Rosa Branca Esteves

Competitive Behaviour-Based Price Discrimination

D.Phil in Economics

Wadham College, University of Oxford

Michaelms 2004

To my parents and Paco

Acknowledgments

As any author of a D.Phil thesis I inevitably owe a debt of gratitude to those that have made this research feasible. Obviously, words are always a less meaningful way of expressing gratitude.

A long-standing thanks is due to Mark Armstrong, a person of great talent, for an extremely constructive and very demanding supervision; and for everything that I have learned with him and that certainly go well beyond the completion of this thesis.

I am also grateful to Paul Klemperer and my college adviser, Valerie Lechene, for their advice and encouragement during my years in Oxford.

I owe a word of gratitude to the *Fundação para a Ciência e Tecnologia* (Grant number SFRH/BD/1099/2000) and the *Universidade do Minho* for the financial support I was awarded with.

Finally, I am eternally grateful to my parents and brother for their constant faith and support. My heartfelt thanks cannot adequately express the debt that I owe to my husband, Paco, whose love, patience and encouragement made these acknowledgments possible. I thank him not only for keeping me focused on the completion of this thesis and alleviating my disappointments, but specially for the time he dedicated to me in one of my worst moments rather than to his own research.

Abstract

Competitive Behaviour-Based Price Discrimination

Rosa B. Esteves—Wadham College, Oxford University D.Phil in Economics—Michaelms 2004

Advances in information technologies have increasingly enabled firms to use consumers' past purchasing data to charge different prices to its own customers and to those customers that in some sense belong to the rival firm. At first glance this new form of price discrimination seems to be lucrative as it allows a firm to generate profitable incremental sales without damaging profits it can extract from its own customer base. However, as *behaviour-based price discrimination* gains popularity many interesting questions arise. Is it, really, in the best interest of firms to recognise customers with different past behaviour and to price discriminate accordingly? Or is it rather in their interest to avoid any possible learning and thereby price discrimination practices? Should consumers hide their true types, i.e., should they behave anonymously? Further, should government regulation restrict information collection and price discrimination practices?

The study of these questions is the study of the profit and welfare effects of behaviour-based price discrimination. This is the central issue of this thesis. With that in mind, this thesis addresses three theoretical models. The first one is based on the hypothesis that the ability of firms to predict the preferences of individual customers for the purpose of price discrimination is less than perfect but is constantly improving due to advances in information technologies. Here the main goal will be to investigate how profits, consumer surplus and welfare evolve as price discrimination is based on more accurate information. The second model is a natural sequel of the former as it tries to model *how* firms might obtain a signal of a consumer's preferences. Whether or not a given consumer bought from the firm previously might be used as an accurate signal of a consumer's preferences. A key issue here will be to examine whether or not it is in the interest of firms to avoid learning and price discrimination and *how* can they attain that goal. Finally, the third model studies the interaction between purely informative advertising and price discrimination based on customers' past behaviour. As without advertising consumers are left out of the market, the welfare effects of price discrimination are guided by *how* will price discrimination affect each firm's advertising decisions in relation to the social optimal level of advertising.

Contents

Chapter 1

General Introduction

> *"Information technology allows for fine-grained observation and analysis of consumer behaviour. This allows for various kinds of marketing strategies that were previously extremely difficult to carry out, at least on a large scale. For example, a seller can offer prices and goods that are differentiated by individual behaviour and/or characteristics."*
>
> Hal Varian (2003), *Economics of Information Technology.*

Economists have long been interested in understanding the profit, consumer surplus and welfare effects of an ancient marketing strategy: *Price Discrimination.*

While it is not new that firms try frequently to segment customers in order to price discriminate, what has dramatically changed, with recent advances in information technologies, is the quality of consumer-specific data now available in many markets and how this information has been used by firms for price discrimination purposes. Specifically, thanks to information technology it is nowadays increasingly feasible for sellers to segment customers on the basis of their purchasing histories and to price discriminate accordingly. This form of price discrimination has been named in the literature as *behaviour-based price discrimination.*

Broadly speaking, the chief aim of this thesis is to contribute to the ongoing debate on the profit, consumer surplus and welfare effects of this novel kind of competitive price discrimination.

1.1 Motivation

In the latter years of the 1990s and in the early years of the new millennium we have witnessed remarkable advances in information technologies. According to Acquisti and Varian (2003), between 1998 and 2003 the cost of a gigabyte of hard disk storage fell from about \$11,500 to less than a dollar. Further, between 2000 and 2004 the world Internet usage raised from about 361 to around 798 million people (*www.internetworldstats.com*) and this number is expected to increase in the following years.[1]

The dramatic reduction of the cost of storing information coupled with the increasing use of the Internet as a medium to conduct business have enabled firms to easily gather, store and access consumer-specific data (e.g. purchase histories), thereby increasing the firms' ability to recognise customers almost instantaneously. As a result of that, the practice of offering different prices to customers with different past purchasing profiles has been widely observed in diverse industries—e.g. long-distance telecommunications, mobile telephones, banks, airlines and grocery stores.[2] *AT&T, MCI, American Airlines* and *Amazon* are examples of firms that in some way have been following this marketing strategy. As Streitfield (2000) observes, "With its detailed records on the buying habits of 23 million of consumers, *Amazon* is perfectly situated to employ dynamic pricing on a massive scale." In fact, in September 2000, the *Washington Post* reported that *Amazon* was using "cookies" (i.e. electronic identifiers)[3] installed on a customer's computer to offer different DVD prices to first-time

[1] This information was collected from the site mentioned on August 2004.

[2] See Rossi, McCulloch and Allenby (1996) for an overview of issues raised by available purchase data (e.g. price discrimination).

[3] Cookies are based on a two-stage process. First, the cookie is stored in the user's computer. The Web server then creates a specific cookie, which is essentially a tagged string of text containing for instance the user's preferences, and it transmits this cookie to the user's computer. The user's Web browser, if cookie-savvy, receives the cookie and stores it in a special file called a cookie list. As a result, personal information (e.g. the user's preferences, past purchases, e-mail, etc.) is formatted by the Web server, transmitted, and saved by the user's computer. During the second stage, the cookie is transferred from the user's machine to a Web server. Whenever a user directs his Web browser to display a certain Web page from the server, the browser will, without the user's knowledge, transmit the cookie containing personal information to the Web server. This definition was partially extracted from the site *www.cookiecentral.com* (accessed on September 2004).

visitors and to loyal or registered customers. Again, in November 2000, the *Register* reported that *Amazon* was targeting regular customers with a special direct e-mail book offer for *Terry Pratchett*'s latest novel, but offered it at a price higher than it was charged on its site.[4] Of course, *Amazon* was trying to attract new customers while squeezing money out of their loyal customers.

In short, firms are increasingly able to collect an enormous amount of data about individual consumers, mine that data quickly to understand preferences and buyer behaviour, and thereby to price discriminate. In the most extreme case, information technology would allow firms to offer what has been termed as "personalised" prices—i.e. firms would set prices on an individual basis.

Obviously, the use of personal information about consumers for price discrimination purposes raises many questions of interest. Guided by old concerns on the effects of price discrimination, recent economic research has been trying to investigate whether such forms of price discrimination act to intensify competition or not, and whether they should be banned or encouraged. This thesis shares the same type of concerns. Broadly speaking, it tries to investigate whether it is the firms or the consumers who will benefit the most from the ability of firms to tailor different prices to consumers with different past purchasing profiles. To that end, three different approaches will be considered in this thesis.

The first approach is based on the hypothesis that the ability of firms to predict the preferences of individual customers for the purpose of price discrimination is far from perfect but is constantly improving due to advances in information technologies. Specifically, it is assumed that based on their records firms can observe a private (less than perfect) signal about each consumer's brand preferences and afterwards quote different prices to consumers with different signals. (When we log on at *Amazon*, its computers try to guess which type of consumer we are, and what price is likely to be accepted or not.) Here the main goal will be to investigate how profits, consumer surplus and welfare evolve as price discrimination is based on more accurate information.

[4] See *www.theregister.co.uk/2000/11/06/amazon_rips_off_regular_customers/*.

The second approach is a natural sequel of the first as it tries to model how firms might obtain a signal of a consumer's preferences. In a repeated interaction, whether or not a given consumer bought from the firm previously might be used as an accurate signal of a consumer's preferences (i.e., a firm might learn to which firm each consumer is loyal). (The *Amazon*'s story of charging different prices to loyal and first-time customers may lend support to this approach.) In this case the extent of learning is endogenously determined by firms' first-period pricing behaviour. Therefore, apart from other relevant questions that will be discussed later on, a key issue here will be to examine whether or not it is in the interest of firms to avoid learning and price discrimination and *how* can they attain that goal.

Finally, the third approach studies the interplay between purely informative advertising and price discrimination based on customers' past behaviour. While in the former two approaches firms reach perfectly and costlessly any potential customer, here firms need to invest in advertising to generate awareness; and in doing so they endogenously segment the market into *captive* and *price-sensitive* consumers. (This hypothesis is particularly relevant in new product markets.) Again, when firms interact more than once, they might be able to learn whether a consumer that was reached by their advertising campaign is a price-sensitive or a captive customer and to tailor their prices accordingly by means of targeted advertising. As without advertising consumers are left out of the market, the welfare effects of price discrimination are also guided by old concerns in the informative advertising literature. That is, do firms over-or-under advertise? Finally, this approach looks also on the following question. How will price discrimination affect each firm's advertising decisions? To the best of our knowledge this is an issue not yet explored in the literature so far.

1.2 Survey of the relevant literature on competitive price discrimination

For a long time economists have been concerned in understanding the economic effects of price discrimination in monopolistic markets.[5] However, because imperfect competition is undoubtedly the most common economic setting, recent research on the field has been concerned with the following issues. Firstly, how are profit, consumer surplus and welfare affected when firms practice some form of price discrimination in imperfectly competitive markets? Secondly, in which circumstances may competitive firms have an incentive to price discriminate or rather to avoid it?

In general, conclusions regarding the previous issues are strongly dependent upon the form of price discrimination, which in turn depends upon the form of consumer heterogeneity and the different instruments available for price discrimination. Basically, the aim of this survey is to clarify the two aforementioned issues in imperfectly competitive markets where firms follow a kind of third-degree price discrimination.

1.2.1 Some useful concepts

Broadly speaking, price discrimination exists when the difference in prices among consumers is not proportional to the difference in marginal costs (e.g. Stigler (1987) and Stole (2003)). Obviously, as in monopoly settings, under imperfect competition price discrimination is only feasible if the following conditions are satisfied: (i) firms must have some market power; otherwise the law of one price applies; (ii) firms must be able to segment consumers, either directly or indirectly (i.e. through the use of self selection mechanisms) and, (iii) firms must be able to prevent resale or, equivalently, arbitrage across differently priced goods must be prevented.[6]

In a recent paper, Fundenberg and Tirole (2000, p. 637) noticed that "...there are

[5] For a survey of price discrimination with monopoly see, for instance, Phlips (1988), Tirole (1988) and Varian (1989).

[6] Some of the mechanisms firms can use to prevent resale can be found in Carlton and Perloff (1994).

more forms of price discrimination than the standard typology suggests..."

The standard typology of price discrimination forms goes back to Pigou (1920) and is the one adopted in almost all textbooks. Following Pigou there are three types of price discrimination. First-degree price discrimination arises when the firm is able to charge a different price per unit of product and per consumer, which under monopoly means that the firm is able to extract all consumer surplus. (We will see below that under competition first-degree price discrimination will in general leave some consumer surplus.) Most recently, economists have defined second-degree price discrimination as the practice of discriminating on the basis of unobserved consumer heterogeneity. Thus, the firm offers a menu of products and prices and consumers self-select into the appropriate niche of the market. Lastly, under third-degree price discrimination, perhaps the most evident form of price discrimination, the firm can discriminate on the basis of observable and verifiable consumer characteristics (e.g. past purchasing decisions, age, gender, geographical location, etc.). In the context of the new information markets, Shapiro and Varian (1999) recover the Pigou's terminologies of price discrimination but give them different designations, respectively: (i) *personalised pricing:* sell to each customer at a possibly different price; (ii) *versioning*: offer a product line and let users choose the version of the product most appropriate for them; and (iii) *group pricing*: set different prices for different groups of consumers.

Latest work on price discrimination has pointed out new ways of categorising different forms of price discrimination. Armstrong and Vickers (2001) notice that price discrimination may take the form of *interpersonal* discrimination or *intrapersonal* discrimination.[7] This classification is based on whether firms discriminate across consumers or across units for the same consumer. Interpersonal price discrimination is present when *different* consumers face different price-marginal cost ratios (i.e., the variation in prices is across consumers). Using the *Amazon*'s story example when it charges different prices to loyal and first-time consumers it is able to sort the consumers into different segments and charge each segment a different price. In doing so, the firm

[7]Because this survey will not focus on intrapersonal price discrimination the interested reader may look at Armstrong and Vickers (2001) for a detailed discussion of this form of price discrimination.

is using interpersonal price discrimination. In contrast, intrapersonal price discrimination is present when the *same* consumer faces different price-marginal cost ratios across the portfolio of goods purchased. When *Amazon* sends a given loyal consumer a special e-mail promotion for a particular book plus a DVD that does not correspond to the price that same consumer would pay if buying separately each of the goods, it is using intrapersonal price discrimination.

As well as categorising price discrimination strategies as either interpersonal or intrapersonal, Stole (2003) claims that it is also useful to categorise price discrimination strategies according to whether the form of consumer heterogeneity is observable or not. In this regard, price discrimination is either *direct* or *indirect.* Direct price discrimination arises when price discrimination is based on some observable demand related characteristic (e.g. third-degree, location based, behaviour-based, etc.). In contrast, indirect price discrimination arises if consumer heterogeneity is not directly observable and firms need to rely on self-selection mechanisms to indirectly separate consumers (e.g. nonlinear pricing).

Finally, within the terminology of direct and interpersonal third-degree price discrimination, we present a new form of price discrimination in which firms offer different deals (e.g. prices, discounts or coupons) to its own customers and to the rivals' previous consumers. Indeed, it is generally the case that firms offer better deals to those consumers that bought from rival firms before. Broadly speaking, the main goal of this marketing strategy is to generate profitable incremental sales without damaging the profits a firm can extract from its own customer base. While Chen (1997) calls this strategy *paying customers to switch*, Fudenberg and Tirole (2000) call it *customer poaching.*

In light of the above, this survey will address the profit, consumer surplus and welfare implications of direct and interpersonal price discrimination in competitive settings. In the subsequent study a dichotomy is established. We begin with those papers that have focused on price discrimination strategies in static frameworks. Within this class of works we will examine models where firms have the required information to price discriminate on an individual basis (i.e., first-degree price discrimination) and on a segment basis (i.e., third-degree price discrimination). Afterwards, we will examine a relevant class of models that have extended third-degree price discrimination to dynamic settings. In this latter case, price discrimination is based on information about consumers' past purchasing behaviour.

As usual in the literature, the evaluation of the economic effects of price discrimination will be discussed in relation to the benchmark case where price discrimination is for any reason not permitted. In other words, we will compare profits, consumer surplus and welfare when departing from a setting where firms set uniform prices to one where price discrimination is introduced.[8]

1.2.2 Competitive price discrimination in static settings

As was once suggested by Louis Phlips (1988, p.18) in his book on *The Economics of Price Discrimination*, "If you can discriminate, it is profitable to do so. (...) a discriminating price policy is at least as profitable as a nondiscriminating one."

While this basic principle is widely accepted for a monopolistic firm with market power we will see that this may no longer be the case when departing from a monopoly setting towards an imperfect competitive one. When a monopolist implements any form of price discrimination its aim is to extract more consumer surplus than it could extract under uniform pricing. In other words, the only effect at work under monopolistic price discrimination is the *surplus extraction effect.* In contrast, when we depart from a monopoly setting to introduce competition into the marketplace this is no longer the case, as each firm needs to take into account the strategic effects of its actions. For that reason, in competitive settings there is another effect at work, namely the *business stealing effect* or following Ulph and Vulkan (2000) the *intensified competition effect.* According to this latter effect in competitive environments price discrimination may act to intensify competition. Whereas the *surplus extraction effect* tends to increase profits

[8]Before proceeding it is fair to say that this survey benefited greatly from Stole's (2003) survey on competitive price discrimination.

the *intensified competition effect* tends to reduce them. Thus, whether profits fall or rise with discrimination depends upon which effect dominates. As will be explained the final outcome depends in turn on the specificity of markets.

The literature on price discrimination in competitive settings is not as extensive as that under monopoly settings. However, one very relevant benchmark model to assess the competitive effects of price discrimination is that proposed by Thisse and Vives (1988). Specifically, they consider a standard Hotelling (1929) duopoly model of spatial price discrimination, where the two firms directly observe the location of each consumer on a linear market. Thus, direct price discrimination is implemented if firms can base their prices on consumer location. Additionally, since firms set their prices on a consumer's location basis this is clearly an example of first-degree price discrimination. In this framework, Thisse and Vives show that price discrimination acts to intensify competition in each location. Like in Lederer and Hurter (1986), Thisse and Vives show that the best price the more distant firm may set in equilibrium is the marginal cost, and that the closest firm needs to provide the same utility level in order to make a sale. As a result of that, they show that *all* prices might fall in relation to uniform pricing. To be precise, if the distribution of consumers is uniform and includes the end points of the market then all prices fall down except the price charged to consumers located exactly at the mill, which remain unchanged.[9] In contrast, price discrimination by a multiplant monopolist would lead to increased prices and profits relatively to uniform pricing.

[9] Although the Thisse and Vives model has been usually referred as a model where under competition price discrimination leads all prices to fall, the reader should take into account that this result is specially true when consumers are uniformly distributed say on the interval $[0,1]$. It is important to say that the Thisse and Vives model does not always reduce all prices with discrimination. The interested reader may consider the following two examples. First, assume each consumer has an observable location x uniformly distributed on the interval $[0,1]$, transport cost is t and marginal production costs are null. It is well known that without discrimination the equilibrium price is t. Second, consider the same assumptions as before but now suppose that consumers are distributed on $[0,1]$ with density $f(x) = 6x(1-x)$. In this case, the reader may confirm that the equilibrium price without discrimination is $\frac{2}{3}t$. Since in both examples prices with discrimination do not depend on the distribution and belong to the interval $[0,t]$ it immediately follows that in the first example the discrimination prices are below the non-discrimination price. In contrast, in the second example, the no discrimination price is between the discrimination prices. Thus, in this latter case price discrimination does not reduce all prices.

Therefore, in this duopoly model moving from uniform pricing to a discriminatory pricing policy leads to more aggressive pricing in every location, and thereby all prices fall as well as profits. Since the intensified competition effect is clearly the dominant one, firms are worse off with the ability to price discriminate. As this model deals with unit inelastic demands and the market is always covered in equilibrium, conclusions regarding the consumer surplus and welfare effects of price discrimination are straightforward and immediate. As prices fall to every consumer, it immediately follows that consumer surplus is higher with price discrimination. However, because all consumers buy one unit of product either under uniform pricing either under discrimination, and all consumers buy from the closest firm in both regimes, welfare is not affected by the ability of firms to price discriminate.

Going back to the second question we have stated in the beginning of this survey, as competitive price discrimination intensifies competition and leads to lower profits, it is natural to wonder whether firms may have an incentive to avoid price discrimination, perhaps through commitments to uniform pricing. This type of concerns was also present in Thisse and Vives's (1988) analysis, who investigate whether firms may find individually optimal to avoid price discrimination by committing publicly to uniform pricing. To address this issue they add an initial stage in which each firm may decide to commit to uniform pricing or not. In the next stage, both firms observe any first stage commitments and set their prices accordingly. Interestingly, they show that when firms commit publicly to any of the two pricing policies, price discrimination is always a dominant strategy, and thereby firms find themselves in the well known Prisoner's dilemma—both firms would like to commit collectively to uniform pricing but individually price discrimination is a dominant strategy. (This result is not a surprise as all else equal each firm prefers to use more instruments rather than fewer (see Phlips (1988).)

More recently, stimulated by advances in information technologies some researchers have extended the Thisse and Vives's (1988) model to analyse the competitive effects that arise when firms can tailor special prices, discounts or coupons either to individual customers or to specific consumer segments.

Ulph and Vulkan (2000), for instance, have recovered the Thisse and Vives's model to investigate whether it is in the interest of firms to set personalised prices (i.e. to first degree price discriminate) in the context of e-commerce markets. The main difference between the two models is that Ulph and Vulkan use a general transport cost technology, which has the linear one used by Thisse and Vives as a special case. Additionally, they replace the location interpretation by any form of product differentiation (e.g. brand loyalty). For the Thisse and Vives reasons, they find that it is generally the case that price discrimination leads to lower profits for both firms. Particularly, they show that if some consumers are very loyal and some do not, then although discrimination can lead to higher prices to most loyal consumers it leads to lower prices to those consumers located in the middle. Even when firms are able to appropriate some consumer surplus due to discrimination, this may not be enough to overcome the negative effects of intensified competition. They show that only when there is a bias towards very strong loyalty will discrimination improve profits. (Only when most consumers are strong loyal to one of the firms will the surplus extraction effect dominate the intensified competition effect, thereby allowing profits to rise.)

Let us turn now to direct and interpersonal third-degree price discrimination in environments of imperfect competition. A relevant article shedding some light on the price, profits and consumer welfare effects of this form of price discrimination in oligopoly models with product differentiation is Corts (1998). Following Corts there are two distinct models of price competition with product differentiation. While some markets exhibit *best-response symmetry* others exhibit *best-response asymmetry.* Best-response symmetry is present when the "strong" and the "weak" markets of each firm coincide.[10] In contrast, best-response asymmetry is present when the weak and the strong market of each firm differ (i.e., when one firm's weak market is the other's strong market and vice-versa). Particularly, he shows that in models exhibiting best-

[10]Consider the existence of two markets 1 and 2 and two firms A and B. Following Stole (2003), market 1 is firm A's strong market (whilst market 2 is its weak market) if, for any uniform price set by firm B, the optimal price in market 1 is always higher than the price in market 2.

response symmetry, the uniform price lies between the two discriminatory prices. Like in monopolistic models of third-degree price discrimination, the welfare effects of price discrimination in models displaying best-response symmetry is in general ambiguous.

Conversely, he demonstrates that when one firm's weak market is the other's strong market unambiguous price, profit and consumer welfare effects may arise. Regarding the effects of price discrimination on prices, he shows that it is possible that all prices fall with price discrimination—which he designates as *all-out competition* result—or all prices rise—which he terms as *all-out prices increases.* As a result of that, with all-out competition, price discrimination clearly leads to lower profits but higher consumer surplus. (With best-response asymmetry, all-out competition will in generally occur if price discrimination intensifies competition and the strategic complementarity of pricing is strong enough.) In contrast, with all-out prices increases, price discrimination has opposite effects: firms are better off while consumers are worse off. Nevertheless, it is important to say that even within best-response asymmetry the welfare effects are not immediately obvious. If with all-out competition aggregate output increases price discrimination is welfare improving. However, as will become clear below it may happen that welfare remains constant or even decreases with all-out competition. This latter result tends to occur in models where there is no role for price discrimination to increase aggregate output and where price discrimination uniquely gives rise to *inefficient shopping* (e.g. when due to price discrimination some consumers buy from the more distant firm).

The Thisse and Vives's (1988) model presented above is, of course, an example of a market exhibiting best-response asymmetry. Indeed, this is in general the rule in models of price discrimination based on location: the far away market from one firm (i.e. its weak market) is the strong market for the other firm. We have seen that as price discrimination strongly intensifies competition in each segment all prices might fall down as well as profits, leading to all-out competition. However, because the market is completely covered and demands are inelastic we have seen that though consumers are clearly better off with discrimination, welfare remains constant. (Here when firms price discriminate all consumers buy from the nearest firm. Thus, there is no inefficient shopping.)

Notwithstanding that this thesis will focus on models that exhibit best-response asymmetry, it is worth presenting the Holmes's (1989) article within the approach of best-response symmetry. He extends the Robinson's (1933) seminal article on third-degree price discrimination with monopoly,[11] to examine the profits and welfare effects of this form of price discrimination in a symmetric duopoly model with product differentiation.[12] Additionally, he provides a useful result to predict in which circumstances will aggregate output (and therefore welfare) increase or decrease with the introduction of price discrimination. Whether aggregate output will rise or fall with discrimination depends not only on the convexity-concavity properties of industry demands—as in monopoly—but also upon the ratio of market to cross-price elasticities, which stresses the effect of competition. Under linear demand functions he finds that when the elasticity ratio condition is satisfied profit as well as output (and welfare) increases. Conversely, when the elasticity ratio condition is violated while the effect on profits is ambiguous, welfare falls down with discrimination. More precisely, if in relation to the strong market, the weak market has a higher cross-price elasticity but lower market elasticity then profits are lower with price discrimination.[13]

Recently, Armstrong and Vickers (2001) propose a new framework to investigate the profits and welfare effects of interpersonal and intrapersonal price discrimination in competitive settings (i.e., they model firms supplying utility directly to consumers). Within the interpersonal approach they discuss a model similar to Holmes (1989). Par-

[11] Robinson (1933) analyses a monopolist which third-degree price discriminates on the basis of weak and strong market segments. As usual under monopoly the price charged to consumers in the strong market is higher than the price charged to consumers in the weak market.

[12] In Holmes (1989) not only is the ranking of weak and strong markets symmetric, but own-price and cross-price elasticities are also identical.

[13] Let ε_i^m be the market elasticity of demand and ε_i^c be cross-price elasticity of demand in market $i = s, w$ where s stands for strong and w stands for weak markets respectively. Then when demands are linear, the elasticity ratio condition is satisfied if

$$\frac{\varepsilon_w^m(p_w)}{\varepsilon_s^m(p_s)} > \frac{\varepsilon_w^c(p_w)}{\varepsilon_s^c(p_s)}.$$

ticularly, they analyse a duopoly model where firms face two separate market segments, with each segment being a Hotelling market with uniformly distributed consumers and with distinct "transport costs". In this framework, they show that if there is enough competition (i.e., low enough transport costs), price discrimination increases profits and reduces consumer surplus and welfare. In this way, they suggest that the Holmes's (1989) result that profits may fall is a consequence of relatively uncompetitive markets. In light of the two previous models, as Stole (2003) notices, it can be said that there is a consensus that in settings exhibiting best-response symmetry it is very often the case that price discrimination increases profits.

Finally, we focus next on the aforementioned form of interpersonal direct third-degree price discrimination in which firms offer different deals (e.g. prices, discounts or coupons) to its own customers and to the rival's consumers.[14] In other words, each firm observes directly to which firm each individual consumer belongs and price discriminates accordingly. Clearly, this type of model exhibits best-response asymmetry.

While much of the work that has been produced in this form of price discrimination deals with dynamic settings (as we will see in the next section), Shaffer and Zhang (2000) propose a static model to investigate whether duopolistic firms should offer better deals to its own customers—i.e. follow a *pay to stay* strategy—or to the rival's customers—i.e. follow a *pay to switch* strategy. The model is static in the sense that they do not investigate how firms obtain the required information for price discrimination.[15] They consider a duopoly model where each firm faces two separate groups of consumers: its loyal customers and the rival's loyal customers. However, as they assume that one group's average loyalty may be higher than the other group's average loyalty, the model allows for asymmetric demands. Additionally, they assume

[14] For models where firms price discriminate among customers with different brand loyalties on the basis of coupons see, for instance, Shaffer and Zhang (1995) and Bester and Petrakis (1996). In particular, both models find that price discrimination leads to a prisoner's dilemma.

[15] We will see in the next section that much of the literature on this kind of price discrimination looks at two-period models, in which consumers' purchases in the initial period reveal their preferences from one of the firms. Conversely, Shaffer and Zhang (2000) exclude any prior competition, and assume that firms are able to infer a consumer's preference on the basis of internal sources of information such as firm's transactions databases or from specialised information vendors. Therefore, it can be said that their model corresponds to the second period of models analysed in the ensuing section.

that firms are asymmetric in size given that, all else equal, one firm has a higher initial market share. When the average loyalty of the two groups is not very dissimilar each firm's more (cross-price) elastic group is the rival's customers group, that is each firm's weak market is the other's strong market. In this case the model exhibits best-response asymmetry and each firm should pay customers to switch (i.e. offer a lower price to the rival's customers). As expected, they show that in a symmetric market (i.e. equal average loyalty across groups) both firms offer lower prices to the rival's customers; price discrimination intensifies competition thereby leading to all-out competition. In contrast, they show that if the average loyalty of the two groups is too dissimilar, then the firm which own customers are not very loyal finds that group the more elastic one, and thereby it should pay customers to stay (i.e., offer a better deal to its own customers). Thus, while one firm charges a lower price to the rival's customers the other charges a lower price to its own customers. The profit effects of price discrimination are strongly dependent upon the relative loyalty and firm's size. Particularly, they show that if the firm, which has a larger baseline market share, follows a pay to stay strategy while the smaller firm follows a pay to switch strategy then it may happen that price discrimination may lessen price competition, thereby increasing the smaller firm's profits or even both firms' profits.

1.2.3 Competitive price discrimination in dynamic settings

This survey has focused so far on competitive price discrimination in static settings. However, as aforementioned a new price discrimination tool has become available for firms, namely each consumer's purchasing history. As firms become able to recognise its previous customers they can offer different prices to old and new customers, which clearly is a kind of dynamic (direct and interpersonal) third-degree price discrimination.

Whereas static settings are a useful framework to investigate how price discrimination, based for instance on consumer-specific information, will affect firms' profits and welfare, they are silent about any prior competition and about the dynamic effects of price discrimination. When firms price discriminate by observing each customer past purchasing behaviour some interesting questions arise. First, how can firms obtain the required information for price discrimination? Second, as firms foresee the future effects of price discrimination enabled by customer recognition, will they change their current pricing behaviour? Are first-period prices above or below the non-discrimination levels? Third, what are the overall profit, consumer surplus, and welfare effects of price discrimination? These and other related issues have given rise to a series of papers in which firms offer different deals to customers with different past behaviour profiles.

Two different approaches have been considered in the literature so far. While in one approach purchase history discloses information about exogenous switching costs, in the other approach purchase history discloses information about a consumer's exogenous brand preference. A common feature in both approaches is that price discrimination can only be implemented after firms have observed the first-period actions of consumers (i.e., in the initial period consumers are anonymous). In what follows it is assumed that firms cannot commit themselves in the first period to long-term prices.[16]

Homogeneous products and exogenous switching costs

The works within the switching costs approach are motivated by those industries where consumers often need to incur a switching cost if they decide to move from one supplier to another (e.g. due to the existence of subscription fees, transactions costs or costs of learning to use a new product).[17] Despite the fact that goods are initially homogeneous, after purchase decisions have been taken, due to the existence of switching costs, consumers are in the next stage of the game partially *locked in* with their initial sellers. In this setting, when firms are able to recognise their own customers and to separate them from the rival's ones, they will try to entice the latter group of customers to switch by offering them a better deal.

The first set of models investigating dynamic third-price discrimination in markets

[16]For an analysis of purchase history pricing with long-term commitment see Caminal and Matutes (1990) and Fudenberg and Tirole (2000, section 5).

[17]For a detailed survey of switching costs and their effects see Farrel and Klemperer (2003). See also, for instance, Klemperer (1987a,b) and Klemperer (1995).

with switching costs are Nilssen (1992), Chen (1997) and Taylor (2003).[18] Nilssen (1992) is the earliest discussion on third-degree price discrimination in a market with switching costs. However, his main goal is to discuss how two different types of switching costs—transaction costs and learning costs—affect the market outcomes.[19] A crucial feature of Nilssen's model is that because there is no uncertainty about the size of switching costs and all consumers are identical, in spite of firms offering discounts to entice consumers to switch no consumer actually switches in equilibrium.

The focus in Chen (1997) is to investigate the competitive effects of paying customers to switch. Particularly, she studies a two-period duopoly model with homogeneous products and where switching costs are uniformly distributed. After first period purchase decisions have been taken, each consumer is partially locked in with his current supplier. In the second period, by observing each consumer's past behaviour, each firm is able to recognise its own customers (i.e., its strong market) and to offer better deals to the rival's previous customers (i.e., its weak market). (This model exhibits clearly best response asymmetry.) As both firms try to entice each other's previous customers the second-period market exhibits *all-out competition*: in comparison to uniform pricing, price discrimination lowers all segment second period prices,[20] raises consumer surplus and lowers profits. Furthermore, as firms realise that they can charge higher discriminating prices to its locked in customers base, they have an incentive to build market share. This in turn leads to first period prices *below* the non-discriminating counterparts. Indeed, in Chen's model first-period prices are even below marginal cost. As a whole, Chen shows that total profits with price discrimination are always lower than if price discrimination were not permitted. In spite of the consumer surplus effect of price discrimination is ambiguous, welfare is clearly higher

[18]The Shaffer and Zhang's (2000) model presented before also studies third-degree price discrimination in markets with switching costs. However, their model corresponds to the second-period of the models presented here.

[19]While transaction costs are incurred every time the consumer switches, learning costs are only incurred the first time the consumer uses a new brand. For a comprehensive discussion of different types of switching costs see Klemperer (1987a) and Klemperer (1995).

[20]An interesting feature of Chen's model is that because a firm's old and new customers are in unconnected markets, second-period prices do not depend on market shares.

without price discrimination. Of course, because aggregate output is constant, welfare is lower with price discrimination because it induces *inefficient switching.*

Finally, Taylor (2003) extends Chen's model to many periods and many firms. Like Chen he finds that price discrimination is bad for profits and welfare. An interesting point in his analysis is that while in a duopolistic market firms earn positive profits on its base of old customers as well as on its base of new customers, in markets with more than two firms, firms earn positive rents on their base of old customers but zero expected profits per new customer attracted.

In short, behaviour-based price discrimination in markets with exogenous switching costs leads firms to charge first period prices *below* their non-discrimination counterparts and then to *raise* their prices once consumers are locked in. (In these models old (loyal) customers pay always higher prices than first time customers.) In general, profits and welfare are lower than if firms were unable to price discriminate between old and new customers.

Differentiated products and exogenous brand preferences

The set of models that have analysed price discrimination based on consumer past behaviour within this second approach assumes that firms offer horizontally differentiated products (rather than homogenous products as in the preceding approach) and that there are no switching costs. Here the fact that a consumer chose a given firm's product in the past reveals to that firm, under certain conditions, that he has a preference for that product. It is worth noting that consumers' preferences should be unchanging (or at least correlated) over time; otherwise purchase history would be completely uninformative. Hence, if preferences are fixed across periods, by observing each consumer past behaviour decisions, each firm identifies in the next period two separate markets: its previous clientele (i.e. its strong market) and the rival's customers (i.e. its weak market). If price discrimination is permitted, each firm can try to poach those customers who have been revealed to prefer the rival's product by offering them lower prices. As mentioned before Fudenberg and Tirole (2000) designate this strategy as customer poaching.

The models that have analysed behaviour-based price discrimination within the exogenous brand preferences approach are Villas-Boas (1999), Fudenberg and Tirole (2000) and Chen and Zhang (2002).[21]

Fudenberg and Tirole (2000) consider a two period version of the classic linear-city model in which consumers' preferences are distributed on an interval between the two firms serving the market. In the second period, firms *recognise* those customers that bought from them before and price discriminate accordingly. Thus, each firm will target the rival's previous customers with lower prices than its old customers. Again because each firm tries to poach each other's customers, price discrimination acts to intensify competition. Although loyal consumers pay higher prices than new customers, price discrimination intensifies competition thereby reducing both segment prices. As a result of that, consumers are better off but firms find themselves in the classic prisoner's dilemma. In contrast to the switching cost approach, Fudenberg and Tirole (2000) find that first-period prices are *above* the non-discrimination counterparts. (It is worth mentioning that if consumers were myopic first period prices would be the same regardless of whether or not customer poaching is permitted.) The intuition for first-period prices above the static levels runs as follows. Given that non myopic consumers realise that second period poaching will give rise to lower second period prices, they become less elastic in period 1. Consequently, firms are able to quote higher first period prices than if poaching were not permitted. Thus, yet in contrast with the exogenous switching cost approach here prices *fall* over time. As a whole, price discrimination is bad for profits and welfare, though good for some consumers. Once more, because aggregate output is constant in both price regimes (i.e. uniform pricing and price discrimination) and because some consumers are effectively poached in equilibrium (i.e. each firm sells to some of the competitor's previous clientele), there is inefficient switching, which clearly is not good for social welfare.

Villas-Boas (1999) studies a related but infinite-period, overlapping-generations

[21] For models of behaviour-based price discrimination in a monopoly setting see Fudenberg and Tirole (1998), Villas-Boas (2003) and Acquisti and Varian (2003).

model in which firms can only price discriminate between loyal and first-time customers, which means that firms cannot distinguish between new and the rival's previous customers. As in Fudenberg and Tirole, he finds that price discrimination enabled by customer recognition leads to lower prices and profits. Further, he shows that as firms foresee the negative effects of price discrimination, which are greater the higher is the size of previous clientele, firms have less incentives to build market share, thereby competing less aggressively in the initial period. As a result of that first period prices are *above* the non-discrimination counterparts. Again because some consumers are in effect poached in equilibrium, price discrimination is bad for welfare and profits, but good for consumers.

Finally, Chen and Zhang (2002) revisit the profitability issue of behaviour-based price discrimination using a discrete version of the Fudenberg and Tirole's two-period model. Specifically, they assume that each firm has an exogenous "loyal" captive segment of consumers who always buy from a particular firm as long as the firm's price is not above their reservation price, and who cannot be induced to switch. They further assume that there is a segment of price-sensitive consumers (or switchers) with a reservation value lower than the loyal customers' one, who always purchase from the firm offering the lowest price. Thus, firms compete only for switchers. In this framework, they show that by observing each customer past behaviour, firms might become able to know whether a consumer is a price sensitive or a captive customer. However, customer recognition will only occur if a firm's first-period price is high enough such that it is not accepted by all consumers. In doing so, they show that price discrimination based on purchase history will allow a firm to increase sales without damaging the profits it can extract from its locked in customers. A general result in their analysis is that second-period profits are higher with price discrimination. Hence, as price discrimination may benefit firms, they find that the pursuit of customer recognition motivates firms to price high in the first period, which in turn induces the rival firm to price less aggressively in that period. As a result, first-period prices are *above* their non-discrimination counterparts. It is worth mentioning that in contrast with Fudenberg and Tirole (2000), in this model first-period prices are above the non-discrimination levels even when consumers are myopic.

So far it can be said that there is a consensus that behaviour-based price discrimination in markets where consumers have an exogenous preference for one of the brands leads firms to charge first period prices *above* their non-discrimination counterparts.[22] As in the exogenous switching cost approach loyal customers will pay higher prices than price-sensitive consumers.[23]

In conclusion, regarding the profit effects of behaviour-based price discrimination, in imperfectly competitive markets exhibiting best-response asymmetry, it can be said that as long as both firms try to poach each other's customers, they find themselves in a classic Prisoner's Dilemma: even though firms would like to collectively commit to non-discrimination, individually price discrimination is a dominant strategy. Finally, when the exclusive effect of price discrimination is to give rise to inefficient switching—i.e. when there is no role for price discrimination to increase aggregate output—any public policy against price discrimination would be socially desirable if the emphasis were aggregate welfare. Of course, if the target were solely consumer surplus, in light of the above, price discrimination could prove to be in some circumstances desirable.

1.3 Outline of the thesis

As stated before the question at the heart of this thesis is to investigate the profit, consumer surplus and welfare effects of behaviour-based price discrimination in imperfectly competitive markets. To assess this question the thesis is divided into three theoretical essays, being each one based on a different market paradigm.

Chapter 2 is entitled *Price Discrimination with Private and Imperfect Informa-*

[22]In contrast, yet within the exogenous brand preferences approach we will see in chapter 3 (in this thesis) that first-period prices might be *below* their non-discrimination levels.

[23]Rossi and Allenby (1993) suggest that firms should offer price reductions to households "that show loyalty toward other brands and yet are price sensitive" (p. 178). Similarly, Bester and Petrakis (1996) recommend that firms should offer discounts to attract "some consumers that are otherwise more inclined to buy a competing brand" (p. 228).

tion. The main goal of this chapter is to examine how profits, consumer surplus and welfare evolve as the firms' ability to recognise customers—and to price discriminate accordingly—gradually improves thanks to developments in information technologies. Specifically, this chapter is based on two key assumptions. First, the accuracy of a firm's information for price discrimination purposes is far from perfect. Second, each firm price discriminates on the basis of its own customer databases (e.g. records of customer purchase histories, information acquired from specialised vendors, etc.), which means that the firms may not observe the same piece of information for a particular consumer. As can be inferred from the preceding survey, apart from Villas-Boas (1999), the literature has basically hitherto focused on the case where firms price discriminate on the basis of the *same* piece of information about consumers. Essentially, this chapter extends the Thisse and Vives (1988) model by assuming that each firm observes a (less than perfect) signal of a consumer's brand preference, and that one firm's signal is not seen by its rival.

The chapter shows that (as expected) when firms rely on very accurate information they behave more aggressively leading all prices and profits to fall. In spite of firms being able to raise their prices to consumers recognised as loyal, even when the signal is highly imperfect it is shown that firms are worse off if they have the ability to price discriminate. In this way, this chapter puts forward that collectively it is not in the interest of firms to improve the quality of their information about customers, however a firm might individually prefer to do so. Further, because consumers are expected to pay lower prices, the chapter proves that consumers are strictly better off when firms recognise them more accurately while profits and welfare are unambiguously worse off.[24] Remarkably, this suggests that any public policy protecting consumer privacy by restricting customer recognition would benefit all competing firms but at the expense of consumer welfare.

[24]In his book *The Economics of E-commerce*, Nir Vulkan (2003) states that "...if firms cannot gain from perfect price discrimination, they have even fewer incentives to charge individual prices when they can discriminate only partially." The results derived in chapter 2 challenge Vulkan's conclusion as they suggest that firms are unequivocally better off when they only have the ability to price discriminate imperfectly.

The final contribution of chapter 2 is to lay out the conceptual issues that arise when firms price discriminate on the basis of public and inaccurate information. The results derived in this extension reinforce the idea that consumers clearly benefit when firms recognise them more accurately and in the same way given that this would lead to more aggressive behaviour and to lower expected prices for everyone. By contrast, profits and welfare move in an opposite direction. Extending the model to public information suggests that it is not in the interest of firms to share their private information with their competitors. However, it is in the best interest of consumers to allow firms to recognise them accurately. Paradoxically, the chapter stresses that any regulation trying to restrict information sharing would most likely benefit firms' profitability and overall welfare but again at the expense of consumers.

Chapter 3, entitled *Pricing with Customer Recognition,* is a natural sequel of the former one as it tries to model *how* firms might obtain a signal of a consumer's preferences. If firms and consumers interact more than once, whether or not a consumer bought from the firm previously might be used as a signal of a consumer's preferences (i.e., firms might learn to which firm each consumer is loyal). When learning is endogenously determined by firms' first-period pricing behaviour, a key issue will be to investigate whether or not it is in the interest of firms to avoid learning and price discrimination and if so *how* can they attain that goal. To address these and other related issues, chapter 3 proposes a stylised theoretical model closely related to Fudenberg and Tirole's (2000) paper. (Nevertheless, we will see that by choosing a discrete framework to model consumers' preferences the results attained in this chapter are quite different from those in Fudenberg and Tirole (2000).) In particular, chapter 3 proposes a model that looks at the dynamic effects of price discrimination based on consumers' past behaviour in a two-period duopoly market where consumers have *limited* brand loyalty from one of the two firms—i.e., consumers are loyal (to a specific degree) to one firm or the other.

The chapter shows that if firms set (approximately) the same first period price, they will share the market in that period and both firms will observe an accurate signal of a consumer's brand loyalty in period 2. In this case, firms will engage in price discrimination in period 2. Otherwise, if one firm's first period price is much lower than the competitor's price, *all* consumers buy from the low price firm, and price discrimination cannot occur in period 2 since nothing is learned by that period. As expected, it is shown that second period prices and profits are clearly higher when nothing is learned; thereby, all else equal, forward looking firms will have an incentive to avoid price discrimination which can be achieved by *not* sharing the market in the first period. In this regard, it is shown that an effective "*weapon*" to preclude all-out competition associated with competitive price discrimination is by not sharing the market in the first period. As a result of that, the chapter shows that first period prices are *below* their non-discrimination counterparts (which is in stark contrast with the behaviour of first-period prices in the extant models within the exogenous brand preference approach). An interesting feature of the model is that it shows that when firms put more weight on future than on current profits, they can *completely* avoid the drawbacks of price discrimination. More precisely, the chapter shows that in this case there exists an *asymmetric* pure strategy equilibrium in period 1, in which one firm sets a low price and gets all consumers, but it is not worth to the competitor to match this low price-firm as its profits would then be low in the second period. Basically, the chapter shows that allowing firms to price discriminate on the basis of purchase history is bad for profits, but good for consumer surplus and welfare.

The final essay of the thesis, entitled *Customer Poaching and Advertising*, is presented in chapter 4. As can be inferred from the title, this chapter proposes a model in which it is possible to examine the interplay between purely informative advertising and price discrimination. This work is motivated by the important role played by informative advertising in competitive markets. Although information technologies have made easier for consumers to obtain relevant information about products, there is evidence that (at least for new products) advertising still plays a very important task.[25] The literature on behaviour-based price discrimination has hitherto focused

[25] According to the new Internet Advertising Revenue Report study, conducted by the New Media

on the assumption that firms can reach costlessly all potential consumers—i.e., on the assumption that there is no role for advertising. This chapter departs from this hypothesis by assuming that without advertising consumers are uninformed and firms have no demand. More precisely, chapter 4 addresses a two-period model with two identical firms advertising a new homogeneous product (and its price) to otherwise uninformed consumers. In period 1, each firm chooses what price to quote in its ads and an intensity of advertising that will be used in the current period as well as in the next one. After firms have sent their ads, *ex-ante* uninformed identical consumers will be differentiated on an informational basis. Some consumers will receive ads from both firms, will be *selective* (or price-sensitive) customers and will always buy from the lowest-priced known firm. Other consumers will receive ads from only one firm, will be *captive* customers and will buy from that firm provided that the price does not exceed the reservation value. Finally, some consumers will remain uninformed because they will receive no ads. In this setting, when a firm is in the market for more than one period, it may learn whether or not a previous contacted consumer bought its product. If a firm achieves that type of learning, it will face in the second period, two separate markets—that of its own customers and that of its rival's customers—and so it may be tempted to poach the rival's previous customers by sending targeted ads with lower prices to that group of customers.[26] Thus, in this chapter apart from its informative role, advertising might also be used as a price discrimination device.

Chapter 4 will revisit with a new perspective some of the old questions that have been the subject of analysis in the price discrimination literature as well as in the informative advertising literature. Is it in the interest of firms to engage in price

Group of PricewaterhouseCoopers (PwC) and sponsored by the Interactive Advertising Bureau (IAB), Web advertising revenue totaled $2.37 billion in the second quarter of 2004, representing a 42.7% increase from last year. Additionally, the report found that for the first six months of 2004, U.S. Internet advertising revenue totaled approximately $4.6 billion—a 39.7% increase over the first six months of 2003. The report found also that consumer advertising was the top category in online advertising spending, accounting for 49% of total revenues for the second quarter in 2004 (in the same quarter in 2003, consumer advertising accounted for 35% of total revenues). This information was obtained from *www.imediaconnection.com/content/4261.asp* (on September 21, 2004)

[26]It is worth noting that this chapter will investigate the effects of behaviour-based price discrimination in a framework where there are neither switching costs nor exogenous brand preferences (as has been the rule in the literature so far).

discrimination? What are the effects on the firms' first-period pricing and advertising decisions if price discrimination is permitted? Do firms have an incentive to invest in more advertising when price discrimination is allowed? What about consumers and social welfare? A very relevant question not analysed in the economic literature so far is the following. Suppose firms can engage in price discrimination: Is there too little or too much advertising? Or, equivalently, do firms select a more efficient level of advertising under price discrimination or under non-discrimination?

Regarding the profit effects of behaviour-based price discrimination in a model where advertising is a *sine qua non* condition for demand, chapter 4 shows that only the firm that advertises the highest price in period 1 will be able to price discriminate, and that poaching clearly benefits the discriminating firm. As a consequence, this chapter identifies *"the race for discrimination effect"*, through which each firm has incentives to price high in period 1. (First-period prices are, therefore, *above* their non-discrimination counterparts.) Due to this strategic effect, it is shown that price discrimination may act to *soften* price competition rather than to intensify it. As a result of that, the model addressed in this chapter shows that both firms may become better off even when only one of them can engage in price discrimination.

Turning to the effects of price discrimination on the efficient properties of advertising, the chapter shows that whether price discrimination leads to too little, too much or even to the social optimal level of advertising, strongly depends on the costs' structure of each advertising-product market. In that way, the chapter puts forward that in markets with low or no advertising costs, allowing firms to price discriminate gives rise to too little advertising. Only in markets with high marginal advertising costs might firms overadvertise from the social point of view. Based on the effects of price discrimination on the efficiency properties of informative advertising, chapter 4 shows that it is possible to establish a simple criterion for predicting whether price discrimination would be desirable or not. Since with no discrimination firms always advertise in a social optimal way, when price discrimination leads firms to select a suboptimal or an excessive level of advertising, welfare falls with discrimination. As a whole, the model addressed in chapter 4 suggests that behaviour-based price discrimination in markets where firms need to invest in advertising to generate awareness is, in general, bad for consumers and welfare, though good for firms.

Finally, chapter 5 offers some concluding remarks.

Chapter 2

Price Discrimination with Private and Imperfect Information

> *"Dynamic pricing is a new version of an old practice: price discrimination. It uses a potential buyer's electronic fingerprint—his record of previous purchases, his address, maybe the other sites he has visited—to size up how likely he is to balk if the price is high. If the consumer looks price-sensitive, he gets a bargain; if he doesn't he pays a premium."*
>
> Paul Krugman (2000)

2.1 Introduction

The last years have witnessed dramatic advances in information technologies that have enhanced the firms' ability to gather, storage and use a vast amount of consumer-specific data. This in turn has led firms to become increasingly able to recognise customers and to tailor different prices to customers with different past purchasing profiles. While this price discrimination practice has been termed as *behaviour-based price discrimination* by the economics literature (e.g. Fudenberg and Tirole (2000)), in the Internet jargon it as been named *dynamic pricing*. In fact, in September 2000, the *Washington Post*, reported that the book retailer *Amazon* was conducting dynamic pricing as it was charging different DVDs prices to first-time visitors and to registered customers. Notwithstanding that the development of information technologies has greatly improved the capability of firms for gathering and using consumer-specific information for price discrimination purposes, customer recognition may, in practice, be imperfect rather than perfect, either due to insufficient and/or imprecise data or inaccurate statistical inferences.

When firms rely on imperfect information to segment the market, they will recognise customers with a less than perfect probability (i.e., imperfectly), which means that some customers will be wrongly recognised. This in turn will lead firms to offer some consumers the wrong intended price. Therefore, the incapability of firms to perfectly predict whether a consumer belongs to one or other segment will affect the profit and welfare effects of price discrimination based on customer recognition. Furthermore, in practice, it is often the case that firms recognise customers on the basis of private information (e.g. on the basis of internal sources of customer information such as firms' transaction databases that record individual customers' purchase histories).[1] Consider for instance a duopoly market where firm A, say, classifies a particular customer on the basis of its private information, firm B does not know for sure how that customer is classified by firm A and what price is firm A offering to him. It is therefore worth pursuing the competitive effects that arise when firms price discriminate on the basis of private and imperfect information.

The issue at the heart of this chapter is basically to investigate how profit, consumer surplus and welfare evolve as the firms' ability to recognise customers—and to price discriminate accordingly—gradually improves. In addition this chapter hopes to shed some light on the following issues. Do firms benefit from price discrimination based on more accurate consumer-specific information? Does a firm have an incentive to exchange its private information with its competitors? Further, is it in the interest of consumers to forbid firms the access and use of their information for pricing? Finally,

[1]Firms can also rely on consumer data acquired from specialised information vendors, such as *Double Click* and *I-Behavior*. These firms collect and sell consumer-specific information that usually includes an individual's purchasing history, interests, income and so on.

should government regulation restrict information collection and information sharing for price discrimination practices?

In order to provide an answer to the previous issues, this chapter addresses a stylised Hotelling duopoly model where two firms A and B sell their products directly to consumers whose brand loyalty towards the right-hand firm (firm A) is indexed by their location along an interval. Each firm's private information comes from the observation of a signal that is not seen by the rival firm, which in this case informs whether a customer is loyal to brand A or loyal to brand B. (Notice that the signal is about brand loyalty, not, say, about a customer's willingness to pay for each firm's product.) To analyse the competitive effects of price discrimination based on imperfect information we assume that the signal may be more or less accurate.[2]

To motivate the model we invite the reader to consider the subsequent example. Two on-line booksellers, say *Amazon* (brand A) and *Barnes&Noble* (brand B), know that potential consumers have different degrees of brand loyalty. Although *Amazon* might not have the capacity to identify the brand preferences of each individual customer it can be able through the lens of its own information (i.e. signal), to predict (with a less than perfect probability) whether a given customer turns out to prefer its brand or the rival's one. Since each firm recognises customers on the basis of its private and imperfect signal, it is uncertain about the signal observed by the rival and thereby about the price the rival is offering to a particular customer. Of course, this will have important effects on prices.

With some probability both firms observe the same type of signal for a particular consumer, for instance that the customer is loyal to *Amazon.* In this case *Barnes&Noble* would have an incentive to offer that customer a better deal in order to entice him to buy its product. Although at a first glance it seems intuitive that *Amazon* would have an incentive to extract more surplus from a customer recognised as loyal, the observation of the same signal by both firms would act to intensify competition, thereby limiting

[2]When we log on say on *Amazon*'s webpage its computers can use specific algorithms to predict (in general with some randomness) which consumers are likely to accept a high price and which are likely to reject it, and then the firm sets prices accordingly.

Amazon's incentive to charge a high price to a customer perceived as loyal. On the other hand, it may also happen that with some probability each firm observes that a particular consumer prefers its own brand (i.e. they observe different signals). In this case both firms offer that customer a price tailored to a loyal customer, which means that the customer is misrecognised by one of the firms. This misrecognition effect would act to soften price competition. Obviously, whether it is more likely that firms observe equal or different signals will depend on the accuracy of their information.[3]

In order to measure the effects of price discrimination on the basis of different information structures, we first analyse the benchmark case where price discrimination is not permitted, either because information is completely inaccurate or because price discrimination is illegal (Section 2.3). Subsequently we look at the equilibrium behaviour of firms when price discrimination is based on private and imperfect information, and thereby on private and imperfect recognition of consumers (Section 2.4). Here we find that when price discrimination is allowed firms will *always* set higher prices to consumers recognised as loyal than to consumers recognised as disloyal and that the level of price dispersion will be greater the more accurate is each firm's information (Proposition 2).

The effects of improvements in information accuracy on equilibrium outcomes is analysed in Section 2.5. Regarding prices, we will see that the price charged to customers recognised as loyal has an inverted-U relationship with the signal's accuracy. We will also see that when the signal's accuracy is relatively low, price discrimination will lead firms to increase the price quoted to perceived loyal customers above the non-discrimination level. In contrast, we will see that the price charged to customers

[3]The model addressed in this chapter might also apply to pricing policies that will be possible through the mobile wireless technologies. Particularly relevant will be the possibility of firms setting their prices according to the geographic location of consumers—i.e., to set different prices to consumers with different location-based information. In general, a typical cell phone that is turned on sends out signals every ten minutes and identifies the location of the nearest cell tower. These signals can then be used to determine a cell phone user's location so that even without the precise auto-location technologies (e.g. GPS), a user's location can be predicted fairly precisely. Thus, after receiving a signal more or less accurate about each individual's location, a firm might be able to price discriminate between consumers located near its store and consumers located closer to a rival's store. To read more on mobile wireless technologies and consumer uses see for instance *www.ftc.gov/bcp/reports/wirelesssummary.pdf*

recognised as disloyal is always below the non-discrimination level, and even lower the more accurate is the signal (Corollary 1). Even though a firm is able to raise its price to customers classified as loyal for low accurate signals we will show that in this case the probability of winning a customer after observing a loyal signal is smaller (Corollary 2). As a result, we will see that firms are strictly worse off under price discrimination, and even worse the more accurate is the signal (Corollary 3). Because more precise information gives rise to less uncertainty in the marketplace, competition will become more intense as the signal's accuracy improves, thereby leading all segment prices to fall. This result suggests that, collectively, firms would prefer not to improve the accuracy of their information, although individually they might prefer to do so.

