1 \) one arrives at \( M_{Z'} \gtrsim 2.4 \text{ TeV} \).

Finally, it has been explicitly verified that varying the DM mass has a small effect on the parameter space constraints. For \( g_l = 0.025 \), the bound on \( M_{Z'} \) increases by roughly 200 GeV as \( m_\chi \) is increased from 10 GeV up to the production threshold, \( M_{Z'}/2 \). It is noted, when the value of \( g_l \) is larger, the corresponding increase in the minimum value of \( M_{Z'} \) is even less significant.

### 2.3.2 Dijet Constraints

In this subsection, we study the restrictions provided by dijet searches performed at the LHC. This process is illustrated at LO by the Feynman diagram in Figure 2.4.
Here, the value of \( g_q \) is varied, while \( g_\chi \) is again set to 1 and \( g_l \) is set to 0, justified by the earlier calculation which showed that in order for \( M_{Z'} \) to not be too tightly constrained \( g_l \) must be very small. Furthermore, in the calculations of \( \sigma B_{2j} \) for dijet events, \( g_l \) enters only in the evaluation of the total width\(^2\). To illustrate that this simplification is indeed appropriate, one finds that for \( g_q = 0.25 \) and \( M_{Z'} = 2 \) TeV, setting \( g_l = 0 \) rather than \( g_l = 0.025 \) underestimates the width by less than 0.2%.

The cross section for \( Z' \) production is calculated in the same way. In evaluating the branching ratio for dijet production, to match the experimental analysis top quarks are not included in the partial width. One obtains

\[
B_{2j} = \frac{M_{Z'} g_q^2}{4\pi \Gamma_{Z'}} \left[ 3 + \left( 1 - \frac{4m_c^2}{M_{Z'}^2} \right)^{1/2} \left( 1 + \frac{2m_c^2}{M_{Z'}^2} \right) \right],
\]

(2.14)

As before, the masses of all light fermions are set to zero, and the DM mass and coupling are chosen to be \( m_\chi = 10 \) GeV and \( g_\chi = 1 \).

Figure 2.5 shows the resulting \( \sigma B_{2j} \) curves, as a function of \( M_{Z'} \), for values of \( g_q \) in the range \([0.1, 1]\), compared to the 95% CL upper limit on dijet production data obtained by the ATLAS experiment \( ^{98} \). Again it is observed that as \( g_q \) is increased, the lower limit on \( M_{Z'} \) increases, however for \( g_q = 0.1 \) there is in fact no constraint from dijet searches. That value is therefore discarded at this stage. For \( g_q = 0.25 \),

\(^2\)It is noted that \( \sigma(pp \rightarrow Z') \) is dependent on \( g_q^2 \). Thus, the full dijet production process scales with \( g_q^4 \) compared to \( g_q^2 g_l^2 \) for the dilepton case.
values of $M_{Z'}$ above 1.5 TeV are allowed, which is a slightly weaker bound than that obtained from dilepton searches for essentially the same choice of parameters. Note that with this choice for the coupling, dijet searches also suggest that a light mediator with $M_{Z'} \lesssim 760$ GeV is allowed. It is again found that for large values of $g_q$, the lower limit on $M_{Z'}$ is significantly higher. Finally, this bound is even more weakly dependent on $m_\chi$ than in the dilepton case.

### 2.3.3 Analysis of Results

Table 2.1 summarises the lower bounds on $M_{Z'}$ arising from dilepton and dijet searches respectively, for various values of $g_l$ and $g_q$. The data shows that constraints from the two types of searches are complimentary. As expected, if one wishes to have a weaker lower limit on $M_{Z'}$ then dilepton (dijet) searches suggest that $g_l$ ($g_q$) should be made smaller. It is also noted that for the choice $g_l = 0.025$, $g_q = 0.25$ the bound on $M_{Z'}$ from dilepton searches is stronger.
Finally, the branching ratios for dilepton, dijet and DM pair production are compared. For $M_{Z'} = 2$ TeV, which satisfies dilepton constraints, all final states are kinematically accessible and terms proportional to $m_f^2/M_{Z'}^2$ in the widths are negligible. The total width is then well approximated by (2.3), and the branching ratios by

$$B_{l^+l^-} \approx \frac{M_{Z'}}{6\pi\Gamma_{Z'}} g_l^2, \quad B_{2j} \approx \frac{5M_{Z'}}{4\pi\Gamma_{Z'}} g_q^2, \quad B_{\bar{\chi}\chi} \approx \frac{M_{Z'}}{12\pi\Gamma_{Z'}} g_\chi^2.$$  (2.15)

For the now well-motivated choice of $g_l = 0.025$, $g_q = 0.25$ and $g_\chi = 1$, with a DM mass of $m_\chi = 10$ GeV, we find that $B_{l^+l^-} \approx 5.9 \times 10^{-4}$, $B_{2j} \approx 0.44$ and $B_{\bar{\chi}\chi} \approx 0.47$. Decays to DM and 2 jets therefore dominate the total width in roughly equal proportion, while dilepton production is extremely rare. Decays to top quarks make up roughly 9% of decays and contributions from $\tau$ and $\nu$ final states are negligible. The implication is that searches for DM pair production are promising strategies to study the properties of DM-SM interactions for the above choice of parameters.

## 2.4 Monojet Searches

For the DM decay mode of the $Z'$, the collider signature will be a large amount of missing transverse energy, $E_T$, corresponding to the DM pair which has escaped the detector, in association with hadronic jets. This section focuses on $E_T + j$ or ‘monojet’ events. A typical diagram corresponding to such a process at LO is shown in Figure 2.6 In order to simulate monojet events at the LHC, the POWHEG BOX is utilised [99][101]. An extension of this framework has been developed to calculate the
cross-sections of monojet events at NLO while also including parton showering (PS) effects employing \textsc{Pythia 6.4} \cite{102}. This allows for the implementation of realistic cuts, and thus for a close comparison to be made between simulated and experimental data \cite{103}.

Results are obtained using the MSTW2008 LO and NLO parton distribution functions (PDFs) \cite{104}, where appropriate. The scale factor, which determines the value of the strong coupling \(\alpha_s(\mu)\) dynamically, is chosen to be \(\mu_F = \mu_R = \xi H_T/2\) where

\[
H_T = \sqrt{m_{\tilde{\chi}\tilde{\chi}}^2 + p_{T,j_1}^2 + p_{T,j_2}}.
\] (2.16)

In this expression, \(m_{\tilde{\chi}\tilde{\chi}}\) is the invariant mass of the DM pair and \(p_{T,j_1}\) is the transverse momentum of the primary jet. The scale is calculated on an event-by-event basis for selected values of the parameter \(\xi\). This facilitates an analysis of the scale uncertainties of the results.

### 2.4.1 Event Selection

In order to correctly identify signal events, various cuts need to be applied in the analysis. These are taken from the latest CMS monojet search \cite{105}. The highest transverse momentum jet \((j_1)\) is required to have \(p_{T,j_1} > 110\) GeV and pseudorapidity \(|\eta_{j_1}| < 2.4\). A second jet \((j_2)\) is permitted, which can for instance arise from initial state radiation (i.e. from the ingoing quark or antiquark in Figure 2.6), with \(p_{T,j_2} > 30\) GeV, \(|\eta_{j_2}| < 4.5\) and an azimuthal separation from the leading jet of
<table>
<thead>
<tr>
<th>Source</th>
<th>Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z(\nu\bar{\nu})+\text{jets}$</td>
<td>$1460 \pm 140$</td>
</tr>
<tr>
<td>$W+\text{jets}$</td>
<td>$501 \pm 36$</td>
</tr>
<tr>
<td>$tt$</td>
<td>$15 \pm 7.7$</td>
</tr>
<tr>
<td>$Z(l^+l^-)+\text{jets}$</td>
<td>$5.2 \pm 2.6$</td>
</tr>
<tr>
<td>Single $t$</td>
<td>$0.9 \pm 0.4$</td>
</tr>
<tr>
<td>QCD multijets</td>
<td>$2.0 \pm 1.2$</td>
</tr>
<tr>
<td>Diboson</td>
<td>$65 \pm 33$</td>
</tr>
<tr>
<td>Total SM</td>
<td>$2050 \pm 150$</td>
</tr>
<tr>
<td>Data</td>
<td>$1830$</td>
</tr>
<tr>
<td>Exp. upper limit</td>
<td>$266$</td>
</tr>
<tr>
<td>Obs. upper limit</td>
<td>$154$</td>
</tr>
</tbody>
</table>

Table 2.2: SM background predictions of the number of events which pass the selection criteria, with an applied cut of $E_T > 450$ GeV, compared to the observed numbers of events. Uncertainties include both statistical and systematic components. The last two rows show the expected and observed upper limits for the contribution of non-SM events passing the cuts, at 95% CL. These data are taken from [105].

$\Delta \phi_{j_1,j_2} < 2.5$ radians. The latter requirement reduces the QCD dijet background. A jet veto is also applied, such that events containing a tertiary jet with $p_{T,j_3} > 30$ GeV and $|\eta_{j_3}| < 4.5$ are rejected. Jets are reconstructed using the FastJet [106] implementation of the anti-$k_t$ clustering algorithm [107] with a radius parameter of $R = 0.5$.

Table 2.2 show predictions for the SM backgrounds. $Z$- and $W$-boson production dominate, where these decay to $\nu\bar{\nu}$ and $l\nu$, respectively. In order to lead to $E_T$ events and pass the selection criteria above, the charged lepton produced in $W$-boson decays must be lost. Diboson production is also significant, where one boson decays to hadronic jets and the other to leptons. To reduce these backgrounds, events with well reconstructed isolated electrons and muons with $p_T > 10$ GeV and taus with $p_T > 20$ GeV can also be rejected.

2.4.2 Results

The yellow contour in Figure 2.7 shows the region in the $M_{Z'}-m_\chi$ plane which is excluded at the 95% CL by the latest monojet results [105]. This search is performed
Figure 2.7: The yellow contour and shading indicates the region in the $M_{Z'}-m_\chi$ plane for a pure vector mediator which is excluded by the latest CMS monojet search \cite{105}, at 95% CL. The green contour and shading represents the region excluded by the LUX direct detection results \cite{71}. The pink dashed contour and shading indicates the region excluded by relic density constraints. The black dashed line corresponds to $M_{Z'} = 2m_\chi$. Above (below) this line, DM is produced off-shell (on-shell). The values of the couplings have been set to $g_\chi = 1$, $g_q = 0.25$ and $g_l = 0$, motivated by the work in Section 2.3.

for seven different cuts on the minimum value of $E_T$. For the case under consideration here, the highest sensitivity is obtained for $E_T > 450$ GeV. The corresponding 95% CL limit on the fiducial cross section is \cite{105}

$$\sigma_{\text{fid}}(pp \rightarrow E_T + j) < 7.8 \text{ fb}. \quad (2.17)$$

The results are obtained at NLO, including PS effects, using the full theory for a pure vector mediator with the couplings set to $g_\chi = 1$, $g_q = 0.25$ and $g_l = 0$. The scale parameter is set to $\xi = 2$ such that the obtained bound is a conservative one. Firstly, it is observed that for light DM values of $M_{Z'} \lesssim 1$ TeV are excluded in the case that DM is produced on-shell. Secondly, we see that constraints from the CMS search are very weak in the case that DM is produced off-shell. Very similar results would be
obtained for an axial vector mediator.

Also shown on Figure 2.7 are the direct detection and relic density constraints for the vector mediator. In this case the DM-nucleon scattering is SI and the cross section at zero momentum transfer is given by

$$\sigma_{N,SI} = \frac{\mu^2_{\chi N}}{\pi M^2_{Z'}} f^2_N,$$  \hspace{1cm} (2.18)

where $N$ denotes either a proton ($p$) or a neutron ($n$), and the effective couplings have the form

$$f_p = g_\chi (2g_u + g_d), \quad f_n = g_\chi (g_u + 2g_d),$$  \hspace{1cm} (2.19)

which are equal for the case under consideration where the quark couplings are universal. Thus we have

$$\sigma_{N,SI} \approx 1.1 \times 10^{-39} \text{ cm}^2 \left( \frac{\mu_{\chi N}}{\text{GeV}} \right)^2 \left( \frac{1 \text{ TeV}}{M_{Z'}} \right)^4.$$  \hspace{1cm} (2.20)

Requiring that the limits placed by LUX on the spin independent cross section [71] are satisfied yields the green excluding curve and shaded region displayed on Figure 2.7.

In Appendix C it is shown that the relic density can be expressed as

$$\Omega_\chi h^2 \approx \frac{1.07 \times 10^9}{\text{GeV}} \frac{x_f}{M_{Pl} \sqrt{g_*}} \left( a + \frac{3b}{x_f} \right),$$  \hspace{1cm} (2.21)

where $a$ and $b$ are the coefficients in the expansion $\sigma v_\chi = a + bv_\chi^2 + \mathcal{O}(v_\chi^4)$, and the other variables are defined in the appendix. For a vector mediator, the annihilation cross section to SM quarks is given by [108]

$$\langle \sigma v_\chi \rangle (\bar{\chi} \chi \rightarrow Z' \rightarrow \bar{q}q) = \frac{2m^2_\chi (g_\chi g_q)^2}{2\pi((M^2_{Z'} - 4m^2_\chi)^2 + \Gamma^2_{Z'} M^2_{Z'})} \times \left( 1 - \frac{4m^2_\chi}{M^2_{Z'}} \right)^{1/2} \left( 2 + \frac{m^2_\chi}{M^2_{Z'}} \right)$$  \hspace{1cm} (2.22)

to leading order in the DM velocity $v_\chi$. This expression is used to replace the coefficient $a$ in (2.21), while setting $b = 0$, and the pink exclusion curve on Figure 2.7 is
Figure 2.8: Differential cross sections for $E_T + j$ events as a function of the amount of missing transverse energy for interactions mediated by a pure vector (left) and axial vector (right), at LO (blue) and NLO (red). The central values are indicated by the solid lines, and the scale uncertainties are represented by the shaded regions. The values of the couplings are $g_\chi = 1$, $g_q = 0.25$ and $g_l = 0$, the mediator mass is $M_{Z'} = 1$ TeV, and the DM mass is $m_\chi = 10$ GeV.

obtained by requiring that the observed relic density $\Omega_\chi h^2 \approx 0.119^{24}$ is not oversaturated. We observe that the parameter space of this model is very tightly bounded when one combines these constraints. While the full analysis is not performed here for the axial vector case it is noted that, while monojet searches place similar restrictions on the parameter space, direct detection constraints are considerably weaker. This is because the axial vector coupling is spin-dependent and therefore does not receive the coherent enhancement due to the size of the target nucleus. A detailed discussion of direct detection constraints on simplified models involving a spin-1 $s$-channel mediator can be found in $^{109}$.

Next, the missing transverse energy distributions are considered. Figure 2.8 shows the differential monojet cross sections as a function of $E_T$, due to interactions mediated by either a pure vector (left) or axial vector (right), at both LO and NLO. The couplings have the same values as used above, and the mediator and DM masses are set to $M_{Z'} = 1$ TeV and $m_\chi = 10$ GeV, respectively. The theoretical uncertainties,
indicated by the shaded bands, are assessed by varying the value of the scale parameter $\xi$ in the range $[1/2, 2]$. The $K$-factor, which is a measure of the enhancement of the cross section due to the inclusion of NLO effect, is defined as follows

$$K = \frac{\sigma(pp \rightarrow E_T + j)_{\xi=[1/2,2]}^{\text{NLO}}}{\sigma(pp \rightarrow E_T + j)_{\xi=1}^{\text{LO}}}.$$  

(2.23)  

It is found for both the vector and axial vector cases that $K \approx 1.2 \pm 0.1$ across the full $E_T$ range; the relative difference between the NLO and LO results is rather independent of the amount of missing transverse energy. It is noted that PS effects have not been included in these results, however it has been verified that they would not change the results dramatically.

Most importantly for this work, it is clear that the vector and axial vector results are essentially indistinguishable. Consequently, it is impossible to determine the Lorentz structure of the interactions by studying data from $E_T + j$ events. While monojet searches are useful for constraining the size of the interactions, if one wishes to understand more deeply the structure of interactions between SM and DM particles, it is necessary to examine more complex final states. This is the focus of the next two chapters.
Chapter 3

Probing the Structure of Dark Matter Couplings

This chapter is based on [110], original work done in collaboration with Ulrich Haisch and Emanuele Re.

The last chapter has shown that while monojet searches can be used to derive bounds on the size of the couplings between DM and the SM, they do not yield much information about the structure of these couplings. More specifically, we have seen that the $E_T$ spectra for vector and axial vector mediators are essentially featureless and indistinguishable, within theoretical uncertainties. The same problem occurs when comparing scalar and pseudoscalar mediators. Furthermore, there are additional parameters of the simplified model that one would like to probe more deeply; the DM mass is perhaps the most important of these.

The implication is that, in order to learn more useful information about a simplified model of DM, one must go beyond monojet searches. This chapter studies a simplified model of fermionic DM which has interactions with the SM mediated by a scalar or pseudoscalar mediator. Searches for two hadronic jets plus some missing transverse energy ($E_T + 2j$) are considered, and simulated results are compared for the two mediators.
3.1 Model Description

In analogy with Section 2.1 a Dirac fermion $\chi$, which represents the DM candidate, is introduced along with a spin-0 $s$-channel scalar ($S$) or pseudoscalar ($P$) mediator. The Lagrangians for the new interactions are then given by

$$\mathcal{L}_S = g_S^\chi (\bar{\chi}\chi)S + \sum_{f=q,\ell} g_S^f \frac{m_f}{v} (\bar{f}f)S,$$

$$\mathcal{L}_P = ig_P^\chi (\bar{\chi}\gamma_5\chi)P + \sum_{f=q,\ell} ig_P^f \frac{m_f}{v} (\bar{f}\gamma_5f)P,$$

where $g_S^\chi$ and $g_P^\chi$ are the scalar/pseudoscalar couplings of the DM particle $\chi$ and SM fermion $f$ to the mediator, and the sum runs over all of the SM quarks and charged leptons. The proportionality of the SM interactions to the fermion mass renders the neutrino interactions negligibly small, and hence they are ignored in (3.1) and (3.2). The presence of the fermion masses is a consequence of again imposing MFV, the motivations for which were discussed in Section 2.2. In this case, the spin-0 mediator mixes with the SM Higgs leading to the Yukawa interactions in the above Lagrangians.

