

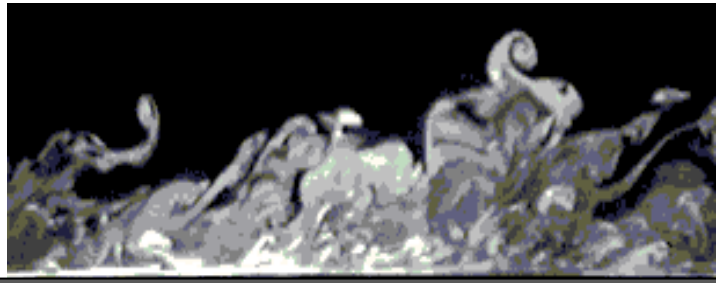
Two-scale Methods for LES/URANS

L. He

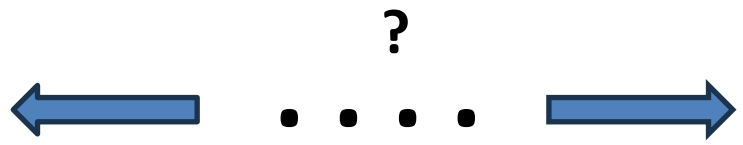
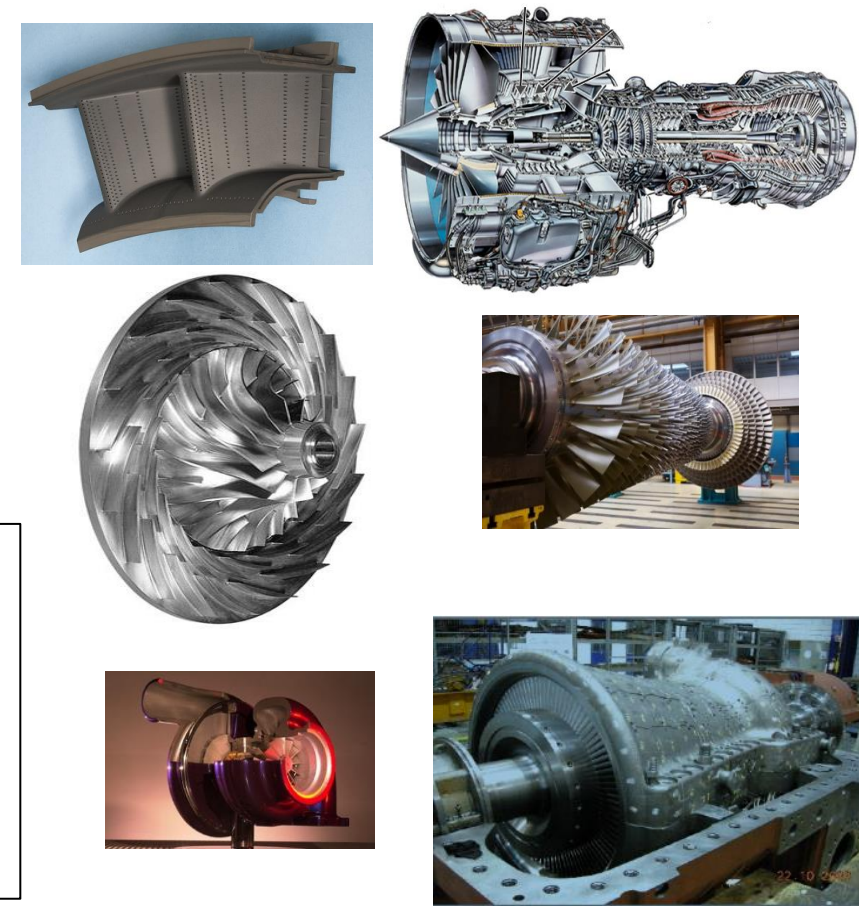
Department of Engineering Science
University of Oxford

Accuracy-Cost Trade-off: *What Choice Can/Should We Make?*

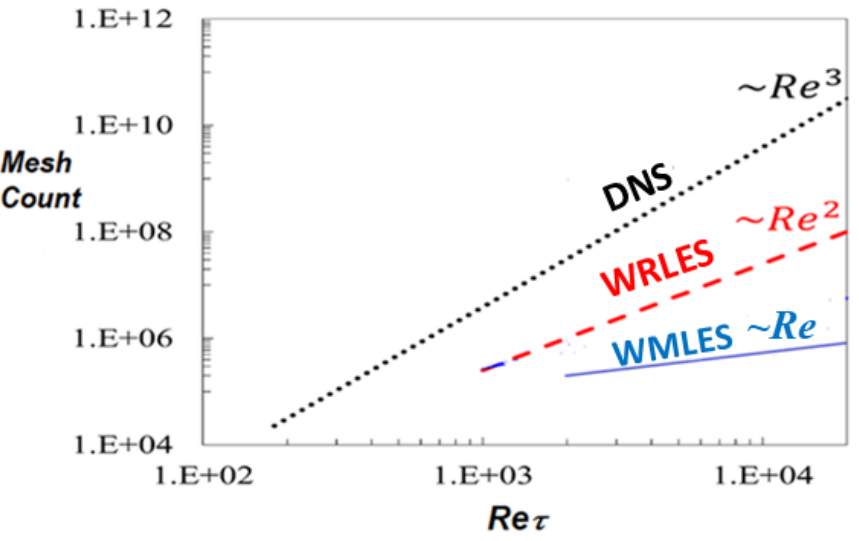
Fundamental



Application

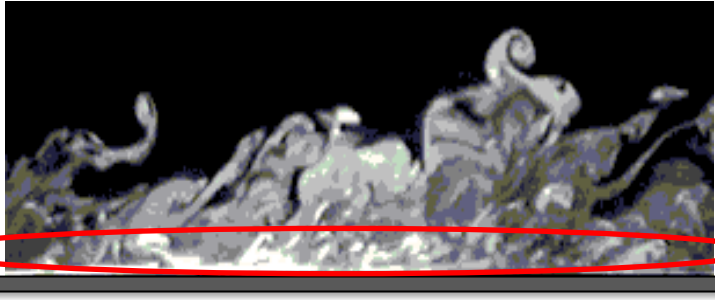


Turbulence-Resolving Options



- How much accuracy gain (vs. RANS) ?
- How much extra cost (vs. RANS) ?

WMLES vs. WRLES

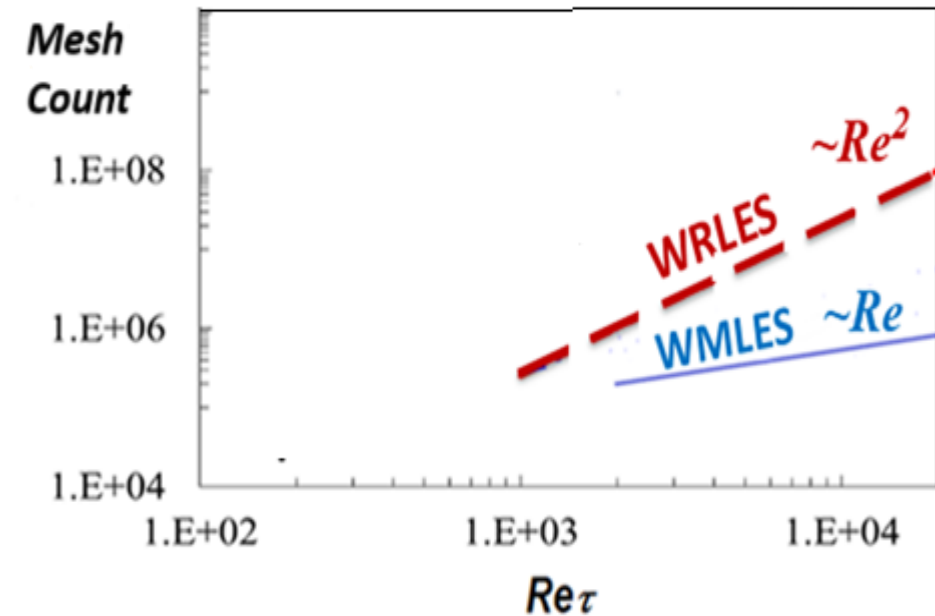


Near-wall Region:

- Major source of high mesh-count
- Active small scale dynamics
- Wide range of length scales & interactions
- High gradient (unsteady & time-mean)
- High sensitivity of bottomline parameters (τ_w, q_w)

Hence:

- **Highly sensitive to empirical model & its change**
- **Any alternative options?**
 - **less empirical than WMLES**
 - **less costly than WRLES**



Outline ('Two-scale': why, how & what)

- **Context & Motivation**

- *Classic View: 'Universal' Inner Scaling.*
- *Recent Findings: 'Dual Behaviour' with Re-dependence.*
- *Two-scale Approach: Aims/Target Settings.*

- **Framework Approach**

- *Perspective: Local Resolution vs. Global Conditioning*
- *Pathway: 'Leveraging **Averaging** for Closure'*
 - Time-Averaging ('physics-learning' from Reynolds)*
 - Space-Time Averaging (Two-scale Mesh domain)*
 - Local-Global Domain Coupling (Two-scale Interaction)*

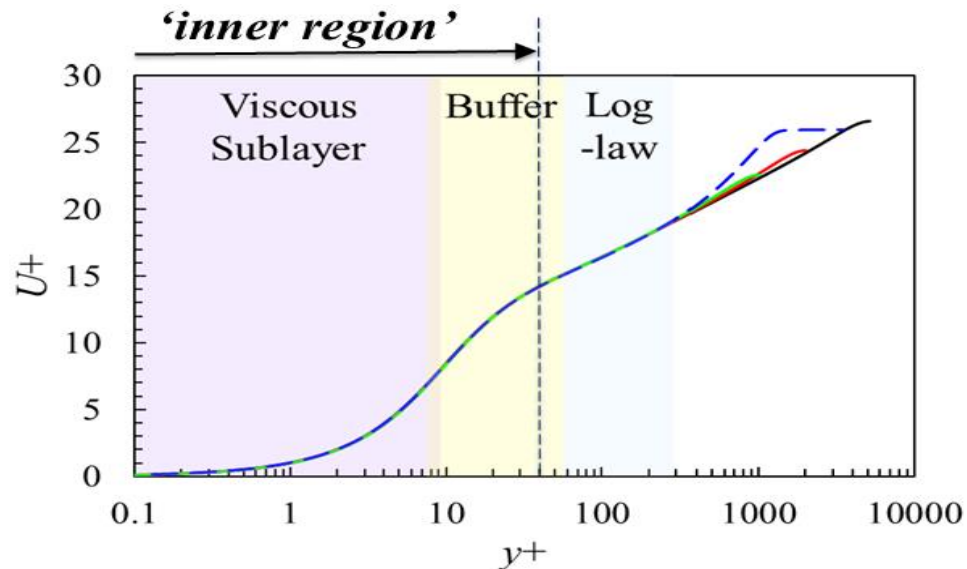
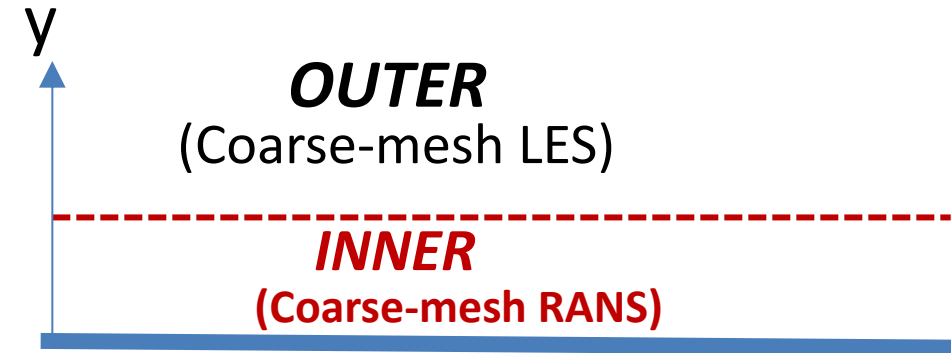
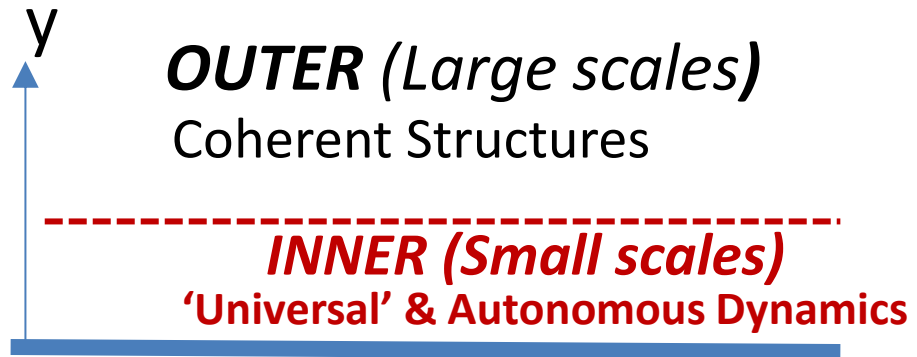
- **Embodiment/Solution Method**

- *Scale-dependent Interface*
- *'Mesh-informed' Source Term Generation & Propagation*
- *Balanced Eddy Viscosity for Robust Coarse-mesh Solution*

- **Supporting Case Studies**

- *Channel Flow, Tripped Boundary Layer (presented in relevant sections)*
- *Compressor Cascade, Turbine Stage (Multi-passages, Multi-rows)*

Nearwall Region (Classic View): ‘Universal’ and ‘Autonomous’

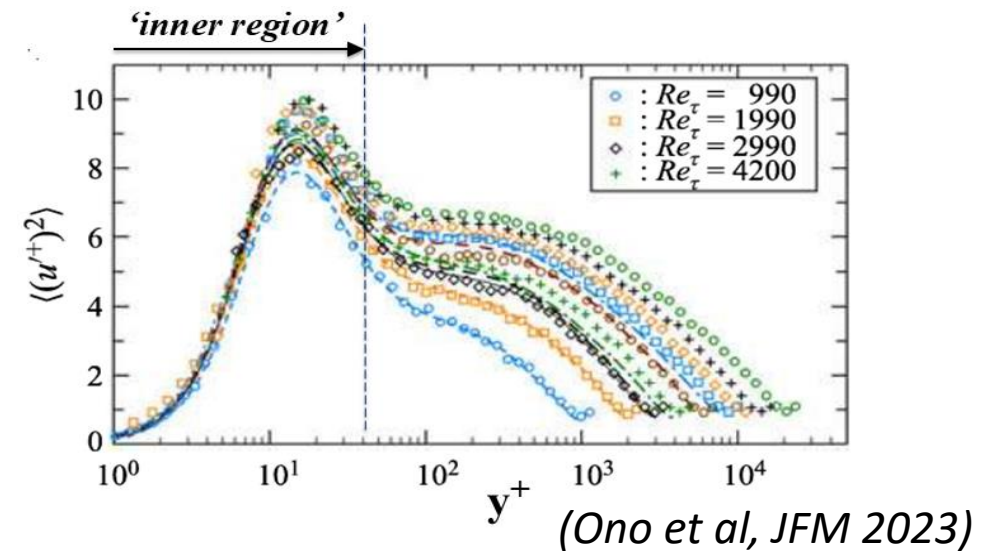
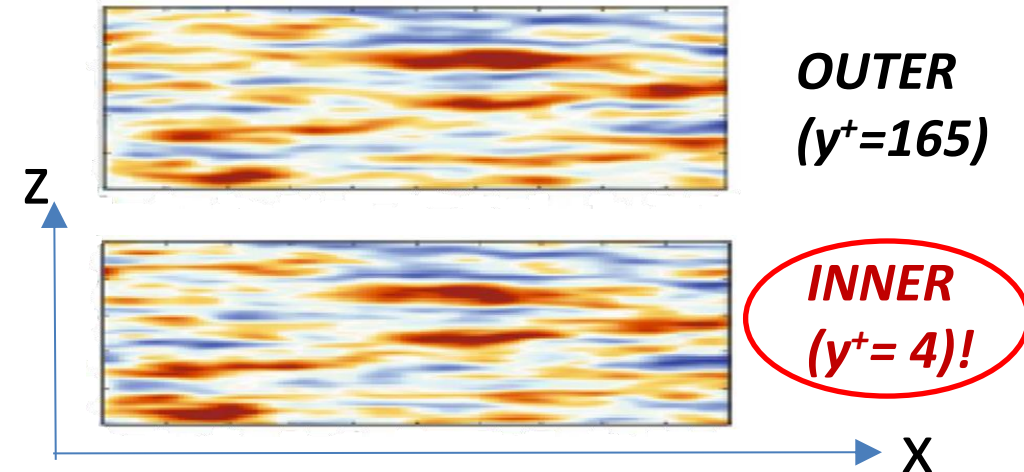


- ‘Inner scaling’: $(U^+, \dots) = f(y^+)$
 - ‘Classical View’: All turbulence statistics follow ‘Inner-scaling’ (independent of Re !) (Marusic et al 2017)
- ➔ **Motivation & Justification for WMLES & other “Scale-Separated” models/methods**