The welfare analysis is carried out in Section 2.6. There we will see that though consumers are better off as price discrimination is based on more accurate information (Corollary 4), profits and welfare are unambiguously worse off with improvements in information precision (Proposition 3).

This chapter will also investigate the effects of price discrimination with public information (Section 2.8). In doing so, we will be able to draw some intuition on whether or not it would be in the interest of firms to exchange their private information with their rivals. Because firms will price more aggressively under public than under private information we will see that all types of consumers are expected to pay lower prices under public information (Corollary 5) which then leads to lower equilibrium profits as well (Corollary 6). Therefore, while it is in the best interest of consumers that firms recognise them on the basis of the same piece of information, it is not in the interest of firms to share their information with rival firms. In other words, moving from private to public information is good for consumers, but bad for profits and welfare (Proposition 6).

The remainder of the chapter is organised as follows. We finish this Introduction with a brief review of the more relevant literature. Then Section 2.2 lays out the formal model with private information. Section 2.7 derives the Bayesian Nash equilibrium with public information. Some privacy issues are discussed in section 2.9 and concluding remarks appear in section 2.10. Finally, an appendix provides those proofs that were omitted from the text.

Related literature The model addressed in this chapter blends features from the literature on price discrimination in imperfectly competitive markets. Particularly, it is closely related to the recent vein of research on behaviour-based price discrimination.

We have seen that the main contribution of this chapter is to add to the previous literature a stylised theoretical model that encompasses situations where firms have some uncertainty about consumer preferences as well as about the information the rival is using for segmenting the market. One way of dealing with this possibility is by introducing some randomness in the accuracy of a firm's private information (i.e., firms do not observe the same piece of information). In doing so, we assume that based for instance on their own records on consumers' past behaviour each firm knows only with some probability whether a consumer "belongs to its turf" or to the rival's. One important implication of this assumption is that markets are no longer completely separate; and therefore one interesting feature of the present third-degree price discrimination model is that it will display a leakage between segments as some consumers will be misrecognised and will receive the wrong intended price.

Thisse and Vives (1988) show that when firms have the *same* piece of information about each individual consumer (i.e. each firm observes a public fully accurate signal of each consumer's brand preference), and firms base their prices on this observed signal (i.e. firms set personalised prices), then each consumer is a completely separate market to be contested. As a result, they show that price discrimination may intensify competition leading all prices and profits to fall down compared to the case where price discrimination is not possible (for instance because consumers have private information about their tastes).[4] Thus, it can be said that the model proposed in this chapter extends the Thisse and Vives's model in the sense that even though a firm cannot observe the true brand preference of each individual consumer, it is able to observe

[4] As mentioned before in Thisse and Vives all prices fall down if consumers are uniformly distributed.

a *noisy* private signal of a consumer's brand preference, and thereby to set its prices accordingly. Indeed, we believe the model we address is a natural way of thinking in situations where firms set personalised prices.

Other models have extended the Thisse and Vives's analysis to frameworks where, although firms are unable to observe the brand preference of individual consumers, they are able to recognise them only as their own customers or as the rival's customers. These models have assumed that by observing the consumers' past purchasing decisions, firms obtain a public and accurate signal of whether a consumer prefers its brand or the rival's one. Price discrimination being feasible, each firm offers a different price to loyal and to the rival's customers. However, because information is fully precise and public there is neither leakage on market segmentation nor uncertainty about the rival's information.

In these group of models market segmentation may be due to the existence of exogenous switching costs (e.g. Chen (1997) and Taylor (2003)), or due to exogenous brand preferences (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000) and Shaffer and Zhang (2000)). As said, these models assume that firms observe the *same* piece of information and thereby that consumers are classified in the right segment. For instance, in Fudenberg and Tirole (2000) after a firm has observed whether or not a consumer bought its product previously, it is able, in period 2, to segment the market into old (i.e. loyal) customers and rival's (i.e. disloyal) customers. Once one firm recognises a customer as a loyal it must be the case that the competitor recognises that customer as a disloyal one. Hence, market segmentation is based on public information thereby implying that firms are not uncertain about the information the rival is using for setting its discriminatory prices. When it comes to evaluate the profitability of price discrimination Fudenberg and Tirole also find that price discrimination leads to a prisoner's dilemma. Because each firm tries to poach each other's customers, price discrimination acts to intensify competition leading all segment prices to fall as well as profits.

Villas-Boas (1999) analyses an in infinite-horizon model with overlapping generations of consumers, where although firms are able to recognise old customers, they are not able to distinguish a first-time customer in the market from one that bought the rival's product previously. In this case, each firm has private information concerning its old customers. That is, firms know with certainty whether a customer is an old one or not, but when they face a new customer they are uncertain about whether that customer is a first-time customer to the market or a rival's previous customer. Even in this context, Villas-Boas shows that each firm's temptation to try to attract the rival's previous customers makes both firms cut prices in relation to the situation where price discrimination cannot occur. Hence he also finds that price discrimination leads to lower equilibrium prices and profits for all competing firms. (This result was also obtained in models within the switching cost approach like Chen (1997) and Taylor (2003).) Because this chapter abstracts from any previous competition leading consumers to have a brand preference from one of the firms, it could be view as the second-period game of say Fudenberg and Tirole's model. Nevertheless it is different from the latter model as it assumes that each firm's signal is not fully accurate and is its private information.

Finally, our analysis has some connections with Chen, Narasimhan and Zhang's (2001) study of individual marketing with imperfect targetability. (They define targetability as a firm's ability to predict the preferences and purchase behaviours of individual consumers.) Specifically, they propose a duopoly model where there are three segments of consumers; some consumers are captive to one or the other firm and the remaining consumers are switchers. Additionally, they assume that each firm has information only about its own captive consumers and switchers, and that each firms identifies (with a less than perfect probability) a given customer as a captive or a switcher. In their model firms compete only for the switchers. When firms can separate consumers and engage in price discrimination they increase the price for captive consumers and reduce the price for switchers. However, when targetability is not perfect it may happen that a captive consumer receives the price tailored to a switcher and vice-versa. In this way, a low level of targetability tends to soften price competition in the marketplace as firms try to avoid that some captive consumers receive a very low price. For this reason they show that profits may increase with improvements in targetability when it departs from a low enough level. Nevertheless, when the firms' targetability becomes higher, firms can separate almost perfectly captive and switchers and, therefore, the ensuing competition for the switchers outweighs the surplus extraction rents from captive consumers. As a result of that, as firms' targetability is sufficiently high further improvements in targetability will intensify competition and lead to a prisoner's dilemma as well.

2.2 The model

Let us suppose a market in which two firms, A and B, produce at zero marginal cost nondurable goods A and B, respectively. On the demand side there is a continuum of consumers with mass normalised to one. Each consumer buys at most one unit of good A or one unit of good B. Further, each consumer reservation price, v is the maximum price any consumer is willing to pay for the good and it is sufficiently high implying that nobody stays out of the market.

Consumers are heterogeneous in brand loyalty because they differ in how much they are willing to pay for their less preferred brand. Brand loyalty is defined as the minimum difference between the prices of two competing brands necessary to induce consumers to buy their less preferred brand. Each consumer's brand loyalty towards brand A is represented by a parameter l which is uniformly distributed on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$. In so doing, all else equal, consumers with positive loyalty prefer brand A while consumers with negative loyalty prefer brand B. Thus, a consumer loyal to brand A buys brand B if and only if the price of brand B plus l is lower than the price of brand A. For equal prices, consumers with loyalty equal to $\frac{1}{2}$ are strongly loyal to brand A, consumers with loyalty equal to $-\frac{1}{2}$ are strongly loyal to brand B and consumers with loyalty equal to zero are just indifferent between the two brands. A consumer of type l bases his buying decision on the maximisation of the net utility. The net utility of a consumer buying his preferred brand is $v - p$ while the net utility of buying his less preferred brand is $v - p - |l|$.

Consumers have private information about their types and firms are not able to observe the degree of brand loyalty of individual customers. Nevertheless, a firm possesses consumer-specific information—acquired either from the firm's transaction databases that record individual customers' purchase histories or from specialised information vendors—that enables it to observe a noisy signal of a consumer's brand preference. In other words, for each consumer each firm observes a signal not seen by the rival, that informs it whether that customer is loyal to brand A (signal α) or loyal to brand B (signal β). Hence, after the signal has been observed each firm identifies (with a less than perfect accuracy) each customer as a loyal or a disloyal. Because each firm's signal is not perfect, customers are recognised imperfectly which means that market segmentation will be also imperfect. A more accurate signal is associated with better information in the sense of Blackwell (1951), which is then reflected in a higher ability of firms to classify correctly potential customers. Price discrimination being permitted, each firm tailors different prices to customers perceived as loyal and to customers perceived as disloyal.

In short, for a given consumer each firm $i = A, B$ observes an independent private binary noisy signal $s_i \in \{\alpha, \beta\}$ about that consumer's brand loyalty. While α informs it that the consumer is loyal to brand A, β tells it that the consumer is loyal to brand B. Further, it is common knowledge that the probability of each signal conditional on each consumer's true brand loyalty l is given by,

$$q(l) = \Pr(s_i = \alpha \mid l), \tag{2.1}$$

$$1 - q(l) = \Pr(s_i = \beta \mid l). \tag{2.2}$$

It is also assumed that $q(l)$ is increasing in l, which reflects the idea that the greater is the degree of brand loyalty for a particular consumer, the higher is the probability of a given firm observing a loyal signal. As an example, if each firm's signal is obtained based on consumer past purchasing profiles, it is more likely that say firm A observes a loyal signal for a consumer that bought many times brand A in the past than for a customer that bought product A few times or did not buy at all product A in the past. Thus, for the sake of simplicity consider that

$$q(l) = \frac{1}{2} + bl, \qquad 0 \leqq b \leqq 1, \tag{2.3}$$

where b measures the signal's accuracy. The signal discloses no information when b is equal to zero, which means that in this case firms have no way to recognise consumers. In contrast, the signal discloses increasingly more accurate information as b approaches 1, thereby allowing firms to better recognise customers. For intermediate values of b some consumers are incorrectly classified by firms or, in other words, some consumers loyal to brand A are *misrecognised* as loyal to brand B and vice-versa.

Updating beliefs After observing a given signal for a particular customer firms update their own beliefs over that customer's true type and form beliefs about the rival's signal for that same customer. The density function of l, denoted by $f(l)$ (which is in this case equal to 1) and $q(l)$ are common knowledge of both firms and thus form the basis of their prior and posterior beliefs. Hence, after observing signal $s_i \in \{\alpha, \beta\}$, and using Bayes rule, each firm's posterior belief about each customer's brand loyalty degree is given by the conditional density function $h_{s_i}(l)$ where [5]

$$h_\alpha(l) = \Pr(l \mid s_i = \alpha) = \frac{\Pr(s_i = \alpha \mid l) f(l)}{\Pr(s_i = \alpha)} = \frac{q(l) f(l)}{\mu_\alpha}, \tag{2.4}$$

$$h_\beta(l) = \Pr(l \mid s_i = \beta) = \frac{\Pr(s_i = \beta \mid l) f(l)}{\Pr(s_i = \beta)} = \frac{[1 - q(l)] f(l)}{\mu_\beta}, \tag{2.5}$$

and

$$\mu_\alpha = \Pr(s_i = \alpha) = \int_{-\frac{1}{2}}^{\frac{1}{2}} q(l) f(l) dl = \frac{1}{2}, \tag{2.6}$$

$$\mu_\beta = \Pr(s_i = \beta) = \int_{-\frac{1}{2}}^{\frac{1}{2}} [1 - q(l)] f(l) dl = \frac{1}{2}. \tag{2.7}$$

[5]Notice that $h_{s_i}(l)$ satisfies the property $\int_{-\frac{1}{2}}^{\frac{1}{2}} h_{s_i}(l) dl = 1$.

Now, given signal $s_i = k$, firm i believes that firm j observed signal $s_j = \alpha$ with probability $\rho_k = \Pr(s_j = \alpha \mid s_i = k)$. Therefore,

$$\rho_\alpha = \Pr(s_j = \alpha \mid s_i = \alpha) = \frac{\Pr(s_j = \alpha, s_i = \alpha)}{\Pr(s_i = \alpha)} = \frac{\lambda_{\alpha\alpha}}{\mu_\alpha} = 2\lambda_{\alpha\alpha} \tag{2.8}$$

$$\rho_\beta = \Pr(s_j = \alpha \mid s_i = \beta) = \frac{\Pr(s_j = \alpha, s_i = \beta)}{\Pr(s_i = \beta)} = \frac{\lambda_{\beta\alpha}}{\mu_\beta} = 2\lambda_{\beta\alpha} \tag{2.9}$$

where

$$\lambda_{kr} = \Pr(s_i = k, s_j = r); \qquad k, r = \{\alpha, \beta\}.$$

Since the signals observed by firms are independent conditional on l we have

$$\Pr(s_i = k, s_j = r \mid l) = \Pr(s_i = k \mid l) \Pr(s_j = r \mid l)$$

and so,

$$\lambda_{kr} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \Pr(s_i = k, s_j = r \mid l) f(l) dl. \tag{2.10}$$

Therefore,

$$\lambda_{\alpha\alpha} = \Pr(s_i = \alpha, s_j = \alpha) = \int_{-\frac{1}{2}}^{\frac{1}{2}} [q(l)]^2 f(l) dl = \frac{1}{4} + \frac{b^2}{12}, \tag{2.11}$$

$$\lambda_{\beta\beta} = \Pr(s_i = \beta, s_j = \beta) = \int_{-\frac{1}{2}}^{\frac{1}{2}} [1 - q(l)]^2 f(l) dl = \frac{1}{4} + \frac{b^2}{12}, \tag{2.12}$$

$$\lambda_{\alpha\beta} = \Pr(s_i = \alpha, s_j = \beta) = \int_{-\frac{1}{2}}^{\frac{1}{2}} q(l) [1 - q(l)] f(l) dl = \frac{1}{4} - \frac{b^2}{12}. \tag{2.13}$$

Since both firms observe signal α with equal probability it follows that $\lambda_{\beta\alpha} = \lambda_{\alpha\beta}$.

Lemma 1. *For any level of the signal's accuracy it follows that $\rho_\alpha \geqslant \rho_\beta$.*

Proof. See the Appendix.

Based on that result once firm A has observed signal α for a given consumer it knows that it is more likely that the rival has observed the same signal for that same consumer. Moreover, the more accurate is the signal the higher is the likelihood of say firm B has observed signal α given that firm A has also observed signal α. (Notice that this probability is equal to $\rho_\alpha = \frac{1}{2} + \frac{b^2}{6}$, which increases with b.) Hence, more accurate signals reduce each firm's uncertainty about the rival's private information.

Finally, firms also form beliefs about the loyalty degree of a given consumer after signals s_i and s_j have been observed. This is given by the conditional density function $g_{kr}(l)$ where,[6]

$$g_{kr}(l) = \Pr(l \mid s_i = k, s_j = r) = \frac{\Pr(s_i = k \mid l)\Pr(s_i = r \mid l)\, f(l)}{\Pr(s_i = k, s_j = r)}.$$

Thus,

$$g_{\alpha\alpha}(l) = \Pr(l \mid s_i = \alpha, s_j = \alpha) = \frac{[q(l)]^2 f(l)}{\lambda_{\alpha\alpha}}, \tag{2.14}$$

$$g_{\alpha\beta}(l) = g_{\beta\alpha}(l) = \Pr(l \mid s_i = \alpha, s_j = \beta) = \frac{q(l)\,[1 - q(l)]\, f(l)}{\lambda_{\alpha\beta}}, \tag{2.15}$$

$$g_{\beta\beta}(l) = \Pr(l \mid s_i = \beta, s_j = \beta) = \frac{[1 - q(l)]^2 f(l)}{\lambda_{\beta\beta}}. \tag{2.16}$$

Finally,

$$G_{kr}(x) = \Pr(l < x \mid s_i = k, s_j = r) = \int_{-\frac{1}{2}}^{x} g_{kr}(l)dl. \tag{2.17}$$

With these preliminaries aside, before proceeding to solving the model we look next at the benchmark case where firms cannot engage in price discrimination.

2.3 Benchmark case: non-discrimination

In order to better evaluate the competitive effects of price discrimination based on imperfect and private information, we first examine the benchmark case where price discrimination cannot occur either because the signal is non-informative (i.e. $b = 0$) or because price discrimination is illegal. Here the model is just a replication of the standard Hotelling equilibrium. Because the game is symmetric both firms set

[6]Notice that $g_{kr}(l)$ satisfies the property $\int_{-\frac{1}{2}}^{\frac{1}{2}} g_{kr}(l)dl = 1$.

the same price. Let p_N denote the equilibrium price under non-discrimination. It is straightforward to verify that being in our case $f(l) \equiv 1$

$$p_N = \frac{1}{2}. \tag{2.18}$$

Thus, p_N is the price quoted by firms when price discrimination is for any reason not permitted.

Equilibrium profits per firm under non-discrimination are denoted by π_N and equal to

$$\pi_N = \frac{1}{4}. \tag{2.19}$$

Consumer surplus under non-discrimination, denoted by CS_N, is equal to

$$CS_N = v - p_N = v - \frac{1}{2}, \tag{2.20}$$

As far as welfare is concerned, it is easy to see that under non-discrimination consumers always buy their most preferred brand, which means that there is no inefficient shopping in equilibrium. Thus, total welfare under non-discrimination (i.e., the first-best level of welfare), denoted by W_N, is equal to

$$W_N = 2\pi_N + CS_N = v. \tag{2.21}$$

2.4 Price discrimination with private and imperfect customer recognition

As mentioned, for each consumer each firm observes a private noisy signal, which allows it to classify (imperfectly) that consumer as a loyal or as a disloyal. Therefore, because the signal is not perfect firms may classify consumers incorrectly— i.e. some consumers will be misrecognised. After recognising a given customer as a loyal or a disloyal, each firm chooses simultaneously two prices, one for customers recognised as loyal and a different one for customers recognised as disloyal. This type of price discrimination was recently followed by *Amazon.* Based on its records, *Amazon* was capable of inferring with a less than perfect probability whether a customer was willing to accept a higher or a lower price, and to offer different prices to first-time customers and to old customers. (Obviously, the *Amazon*'s goal behind this pricing strategy was to attract first-time customers, by offering them lower prices, while charging more to its loyal customers.)

The model developed in this section aims to analyse price discrimination in markets where customer recognition is imperfect and private information of each firm. Due to imperfect recognition some consumers will be offered the wrong intended price—i.e. there will exist some leakage between market segments—which clearly will reduce the efficacy of price discrimination. Additionally, because each firm classifies customers based on its private information, the way each firm segments the market is also its private information. As a result of that, each firm is uncertain about what price is offered by its competitor to a particular customer.

We now turn to solving the model for the case in which price discrimination is legal. Formally, upon observing signal $s_i \in \{\alpha, \beta\}$ firm A chooses $p^A_{s_A} \in \{p^A_\alpha, p^A_\beta\}$ and firm B chooses $p^B_{s_B} \in \{p^B_\alpha, p^B_\beta\}$. For the sake of simplicity, assume $s_A = k$ and $s_B = r$, where $k, r = \{\alpha, \beta\}$. Because firms have private information, we will use the Bayesian Nash equilibrium as the solution concept. Equilibrium prices are obtained by solving the ensuing maximisation problems:

$$\underset{p^A_{s_A}}{Max}\ E\left[\pi^A \mid s_A = k\right] \tag{2.22}$$

and

$$\underset{p^B_{s_B}}{Max}\ E\left[\pi^B \mid s_B = r\right] \tag{2.23}$$

where

$$E\left(\pi^A \mid s_A = \alpha\right) = p^A_\alpha \sum_{r \in \{\alpha,\beta\}} \Pr\left(p^A_\alpha < p^B_r + l \mid s_A = \alpha, s_B = r\right) \Pr\left(s_B = r \mid s_A = \alpha\right)$$

$$= p_\alpha^A \left\{ \rho_\alpha \left[1 - G_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right)\right] + (1-\rho_\alpha)\left[1 - G_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right)\right]\right\}, \tag{2.24}$$

and,

$$E\left(\pi^A \mid s_A = \beta\right) = p_\beta^A \sum_{r\in\{\alpha,\beta\}} \Pr\left(p_\beta^A < p_r^B + l \mid s_A = \beta, s_B = r\right) \Pr\left(s_B = r \mid s_A = \beta\right)$$

$$= p_\beta^A \left\{ \rho_\beta \left[1 - G_{\beta\alpha}(p_\beta^A - p_\alpha^B)\right] + \left(1-\rho_\beta\right)\left[1 - G_{\beta\beta}(p_\beta^A - p_\beta^B)\right]\right\}. \tag{2.25}$$

Symmetric expressions hold for firm B.

Considering for instance the perspective of firm A, first-order conditions for this firm are:

$$\frac{\partial E\left(\pi^A \mid s_A = \alpha\right)}{\partial p_\alpha^A} = \rho_\alpha \left[1 - G_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right)\right] + (1-\rho_\alpha)\left[1 - G_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right)\right]$$
$$-p_\alpha^A \left[\rho_\alpha g_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) + (1-\rho_\alpha)\, g_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right)\right] = 0$$
$$\frac{\partial E\left(\pi^A \mid s_A = \beta\right)}{\partial p_\beta^A} = \rho_\beta \left[1 - G_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right)\right] + \left(1-\rho_\beta\right)\left[1 - G_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)\right]$$
$$-p_\beta^A \left[\rho_\beta g_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right) + \left(1-\rho_\beta\right) g_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)\right] = 0$$

From both conditions we obtain:

$$p_\alpha^A = \frac{\rho_\alpha \left[1 - G_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right)\right] + (1-\rho_\alpha)\left[1 - G_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right)\right]}{\rho_\alpha g_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) + (1-\rho_\alpha)\, g_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right)} \tag{2.26}$$

and

$$p_\beta^A = \frac{\rho_\beta \left[1 - G_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right)\right] + \left(1-\rho_\beta\right)\left[1 - G_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)\right]}{\left[\rho_\beta g_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right) + \left(1-\rho_\beta\right) g_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)\right]}. \tag{2.27}$$

Similar expressions hold for firm B. Second-order conditions are also satisfied as can be seen in the proof of Proposition 1 in the Appendix provided.

Competitive Equilibrium Since the game presented is symmetric one must look at an equilibrium solution where $p_\alpha^A = p_\beta^B$ and $p_\beta^A = p_\alpha^B$. In doing so, the above system is reduced to two equations with two unknowns. Denoting as p_L the price quoted by firms to customers recognised as loyal and as p_D the price quoted by firms to customers recognised as disloyal we have $p_L = p_\alpha^A = p_\beta^B$ and $p_D = p_\beta^A = p_\alpha^B$. Unfortunately since G is a cubic function the model does not allow for closed-form solutions. However, imposing the condition $y = p_L - p_D$ gives the answer implicitly. We get the following result.

Proposition 1. *When firms price discriminate on the basis of private and imperfect information, the symmetric Bayesian Nash equilibrium in prices is given by*

$$p_L = \frac{\frac{1}{4} + \frac{1}{8}b - \frac{1}{4}y - \frac{1}{2}by^2 - \frac{1}{3}b^2y^3}{\frac{1}{2} + by + b^2y^2}, \tag{2.28}$$

and

$$p_D = \frac{\frac{1}{4} - \frac{1}{8}b + \frac{1}{4}y + \frac{1}{2}by^2 + \frac{1}{3}b^2y^3}{\frac{1}{2} + by + b^2y^2}. \tag{2.29}$$

Proof. See the Appendix.

Notice that the symmetric interior solution presented in Proposition 1 is the only possible equilibrium solution. Despite the cubic equation in y, it is easy to see that there is only one real root for all values of $b \in [0,1]$. Therefore, the symmetric equilibrium derived is unique. Further,

Proposition 2. *(i) When the signal is non-informative (i.e. $b = 0$), firms are not able to recognise customers and, thus, price discrimination is unfeasible. Equilibrium prices are $p_L = p_D = \frac{1}{2}$.*

(ii) For any informative signal (i.e. $b > 0$), customers recognised as loyal face higher prices than customers recognised as disloyal, that is $p_L > p_D$.

(iii) The more informative is the signal the greater is the difference between the two discriminatory prices. In other words, $p_L - p_D$ increases with b.

Proof. Using (2.28) and (2.29) part (i) is easily obtained. To prove (ii) and (iii) Figure 1 plots $y = p_L - p_D$ as an implicit function of b. More precisely, y is defined implicitly as follows: $y + 2by^2 + \frac{5}{3}b^2y^3 - \frac{1}{4}b = 0$.

Figure 1: Relationship between $y = p_L - p_D$ and the signal's accuracy

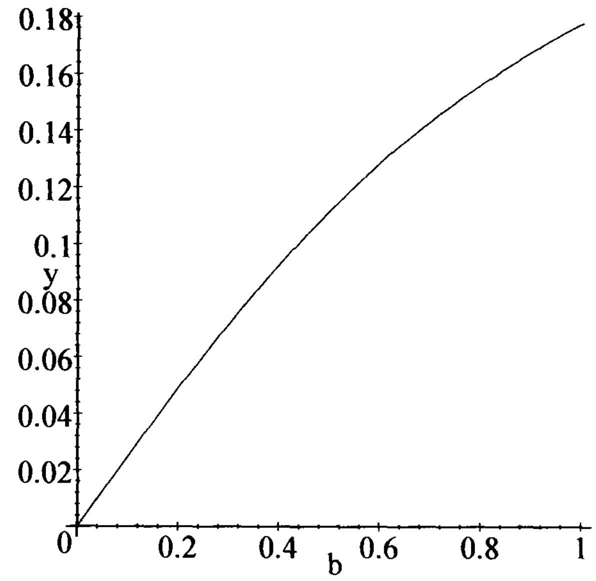


Figure 1 shows that, in equilibrium, for any level of the signal's accuracy (i.e. $0 < b \leqq 1$), $y > 0$ which then implies that $p_L > p_D$. If the level of price dispersion is measured by the difference between prices we can say that price discrimination gives rise to price dispersion and further that the level of price dispersion will be greater the more accurate is each firm's private signal. This result is interesting as it predicts that the level of price dispersion is expected to be greater in more "segmented" markets, which occurs when $b \approx 1$.

Proposition 2 claims that consumers recognised as loyal will pay a higher price than consumers recognised as disloyal. It is well known that when firms are able to perfectly distinguish loyal and disloyal customers they will charge more to their loyal customers as a way to extract more consumer surplus. (This is the *surplus extraction effect.*) Conversely, they will set lower prices to those customers that prefer the competitor's product in order to entice those customers to buy its product. (This is the *business stealing effect.*) Going back to the *Amazon*'s DVDs story, we can see that our result lends support to *Amazon*'s behaviour of charging more to customers perceived as loyal than to those perceived as disloyal (or first-time customers). Indeed, the practice of charging more to loyal than to disloyal customers is well documented in the literature on competitive price discrimination (e.g. Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000), to name few.) The reason is that each firm will charge less to those customers that, from its perspective, are more price-sensitive, which in general are those customers recognised as disloyal.

Proposition 2 also predicts that the gap between loyal and disloyal prices is greater as the accuracy of information increases. In order to offer an explanation for the behaviour of this gap as b increases we should take into account that when firms price discriminate on the basis of imperfect and private information they are uncertain not only about the true type of each individual consumer but also about the rival's private signal. Obviously, the level of uncertainty is greater under less accurate signals. Additionally, when information is more imprecise it is also more likely that firms misrecognise consumers, which then increases the likelihood of firms offering the wrong intended price to each individual consumer. In contrast, as information's accuracy increases it is less likely that firms receive the wrong signal and it is more likely that both firms observe the same signal for a particular consumer. (Notice that the probability of a customer be in fact loyal say to firm A, after a signal α has been observed, is given by $h_\alpha(l)$ (see equation (2.4)) which is increasing in b; and that the probability of both firms observing the same signal, i.e. λ_{kk} given by equation (2.11) is increasing in b, meaning that as information accuracy increases it is also more likely that firms classify customers in the same way.)

When information is extremely inaccurate it is more likely that a customer recognised as disloyal may turn out to be a true loyal and vice-versa. Clearly, in this case a firm has less incentives to reduce the price to a customer that generates a disloyal signal and to increase the price to a customer generating a loyal signal. On the other hand, when information is more imprecise it is also less likely that both firms receive equal signals. That is, when say firm A receives signal α there is a good chance that its rival receives signal β, which then means that the consumer receives the same price by both firms. As a result, when b is small firms are more uncertain about each consumer's type and the rival's private information which then leads to a smaller gap between the loyal and disloyal price.

Conversely, as information becomes more precise each firm acknowledges that it is more likely that each consumer generates a correct signal and that the rival receives the same signal as well. As a result of that, each firm has more incentives to charge a price high enough to be accepted by a consumer recognised as loyal and a price low enough to be accepted by a consumer recognised as disloyal. So it is clear that the gap between loyal and disloyal prices increases as information precision improves.

2.5 Effects of improvements in the signal's accuracy

The signal allows firms to infer each consumer's brand loyalty as long as b is greater than zero. As the signal becomes more informative firms are more capable of classifying customers correctly and they are less uncertain about the rival's signal. Now our aim is to shed some light on the following issues. First, what are the competitive effects of price discrimination with imperfect and private information? How do prices and profits evolve as the firms' ability to recognise customers improves? Second, in e-commerce markets there is a concern on the part of firms to gather and storage detailed consumer-specific databases for pricing and other marketing purposes (e.g. advertising). But, is it in the best interest of firms to invest in more accurate information if the goal is solely to implement price discrimination? We look at these and related issues in what ensues.

2.5.1 Prices

It is well known that when a firm can engage in price discrimination it seeks to extract more surplus from less elastic consumers (in this case loyal consumers) while it tries to entice more elastic consumers (in this case disloyal consumers) to buy its product offering them better deals. When a firm has the required information to segment the market perfectly, it can set a segment specific price without reference to its pricing behaviour in any other segment. However, this is no longer the case when firms segment the market on the basis of imprecise information. When firms price discriminate on the basis of imperfect and private information they are uncertain not only about which firm each consumer really prefers but also about the rival's private information. In addition, we have seen that under inaccurate information there is a good chance that firms classify consumers incorrectly. This will give rise to a *misrecognition effect* which is not present in the extant literature on behaviour-based price discrimination. Consider the following example. Suppose that a particular consumer is loyal to brand A. Under imperfect and private information four scenarios are possible. With some probability both firms observe signal α and the consumer is correctly classified as a loyal consumer to brand A by both firms. There is also a chance that firm A observes signal α while firm B observes signal β, in which case the consumer is misrecognised by firm B. Similarly, with some probability firm A observes signal β while its rival observes signal α, which then means that the consumer is correctly recognised by firm B but he is misrecognised by firm A. Finally, there is a chance that both firms observe the wrong signal, that is signal β, in which case the consumer is misrecognised by both firms. Summing up, under private information the "misrecognition effect" is present either when both firms misrecognise customers (i.e. when both receive the wrong signal) either when only one of them misrecognises (i.e. when both receive different signals). (We will see that under public information misrecognition only occurs when both firms observe the wrong signal for a given consumer.) Obviously, the "misrecognition effect" will have important implications on the equilibrium discriminatory prices. Using the equilibrium solutions presented above, we represent in Figure 2 the following equilibrium prices: p_D (dotted line), p_L (solid line) and p_N (bold line). Based on this Figure we may establish the following result.

Corollary 1. *(i) The price charged to a customer classified as disloyal is always below the non-discrimination price, i.e.* $p_D < p_N$.

(ii) When a firm observes a signal which is not too accurate it charges a customer recognised as loyal a price that is above the non-discrimination price. Conversely, when the signal's accuracy is sufficiently high the price charged to a customer recognised as loyal is below the non-discrimination price.

Figure 2: Relationship between prices and the signal's accuracy

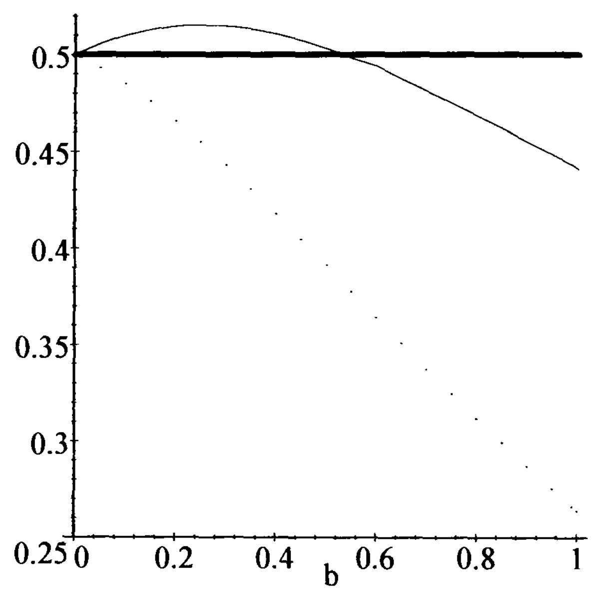


Figure 2 illustrates the relationship between each firm's prices with and without discrimination and the signal's accuracy. There we see that the price charged to customers recognised as loyal has an inverted-U shaped relationship with the signal's accuracy. Specifically, we can see that a firm charges a consumer identified as loyal a price higher than its non-discrimination counterpart when the signal's accuracy is (approximately) below 0.5 and after that point it charges that consumer a price lower than the non-discrimination level.[7] On the contrary, a consumer recognised as disloyal *always* faces a price below the non-discrimination level, and this price is lower the more accurate is the signal.

In order to explain these findings it is important to have in mind that there are different effects at work when price discrimination is based on imperfect and private information. There is the "*surplus extraction effect*" through which price discrimination allows a firm to extract greater surplus from those consumers who are willing to pay more from its product (i.e. its loyal customers). In a competitive setting there is also the "*business stealing effect*" as the ability to set discriminatory prices gives firms an incentive to reduce the price to disloyal consumers in order to poach them from rival firms. Apart from these two effects that are well documented in the extant literature,

[7]Numerical analysis shows that p_L reaches its maximum value approximately at 0.5154 for $b \cong 0.25$ and is equal to the non-discrimination level for $b \cong 0.5$. When $b = 1$ the price charged to a perceived loyal customer falls to 0.44 (lower than the non-discrimination level).

our analysis suggests that when price discrimination is based on inaccurate information there is a new effect at work, namely the *"misrecognition effect"*. When information is imperfect firms may classify consumers incorrectly thereby leading them to sometimes offer the wrong intended price. We will see below that one important implication of this effect is to soften price competition into the marketplace. Hence, the discussion of the relationship between the quality of information and the price to a loyal and a disloyal consumer will be based on the balance among these three effects.

Corollary 1 predicts that the price to a disloyal consumer is always below its non-discrimination counterpart and that this price decreases as information's accuracy increases. The intuition for this finding is straightforward. It is easy to see that when misrecognition of consumers is more likely a firm has less incentives to decrease its price to a consumer with a disloyal signal, as there is a good chance that the consumer turns out to be a loyal consumer. Because the "misrecognition effect" is larger for less accurate information it reduces a firm's incentive to reduce the price after a disloyal signal has been observed. However, as information's accuracy increases it becomes less likely that firms observe a wrong signal for a given consumer. In other words, better information means that it is more likely that a given consumer turns out to be a disloyal one after a disloyal signal has been observed. In consequence, as the quality of information improves firms have more incentives to quote a low price after a disloyal signal has been received. Hence, as the quality of information increases the "misrecognition effect" becomes smaller leading the "business stealing effect" to become larger. In this way, the price to a consumer recognised as disloyal decreases as information's precision increases.

Perhaps one of the most striking findings of this chapter is that there is an inverted-U shaped relationship between the quality of information and the price to a consumer recognised as loyal. Obviously, in order to explain this non-monotonic relationship we must have in mind that there have to be opposing forces at work, with a balance between the forces that changes as the parameter b increases.

Apart from strategic reasoning, when a firm receives a loyal signal for a given consumer it has an incentive to raise its price as a way of appropriating greater surplus from that consumer. When information is imperfect the expected surplus that a firm can extract after a loyal signal has been observed depends on the probability that a correct signal of a consumer type has been observed. Thus, the firm expects to extract greater surplus when the probability of classifying incorrectly a consumer decreases. Because misrecognition decreases as b increases the "surplus extraction effect" becomes larger as the quality of information improves.

Considering now the strategic interaction between firms it follows that the lower is the level of information's precision the greater is the probability of firms receiving different signals. Suppose that a consumer is loyal to firm A and that firm A receives signal α while firm B receives signal β. In this case firm B misrecognises that consumer and thus it offers him the wrong intended price. Clearly, when firm A believes that it is more likely that its rival misrecognises one of its loyal customers, firm A has more incentives to quote a high price to a customer recognised as loyal. However, as the quality of information increases, although firm A realises that it is more likely that it receives a correct signal it also acknowledges that it is less likely that the rival firm misrecognises one of its loyal customers. Hence, as information accuracy increases the "misrecognition effect" becomes smaller, firms compete more aggressively for each consumer leading the "business stealing effect" to become increasingly larger.

Summing up, for sufficiently low levels of the signal's accuracy (i.e. $b \lessapprox 0.25$) the main implication of the "misrecognition effect" is to reinforce the "surplus extraction effect" thereby leading the price to a loyal consumer to increase with improvements in information's accuracy. (Increases in b reduce the probability of firms receiving a wrong signal thereby allowing the "surplus extraction effect" to be the dominant one.) However, as information becomes increasingly more precise, despite the fact that firms are more likely to receive a correct signal it is also more likely that both firms receive the same signal (i.e. the "misrecognition effect" becomes increasingly smaller). As the "misrecognition effect" becomes weaker and weaker firms compete more aggressively in prices leading the "business stealing effect" to become larger. This explains why the price to a customer recognised as loyal is a decreasing function of the quality of information for $b \gtrapprox 0.25$. Nevertheless, the "misrecognition effect" allows the "surplus extraction effect" to dominate the "business stealing effect" when $0.25 \lessapprox b \lessapprox 0.5$. Conversely, when information becomes increasingly more accurate (i.e. $b \gtrapprox 0.5$) the role of the "misrecognition effect" becomes so small that the "business stealing effect" dominates the "surplus extraction effect". For this reason, the price to a loyal consumer becomes smaller than its non-discrimination counterpart.

As expected a very accurate signal *intensifies* price competition thereby explaining why both prices are below the non-discrimination level. This latter result is not new. We know from the previous literature that when each firm's strong market is the rival's weak market, price discrimination acts to intensify competition thereby lowering all segment prices (e.g. Thisse and Vives (1988), Chen (1997), Villas-Boas (1999) and Fudenberg and Tirole (2000)). The novelty of our analysis is that it suggests that price discrimination which is based on relatively imprecise information will in general allow firms to appropriate a higher proportion of surplus from each consumer recognised as loyal, which in some way may seem counterintuitive. While intuition suggests that for a monopolist more precise information is always better as it increases its capability to extract more consumer surplus, we can see that in a duopolistic setting information relatively inaccurate may act to soften price competition, thus allowing firms to appropriate more surplus from loyal consumers.

In short, it can be said that when information is not too accurate the "misrecognition effect" allows the "surplus extraction effect" to dominate the "business stealing effect", which then permits firms to safely extract more surplus from each consumer recognised as loyal. However, as information becomes more and more accurate the "misrecognition effect" plays a small role and then the "business stealing effect" becomes the dominant force. As a result of that, price discrimination leads to all-out competition.

2.5.2 Probability of winning a customer

When say *Amazon* infers that a particular customer is a loyal one it may be interested in determining what is the probability of winning that customer with a loyal tailored price, taken into account that say *Barnes&Noble* also offers that customer a price based on its beliefs about that customer's brand preference. This section looks at this issue and on *how* this probability evolves as information becomes more accurate. We first analyse the case in which price discrimination is allowed and afterwards we turn to the situation where discrimination is illegal.

Price discrimination is allowed Due to symmetry let γ_L and γ_D denote the probability of a firm winning a customer with a loyal and a disloyal signal, respectively, when firms are allowed to set a different price for different signals. Using, for instance, the perspective of firm A,

$$\begin{aligned}\gamma_L &= \Pr(\text{firm } A \text{ wins customer} \mid s_A = \alpha) \\ \gamma_D &= \Pr(\text{firm } A \text{ wins customer} \mid s_A = \beta).\end{aligned}$$

Thus it follows that,

$$\gamma_L = \sum_{r\in\{\alpha,\beta\}} \Pr\left(p_\alpha^A < p_r^B + l \mid s_A = \alpha, s_B = r\right) \Pr(s_B = r \mid s_A = \alpha)$$

$$= [1 - G_{\alpha\alpha}(y)]\,\rho_\alpha + [1 - G_{\alpha\beta}(0)]\,(1-\rho_\alpha),$$

and

$$\gamma_D = \sum_{r\in\{\alpha,\beta\}} \Pr\left(p_\beta^A < p_r^B + l \mid s_A = \beta, s_B = r\right) \Pr(s_B = r \mid s_A = \beta)$$

$$= [1 - G_{\beta\alpha}(0)]\,\rho_\beta + [1 - G_{\beta\beta}(-y)]\left(1-\rho_\beta\right).$$

After some algebra one gets that the probability of firms winning a customer after observing a loyal and disloyal signal is respectively given by

$$\gamma_L = \frac{1}{2} + \frac{1}{4}b - \frac{1}{2}y - by^2 - \frac{2}{3}b^2y^3, \tag{2.30}$$

and

$$\gamma_D = \frac{1}{2} - \frac{1}{4}b + \frac{1}{2}y + by^2 + \frac{2}{3}b^2y^3. \tag{2.31}$$

Hence, γ_L represents the probability of winning a customer with a loyal signal (or with a loyal price) and γ_D represents the probability of winning a customer with a disloyal signal (or with a disloyal price). Based on these probabilities one gets the subsequent result.

Corollary 2. *The greater is the accuracy of each firm's private signal the greater is the probability of a firm winning a customer with a loyal price and the lower is the probability of a firm winning a customer with a disloyal price.*

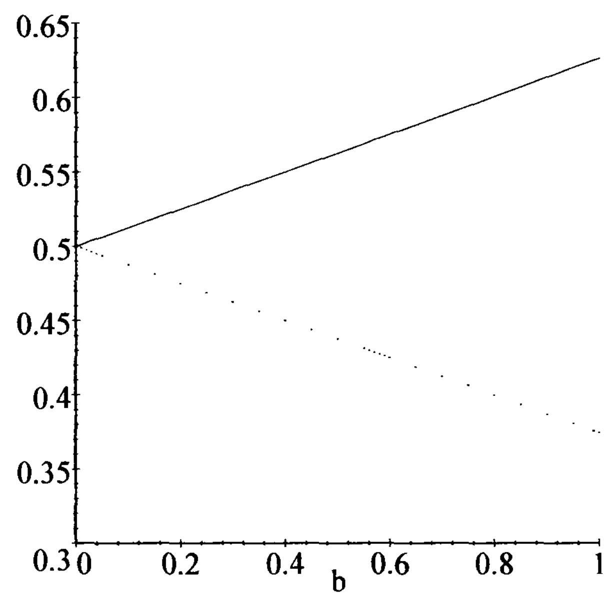
It is straightforward to verify that for any $b > 0$ it immediately follows that $\frac{1}{4}b > \frac{1}{2}y + by^2 + \frac{2}{3}b^2y^3$.[8] Therefore, while γ_L is strictly increasing in b, γ_D is strictly decreasing. Figure 3 which represents γ_L (solid line) and γ_D (dotted line) also confirms this result. As expected it can be seen that when the signal is non-informative—i.e. when $b = 0$—price discrimination is not feasible and so the previous probabilities are just equal to 0.5. The higher is b the greater is the likelihood of firms facing in fact a loyal customer after observing a loyal signal and, therefore, the higher is the probability of firms winning a customer with a loyal signal. (Recall also that as b is increasingly higher, firms also decrease the price to customers recognised as loyal). Further, as the signal becomes more exact the smaller is the probability of firms winning a customer with a disloyal signal (and with a disloyal price). For a very accurate signal firms are more certain

[8] Recall that the condition that defines y as an implicit function of b is

$$y + 2by^2 + \frac{5}{3}b^2y^3 - \frac{1}{4}b = 0.$$

about each customer's type and about the rival's private signal. Thus, they are forced to reduce the price for customers perceived as loyal which increases the probability of firms winning a customer with a loyal price. The reverse happens for the probability of winning a customer with a disloyal price.

Figure 3: Probability of winning a customer



Price discrimination is illegal In this situation we would observe $p_A^\alpha = p_A^\beta$, $p_B^\alpha = p_B^\beta$ and as result $y = 0$. Thus, using (2.30) and (2.31) and making $y = 0$ one obtains the probability of a firm winning a customer conditional on each signal. Again symmetry allows us to ease notation and to denote as γ_L^N and γ_D^N the probability that each firm wins a customer after observing a loyal and a disloyal signal, respectively, when discrimination is not allowed. We find that:

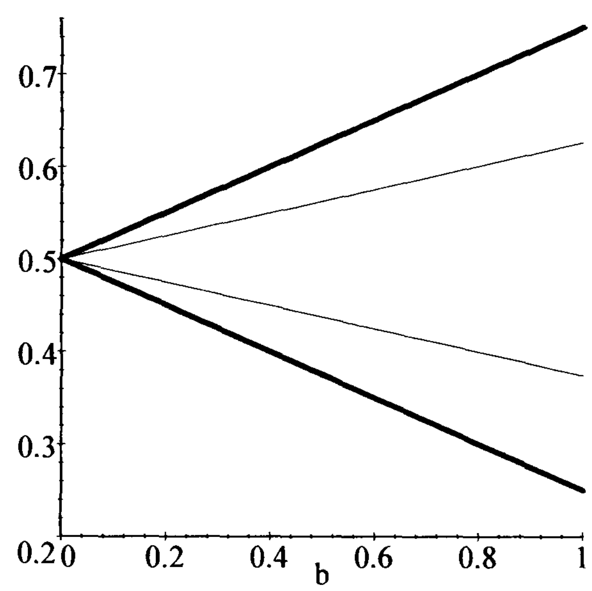
$$\gamma_L^N = \frac{1}{2} + \frac{1}{4}b \tag{2.32}$$

and

$$\gamma_D^N = \frac{1}{2} - \frac{1}{4}b. \tag{2.33}$$

From Figure 4 and from the comparison between equations (2.30) and (2.32) it is easy to see that the probability of a firm winning a customer with a loyal signal, when discrimination is illegal (bold upward line), is always above the probability of winning a customer with a loyal signal when discrimination is allowed (thin upward line). This happens because under non-discrimination consumers have no incentive to swap brands. Conversely, the probability of a firm winning a customer upon observing a disloyal signal, under discrimination (bold downward line), is always higher than the probability of winning a customer upon observing a disloyal signal when discrimination is not allowed (thin downward line). Given that under discrimination a firm tries to attract those customers who prefer the rival's product by offering them a lower price it is obvious that the probability of winning a customer after observing a disloyal signal should be higher than its no-discrimination counterpart.

Figure 4: Probability of winning a customer with/without discrimination



In a context in which discrimination is not allowed, firms set the price $\frac{1}{2}$ regardless of observing signal α or β. Therefore, the probability of winning a customer with a loyal signal only depends on b and due to our assumptions the greater is b the greater is that probability. Finally, notice that obviously the probability of a firm winning a customer with the non-discriminatory price is always equal to $\frac{1}{2}$.

2.5.3 Expected number of *Inefficient Shoppers*

We define as *inefficient shoppers* those customers that buy their less preferred brand thereby incurring a disutility (a kind of "transport cost").[9] When firms are allowed to

[9] Because our model abstracts from any previous competition we think it is more convenient to adopt the term inefficient shoppers rather than the usual "switchers" which is more indicate to situations in

price discriminate, some consumers might have an incentive to swap to their less preferred brand. This explains why in Figure 4 above there is a vertical difference between the probability of winning a customer with a loyal signal under non-discrimination and under discrimination when we allow b to vary. Thus, for a given level of the signal's accuracy, the vertical difference between these two probabilities gives the number of consumers that are expected to buy the wrong brand in equilibrium. The expected number of *inefficient shoppers* denoted as EIS is then given by

$$EIS = \left(\gamma_L^N - \gamma_L\right)\left[\Pr\left(s_A = \alpha\right) + \Pr\left(s_B = \beta\right)\right]$$

or,

$$EIS = \frac{1}{2}y + by^2 + \frac{2}{3}b^2y^3. \tag{2.34}$$

The number of customers who buy inefficiently in equilibrium under price discrimination depends on the informativeness of the signal and on the difference between prices, that is y. As expected, when discrimination is not allowed *every* consumer buys his most preferred brand, thereby implying that no consumer buys inefficiently.

However, when price discrimination is permitted it is easy to see that more accurate information gives rise to more inefficient shopping in equilibrium. (Notice that in Figure 4 the higher is b, the greater is the expected number of customers who buy the wrong brand.) We have seen that more accurate information leads to more intense competition, and to a larger difference between p_L and p_D (i.e. y is higher). As a result of that, those customers with a smaller brand loyalty degree—i.e. those located in the middle—will have more incentives to buy the wrong brand. It is easy to see that when the signal reaches its maximum level of accuracy, i.e. when $b = 1$, approximately 12.5% of customers buy inefficiently. In consequence, moving from non-discrimination to discrimination leads some consumers to buy their less preferred brand. Moreover, the better is the quality of each firm's information, the greater will be the number of consumers that are expected to buy inefficiently.

---

which consumers choose one brand in one period and a different one in the other period.

It is worth emphasizing that while in Thisse and Vives (1988) price discrimination does not lead to inefficient shopping the same does not happen in the current model. In Thisse and Vives (1988) both firms observe the brand loyalty degree of each individual consumer. Thus, in their model each individual consumer is treated as a submarket and firms can engage in first-degree price discrimination. Even though firms try to entice those consumers with a preference from the rival, no consumer is in fact poached in equilibrium. Here, in contrast, firms cannot know the brand loyalty degree of each individual consumer, as they can only at best separate consumers into two different segments. For that reason, as happens in the second-period of Fudenberg and Tirole's (2000) model, price discrimination in the present model always allows firms to attract some consumers that prefer the rival's brand.

2.5.4 Profits

This section assesses the profitability of price discrimination in markets where firms have private and imperfect information and looks at the following question. Does a firm benefit from being able to price discriminate on the basis of more precise information? Answering this question will shed some light on whether a firm should have an incentive to invest in more accurate consumer-specific information for price discrimination purposes.

We start by analysing how profits respond to improvements in the signal's accuracy or, equivalently, how price discrimination based on more informative signals (and better recognition of consumers) affects each firm's expected profit. Since the equilibrium is symmetric let $E\pi_L$ and $E\pi_D$ represent, respectively, each firm expected profit from perceived loyal and disloyal customers. Then it follows that:

$$E\pi_L = p_L\left(\frac{1}{2}+\frac{1}{4}b-\frac{1}{2}y-by^2-\frac{2}{3}b^2y^3\right), \tag{2.35}$$

$$E\pi_D = p_D\left(\frac{1}{2}-\frac{1}{4}b+\frac{1}{2}y+by^2+\frac{2}{3}b^2y^3\right). \tag{2.36}$$

Because each firm recognises a customer as being loyal or disloyal with probability equal to $\frac{1}{2}$ (remember that $\Pr(s_i = \alpha) = \Pr(s_i = \beta) = \frac{1}{2}$) each firm expected aggregate profit, denoted by $E\Pi$ is just equal to

$$E\Pi = \frac{1}{2}\left(E\pi_L + E\pi_D\right). \tag{2.37}$$

Figure 5 illustrates the relationship between each firm's profit and the level of accuracy of the signal. There we see that as competing firms observe a more accurate signal, the profit from perceived loyal customers—i.e. $E\pi_L$ (solid line)—has an inverted-U shaped relationship with the signal's accuracy. Conversely, we observe that the profit that comes from perceived disloyal customers—i.e. $E\pi_D$ (dotted line)—decreases as the signal becomes more informative.

Figure 5: Expected equilibrium profits from loyal and disloyal

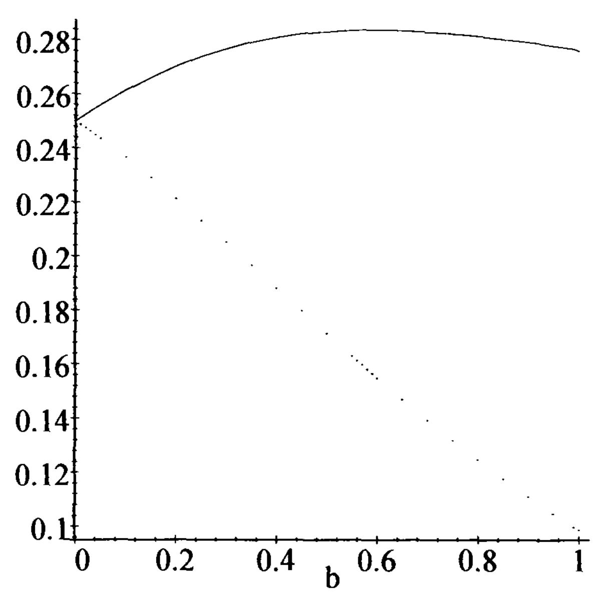


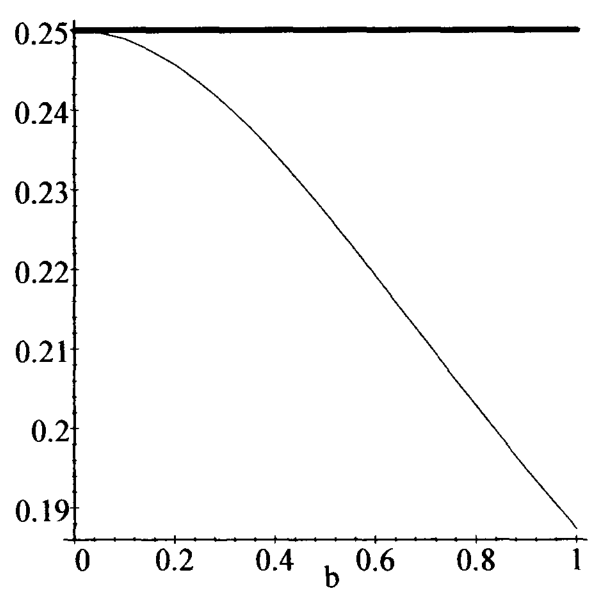
Figure 6 illustrates the behaviour of expected aggregate profits with price discrimination (thin line) and without discrimination (bold line) and allows us to examine whether or not firms benefit from price discrimination as their ability to recognise customers gradually improves. Based on both figures we may establish the following result:

Corollary 3. *(i) As the signal becomes more informative, the profit that comes from selling to customers classified as loyal first increases and then decreases. Con-*

versely, as the signal's accuracy improves the profit that comes from selling to customers classified as disloyal decreases.

(ii) Price discrimination is always bad for profits, and worse as the information's accuracy improves.

Figure 6: Expected aggregated profits



Obviously, the relationship between the profit a firm earns either from customers recognised as loyal or from consumers recognised as disloyal and the accuracy of its information is strongly related to the relationship between prices and the signal's accuracy. In fact, we have seen before that when a firm's signal is not very accurate—i.e. b approximately below 0.5—a firm is able to extract more surplus from customers recognised as loyal which clearly is good for profits. However, when the signal becomes increasingly more accurate firms compete more aggressively which leads to lower prices and profits. Similarly, because the price charged to customers perceived as disloyal always falls down as the accuracy of information improves, the same happens to profits from perceived disloyal customers.

