

COMPETING SALES CHANNELS WITH CAPTIVE CONSUMERS*

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We study strategic interactions in markets in which firms sell to consumers both directly and via a competitive channel, such as a price comparison website or marketplace, where multiple sellers' offers are visible at once. We ask how a competitive channel's size influences market outcomes when some consumers have limited price information. A bigger competitive channel means that more consumers compare prices, increasing *within*-channel competition. However, we show that such seemingly pro-competitive developments can raise prices and harm consumers by weakening *between*-channel competition.

Many goods and services can be purchased either through a seller's *direct channel* (e.g., their own store or website) or through a *competitive channel* where multiple sellers' prices can be considered simultaneously. Competitive channels (CCs) take a variety of forms; in the utilities and financial services industries, price comparison websites allow consumers to view prices for products such as broadband or insurance. Figure 1 shows an example. For durable goods, online marketplaces such as eBay are commonly used by sellers as an additional sales channel, where they compete alongside other sellers of the same product. Offline, marketplaces such as shopping malls also allow consumers to compare multiple sellers' offers in one location. Across various markets, advisers and brokers also commonly serve as competitive channels for consumers.

Competitive channels facilitate price comparison and so strengthen competition between sellers. But the operators of competitive sales channels are economic actors too, and their incentives need not align with the interests of consumers.¹ Such concerns have led competition authorities in various jurisdictions to more closely scrutinise industries where CCs play a significant role, including the fear that CCs may distort competition between sellers in harmful ways.² This paper contributes to the ongoing debate by studying the implications of competition between, and within, sales channels. We recognise the strategic agency of the competitive channel and examine when and how the direct sales channel serves to discipline a CC's behaviour and vice versa.

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This paper was received on 24 August 2020 and accepted on 10 September 2021. The Editor was Gilat Levy.

The authors are grateful to the Editor, Gilat Levy, and anonymous referees for their helpful suggestions and comments. We thank Mark Armstrong, Dan Bernhardt, Sushil Bikhchandani, Alessandro Bonatti, Alexandre de Cornière, Jacques Crémer, Anna D'Annunzio, Maarten Janssen, Andrea Mantovani, José Moraga-González, Alexei Parakhonyak, Martin Peitz, Markus Reisinger, Andrew Rhodes, Daniel Sgroi, Shiva Shekhar, Sandro Shelegia, Anton Sobolev and Christopher Wilson for useful conversations; and participants at various conferences and seminars.

Taylor is grateful for financial support from the Carnegie Corporation of New York.

¹ Some recent work highlights practices that reduce the benefits of CCs, such as most favoured nation clauses (MFNs, e.g., Edelman and Wright, 2015; Johnson, 2017) and high commission levels (e.g., Ronayne, 2021). An MFN is a contractual clause requiring that a given sales channel (typically a CC) has the lowest price in the market. Such clauses have been banned in many jurisdictions following evidence that they suppress competition.

² For examples, see work by, or commissioned by, the OECD (Hviid, 2015), the EU (EU Competition Authorities, 2016) and the UK's Competition and Markets Authority (CMA, 2017c).




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Fig. 1. Example of a Competitive Sales Channel: Comparing Broadband Internet Prices on *comparethemarket.com*.

The relative importance of direct and competitive channels has varied across markets and over time. Indeed, technology has facilitated the emergence and rapid growth of new kinds of competitive channels. Moreover, the operators of such channels often invest heavily in marketing, leading many to become household names in their own right, from important regional brands like Compare The Market in the UK, to global platforms such as eBay.

Unlike existing work, we introduce a model that allows both competitive and direct channels to vary in their market power, or ‘size’. We use our model to study the effects of variations in the relative size of the competitive channel and show how this is an important determinant of market prices and consumer surplus. The central tension our work identifies is that of *within-* versus *between-*channel competition: competitive channels offer environments within which competition can be intense; but when they themselves become dominant, they can exploit their power in ways that relax the competitive constraint channels impose on each other. This can make consumers worse off, even as they become more informed about competing price offers.

We bring out this tension with a model in which the key component is consumers’ (limited) price information. Heterogeneity of consumer information is arguably a core feature of the modern retail environment’s extensive and crowded consumer markets. In our model, if all consumers had all available price information, price would equal the marginal cost of production and the law of one price would realise in a symmetric equilibrium. At the other extreme, if all consumers

were captive, that is, aware of prices in only one location (be it a seller or a CC), the only price on the market would be the monopoly price. If, however, there is heterogeneity in information, with some consumers captive while others compare prices, richer results can be obtained. It is in these intermediate cases that we operate, characterise equilibrium, and deliver our results.

To be more precise, the actors in our model are sellers, a competitive sales channel and a mass of consumers. Sellers set prices: one for consumers who buy direct and, if they wish to, another for consumers who buy via the competitive channel. The competitive channel sets a fee that is paid by a seller each time it makes a sale through the CC. We adopt a clearinghouse framework à la Varian (1980), a paradigm set up precisely to examine the implications of heterogeneity in consumers' information about prices. Specifically, some consumers are 'shoppers' who buy at the lowest price available anywhere in the market. The remainder of the consumers are 'captives' and check only one sales channel, either a seller's direct channel or the competitive channel, and buy at the best price they see there. Captives can be thought of as being directed by brand marketing, facing high search or information frictions, suffering from default bias, finding a channel particularly salient, or simply being relatively inactive or naive. The number of captives a seller or CC has is a measure of its market power or size. We refer collectively to the masses of captives and shoppers as the 'market composition'.

We first provide a complete characterisation of the equilibrium prices and fees for any given market composition. We show that equilibrium strategies have a sharp, dichotomous structure, falling into one of two qualitatively distinct regimes: low fee and high fee. In the case of a relatively small CC (i.e., a CC with few captives), the CC sets a *low fee* and hosts the lowest prices in the market. Shoppers buy through the competitive channel, while sellers set a high direct price to exploit their captive consumers. By contrast, a relatively large CC demands a *high fee*, exploiting its position as bottleneck provider of access to its captive consumers. To avoid paying the high fee, sellers compete for shoppers via direct prices, undercutting the prices on the competitive channel. In this regime it is direct prices that are the lowest in the market.

We analyse the effects of market composition on equilibrium prices and consumer surplus. As the competitive channel becomes larger, we identify two effects. The direct effect is that more consumers are subjected to the (frictionless) CC where prices are easily compared, which tends to increase consumer surplus. But there is a countervailing effect that arises through firms' endogenous competitive responses. When a seller has relatively few captive consumers, it is more focused on attracting shoppers, and therefore is more willing to undercut the prices on the CC with its direct price. It eventually becomes so difficult for the CC to deter undercutting that it gives up and switches to the high-fee regime in which prices are higher, and focuses on selling to its own captive base. This means that seemingly pro-competitive changes in market composition that result in more consumers comparing more prices (such as uninformed consumers becoming informed or adopting a price comparison technology) can increase prices, harming consumers.

A novel feature of our model is that some consumers are exogenously captive to the CC. This enables us to cleanly showcase the model's central mechanisms, but the exogeneity is not crucial. In Subsection 4.1 we show that our qualitative results hold even if consumers can leave the CC at zero cost, provided that some consumers visit the CC first. For instance, some consumers may initially be aware of only the CC, but, by visiting the CC, learn about other firms they see listed there. This gives rise to the possibility of 'showrooming', whereby consumers learn which firms are in the market by seeing them on the CC, and then buy direct if cheaper. In equilibrium, the CC sets its fee to optimally deter showrooming, resulting in consumers who are endogenously

‘captive’. Compared to our baseline model, this leads to quantitatively different equilibrium strategies (e.g., the exact price and fee levels predicted) but no qualitative departures, even in the case that consumers can showroom for free.

The rest of the paper proceeds as follows: after reviewing the related literature, Section 1 introduces the model, Section 2 characterises equilibrium, Section 3 demonstrates the importance of market composition for equilibrium outcomes, Subsection 4.1 allows for the possibility of showrooming, Subsections 4.2 and 4.3 allow for more general CC fee structures and Section 5 concludes.

Related literature and contribution

Our model builds on those of the established literature on price clearinghouses. In such models, a fraction of consumers only consider one firm’s price, while the remainder use a ‘clearinghouse’ to compare firms’ prices. Foundational papers in this literature include Shilony (1977), Rosenthal (1980) and Varian (1980); a survey can be found in Baye *et al.* (2006). The sustained popularity of the paradigm has led to many extensions and applications, including consumer search (e.g., Stahl, 1989; Janssen and Moraga-González, 2004), product substitutability (e.g., Inderst, 2002), competition with boundedly rational consumers (e.g., Piccione and Spiegel, 2012; Inderst and Obradovits, 2020; Heidhues *et al.*, 2021), price discrimination (e.g., Armstrong and Vickers, 2019; Fabra and Reguant, 2020), and generalisations allowing for firm heterogeneity (e.g., Baye *et al.*, 1992; Moraga-González and Wildenbeest, 2012; Shelegia and Wilson, 2021) and richer patterns of consumer comparisons (e.g., Armstrong and Vickers, 2021; Johnen and Ronayne, 2021).

Unlike those papers, we consider a market with competitive channel that is a profit maximiser in its own right. We propose a new way to model competition not only within, but also between different sales channels, enabling several contributions relative to the literature. First, we show how inter-channel competition affects equilibrium prices for a given market composition. Second, we study the welfare effects of a change in market composition (with a particular focus on the effects of a CC whose importance grows). Third, we endogenise consumers’ decisions to stick with the CC or undertake costly showrooming, and show how equilibrium fees and prices respond to this source of discipline.

In addition, deriving the entire set of equilibrium strategies in clearinghouse models can be complex. Owing to their continued relevance, recent work has made progress in unearthing equilibrium strategies and identifying the conditions for uniqueness under different assumptions on consumer information (e.g., Arnold and Zhang, 2014; Armstrong and Vickers, 2021; Johnen and Ronayne, 2021), but not in settings with price discrimination or a commission-charging clearinghouse. In such a setting, we find that (when there are two sellers and a CC) equilibrium strategies are unique for generic parameter values.

In a symmetric setup with a monopolist intermediary, Baye and Morgan (2001) were first to allow a more active role for a clearinghouse. They modelled a two-sided environment in which a clearinghouse sets fixed advertising and subscription fees in order to get both sellers and consumers on board, respectively. We provide a new model with features that reflect empirical characteristics of many modern competitive channels: (i) CCs that charge per-unit commissions rather than lump-sum fees, (ii) sellers that can set different prices for direct and CC sales and (iii) consumers that only consider prices on the competitive channel. This allows us to study a new set of strategic considerations; whereas Baye and Morgan are mostly concerned with the platform’s

problem of getting consumers and firms on board, the main trade-offs in our model revolve around simultaneous competition between and within the different sales channels available to firms.