To see more clearly why this occurs, recall that MFV dictates that interactions between quark fields and the new mediator should either be invariant under $G_q$, or break it only via the Yukawa matrices $Y^u$ and $Y^d$. Noting that the quark field transformation properties under $SU(3)_q \otimes SU(3)_u \otimes SU(3)_d$ are

$$q \sim (3,1,1), \quad u \sim (1,3,1), \quad d \sim (1,1,3),$$

the bilinears $\bar{q}u$ and $\bar{q}d$ break $SU(3)_q \otimes SU(3)_u$ and $SU(3)_q \otimes SU(3)_d$, respectively. This means the only choice is to introduce the Yukawa matrices such that, in the gauge eigenstate bases, we have

$$\mathcal{L} \supset - \sum_{i,j} \left( (Y^u)_{ij} \bar{q}_i H u_j + (Y^d)_{ij} \bar{q}_i \tilde{H} d_j + \text{h.c.} \right).$$
Here, the sum runs over the three quark generations, the presence of the SM Higgs doublet, $H$, is required to ensure \( SU(2)_L \otimes U(1)_Y \) gauge invariance, and $\tilde{H}_a = \epsilon_{ab} H_b$ with $a, b = 1, 2$. If the Yukawa matrices are promoted to be non-dynamical fields with the transformation properties

$$Y^u \sim (3, \bar{3}, 1), \quad Y^d \sim (3, 1, \bar{3}),$$

then the interactions \([3.4]\) are invariant under \( G_q \). Rewriting these terms in the mass eigenstate basis gives

$$\mathcal{L} \supset - \sum_i \left( \frac{m^u_i}{v} \bar{u}_i u_i + \frac{m^d_i}{v} \bar{d}_i d_i \right) h.$$  

This derivation shows that in MFV simplified models with a new spin-0 $s$-channel mediator the couplings between the mediator and SM fermions are necessarily of Yukawa type.

The factors $g^{S,P}_f$ which scale these Yukawa couplings are permitted in the MFV framework to be different for up-type quarks, down-type quarks and leptons, and are the same across the three generations in each case. This reduces the set of couplings on the right-hand sides of \([3.1]\) and \([3.2]\) to \( \{ g^S_u, g^S_d, g^S_l \} \). Also, it is noted that the DM mass is unlikely to be generated by EWSB and rather by some unknown mechanism(s), so its couplings to the mediators are simply parametrised by $g^S_\chi$. Finally, in order to avoid the severe experimental bounds from electric dipole moments (EDMs) (see Appendix D), the spin-0 mediators $S, P$ are taken to be CP eigenstates and it is additionally assumed that all couplings are real.

It is clearly necessary to consider the possible decays of the mediators. For the scalar case the total width is given by

$$\Gamma_S = \sum_{f = u, d} N^f_c (g^S_f)^2 m^2_f M_S \left( 1 - \frac{4m^2_f}{M_S^2} \right)^{3/2} + \frac{(g^S_\chi)^2 M_S}{8\pi} \left( 1 - \frac{4m^2_\chi}{M_S^2} \right)^{3/2} + \frac{\alpha^2 (g^S_\chi)^2 M_S^2}{32\pi^3 v^2} \left| F_S \left( \frac{4m^2_\chi}{M_S^2} \right) \right|^2,$$  

\(3.7\)
with the replacements $3/2 \rightarrow 1/2$ in the exponents and $S \rightarrow P$ for the pseudoscalar case. $N_f^f$ is the number of colours associated with the fermion $f$ (3 for quarks and 1 for leptons), $\alpha_s$ is the QCD coupling, and

$$F_S(\tau) = \tau \left[ 1 + (1 - \tau) \arctan^2 \left( \frac{1}{\sqrt{\tau - 1}} \right) \right], \quad F_P(\tau) = \tau \arctan^2 \left( \frac{1}{\sqrt{\tau - 1}} \right). \quad (3.8)$$

The first two terms in (3.7) correspond to decays to SM fermions and DM respectively, provided these states are kinematically accessible. The final term corresponds to decays to gluons via top quark loops. Contributions from other quarks at loop level are ignored since their Yukawa couplings are much smaller than that of the top quark. Decays to pairs of photons and other kinematically accessible SM particles also arise, but their partial widths are smaller than $\Gamma_{S,P \rightarrow gg}$ and these contributions are therefore also neglected.

To examine more closely the individual decay channels, we compare the branching ratios to $t\bar{t}$ and $b\bar{b}$, assuming that top quark pair production is kinematically available,

$$\frac{\mathcal{B}(S \rightarrow t\bar{t})}{\mathcal{B}(S \rightarrow b\bar{b})} = \left( \frac{g_u^S}{g_d^S} \right)^2 \left( \frac{m_t}{m_b} \right)^2 \left[ \frac{1 - 4m_t^2/M_S^2}{1 - 4m_b^2/M_S^2} \right]^{3/2}. \quad (3.9)$$

Clearly, due to the presence of the quark masses, decays to top quarks will dominate over the other SM fermions unless $M_S < 2m_t$ or $g_d^S \gg g_u^S$. The latter scenario can for instance be realised in a two-Higgs-doublet model, however such extensions to the SM are not considered in this work.

The branching ratios for $t\bar{t}$, $b\bar{b}$, $\chi\bar{\chi}$ and $gg$ are shown in Figure 3.1 for a DM mass of $m_\chi = 50$ GeV and with all couplings set to 1. The remaining SM fermions are ignored since their branching ratios will be even smaller than that for $b\bar{b}$. We see that above the threshold for DM pair production this channel dominates for $M_S \lesssim 700$ GeV, beyond which the $t\bar{t}$ branching ratio becomes larger. It is noted, however, that even for large values of $M_S$ the $\chi\bar{\chi}$ branching ratio is still significant: with $M_S = 1$ TeV we find that $\mathcal{B}(S \rightarrow \chi\bar{\chi}) \approx 0.44$. This highlights that missing energy searches are a well motivated approach to studying this model. For $M_S > 2m_\chi$, the $\chi\bar{\chi}$ and $t\bar{t}$
Figure 3.1: Branching ratios for decays of the scalar mediator $S$ to $t\bar{t}$ (orange), $b\bar{b}$ (blue), $\chi\bar{\chi}$ (purple) and $gg$ (yellow). All couplings are set to 1 and $m_\chi = 50$ GeV. See text for further details.

channels form by far the largest contribution to the total width, meaning that decays to lighter quarks, leptons and gluons can be safely neglected.

All SM fermions except the top quark have very small Yukawa couplings, explaining the above observations. Furthermore, this means that adjusting the size of the coupling constants has a small effect on the branching ratios. Varying $g_q$, $g_l$ and $g_\chi$ between 0.1 and 10 in turn, it is found that the combined $t\bar{t}$ and $\chi\bar{\chi}$ channels comprise the dominant contributions to the total width for all values of $M_S$, away from the $\chi\bar{\chi}$ production threshold.

In conclusion, it is clear that the relevant interactions to consider in this model are those of the mediator to the top quark and to DM. If the mediator mass is large compared to the invariant mass of the DM pair, it can be integrated out to generate the effective operators

\[ O_S = \frac{m_t}{\Lambda_S^2} \bar{t}t\bar{\chi}\chi, \quad O_P = \frac{m_t}{\Lambda_P^2} \bar{t}\gamma_5 t\bar{\gamma}_5\gamma_5\chi, \]  

(3.10)

at tree level, as well as contact terms consisting of four DM or top quark fields, which
Figure 3.2: Typical diagrams leading to $E_T + 2j$ events in proton collisions. The black squares indicate insertions of the operators $O_{S,P}$ given in (3.10) are not considered further. In terms of fundamental parameters, the suppression scales are then given by

$$\Lambda_S = \left(\frac{vM_S^2}{g_S^2 g_t^2}\right)^{1/3},$$

with $S \rightarrow P$ for the pseudoscalar case. Examples of diagrams which give rise to the $E_T + 2j$ signal are shown in Figure 3.2.

Data from $E_T + j$ [111] and $E_T + tt$ [112] searches constrains the suppression scale to be larger than approximately 150 GeV (170 GeV) in the scalar (pseudoscalar) case for light DM. These values are small compared to typical LHC energies, and motivate a study of the validity of the EFT approach, which is presented in Subsection 3.5.3.

At this point a discussion of the direct and indirect detection constraints on this model is warranted. Scattering of DM on nuclei via a scalar mediator is SI and

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1Bounds have been calculated using more recent data [113] but, within scale uncertainties, they are unchanged compared to the values given here.
Figure 3.3: A contour plot of the 95% CL upper limits on $g \chi g_q$ from the LUX \cite{71} and CDMS-lite \cite{117} direct detection searches, as a function of the mediator mass $M_S$ and DM mass $m_\chi$. For further details see \cite{116}.

therefore strong bounds can be obtained from direct searches. In this case, the DM-nucleon scattering cross section at zero momentum transfer is given by

$$\sigma_{N,SI} = \frac{\mu_{\chi N} m_N^2}{\pi} \left( \frac{g_\chi g_q}{v M_S^2} \right)^2 f_N^2,$$

(3.12)

where, as before, $N = n, p$, $\mu_{\chi N}$ is the DM-nucleon reduced mass, $m_N$ is the nucleon mass. The form factor is

$$f_N = \sum_{q=u,d,s} f_N^q + \frac{2}{27} f_N^{TG} \sum_{Q=c,b,t} 1,$$

(3.13)

where $f_N^{TG} = 1 - \sum_{q=u,d,s} f_N^q$ and \cite{114,115}

$$f_p^u = (20.8 \pm 1.5) \times 10^{-3}, \quad f_d^u = (41.1 \pm 2.8) \times 10^{-3},$$

$$f_n^u = (18.9 \pm 1.4) \times 10^{-3}, \quad f_d^u = (45.1 \pm 2.7) \times 10^{-3},$$

(3.14)

$$f_N^s = 0.043 \pm 0.011.$$

The plot in Figure 3.3 shows the direct detection upper bounds on the combination $g_\chi g_q$ as a function of $M_S$ and $m_\chi$ \cite{116}.
In the case of a pseudoscalar mediator, the DM-nucleon scattering is SD and velocity suppressed, meaning that direct detection bounds are very weak. On the other hand, bounds can be obtained from indirect searches since the annihilation cross section in this case is not velocity suppressed. For SM fermions in the final state, to leading order in the DM velocity $v_\chi$ we have

$$\langle \sigma v_\chi \rangle(\chi \bar{\chi} \to P \to f \bar{f}) = N_c f \frac{(g_P^\chi g_P^f)^2 (m_\chi m_f)^2}{2\pi [(M_P^2 - 4m_\chi^2)^2 + M_P^2 \Gamma_P^2]} \left(1 - \frac{m_f^2}{m_\chi^2}\right) . \quad (3.15)$$

The plot in Figure 3.4 shows the indirect detection upper bounds on the combination $g_\chi g_q$ as a function of $M_P$ and $m_\chi$. [116]

Relic density constraints on this model are considered in Subsection 3.5.3

### 3.2 Kinematic Study

This section motivates the study of events containing 2 hadronic jets in the final state as a means to probe the Lorentz structure of the SM-DM couplings. This work also highlights event selection criteria which can be applied in the analysis in order
Figure 3.5: Example of a VBF-like diagram corresponding to production of a scalar (S) or pseudoscalar (P) in association with 2 jets in proton-proton collisions. The shaded circle indicates insertion of $\mathcal{O}_G$ ($\tilde{\mathcal{O}}_G$) in the scalar (pseudoscalar) case, where these operators are given in (3.16).

to enhance the distinction between the $\mathcal{O}_S$, $\mathcal{O}_P$ and SM predictions. The following calculation is greatly simplified by taking the heavy top quark limit, $m_t \to \infty$. The impact of this approximation is the topic of discussion in Subsection 3.5.1. The top quark loops are then integrated out, and their effect is described by the following operators

$$\mathcal{O}_G = \frac{\alpha_s}{12\pi \Lambda_S^3} G^a_{\mu\nu} G^{a,\mu\nu} \bar{\chi} \chi, \quad \mathcal{O}_{\tilde{G}} = \frac{\alpha_s}{8\pi \Lambda_P^3} \tilde{G}^a_{\mu\nu} \tilde{G}^{a,\mu\nu} \bar{\chi} \gamma_5 \chi, \quad (3.16)$$

where $G^a_{\mu\nu}$ is the gluon field strength tensor and $\tilde{G}^{a,\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma}$ is its dual.

To study the kinematics of $E_T + 2j$ events, it is particularly insightful to consider diagrams such as the one displayed in Figure 3.5, which bear a close resemblance to Higgs boson production via vector boson fusion (VBF). By analysing this process for the case of a scalar and pseudoscalar Higgs boson, it was found in [118] that the differential cross section is related to the azimuthal angular separation of the two jets, $\Delta \phi_{j_1j_2}$, by

$$d\sigma_{S/P} \sim F_1 \pm |F^-_g| \cos 2\Delta \phi_{j_1j_2}, \quad (3.17)$$

where the $+$ ($-$) sign is for production of a scalar (pseudoscalar). The coefficient functions $F_1$ are dependent on the whether the incoming partons are quarks or gluons, while the function $F^-_g$ is not. Up to a prefactor, which is the same for all coefficient
functions, we have

\[ F_1[qg] \propto \left( 1 + \sin^2 \frac{\theta_{13}}{2} \right) \left( 1 + \sin^2 \frac{\theta_{24}}{2} \right), \quad (3.18) \]

\[ F_1[qg] \propto \frac{1}{\cos^2 \frac{\theta_{13}}{2}} \left( 1 + \sin^2 \frac{\theta_{13}}{2} \right) \left( 1 + 3 \sin^2 \frac{\theta_{24}}{2} \right)^2, \quad (3.19) \]

\[ F_1[qg] \propto \frac{1}{\cos^2 \frac{\theta_{13}}{2} \cos^2 \frac{\theta_{24}}{2}} \left( 1 + 3 \sin^2 \frac{\theta_{13}}{2} \right)^2 \left( 1 + 3 \sin^2 \frac{\theta_{24}}{2} \right)^2, \quad (3.20) \]

\[ F_9^- \propto \left( 1 - \sin^2 \frac{\theta_{13}}{2} \right) \left( 1 - \sin^2 \frac{\theta_{24}}{2} \right), \quad (3.21) \]

with analogous expressions for \( q \to \bar{q} \) and the angle indices swapped for \( qg \to gq \).

The \( \theta_{ij} \) are the polar angles which the outgoing parton \( j \) makes with the trajectory of the incoming parton \( i \). For example, parton \( i \) continues along the same path as parton \( j \) if \( \theta_{ij} = 0 \), while they are in opposite directions if \( \theta_{ij} = \pi \).

The second term in (3.17) is proportional to the azimuthal angular difference between the two tagging jets. The difference in the sign of this term between the scalar and pseudoscalar case will lead to contrasting distributions for the differential cross section as a function of \( \Delta \phi_{j_1j_2} \). In the scalar (pseudoscalar) case a cosine-like (sine-like) spectrum is expected.

Evidently, the relative size of the coefficient functions is important: the \( \Delta \phi_{j_1j_2} \) dependence will be difficult to observe if \( F_9^- \ll F_1 \). It is therefore desirable that the event selection criteria are chosen such that \( F_9^- \) is enhanced relative to \( F_1 \). Examining the dependence of these functions on the polar angles, it is found that \( F_9^- \) is maximised and \( F_1 \) minimised for \( \theta_{ij} \to 0 \). In this case, we have that \( F_9^- \to 1 \) and \( F_1 \to 1 \), such that \( d\sigma_{S/P} \sim 1 \pm \cos 2\Delta \phi_{j_1j_2} \).

Physically, small values of \( \theta_{ij} \) correspond to the situation in which the external partons experience a very small energy loss, and the jets are produced almost back-
Figure 3.6: *Top panel:* plots of the polar angle dependence of $F_1[qq]$ (orange), $F_1[qg]$ (blue), $F_1[gg]$ (yellow), and $F_9^-$ (purple). The expressions are given in (3.18) to (3.21) although for this plot we have set $\theta_{13} = \theta_{24} = \Theta$ for simplicity. *Bottom panel:* plots of the ratio of the $F_1$ functions to $F_9^-$. The pseudorapidity is given on the top axis, and the red shaded region is excluded by the $|\eta_{j_1}| < 2.4$ cut.

to-back along the collision axis. The pseudorapidity is related to the angle $\Theta$ between the collision axis and an emitted jet by

$$\eta = -\ln \left[ \tan \left( \frac{\Theta}{2} \right) \right], \quad (3.22)$$

and therefore small values of $\theta_{ij}$ are equivalent to the jets having a large $|\eta|$. In the event selection, however, the jet with the highest transverse momentum is required to have $|\eta_{j_1}| < 2.4$. Despite this restriction the distributions should be clearly observed;

\(^3\)The use of $\Theta$ in (3.22) is deliberate: defining the collision axis to be in the $z$-direction, $\Theta$ is measured relative to the $z$-axis. Since the incoming partons are travelling in opposite directions, one of the angles $\theta_{ij}$ is measured from the $z$-axis and the other from the $-z$-axis. Ultimately, we are only concerned here with the magnitude of $\eta$ and not its sign.
$|\eta| = 2.4$ corresponds to a polar angle of roughly $10^\circ$, and setting $\theta_{13} = \theta_{24} = 10^\circ$ in (3.18) to (3.21) gives $F_1[qg] \approx 1.0$, $F_1[gg] \approx 1.1$, $F_1[gg] \approx 1.1$ and $F_9^- \approx 1.0$. Hence the two terms in (3.17) are of comparable size. Figure 3.6 shows how the values of the functions in (3.18)–(3.21) vary with $\Theta = \theta_{13} = \theta_{24}$. It is observed that $F_1[gg]$, which is the most rapidly increasing of the $F_1$ functions, is less than an order of magnitude larger than $F_9^-$ up to $\Theta \approx 48^\circ$ or $\eta \approx 0.8$. Consequently, the $\Delta \phi_{j_1 j_2}$ distribution should be clearly observable over a relatively wide range of allowed values of the jet pseudorapidities.