Time-mean velocity profiles at different Re (Chen 2022)

Nearwall Region (Recent Finding): 'Dual' Re-dependent Behaviour

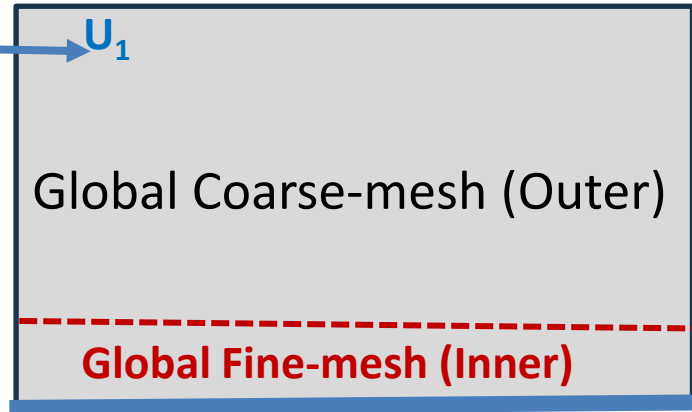
- **New findings (via Experiment & DNS for High Re):**
 - '**Foot-printing**' of Outer Large-scales on Inner part & scale '**modulation**' dynamics (Marusic et al 2010s)
- **Near Wall Inner Region: 'Dual' Characteristics**
 - '**Inner-scaling**' is only a partial descriptor
 - '**Missing part** arising from outer scales depends on Re
- **Implications:**
 - Extra challenges for WMLES
 - Impetus for new models/methods development (need to capture/resolve inner-outer scale interaction)



Toward Two-scale Framework:


Problem Deconstruction → *Solution Construction*

Global Fine-Mesh (Full WRLES)

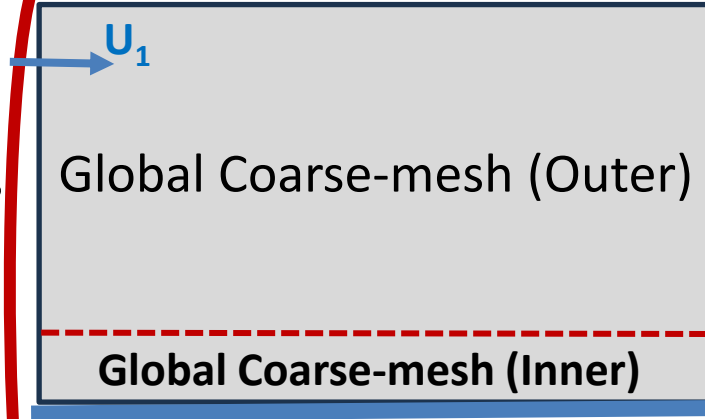


Very Costly !

*Affordable
alternative?*



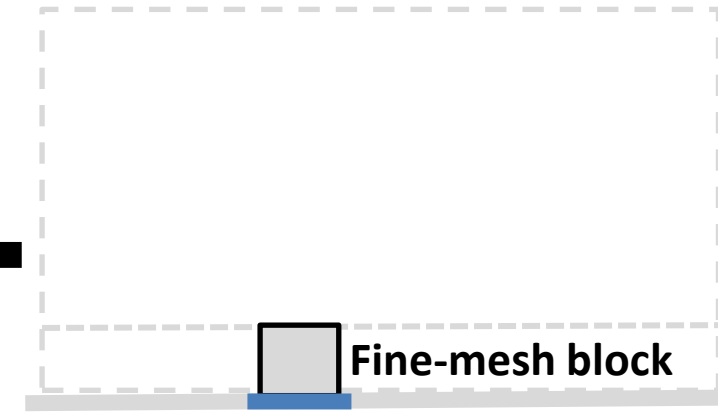
Global Coarse-Mesh



well-conditioned,
poorly-resolved !



Local Fine-Mesh (Inner)

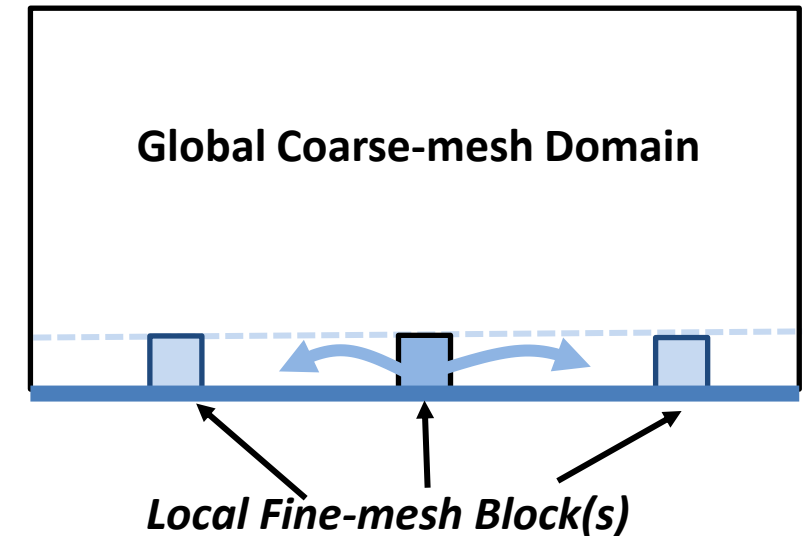


well-resolved,
ill conditioned !

Aim: to couple two sub-solutions to correct each other ?

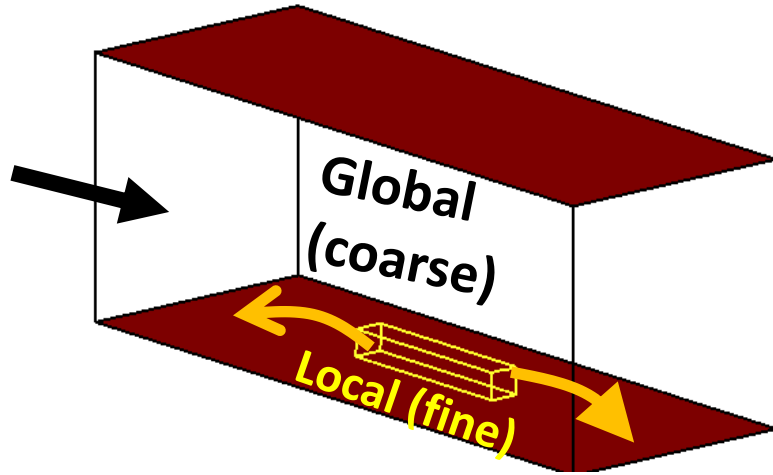
Targeted Attributes for 'Coarse-mesh based Wall-resolved LES'

- Near-wall small-scale dynamics resolved (cf. wall-resolved **WRLES**);
- Outer large-scale dynamics resolved (cf. wall-modelled **WMLES**);
- Global Coarse-mesh (for both Outer and Near-wall regions);
- Local Fine-mesh (discretely embedded near-wall small blocks)
 - ➔ correct global **under-resolved** coarse-mesh 'on-the fly' :
 - a) BC errors on the wall boundary;
 - b) Truncation errors in the interior region.
- Capture scale fluence/interactions in nearwall region
 - a) Local-Global interface transparent to outer flow 'footprints'
 - b) Full range of turbulence spectrum without 'scale-separation'.

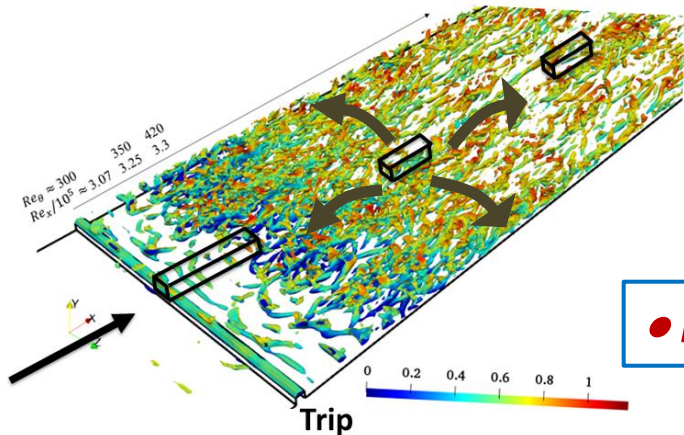


Solution Settings for Locally-embedded Fine-mesh Block(s)

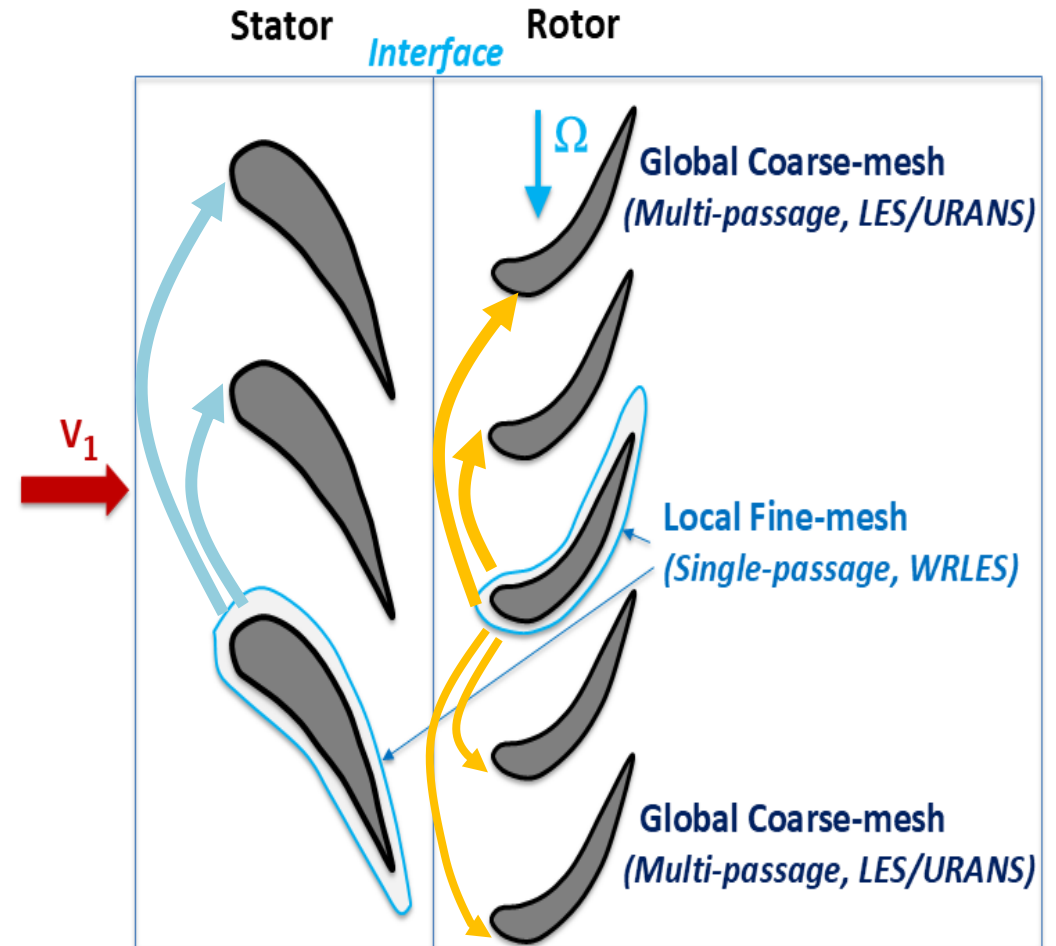
Channel Flow (only one fine-mesh block)
(streamwise & spanwise homogeneous mean flow)



Boundary Layer (streamwise fine-mesh blocks)
(spanwise homogeneous mean flow)



Multi-passage Multi-row Turbomachinery



• How to Create & Propagate Local High-resolution Effect Globally for Interaction?

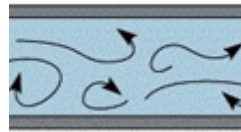
Learning from Osborne Reynolds: ‘Averaging for a Purpose’

- Revisit of Time-Averaging (‘Reynolds-averaging’)



Time–Averaging for Incompressible Flow (Mass & Momentum eqs.)

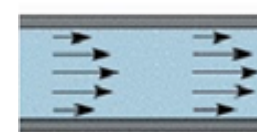
Unsteady



$$\frac{\partial U}{\partial t} + \mathbf{R}(U) = 0 \quad (\text{closed})$$



Time-mean



$$\mathbf{R}(\bar{U}) = \text{LRST} \quad (\text{open due to nonlinearity})$$

↑
(Lumped Reynolds Stress Terms)

- A Common form (*LRST* as input): $\mathbf{R}(\bar{U}) = \text{LRST}$
Given *Lumped Reynolds Stress Terms* (regardless of how) → the target ‘coarse grained’ time-averaged flow solution
- An Uncommon form (\bar{U}_0 as input): $\text{LRST} = \mathbf{R}(\bar{U}_0)$
Given time-average solution \bar{U}_0 (regardless of how) → *LRST* (in force/volume: *one scalar per one momentum eq.*)

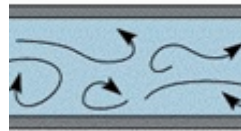
• **What matters is *Lumped Reynolds Stress Terms*, not Reynolds Stresses themselves!**

'Direct Mode' and 'Inverse Mode' of Reynolds-averaging

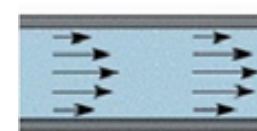
Time-Averaging for Compressible Flow (Mass, Momentum & Energy eqs.)



Unsteady



Time-mean



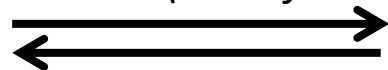
$$\frac{\partial U}{\partial t} + \mathbf{R}(U) = 0 \text{ (closed)}$$

$$\mathbf{R}(\bar{U}) = \mathbf{LRST} \text{ (open)}$$

- 'Direct Mode' (commonly used!): $\mathbf{R}(\bar{U}) = \mathbf{LRST}$
 Given Lumped Reynolds Stress & flux Terms (regardless of where/how) \rightarrow Target 'coarse grained' RANS solution.
- 'Inverse Mode' (hardly used!): $\mathbf{LRST} = -\mathbf{R}(\bar{U}_0)$
 Given a time-mean field (\bar{U}_0) (regardless of how) \rightarrow \mathbf{LRST} (5 scalar unknowns for 5 eqs),
 (can use \mathbf{LRST} to fully recover \bar{U}_0 , e.g. for vortex shedding (Ning & He 2001), but any practical point?)

Source Term

'Direct' (lots of use!)



'Target Solution'

'Inverse' (any use?)