More interesting is, however, to compare expected aggregate profits as the signal improves with those under non-discrimination. The extant literature has shown that in markets exhibiting best-response asymmetry, price discrimination might lead to all-out competition (e.g. Thisse and Vives (1988), Chen (1997) and Fudenberg and Tirole (2000)). However, this conclusion was obtained in the extreme case where firms observe the same piece of information and know for sure whether a customer is a loyal or a disloyal one. That is, this result was obtained in models where firms price discriminate on the basis of a perfect segmentation of the market (either on an individual basis, e.g. Thisse and Vives (1988); or on a group basis, e.g. Fudenberg and Tirole (2000)). The analysis developed so far adds to the existing literature some new insights by analysing a market where competing firms are neither able to infer perfectly each customer's type neither to know with certainty what is the rival's information about that same customer. For instance, with some probability each firm's strong perceived market may coincide, thereby soften price competition. (This is more likely to occur when b is smaller.) However, even in this case, we find that price discrimination is bad for profits. The reason is that although firms can charge higher prices to consumers recognised as loyal when the signal is not too precise, it also happens that the probability of winning a customer with this price is smaller. Thus, the expected surplus extraction benefit is not enough to overcome the reduction in profits that result from a lower price being charged to a consumer recognised as disloyal. Because more accurate information turns out into more aggressive pricing, the model shows that firms are strictly better off when price discrimination is based on highly inaccurate information.

It is further worth stressing that our findings are also different from Chen, Narasimhan and Zhang (2001). In their model there are only three types of consumers, each firm has a captive group of consumers and firms only compete for switchers. Thus, they find that both firms might benefit from price discrimination based on more accurate information (which they designate as targetability) when the level of information precision is low. Specifically, they show that when firms depart from low levels of targetability, profits will increase as targetability improves. The intuition for their result is as follows. When targetability is not too high, improvements in it allow firms to extract more surplus from their captive customers as they are increasingly able to identify them. At the same time, because targetability remains at a relatively low level, firms can not fully separate switchers from captive customers which clearly softens price competition for the switchers. Thus, while in their model competing firms may benefit from more accurate information, here this is never the case.

As in the previous literature, in the present model when firms try to attract each other's customers, the prisoner's dilemma result always ensues. Firms would like to collectively commit to uniform pricing but individually each firm prefers to price discriminate. However, we can say that here the information technology available for each firm acts as a restriction to more aggressive price discrimination, which of course is good for profits. Notice that when the information technology is fully uninformative there is a credible kind of commitment to uniform pricing, which benefits all competing firms.

In sum, it can be said that although individually a firm could have an incentive to invest in more accurate information, as long as all competing firms try to improve the quality of their information, all firms will end up worse off. Thus, our analysis and earlier work put forward that competing firms could all benefit from regulatory policies protecting consumer privacy, which would limit the firms' ability to recognise customers (and thereby to price discriminate).

2.6 Welfare analysis

In this section we investigate how welfare changes when the quality of information available for price discrimination improves. Throughout this section it is assumed that whenever b is greater than zero firms are allowed to price discriminate. Moreover, in order to evaluate the effects of improvements in the signal's accuracy on welfare, we carry out some simulations assuming without any loss of generality that $v = 2$.

As usual expected social welfare is defined as the sum of expected industry profit and expected consumer surplus. Hence,

$$EW = 2E\Pi + ECS, \tag{2.38}$$

where $E\Pi$ denotes each firm's expected aggregate profit and ECS denotes expected consumer surplus.

Lemma 2. *Expected consumer surplus is given by:*

$$ECS = v - p_L\gamma_L - p_D\gamma_D - \frac{1}{4}y^2 - \frac{2}{3}by^3 - \frac{1}{2}b^2y^4. \tag{2.39}$$

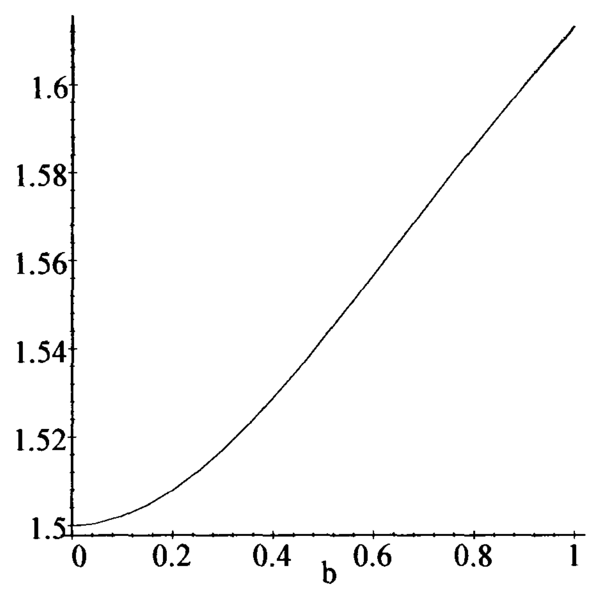
Proof. See the Appendix.

As expected, when $b = 0$, expected consumer surplus is equal to consumer surplus with non-discrimination, that is $ECS = v - p_N$.

Corollary 4. *(i) As each firm relies on a more informative signal for the purpose of price discrimination, consumers as a whole are better off.*

(ii) Each individual consumer is increasingly better off as price discrimination is based on more accurate information.

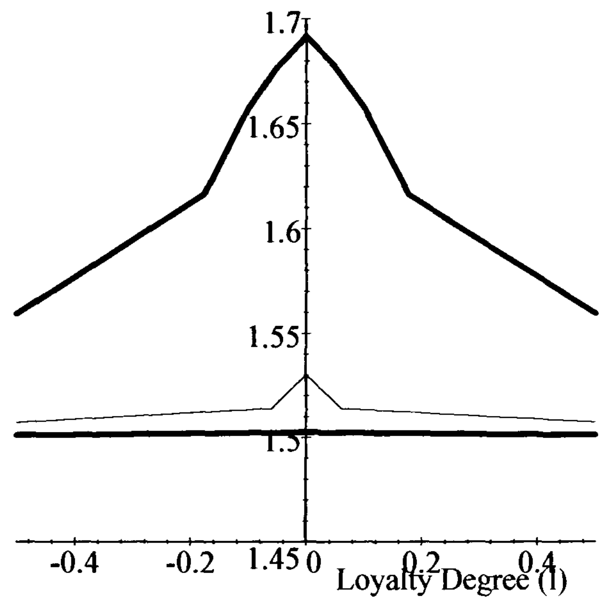
Figure 7: Expected consumer surplus



Part (i) can be confirmed by using Figure 7, which illustrates the relationship between expected consumer surplus and the signal's accuracy.

Likewise Figure 8 confirms part (ii). This figure shows expected consumer surplus per consumer for a fixed level of accuracy. Particularly the figure is plotted for $b = 0.1$ (lower bold line); $b = 0.25$ (thin line) and $b = 1$ (upper bold line).

Figure 8: Expected consumer surplus per consumer



It is worth remembering that when discrimination is not allowed or when $b = 0$, expected consumer surplus per consumer is constant regardless of l. In particular, for $v = 2$ it is easy to see that expected consumer surplus is in this case equal to 1.5 for every consumer. Although ECS remains approximately at the same level for tiny increases in b, when the signal becomes increasingly informative we observe that ECS increases with b. We have seen that as the signal becomes more informative price competition on a segment basis is more intense and thereupon prices fall. As a result, consumers can buy at better deals.

Despite consumers as a whole being unequivocally better off with price discrimination, we know that for signals that are not too accurate ($b \lesssim 0.5$) some consumers are expected to face a price higher than its non-discrimination counterpart. At a first glance this suggests that not all consumers benefit from price discrimination when the signal's precision is relatively low. Nevertheless, Figure 8 presented above shows that even in the range where loyal customers pay a price higher than under non-discrimination, expected consumer surplus is higher than under non-discrimination. Notice that p_L reaches its maximum value when $b = 0.25$ and even in this case expected consumer surplus for the most loyal customers $l = \left\{-\frac{1}{2}, \frac{1}{2}\right\}$ is higher than under non-discrimination. The intuition is as follows. In spite of for small levels of the signal's accuracy loyal customers are expected to pay higher prices, it is also true that with some positive probability they will be misrecognised (i.e. they will be classified as disloyal) and therefore they will buy the product at the lowest price p_D. Although a firm tends to charge higher prices to perceived loyal customers when b is low, it is also true that it is more likely that the firm mistakenly recognises a true loyal customer as a disloyal when b is low. As a result, expected consumer surplus is always above the non-discrimination level.

Using (2.37) and (2.39) it ensues that expected social welfare is equal to:

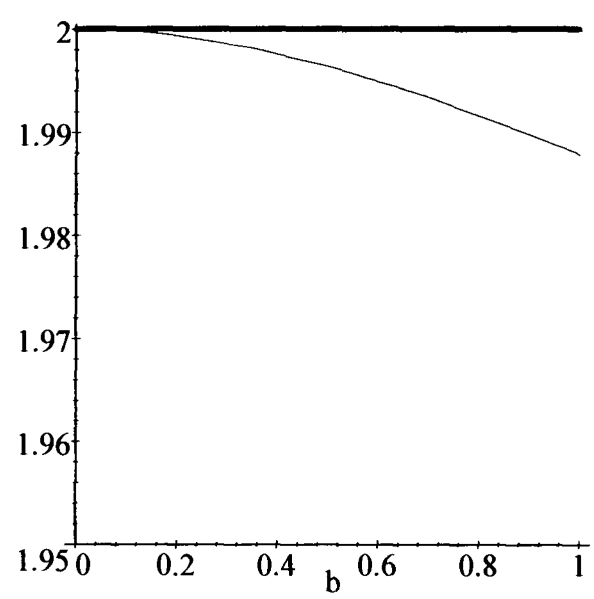
$$\begin{aligned} EW &= p_L\gamma_L + p_D\gamma_D + \left(v - p_L\gamma_L - p_D\gamma_D - \frac{1}{4}y^2 - \frac{2}{3}by^3 - \frac{1}{2}b^2y^4\right) \qquad (2.40) \\ &= v - \left(\frac{1}{4}y^2 + \frac{2}{3}by^3 + \frac{1}{2}b^2y^4\right). \end{aligned}$$

As expected, expected aggregate welfare is equally given by v minus the disutility incurred by those consumers who buy inefficiently. It immediately follows that welfare is lower the higher is b. Notice that the higher is b, the higher is y, the higher is $\left(\frac{1}{4}y^2 + \frac{2}{3}by^3 + \frac{1}{2}b^2y^4\right)$ and the lower is EW. Further, because welfare is under non-discrimination equal to v, it is clear-cut that price discrimination is bad for welfare. Thus, we may establish the following proposition.

Proposition 3. *Price discrimination is always good for consumers although bad for profits and overall welfare. Moreover, consumers are increasingly better off while firms and welfare are increasingly worse off as price discrimination is based on more precise information.*

Figure 9 illustrates the relationship between aggregate expected welfare and the signal accuracy with discrimination, given by EW (thin line). It also represents aggregate welfare with no-discrimination (bold line). There we observe that welfare is unambiguously lower with price discrimination. Furthermore, the negative effect of price discrimination on overall welfare is greater the higher is the level of informativeness of each firm's signal. As firms become increasingly able to recognise customers, and to segment them with less mistakes (i.e., less imperfectly), the stronger is the damage of price discrimination on welfare. In fact, welfare reaches its minimum value when the signal's accuracy reaches its maximum level. The reason is that the expected number of consumers that buy inefficiently in equilibrium reaches its maximum value when $b = 1$. Because in the present model there is no role for price discrimination to increase aggregate output, variations in welfare are uniquely explained by the costs incurred by those consumers that buy the wrong brand. Thus, improvements in the information required for price discrimination leads to more inefficient shopping which clearly is not good for welfare.[10]

Figure 9: Expected welfare with and without price discrimination



In short, as information accuracy improves profits and welfare move in the same direction but consumer surplus moves in an opposite direction. Then, any advice to a regulatory authority should take into account whether the target is welfare or solely consumer surplus. Restrictions protecting consumer privacy and limiting the efficacy of price discrimination would benefit profits and welfare but at the expense of consumer welfare.

[10]Notice that this result is in contrast with that achieved in Thisse and Vives (1988). The reason is that in their model price discrimination based on perfect information gives rise to no welfare loss since no consumer actually switches in equilibrium. Here because firms can only segment consumers into loyal or disloyal (as in Fudenberg and Tirole (2000)) there are always some consumers who buy inefficiently. Therefore, here price discrimination based on better information increases the welfare loss.

2.7 Price discrimination with public information

The analysis developed so far has assumed that firms price discriminate on the basis of private information. One interesting issue is to investigate how profits evolve as we depart from a setting where price discrimination is based on private information to a setting where price discrimination is based on public information—i.e. a framework where both firms have the same information. Looking at this issue will also shed light on whether a firm benefits or not from exchanging its private information with its rival for price discrimination purposes.

With that in mind we now extend the model to the case in which both firms observe a public signal about each consumer's loyalty. The signal is "public" in the sense that its actual realization is common knowledge to both firms. This means that although firm A observes s_A and firm B observes s_B each firm knows the signal of each other. Alternatively, this public information case could also fit a situation where both firms observe two public signals about the same consumer's loyalty rather than one. It can fit as well the case where firms exchange its customer information with its competitor. To ease notation we denote by s the public signal observed by firms where $s = (s_A = k, s_B = r)$ and $(k, r) = \{(\alpha, \alpha), (\beta, \beta), (\alpha, \beta), (\beta, \alpha)\}$. Again, α informs that the consumer is loyal to firm A while β informs that the consumer is loyal to firm B. In this scenario firms may observe a *biased* signal in favour of one firm, that is two signals informing that the consumer is either loyal to firm A or loyal to firm B (i.e. (α, α) or (β, β)); or, alternatively, firms may observe a *non-biased* signal meaning that one of the signals reveals that the consumer prefers A, while the other reveals that the consumer prefers B (i.e. (α, β) or (β, α)).

Since now signals are public firms only need to update their beliefs about each consumer's brand loyalty degree after observing signal s. Thus, using our previous computations $g_{rk} = \Pr(l \mid s = (r, k))$ is now the density function of l conditional on the public signal observed by firms.

2.7.1 Competitive equilibrium

Given that firms may observe four different types of signals, price discrimination being allowed, each firm chooses simultaneously a different price for each realization of the signal. Hence, firm i chooses $p_s^i \in \{p_{\alpha\alpha}^i, p_{\beta\beta}^i, p_{\alpha\beta}^i, p_{\beta\alpha}^i\}$, $i = A, B$. These prices are the solution to the following problem

$$\underset{p_s^i \geq 0}{Max}\ E\left[\pi^i \mid s\right], \tag{2.41}$$

where, for instance for firm A, one gets

$$\begin{aligned} E\left[\pi^A \mid s = (\alpha,\alpha)\right] &= p_{\alpha\alpha}^A \Pr\left(p_{\alpha\alpha}^A < p_{\alpha\alpha}^B + l \mid s = (\alpha,\alpha)\right) \\ &= p_{\alpha\alpha}^A \left[1 - G_{\alpha\alpha}\left(p_{\alpha\alpha}^A - p_{\alpha\alpha}^B\right)\right], \end{aligned}$$

$$\begin{aligned} E\left[\pi^A \mid s = (\beta,\beta)\right] &= p_{\beta\beta}^A \Pr\left(p_{\beta\beta}^A < p_{\beta\beta}^B + l \mid s = (\beta,\beta)\right) \\ &= p_{\beta\beta}^A \left[1 - G_{\beta\beta}\left(p_{\beta\beta}^A - p_{\beta\beta}^B\right)\right], \end{aligned}$$

$$\begin{aligned} E\left[\pi^A \mid s = (\alpha,\beta)\right] &= p_{\alpha\beta}^A \Pr\left(p_{\alpha\beta}^A < p_{\alpha\beta}^B + l \mid s = (\alpha,\beta)\right) \\ &= p_{\alpha\beta}^A \left[1 - G_{\alpha\beta}\left(p_{\beta\beta}^A - p_{\beta\beta}^B\right)\right], \end{aligned}$$

$$\begin{aligned} E\left[\pi^A \mid s = (\beta,\alpha)\right] &= p_{\beta\alpha}^A \Pr\left(p_{\beta\alpha}^A < p_{\beta\alpha}^B + l \mid s = (\beta,\alpha)\right) \\ &= p_{\beta\alpha}^A \left[1 - G_{\alpha\beta}\left(p_{\beta\alpha}^A - p_{\beta\alpha}^B\right)\right]. \end{aligned}$$

Symmetric expressions hold for firm B. Due to symmetry we are looking for a Bayesian Nash equilibrium where $p_{\alpha\alpha}^A = p_{\beta\beta}^B$, $p_{\beta\beta}^A = p_{\alpha\alpha}^B$ and $p_{\alpha\beta}^A = p_{\alpha\beta}^B = p_{\beta\alpha}^A = p_{\beta\alpha}^B$. Following the same reasoning as before and once again due to symmetry we represent as p_L^{pub} and p_D^{pub} the equilibrium prices firms charge after observing a double loyal and a double disloyal signal, respectively, in the public information context. Likewise we represent as p_{NB}^{pub} the price firms charge after observing a non-biased signal. Again, $p_L^{pub} = p_{\alpha\alpha}^A = p_{\beta\beta}^B$ and $p_D^{pub} = p_{\beta\beta}^A = p_{\alpha\alpha}^B$.

Notice that in the public information framework each firm may classify customers into three different segments. The segment of perceived strong loyal customers, the segment of perceived strong disloyal customers and the segment of consumers to whom firms have observed a non-biased signal. Under price discrimination each firm tailors a different price for each different type of signal. After some algebra we may establish the following proposition.

Proposition 4. *The Bayesian Nash equilibrium in prices when information is public or, equivalently, when firms exchange their information is given by*

$$p_L^{pub} = \frac{\frac{1}{8} + \frac{1}{8}b + \frac{1}{24}b^2 - \frac{1}{4}z - \frac{1}{2}bz^2 - \frac{1}{3}b^2z^3}{\left(\frac{1}{2} + bz\right)^2}, \tag{2.42}$$

$$p_D^{pub} = \frac{\frac{1}{8} - \frac{1}{8}b + \frac{1}{24}b^2 + \frac{1}{4}z + \frac{1}{2}bz^2 + \frac{1}{3}b^2z^3}{\left(\frac{1}{2} + bz\right)^2}, \tag{2.43}$$

where $z = p_L^{pub} - p_D^{pub}$; *and*

$$p_{NB}^{pub} = \frac{1}{2} - \frac{1}{6}b^2. \tag{2.44}$$

Proof. See the Appendix.

Additionally,

Proposition 5. *(i) When the signal is fully inaccurate* ($b = 0$) *price discrimination is unfeasible. Thus,* $p_L^{pub} = p_D^{pub} = p_{NB}^{pub} = \frac{1}{2}$.

(ii) For an informative signal ($b > 0$) *the price charged to customers perceived as loyal is always higher than the price charged to customers perceived as disloyal. That is,* $p_L^{pub} > p_D^{pub}$.

(iii) As the public signal becomes increasingly more accurate, the higher is the difference between p_L^{pub} *and* p_D^{pub}.

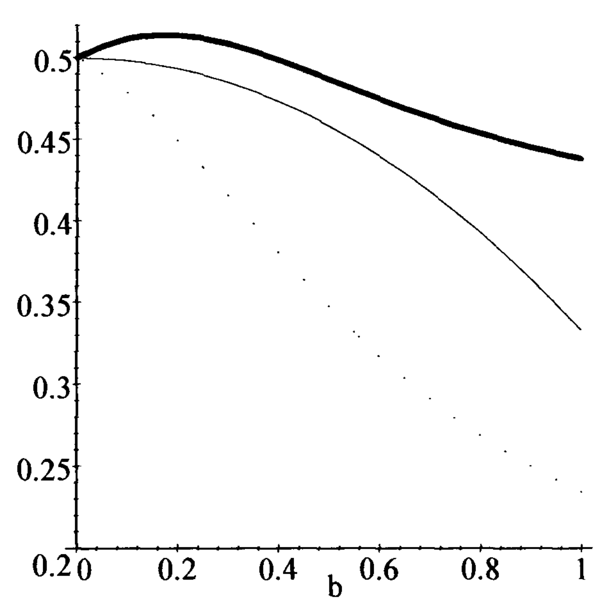
(iv) The more accurate is the public signal, the lower is the price charged to those customers to whom firms observe a non-biased signal.

(v) For any level of the signal's informativeness, $p_L^{pub} > p_{NB}^{pub} > p_D^{pub}$.

Proof. Part (i) can be easily proved using (2.42), (2.43) and (2.44) and making $b = 0$. Part (iv) is also clear looking at (2.44). Part (iii) is proved by plotting $z = p_L^{pub} - p_D^{pub}$ as an implicit function of b. This is done in Figure 11. There the reader can observe that z is positive and increases with b. Finally, Figure 10 proves part (v) by plotting p_L^{pub} (bold line), p_{NB}^{pub} (thin line) and p_D^{pub} (dots line). Figure 10 shows that in fact $p_L^{pub} > p_{NB}^{pub} > p_D^{pub}$. *Q.E.D.*

Intuition suggests that when firms are more certain about the loyalty of a particular consumer they have more incentives to charge that consumer a higher price. The same reasoning applies when firms observe a double disloyal signal. Obviously when firms observe a non-biased signal it is equally likely that the consumer can turn out to be a truly loyal or disloyal. So, the price charged to a consumer with that type of signal should be below p_L^{pub} but above p_D^{pub}.

Figure 10: Behaviour of equilibrium prices under public information



As before, it is interesting to look at the relationship between p_L^{pub} and the accuracy of information. When information is not very accurate, an increase in the signal's accuracy has the following effects on p_L^{pub}. First, when a firm has some ability to recognise a loyal consumer it can extract more surplus from him. Second, when the level of accuracy raises but remains relatively low it is still highly likely that a firm mistakenly recognises a true loyal as a disloyal consumer. Due to this latter effect a firm has less incentives to charge a low price to a consumer perceived as disloyal. Of course, this acts to soften price competition. For that reason a firm is able to increase the price to customers recognised as loyal for relatively low accurate signals. However, as the information becomes more and more accurate firms have less doubts about each consumer's type and therefore they behave more aggressively.

Expected equilibrium profits per recognised segment and per firm under public information are denoted by

$$E\pi_L^{pub} = \frac{1}{\lambda_{\alpha\alpha}} \left(p_L^{pub}\right) \left(\frac{1}{8} + \frac{1}{8}b + \frac{1}{24}b^2 - \frac{1}{4}z - \frac{1}{2}bz^2 - \frac{1}{3}b^2z^3\right), \tag{2.45}$$

$$E\pi_D^{pub} = \frac{1}{\lambda_{\beta\beta}} \left(p_D^{pub}\right) \left(\frac{1}{8} - \frac{1}{8}b + \frac{1}{24}b^2 + \frac{1}{4}z + \frac{1}{2}bz^2 + \frac{1}{3}b^2z^3\right), \tag{2.46}$$

$$E\pi_{NB}^{pub} = \frac{2}{\lambda_{\alpha\beta}} \left(p_{NB}^{pub}\right) \left(\frac{1}{8} - \frac{1}{24}b^2\right) = p_{NB}^{pub} \left(\frac{1}{4} - \frac{1}{12}b^2\right), \tag{2.47}$$

and due to symmetry each firm's expected aggregated profit under public information denoted by $E\Pi^{pub}$ is equal to:

$$E\Pi^{pub} = E\pi_L^{pub} \Pr(\text{double loyal}) + E\pi_D^{pub} \Pr(\text{double disloyal}) + E\pi_{NB}^{pub} \Pr(\text{non-biased}).$$

2.8 Private versus public information: a comparative analysis

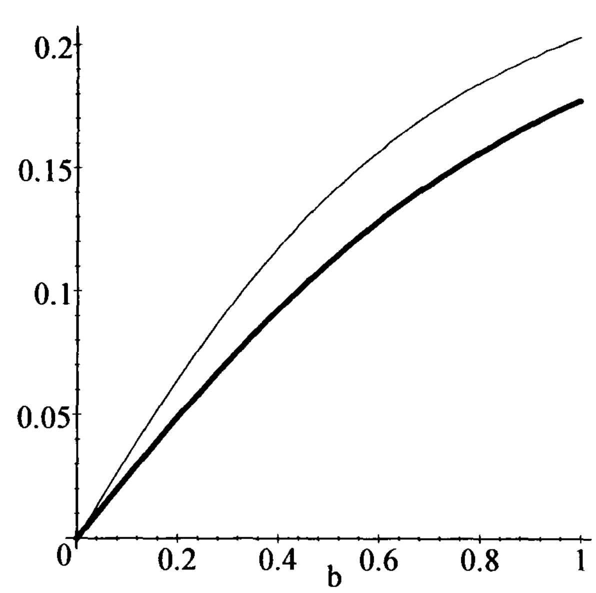
This section aims to examine whether firms, consumers and welfare benefit from price discrimination based on public rather than on private information. In this regard, in what follows we compare equilibrium prices, profits, consumer surplus and welfare attained with private and public information.

2.8.1 Comparison between equilibrium prices

In order to compare the prices firms set in equilibrium upon observing a loyal and a disloyal signal, under both information settings, the following diagram plots the difference between the highest and the lowest price under private—namely, y (bold line)—and under public information—namely, z (thin line). Throughout the comparative analysis bold lines will be used for private information.

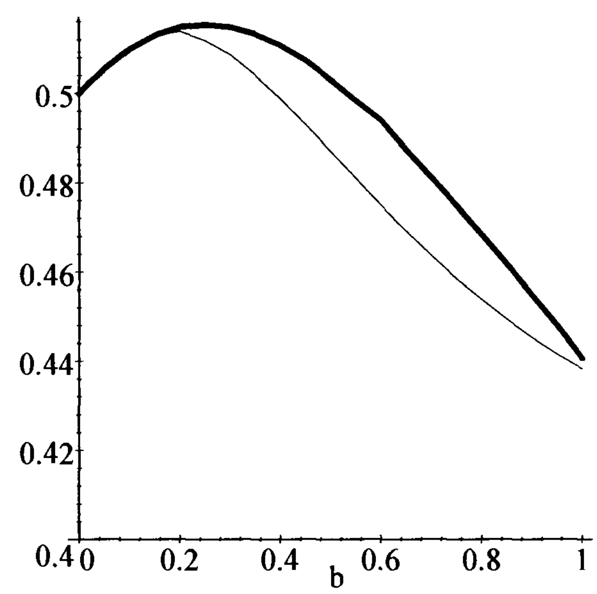
Figure 11 shows that the range between the highest (loyal) and the lowest (disloyal) price with public information is larger than with private information. If we measure price dispersion by the range of prices we can say that the level of price dispersion is greater under public than under private information. As we expect more aggressive behaviour under public than under private information, Figure 11 seems to suggest that price dispersion will reach its maximum level under public information when signals are extremely accurate (i.e. when $b \approx 1$). In other words, an increase in competition will increase price dispersion.

Figure 11: Comparison between y and z in both information settings



Next we look at the behaviour of the highest and the lowest price under both information regimes. Figure 12 illustrates the relationship between the price charged to customers recognised as loyal under private information (bold line) and under public information (thin line).

Figure 12: Price to loyal customers

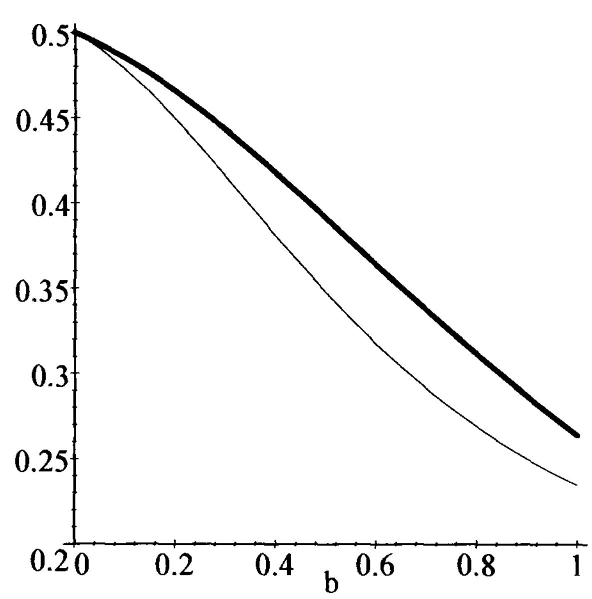


As in the private information setting we observe that there is a non-monotonic relationship between the quality of information and the price to a loyal consumer in the public information framework. In order to explain this finding it is helpful to have in mind our previous discussion (see subsection 2.5.1). There we have seen that for low levels of the signal's accuracy, firms recognise that there is a good chance that a consumer turns out to be a disloyal one after a loyal signal has been observed and vice-versa. This kind of uncertainty is as well present in the public information case, which thereby means that with some probability both firms may misrecognise customers. Notice that the situation where one firm misrecognises customers while the other does not is absent from the public information analysis, so the "misrecognition effect" is only present when both firms misrecognise customers. For that reason, the "misrecognition effect" plays a smaller role under public than under private information. In fact, in the public information game the "misrecognition effect" only acts to reinforce the "surplus extraction effect" when the signal's accuracy departs from an extremely low level.

For sufficiently low levels of information's precision we find that increases in b reduce the probability of firms receiving a wrong signal thereby allowing the "surplus extraction effect" to become stronger. Put differently, as the probability of misrecognition decreases the "surplus extraction effect" increases, allowing firms to charge more to consumers recognised as loyal. (This explains why the price to a loyal consumer increases as the quality of information improves when $b \lesssim 0.2$.) However, as information becomes more and more accurate, firms are more likely to receive a correct signal, the role of the "misrecognition effect" is increasingly smaller leading firms to compete more aggressively in prices. The "business stealing effect" becomes larger explaining why the price to a customer recognised as loyal is a decreasing function of the quality of information when $b \gtrapprox 0.2$. In this way, as the signal is increasingly more precise (i.e. when $b \gtrapprox 0.4$) the "business stealing effect" dominates the "surplus extraction effect" and in consequence the price to a loyal consumer becomes smaller than its non-discrimination counterpart.

Figure 13 illustrates the relationship between the price to customers recognised as disloyal under private information (bold line) and under public information (thin line).

Figure 13: Price to disloyal customers



Numerical analysis allows us to establish the following corollary.

Corollary 5. *(i) Moving from private to public information reduces the price to customers perceived as disloyal.*

(ii) When the signal's accuracy is not too low (i.e. $b \gtrsim 0.2$) the price to a customer perceived as loyal is higher under private than under public information.

This result claims that for sufficiently accurate signals (i.e. $b \gtrsim 0.2$) the prices to loyal and disloyal consumers are lower under public than under private information.

Although our analysis offers no clear cut results with respect to the comparison between the price to a loyal consumer in both information regimes when the signal's accuracy is too low, it shows that as information's accuracy becomes increasingly higher consumers perceived as loyal are expected to pay higher prices under private than under public information. Numerical analysis shows that while under private information the price to a loyal consumer reaches the no-discrimination level for $b \cong 0.5$ under public information this happens for $b \cong 0.4$. Similarly, we find that whilst under private information the price to a loyal consumer reaches its maximum value approximately at 0.5154 for $b \cong 0.25$ under public information the price to a loyal consumer reaches its maximum value approximately at 0.5138 for $b \cong 0.2$.

The intuition is as follows. We have seen that in the private information setting firms are uncertain about each consumer's true type as well as about the rival's private information. The latter type of uncertainty disappears in the public information framework. Whilst under private information a given consumer may be recognised differently by both firms, leading the "misrecognition effect" to soften price competition; under public information this is no longer the case because firms always classify consumers in the same way. As a consequence, at least for sufficiently accurate signals, we expect firms to compete more aggressively in the public information game which then leads the price to customers recognised as loyal as well as the price to customers recognised as disloyal to become smaller than their private information counterparts.

2.8.2 Comparison between firm's expected profits

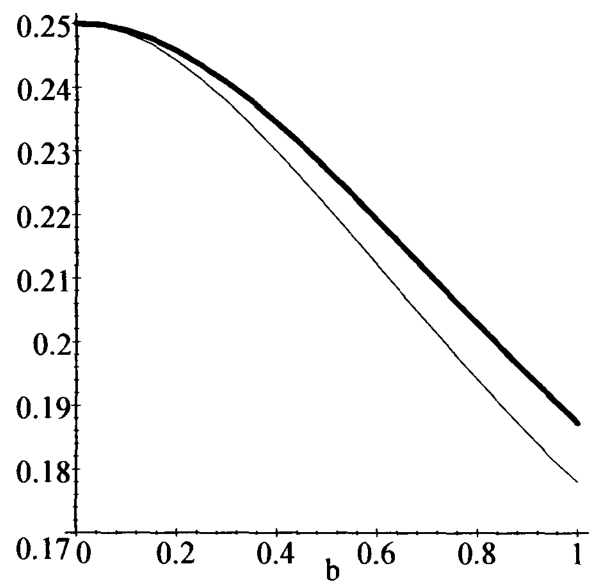
Since firms will play more aggressively when they have access to the same information about potential customers the next result ensues.

Corollary 6. *Firms are better off when price discrimination is based on private information rather than on public information.*

Figure 14 shows how aggregate expected profits behave as the signal becomes more precise under private information (bold line) and under public information (thin line).

Given that under non-discrimination each firm's profits are equal to 0.25 it is easy to see that price discrimination is always bad for profits. And, it is even worse when both firms have the same information. Of course this result is not a surprise. The existing literature is all based on public information (e.g. Thisse and Vives (1988), Chen (1997), Fudenberg and Tirole (2000), to name a few) and a common finding is that price discrimination leads in general to a prisoner's dilemma. The same happens in the present model.

Figure 14: Expected aggregated profit



On the basis of the previous corollary we may establish the following result which sheds some light on the incentives firms could have to share their private information with their competitors.

Corollary 7. (Information sharing) *Under symmetry a firm would have no incentive to exchange its private consumer-specific information with its competitor.*

Intuitively, the model seems to suggest that when all firms exchange their information with their competitors for price discrimination purposes they become worse off. In this way, any regulatory policy restricting firms to disclose private information about their customers to other firms would benefit firms' profitability at the expense of consumer welfare.

2.8.3 Comparison between the expected number of *inefficient shoppers*

Lemma 3 The expected number of *inefficient shoppers* under public information denoted by EIS^{pub} is given by:

$$EIS^{pub} = \left(\gamma_L^{N,pub} - \gamma_L^{pub}\right) = \frac{1}{2}z + bz^2 + \frac{2}{3}b^2z^3. \tag{2.48}$$

Proof. See the Appendix.

Corollary 8. *The number of customers that buy the wrong brand is higher under public than under private information, and it is increasingly higher the more informative is the signal.*

Proof. See the Appendix.

This result claims that the degree of inefficiency is greater in the public information than in the private information setting. The intuition is as follows. On the one hand, because price discrimination leads to more intense competition and to lower prices to consumers recognised as disloyal in the public information case, it is obvious that those consumers that are not extremely loyal to one of the brands (i.e. those located in the middle) will have more incentives to buy the wrong brand under public information. On the other hand, the groups of consumers that generate inefficiency in both information frameworks are those who generate two identical signals as in this case they will receive a different price from the two firms. (Notice that consumers who generate different signals will always buy efficiently under public and under private information because they will receive the same price from the two firms.) So it is clear that the level of inefficiency is greater in the public information than in the private information game.

2.8.4 Comparison between overall welfare

Lemma 4. Expected consumer surplus with public information denoted as ECS^{pub} is given by

$$ECS^{pub} = v - p_{NB}^{pub}\gamma_{NB}^{pub} - p_L^{pub}\gamma_L^{pub} - p_D^{pub}\gamma_D^{pub} - \frac{1}{4}z^2 - \frac{2}{3}bz^3 - \frac{1}{2}b^2z^4.$$

Proof. See the Appendix.

Lemma 5. Expected welfare with public information denoted as EW^{pub} is equal to

$$EW^{pub} = v - \frac{1}{4}z^2 - \frac{2}{3}bz^3 - \frac{1}{2}b^2z^4. \tag{2.49}$$

Proof. See the Appendix.

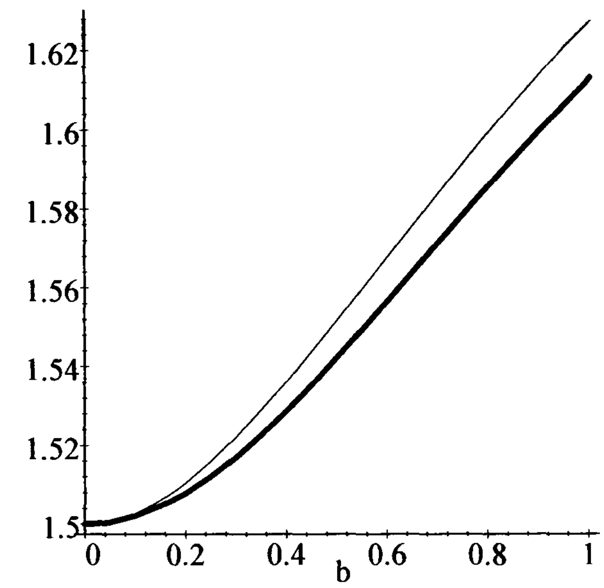
The next proposition summarizes the welfare effects of price discrimination with public information.

Proposition 6. *For any level of the signal's accuracy, moving from private to public information has the following effects: (i) consumer surplus increases; (ii) profits fall, and (iii) welfare falls.*

Proof. See the Appendix for the proof of (iii). Part (i) is proved graphically.

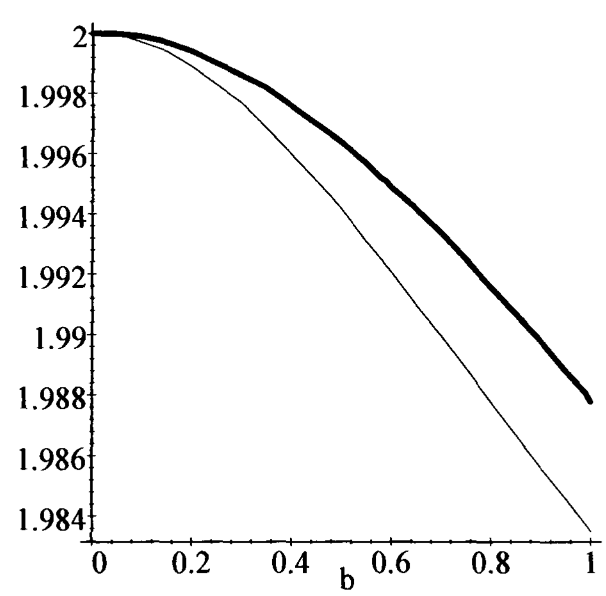
Figure 15 shows expected consumer surplus with private (bold line) and public information (solid thin line). Likewise Figure 16 shows welfare with private (bold line) and public information (solid thin line). (Again these figures are plotted for the case where $v = 2$.)

Figure 15: Expected consumer surplus



The intuition behind Proposition 6 is straightforward. Because consumers are expected to pay lower equilibrium prices under public information, they are strictly better off when we move from private to public information. By contrast, welfare moves in an opposite direction as well as profits. Lower prices lead to more inefficient shopping which clearly reduces overall welfare when moving from private to public information.

Figure 16: Expected welfare



To summarise, allowing firms to have access to public information or, equivalently, to exchange consumer-specific information for price discrimination purposes, intensifies price competition in the marketplace which is good for consumers although bad for profits and overall welfare. Regarding consumers, they are able to buy products at lower prices when firms have very accurate information about them and, moreover, when all firms have the same information. As far as firms is concerned, we realise that they do not benefit from being able to recognise customers more perfectly and, further, they become even worse off when the rival observes the same piece of information. This strengthens the conclusion that in competitive markets, apart from other issues underpinning privacy concerns, allowing firms to recognise customers more accurately and to collect and use consumer-specific information for price discrimination purposes may in fact be in the best interest of consumers. Conversely, our analysis puts forward that it is not in the best interest of firms to improve their ability to recognise consumers, at least when both firms follow the same strategy. Further, when firms have access to private information about customers, they should always guard it and never share their information with their competitors.

The previous literature on behaviour-based price discrimination has been based on the following premises: (i) each firm has *accurate* information on whether a customer prefers its brand or the rival's (i.e. there is no misrecognition of consumers); and (ii) firms have the *same* information (i.e. there is no uncertainty about the rival's information for segmentation purposes). Models based on these assumptions and on unit demands have found that when each firm's weak market is the rival's strong market, price discrimination gives merely rise to inefficient switching thereby leading welfare to fall down (e.g. Chen (1997) and Fudenberg and Tirole (2000)).

The novelty of our analysis is that it offers some criterion to assess *how* welfare evolves as price discrimination is based on more precise information as well as on private rather than on public information. Paradoxically, we have found that welfare is unambiguously greater when price discrimination is based on private information and also on more inaccurate information. Obviously, we need to be extremely careful in drawing any public policy on the basis of this result. If aggregate output increased with either more accurate information or when moving from private to public information, then the effects of such changes would need to be taken into account. Nevertheless, we believe that the model addressed in this chapter is useful as it provides clear intuitions about the profit, consumer surplus and welfare effects of price discrimination based on different information structures.

2.9 Privacy issues

In recent years advances in information technologies have provided unprecedented opportunities for the collection and sharing of information about consumers. As a result of that, privacy issues have increasingly attracted the attention of businesses, regulators, politicians and privacy advocates. In the context of e-commerce markets, in order to respond to consumers' concerns about privacy, many Web sites are nowadays posting privacy policies that describe how consumers' personal information is collected, used, shared and secured.[11]

The *Federal Trade Commission* has also launched consumer privacy initiatives where the focus has basically been on rights over the collection, storage, exchange and sale of personal information.[12] Further, the FTC has held public workshops to explore how businesses merge and exchange detailed consumer information—i.e., compilations of identifying information, preference information, purchasing habits, and other information related to a particular consumer—and how such information is used commercially. A key part of the FTC's privacy program is "...making sure companies keep the promises they make to consumers about privacy and, in particular, the precautions they take to secure consumers' personal information." For instance, when *Toysmart.com* went to bankrupt, the FTC prevented that firm from selling its databases on consumer information collected under the company's privacy policy establishing that "...your information is never shared with a third party."[13]

Our analysis has put forward that it is, indeed, in the interest of firms, firstly, not to share their private information with their competitors and, secondly, to hon-

[11] On *Amazon*'s webpage the reader can confirm that its Privacy Notice currently states that "...information about our customers is an important part of our business, and we are not in the business of selling it."

[12] See *www.ftc.gov/privacy/index.html*

[13] See *www.ftc.gov/os/2000/07/toyexh1.pdf.*

our promises established in their privacy policies regarding the exchange of consumer-specific information. In addition, the present model has stressed that when personal information about consumers is used for price discrimination purposes, any public policy restricting information sharing would act in favour of firms and welfare but against consumers.[14] Nevertheless, it is fair to say that any conclusion about consumer privacy and proprietary rights of consumer information and how this information should be used is far more complex than our approach.

2.10 Conclusions

This chapter has tried to provide a more complete picture of the prices, profits and social welfare effects of price discrimination as information technologies gradually improve firms' ability to recognise different types of consumers. Our main innovation was to extend the previous literature by allowing firms to price discriminate on the basis of imperfect and private information. We have seen that while imperfect information tends to lead firms to misrecognise customers, private information gives rise to some uncertainty about the rival's information for price discrimination. Besides a customer may be wrongly recognised by some firm, he can also be recognised in a different way by the two firms. In this way, it was shown that the economic implications of price discrimination are in fact affected by the misrecognition of customers as well as by the inability of firms to perfectly predict how the rival is segmenting customers.

In the private information setting, it was shown that customers recognised as loyal pay *always* a higher price than customers recognised as disloyal. This result is in consonance with *Amazon*'s practice of charging more to old than to first-time customers. It was also shown that customers recognised as loyal are expected to pay prices above the non-discrimination level when information is not too accurate while they are expected to pay prices below their non-discrimination counterparts when each firm relies

[14] In March 2001 the FTC held a workshop on consumer profiling and data exchange. The Comission's own survey results indicate that 82% of respondents agreed that the government should regulate how online sellers use personal information.

on very precise information. By contrast, it was found that customers recognised as disloyal are expected to pay lower prices under price discrimination, and increasingly lower prices the more accurate is each firm's signal. Basically, the model addressed in this chapter has stressed that as the firms' ability to recognise customers gradually improves every consumer is expected to face lower prices as a result of more aggressive behaviour.

In light of the above it was shown that price discrimination is good for consumers, but bad for profits and welfare. Moreover, it was shown that the better is the quality of each firm's information, the greater is the consumer surplus and the lower are profits and overall welfare. The model has put forward that collectively it is not in the interest of firms to improve the quality of their information about customers, even though individually a firm might prefer to do so. Remarkably, we have seen that any public policy protecting consumer privacy, by restricting customer recognition, would benefit all competing firms at the expense of consumer welfare.

This chapter has also analysed price discrimination under public information. The aim was basically to offer some intuition about the competitive effects of price discrimination when firms observe the same information about consumers. Extending the model in this direction has proved to be helpful to understand whether or not firms might have an incentive to share their private information with their rivals.

From the comparison of the equilibrium outcomes with private and public information we have found that a consumer recognised as disloyal is expected to pay a lower price under public information and the same is true for a consumer recognised as loyal when the signal's precision is sufficiently high. As a result of that, consumers are expected to become better off under public than under private information. This is the result of more intense competition when departing from private to public information. Of course, profits are lower under public information. Because more consumers are expected to buy the "wrong" brand under public information (due to lower prices), it was shown that moving from private to public information leads welfare to fall.

Therefore, our analysis has advocated that under symmetry it is not in the interest of firms to share their private information with their competitors. This in turn means that it is in the interest of firms honouring their privacy notices promising not to share data they have been collecting. On the contrary, this model has strengthened the idea that it is in the best interest of consumers to allow firms to recognise them (more) accurately. Interestingly, any regulation trying to restrict information sharing would most likely benefit firms' profitability and overall welfare but again at the expense of consumers.

In conclusion, notwithstanding the model addressed in this chapter is far from covering all complex aspects of real markets, it has tried to offer a closer approximation of reality where the quality of consumer-specific information that firms have been using to implement their pricing strategies is increasingly improving thanks to advances in information technologies. Albeit the results derived in this chapter should be interpreted carefully, we believe that the analysis was worth pursuing as it provides clear intuitions about the profit, consumer surplus and welfare effects of price discrimination based on different information structures.

2.11 Appendix

This appendix collects the proofs and computations that were omitted from the text.

Proof of Lemma 1.

If $\rho_\alpha \geqslant \rho_\beta$ one must observe that:

$$\begin{aligned} 2\lambda_{\alpha\alpha} &\geq 2\lambda_{\beta\alpha} \\ \frac{1}{4}+\frac{b^2}{12} &\geq \frac{1}{4}-\frac{b^2}{12} \\ b &\geq 0 \end{aligned}$$

which is true $\forall b \in [0,1]$. *Q.E.D.*

Proof of Proposition 1.

From the first-order conditions for both firms we obtain:

$$p_\alpha^A = \frac{\rho_\alpha \left[1 - G_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right)\right] + \left(1-\rho_\alpha\right)\left[1 - G_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right)\right]}{\rho_\alpha g_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) + \left(1-\rho_\alpha\right) g_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right)}, \tag{2.50}$$

$$p_\beta^A = \frac{\rho_\beta \left[1 - G_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right)\right] + \left(1-\rho_\beta\right)\left[1 - G_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)\right]}{\left[\rho_\beta g_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right) + \left(1-\rho_\beta\right) g_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)\right]}. \tag{2.51}$$

$$p_\alpha^B = \frac{\rho_\alpha G_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) + \left(1-\rho_\alpha\right) G_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right)}{\rho_\alpha g_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) + \left(1-\rho_\alpha\right) g_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right)} \tag{2.52}$$

and

$$p_\beta^B = \frac{\rho_\beta G_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right) + \left(1-\rho_\beta\right) G_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)}{\rho_\beta g_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right) + \left(1-\rho_\beta\right) g_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)}. \tag{2.53}$$

Considering for instance the perspective of firm A, second-order partial derivatives with respect to both prices are

$$\begin{aligned}\frac{\partial^2 E\left(\pi^A \mid s_A = \alpha\right)}{\partial p_\alpha^{A2}} &= -2\rho_\alpha g_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) - 2\left(1-\rho_\alpha\right) g_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right) \\ &\quad -p_\alpha^A \rho_\alpha g'_{\alpha\alpha}\left(p_\alpha^A - p_\alpha^B\right) - p_\alpha^A\left(1-\rho_\alpha\right) g'_{\alpha\beta}\left(p_\alpha^A - p_\beta^B\right),\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 E\left(\pi^A \mid s_A = \beta\right)}{\partial p_\beta^{A2}} &= -2\rho_\beta g_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right) - 2\left(1-\rho_\beta\right) g_{\beta\beta}\left(p_\beta^A - p_\beta^B\right) \\ &\quad -p_\beta^A \rho_\beta g'_{\beta\alpha}\left(p_\beta^A - p_\alpha^B\right) - p_\beta^A\left(1-\rho_\beta\right) g'_{\beta\beta}\left(p_\beta^A - p_\beta^B\right)\end{aligned}$$

and,

$$\frac{\partial^2 E\left(\pi^A \mid s_A = \alpha\right)}{\partial p_\alpha^A \partial p_\beta^A} = 0.$$

Symmetric conditions hold for firm B. Due to symmetry we are looking for an equilibrium where $p_\alpha^A = p_\beta^B = p_L$ and $p_\beta^A = p_\alpha^B = p_D$. Thus, making $y = p_L - p_D$, and using without loss of generality the perspective of firm A, one gets:

$$p_L = p_A^\alpha = \frac{\rho_\alpha\left[1 - G_{\alpha\alpha}(y)\right] + \left(1-\rho_\alpha\right)\left[1 - G_{\alpha\beta}(0)\right]}{\rho_\alpha g_{\alpha\alpha}(y) + \left(1-\rho_\alpha\right) g_{\alpha\beta}(0)} \tag{2.54}$$

$$p_D = p_\beta^A = \frac{\rho_\beta \left[1 - G_{\beta\alpha}(0)\right] + \left(1-\rho_\beta\right)\left[1 - G_{\beta\beta}(-y)\right]}{\left[\rho_\beta g_{\beta\alpha}(0) + \left(1-\rho_\beta\right) g_{\beta\beta}(-y)\right]}. \tag{2.55}$$

Then second-order partial derivatives for firm A are then equal to:

$$\frac{\partial^2 E\left(\pi^A \mid s_A = \alpha\right)}{\partial p_\alpha^{A2}} = -2\rho_\alpha g_{\alpha\alpha}(y) - 2\left(1-\rho_\alpha\right) g_{\alpha\beta}(0) - p_L \rho_\alpha g'_{\alpha\alpha}(y) - p_L\left(1-\rho_\alpha\right) g'_{\alpha\beta}(0)$$

and

$$\frac{\partial^2 E\left(\pi^A \mid s_A = \beta\right)}{\partial p_\beta^{A2}} = -2\rho_\beta g_{\beta\alpha}(0) - 2\left(1-\rho_\beta\right) g_{\beta\beta}(-y) - p_D \rho_\beta g'_{\beta\alpha}(0) - p_D\left(1-\rho_\beta\right) g'_{\beta\beta}(-y).$$

Since,

$$\begin{aligned}
1 - G_{\alpha\alpha}(y) &= \frac{1}{\lambda_{\alpha\alpha}} \int_y^{\frac{1}{2}} \left(\frac{1}{2} + bl\right)^2 dl \\
&= \frac{1}{\lambda_{\alpha\alpha}} \left(\frac{1}{8} + \frac{1}{8}b + \frac{1}{24}b^2 - \frac{1}{4}y - \frac{1}{2}by^2 - \frac{1}{3}b^2y^3\right),
\end{aligned}$$

$$\begin{aligned}
1 - G_{\alpha\beta}(0) &= \frac{1}{\lambda_{\alpha\beta}} \int_0^{\frac{1}{2}} \left(\frac{1}{2} + bl\right)\left(\frac{1}{2} - bl\right) dl \\
&= \frac{1}{\lambda_{\alpha\beta}} \left(\frac{1}{8} - \frac{1}{24}b^2\right),
\end{aligned}$$

$$g_{\alpha\alpha}(y) = \frac{1}{\lambda_{\alpha\alpha}} \left(\frac{1}{2} + by\right)^2,$$

$$g_{\alpha\beta}(0) = \frac{1}{\lambda_{\alpha\beta}} \left(\frac{1}{4}\right),$$

$$\begin{aligned}
1 - G_{\beta\beta}(-y) &= \frac{1}{\lambda_{\beta\beta}} \int_{-y}^{\frac{1}{2}} \left(\frac{1}{2} - bl\right)^2 dl \\
&= \frac{1}{\lambda_{\beta\beta}} \left(\frac{1}{8} - \frac{1}{8}b + \frac{1}{24}b^2 + \frac{1}{4}y + \frac{1}{2}by^2 + \frac{1}{3}b^2y^3\right),
\end{aligned}$$

$$
\begin{aligned}
1 - G_{\beta\alpha}(0) &= \frac{1}{\lambda_{\beta\alpha}} \int_0^{\frac{1}{2}} \left(\frac{1}{4} - b^2 l^2\right) dl \\
&= \frac{1}{\lambda_{\beta\alpha}} \left(\frac{1}{8} - \frac{1}{24} b^2\right),
\end{aligned}
$$

$$
g_{\beta\alpha}(0) = \frac{1}{\lambda_{\beta\alpha}} \left(\frac{1}{4}\right),
$$

and

$$
g_{\beta\beta}(-y) = \frac{1}{\lambda_{\beta\beta}} \left(\frac{1}{2} + by\right)^2.
$$

It is now easy to check that second-order conditions for a maximum are satisfied. Using the expressions derived above and the fact that

$$
g'_{\alpha\alpha}(y) = \frac{2b}{\lambda_{\alpha\alpha}} \left(\frac{1}{2} + by\right)
$$

$$
g'_{\alpha\beta}(0) = g'_{\beta\alpha}(0) = 0
$$

$$
g'_{\beta\beta}(-y) = -\frac{2b}{\lambda_{\beta\beta}} \left(\frac{1}{2} + by\right)
$$

it is straightforward to observe that:

$$
\begin{aligned}
\frac{\partial^2 E\left(\pi^A \mid s_A = \alpha\right)}{\partial p_\alpha^{A2}} &= -2\rho_\alpha g_{\alpha\alpha}(y) - 2\left(1 - \rho_\alpha\right) g_{\alpha\beta}(0) - p_L \rho_\alpha g'_{\alpha\alpha}(y) \\
&= -4\lambda_{\alpha\alpha} \frac{1}{\lambda_{\alpha\alpha}} \left(\frac{1}{2} + by\right)^2 - 4\left(\lambda_{\alpha\beta}\right) \frac{1}{\lambda_{\alpha\beta}} \left(\frac{1}{4}\right) - p_L 2\lambda_{\alpha\alpha} \frac{2b}{\lambda_{\alpha\alpha}} \left(\frac{1}{2} + by\right) \\
&= -4\left(\frac{1}{2} + by\right)^2 - 1 - 4bp_L \left(\frac{1}{2} + by\right) < 0.
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial^2 E\left(\pi^A \mid s_A = \beta\right)}{\partial p_\beta^{A2}} &= -2\rho_\beta g_{\beta\alpha}(0) - 2\left(1 - \rho_\beta\right) g_{\beta\beta}(-y) - p_D \left(1 - \rho_\beta\right) g'_{\beta\beta}(-y) \\
&= -4\lambda_{\beta\alpha} \frac{1}{\lambda_{\beta\alpha}} \left(\frac{1}{4}\right) - 4\lambda_{\beta\beta} \frac{1}{\lambda_{\beta\beta}} \left(\frac{1}{2} + by\right)^2 - p_D \left(2\lambda_{\beta\beta}\right) \left(-\frac{2b}{\lambda_{\beta\beta}} \left(\frac{1}{2} + by\right)\right.
\end{aligned}
$$

$$= -1 - 4\left(\frac{1}{2} + by\right)^2 + 4bp_D\left(\frac{1}{2} + by\right)$$
$$= -1 - 4\left(\frac{1}{2} + by\right)\left(\frac{1}{2} + b\left(y - p_D\right)\right).$$

Despite the fact that $b\left(y - p_D\right) < 0$ we find that for $0 \leq b \leq 1$, $0.41 \leq \frac{1}{2} + b\left(y - p_D\right) \leq 0.5$ thus $\frac{\partial^2 E\left(\pi^A | s_A = \beta\right)}{\partial p_\beta^{A2}} < 0$. In sum, since $\frac{\partial^2 E\left(\pi^A | s_A = \alpha\right)}{\partial p_\alpha^{A2}} < 0$ and $\left(\frac{\partial^2 E\left(\pi^A | s_A = \alpha\right)}{\partial p_\alpha^{A2}}\right)\left(\frac{\partial^2 E\left(\pi^A | s_A = \beta\right)}{\partial p_\beta^{A2}}\right) - \left(\frac{\partial^2 E\left(\pi^A | s_A = \alpha\right)}{\partial p_\alpha^A \partial p_\beta^A}\right)^2 > 0$ it follows that second-order conditions for a maximum are as well satisfied.