Next, the related effect of imposing a cut on the invariant mass of the dijet system, $m_{j_1 j_2}$, is examined. We follow the kinematic analysis performed in [118] of VBF processes similar to that shown in Figure 3.5. The setup is shown in Figure 3.7, where the angles $\theta_{ij}$ are the same as those used above. The invariant mass of the two jets is

$$m_{j_1 j_2}^2 = (k_3 + k_4)^2,$$  

(3.23)

where $k_3$ and $k_4$ are the 4-momenta of the outgoing partons. In order to evaluate the right-hand side of (3.23) it is necessary to bring $k_3$ and $k_4$ into the same inertial frame. To this end, they are first expressed in the $q_1$ and $q_2$ Breit frames, (I) and (II) (defined by $q_1' = \sqrt{-q_1 \cdot q_1} \equiv Q_1$ and $q_2' = -\sqrt{-q_2 \cdot q_2} \equiv -Q_2$, such that the internal gluons travel in opposite directions parallel to the $z'$-axis) and then Lorentz transformed into the rest frame of $S, P$ (III). In the $q_1$ Breit frame (I)

$$q_1 = k_1 - k_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ Q_1 \end{pmatrix}, \quad k_3 = \frac{Q_1}{2 \cos \theta_1} \begin{pmatrix} 1 \\ \sin \theta_1 \cos \phi_1 \\ \sin \theta_1 \sin \phi_1 \\ -\cos \theta_1 \end{pmatrix},$$

(3.24)

where $0 < \theta_1 < \pi/2$ and $0 < \phi_1 < 2\pi$. In the $q_2$ Breit frame (II)

$$q_2 = k_2 - k_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -Q_2 \end{pmatrix}, \quad k_4 = \frac{-Q_2}{2 \cos \theta_2} \begin{pmatrix} 1 \\ \sin \theta_2 \cos \phi_2 \\ \sin \theta_2 \sin \phi_2 \\ -\cos \theta_2 \end{pmatrix},$$

(3.25)
where $\pi/2 < \theta_2 < \pi$ and $0 < \phi_2 < 2\pi$. The Lorentz boost factors from the Breit frames to the $S, P$ rest frame are given by

$$\beta_1 = \left(1 - \frac{Q_1^2 - Q_2^2}{M^2}\right)/\beta \quad \text{and} \quad \beta_2 = -\left(1 - \frac{Q_2^2 - Q_1^2}{M^2}\right)/\beta,$$

(3.26)

where $\beta = \left[1 + 2\frac{Q_1^2 + Q_2^2}{M^2} + \frac{(Q_1^2 - Q_2^2)^2}{M^4}\right]^{1/2}$. 

(3.27)

Applying the Lorentz transformations along the $z'$-axis and, one obtains

$$k'_3 = \frac{M}{4} \begin{pmatrix}
\frac{\beta}{\cos\theta_1} - 1 + \frac{Q_1^2 - Q_2^2}{M^2} \\
\frac{2Q_1}{M} \tan\theta_1 \cos\phi_1 \\
\frac{2Q_1}{M} \tan\theta_1 \sin\phi_1 \\
-\beta + \frac{1}{\cos\theta_1} \left(1 - \frac{Q_1^2 - Q_2^2}{M^2}\right)
\end{pmatrix},$$

(3.28)

$$k'_4 = -\frac{M}{4} \begin{pmatrix}
\frac{\beta}{\cos\theta_2} + 1 - \frac{Q_1^2 - Q_2^2}{M^2} \\
\frac{2Q_2}{M} \tan\theta_2 \cos\phi_2 \\
\frac{2Q_2}{M} \tan\theta_2 \sin\phi_2 \\
-\beta - \frac{1}{\cos\theta_2} \left(1 - \frac{Q_1^2 - Q_2^2}{M^2}\right)
\end{pmatrix}.$$
Inserting these into (3.23) one finds
\[ m_{j_1j_2}^2 = \frac{M^2}{16} \left( \beta (\sec \theta_1 - \sec \theta_2) - 2 \right)^2 \]
\[ - \frac{M^2}{16} \left[ \left( 1 - \frac{Q_1^2 - Q_2^2}{M^2} \right) \sec \theta_1 + \left( 1 + \frac{Q_1^2 - Q_2^2}{M^2} \right) \sec \theta_2 \right]^2 \]
\[ - \frac{1}{4} (Q_1 \tan \theta_1 \cos \phi_1 - Q_2 \tan \theta_2 \cos \phi_2)^2 \]
\[ - \frac{1}{4} (Q_1 \tan \theta_1 \sin \phi_1 - Q_2 \tan \theta_2 \sin \phi_2)^2. \] (3.30)

In the case that \( M \gg Q_1, Q_2 \), the dijet invariant mass is maximised when \( \theta_1 \) and \( \theta_2 \) simultaneously go to \( \pi/2^- \) (from below) and \( \pi/2^+ \) (from above), respectively. This corresponds to \( \theta_{13}, \theta_{24} \to 0 \), i.e. what has been shown is that imposing a strict cut on \( m_{j_1j_2} \) forces the jets to be more back-to-back, along the collision axis.

In summary, it has firstly been observed that studying the azimuthal angle distributions of the tagging jets in \( E_T + 2j \) events will lead to distinct distributions depending on whether the mediator is a scalar or pseudoscalar. Secondly, we saw that these distributions will be more pronounced if the jets are travelling in opposite directions close to the collision axis. It was then found that this signal can be enhanced by applying a cut on the minimum value of \( m_{j_1j_2} \).

### 3.3 Event Selection

In this analysis proton-proton collisions with centre-of-mass energy \( \sqrt{s} = 14 \text{ TeV} \) are considered. Event selection criteria are used corresponding to CMS monojet searches at \( \sqrt{s} = 8 \text{ TeV} \) [119].

Events containing more than two jets with pseudorapidity \(|\eta| < 4.5\) and transverse momentum \( p_T > 30 \text{ GeV} \) are rejected. The azimuthal angular separation of the two tagging jets is required to be \( \Delta \phi_{j_1j_2} < 2.5 \) in order to reduce the background due to QCD dijet events. We also require that the leading jet fulfils \(|\eta_{j_1}| < 2.4\), \( p_{T,j_1} > 110 \text{ GeV} \) and only events with \( E_T > 350 \text{ GeV} \) are kept. The effect of imposing
strict restrictions on the minimum values of $p_{T,j}$ and $E_T$ are considered in the results. Finally, motivated by the work in the previous section, a cut is applied to the invariant mass of the two-jet system, $m_{j_1j_2} > 600$ GeV, in order to more clearly distinguish the scalar and pseudoscalar mediated $E_T^* + 2j$ events from each other, and from the SM background.

### 3.4 Results

The calculation is performed using the GGFLO sub-package of VBFNLO \cite{120,122} with appropriate modifications made to the $pp \rightarrow H/A + 2j$ process. This process is implemented in the package utilising the analytical LO results of \cite{123,124} in the scalar Higgs ($H$) case, and \cite{125} in the pseudoscalar Higgs ($A$) case. The MSTW2008LO PDF set is used \cite{104} and jets are reconstructed using the anti-$k_t$ jet clustering algorithm \cite{107} with $R = 0.5$, in accordance with the CMS analysis \cite{119}.

We employ the EFT approach with $\Lambda_{S,P} = 150$ GeV and $m_\chi = 50$ GeV, a choice that will be discussed and justified in Subsection 3.5.3. For these parameters and with the above selection criteria applied, the central values of the cross sections for the $E_T^* + j$ and $E_T^* + 2j$ signals in the scalar (pseudoscalar) case are 675 fb and 205 fb (1119 fb and 338 fb), respectively. The SM background predictions are 1289 fb and 330 fb. For perspective, the CMS analysis \cite{119} excludes excesses in the monojet cross section with signal-over-background ratios of $S/B \gtrsim 0.15$ at the 95\% CL. The values suggest that the monojet signal with $\Lambda_{S,P} = 150$ GeV and $m_\chi = 50$ GeV should be easily observable at the 14 TeV LHC.

The upper panel of Figure 3.8 shows the normalised differential cross section as a function of $\Delta \phi_{j_1j_2}$ for insertions of $O_S$ (red), $O_P$ (blue), and for the dominant SM background (green), due to the process $pp \rightarrow Z(\bar{\nu}\nu) + 2j$. This process is calculated at LO using the POWHEG BOX \cite{99,101} with the same event selection criteria applied. In the case of $O_S$ the distribution has a maximum and $\Delta \phi_{j_1j_2} = 0$ and a minimum at
Figure 3.8: Top panel: normalised $\Delta \phi_{j_1j_2}$ distributions for the insertion of $\mathcal{O}_S$ (red), $\mathcal{O}_P$ (blue), and the dominant SM background, $pp \rightarrow Z(\bar{\nu}\nu) + 2j$ (green). Here, the applied cuts include $E_T > 350$ GeV and $p_{T,j_1} > 110$ GeV. Bottom panel: ratio $R$ between the results obtained in the heavy top quark approximation and the full calculation.

$\Delta \phi_{j_1j_2} \approx \pi/2$, whereas for $\mathcal{O}_P$ the positions of the peak and trough are reversed. This is what was expected from the work in Section 3.2. The SM background, meanwhile, has a minimum at $\Delta \phi_{j_1j_2} = 0$ and a maximum around $\Delta \phi_{j_1j_2} = 2.5$.

Motivated by the work in [124] the choice of factorisation scale is $\mu_F = \xi \sqrt{p_{T,j_1}p_{T,j_2}}$, and the factor of $\alpha_s^4$ which enters the calculation of the cross sections is replaced with $\alpha_s(\xi p_{T,j_1})\alpha_s(\xi p_{T,j_2})\alpha_s^2(\xi m_{\tilde{\chi}})$. The scale uncertainties in the results are then assessed by varying $\xi$ in the range $[1/2, 2]$ for every event generated in the Monte Carlo simulation. The induced uncertainties are around $+80\% -40\%$ for the total cross sections and $+5\% -5\%$ for the normalised $\Delta \phi_{j_1j_2}$ distributions. The conclusion is that even after considering scale uncertainties the azimuthal angle distributions corresponding to $\mathcal{O}_S$ and $\mathcal{O}_P$ are still clearly distinguishable from each other, and from the dominant SM background.
3.5 Interpretation and Discussion

In Subsection 3.5.1 we examine the validity of the heavy top quark limit, demonstrating that our results are still valid after applying this simplification. Then, in Subsection 3.5.2 the impact of applying stricter cuts to the missing transverse energy and the transverse momentum of the leading jet is studied, followed by a discussion of corrections due to higher order QCD effects, parton showering and hadronisation. Finally, in Subsection 3.5.3 the validity of the EFT approach is considered, and results are obtained from a full theory calculation due to a simple ultraviolet (UV) completion described by the $s$-channel model given by (3.1) and (3.2).

3.5.1 Validity of Heavy Top Quark Limit

At this point a discussion of the validity of the heavy top quark limit is warranted. In the bottom panel of Figure 3.8 the dashed lines indicate the ratio between the results in the $m_t \to \infty$ limit and the exact predictions. This shows that, while the approximation describes the full results with an accuracy of roughly $\pm20\%$ or better, the amplitudes of the distributions are reduced. The behaviour observed here is in contrast to the monojet case, in which the heavy top quark limit is not a good approximation [111] since the high-$p_T$ jet is able to resolve the sub-structure of the top quark loop. It is also found that this limit overestimates the $E_T+2j$ cross section by a factor of roughly 7 (10) in the $\mathcal{O}_S$ ($\mathcal{O}_P$) case.

3.5.2 Impact of Cuts on $E_T$ and $p_{T,j}$, NLO QCD Corrections, Parton Showering and Hadronisation Effects

In Figure 3.9 the normalised $\Delta\phi_{j_1,j_2}$ for $\mathcal{O}_S$ and $\mathcal{O}_P$ are again shown, but this time with stricter cuts applied to the missing transverse energy and the transverse momentum of the primary jet: $E_T > 500$ GeV and $p_{T,j_1} > 350$ GeV. The corresponding $E_T+j$ and $E_T+2j$ cross sections are 214 fb and 87 fb (344 fb and 141 fb) for $\mathcal{O}_S$ ($\mathcal{O}_P$), and 246 fb
Figure 3.9: Top panel: normalised $\Delta\phi_{j_1j_2}$ distributions for the insertion of $\mathcal{O}_S$ (red), $\mathcal{O}_P$ (blue), and the dominant SM background, $pp \rightarrow Z(\bar{\nu}\nu) + 2j$ (green). Here, the applied cuts include $E_T > 500$ GeV and $p_{T,j_1} > 350$ GeV. Bottom panel: ratio $R$ between the results obtained in the heavy top quark approximation and the full calculation.

and 92 fb for the dominant SM backgrounds. First, it is observed that the $m_t \to \infty$ approximation still provides an acceptable description of the full results. Second, the $\Delta\phi_{j_1j_2}$ distributions are somewhat less pronounced than in the case where the restrictions on $E_T$ and $p_{T,j_1}$ were less strict. Again considering the kinematic analysis performed in Section 3.2, it is clear why this is observed. The larger the $p_T$ of a jet, the greater the angle between the jet and the collision axis. This in turn will reduce the size of the term proportional to $\cos 2\Delta\phi_{j_1j_2}$ in (3.17) relative to the first term. In conclusion, the cuts on $E_T$ and $p_{T,j_1}$ should be as loose as possible in order to enhance the distinction between the $\Delta\phi_{j_1j_2}$ spectra and determine the Lorentz structure of the couplings between DM and top quarks.

Next, we consider whether or not including the effect of including higher order QCD corrections could potentially wash out the shapes of the observed distributions.
It has been shown (see e.g. \[126–128\]) for the analogous process \(pp \rightarrow H/A + 2j\) that the shape of lowest order distributions are unchanged with the inclusion of NLO QCD corrections, as well as PS and hadronisation effects. This is verified for PS corrections in the case of \(pp \rightarrow E_T + 2j\) by showering our LO results with \textsc{Pythia} 6.4 [102]. The \(\Delta \phi_{j_1,j_2}\) distributions undergo relative shifts of at most roughly \(\pm 8\%\) and the amplitudes are slightly reduced, but the spectra are not distorted. Overall, the shapes of these distributions are stable under these corrections.

### 3.5.3 Restrictions on Parameters and Validity of Effective Field Theory Approach

The results that have been obtained thus far have all been calculated in the EFT framework. This approach is particularly simple because it enables the details of the interaction between DM and top quarks to be completely encoded within the suppression scales \(\Lambda_{S,P}\), and one is not required to specify the mediator’s mass, width, nor the size of its couplings. The bounds on \(\Lambda_{S,P}\) are, however, rather weak [111,112] and this raises serious concerns about the applicability of the EFT approach (see e.g. [129–134] for similar discussions).

In this subsection the validity of the approach is considered and we examine the circumstances in which it breaks down. It is necessary to specify a UV completion for the theory, which is assumed to be given by (3.1) and (3.2) such that the effective interactions (3.10) are generated by integrating out a colourless spin-0 s-channel mediator \(S\) or \(P\).

We follow the procedure in [134] to determine the minimum value of the couplings, given by \((g^{S,P}_\chi g^{S,P}_t)^{1/2} = M_{S,P}(\Lambda_{S,P}^3/v)^{1/2}\), for which the EFT is valid. First, the limits on the suppression scales \(\Lambda_{S,P}\) are calculated as a function of \(m_\chi\). The computations are done following the MC methods outlined in [111,135], which are implemented into \textsc{Mcfm} [136]. For concreteness, the analysis is again based on the \(\sqrt{s} = 8\) TeV CMS search with an integrated luminosity of 19.5 fb\(^{-1}\) [119]. The standard event selection
EFT applies non-perturbative \( W_{ch^2} > 0.119 \)

Figure 3.10: **Left panel:** bounds on \((g^S_\chi g^S_\gamma)^{1/2}\) as a function of \(m_\chi\). The red curve and band indicates the minimum value of \((g^S_\chi g^S_\gamma)^{1/2}\) for which the LHC bounds on \(\Lambda_S\) hold. For \((g^S_\chi g^S_\gamma)^{1/2} > 4\pi\), marked by the blue dashed line, the theory is no longer perturbative since the couplings are too large. The green dashed line indicates where the DM relic density agrees with observation. DM is overproduced to the left of this line. The region of parameter space in which all constraints are satisfied is shaded in yellow. **Right panel:** analogous bounds on \((g^P_\chi g^P_\gamma)^{1/2}\). See text for further details.

criteria outlined in Section 3.3 are utilised. To find the limits on \(\Lambda_{S,P}\) we use \texttt{MCFM} to obtain the cross section for a range of values of \(m_\chi\) in the EFT with \(\Lambda_{S,P}\) set to 100 GeV. The CMS search places a 90\% CL upper limit of 882 events due to new physics contributions with the \(E_T > 350\) GeV requirement [119]. Thus, the lower limit on the suppression scale is given by

\[
\Lambda_{S,P}^{\text{min}} = \left( \frac{19.5 \sigma}{882 \text{ fb}} \right)^{1/6} \times 100 \text{ GeV},
\]

where \(\sigma\) is the top quark loop induced \(pp \rightarrow E_T + j\) cross section, in fb, for a given value of \(m_\chi\). This cross section is then calculated in the full theory as a function of both \(m_\chi\) and \(M_{S,P}\). For each value of \(m_\chi\), the minimum value of \((g^S_\chi g^S_\gamma)^{1/2}\) consistent with an EFT description is calculated by requiring that there is a better than 20\% agreement between the full theory results and those obtained in the EFT. These calculations make use of the relation (3.11) between \(M_{S,P}, \Lambda_{S,P}\) and \(g^S_\chi g^S_\gamma, M^S_\gamma\).