Space-Time Averaging (Two-scale Framework)

- Setting the Target for Coarse-mesh Solution

- **Primary Intent:** Use Global coarse-mesh (under-resolved!) in nearwall region & Use Local fine-mesh solution to mitigate/correct coarse-mesh errors

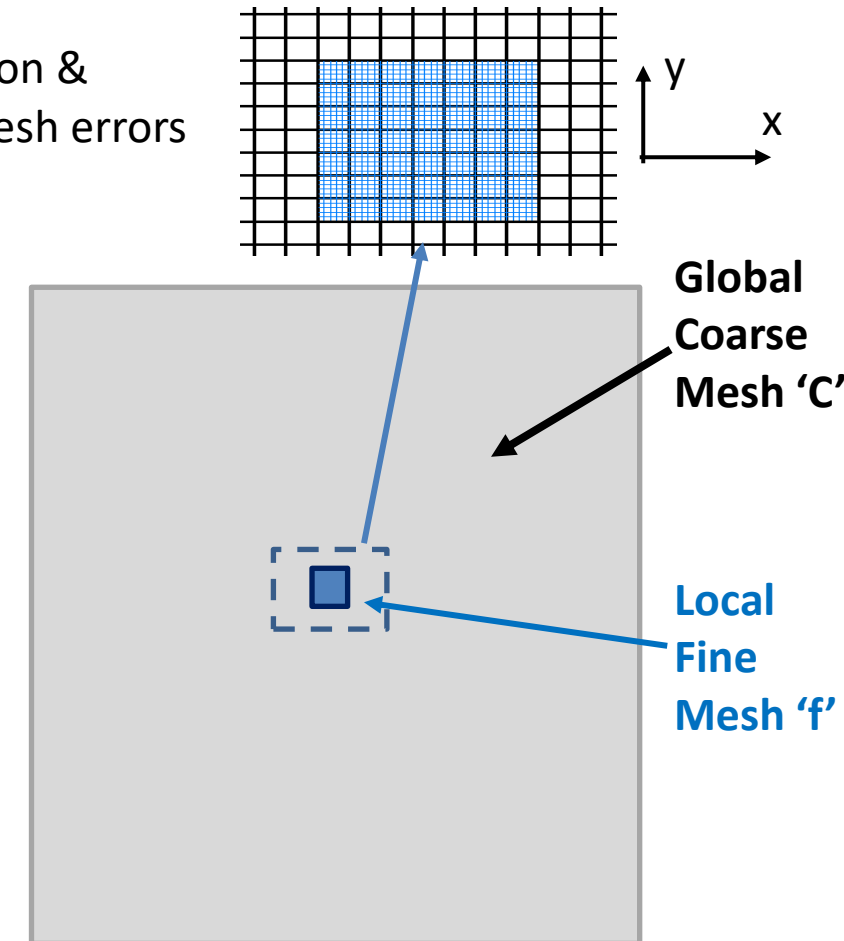
- **Dual meshing:** embedding fine-mesh 'f' in coarse-mesh 'C' (creating the fine-mesh by sub-dividing the coarse-mesh)

- **Local spatial-averaging:** volumetric average for each coarse-mesh cell

$$\tilde{U}_f = \frac{1}{V_C} \sum U_f \Delta v_f$$

- **Space-Time Averaging (STA):** time-average of local spatially averaged

$$\bar{\tilde{U}}_f = \frac{1}{T} \sum \tilde{U}_f \Delta t \quad \bar{\tilde{U}}_f - \text{Target for Corresponding Coarse-mesh}$$



Full Solution by Interaction between Parts (Two-scale Framework)

- **Coupled System consisting of Two Sub-systems/Domains:**

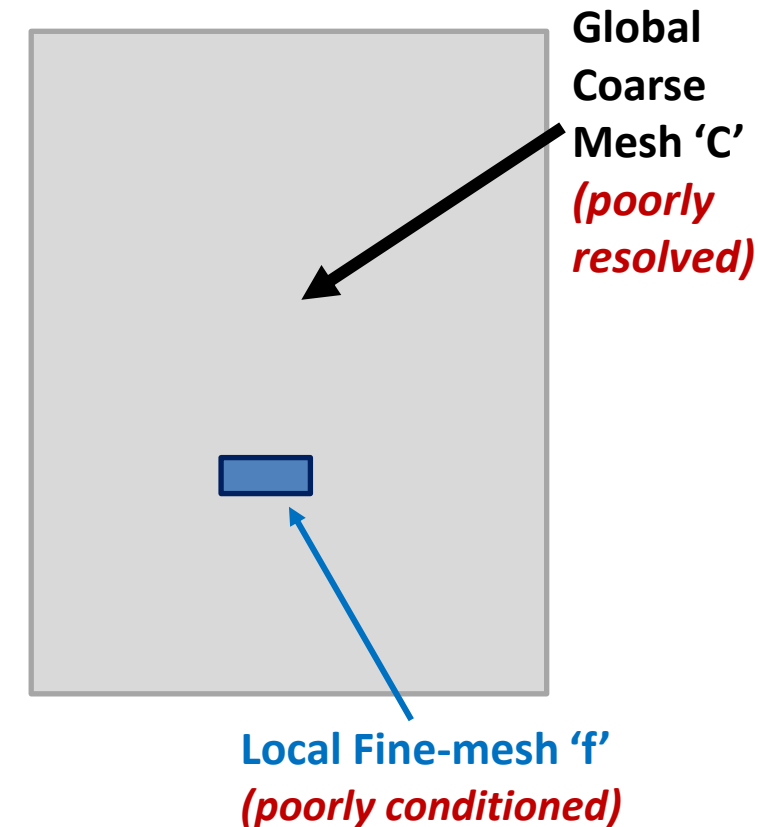
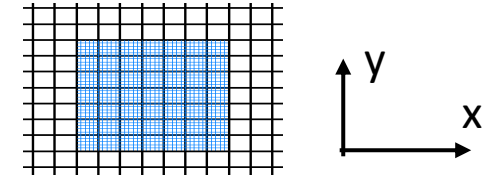
- **Local fine-mesh 'f'** ('Inverse Mode'): $S_{ST} = R_C(\tilde{U}_f)$

- **Global coarse-mesh 'C'** ('Direct Mode'): $\frac{\partial U_C}{\partial t} + R_C(U_C) = S_{ST}$

(Source terms to 'fight' Space-Time errors: driving \bar{U}_C to \tilde{U}_f)

- **Interactive Solution** (make two domains correct/'train' each other!)

- **Local fine-mesh:** subject to *improved conditioning*
- **Global coarse-mesh:** subject to *improved resolution*



1). Local Fine- Global Coarse Mesh Interface :

How can inner fine-mesh block 'see' outer large scales?

(**'foot-prints'** impeded by direct periodicity, widely used in 'MFU')

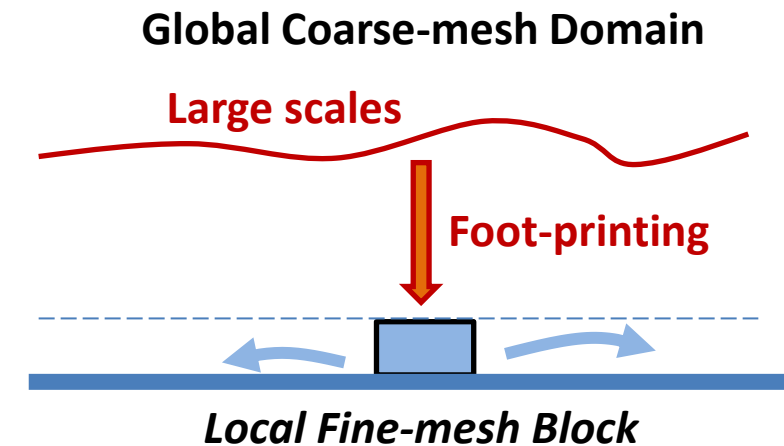
2). Source Term Generation & Propagation:

How to deal with to nonuniformity in global flow and mesh ?

3). Coarse-mesh Solution driven by Given Source Terms:

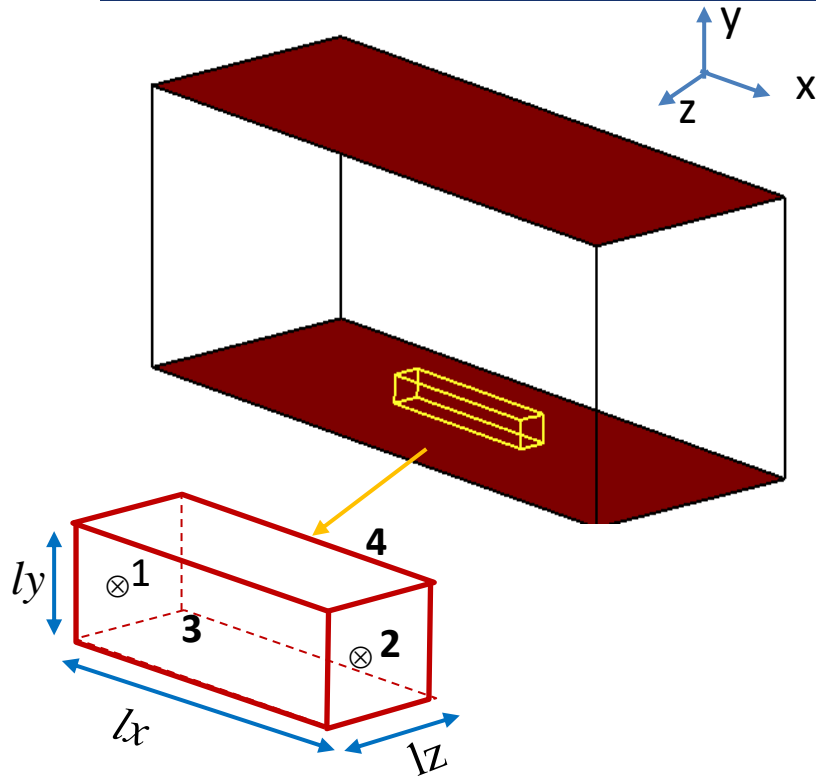
How to improve its robustness & convergence ?

(cf. convergence issues for Reynolds Stress Model based RANS)



1). Fine-Coarse Interface: *How Can Local Fine-block 'SEE' Global Large Scales?*

-Scale-dependent Interface Treatment (He, 2018)

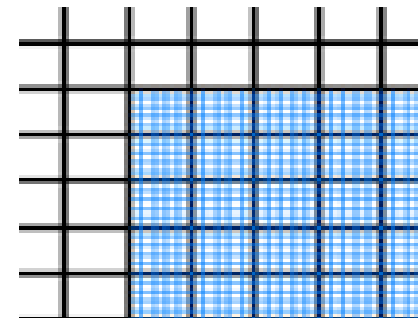


- **Prior Minimum Flow Unit (MFU):** restricted by *the Direct Periodicity (in two wall parallel directions, X, Z)*

- **Dual-mesh based Two-scale Decomposition:**

$$U(x, t) = U_c(x, t) + \Delta U_f(x, t)$$

Coarse base Fine perturbation



Coarse 'C' base: Direct interfacing between two sides;

Fine 'f' perturbation: Periodic ('recycling' fine-mesh Δ only)

- **'Scale-dependent' Interfacing** (e.g. for pairing faces '1' and '2'):

Direct for Coarse base 'C'

$$[U]_1 = [U_c]_1 + [\Delta U_f]_2$$

Periodic for fine perturbation 'f'

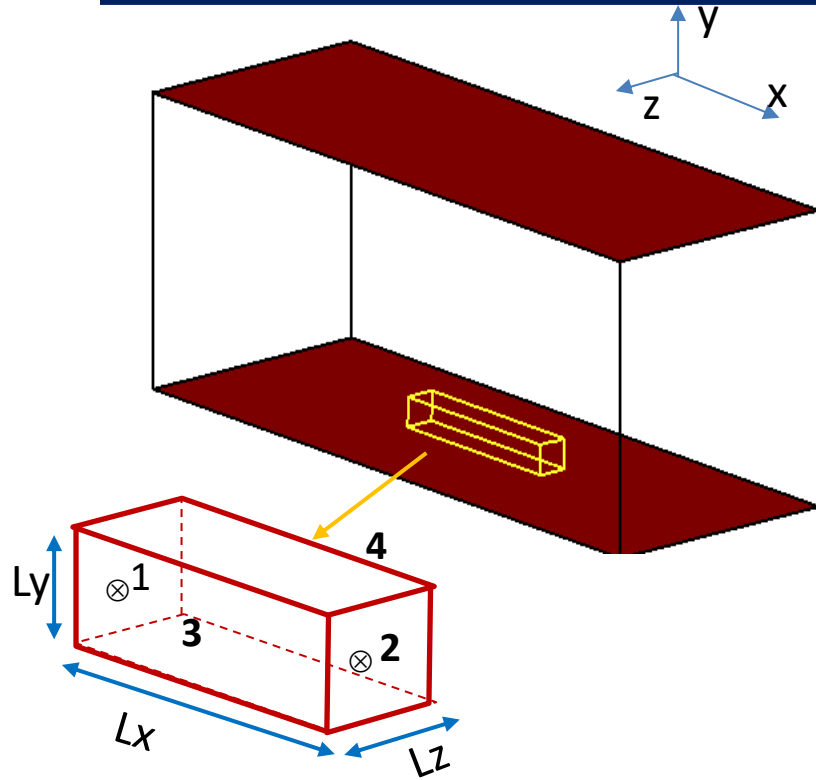
$$[U]_2 = [U_c]_2 + [\Delta U_f]_1$$

$l_y^+ \sim 3\sqrt{Re_\tau}$ - start of log-law region

$l_x^+ \sim 3500$

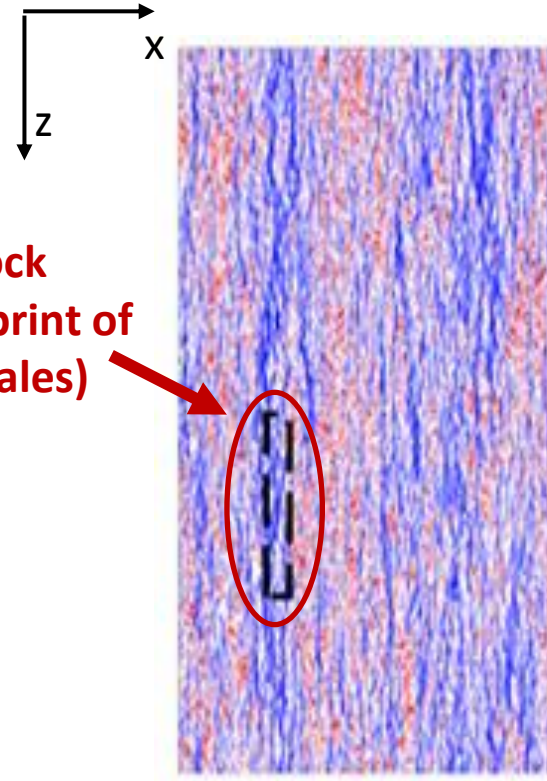
$l_z^+ \sim 350$

Local Fine-mesh Block Needs to 'See' Outer Flow 'Foot-prints'

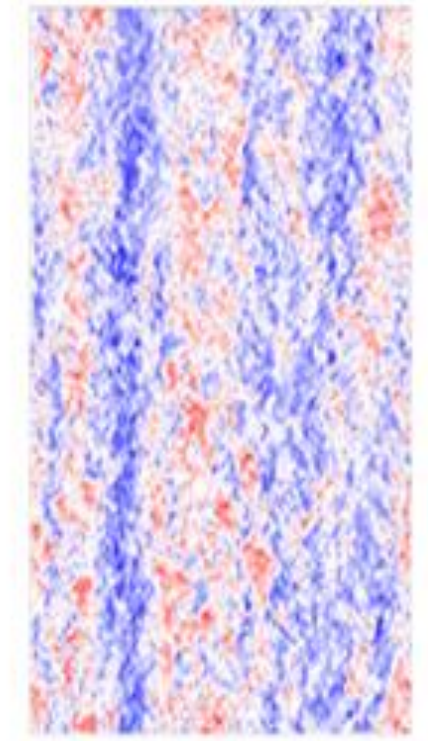


Velocity Disturbances on Two Wall-parallel cut-planes
(Chen & He, 2022)

Fine-mesh block
(‘seeing’ footprint of
outer large-scales)

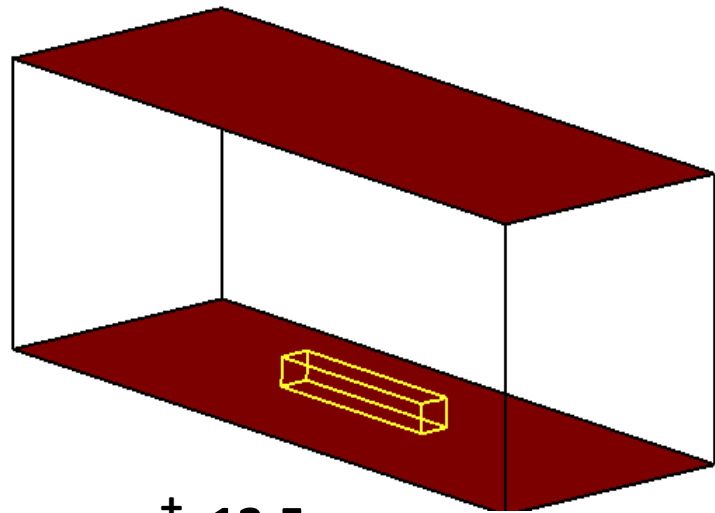


Near-wall ($y^+=13.5$)

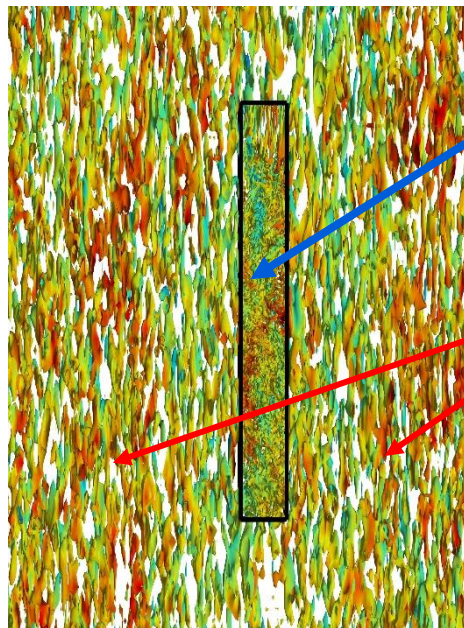


Outer ($y^+=175$)

Any 'Spectral Gap' (Scale-Separation) between Two Scales? (Chen & He, 2022)