After some algebra, we find that the prices firms set after observing a loyal and a disloyal signal are, respectively, as follows:

$$p_L = \frac{\frac{1}{4} + \frac{1}{8}b - \frac{1}{4}y - \frac{1}{2}by^2 - \frac{1}{3}b^2y^3}{\frac{1}{2} + by + b^2y^2} \tag{2.56}$$

$$p_D = \frac{\frac{1}{4} - \frac{1}{8}b + \frac{1}{4}y + \frac{1}{2}by^2 + \frac{1}{3}b^2y^3}{\frac{1}{2} + by + b^2y^2}. \tag{2.57}$$

Q.E.D.

Proof of Lemma 2.

For each individual consumer with brand loyalty parameter $l \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ each firm observes a binary signal. Representing by $(s_A = i, s_B = j)$ the signal observed by the two firms for a particular consumer, where $i, j \in \{\alpha, \beta\}$, we may analyse four possible scenarios with respective equilibrium prices:

$(s_A = i, s_B = j)$	$\left(p_i^A, p_j^B\right)$	$\Pr(s_A = i, s_B = j \mid l)$
$(s_A = \alpha, s_B = \alpha)$	(p_L, p_D)	$\Pr(s_A = \alpha, s_B = \alpha \mid l) = (q(l))^2$
$(s_A = \alpha, s_B = \beta)$	(p_L, p_L)	$\Pr(s_A = \alpha, s_B = \beta \mid l) = q(l)\,(1 - q(l))$
$(s_A = \beta, s_B = \alpha)$	(p_D, p_D)	$\Pr(s_A = \beta, s_B = \alpha \mid l) = (1 - q(l))\,q(l)$
$(s_A = \beta, s_B = \beta)$	(p_D, p_L)	$\Pr(s_A = \beta, s_B = \beta \mid l) = (1 - q(l))^2$

Therefore,

$$ECS = (v - p_L)\int_{-\frac{1}{2}}^{\frac{1}{2}} (1 - q(l))\,q(l)f(l)dl + (v - p_D)\int_{-\frac{1}{2}}^{\frac{1}{2}} (1 - q(l))\,q(l)f(l)dl$$

$$
\begin{aligned}
&+\int_0^{\frac{1}{2}} \max\{v-p_L, v-p_D-l\}\,(q(l))^2 f(l)dl \\
&+(v-p_D)\int_{-\frac{1}{2}}^{0} (q(l))^2 f(l)dl \\
&\int_{-\frac{1}{2}}^{0} \max\{v-p_L, v-p_D+l\}\,(1-q(l))^2 f(l)dl \\
&+(v-p_D)\int_0^{\frac{1}{2}} (1-q(l))^2 f(l)dl.
\end{aligned}
$$

After some algebra and using the fact that $y = p_L - p_D$, one finds that

$$
\begin{aligned}
ECS \;=\;& (v-p_L)\left(\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(\frac{1}{2}-bl\right)\left(\frac{1}{2}+bl\right)dl+\int_y^{\frac{1}{2}}\left(\frac{1}{2}+bl\right)^2 dl+\int_{-\frac{1}{2}}^{-y}\left(\frac{1}{2}-bl\right)^2 dl\right) \\
&+(v-p_D)\left(\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(\frac{1}{2}-bl\right)\left(\frac{1}{2}+bl\right)dl+\int_{-\frac{1}{2}}^{y}\left(\frac{1}{2}+bl\right)^2 dl+\int_{-y}^{\frac{1}{2}}\left(\frac{1}{2}-bl\right)^2 dl\right) \\
&-\int_0^y l\left(\frac{1}{2}+bl\right)^2 dl+\int_{-y}^{0} l\left(\frac{1}{2}-bl\right)^2 dl,
\end{aligned}
$$

from which we get

$$
\begin{aligned}
ECS \;=\;& v-p_L\left(\frac{1}{2}+\frac{1}{4}b-\frac{1}{2}y-by^2-\frac{2}{3}b^2y^3\right)-p_D\left(\frac{1}{2}-\frac{1}{4}b+\frac{1}{2}y+by^2+\frac{2}{3}b^2y^3\right) \\
&-\frac{1}{4}y^2-\frac{2}{3}by^3-\frac{1}{2}b^2y^4.
\end{aligned}
$$

Q.E.D.

Proof of Proposition 4.

Given that firms may observe four different types of signals, upon observing signal $s=(i,j)$, being price discrimination allowed each firm chooses simultaneously a different price for the signal observed. Hence, firm i chooses $p_s^i \in \{p_{\alpha\alpha}^i, p_{\beta\beta}^i, p_{\alpha\beta}^i, p_{\beta\alpha}^i\}$, $i = A, B$. These prices are the solution to the following problem

$$
\underset{p_s^i \geq 0}{Max}\; E\left[\pi^i \mid s\right], \tag{2.58}
$$

Considering for example, the perspective of firm A, first-order conditions for firm A are:

$$\frac{\partial E\left(\pi^{A}\mid s=(\alpha,\alpha)\right)}{\partial p_{\alpha\alpha}^{A}}=\left[1-G_{\alpha\alpha}\left(p_{\alpha\alpha}^{A}-p_{\alpha\alpha}^{B}\right)\right]-\left(p_{\alpha\alpha}^{A}\right)g_{\alpha\alpha}\left(p_{\alpha\alpha}^{A}-p_{\alpha\alpha}^{B}\right)=0,$$

$$\frac{\partial E\left(\pi^{A}\mid s=(\beta,\beta)\right)}{\partial p_{\beta\beta}^{A}}=\left[1-G_{\beta\beta}\left(p_{\beta\beta}^{A}-p_{\beta\beta}^{B}\right)\right]-\left(p_{\beta\beta}^{A}\right)g_{\beta\beta}\left(p_{\beta\beta}^{A}-p_{\beta\beta}^{B}\right)=0,$$

$$\frac{\partial E\left(\pi^{A}\mid s=(\alpha,\beta)\right)}{\partial p_{\alpha\beta}^{A}}=\left[1-G_{\alpha\beta}\left(p_{\alpha\beta}^{A}-p_{\alpha\beta}^{B}\right)\right]-\left(p_{\alpha\beta}^{A}\right)g_{\alpha\beta}\left(p_{\alpha\beta}^{A}-p_{\alpha\beta}^{B}\right)=0,$$

$$\frac{\partial E\left(\pi^{A}\mid s=(\beta,\alpha)\right)}{\partial p_{\beta\alpha}^{A}}=\left[1-G_{\beta\alpha}\left(p_{\beta\alpha}^{A}-p_{\beta\alpha}^{B}\right)\right]-\left(p_{\beta\alpha}^{A}\right)g_{\beta\alpha}\left(p_{\beta\alpha}^{A}-p_{\beta\alpha}^{B}\right)=0.$$

Second-order conditions are as well satisfied.

Due to symmetry we are looking for an equilibrium where $p_{\alpha\alpha}^{A}=p_{\beta\beta}^{B}$, $p_{\beta\beta}^{A}=p_{\alpha\alpha}^{B}$ and $p_{\alpha\beta}^{A}=p_{\alpha\beta}^{B}=p_{\beta\alpha}^{A}=p_{\beta\alpha}^{B}$. Making $z=p_{\alpha\alpha}^{A}-p_{\alpha\alpha}^{B}$ we have

$$\left[1-G_{\alpha\alpha}(z)\right]-\left(p_{\alpha\alpha}^{A}\right)g_{\alpha\alpha}(z)=0$$

from which it follows:

$$p_{\alpha\alpha}^{A}=\frac{1-G_{\alpha\alpha}(z)}{g_{\alpha\alpha}(z)}. \tag{2.59}$$

We also have

$$\left[1-G_{\beta\beta}(-z)\right]-\left(p_{\beta\beta}^{A}\right)g_{\beta\beta}(-z)=0$$

from which we obtain

$$p_{\beta\beta}^{A}=\frac{1-G_{\beta\beta}(-z)}{g_{\beta\beta}(-z)}. \tag{2.60}$$

Finally from

$$\left[1-G_{\alpha\beta}(0)\right]-\left(p_{\alpha\beta}^{A}\right)g_{\alpha\beta}(0)=0$$

we get

$$p_{\alpha\beta}^{A}=p_{\beta\alpha}^{A}=\frac{1-G_{\alpha\beta}(0)}{g_{\alpha\beta}(0)}. \tag{2.61}$$

As before,

$$1-G_{\alpha\alpha}(z)=\frac{1}{\lambda_{\alpha\alpha}}\left(\frac{1}{8}+\frac{1}{8}b+\frac{1}{24}b^2-\frac{1}{4}z-\frac{1}{2}bz^2-\frac{1}{3}b^2z^3\right),$$

$$1-G_{\alpha\beta}(0)=\frac{1}{\lambda_{\alpha\beta}}\left(\frac{1}{8}-\frac{1}{24}b^2\right),$$

$$g_{\alpha\alpha}(z)=\frac{1}{\lambda_{\alpha\alpha}}\left(\left(\frac{1}{2}+bz\right)^2\right),$$

$$g_{\alpha\beta}(0)=\frac{1}{\lambda_{\alpha\beta}}\left(\frac{1}{4}\right),$$

$$1-G_{\beta\beta}(-z)=\frac{1}{\lambda_{\beta\beta}}\left(\frac{1}{8}-\frac{1}{8}b+\frac{1}{24}b^2+\frac{1}{4}z+\frac{1}{2}bz^2+\frac{1}{3}b^2z^3\right),$$

$$1-G_{\beta\alpha}(0)=\frac{1}{\lambda_{\beta\alpha}}\left(\frac{1}{8}-\frac{1}{24}b^2\right),$$

$$g_{\beta\alpha}(0)=\frac{1}{\lambda_{\beta\alpha}}\left(\frac{1}{4}\right),$$

and

$$g_{\beta\beta}(-z)=\frac{1}{\lambda_{\beta\beta}}\left(\left(\frac{1}{2}+bz\right)^2\right),$$

Rearranging and using the fact $p_L^{pub}=p_{\alpha\alpha}^A=p_{\beta\beta}^B$ and $p_D^{pub}=p_{\beta\beta}^A=p_{\alpha\alpha}^B$ we find that:

$$p_L^{pub}=\frac{\frac{1}{8}+\frac{1}{8}b+\frac{1}{24}b^2-\frac{1}{4}z-\frac{1}{2}bz^2-\frac{1}{3}b^2z^3}{\left(\frac{1}{2}+bz\right)^2},\tag{2.62}$$

$$p_D^{pub}=\frac{\frac{1}{8}-\frac{1}{8}b+\frac{1}{24}b^2+\frac{1}{4}z+\frac{1}{2}bz^2+\frac{1}{3}b^2z^3}{\left(\frac{1}{2}+bz\right)^2},\tag{2.63}$$

$$p_{NB}^{pub}=\frac{1}{2}-\frac{1}{6}b^2.\tag{2.64}$$

Q.E.D.

Proof of Lemma 3.

Due to the normalisation of consumers to unit, the expected number of consumers that buy the product at the highest price under public information (p_L^{pub}) is given by,

$$\begin{aligned}\gamma_L^{pub} &= \Pr(A \text{ wins} \mid s = (\alpha,\alpha)) \Pr(s = (\alpha,\alpha)) + \Pr(B \text{ wins} \mid s = (\beta,\beta)) \Pr((s = (\beta,\beta)) \\ &= \frac{1}{4} + \frac{1}{4}b + \frac{1}{12}b^2 - \frac{1}{2}z - bz^2 - \frac{2}{3}b^2 z^3.\end{aligned}$$

Similarly, the expected number of consumers that buy the product at the lowest price (p_D^{pub}) is given by

$$\begin{aligned}\gamma_D^{pub} &= \Pr(A \text{ wins} \mid s = (\beta,\beta)) \Pr(s = (\beta,\beta)) + \Pr(B \text{ wins} \mid s = (\alpha,\alpha)) \Pr(s = (\alpha,\alpha) \\ &= \frac{1}{4} - \frac{1}{4}b + \frac{1}{12}b^2 + \frac{1}{2}z + bz^2 + \frac{2}{3}b^2 z^3,\end{aligned}$$

and the number of consumers that pay the non-biased price equals

$$\begin{aligned}\gamma_{NB}^{pub} &= \Pr(A \text{ wins} \mid s = (\alpha,\beta)) \Pr(s = (\alpha,\beta)) \\ + \Pr(A \text{ wins} &\mid s = (\beta,\alpha)) \Pr(s = (\beta,\alpha)) + \Pr(B \text{ wins} \mid s = (\alpha,\beta)) \Pr(s = (\alpha,\beta)) \\ + \Pr(B \text{ wins} &\mid s = (\beta,\alpha)) \Pr(s = (\beta,\alpha)) = \frac{1}{2} - \frac{1}{6}b^2.\end{aligned}$$

Under non-discrimination the expected number of consumers that pay p_L^{pub}, p_D^{pub} and p_{NB}^{pub} is

$$\gamma_L^{N,pub} = \frac{1}{4} + \frac{1}{4}b + \frac{1}{12}b^2,$$

$$\gamma_D^{N,pub} = \frac{1}{4} - \frac{1}{4}b + \frac{1}{12}b^2,$$

$$\gamma_{NB}^{N,pub} = \frac{1}{2} - \frac{1}{6}b^2.$$

Thus, the expected number of *inefficient shoppers* under public information denoted by EIS^{pub} is given by:

$$EIS^{pub} = \left(\gamma_L^{N,pub} - \gamma_L^{pub}\right) = \frac{1}{2}z + bz^2 + \frac{2}{3}b^2z^3. \; Q.E.D. \tag{2.65}$$

Proof of Corollary 8.

Using (2.34) and (2.48) and the fact that $z > y$ it follows that

$$EIS^{pub} - EIS = \frac{1}{2}(z-y) + b\left(z^2 - y^2\right) + \frac{2}{3}b^2\left(z^3 - y^3\right) > 0.$$

Q.E.D.

Proof of Lemma 4.

When the signal observed by firms to each customer is public we may analyse the following scenarios:

$s = (i,j)$	(p_i^A, p_j^B)	$\Pr(s = (i,j) \mid l)$
$s = (\alpha,\alpha)$	$\left(p_L^{pub}, p_D^{pub}\right)$	$\Pr(s = (\alpha,\alpha) \mid l) = (q(l))^2$
$s = (\alpha,\beta)$	$\left(p_{NB}^{pub}, p_{NB}^{pub}\right)$	$\Pr(s = (\alpha,\beta) \mid l) = q(l)(1-q(l))$
$s = (\beta,\beta)$	$\left(p_D^{pub}, p_L^{pub}\right)$	$\Pr(s = (\beta,\alpha) \mid l) = (1-q(l))^2$
$s = (\beta,\alpha)$	$\left(p_{NB}^{pub}, p_{NB}^{pub}\right)$	$\Pr(s = (\beta,\beta) \mid l) = (1-q(l))\,q(l)$

Therefore, expected consumer surplus under public information, denoted by ECS^{pub} is

$$\begin{aligned} ECS \;=\; & \left(v - p_{NB}^{pub}\right)\int_{-\frac{1}{2}}^{\frac{1}{2}} (1-q(l))\,q(l)f(l)dl + \int_0^{\frac{1}{2}} \max\left(v - p_L^{pub}, v - p_D^{pub} - l\right)(q(l))^2 f(l)dl \\ & + \left(v - p_D^{pub}\right)\int_{-\frac{1}{2}}^{0} (q(l))^2 f(l)dl + \int_{-\frac{1}{2}}^{0} \max\left(v - p_L^{pub}, v - p_D^{pub} + l\right)(1-q(l))^2 f(l)dl \\ & + \left(v - p_D^{pub}\right)\int_0^{\frac{1}{2}} (1-q(l))^2 f(l)dl. \end{aligned}$$

Using previous computations and using the fact that $z = p_L^{pub} - p_D^{pub}$, after some algebra:

$$ECS^{pub} = v - p_{NB}^{pub}\gamma_{NB}^{pub} - p_L^{pub}\gamma_L^{pub} - p_D^{pub}\gamma_D^{pub} - \frac{1}{4}z^2 - \frac{2}{3}bz^3 - \frac{1}{2}b^2z^4. \tag{2.66}$$

Q.E.D.

Proof of Lemma 5.

Given that welfare is equal to industry profit plus consumer surplus one gets,

$$\begin{aligned} EW^{pub} &= p_L^{pub}\gamma_L^{pub} + p_D^{pub}\gamma_D^{pub} + p_{NB}^{pub}\gamma_{NB}^{pub} \\ &\quad + v - p_{NB}^{pub}\gamma_{NB}^{pub} - p_L^{pub}\gamma_L^{pub} - p_D^{pub}\gamma_D^{pub} - \frac{1}{4}z^2 - \frac{2}{3}bz^3 - \frac{1}{2}b^2z^4. \end{aligned}$$

Thus,

$$EW^{pub} = v - \frac{1}{4}z^2 - \frac{2}{3}bz^3 - \frac{1}{2}b^2z^4. \tag{2.67}$$

Q.E.D.

Proof of Proposition 6.

Here we prove analytically that $EW^{pub} < EW$. Using (2.40) and (2.49) and the fact that $z > y$ it follows that

$$EW^{pub} - EW = -\frac{1}{4}\left(z^2 - y^2\right) - \frac{2}{3}b\left(z^3 - y^3\right) - \frac{1}{2}b^2\left(z^4 - y^4\right) < 0.$$

Q.E.D.

Chapter 3

Pricing with Customer Recognition

3.1 Introduction

Beginning with Stigler's (1961) seminal paper, information has been brought to the forefront of the economics world. Up to then information occupied, in Stigler's words a "... slum dwelling in the town of economics" (p. 213). Today, more than ever before, information plays an increasingly important role in all society spheres. In particular, the development of information technologies, in which the Internet is the most prominent, has offered firms and consumers new and easy ways to collect, analyse and use information in their own interests. With a single mouse click consumers can now find easily the products they are looking for and to compare prices. But firms can also now have far more information about their customers, which means that they are increasingly able to recognise customers (non-anonymous consumers), through, for instance, their past behaviour, and to price discriminate accordingly. Considering exclusively the use of consumer past purchasing data for price discrimination purposes a key question arises: *Are the firms or the consumers who will benefit the most?*

Yet related to this question are the following important issues. Is it in the interest of firms to recognise customers and to price discriminate accordingly? Or, is it rather in their interest to avoid learning and price discrimination? Should government regulation be oriented in a way that makes information collection more difficult and restricts price discrimination practices? Or, should consumers make themselves anonymous by using special tools or neglecting firms the access and/or use of their information for pricing?

The purpose of this chapter is, therefore, to contribute to the analysis of these issues. With that in mind, it addresses a stylized model that captures pricing policies found in e-commerce markets. Despite price discrimination based on purchase history has been widely observed in off-line subscription markets—e.g. mobile telephones, long-distance telecommunications and credit cards—it has also been implemented in online markets. In fact, it is well known that the book retailer *Amazon* has been charging different DVDs prices to returning (loyal) and first-time (disloyal) customers. More precisely, this chapter looks at a two-period duopoly model where consumers are either loyal (to a specific degree) to one firm or to the other, and where preferences are fixed across periods. In a recent study Brynjolfsson and Smith (2000b) have found that *Amazon*'s loyal customers are willing to pay around 6.8 percent more before they consider switching to another seller. For that reason, it can be said that there is evidence that in online markets it is likely that consumers' loyalty is limited, which means that though firms may have some advantage over their competitors due to brand loyalty, all consumers may in spite of that be induced to switch. This lends support to the choice of a discrete framework to model consumers' preferences. In the first period firms have not the required information to price discriminate because consumers are anonymous. However, by observing the consumers first period purchasing decisions (whether or not a consumer went to the firm in that period), firms might obtain an accurate *signal* of a consumer's brand loyalty. When a firm learns to which firm each consumer is loyal it will be able to set second period prices accordingly.

In order to measure the dynamic effects of behaviour-based price discrimination in a duopoly, we first analyse the benchmark case where price discrimination is not permitted either because consumers are anonymous or because price discrimination is illegal (Section 3.3.2). Because consumers are loyal to a specific degree to one firm or the other, we show that when price discrimination is for any reason absent the duopoly equilibrium is in mixed strategies, i.e. there is *price dispersion* (Proposition 3).

As Hal Varian (1980) quipped two decades ago, "...the "law of one price" is not law at all" (p. 651). Although in the earliest days of Internet commerce, many scholars predicted that it would lead to a frictionless market, in which prices would converge to the perfect competition levels, several studies have shown that price dispersion is persistent in online markets (Brynjolfsson and Smith (2000a), Baye, Morgan and Sholton (2002), Pan, Ratchford and Shankar (2001), to name few).[1] Brynjolfsson and Smith (2000a) claim that online price dispersion is mainly due to retailers heterogeneity with respect to brand loyalty, trust, and awareness. Thus, this chapter lends support to empirical work that has been explaining price dispersion on the basis of brand loyalty. The reason is that since loyal customers to a firm may after all be induced to switch there is a tension between the firm's incentives to price low, in order to undercut the rival's price—and, serve the entire market—, and to price high and sell exclusively to its loyal segment of customers. This tension coupled with each firm's attempt to preclude the rival from systematically predicting its price results in an equilibrium displaying price dispersion. Additionally, we find that we expect to observe greater levels of price dispersion in product markets where consumers are more loyal.

We then analyse a two-period interaction model where price discrimination can only occur in the second period, if firms become able to recognise customers (Section 3.4). Here we find that if firms set approximately the same first period price, they will share the market in that period and both firms will observe an accurate signal of a consumer's brand loyalty in period 2. In this case, firms will engage in price discrimination in period 2, i.e. they will offer lower prices to those customers who have previously purchased from the rival. Otherwise, if one firm's first period price is much lower than the competitor's price, the market will be entirely served by the low price firm; and price discrimination cannot occur in period 2 since nothing is learned by that period. As expected, we show that second period prices and profits are higher under no discrimination (i.e., under no recognition) than under discrimination. Because

[1] The site *www.nash-equilibrium.com* maintained by Baye, Morgan and Sholton provides both current and historical data on price dispersion in online markets.

one firm's "strong" market (i.e., its loyal segment) is the other's "weak" market (i.e., its disloyal segment) price discrimination would act to intensify competition in each segment, leading all prices and profits to fall. When firms are forward looking we will see that the negative effects of price discrimination will give them an incentive *not* to share the market in the first period (Proposition 6). Obviously, this latter result is closely related to the implications of discrimination enabled by customer recognition on first-period prices (Section 3.5.2). Given that firms try not to share the market in the first period, we find that first period prices are below the static or no-discrimination levels (Proposition 7). Since each consumer brand loyalty remains a mystery in period 1, first period equilibrium prices remain dispersed. One question not merely academic is whether the price dispersion observed in a static game is affected by the ability of firms to price discriminate. We show that while if price dispersion is measured by the range of prices, price discrimination has no effect on the level of price dispersion, if it is measured by the variance of prices, price discrimination tends to increase the level of price dispersion in first period prices (Corollary 2).

Given that when profits are lower with price discrimination, firms have an incentive to avoid it by not sharing first period demand, one interesting issue that can be explored in the context of the present model is whether or not there is an *asymmetric* pure strategy equilibrium in the initial period, where the same firm serves the entire market. (We look at this issue in Section 3.6.) Here we find that when the future period becomes increasingly more important than the current one, there is indeed an asymmetric equilibrium in which one firm sets a low price and gets all consumers while the rival finds not profitable to match the low price firm because its profits will then be low in the second period due to discrimination (Proposition 9). This result is interesting since in some sense it seems to suggest that under certain conditions the uniform pricing can be sustained without any explicit collective action.

We finish our analysis by shedding some light on the welfare and consumer surplus effects of this form of price discrimination (Section 3.7). In short, allowing firms to price discriminate on the basis of consumer past purchasing data is bad for profits, but good for consumers and welfare (Proposition 10). So, going back to our initial question, we may anticipate that *it is the consumers and not the firms who will benefit the most.*

The remainder of the chapter is organised as follows. We finish this Introduction with a brief review of the more relevant literature. Section 3.2 presents the model. Some privacy issues are discussed in section 3.8 and the main conclusions are given in section 3.9. Finally, an appendix provides those proofs that were omitted from the text.

Related literature The model presented in this chapter combines two strands of research. One is the literature on competitive price discrimination, particularly, the recent literature on behaviour-based price discrimination.[2] The other one is the earliest literature on mixed pricing in oligopoly (e.g. Shilony (1977), Varian (1980) and Narasimhan (1988)), which in some sense we may say that tends to generate price discrimination *across* firms but not at the firm level. With regard to the latter strand of the literature, the existence of an equilibrium displaying price dispersion arises due to consumer heterogeneities, which in Varian (1980) stem from differences in consumers' cost of acquiring price information.[3] Differently, in Shilony (1977) and Narasimhan (1988) firms set their prices according to a distribution function defined over a range of prices because some consumers have a brand preference for a particular firm's product (e.g. brand loyalty, switching costs, etc.) and will be willing to pay a premium even if that firm does not offer the lowest price.[4]

[2]For a survey of competitive price discrimination see, for example, Stole (2003).

[3]In particular, Varian (1980) considers a monopolistically competitive market in which some consumers are perfectly informed about all prices and will buy from the lowest price firm. The remaining consumers are uninformed, and so will buy at random. In this setting, he shows that the unique equilibrium is in mixed strategies. The existence of uninformed consumers means that each firm has a fraction of captive consumers. A general feature of models where some consumers are captive is that sometimes firms can set the "monopoly" price. For other models with captive and price-sensitive consumers see, for instance, Narasimhan (1988), Raju, *et al.* (1990), Chen and Zhang (2002) and Iyer, *et al.* (2003).

[4]In other models firms may set random prices in equilibrium due to costly advertising, which tends to generate imperfectly informed consumers. In this vein see for instance Butters (1977), Stegeman (1991), Stahl (1994) and chapter 4 in this thesis.

Our model blends features from this literature mainly from Shilony (1977) who studies a static oligopolistic market where consumers can purchase costlessly from neighbourhood firms, but incur a transport cost when buying from more distant firms. Thus, the market is segmented according to location. An important feature of this model, and so the similarity with ours, is that, if the reservation value is high enough, even though each firm has an advantage on its local market, this segment of consumers is not fully captive.[5] In this setting, he shows that any pure strategy equilibrium in prices would allow one of the two firms to gain either by undercutting the rival or by increasing its price by some small amount. Thus, he shows that there is a symmetric mixed strategy equilibrium. The present model differs from Shilony's model by extending his model to a dynamic framework in which second period pricing may embody mixed pricing or behaviour-based price discrimination. This extension will allow us to analyse *how* price discrimination based on purchase history, will affect the level of prices and the degree of price dispersion generated in the context of static models.

Finally, the model addressed in this chapter is also related to the strand of literature on competitive price discrimination, specifically, to those models where it is possible to say that one firm's "strong" market is the other's "weak" market, and vice-versa. (In other words, using Corts's (1998) terminology the present model exhibits best-response asymmetry.[6]) A very relevant model to understand *why* discrimination may lead to lower prices and profits in competitive settings is that by Thisse and Vives (1988). They examine price competition in a spatial model where firms observe the "location" of each individual consumer. In this setting the strong (close) market for one firm is the weak (far away) market for the other firm. Then, if firms can set their prices

[5] An important difference between Shilony's model and Varian (1980) and Narasimhan (1988) is that while in these two latter models each firms has a captive group of customers (e.g. uninformed or loyal) and both compete from price-sensitive (e.g. informed or switchers) in Shilony neither firm has a group of captive customers. As a result, while in Varian and Narasimhan firms may set sometimes in equilibrium the monopoly price, the same does not happen in Shilony.

[6] Best-response symmetry arises when the weak and the strong market of each firm coincide. In this regard, see for instance Holmes (1989). An important feature of this type of models is that the uniform price is in general below the strong's market price but above the weak's market price. Conversely, with best response-asymmetry it may happen that all prices fall down or rather that all prices rise.

according to each consumer's location, price discrimination will intensify competition because each consumer is a market to be contested. As a result, *all* prices fall; firms are clearly worse off while consumers are better off. (Because they assume that the market is always covered and all consumers have a unit demand, welfare is constant.) In addition, by adding an initial stage where firms decide simultaneously whether to commit to uniform pricing or not in the next stage, they show that discrimination is a dominant strategy. Thus, firms are trapped in the classic prisoner's dilemma.

In the last years a sequence of papers has emerged on dynamic third-degree behaviour-based price discrimination. As the terminology suggests, this type of price discrimination may be implemented if each firms learns—by observing each customer behaviour in the initial game—*what* firm he or she prefers in the second period.[7] In general, firms will offer lower prices to those customers that bought from the rival before (its weak market) than to its old customers (its strong market).[8]

Two approaches have been considered so far. Whereas in one approach purchase history discloses important information about exogenous switching costs (e.g. Chen (1997) and Taylor (2003)) in the other one it discloses information about product preferences (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000) and Chen and Zhang (2002)).[9] The works within the switching costs approach are motivated by those industries where consumers often incur a switching cost if they decide to move from one supplier to another (as for example, due to the existence of subscription fees, costs of transactions or costs of learning to use a new product). Despite the fact that goods are initially homogeneous, after purchase decisions have been taken, due to the existence of switching costs, consumers are in the next stage of the game partially locked in with their sellers. In this setting, when firms are able to recognise their own customers and separate them from the rival's ones, they will try to entice the latter group of cus-

[7] Here, unlike Thisse and Vives (1988), firms are unable to distinguish the preferences of individual consumers, they are only able to identify customers as either its own customers or the rival's customers.

[8] This strategy has been designated as "paying customers to switch" by Chen (1997) and as "customer poaching" by Fudenberg and Tirole (2000).

[9] In chapter 4 in this thesis we will see that purchase history may disclose information about consumers' awareness.

tomers to switch by offering them a better deal. Roughly speaking, because both firms try to poach each other's customers, price discrimination leads to second period prices and profits lower than their uniform counterparts (for the Thisse and Vives reasons). Further, as firms realise that higher prices can be charged to locked in customers in the next stages, they have an incentive to increase first-period market share at the expense of *lower* first-period prices (indeed, in Chen's model firms price below marginal cost in period 1).

Specifically, the model we are keen to analyse has strong connections to the approach where purchase history discloses important information about consumers' preferences. The more relevant papers in this approach are those by Villas-Boas (1999) and Fudenberg and Tirole (2000).[10] While of growing importance in the context of new information markets, this kind of price discrimination has received little theoretical attention to date. Being the past purchasing history a requirement to the practice of price discrimination both models are designed in repeated purchasing settings. Both papers study the market for a horizontally differentiated product where there are no explicit costs of changing suppliers. In particular, Fudenberg and Tirole (2000) analyse a two period version of the classic linear-city model in which consumers' preferences are distributed on an interval between the two firms serving the market. In the second period, firms *recognise* those customers that bough from them previously and price discriminate accordingly. Thus, each firm will target the rival's previous customers (i.e. its weak market) with lower prices than its old customers (i.e. its strong market). Again as both firms try to poach each other's customers, price discrimination acts to intensify competition leading to second period prices and profits lower than the uniform levels (the same happens in Villas-Boas (1999)).[11]

In contrast with the switching costs approach, first period prices under customer recognition within this second approach will be above the static levels. In fact, Fu-

[10] For a model with customer recognition with personalised prices see Shaffer and Zhang (2001). For models of customer recognition under a monopoly setting look at Fudenberg and Tirole (1998) and Villas-Boas (2001).

[11] For a model where customer recognition may benefit firms see for example chapter 4 in this thesis and Chen and Zhang (2002).

denberg and Tirole (2000) claim that "...the implications of poaching are the reverse of those in models with switching costs: here prices are *above* the static levels in the first period" (p. 643). (Further, when consumers are myopic and consumers' tastes are uniformly distributed they show that price discrimination has no effect on first period prices.) For the reasons explained before, we complement Fudenberg and Tirole's model by modeling consumers' preferences in a discrete framework rather than in a continuous one. Interestingly, in this new setting, we find that price discrimination under customer recognition leads to first period prices *below* the static levels—as in the switching costs approach—which is in contrast with Fudenberg and Tirole's conclusion. (However, the intuition behind our result is quite different from that under the switching costs approach.) Yet in contrast with Fudenberg and Tirole, who find that firms are always able to recognise customers by the second period, we find that sometimes firms learn nothing which means that customer recognition as well as price discrimination cannot occur by period 2. Furthermore, while in their model price discrimination tends to give rise to inefficient switching in equilibrium, here we find that when firms price discriminate no consumer actually switches in equilibrium. (In other words, in our model there is no poaching in equilibrium.)

Villas-Boas (1999) studies a related but infinite-period, overlapping-generations model in which firms can only discriminate between returning and non-returning customers, which means that firms cannot distinguish new and rival's customers. Like Fudenberg and Tirole (2000) he finds that first period prices are above the static levels. The intuition is that a higher first period market share is bad for future profits because it leads the rival to offer lower poaching prices. When firms are forward looking they realise that they will be negatively affected by today's demand, and thus they have an incentive to play less aggressively in the current period. This in turn leads to first period prices above the static levels. (Nonetheless, the model remains yielding an equilibrium that converges to a 50-50 division of the market.) With a different perspective, the model addressed in this chapter also looks on the dynamics of market share. As explained we will see that when firms are forward looking they realise that a 50-50 division of the market will be bad for future profits, and thus they reduce the probability of sharing the initial market. Indeed, we will see that there is sometimes an equilibrium yielding a 100-0 division of the market.

3.2 The model

Let us suppose a market in which two firms, A and B, produce at zero marginal cost nondurable goods A and B, respectively.[12] There are two periods, 1 and 2. On the demand side, there is a large number of consumers, normalised to one, who each desire to purchase in each period either one unit of good A or one unit of good B. A consumer's reservation price, v, is the maximum price any consumer will be willing to pay for the good.

As mentioned in the Introduction, there is evidence that brand loyalty is one important factor behind price competition in the new media markets, particularly in e-commerce markets. But, there is also empirical support that consumers' loyalty in these markets is limited. This suggests that albeit loyalty gives in fact an advantage to a particular seller, it is perhaps less likely that consumers are completely captive to a given firm due to loyalty (at least in some product markets). With that in mind, we assume that an equal number of consumers are either loyal (to a specific degree) to one firm or the other. (In other words, the market consists of two segments of equal size.) One way of modelling this segmentation assumption is based on a discrete version of the Hotelling classic model. On the line segment $[0, 1]$, placing firm A at zero and firm B at one, each consumer's type (i.e. degree of loyalty) is given by his "location" in one of the two points $\{x_A, x_B\}$ where the subscript $i = \{A, B\}$ in x means that the consumer is closer to firm i in the line segment. In the brand loyalty interpretation consumers face a kind of "transport cost " when buying the brand they like less.[13] The

[12] The assumption of zero marginal costs can be relaxed without altering the basic nature of the results derived throughout the model.

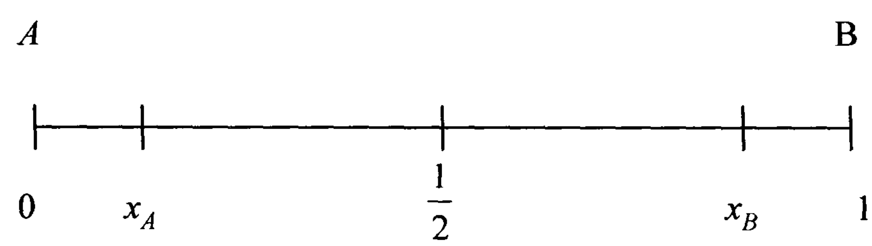
[13] It is worth noting that even though we consider that the market is segmented according to brand loyalty, the model can equally embrace any other alternative segmentation (e.g. location, search costs and switching costs)

assumption that x_A and x_B may take, respectively, any value in the interval $\left[0, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, 1\right]$ will allow us to solve the model for different degrees of brand loyalty. In doing so, it can be said that a consumer located at x_A is loyal to brand A whilst a consumer located at x_B is loyal to brand B. Consumers are more loyal when their location is biased towards the two end points of the line. In addition, it is assumed that consumers' preferences are fixed across periods.[14]

In sum, the model addressed in this chapter is based on the following assumptions:

Assumption 1 (Symmetry) The location of consumers is symmetric implying that $x_A = 1 - x_B$. Given the two firms' location, tx_i is the "transport cost" of choosing brand A while $t(1 - x_i)$ is the "transport cost" of choosing brand B.

Figure 1: Possible configuration of the market



Assumption 2 Due to symmetry when customers buy their most preferred brand they bear a "transport cost" equal to $\alpha = tx_A = t(1 - x_B)$. Similarly, when they buy their least preferred brand they bear a transport cost equal to $\beta = t(1 - x_A) = tx_B$. Given assumption 1 it is always true that $\alpha \leqq \beta$. This inequality holds as an equality only when $x_A = x_B = \frac{1}{2}$, i.e. when consumers are just indifferent between the two brands.

[14]The assumption of fixed-preferences across periods is a requirement in models where firms can price discriminate based on past behaviour. It is important to notice that if preferences were not fixed from period to period, the knowledge of a consumer's first period choice would provide no information about his second period preferences. In this case firms could not engage in price discrimination. For an analysis where preferences change over time, see for instance, Caminal and Matutes (1990) and Fudenberg and Tirole (2000, section 6).

Assumption 3 (Degree of brand loyalty) The difference in "transport costs" between buying the least and the most preferred brand is given by

$$\gamma = \left\{ \begin{array}{ll} t(1-x_A) - tx_A = t\,(1-2x_A) & \text{for consumers located at } x_A \\ tx_B - t(1-x_A) = t\,(2x_B - 1) & \text{for consumers located at } x_B \end{array} \right\},$$

where $\gamma \geq 0$ measures de degree of a consumer's brand loyalty, which thereby can be defined as the minimum difference between the prices of the two competing brands necessary to induce consumers to buy the wrong brand.

Assumption 4 It is assumed that v is sufficiently high so that all consumers will buy one of the goods in both periods. More precisely, it is assumed that $v > \left(2 + \sqrt{2 + 2\delta^2 - 2\delta\sqrt{2}}\right)\gamma$, where δ is the discount factor used by both firms.

3.2.1 Consumer behaviour

In order to isolate the strategic effects of pricing based on customer recognition it is assumed that consumers are myopic.[15] This means that in period 1 consumers do not anticipate that next period's prices may depend on their current behaviour. After firms have set their prices consumers shop for the better bargain.

In each period the net utility of a consumer located at x_i purchasing good A at price p_A is $v - tx_A - p_A$ while the net utility of purchasing good B at price p_B is $v - t\,(1 - x_A) - p_B$. Thus, a consumer loyal to firm A—i.e. located at x_A—buys good A whenever

$$v - tx_A - p_A > v - t\,(1 - x_A) - p_B$$

or, if

$$p_A < p_B + \gamma. \tag{3.1}$$

[15] Although the assumption of myopic consumers may at the first glance seem to be restrictive it does not make a difference in the framework of the current model. In other words, it will become clear for the reader that forward-looking consumers will behave in the same way.

Similarly, a consumer loyal to brand B—i.e. located at x_B—buys good A whenever

$$p_A < p_B - \gamma. \tag{3.2}$$

Reversing the above inequalities one gets the conditions under which consumers located at x_i buy brand B.

3.2.2 Firm behaviour

In each period firms act simultaneously. In period 1 consumers are *anonymous* because firms have no information about the degree of brand loyalty of individual customers. For that reason, firms are forced to quote the same price for all consumers, which we denote as p_A and p_B. However, in period 2, whether or not a consumer went to the firm in the initial period may be used as a *signal* of that consumer's brand preference. In this manner, if firms have the required information to implement behaviour-based price discrimination, they will set a different price to loyal and disloyal customers. Otherwise, when nothing is learned from the initial period, firms quote once more a single price to all consumers. In addition, as mentioned it is assumed that firms discount future profits using a common discount factor δ.

3.3 Benchmark cases

To better evaluate the effects of price discrimination based on purchase history in a duopoly, the following static benchmark cases are analysed before proceeding. We start looking at the case where consumers are *non-anonymous*, thereby allowing firms to implement price discrimination. After that, we analyse the case in which consumers are *anonymous* and price discrimination is unfeasible (which corresponds to our initial period). The results derived in this latter case, will also be used when price discrimination is not permitted due to legal restrictions.

3.3.1 *Non-anonymous* consumers and price discrimination

This special case is akin to Thisse and Vives (1988) paper on price discrimination, where firms base their prices on the observation of a consumer's brand preference ("location"). As in Thisse and Vives, it is assumed that firms are able to price discriminate directly on the consumer's location x_i. (The main difference is that here there are only two types of consumers.) Let $p_i(x_i)$ denote the price charged by firm i to consumers located at x_i. In equilibrium firms compete for consumers at each of the two points on the line. As a result, the lowest price the more distant firm may offer is the marginal cost price, which in this case is equal to zero. And then, to avoid being poached by its competitor, the closer firm needs to offer a price generating the same level of utility as its rival. It immediately follows that in equilibrium for those consumers located, for instance at x_A, firm B sets price $p_B(x_A) = 0$ and thus $p_A(x_A)$ must satisfy

$$v - tx_A - p_A(x_A) = v - t(1 - x_A)$$

from which we obtain

$$p_A(x_A) = t(1 - 2x_A) = \gamma.$$

The same reasoning is applied to those consumers located closer (i.e. loyal) to firm B. Summing up, when firms are able to recognise customers (non-anonymous consumers) and price discrimination is allowed it is straightforward to verify that Bertrand competition in each segment of customers leads equilibrium prices to:

$$p_i(x_i) = \gamma, \tag{3.3}$$

and

$$p_j(x_i) = 0, \text{ for } i \neq j. \tag{3.4}$$

Equilibrium profit per firm when it recognises customers and price discriminates accordingly is

$$\pi_{Disc} = \frac{1}{2}\gamma. \tag{3.5}$$

3.3.2 *Anonymous* consumers and non-discrimination

Firms cannot engage in price discrimination either when they do not observe whether a consumer is located at x_A or at x_B—i.e. if consumers are anonymous—or when price discrimination is illegal, even if feasible. The results derived in this benchmark case will be used indifferently, throughout the chapter, whenever firms cannot engage in price discrimination either due to unfeasible reasons or legal restrictions.

In this case, using (3.1) and (3.2) firm i's demand is given by,

$$D_i = \left\{ \begin{array}{lc} 0 \ \ if & p_i > p_j + \gamma \\ \frac{1}{2} \ \ if & p_j - \gamma \leqq p_i \leqq p_j + \gamma \\ 1 \ \ if & p_i < p_j - \gamma \end{array} \right\}. \tag{3.6}$$

Firm i's profit can be written as $\pi_i(p_i, p_j) = p_i D_i$, or equivalently,

$$\pi_i(p_i, p_j) = \left\{ \begin{array}{ccc} 0 & if & p_i > p_j + \gamma \\ \frac{1}{2}p_i & if & p_j - \gamma \leqq p_i \leqq p_j + \gamma \\ p_i & if & p_i < p_j - \gamma \end{array} \right\}. \tag{3.7}$$

Equilibrium analysis

For obvious reasons in equilibrium each firm must make a nonnegative profit, which means that $p_i \geq 0$. A Nash-Bertrand equilibrium is the pair of prices (p_i^*, p_j^*) such that, for a given p_j^*, firm i chooses p_i to maximise π_i.

Proposition 1. *(i) There is no equilibrium where both firms quote the marginal cost price. Moreover, (ii) there is no equilibrium where both firms set the "monopoly" price v.*

Proof. See the Appendix.

It is evident that the marginal cost price cannot be an equilibrium of this game since by charging price γ each firm can always guarantee itself a profit equal to $\frac{1}{2}\gamma$.

Similarly, as the reader can see in the appendix provided, when both firms set price v, it is always profitable for a given firm to decrease slightly its price to $v - \varepsilon$ and capture the remaining customers. Therefore, given assumption 4, we may establish the following result.

Proposition 2. *When customers are anonymous there is no Nash equilibrium in pure strategies.*

Proof. See the Appendix.

The intuition for this result is as follows. Despite the fact that each firm can always guarantee itself a profit equal to $\frac{1}{2}\gamma$, by selling at price γ only to its loyal segment, the lure of getting the additional demand that comes from the disloyal segment is responsible for the failure of a pure strategy equilibrium in prices. In other words, the presence of a positive fraction of disloyal customers creates a tension between the firm's incentives to price low in order to attract this latter set of customers and to price high in order to extract rents from its loyal customers. This tension results in an equilibrium displaying price dispersion. In what ensues it will be shown that when price discrimination is not feasible, the unique equilibrium has each firm setting its price at random, in order to prevent the rival from systematically predicting its price, which in turn makes undercutting less likely. In fact, a key point in models where firms set random prices when they act simultaneously is that it is not optimal for firms to have any predictable pricing strategy. If a firm sets a high price to extract surplus from say its loyal customers, it is at a competitive disadvantage in relation to its rival, who can take advantage of that predictability by undercutting that price in order to sell to all consumers. Likewise, setting a low price in an attempt to grab all the market would lead to ruinous competition. Thus, in equilibrium firms need to be *unpredictable* which can only be achieved by setting their prices at random. In this regard, in a recent empirical paper, Baye, Morgan and Sholten (2004) provide evidence that those firms that adopted predictable pricing strategies were driven out of the market. For that reason, they claim that unpredictability in prices—i.e., setting prices at random—is widely used and is an effective way of avoiding aggressive price competition in online markets.

Mixed Strategy Nash Equilibrium In any mixed strategy equilibrium firms engage in a randomised pricing strategy. Suppose that firm i selects a random price from the distribution function $F_i(p) = \Pr(p_i \leq p)$ where $F_i(p)$ yields the probability that firm i will choose a price at or below p. In a symmetric mixed strategy equilibrium, both firms follow the same pricing strategy and, thus, for the sake of simplicity we can write $F_i(p) = F_j(p) = F(p)$. If $p_{\min}$ and $p_{\max}$ are, respectively, the lowest and the highest price charged by firms with positive density in equilibrium, then the equilibrium support of prices in which $F(p)$ is defined, is given by $p \in [p_{\min}, p_{\max}]$. In equilibrium all prices that belong to the equilibrium support must yield the same expected profit, given the behaviour of the other firm. The derivation of the mixed strategy equilibrium follows a reasoning similar to that applied in Shilony (1977) and Varian (1980). From the previous propositions it can be inferred that:

Claim 1. *In equilibrium we must observe $p_{\min} \geqq \gamma$ and $p_{\max} \leqq v$.*

Basically this implies that setting a price higher than the reservation value is a dominated strategy because firms would get no demand. Likewise, setting a price lower than γ is also a dominated strategy because each firm can guarantee itself at least a profit equal to $\frac{1}{2}\gamma$ by charging price γ.

Let us now determine the expected profit for a representative firm i. When firm i chooses any price that belongs to the equilibrium support of prices, and firm j uses $F(p)$, firm i's expected profit is always equal to a constant which we denote by K. Since a firm can always guarantee itself a profit equal to $\frac{1}{2}\gamma$, it immediately follows that $K \geq \frac{1}{2}\gamma$.

When firm i charges price p two events are relevant. On the one hand, it may occur that p is low enough that allows firm i to get all consumers. This event happens with probability $[1 - F(p+\gamma)]$. On the other hand, it may be that p is such that both firms share the market. This event occurs with probability $[F(p+\gamma) - F(p-\gamma)]$.

The case where p is too high that precludes firm i from getting any demand is not relevant because in that specific case firm i realises no profit. Hence, firm i's expected profit is

$$\begin{aligned} E\pi(p) &= p\left[1 - F(p+\gamma)\right] + \frac{1}{2}p\left[F(p+\gamma) - F(p-\gamma)\right] \\ &= p\left(1 - \frac{1}{2}F(p+\gamma) - \frac{1}{2}F(p-\gamma)\right). \end{aligned}$$

Given that in equilibrium expected profit is always equal to a constant K it follows that,

$$p\left(1 - \frac{1}{2}F(p+\gamma) - \frac{1}{2}F(p-\gamma)\right) = K. \tag{3.8}$$

Claim 2. *Assume that*

$$0 \leqq p_{\min} < p_{\max} - \gamma \leqq p_{\min} + \gamma < p_{\max} \leqq v. \tag{3.9}$$

From claim 2 it follows that:

Lemma 1. *The distribution function $F(p)$ is as follows:*

$$F(p) = \left\{ \begin{array}{ccc} 0 & if & p < p_{\min} \\ 1 - \frac{2K}{p+\gamma} & if & p_{\min} \leqq p \leqq p_{\max} - \gamma \\ 1 - \frac{2K}{p_{\max}} & if & p_{\max} - \gamma \leqq p < p_{\min} + \gamma \\ 2 - \frac{2K}{p-\gamma} & if & p_{\min} + \gamma \leqq p < p_{\max} \\ 1 & if & p \geqq p_{\max} \end{array} \right\}. \tag{3.10}$$

Proof. See the Appendix.

For any price p such that $p_{\max} - \gamma \leqq p < p_{\min} + \gamma$, one observes that $F(p)$ will be flat in this range and thus $f(p) = \frac{\partial F}{\partial p} = 0$. However, discontinuities cannot arise in equilibrium because if $F(p)$ were discontinuous, a firm could benefit by infinitesimally undercutting at the point of discontinuity.

Lemma 2. *$F(p)$ is continuous with no gap in the support of prices whenever $p_{\max} - p_{\min} = 2\gamma$.*

The proof of this lemma is straightforward. If $p_{\max} - \gamma = p_{\min} + \gamma$, F has no flat range and all prices in the support of F are charged with positive density. Therefore, we need to impose that $p_{\max} - p_{\min} = 2\gamma$. For that reason, we are looking for a support of equilibrium prices that satisfies the condition $p_{\max} - p_{\min} = 2\gamma$.

Lemma 3. *$F(p)$ is an increasing function of p.*

Taking the derivative of $F(p)$ with respect to p it is easy to see that, at any p, $\frac{\partial F}{\partial p} = f(p) > 0$.

Lemma 4. *From $F(p_{\min}) = 0$ and $F(p_{\max}) = 1$ one gets*

$$p_{\min} = 2K - \gamma \tag{3.11}$$

and

$$p_{\max} = 2K + \gamma. \tag{3.12}$$

Based on the above results, we may establish the following proposition which characterises the mixed strategy Nash equilibrium of this game.

Proposition 3. *Under anonymous consumers the Nash equilibrium in prices is described as follows:*

(i) Each firm chooses a price randomly from the nondegenerate distribution function[16]

$$F(p) = \left\{ \begin{array}{ccc} 0 & if & p < p_{\min} \\ 1 - \frac{p_{\min}+\gamma}{(p+\gamma)} & if & p_{\min} \leqq p \leqq p_{\min} + \gamma \\ 2 - \frac{p_{\min}+\gamma}{(p-\gamma)} & if & p_{\min} + \gamma \leqq p < p_{\max} \\ 1 & if & p \geqq p_{\max} \end{array} \right\}, \tag{3.13}$$

[16] *A distribution function $F(x)$ is degenerate if it assigns probability one to a single x, and nondegenerate otherwise.*

with density function $f(p) = \frac{\partial F(p)}{\partial p}$.[17] *The minimum and maximum prices are in equilibrium, respectively equal to*

$$p_{\min} = \sqrt{2}\gamma, \tag{3.14}$$

and

$$p_{\max} = \left(2+\sqrt{2}\right)\gamma. \tag{3.15}$$

Because prices are bounded, $p_{\max} \leqq v$, *and so one needs to impose*

$$\left(2+\sqrt{2}\right)\gamma \leqq v.$$

which is true under assumption 4.

(ii) Firm i's expected profit is in equilibrium equal to

$$K = \frac{1}{2}\left(1+\sqrt{2}\right)\gamma. \tag{3.16}$$

Proof. See the proof of (ii) in the Appendix.

Proposition 3 shows that loyalty is in fact a relevant variable to a better understanding of Internet markets, particularly in regard to the level of prices and profits as well as the persistence of price dispersion, even for what would seem to be homogeneous products (e.g. DVDs, books, etc.). Regarding prices and profits, as expected, we observe that a higher degree of consumers' loyalty gives rise to higher prices and profits. (Notice that both the minimum and the maximum prices are increasing in the degree of brand loyalty γ.) Conversely, as loyalty vanishes (i.e. $\gamma = 0$ or $x_A = x_B = \frac{1}{2}$) consumers are indifferent between firms, there is no brand premium, and therefore

[17] *Notice that for the equilibrium support of prices we verify that:*

$$\int_{p_{\min}}^{p_{\max}} f(p)dp = \int_{\sqrt{2}\gamma}^{(2+\sqrt{2})\gamma} f(p)dp = \left(1+\sqrt{2}\right)\gamma\left(\frac{\left(2+\sqrt{2}\right)\gamma-\left(1+\sqrt{2}\right)\gamma}{\left(2+\sqrt{2}\right)\left(1+\sqrt{2}\right)\gamma^2}\right) + \left(1+\sqrt{2}\right)\gamma\left(\frac{\left(1+\sqrt{2}\right)\gamma-\left(\sqrt{2}\right)\gamma}{\left(1+\sqrt{2}\right)\left(\sqrt{2}\right)\gamma^2}\right) = 1.$$

firms no longer have an advantage over their competitors. The Bertrand outcome is, evidently, the only possible equilibrium solution; firms quote the marginal cost price, make zero profits and price dispersion disappears.

The following result summarises some statistics that have been used to measure price dispersion.

Corollary 1. (Level of price dispersion)

(i) The range of equilibrium prices is $R(p) = 2\gamma$.

(ii) The variance in prices is equal to $Var(p) = 0.288\gamma^2$

Proof. See the proof of (ii) in the Appendix.

The range of prices—the difference between the highest and the lowest price—has been widely used in recent work on Internet price dispersion (e.g. Bynjolfsson and Smith (2000a,b) Baye and Morgan (2003), Baye, Morgan and Sholten (2004)). It is easy to see that here the range of prices is equal to 2γ. Thus, the model developed so far predicts that price dispersion, measured either by the range or variance, tends to be greater in product-markets where customers are more loyal. This result lends support to recent empirical research showing that observed price dispersion is mainly due to perceived differences among retailers related to branding, trust and awareness (e.g. Bynjolfsson and Smith (2000a) and Ward and Lee (2000)).[18]

Finally, for future reference, whenever firms cannot engage in price discrimination, either because they cannot recognise customers or because price discrimination is illegal, each firm expected profit, denoted by $E\pi_{ND}$, is equal to

$$E\pi_{ND} = \frac{1}{2}\left(1 + \sqrt{2}\right)\gamma. \tag{3.17}$$

[18] As we have mentioned elsewhere despite we have been interpreting γ as a brand loyalty parameter, it may equally be interpreted as for example a cost of acquiring information. In other words, if customers were perfectly informed about their most preferred firm but would have to bear a cost to obtain information from other firms (due to the existence of search costs) our model would predict that the level of price dispersion would be greater in markets where it is more difficult to acquire such information (i.e., in markets with higher search costs).