Some papers study competition within platform-based sales channels, but assume these are the only sales channels (e.g., Hagiu, 2009; Belleflamme and Peitz, 2010; Boik and Corts, 2016; Karle *et al.*, 2020).³ In contrast, we allow firms to sell through multiple channels and study the impact of market composition on equilibrium prices and consumer surplus.

Our work is also distinct from classical studies of vertical relations between firms. Specifically, our model involves both vertical and horizontal dimensions to competition because, as well as serving as an essential gatekeeper for access to some consumers, the CC competes with sellers' direct channels. The implicit heterogeneity in consumer search frictions is a key component of our analysis, and is typically not built into studies of vertical markets. A small number of papers have studied the interaction, but do so in classic wholesaler-retailer settings (see, e.g., Lal and Villas-Boas, 1996; Janssen, 2020; Janssen and Shelegia, 2020). The study of competitive channels and retailers is distinct because a CC is a technology enabling comparison between sellers and so, by its nature, directly impacts consumers' information and competitive outcomes. Recent literature (e.g., Hagiu *et al.*, 2020; Anderson and Bedre-Defolie, 2021; Etro, 2021) considers vertical integration in platform markets.⁴ These authors considered a different setting in which the platform is also active as a seller in its own marketplace.

Our work is also related to literature on the coexistence of retailers and direct sales by a manufacturer (e.g., Chiang *et al.*, 2003; Kumar and Ruan, 2006; Arya *et al.*, 2007). Unlike these papers, we focus on cases where the seller sets the price for both direct and intermediated sales, and introduce a model that allows us to study the relative size of the two channels and the implications of that for competitive outcomes.

1. Model

Two sellers produce a homogeneous good at a common constant marginal cost, normalised (without further loss of generality) to zero. Sales can be made both directly and through a competitive channel. Sellers are indexed $i = 1, 2$, and the competitive channel, $i = 0$. The CC chooses a commission fee, $c \geq 0$, which a seller must pay for each sale that takes place via the CC.⁵

Given c , sellers choose a direct price, $p_i^d \geq 0$, whether to list on the CC, and if so, a price to list on the CC, $p_i \geq 0$. Choosing not to list on the CC is formally equivalent to listing a price above all consumers' willingness to pay, so we subsume the decision whether to list into a seller's choice of price.

There is a mass of consumers of measure $\mu + \lambda_0 + \lambda_1 + \lambda_2$ who wish to buy one unit of the good and have willingness to pay $v > 0$. There are two types of consumer: shoppers and captives.

³ Another recent paper that allows competition between sales channels is Shen and Wright (2019), who asked why sellers do not always undercut intermediaries to ensure sales take place direct. Their answer is that, when sellers set the fee paid to intermediaries, it is cheaper to attract consumers by raising their fee than by lowering their price. In contrast, we study settings where fees are set by the intermediary and show how undercutting incentives depend on the market composition.

⁴ We considered related issues in the context of a vertically integrated CC in an earlier working paper (see Ronayne and Taylor, 2020, Subsection 6.2).

⁵ Alternatively, we could have assumed that the fee is proportional to a seller's revenue, or that the CC discriminates across sellers with $c_1 \neq c_2$. However, in Subsections 4.2 and 4.3, we show that ignoring these possibilities is without loss of generality.

Shoppers are of mass $\mu > 0$. They are informed of all prices and buy at the lowest price available. Sellers each have a mass, $\lambda_i > 0$, of captive consumers who shop directly, e.g., on the seller's website or at its physical store. Without loss of generality, we index the sellers such that $\lambda_1 \leq \lambda_2$. The CC also has a mass of captive consumers, $\lambda_0 > 0$, who shop exclusively via the competitive channel, buying at the lowest price listed there. If there is a tie for the lowest price where such prices are either all direct prices or all available through the CC, then shoppers (and, in the latter case, CC captives also) buy at the lowest price from one of the tied sellers at random. If there is a tie in the lowest price where some of these prices are direct and some are via the CC, shoppers complete the purchase from one of the CC prices with probability $r_0 \in [0, 1]$ and one of the direct prices with probability $1 - r_0$.⁶ We refer to $\lambda = (\lambda_0, \lambda_1, \lambda_2, \mu)$ as the market composition. The solution concept is subgame-perfect Nash equilibrium. The game's timing is as follows:

- ($t = 1$) CC sets commission fee c ;
- ($t = 2$) sellers observe c and set prices p_i^d and p_i , with $p_i > v$ corresponding to a decision not to list on the CC;
- ($t = 3$) consumers shop.

1.1. Discussion of Model Assumptions

1.1.1. CC captivity

Our key novel modelling assumption is that there exist at least some consumers who are captive to the competitive channel (i.e., $\lambda_0 > 0$).⁷ Captivity has many potential causes (search frictions, loyalty, biases, naivety, etc.), but at its core it is a recognition that some consumers make some purchases with limited information. It is that informational limitation that we argue can apply to any retail location, be it a seller's own store or website, or a competitive channel's.

Captivity to the CC is also our measure of its market power, or 'size'. Because we measure the size of sellers similarly (by the number of consumers captive to their direct channel), we have a commensurate scale by which to examine the balance of power in markets where both actors are present.

Evidence on consumer behaviour supports CC captivity as an important force in relevant markets. For example, the UK's Competition and Markets Authority observed:

Our qualitative research found loyalty to one or two comparison sites was strong for some users. [...] users were reluctant to use new sites after they had spent time learning how to use one. Some had set up user accounts with particular comparison sites and were reluctant to enter their details on other sites.

(CMA, 2017b, p. 11)

Supporting this qualitative observation, survey results show that 'brand loyalty' is one of the most commonly cited reasons why 30% of price comparison website (PCW) users consulted only one PCW and no other source of price quotes (CMA, 2017a).

Of course, this survey evidence is observational. If CC prices were to increase, then a consumer who initially appeared captive might start to look elsewhere. In Subsection 4.1 we extend the

⁶ The between-channel tie-breaking rule, r_0 , does of course not appear in previous work in which each seller has only one sales channel. Instead of fixing some value for r_0 , we solve for it as part of equilibrium (à la Simon and Zame, 1990). In some off-path subgames, equilibrium only exists for some values of r_0 . However, the on-path equilibrium strategies of the whole game hold for any $r_0 \in [0, 1]$.

⁷ In contrast, the assumption that there are consumers who consider only one seller has been widely used since models such as that of Varian (1980); see the literature review for more detail.

model to accommodate this intuition by allowing the λ_0 consumers to incur a cost to break their captivity and check the direct prices of sellers listed on the CC. If CC prices rise sufficiently, then they will find it worthwhile to do so. This imposes a new competitive constraint on the CC, but does not qualitatively affect our main findings, even as the costs of search vanish.

One consequence of the CC having market power is that it sometimes hosts higher prices than sellers' direct channels. As an example of evidence in line with that possibility, Ennis *et al.* (2020) found that, by 2017, Europe's largest two hotel stay comparison sites' prices were higher than direct channel prices (by at least 5%) in about 50% of cases (for similar results, see Hunold *et al.*, 2020).⁸ A European Commission investigation (EU Competition Authorities, 2016) found that approximately 40% of surveyed hotels had undercut the CC-listed price with their direct price.

1.1.2. Price discrimination across channels

We allow firms to set $p_i \neq p_i^d$. Of course, the ability to discriminate does not imply it occurs; indeed, retail chains often do not price discriminate much across their stores (DellaVigna and Gentzkow, 2019).⁹ However, price discrimination across *channels* is common in many of the markets relevant to our study. For example, Ennis *et al.* (2020) and Hunold *et al.* (2020) reported marked discrimination between prices on hotels' websites and via leading online travel agents, while the UK's competition authority reported that consumers often 'get a better deal elsewhere' than the comparison site for various services (CMA, 2017a), implying differential pricing.

1.1.3. Unit demand

We model consumers with unit demand, which allows for greater analytic simplicity. It is straightforward to extend the model to accommodate downward-sloping demand. For example, suppose that each consumer demands $q(p)$ units at price p , such that (i) over $p \in [0, \bar{p}]$, q is continuous and $q' < 0$, and (ii) revenue, $r(p) = pq(p)$, is increasing over $p \in [0, \bar{p}]$ with a unique maximum at \bar{p} (i.e., \bar{p} is the monopoly price). Without loss of generality and to ease comparability, let $r(\bar{p}) \equiv v$. There is then a one-to-one mapping between p and r over $[0, v]$, so the problem of choosing $r \in [0, v]$ is strategically equivalent to that we present in which firms choose $p \in [0, v]$. As such, the dichotomous structure of equilibrium strategies and the pivotal condition we uncover (reported in Proposition 1) that dictates which of the two regimes prevails are unchanged.

1.1.4. Other assumptions

We conduct our main analysis with $n = 2$ sellers because this facilitates the cleanest and simplest exposition of the forces at play in our model. However, analogous equilibria with equivalent payoffs exist in the game with $n > 2$ sellers; details are provided in Appendix A.¹⁰

⁸ This evidence should be treated as indicative because of the difficulty in establishing like-for-like comparisons (e.g., hotels may list different kinds of rooms on competitive and direct channels, and constructing representative samples of direct channels is difficult).

⁹ Moreover, some competitive channels have adopted MFNs that restrict sellers not to offer consumers lower prices elsewhere. In an earlier working paper, we extend our analysis to show how those clauses can harm consumers by weakening between-channel competition (Ronayne and Taylor, 2020, Subsection 6.1).

¹⁰ As in Baye *et al.* (1992), there exist a multiplicity of payoff-equivalent equilibria when $n > 2$.

Our assumption that the CC uses per-sale fees is consistent with modern practice in the industry (see Subsection 4.2 for examples and discussion). In Subsection 4.2 we allow the CC to use a combination of per-unit and ad valorem fees, and in Subsection 4.3 we allow it to discriminate and set different fees for each seller. Neither modification to the model changes the results of the analysis.

2. Equilibrium

2.1. Pricing Subgames

We begin by studying the best responses of sellers at stage $t = 2$ for a given choice of c by the competitive channel. In equilibrium, competition à la Bertrand in prices on the frictionless CC implies that sellers make zero profits from any sales made there.

LEMMA 1 (CC PRICING). *In any equilibrium, $\min_i p_i = c$. If $c < v$ then in any equilibrium, $p_1 = p_2 = c$.*

The proof is given in Appendix A. Lemma 1 means that every equilibrium of the subgame starting at $t = 2$ is payoff equivalent to one with $p_1 = p_2 = c$, all else equal. We therefore continue by taking $p_1 = p_2 = c$ as given. Since $p_i > v$ is formally equivalent to not listing, Lemma 1 implies that there must be at least one firm listing on the CC in equilibrium. This is because listing provides an extra source of demand without constraining a firm's ability to serve other consumers via their direct channel at whatever price it wishes.