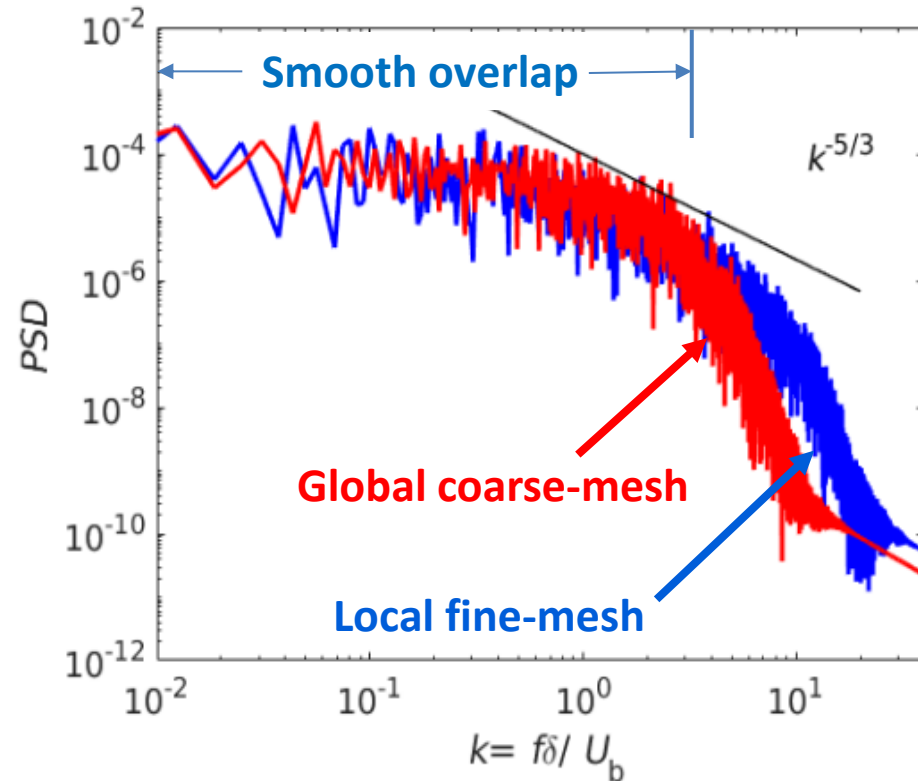


$y^+ = 13.5$



Local Near-wall
Fine-mesh Block

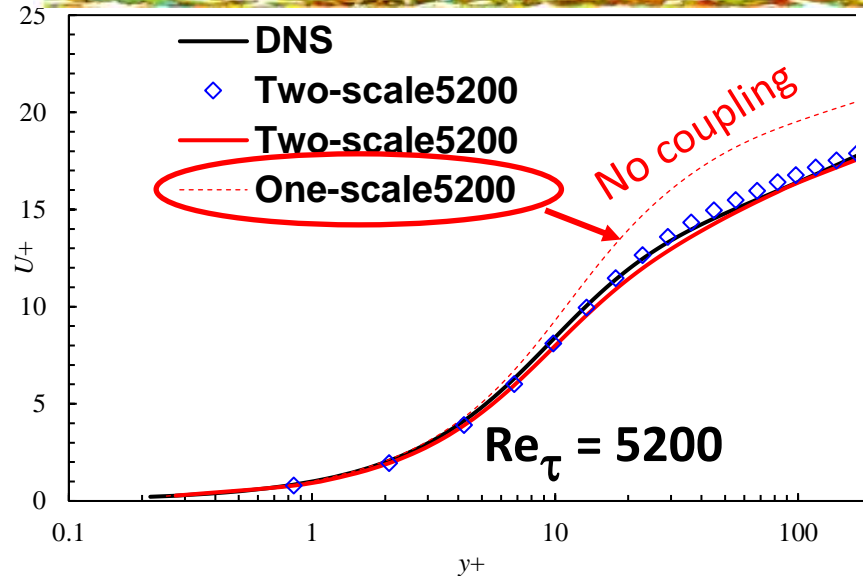
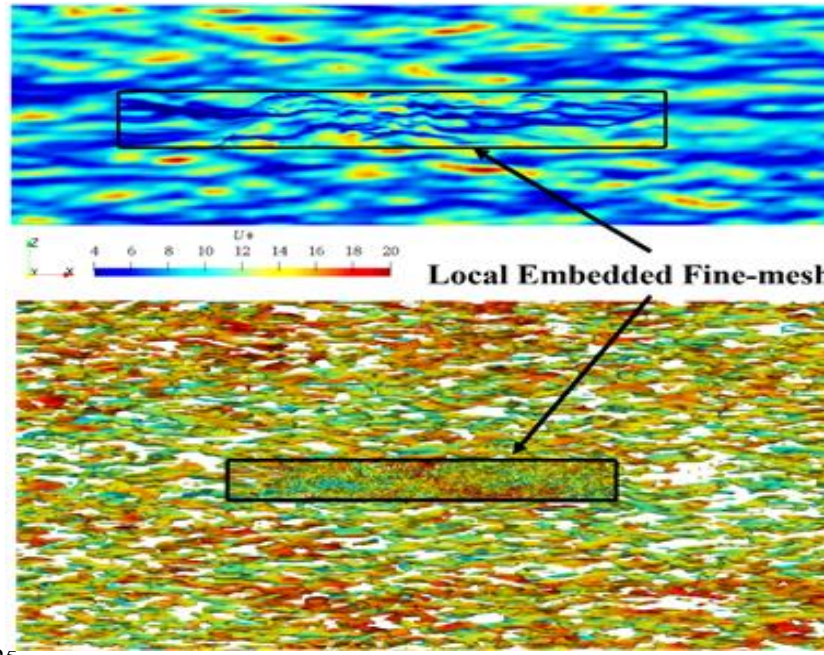
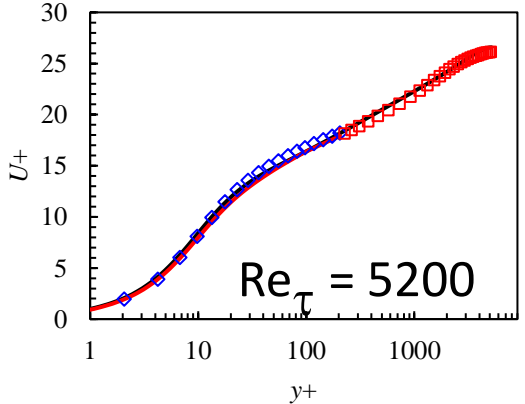
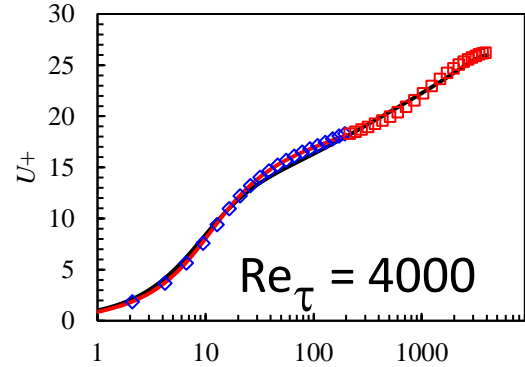
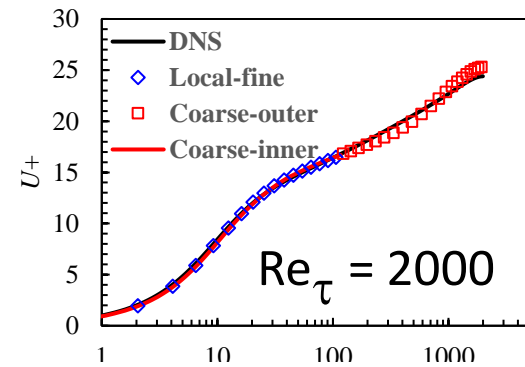
Global Coarse-mesh
Near-wall region



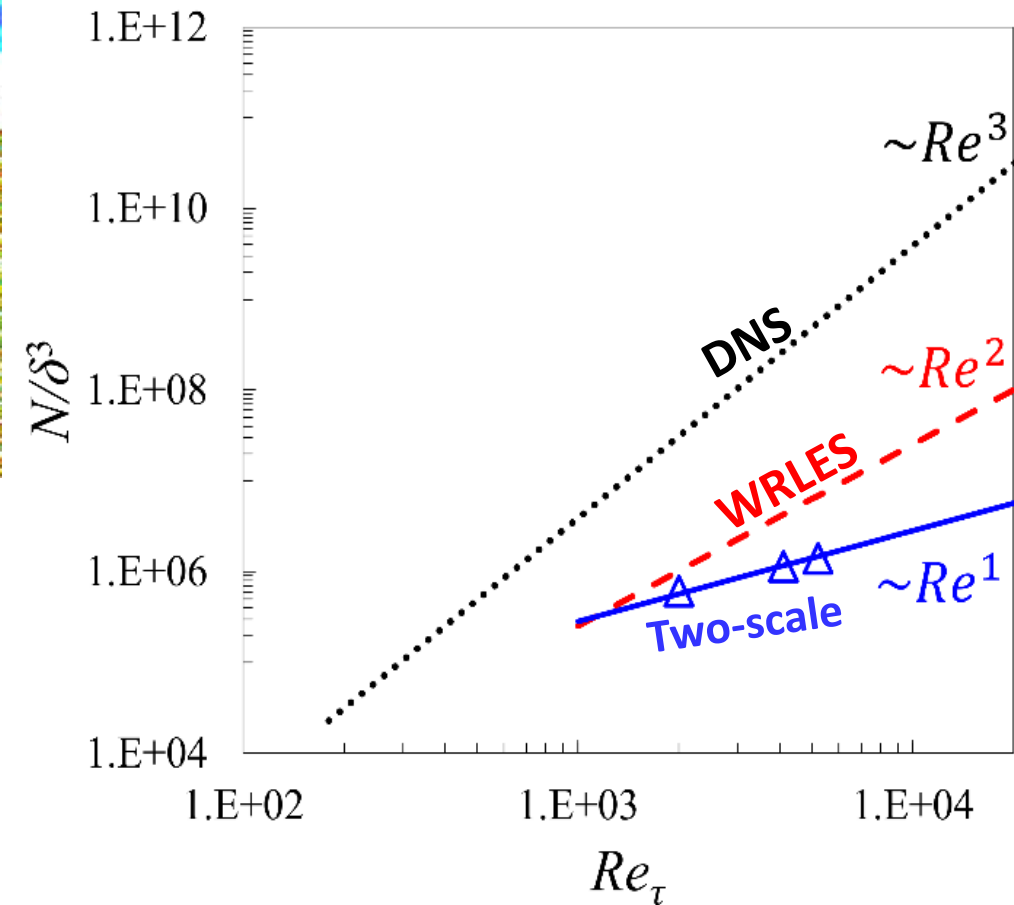
- Spectral gap/separation neither assumed, nor observed between the two domains
(though the coarse-mesh is clearly under-resolved)

Does Two-scale Method Work (Canonical setting)?

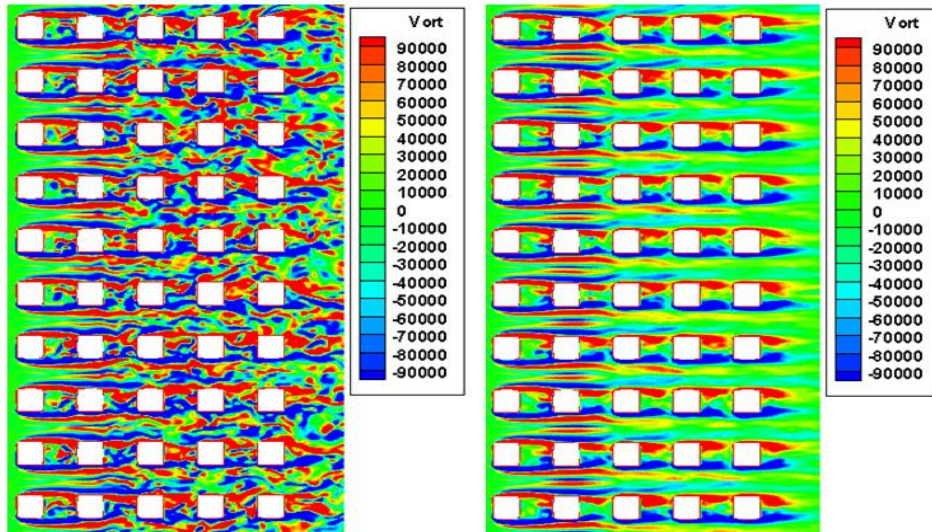
Channel Flow at Different Reynolds numbers (Chen & He 2022)



Mesh Count- Re Scaling

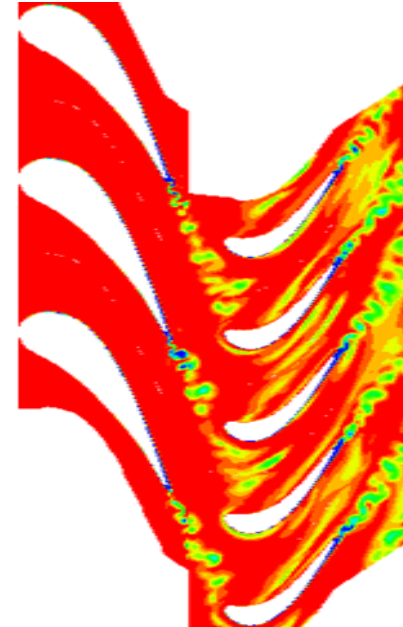


2). Generate and Propagate Local fine-mesh info to Non-uniform Global Coarse-mesh? - 'Averaging to enable High-fidelity Solution Mapping'

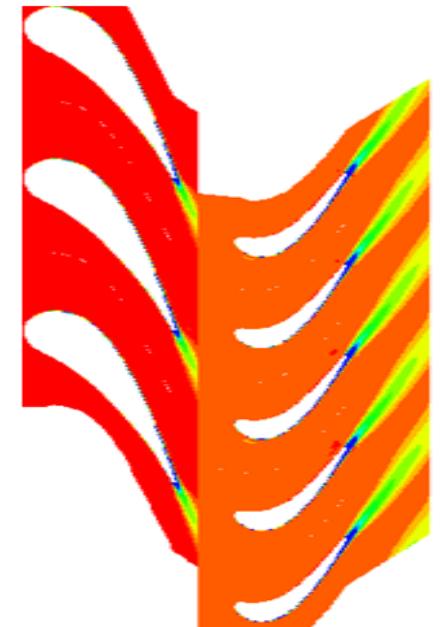


a) Instantaneous

b) Time-averaged



Instantaneous



Time-averaged

- Time-averaging → Block-Block Similarity (*'Blockwise Mappability'*)
- Only need a few local fine-mesh blocks for mapping local fine-mesh effect to global domain

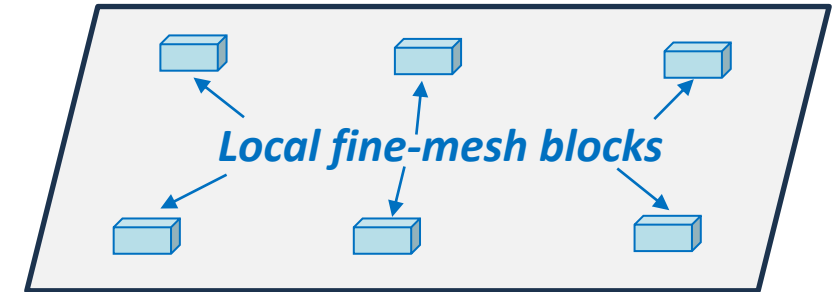
'Block-spectral Mapping' (He 2013, 2018)

(the spectral mapping of time-invariant flow variables/source terms)

- Mesh Pointwise Block-Block spectrum for (i,j,k):

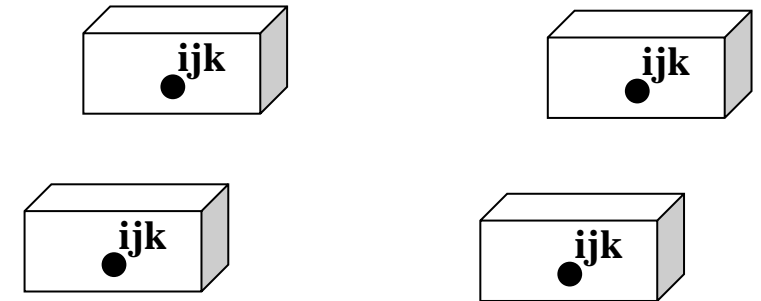
$$W_{ijk} = (W_{ijk})_0 + \sum C_{ijk} F_{basis}(\theta)$$

(W_{ijk} - time-invariant flow variables/source terms)



- Spatial Smoothness of Time-invariant Variables:

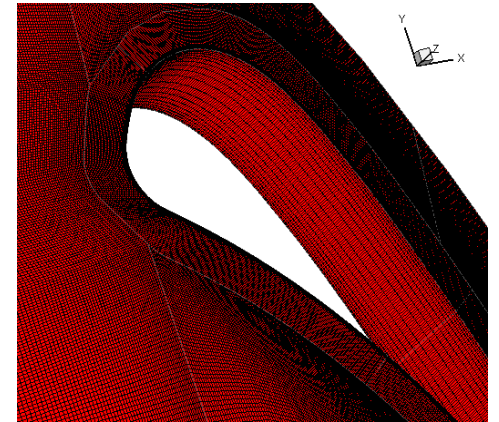
- create a spectrum from a small set of local fine-mesh blocks
- map to inhomogeneous global coarse-mesh domain



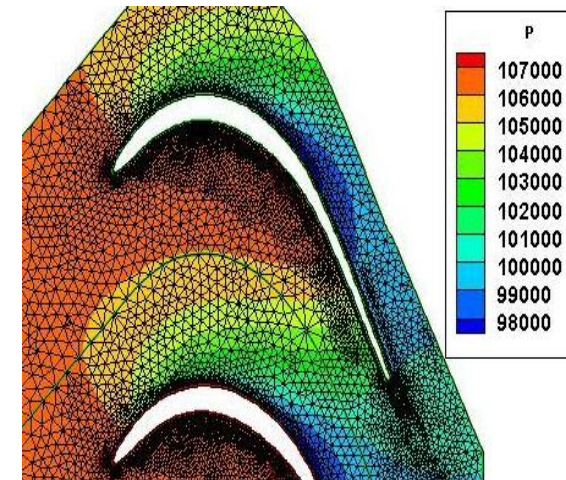
'Mesh-Dependent' Errors need 'Mesh-Informed' Source-terms Correction

- Basic conflict of Two-scale LES (as all WMLES!):
 - to use Coarse-mesh in near-wall region
 - inherently under-resolved flow field
 - associated numerical errors are inevitably 'Mesh-dependent'
 - the corrective source terms need to be 'Mesh-informed'

(cf. some current 'WMLES' (coarse-mesh LES with just a 'trained' wall-function))



- Two Source Term Production Sequences:
 - 'Generation-Propagation' (Uniform mesh) :
 - i) Generate source terms in the fine-mesh block
 - ii) Propagate the generated source term to global domain
 - 'Propagation-Generation' (Complex mesh)
 - i) Propagate Local fine-mesh STA solution $\overline{\tilde{U}}_f$ to Global coarse-mesh
 - ii) Generate source terms using the propagated target in global domain



3) Robustness of Source-term Driven Solution: Problem & Mitigation

Problem:

- Convergence difficulties often observed previously for Reynolds Stress Transport Model (RSTM) RANS.
- Issues also seen when using the Reynolds Stresses from LES/DNS (*Zhang & He 2018, Wu et al 2019*).
 - ➔ All arise from using '**Source-Terms**' (regardless of where/how they are obtained).