The red curves and bands shown in Figure 3.10 show the results of this calculation.
The width of the bands represents the dependence on the width of the mediators, which are varied in the range $\Gamma_{S,P}/M_{S,P} \in [1/(8\pi), 1/3]$. It is observed that the couplings of the mediator to top quarks and DM must be strong in order for the EFT to be valid, and increasingly large values are required if the DM mass is at or above the weak scale. Furthermore, the theory becomes non-perturbative if $m_\chi \gtrsim 490$ GeV ($m_\chi \gtrsim 580$ GeV) in the case of $O_S$ ($O_P$), indicated by the intersection of the blue dashed lines with the red curves. It is important to note that for light DM the values of $M_{S,P}$ for which the EFT is valid are below 1 TeV. For example, for $m_\chi = 50$ GeV the EFT limits correspond to $M_S \approx 370$ GeV and $M_P \approx 310$ GeV, assuming that $\Gamma_{S,P}/M_{S,P} = 1/3$.

The DM relic density $\Omega_\chi h^2$ is sensitive to the full particle content of the underlying theory, and is therefore more model-dependent than the monojet cross section. It is assumed that the couplings and particle contents of this model are completely specified by (3.1) and (3.2). In order to calculate the relic density, we use formulae for the annihilation cross sections given in [111]. To LO in the DM velocity $v_\chi$ we have

$$\langle \sigma_\chi^S v_\chi \rangle_{\bar{\chi} \chi \rightarrow t\bar{t}} = \frac{3v_\chi^2 m_\chi^4}{8\pi^3 \Lambda_S^6} z_t (1 - z_t)^{3/2},$$  \hfill (3.32)

$$\langle \sigma_\chi^P v_\chi \rangle_{\bar{\chi} \chi \rightarrow t\bar{t}} = \frac{3m_\chi^4}{2\pi \Lambda_P^6} z_t (1 - z_t)^{1/2},$$  \hfill (3.33)

for annihilation to top quarks, and

$$\langle \sigma_\chi^S v_\chi \rangle_{\bar{\chi} \chi \rightarrow g\bar{g}} = \frac{v_\chi^2 \alpha_s^2 m_\chi^4}{8\pi^3 \Lambda_S^6} |F_S(z_t)|^2,$$  \hfill (3.34)

$$\langle \sigma_\chi^P v_\chi \rangle_{\bar{\chi} \chi \rightarrow g\bar{g}} = \frac{\alpha_s^2 m_\chi^4}{2\pi^3 \Lambda_P^6} |F_P(z_t)|^2,$$  \hfill (3.35)

for annihilation to gluons. In the above, $z_t \equiv m_t^2/m_\chi^2$, and annihilation to top quarks only occurs when $z_t < 1$ (i.e. when the final state is kinematically accessible). The form factors are given in (3.8).
The relic density can be expressed as (see Appendix C)

\[ \Omega \chi h^2 \approx \frac{1.07 \times 10^9 \text{GeV}}{M_{\text{Pl}} \sqrt{g_* (a + b \chi^2)}} \],

(3.36)

where \( a \) and \( b \) are the coefficients in the expansion \( \sigma v_\chi = a + (1 + \mathcal{O}(v_\chi^4))v_\chi^2 \), and the other variables are defined in the appendix. We use (3.32) to (3.35) to replace \( a \) and \( b \) in (3.36) and solve for \( \Lambda_{S,P} \). Under the requirement that the observed relic density \( \Omega \chi h^2 \approx 0.119 \) [24] is reproduced, we now have the suppression scales as a function of the DM mass. Finally, the relation (3.11) can be used to find \( (g_{S,P}^S g_{S,P}^S)^{1/2} \) as a function of \( m_\chi \). This is indicated by the green dashed curves on Figure 3.10.

DM is overproduced to the left of these curves and underproduced to the right. The model under consideration is a simplified one, and it is likely that there exist other particles which contribute to the dark sector. For this reason, it is not considered an issue if the DM in this model does not fully saturate the observed relic density. This means that we are not restricted to considering points in parameter space that sit on the green curves, but rather the regions to the right. Furthermore, the strong model dependence of the calculation of \( \Omega \chi h^2 \) means that the constraints are loosened if DM has large annihilation cross sections to additional SM particles or (more likely) other dark sector states. The effect of including these final states would be to shift the green curves further to the left, opening up the allowed region of parameter space.

This reduces the tension between LHC monojet limits and relic density constraints.

Combining together the constraints discussed in this subsection establishes bounds on the allowed values for the DM mass and couplings. These are

\[ \mathcal{O}_S : \quad 40 \text{ GeV} \lesssim m_\chi \lesssim 470 \text{ GeV}, \quad 3.9 \lesssim (g_{S,P}^S g_{S,P}^S)^{1/2} < 4\pi, \]

(3.37)

\[ \mathcal{O}_P : \quad 10 \text{ GeV} \lesssim m_\chi \lesssim 580 \text{ GeV}, \quad 2.2 \lesssim (g_{S,P}^S g_{S,P}^S)^{1/2} < 4\pi. \]

(3.38)

It is evident now that the choice of \( \Lambda_{S,P} = 150 \text{ GeV} \) and \( m_\chi = 50 \text{ GeV} \) used to obtain the results in Section 3.4 ensures firstly that the EFT approach is valid and secondly that DM is not overproduced.
Finally, we check that the $\Delta \phi_{j_1j_2}$ distributions remain a good observable to probe the structure of the DM-SM couplings beyond the EFT approach. We study the same UV completion as discussed above — that is, the theory is described by (3.1) and (3.2) such that interactions between top quarks and DM proceed via a purely scalar or pseudoscalar $s$-channel mediator. Our choice of parameters is $g^{S,P}_X = g^{S,P}_t = 1$, $m_\chi = 200 \text{ GeV}$ and $M_{S,P} = 500 \text{ GeV}$ which, it is noted, satisfy the relic density constraints and perturbativity limit. At this stage, it is verified that the model does not lead to observable excesses in resonance searches with $\tilde{t}t$ and dijet final states. Numerically, the $\tilde{t}t$ cross section is enhanced by $\mathcal{O}(1\%)$ due to contributions from the one-loop processes $gg \to S, P \to \tilde{t}t$. The dijet cross section, meanwhile, receives negligible contributions from the two-loop processes $gg \to S, P \to gg$.

The signal strength is dependent on the total width of the mediators. From (3.7) and (3.8), with our choice of couplings we have $\Lambda_S/M_S \approx 3.1\%$ and $\Lambda_P/M_P \approx 6.4\%$, i.e. the resonances are narrow. Applying the standard event selection criteria, the total cross sections are

$$
\sigma(pp \to S(\bar{\chi}\chi) + j) \approx 9 \text{ fb}, \quad \sigma(pp \to P(\bar{\chi}\chi) + j) \approx 25 \text{ fb}
$$

and

$$
\sigma(pp \to S(\bar{\chi}\chi) + 2j) \approx 5 \text{ fb}, \quad \sigma(pp \to P(\bar{\chi}\chi) + 2j) \approx 16 \text{ fb},
$$

and it is recalled that the cross sections due to the dominant SM backgrounds, $pp \to Z(\bar{\nu}\nu) + j$ and $pp \to Z(\bar{\nu}\nu) + 2j$, are 1289 fb and 330 fb respectively. At the 14 TeV LHC run with an integrated luminosity of 300 fb$^{-1}$ we should see more than 1000 $E_T + 2j$ signal events, allowing for a measurement of the $\Delta \phi_{j_1j_2}$ distribution.

Figure 3.11 shows predictions for the $\Delta \phi_{j_1j_2}$ distributions calculated in the full theory. We see that the approximate sine- and cosine-like shapes are present in the simplified model as well. In fact, the predicted distributions from a full theory calculation are even more clearly distinct from one another.

It is also shown that taking the heavy top quark limit again does not wash out the shapes of the distributions. Furthermore, this approximation reproduces the total
Figure 3.11: Top panel: normalised $\Delta \phi_{jj'}$ distributions for the full theory described by the Lagrangians (3.1) (red) and (3.2) (blue), and the dominant SM background, $pp \rightarrow Z(\bar{\nu}\nu) + 2j$, (green). Here, the standard selection criteria in Section 3.3 are used with $E_T > 350$ GeV and $p_T,j_1 > 110$ GeV. Bottom panel: ratio $R$ between the results obtained in the heavy top quark approximation and the full calculation.

Cross sections with considerably better accuracy in the full theory compared to the EFT. In this case, the $m_t \rightarrow \infty$ limit overestimates the exact cross sections by a factor of roughly 1.4 for both scalar and pseudoscalar interactions (compared to the factors of 7 (10) found for $O_S$ ($O_P$)). This is because in the full theory the $pp \rightarrow E_T + 2j$ cross section is dominated by invariant masses of the DM pair close to $M_{S,P}$, i.e. when the mediator is close to being on-shell. In the EFT case, however, the momentum transfer to the DM pair is on average much larger. Since the quality of the heavy top quark approximation decreases as the mediator becomes more off-shell [111], the full theory calculation is reasonably well described by taking $m_t \rightarrow \infty$, but the EFT cross section is overestimated significantly.

To summarise, it has been found that the strong angular correlation of the tagging jets in $E_T + 2j$ events is present irrespectively of whether or not the calculation is
performed in an EFT regime, with or without taking the heavy top quark limit. This
is not the case for predictions of the monojet cross section, which are highly model-
dependent (see for instance [134]). Combined with the stability of the observable
under various corrections, as discussed in Subsection 3.5.2 this shows that measure-
ments of the $\Delta\phi_{j_1j_2}$ distributions therefore provide a unique and robust way to study
the structure of DM couplings to the SM.
Chapter 4

Constraints on Gauge Boson Couplings to Dark Matter

This chapter is based on [137], original work done in collaboration with Ulrich Haisch and Andreas Crivellin.

In the previous chapter we considered a simple model of fermionic DM whose interactions with the SM are mediated by a new scalar or pseudoscalar. It was shown that by studying the azimuthal angular difference between the tagging jets in $pp \rightarrow \not{E}_T + 2j$ events it is possible to disentangle whether the interaction proceeds via scalar or pseudoscalar interactions.

It is natural to now consider the generality of this result: if one were to study a more complex theory of DM could this observable still be used to determine the Lorentz structure of the interactions? This question is now examined for a theory in which DM couples to the SM gauge bosons. Section 4.1 sets out the model under consideration, illustrating the types of processes which would lead to signal events at the LHC. In Section 4.2 various signal processes are considered which involve a large amount of missing transverse energy in association with certain SM particles. The details of the recent ATLAS and CMS searches for signals in these channels are presented, along with the corresponding 95% CL bounds on the fiducial cross sections. These bounds are then used in Section 4.3 to derive limits on the size of
the suppression scales in the effective model, highlighting which search strategies can most strongly constrain the effective operators. This prompts a discussion of the validity of the EFT approach, in which possible UV completions are considered. The section concludes with a study of future prospects for the bounds on the suppression scales. In Section 4.4, the focus turns to $E_T + 2j$ final states, and we examine the azimuthal angular separation of the two tagging jets for interactions featuring a CP-even or -odd operator. Prospects for measurement of this observable at the 14 TeV LHC are then studied.

4.1 Model Description

In this chapter we consider interactions in which the mediating particles are heavy enough to be integrated out, such that the EFT framework is applicable. It was shown in the previous chapter that, for the case of a scalar or pseudoscalar mediator which couples SM quarks to DM, the shapes of the azimuthal angle distributions of the jets in $E_T + 2j$ events are in fact rather insensitive to whether the calculation is performed using the full theory or in the EFT limit. In Section 4.4 we see again that the CP-even and -odd operators lead to $\Delta \phi_{j_1j_2}$ spectra which are distinct from one another, and from the SM background, without needing to perform the calculations using a full theory specified by some UV completion.

In this model, the following effective Lagrangian is considered

$$\mathcal{L}_{\text{eff}} = \sum_{k=B,W,B,\tilde{W}} \frac{C_k(\mu)}{\Lambda^3} O_k,$$

(4.1)

where $\Lambda$ is the scale of new physics, i.e. where the mediating particles are removed and the operators generated. The $C_k(\mu)$ are the Wilson coefficients, evaluated at the scale $\mu$, corresponding to the following four $SU(2)_L \times U(1)_Y$ gauge-invariant dimension-7 operators

$$O_B = \bar{\chi}\chi B_{\mu\nu}B^{\mu\nu}, \quad O_W = \bar{\chi}\chi W^{\dagger}_{\mu\nu}W^{i,\mu\nu},$$

$$O_{\tilde{B}} = \bar{\chi}\chi B_{\mu\nu}\tilde{B}^{\mu\nu}, \quad O_{\tilde{W}} = \bar{\chi}\chi W^{\dagger}_{\mu\nu}\tilde{W}^{i,\mu\nu}.$$
Here, $B_{\mu\nu}$ and $W_{\mu\nu}^i$ are the $U(1)_Y$ and $SU(2)_L$ field strength tensors, respectively, given by

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$
$$W_{\mu\nu}^i = \partial_\mu W_{\nu}^i - \partial_\nu W_{\mu}^i + g_w \epsilon^{ijk} W_{\mu}^j W_{\nu}^k,$$

while $\tilde{B}_{\mu\nu} = 1/2 \epsilon^{\mu\nu\rho\sigma} B_{\rho\sigma}$ and $\tilde{W}_{i,\mu\nu} = 1/2 \epsilon^{\mu\nu\rho\sigma} W_{i,\rho\sigma}$ are their duals. Finally, $g_w$ is the weak coupling constant.

While there are additional operators that could be considered, those in (4.2) are specifically chosen because they are the only ones which lead to velocity-suppressed annihilation rates of DM to photon pairs [138–141]. Replacing $\bar{\chi}\chi$ with the pseudoscalar DM current $\bar{\chi}\gamma_5\chi$, on the other hand, leads to unsuppressed annihilation rates to photon pairs. Consequently, those operators in (4.2) are of particular interest here because they are the least constrained by direct and indirect detection experiments, and the strongest bounds can therefore be obtained from collider searches.

Interactions between DM and the SM gauge bosons arising due to these operators have been constrained using 7 TeV and 8 TeV LHC data from searches for invisible decays of the Higgs boson in the VBF ($E_T + 2j$) mode [142], as well as in the mono-Z ($E_T + Z$) [143,144], mono-W ($E_T + W$) [145] and mono-photon ($E_T + \gamma$) [146] channels. Examples of Feynman diagrams that give rise to these signals are shown in Figure 4.1.

In the next section, the existing constraints are updated using more recent mono-photon, mono-W/Z ($\rightarrow$ hadrons) and invisible VBF Higgs decay data. The study is also extended to include the $E_T + W(\rightarrow$ leptons) channel and monojet searches.

### 4.2 Search Channels

In this section the LHC searches for $E_T$ that will be used to constrain the operators in (4.2) are reviewed. For each channel the important event selection criteria are listed, along with the value of the fiducial cross section.
Figure 4.1: Example Feynman diagrams of DM pair production induced by the operators in equation (4.2), represented by the black squares. The corresponding search channels are $E_T + \gamma$ (top left), $E_T + Z(\rightarrow l^+l^-)$ (top right), $E_T + W/Z(\rightarrow j)$ (bottom left) and $E_T + 2j$ (bottom right). The label $V$ indicates all possible $\gamma$, $Z$- or $W$-boson exchanges.