Now consider the sellers' choice of direct price. Suppose that the lowest price in the market, at which the shoppers buy, is on the competitive channel with probability 1. This implies that sellers serve only captive consumers through their direct channel and, therefore, that $p_i^d = v$ with corresponding profit $\pi_i = v\lambda_i$. The best deviation for the seller would be to set p_i^d just low enough to induce shoppers to buy direct. But this requires the seller to undercut not only its rival's direct price, but also the prices listed on the CC (i.e., to set $p_i^d \leq c$). The best such deviation yields profit $c(\lambda_i + \mu)$ and is, therefore, not profitable if

$$c \leq \underline{c}_i \equiv \frac{v\lambda_i}{\lambda_i + \mu}. \quad (1)$$

Seller 1 finds such a deviation profitable for lower levels of c than seller 2 (i.e., $\underline{c}_1 \leq \underline{c}_2$), because seller 1 has fewer captives and therefore loses less from a reduction in its direct price. If (1) is satisfied for both sellers, then the unique equilibrium behaviour is for sellers to set the monopoly price on their direct channel and allow the CC to serve the shoppers.

LEMMA 2 (DIRECT PRICING 1). *Suppose that $0 \leq c \leq \underline{c}_1$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, $p_1^d = p_2^d = v$ and any $r_0 \in [0, 1]$. Profits are $\pi_0 = c(\lambda_0 + \mu)$, $\pi_1 = v\lambda_1$ and $\pi_2 = v\lambda_2$. When $0 \leq c < \underline{c}_1$, there are no other equilibria.*

Now suppose that $\underline{c}_1 < c < \underline{c}_2$. In this range, seller 1 finds it worthwhile to undercut the CC prices in order to attract shoppers: the resulting increase in demand more than compensates it for foregone monopoly rents on its captive consumers. Seller 2, on the other hand, earns more from monopoly pricing on its captives and is unwilling to undercut CC prices. This gives rise to a subgame equilibrium in which sellers pursue asymmetric strategies, with shoppers buying directly from seller 1.

LEMMA 3 (DIRECT PRICING 2). *Suppose that $\underline{c}_1 \leq c \leq \underline{c}_2$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, $p_1^d = c$, $p_2^d = v$ and $r_0 = 0$.¹¹ Profits are $\pi_0 = c\lambda_0$, $\pi_1 = c(\lambda_1 + \mu)$ and $\pi_2 = v\lambda_2$. When $\underline{c}_1 < c \leq \underline{c}_2$, there are no other equilibria.*

For $c > \underline{c}_2$, both sellers prefer to undercut the CC and fight for shoppers using direct prices. Because more than one seller now uses a single variable (direct price) to trade off competition for shoppers against sure sales to captives, equilibrium seller strategies are mixed. The strategies, which take the CC's fee into account, are given by Lemma 4. Here, shoppers buy directly from whichever seller charges the lowest direct price.

LEMMA 4 (DIRECT PRICING 3). *Suppose that $\underline{c}_2 < c \leq v$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = c$, and p_1^d, p_2^d mixed over supports $[\underline{p}, c]$ and $[\underline{p}, c] \cup v$, respectively, via the strategies*

$$F_1(p) = \begin{cases} \frac{\mu p - \lambda_2(v - p)}{\mu p} & \text{for } p \in [\underline{p}, c), \\ 1 & \text{for } p \geq c, \end{cases} \quad (2)$$

$$F_2(p) = \begin{cases} \frac{\mu p - \lambda_2(v - p)}{\mu p} \frac{\mu + \lambda_1}{\mu + \lambda_2} & \text{for } p \in [\underline{p}, c), \\ \frac{\mu c - \lambda_2(v - c)}{\mu c} \frac{\mu + \lambda_1}{\mu + \lambda_2} & \text{for } p \in [c, v), \\ 1 & \text{for } p \geq v, \end{cases} \quad (3)$$

where $\underline{p} = v\lambda_2/(\lambda_2 + \mu)$ and $r_0 = 0$. When $c = v$, any $r_0 \in [0, 1]$ is supported. Profits are $\pi_0 = c\lambda_0$, $\pi_1 = [v\lambda_2(\lambda_1 + \mu)]/(\lambda_2 + \mu)$ and $\pi_2 = v\lambda_2$. When $\lambda_1 < \lambda_2$, there are no other equilibrium pricing strategies.¹²

Lemmas 2–4 characterise equilibrium when there are $n = 2$ sellers. When $n > 2$ and $\lambda_1 \leq \dots \leq \lambda_n$, there are analogous equilibria of these subgames that leave seller 1's and seller 2's strategies unchanged while each $i > 2$ plays $p_i = c$, $p_i^d = v$. The profits of 0, 1 and 2 are identical to those with $n = 2$, as is consumer surplus (because the extra $\lambda_3 + \dots + \lambda_n$ consumers each have their surplus fully extracted). More details are provided in Appendix A.

2.2. Competitive Channel Fee Setting

Now we solve the game starting at $t = 1$. Accordingly, consider the incentives of the competitive channel when it chooses its fee level, c . For $c \in (0, v]$, the CC makes positive profit from the fees paid by sellers for purchases made by its λ_0 captives. In addition, when c is in the range of Lemma 2, direct prices exceed those listed on the CC, so shoppers also buy through the

¹¹ If $\underline{c}_1 < c < v$ then we must have $r_0 = 0$ in equilibrium. If $r_0 > 0$, sellers' profits are downwards discontinuous at $p_i^d = c$, leaving them wishing to play $\max\{p_i^d : p_i^d < c\}$, which is not well defined. Thus, $r_0 = 0$ can be thought of as a tool that allows firms to undercut the CC in a technically well-defined way (cf., Simon and Zame, 1990). It will turn out that these subgames are off the equilibrium path. On the equilibrium path of the whole game we have $c \in \{\underline{c}_1, v\}$ and any $r_0 \in [0, 1]$ can be supported.

¹² See the Online Appendix for the proof of uniqueness when $\lambda_1 < \lambda_2$, and the case of $\lambda_1 = \lambda_2$ (and $c < v$) in which equilibrium pricing can be slightly different.

competitive channel. For higher levels of c , at least one seller sets a direct price lower than all prices on the competitive channel, leaving the CC selling only to its captives. The CC therefore faces a choice between (i) a *low-fee regime* where it facilitates sales to shoppers and its captives and (ii) a *high-fee regime*, defined as a situation where it facilitates sales only to its captives. By Lemmas 2–4, the highest c such that the CC sells to shoppers and its captives is \underline{c}_1 , whereas the highest c such that it sells only to its captives is v . Hence, the CC prefers (i) to (ii) if and only if

$$\frac{v\lambda_1}{\lambda_1 + \mu}(\lambda_0 + \mu) \geq v\lambda_0 \iff \lambda_0 \leq \lambda_1.$$

Our model reveals that a comparison of channel size is central to a competitive channel's trade-off. If the CC is large relative to the direct channel, it chooses to set a high fee and generate profit through sales to its captives. When it is relatively small, it sets a low fee in order to compete with the direct channel for shoppers. This reflects the ways that the market composition affects the CC's profits. A higher λ_1 increases \underline{c}_1 and hence profits for the CC in the low-fee regime. On the other hand, a higher λ_0 increases profits in both regimes, but increases profit by more in the high-fee regime because that is where the marginal revenue is higher. In line with this prediction, analysis in the insurance industry by the UK's competition authority finds that larger sellers tend to be associated with lower CC commissions, while commissions tend to be higher for larger CCs (CMA, 2017b). Proposition 1 formalises these forces and states the equilibrium.

PROPOSITION 1 (EQUILIBRIUM). *Equilibrium strategies are either those of the low- or the high-fee regime.*

- (1) When $\lambda_0 < \lambda_1$, the low-fee equilibrium results: the competitive channel sets $c = \underline{c}_1 = v\lambda_1/(\lambda_1 + \mu)$ and the sellers price in accordance with Lemma 2, i.e., $p_1 = p_2 = \underline{c}_1$, $p_1^d = p_2^d = v$.
- (2) When $\lambda_0 > \lambda_1$, the high-fee equilibrium results: the competitive channel sets $c = v$ and the sellers price in accordance with Lemma 4, i.e.,

$$F_1(p) = \begin{cases} \frac{\mu p - \lambda_2(v - p)}{\mu p} & \text{for } p \in [\underline{p}, v), \\ 1 & \text{for } p \geq v, \end{cases}$$

$$F_2(p) = \begin{cases} \frac{\mu p - \lambda_2(v - p)}{\mu p} \frac{\mu + \lambda_1}{\mu + \lambda_2} & \text{for } p \in [\underline{p}, v), \\ 1 & \text{for } p \geq v, \end{cases}$$

where $\underline{p} = \underline{c}_2 = v\lambda_2/(\lambda_2 + \mu)$.

When $\lambda_0 = \lambda_1$, both low- and high-fee equilibria exist.

3. Comparative Statics

Proposition 1 shows market composition to be an important determinant of equilibrium outcomes. In practice, composition varies both across markets and over time, with the reach of competitive channels growing in various markets. For example, UK consumers are around twice as likely to rely on a price comparison website when buying motor insurance than when buying home broadband. Over time, the sites have grown to the point that 85% of consumers have used such a site (CMA, 2017a).

In this section we consider the effect on prices and consumer surplus of a change in market composition from $\lambda = (\lambda_0, \lambda_1, \lambda_2, \mu)$ to some other $\lambda' = (\lambda'_0, \lambda'_1, \lambda'_2, \mu')$. Because much of this section is concerned with the effects of shifting consumers between groups, it is convenient to use the notation $\Delta(\alpha, \beta) \in \mathbb{R}$ to denote the size of a reduction in α and increase in β , such that $\alpha + \beta$ remains constant, *ceteris paribus*. For example, $\Delta(\lambda_1, \mu) = 1$ denotes the exercise of informing mass 1 of firm 1's captive consumers of all prices, turning them into shoppers.

We first show how the presence of competing sales channels changes the standard intuition from clearinghouse models of price competition, before analysing the effects of a growing competitive channel.

Key to our analyses is the surplus derived by consumers across the different fee regimes. Given the optimal commission levels reported in Proposition 1, consumer surplus in the low- and high-fee regimes (CS_{LFR} and CS_{HFR} , respectively) is found by subtracting profits from total welfare, $CS = v(\lambda_0 + \lambda_1 + \lambda_2 + \mu) - \pi_0 - \pi_1 - \pi_2$.¹³

$$CS = \begin{cases} CS_{\text{LFR}} \equiv v\mu \frac{\lambda_0 + \mu}{\lambda_1 + \mu} & \text{for } \lambda_0 \leq \lambda_1, \\ CS_{\text{HFR}} \equiv v\mu \frac{\lambda_1 + \mu}{\lambda_2 + \mu} & \text{for } \lambda_0 \geq \lambda_1. \end{cases} \quad (4)$$

3.1. Competition within versus between Channels

It is important, both for policy and more generally, to understand the effects of consumers' information on market outcomes. While an individual consumer's own welfare of course improves if they become aware of more of the alternatives available on the market *ceteris paribus*, it is not obvious (i) how firms respond to broader shifts in consumer awareness, (ii) how the information of some consumers affects others, and (iii) how the presence of a competitive channel alters any effects. In this section we shed light on these issues.