Mitigation:

- **Eddy-viscosity** shown to stabilize numerically source-term driven solutions (*Ning & He 2001, He 2018*)
- '**Implicit**' (rather than '**Explicit**') way for adding Reynolds stress sources seems more stable (*Wu et al, 2019*)
(noted however that the '*implicit*' implementation was in fact through '**Eddy-viscosity**'!)
- ➔ **Eddy-viscosity** holds the key to the damping at work regardless, physically or numerically.

3) Balanced Eddy Viscosity (BEV) for Robust Source-Driven Global Solution

- BEV aimed enhance convergence with minimal net-effect on reaching the final STA target (He 2018)
- A simple form of Smagorinsky SGS model sets the limit relative to mesh-size (He 2023)

-Original Coarse-mesh eq. for U_C for given \bar{U}_f

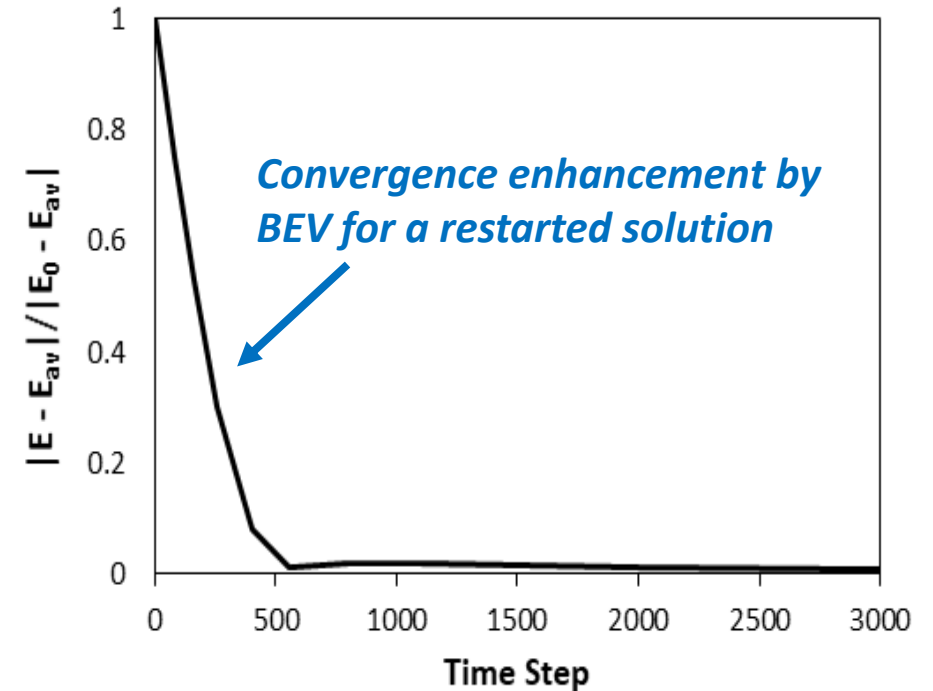
$$\frac{\partial U_C}{\partial t} + R(U_C) = R(\bar{U}_f)$$

- Augmented Coarse-mesh eq. with BEV

$$\frac{\partial U_C}{\partial t} + R(U_C) = R(\bar{U}_f) + \mathbf{LD}(U_C) - \mathbf{LD}(\bar{U}_f)$$

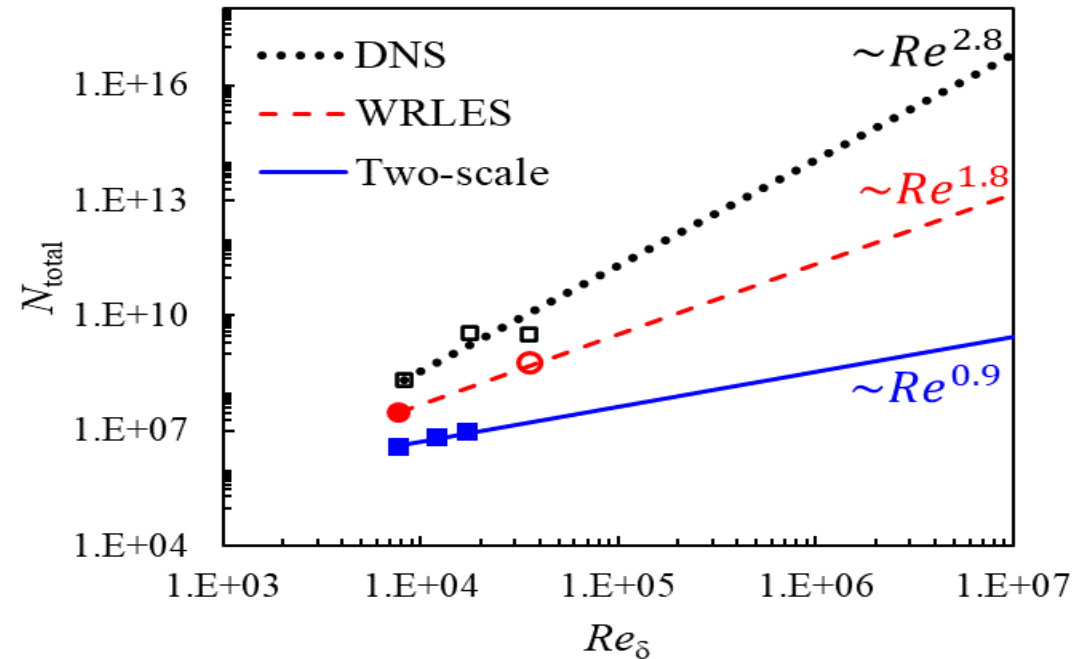
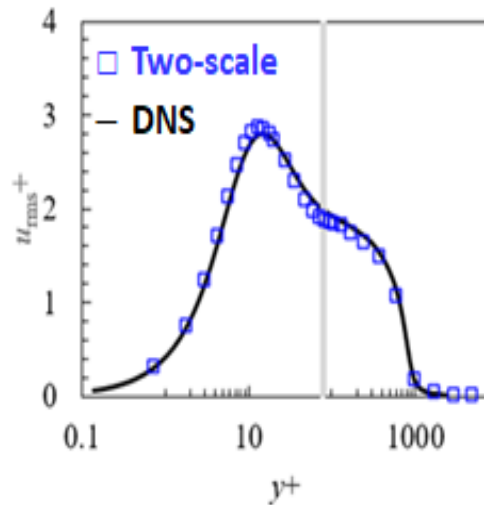
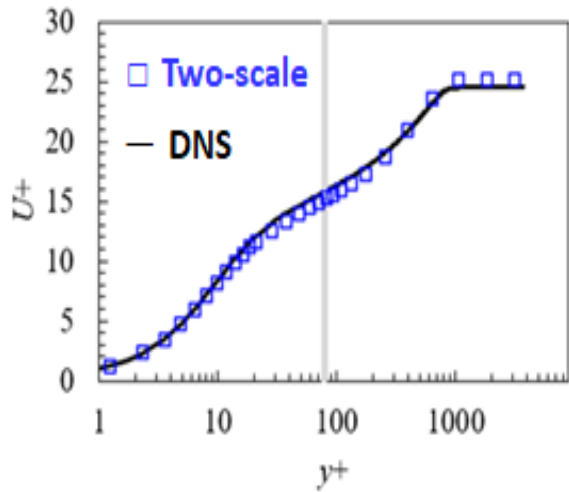
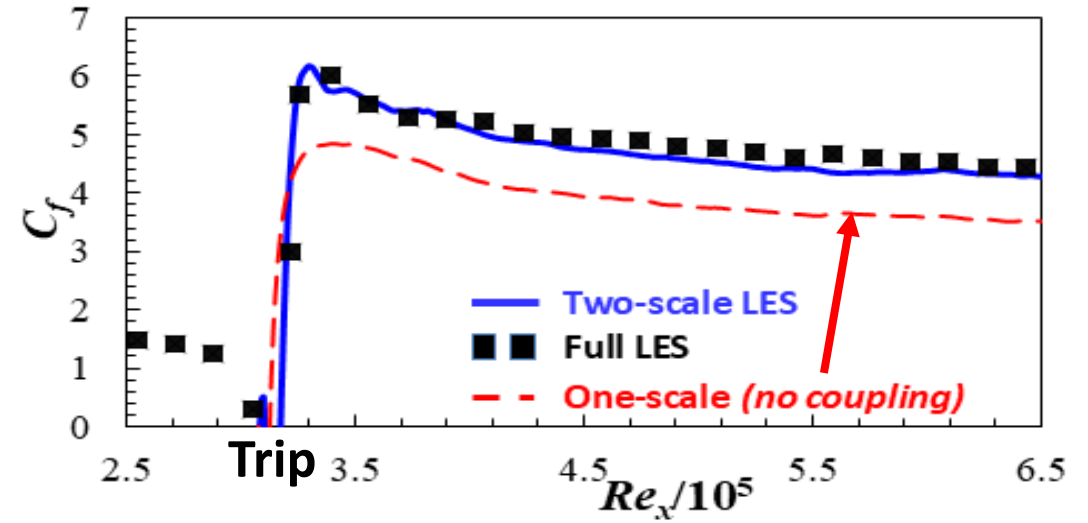
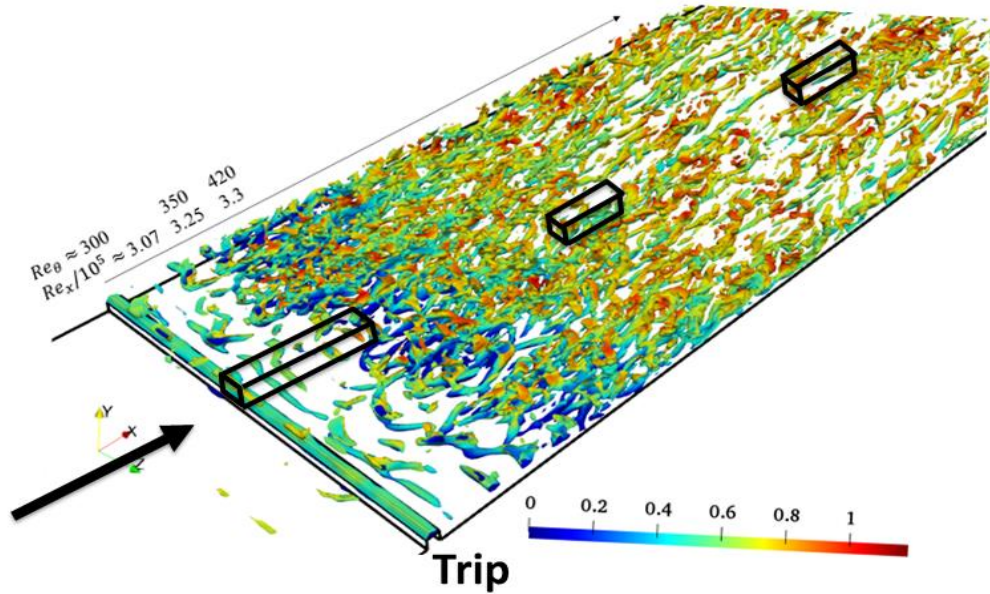
(LD - Linear Dissipation terms)

→ When converging, $U_C \rightarrow \bar{U}_f$, two **BEV terms** balance out.

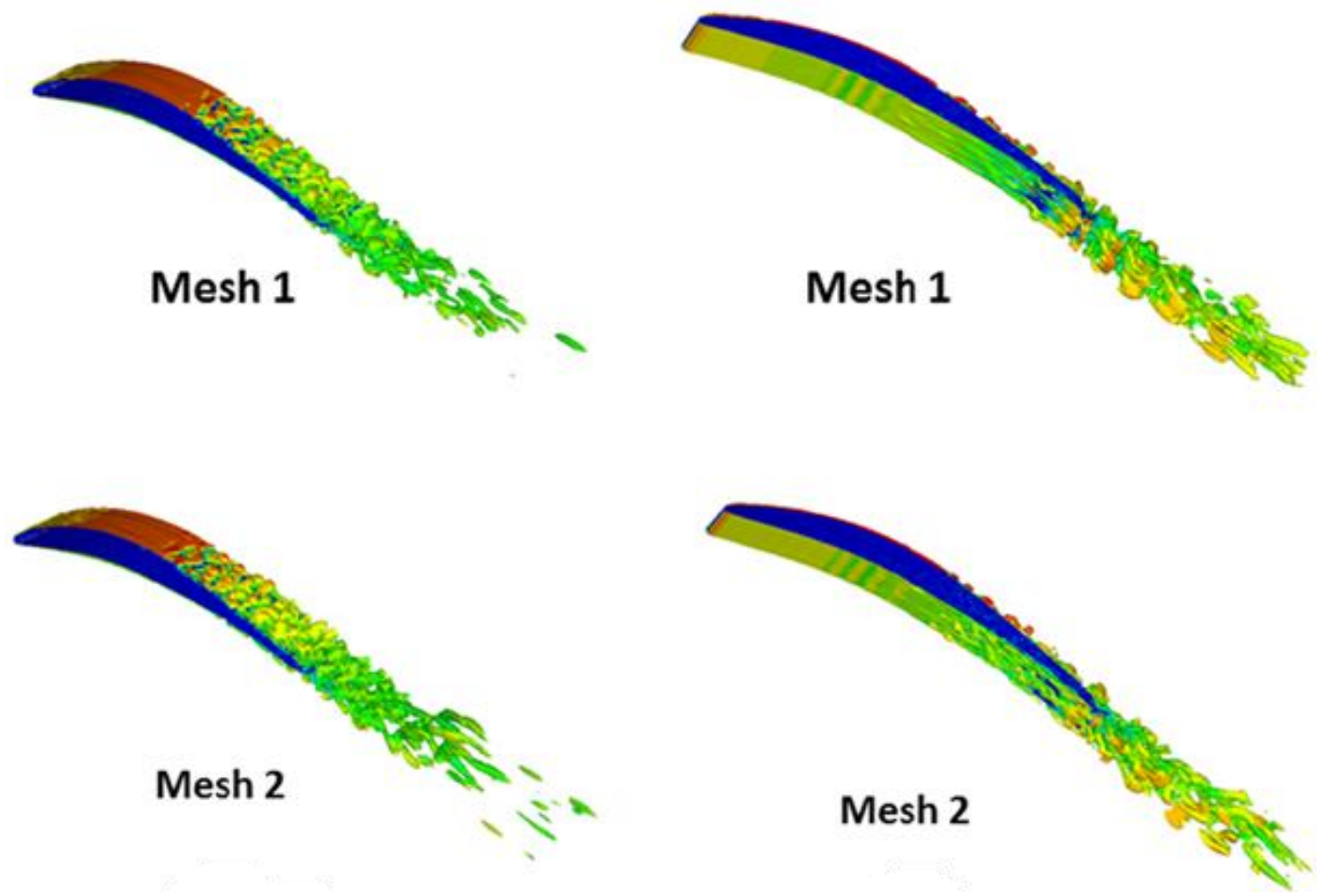
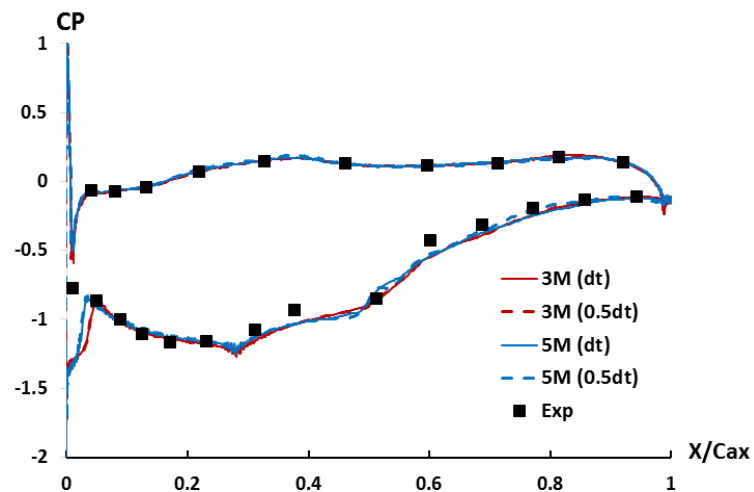
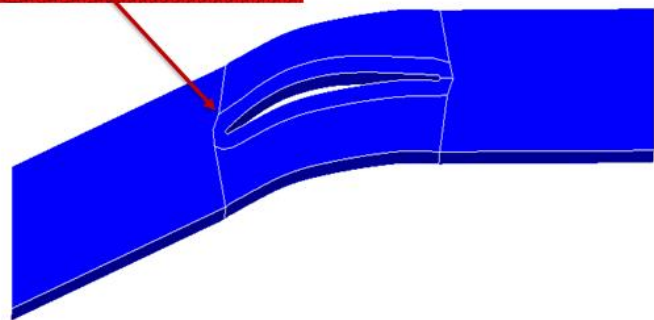
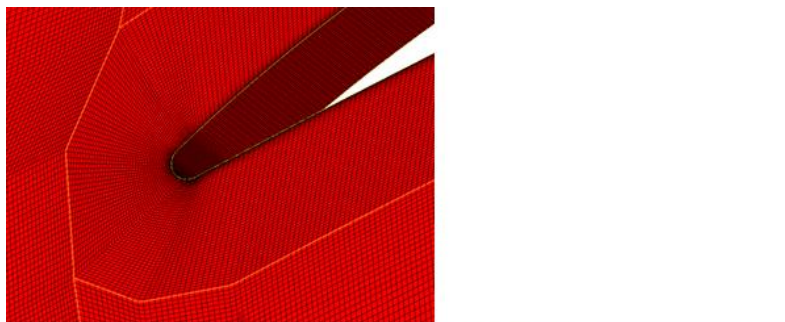