4.2.1 Monophoton

Both the CMS [147] and ATLAS [148] collaborations have recently conducted searches for missing transverse energy in association with a photon. Stronger restrictions were obtained by CMS with 19.6 fb$^{-1}$ of 8 TeV data, so those results are used here. The important cuts that are applied in the analysis are

$$E_T > 140 \text{ GeV}, \quad |\eta_\gamma| < 1.4442,$$

where $\eta_\gamma$ is the pseudorapidity of the photon. The dominant SM background for this process is $pp \rightarrow Z(\bar{\nu}\nu) + \gamma$ which is predicted to contribute roughly 90% of the total background. The CMS search performs a measurement of the signal cross section for a range of cuts on the photon transverse energy, $E_{T,\gamma}$. The $E_T + \gamma$ signal has a rather hard $E_{T,\gamma}$ spectrum due to the higher dimensional nature of the operators (4.2), so it is found that the most stringent constraint is obtained for the most severe cut of
\( E_{T,\gamma} > 700 \text{ GeV.} \) In this case, the 95\% CL bound on the fiducial cross section is

\[ \sigma_{\text{fid}}(pp \rightarrow E_T + \gamma) < 0.22 \text{ fb.} \]  \hspace{1cm} (4.5)

4.2.2 Mono-\( Z \) (Leptonic)

The ATLAS results \[144\] for 20.3 fb\(^{-1}\) of 8 TeV data are used to constrain the \( E_T + Z(l^+l^-) \) search channel. The relevant cuts in this case are

\[ p_{T,l} > 20 \text{ GeV}, \quad |\eta_l| < 2.5, \quad m_{ll} \in [76, 106], \]

\[ |\eta_{ll}| < 2.5, \quad \frac{|p_{T,ll} - E_T|}{p_{T,ll}} < 0.5, \quad \Delta \phi_{E_T,p_{T,ll}} > 2.5 \]  \hspace{1cm} (4.6)

where \( m_{ll}, \eta_{ll} \) and \( p_{T,ll} \) are the invariant mass, pseudorapidity and transverse momentum, respectively, of the dilepton system. \( \Delta \phi_{E_T,p_{T,ll}} \) is the azimuthal angle between the \( E_T \) and \( p_{T,ll} \) vectors, and this cut is imposed to suppress events in which the missing transverse energy originates from mismeasured jets. Decays of the \( Z \)-boson to electron and muon pairs are considered in this analysis. The dominant SM background process is \( pp \rightarrow ZZ \rightarrow \bar{\nu}\nu l^+l^- \), which is irreducible and predicted to contribute roughly 90\% of the total background. The measurement is taken for four values of \( E_{T,\text{min}} \), and the 350 GeV cut leads to the best bound on the fiducial cross section. At the 95\% CL this is

\[ \sigma_{\text{fid}}(pp \rightarrow E_T + Z(l^+l^-)) < 0.27 \text{ fb.} \]  \hspace{1cm} (4.7)

4.2.3 Mono-\( W \) (Leptonic)

Both the CMS \[149\] and ATLAS \[150\] collaborations have recently conducted searches for missing transverse energy in association with a single lepton, corresponding to events containing a single \( W \)-boson which decays to a charged lepton and a neutrino. In this case, the ATLAS search for the \( E_T + \mu\bar{\nu}_\mu \) final state provides the strongest constraints, using 20.3 fb\(^{-1}\) of 8 TeV data. The relevant selection criteria are

\[ p_{T,\mu} > 45 \text{ GeV}, \quad |\eta_\mu| \in [0, 1] \cup [1.3, 2], \]  \hspace{1cm} (4.8)
and the transverse mass, which also has an applied cut, is defined as

\[ m_T = \sqrt{2p_{T,\mu}E_T(1 - \cos \Delta \phi_{E_T, p_{T,\mu}})}, \quad (4.9) \]

Here, \( \Delta \phi_{E_T, p_{T,\mu}} \) is the azimuthal angle between the \( E_T \) and \( p_{T,\mu} \) vectors. The dominant SM background process is \( W \)-boson production with decays to the same final state, \( pp \rightarrow W \rightarrow \mu\nu_{\mu} \), and this is predicted to contribute roughly 90% of the total background. Several values of \( m_{T,\text{min}} \) are considered in the ATLAS analysis, and the strictest of these cuts, \( m_T > 843 \) GeV, leads to the best bound on the fiducial cross section. At the 95% CL this is

\[ \sigma_{\text{fid}}(pp \rightarrow E_T + W(\mu\nu_{\mu})) < 0.54 \text{ fb}. \quad (4.10) \]

### 4.2.4 Mono-\( W/Z \) (Hadronic)

The ATLAS collaboration \[151\] has searched for \( E_T \) events which feature a hadronically decaying \( W \)- or \( Z \)-boson. The results are obtained for 20.3 fb\(^{-1}\) of 8 TeV data. These decays typically form a jet with a mass consistent with the gauge boson which generated it. Jet candidates are reconstructed using the Cambridge/Aachen (C/A) algorithm \[152\] with a radius parameter of \( R = 1.2 \) and subjected to a mass-drop filtering procedure \[153,154\]. This ‘large radius’ jet is intended to contain the hadronic products of both quarks in the gauge boson decay. Events are required to have at least one of these C/A jets with

\[ p_{T,j} > 250 \text{ GeV}, \quad |\eta_j| < 1.2, \quad m_j \in [50, 120] \text{ GeV}, \]

\[ \sqrt{\gamma} = \min(p_{T,j_1}, p_{T,j_2}) \sqrt{\frac{(\Delta \phi_{j_1, j_2})^2 + (\Delta \eta_{j_1, j_2})^2}{m_j}} < 0.4. \quad (4.11) \]

Here, \( m_j \) is the calculated mass of the large radius jet and \( \sqrt{\gamma} \) is a measure of the momentum imbalance of the two leading subjets, \( j_1 \) and \( j_2 \). These subjets are reconstructed using the anti-\( k_t \) clustering algorithm \[107\] with a radius parameter of \( R = 0.4 \), and are hence referred to as ‘narrow’ jets. \( \Delta \phi_{j_1, j_2} \) and \( \Delta \eta_{j_1, j_2} \) are the
azimuthal angle separation and pseudorapidity separation of the narrow jets, respectively. The dominant SM background is $Z \rightarrow \bar{\nu}\nu$ production with jets from initial state radiation. It is predicted to contribute roughly 60% of the total number of events. The next largest contribution to the background in this case is production of $W$- and $Z$-bosons which contain charged leptons in their decay products. These charged leptons fail identification requirements, or the $\tau$ leptons decay hadronically. The contribution from these processes is predicted to form roughly 25% of the total number of events. It is found that the most stringent constraints are provided by using the harder cut, $E_T > 500$ GeV. In this case, the 95% CL limit on the fiducial cross section is

$$\sigma_{\text{fid}}(pp \rightarrow E_T + W/Z(\text{hadrons})) < 2.2 \text{ fb.} \quad (4.12)$$

### 4.2.5 Monojet

Monojet data can also be used to constrain the operators (4.2) since the corresponding searches permit an additional jet to be present. The CMS search [105] is considered here, which utilises 19.7 fb$^{-1}$ of 8 TeV data. Jets are reconstructed using the anti-$k_t$ algorithm [107] with a radius parameter of 0.5. The relevant cuts are

$$p_{T,j_1} > 110 \text{ GeV}, \quad |\eta_{j_1}| < 2.4,$$

$$p_{T,j_2} > 30 \text{ GeV}, \quad |\eta_{j_2}| < 4.5,$$

$$\Delta \phi_{j_1 j_2} < 2.5.$$ \quad (4.13)

where $\Delta \phi_{j_1 j_2}$ is the azimuthal angular separation of the two leading jets. Additionally, a jet veto is imposed such that events are rejected if they contain a tertiary jet with $p_{T,j_3} > 30$ GeV and $|\eta_{j_3}| < 4.5$. The dominant SM background is due to $pp \rightarrow Z(\bar{\nu}\nu) +$ jets, which is predicted to contribute roughly 70% of the total number of events. The next largest background is $pp \rightarrow W(l\nu) +$ jets, where the charged lepton does not get rejected by the lepton veto. The predicted contribution from this process is roughly 25% of the total data. Seven values of the $E_{T,\text{min}}$ cut are
considered in the range $[250, 550]$ GeV and it is found that the best bound is obtained for $E_T > 500$ GeV. The corresponding 95% CL limit on the fiducial cross section is

$$\sigma_{\text{fid}}(pp \rightarrow E_T + j) < 6.1 \text{ fb.} \quad (4.14)$$

### 4.2.6 VBF Higgs Boson Invisible Decays

The CMS collaboration [155] have conducted searches for invisible decays of the Higgs boson in the VBF channel, utilising 19.5 fb$^{-1}$ of 8 TeV data. Events contain two jets in the final state which are reconstructed using the anti-$k_t$ clustering algorithm [107] with a radius parameter of 0.5. The relevant imposed cuts are

$$p_{T,j1}, p_{T,j2} > 50 \text{ GeV}, \quad |\eta_{j1}|, |\eta_{j2}| < 4.7, \quad \eta_{j1} \cdot \eta_{j2} < 0,$$

$$\Delta \eta_{j1,j2} > 4.2, \quad m_{j1,j2} > 1100 \text{ GeV}, \quad \Delta \phi_{j1,j2} < 1.0,$$  

where $m_{j1,j2}$ is the invariant mass of the two-jet system. In addition a jet veto is imposed such that events are rejected if they contain a tertiary jet with $p_{T,j3} > 30$ GeV and pseudorapidity between those of the two tagging jets. The dominant SM background is $pp \rightarrow W(l\nu) + 2j$, where the final state charged lepton fails identification requirements, or the $\tau$ leptons decay hadronically. This is predicted to contribute roughly 55% of the total background. The next largest background is $pp \rightarrow Z(\bar{\nu}\nu) + 2j$ which is predicted to contribute roughly 30% of the total background. The cut on the missing transverse energy is $E_T > 130$ GeV and the 95% CL limit on the fiducial cross section is

$$\sigma_{\text{fid}}(pp \rightarrow E_T + 2j) < 6.5 \text{ fb.} \quad (4.16)$$

### 4.3 Constraints on Suppression Scales

#### 4.3.1 Event Generation

Appendix E explains in detail how the cross sections for the $E_T$ signals associated with the operators (4.2) are calculated. In brief, the operators are first implemented...
into FeynRules [156][157], a Mathematica [158] package which determines the Feynman rules for each operator and generates a UFO output [159]. This file is then imported into MadGraph 5 [95][96] which performs the event generation at LO, using the CTEQ6L1 PDFs [160]. PYTHIA 8 [161] is utilised to include PS and hadronisation effects, and jets are reconstructed using FastJet 3 [106]. Delphes 3 [162] is used as a fast detector simulation to estimate the reconstruction efficiencies for the various $E_T$ signals. It is found that the efficiencies are around 70% for the monophoton case, 60% for the leptonic mono-$Z$ and mono-$W$ cases, and 65% for the hadronic mono-$W/Z$ case. These findings agree with [148] for $E_T + \gamma$, [144] for $E_T + Z(l^+l^-)$, [143] for $E_T + W(\mu\nu)$, and [151] for $E_T + W/Z$(hadrons). For the monojet case and invisible decays of the Higgs boson in the VBF mode, reconstruction efficiencies of around 95% are found.

This MC implementation has been validated by reproducing the numerical results of [144][146] within theoretical uncertainties. The errors have been assessed by studying the scale ambiguities of the results. The default dynamical scale choice in MadGraph 5 was used, varying the scale factor in the range $[1/2, 2]$. For the monophoton and leptonic mono-$Z$ and mono-$W$ cases, the cross sections vary by roughly ±15%, while for hadronic mono-$W/Z$, monojet and VBF Higgs boson invisible decays, the relative shifts are around ±20%. Note that these errors are smaller than those obtained in [103][111][113][163] since the tree-level $E_T$ cross sections considered do not explicitly depend on the strong coupling, $\alpha_s$. The uncertainties given here therefore only reflect ambiguities related to changes in the factorisation scale, but not renormalisation scale.

### 4.3.2 Dependence on a Single Wilson Coefficient

We first consider the bounds on the new physics scale $\Lambda$ that are obtained for only one of the Wilson coefficients being non-zero. In Figure 4.2 these limits are presented...
Figure 4.2: Left panel: lower limits on the new physics scale $\Lambda$ for the case that $C_B(\Lambda) = 1$, $C_W(\Lambda) = 0$. The DM particles are taken to be Dirac fermions and $C_B'(\Lambda) = C_W'(\Lambda) = 0$. The coloured curves correspond to the limits arising from the latest monophoton (red), $E_T + Z(l^+l^-)$ (orange), $E_T + W(\mu\nu)$ (yellow), $E_T + W/Z$ (hadrons) (purple), monojet (blue) and VBF $h \rightarrow$ invisible (grey) searches. The width of the bands reflect the associated scale uncertainties. Right panel: Corresponding bounds for the case that $C_B(\Lambda) = 0$, $C_W(\Lambda) = 1$ for the case that $C_B'(\Lambda) = C_W'(\Lambda) = 0$, with $C_B(\Lambda) = 1$, $C_W(\Lambda) = 0$ (left panel) and $C_B(\Lambda) = 0$, $C_W(\Lambda) = 1$ (right panel) for the Wilson coefficients evaluated at $\Lambda$. These predictions correspond to Dirac fermion DM and the widths of the coloured bands reflect the impact of scale variations.

For $C_B(\Lambda) = 1$, $C_W(\Lambda) = 0$ it is found that the strongest constraint on $\Lambda$ is provided by the monophoton search $[147]$ over the majority of the parameter space. Numerically, the scale has to satisfy $\Lambda \gtrsim 510$ GeV for $m_\chi \lesssim 100$ GeV in order to satisfy the 95\% CL requirement $[4.5]$. On the other hand, for $C_B(\Lambda) = 0$, $C_W(\Lambda) = 1$ the monojet search $[105]$ imposes the strongest restriction on $\Lambda$. To satisfy the 95\% CL requirement $[4.14]$ we require $\Lambda \gtrsim 600$ GeV for $m_\chi \lesssim 100$ GeV. The shown limits also hold in the case that $C_B(\Lambda) = C_W(\Lambda) = 0$, with $C_B(\Lambda) = 1$, $C_W(\Lambda) = 0$ or $C_B(\Lambda) = 0$, $C_W(\Lambda) = 1$, while for Majorana DM the bounds are roughly 12\% stronger. The flatness of the curves for low values of the DM mass is due to the fact that the cross section is proportional to $(1 - 4m_\chi^2/s)^{1/2}$, which tends to 1 for light
DM but causes a suppression of the cross section for heavier DM, explaining why the bounds weaken in that region. Finally, it is noted that $E_T + W(\mu\nu_\mu)$ does not provide any constraint on scenarios in which $C_W(\Lambda) = C_{\tilde{W}}(\Lambda) = 0$.

To better understand the restrictions imposed by the various search channels, we consider the Feynman rules associated with the operators $O_B$ and $O_W$. In momentum space, the resulting interactions between pairs of DM particles and SM gauge bosons are required, due to gauge invariance, to take the form

$$\frac{4i}{\Lambda^3} g_{V_1 V_2} (p_1^{\mu_1} p_2^{\mu_2} - g^{\mu_1 \mu_2} p_1 \cdot p_2), \quad (4.17)$$

where $p_i$ ($\mu_i$) denotes the 4-momentum (Lorentz index) of the vector field $V_i$ and for simplicity the spinors associated with the DM fields have been dropped. The couplings $g_{V_1 V_2}$ can be derived by re-expressing the operators (4.2) in terms of the physical fields, using

$$A_{\mu\nu} = \frac{1}{\sqrt{g_w^2 + g_w'^2}} (g_w' W_3^\mu W_{\mu\nu} + g_w B_{\mu\nu})$$

$$W_{\mu\nu}^\pm = \frac{1}{\sqrt{2}} (W_1^\mu \mp i W_2^\mu)$$

$$Z_{\mu\nu} = \frac{1}{\sqrt{g_w^2 + g_w'^2}} (g_w W_3^\mu W_{\mu\nu} - g_w' B_{\mu\nu}). \quad (4.18)$$

In the above expressions, $g_w$ and $g_w'$ are the couplings constants corresponding to the massless $SU(2)_L$ and $U(1)_Y$ gauge bosons, respectively. These are related to the sine ($s_w$) and cosine ($c_w$) of the weak mixing angle via

$$c_w = \frac{g_w}{\sqrt{g_w^2 + g_w'^2}}, \quad s_w = \frac{g_w'}{\sqrt{g_w^2 + g_w'^2}}. \quad (4.19)$$

Using these relations and inserting (4.18) into the expressions for the CP-even operators in (4.2) yields

$$\mathcal{L}_{\text{eff},B} = \frac{C_B(\mu)}{\Lambda^3} \bar{\chi} \chi \left( c_w^2 A_{\mu\nu} A^{\mu\nu} + s_w^2 Z_{\mu\nu} Z^{\mu\nu} - 2c_w s_w A_{\mu\nu} Z^{\mu\nu} \right),$$

$$\mathcal{L}_{\text{eff},W} = \frac{C_W(\mu)}{\Lambda^2} \bar{\chi} \chi \left[ \left( s_w^2 A_{\mu\nu} A^{\mu\nu} + c_w^2 Z_{\mu\nu} Z^{\mu\nu} + 2c_w s_w A_{\mu\nu} Z^{\mu\nu} \right) + 2W_{\mu\nu}^\pm W^{\mu\nu} \right]. \quad (4.20)$$
Since the full Lagrangian involves a sum of these two expressions, the couplings to the physical fields can now simply be read off as
\begin{align}
g_{AA} &= c_w^2 C_B(\Lambda) + s_w^2 C_W(\Lambda), \\
g_{AZ} &= -s_w c_w (C_B(\Lambda) - C_W(\Lambda)), \\
g_{ZZ} &= s_w^2 C_B(\Lambda) + c_w^2 C_W(\Lambda), \\
g_{WW} &= C_W(\Lambda). \quad (4.21)
\end{align}

These do not coincide with the expressions given in \[143, 145, 146\]. From (4.21), in the case of the coupling \(g_{AA}\) to two photons, the Wilson coefficient \(C_B(\Lambda)\) enters compared to \(C_W(\Lambda)\) with a relative factor of \(c_w^2/s_w^2 \approx 3.3\). On the other hand, for the case of the couplings to \(Z\)-boson pairs, \(g_{ZZ}\), the dependence on \(c_w\) and \(s_w\) is reversed. This explains why the limit on the new physics scale \(\Lambda\) from monophoton searches is stronger in the left panel of Figure 4.2 than that obtained in searches for \(E_T + Z(l^+l^-)\), \(E_T + W/Z\) (hadrons), monojet and VBF \(h \rightarrow\) invisible.

It is also important to note that channels which contain leptons in the final state typically lead to weaker bounds on \(\Lambda\) compared to modes which involve hadrons. This is simply because the branching ratios for SM gauge bosons decaying to leptons are considerably smaller than those for hadronic decays. Numerically,
\begin{align}
\mathcal{B}(Z \rightarrow l^+l^-) &\approx 0.07, \\
\mathcal{B}(W \rightarrow \mu \nu) &\approx 0.11, \\
\mathcal{B}(Z \rightarrow \text{hadrons}) &\approx 0.7, \\
\mathcal{B}(W \rightarrow \text{hadrons}) &\approx 0.68.
\end{align}

This strong suppression in the leptonic decay channels overcompensates the higher detection efficiencies of final states which involve leptons, and consequently the LHC searches for \(E_T + \text{hadrons}\) are superior to those looking for \(E_T + \text{leptons}\) signals.

Finally, it is observed that the monojet data is evidently more constraining than the VBF \(h \rightarrow\) invisible search, despite the fact that they consider the same final state, \(E_T + 2j\). This is because the phase space in the two cases is quite different due to the contrasting sets of event selection criteria that are used. The cut which has the largest
impact on the results here is the rather loose $E_T > 130$ GeV restriction imposed in the VBF $h \to$ invisible search. This selection is tailored for a Higgs boson of 125 GeV but fares less well if one is attempting to probe higher-dimensional operators of the form (4.2). Since these operators produce a rather hard $E_T$ spectrum, more severe requirements on the minimum value of the missing transverse energy lead to a clearer separation between signal and SM background.