In standard clearinghouse models (e.g., Narasimhan, 1988), consumer surplus increases as consumers become more informed (in the sense that some captive consumers are turned into shoppers). Conditional on remaining within the same regime, the same is true here. Indeed, from (4), consumer surplus in each regime has the form $v\mu(\lambda_i + \mu)/(\lambda_j + \mu)$, which increases in $\Delta(\lambda_i, \mu)$ and $\Delta(\lambda_j, \mu)$. This happens for different reasons in each regime. In the high-fee regime, more shoppers leads firms to compete more fiercely in price within the direct channel, benefiting seller captives and existing shoppers (a standard clearinghouse result).¹⁴ In the low-fee regime, the CC lowers its fee in response to an increase in shoppers to fend off the increased temptation of sellers to undercut CC prices and win the shoppers for themselves. This benefits CC captives and existing shoppers. In either case, more information for some consumers benefits others.

The discussion so far is based on within-regime forces. However, a change in market composition can change the regime, with consequences summarised by Proposition 2.

PROPOSITION 2 (BETWEEN-REGIME EFFECTS). *It holds that $CS_{\text{HFR}} \leq \lim_{\lambda_0 \uparrow \lambda_1} CS_{\text{LFR}}$, with a strict inequality when $\lambda_1 < \lambda_2$. Therefore, for generic λ and λ' such that $\lambda_1 - \lambda_0 > 0 > \lambda'_1 - \lambda'_0$, there exists an $\epsilon > 0$ such that λ yields higher consumer surplus than λ' if $\|\lambda - \lambda'\| < \epsilon$.*

¹³ Proposition 1 reports two equilibria at $\lambda_0 = \lambda_1$ (but uniqueness otherwise); hence, the two weak inequalities in (4).

¹⁴ Armstrong (2015) terms this phenomenon a 'search externality'.

Proposition 2 says that, for generic parameters, consumer surplus discontinuously decreases as the market tips from the low- to the high-fee regime. Between-channel competition in the low-fee regime forces the CC to keep prices (and hence c) low to prevent undercutting. When it ceases to be worthwhile for the CC to compete for shoppers, the disciplining effect of between-channel competition disappears and the CC's fee and prices increase. Instead, only within-channel competition remains to discipline prices in the high-fee regime. But because of the information frictions present in the direct channel, the within-channel competition that takes place there is insufficient to offset the loss of competition between channels. The result implies that the interplay of within- and between-channel competition can overturn the standard intuition for the effects of informing consumers. To highlight this phenomenon, we obtain the following result as an immediate corollary to Proposition 2.

COROLLARY 1. *Consumer surplus is non-monotonic in $\Delta(\lambda_1, \mu)$.*¹⁵

In standard models, a higher shopper-to-captive ratio would unambiguously reduce prices (by intensifying within-channel competition). We find that the same is not true once competition between channels is also considered. Informing a seller's captives so that there are more shoppers (i) increases the seller's payoff from undercutting prices on the CC to serve shoppers directly, and (ii) reduces the benefit of charging high direct prices that serve only captives. Thus, the CC finds it more difficult to deter undercutting, forcing it to set a lower fee. Eventually, it is so costly to deter undercutting that the CC switches to a higher fee and focuses solely on extracting surplus from its own captive base. As seen in Proposition 2, the result is a fall in consumer surplus. Thus, while the standard pro-competitive effect of consumer information is active in our model, technologies or interventions that promote transparency or awareness can also have anti-competitive effects by changing the balance of power between competing sales channels.

We close this section by examining how a CC's market power affects within- and between-channel competition by comparing outcomes in a market with a strong CC to one with a CC with no market power. Suppose that $\lambda_0 > \lambda_1$ (so that the high-fee regime prevails) and consider the effect of completely removing the λ_0 CC captives from the market.¹⁶ Because those consumers were getting zero surplus in equilibrium to begin with, there is no direct effect on overall consumer surplus. But there is an indirect equilibrium effect. Indeed, once $\lambda_0 = 0$ we switch to the low-fee regime, resulting in lower prices for shoppers but higher prices for seller captives. This happens because competition *between* channels is strengthened (the CC fights for shoppers when it has no captives), while competition *within* the direct channel is weakened. From (4), we see that the overall effect is to increase consumer surplus if $\lambda_2 > \lambda_1(2 + \lambda_1/\mu)$. Intuitively, if sellers are highly asymmetric then there is already little competition in direct prices in the high-fee regime, so switching regime does not cause these prices to increase much. If sellers are more symmetric however, direct channel competition in the high-fee regime is fierce and a switch to the low-fee regime reduces surplus by causing direct prices to increase significantly. We thus see that CC market power can distort competition in non-trivial ways, with wider implications for surplus. We explore this issue in more detail in the next subsection.

¹⁵ When $\lambda_0 < \lambda_1$, consumer surplus is also non-monotonic in $\Delta(\lambda_2, \mu)$.

¹⁶ From a consumer surplus point of view, this exercise is equivalent to allowing the CC to observe whether a consumer is in λ_0 or in μ and to set a different c for each group, allowing firms to also set a different p_i for each. The CC would then set $c_0 = v$ for the CC captives and extract their full surplus, with c_μ for shoppers being optimally set as if $\lambda_0 = 0$.

3.2. A Growing Competitive Channel

The rise in the presence and power of competitive channels around the world stresses the importance and urgency to study the impact on competitive outcomes. Towards that end, we now build on Proposition 2 to examine the effects of a growing CC on consumer surplus. To do so, it is useful to express the relationship between any λ and λ' as

$$\begin{aligned}\lambda'_1 &= \lambda_1 - \Delta(\lambda_1, \lambda_0), & \lambda'_2 &= \lambda_2 - \Delta(\lambda_2, \lambda_0), & \mu' &= \mu - \Delta(\mu, \lambda_0), \\ \lambda'_0 &= \lambda_0 + \sum_{x \in \{\lambda_1, \lambda_2, \mu, \emptyset\}} \Delta(x, \lambda_0).\end{aligned}$$

Expressing λ' in this way recognises that a CC's captive audience can grow through market expansion (higher $\Delta(\emptyset, \lambda_0)$), through its adoption by consumers who would otherwise be captive to seller i (higher $\Delta(\lambda_i, \lambda_0)$) or through adoption by shoppers (higher $\Delta(\mu, \lambda_0)$). The following proposition describes how the different sources of CC growth influence consumer surplus.

PROPOSITION 3 (CONSUMER SURPLUS AND CC GROWTH). *Consumer surplus is non-monotonic in the growth of the competitive channel whenever that growth reduces the power of the direct channel or expands the market, and is decreasing if it reduces the number of shoppers. Specifically, consumer surplus is*

- (1) *non-monotonic in $\Delta(\lambda_i, \lambda_0)$;*
- (2) *non-monotonic in $\Delta(\emptyset, \lambda_0)$ (if $\lambda_1 < \lambda_2$; else non-decreasing);*
- (3) *decreasing in $\Delta(\mu, \lambda_0)$.*

Points (1) and (2) of Proposition 3 reveal that increasing the share of consumers exposed to the frictionless competition hosted on the CC can lower overall surplus. These types of CC growth have three effects: (i) consumers compare more prices; (ii) in the case of $\Delta(\lambda_i, \lambda_0) > 0$, sellers have fewer captives and are more willing to fight for shoppers; and (iii) the CC has more captive consumers to exploit. The first and second effects, in isolation, tend to make the market more competitive. But the second and third effects leave the CC less interested in between-channel competition, and more inclined to set high fees. Proposition 3 establishes that these latter, anti-competitive effects can dominate, to the detriment of consumers. An example of point (1) of Proposition 3 is illustrated in Figure 2 (an illustration of point (2) would look similar). Point (3) of Proposition 3 follows because, although CC captives cause fierce pricing *within* the CC channel, shoppers also exert competitive pressure *between* channels, making shoppers a more effective competitive discipline.

4. Robustness

4.1. Imperfectly Captive 'Showrooming' CC Users

One common justification for the assumption that some consumers are captive is that they are uninformed about what other options are available in the market. When a consumer is captive to a direct channel, they see only that seller's price—there is no cue or prompt that would help them become aware of or think about other alternatives. However, the CC captives we introduce are different: when a CC captive visits the competitive channel, they will typically be presented with a

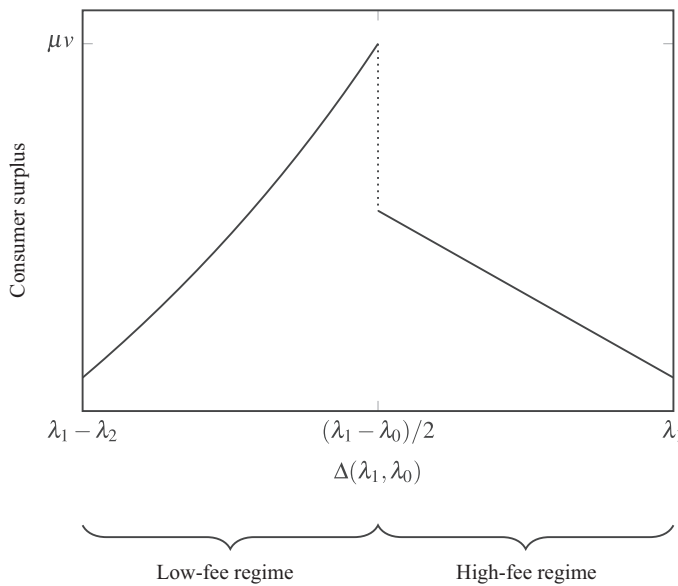


Fig. 2. Effect of $\Delta(\lambda_1, \lambda_0)$ on Consumer Surplus, *Ceteris Paribus*.

list of many sellers. Seeing this list may make a consumer aware of sellers they did not previously know about or had not previously considered. This suggests that visiting the competitive channel could prompt CC-captive consumers to become more active and look at sellers' direct channels (a phenomenon sometimes known as showrooming).¹⁷

In this section we relax the assumption that consumers are captive to the CC by allowing them to check the direct prices of sellers—the identity of whom they discovered via the CC. One might expect this to be a source of discipline for the CC, which would have to keep its hosted prices (and thus its fee) low to avoid driving consumers to sellers' direct sales channels. This is exactly what we find. However, the dichotomous structure of the baseline model's prediction is preserved; equilibrium falls into either a low- or high-fee regime depending on the relative size of the CC. The main difference to the baseline is quantitative rather than qualitative, in the sense that the high-fee regime has $c < v$ because the CC must keep prices low enough to remain competitive with the direct channel. This analysis shows that the earlier stark result – that CC captives pay the monopoly price in the high-fee equilibrium – is not crucial. Rather, CC prices increase with the inconvenience of checking direct prices, with monopoly prices emerging when the inconvenience is large enough. We now provide the analysis supporting this overview.