From the comparison between (3.5) and (3.17) we may establish part (i) of the following proposition.

Proposition 4. *(i) Firms are better off when they cannot engage in price discrimination; either because customers are anonymous or because price discrimination is illegal.*

(ii) Consumers pay strictly lower prices under discrimination than under no-discrimination.

The intuition for lower profits with discrimination is just the same as in Thisse and Vives (1988). When discrimination is introduced, firms compete aggressively at the two "locations", leading all prices to fall down, and thereby profits. It is easy to see that the price consumers pay with discrimination given by (3.3) is always below the minimum price they expect to pay under no-discrimination given by (3.14). Moreover, under discrimination all consumers buy from their most preferred firm, which is socially desirable. In contrast, under no-discrimination with some positive probability some consumers will buy from the "wrong" firm, which clearly is not good for welfare. Hence, when customers are non-anonymous price discrimination seems to be welfare improving (at least in a static context).

3.4 Duopoly competition under customer recognition

We have seen that price discrimination is only feasible if firms learn something from the initial period—specifically, if they recognise loyal and disloyal customers by period 2. However, learning can only occur if two conditions hold. First, if a consumer reveals his type through a purchase. (This means that neither firm can recognise a consumer's type in period 1.) Second, if a firm sells its product only to its loyal customers—i.e., if firms share the market in period 1. On the one hand, if both firms set (approximately) the same price in the initial period then they will share the market, and each firm will have an accurate *signal* of a consumer's brand preference (i.e., they will know to each firm each consumer is loyal) in the second period. On the other hand, if one firm's first-period price is much lower than the rival firm's, all consumers go to that firm, and nothing is learned by period 2. Altogether, the only way firms have to pursue customer recognition is through the selection of a first-period price sufficiently high to separate customers but, at the same time, low enough to avoid lose their loyal customers. Throughout this section it is assumed that when feasible price discrimination is permitted. That is to say, if the conditions for behaviour-based discrimination are met price discrimination is not illegal.

As stated earlier, in order to isolate the strategic effects of price discrimination based on customer recognition it is assumed that consumers are myopic.[19] Thus, from the consumers' perspective both periods are independent. Conversely, in the light of the above, the two periods are not independent from a firm's perspective. In period 1, because each consumer's type is a mystery, each firm sets a single first period price, which is denoted as p_i^1. If in period 2, after observing consumers' past behaviour, firms are be able to distinguish perfectly loyal and disloyal customers (i.e. those customers that bought previously from the rival) they can offer different prices to loyal and disloyal customers. Let p_{iL}^2 and p_{iD}^2 denote firm i's second period prices to loyal and disloyal customers, respectively. Otherwise, if nothing is learned by the second period, both firms charge again a single price. In each period firms set their prices simultaneously. As usual, to derive the subgame perfect equilibrium, the game is solved working backward from the second period.

3.4.1 Second-period pricing

Depending on first period prices and correspondent market shares, two scenarios are possible in period 2: (i) it may be that both firms know to which firm any consumer is loyal, which happens when firms share the market in period 1, i.e. when $(D_i, D_j) =$

[19]For an analysis of behaviour-based targeted pricing with strategic consumers see Chen and Zhang (2002).

$\left(\frac{1}{2},\frac{1}{2}\right)$, $i,j=A,B$; or, (ii) it may be that, because nothing is learned, both firms do not know the brand preference of each customer. This happens when $(D_i, D_j) = (0,1)$, $i,j=A,B$ and $i \neq j$. The case where one firm recognises consumers while the other does not, cannot arise in equilibrium. So, there is no subgame where only one firm engages in price discrimination.[20]

Subgame 1: Both firms recognise customers in period 2

As previously referred firms are in this subgame if p^1 is such that both firms share first-period demand. Now let $G_i(p^1) = \Pr(p_i^1 \leq p^1)$ represent the distribution function of firm i's first period prices in the overall game. To simplify notation, as in a symmetric equilibrium $G_i(p^1) = G_j(p^1) = G(p^1)$ we will use $G(p^1)$ as the cumulative distribution function of first-period prices. For a given price p^1 chosen by firm i in period 1, and assuming that firm j sets its price according to $G(p^1)$, the probability of both firms split the market in period 1 is given by $\left[G(p^1+\gamma) - G(p^1-\gamma)\right]$. Because in this subgame both firms can engage in price discrimination based on customer recognition, each firm sets price p_{iL}^2 to customers recognised as loyal and price p_{iD}^2 to customers who bought previously from the rival (recognised as disloyal). In so doing, this subgame is similar to the benchmark case with non-anonymous consumers and price discrimination. Hence, it is easy to see that as firms are symmetric, each firm's equilibrium price to loyal and disloyal customers is respectively given by

$$p_L^2 = \gamma, \tag{3.18}$$

and

$$p_D^2 = 0. \tag{3.19}$$

[20]The next chapter analyses a model where only one firm will be able to recognise customers by the second period, and thereby to engage in price discrimination.

Second period profit under discrimination, denoted by π^2_{Disc} is

$$\pi^2_{Disc} = \frac{1}{2}\gamma. \tag{3.20}$$

In sum, with probability $[G(p^1 + \gamma) - G(p^1 - \gamma)]$, each firm's second period profit is given by π^2_{Disc}.

Subgame 2: Neither firm recognises customers in period 2

This subgame occurs when in the initial period one firm gets the entire market while the other gets nothing. With no loss of generality take the behaviour of firm A in the first period. If firm A's price p^1 is much lower than the rival firm's, all consumers go to firm A. This happens with probability $[1 - G(p^1 + \gamma)]$. Otherwise, if firm A's first-period price p^1 is much higher than the firm B's, no consumer goes to firm A. Obviously, this happens with probability $G(p^1 - \gamma)$. In any case both firms learn nothing by the second period. Thus, the second period pricing game is a replication of the static benchmark case with anonymous consumers and no-discrimination. If we let second period expected equilibrium profit per firm be now denoted by $E(\pi^2_{NR})$ where NR stands for non recognition of customers, it follows from (3.17) that:

$$E(\pi^2_{NR}) = \frac{1}{2}\left(1 + \sqrt{2}\right)\gamma. \tag{3.21}$$

Thus, with probability $[1 - G(p^1 + \gamma) + G(p^1 - \gamma)] = 1 - [G(p^1 + \gamma) - G(p^1 - \gamma)]$ each firm's second period profit is given by $E(\pi^2_{NR})$.

As mentioned earlier, firms are clearly worse off when they can engage in price discrimination. It is easy to see that $\pi^2_{Disc} - E(\pi^2_{NR}) = -\frac{\sqrt{2}}{2}\gamma$. Thus we can say that the damage of price discrimination is equal to $\frac{\sqrt{2}}{2}\gamma$, which is greater the more loyal consumers are.

3.4.2 First-period pricing

To evaluate the overall impact of price discrimination based on customer recognition, we now consider the equilibrium first-period pricing. Because each firm is forward looking ($\delta > 0$) it takes today's pricing decisions rationally anticipating *how* will they affect second period pricing and profit. Firstly, firms are, of course, aware that first period pricing affects current profits. Secondly, they also acknowledge that first period prices strongly affect the feasibility of price discrimination and thus future profits. This will give rise to a trade-off between direct effects on current profits and strategic effects on future profits.

Since in period 1 consumers are anonymous, for the same reasons as in the benchmark case with anonymous consumers, there is no equilibrium in pure strategies. In order to find the cumulative distribution of prices that characterises first-period equilibrium pricing, denoted by $G(p^1)$, we will follow the same reasoning applied in that benchmark case. Let $E\pi^1$ denote each firm's first period expected profit. As previously discussed, if in period 1 firm i charges price $p^1 \in [p^1_{\min}, p^1_{\max}]$, it sells to the entire market if $p^1_j > p^1 + \gamma$, which happens with probability $[1 - G(p^1 + \gamma)]$, and yields a first period profit p^1; it sells nothing if $p^1_j < p^1 - \gamma$, which happens with probability $G(p^1 - \gamma)$ and yields a zero economic profit and, finally; it sells only to its loyal segment if $p^1 - \gamma < p^1_j < p^1 + \gamma$, which in turn happens with probability $[G(p^1 + \gamma) - G(p^1 - \gamma)]$ and yields a first period profit $\frac{1}{2}p^1$. Based on this reasoning, it ensues that

$$E\pi^1 = p^1 \left[1 - G\left(p^1 + \gamma\right)\right] + \frac{1}{2}p^1 \left[G\left(p^1 + \gamma\right) - G\left(p^1 - \gamma\right)\right]. \tag{3.22}$$

As stated, firms take into account that first period pricing will also affect their second period profit. With probability $1 - [G(p^1 + \gamma) - G(p^1 - \gamma)]$, neither firm recognises customers and then each firm's expected second period profit is $E(\pi^2_{NR}) = \frac{1}{2}\left(1 + \sqrt{2}\right)\gamma$. With probability $[G(p^1 + \gamma) - G(p^1 - \gamma)]$, both firms know to which firm each consumer is loyal, and price discrimination is implemented in period 2. In this case second period profit is equal to $\pi^2_{Dis} = \frac{1}{2}\gamma$. Summing up, being $E\pi$ each firm's overall expected profit, it follows that

$$E\pi = E\pi^1 + \delta E\pi^2,$$

where

$$E\pi^2 = \left\{1 - \left[G\left(p^1+\gamma\right) - G\left(p^1-\gamma\right)\right]\right\} E\left(\pi^2_{NR}\right) + \left[G\left(p^1+\gamma\right) - G\left(p^1-\gamma\right)\right] \pi^2_{Dis}.$$

Thus,

$$E\pi = E\pi^1 + \delta E\left(\pi^2_{NR}\right) + \delta\left[G\left(p^1+\gamma\right) - G\left(p^1-\gamma\right)\right]\left[\underbrace{\pi^2_{Disc} - E\left(\pi^2_{NR}\right)}_{\text{Damage of price discrimination}}\right]. \tag{3.23}$$

For the sake of simplicity, suppose that the overall expected profit is equal to a constant C. Since in a mixed strategy equilibrium any price chosen from a firm's price support, $p^1 \in [p^1_{\min}, p^1_{\max}]$, should generate the same expected profit, using (3.20), (3.21) and (3.22) yields,

$$\begin{aligned} C &= p^1\left[1 - G\left(p^1+\gamma\right)\right] + \delta\frac{1}{2}\gamma\left(1+\sqrt{2}\right) \\ &\quad + \frac{1}{2}\left[G\left(p^1+\gamma\right) - G\left(p^1-\gamma\right)\right]\left(p^1 - \delta\sqrt{2}\gamma\right). \end{aligned} \tag{3.24}$$

Lemma 5. *Given claim 2 and the condition that precludes the existence of any flat range on $G\left(p^1\right)$, which is as before $p^1_{\max} - p^1_{\min} = 2\gamma$, one finds that,*

$$G(p^1) = \left\{\begin{array}{ccc} 0 & if & p^1 < p^1_{\min} \\ 1 - \frac{2C - \delta\left(1+\sqrt{2}\right)\gamma}{p^1 + \left(1-\delta\sqrt{2}\right)\gamma} & if & p^1_{\min} \leqq p^1 \leqq p^1_{\max} - \gamma \\ 2 - \frac{2C - \delta\left(1-\sqrt{2}\right)\gamma}{p^1 - \left(1-\delta\sqrt{2}\right)\gamma} & if & p^1_{\max} - \gamma \leqq p^1 < p^1_{\max} \\ 1 & if & p^1 \geqq p^1_{\max} \end{array}\right\} \tag{3.25}$$

Proof. See the Appendix.

Lemma 6. *$G(p^1)$ is increasing in p^1 whenever $C > \frac{1}{2}\delta\left(1+\sqrt{2}\right)\gamma$.*

Proof. See the Appendix.

Lemma 7. *From the conditions $G\left(p^1_{\text{min}}\right) = 0$ and $G\left(p^1_{\text{max}}\right) = 1$ it follows that*

$$p^1_{\text{min}} = 2C - (1+\delta)\,\gamma \tag{3.26}$$

and

$$p^1_{\text{max}} = 2C + (1-\delta)\,\gamma \quad \text{where} \quad p^1_{\text{max}} \leqq v. \tag{3.27}$$

Lemma 8. *From the continuity of $G(p_1)$ at $p_1 = p_{1\,\text{max}} - \gamma$ we obtain that*

$$C = \frac{1}{2}\left((1+\delta) + \sqrt{\left(2 + 2\delta^2 - 2\delta\sqrt{2}\right)}\right)\gamma. \tag{3.28}$$

Proof. See the Appendix.

Notice that $C = \frac{1}{2}\left((1+\delta) + \sqrt{\left(2 + 2\delta^2 - 2\delta\sqrt{2}\right)}\right)\gamma$ satisfies lemma 6. Hence, we may now provide a complete characterisation of the subgame perfect equilibrium of this game.

Proposition 5. *There is a symmetric mixed strategy subgame perfect Nash equilibrium in which, (i) each firm quotes a first-period price randomly chosen from the distribution*

$$G(p^1) = \left\{\begin{array}{ccc} 0 & if & p^1 < p^1_{\text{min}} \\ 1 - \frac{p^1_{\text{min}} + \left(1-\delta\sqrt{2}\right)\gamma}{p^1 + \left(1-\delta\sqrt{2}\right)\gamma} & if & p^1_{\text{min}} \leqq p^1 \leqq p^1_{\text{min}} + \gamma \\ 2 - \frac{p^1_{\text{min}} + \left(1+\delta\sqrt{2}\right)\gamma}{p^1 - \left(1-\delta\sqrt{2}\right)\gamma} & if & p^1_{\text{min}} + \gamma \leqq p^1 < p^1_{\text{max}} \\ 1 & if & p^1 \geqq p^1_{\text{max}} \end{array}\right\}, \tag{3.29}$$

with density function $g(p^1) = \frac{\partial G(p^1)}{\partial p^1} > 0$. *The minimum and maximum prices are respectively given by*

$$p^1_{\min} = \left(\sqrt{2 + 2\delta^2 - 2\delta\sqrt{2}}\right)\gamma \tag{3.30}$$

and

$$p^1_{\max} = \left(2 + \sqrt{2 + 2\delta^2 - 2\delta\sqrt{2}}\right)\gamma. \tag{3.31}$$

Because prices are bounded, $p^1_{\max} \leqq v$, *implies that* $\left(2 + \sqrt{2 + 2\delta^2 - 2\delta\sqrt{2}}\right)\gamma \leqq v$ *(which satisfies Assumption 4).*

(ii) Each firm earns expected overall equilibrium profits equal to

$$E\pi = \frac{1}{2}\left((1+\delta) + \sqrt{\left(2 + 2\delta^2 - 2\delta\sqrt{2}\right)}\right)\gamma. \tag{3.32}$$

It is worth noting that the model developed so far predicts that customer recognition, and thus price discrimination, only occurs *sometimes*—i.e., when firms share the market in the first period. Otherwise, when they do not share the market, nothing is learned, in which case there is neither customer recognition nor price discrimination in period 2. This model is different from Villas-Boas (1999) and Fudenberg and Tirole (2000), in which customer recognition through learning *always* occurs by the second period. Because in these latter models firms have always a positive market share in period 1, they are always able to distinguish their own and the rival's customers, and to price discriminate accordingly. Thus, the present model sheds some light on the dynamics of market share, specifically on the incentives firms might have to pursue a 100-0 division of the market. We look at this and related issues in the subsequent sections.

3.5 Implications of price discrimination based on customer recognition

When firms are myopic ($\delta = 0$) the future does not affect, of course, current behaviour and, as a consequence, first-period prices and profits are just equal to those in the static benchmark case with anonymous consumers. However, when firms are forward looking ($\delta > 0$) they take into account the impact of first period pricing on overall profit. First, it affects current profit. Second, it may give firms the opportunity to recognise customers, to price discriminate accordingly in the future, and to become worse off. Obviously, when a firm acknowledges this kind of inter-period dependence it may have incentives to change its first-period pricing in relation to its myopic or static counterpart. Some interesting questions arise in this context. How will the ability of firms to foresee the negative effects of price discrimination affect their first-period prices and demand? Will they set higher or lower initial prices? What is the overall effect of customer recognition on profits? The aim of this section is, therefore, to study *how* price discrimination enabled by customer recognition will affect the equilibrium outcomes in relation to their non-discrimination counterparts.

3.5.1 First-period demand

We have seen that when firms are forward looking they realise that halve the market in the current period will clearly hurt the firm in the future. In this context, one important issue is the following. Has a firm an incentive to *forgo* current demand as an effective way to avoid discrimination tomorrow? (Remember that when firms do not share demand, one of them has no demand at all.)

Proposition 6. *For all positive γ : (i) Either when firms are myopic or when price discrimination cannot occur firms share first period demand with probability equal to* 0.8.

(ii) When firms are forward looking they recognise the negative effects of price dis-

crimination and, therefore, they reduce the probability of sharing first-period demand. In particular, when $\delta = 1$, firms share first period demand with probability equal to 0.71.

Proof. See the Appendix.

The intuition underlying proposition 6 is straightforward. When firms foresee that by learning they become trapped in a classic prisoner's dilemma tomorrow, they have incentives to avoid learning, which can be achieved by *not* sharing the market today. This in turn gives rise to a reduction in the equilibrium probability of sharing the market in period 1. If we consider, for instance, that firms discount future profits using a unit discount factor, proposition 6 predicts that in equilibrium firms will share first period market with probability equal to 0.71, smaller than the probability of sharing first period market when discrimination is not permitted. (Notice, however, that as δ declines the drawback of price discrimination shrinks, weakening the incentives to reduce the probability of sharing first period demand.)

This result shows that forward looking firms try to preclude the all-out competition result associated with price discrimination by not sharing the market in the first period. In doing so, the model predicts that as δ increases it is less likely to observe price discrimination enabled by customer recognition.[21] While when firms are myopic price discrimination is implemented with probability equal to 0.8, when they are forward looking and $\delta = 1$ price discrimination only occurs with probability equal to 0.71.

The extant models on this form of price discrimination show that even when firms change their first-period behaviour due to the negative effects of price discrimination, they cannot avoid price discrimination (Villas-Boas (1999) and Fudenberg and Tirole (2000)). In fact, Villas-Boas (1999) demonstrates that as firms realise that a greater previous market share is bad for future profits—because the rival will play more aggressively in the next period—they become less aggressive in trying to gain market

[21] Although we only present results for extreme values of δ, we checked that the probability of demand sharing is monotone decreasing in δ for $\delta \in [0, 1]$. In this range the higher is δ the lower is the probability of firms sharing demand.

share in the initial period. Nonetheless, in equilibrium both firms remain sharing first period demand and price discrimination is present anyway. Here, in contrast, we find that when price discrimination is permitted and firms are forward looking, there is a bias towards asymmetric outcomes, where the initial market may be entirely served by the same firm. Thus, while in the extant models firms have no effective away of precluding the negative effects of discrimination in the next stages of the game, here it happens that *sometimes* firms may strategically *forgo* any previous market share as an effective "weapon" for avoiding all-out competition in the next period.

3.5.2 Prices

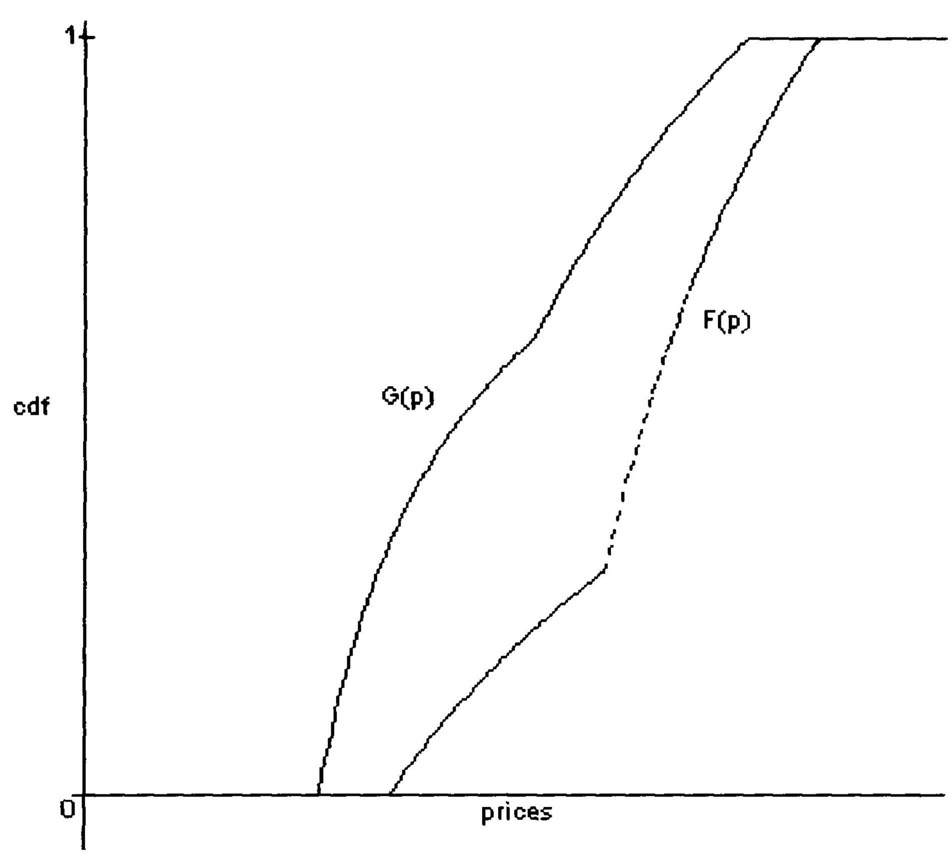
Second-period prices When learning is achieved by the second period, price discrimination based on customer recognition acts to intensify competition in each "location", which leads all prices to fall (see end of section 3.3.2). This is what Corts (1998) designates as the *all-out competition* result. The reason is that in period 2, under customer recognition, one firm's strong market (its loyal customers) is the rival's weak market (its disloyal customers). When discrimination is permitted, each firm tries to poach the rival's previous clientele by quoting the marginal cost price. And, then, to prevent poaching each firm is forced to reduce the price to its loyal customers as well. As a result of that all prices fall.

First-period prices We have seen that when nothing is learned, firms are better off in the second period and so, all else equal, firms have an incentive not to share the market in the first-period. Obviously, this kind of behaviour can be only implemented by changing first-period pricing in relation to the non-discrimination (or myopic) case.

Proposition 7. *When firms are forward looking, first-period prices are below the static or no-discrimination levels.*

Comparing equations (3.14) and (3.15) respectively with (3.30) and (3.31) it is easy to see that in fact as long as $\delta \leq \sqrt{2}$ the minimum and the maximum price of the equilibrium distribution function falls down. In particular, if $\delta = 1$, the support of current prices moves down from $[1.414\gamma, 3.414\gamma]$ in the benchmark case where there is no price discrimination to $[1.082\gamma, 3.082\gamma]$ when there is price discrimination. Likewise, by comparing the equilibrium distribution functions in the overall game where discrimination is permitted with that in the benchmark case with non-discrimination we find that in the range $0 < \delta \leq 1$, $F(p)$ first-order stochastically dominates $G(p)$. Figure 2 illustrates both distribution functions for the special case where $\delta = 1$.

Figure 2: Cumulative distribution functions for $\delta = 1$



From the first order stochastic dominance of $F(p)$ over $G(p)$ it follows that the average first-period price under discrimination is lower than its static counterpart. In fact, while under non-discrimination the average price is approximately 2.54γ (see proof of corollary 1 in the appendix provided) under discrimination the average first period price falls down to 1.87γ (see proof of corollary 2 in the Appendix). Thus, when discrimination is introduced consumers are expected to pay lower first period prices as well.

From Figure 2 it can also be inferred that when price discrimination is permitted, firms charge low first period prices more frequently than high prices. Conversely, in the static or in the non-discrimination case, firms charge high prices more often than low prices. The intuition for first-period prices lower than their static counterparts is as follows. Forward looking firms have an incentive not to learn at all, which is achieved by not sharing the market in period 1. In order to attain that goal a firm needs to set its price far away from the rival. However, because each firm prefers to get the whole market than to get nothing at all, this may give an incentive to price in a more aggressive way, thereby leading to first-period prices below the static levels.

This result is in contrast with that achieved in Villas-Boas (1999) and Fudenberg and Tirole (2000) where first period prices are *above* the static levels. As aforementioned, the intuition in Villas-Boas is that when firms are forward looking they anticipate that future profits will be negatively affected by the size of initial market share—because higher first period market share leads to more aggressive behaviour in the second period—and, as a result, they compete less aggressively in the first period, thereby charging higher prices. (This result is valid either with myopic or non-myopic consumers.) In Fudenberg and Tirole (2000) the intuition is that as consumers anticipate lower future prices due to poaching, they become less price sensitive in the initial period, and then firms can set higher first period prices than if poaching were not permitted. However, it is important to stress that, in Fudenberg and Tirole, if consumers' preferences are uniformly distributed and consumers are myopic price discrimination has no effect on first period prices (i.e., first period prices are equal to their static counterparts).

While Fudenberg and Tirole claim that the effects of behaviour based price discrimination on first period prices, in the brand preference approach, are the reverse of those effects in the switching costs approach, here we find that this form of price discrimination may lead to lower first period prices as happens in the switching costs approach. Nevertheless, the intuition for our result is quite different from the intuition for first period prices lower than the static levels in the context of switching costs. In

Chen (1997) the presence of switching costs allows firms to lock-in their old customers and to offer new customers a lower price, in order to entice them to switch from the rival. Thus, firms have an incentive to increase first-period market share as a way to increase their base of locked-in (i.e., captive) customers. This explains why in Chen's model firms price below the static levels in the first period (indeed, they price below marginal cost) and afterwards firms raise their prices once consumers are locked-in.

3.5.3 Price dispersion

As stated, the static model with anonymous consumers gives support to empirical research explaining the existence of price dispersion based on brand loyalty. However, once we move to a repeated interaction framework in which firms may engage in price discrimination, one natural question that arises is whether or not price discrimination affects the level of current price dispersion.

Corollary 2. *When price discrimination is introduced and $\delta = 1$ we find that:*
(i) the range of prices is $R(p)_{Disc} = 2\gamma$
(ii) the variance of prices is $Var(p)_{Disc} = 0.373\gamma^2$

This result shows that when firms are allowed to price discriminate, first period prices remain dispersed—i.e., price dispersion is ubiquitous. In fact, price dispersion only disappears when consumers are indifferent between both firms (i.e. $\gamma = 0$). As before the level of price dispersion is greater the more loyal customers are.

When we compare corollary 1 and 2 we observe that the range of prices is constant and equal to 2γ. However, if we measure price dispersion using the variance of prices we conclude that the level of first period price dispersion under discrimination is higher than if discrimination were banned. Thus, while an absolute measure of price dispersion hides the impact of discrimination on the level of price dispersion, the variance of prices illustrates that price dispersion is higher when discrimination is permitted. If the level of price dispersion is measured by the variance of prices, from corollary 1 and 2 it follows that, all else equal, there is a 29.5% increase on the level of first-period price dispersion, when moving from no-discrimination to discrimination.[22] This suggests that a greater level of observed price dispersion in period 1 is, all else equal, explained by the strategic effects associated with price discrimination. Notwithstanding firms' heterogeneities (e.g. brand loyalty) are in fact an important force behind observed price dispersion, some other factors may as well account for some fraction of price dispersion, as happens here with price discrimination.[23]

3.5.4 Is there any poaching in equilibrium?

If firms know to which firm any consumer is loyal in the second period, they will try to poach the rival's previous clientele—or, equivalently, those customers that have revealed to have some preference from the rival through their purchase in period 1. This is the reasoning behind the literature on behaviour-based price discrimination.

Broadly speaking, although with frameworks slightly different, Villas-Boas (1999) and Fudenberg and Tirole (2000) show that poaching occurs in equilibrium. (The same happens in Chen (1997) under the switching costs approach.) In contrast, the model addressed in this chapter shows that even though firms decrease prices to the rival's customers (indeed, they quote the marginal cost price), in order to entice them to switch, in equilibrium there is no poaching at all. The intuition is as follows. Given that in the former two referred models there is a continuum of consumers' types, firms only poach those consumers that have a weak preference from the rival's product (those in the middle of the line segment), but not all of the rival's previous clientele. Here, because there are only two segments of customers, if poaching were to occur in equilibrium one of the firms would get no demand. Thus, in order to avoid being completely poached by the rival, each firm has an incentive to decrease the price to its loyal customers to such a level that makes these customers just indifferent between

[22]Notice that $\frac{Var(p)_{Disc}}{Var(p)} - 1 = \frac{0.373}{0.288} - 1 = 0.295$.

[23]In this regard, using data collected over an 18 month period from the price comparison site Shopper.com, Baye, Morgan and Sholten (2002), note that in spite of observed price dispersion being mainly due to retailers heterogeneities (i.e. branding, reputation and trust), 28% of the observed dispersion is not explained by these heterogeneities.

buying the right and the wrong brand.[24] So, there is no poaching at all in equilibrium.

3.5.5 Profits

In this section we revisit the profitability issue of behaviour-based price discrimination. In light of the above, it is evident that price discrimination is bad for second period profits. (Remember that the loss in second period profits due to discrimination is equal to $\frac{\sqrt{2}}{2}\gamma$). However, we have also seen that foreseeing how bad is price discrimination for future profits, a firm may find itself punishing current profits, as a way to avoid price discrimination in the future (this happens when it sells nothing in that period). Thus, a complete picture of the profit implications of price discrimination enabled by customer recognition can only be drawn after looking on overall profit and first-period profit.

Proposition 8. *(i) Price discrimination enabled by customer recognition hurts not only second period profit, but also first period profit because it gives firms incentives to reduce first-period prices in order to reduce the probability of sharing the market in that period.*

(ii) Price discrimination based on customer recognition makes firms become worse off because overall expected profit is lower than if price discrimination were not allowed.

To prove part (ii) notice that if discrimination were not permitted, second period expected profit would be a replication of first period profit under anonymous consumers, namely $\frac{1}{2}\left(1+\sqrt{2}\right)\gamma$. Thus, in this case, overall expected profit when discrimination is not permitted would be equal to $\frac{1}{2}(1+\delta)\left(1+\sqrt{2}\right)\gamma$. In contrast, when discrimination occurs with some probability in period 2, overall expected profit is equal to $\frac{1}{2}\left((1+\delta)+\sqrt{\left(2+2\delta^2-2\delta\sqrt{2}\right)}\right)\gamma$. Then, it is easy to see that, apart from the case where $\delta=0$, expected overall profit when discrimination is permitted is always below its non-discrimination counterpart.

[24]In Thisse and Vives (1988) even though a firm tries to entice those consumers with a brand preference from the rival by offering them lower prices, no consumer is actually poached in equilibrium. A similar finding is obtained in Nilssen (1992).

Moreover, expected first period profit in the overall game, $E\pi^1$, is also lower when there is price discrimination. $E\pi^1$ can be easily derived from $E\pi$ given by equation (3.23). That is,

$$
\begin{aligned}
E\pi^1 &= E\pi - \delta E\left(\pi^2_{NR}\right) - \delta\left[G\left(p^1+\gamma\right) - G\left(p^1-\gamma\right)\right]\left[\pi^2_{Disc} - E\left(\pi^2_{NR}\right)\right] \\
E\pi^1 &= \frac{1}{2}\left((1+\delta) + \sqrt{\left(2+2\delta^2-2\delta\sqrt{2}\right)}\right)\gamma - \frac{1}{2}\delta\left(1+\sqrt{2}\right)\gamma \\
&\quad -\frac{1}{2}\delta \Pr(\text{Demand sharing})\left(-\sqrt{2}\right)\gamma.
\end{aligned}
$$

(Remember that first period expected profit under non-discrimination is $E\pi_{ND} = \frac{1}{2}\left(1+\sqrt{2}\right)\gamma$.) Thus, if we let $q = \Pr(\text{Demand sharing})$, it follows that

$$
E\pi^1 - E\pi_{ND} = \frac{1}{2}\gamma\left[\sqrt{\left(2+2\delta^2-2\delta\sqrt{2}\right)} - (1+\delta(1-q))\left(\sqrt{2}\right)\right].
$$

Thus, it is easy to see that for $0 < \delta \leq 1$, the above difference is negative and so first-period profit is clearly below its non-discrimination counterpart. If for instance $\delta = 1$, firms reduce the probability of sharing the market in period 1 from 0.8 (under non-discrimination) to 0.71, and then it follows that expected profit in period 1 is $E\pi^1 \cong 0.836\gamma$, lower than first period profit when there is no price discrimination, which is equal to 1.207γ. Consequently, price discrimination based on customer recognition is bad not only for second period profit but also for first period profit. The reason for this finding is, of course, the reduction in first period prices as well as the reduction in the probability of sharing the market in that period. Indeed, under discrimination, with some probability one of the firms may realise zero economic profits in period 1.

In addition, the model predicts that the more loyal consumers are the higher is: (i) firm and industry profit; (ii) the level of observed price dispersion in the market; and (iii) the negative effect of price discrimination on profits. This gives support to those claiming that the profitability of any business activity online will be strongly dependent on the ability of retailers to differentiate themselves from competitors.

3.6 Asymmetric equilibrium

As is by now a familiar theme in the present model, when price discrimination leads to lower profits for all firms, they may wish not to share the market in the first period as a way to refrain from more aggressive behaviour in the second period. Indeed, the incentive to not halve the market in period 1 will be stronger the more important is the future for firms ($\delta \to 1$). (Notice that the probability of demand sharing is a decreasing function of δ for $0 < \delta \leq 1$.) Thus, the model predicts that there is a bias towards asymmetric outcomes in period 1, and that this type of outcomes will be more frequent as the future becomes increasingly important. In this context, the aim of this section is to investigate whether or not there is an asymmetric pure strategy equilibrium through which price discrimination would be completely driven out of the market in the next period?

With that in mind, we allow δ to be higher than one. In this new setting, we find the following result:

Proposition 9. *(i) If $\delta < 1.06$ there is no asymmetric pure strategy equilibrium.*

(ii) If $\delta \geqq 1.06$ there exists an asymmetric pure strategy equilibrium in the first period with prices $\left(p_i^1, p_j^1\right)$ such that firm i sets a low price and gets all the consumers whilst firm j gets nothing. For a given p_i^1, it must then be the case that $p_j^1 > p_i^1 + \gamma$. The equilibrium is defined as follows:

(a) For $1.06 \leqq \delta < \sqrt{2}$ and $p_j^1 > p_i^1 + \gamma$,

$$p_i^1 = \left(\sqrt{2}\delta - 1\right)\gamma \tag{3.33}$$

and overall profits per firm are given by

$$\pi_i = (2.621\delta - 1)\,\gamma, \tag{3.34}$$

$$\pi_j = 1.207\delta\gamma, \tag{3.35}$$

(b) For $\delta \geqq \sqrt{2}$ and $p_j^1 > p_i^1 + \gamma$,

$$p_i^1 = \gamma,$$

and overall profits per firm are given by

$$\pi_i = (1 + 1.207\delta)\,\gamma, \tag{3.36}$$

$$\pi_j = (1.207\delta)\,\gamma. \tag{3.37}$$

Interestingly, proposition 9 shows that even though firms are symmetric, there is sometimes an asymmetric pure strategy equilibrium where one firm quotes a low price and gets all consumers in the initial period and where the rival firm has no incentive to match this low-price firm because its profits will then be low in the second period. Nevertheless, the asymmetric equilibrium will only be sustained when firms put more weight on future than on current profits—i.e., when δ is high enough. Otherwise, firms would not have a sufficient incentive to sacrifice current profits; and thereby, the equilibrium would be the one defined in proposition 5.

The general conclusion is that when firms value future more than current profits, they are increasingly willing to sacrifice current profits as a way to fight against the drawbacks of price discrimination. First, they are willing to reduce first period prices. Second, when δ is high enough, in order to completely avoid the negative effects of discrimination, one of the firms is even willing to forgo first period demand and profits. Despite there is no explicit agreement between firms to restrict discrimination, the model suggests that, under certain conditions, when one firm quotes a low enough price it will not be undercutting by its rival because the latter would not find it profitable to do so. This suggests that under certain conditions *not* sharing first period market is in fact an effective "*weapon*" for avoiding all-out competition in the next periods.

The literature on competitive price discrimination has pointed out that when profits are lower with discrimination firms would become better off by colluding and committing not to discriminate. However, in the absence of any collective commitment, it is generally the case that even when one firm unilaterally commits to uniform pricing, the other firm finds it profitable to discriminate. In this regard, Thisse and Vives (1988) show that when one firm commits to uniform pricing, price discrimination is a dominant strategy to the other firm. The reason is that once one firm is committed to uniform pricing, the other firm is better off unconstrained in its pricing strategy. Thus, in this case an unilateral commitment would not solve the prisoner's dilemma. One important implication of proposition 9 is that it highlights that a uniform pricing policy might arise without any collective action. In other words, uniform pricing in the next stage of the game might arise on the basis of unilateral actions. Specifically, the choice of a *low* enough price by a certain firm could be interpreted as a credible kind of commitment to non-discrimination in the next stage of the game. For a high enough δ, the choice of a low price by a given firm would solve the prisoner's dilemma because at such a price, it would not be worth for the other firm to undercut that price; and thus this latter firm would also be committed to a non-discriminatory pricing in the second period.

3.7 Welfare analysis

This section investigates the effects of price discrimination through customer recognition on consumer surplus and total welfare. Given that customer recognition may only occur with some positive probability in the symmetric equilibrium analysed, the welfare analysis will only focus on that type of equilibrium. In addition, to simplify the analysis, throughout this section it is assumed that $\delta = 1$.

In each period welfare is given by the sum of consumer surplus and industry profit. Welfare is equivalently given by

$$\text{Welfare} = v - \text{Expected “transport cost”}.$$

In the social optimal solution each consumer buys the product from his or her most preferred firm in each period. This happens exclusively when firms share the market in period 1. Otherwise, one group of consumers buy from the wrong firm thereby supporting an extra transport cost of $\gamma = \beta - \alpha$.

We start looking at the second period of the game. Two possible situations may occur: (i) there is price discrimination enabled by customer recognition; and, (ii) there is no price discrimination either because it is illegal or because firms do not recognise customers.

When customer recognition occurs and price discrimination is allowed in the second period all consumers buy from the correct firm and, thus, welfare is just equal to

$$w^2_{Disc} = v - \alpha. \tag{3.38}$$

Conversely, if price discrimination does not occur in period 2, firms set their prices randomly and therefore with some positive probability one group of consumers buy from the wrong firm. Since the second period equilibrium equals in this case the static non-discrimination one, the probability that all consumers buy from their most preferred firm is equal to 0.8. Therefore, welfare is now equal to

$$w^2_{ND} = v - ETC,$$

where ETC is the expected transport cost supported by all consumers. In this case it is equal to

$$\begin{aligned} ETC &= \alpha \Pr(\text{Demand sharing}) + \frac{1}{2}(\alpha+\beta)(1-\Pr(\text{Demand sharing})) \\ &= 0.8\alpha + \frac{1}{2}(\alpha+\beta)(0.2) = 0.9\alpha + 0.1\beta. \end{aligned}$$

Thus,

$$w^2_{ND} = v - 0.9\alpha - 0.1\beta. \tag{3.39}$$

When price discrimination is allowed, expected overall welfare, denoted by W, is equal to the sum of welfare in both periods, that is,

$$\begin{aligned} W_{Disc} &= w^1 + \Pr\left(\text{Demand sharing in period 1}\right) w^2_{Disc} \\ &\quad + \left(1 - \Pr\left(\text{Demand sharing in period 1}\right)\right) w^2_{NR}. \end{aligned}$$

As for $\delta = 1$, $\Pr\left(\text{Demand sharing in period 1}\right) = 0.71$, expected transport cost in period 1 is equal to

$$ETC = 0.71\alpha + \frac{1}{2}\left(\alpha + \beta\right)\left(0.29\right) = 0.855\alpha + 0.145\beta.$$

Therefore, first-period welfare is equal to

$$w^1_{Disc} = v - ECT = v - 0.855\alpha - 0.145\beta. \tag{3.40}$$

Using (3.38), (3.39) and (3.40) expected overall welfare when price discrimination is allowed in period 2 equals,

$$W_{Disc} = 2v - 1.826\alpha - 0.174\beta. \tag{3.41}$$

Using (3.32), overall consumer surplus under discrimination, denoted by CS_{Disc}, is given by

$$CS_{Disc} = W_{Disc} - 2E\pi = 2v + 1.256\alpha - 3.256\beta. \tag{3.42}$$

In contrast, if there is a ban on price discrimination, firms do not reduce the probability of sharing the market in period 1 since the negative effect of price discrimination on profits is absent. In this case, $\Pr\left(\text{Demand sharing in period 1}\right) = 0.8$ and then,

$$w^1_{ND} = v - 0.9\alpha - 0.1\beta. \tag{3.43}$$

Overall welfare when discrimination is not permitted, W_{ND}, is now equal to:

$$W_{ND} = w^1_{ND} + w^2_{ND}.$$

Using (3.39) and (3.43) it follows that,

$$W_{ND} = 2v - 1.8\alpha - 0.2\beta. \tag{3.44}$$

Using (3.17), overall consumer surplus when price discrimination is not allowed is equal to:

$$CS_{ND} = W_{ND} - 2\left(2E\pi_{ND}\right) = 2v + 3.028\alpha - 5.028\beta. \tag{3.45}$$

After comparing consumer surplus, industry profit and welfare when price discrimination is permitted and when it is banned it is possible to establish the following proposition.

Proposition 10. *Allowing firms to price discriminate on the basis of past purchase history is bad for profits, but good for consumers and welfare.*

Proof. This is simply proved by calculating the variation in consumer surplus, industry profits and total welfare when moving from a framework where price discrimination is illegal to one where price discrimination is permitted. For $\delta = 1$, we find that $\Delta CS = CS_{Disc} - CS_{ND} = 1.772\gamma$, $\Delta\Pi = \Pi_{Disc} - \Pi_{ND} = -1.746\gamma$ and $\Delta W = W_{Disc} - W_{ND} = 0.26\gamma$. Here, as in other models with inelastic demands there is no role for price discrimination to increase aggregate output. Obviously, in this case, discrimination is welfare enhancing because it leads to more efficient shopping. In other words, welfare increases because "expected transport cost" falls down with discrimination, $\Delta ETC = -0.26\gamma$. *Q.E.D.*

When all-out competition is present, it immediately follows that prices and profits are lower which clearly is bad for firms but good for consumers. As far as welfare is concerned, we have seen that when discrimination is introduced it is more likely that some consumers buy the wrong brand in period 1 (due to the reduction on the probability of demand sharing). However, discrimination always leads consumers to buy in period 2 from the firm they like most, which is socially desirable. As a whole, price discrimination tends to reduce expected "transport costs", allowing welfare to increase. This result is in contrast with those models where poaching decreases welfare because it leads to inefficient switching (e.g. Chen (1997) and Fudenberg and Tirole (2000)). As in these models without discrimination consumers always buy from the right firm, a ban on price discrimination would be socially desirable. Here, in contrast, because random pricing tends to generate some inefficient shopping, price discrimination should be encouraged as it would lead to more efficient shopping.

Therefore, the welfare results derived so far suggest that one must be careful in drawing any public policy in favour or against price discrimination practices. In particular, it seems that conclusions regarding the welfare implications of behaviour-based price discrimination depend upon the form of consumer heterogeneity as well as on the available information for pricing. When the market tends to generate price dispersion, as is usually the case in Internet markets, any attempt of firms to price discriminate based on purchase history is expected to be welfare enhancing as long as it leads to more efficient shopping.

3.8 Implications for privacy issues

The rapid changes in technology, the increasing use of Internet and e-commerce and the development of more sophisticated methods for collecting, analysing and using consumer's information have made privacy a hot topic. Indeed, privacy issues have increasingly attracted the attention of the media, businesses, legislators, regulators, politicians, and privacy advocates. This is illustrated by the new *opt-in* regulation imposed by the European Union since October 2003. This new directive requires firms to seek some form of permission from individuals before collecting and using their private information for marketing purposes (e.g. pricing and advertising). Regarding consumers although they have become more worry about the protection of their private information, it is also true that some consumers behave in a manner that demonstrates their willingness to disclose and allow the use of their private information in exchange of better deals, discounts, free services, convenience, and other benefits.

Even though there are, of course, many more questions underpinning privacy concerns, this chapter aims to analyse *uniquely* the use of consumers' information for price discrimination purposes, in order to shed some light on the following questions. Should government regulation be oriented in a way that makes information collection more difficult and restricts price discrimination practices? Or, should consumers make themselves anonymous by using special tools or neglecting firms the access and/or use of their information for pricing?[25]

The stylised model developed in this chapter has hoped to clarify some of these issues. Basically, it predicts that competitive price discrimination based on past purchasing data is in fact good for consumers and welfare; and thus it is socially desirable to allow firms to discriminate. For that reason, in markets that could be well represented by the features of the current model the advise would be that there is no need for any regulation restricting the practice of discrimination based on purchasing history. The consumer surplus analysis suggests as well that there is no reason for consumers to make themselves anonymous online, since they are clearly better off when firms do recognise them. Also related to this issue, we can compare the total price consumers are expected to pay when they buy anonymously with that they are expected to pay when firms are able to recognise them and to price discriminate. In some sense it can be said that the *value of recognition,* say VR is

$$VR = 2E\left(p\right)_{ND} - \left[E\left(p^{1}\right)_{Disc} + q\left(p_{Disc}\right) + \left(1-q\right)E\left(p\right)_{ND}\right]$$

where $q =$Prob(Demand sharing). It is obvious that as far as prices is concerned, consumers do really benefit from being recognised, since in comparison to the anonymous framework they are expected to pay less. If say $\delta = 1$, $q = 0.71$ and therefore by behaving non-anonymously consumers are expected to save $2\left(2.54\gamma\right) - [1.87\gamma + 0.71\left(0.5\gamma\right)$ $+ 0.29\left(2.54\gamma\right)] \simeq 2.12\gamma$.

[25] Consumers may make themselves anonymous online by using special intelligent agents, by refusing or delecting cookies (electronic identifiers) from their computers or by shopping using other computers.

3.9 Conclusions

This chapter has studied the dynamic effects of price discrimination based on consumers' past purchasing data in a two-period duopoly market where consumers have *limited* brand loyalty from one of the two firms. In this setting, it was shown that when firms cannot offer different prices to customers with different brand preferences—either because consumers are anonymous or because discrimination is illegal—they set random prices in equilibrium, which in turn results in an equilibrium displaying price dispersion in both periods. This pricing strategy seems to be in accordance with evidence collected by recent empirical studies on Internet pricing, which point out brand loyalty as one of the main drivers of observed price dispersion.

When price discrimination enabled by learning is allowed, it was shown that as firms foresee the negative effects of price discrimination they have an incentive to avoid customer recognition. In this regard, it was found that an effective "*weapon*" to preclude all-out competition associated with competitive price discrimination is *not* share the market in the first period. In consequence of this strategic behaviour, it was proved that when $\delta \leq 1$ firms share the market less frequently which in turn means that firms *sometimes* avoid price discrimination in the next period. It was also shown that first period prices are *below* their static or the non-discrimination counterparts. This latter result is quite the reverse of that achieved in the extant models where purchase history discloses information about brand preferences (e.g. Villas-Boas (1999) and Fudenberg and Tirole (2000)).

Interestingly, it was found that when firms put more weight on future than on current profits, they can *completely* avoid the drawbacks of price discrimination. More precisely, it was proved that in this case there exists an *asymmetric* pure strategy equilibrium in the first period, in which one firm sets a low price and gets all consumers, but it is not worth to the competitor to match this low price-firm as its profits would then be low in the second period.

Although the model addressed is at best a crude approximation of real Internet markets, we believe it offers a suitable theoretical ground to contribute to the ongoing debate on the profits, consumer surplus and welfare effects of price discrimination based on customer recognition. Basically, it was shown that allowing firms to price discriminate on the basis of purchase history is bad for profits, but good for consumer surplus and welfare. Thus, in those markets that could be reasonably well represented by the features of the current model it seems that it is the consumers and not the firms that will benefit the most from price discrimination enabled by customer recognition.

3.10 Appendix

This appendix collects the proofs that were omitted from the text.

Proof of Proposition 1.

(i) By contradiction suppose that $(p_A^* = 0, p_B^* = 0)$ is a Nash equilibrium with corresponding equilibrium profits $\pi_A^* = 0$ and $\pi_B^* = 0$. With no loss of generality consider the perspective of firm B. In this case all consumers located at x_B buy from firm B. Hence, firm B would be better off deviating and increasing its price to $0 < p_B^d \leqq t\,(2x_B - 1) = \gamma$ and selling only to its loyal customers. This deviation would be profitable because $\pi_B^d = \frac{1}{2}\gamma > \pi_B^*$. A contradiction.

(ii) By contradiction suppose that (v, v) is a Nash equilibrium with corresponding equilibrium profits $\frac{v}{2}$ (because both firms split the market). Any price lower than v but greater than or equal to $v - \gamma$ is dominated by v as it would give firm i the same demand and lower profits. However, if firm i chooses to undercut its rival charging a price $(v - \gamma - \varepsilon)$ it would get all consumers and its profits would be equal to $(v - \gamma - \varepsilon)$. This deviation would not be profitable for firm i as long as $\gamma > \frac{v}{2}$. Hence, (v, v) would be a pure strategy Nash equilibrium if and only if $\gamma > \frac{v}{2}$. However, this condition contradicts assumption 4. *Q.E.D.*

Proof of Proposition 2.

By contradiction suppose that (p_A^*, p_B^*) is a Nash equilibrium with corresponding equilibrium profits π_A^* and π_B^*.

(i) With no loss of generality suppose $p_A^* > p_B^* + \gamma$. Hence, from (3.6) firm A has no demand and $\pi_A^* = 0$. Nonetheless, firm A can deviate and increase its profit by quoting a lower price $p_A^d = p_B^* + \gamma$ and making a profit $\pi_A^d = \frac{1}{2}(p_B^* + \gamma) > 0$. A contradiction.

(ii) With no loss of generality suppose $p_A^* < p_B^* + \gamma$. In this case, firm A can deviate and increase its profit by slightly increasing its price to $p_A^d = p_B^* + \gamma$. In this case $\pi_A^d = \frac{1}{2}(p_B^* + \gamma) > \pi_A^*$. A contradiction.

(iii) With no loss of generality suppose $p_A^* < p_B^* - \gamma$. Then , $p_B^* > p_A^* + \gamma$ and as firm A does in (i) and (ii) there is a profitable deviation for firm B, implying that $\pi_B^d > \pi_B^*$. A contradiction. *Q.E.D.*

Proof of Lemma 1.

For $p_{\min} + \gamma \leqq p < p_{\max}$, $F(p+\gamma) = 1$, so using (3.8) yields

$$p\left(\frac{1}{2} - \frac{1}{2}F(p-\gamma)\right) = K$$

$$F(p-\gamma) = 1 - \frac{2K}{p} \qquad \text{for} \qquad p_{\min} + \gamma \leqq p < p_{\max}$$

or,

$$F(p) = 1 - \frac{2K}{p+\gamma} \qquad \text{for} \qquad p_{\min} \leqq p < p_{\max} - \gamma. \tag{3.46}$$

For $p_{\min} \leqq p < p_{\max} - \gamma$, $F(p-\gamma) = 0$, so using (3.8) yields

$$p\left(1 - \frac{1}{2}F(p+\gamma)\right) = K$$

$$F(p+\gamma) = 2 - \frac{2K}{p} \qquad \text{for} \qquad p_{\min} \leqq p < p_{\max} - \gamma$$

or,

$$F(p) = 2 - \frac{2K}{p-\gamma} \qquad \text{for} \qquad p_{\min} + \gamma \leqq p < p_{\max}. \tag{3.47}$$

Finally, for $p_{\max} - \gamma \leqq p < p_{\min} + \gamma$,

$$F(p) = 1 - \frac{2K}{p_{\max}}. \tag{3.48}$$

Q.E.D.

Proof of Proposition 3.

We only need to prove part (ii). From the continuity of $F(p)$ at $p = p_{\max} - \gamma$ and using the fact that $p_{\max} = 2K + \gamma$ one gets

$$2 - \frac{2K}{2K - \gamma} = 1 - \frac{2K}{2K + \gamma}$$

from which we obtain

$$K = \frac{1}{2}\left(1 + \sqrt{2}\right)\gamma.$$

Q.E.D.

Proof of part (ii) in Corollary 1.

The average price consumers expect to pay in the static or no-discrimination mixed strategy equilibrium is given by

$$\begin{aligned} E(p) &= \int_{\text{sup}} pf(p)dp = \left(1 + \sqrt{2}\right)\left(\int_{(\sqrt{2})\gamma}^{(1+\sqrt{2})\gamma} \frac{p}{(p+\gamma)^2}dp + \int_{(1+\sqrt{2})\gamma}^{(2+\sqrt{2})\gamma} \frac{p}{(p-\gamma)^2}dp\right) \\ &\cong 2.54\gamma. \end{aligned}$$

The variance of prices is $Var(p) = E(p^2) - [E(p)]^2$. Given that

$$E(p^2) = \int_{\sqrt{2}\gamma}^{(2+\sqrt{2})\gamma} p^2 f(p)dp \cong 6.74\gamma^2$$

it follows that

$$Var(p) = 6.74\gamma^2 - (2.54\gamma)^2 = 0.288\gamma^2.$$

Q.E.D.

Proof of Lemma 5.

Considering again claim 2, for $p^1_{\min} + \gamma \leqq p^1 < p^1_{\max}$, one gets $G(p^1 + \gamma) = 1$, and thus (3.24) becomes

$$C = \delta \frac{1}{2} \gamma \left(1 + \sqrt{2}\right) + \frac{1}{2} \left[1 - G\left(p^1 - \gamma\right)\right] \left(p^1 - \delta\sqrt{2}\gamma\right),$$

which in turn yields

$$G\left(p^1\right) = 1 - \frac{2C - \delta\left(1 + \sqrt{2}\right)\gamma}{p^1 + \left(1 - \delta\sqrt{2}\right)\gamma} \quad \text{for} \quad p^1_{\min} \leqq p^1 < p^1_{\max} - \gamma. \tag{3.49}$$

For $p^1_{\min} \leqq p^1 < p^1_{\max} - \gamma$, one observes that $G(p^1 - \gamma) = 0$, and

$$C = p^1 \left[1 - G\left(p^1 + \gamma\right)\right] + \frac{1}{2}\delta\gamma\left(1 + \sqrt{2}\right) + \frac{1}{2} G\left(p^1 + \gamma\right)\left(p^1 - \delta\sqrt{2}\gamma\right).$$

It follows then that,

$$G\left(p^1\right) = 2 - \frac{2C - \delta\left(1 - \sqrt{2}\right)\gamma}{p^1 - \left(1 - \delta\sqrt{2}\right)\gamma} \quad \text{for} \quad p^1_{\min} + \gamma \leqq p^1 < p^1_{\max}. \tag{3.50}$$

In order to preclude the existence of any flat range in the distribution function, it must be that $p^1_{\max} - \gamma = p^1_{\min} + \gamma$, or $p^1_{\max} - p^1_{\min} = 2\gamma$. *Q.E.D.*

Proof of Lemma 6.