Instantaneous entropy function E relative to time-averaged E_{av} (BEV introduced at $N = 0$)

Two Scale Solution for Tripped Turbulent Boundary Layer (Chen & He, JFM 2023)

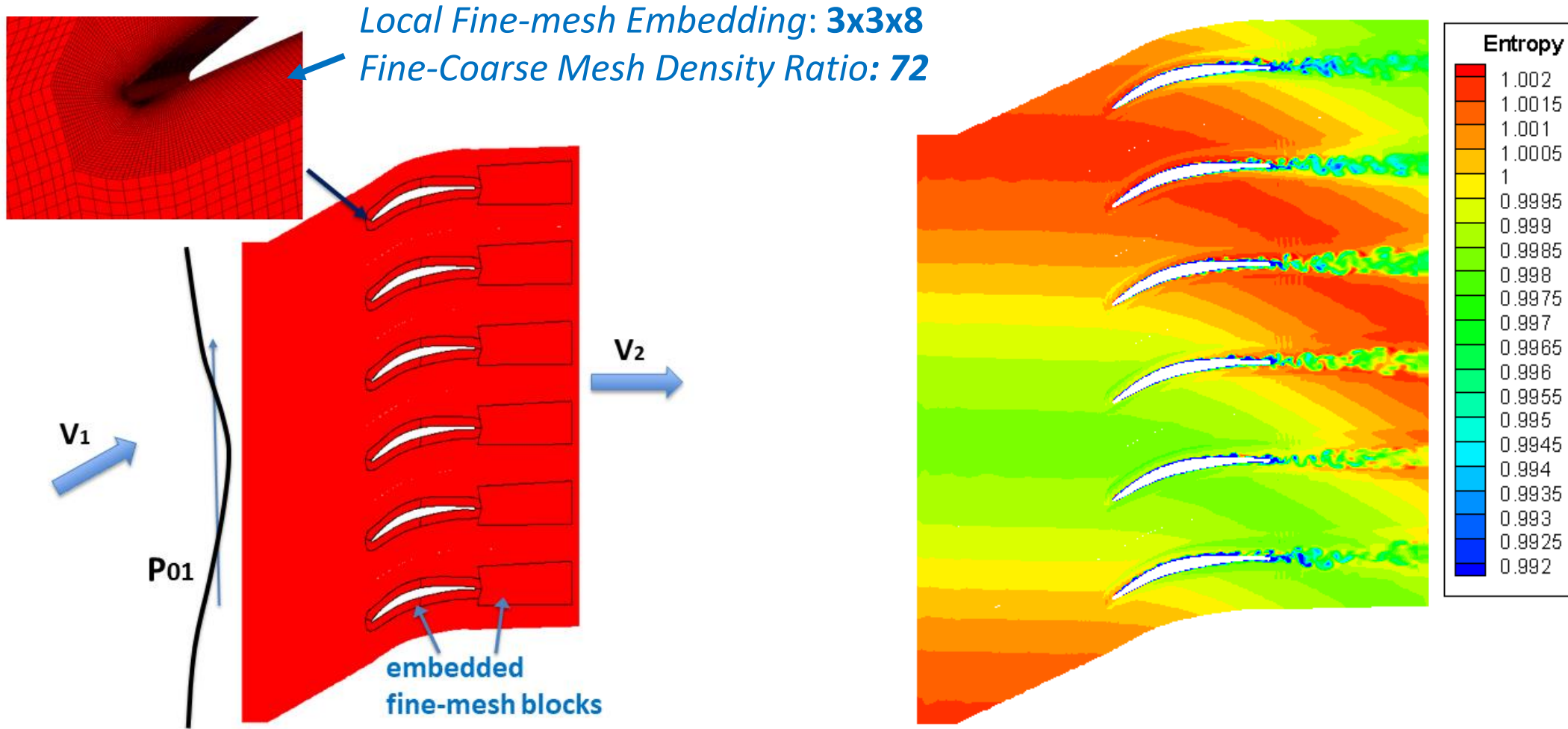


Durham Oscillating Compressor Cascade ($Re=2.5 \times 10^5$)



Mesh 1: 3M for 10% span
Mesh 2: 5M for 10% span

Large Domain & Long Disturbance: Multi-passages with Inlet Distortion

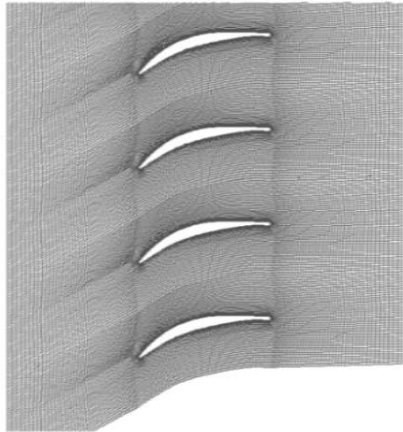


Direct Solution Setup (all 6 passages with fine-mesh)

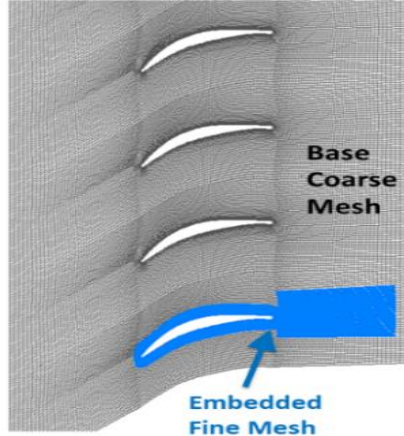
Instantaneous Entropy function ($e^{-\Delta S/R}$)

Multi-passages with Inlet Distortion

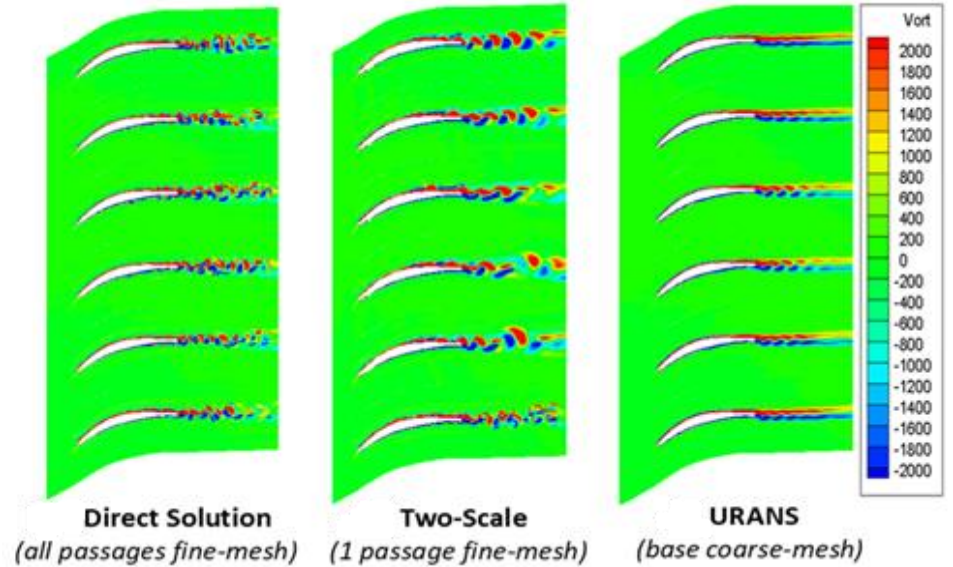
Base Coarse-mesh



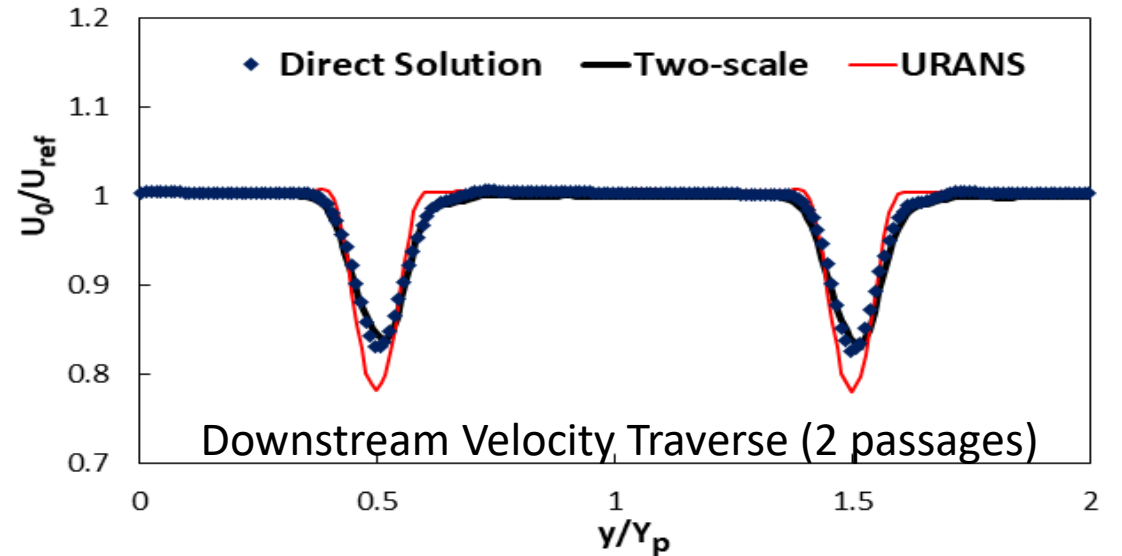
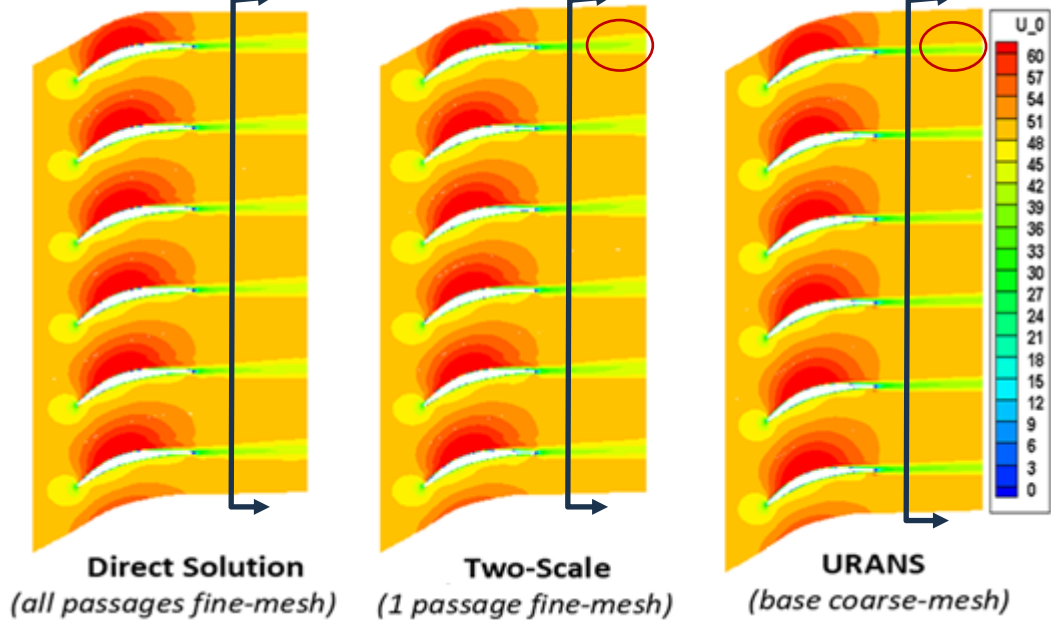
Single-passage Fine-mesh



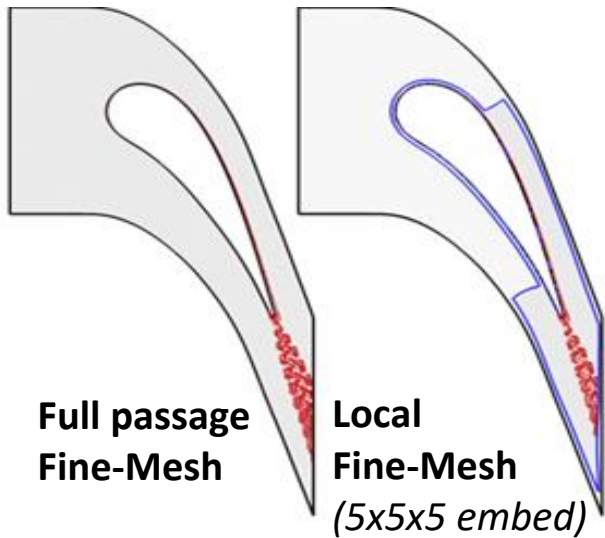
Unsteady Vorticity



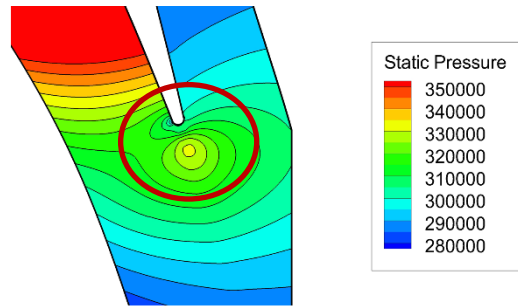
Time Averaged Velocity



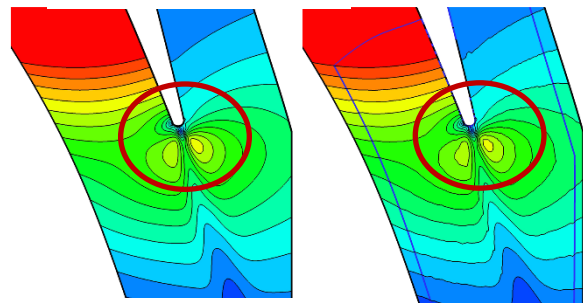
Turbine Blade Flow: Attention to Trailing Edge *(He and Yi 2017)*