Formally, the model is modified as follows: the μ , λ_1 and λ_2 consumers behave as before. After seeing the prices on the CC, the λ_0 consumers choose whether to check direct channel prices before buying (i.e., to showroom). Checking the direct prices costs $\sigma \geq 0$: if a λ_0 consumer pays σ , they learn both p_1^d and p_2^d and can buy at any of $\{0, 1, 2\}$; if they do not pay σ , they learn

¹⁷ See Wang and Wright (2020) for an analysis of how showrooming interacts with the use of most favoured nation clauses.

neither and can only buy on the CC (i.e., at $\{0\}$).¹⁸ Finally, we assume that players coordinate on equilibria without showrooming, where those exist.¹⁹

4.1.1. *Equilibrium in the pricing subgames when consumers can showroom*

As a first step in the analysis, if CC-captive consumers expect $\min_i p_i^d \geq \min_i p_i$ then they prefer not to check any direct prices. Moreover, we saw in Section 2 that sellers indeed set prices such that $\min_i p_i^d \geq \min_i p_i$ if $c \leq \underline{c}_2$ and the λ_0 consumers do not check direct prices. Thus, the analysis in the baseline model (Lemmas 2 and 3) survives unchanged as an equilibrium in subgames following $c \leq \underline{c}_2$. We therefore focus on how showrooming alters the analysis in the case of higher fees, in which consumers can have a strict incentive to check direct prices.

Let $c \in [\underline{c}_2, v]$, and begin by supposing that consumers are not expected to showroom (meaning that they behave as in the baseline model, and so sellers' pricing incentives are the same as they were there). From Lemma 4, the expected gain to showrooming, $c - E(\min p_i^d | c)$, is increasing in c and goes to zero when $c \rightarrow \underline{c}_2$.²⁰ Thus, for any positive σ , the CC can find a $c > \underline{c}_2$ that gives each consumer a strict incentive not to showroom and pricing is as in Lemma 4. Let $\tilde{c}(\sigma) \equiv \max\{c : c - E(\min p_i^d | c) \leq \sigma\}$ denote the maximum such c consistent with no showrooming.

As c is increased above $\tilde{c}(\sigma)$, the subgame equilibrium of Lemma 4 is no longer sustainable because CC captives would showroom. Note that those consumers become functionally equivalent to shoppers once they showroom. Therefore, the analysis can follow a version of the baseline model in which additional showroomers correspond to a reduction in the mass of CC captives and an increase in the mass of shoppers. Now observe, from Lemma 4, that firms' direct prices fall (in the first-order stochastic dominance sense) as μ increases, meaning that showrooming becomes more attractive when more consumers are expected to check direct prices. Thus, in these subgames, there is no equilibrium in which some, but not all, potential showroomers showroom. As such, equilibrium for subgames following $c > \tilde{c}(\sigma)$ is found from Lemma 4 by replacing μ with $\tilde{\mu} = \mu + \lambda_0$ and reducing λ_0 to 0. However, this cannot be an equilibrium of the entire game because the CC serves no consumers and would prefer to deviate to a lower c .

4.1.2. *Equilibrium of the overall game*

Thus, the CC chooses between a low-fee regime in which it serves shoppers and sets $c = \underline{c}_1$ for profit $\pi_0 = \underline{c}_1(\mu + \lambda_0)$, and a high-fee regime in which it serves only CC captives and sets the highest fee that deters showrooming, $c = \tilde{c}(\sigma)$, for profit $\pi_0 = \tilde{c}(\sigma)\lambda_0$. The CC prefers the high-fee regime if $\lambda_0 > \tilde{\lambda}(\sigma) \equiv \mu \underline{c}_1 / [\tilde{c}(\sigma) - \underline{c}_1] \geq \lambda_1$,²¹ so we again find that the regime switches when the CC is large enough. The threshold at which it prefers the high-fee regime is higher than in our baseline analysis because it must now attenuate its fee (reducing its profit) in order to suppress showrooming. Proposition 4 summarises.

¹⁸ Instead, we could assume that λ_0 consumers face two costs, one for each of the direct prices. This complicates the formal analysis without changing the qualitative results from the single-cost assumption. As such, the search configuration is not crucial. What matters is that there is some cost to revealing *at least one* direct price, a friction we consider to be at the heart of showrooming.

¹⁹ The game has a coordination dimension because of the way in which the number of consumers who showroom influences firms' pricing incentives, which affects the incentive to showroom. By restricting attention to equilibria without showrooming we find an equilibrium that is essentially unique.

²⁰ Throughout the remainder of this subsection, $E(\min p_i^d | c)$ is computed using (2) and (3).

²¹ Here $\tilde{\lambda}(\sigma)$ does not depend on λ_0 and therefore defines a unique threshold for λ_0 .

PROPOSITION 4 (EQUILIBRIUM WITH THE THREAT OF SHOWROOMING). *Equilibrium strategies are either those of the low- or the high-fee regime. In the latter, c is set to deter showrooming.*

- (1) *When $\lambda_0 < \tilde{\lambda}(\sigma)$, the low-fee regime results: the competitive channel sets $c = c_1$ and equilibrium play is otherwise as in Lemma 2.*
- (2) *When $\lambda_0 > \tilde{\lambda}(\sigma)$, the high-fee regime results: the competitive channel sets $c = \tilde{c}(\sigma) \in [\underline{c}_2, v]$ and equilibrium play is otherwise as in Lemma 4.*

When $\lambda_0 = \tilde{\lambda}(\sigma)$, both low- and high-fee equilibria exist.

The gain to showrooming within the high-fee regime, $c - E(\min p_i^d | c)$, is increasing in c , meaning that $\tilde{c}'(\sigma) > 0$. The higher σ is, the more the CC can increase c before the captives are driven to check direct prices. This also implies that we are more likely to see a high-fee regime when showrooming is costly (i.e., $\tilde{\lambda}'(\sigma) < 0$) because such a regime is then more profitable for the CC.

Once σ grows large enough,²² $\tilde{c}(\sigma) = v$ (and $\tilde{\lambda}(\sigma) = \lambda_1$): if showrooming is prohibitively costly then consumers will never showroom, meaning we are back in the baseline model. At the other extreme, we have $\lim_{\sigma \rightarrow 0} \tilde{c}(\sigma) = \underline{c}_2$ (and $\tilde{\lambda}(0) = \lambda_1(\lambda_2 + \mu)/(\lambda_2 - \lambda_1) > \lambda_1$). When showrooming costs vanish, the competitive constraint imposed by showrooming on the CC is at its strongest and the high-fee regime's fee is at its minimum. Nevertheless, even as the friction vanishes, we still have both a low-fee regime (in which the CC serves shoppers) and a high-fee regime (in which the CC gives up on serving shoppers to charge the maximum fee it can get away with over the CC captives).

Last, we look at the effect of switching between regimes on consumer surplus. Consumer surplus in the high-fee regime is found by subtracting profits from the total available surplus, i.e., $CS_{\text{HFR}} = v(\lambda_0 + \lambda_1 + \lambda_2 + \mu) - \pi_1 - \pi_2 - \tilde{c}(\sigma)\lambda_0$, while consumer surplus in the low-fee regime is as before, i.e., $CS_{\text{LFR}} = v\mu(\lambda_0 + \mu)/(\lambda_1 + \mu)$. One can check that the difference, $CS_{\text{LFR}} - CS_{\text{HFR}}$, evaluated at $\lambda_0 = \tilde{\lambda}(\sigma)$, is independent of σ . The case with σ prohibitive was covered in Section 3, where we found that expression to be downwards discontinuous at the point of regime change. Thus, the non-monotonicity of consumer surplus identified in Proposition 3 continues to hold with the introduction of imperfectly-captive CC users.

4.2. Percentage Fees and Positive Marginal Cost

In practice, competitive channels typically use per-sale fees.²³ Sometimes these are in the form of flat commission rates, independent of the sale price; sometimes they are ad valorem, usually a percentage of the sale price; and sometimes there is a combination of the two.²⁴ Our baseline model employs a flat fee, c , rather than an ad valorem commission. In this subsection we instead

²² In particular, if $\sigma \geq v - E(\min p_i^d | c = v)$, so that even consumers who have all of their surplus extracted on the CC do not wish to check direct prices.

²³ In a clearinghouse context, Baye *et al.* (2011) allowed intermediaries to choose between any combination of fixed or per-sale payment and found that they optimally set the fixed component to zero.

²⁴ Amazon (<https://sellercentral.amazon.com/gp/help/external/200336920/>) and eBay (<http://pages.ebay.com/help/sell/storefees.html#fvf>), both last accessed 24 August 2021), charge both percentage and flat per-sale fees to sellers, making the majority of their revenue through the latter. Travel reservation websites such as Booking.com and Expedia are understood to charge their sellers 15%–25% of the sale price; see, e.g., Daily Mail (2015). Price comparison sites in the UK are reported to charge flat per-sale fees; see, e.g., BBC (2013). Shopping Malls often collect rent from the stores they host as a percentage of their sales (Gould *et al.*, 2005).

allow for arbitrary combinations of flat and percentage fees. To do so, denote the competitive channel's choice of flat and a percentage commission by $c \geq 0$ and $\tau \in [0, 1]$, respectively. We also allow for a positive marginal cost of production, $m \in [0, v]$, which sellers incur for each unit sold.²⁵

In our framework, the price set by sellers on the competitive channel is a sufficient statistic to characterise equilibrium, and that price is determined by c , τ and m . In other words, it does not matter what the exact levels of c and τ are, what matters is the price on the CC, p , that they generate. Indeed, one p corresponds to many different (c, τ) pairs. Furthermore, in equilibrium, the CC's profits are not affected by (c, τ) other than through p , and hence the CC is indifferent between any of the (c, τ) pairs corresponding to that p . For a given p , sellers also have the same equilibrium profit, and consumers have the same surplus. Therefore, the analysis of the baseline model is unchanged under the alternate assumption that commissions are a percentage of the sale price. For completeness, the equilibrium is stated in terms of p in Appendix C.1.

We now make this equivalence between variables explicit. Bertrand competition between sellers on the CC ensures sellers make zero profit through sales there: $p(1 - \tau) - c - m = 0 \Leftrightarrow p = (c + m)/(1 - \tau)$. This identity shows how the baseline analysis of $\tau = 0$ is without loss. For some m , consider arbitrary flat and percentage commissions (c, τ) . The induced CC prices are the same as those with alternative fee structure

$$(c', \tau') = \left(\frac{c + m\tau}{1 - \tau}, 0 \right).$$

Because p is a sufficient statistic for characterising equilibrium strategies, equilibrium under the two fee structures is equivalent and there is no loss of generality from assuming that $\tau = 0$.

4.3. Commission Discrimination

Competitive channels often charge different sellers the same commission levels. For example, one can easily check the standard seller fees charged by eBay and Amazon on their respective websites.²⁶ In other markets, e.g., the hotel reservation industry, although the fees are set in private it is understood there is a common base commission.²⁷ However, our framework allows sellers to have different levels of market power. This suggests that our model should not constrain the competitive channel to charge the same commission level to all sellers. Here, we show that this assumption is without loss of generality.