The derivative of $G(p^1)$ with respect to p^1 for the range $p^1_{\min} \leq p \leq p^1_{\max} - \gamma$ is given by $\frac{\partial G(p^1)}{\partial p^1} = \frac{2C - \delta\left(1+\sqrt{2}\right)\gamma}{\left(p^1 + \left(1 - \delta\sqrt{2}\right)\gamma\right)^2}$ which is positive if and only if $2C > \delta\left(1 + \sqrt{2}\right)\gamma$. Similarly, the derivative of $G(p^1)$ with respect to p^1 for the range $p^1_{\max} - \gamma \leq p \leq p^1_{\max}$ is given by $\frac{\partial G(p^1)}{\partial p^1} = \frac{2C - \delta\left(1-\sqrt{2}\right)\gamma}{\left(p^1 - \left(1 - \delta\sqrt{2}\right)\gamma\right)^2}$ which is positive if and only if $2C > \delta\left(1 - \sqrt{2}\right)\gamma$. Together, both conditions are satisfied whenever $C > \frac{1}{2}\delta\left(1 + \sqrt{2}\right)\gamma$. *Q.E.D.*

Proof of Lemma 8.

From the continuity of $G(p_1)$ at $p_1 = p_{1\max} - \gamma$ it follows that

$$1 - \frac{2C - \delta\left(1+\sqrt{2}\right)\gamma}{p_{1\max} - \gamma + \left(1 - \delta\sqrt{2}\right)\gamma} = 2 - \frac{2C - \delta\left(1-\sqrt{2}\right)\gamma}{p_{1\max} - \gamma - \left(1 - \delta\sqrt{2}\right)\gamma}.$$

Using the fact that $p_{1\max} - \gamma = 2C - \delta\gamma$ yields

$$1 - \frac{2C - \delta\left(1+\sqrt{2}\right)\gamma}{2C - \delta\gamma + \left(1 - \delta\sqrt{2}\right)\gamma} = 2 - \frac{2C - \delta\left(1-\sqrt{2}\right)\gamma}{2C - \delta\gamma - \left(1 - \delta\sqrt{2}\right)\gamma},$$

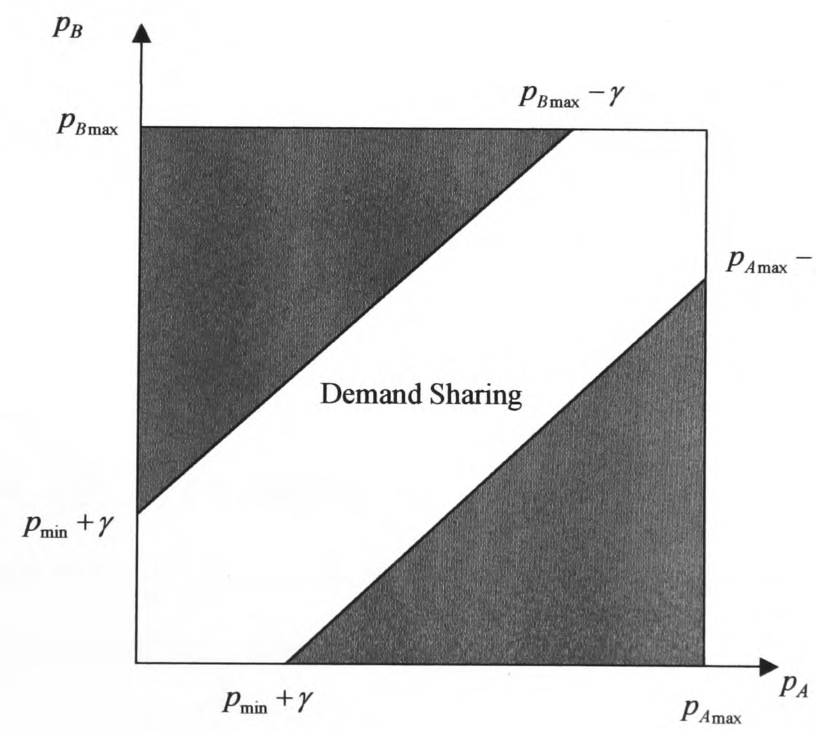
from which we get the solution

$$C = \frac{1}{2}\left((1+\delta) + \sqrt{\left(2 + 2\delta^2 - 2\delta\sqrt{2}\right)}\right)\gamma. \tag{3.51}$$

Q.E.D.

Proof of Proposition 6.

The next figure shows the area under which both firms share the market in the first period.



Because the model is symmetric both firms have the same support of prices. Then

$$\Pr(\text{Demand Sharing}) = 1 - 2\int_{p_{\min}+\gamma}^{p_{\max}} \left(\int_{p_{\min}}^{p_A-\gamma} f(p_B)\, dp_B \right) f(p_A)\, dp_A \tag{3.52}$$

Either when there is no price discrimination either when firms are myopic ($\delta = 0$) one gets,

$$\Pr(\text{Demand Sharing}, \delta = 0) = 1 - 2\int_{p_{\min}+\gamma}^{p_{\max}} \left(1 - \frac{p_{\min}+\gamma}{p_A}\right) \frac{p_{\min}+\gamma}{(p_A-\gamma)^2} dp_A$$

Thus, for $p_{\min} = 1.414\gamma$ and $p_{\max} = 3.414\gamma$ it follows that

$$\begin{aligned} \Pr(\text{Demand Sharing}, \delta = 0) &= 1 - 2\int_{2.414\gamma}^{3.414\gamma} \left(1 - \frac{2.414\gamma}{p_A}\right) \frac{2.414\gamma}{(p_A-\gamma)^2} dp_A \\ &= 0.8. \end{aligned}$$

When firms are forward looking and $\delta = 1$,

$$\Pr(\text{Demand Sharing}, \delta = 1) = 1 - 2\int_{p_{\min}+\gamma}^{p_{\max}} \left(1 - \frac{p_{\min} - 0.414\gamma}{p_A - 1.414\gamma}\right) \frac{p_{\min} + 2.414\gamma}{(p_A + 0.414\gamma)^2} dp_A$$

Since $p_{\min} = 1.082\gamma$ and $p_{\max} = 3.082\gamma$ it follows that

$$\begin{aligned} \Pr(\text{Demand Sharing}, \delta = 1) &= 1 - 2\int_{2.082\gamma}^{3.082\gamma} \left(1 - \frac{0.668\gamma}{p_A - 1.414\gamma}\right) \frac{3.496\gamma}{(p_A + 0.414\gamma)^2} dp_A \\ &= 0.71. \end{aligned}$$

Q.E.D.

Proof of part (ii) in Corollary 2.

The average first period price consumers expect to pay in the mixed strategy equilibrium under discrimination, when $\delta = 1$ is given by

$$
\begin{aligned}
E\left(p^1\right)_{Disc} &= \int_{\text{sup}} p^1 g(p^1)dp^1 = \left(\int_{1.082\gamma}^{2.082\gamma} \frac{0.668\gamma p^1}{\left(p^1 - 0.414\gamma\right)^2} dp^1 + \int_{(2.082)\gamma}^{(3.082)\gamma} \frac{3.496\gamma p^1}{\left(p^1 + 0.414\gamma\right)^2} dp^1\right) \\
&\cong 1.87\gamma.
\end{aligned}
$$

The variance of prices is $Var\left(p\right) = E\left(p\right) - \left[E\left(p\right)\right]^2$. Given that

$$
\begin{aligned}
E\left(\left(p^1\right)^2\right)_{Disc} &= \int_{\text{sup}} \left(p^1\right)^2 g(p^1)dp^1 = \int_{1.082\gamma}^{2.082\gamma} \frac{0.668\gamma \left(p^1\right)^2}{\left(p^1 - 0.414\gamma\right)^2} dp^1 \\
+ \int_{2.082\gamma}^{3.082\gamma} \frac{3.496\gamma \left(p^1\right)^2}{\left(p^1 + 0.414\gamma\right)^2} dp^1 &\cong 3.87\gamma^2,
\end{aligned}
$$

it follows that

$$
Var\left(p^1\right)_{Disc} = 3.87\gamma^2 - \left[1.87\gamma\right]^2 \cong 0.373\gamma^2.
$$

Q.E.D.

Proof of Proposition 8.

Suppose there exists an asymmetric pure strategy equilibrium with prices $\left(p_i^1, p_j^1\right)$ such that firm i serves the entire market in period 1 while firm j gets no demand. For a given p_i^1, it must be the case that $p_j^1 > p_i^1 + \gamma$. Because the market is entirely served by the same firm in the initial period, there is no learning by the second period. In this situation, both firms set their prices at random as in the static case with anonymous consumers. Thus, overall equilibrium profits per firm are given by,

$$
\pi_i = p_i^1 + \delta\left(1.207\gamma\right)
$$

and

$$
\pi_j = 0 + \delta\left(1.207\gamma\right).
$$

This is an equilibrium if and only if firm j has no incentives to deviate. When firm j deviates two possible situations may occur: (i) firm j sets a price that allows both firms to share first period demand, that is $p_j^1 = p_i^1 + \gamma$; or (ii) firm j grabs the entire market to firm i, which is attained by setting $p_j^1 = p_i^1 - \gamma - \varepsilon$. In the former case both firms recognise customers and in period 2 they price discriminate accordingly. The latter case is identical to the case where the market is entirely served by firm i. Thus, let π_j^d denote firm j's profit when it deviates.

When firm j sets such a price that $p_j^1 = p_i^1 + \gamma$ it shares the market with firm i and its profit from deviation is

$$\pi_j^d = \frac{1}{2}\left(p_i^1 + \gamma\right) + \delta\left(\frac{1}{2}\gamma\right).$$

Likewise, when firm j sets a such a price that $p_j^1 = p_i^1 - \gamma - \varepsilon$ it grabs the entire market from firm i. In this case its profit from deviation is

$$\pi_j^d = p_i^1 - \gamma - \varepsilon + \delta\left(1.207\gamma\right).$$

Summing up, firm j has no incentives to pursue any deviation as long as $\pi_j = \delta\left(1.207\gamma\right) > \pi_j^d$, i.e. if the two following conditions hold:

$$\delta\left(1.207\gamma\right) \geqq \frac{1}{2}\left(p_i^1 + \gamma\right) + \delta\left(\frac{1}{2}\gamma\right)$$

and

$$\delta\left(1.207\gamma\right) \geqq p_i^1 - \gamma - \varepsilon + \delta\left(1.207\gamma\right)$$

or, equivalently if,

$$p_i^1 \leqq 1.414\delta\gamma - \gamma \tag{3.53}$$

and

$$p_i^1 \leqq \gamma + \varepsilon \lessapprox \gamma \text{ for } \varepsilon \text{ suffieciently small.} \tag{3.54}$$

Since the equilibrium price must be equal or above marginal cost, $p_i^1 \geq 0$, this implies that if the asymmetric equilibrium exists it must be the case that $\delta \geq 0.707$.

From (3.53) and (3.54) it follows that

$$1.414\delta\gamma - \gamma \leqq \gamma,$$

if $\delta \leqq 1.414$. In sum, for $0.707 < \delta \leqq 1.414$ firm i's first period price is given by $p_i^1 = 1.414\delta\gamma - \gamma$ whilst for $\delta > 1.414$ firm i's first period price is equal to γ.

Thus, when $0.707 < \delta \leqq 1.414$ overall equilibrium profits for each firm are respectively,

$$\begin{aligned}\pi_i &= 2.621\delta\gamma - \gamma \\ \pi_j &= 1.207\delta\gamma\end{aligned}$$

while when $\delta > 1.414$, overall equilibrium profits are

$$\begin{aligned}\pi_i &= (1 + 1.207\delta)\,\gamma \\ \pi_j &= (1.207\delta)\,\gamma.\end{aligned}$$

Finally, to finish the proof we need to verify that firm i has also no incentives to increase its price and share the market with firm j. Given that p_i^1 is such that allows firm i to serve the entire market, firm i may increase its price by 2γ and share the market with firm j. Then, its profit from deviation would equal

$$\pi_i^d = \frac{p_i^1 + 2\gamma}{2} + \delta\left(\frac{1}{2}\gamma\right).$$

This deviation would not be profitable if

$$\frac{p_i^1 + 2\gamma}{2} + \delta\left(\frac{1}{2}\gamma\right) \leqq p_i^1 + \delta\,(1.207\gamma)$$

that is, if

$$p_i^1 \geqq 2\gamma - 1.414\delta\gamma.$$

When $p_i^1 = 1.414\delta\gamma - \gamma$ the previous condition is satisfied if and only if

$$\begin{aligned} 1.414\delta\gamma - \gamma &\geqq 2\gamma - 1.414\delta\gamma \\ \delta &\geqq 1.06, \end{aligned}$$

otherwise, when $p_i^1 = \gamma$ the above condition is satisfied if and only if

$$\begin{aligned} \gamma &> 2\gamma - 1.414\delta\gamma \\ \delta &> 0.707 \text{ which is true given that } \delta > 1.414. \end{aligned}$$

To summarise, we have that: (i) if $\delta < 1.06$ there is no asymmetric equilibrium in pure strategies; whilst (ii) if $\delta \geqq 1.06$ there is an asymmetric equilibrium in which:

(a) for $1.06 \leqq \delta < \sqrt{2}$ and $p_j^1 > p_i^1 + \gamma$, one gets

$$p_i^1 = \left(\sqrt{2}\delta - 1\right)\gamma$$

and overall profits per firm are given by

$$\begin{aligned} \pi_i &= (2.621\delta - 1)\,\gamma \\ \pi_j &= 1.207\delta\gamma \end{aligned}$$

(b) otherwise, for $\delta \geqq \sqrt{2}$ and $p_j^1 > p_i^1 + \gamma$, one gets

$$p_i^1 = \gamma$$

and overall profits per firm are given by

$$\begin{aligned} \pi_i &= (1 + 1.207\delta)\,\gamma \\ \pi_j &= (1.207\delta)\,\gamma. \end{aligned}$$

Q.E.D.

Chapter 4

Customer Poaching and Advertising

4.1 Introduction

In many markets firms need to invest in advertising to inform potential consumers about the existence and price of their new products. Indeed, the informative view of advertising claims that the primary role of advertising is to transmit information to otherwise uninformed consumers.[1] However, as e-commerce retailing grows, firms are increasingly able to recognise customers with different past purchasing histories and to implement *behaviour-based* price discrimination by means of a kind of *behaviour-based* informative advertising. If a firm can distinguish its old customers and its rival's customers it may want to send targeted advertising messages (henceforth, ads) with differentiated content—like prices—to customers with different past behaviour. Therefore, apart from its informative role, advertising might have an additional role: it may be used by firms as a price discrimination device.

In this regard, consider the following example. An outdoor recreation seller designs an online advertising campaign for a particular camping equipment. After receiving one of the seller's ads, a certain customer visits the seller's web site and follows the usual

[1] In contrast, the persuasive view of advertising holds that the main role of advertising is to increase a consumer's willingness to pay for the advertised product. In this sense, firms may use advertising to alter consumers' tastes and to increase brand loyalty. For a review of models in the persuasive view see Bagwell's (2003) comprehensive survey on the "The Economic Analysis of Advertising."

registration process. Empowered with a customer's relevant information, as whether he bought or not and his email, the seller may in the next time send a tailored email with almost no cost. When the seller realises that a certain reached customer was in its site but did not buy, it may design a campaign to entice that customer back with a special email offer.[2,3]

In spite of the fact that economists have long been concerned with understanding the competitive and welfare effects of either price discrimination or informative advertising, little theoretical attention has been dedicated to the interaction between informative advertising and price discrimination. To fill that gap, this chapter proposes a simple two-period game-theoretical model in which all information about price and product availability flows through advertising,[4] and where advertising may also be used as a way to implement behaviour-based price discrimination in the next period of the game. In doing so, we will revisit with a new perspective some of the old questions that have been the subject of analysis in the price discrimination as well as in the informative advertising literature.

On the one hand, the effect of price discrimination on prices, profits and welfare has long been of interest to economists. Hence, will the firm's temptation to poach the rival's previous customers lead to lower or higher profits? What are the effects on the firms' first-period pricing and advertising decisions if price discrimination is permitted in period 2? Do firms have an incentive to invest in more advertising when price discrimination is allowed? What about consumers and social welfare?

On the other hand, the welfare properties of advertising have been the subject of an ongoing debate that goes back to Kaldor's (1950) classical paper. Thus, this chapter explores the impact of price discrimination on the efficient properties of purely

[2] *Amazon,* for instance, was targeting its old customers with an email book offer for Terry Pratchett's latest book, but offered it at a higher price than it was charging to new customers.

[3] *CDnow*, an online vendor of music albums, emails certain buyers (e.g. first-time buyers) a special Web site address with lower prices. (Kotler (2000), p. 476).

[4] This assumption is in line with the stream of the informative advertising which considers that in the absence of advertising potential buyers do not purchase the advertised good. Implicitly, this means that consumers are passive, which may be due to high search costs in relation to the expected surplus generated by the good in question. For models in this vein see, for instance, Butters (1977), Grossman and Shapiro (1984), Stegeman (1991) and Stahl (1994).

informative advertising. When firms can engage in price discrimination by means of advertising, is there too little or too much advertising? Do firms select a more efficient level of advertising under non-discrimination or under discrimination?

To show that the analysis is worth pursuing we invite the reader to consider the following thought experiments. We start with a static game with no-discrimination. Suppose that two online firms are offering a new homogeneous product and that advertising is the consumers' sole source of information about product's existence and price. In order to reach a given fraction of the market, firms need to invest in advertising. At the same time firms decide what price to advertise. An interesting issue to look at is the following. If advertising costs were null, what fraction of the market would firms decide to inform? What price would firms set? As the reader may correctly guess, in this simple example, when advertising is costless, firms choose an advertising campaign with full market coverage. In fact, this intuitive result is obtained in all extant models in which the unique purpose of advertising is to convey information about the firms' product existence, regardless of the number of firms and the advertising cost function (e.g. Butters (1977), Grossman and Shapiro (1984), Stegeman (1991), Stahl (1994), Iyer, et al (2003)). Obviously, the competitive implications of full market reach are well known: in a homogeneous product market, as all consumers become perfectly informed the equilibrium approaches the classic Bertrand competition outcome; firms set the marginal cost price, profits go to zero and consumers are clearly better off than under imperfect reach.

Now we extend the previous simple example to a two-period game. This is basically the model formulated in this chapter. As before, there are two identical online firms advertising a new homogeneous product (and its price) to otherwise uninformed consumers. In period 1, each firm chooses what price to quote in its ads and an intensity of advertising that will be used in the current period as well as in the next one. After firms have sent their ads, *ex-ante* uninformed identical consumers will be differentiated on an informational basis. (By investing in advertising, firms endogenously determine the level of informational differentiation.) More precisely, there will be three types of consumers. Some consumers will receive ads from both firms, will be *selective* customers and will always buy from the lowest-priced known firm. Other consumers will receive ads from only one firm, will be *captive* customers and will buy from that firm provided that the price does not exceed the reservation value. Finally, some consumers will remain uninformed because they will receive no ads. In this setting, when a firm is in the market for more than one period, it might learn whether a previous contacted consumer bought its product or bought instead the rival's. If a firm achieves that type of learning, it will face in the second period, two separate markets—that of its own customers and that of its rival's customers—and so it may be tempted to poach the rival's previous customers by sending ads with lower prices to that group of customers.

To illustrate the relevance of studying the interaction between advertising and price discrimination we start from a setting where discrimination is not permitted in period 2. Afterwards, we extend this benchmark case, by allowing firms to engage in price discrimination. If advertising were costless, what intensity of advertising would firms select under non-discrimination? And, what under discrimination? As in the simple static example presented above, we show that when price discrimination is banned in period 2 and advertising costs are null firms choose full market reach (Proposition 3). In contrast, we show that, regardless of the advertising cost function, when firms can engage in price discrimination and advertising has no cost, forward-looking firms never select in equilibrium full market coverage (Proposition 8). As we can see when advertising costs are null and price discrimination is not permitted, consumers become perfectly informed either in a static version of the game either in a dynamic one. Conversely, when discrimination is introduced in a game where advertising costs are null, the full information outcome is no longer an equilibrium. As a consequence, by choosing an advertising reach lower than 100% under discrimination, each firm earns positive profits. It is worth emphasising that this novel result is in this case exclusively due to the introduction of price discrimination. Hence, by looking at the simplest case where advertising has no cost, it becomes clear that it is of particular relevance to study how advertising, which serves to inform potential consumers, is used in conjunction with price discrimination.

Our goal in this chapter is twofold. As mentioned, one of our goals is to analyse the effects of price discrimination in competitive markets in which advertising is needed to generate demand. Secondly, we try to lay out the conceptual issues that arise in analysing poaching in markets where consumers are imperfectly informed and where advertising is used by firms as a long-run variable. In this regard, this chapter offers a new ground to assess the competitive and welfare effects of behaviour-based price discrimination.

With that in mind, we address a duopoly model similar to the previous two-period example but with a general advertising cost function, in which the costless advertising situation occurs as a special case (Section 4.3). In order to evaluate the effects of price discrimination, we start by analysing in Section 4.4 the benchmark framework in which price discrimination is banned, even if feasible, in period 2. The fundamental issue here is to determine the level of advertising selected by firms in period 1 as well as prices. It is shown that as in other models with imperfectly informed consumers the unique equilibrium in prices is characterised by price dispersion (e.g. Butters (1977), Salop and Stiglitz (1977), Varian (1980) and Stahl (1994), to name few). In addition, as referred, it is shown that as advertising costs go to zero, firms tend to choose full market reach, which drives prices to marginal cost and profits to zero.

We then analyse the case in which whether feasible price discrimination is permitted in period 2 (Section 4.5). Here, we find that only the highest priced firm in period 1 will engage in price discrimination. Additionally, we find that discrimination clearly benefits the firm that can engage in price the discrimination and, more interestingly, we show that the same might happen to the non-discriminating firm. Each firm has, then, incentives to pursue price discrimination which is achieved by setting high first-period prices. In fact, we show that price discrimination leads firms to charge on average higher first-period prices than under non-discrimination. A similar result is obtained in Fudenberg and Tirole (2000). This latter model shows that being price discrimination permitted, in period 2 firms will offer lower prices to those customers that have revealed a preference for the rival's product. As consumers rationally anticipate customer poaching, they become less price sensitive for a firm's product in period 1, which allows firms to set higher first period prices than if poaching were not permitted. However, the intuition for our similar finding is quite different. Broadly speaking, as each firm realises that price discrimination is good for second period profits, it seeks to gain the discriminating position by quoting a high first-period price. As prices are strategic complements, firms compete less aggressively in period 1, which leads on average to first-period prices above their non-discrimination counterparts.

Other noteworthy result is the fact that under discrimination, as advertising costs go to zero, first-period prices remain dispersed and the minimum price of the support of equilibrium prices does not go to marginal cost, which is in contrast with the marginal cost price result under non-discrimination (Proposition 9). Obviously, the reason is that with discrimination costless advertising is not followed by full market reach.

The profitability of price discrimination based on consumer purchase history when consumers are imperfectly informed is also analysed (Subsection 4.6.3). Our main finding here is that competing firms may benefit from behaviour-based price discrimination. Specifically, it is shown that at least when advertising costs are not too high price discrimination increases overall expected profits regardless of the advertising technology considered (Corollary 4). This result is in contrast with the prisoner's dilemma result obtained in the existing literature (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000) and chapter 3 in this thesis). On the one hand, because in the present model only one firm can engage in price discrimination, price competition in period 2 is less intense than in models where both firms try to poach each other's customers. On the other hand, the benefit of price discrimination in the second period motivates a firm to quote a high first-period price and moderates also price competition in period 1. As a result of lessened price competition, firms are expected to gain with poaching.

Also worth noting is the relationship between advertising costs and profits. In contrast to what happens in the extant models of informative advertising (e.g. Grossman and Shapiro (1984)) or in our benchmark case, where profits first increase with increases in advertising costs and then decrease; we show that with discrimination profits increase with decreases in advertising costs (Proposition 10).

Finally, we revisit with a new perspective the welfare issues associated with price discrimination based on customer recognition and informative advertising (Section 4.7). To our best knowledge there is no analytical research that examines the effects of price discrimination on the efficient properties of informative advertising. In this regard, under no discrimination, we show that firms select the social optimal level of advertising (Proposition 11). Interestingly, under price discrimination we show that, for any advertising cost function, private and social incentives to advertise only coincide if the advertising cost is such that either the firms either the regulator choose an intensity of advertising equal to 50%. Otherwise firms under or over advertise (Proposition 12).

A recurring policy question in the price discrimination literature is whether to encourage or ban price discrimination practices. A key point to understand this question in the context of competitive markets in which, without advertising, consumers are left out of the market, is by looking at the impact of discrimination on the firms' advertising choices. Our main finding, concerning the welfare effects of price discrimination, is that regardless of the advertising technology used by firms, permitting price discrimination is in general bad for consumers and overall welfare, though good for firms.

To summarise, the rest of the chapter is organised as follows. A brief survey of the relevant literature as well as the chapter main contributions appear in the next section. Section 4.3 presents the model and is followed by section 4.4 which provides some preliminary results associated with the benchmark case where discrimination is not permitted. Section 4.5 looks at the subgame perfect equilibrium when firms are permitted to engage in price discrimination in period 2. The competitive effects of price discrimination based on customer recognition in markets where advertising is a *sine qua non* condition for demand existence are discussed in Section 4.6. Issues as the efficient properties of advertising and the welfare effects of price discrimination are addressed in Section 4.7. Finally, Section 4.8 concludes and an Appendix presents the proofs that were omitted from the text.

4.2 Related literature

The model addressed in this chapter blends features from the following strands of the literature. One is the extensive literature on informative advertising and price competition in homogeneous product markets. The other strand is the recent literature on behaviour-based price discrimination in competitive markets.

Informative advertising According to this view, advertising plays an important role as it conveys relevant information—like product existence, availability and price—to otherwise uninformed customers. As Nelson (1974) points out, advertising can serve as a tool for transmitting this information to consumers and therefore should not be considered as an unnecessary activity. Whilst in some models advertising is a *sine qua non* condition for demand existence, in others the main purpose of advertising is to transmit information about price. Our model is mainly related to the former type of advertising models. Hence, in what follows, we will focus on those models in which without advertising consumers are left out of the market.[5]

In his seminal article, Stigler (1961) quoted that "*Price dispersion is a manifestation—and, indeed it is a measure of ignorance in the market.*" In that context, Stigler holds that advertising is a valuable source of information for consumers since it may reduce such ignorance. Stimulated by Stigler, Butters (1977) provides the first attempt to study how sellers can influence buyer information about product existence and price by advertising. Firms consciously make advertising decisions, but consumers passively receive advertisements.[6] He shows that in a monopolistically competitive model with homogeneous product, if firms must bear the cost of providing information to otherwise

[5]Within the stream of models where consumers are imperfectly informed only about price see, for instance, Bester and Petrakis (1995) and Moraga-González and Petrakis (1999). Basically, both works provide a duopoly model where though consumers know the existence of both firms, they are only informed about the price offered by the neighbourhood firm. The main role of advertising is, therefore, to transmit information about prices to consumers located at distant locations.

[6]For models in which consumers may obtain price information through advertising and costly search see, for instance, Robert and Stahl (1993). For models with no advertising and where consumers obtain relevant information (as prices) through sequential or fixed-sample search see for instance Stahl (1989,1996).

uninformed customers, the equilibrium in prices is characterised by price dispersion. In other words, as firms cannot provide full market coverage, the existence of imperfectly informed consumers leads firms to advertise different prices with different intensities.[7] Additionally, also stimulated by early controversial debates on the welfare effects of advertising (e.g. Kaldor (1950)), Butters investigates whether there is too much or too little advertising. Remarkably, he finds that the market equilibrium level of advertising is socially optimal. This puzzling result was confirmed by Stahl (1994) who extended the latter model to oligopolistic markets and with more general demand curves and advertising technologies. Variations on Butters's (1977) model such as the introduction of product differentiation (Grossman and Shapiro (1984)), or heterogeneity among buyers (Stegeman (1991)) were shown to easily offset this result and helped establish the idea that increased competition stimulated additional advertising (the *business stealing effect*), while the incapability of the firm to appropriate the social surplus it generates acts as a deterrent to advertising (the *nonappropriability of social surplus effect,* Tirole (1988)). In addition, apart from Grossman and Shapiro (1984) who find that in a differentiated-product industry there is a single price equilibrium, those authors that have extended the Butters's model without departing from the homogeneous product assumption also obtain equilibrium outcomes displaying price dispersion (e.g. Stegeman (1991) and Stahl (1994)).

Notwithstanding the economic literature on informative advertising has in general ignored the ability of firms to target advertising to specific groups of consumers in a market, the role of targeting as a mechanism that can improve the reach of the firm—and thus more efficiently bring information to selected market segments—has not been ignored in more recent studies in this field.[8]

[7]In other models displaying price dispersion advertising is treated as exogenous (e.g. Salop and Stiglitz (1977), Shilony (1977), Varian (1980) and Narasimhan (1988)). Equilibrium price dispersion of advertised prices arises due to consumers heterogeneities, which in Salop and Stiglitz (1977) and Varian (1980) is the result of differences in consumers' costs to obtain information. In Shilony (1977) and Narasimhan (1988), heterogeneities arise because some consumers have a brand preference for a particular firm.

[8]For studies of targeted advertising in monopolistic markets see Hernández-Garcia (1997) and Esteban, Gil and Hernández-Garcia (2001). These studies have concentrated on the ability of the monopolist to target ads to those consumers with a higher valuation for the good. For models with

However, the ability of a firm to target its ads opens up the possibility of sending messages with differentiated prices to different types of consumers. Price discrimination by means of targeted advertising becomes a possibility. To our knowledge the only works analysing together informative targeted advertising and price discrimination are Esteves and Guimarães (2001) and Iyer, *et al.* (2003). The former work addresses the welfare properties of informative advertising in a monopolistic model with two types of consumers—high and low valuation—and where price discrimination may be implemented by means of targeted advertising. They show that when targeting is imperfect the monopolist will overadvertise to the highest valuation group of consumers in detriment of the lowest valuation group. Only when targeting is perfect will the seller behave in a socially desirable way. Iyer, *et al.* (2003) examine a product differentiated market where advertising is also needed for demand and where price discrimination may be implemented through advertising. They assume that some consumers are loyal (or captive) to one firm, some other consumers are loyal to the other firm and the rest are comparison shoppers (or selective customers). Further, each firm can either inform the whole market, or target ads to distinct consumer groups. In this framework, they show that the unique equilibrium in prices, either with mass or targeted advertising, exhibits price dispersion. A key issue in their analysis is the profitability of price discrimination by means of advertising. When price discrimination is feasible, firms can quote the reservation price to their captive segment and a different one to the group of selective customers. They show that this policy gives rise to increased price competition in the latter group of consumers such that the gains realised in the captive group are eliminated by reduced profits from the selective group of customers. Remarkably, they find that price discrimination has no effect on firms' profits.

We contribute to the strand of literature on informative advertising in several respects. Firstly, we analyse a dynamic duopoly version of Butters model. The informative role of advertising has in general been analysed using static models.[9] However,

coupon targeting in duopoly settings see, for instance, Shaffer and Zhang (1995), Bester and Petrakis (1996) and Moraga-González and Petrakis (1999).

[9] An exception to static models is Roy (2001) who studies a two-period duopoly game where in

static models cannot tell us anything about the effects of using advertising as a long-run variable. Evidently, in real markets, when considering what level of advertising to provide, firms take into account the possibility of building a long term relationship with potential customers, which in turn leads firms to anticipate possible strategic effects. With that in mind, we address a two-period model where advertising is used as a long-run variable in the sense that by reaching a certain fraction of potential customers today, a firm determines also the fraction of customers that will be reached by its advertising campaign in the future. Moreover, when a firm anticipates the effects of price discrimination on current and future profits, it might change its advertising decision in relation to what would happen under a static analysis. Hence, by incorporating price discrimination into a model where advertising is used as a long-run variable we are able to understand *how* today's advertising decisions affect tomorrow's pricing and profits and also *how* price discrimination affects today's advertising decisions.

Secondly, our framework entails an endogenous partition of consumers into captive and selective. Therefore, it can be seen as an extension of Iyer, *et al* (2003) static model in which the proportion of captive and selective customers is exogenously given. In addition, by adopting a general advertising technology according to which firms may inform any fraction of the market we are able to yield novel insights on the welfare properties of informative advertising. It is worth noting that because Iyer, *et al.* (2003) consider an advertising technology that informs all of a given segment or none of it, they are silent about the welfare effects of price discrimination in markets where firms may inform any fraction of the market, as is usually the case.

Behaviour-based price discrimination We have seen elsewhere that the extant literature on this form of dynamic third-degree price discrimination has considered that purchase history is useful in disclosing important information about exogenous

period one firms send ads to consumers to generate awareness and then select their prices. Hence, the timing of the game suggests that advertising has, in fact, a long run nature. Advertising is used by firms to segment the market and to commit to not invade the "natural" market of the rival. Interestingly, he finds that the two firms divide the whole market into mutually exclusive captive segments, within which each firm operates as a local monopolist.

switching costs (e.g. Chen (1997) and Taylor (2003)) and product preferences (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000), Chen and Zhang (2002) and chapter 3 in this thesis). In this context, two approaches have been taken in the literature to model price discrimination based on purchase history in imperfectly competitive markets. Broadly speaking, authors like Chen (1997) and Taylor (2003) assume that despite goods being initially homogeneous, after purchase decisions have been taken, due to the existence of switching costs, consumers are in the next stage of the game partially locked in with their sellers. In this setting, when firms are able to recognise their own customers and separate them from the rival's ones, they may try to entice the latter group of customers to switch. A main finding in this type of models is that as firms realise that higher prices can be charged to locked in customers in the next stages, they have incentives to increase first-period market share at the expense of lower first-period prices (indeed, in Chen's model firms price below marginal cost in period 1). As a whole, in the switching-cost approach, price discrimination unambiguously makes firms worse off.

Authors like Fudenberg and Tirole (2000) and Villas-Boas (1999) were the pioneers to investigate the dynamics of behavior-based price discrimination in the horizontal differentiated products approach. In particular, Fudenberg and Tirole show that in the second period a firm always has the incentive to offer a lower price to the rival's previous customers, who have revealed, through their purchase in period 1, their preference for the rival's firm. As both firms try to poach each other's customers, competition in the second-period is more intense and all prices fall. When consumers are non-myopic, second-period poaching leads firms to compete less aggressively in the first-period, which translates into first-period prices above their non-discrimination counterparts.[10] Similar results are obtained by Villas-Boas (1999) who studies a related but infinite-period, overlapping-generations model in which firms can only discriminate between returning and non-returning customers. Regarding the profitability of poaching both

[10]It is worth to say that in Fudenberg and Tirole (2000), poaching leads to higher first-period prices because consumers are forward looking. Conversely, when consumers are myopic, first-period prices are the same regardless of whether or not poaching is permitted.

studies show that firms are clearly worse off when discrimination is permitted.

Yet within the brand-preference approach, Chen and Zhang (2002) revisit the profitability issue of behaviour-based price discrimination using a discrete version of the Fudenberg and Tirole's (2000) model. They assume that each firm has an exogenous "loyal" (i.e., captive) segment of the market, and that they compete for the remaining consumers, who are price-sensitive (i.e., selective) consumers with a reservation value lower than the loyals' one. In this context, they show that customer recognition only occurs if a firm's first-period price is high enough such that it is not accepted by all consumers. Hence, as price discrimination may benefit firms, they find that the pursuit of customer recognition motivates firms to compete less aggressively in period 1, which leads to higher first-period prices than if price discrimination was not allowed.

This chapter contributes to the literature in this field by analysing the dynamic effects of such form of price discrimination following a different approach. In many markets advertising and pricing decisions are taken together. Especially in new product markets, advertising may be the consumers' sole source of information and thus, after advertising decisions have been taken, *ex-ante* identical consumers become imperfectly informed. Unlike Chen and Zhang's (2002) model, in which the partition of customers is exogenous, we allow firms to endogenously segment customers into captive and selective (price sensitive). Moreover, unlike the switching costs approach, some consumers are locked in with a certain firm not due to switching costs but because they ignore the other firm. Finally, the existing models on behaviour-based price discrimination have assumed that firms can perfectly reach and with no cost all potential customers in a market. While this assumption may fit well some cases, it may be inappropriate in others. Then, it is relevant to depart from this assumption and to reevaluate the competitive effects of price discrimination based on customer recognition in markets where advertising plays an important informative role.

4.3 The model

There are two firms A and B, who produce a homogeneous good at a constant marginal cost which without loss of generality is assumed to be zero. There are two periods, 1 and 2, and the firms act to maximise the expected discounted value of their profits, using a common discount factor $\delta \leq 1$.

On the demand side of the market there is a large number of identical potential buyers, with mass normalised to one, who desire to buy at most one unit of the good in each period. Consumers have a common reservation price v, and they are, *a priori*, ignorant about the existence of firms' products and prices. Consumers will be actual buyers if they become aware of firms' product existence and if the price is not higher than v. In spite of the fact that consumers may in the real world acquire information in several ways, it is assumed that advertising is the consumers' sole source of information about the sellers' product existence and price.[11] Accordingly, a potential consumer cannot be an actual buyer unless firms invest in advertising. It is well known that, for new products, awareness is the first stage in creating demand for a product.

Therefore, following the informative view, advertising is a necessary activity for transmitting relevant information to otherwise uninformed consumers. Like in Butters (1977) each firm sends advertising messages which contain truthful[12] and full information about the existence of its products and price. Each consumer strategy is quite simple. Having received ads from the firms, each consumer acquires one unit of the product from the firm which advertises the lowest price, whenever that price is below v. In the event that a consumer is indifferent between the two firms, he selects one of the firms through a random process, assigning an equal chance to any of the firms that contacted him.

As mentioned, after firms have sent their ads there are three kinds of customers in each period. Some consumers are *captive* to a given firm because they are only aware

[11] One alternative interpretation is to assume that search costs are prohibitively high.

[12] This is guaranted by FTC regulation that prohibits advertisers from making false and deceptive statements about their products (*www.ftc.gov/bcp/conline/pubs/buspubs/ad-faqs.htm*).

about that firm. Other informed consumers, however, receive ads from both firms and are *selective*. Since these latter consumers have full information about each firm's product and price, they buy from the firm that offers them the highest surplus. Finally, the remaining consumers receive no ad from either firm, are *uninformed*; excluded from the market; and realise zero utility. Thereupon, by investing in advertising, firms endogenously determine the partition of *ex-ante* identical consumers into *captive* and *selective*.

In addition, because firms are in the market for more than one period, in the end of period 1 they may learn whether a previous contacted consumer is a old captive or a selective that bought from the rival previously. When a firm achieves that type of learning it will face, in the second period, two separate markets—that of its own captive customers and that of its rival's selective customers—and so profit maximisation requires that selective customers should be offered a lower second period price. Here, the fact that an informed consumer did not buy from the firm in the past reveals to that firm that he is a selective one—i.e. he received also an ad from the rival with a lower price. Likewise, this allows the firm to infer that all consumers that bought its product must be captive. Conversely, when the firm sells its product to all consumers that received one of its ads it learns nothing, thereby price discrimination is unfeasible in the next period.

Advertising in this model is used as a variable with a long-run nature. On the one hand, we allow firms to reach in period 2 the same consumers they reached in period 1. On the other hand, price discrimination being permitted and feasible, firms may implement behaviour-based price discrimination by means of a kind of behaviour-based targeted advertising—i.e., consumers with different past behaviour will receive targeted ads with different prices.

The game proceeds as follows. In the first-period firms choose advertising intensities and prices simultaneously and non-cooperatively. (Notice that each firm puts the same price in each of its period 1 ads.) In the second-period, firms observe each other's choice of first-period advertising intensity and price. In the no-discrimination benchmark analysis we assume that firms are constrained to choosing the same advertising intensities and prices. In contrast, in the price discrimination analysis we assume that firms are constrained to choosing the same advertising intensities but can choose different prices. (Thus, being price discrimination permitted, in the second-period firms send ads quoting different prices to informed customers with different past behaviour.) Thus, in period 2 each firm is able to reach again those consumers that were reached by its advertising campaign in period 1. Further, to simplify the analysis, firms reach the same customers with no additional cost, which means that advertising costs are sunk in period 1.[13]

Before proceeding we look next at the different advertising cost functions that have been used in the literature and that will be used throughout the analysis.

Advertising technologies Advertising is a costly activity for firms. The cost of reaching fraction ϕ of consumers is given by the function $A(\phi) = \alpha\eta(\phi)$. Following the extant literature on informative advertising (e.g. Grossman and Shapiro (1984)) it is assumed that the cost of reaching consumers increases at an increasing rate, which formally can be written by $\frac{\partial A(\phi)}{\partial \phi} = A_\phi > 0$ and $\frac{\partial^2 A(\phi)}{\partial \phi^2} = A_{\phi\phi} \geq 0$. The latter condition means that it is increasingly expensive to inform an additional customer, or equivalently, to reach a higher proportion of customers. Several justifications support this assumption. One justification has to do with the advertising technology itself. In other words, if ads are sent randomly at a fixed cost per ad, then the probability of reaching a consumer not yet informed decreases with the amount already advertised. Examples are the Butters's urn technology and the *Constant Reach Independent Readership* (CRIR) technology proposed by Grossman and Shapiro (1984). Other justifications are the existence of different predispositions to view ads on the part of the target pop-

[13]This assumption fits well the following cases. After firms have sent their ads, a reached consumer will be directed to the firm's website. If he is interested in the firm's product, he may be asked to fill a registration form in which the email may be one of the requirements. The next time, empowered with the consumers' email accounts, the firm may send its ads by email with almost no cost.

Additionally, firms have nowadays access to syndicated vendors of information about potential consumers. Hence, firms may decide to buy external databases with a certain market coverage for advertising proposes. In this case, the cost of advertising is entirely borne in the beginning of the game, since thereafter firms can contact the same consumers with no additional cost.

ulation and the possibility of media saturation. It is also assumed that $A(0) = 0$, or in words, there are no fixed costs in advertising. These assumptions hold for all specific advertising cost functions that have been used in the literature, including the CRIR technology proposed by Grossman and Shapiro (1984), the Butters's technology, and the Quadratic Advertising cost function proposed in Tirole (1988). Finally, in order to advertising be viable it is assumed that $A_\phi(0) < v$. In a two-period game this latter assumption translates into $A_\phi(0) < v(1+\delta)$.

Different types of online advertising—banner ads, sponsorship advertising, keyword-related search advertising, and so on—may reasonably be represented by the Butters or even by the quadratic technology. Despite this latter technology is not based upon an underlying technology of message production, as happens with the Butters's technology, it has the advantage of being extremely simple to manipulate algebraically. It is given by $A(\phi) = \alpha\eta(\phi)$ where $\eta(\phi) = \phi^2$. When the classic urn technology proposed by Butters is the most adequate the advertising cost function is $A(\phi) = \alpha\eta(\phi)$ where $\eta(\phi) = \ln\left(\frac{1}{1-\phi}\right)$. Butters (1977) notes that messages are sent out in a purely random fashion at a fixed cost per unit. If L messages are sent to M buyers and both L and M are large, then the fraction of buyers who do not receive any ad is $1-\phi = (1-\frac{1}{M})^L \simeq e^{-\frac{L}{M}}$. If each ad has a fixed cost of λ then the total advertising cost is λL, or in terms of ϕ, $\lambda M \ln\left(\frac{1}{1-\phi}\right)$. If we let α equal λM we obtain that the cost of informing ϕ consumers is $A(\phi) = \alpha \ln\left(\frac{1}{1-\phi}\right)$. Because, in our model the number of buyers is normalised to one, α can be identified with the cost per ad. In what follows, whenever a functional form will be needed we will make use of one of these technologies.

4.4 Benchmark case: non-discrimination

Before proceeding to the study of poaching with advertising, we first establish the benchmark case in which, even if feasible in period 2, price discrimination is not permitted. (As mentioned each firm puts the same price in each of its period 1 ads.) Throughout the chapter we will use this benchmark case to assess the competitive and welfare effects of price discrimination enabled by informative advertising. To solve for the equilibrium without discrimination we start looking at the behaviour of firms in a static game.

Since the product is homogeneous, buyers care only about the price. It can be said that when consumers know the two firms' prices, they are indifferent between firms if they quote the same price, otherwise, they prefer the lowest-priced firm. As afore mentioned after firms have sent their ads independently (i.e., advertising reach is independent for each firm), a proportion ϕ_i and ϕ_j of customers is reached and then firm i's potential demand is made of a group of captive (locked-in) customers, namely $\phi_i\left(1-\phi_j\right)$ and of a group of selective customers, namely $\phi_i\phi_j$. It follows that because a firm is not able to distinguish a captive from a selective customer, it faces a trade-off between quoting a lower price and competing for the segment of selective customers; or setting a higher price and extract surplus from its captive segment. In this context, we may establish the next proposition.

Proposition 1. *There is no symmetric Nash equilibrium in pure strategies in prices.*[14]

Proof. See the Appendix.

Although a pure strategy equilibrium in prices fails to exist there is a symmetric Nash equilibrium in prices involving mixed strategies. In what ensues, we follow a reasoning similar to Butters (1977) to show the existence of such an equilibrium.[15] Suppose that firm i selects a random price from the distribution function $F_i(p)$ with support $[p_{\min}, v]$. Since the marginal cost of production is assumed to be null, then no ads will be sent specifying prices below the marginal cost of advertising or above v. Given firm B's price and advertising strategies, firm A's expected profit is equal to

$$E\pi_A = p\phi_A\left(1-\phi_B\right) + p\phi_A\phi_B\left[1-F_B(p)\right] - A(\phi_A). \tag{4.1}$$

[14] More precisely, a price equilibrium in pure strategies fails to exist unless firms' advertising gives rise to perfect information, that is to say unless $\phi_i = \phi_j = 1$.

[15] For a similar derivation of the mixed strategy equilibrium in prices, but with exogenous captive and selective consumers see for example Shilony (1977), Varian (1980) and Narasimhan (1988).

In equilibrium firm A should be indifferent between quoting any price that belongs to the equilibrium support. When it chooses price v it knows that this price will be only accepted by a consumer who is not aware of any other price (i.e. a captive consumer). In this case its expected profit is equal to $v\phi_A(1-\phi_B)$. This implies that in equilibrium we must observe,

$$p(1-\phi_B)+p\phi_B[1-F_B(p)]=v(1-\phi_B).$$

Solving for $F_B(p)$, we get

$$F_B(p)=1-\frac{(v-p)(1-\phi_B)}{p\phi_B}. \tag{4.2}$$

From $F_B(p_{\max})=1$ and $F_B(p_{\min})=0$ it follows that $p_{\max}=v$ and $p_{\min}=v(1-\phi_B)$.

The equilibrium level of advertising maximises (4.1) with respect to ϕ_A. Plugging the expression that defines $F_B(p)$, the first-order condition is

$$v(1-\phi_B)=A_\phi.$$

Therefore, we obtain that under symmetry

$$v(1-\phi)=A_\phi \text{ or, equivalently, } p_{\min}=A_\phi. \tag{4.3}$$

It is worth noting that the condition that defines the equilibrium level of advertising is simply the same we would obtain by solving the Butters's static model with only two firms. Basically, the equilibrium condition for advertising, $v(1-\phi)=A_\phi$, stipulates that in equilibrium firms should advertise up to the point where the cost of the last ad sent equals the expected revenue of a sale at the highest price to an uninformed consumer.

Turning now to the two-period game without discrimination, as we assume that in the second-period firms are constrained to choosing the same advertising intensities and *prices* we find that in period 2 expected profits say for firm A are again equal to

$v\phi_A(1-\phi_B)$. Therefore, overall expected profit for that firm is

$$p\phi_A(1-\phi_B)+p\phi_A\phi_B[1-F_B(p)]-A(\phi_A)+\delta v\phi_A(1-\phi_B). \tag{4.4}$$

Once more, in equilibrium, firm A should be indifferent between quoting any price that belongs to the equilibrium support. When it chooses price v its expected profit is now equal to

$$v(1+\delta)\phi_A(1-\phi_B)-A(\phi_A).$$

In equilibrium we must observe

$$\begin{aligned} p\phi_A(1-\phi_B)+p\phi_A\phi_B[1-F_B(p)]-A(\phi_A)+\delta v\phi_A(1-\phi_B) &= v(1+\delta)\phi_A(1-\phi_B)-A(\phi_A) \\ p(1-\phi_B)+p\phi_B(1-F_B(p)) &= v(1-\phi_B), \end{aligned}$$

from which it follows that

$$F_B(p)=1-\frac{(v-p)(1-\phi_B)}{p\phi_B}, \tag{4.5}$$

which is the same expression we have obtained in the static game. The equilibrium level of advertising maximises (4.4) with respect to ϕ_A. From the first-order condition it follows that the equilibrium level of advertising is implicitly given by,[16]

$$v(1+\delta)(1-\phi_B)=A_\phi.$$

To summarise, if we let the superscript *nd* identify the no-discrimination case, it ensues that under symmetry:

Proposition 2. *In the benchmark case without discrimination,*

(i) there exists a symmetric mixed strategy subgame perfect Nash equilibrium in prices described as follows. In each period, each firm chooses a price randomly from

[16] Second-order conditions also hold since $-A_{\phi\phi}<0$.

the distribution

$$F^{nd}(p) = \left\{ \begin{array}{ccc} 0 & if & p < v\left(1-\phi^{nd}\right) \\ 1-\frac{(v-p)\left(1-\phi^{nd}\right)}{p\phi^{nd}} & if & v\left(1-\phi^{nd}\right) \leq p \leq v \\ 1 & if & p > v \end{array} \right\} \tag{4.6}$$

with minimum equilibrium price equal to

$$p^{nd}_{\min} = v\left(1-\phi^{nd}\right). \tag{4.7}$$

(ii) There is a symmetric Nash equilibrium in pure strategies in which each firm chooses an advertising reach $\phi^{nd} \in (0,1)$ *defined by*

$$v\,(1+\delta)\left(1-\phi^{nd}\right) = A_{\phi}\left(\phi^{nd}\right) \tag{4.8}$$

where $A_{\phi}(0) \leq v\,(1+\delta)$.

(iii) Each firm earns overall expected equilibrium profits equal to

$$E\Pi^{nd} = v\,(1+\delta)\,\phi^{nd}\left(1-\phi^{nd}\right) - A\left(\phi^{nd}\right) = \phi^{nd}A_{\phi} - A\left(\phi^{nd}\right) \tag{4.9}$$

The next corollary summarises the effects of changes in advertising costs on the equilibrium outcomes when firms cannot engage in price discrimination.

Proposition 3. (Effects of advertising costs)

(i) As advertising becomes costless, i.e. as α *approaches 0,* $\phi^{nd} \rightarrow 1$, $p^{nd}_{\min}$ *and expected profits approach zero and* F^{nd} *converges to unit mass at zero;*

(ii) As $A_{\phi}(0)$ *approaches* $v\,(1+\delta)$, ϕ^{nd} *and expected profits approach zero,* $p^{nd}_{\min}$ *approaches* v *and* F^{nd} *converges to unit mass at* v.

(iii) Apart from the costless advertising case, the equilibrium level of price dispersion, measured by the range of advertised prices, is greater in markets with lower

advertising costs.

Proof. See the Appendix.

The intuition for this result is straightforward. As advertising becomes cheaper, firms invest in more advertising. The group of selective buyers increases and firms compete more aggressively on prices. In fact, when the advertising cost is null the equilibrium approaches the Bertrand outcome. Conversely, as advertising costs rise, advertising decreases and higher prices are more likely.

Part (iii) of proposition 3 suggests that in fact even in homogeneous product markets, where price is the dominant factor in consumers' buying decisions, price dispersion may remain persistent and substantial in online markets. Bynjolfsson and Smith (2000a) argue that the observed price dispersion in online markets is largely explained by awareness, branding and trust—factors greatly affected by the promotion activities of online sellers, such as advertising. Clearly, our model gives support to the explanation of price dispersion based on awareness. One of the more widely used measures of price dispersion for online markets has been the range of prices (e.g. Brynjolfsson and Smith (2000a,b) and Baye and Morgan (2001)) which in this case is given by $v - p^{nd}_{\min}$. Based on this measure it is easy to establish a relationship between advertising costs and the level of observed price dispersion. Remarkably, Proposition 3 predicts that we should observe higher levels of price dispersion in markets characterised by low costs of information transmission. This result is in contrast with the usual conclusion drawn about the relationship between the competitiveness of markets and price dispersion. While it is a common practice to associate more price dispersion with less competitive markets, in our model we find that greater price dispersion is expected in markets where advertising is cheaper. A similar outcome is obtained by Baye and Morgan (2001), who show that price dispersion is greater as it becomes less costly for firms to list prices at price-comparison sites. As it becomes less costly to provide information to customers, more consumers will be informed. Thus, as more and more people have access to more information prices tend to fall. It is then easy to see that, as an immediate consequence, there is a reduction of the lower bound of the distribution of prices, which gives rise to a wider range in advertised (or listed) prices. Hence, this result challenges the common view that price dispersion should decrease as more consumers become informed. Nevertheless, it lends support to empirical and theoretical findings showing that price dispersion is persistent online even when search costs (in this case advertising costs) are significantly reduced and more consumers become better informed about prices (e.g. Brynjolfsson and Smith (2000a) and Baye, Morgan and Sholton (2002), Baye and Morgan (2001)).

The ensuing section extends the model analysed so far by allowing firms, if feasible, to price discriminate in period 2.

4.5 Equilibrium analysis of poaching with advertising

Price discrimination will be feasible in the second period of the game if two conditions hold. First, if at least one of the firms learns something from the previous period, that is to say, if at least one firm has the ability to distinguish an old captive customer from a selective one that bought from the rival before. Second, if the firm can separate customers with different past behaviour and target ads with different prices to different types of customers.

Considering that firms are forward-looking $(\delta > 0)$, the two periods are not independent from a firm's perspective. When choosing its price and advertising strategies in period 1, each firm anticipates the resulting Nash equilibrium in prices in period 2. As usual, to derive the subgame perfect equilibrium, the game is solved working backward from the second period.

4.5.1 Second-period pricing game

Depending on first period pricing and advertising decisions, two scenarios are possible in period 2 for each firm. In one scenario, firm i $(i = A, B)$ is the lowest-priced firm in period 1, thereby it sells to its captive group of customers as well as to the entire group of selective customers. In this manner, first-period demand for the highest-priced firm—in this case firm j $(i \neq j)$—comes exclusively from its captive segment. Since the lowest-priced firm sells to all consumers that received one of its ads, it learns nothing and, in consequence, it cannot engage in price discrimination in period 2. In opposition, the highest-priced firm in period 1 learns that *some* consumers that received one of its ads did not buy its product. Likewise, it also infers that those customers that bought its product did not receive any of the rival's ads and, for that reason, they must be captive. Based on this reasoning, the highest-priced firm in period 1 is able to sort out customers into captive customers and selective customers that bought from the rival previously. Accordingly, this latter firm will try to poach some of the rival's previous customers and exploit its captive customers. Assuming that it can reach again the same customers, it will tailor ads with different content—i.e., different prices—to old and selective customers that bought from the competitor before. Summing up, the highest-priced firm in period 1 will be the discriminating firm in period 2.

For a given price p_i chosen by firm i in period 1, firm i is the lowest-priced firm in period 1 (or, the non-discriminating firm in period 2) with probability equal to $\Pr(p_i \leq p_j) = 1 - F_j(p_i)$; and it is the highest-priced firm in period 1 (or, the discriminating firm in period 2) with probability equal to $\Pr(p_i > p_j) = F_j(p_i)$.