Single-Passage Domain

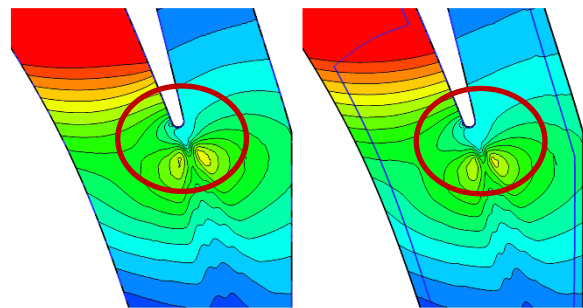


Steady RANS



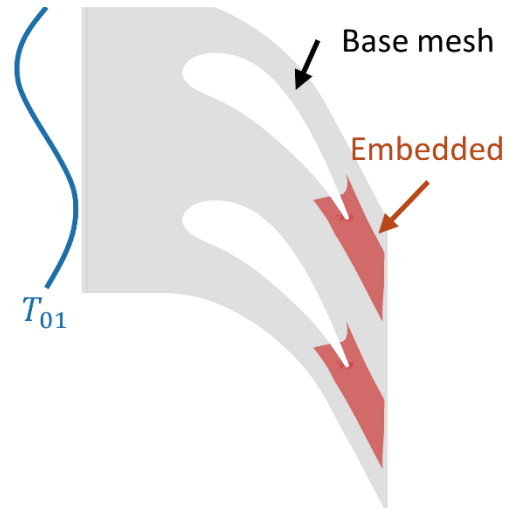
Direct URANS

Two-scale URANS

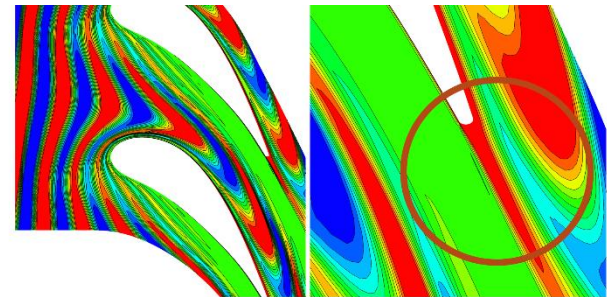


Direct LES

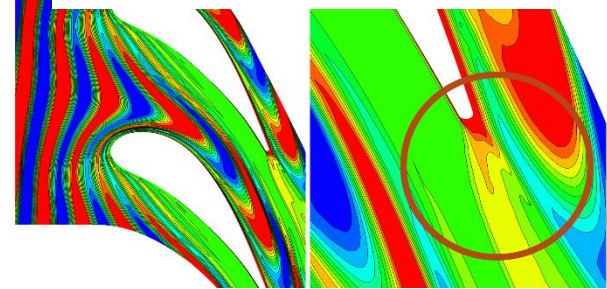
Two-scale LES



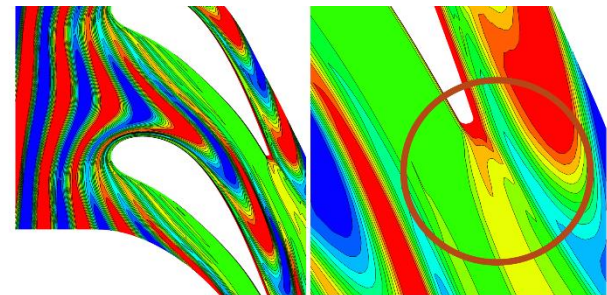
Two-Passage Domain
(inlet rotating distortion)



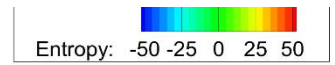
URANS



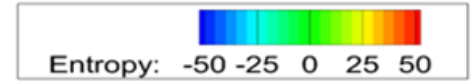
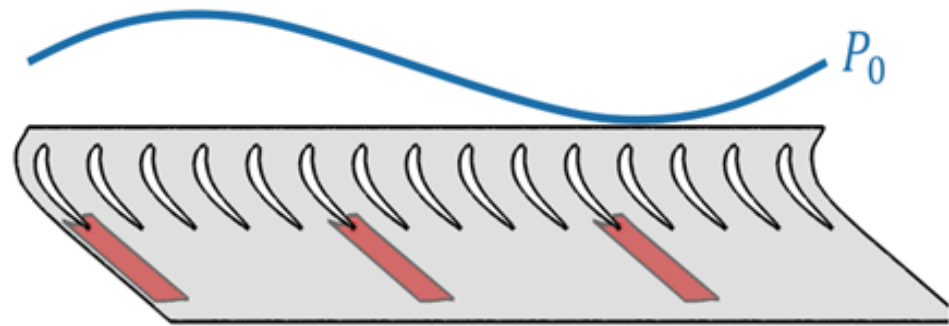
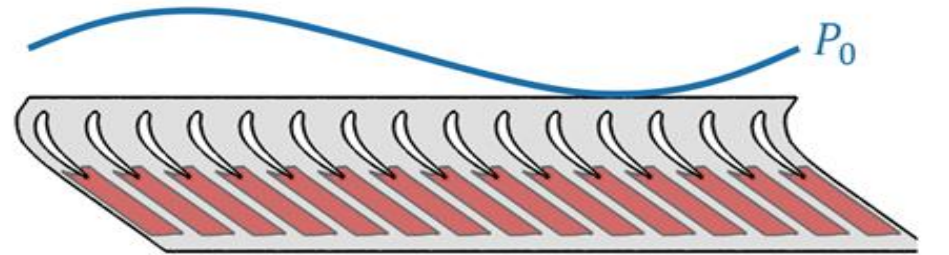
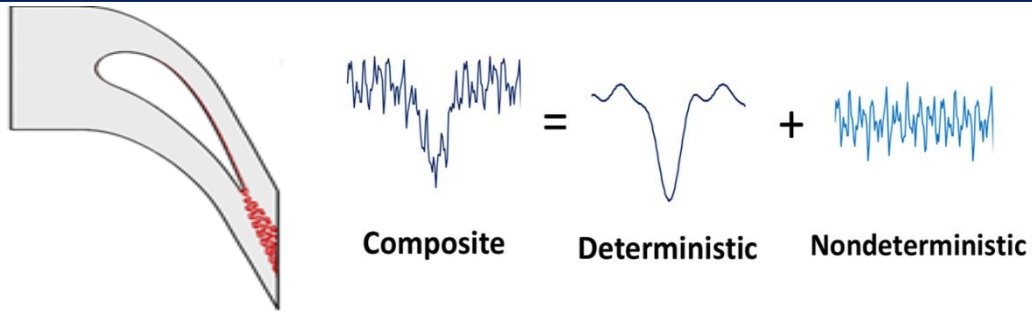
Direct LES (Ensemble-averaged)



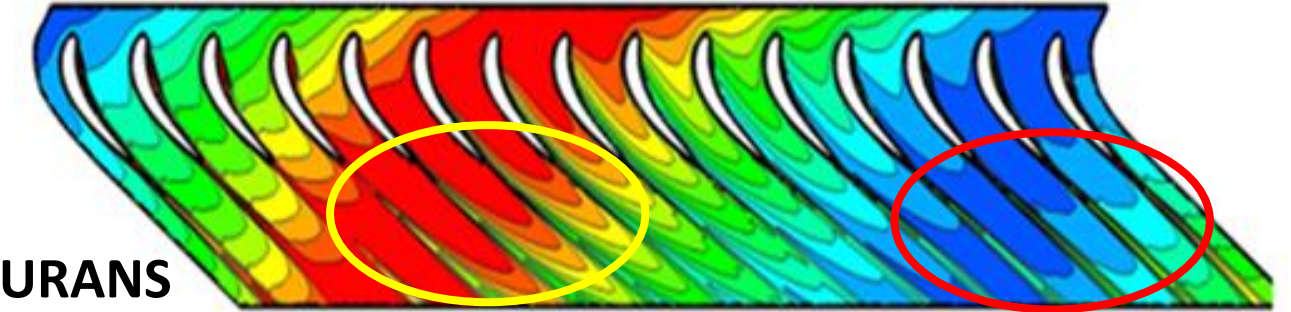
Two-scale LES



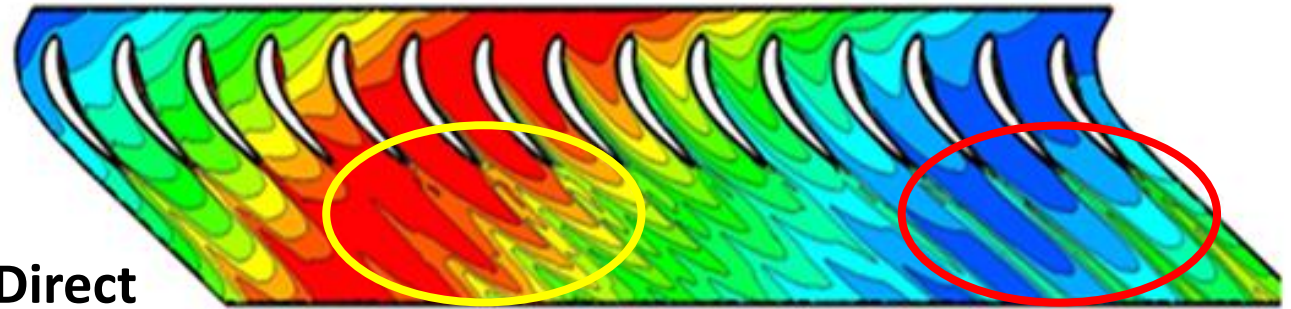
Two-scale Ensemble-Averaging for URANS/LES (He & Yi 2017)



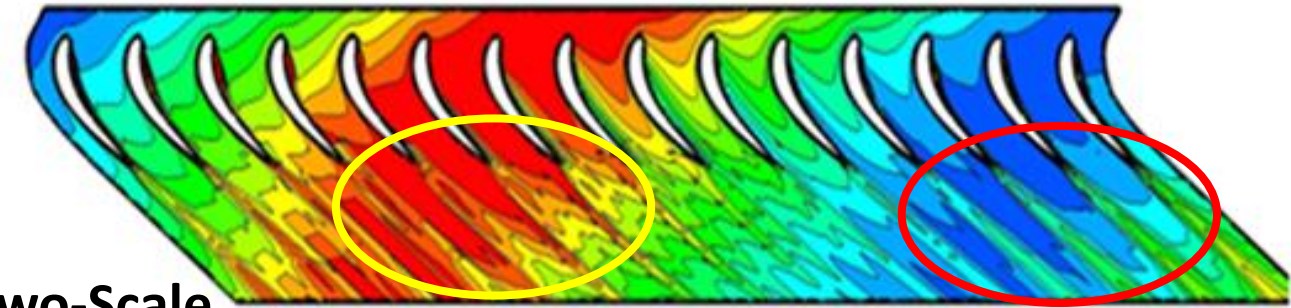
URANS



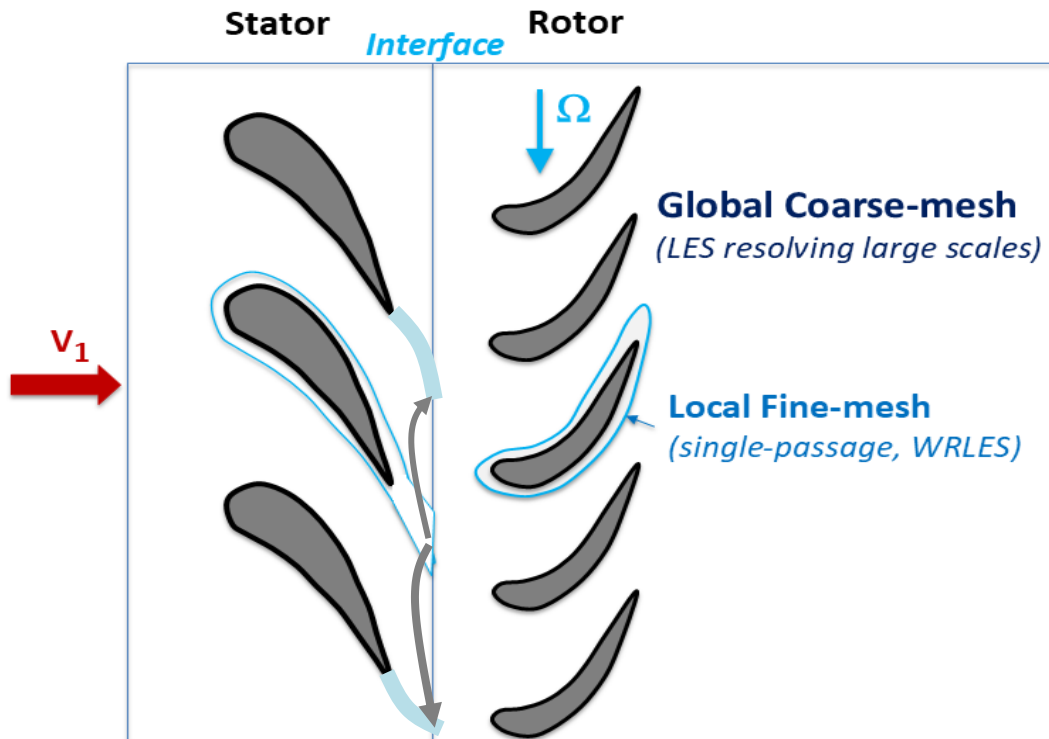
Direct



Two-Scale

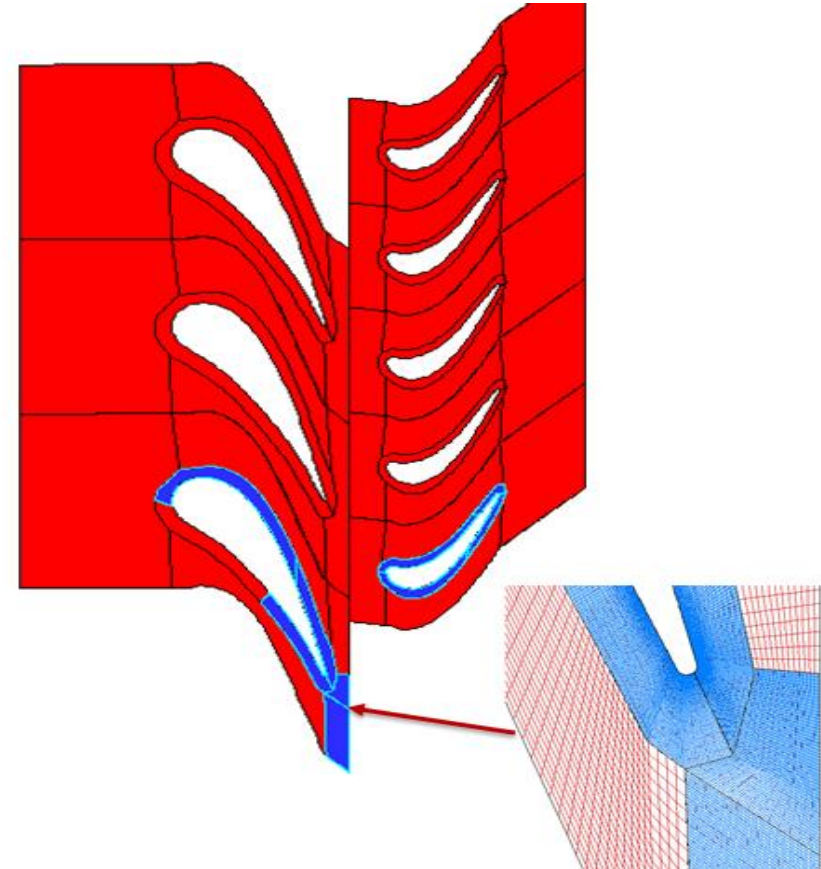


Multi-passages & Multi-rows: *Turbine Stage*



'Harmonic Construction' of Multi-passage Wakes (He 2021):

- Spatial FT at Interface for each upstream blade
- Construction of Downstream Characteristic Variables:
Low harmonics (own passage) + High harmonics (fine-mesh passage)