4.3.3 Dependence on Two Wilson Coefficients

In the previous subsection the limits on the scale $\Lambda$ were considered as a function of the DM mass $m_\chi$, using values of 1 and 0 for the Wilson coefficients. We now analyse the impact of fixing the DM mass at $m_\chi = 100$ GeV, again setting $C_{\tilde{B}}(\Lambda) = C_{\tilde{W}}(\Lambda) = 0$, 83
and varying $C_B(\Lambda)$ and $C_W(\Lambda)$ between -1 and 1. The panels of Figure 4.3 show contours of constant $\Lambda$ in the $C_B(\Lambda)$–$C_W(\Lambda)$ plane. The first observation is that only the monophoton signal depends more strongly on $C_B(\Lambda)$ than $C_W(\Lambda)$, while this situation is reversed for all other $E_T$ channels considered. The reason for this is evident from examining (4.21): only for the coupling $g_{AA}$ is the coefficient of $C_B(\Lambda)$ larger than that of $C_W(\Lambda)$. Secondly, with the exception of the monophoton case, the major axes of the elliptic contours in all panels are closely aligned with the $C_B(\Lambda)$ axis, suggesting that interference effects between contributions from the two operators $O_B$ and $O_W$ are small. Thirdly, and best illustrated by the shading of the contours, the monophoton and monojet results provide the best bounds in the entire $C_B(\Lambda)$–$C_W(\Lambda)$ plane. This is further illustrated by the left panel of Figure 4.4 in which the overlaid numbers indicate the search strategy that provides the strongest bound on $\Lambda$ in that area of the $C_B(\Lambda)$–$C_W(\Lambda)$ plane. The numbers 1 and 5 correspond to the monophoton and monojet channels, respectively. It is found that if the ratio of the Wilson coefficients satisfies $|C_B(\Lambda)/C_W(\Lambda)| \gtrsim 1.5$ then the bound (4.5) gives the strongest constraint, while in the remainder of the $C_B(\Lambda)$–$C_W(\Lambda)$ plane the bound (4.14) is more restrictive. When all $E_T$ searches are combined, the obtained $\Lambda$ contours are those shown in the right panel of Figure 4.4.

### 4.3.4 Validity of EFT Approach

Finally, it is noted that the values of $\Lambda$ which are excluded by the data presented here are small compared to typical LHC energies. This raises concerns about the validity of the EFT approach. In order to go beyond this description, it is necessary to specify a UV completion where the operators in (4.2) arise from a renormalisable theory after integrating out the heavy degrees of freedom which mediate the interactions. We first consider a counter-example, in which there is a real scalar mediator $S$ and a fermion
Figure 4.4: Combination of the bounds on the new physics scale $\Lambda$ from all search channels in the $C_B(\Lambda)$–$C_W(\Lambda)$ plane, employing $m_\chi = 100$ GeV and $C_B(\Lambda) = C_W(\Lambda) = 0$. Left panel: the overlaid numbers indicate the search strategy that provides the leading constraint on $\Lambda$, with 1 (5) indicating the monophoton (monojet) channel. Right panel: the contours represent the bounds on $\Lambda$ in units of GeV.

DM singlet $\chi$ which couple via

$$\mathcal{L} \supset -y_\chi \bar{\chi} \chi S - \mu_p S|H|^2 - \lambda_p S^2 |H|^2. \quad (4.22)$$

Here, $y_\chi$ is a Yukawa coupling, $\mu_p$ and $\lambda_p$ provide a portal between the SM and the dark sector, and $H$ is the SM Higgs doublet. After EWSB, $S$ and $h$ (the neutral component of $H$, i.e. the SM Higgs boson) mix into mass eigenstates $h_1$ and $h_2$,

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h \\ S \end{pmatrix}, \quad (4.23)$$

where $\theta$ is the corresponding mixing angle. The Lagrangian then becomes

$$\mathcal{L} \supset \left( \frac{2M_h^2}{v} W^+ W^- + \frac{M_Z^2}{v} Z^+ Z^- - \sum_{f=q,t} \frac{m_f}{v} \bar{f} f \right) (h_1 \cos \theta + h_2 \sin \theta) - y_\chi \bar{\chi} \chi (h_2 \cos \theta - h_1 \sin \theta), \quad (4.24)$$

where the sum runs over all SM fermions and $v \approx 246$ GeV is the Higgs boson VEV.

In (4.24) $h_1$ is the scalar particle that has been observed at the LHC, while $h_2$ is a new scalar. For simplicity, we assume that $M_{h_1} < 2m_\chi < M_{h_2}$ such that there is only one relevant contribution to the process $W^+ W^- \rightarrow \bar{\chi} \chi$, shown in Figure 4.5.
The amplitude corresponding to this diagram is

\[ A = -\frac{M_W^2 y_\chi}{v} \sin \theta \cos \theta \times \frac{1}{q^2 - M_{h_2}^2} \epsilon(p_1) \cdot \epsilon(p_2) \bar{\chi} \chi, \tag{4.25} \]

where \( p_1 \) and \( \epsilon(p_1) \) (\( p_2 \) and \( \epsilon(p_2) \)) are the four-momentum and polarization vector of the \( W^+ \) (\( W^- \)), and \( q = p_1 + p_2 \). Assuming that \( M_{h_2} \gg q^2 \), and stripping off the external fields, we arrive at the amplitude for the point-like interaction

\[ A \left[ \begin{array}{c} W_+ \cr W_- \end{array} \right] = g_{\mu\nu} M_W^2 y_\chi \frac{\sin(2\theta)}{2v} \frac{1}{M_{h_2}^2}. \tag{4.26} \]

However, for the same process the operator \( O_W \) in \([4.2]\) would lead to an amplitude proportional to \([4.17]\), i.e. with a different Lorentz structure. Thus, the theory examined above cannot be a valid UV completion for the EFT model under consideration in this chapter.

In fact, operators such as those in \([4.2]\) arise in models of Rayleigh DM (see e.g. \([164, 166]\) for a more detailed discussion of these models). In particular, a valid UV completion which does induce the operators under consideration includes two new particles in addition to the DM candidate \( \chi \): a Dirac fermion \( \psi \) and a charged scalar \( \varphi \) which are both \( SU(2) \) doublets and heavier than \( \chi \). These couple to \( \chi \) via a
The model is described by (4.27) and contains a new Dirac fermion $\psi$ and charged scalar $\varphi$, which act as messenger particles, as well as the DM fermion $\chi$ \cite{165}.

Yukawa coupling denoted by $\lambda$, and the Lagrangian for this model is \cite{165}

$$
\mathcal{L} \supset \bar{\chi}(i\partial - m_\chi)\chi - \frac{1}{2} \delta m \chi^C\chi + \bar{\psi}(i\gamma - M_\psi)\psi
+ (D^\mu\varphi)\gamma D_\mu\varphi - M\varphi\varphi\gamma + \lambda\bar{{\psi}}\chi\varphi + \text{h.c.},
$$

where $D_\mu = \partial_\mu - igW^a_\mu\tau^a - ig'B_\mu/2$ is the covariant derivative associated with the $SU(2)_L \otimes U(1)_Y$ gauge bosons $W^a_\mu$ and $B_\mu$, respectively, and the $\tau^a$ are the $SU(2)_L$ generators obeying $\text{tr}(\tau^a\tau^b) = \delta^{ab}/2$ and related to the Pauli matrices by $\tau^a = \sigma^a/2$.

The second term in (4.27) corresponds to the Majorana mass splitting of the DM particle and, in the Weyl basis, $\chi^C = C\gamma^0\chi^*$. The properties of the charge conjugation matrix $C$ are summarised in Appendix A.

This model gives rise to DM interactions involving one or two gauge bosons. The latter scenario is of interest in this chapter as these interactions give rise to the operators (4.2) in the case that $m_\chi \ll M_\varphi, M_\varphi$ and the messenger states can be integrated out. Examples of one-loop Feynman diagrams that are generated in the model described by (4.27) are displayed in Figure 4.6.

If the new mediators are light, the high-$p_T$ gauge bosons which participate in the $E_T$ processes under consideration are able to resolve the substructure of the loops. This generically suppresses the cross sections compared to the EFT predictions \cite{111}. 

Figure 4.6: Examples of Feynman diagrams in models of Rayleigh DM corresponding to a possible UV completion of the EFT considered in this chapter. The model is described by (4.27) and contains a new Dirac fermion $\psi$ and charged scalar $\varphi$, which act as messenger particles, as well as the DM fermion $\chi$ \cite{165}. 
Consequently, the bounds on the interaction strengths between DM and SM gauge bosons will be somewhat weakened. Furthermore, the light mediators may be produced on-shell in $pp$ collisions, leading to pair production of the charged components of $\psi$ and $\varphi$. Events such as these could lead to signals consisting of charged particle tracks that disappear midway through the detector, at which point the charged particles decay \[166\]. In this case, the strongest bounds on the couplings could be obtained from collider searches for direct production of the states $\psi$ and $\varphi$, rather than $E_T$ searches.

4.3.5 Future Sensitivity

It is insightful to consider how the obtained limits on the new physics scale $\Lambda$ might improve at the 14 TeV LHC. We consider here only the monojet signal, applying the event selection criteria that have been used in the sensitivity study by ATLAS [167]. The most relevant cuts are

\[
\begin{align*}
    p_{T,j_1} &> 300 \text{ GeV}, \quad |\eta_{j_1}| < 2.0, \\
    p_{T,j_2} &> 50 \text{ GeV}, \quad |\eta_{j_2}| < 3.6, \\
    \Delta\phi_{j,E_T} &> 0.5,
\end{align*}
\]

where $\Delta\phi_{j,E_T}$ is the azimuthal angular separation between either jet and the missing transverse energy. This cut is imposed to suppress the background from misreconstructed multijet events. Jets are reconstructed using the anti-$k_t$ algorithm [107] with a radius parameter of $R = 0.4$. There is a veto applied to events which feature a third jet with $p_{T,j_3} > 50$ GeV and $|\eta_{j_3}| < 3.6$, and the missing transverse energy cut is $E_T > 800$ GeV. The $p_{T,j_1}$, $p_{T,j_2}$ and $E_T$ cuts are stricter here compared to (4.13) to avoid pile-up and enhance the signal-over-background ratio.

The dominant SM background is production of jets in association with a $Z$-boson which decays to a neutrino pair. The predicted fiducial cross section for this process is $\sigma_{\text{fid}}(pp \to Z(\bar{\nu}\nu) + 2j) = 0.5$ fb [167], assuming a total systematic uncertainty on the SM background of 5%. For the choice $C_B(\Lambda) = 0$, $C_W(\Lambda) = 1$ with


$C_B(\Lambda) = C_W(\Lambda) = 0$, it is found that with 25 fb$^{-1}$ of data, corresponding to the first year of running after the LHC upgrade to 14 TeV, one may be able to set a 95% CL bound of $\Lambda \gtrsim 1.3$ TeV for $m_\chi \lesssim 100$ GeV. This corresponds to an improvement of the bound on $\Lambda$ by more than a factor of 2 compared to the limit found earlier in this section for the 8 TeV LHC. With 300 fb$^{-1}$ and 3000 fb$^{-1}$ of accumulated data, the bound is only slightly enhanced to $\Lambda \gtrsim 1.4$ TeV. This highlights that at 14 TeV the sensitivity of $E_T^\text{+ jets}$ searches will soon become limited by the systematic uncertainties associated with the irreducible SM background. To what extent this limitation can be evaded by an optimisation of the monojet search strategy, or a better understanding of the $pp \rightarrow Z(\bar{\nu}\nu) + 2j$ background, would require a dedicated study.

### 4.4 Analysis of Jet-Jet Azimuthal Angular Correlations

Until this point the observables that have been analysed are insensitive to whether the $E_T$ signal is generated by an insertion of the effective operators $O_B (O_W)$ or $O_B^\sim (O_W^\sim)$. In Chapter 3 we saw that studying the azimuthal angle distributions of the tagging jets in $E_T^\text{+ 2j}$ events, where the interaction proceeds via a top quark loop in that simplified model, allows one to discern whether the DM-SM interactions proceed via a scalar or pseudoscalar mediator. This is also been examined for the case of Higgs boson production via VBF (see e.g. [118,168,169]) and for DM interactions with SM gauge bosons [142]. For the analysis here, in addition to the event selection criteria (4.28) the following VBF-like cuts are employed

\begin{equation}
\eta_{j_1} \cdot \eta_{j_2} < 0, \quad \Delta \eta_{j_1,j_2} > 2, \quad m_{j_1,j_2} > 1100 \text{ GeV}. \quad (4.29)
\end{equation}

These cuts are motivated by the kinematic analysis conducted in Section 3.2; they help to sculpt the angular correlations between the jets, and the invariant mass threshold also improves the signal-over-background ratio.
Figure 4.7: Azimuthal angle distributions at the 14 TeV LHC. The signal curves correspond to $C_B(\Lambda) = 0$, $C_W(\Lambda) = 1$ (red) and $C_\tilde{B}(\Lambda) = 0$, $C_\tilde{W}(\Lambda) = 1$ (blue), and both use $\Lambda = 1$ TeV and $m_\chi = 100$ GeV. For comparison the prediction of the dominant background process $pp \to Z(\bar{\nu}\nu) + 2j$, employing the same event selection criteria, is also shown (black).

To understand why the operators $O_B$ ($O_W$) and $O_\tilde{B}$ ($O_\tilde{W}$) lead to contrasting azimuthal angle distributions, we refer back to the kinematic analysis of Section 3.2, which showed that the differential cross section in loop-induced VBF-type processes is related to $\Delta \phi_{j_1j_2}$ by

$$d\sigma \sim F_1 \pm |F_9^-| \cos 2\Delta \phi_{j_1j_2}, \quad (4.30)$$

where the $+$ ($-$) sign is obtained for the CP-even (CP-odd) operator, and the $F_1$ and $F_9^-$ coefficient functions are discussed in depth in Section 3.2. Consequently, it is expected that the $\Delta \phi_{j_1j_2}$ spectra for $O_B$ and $O_W$ ($O_\tilde{B}$ and $O_\tilde{W}$) feature a peak (trough) around $\Delta \phi_{j_1j_2} = 0$ and a trough (peak) around $\Delta \phi_{j_1j_2} = \pi/2$.

Figure 4.7 shows the $\Delta \phi_{j_1j_2}$ spectra for the choices $C_B(\Lambda) = 0$, $C_W(\Lambda) = 1$ (red curve) and $C_\tilde{B}(\Lambda) = 0$, $C_\tilde{W}(\Lambda) = 1$ (blue curve). The dominant SM background process, $pp \to Z(\bar{\nu}\nu) + 2j$ is also shown for comparison (black curve). All predictions are obtained for the 14 TeV LHC and employ $\Lambda = 1$ TeV and $m_\chi = 100$ GeV. The
fiducial cross sections amount to 1.0 fb, independently of whether the insertion of $O_W$ or $O_{W'}$ is considered. The expected modulations of the distributions due to (4.30) are clearly observed, while the background is seen to be rather flat up to $\Delta \phi_{j_1j_2} \approx 2.6$, beyond which it rapidly drops to zero. The fiducial cross section for the background is 0.35 fb, implying a signal-over-background ratio of $S/\sqrt{B} \approx 8.4$, 29 and 93 for 25 fb$^{-1}$, 300 fb$^{-1}$ and 3000 fb$^{-1}$ of data, respectively.

These values of $S/\sqrt{B}$ suggest that running the 14 TeV LHC for a couple of years should yield a sufficient number of events to analyse the jet-jet azimuthal angular correlations. To further quantify this statement, a toy MC is used to generate event samples for both signals and background corresponding to 300 fb$^{-1}$ and 3000 fb$^{-1}$ of integrated luminosity. The resulting differential cross sections are fitted to

$$\frac{1}{\sigma} \frac{d\sigma}{d\Delta \phi_{j_1j_2}} = \sum_{n=0}^{2} a_n \cos (n\Delta \phi_{j_1j_2}).$$

(4.31)

The coefficient $a_0$ is fixed by the normalisation of the $\Delta \phi_{j_1j_2}$ spectrum, and the ratio $r_1 = a_1/a_0$ turns out to be rather insensitive to which type of higher-dimensional interaction is considered. In contrast, the combination $r_2 = a_2/a_0$ is a measure of the CP nature of the interactions that lead to the final state $[168]$. By comparison with (4.30) it is expected that this ratio is positive (negative) for the insertion of $O_B$ and $O_W$ ($O_{\tilde{B}}$ and $O_{\tilde{W}}$). It is stressed that by considering normalised $\Delta \phi_{j_1j_2}$ distributions, the theoretical uncertainties are reduced and the predictions become fairly independent of EFT assumptions.

The results for this toy MC are presented in Figure 4.8. The left (right) panel corresponds to 300 fb$^{-1}$ (3000 fb$^{-1}$) of data collected at the 14 TeV LHC. The expected azimuthal angle distributions for the signal plus background predictions are coloured red (blue) for $O_W$ ($O_{\tilde{W}}$). For comparison the SM-only result, divided by a factor of 3 for clarity, is also shown (grey). The solid curves illustrate the best fits to (4.31), restricting the azimuthal angular separation $\Delta \phi_{j_1j_2}$ to the range $[0, 2.5]$. For 300 fb$^{-1}$
Figure 4.8: Normalised $\Delta \phi_{j_1j_2}$ distributions for 300 fb$^{-1}$ (left) and 3000 fb$^{-1}$ (right) of 14 TeV LHC data. The red (blue) histogram shows the signal plus background prediction for $\mathcal{O}_W$ ($\mathcal{O}_{W'}$). The grey bar chart represents the expected SM background which, for better visibility, has been rescaled by a factor of 1/3. The solid curves represent the best fits of the form $a_0 + a_1 \cos \Delta \phi_{j_1j_2} + a_2 \cos 2 \Delta \phi_{j_1j_2}$. See text for further details.