Subgame 1: Firm i is the discriminating firm

We have seen that firm i is the discriminating firm in period 2 if it is the highest priced firm in period 1. While firm i's first-period demand embraces uniquely its captive customers, namely $\phi_i(1-\phi_j)$, firm j's demand comprises its captive customers as well as the selective group of customers, that is $\phi_j(1-\phi_i)+\phi_i\phi_j$. In this case, in the second stage of the game firm i is able to recognise old customers and selective customers that bought from j before, and to entice the latter customers to switch, by offering them a possibly lower price. In doing so, firm i will advertise a different price to its old captive customers and to some of the rival's previous customers (only the selective customers). Let us denote by p_i^o firm i's price to its old customers and p_i^r firm i's price to the rival's customers. Regarding firm j's pricing strategy, because it is neither able to reach any of the rival's previous customers nor to recognise different types of customers, it is forced to advertise the same price for all customers that were reached by its advertising campaign in period 1. We denote by $\widehat{p}_j$ firm j's non-discrimination price.

Corollary 1. *The discriminating firm will charge its old captive customers the highest possible price, $p_i^o = v$, regardless of the price it charges to some of the rival's customers.*

The proof of this result is quite simple. Firm i's ability to fully separate its captive customers from selective customers that bought from the rival before, together with firm j's incapability to reach any of firm i's captive customers, allow firm i to charge its captive customers their reservation price, without fearing any poaching attempt by its rival.

Then, firm i's profit from its old captive customers, denoted by π_i^o is equal to

$$\pi_i^o = v\phi_i \left(1 - \phi_j\right). \tag{4.10}$$

Since consumers remain anonymous to firm j, it is forced to advertise the same price for all consumers. Thus, if we look on second-period price competition for this group of consumers, which includes all selective consumers and firm j's captive consumers we may establish the following proposition.

Proposition 4. *There is no pure strategy equilibrium regarding the group of the non-discriminating firm's previous customers.*

Proof. See the Appendix.

The intuition behind the above result runs as follows. Considering price competition for the segment of firm j's old customers, we realise that firm i is able to build market share, by offering a better price to the group of selective customers that bought from firm j before, without damaging the profit it can extract from its captive segment. When deciding what price to target in order to poach the entire group of selective customers, firm i takes into account its rival's pricing behaviour. Even though firm j can always guarantee itself a profit equal to $v\phi_j(1-\phi_i)$, by selling at the highest price only to its captive segment, the lure of getting the additional demand that comes from the selective group is responsible for the failure of a pure strategy equilibrium. In other words, the presence of a positive fraction of selective consumers creates a tension between the firm's incentives to price low in order to attract the selective customers and to price high in order to extract rents from its locked in customers. This tension results in an equilibrium displaying price dispersion. The proof of such an equilibrium is by construction.

Let $G_i^r(\widehat{p}_j) = \Pr(p_i^r \leqq \widehat{p}_j)$ be the probability of firm i quote a price no higher than $\widehat{p}_j$. Additionally, let $p_{i\,\min}^r$ and $p_{i\,\max}^r$ be, respectively, the lowest and the highest price advertised by firm i with positive density in equilibrium. Then, the equilibrium support of prices in which $G_i^r(\widehat{p}_j)$ is defined is $[p_{i\,\min}^r, p_{i\,\max}^r]$. Similarly let $\widehat{G}_j(p_i^r) = \Pr(\widehat{p}_j \leqq p_i^r)$, with support $[\widehat{p}_{j\,\min}, v]$. In a mixed strategy equilibrium, given the distribution of advertised prices by its competitor, each firm must be indifferent among prices advertised with positive density. That is, all prices that belong to the equilibrium support must yield the same expected profit, given the behaviour of the other firm.

As previously mentioned, firm j can guarantee itself a profit equal to $v\phi_j(1-\phi_i)$ by focusing only on its captive group. Thus, in equilibrium firm j must be indifferent between quoting price v (and sell only to its captive customers) and quoting a lower price with the aim of serving again captive and selective consumers. Formally, the minimum price the non-discriminating firm, in this case firm j, will be willing to advertise in equilibrium must satisfy the following condition:

$$\widehat{p}_{j\,\min}\left(\phi_j(1-\phi_i)+\phi_i\phi_j\right) = v\phi_j(1-\phi_i),$$

from which we get that:

$$\widehat{p}_{j\,\min} = v\,(1-\phi_i)\,. \tag{4.11}$$

What is then the minimum price firm i can charge in order to grab the entire group of selective consumers? It is a dominated strategy for firm i to set a price below $\widehat{p}_{j\,\min}$. Quoting $p_i^r = \widehat{p}_{j\,\min}$ allows firm i to poach the entire group of selective consumers and makes firm j indifferent. Therefore, firm i's minimum price for its rival's previous customers is equal to $\widehat{p}_{j\,\min}$ and its expected profit is also equal to $\widehat{p}_{j\,\min}\phi_i\phi_j = v\,(1-\phi_i)\,\phi_i\phi_j$. Thus, in equilibrium for the non-discriminating firm we must observe:

$$\widehat{p}_j\phi_j\,(1-\phi_i) + \widehat{p}_j\phi_i\phi_j\,[1 - G_i^r(\widehat{p}_j)] = v\phi_j\,(1-\phi_i)\,,$$

from which we obtain:

$$G_i^r(\widehat{p}_j) = 1 - \frac{(v-\widehat{p}_j)\,(1-\phi_i)}{\widehat{p}_j\phi_i}, \tag{4.12}$$

with $G_i^r(\widehat{p}_{j\,\min}) = 0$ and $G_i^r(v) = 1$.

For the discriminating firm i, in equilibrium we must observe:

$$p_i^r\phi_i\phi_j\left[1 - \widehat{G}_j\,(p_i^r)\right] = \widehat{p}_{j\,\min}\phi_i\phi_j$$

which yields,

$$\widehat{G}_j\,(p_i^r) = 1 - \frac{\widehat{p}_{j\,\min}}{p_i^r} = 1 - \frac{v\,(1-\phi_i)}{p_i^r}, \tag{4.13}$$

with $\widehat{G}_j\,(p_i^r = \widehat{p}_{j\,\min}) = 0$ and $\widehat{G}_j\,(v) = 1 - (1-\phi_i) < 1$. In consequence, firm j has a mass point at v.

Proposition 5. *When one firm can engage in price discrimination whilst the other cannot, competition within the group of the non-discriminating firm's previous customers, gives rise to an asymmetric mixed strategy equilibrium in prices.*

(i) The non-discriminating firm chooses a price randomly from the distribution

$$\widehat{G}_j\left(p_i^r\right) = \left\{ \begin{array}{ccc} 0 & for & p_i^r < \widehat{p}_{j\min} \\ 1 - \frac{v(1-\phi_i)}{p_i^r} & for & \widehat{p}_{j\min} \leqq p_i^r \leqq v \\ 1 & for & p_i^r > v \end{array} \right\} \tag{4.14}$$

with support $\left[\widehat{p}_{j\min}, v\right]$, *and the discriminating firm chooses a price randomly from the distribution*

$$G_i^r(\widehat{p}_j) = \left\{ \begin{array}{ccc} 0 & for & \widehat{p}_j < \widehat{p}_{j\min} \\ 1 - \frac{(v-\widehat{p}_j)(1-\phi_i)}{\widehat{p}_j\phi_i} & for & \widehat{p}_{j\min} \leqq \widehat{p}_j \leqq v \\ 1 & for & \widehat{p}_j \geqq v \end{array} \right\} \tag{4.15}$$

with support $\left[\widehat{p}_{j\min}, v\right)$ *where*

$$\widehat{p}_{j\min} = v\left(1 - \phi_i\right). \tag{4.16}$$

The non-discriminating firm has a mass point at v *equal to*

$$m_j = \left(1 - \phi_i\right). \tag{4.17}$$

(ii) Expected profit for the discriminating firm, coming from poached customers equals

$$\pi_i^r = v\left(1 - \phi_i\right)\phi_i\phi_j, \tag{4.18}$$

and expected profit for the non-discriminating firm equals

$$\widehat{\pi}_j = v\phi_j\left(1 - \phi_i\right). \tag{4.19}$$

Therefore, total second-period expected profit for the discriminating firm equals

$$\pi_i = \pi_i^o + \pi_i^r = v\phi_i\left(1 - \phi_j\right) + v\left(1 - \phi_i\right)\phi_i\phi_j. \tag{4.20}$$

It can be said that in this equilibrium the non-discriminating firm uses a "Hi-Lo" pricing strategy. To squeeze more surplus from its locked in customers, it charges the highest price v, with probability m_j; which increases with the size of that segment. However, in order to avoid being poached it quotes occasionally a low price.

Corollary 2. *From the equilibrium distribution functions defined by (4.14) and (4.15) it follows that:*

(i) $\widehat{G}_j(p) < G_i^r(p)$, *that is,* $\widehat{G}_j(p)$ *first-order stochastically dominates* $G_i^r(p)$;

(ii) $E(\widehat{p}) > E(p^r)$; *and*

(iii) the mass point m_j *is decreasing in* ϕ_i.

Proof. See the Appendix.

In words, part (i) states that $\widehat{p}_j$ is stochastically larger than p_i^r because it assumes large values with higher probability. As a consequence, regarding only price competition between firms due to poaching, (ii) says that on average the non-discriminating firm charges higher prices than its competitor. Indeed, the discriminating firm has an advantage over its rival. It sets lower prices on average because it is able to entice some of the rival's customers, by offering them a low price, without damaging the profit that comes from its locked in segment. On the contrary, when the non-discriminating firm decreases its price, in order to prevent poaching, it damages the profit that comes from its captive segment. As the discriminating seller has less to lose it is more aggressive and, therefore, it charges on average lower prices.

Not surprisingly, part (iii) states that the probability of the non-discriminating firm will charge the reservation price v is higher the greater is its captive group of customers. Notice that there exists a negative relationship between a firm's captive segment size and the rival's advertising reach. Clearly, when the rival firm increases its advertising reach, the group of captive customers falls down, as more consumers will be aware of both firms. For that reason, when the discriminating firm increases its advertising reach in period 1, it is less likely that the non-discriminating firm will quote price v.

Corollary 3. *With symmetric first period advertising intensities, the firm that can engage in price discrimination earns higher profits than the firm that cannot.*

From (4.19) and (4.20) it is trivial to see that under symmetry (i.e. $\phi_i = \phi_j$) it follows that $\pi_i > \widehat{\pi}_j$.

Subgame 2: Firm i is the non-discriminating firm

In this subgame firm i is the firm offering the better deal in period 1 and, therefore, it captures all informed consumers except those that are locked in with firm j. Now firm i is not able to recognise consumers in period 2 and, for that reason, it is not able to price discriminate. Conversely, firm j is now the price discriminating firm. It immediately follows that in comparison to the previous subgame firms interchange positions with each other. Using our previous computations we have that expected second-period profit for the non-discriminating firm, now firm i, is equal to

$$\widehat{\pi}_i = v\phi_i\left(1 - \phi_j\right). \tag{4.21}$$

Likewise, expected second-period profit for the discriminating firm, now firm j, is equal to

$$\pi_j = \pi_j^o + \pi_j^r = v\phi_j\left(1 - \phi_i\right) + v\left(1 - \phi_j\right)\phi_i\phi_j \tag{4.22}$$

with $\pi_j^o = v\phi_j\left(1 - \phi_i\right)$ and $\pi_j^r = v\left(1 - \phi_j\right)\phi_i\phi_j$.

Looking at the difference between a firm's second-period profit with and without discrimination we have

$$\pi_i - \widehat{\pi}_i = v\left(1 - \phi_i\right)\phi_i\phi_j > 0.$$

Since the above difference is positive we have a measure of the benefit of engaging in price discrimination. Obviously, this benefit will give a firm an incentive to pursue price discrimination when devising its pricing and advertising behaviour in the first-stage of the game. Remarkably, the benefit of engaging in price discrimination motivates a firm to be the highest priced firm in period 1. In fact, we will see below that the race between firms for becoming the discriminating firm, may result in higher first-period prices in comparison to the non-discrimination case.

To examine how a firm's advertising decision in period 1 affects the benefit of price discrimination in period 2, notice that $\frac{\partial(\pi_i - \widehat{\pi}_i)}{\partial \phi_i} = v(1-2\phi_i)\phi_j$ through which we obtain that the benefit of engaging in price discrimination increases with ϕ_i until reach its maximum value at $\phi_i = 0.5$, and then decreases.

We may now compute the overall expected profit for a representative firm in the second-stage of the game. As seen before firm i is the discriminating firm in period 2 if it advertises the highest price in period 1, which occurs with probability equal to $\Pr(p_i \geq p_j) = F_j(p_i)$. With the remaining probability firm i advertises the lowest price, whereby it is the non-discriminating firm in period 2. This occurs with probability equal to $[1 - F_j(p_i)]$. Hence, firm i's total expected profit in period 2 denoted by π_i^2 is equal to

$$\pi_i^2 = F_j(p_i)\pi_i + [1 - F_j(p_i)]\widehat{\pi}_i = \widehat{\pi}_i + F_j(p_i)\left[\underbrace{\pi_i - \widehat{\pi}_i}_{\text{Benefit of price discrimination}}\right],$$

which simplifies to

$$\pi_i^2 = v\phi_i(1-\phi_j) + F_j(p_i)v(1-\phi_i)\phi_i\phi_j. \tag{4.23}$$

4.5.2 First-period pricing and advertising

In period 1, a firm makes its pricing and advertising decisions, rationally anticipating how such decisions will affect the firm's profit in the second period. We have seen that the benefit of engaging in price discrimination in period 2 may motivate a firm to price high in period 1. With that in mind, one can conjecture whether the monopoly price v is a pure strategy equilibrium of this game. In fact, given the similarity of this game with that in the benchmark case, it seems plausible to infer that if a pure strategy equilibrium exists, it must be at $(p_A, p_B) = (v, v)$.

Proposition 6. *There is no subgame perfect Nash equilibrium in which both firms set the monopoly price. Moreover, there is no subgame perfect Nash equilibrium in pure strategies.*

Proof. See the Appendix.

If there was such a pure strategy equilibrium in period 1, both firms would quote the same first period price and therefore they would share equally the group of selective consumers. In period 2, within the group of old customers, no firm would be able to distinguish the fraction of selective and captive customers. However, each firm would be able to recognise those selective customers that bought from the rival before. As a result, in the second period both firms could engage in price discrimination, as they could advertise a different price for old and the rival's previous (selective) customers. As the reader can see in the appendix provided, when both firms advertise price v in period 1, it is always profitable for a given firm to decrease slightly its price to $v - \varepsilon$ and capture the remaining selective customers. The increase in first-period profit due to deviation is higher than the decrease in second-period profit as a result of becoming the non-discriminating firm. For that reason, the monopoly price is not a pure strategy of this game.

Nonetheless, there is a mixed strategy equilibrium. As before, the existence of such an equilibrium is proved by construction. Overall expected profit for firm i when it charges first-period price p_i, uses a discount factor equal to δ and its competitor charges first-period price equal to p_j according to $F_j(p_i)$ is equal to:

$$E\Pi_i = \underbrace{p_i\phi_i\left(1-\phi_j\right) + p_i\phi_i\phi_j\left[1-F_j\left(p_i\right)\right] - A(\phi_i)}_{\text{First-period expected profit}} + \delta\underbrace{\left[v\phi_i\left(1-\phi_j\right) + F_j\left(p_i\right)v\left(1-\phi_i\right)\phi_i\phi_j\right]}_{\text{Second-period expected profit}}.$$

Following a similar reasoning as before, in equilibrium firm i must be indifferent between quoting any price that belongs to its equilibrium support, where $p_i \in [p_{i\,\min}, v]$. When it chooses the highest possible price v in period 1, its overall expected profit is equal to

$$
\begin{aligned}
& v\phi_i\left(1-\phi_j\right)+v\phi_i\phi_j\left[1-F_j\left(v\right)\right]-A(\phi_i)+\delta\left[v\phi_i\left(1-\phi_j\right)+F_j\left(v\right)v\left(1-\phi_i\right)\phi_i\phi_j\right] \\
= \; & v\phi_i\left(1-\phi_j\right)\left(1+\delta\right)-A(\phi_i)+\delta v\left(1-\phi_i\right)\phi_i\phi_j.
\end{aligned}
$$

Thus, in equilibrium we must observe

$$
\begin{aligned}
& p_i\phi_i\left(1-\phi_j\right)+p_i\phi_i\phi_j\left[1-F_j\left(p_i\right)\right]-A(\phi_i)+\delta\left[v\phi_i\left(1-\phi_j\right)+F_j\left(p_i\right)v\left(1-\phi_i\right)\phi_i\phi_j\right] \\
= \; & v\phi_i\left(1-\phi_j\right)\left(1+\delta\right)-A(\phi_i)+\delta v\left(1-\phi_i\right)\phi_i\phi_j,
\end{aligned}
$$

from which we obtain

$$
F_j\left(p_i\right)\phi_i\phi_j\left(\delta v\left(1-\phi_i\right)-p_i\right)=\left(v-p_i\right)\phi_i\left(1-\phi_j\right)+\delta v\left(1-\phi_i\right)\phi_i\phi_j-p_i\phi_i\phi_j.
$$

Solving for $F_j\left(p_i\right)$ we obtain

$$
F_j\left(p_i\right)=1-\frac{\left(v-p_i\right)\left(1-\phi_j\right)}{\phi_j\left(p_i-\delta v\left(1-\phi_i\right)\right)}. \tag{4.24}
$$

As expected if the discount factor is zero, we get the same distribution function as in the static case (see equation (4.2)). From the conditions which establish that $F_j\left(p_{i\min}\right)=0$ and $F_j\left(p_{i\max}\right)=1$, we have:

$$
p_{i\min}=v\left(1-\phi_j\right)+\delta v\phi_j\left(1-\phi_i\right) \tag{4.25}
$$

and

$$
p_{i\max}=v. \tag{4.26}
$$

Looking now at the advertising strategy of firms in period 1, firm i maximises its overall expected profit with respect to ϕ_i. Plugging into firm i's overall expected profit the expression that defines $F_j(p_i)$ we obtain:

$$\begin{aligned} E\Pi_i &= \left[p_i\phi_i\left(1-\phi_j\right)+p_i\phi_i\phi_j\left(\frac{(v-p_i)\left(1-\phi_j\right)}{\phi_j\left(p_i-\delta v\left(1-\phi_i\right)\right)}\right)\right]-A(\phi_i) \\ &+\delta\left[v\phi_i\left(1-\phi_j\right)+\left(1-\frac{(v-p_i)\left(1-\phi_j\right)}{\phi_j\left(p_i-\delta v\left(1-\phi_i\right)\right)}\right)v\left(1-\phi_i\right)\phi_i\phi_j\right] \end{aligned}$$

Therefore, first-order condition with respect to ϕ_i is given by

$$\begin{aligned} A_\phi &= -p_i\left(-1+\phi_j\right)v\frac{-p_i\delta+2p_i\delta\phi_i+\delta^2v-2\delta^2v\phi_i+\delta^2v\phi_i^2+p_i-\delta v}{\left(p_i-\delta v+\delta v\phi_i\right)^2} \\ &-\delta v\left(-1+\phi_j\right)-\delta 2v\phi_j\phi_i+\delta v\phi_j \\ &+\delta\left(-v+p_i\right)\left(-1+\phi_j\right)v\frac{-2\delta v\phi_i+\delta v\phi_i^2+2\phi_i p_i-p_i+\delta v}{\left(p_i-\delta v+\delta v\phi_i\right)^2} \end{aligned}$$

which simplifies to

$$v\left(1-\phi_j\right)+v\delta\left(1-2\phi_j\phi_i\right)=A_\phi. \tag{4.27}$$

Second order condition also holds, since $\frac{\partial^2 E\Pi_i}{\partial\phi_i^2}=-2v\delta\phi_j-A_{\phi\phi}<0$. In sum, the following proposition characterises the symmetric subgame perfect equilibrium that arises when price discrimination is permitted in the second-stage of the game.

Proposition 7. *There is a symmetric mixed strategy subgame perfect Nash equilibrium in which,*

(i) each firm advertises a first-period price randomly chosen from the distribution

$$F^*(p)=\left\{\begin{array}{ccc} 0 & for & p\leqq p^*_{\min} \\ 1-\frac{(v-p)(1-\phi^*)}{\phi^*(p-\delta v(1-\phi^*))} & for & p^*_{\min}\leqq p\leqq v \\ 1 & for & p\geqq v \end{array}\right\} \tag{4.28}$$

with minimum equilibrium price equal to

$$p^*_{\min}=v\left(1-\phi^*\right)+\delta v\phi^*\left(1-\phi^*\right); \tag{4.29}$$

(ii) each firm selects an advertising reach, denoted ϕ^*, implicitly defined by

$$v(1-\phi^*) + v\delta\left(1 - 2(\phi^*)^2\right) = A_\phi(\phi^*) \tag{4.30}$$

where $A_\phi(0) \leq v(1+\delta)$.

(iii) Each firm earns expected overall equilibrium profits equal to

$$E\Pi^* = v\phi^*(1-\phi^*)(1+\delta(1+\phi^*)) - A(\phi^*) = \phi^* A_\phi - A(\phi^*) + \delta v(\phi^*)^3. \tag{4.31}$$

Proposition 8. *When marginal advertising costs go to zero forward-looking firms do not select full market coverage. To be exact, in this case, the equilibrium level of advertising is* $\phi^* = \frac{1}{4\delta}\left(\sqrt{1+8\delta+8\delta^2} - 1\right) < 1.$

The proof of the above result is trivial. Given any general advertising technology, with form $A(\phi) = \alpha\eta(\phi)$, when α goes to zero, A_ϕ goes to zero as well. In that way, the right-hand side of (4.30) is equal to zero, thus, by solving for ϕ we obtain the result.

Interestingly, comparing proposition 3 and 8, we can see that unlike the advertising behaviour of firms in the non-discrimination case, when price discrimination is permitted and advertising becomes costless, firms never choose full market coverage. In particular, considering, for instance, that $\delta = 1$ and that advertising costs are null, if we move from non-discrimination to discrimination we find that firms reduce their advertising reach from 100% to 78%. This is a novel result since the extant models on informative advertising[17] find that when firms can advertise their existence costlessly they provide full information. Obviously, as this outcome is in our case entirely due to the introduction of price discrimination, it sheds some light on the dynamic effects of price discrimination on firms' advertising decisions. One possible explanation for this outcome is as follows. Each firm realises that the benefit of price discrimination is higher if its advertising reach is not too high, since in that way its rival will play less aggressively in period 2. Thus, we may say that by not selecting a very high market

[17] See, for instance, Butters (1977), Grossman and Shapiro (1984), Stegeman (1991), Stahl (1994) and Iyer, *et al.* (2003).

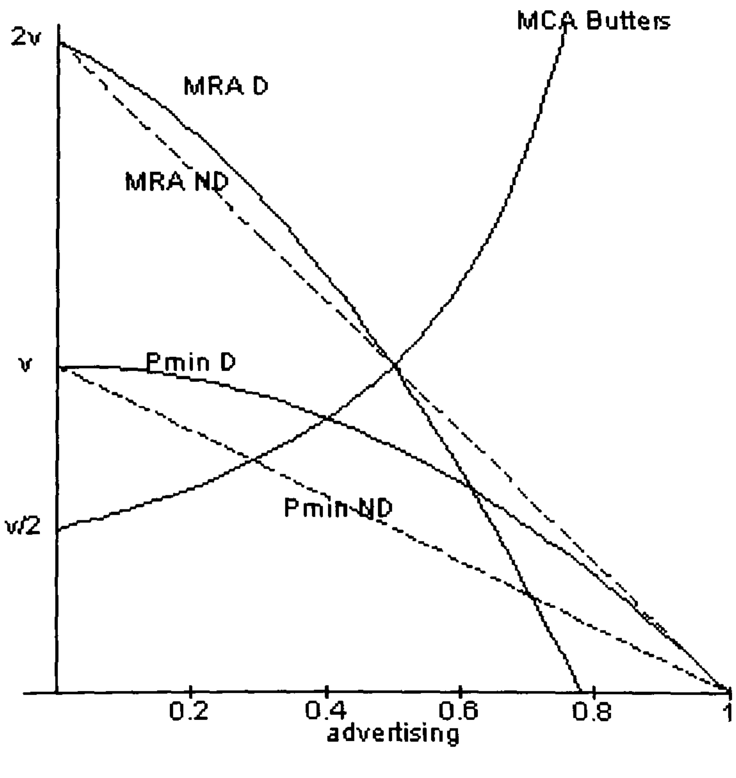
reach in period 1, each firm can commit to play less aggressively in the next stage of the game.

4.6 Competitive implications of price discrimination

In this section we analyse how the permission of price discrimination affects the equilibrium outcomes—i.e., advertising intensity, prices and profits—in comparison to the situation in which price discrimination is banned.

Figure 1 illustrates the solutions to equations (4.8) and (4.30) and to equations (4.7) and (4.29), for the special case in which $\delta = 1$ and $\alpha = \frac{v}{2}$, and using the urn technology proposed by Butters.

Figure 1: Equilibrium solutions with and without discrimination with Butters's technology



The downward-sloping curves "MRA D" and "MRA ND" are respectively the marginal revenue of advertising with discrimination and with no discrimination. The upward-sloping curve "MCA" is the marginal cost of advertising for the Butters's technology. The intersection between "MRA D" and "MCA" provides the equilibrium level of advertising with price discrimination and, similarly the intersection between "MRA ND" and "MCA" provides the equilibrium level of advertising with non discrimination. Additionally, the curves "Pmin D" and "Pmin ND" show respectively the behaviour of the minimum price selected by firms in period 1 when discrimination is permitted and when it is banned. In that manner, given the equilibrium level of advertising in each case, we may determine $p_{\min}$ and also the support of equilibrium prices with and without discrimination. This figure is helpful in seeing mainly (i) the effects of moving from non-discrimination to discrimination, (ii) the effects of the advertising intensity on the equilibrium outcomes; and (iii) the effects of changes in the exogenous parameters, namely α, v and δ.

It is worth noting that although the above figure is plotted for the case where $A(\phi) = \alpha\eta(\phi)$ with $\eta(\phi) = \ln\left(\frac{1}{1-\phi}\right)$, we could analyse the equilibrium outcomes for any other advertising technology.

4.6.1 Advertising decisions

Figure 1 shows that the left-hand side of equation (4.30)—i.e., the marginal revenue of advertising—is strictly decreasing and that shifts in that curve arise when there is a change in v or in δ. Concretely, the effect of an increase in v is to shift the curve upwards whilst the effect of a decrease in δ, from one to zero, is to shift the curve downwards until overlap the "Pmin ND" which is exactly equal to the MRA in the static game (recall that in static case the equilibrium level of advertising is given by $v(1-\phi) = A_\phi$). The right-hand side of equations (4.8) and (4.30)—i.e., the marginal cost of advertising—is the same with and without discrimination, and whatever the advertising technology considered. Shifts in the latter curve are only due to changes in α. Thus, we can see that an increase (decrease) in marginal costs makes that curve move upwards (downwards) and gives rise to less (more) advertising in equilibrium. To be precise, static comparative analysis shows that

$$\frac{\partial \phi^*}{\partial \alpha} = -\frac{A_{\phi\alpha}}{v + 4\delta v \phi + A_{\phi\phi}} < 0 \text{ and } \frac{\partial \phi^{nd}}{\partial \alpha} = -\frac{A_{\phi\alpha}}{v + \delta v + A_{\phi\phi}} < 0$$

because $A_{\phi\alpha} > 0$ and $A_{\phi\phi} \geq 0$.

Turning now to the comparison between the equilibrium advertising decisions with and without discrimination, we can observe in Figure 1 that firms advertise less under discrimination for marginal advertising costs not too high; whilst they advertise more for high advertising costs. Clearly, price discrimination has no effect on advertising decisions when $\phi^* = \phi^{nd}$. From Figure 1 it follows that both advertising intensities coincide when the marginal revenue of advertising with and without discrimination cross. Indeed, using (4.8) and (4.30) the reader can easily check that for $\delta > 0$, regardless of v and δ it follows that the latter condition is satisfied whenever $\phi^* = \phi^{nd} = 0.5$ (which obviously occurs for a specific α). Otherwise, when α is such that $\phi^* > 0.5$ it follows that $\phi^* < \phi^{nd}$. Likewise, when α is such that $\phi^* < 0.5$ it ensues that $\phi^* > \phi^{nd}$. Considering for example the Butters's technology, we obtain that $\phi^* \geq \phi^{nd}$ *if* $\alpha \geq \frac{1}{4}v(1+\delta)$ and $\phi^* < \phi^{nd}$ otherwise. Similarly, if $\eta(\phi) = \phi^2$ then $\phi^* \geq \phi^{nd}$ *if* $\alpha > \frac{1}{2}v(1+\delta)$; otherwise $\phi^* < \phi^{nd}$. Thus, the comparison is ambiguous. When advertising is costly, then there is more advertising with discrimination; when advertising is cheap, the reverse happens.

4.6.2 Prices

This subsection examines the impact of poaching on second and first period prices.

Second-period prices When price discrimination is permitted, the discriminating firm increases (or at least does not reduce) the price for its captive consumers. Thus, those consumers that bought from the highest priced firm in period 1—the discriminating firm in period 2—are expected to pay higher first and second period prices. From a comparison between F^* and G^r we observe that $F^* < G^r$ which means that first-period price is stochastically larger than p^r as it assumes large values with higher probability.[18] From the latter result it follows that the group of selective consumers will pay on average a lower price in period 2. Finally, regarding the group of captive consumers that bought from the lowest priced firm before the conclusion is not so clear-cut. We have seen that the non-discriminating firm uses a "Hi-Lo" strategy in period 2. With probability equal to m_j, its locked in customers will pay a higher second period price, namely v. Otherwise, because no general stochastic order is possible to establish between F^* and $\widehat{G}$ this set of consumers may pay a higher or lower second period price.

First-period prices The benefit of price discrimination gives rise to what we designate as the "*race for discrimination effect*". This effect is related to the idea that the benefit of price discrimination motivates a firm to advertise a high price in the first-stage of the game in order to secure the discriminating position in the second-stage. As the future becomes more important the higher is that benefit and so the stronger is the "*race for discrimination effect*". Conversely, as δ declines the benefit of price discrimination shrinks, weakening the incentives to set high prices in period 1 (notice that $\frac{\partial p_{\min}}{\partial \delta} > 0$). Thus, we can say that the benefit of price discrimination gives rise to the "*race for discrimination effect*" through which first-period prices are expected to be above the non-discrimination levels. Furthermore, because prices are strategic complements, as one firm raises its price it induces the rival firm to increase its price as well. In other words, the "*race for discrimination effect*" tends to soften first period price competition. In consequence, this effect suggests that when firms can engage in price discrimination they tend to charge higher first period prices than if discrimination were banned. A similar result is obtained by Chen and Zhang (2002), in a model where the market is exogenously divided into captive customers and switchers and where firms can reach perfectly all of them. In particular, they assume that switchers have a lower

[18] Because ϕ is the same in period one and two, F^* first-order stochastically dominates G^r if

$$1 - \frac{(v-p)(1-\phi)}{\phi(p-\delta v(1-\phi))} \leq 1 - \frac{(v-p)(1-\phi)}{p\phi}$$

i.e., if $-\delta v(1-\phi) \leq 0$, which is always tue.

reservation price than captive customers and that a firm is only able to distinguish a switcher from a captive customer if it sells exclusively to its captive customers in period 1, that is if its price is high enough. They show that in period 2 it may happen that only one firm discriminates or both of them discriminate. Analogously to our model, they show that the benefit of price discrimination in the second period motivates a firm to increase first-period prices in order to obtain the required information for price discrimination.

Nevertheless, it is worth noting that in our model, apart from the "*race for discrimination effect*", there is also the effect of advertising on first-period prices. We have seen that price discrimination affects in general advertising decisions, which in turn affect first-period advertised prices. This is the "*advertising effect*" on first-period prices. If price discrimination had no effect on advertising (i.e., if $\phi^* = \phi^{nd}$), this latter effect would play no role; in which case $F^*(p)$ would dominate stochastically $F^{nd}(p)$, and first-period prices would move upwards (exclusively due to the former effect).[19] Because there is a negative relationship between the advertising reach and prices,[20] this effect is expected to reinforce the "*race for discrimination effect*" when price discrimination gives rise to less advertising, and so to higher prices. Even though it is not possible to establish a general stochastic ordering between $F^*(p)$ and $F^{nd}(p)$, Figure 1 shows that for not very high marginal advertising costs price discrimination leads firms to select a lower advertising reach and higher first-period prices relatively the non-discrimination case. Thus, for marginal advertising costs such that $\phi^* < \phi^{nd}$ first-period prices are clearly *above* their non-discrimination counterparts.

The more complex case occurs when price discrimination gives rise to more advertising (this is particularly the case when α is too high). In this situation the "*advertising effect*" would act in favour of lower first period prices due to more advertising. (That is,

[19]Because in this case we would have $\phi^* = \phi^{nd} = \phi$, then F^* first-order stochastically dominates F^{nd} if

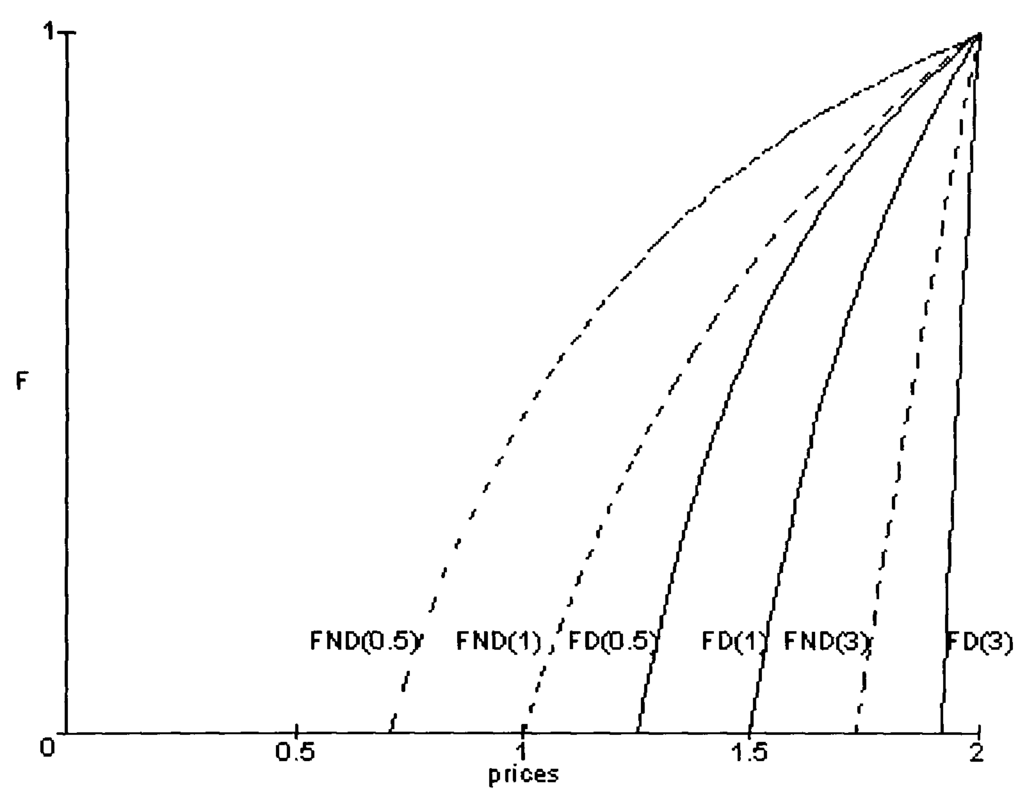
$$1 - \frac{(v-p)(1-\phi)}{\phi(p - \delta v(1-\phi))} \leq 1 - \frac{(v-p)(1-\phi)}{p\phi}$$

i.e., if $-\delta v(1-\phi) \leq 0$, which is always tue.

[20]More precisely notice that $\frac{\partial p_{\min}}{\partial \phi^*} < 0$ and $\frac{\partial F^*}{\partial \phi^*} > 0$.

when $\phi^* > \phi^{nd}$ although firms have an incentive to raise prices they also invest in more advertising—which, in turn, has a negative effect on prices.) Obviously, whether first period prices are above or below their non-discrimination counterparts, when α is too high, strongly depends on which effect dominates. As a general rule, it could be said that when the "*race for discrimination effect*" dominates we expect first-period prices to be on average above the non-discrimination levels, otherwise we expect first-period prices to fall when discrimination is introduced.

Figure 2: Distribution functions with and without price discrimination



In order to investigate whether first period prices are above or below their non-discrimination counterparts, Figure 2 illustrates the behaviour of $F^*(p)$ and $F^{nd}(p)$, respectively given by "FD(α)" and "FND(α)", for the Butters's technology and for different values of α. It is plotted for $v = 2$, $\delta = 1$ and for $\alpha = \{0.5, 1, 3\}$.

Interestingly, looking at Figure 2 we realise that for the Butters's technology $F^*(p)$ stochastically dominates $F^{nd}(p)$. In other words, we conclude that first-period prices with discrimination are on average above their non-discrimination counterparts, even when α is too high. (The same pattern was also obtained for other advertising cost functions as for instance the quadratic technology, as well as for higher values for α).

This suggests that the "*race for discrimination effect*" is the dominant one even when advertising costs are high.

If price dispersion is measured by the range of prices, and if the lower bound of the support of equilibrium prices moves upwards when we depart from non-discrimination to discrimination, then the level of price dispersion tends to be lower when price discrimination is allowed. Going back to Figure 1, the reader can easily confirm that, regardless of the advertising technology considered, even when $\phi^* > \phi^{nd}$ (due to high values for α) it is highly likely that $p^*_{\min} > p^{nd}_{\min}$. Since the benefit of price discrimination gives rise to the "*race for discrimination effect*" firms play less aggressively in period 1, competition is less intense and therefore the level of price dispersion falls. (Recall that in the previous chapters price discrimination increases price dispersion because it leads firms to behave more aggressively.)

To summarise, when α is not too high, price discrimination does not give rise to more advertising and so first-period prices are on average *above* the non-discrimination levels, thereby reducing the level of observed price dispersion. For high values of α, although price discrimination leads firms to advertise more, we observe that the decrease in prices due to more advertising is not enough to offset the increase in prices that results from the firm's incentives to price high. Thus, even in this case first-period prices tend to be above their non-discrimination counterparts.

Fudenberg and Tirole (2000) and Villas-Boas (1999) have also found that when behaviour-based price discrimination is permitted, first-period prices are above the non-discrimination levels. However, the intuition behind their result is different from ours. In their models as consumers anticipate poaching, they become less price sensitive in period 1. As a result of that, firms are able to quote higher first period prices than if poaching were not permitted. Here, in contrast, first period prices are above their non-discrimination counterparts due to a strategic behaviour of competing forward looking firms, i.e., due to the "*race for discrimination effect*".

Finally, the next proposition summarises some of the effects of changes in α on first-period prices. Advertising costs affect the distribution of prices through ϕ^*. Since increases in α make ϕ^* fall, the price distribution shifts towards higher prices (as is illustrated in Figure 2).[21]

Proposition 9. *Regardless of the advertising technology considered, as advertising becomes cheaper, the minimum price in the equilibrium support is smaller and lower prices are more likely. However, as advertising becomes costless, that is to say as α approaches zero,*

$$p^*_{\min} \to v\left[1 - \frac{1}{4\delta}\left(\sqrt{1+8\delta+8\delta^2} - 1\right)\right]\left(\frac{3}{4} + \frac{1}{4}\sqrt{1+8\delta+8\delta^2}\right) > 0.$$

The key difference between proposition 9 and part (i) of proposition 3 is the following. Whereas under non-discrimination as α goes to zero, the minimum advertised price goes as well to zero (i.e., goes to marginal cost), under price discrimination the minimum price that firms advertise in period 1 is always greater than the marginal cost. As we have seen in proposition 8, when marginal advertising costs go to zero, firms do not select full market reach as happens under non-discrimination. Therefore, whatever the advertising technology considered, as long as $\delta > 0$, the classic Bertrand equilibrium is never a solution of the price discrimination game.

4.6.3 Profitability

The profitability issue of price discrimination is one of the major concerns when it comes to assess the competitive effects of this practice. We revisit this issue in what

[21] More precisely, it is easy to see that

$$\frac{\partial F^*}{\partial \alpha} = \underbrace{\frac{\partial F^*}{\partial \phi^*}}_{(+)}\underbrace{\frac{\partial \phi^*}{\partial \alpha}}_{(-)} < 0.$$

and

$$\frac{\partial p_{\min}}{\partial \alpha} = \frac{\partial p_{\min}}{\partial \phi^*}\frac{\partial \phi^*}{\partial \alpha} > 0.$$

ensues.

Regarding the firm which is able to price discriminate in period 2, if we compare its profit with non-discrimination, namely $v\phi^{nd}\left(1-\phi^{nd}\right)$ with its second-period profit with discrimination given, under symmetry, by $v\phi^{*}\left(1-\phi^{*}\right)\left(1+\phi^{*}\right)$ (see equation (4.20)) it is evident that when $\phi^{*}=\phi^{nd}$, price discrimination makes that firm clearly better off. The reader can easily verify that when (i) $0.5<\phi^{*}<\phi^{nd}$ or when (ii) $\phi^{nd}<\phi^{*}<0.5$, second-period profit under price discrimination is always above its non-discrimination counterpart. Interestingly, going back to Figure 1 and to our discussion in subsection 4.6.1 it immediately follows that the previous two conditions are always met in equilibrium. We have seen that when α is such that $\phi^{*}>0.5$ then it follows that in equilibrium $\phi^{*}<\phi^{nd}$. Likewise, we have seen that when α is such that $\phi^{*}<0.5$ it ensues that in equilibrium $\phi^{*}>\phi^{nd}$. Therefore, the discriminating firm is clearly better off under price discrimination. This is not, of course, a surprising result. It is well known that price discrimination is privately profitable for each firm: given the rival's strategy, each firm is better off if it is free to adjust its prices to different groups of customers (e.g. Thisse and Vives (1988), Corts (1998) and Shaffer and Zhang (2000)).

What is perhaps more interesting is the fact that depending on the level of advertising selected in the first-stage of the game, the non-discriminating firm might also become better off in period 2. The non-discriminating firm would face no effect on its second period profit if each firm's advertising decision were not affected by discrimination. However, as said, this is not usually the case. Specifically, under symmetry, and denoting by ϕ^{*} each firm's first-period advertising decision when discrimination is permitted, from the comparison between $v\phi^{nd}\left(1-\phi^{nd}\right)$ and $v\phi^{*}\left(1-\phi^{*}\right)$ (see equation (4.19)) it immediately follows that the non-discriminating firm might as well gain when its rival discriminates. On the one hand, if $\phi^{nd}<\phi^{*}<0.5$, the size of captive customers is greater than the size of selective customers, and it is even higher under poaching in period 2. With discrimination, the non-discriminating firm would face a greater captive segment in period 2. In consequence, its second-period expected profit would move upwards. On the other hand, the non-discriminating firm might also become better off when $0.5 < \phi^* < \phi^{nd}$. In this case, although the size of captive customers is smaller than the size of selective customers, moving from no discrimination to discrimination increases the size of captive customers at the expense of the size of selective customers. Clearly, this benefits the nondiscriminating firm.

A key finding is, therefore, that whenever price discrimination increases the size of captive customers, the non-discriminating firm will also become better off when its rival discriminates. Since the two conditions are met in equilibrium for any α it immediately follows that the non-discriminating firm might also gain when its rival discriminates. It should be emphasised that a feature of the extant models on customer poaching, is that price discrimination in period 2 tends to intensify competition and to decrease equilibrium profits. In contrast, because in our model only one firm can engage in customer poaching, price discrimination might *soften* price competition rather than intensify it. This happens when price discrimination gives rise to an increase in the proportion of locked in customers.

We have seen that in period 2 both firms might become better off when one of them is able to discriminate. However, in order to analyse whether the permission of price discrimination is good for firms we need to look rather at overall expected profit. Intuitively, as first-period prices tend to be above their non-discrimination counterparts we may anticipate that we expect first-period profits to be also above the non-discrimination levels. Obviously, this seems to suggest that overall expected profits would move upwards when price discrimination is introduced. Recall that from (4.9) when price discrimination is banned overall expected profit is equal to

$$E\Pi^{nd} = v\left(1+\delta\right)\phi^{nd}\left(1-\phi^{nd}\right) - A\left(\phi^{nd}\right), \tag{4.32}$$

while when price discrimination is allowed overall expected profit is equal to

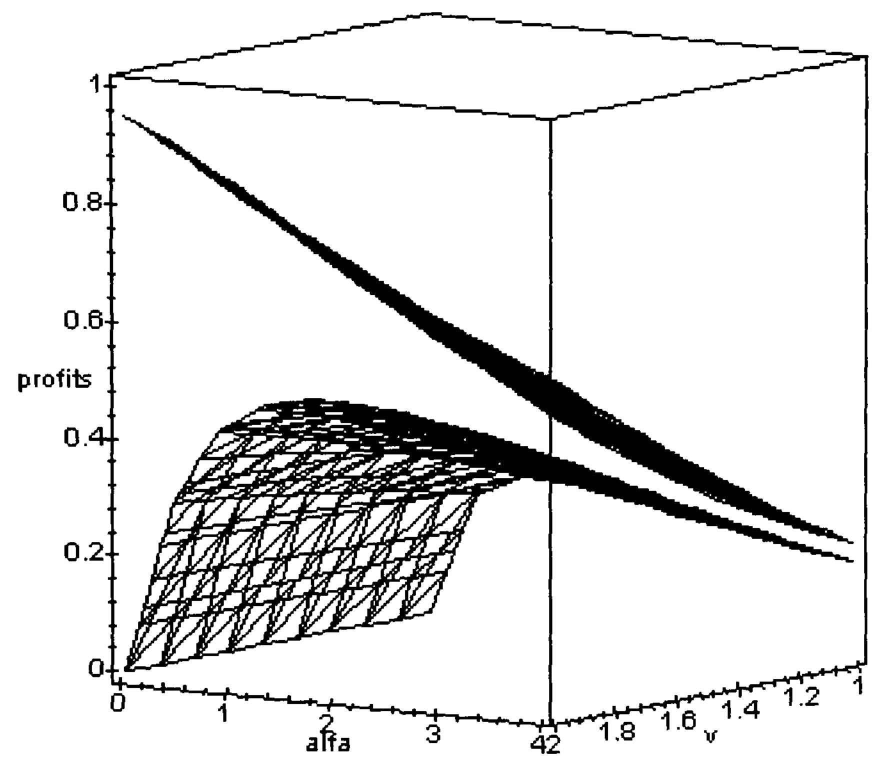
$$E\Pi^{*} = v\left(1+\delta\right)\phi^{*}\left(1-\phi^{*}\right) + \delta v\phi^{*}\left(1-\phi^{*}\right) - A\left(\phi^{*}\right). \tag{4.33}$$

It is immediate that if $\phi^* = \phi^{nd}$ overall expected profit under discrimination is always above overall expected profit under non-discrimination. Additionally, the reader can easily verify that the first term in (4.32) and (4.33) has an inverted-U relationship with the intensity of advertising, reaching its maximum value at $\phi^{nd} = 0.5$ and $\phi^* = 0.5$. From our previous discussion it has become clear that depending on α in equilibrium we may obtain (i) $\phi^* = \phi^{nd} = 0.5$; (ii) $0.5 < \phi^* < \phi^{nd}$ or finally (iii) $\phi^{nd} < \phi^* < 0.5$. The case in (i) was already discussed. Let us now compare overall equilibrium profits when $0.5 < \phi^* < \phi^{nd}$. In this case it follows that the first term in (4.33) is greater than the first term in (4.32). On the other hand because $\phi^* < \phi^{nd}$ it immediately follows that $A(\phi^*) < A\left(\phi^{nd}\right)$. If we further take into account that the second term in (4.33) is positive it follows that $E\Pi^*$ is strictly higher than $E\Pi^{nd}$. Thus, in this case price discrimination increases overall profits. In sum, when α is such that $\phi^* \leq \phi^{nd}$ overall equilibrium profits under discrimination are always above their non-discrimination counterparts, regardless of the advertising technology considered.

As previously mentioned, the more complex situation occurs when price discrimination leads firms to increased advertising. This happens when α is such that $\phi^{nd} < \phi^* < 0.5$. Again it is easy to confirm that for this range the first term in (4.33) is larger than the first term in (4.32). We already know that the second term in (4.33) is also positive. However, since $\phi^* > \phi^{nd}$ it follows that $A(\phi^*) > A\left(\phi^{nd}\right)$. This means that overall profits under discrimination will be above their non-discrimination counterparts as long as the increase in advertising costs does not offset the increase in sales revenue when moving from non-discrimination to discrimination. (The reader may observe in Figure 1 that when $\phi^{nd} < \phi^* < 0.5$ the difference between ϕ^{nd} and ϕ^* is not extremely high, which seems to suggest that the increase in advertising costs will not be too high.)

Nevertheless, in order to offer a more complete picture about the impact of price discrimination on overall expected profits when moving from non-discrimination to discrimination, Figure 3 plots overall profit with price discrimination (upper function) and without discrimination (lower function), using the quadratic technology, for different values of α and v and considering that $\delta = 1$.

Figure 3: Overall profit with discrimination (upper function) and without discrimination (lower function) using the Quadratic Technology



Interestingly we find that in this special case firms are clearly better off with price discrimination as overall expected profits with discrimination are above those under non-discrimination, even when α is high. This confirms our previous intuition that the increase in advertising costs is not enough to offset the increase in sales revenue that results when departing from non-discrimination to discrimination. The same pattern is valid for the Butters's technology. However, due to the complexity of working algebraically with the latter technology, neither a figure nor a technical proof is provided. Nonetheless, all numerical simulations carried out for this technology produced the same result. Table 1 displays overall expected profit with and without discrimination for the case in which $\delta = 1$.

We are now in conditions to establish the following result.

Corollary 4. *Regardless the advertising technology considered, overall equilibrium profit with discrimination is above its non-discrimination counterpart, at least for advertising costs not too high (i.e. when α is such that $\phi^* \leq \phi^{nd}$).*

Table 1: Overall profits with and without discrimination using the Butters's technology

v	α	Overall Profit with Disc	Overall Profit without Disc
1.5	0	0.714	0
1.5	0.5	0.523	0.278
1.5	1	0.400	0.183
1.5	1.5	0.178	0.101
2	0	0.950	0
2	0.5	0.765	0.393
2	1	0.557	0.306
2	1.5	0.378	0.213
2	2	0.235	0.134

The previous result is in contrast with that achieved in other models on behaviour-based price discrimination, in which firms are clearly worse off if price discrimination is permitted (Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000) and chapter 3 in this thesis). However, it is closely related to Chen and Zhang's (2002) result, according to which price discrimination may benefit competing firms. How can we therefore explain these different outcomes?

Broadly speaking, and as is suggested by Ulph and Vulkan (2000), the profitability of price discrimination can be explained on the basis of two different effects: the *"enhanced surplus extraction effect"* and the *"intensified competition effect"*. The former effect claims that price discrimination benefits firms since it allows them to extract greater surplus from consumers. (This is the only effect at work under a monopoly setting.) Conversely, the latter effect holds that in competitive settings it may happen that price discrimination leads all firms to target each other's customers with lower prices. Clearly, this would intensify competition, resulting in lower profits for every firm. The overall effect of price discrimination on profits depends crucially on which of both effects dominates.

Two important common features among those models where firms are negatively affected by price discrimination are that *both* firms engage in price discrimination and that one firm's weak market is in general the rival's strong market (i.e., there is *best-response asymmetry*). In this set of models, the *"intensified competition effect"* dominates and profits fall down. Indeed, in some of these models due to best-response asymmetry we observe *all-out competition*, through which all prices fall down as a result of more aggressive pricing (e.g. Fudenberg and Tirole (2000)). On the contrary, because in our model only one firm discriminates, the *"intensified competition effect"* is less strong. Furthermore, in our model there is another important effect at work: namely, the *"race for discrimination effect"*. As explained, this effect reflects a firm's strategic incentive to raise its first period prices in the pursuit of becoming the discriminating firm. For that reason, price discrimination may act to *soften* price competition in period 1, permitting profits to rise. Altogether, it can be said that in the context of our model behaviour-based price discrimination will in general be good for all competing firms.

Finally, the impact of the exogenous parameters on profits is also a question of interest. As expected, profits are increasing in consumers' willingness to pay, v. More interesting is the response of firms' profits to variations in advertising costs.

Advertising costs and profits From Figure 3 and Table 1 it is possible to infer that the impact of advertising costs on profits, should be evaluated taking into account whether or not price discrimination is allowed.

Proposition 10. (Advertising costs and profits)

For the Quadratic Technology it follows that, when $\delta = 1$:

(i) under non-discrimination, sellers benefit from advertising costs increases if advertising costs are low;

(ii) under discrimination, sellers are always better off with decreases in advertising costs.

Proof. See the Appendix.

As expected, part (i) confirms a well known result in the extant literature on informative advertising: firms' profits may increase with advertising costs (e.g. Grossman and Shapiro (1984), Stahl (1994) and Bester and Petrakis (1996)). In general, whilst an increase in advertising costs has a negative *direct effect* on profits, there is as well a *strategic effect*: as advertising costs increase, firms respond with less advertising, permitting prices to rise. When the strategic effect dominates, profits may increase with advertising costs. In particular, in the appendix provided it is shown that, for a quadratic technology, firms benefit from advertising costs increases if advertising costs are relatively low (i.e., $\alpha < \frac{v}{2}(1+\delta)$). The reason is that low advertising costs induce more intense competition which affects negatively prices and profits. Conversely, as costs become higher and higher, advertising shrinks so much that demand declines as well as profits. Expressed differently, if we depart from a situation where ϕ^{nd} is low (α is high), we observe that additional advertising is more likely to increase the fraction of locked in customers than the fraction of selective customers. It turns out that the probability of reaching an uninformed buyer is high and, then, firms have more incentives to focus on the group of captive consumers, thereby quoting high prices. However, as α becomes increasingly smaller, and advertising becomes increasingly higher, more buyers will be aware of both firms' prices. As the size of selective consumers increases the danger of losing those consumers to the other firm is greater and, therefore, low prices are more likely. An increase in α induces less advertising, relaxes price competition and allows firms to increase prices by enough to offset the direct effect on profits.

Interestingly, proposition 10 claims that (at least for the quadratic technology) when price discrimination is allowed profits and advertising costs move in opposite directions. That is, an increase in advertising costs is always bad for profits. Following the theory, this suggests that the direct effect is stronger than the strategic effect. When α is high (ϕ is low), the arguments presented above are valid to explain why profits decrease with increases in α. However, when α is low (ϕ is high) and we move from non-discrimination to discrimination we find that the relationship between profits and advertising costs is reversed—i.e., profits no longer increase with increases in advertising costs. The reason is that while under no discrimination lower and lower advertising costs tend to push firms to the Bertrand outcome, with price discrimination the Bertrand result is far from being implemented. Expressed differently, under non-discrimination an increase in α, when α is low, has the strategic effect of avoiding more aggressive behaviour, thereby increasing firms' profits; in contrast under price discrimination the strategic effect plays a very small role as full market coverage is never provided in equilibrium. The direct effect dominates and profits are clearly negatively affected by increases in advertising costs.

4.7 Welfare issues

This section evaluates the welfare effects of price discrimination enabled by informative advertising. First, we examine the conventional question of whether there is too little or too much informative advertising. With that in mind, we draw a comparison between the level of advertising that would be selected by an advertising regulator whose aim would be to maximise total welfare, and compare that level with the market equilibrium. Second, we discuss how the transition from non-discrimination to discrimination affects aggregate welfare as well as sellers and consumers' surplus. Although prices play no welfare role here—due to the unit demand assumption, no dropping out of consumers and no loyalty costs—allowing discrimination affects advertising decisions and thereby the efficiency properties of the equilibrium advertising level. In this way, given that the regulator would select the same level of advertising, regardless of discrimination, we also evaluate whether the introduction of price discrimination enhances or not the efficiency properties of sellers' advertising decisions. To simplify the analysis, throughout this section it is assumed that $\delta = 1$.