Formally, we allow the competitive channel to set two fee levels in $t = 1$, c_1 and c_2 , which are the commissions that sellers 1 and 2 respectively pay upon a sale through the CC. In Appendix C.2, we prove that the CC optimally sets $c_1 = c_2$. For the intuition, suppose that $c_i < c_j$. Prices via the CC are competed à la Bertrand down to c_j , but i can keep undercutting as $c_i < c_j$, and so secures all the sales through the CC at a cost of c_i . This means that the CC's income is solely determined by c_i , but prices on the CC are set by c_j . Therefore, the CC wants either to increase c_i to receive more per sale over the same number of sales, or to decrease c_j to lower CC prices and make undercutting more costly. Thus, when the CC can set discriminatory fee levels, it chooses not to.

²⁵ Note that it can be shown that $m + c > 0$ must hold in equilibrium. Briefly, if $m + c = 0$, then sellers' net marginal cost is zero and Bertrand competition on the CC drives prices there to zero, implying that the CC earns zero profit. The CC will therefore always choose a strictly positive c if $m = 0$.

²⁶ Amazon: https://sellercentral.amazon.com/gp/help/external/200336920/ref=asus_soa_p_fees?Id=NSGoogle. eBay: <http://pages.ebay.com/help/sell/storefees.html#fvf>, last accessed 24 August 2021.

²⁷ For example, 15% on booking.com: <http://blog.directpay.online/booking-com>, last accessed 24 August 2021.

5. Conclusion

We studied an environment in which selling occurs both directly and via a competitive sales channel (CC). A CC is a platform (such as a price comparison website, online marketplace or shopping mall) that lists multiple sellers' prices. In our model, the CC is a strategic actor in its own right, just as sellers are. Furthermore, our framework features a measure of each actor's market power, or size, which is commensurate across types of actor - both sellers and competitive channel operators.

A key question for our model is whether the most-informed (and hence most price-sensitive) consumers buy directly from the seller or via a competitive sales channel; a tension that causes equilibrium to fall into one of two regimes. In the first regime, the CC attracts those consumers by demanding a low fee that results in it hosting the lowest prices on the market. Sellers, meanwhile, focus on serving their captive consumers at high prices through their direct channel. In the second regime, the situation is reversed in the sense that fees (and hence prices) on the CC are high, while sellers undercut it and compete with each other for the most sensitive consumers in their direct prices. Which regime occurs in equilibrium depends on how much power the CC holds over consumers, relative to the power sellers hold in their direct channels. The more consumers rely on the CC, the more tempting it is for the CC to increase its commission and exploit its position as a bottleneck gatekeeper. Moreover, as sellers' captive audience shrinks, price-sensitive shoppers account for a higher share of their potential demand, which increases the temptation to undercut prices on the CC and serve these consumers directly.

These equilibrium dynamics have implications for consumer welfare. Suppose that consumers become less captive to individual firms and start comparing prices across firms more often (either by using the competitive channel, or by independently becoming more informed shoppers). The direct and immediate effect of more widespread price comparisons is to increase the intensity of competition between sellers and cause them to lower their prices, i.e., competition *within* channels is strengthened. However, there is a countervailing effect, namely that the equilibrium regime can change in such a way that prices on the CC increase, making it a less effective discipline on prices more generally, i.e., competition *between* channels is weakened. In contrast to models that neglect the role of inter-channel competition, this countervailing effect means that consumer surplus can fall, even as consumers become informed of a greater number of prices.

Appendix A

Allow there to be any number of sellers, n , indexed $i = 1, \dots, n$ without loss of generality such that $\lambda_i \leq \lambda_j \Leftrightarrow i \leq j$. After Section A.1, we set $n = 2$ unless otherwise specified. We now define the tie-break parameter, r_0 . Suppose that the lowest price on the market is p , set by the set of sellers $T \neq \emptyset$ on the CC and $D \neq \emptyset$ via direct prices. There are then $|T| + |D|$ sources of the lowest price on the market. Let $r_0 \in [0, 1]$ such that if

- (1) $i \notin T, i \notin D$, the probability that i sells to a shopper is 0;
- (2) $i \in T, i \notin D$, the probability that i sells to a shopper is $r_0/|T|$;
- (3) $i \notin T, i \in D$, the probability that i sells to a shopper is $(1 - r_0)/|D|$;
- (4) $i \in T, i \in D$, the probability that i sells to a shopper is $((1 - r_0)/|D|) + (r_0/|T|)$.

In words, when at least one direct price and at least one competitive channel price are tied and are the lowest on the market, r_0 is the probability with which shoppers buy at one of the lowest prices listed on the competitive channel. For all the results that follow, if equilibrium restricts the value of r_0 , the restriction is reported.

A.1. Equilibrium Derivation

We first prove Lemma 1 via some constructive intermediate results. Denote the (possibly degenerate) distribution of the *lowest* price listed on the CC as G_{\min} . Denote the minimum and maximum of the support of G_{\min} by \underline{s} and \bar{s} , respectively.

LEMMA A1. *It holds that $\underline{s} \geq c$.*

PROOF. Suppose that $\underline{s} < c$. Then some seller, i , has a positive probability of serving a positive mass of consumers via the CC at price $p_i < c$. A deviation to $p_i > v$ (or, equivalently, to not listing a price on the CC), holding p_i^d fixed, cannot reduce i 's demand via its direct channel, but reduces demand via the CC to zero. Since each consumer served via the CC at $p_i < c$ incurred a loss, the deviation is profitable. \square

LEMMA A2. *When $c < v$, $\underline{s} = c$.*

PROOF. By Lemma A1, $\underline{s} \geq c$. Suppose that $\underline{s} > c$. Then all sellers must make positive profits via the CC. If not, the i with zero profit could deviate to $p_i = \underline{s} - \epsilon$, ϵ small (in the case where $p_i^d = \underline{s}$ and there is a positive probability this is the cheapest direct price, then in addition consider i deviating to $p_i^d = \bar{s} - 2\epsilon$). As sellers make positive profits through the CC and must be indifferent between all prices they play, all prices listed are no greater than v . Consider the highest price in the union of all the supports of all sellers, \bar{p} , where $\bar{s} \leq \bar{p} \leq v$. If there is a positive probability of a tie at \bar{p} , any seller i would shift the associated probability mass to $\bar{p} - \epsilon$ (and wherever $p_i^d = \bar{p}$ and there is a positive probability this is the cheapest price, then in addition i deviates to $p_i^d = \bar{p} - 2\epsilon$). If there is a zero probability of a tie, any seller playing \bar{p} makes zero profit, a contradiction. \square

LEMMA A3. *When $c < v$, $\bar{s} = c$.*

PROOF. From Lemma A2, $\underline{s} = c$. Therefore, some seller i has c in its support and makes zero profit via the CC when it plays $p_i = c$ (or vanishing profit as $p_i \downarrow c$). If $\bar{s} > c$, then, when i is called upon to play $p_i = c$ (or $p_i \downarrow c$), it can instead play $\bar{s} - \epsilon$ for $\epsilon > 0$ small and net a strictly higher profit. Such an increase in p_i cannot reduce demand through i 's direct sales channel, but would strictly increase expected profit through the CC. \square

LEMMA A4. *When $c < v$, at least two sellers list $p = c$ with probability 1.*

PROOF. Lemmas A2 and A3 show that $\bar{s} = \underline{s} = c$. This implies that there is at least one seller, say i , which lists c on the CC with probability 1. Suppose that i was the only firm to do so. If so, i makes zero profit through the CC, yet could profitably deviate to some slightly higher p_i at which there is a positive probability of having the lowest listed price. Such an increase in p_i cannot reduce demand through i 's direct sales channel, but would strictly increase expected profit through the CC. \square

LEMMA A5. *When $c = v$, at least one seller lists $p = v$ with probability 1.*

PROOF. By Lemma A1, every seller sets $p_i \geq c$. When $c = v$, sellers are indifferent between any $p \geq v$. However, suppose that there is no seller that plays $p = v$ with probability 1. This implies that CC profit is lower than $v\lambda_0$. This cannot be an equilibrium because the CC could slightly reduce c to induce at least two sellers to list $p = c$ with probability 1 (by Lemma A4), which increases CC profit. \square

Proof of Lemma 1. Follows from Lemmas A1–A5. \square

Proof of Lemma 2. Sellers earn zero profit from consumers served through the CC in any equilibrium (Lemma 1). If a seller charges $p_i^d = v$, it makes total profit of $\pi_i = v\lambda_i$. Prices $p_i^d \in (c, v)$ are dominated by v . The highest profit from setting any other value of p_i^d is from $p_i^d = c - \epsilon$ as $\epsilon \downarrow 0$ (conditional on all other direct prices being the same or higher), which in the limit generates $\pi_i' = c(\lambda_i + \mu)$. Because $\pi_i' - \pi_i$ is decreasing in i , it suffices to check that seller 1 does not find this deviation profitable, $\pi_1 \geq \pi_1' \Leftrightarrow c \leq v\lambda_1/(\lambda_1 + \mu)$. For $c < \underline{c}_1$, given that profit $v\lambda_i$ is the best a seller can ever do, uniqueness follows because a seller can always obtain such profit (only) by $p_i^d = v$.²⁸ Let $n = 2$ for the lemma. \square

Proof of Lemma 3. Sellers can guarantee themselves profit of $v\lambda_i$ by setting $p_i^d = v$; no greater profit can be earned by serving only captives via the direct channel (by Lemma 1). Attracting shoppers to the direct channel would require $p_i^d \leq c$, which is not profitable by definition when $c \leq \underline{c}_i$. Thus, in any equilibrium with $c < \underline{c}_2$, we must have $p_i^d = v$ for all $i \geq 2$. Therefore, when $c < \underline{c}_2$, seller 1 chooses a direct price of either c or v (all other direct prices are dominated). When $\underline{c}_1 < c < \underline{c}_2$, seller 1 strictly prefers c because $c > \underline{c}_1$. Note that $r_0 = 0$; if $r_0 > 0$, seller 1 strictly prefers direct prices arbitrarily below c to c , which are not well defined. This leaves us with a unique equilibrium. Let $n = 2$ for the lemma.

When $c = \underline{c}_2$, any firm i such that $\lambda_i = \lambda_2$ is indifferent between $p_i^d = v$ and $p_i^d = c$, so long as (i) no other firm plays a direct price of c with positive probability and (ii) $r_0 = 0$, $p_i^d = c$ (all other direct prices are strictly dominated by v). When $\underline{c}_1 < c = \underline{c}_2$, playing $p_i^d = c$ for $i \geq 2$ would therefore require seller 1 to play a price $p_1^d \leq c$ with probability 0, but this is not possible in equilibrium because $c > \underline{c}_1$; hence, $p_i^d = v$ for $i \geq 2$. This again leaves us with the unique equilibrium. Let $n = 2$ for the lemma. Footnote 28 explains how uniqueness fails when $\underline{c}_1 = c \leq \underline{c}_2$. \square

Proof of Lemma 4. We show that the strategies of Lemma 4 constitute an equilibrium when $\lambda_1 < \lambda_2$. For the uniqueness proof and the special case $\lambda_1 = \lambda_2$, see the Online Appendix.