In the case of 300 fb$^{-1}$ of data, the obtained central values and uncertainties for $r_2$ are

\[
(r_2)_{W+SM} = 0.15 \pm 0.10,
\]
\[
(r_2)_{W'+SM} = -0.45 \pm 0.14,
\]
\[
(r_2)_{SM} = -0.12 \pm 0.22.
\]

In the case of 3000 fb$^{-1}$ of data, these read

\[
(r_2)_{W+SM} = 0.18 \pm 0.03,
\]
\[
(r_2)_{W'+SM} = -0.40 \pm 0.04,
\]
\[
(r_2)_{SM} = -0.13 \pm 0.07.
\]

As expected, $r_2$ is indeed positive (negative) for $\mathcal{O}_W$ ($\mathcal{O}_{W'}$). Defining a significance as

\[
s_k = \frac{(r_2)_{k+SM} - (r_2)_{SM}}{(\Delta r_2)_{k+SM}},
\]

where $(\Delta r_2)_{k+SM}$ is the uncertainty on $(r_2)_{k+SM}$, we find

\[
\begin{align*}
300 \text{ fb}^{-1} : & \quad s_W = 2.7, \quad s_{W'} = -2.4, \\
3000 \text{ fb}^{-1} & \quad s_W = 10.3, \quad s_{W'} = -6.8.
\end{align*}
\]
This toy MC study for 300 fb$^{-1}$ (3000 fb$^{-1}$) of data therefore suggests that a distinc-
tion between the azimuthal angle distributions corresponding to $O_W$ and $O_{\tilde{W}}$ at the
5σ (17σ) level should be possible at the 14 TeV LHC.

It is emphasised that this study assumes a perfect detector and the cuts (4.29) have not been optimised to achieve the best significance. Once the data has been
gathered, it will become an experimental challenge to ascertain how stringent the
VBF-like selection criteria can be made in order to extract the most information on
the jet-jet azimuthal angular correlations for a given limited sample size.

The work presented in this chapter lead essentially to two key results. Firstly,
the suppression scale of the four dimension-7 operators that were considered can be
constrained by recent LHC searches for missing transverse energy in association with
various SM particles. The leading constraints come from monophoton or monojet
searches, depending on the choice of parameters, and for light DM and Wilson co-
efficients $|C_k(\Lambda)| \sim 1$ it was found that $\Lambda \gtrsim 600$ GeV. With 25 fb$^{-1}$ of 14 TeV
LHC data, monojet searches should be able to improve this bound to approximately
1.3 TeV. Further progress, however, will be hindered by the imperfect understanding
of irreducible LHC backgrounds such as $pp \rightarrow Z(\bar{\nu}\nu) + \text{jets}$. Secondly, by studying
the $\Delta\phi_{j_1,j_2}$ distributions for $E_T + 2j$ events, it was shown that it is possible to dis-
cern whether the interaction proceeds via the CP-even ($O_{B,W}$) or CP-odd ($O_{\tilde{B},\tilde{W}}$)
operators. Furthermore, these spectra are clearly distinct from the SM background,
and the predictions are only weakly dependent on the assumptions underlying the
EFT description. The toy MC study indicated that with 300 fb$^{-1}$ of 14 TeV LHC
data a distinction between the new-physics and SM-only hypotheses can be achieved
at a statistically significant level, and that the sensitivity of these searches is greatly
improved with 3000 fb$^{-1}$ of integrated luminosity.
Chapter 5

Discussion and Conclusions

This thesis began by reviewing the extensive evidence which suggests that there is a large amount of missing mass in the Universe. Undeniably, one of the largest mysteries of modern physics is the constitution of DM. Recent evidence, in particular, strongly indicates that the solution lies in the study of particle physics. Combined with the many unsolved problems of that field, and the potential for a DM candidate to address some of those issues, the search for particle DM is therefore extremely well motivated.

The approach considered in this work involves the construction of simplified DM models. In all cases, a fermionic DM particle $\chi$ is introduced which participates in certain new interactions with SM particles. It is clear that typically the leading collider signatures for production of DM via these interactions are SM particles in association with a large amount of missing transverse energy. Consequently, information about the DM-SM couplings is extracted by studying the SM final states of such events.

In Chapter 2, the model under consideration featured a new spin-1 s-channel mediator, a $Z'$ boson, which couples DM to the SM fermions. A key component of many simplified models is minimal flavour violation (MFV). The hypothesis was explained and it was explicitly shown that without enforcing MFV simplified models can give rise to large flavour-changing neutral currents (FCNCs) at tree-level. To bring the contributions in line with experimental constraints would require the mediator to be extremely heavy ($O(10^4)$ TeV, if the flavour structure is generic) and the $Z'$ could
not be produced at the LHC. The MFV framework, on the other hand, specifies that only Yukawa interactions are permitted to break the global flavour symmetry of the quark sector, preventing the presence of terms in the Lagrangian that can lead to tree-level FCNCs.

We next saw that by considering experimental searches for excesses in the dilepton and dijet channels constraints can be derived on the sizes of the SM couplings. The coupling to leptons in particular must be very small if one wishes to have a mediator which is not too heavy. The light quark couplings, on the other hand, are not so strongly constrained. This means that the $Z'$ should be produced in $pp$ collisions at the LHC. Furthermore, decays to DM pairs were found to make up just under 50% of the total width, implying that searches for missing transverse energy are a well motivated strategy for studying the model.

Consequently, monojet searches were then used to constrain the $M_{Z'}-m_\chi$ region of parameter space. It was found that for on-shell production of light DM, mediator masses below roughly 1 TeV are excluded. These results, however, are essentially identical irrespective of whether the mediating particle is a vector or axial vector. The chapter concluded with a comparison between the $E_T$ distributions corresponding to these two cases, and it was seen that these are impossible to distinguish from one another. So, while present monojet searches are certainly useful for deriving bounds on the size of interactions between SM and DM particles, they do not yield sufficient information to determine the structure of the couplings.

This motivated a study of more complex final states. Chapter 3 focused on events which were required to contain two jets in the final state, along with missing transverse energy. The model considered this time featured a spin-0 $s$-channel mediator. Enforcing MFV in this case leads to the presence of Yukawa couplings in the Lagrangian, which in turn means that the most relevant tree-level interactions with the SM are typically those featuring top quarks. Calculations were performed in an ef-
effective field theory (EFT) regime, whereby the mediator is integrated out and DM undergoes effective interactions directly to top quarks, and plots of the differential cross section as a function of the azimuthal angular separation were presented. It was found, as expected from the kinematic analysis, that the distributions corresponding to the scalar and pseudoscalar operators were clearly distinguishable from each other and from the dominant SM background.

The stability of this observable was analysed by studying the effects of varying the $E_T$ and $p_{T,j}$, cuts, QCD corrections, parton showering and hadronisation. The results showed that the spectra are still clearly distinct when considering all of these issues. The validity of the heavy top quark approximation and EFT description were also considered, and the spectra were again produced with the calculation performed in the full simplified model, involving the mediator and top quark loops. The obtained distributions demonstrated the expected shapes and, in fact, were slightly more distinct from one another in this case. This is in contrast to predictions of the monojet cross section, which are highly model dependent. The overall conclusion is that measurements of the azimuthal angular separation in $E_T + 2j$ events is a very robust observable for probing the structure of DM-SM interactions.

It is not unlikely that the interactions which couple SM and DM particles are not as simple as the tree-level exchange of a new mediator. In Chapter 4, our attention shifted to a scenario in which DM undergoes effective interactions with the SM gauge bosons. Using data from recent searches by the ATLAS and CMS collaborations for a variety of final states containing missing transverse energy, bounds were derived on the size of the suppression scale, $\Lambda$. It was found that across the Wilson coefficient $(C_B(\Lambda) - C_W(\Lambda))$ plane the strongest bounds on $\Lambda$ were provided by either monojet or monophoton searches, depending essentially on which of the Wilson coefficients was larger in magnitude. This was understood by examining the Feynman rules and the event selection criteria applied in the analysis. The results showed that for DM
masses of $m_{\chi} \lesssim 100$ GeV and Wilson coefficients $|C_k(\Lambda)| \simeq 1$ the present 8 TeV LHC searches allow values of $\Lambda \lesssim 600$ GeV to be excluded. Meanwhile, for smaller values of $|C_k(\Lambda)|$ this bound is less strict.

The smallness of the derived bounds on the suppression scale raises concerns about the validity of the EFT approach, and prompted a discussion about a possible UV completion. We saw that the simple tree-level exchange of a new scalar mediator does not lead to the correct Lorentz structure for the interactions, while a possible scenario involves DM coupling to the SM gauge bosons via a loop containing two new exchange particles, as in models of Rayleigh DM. It is interesting to note that if these additional states are light, they could be pair produced in $pp$ collisions at the LHC, potentially with distinct collider signatures. Direct searches for these particles could therefore give rise to stricter constraints than missing energy searches.

Considering then the azimuthal angular separation of the jets in $E_T + 2j$ events, the spectra due to the insertion of $O_W(\Lambda)$ and $\tilde{O}_W(\Lambda)$ were compared. Once again, it was observed that the distributions are clearly distinguishable from each other and from that of the dominant SM background. This demonstrates the scope of this observable to probe the Lorentz structure of quite different types of interactions between SM and DM particles.

Looking ahead to prospects at the 14 TeV LHC run, it was shown that the bounds on the suppression scales entering the gauge boson couplings will be improved dramatically once 25 fb$^{-1}$ of data has been collected. With more data, however, this bound does not improve significantly due to the systematic uncertainties associated with the irreducible SM background. To make further progress in this regard will therefore require at least a better understanding of the background in addition to an optimisation of the monojet search strategy. In the case of the $\Delta\phi_{j_1,j_2}$ distributions, on the other hand, it was demonstrated that the distinction between the spectra should be enhanced considerably as more data is collected at the 14 TeV LHC. This
will also be true for the simplified model considered in Chapter 3.

In summary, it is evident that the azimuthal separation of the jets in $E_T + 2j$ events is a powerful probe of the Lorentz structure of DM interactions. It is therefore imperative that, in the event of a discovery, ATLAS and CMS study these distributions for final states beyond $E_T + j$. 
Appendix A

Absence of a Vector Coupling for Majorana Fermions

In the case that the DM particles are Majorana fermions, the vector coupling vanishes. This is verified by the calculation presented in this appendix.

Herein the gamma matrices are represented in the Weyl basis, i.e.

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu} \\ \bar{\sigma}^{\mu} & 0 \end{pmatrix}, \quad \sigma^{\mu} \equiv (\sigma^{0}, \sigma^{i}), \quad \bar{\sigma}^{\mu} \equiv (\sigma^{0}, -\sigma^{i}), \quad (A.1)$$

where $\sigma^{0}$ is the $2 \times 2$ identity matrix, and $\sigma^{i}$ are the Pauli matrices,

$$\sigma^{0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^{1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^{2} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^{i} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (A.2)$$

A field $\psi$ is a Majorana field if $\psi = \psi^{C}$, where the charge conjugate field is defined as $\psi^{C} = C\bar{\psi}^{T} = C\gamma^{0}\psi^{*}$. Here, we define the charge conjugation matrix as $C = i\gamma^{2}\gamma^{0}$.

It has the following properties

$$C^{\dagger} = C^{-1}, \quad C^{T} = -C, \quad C(\gamma^{\mu})^{T}C^{-1} = -\gamma^{\mu}, \quad \gamma^{0}C^{\dagger} = -\gamma^{0}. \quad (A.3)$$

Consider the vector current $\bar{\psi}^{C}\gamma^{\mu}\psi^{C}$. Using the above relations, and the algebra of
spinors and the Dirac matrices, this can be manipulated as follows

\[
\bar{\psi}^C \gamma^\mu \psi^C = (\psi^C)^\dagger \gamma^0 \gamma^\mu \psi^C \\
= (C^* \gamma^0 \psi^*)^\dagger \gamma^0 \gamma^\mu C^\dagger \gamma^0 \psi^* \\
= \psi^T \gamma^0 C^\dagger \gamma^0 \gamma^\mu C^\dagger \gamma^0 \psi^* \\
= -\psi^T C^\dagger (\gamma^0)^2 \gamma^\mu C \gamma^0 \psi^* \\
= -\psi^T C^{-1} \gamma^\mu C \bar{\psi}^T \\
= \psi^T (\gamma^\mu)^T \bar{\psi}^T.
\] (A.4)

In index notation, the final expression is

\[
\psi^\alpha (\gamma^\mu)_{\beta \alpha} \bar{\psi}^\beta = -\bar{\psi}^\beta (\gamma^\mu)_{\beta \alpha} \psi^\alpha,
\] (A.5)

where the minus sign arises due to the fact that the spinors fields anticommute. The final expression is equivalent to \(-\bar{\psi} \gamma^\mu \psi\). Now, since for a Majorana field \(\psi = \psi^C\), this calculation has shown that

\[
\bar{\psi} \gamma^\mu \psi = -\bar{\psi} \gamma^\mu \psi,
\] (A.6)

and therefore this must be zero and the vector current vanishes.
Appendix B

Calculation of $\sigma(pp \rightarrow Z')$ in MadGraph

The version used to generate events was MadGraph5_aMC@NLO v2.2.3 with the simplified DM model file obtained from [170]. This model introduces DM particles and mediators according to a simplified model consistent with that examined in Chapter 2. More information can be found at the referenced web page. The model file should be downloaded and placed in the MadGraph ‘models’ directory.

MadGraph is launched from the main directory with

$ bin/mg5_aMC

From the interface, the model can be loaded by entering

MG5_aMC> import model DMsimp_UFO

To generate the $Z'$ production process,

MG5_aMC> generate p p > y1

This produces the required diagrams, which can now be exported to a folder in the MadGraph directory by entering

MG5_aMC> output [directory name]
where [directory name] should be replaced with a title of the user’s choice. Exiting MadGraph and navigating to the newly created directory, the user will find a subdirectory named ‘Cards’. Here, the file param_card.dat can be modified to adjust the model parameters used in generating events, while the file run_card.dat controls the run parameters (such as beam energy and type, PDF set, renormalisation and factorisation scales and cuts). Once the desired changes have been made, events can be generated by running (from the process directory)

```
$ bin/generate_events
```

Results are then displayed in crossx.html.

For the work in Chapter 2, the cross section was calculated for values of $M_{Z'}$ between 200 GeV and 5 TeV, at intervals of 100 or 200 GeV. All vector couplings at this stage were set to 1 while the axial vector couplings were set to 0. This is justified since we are only considering the vectorial $Z'$ model, and since $g_x$ and $g_t$ do not enter the process $pp \to Z'$. We wish to vary $g_q$ but, since the process is directly proportional to $g_q^2$, it was set to 1 in the input parameters for MadGraph and then the resulting cross sections could simply be multiplied by the desired value of $g_q^2$ in the analysis. The proton beam energies were set to 4 TeV, such that the total centre-of-mass collision energy was 8 TeV, the NN23LO1 PDF set was used [171], and 10000 events were generated for each parameter point. The cross section data was then imported into Mathematica [158], in which the analysis was performed and the plots generated.
Appendix C

Calculation of the Dark Matter Relic Density

It is essential that any viable DM candidate is able to reproduce the observed relic density, $\Omega_\chi h^2 \approx 0.119$ [24]. Here is presented a standard approach to determining the relic density of DM. This follows the calculations presented in e.g. [38, 172–176] in which more complete discussions are available.

Typically, it is assumed that in the early Universe (i) DM particles were produced in collisions between the particles that constituted the thermal plasma, and (ii) the temperatures were sufficiently high ($T \gg m_\chi$) such that DM particles annihilated to produce SM particles (and vice versa) frequently. In other words, DM was in thermal equilibrium. The rates of production and annihilation were then equal,

$$\Gamma_{\text{prod}} = \Gamma_{\text{ann}} = \langle \sigma_{\text{ann}} v \rangle n_{\text{eq}}, \quad (C.1)$$

where $\sigma_{\text{ann}}$ is the total annihilation cross section for DM particles, $v$ is the relative velocity of annihilating particles, $n_{\text{eq}}$ is the equilibrium DM number density (per comoving volume), and the angled brackets indicate thermal averaging. We assume that the interactions are strong enough that DM remained in thermal equilibrium after $T$ dropped below $m_\chi$. Their number density was then given by

$$n_{\text{eq}} = g_X \left( \frac{m_\chi T}{2\pi} \right)^{3/2} e^{-m_\chi/T}, \quad (C.2)$$
where $g_\chi$ is the number of degrees of freedom. At this point it is important to note that we assume no asymmetry between DM particles and antiparticles, i.e. $n_\chi = n_{\bar{\chi}}$. Equation (C.2) shows that while DM remains in thermal equilibrium, the number density decreased as the Universe cooled or, equivalently, as the Universe expanded. Consequently, annihilations become less frequent and eventually stop once $\Gamma_{\text{ann}}$ drops below the expansion rate of the Universe $H(t) = \dot{a}(t)/a(t)$, where $a(t)$ is the scale factor. From this point, referred to as ‘freeze-out’, the DM number density remained constant. To determine the current density of DM particles, we begin by writing the Boltzmann equation,

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{ann}}v \rangle (n^2 - n_{\text{eq}}^2).$$

(C.3)

This can be re-expressed in terms of $Y \equiv n/s$ where $s$ is the entropy density. Assuming that entropy is not produced, the total entropy $S = a^3(t)s$ is constant and

$$\frac{ds}{dt} = -3 \frac{da}{dt}s = -3Hs.$$  

(C.4)

Thus, one obtains

$$\frac{dY}{dt} = \frac{1}{s} \frac{dn}{dt} + 3H \frac{n}{s}. $$

(C.5)

Combining this with (C.3) the Boltzmann equation can be re-written in terms of $Y$ as

$$\frac{dY}{dt} = -s \langle \sigma_{\text{ann}}v \rangle (Y^2 - Y_{\text{eq}}^2).$$

(C.6)

We can now apply a change of variables to describe the evolution in terms of temperature rather than time. Setting $x \equiv m_\chi/T$ gives

$$\frac{dY}{dx} = m_\chi \frac{1}{x^2} \frac{ds}{dT} \langle \sigma_{\text{ann}}v \rangle (Y^2 - Y_{\text{eq}}^2).$$

(C.7)

In a radiation dominated universe, the Friedmann equation is

$$H^2 = \frac{8\pi G}{3} \rho,$$

(C.8)
where $G$ is the gravitational constant. The entropy and energy densities, $\rho$ and $s$, can be written in terms of effective degrees of freedom $g_{\text{eff}}$ and $h_{\text{eff}}$ as

$$s = h_{\text{eff}}(T)\frac{2\pi^2}{45}T^3, \quad \rho = g_{\text{eff}}(T)\frac{\pi^2}{30}T^4.$$ (C.9)

Inserting these into (C.7) yields

$$\frac{dY}{dx} = -\frac{m_\chi M_{\text{Pl}}}{x^2} \sqrt{\frac{\pi g_*}{45}} \langle \sigma_{\text{ann}} v \rangle (Y^2 - Y_{\text{eq}}^2),$$ (C.10)

where

$$\sqrt{g_*} = \frac{h_{\text{eff}}}{\sqrt{g_{\text{eff}}}} \left(1 + \frac{T}{3h_{\text{eff}}} \frac{dh_{\text{eff}}}{dT}\right).$$ (C.11)

The thermally averaged annihilation cross section can be expanded in powers of $v^2$ giving $\langle \sigma_{\text{ann}} v \rangle = a + b\langle v^2 \rangle + \mathcal{O}(\langle v^4 \rangle) \approx a + 6b/x$. To calculate the relic density at present, we require the value of $Y$ as measured today, $Y_0$. This can be obtained by integrating (C.10) and is approximately given by

$$Y_0 \approx \sqrt{\frac{45}{\pi g_*}} \frac{x_f}{m_\chi M_{\text{Pl}}(a + 3b/x_f)},$$ (C.12)

where $x_f = m_\chi/T_f$ for the freeze-out temperature $T_f$. The relic density is then $\Omega_\chi = \rho_\chi^0/\rho_{\text{crit}}$, with $\rho_\chi^0 = Y_0 s_0 m_\chi$ and $\rho_{\text{crit}} = 3H^2 M_{\text{Pl}}^2/(8\pi)$. Putting everything together, one obtains

$$\Omega_\chi h^2 \approx \frac{1.07 \times 10^9}{\text{GeV}} \frac{x_f}{M_{\text{Pl}} \sqrt{g_*}(a + 3b/x_f)}.$$ (C.13)

In the above formula, $M_{\text{Pl}} = 1.22 \times 10^{19}$ GeV denotes the Planck mass, $g_* \in [80, 90]$ is the number of relativistic degrees of freedom at freeze-out, and $x_f \in [20, 30]$.
Appendix D

Constraints on Simplified Spin-0 Dark Matter Models from Electric Dipole Moments

In Chapter 3 an EFT approach is introduced whereby the heavy spin-0 mediator of the simplified model can be integrated out to yield effective operators that couple DM to SM fermions. It is also possible to write down composite operators that contain both a scalar and pseudoscalar current. In this appendix, it is shown that such operators are strongly constrained since they contribute to the EDM of the electron. The methods presented here follow those of \[177,178\].