Since production costs are assumed to be zero, total welfare is equal to the value of the good for all buyers that enter the market in both periods minus total advertising costs, that is

$$W = 2v\left[1 - (1-\phi)^2\right] - 2A(\phi). \tag{4.34}$$

4.7.1 Do firms under-or-over advertise?

As aforementioned to address the question of whether the advertising choice is excessive or inadequate, we determine the level of advertising that would be selected by an advertising regulator whose aim would be to maximise total welfare, and compare that level with the non-cooperative equilibrium. In a homogeneous product market with competition there are mainly two effects behind divergences between the equilibrium and the social optimal level of advertising. Firstly, an ad that reaches any consumer that would be otherwise uninformed generates a sale whatever the price. From a social point of view this action is desirable because more consumers can enter the market, thereby increasing total welfare. Nonetheless, due to price competition, firms cannot extract all the surplus generated by advertising, because they are not able to charge the highest price. As a result, due to the *"nonappropriability of social surplus effect"* firms tend to underadvertise.

Secondly, when a firm decides to invest in more advertising it does not take into account for the profit reduction at the rival firm as it increases its own advertising level and poaches some of the rival's customers. When this latter effect—the "*business stealing effect*"—is the dominant one firms tend to overadvertise. In this case, more advertising may be socially undesirable if it merely leads to a swap of consumers between firms.

In what follows, it is assumed that the regulator has access to the same advertising technology as firms and that maximises W, given by (4.34), with respect to ϕ. From the first-order condition, the social optimal level of advertising, denoted by ϕ^w, is implicitly given by:

$$2v\left(1-\phi^w\right)=A_\phi. \tag{4.35}$$

Recall, that in the overall game with price discrimination the equilibrium level of advertising, ϕ^*, is implicitly given by

$$v\left(1-\phi^*\right)+v\left(1-2\phi^{*2}\right)=A_\phi; \tag{4.36}$$

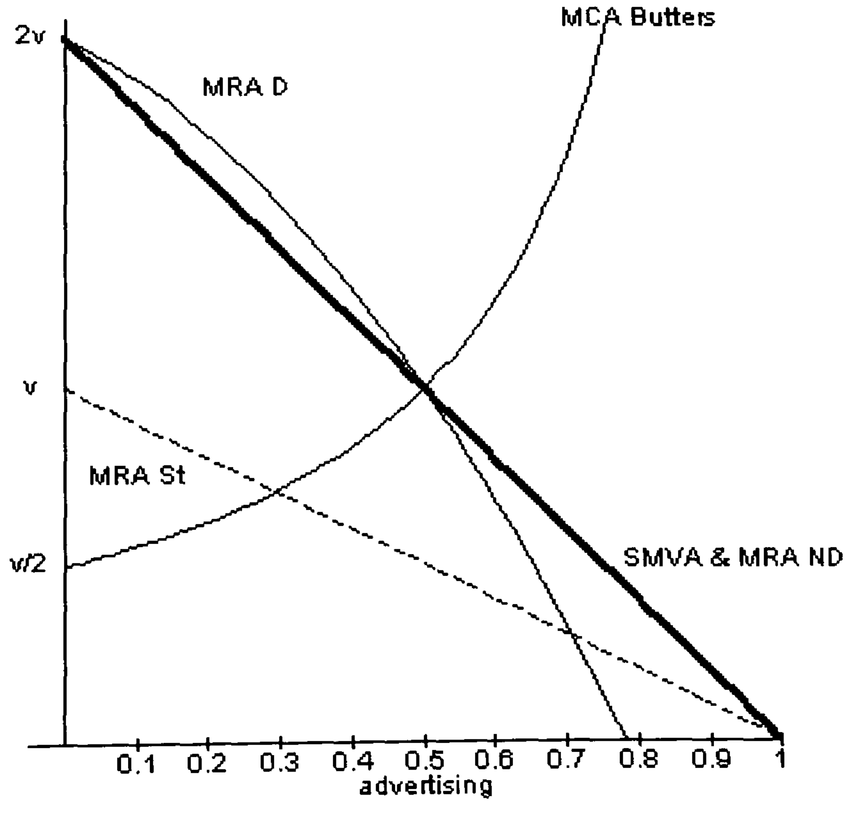
whilst without discrimination the equilibrium level of advertising, ϕ^{nd}, is implicitly given by

$$2v\left(1-\phi^{nd}\right)=A_{\phi}. \tag{4.37}$$

It immediately follows that the expression that defines the social optimal level of advertising is equal to the expression that defines the equilibrium level of advertising under non-discrimination.

Figure 4 is based on Figure 1 except that now it is introduced the left-hand side of equation (4.35) which is the social marginal value of advertising net of advertising costs, namely "SMVA" and it is removed the equation that defines the minimum equilibrium price. In addition, it is also represented the MRA with and without discrimination for $\delta=0$, given by "MRA St". As in Figure 1, $\alpha=\frac{v}{2}$ and the advertising technology is that proposed by Butters. We can see below that as δ decreases, the marginal revenue of advertising with and without discrimination as well as the social marginal value of advertising move downward until reach the marginal revenue in the static (or myopic) case given by "MRA St".

Figure 4: Efficiency of advertising decisions with Butters's technology



We may now establish the following propositions.

Proposition 11. (Efficiency properties with no-discrimination)

Suppose there is a ban on price discrimination. Then, firms select the social optimal level of advertising.

Proof. See the Appendix.

Proposition 12. (Efficiency properties with discrimination)

Regardless of the advertising technology considered, when price discrimination is permitted and $\delta = 1$, firms select the social optimal level of advertising if α is such that $\phi^* = \phi^w = 0.5$, *underadvertise if* α is such that $0.5 < \phi^* < \phi^w$ *and overadvertise if* α is such that $\phi^w < \phi^* < 0.5$.

Proof. See the Appendix.

Notwithstanding we illustrate these results by means of the Butters's technology, both results are quite robust for any advertising technology, because we only need to look at MRA and SMVA. Proposition 11 can be simply proved using (4.35) and (4.37). Since the left-hand side of both equations coincide one gets the result. A similar result was for the first-time obtained in Butters's (1977) seminal paper. He showed that in a static monopolistically competitive setting the market equilibrium level of advertising is socially optimal. The same happens always in a *static* version of our model as well as in a two-period version without discrimination, but with only two-firms and for any advertising technology. Although Butters is unable to offer an intuition for his welfare finding, Bagwell (2003) offers some explanation for that result. Thus, the intuition for our result is partially based on Bagwell (2003). It runs as follows. Since a consumer that receives an ad today will enter the market in both periods, welfare only increases if an additional ad is received by an uninformed consumer. That is, the social benefit of an additional ad is equal to $v(1+\delta)$ times the probability of that consumer receives no other ad. In a mixed strategy equilibrium in prices the overall private benefit to a firm of sending an ad at any price $p \in [p_{\min}, v]$ is constant and equal to $v(1+\delta)$ times the probability that the consumer receives no other ad. It is therefore evident that in a dynamic game without discrimination private and social benefits coincide.

New theoretical findings arise when the model is extended to allow price discrimination by means of advertising in period 2. As far as our knowledge is concerned this is the first attempt to evaluate the advertising behaviour of firms in a market where price discrimination may occur at the firm level.[22] Interestingly, when $\delta = 1$ we get that, from the social point of view, firms may supply too less, too much or even the efficient level of advertising. To understand this outcome let us look separately on the effect of advertising on firms and consumers surplus. From the sellers overall expected profit, given by (4.31) we obtain that when $\delta = 1$,

$$\frac{\partial E\Pi^*}{\partial \phi} = 2v(1-\phi) - 3v\phi^2 - A_\phi.$$

Evaluating at the non-cooperative equilibrium level of advertising , where $A_\phi = v(1-\phi) + v\left(1-2\phi^2\right)$, it follows that at $\phi = \phi^*$

$$\frac{\partial E\Pi^*}{\partial \phi} = -\phi^* v(1+\phi^*) < 0.$$

Hence, we conclude that from the firms' point of view there is too much advertising. When a firm decides to invest in more advertising it does not take into account for the profit reduction at the rival firm as it increases its own advertising level and poaches some of the rival's customers. Due to this "*business stealing effect*" firms tend to overadvertise.

Conversely, from the consumers' point of view we will see below that there is underadvertising. Expected consumer surplus denoted by ECS is given by total welfare minus industry profits. In the game with price discrimination, expected consumer surplus, when $\delta = 1$, is equal to

$$ECS^* = W^* - 2E\Pi^* = 2v\left(\phi^*\right)^3.$$

[22]When firms choose a price randomly from a continuous distribution function (as happens in our benchmark case) we may say that the market exhibits price discrimination across firms. Nevertheless, price discrimination does not occur at the firm level.

Thus, when $\phi = \phi^*$
$$\frac{\partial ECS^*}{\partial \phi} = 6v(\phi^*)^2 > 0.$$
It is obvious that for $\phi = \phi^* < 1$, there is always too little advertising from the consumers' point of view. When firms are not able to appropriate all the surplus generated by advertising they tend to underadvertise.

Given that consumers desire more advertising and firms desire less advertising, the net social effect is not immediately obvious. Adding the two previous effects we obtain that at $\phi = \phi^*$
$$\frac{\partial ECS^*}{\partial \phi} + 2\frac{\partial E\Pi^*}{\partial \phi} = 2\phi^* v(2\phi^* - 1).$$
Therefore, the net social effect is equal to zero if and only if $\phi^* = 0.5$. This intensity of advertising is efficient from the social point of view. However, the net effect is positive when α is such that $\phi^* > 0.5$, which means that firms advertise too less. (Indeed, Figure 4 shows that when α is such that $\phi^* > 0.5$ it happens that $\phi^* < \phi^w$.) The reason is that in this case the *"nonappropriability of social surplus effect"* dominates. (Recall also that firms charge on average lower prices when ϕ is high (α is low).) Finally, when $\phi^* < 0.5$, it follows that $\phi^* > \phi^w$ (see Figure 4) so firms overadvertise. In particular, considering the Butters's technology, we can see in Figure 4 that when $\delta = 1$ and $\alpha = \frac{v}{2}$, $\phi^* = \phi^w = 0.5$. For higher values of α firms overadvertise whilst for smaller values of α they underadvertise.

To summarise, price discrimination might lead firms to provide too little, too much or the social level of advertising. The intuition is as follows. We have seen that under symmetry the benefit of price discrimination reaches its maximum when the fraction of captive consumers reaches its maximum as well, which occurs when $\phi = 0.5$. Further, the benefit decreases with increases in ϕ when $\phi > 0.5$, while it increases with increases in ϕ when $\phi < 0.5$. This means that price discrimination gives firms an incentive to increase the fraction of captive consumers in the market. Therefore, this strategic behaviour explains why firms change their advertising behaviour when moving from no-discrimination to discrimination.

4.7.2 Welfare effects of price discrimination

A recurrent policy question in the extant literature on price discrimination is whether to restrict or encourage the practice of setting discriminatory prices. We have already noted that in the framework of this model there is a unique source of social inefficiency—namely, the advertising intensity selected by firms—which endogenously determines the fraction of consumers that will enter the market. In this context, it immediately follows that a key ingredient to the understanding of the welfare effects of price discrimination is the impact of price discrimination on firms' advertising choices.

When firms can engage in price discrimination welfare is equal to,

$$W^{*} = 2v\left[1 - (1-\phi^{*})^{2}\right] - 2A\left(\phi^{*}\right),$$

whilst when there is a ban on price discrimination it is equal to

$$W^{nd} = 2v\left[1 - \left(1-\phi^{nd}\right)^{2}\right] - 2A\left(\phi^{nd}\right).$$

Proposition 13. *Regardless of the advertising technology considered, when* α *is such that* $\phi^{*} \neq 0.5$, *welfare falls with price discrimination.*

Proof. See the Appendix.

The intuition for this result is straightforward if we take into account our previous findings. We have seen that a ban on price discrimination leads firms to *always* select the social optimal level of advertising. Therefore, apart from the case in which α is such that $\phi^{*} = 0.5$, the permission of price discrimination introduces a distortion into the level of advertising provided by firms, thereby reducing total welfare. Thus, proposition 13 provides a simple criterion for predicting whether price discrimination will be desirable or not in a market where advertising is the consumers' sole source of information. The main conclusion is that apart from the special case in which $\phi^{*} = 0.5$, from the social point of view, price discrimination should be banned.

Because advertising choices are ultimately dependent on advertising costs, any policy drawn in favour or against discrimination, should also take into account the costs' structure of each advertising-product market. As a whole we observe that as advertising costs rise welfare falls $\left(\frac{\partial W}{\partial \alpha} < 0\right)$. Figure 4 illustrates as well this result. When advertising is costless (i.e., $\alpha = 0$) under non-discrimination firms always select the social optimal level of advertising. Thus, allowing price discrimination clearly introduces a distortion on the intensity of advertising supplied, thereby reducing welfare. For other values of α we may say that when advertising is cheap firms advertise too little and thereby from the social point of view too many consumers are left out of the market. On the contrary, when advertising costs are high firms provide too much advertising from the social point of view and less advertising would be welfare enhancing. Only when the advertising cost is such that $\phi^* = 0.5$ will price discrimination introduce no welfare distortion. (Considering, for instance, the Butters's technology and $\delta = 1$ this happens in the special case where $\alpha = \frac{v}{2}$.)

We have seen that it is generally the case that profits rise when firms may add price discrimination to their pricing strategies in period 2, and that the raise in profits is greater in industries with low advertising costs. Together, the profit and welfare effects of price discrimination, suggest that consumer surplus falls when price discrimination is introduced.

In sum, behaviour-based price discrimination in markets where firms need to invest in advertising to generate awareness is in general bad for consumers and overall welfare, though good for firms. This is always the case in markets with low advertising costs.

4.8 Conclusions

This chapter has been a first look at the dynamic effects of customer poaching in homogeneous product markets where advertising plays two major roles: it is used by firms as a way to transmit relevant information to otherwise uninformed consumers and it is used as a price discrimination device. When a firm can recognise customers with different past purchasing histories it may send them targeted ads with different prices. In particular, it was shown that only the firm that advertises the highest price in the first period will be able to engage in price discrimination, and that poaching clearly benefits the discriminating firm. When a firm foresees that benefit, it was identified "*the race for discrimination effect*", through which each firm has an incentive to price high in the initial period. Mainly on the basis of this strategic effect, it was proved that price discrimination may act to *soften* price competition rather than to intensify it. As a result of that, it was shown that, regardless of the advertising technology considered, *all* firms may become better off even when only one of them can engage in price discrimination.

By proposing a framework in which advertising is the consumers' sole source of information and where price discrimination is implemented by means of advertising, this chapter has allowed to analyse the important interaction that does exist between advertising decisions and price discrimination. As expected, it was found that, for the special case in which advertising is costless and discrimination is not permitted, the market provides full market coverage and firms realise no economic profit. Conversely, departing from non-discrimination to discrimination has proved to change dramatically that result—firms no longer choose full market coverage—which in turn translates into positive profits. Furthermore, it was shown that when advertising is costly, then there is more advertising with discrimination than with no discrimination; when advertising is cheap, the reverse happens.

The welfare effects of informative advertising and price discrimination were also analysed with a new perspective. In fact, to the best of our knowledge, this chapter has offered a first attempt to evaluate the effects of price discrimination on the efficiency properties of advertising. Interestingly, as in Butters (1977), it was shown that, when price discrimination is not permitted the market provides the same level of advertising that would be provided by an advertising regulator. In contrast, when firms can engage in price discrimination it was found that firms might provide too little, too much or even the social optimal level of advertising. Obviously, the effect of price discrimination on the efficiency properties of advertising is strongly related to the costs' structure of each advertising-product market. In this way, this chapter has predicted that in markets with low or no advertising costs, allowing firms to price discriminate leads them to provide too little advertising. Only in markets with very high marginal advertising costs might firms overadvertise from the social point of view.

Based on the effects of discrimination on the efficiency properties of informative advertising, it was possible to establish a simple criterion for predicting whether price discrimination would be desirable or not. In this regard, it was found that in general behaviour-based price discrimination is clearly bad for overall welfare and consumer surplus, though good for firms.

In light of the above, this chapter has tried to contribute to the ongoing debate on the implications of new forms of price discrimination, only made possible in the context of new media markets. It is obvious that the specificity of each market remains playing an important influence on the conclusions derived. A special limitation of the stylized model addressed in this chapter is the assumption that advertising is the consumers' sole source of information. Although this assumption may at first sight seem odd in the context of online markets, it helped us to isolate the effects of price discrimination on the advertising decisions of firms. Evidently, while in new product markets this assumption might not be very restrictive; in other markets it might be inadequate. Allowing consumers to obtain information through advertising and costly search could be a natural extension, bringing new insights to the analysis developed so far. It is known that one popular type of online advertising is based on *keyword-related* search. Indeed, consumers seem to be more motivated to buy a certain product/service if ads are targeted on that basis. Thus, one interesting issue would be to analyse the advertising and pricing behaviour of firms in markets where they can target ads and prices on the basis of consumers' search behaviour. In addition, it would be also worth pursuing to study the implications of price discrimination in markets where it could be possible to establish a correlation between patterns of search and each consumer's brand preference. Finally, another natural extension would be to depart from the product homogeneity assumption.

4.9 Appendix

This appendix collects the proofs that were omitted from the text.

Proof of Proposition 1.

Suppose (p_A^*, p_B^*) is an equilibrium in pure strategies. Then, by definition there is no such p_i, $i = A, B$, such that $\pi_i\left(p_i, p_j^*\right) > \pi_i\left(p_i^*, p_j^*\right)$. The proof proceeds by contradiction.

(i) If $p_A^* = p_B^*$,

$$\pi_A\left(p_A^*, p_B^*\right) = \phi_A\left(1 - \phi_B\right) p_A^* + \frac{1}{2}\phi_A \phi_B p_A^*. \tag{4.38}$$

If firm A deviates and charges $p_A = p_A^* - \varepsilon$, with $\varepsilon > 0$ then its profit from deviation is

$$\pi_A\left(p_A, p_B^*\right) = \phi_A\left(1 - \phi_B\right)\left(p_A^* - \varepsilon\right) + \phi_A \phi_B\left(p_A^* - \varepsilon\right). \tag{4.39}$$

From (4.38) and (4.39),

$$\pi_A\left(p_A, p_B^*\right) > \pi_A\left(p_A^*, p_B^*\right) \text{ if } \varepsilon < \frac{1}{2}\phi_B p_A^*.$$

since $\phi_B > 0$ such an ε always exists. A contradiction. *Q.E.D.*

(ii) Without loss of generality let $p_A^* < p_B^*$ then

$$\pi_A\left(p_A^*, p_B^*\right) = \phi_A\left(1 - \phi_B\right) p_A^* + \phi_A \phi_B p_A^*.$$

Let $p_A = p_A^* + \varepsilon < p_B^*$, then firm A's profit from deviation is

$$\pi_A\left(p_A, p_B^*\right) = \phi_A\left(1 - \phi_B\right)\left(p_A^* + \varepsilon\right) + \phi_A \phi_B\left(p_A^* + \varepsilon\right)$$

from which it is easy to see that there is a ε such that the deviation is profitable. A contradiction. *Q.E.D.*

(iii) If $p_A^* > p_B^*$, then $p_B^* < p_A^*$ and as firm A does in (ii) there is a profitable deviation for firm B. A contradiction. $Q.E.D.$

Proof of Proposition 3.

From equation (4.8) it follows that as $\alpha \to 0$ the right-hand side of that equation goes to zero as well. Therefore, for any advertising cost function with $A(\phi) = \alpha\eta(\phi)$, $\phi^{nd} \to 1$. As $\phi^{nd} \to 1$ from equations (4.6), (4.7) and (4.9) one obtains that $p_{\min}^{nd} \to 0$ and $E\Pi^{nd} \to 0$ and $F^{nd} = 1$. This proves part (i). Similarly, as $\alpha \to v(1+\delta)$, since $p_{\min}^{nd} \to v$, F^{nd} converges clearly to a unit mass at v. Finally, if price dispersion is measured by the range of equilibrium prices, it is equal to $v - p_{\min}^{nd} = v\phi^{nd}$. Then since $\frac{\partial\phi^{nd}}{\partial\alpha} = -\frac{A_{\phi\alpha}}{v(1+\delta)+A_{\phi\phi}} < 0$, it follows that the level of price dispersion is higher the lower is α. This completes the proof. $Q.E.D.$

Proof of Proposition 4.

Suppose $\left(p_i^{r*}, \widehat{p}_j^*\right)$ is an equilibrium in pure strategies. Then, by definition there is no such p_i^r, $i = A, B$, such that $\pi_i^r\left(p_i^r, \widehat{p}_j^*\right) > \pi_i^r\left(p_i^{r*}, \widehat{p}_j^*\right)$. The proof proceeds by contradiction.

(i) If $p_i^{r*} = \widehat{p}_j^*$, then

$$\pi_i^{r*} = \frac{1}{2}\phi_i\phi_j p_i^{r*}. \tag{4.40}$$

If firm i deviates and quotes $p_i^r = p_i^{r*} - \varepsilon$, with $\varepsilon > 0$ its profit from deviation is

$$\pi_i^r = \phi_i\phi_j\left(p_i^{r*} - \varepsilon\right).$$

It is then trivial to see that there exists such an ε that makes the deviation profitable. A contradiction. $Q.E.D.$

(ii) Let $p_i^{r*} < \widehat{p}_j^*$ then

$$\pi_i^{r*} = \phi_i\phi_j p_i^{r*}. \tag{4.41}$$

Let $p_i^r = p_i^{r*} + \varepsilon < \widehat{p}_j^*$, then firm i's profit from deviation is

$$\pi_i^r = \phi_i\phi_j\left(p_i^{r*} + \varepsilon\right)$$

from which it is straightforward to see that the deviation is profitable. A contradiction. *Q.E.D.*

Proof of Corollary 2.

To prove part (i) take as given the level of advertising selected by firms in period 1. Thus, $G^r(p) - \widehat{G}(p) = (1-\phi)\frac{v\phi - v + p}{p\phi} > 0$. This condition is satisfied if and only if $p \geq v(1-\phi)$. Since $v(1-\phi) = \widehat{p}_{\min}$ we get that $G^r(p) - \widehat{G}(p) > 0$ as long as $p \geq \widehat{p}_{\min}$, which is true for the equilibrium support of prices. When (i) holds result (ii) follows. *Q.E.D.*

Proof of Proposition 6.

We prove this result by contradiction. Suppose that $(p_A, p_B) = (v, v)$ is a subgame perfect equilibrium of this game. Because both firms have the same price in period 1, they share equally the selective group of consumers. In period 2, within the group of old customers, each firm is not able to distinguish the fraction of selective and captive customers. However, each firm is able to recognise those selective customers that bought from the rival before. As a result, being price discrimination permitted both firms will offer a different price for old and the rival's previous customers. As before, p_i^o is the price charged by firm i to its old customers whilst p_i^r is the price charged by firm i to the rival's customers. Again there is no pure strategy equilibrium and thus we need to prove the existence of a mixed strategy equilibrium. Each firm's prices are selected according to the distribution functions $G_i^r\left(p_j^o\right)$ and $G_i^o\left(p_j^r\right)$.

When say firm A decides what price to charge to the rival's previous customers it takes into account that firm B is not willing to charge a price yielding an expected profit below $v\phi_B(1-\phi_A)$ (even in the case where it is able to sell to both groups of old customers). Therefore, it easy to see that the minimum price firm B is willing to quote, say $p_{B\min}^o$ is

$$p_{B\min}^o\left(\phi_B(1-\phi_A) + \frac{1}{2}\phi_A\phi_B\right) = v\phi_B(1-\phi_A)$$

or,

$$p^o_{B\min} = \frac{2v(1-\phi_A)}{2-\phi_A}.$$

It is also easy to see that it is a dominated strategy for firm A to set a price below $p^o_{B\min}$. Quoting $p^r_A = p^o_{B\min}$ allows firm A to grab the entire group of selective customers that bought from B before whilst makes firm B indifferent. Therefore, firm A's minimum price to the rival's customers is $p^o_{B\min}$ and its expected profit from that group is equal to $p^o_{B\min}\left(\frac{1}{2}\phi_A\phi_B\right)$. The derivation of the mixed strategy equilibrium follows the same steps as before. In equilibrium we must observe

$$p^r_A\left(\frac{1}{2}\phi_A\phi_B\right)\left[1-G^o_B\left(p^r_A\right)\right] = 2v\left(\frac{1-\phi_A}{2-\phi_A}\right)\left(\frac{1}{2}\phi_A\phi_B\right)$$

which leads to

$$G^o_B\left(p^r_A\right) = 1 - \frac{2v(1-\phi_A)}{p^r_A(2-\phi_A)} \tag{4.42}$$

with $G^o_B\left(p^r_A = p^o_{B\min}\right) = 0$ and $G^o_B\left(p^r_A = v\right) = 1 - \frac{2v(1-\phi_A)}{v(2-\phi_A)}$. Thus, firm B has a mass point at v within the group of its old customers.

Looking now at firm A's pricing strategy for its old customers, we know that firm A's expected profit from its old customers group is equal to $v\phi_A(1-\phi_B)$. Thus in equilibrium we must observe

$$p^o_A\phi_A(1-\phi_B) + p^o_A\left(\frac{1}{2}\phi_A\phi_B\right)\left[1-G^r_B\left(p^o_A\right)\right] = v\phi_A(1-\phi_B)$$

which yields,

$$G^r_B\left(p^o_A\right) = 1 - \frac{2\left(v-p^o_A\right)(1-\phi_B)}{p^o_A\phi_B}. \tag{4.43}$$

From $G^r_B\left(p^o_A\right) = 0$ we obtain $p^o_{A\min} = \frac{2v(1-\phi_B)}{2-\phi_B}$ and from $G^r_B\left(p^o_A\right) = 1$ we obtain $p^o_{A\max} = v$.

Second-period expected profit for firm A is therefore

$$\begin{aligned}\pi_A^2 &= \pi_A^o + \pi_A^r = v\phi_A(1-\phi_B) + 2v\left(\frac{1-\phi_A}{2-\phi_A}\right)\left(\frac{1}{2}\phi_A\phi_B\right) \\ &= v\phi_A\left(\frac{2-\phi_A-\phi_B}{2-\phi_A}\right).\end{aligned}$$

In sum, overall profit when both firms play (v,v) in period 1 is equal to

$$\Pi = v\left(\phi_A(1-\phi_B) + \frac{1}{2}\phi_A\phi_B\right) + \delta v\phi_A\left(\frac{2-\phi_A-\phi_B}{2-\phi_A}\right) - A(\phi_A).$$

Under symmetric decisions in advertising we obtain:

$$\Pi = v\left(\phi(1-\phi) + \frac{1}{2}\phi^2\right) + \delta 2v\phi\left(\frac{1-\phi}{2-\phi}\right) - A(\phi).$$

Now we need to prove that no firm has an incentive to deviate from the equilibrium proposed. Taking as given its rival pricing and advertising strategy in period 1, if say firm A decides to undercut its rival, by quoting a price slightly below v, it is able to sell to the entire group of selective consumers in that period but it is not allowed to discriminate in period 2. In this situation, firm A's first-period profit from deviation is $(v-\varepsilon)(\phi_A(1-\phi_B)+\phi_A\phi_B) \approx v(\phi_A(1-\phi_B)+\phi_A\phi_B)$ for ε infinitely small. In period 2 firm A is the non-discriminating firm with certainty and its expected profit is then $v\phi_A(1-\phi_B)$.

Summing up, firm A's profit from deviation is equal to

$$\Pi^d = v(\phi_A(1-\phi_B)+\phi_A\phi_B) + \delta v\phi_A(1-\phi_B) - A(\phi_A).$$

Under symmetry we have:

$$\Pi^d = v\left(\phi(1-\phi)+\phi^2\right) + \delta v\phi(1-\phi) - A(\phi).$$

This deviation is profitable as long as $\Pi^d - \Pi > 0$. Since,

$$\Pi^d - \Pi = \underbrace{v\phi^2 - \frac{1}{2}v\phi^2}_{\text{Increase in first-period profit due to deviation}} + \underbrace{\delta v\phi(1-\phi)\left(1 - \frac{2}{2-\phi}\right)}_{\text{Decrease in second-period profit due to deviation}},$$

the deviation is profitable as long as the increase in first-period profit overcomes the decrease in second-period profits. The above condition simplifies to

$$\Pi^d - \Pi = \frac{1}{2}v\phi^2 \frac{2 - 2\delta + 2\delta\phi - \phi}{2-\phi}.$$

It is then easy to see that since $\phi \in (0,1)$, $\Pi^d - \Pi > 0$ as long as $2 - 2\delta + 2\delta\phi - \phi > 0$. For $\delta \in (0,1)$ the previous condition is always positive. Consequently, firm A can deviate and increase its overall profit by slightly reducing its price below v. A contradiction. *Q.E.D.*

Proof of Proposition 10.

To prove part (i), for algebraically reasons we use the quadratic technology. From (4.9) we have

$$E\Pi^{nd} = \phi^{nd} A_\phi - A\left(\phi^{nd}\right),$$

from which it follows that the derivative of overall profit with respect to α is

$$\frac{\partial E\Pi^{nd}}{\partial \alpha} = \phi^{nd} A_{\phi\phi}\phi_\alpha^{nd} + \phi^{nd} A_{\phi\alpha} - A_\alpha$$

where

$$\phi_\alpha^{nd} = \frac{\partial \phi^{nd}}{\partial \alpha} = -\frac{A_{\phi\alpha}}{v(1+\delta) + A_{\phi\phi}} < 0.$$

When the advertising technology is quadratic, $A(\phi) = \alpha\phi^2$, we obtain $\frac{\partial \phi^{nd}}{\partial \alpha} = \left(-\frac{2\phi}{v(1+\delta)+2\alpha}\right)$ and $\phi^{nd} = \frac{v}{v+2\alpha}$ and thus,

$$\begin{aligned} \frac{\partial E\Pi^{nd}}{\partial \alpha} &= \phi^{nd} A_{\phi\phi}\phi_\alpha^{nd} + \phi^{nd} A_{\phi\alpha} - A_\alpha \\ & \left(2\alpha\phi^{nd}\right)\phi_\alpha^{nd} + \left(2\phi^{nd}\right)^2 - \left(\phi^{nd}\right)^2 \end{aligned}$$

$$= \phi^2 \frac{(-2\alpha + v(1+\delta))}{v + v\delta + 2\alpha}.$$

Therefore $\frac{\partial E\Pi^{nd}}{\partial \alpha} > 0$ if and only if $\alpha < \frac{v}{2}(1+\delta)$. Otherwise, $\frac{\partial E\Pi^{nd}}{\partial \alpha} < 0$. $Q.E.D.$

To prove part (ii), from (4.31) we have

$$E\Pi^* = \phi^* A_\phi - A(\phi^*) + \delta v (\phi^*)^3.$$

Therefore, the derivative of overall profit with respect to α, is

$$\frac{\partial E\Pi^*}{\partial \alpha} = \phi^* A_{\phi\phi}\phi^*_\alpha + \phi^* A_{\phi\alpha} - A_\alpha + 3\delta v (\phi^*)^2 \phi^*_\alpha$$

where

$$\phi^*_\alpha = \frac{\partial \phi^*}{\partial \alpha} = -\frac{A_{\phi\alpha}}{v(1+4\delta\phi) + A_{\phi\phi}} < 0.$$

When $A(\phi) = \alpha\phi^2$ it follows that

$$\begin{aligned} E\Pi^*_\alpha &= \phi^2 + \left(2\alpha\phi + 3\delta v\phi^2\right)\left(-\frac{2\phi}{v(1+4\delta\phi) + 2\alpha}\right) \\ &= -\phi\left(\frac{v\phi - 2\delta v\phi^2 - 2\alpha\phi}{v + 4\delta v\phi + 2\alpha}\right). \end{aligned}$$

Thus, $\frac{\partial E\Pi^*}{\partial \alpha} < 0$ if $v\phi - 2\delta v\phi^2 - 2\alpha\phi > 0$. Using (4.30) and the quadratic technology it follows that $v(1-\phi) + v\delta\left(1 - 2\phi^2\right) = 2\alpha\phi$, and therefore $\frac{\partial E\Pi^*}{\partial \alpha} < 0$ if $v(1+\delta-2\phi) > 0$ which for $\delta = 1$ is always true. $Q.E.D.$

Proof of Proposition 11.

Evaluating the advertising regulator's first order condition given by (4.35) at $\phi = \phi^{nd}$ it follows that

$$\frac{\partial W}{\partial \phi} = 2v\left(1 - \phi^{nd}\right) - A_\phi\left(\phi^{nd}\right).$$

Using the fact that $2v\left(1 - \phi^{nd}\right) = A_\phi\left(\phi^{nd}\right)$, and plugging into the above expression it follows that $\frac{\partial W}{\partial \phi} = 0$, i.e., welfare is maximised when $\phi = \phi^{nd}$. The reader can easily observe that this conclusion is valid for every δ. For all δ the advertising regulator's first order condition would be

$$\frac{\partial W}{\partial \phi} = v\left(1+\delta\right)\left(1-\phi\right) - A_{\phi}\left(\phi\right).$$

whilst the equilibrium level of advertising without discrimination would be given by $v\left(1+\delta\right)\left(1-\phi^{nd}\right) = A_{\phi}\left(\phi^{nd}\right)$. Again when we evaluate the advertising regulator's first order condition at $\phi = \phi^{nd}$ it follows that $\frac{\partial W}{\partial \phi} = 0$. Therefore, with no discrimination firms select the social optimal level of advertising. This completes the proof. *Q.E.D.*

Proof of Proposition 12.

Evaluating the advertising regulator's first order condition given by (4.35) at $\phi = \phi^*$ it follows that

$$\frac{\partial W}{\partial \phi} = 2v\left(1-\phi^*\right) - A_{\phi}\left(\phi^*\right).$$

Using the fact that when $\delta = 1$, $A_{\phi}\left(\phi^*\right) = v\left(1-\phi^*\right) + v\left(1-2\phi^{*2}\right)$ and plugging this expression into the above condition we obtain that

$$\frac{\partial W}{\partial \phi} = v\phi^*\left(2\phi^* - 1\right). \tag{4.44}$$

In this case it immediately follows that ϕ^* maximises welfare when $\phi^* = 0.5$. Otherwise, when $\phi^* > 0.5$ it follows that $\frac{\partial W}{\partial \phi} > 0$ and so welfare increases if ϕ^* becomes increasingly higher. This implies that $\phi^* < \phi^w$. Similarly, when $\phi^w < \phi^* < 0.5$ welfare increases if ϕ^* becomes smaller. This completes the proof. *Q.E.D.*

Proof of Proposition 13.

From Proposition 11 it follows that welfare is maximised at $\phi = \phi^{nd.}$. In contrast, from Proposition 12 it follows that when $\delta = 1$, $\frac{\partial W}{\partial \phi} = 0$ at $\phi = \phi^* = 0.5$ while $\frac{\partial W}{\partial \phi} \gtrless 0$ when $\phi^* \gtrless 0.5$. Thus apart from the case in which $\phi^* = 0.5$, welfare always falls when moving from no discrimination to discrimination. *Q.E.D.*

Chapter 5

Conclusions

> *"As with technology itself, the innovation comes not in the basic building blocks, the components of economic analysis, but rather the ways in which they are combined."*
>
> Hal Varian (2003), *Economics of Information Technology.*

Advances in information technologies have unequivocally enabled firms to use consumer past purchasing data to design and implement new marketing practices, namely to tailored different prices and ads to consumers with different past behaviour. Indeed, many firms in diverse industries have been trying to offer special promotions (e.g. lower prices, discounts or coupons) to those customers that in some sense belong to the rival firm. At the first glance, this marketing strategy seems to be lucrative as it allows a firm to generate profitable incremental sales without damaging profits it can extract from its own customer base. However, as behaviour-based price discrimination gains popularity many interesting questions arise. Is it really in the best interest of firms to recognise customers with different past behaviour and to price discriminate accordingly? Or, is it rather in their interest to avoid any possible learning and thereby price discrimination practices? Should consumers hide their true types, i.e., should they behave anonymously? Further, should government regulation restrict information collection and price discrimination practices?

The study of these questions is the study of the profit and welfare effects of price discrimination. The core objective of this thesis has been therefore to contribute to the ongoing debate on the profit, consumer surplus and welfare effects of behaviour-based price discrimination in imperfectly competitive markets. To attain that goal three stylised theoretical models were proposed where the concern was to provide situations closer enough to markets where information technologies play an increasingly important role (e.g. e-commerce, m-commerce and wireless markets). Obviously, reality is extremely complex and so economic models cannot usually afford to capture all possible dimensions. Nevertheless, in the end theories are valuable when they provide clear intuitions about particular economic phenomena.

The model addressed in chapter 2 is particularly relevant to understand *how* profits and welfare evolve, as further improvements in information technologies increase the firms' capability to better recognise customers. The analysis carried out in this chapter has suggested that although a firm may, individually, prefer to acquire more accurate information for price discrimination purposes, when all firms follow the same strategy they all become worse off. In other words, firms find themselves trapped in the classic prisoner's dilemma. At the same time the analysis has put forward that any kind of restriction on the accuracy of each firm's information (due to inaccurate statistical inferences and consumer or legal restrictions on information collection) would be profit enhancing as it would be an effective *weapon* to fight against the *all-out competition* result. Because behaviour-based price discrimination based on highly precise information leads to lower prices to all consumers, any action restricting each firm's information accuracy would act against consumers. Apart from other relevant concerns related to privacy issues absent from our analysis, the general conclusion is that it is in the best interest of consumers do not behave anonymously and to allow firms to recognise them more accurately. Any general conclusion regarding welfare should be interpreted carefully as in the model price discrimination gives no rise to any market expansion. However, our analysis has suggested that as firms rely on more accurate information welfare decreases with price discrimination because more consumers are expected to buy the "wrong" brand. In this context, any public policy against or in favour of price discrimination should take into account whether the target is overall welfare or only consumer welfare. Notwithstanding the clear intuitions provided by the current analysis, we should take into account that reality is frequently more asymmetric than our model has assumed. Therefore, it would be worth pursuing to extend the analysis to an asymmetric market. For instance, it would be instructive to consider that one firm has access to more accurate information than its rival. It would also be interesting to analyse the pricing behaviour of firms under asymmetric customer distributions, which would allow for asymmetries in firms' size. Finally, perhaps the more interesting extension would be to introduce an information vendor selling consumer-specific information to competing firms, which could then engage, if permitted, in price discrimination practices. In this setting it would be interesting to investigate whether the information vendor would choose to sell consumer-specific information exclusively to one firm or rather to both? What level of information's accuracy should be provided in each case? This extension would be valuable to understand the equilibrium interaction between an information vendor (such as, for instance, *Double Click* and *I-Behavior*) and competing firms selling their products directly to consumers.

When the goal is to understand how the firms' pricing behaviour in a first interaction will affect the information available for price discrimination in next periods then the framework presented in chapter 3 is the most adequate. It is also especially useful to clarify some aspects related to observed price dispersion in Internet markets. Despite the model has explained price dispersion on the basis of brand loyalty it could easily embrace any other explanation (e.g. brand awareness, search costs, etc.).

As is by now a familiar theme, when firms are able to learn to which firm each consumer is loyal and to price discriminate accordingly, all prices and profits might fall. In consequence of that, when a firm realises that it will be negatively affected by price discrimination it will have an incentive to avoid learning (and, therefore, price discrimination). Remarkably, we have found that, in the context of a discrete model, one effective way to preclude price discrimination is by *not* sharing the market in the first period. Indeed, chapter 3 has proven that there exists sometimes an *asymmetric* equilibrium through which the market is entirely served by the same firm in the initial game. This seems to suggest that we could observe a firm quoting a low price and getting all consumers, but not being worth for the rival to match this low-price firm since its future profits would then be lower. Of course, it would be worth pursuing to empirically test the model in order to confirm whether or not it captures a real phenomenon. In a recent study for a consumer electronics market at the price comparison site *Shopper.com*, Baye, Morgan and Sholton (2004) show that at each point in time a (stationary) distribution of prices across firms will be observed and that the identity of the firm offering the lowest price will vary unpredictably over time.

Another key feature of chapter 3 is that it suggests that in markets where firms follow an unpredictable pricing strategy (i.e., they set random prices), as is frequently the case in Internet markets, the ability of firms to price discriminate on the basis of data on past purchasing behaviour appears to be welfare enhancing. The reason is that while in the non-discrimination mixed price equilibrium some consumers buy inefficiently, with price discrimination *all* consumers buy the "right" brand (a similar result is obtained in Thisse and Vives (1988)). Thus, whilst in the extant models on behaviour-based price discrimination (e.g. Chen (1997) and Fudenberg and Tirole (2000)), price discrimination leads some consumers to buy the "wrong" brand (i.e., there is inefficient switching) here it happens quite the reverse.

In sum, while there seems to exist some general consensus regarding the profit effects of behaviour-based price discrimination in markets exhibiting best-response asymmetry, general conclusions regarding the welfare effects should be drawn carefully. In particular, it is extremely important to take into account the specificity of each market regarding the type of consumer heterogeneity and the information available for price discrimination.

Finally, when the aim is to investigate price discrimination allowed by past purchasing data in markets where firms need to invest in advertising to generate awareness and to reach potential consumers, then the analysis of chapter 4 becomes key. This analysis is particularly pertinent in new product markets where advertising is clearly a first step in creating demand for a product. To our knowledge there is no theoretical work so far analysing *how* price discrimination affects the level of advertising provided by firms. Furthermore, in chapter 4 advertising is used as a long-run variable as it determines the proportion of consumers that will be reached by firms currently as well as in future periods. By investing in advertising firms are also able to endogenously differentiate an *ex-ante* homogenous product market. In this regard, chapter 4 has analysed behaviour-based price discrimination following a new perspective, as there are neither switching costs (as in Chen (1997)) nor exogenous brand preferences (as in Fudenberg and Tirole (2000)). In this setting, it was shown that *only* one firm—the high price firm in the initial period—will be able to recognise customers and to send ads with different prices to its previous customers and to the rival's previous (price-sensitive) customers. Additionally, it was proved that the discriminating firm is strictly better off with price discrimination and that the same might also happen to the non-discriminating firm. Interestingly, it was shown that the benefit of becoming the discriminating gives firms an incentive to price high in the initial period—the *"race for discrimination effect"*—and, as a result of this less aggressive behaviour, all firms might become better off when price discrimination is permitted. Because price discrimination tends to increase prices, consumers become worse off.

When we turn to analyse how price discrimination affects the firms' advertising decisions in relation to the case where discrimination is banned, it was found that firms might provide too little, too much or even the social optimal level of advertising. Obviously, the effect of price discrimination on the efficiency properties of advertising is strongly related to the costs' structure of each advertising-product market. In this way, chapter 4 has predicted that in markets with low or no advertising costs, allowing firms to price discriminate leads them to provide too little advertising. Only in markets with very high marginal advertising costs might firms overadvertise from the social point of view. Based on the effects of discrimination on the efficiency properties of informative advertising, the model addressed in chapter 4 has suggested that behaviour-based price discrimination in markets where firms need to invest in advertising to generate awareness is, in general, bad for consumers and welfare, though good for firms.

Basically, the analysis carried out in chapter 4 highlights the importance of analysing together the pricing and advertising decisions of firms, which constitute the main variables of a firm's marketing strategy. While the economic literature has hitherto focused on the use of consumers' past purchasing data for price discrimination, firms use unequivocally this data to formulate special advertising campaigns. Therefore, the analysis of chapter 4 is an early contribution to understanding *how* price discrimination and advertising interact. Obviously, further research is needed to provide a more complete picture. There are several extensions that we have not considered in this thesis, but which are nonetheless important. First, it would be useful to extend the analysis of chapter 4 to markets where firms offer differentiated products. Yet within this setting it would be instructive to allow firms to reach more accurately its old customers than the rival's customers. Another interesting issue would be to investigate whether a firm should select a higher level of advertising to its old customers or to the rival's ones. It would also be useful to examine how each group's advertising intensity would be affected by price discrimination.

We have assumed that consumers are passive recipients of advertising. While some consumers are in fact happy to receive well targeted ads some other consumers do bear a cost each time they receive an email from a specific firm. Stimulated by privacy issues it would be worth pursuing to formulate a model in which data on consumers' past behaviour could affect consumers' welfare through price discrimination as well as through advertising. This could be done by assuming a kind of disutility associated with firms' ads and/or by assuming that consumers might behave strategically regarding, for instance, the ads received by the firms they like less. Obviously, this new framework would have important implications on the profit and welfare effects of behaviour-based price discrimination.

These and other related questions are, nevertheless, left for future research.

Bibliography

[1] Acquisti, A. and Varian, H. (2003), "Conditioning Prices on Purchase History." Working paper, Berkeley University.

[2] Armstrong, M. and Vickers, J. (2001), "Competitive Price Discrimination." *RAND Journal of Economics,* 32, 579-605.

[3] Bagwell, K. (2003), "The Economic Analysis of Advertising." In Armstrong, M. and R. Porter, eds., *The Handbook of Industrial Organization*, Vol.3, Amsterdam: North-Holland, forthcoming.

[4] Baye, M. and Morgan, J. (2001), "Information Gatekeepers on the Internet and Competitiveness of Homogeneous Product Markets." *American Economic Review,* 91, 454-474.

[5] Baye, M. and Morgan, J. (2003), "Price Dispersion in the Lab and on the Internet: Theory and Evidence." *RAND Journal of Economics,* forthcoming.

[6] Baye, M., Morgan, J. and Sholton, P. (2002), "Persistent Price Dispersion in Online Markets." *The New Economy,* University of Chicago Press, forthcoming.

[7] Baye, M., Morgan, J. and Sholton, P. (2004), "Temporal Price Dispersion: Evidence from an Online Consumer Electronics Market." *Journal of Interactive Marketing*, forthcoming.

[8] Bester, H. and Petrakis, E. (1995), "Price Competition and Advertising in Oligopoly." *European Economic Review,* 39, 1075-1088.

[9] Bester, H. and Petrakis, E. (1996), "Coupons and Oligopolistic Price Discrimination." *International Journal of Industrial Organization,* 14, 227-242.

[10] Blackwell, D. (1951), "Comparison of Experiments." In Neyman, J., ed., *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability,* University of California Press.

[11] Brynjolfsson, E. and Smith, M. (2000a), "Frictionless Commerce? A Comparison of Internet and Conventional Retailers." *Management Science*, 46, 563-585.

[12] Brynjolfsson, E. and Smith, M. (2000b), "The Great Equalizer? Consumer Choice Behavior and Internet Shopbots." Working paper, MIT.

[13] Butters, G. (1977), "Equilibrium Distribution of Sales and Advertising Prices." *Review of Economic Studies*, 44, 465-491.

[14] Caminal, R. and Matutes, C. (1990), "Endogenous Switching Costs in a Duopoly." *International Journal of Industrial Organization*, 8, 353-373.

[15] Carlton, D. and Perloff, J. (1994), *Modern Industrial Organization.* Scott, Foresman and Company.

[16] Chen, Y. (1997), "Paying Customers to Switch." *Journal of Economics and Management Strategy,* 6, 877-897.

[17] Chen,Y., Narasimhan, C. and Zhang, Z. (2001), "Individual Marketing with Imperfect Targetability." *Marketing Science*, 20, 23-41.

[18] Chen, Y. and Zhang, Z. (2002), "The Benefit of Behaviour-Based Targeted Pricing with Strategic Consumers." Working paper, New York University.

[19] Corts, K. (1998), "Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment." *RAND Journal of Economics*, 29, 306-323.

[20] Dixit, A. and Norman, V. (1978), "Advertising and Welfare." *Bell Journal of Economics*, 9, 1-17.

[21] Esteban, L., Gil, A. and Hernández, J. (2001), "Informative Advertising and Optimal Targeting in a Monopoly." *Journal of Industrial Economics*, 69, 161-180.

[22] Esteves, R. and Guimarães, P. (2001), "Price Discrimination and Targeted Advertising: A Welfare Analysis." Working paper, University of Minho.

[23] Farrel, J. and Klemperer, P. (2003), "Coordination and Lock-in: Competition with Switching Costs and Network Effects." In Armstrong, M. and R. Porter, eds., *The Handbook of Industrial Organization*, Vol.3, Amsterdam: North-Holland, forthcoming.

[24] Fudenberg, D. and Tirole, J. (1998), "Upgrades, Tradeins, and Buybacks." *RAND Journal of Economics,* 29, 235-258.

[25] Fudenberg, D. and Tirole, J. (2000), "Customer Poaching and Brand Switching." *RAND Journal of Economics,* 31, 634-657.

[26] Galeotti, A. and Moraga-González, J. (2003), "Strategic Targeted Advertising." Working paper, Tinbergen Institute.

[27] Grossman, G. and Shapiro, C. (1984), "Informative Advertising with Differentiated Products." *Review of Economic Studies*, 51, 63-81.

[28] Guimarães, P. (1995), "A Simple Model of Informative Advertising." Working paper, University of Minho.

[29] Hernandez-Gárcia, J. (1997), "Informative Advertising, Imperfect Targeting and Welfare." *Economics Letters,* 55, 131-137.

[30] Holmes, T. (1989), "The Effects of Third-Degree Price Discrimination in Oligopoly." *American Economic Review,* 79, 244-250.

[31] Hotelling, H. (1929), "Stability in Competition." *Economic Journal*, 39, 41-57.

[32] Iyer, G., Soberman, D. and Villas-Boas, M. (2003), "The Targeting of Advertising." Working paper, Berkeley University.

[33] Kaldor, N. (1950), "Economic aspects of advertising." *Review of Economic Studies,* 18, 1-27.

[34] Klemperer, P. (1987a), "Markets with Consumer Switching Costs." *The Quarterly Journal of Economics*, 102, 375-394.

[35] Klemperer, P. (1987b), "The Competitiveness of Markets with Switching Costs." *RAND Journal of Economics*, 18, 138-150.

[36] Klemperer, P. (1995), "Competition when Consumers have Switching Costs." *Review of Economic Studies,* 62, 515-539.

[37] Kotler, P. (2000), *Marketing Management.* Prentice Hall.

[38] Kotler, P. and Armstrong, G. (1998), *Marketing: An Introduction.* Englewood Cliffs: Prentice Hall.

[39] Krugman, P. (2000), "What Price Fairness?" *The New York Times,* October 4.

[40] Lederer, P. and Hurter, A. (1986), "Competition of Firms: Discriminatory Pricing and Location." *Econometrica*, 54, 623-640.

[41] Moraga-González, J. and Petrakis, E. (1999), "Coupon Advertising under Imperfect Price Information." *Journal of Economics and Management Strategy*, 8, 523-544.

[42] Narasimhan, C. (1988), "Competitive Promotional Strategies." *Journal of Business*, 61, 427-449.

[43] Nelson, P. (1970), "Information and Consumer Behavior." *Journal of Political Economy*, 78, 311-329.

[44] Nelson, P. (1974), "Advertising as Information." *Journal of Political Economy,* 82, 729-754.

[45] Nilssen, T. (1992), "Two Kinds of Consumer Switching Costs." *RAND Journal of Economics*, 23, 579-589.

[46] Pan, X., Ratchford, B. and Shankar, V. (2001), "Do Drivers of Online Price Dispersion Change as Online Markets Grow?" Working paper, eBusiness Research Center.

[47] Phlips, L. (1988), *The Economics of Price Discrimination.* Cambridge University Press.

[48] Pigou, A. (1920), *The Economics of Welfare.* London: Macmillan

[49] Raju, J., Srinivasan, V. and Lal, R. (1990), "The Effects of Brand Loyalty on Competitive Price Promotional Strategies." *Management Science,* 36, 376-394.

[50] Robert, J. and Stahl, D. (1993), "Informative Price Advertising in a Sequential Search Model." *Econometrica*, 61, 657-686.

[51] Robinson, J. (1933), *The Economics of Imperfect Competition.* London: Macmillan.

[52] Rossi, P. and Allenby, G. (1993), "A Bayesian Approach to Estimating Household Parameters." *Journal of Marketing Research*, 30, 171-182.

[53] Rossi, P., McCullock , R. and Allenby, G. (1996), "The Value of Purchase History Data in Targeting Marketing." *Marketing Science*, 15, 321-340.

[54] Roy, S. (2000), "Strategic Fragmentation of a Market." *International Journal of Industrial Organization,* 18, 1279-1290.

[55] Salop, S. and Stiglitz, J. (1977), "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion." *Review of Economic Studies*, 45, 493-510.

[56] Shaffer, G. and Zhang, Z. (1995), "Competitive Coupon Targeting." *Marketing Science,* 14, 395-416.

[57] Shaffer, G. and Zhang, Z. (2000), "Pay to Switch or Pay to Stay: Preference-Based Price Discrimination in Markets with Switching Costs." *Journal of Economics and Management Strategy,* 9, 397-424.

[58] Shaffer, G. and Zhang, Z. (2001), "Competitive One-to-One Promotions." Working paper, University of Rochester.

[59] Shapiro, C. (1980), "Advertising and Welfare: A Comment." *Bell Journal of Economics*, 11, 749-752.

[60] Shapiro, C. and Varian, H. (1999), *Information Rules.* Boston, Harvard Business Press.

[61] Shilony, Y. (1977), "Mixed Pricing in Oligopoly." *Journal of Economic Theory*, 14, 373-388.

[62] Soberman, D. (2002), "Informative Advertising: An Alternative Viewpoint and Implications." Working paper, Insead.

[63] Stahl, D (1989), "Oligopolistic Pricing with Sequential Consumer Search." *American Economic Review,* 79, 700-712.

[64] Stahl, D. (1994), "Oligopolistic Pricing and Advertising." *Journal of Economic Theory*, 64, 162-177.

[65] Stahl, D. (1996), "Oligopolistic Pricing with Sequential Consumer Search and Heterogeneous Search Costs." *International Journal of Industrial Organization,* 14, 243-268.

[66] Stegeman, M. (1991), "Advertising in Competitive Markets." *American Economic Review*, 81, 210-223.

[67] Stigler, G. (1961), "The Economics of Information." *Journal Political Economy*, 69, 213-225.

[68] Stigler, G. (1987), *A Theory of Price.* New York. MacMillan

[69] Stole, L. (2003), "Price Discrimination in Competitive Environments." In Armstrong, M. and R. Porter, eds., *The Handbook of Industrial Organization*, Vol.3, Amsterdam: North-Holland, forthcoming.

[70] Streitfield, D. (2000), "On the Web Price Tags Blur: What You Pay Could Depend on Who You Are." *The Washington Post*, September 27.

[71] Taylor, C. (2003), "Supplier Surfing: Competition and Consumer Behaviour in Subscription Markets." *RAND Journal of Economics*, 34, 223-246.

[72] Thisse, J. and Vives, X. (1988), "On the Strategic Choice of Spatial Price Policy." *American Economic Review,* 78, 122-137.

[73] Tirole, J. (1988), *The Theory of Industrial Organization.* Cambridge. The MIT Press.

[74] Ulph, D. and Vulkan, N. (2000), "Electronic Commerce and Competitive First-Degree Price Discrimination." Working paper, UCL and University of Bristol.

[75] Varian, H. (1980), "A Model of Sales." *American Economic Review*, 70, 651-659.

[76] Varian, H. (1989), "Price Discrimination." In R. Schmalensee and R. Willig, eds., *The Handbook of Industrial Organization*, Vol.1, Amsterdam: North-Holland.

[77] Varian, H. (2003), "Economics of Information Technology." Working paper, University of California, Berkeley.

[78] Villas-Boas, M. (1999), "Dynamic Competition with Customer Recognition." *RAND Journal of Economics,* 30, 604-631.

[79] Villas-Boas, M. (2001), "Price Cycles in Markets with Customer Recognition." *RAND Journal of Economics,* forthcoming.

[80] Vulkan, N. (2003), *The Economics of E-commerce: A Strategic Guide to Understanding and Designing the Online Marketplace.* Princeton University Press.

[81] Ward, M. and Lee, M. (2000), "Internet Shopping, Consumer Search and Product Branding." *Journal of Product and Brand Management*, 9, 6-18.