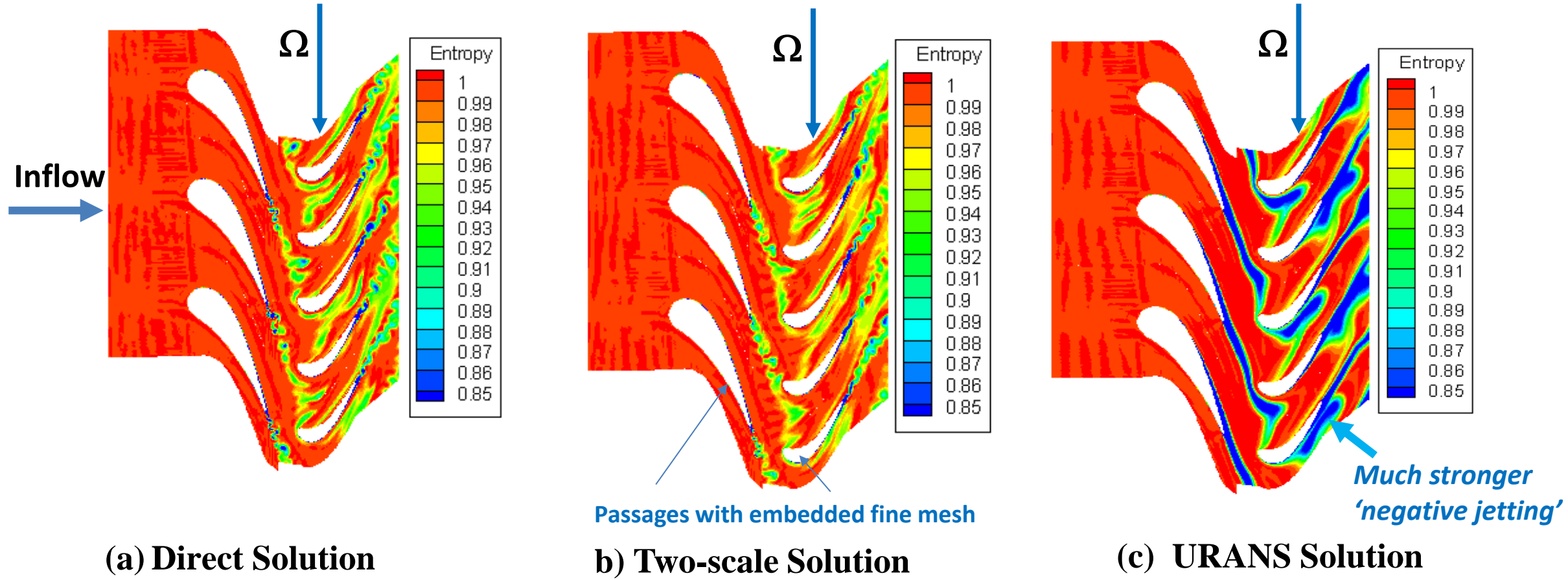


Local Fine-mesh Embedding: 4x4x10
Fine-Coarse Mesh Density Ratio: 160

Transonic Turbine Stage at Realistic Mach & Re (He, 2021)

(3 NGV/5 rotor blades counts are taken to enable the direct solution)

Instantaneous Entropy Function ($e^{-\Delta S/R}$)

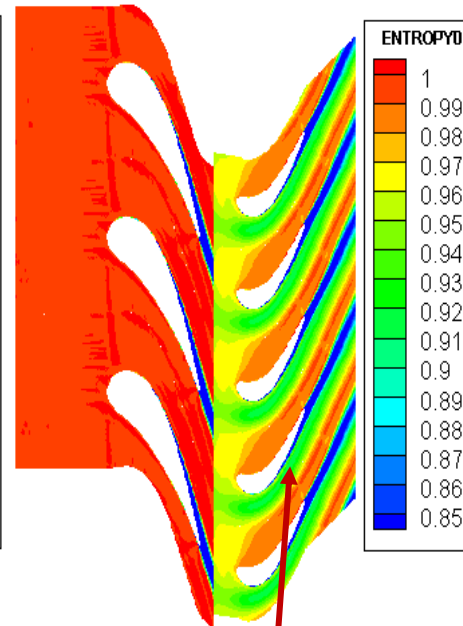
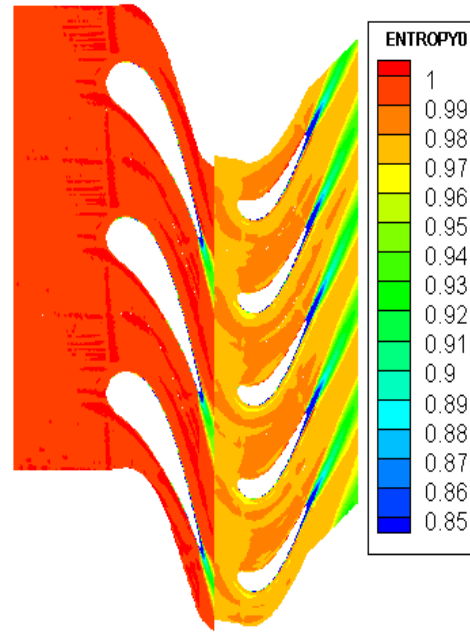
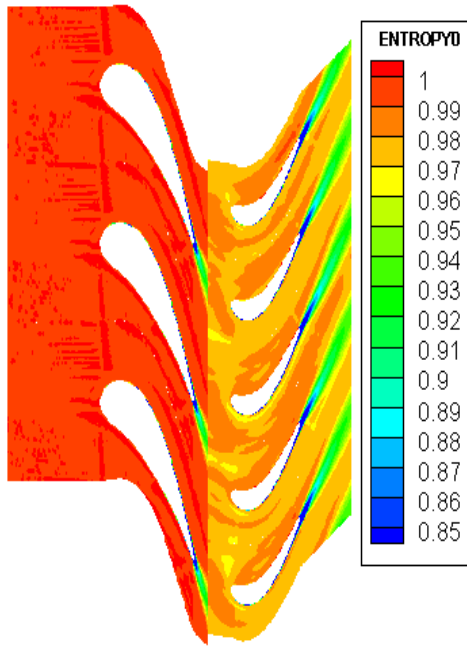


Transonic Turbine Stage, Time-Averaged Entropy ($e^{-\Delta S_0/R}$)

Direct Solution
(multi-passage fine-mesh)

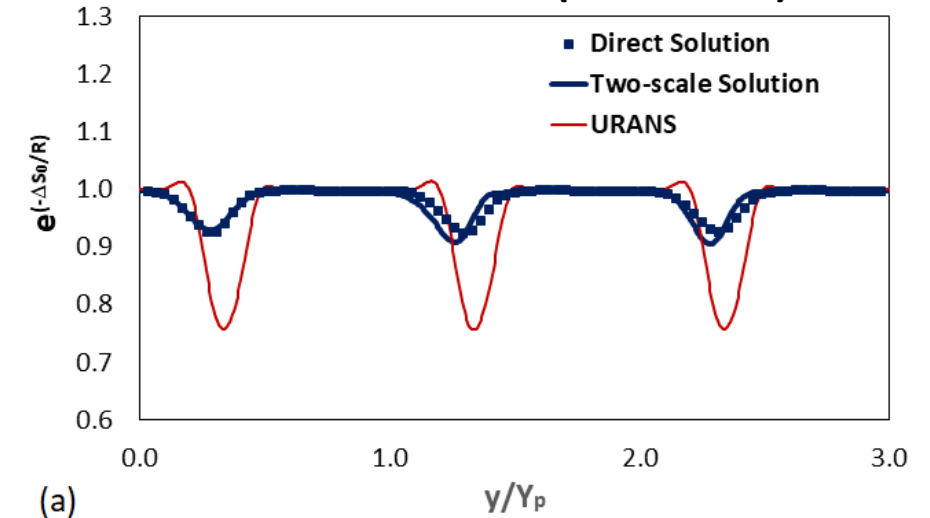
Two-scale Solution
(single-passage fine-mesh)

URANS Solution
(base coarse-mesh)

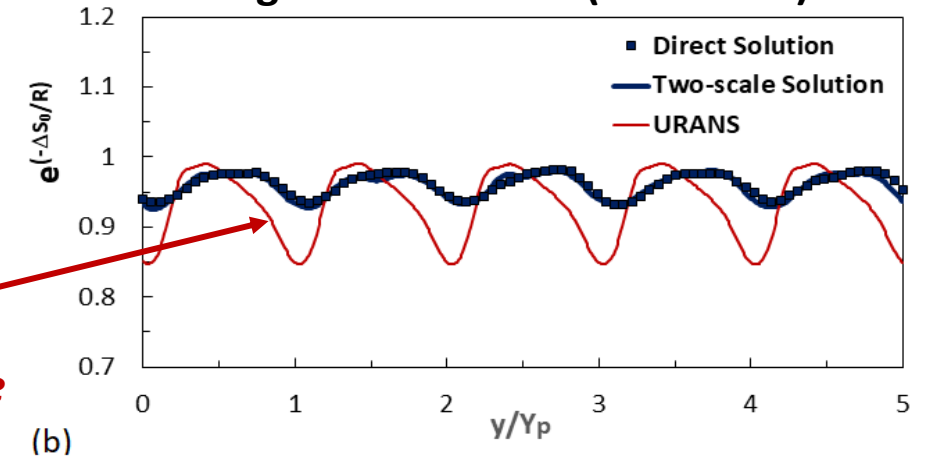


**Over-predicted
'negative jetting'
on rotor suction side**

NGV Loss Traverse (Stator Exit)



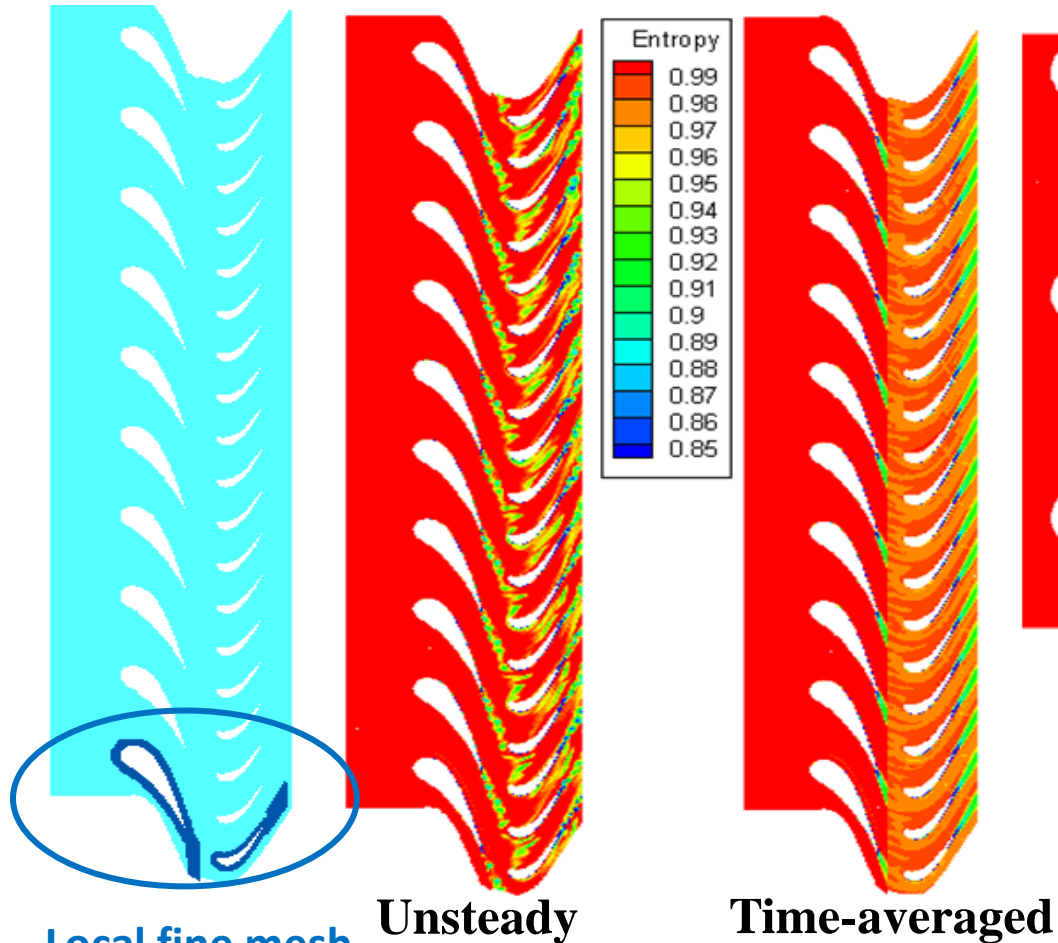
Stage Loss Traverse (Rotor Exit)



Transonic Turbine Stage (10 Stator + 19 Rotor Blades)

Two-scale vs. URANS (He 2021)

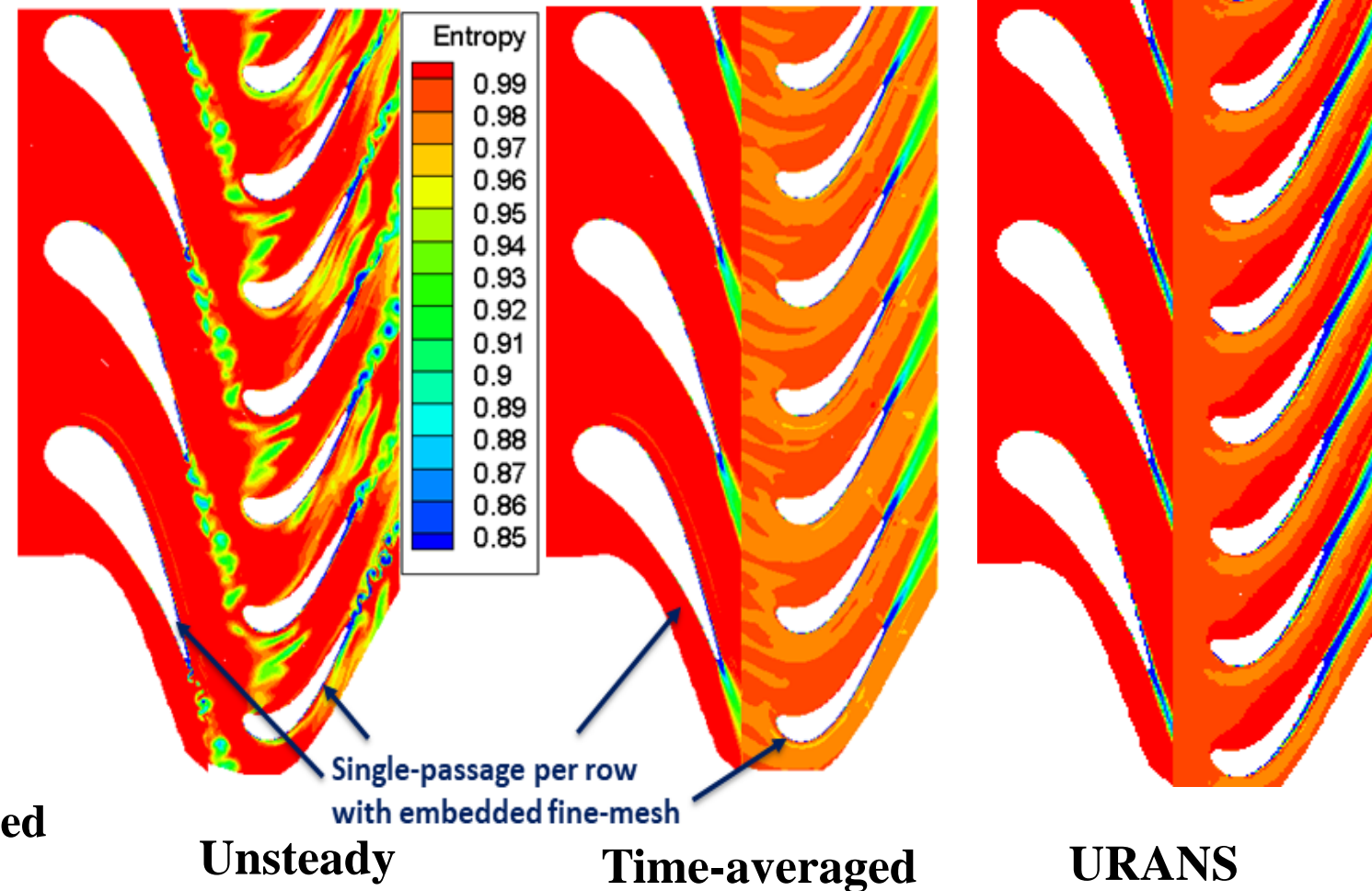
Whole Domain View



Local fine mesh

1 fine-mesh blade/row
fine/coarse mesh density: 128

Local View (around embedded passages)



Unsteady

Time-averaged

URANS

Closing Remarks

- **Motivating Considerations for Efficient Nearwall Turbulence-Resolving Methods**
 - *Resolving multiscale outer-inner interactions (per recent findings on wall-bound turbulence).*
 - *Correcting inherent coarse-mesh under-resolution errors via 'mesh-informed' source terms.*
- **Two-scale Method ('Interactive Coarse-Mesh LES')**
 - *Space-Time Averaging: in 'Direct' & 'Inverse' modes in Global and Local domains respectively*
 - *Source Terms enabled coupling between two domains:
Local: providing source-terms for the global while conditioned by global coarse-mesh;
*Global: providing conditioning for the local while driven by local fine-mesh source terms.**
- **Concept-proof and Demonstration Results**
 - *$O(10^2)$ local mesh refinement demonstrated.*
 - *Mesh Count-Re scaling: $\sim Re^2$ (WRLES) $\rightarrow \sim Re^1$ (Two-scale LES)*