From (3.1) and (3.2) it is evident that, in the case of a mixed mediator $\varphi$, the Lagrangian would in general contain the following interaction terms with a given SM fermion $f$

$$
\mathcal{L} \supset -\frac{m_f}{v} (g_f^S \bar{f} f + ig_f^P \bar{f} \gamma_5 f) \varphi. \tag{D.1}
$$

This generates an effective Lagrangian, leading to an EDM for the electron,

$$
\mathcal{L}_{\text{eff}}^e = -d_e \frac{i}{2} \bar{e} \sigma^{\mu\nu} \gamma_5 e F_{\mu\nu}, \tag{D.2}
$$

where $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$ and $F_{\mu\nu}$ is the field strength tensor of QED. The corresponding two-loop diagrams of Barr-Zee type\[^1\] are shown in Figure D.1. Due to the presence

\[^1\]One-loop contributions are suppressed by additional powers of the electron Yukawa coupling and the electron mass, rendering them negligibly small \[179\].
of Yukawa couplings in (D.1), the contribution from internal top quark loops will dominate those from other SM fermions, which can therefore be safely ignored. The graph involving a photon exchange leads to the contribution

$$\frac{d_e}{e} = \frac{16}{3} \frac{\alpha}{(4\pi)^3} \sqrt{2} G_F m_e \left[ g_e^S g_t^P f_1(x_{t/\phi}) + g_e^P g_t^S f_2(x_{t/\phi}) \right], \quad (D.3)$$

where $x_{t/\phi} = m_t^2/M_{\phi}^2$ and the loop functions $f_{1,2}(x)$ can be written in the following way

$$f_1(x) = \frac{2x}{\sqrt{1 - 4x}} \left[ \text{Li}_2 \left( 1 - \frac{1 - \sqrt{1 - 4x}}{2x} \right) - \text{Li}_2 \left( 1 - \frac{1 + \sqrt{1 - 4x}}{2x} \right) \right], \quad (D.4)$$

$$f_2(x) = (1 - 2x) f_1(x) + 2x (\ln x + 2).$$

with $\text{Li}_2(x) = - \int_0^x du \ln(1 - u)/u$ the usual dilogarithm. It is noted that, in the limit $x \to 0$ these loop functions scale as $f_{1,2} \sim x \ln^2 x$. In addition to (D.3), two-loop diagrams involving the exchange of a virtual $Z$ boson lead to a correction to $d_e/e$. Due to charge conjugation invariance only the vector couplings of the $Z$ boson enter the Barr-Zee expression for the electron EDM $[179]$. As a result the $Z$-boson contribution is suppressed by $(1 - 4\sin^2 \theta_w)(1 - \frac{8}{3} \sin^2 \theta_w) \approx 0.03$ with $\theta_w$ the weak mixing angle, and thus it is neglected in what follows.$^2$

$^2$The analytic result for the $Z$-boson contribution to $d_e/e$ proportional to the combination $\tilde{g}_e g_t$ of couplings has recently been given in [180].
Evaluating the prefactor in (D.3) with $\alpha = \alpha(0) \approx 1/137$, $G_F \approx 1.166 \times 10^{-5}$ GeV$^{-2}$ and $m_e \approx 5.11 \times 10^{-4}$ GeV [37], one obtains

$$\frac{d_e}{e} = 3.26 \times 10^{-27} \text{ cm} \left[ g^S_e g^P_l f_1(x_{t/\phi}) + g^P_e g^S_l f_2(x_{t/\phi}) \right].$$

The present 90% CL limit [181]

$$\left|\frac{d_e}{e}\right| < 8.7 \times 10^{-29} \text{ cm},$$

thus implies that

$$\left|g^S_e g^P_l f_1(x_{t/\phi}) + g^P_e g^S_l f_2(x_{t/\phi})\right| < 2.7 \times 10^{-2}.$$  

The limit (D.7) provides severe restrictions on the parameter space of our simplified DM model. In fact, there are two ways to avoid it: first, by making the mediator $\phi$ very heavy or, second, by choosing the products $g^S_e g^P_l$ and $g^P_e g^S_l$ of the couplings to be sufficiently small. Both options are illustrated in Figure D.2. The left panel shows the value of $|d_e/e|$ as a function of the mediator mass $M_\phi$ fixing the couplings $g^S_e g^P_l = g^P_e g^S_l = g^P_t g^P_l = 1$. The part of the parameter space that is excluded at 90% CL by the present bound (D.6) on the magnitude of the electron EDM is shown in grey. From the orange curve, we observe that for this choice of couplings the mediator mass has to satisfy $M_\phi \gtrsim 11.8$ TeV in order not to violate the constraint imposed by $|d_e/e|$. To illustrate the prospects of the constraints the future hypothetical bound $|d_e/e| \lesssim 3 \times 10^{-31} \text{ cm}$ [182] is indicated by the dotted black line. It constitutes a factor of 290 improvement over the current best limit (D.6) and represents the ultimate reach of existing electron EDM experiments such as ACME [183]. One sees that by achieving such a sensitivity one would push the bound on the mediator mass up to $M_\phi \gtrsim 350$ TeV. The right panel depicts the allowed parameter space in the $|g^S_e| - |g^P_l|$ plane, employing $g^P_e = g^S_l = 0$ and $M_\phi = 300$ GeV (orange contour). Numerically, it is found that for this parameter choice the limit (D.6) translates into $|g^S_e g^P_l| < 1.8 \times 10^{-2}$, while an improvement of the present bound by a factor of 290
Figure D.2: Left panel: prediction for $|d_e/e|$ as a function of the mediator mass $M_\phi$ assuming $g^S_e = g^P_e = g^S_i = g^P_i = 1$ (orange curve). The grey region indicates the parameter space excluded by the present constraint (D.6) on the electron EDM, while the dotted black line illustrates the ultimate reach on $|d_e/e|$ achievable at existing experiments such as ACME [182, 183]. Right panel: allowed parameter space in the $|g^S_e| - |g^P_i|$ plane for $g^P_i = g^S_i = 0$ and $M_\phi = 300$ GeV. The orange contour indicates the present-day limit, while the dotted black line corresponds to an assumed future bound on the electron EDM of $|d_e/e| \lesssim 3 \times 10^{-31}$ cm.

would lead to $|g^S_e g^P_i| \lesssim 6 \times 10^{-5}$. The latter constraint is indicated in the right panel by the dotted black contour.

From (D.7) it is clear that the electron EDM constraint on this simplified DM model vanishes in the scenario that the mediator does not couple to electrons, i.e. $g^S_e g^P_i = 0$. In this case, one can instead derive constraints on the model by considering the EDM of the neutron. This is not studied here since it is not the case in Chapter 3 that the electron couplings are explicitly set to zero. In [177] the neutron EDM constraints on the couplings of a CP-mixed Higgs boson are examined, and the obtained bound is found to be weaker by a factor of roughly 10 compared to that for the electron EDM.
Appendix E

Generating Events Using FeynRules, MadGraph, PYTHIA and Delphes

This appendix sets out the details of how the cross sections corresponding to the various $E_T$ signals considered in Chapter 4 were calculated. The process first utilises FeynRules \[156\,157\], a Mathematica \[158\] package which computes the amplitudes for process in a given particle physics model. To do this, the package requires a Lagrangian in the form of a model file. The SM Lagrangian is pre-packaged, and the FeynRules website has some downloadable beyond-the-SM model files. For other new physics models, the user must input the appropriate terms into a custom file.

For the operators (4.2) the additional DM fermion fields must be specified:

```mathematica
M$ClassesDescription = {
  F[5] == {
    ClassName -> x,
    SelfConjugate -> False,
    Mass -> {Mx, 10},
    Width -> 0,
    QuantumNumbers -> {Q -> 0, LeptonNumber -> 0},
    PropagatorLabel -> "x",
    PropagatorType -> Straight,
    PropagatorArrow -> Forward,
    PDG -> {19},
    Partic
```
Next, the couplings given by \([4.2]\) are input as

\[
\text{MParameters} = \{ \\
\quad c_1 = \{ \\
\quad \text{ParameterType} \rightarrow \text{External}, \\
\quad \text{BlockName} \rightarrow \text{DMCOUPLING}, \\
\quad \text{OrderBlock} \rightarrow 1, \\
\quad \text{Value} \rightarrow 1.0, \\
\quad \text{InteractionOrder} \rightarrow \{\text{DM}, 1\}, \\
\quad \text{TeX} \rightarrow \text{Subscript}[c,1], \\
\quad \text{Description} \rightarrow "\text{Coupling (bar x x)} \\
\quad \quad B_{\mu \nu} B^{\mu \nu}" \\
\}, \\
\quad c_2 = \{ \\
\quad \text{ParameterType} \rightarrow \text{External}, \\
\quad \text{BlockName} \rightarrow \text{DMCOUPLING}, \\
\quad \text{OrderBlock} \rightarrow 2, \\
\quad \text{Value} \rightarrow 1.0, \\
\quad \text{InteractionOrder} \rightarrow \{\text{DM}, 1\}, \\
\quad \text{TeX} \rightarrow \text{Subscript}[c,2], \\
\quad \text{Description} \rightarrow "\text{Coupling (bar x x)} \\
\quad \quad W_{\mu \nu}^i W^{i, \mu \nu}" \\
\}, \\
\quad c_3 = \{ \\
\quad \text{ParameterType} \rightarrow \text{External}, \\
\quad \text{BlockName} \rightarrow \text{DMCOUPLING}, \\
\quad \text{OrderBlock} \rightarrow 3, \\
\quad \text{Value} \rightarrow 1.0, \\
\quad \text{InteractionOrder} \rightarrow \{\text{DM}, 1\}, \\
\quad \text{TeX} \rightarrow \text{Subscript}[c,3], \\
\quad \text{Description} \rightarrow "\text{Coupling (bar x gamma_5 x)} \\
\quad \quad B_{\mu \nu} \tilde{B}^{\mu \nu}" \\
\}, \\
\quad c_4 = \{ \\
\quad \text{ParameterType} \rightarrow \text{External}, \\
\quad \text{BlockName} \rightarrow \text{DMCOUPLING}, \\
\quad \text{OrderBlock} \rightarrow 4, \\
\quad \text{Value} \rightarrow 1.0, \\
\quad \text{InteractionOrder} \rightarrow \{\text{DM}, 1\}, \\
\quad \text{TeX} \rightarrow \text{Subscript}[c,4], \\
\quad \text{Description} \rightarrow "\text{Coupling (bar x gamma_5 x)} \\
\quad \quad \tilde{W}_{\mu \nu}^i i \tilde{W}^{i, \mu \nu}" \\
\}, \\
\quad c_5 = \{ \\
\quad \text{ParameterType} \rightarrow \text{External}, \\
\quad \text{BlockName} \rightarrow \text{DMCOUPLING}, \\
\quad \text{OrderBlock} \rightarrow 5, \\
\quad \text{Value} \rightarrow 1.0, \\
\quad \text{InteractionOrder} \rightarrow \{\text{DM}, 1\}, \\
\quad \text{TeX} \rightarrow \text{Subscript}[c,5], \\
\quad \text{Description} \rightarrow "\text{Coupling (bar x x)} \\
\quad \quad B_{\mu \nu} \tilde{B}^{\mu \nu}" \\
\\},
Note that the four operators involving the CP-odd current $\bar{\chi}_5 \gamma_5 \chi$ are provided, although they were not considered in Chapter 4 for reasons given therein. Finally, the terms in the DM Lagrangian are specified by

\begin{align*}
\text{Lc1} & := c1{xbar.x}*FS[B, mu, nu]*FS[B, mu, nu]; \\
\text{Lc2} & := c2{xbar.x}*FS[Wi, mu, nu, i]*FS[Wi, mu, nu, i]; \\
\text{Lc3} & := c3{xbar.Ga[5].x}*FS[B, mu, nu]*Dual[FS][B, mu, nu]; \\
\text{Lc4} & := c4{xbar.Ga[5].x}*FS[Wi, mu, nu, i]*Dual[FS][Wi, mu, nu, i]; \\
\text{Lc5} & := c5{xbar.x}*FS[B, mu, nu]*Dual[FS][B, mu, nu]; \\
\text{Lc6} & := c6{xbar.x}*FS[Wi, mu, nu, i]*Dual[FS][Wi, mu, nu, i]; \\
\text{Lc7} & := c7{xbar.Ga[5].x}*FS[B, mu, nu]*FS[B, mu, nu]; \\
\text{Lc8} & := c8{xbar.Ga[5].x}*FS[Wi, mu, nu, i]*FS[Wi, mu, nu, i]; \\
\text{Lx} & := \text{Lc1} + \text{Lc2} + \text{Lc3} + \text{Lc4} + \text{Lc5} + \text{Lc6} + \text{Lc7} + \text{Lc8};
\end{align*}
FeynRules can be launched from a Mathemtica notebook by entering

```mathematica
$FeynRulesPath = SetDirectory[ <FeynRules Directory> ];
<< FeynRules';
```

where `<FeynRules Directory>` is the path of the user's FeynRules package. The appropriate models are loaded using

```mathematica
LoadModel["SM.fr","DMEW.fr"]
```

where, in this case, `SM.fr` is the SM model file and `DMEW.fr` is the model file containing the DM inputs given above. This assumes the model files are in the FeynRules directory. MadGraph accepts as input a Universal FeynRules Output (UFO) file \[159\], which can be simply produced by executing

```mathematica
WriteUFO[LGauge, LHiggs, LFermions, LYukawa, LGhost, Lx];
```

To shower MadGraph events with PYTHIA and run the through the fast-detector simulation Delphes, one first has to install PYTHIA and Delphes. This can be achieved by using the MadGraph command line interface:

```
MG5_aMC> install pythia-pgs
MG5_aMC> install Delphes
```

These commands automatically download PYTHIA and Delphes and install the packages for later use.

After generating a process and exporting it to a folder, as described in Appendix[13] one then has to adapt the files `pythia_card.dat` and `delphes_card.dat`. These files control the running of PYTHIA and Delphes, respectively, and are both located in the subdirectory named ‘Cards’. To simulate the $E_T$ signals in Chapter[4] the default settings of PYTHIA as specified in `pythia_card_default.dat` were used, while the file `delphes_card_ATLAS.dat` or `delphes_card_CMS.dat` was chosen depending on
whether a given $E_T$ signature was searched for by ATLAS or CMS. Once **PYTHIA** and **Delphes** are correctly set up, one simply executes

```
$ bin/generate_events
```

One is then presented with options for what packages one wants to run on the generated events:

The following switches determine which programs are run:

1. Run the pythia shower/hadronization: `pythia=OFF`
2. Run PGS as detector simulator: `pgs=OFF`
3. Run Delphes as detector simulator: `delphes=OFF`
4. Decay particles with the MadSpin module: `madspin=OFF`
5. Add weight to events based on coupling parameters: `reweight=OFF`

Either type the switch number (1 to 5) to change its default setting, or set any switch explicitly (e.g. type 'madspin=ON' at the prompt).

Type '0', 'auto', 'done' or just press enter when you are done.

```
[0, 1, 2, 3, 4, 5, auto, done, pythia=ON, ... ]
```

**PYTHIA** and **Delphes** are invoked by pressing 1, 3 and enter twice. The **PYTHIA** output of each run can be found in `tag_1_pythia_events.lhe`, while **Delphes** writes its results into `tag_1_delphes_events.lhco`. Both files are text based and contain the full event information. These files can then be analysed with a **Mathematica** code that implements the necessary cuts for each $E_T$ search.
References


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