Suppose that $n = 2$. When seller 1 sets $p_1^d \in [\underline{p}, c]$, $\pi_1(p_1^d) = p_1^d\{\lambda_1 + \mu[1 - F_2(p_1^d)]\}$. Firm 1's profits when $p_1^d = \underline{p} \equiv v\lambda_2/(\lambda_2 + \mu)$ are $\pi_1(\underline{p}) = [(\lambda_1 + \mu)v\lambda_2]/(\lambda_2 + \mu)$. Setting $\pi_1(p_1^d) = \pi_1(\underline{p})$ gives F_2 below. Similarly, when seller 2 sets $p_2^d \in [\underline{p}, c]$, its profit is $\pi_2(p_2^d) = p_2^d\{\lambda_2 + \mu[1 - F_1(p_2^d)]\}$. When $p_2^d = v$, $\pi_2(v) = v\lambda_2$. Setting $\pi_2(p_2^d) = \pi_2(v)$, gives F_1 :

$$F_1(p) = \frac{\mu p - \lambda_2(v - p)}{\mu p}, \quad F_2(p) = \frac{\mu p - \lambda_2(v - p)}{\mu p} \frac{\mu + \lambda_1}{\mu + \lambda_2}.$$

By construction, F_2 ensures that seller 1 is indifferent over every $p_1^d \in [\underline{p}, c]$ and F_1 makes seller 2 indifferent over every $p_2^d \in [\underline{p}, c) \cup v$. Note that seller 1's strategy includes an atom at c when $c < v$. If $r_0 > 0$, F_1 cannot be part of an equilibrium because seller 1 could profitably

²⁸ When $c = \underline{c}_1$, $i: \lambda_i = \lambda_1$ is indifferent between $p_i^d = v$ and $p_i^d = c$. Therefore, there are equilibria in which $\Pr(p_i^d = c) \equiv \alpha \in [0, 1]$ and $\Pr(p_i^d = v) = 1 - \alpha$, while $p_j^d = v$ for $j \neq i$. (Note that $\alpha > 0$ cannot be part of the equilibrium of the whole game: if so, 0 would slightly lower c to get discretely higher sales.)

deviate downwards slightly in its direct price. Hence, $r_0 = 0$. When $c = v$, there is no atom, CC prices are never the lowest on the market, and so any $r_0 \in [0, 1]$ can be supported.

We now check no seller profits by deviating outside its support. Deviations to $p_i^d > v$ are not profitable. Neither seller can profit from a deviation to $p_i^d < \underline{p}$: this would result in the same demand as $p_i^d = \underline{p}$ but at a lower price. Any $p_i^d \in (c, v)$ is greater than the lowest price on the CC and therefore only attracts captives. Such prices are dominated by $p_i^d = v$. Since seller 1 has a mass point at c , $p_2^d = c$ induces a tie and yields strictly lower expected profit than $p_2^d = c - \epsilon$ (for $\epsilon > 0$ small). Lastly, we check that seller 1 cannot profit from a deviation to $p_1^d = v$. We have $\pi_1(v) > \pi_1(\underline{p}) \Leftrightarrow v\lambda_1 > [(\lambda_1 + \mu)v\lambda_2]/(\lambda_2 + \mu)$, which fails because $\lambda_1 < \lambda_2$. This gives the strategies stated in Lemma 4.

When $n > 2$, there is an equilibrium where, in addition to the strategies derived above, sellers $i > 2$ set $p_i = c$, $p_i^d = v$ and earn profit $v\lambda_i$. We now show that these sellers have no profitable deviations. A deviation to $p_i^d \in (c, v)$ fails to attract any shoppers and is not profitable. A deviation to $p_i^d = c$ induces ties with seller 1's mass point, and are therefore dominated by deviations to $p_i^d = c - \epsilon$ for some small ϵ . Deviations to $p_i^d \in [\underline{p}, c)$ yield profit $p_i^d\{\lambda_i + \mu[1 - F_1(p_i^d)][1 - F_2(p_i^d)]\}$. We observe that (i) seller i earns lower profit from $p_i^d \in [\underline{p}, c)$ than seller 2, and (ii) $v\lambda_i \geq v\lambda_2$. Since seller 2 is indifferent between $p_2^d \in [\underline{p}, c)$ and $p_2^d = v$, seller $i > 2$ must strictly prefer the latter. \square

Proof of Proposition 1. When $c \in [0, \underline{c}_1)$, $\pi_0 = c(\lambda_0 + \mu)$ by Lemma 2. The equilibrium fee, c , cannot fall in this range: if it did, the CC would have a strictly profitable deviation to $c + \epsilon < \underline{c}_1$ for $\epsilon > 0$. When $c \in (\underline{c}_1, v)$, $\pi_0 = c\lambda_0$ by Lemmas 3 and 4. The equilibrium fee, c , cannot fall in this range: if it did, the CC would have a strictly profitable deviation to $c + \epsilon < v$ for $\epsilon > 0$. Therefore, the only values of c possible in equilibrium are \underline{c}_1 and v .

When $c = \underline{c}_1$, either $\pi_0 = \underline{c}_1(\lambda_0 + \mu)$ by Lemma 2 or $\pi_0 = \underline{c}_1\lambda_0$ by Lemma 3. However, the prices of Lemma 3 cannot follow $c = \underline{c}_1$ in equilibrium: if it did, the CC would have a strictly profitable deviation to $c - \epsilon$ for some small $\epsilon > 0$. When $c = v$, Lemma 4 applies and $\pi_0 = v\lambda_0$. We are left with two equilibrium candidates: (i) set $c = \underline{c}_1$ and firms price in accordance with Lemma 2 or (ii) set $c = v$ and firms price in accordance with Lemma 4. Case (i) constitutes an equilibrium if the CC does not profit from a deviation to v , i.e., if $\underline{c}_1(\lambda_0 + \mu) \geq v\lambda_0 \Leftrightarrow \lambda_1 \geq \lambda_0$. Case (ii) constitutes an equilibrium if the CC does not profit from a deviation to any $c \in [0, \underline{c}_1]$. This is ensured when $v\lambda_0 \geq \underline{c}_1(\lambda_0 + \mu) \Leftrightarrow \lambda_0 \geq \lambda_1$. \square

Appendix B. Comparative Statics

Proof of Proposition 2. The first statement follows by inspection of (4). For the second, consider the change in consumer surplus from λ to λ' such that $\lambda'_1 < \lambda'_2$:

$$\frac{v(\mu - \Delta(\lambda_\mu, \lambda_0))(\lambda_1 - \Delta(\lambda_1, \lambda_0) + \mu - \Delta(\lambda_\mu, \lambda_0))}{\lambda_2 - \Delta(\lambda_2, \lambda_0) + \mu - \Delta(\lambda_\mu, \lambda_0)} - \frac{(\lambda_0 + \mu)v\mu}{\lambda_1 + \mu}. \quad (\text{B1})$$

Here, as defined in the main text prior to Corollary 1, when we move from λ to λ' , $\Delta(\alpha, \beta) \in \mathbb{R}$ denotes the size of a reduction in α and an increase in β such that $\alpha + \beta$ remains constant. We have $\|\lambda - \lambda'\| = \sqrt{(\lambda_0 - \lambda'_0)^2 + \Delta(\lambda_1, \lambda_0)^2 + \Delta(\lambda_2, \lambda_0)^2 + \Delta(\lambda_\mu, \lambda_0)^2} < \epsilon$, which implies that $|\lambda_0 - \lambda'_0|$, $|\Delta(\lambda_1, \lambda_0)|$, $|\Delta(\lambda_2, \lambda_0)|$ and $|\Delta(\lambda_\mu, \lambda_0)|$ must all be small for small ϵ . Moreover, this observation along with $\lambda_1 - \lambda_0 > 0 > \lambda'_1 - \lambda'_0$ implies that $\lambda_1 - \lambda_0$ must

approach zero with ϵ . Letting $\lambda_0 \rightarrow \lambda_1$ and $\Delta(\lambda_1, \lambda_0), \Delta(\lambda_2, \lambda_0), \Delta(\lambda_\mu, \lambda_0) \rightarrow 0$, (B1) becomes $[(\lambda'_1 + \mu)/(\lambda'_2 + \mu) - 1]v\mu < 0$. \square

Proof of Proposition 3. Part 1. Index the two sellers i and j rather than 1 and 2 to distinguish between their identities and their relative sizes. For seller i and any λ , there exists some Δ_i^* such that $\lambda'_0 = \lambda'_i$. If $\lambda'_i \leq \lambda_j$, consider values $\Delta(\lambda_i, \lambda_0)$ close to Δ_i^* . Non-monotonicity of consumer surplus in $\Delta(\lambda_i, \lambda_0)$ follows: if $\lambda'_i < \lambda_j$,

$$CS = \begin{cases} v\mu \frac{\lambda_0 + \Delta(\lambda_i, \lambda_0) + \mu}{\lambda_i - \Delta(\lambda_i, \lambda_0) + \mu} & \text{for } \Delta(\lambda_i, \lambda_0) < \Delta_i^*, \text{ increasing in } \Delta(\lambda_i, \lambda_0), \\ v\mu \frac{\lambda_0 + \Delta(\lambda_i, \lambda_0) + \mu}{\lambda_j + \mu} & \text{for } \Delta(\lambda_i, \lambda_0) > \Delta_i^*, \text{ decreasing in } \Delta(\lambda_i, \lambda_0); \end{cases}$$

if $\lambda'_i = \lambda_j$,

$$CS = \begin{cases} v\mu \frac{\lambda_0 + \Delta(\lambda_i, \lambda_0) + \mu}{\lambda_j + \mu} & \text{for } \Delta(\lambda_i, \lambda_0) < \Delta_i^*, \text{ increasing in } \Delta(\lambda_i, \lambda_0), \\ v\mu \frac{\lambda_i - \Delta(\lambda_i, \lambda_0) + \mu}{\lambda_j + \mu} & \text{for } \Delta(\lambda_i, \lambda_0) > \Delta_i^*, \text{ decreasing in } \Delta(\lambda_i, \lambda_0). \end{cases}$$

If $\lambda'_i > \lambda_j$, $\exists \Delta_i^+ : \lambda'_0 = \lambda_j < \lambda'_i$. Consider $\Delta(\lambda_i, \lambda_0)$ close to Δ_i^+ . Non-monotonicity follows:

$$CS = \begin{cases} v\mu \frac{\lambda_0 + \Delta_i^+ + \mu}{\lambda_j + \mu} = v\mu & \text{for } \Delta(\lambda_i, \lambda_0) \uparrow \Delta_i^+, \\ v\mu \frac{\lambda_j + \mu}{\lambda_i - \Delta_i^+ + \mu} < v\mu & \text{for } \Delta(\lambda_i, \lambda_0) \downarrow \Delta_i^+. \end{cases}$$

Part 2. Denote by Δ_\emptyset^* the value of $\Delta(\emptyset, \lambda_0)$ such that $\lambda'_0 = \lambda_1$ and consider the value of $\Delta(\emptyset, \lambda_0)$ close to Δ_\emptyset^* . The jump discontinuity in consumer surplus is of size

$$\lim_{\Delta(\emptyset, \lambda_0) \downarrow \Delta_\emptyset^*} CS - \lim_{\Delta(\emptyset, \lambda_0) \uparrow \Delta_\emptyset^*} CS = \frac{(\lambda_1 - \lambda_2)v\mu}{\lambda_2 + \mu},$$

which is strictly negative for $\lambda_1 < \lambda_2$. When $\lambda_1 = \lambda_2$, consumer surplus is increasing in $\Delta(\emptyset, \lambda_0)$ up to Δ_\emptyset^* , then remains constant thereafter.

Part 3. For $\Delta(\lambda_\mu, \lambda_0)$ such that $\lambda'_0 < \lambda_1$, from (4), consumer surplus is decreasing in $\Delta(\lambda_\mu, \lambda_0)$. For $\Delta(\lambda_\mu, \lambda_0)$ such that $\lambda'_0 > \lambda_1$, from (4), consumer surplus is decreasing in $\Delta(\lambda_\mu, \lambda_0)$. Denote the value of $\Delta(\lambda_\mu, \lambda_0)$ such that $\lambda'_0 = \lambda_1$ as Δ_μ^* and observe that the jump discontinuity is weakly negative; hence, consumer surplus is everywhere decreasing in $\Delta(\lambda_\mu, \lambda_0)$:

$$\lim_{\Delta(\lambda_\mu, \lambda_0) \downarrow \Delta_\mu^*} CS - \lim_{\Delta(\lambda_\mu, \lambda_0) \uparrow \Delta_\mu^*} CS = \frac{(\lambda_1 - \lambda_2)v\mu}{\lambda_2 + \mu}. \quad \square$$

Appendix C. Robustness

C.1. Percentage Fees and a Marginal Cost of Production

Through its selection of fees (flat or ad valorem), the CC determines the equilibrium price available through it, p . As such, in this section it is convenient to write as if the CC is directly determining p . We state the equilibria of the subgames starting at $t = 2$ in terms of p in the same order as Lemmas 2, 3 and 4. In the baseline model, c_i was the threshold fee level such that seller i would prefer to sell to shoppers and their captives directly, but at the CC price, and sell only to their captives directly at the monopoly price. The corresponding expressions in terms of p ,

while allowing for a marginal cost of production, $m \in [0, v]$, are given below for $i = 1, 2$. Let $p_i = (v\lambda_i + m\mu)/(\lambda_i + \mu)$.

LEMMA C1 (DIRECT PRICING 1'). Suppose that $0 \leq p \leq p_1$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = p$, $p_1^d = p_2^d = v$, and any $r_0 \in [0, 1]$. The resulting equilibrium profits are $\pi_0 = (p\tau + c)(\lambda_0 + \mu) = (p - m)(\lambda_0 + \mu)$,²⁹ $\pi_1 = (v - m)\lambda_1$, $\pi_2 = (v - m)\lambda_2$. When $0 \leq p < p_1$, this equilibrium is unique.

LEMMA C2 (DIRECT PRICING 2'). Suppose that $p_1 \leq p \leq p_2$. An equilibrium of the subgame starting at $t = 2$ has $p_1 = p_2 = p$, $p_1^d = p$, $p_2^d = v$ and $r_0 = 0$. The resulting equilibrium profits are $\pi_0 = (p - m)\lambda_0$, $\pi_1 = (p - m)(\lambda_1 + \mu)$ and $\pi_2 = (v - m)\lambda_2$. When $p_1 < p < p_2$, this equilibrium is unique.

LEMMA C3 (DIRECT PRICING 3'). Suppose that $p_2 \leq p \leq v$. An equilibrium of the subgame starting at $t = 2$ has $p_1, p_2 = p$, p_1^d and p_2^d mixed over $[p, p]$ and $[p, p] \cup v$, respectively, via

$$F_1(p^d) = \begin{cases} 1 - \frac{(v - p^d)\lambda_2}{(p^d - m)\mu}, & p^d \in [p, v], \\ 1, & p^d \geq v, \end{cases}$$

$$F_2(p^d) = \begin{cases} 1 - \frac{(p - m)\mu - (p^d - p)\lambda_1}{(p^d - m)\mu}, & p^d \in [p, v], \\ \frac{(p - m)\mu - (p - p)\lambda_1}{(p - m)\mu}, & p^d \in [p, v], \\ 1, & p^d \geq v, \end{cases}$$

where $p = p_2$ and $r_0 = 0$. Profits are $\pi_0 = (p - m)\lambda_0$, $\pi_1 = [(v - m)\lambda_2(\lambda_1 + \mu)]/(\lambda_2 + \mu)$ and $\pi_2 = (v - m)\lambda_2$. When $p = v$, any $r_0 \in [0, 1]$ can be supported. When $p_2 < p \leq v$, this equilibrium is unique.

PROPOSITION C1. When $\lambda_0 \leq \lambda_1$, there is a low-fee equilibrium: the CC sets $p = p_1$ and the sellers price in accordance with Lemma 2. When $\lambda_0 \geq \lambda_1$, there is a high-fee equilibrium: the competitive channel sets $p = v$ and the sellers price in accordance with Lemma 4.

C.2. Commission Discrimination

PROPOSITION C2. Suppose that the CC can charge fee c_1 to seller 1 and c_2 to seller 2. It will choose $c_1 = c_2$ in any equilibrium.

PROOF. If the CC offers fees $c_i < c_j \leq v$, Bertrand competition on the CC implies that all sales via the CC go to firm i at a price of $p_i = c_j$.

Part (i). Suppose that $\min\{p_1^d, p_2^d\} \leq \min\{p_1, p_2\}$ with probability 1 in the pricing subgame and $c_i < c_j \leq v$. Then $\pi_0 = c_i\lambda_0$.³⁰ There is a profitable deviation to $c_1 = c_2 = v$.

Part (ii). Suppose that $\min\{p_1^d, p_2^d\} \leq \min\{p_1, p_2\}$ with probability 0 and $c_i < c_j \leq v$. We show that the CC can serve the same mass of consumers at a higher fee by reducing c_j . Indeed, firm i earns a profit of $v\lambda_i + (c_j - c_i)(\lambda_0 + \mu)$ (assuming that it sets $p_i^d = v$, which is optimal conditional on not undercutting the CC). The best deviation would be to $p_i^d = c_j$ to undercut

²⁹ Here and below we use the fact that $p(1 - \tau) - c - m = 0$ because firms compete away all profit on the CC.

³⁰ If $\min\{p_1^d, p_2^d\} = \min\{p_1, p_2\}$ then the CC's profit is $c_i(\lambda_0 + r_0\mu)$. But we must have $r_0 = 0$ in such an equilibrium or else a firm would want to reduce p_i^d to break the tie.

the CC, yielding profit $c_j(\lambda_i + \mu) + (c_j - c_i)\lambda_0$. The deviation is not profitable if $c_i \leq \tilde{c}_i \equiv [\lambda_i(v - c_j)]/\mu$. Similarly, firm j earns $v\lambda_j$. A deviation to $p_j^d = c_j$ yields profit $c_j(\lambda_j + \mu)$ and is not profitable if $c_j \leq \underline{c}_j$. Thus, the best that the CC can do, conditional on deterring undercutting, is to solve $\max_{c_i, c_j} c_i(\mu + \lambda_0)$, such that $c_i \leq c_j$, $c_i \leq \tilde{c}_i$, $c_j \leq \underline{c}_j$. At least one of $c_i \leq \tilde{c}_i$ and $c_j \leq \underline{c}_j$ must bind and both are slackened by a reduction in c_j .

Part (iii). Lastly, if $c_i < c_j$, we show that it cannot be the case that $\min\{p_1^d, p_2^d\} \leq \min\{p_1, p_2\}$ with probability in $(0, 1)$. Indeed, this implies that both firms put positive mass on direct prices below c_j and positive mass on prices above. Direct prices in (c_j, v) never serve shoppers and are dominated by v . Standard arguments then imply that both firms must share a support, $[\underline{p}, c_j] \cup \{v\}$. To be indifferent between $p_j^d = v$ and $p_j^d = \underline{p}$, j must have $\underline{p}(\lambda_j + \mu) = v\lambda_j \Leftrightarrow \underline{p} = \underline{c}_j$, which does not depend on c_i or c_j . Similarly, i is indifferent if $\underline{p} = [\lambda_i v + \mu(c_j - c_i)]/(\lambda_i + \mu)$. For the two values of \underline{p} to coincide, we require $(\lambda_j - \lambda_i)v = (\lambda_j + \mu)(c_j - c_i)$, which implies that $\lambda_j > \lambda_i$ (i.e., $j = 2$ and $i = 1$).

To be indifferent between \underline{p} and a $p_2^d < c_2$, seller 2 must have $\underline{p}(\lambda_2 + \mu) + \lambda_0(c_2 - c_1) = p_2^d\lambda_2 + \lambda_0(c_2 - c_1) + \mu p_1^d(1 - F_1(p_2^d))$, and the case for seller 1 follows similarly, i.e.,

$$F_1(p_2^d) = \frac{(\lambda_2 + \mu)(p_2^d - \underline{p})}{p_2^d \mu} = \frac{(\lambda_2 + \mu)(p_2^d - \underline{c}_2)}{p_2^d \mu},$$

$$F_2(p_1^d) = \frac{(\lambda_1 + \mu)(p_1^d - \underline{p})}{p_1^d \mu} = \frac{(\lambda_1 + \mu)(p_1^d - \underline{c}_2)}{p_1^d \mu}.$$

The CC's profit is $\pi_0 = c_1[1 - F_1(c_2)][1 - F_2(c_2)]\mu + c_1\lambda_0$.³¹ Observe that F_1 and F_2 do not depend on c_1 , so π_0 is linear in c_1 and maximised when $c_1 \uparrow c_2$. Letting $c_1 = c_2 = c$, it is easily verified that π_0 is convex and maximised at either $c = v$ (inducing firms to undercut with probability 1) or at $c = \underline{c}_1$ (inducing firms to undercut with probability 0). Profit when $c_1 \neq c_2$ must be strictly less so that the CC has a profitable deviation to the case considered in either part (i) or part (ii). \square

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Additional Supporting Information may be found in the online version of this article:

Online Appendix

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³¹ A tie between CC and direct prices must be broken in the direct channel's favour; otherwise, a seller would wish to undercut the tie with its direct price.

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